An Engineering Grey-Box Approach to Macroeconomic Scenario Modelling

Time- and Frequency Domain Analysis of Macroeconomic Systems

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Master of Science Thesis

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Abstract

This thesis is a first effort to develop a grey-box model of a macroeconomic system. This is in contrast to current black-box modelling approaches. These black-box modelling approaches result in models where the variables and parameters have no economic interpretation. Companies who develop macroeconomic scenario models such as Ortec Finance, have identified this as a major limitation.

In this thesis, an economic-engineering approach is taken. Engineers use the laws of physics to develop grey-box models. Economic-engineering theory is based on the analogies between the dynamics of economic phenomena on the one hand and mechanical phenomena on the other hand. By applying these analogues, a structural method is developed that translates the National accounting and the circular flow theory of macroeconomics into the laws of physics and engineering concepts.

The engineering approach leads to three main contribution: 1) A bond graph model for the United States economy. 2) A linear time-invariant state-space model derived from the bond graph model to perform time domain analysis and 3) The Laplace transformation of the model to perform valuation and analysis in the frequency domain. In the financial industry, the use of the frequency domain for evaluating and modelling economic systems is a relatively new development. This thesis demonstrates the potential of an economic-engineering approach to formally use both the time- and frequency domain for macroeconomic scenario analysis and modelling, making them intuitive to both engineers and economists.

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Preface and Acknowledgements

I am grateful to have had the opportunity to work on such an interesting and complex project for my master thesis. When I started my master in Systems and Control Engineering, I would have never thought that I would apply it to model an entire economy. The process of working on this thesis has made me push myself to my limits.

I want to thank my supervisor dr.ir.M.B.Mendel for this unique opportunity, his involvement, and his every available assistance throughout the period of this thesis. I also want to thank Ortec Finance for offering me the opportunity to work on this project at their company. Special thanks to dr. S. Wolters and MSc. E. Hennink for their immense support, guidance, and involvement in the project. Also a special thanks to ir. C. Hutters for his guidance and help in reviewing my work. Finally, I want to thank the economic-engineering group as a whole for showing interest and engagement in my work.

Chapter 1

Introduction

This thesis is divided into four main parts. In Part 1, Ortec Finance's modelling approach and the resulting scenario models are evaluated. In Part 2, the used macroeconomic theory an economic-engineering approach are presented. In Part 3, a modelling method and the resulting grey-box model are developed, and in Part 4, the model is qualitatively validated and system and control engineering tools are used to perform scenario analysis in the time and frequency domain to demonstrate the capabilities of the grey-box model.

1-1 Part 1: Dynamic Factor Models as the Black-Box Models

Economic modelling and forecasting are of critical importance in the financial industry [51, 70, 82]. The balance sheet and investment decisions of pension funds, asset managers and investment companies depend on the dynamics and the forecasted economic variables [77]. This is similar to how a control engineer would be able to obtain a more accurate control policy for a model that better describes the system. Companies such as Ortec Finance (OF) use a large scale Dynamic Factor Model (DFM) [78] to provide financial risk advisory services to financial institutions such as pension funds and insurance companies. These models are used to simulate sample paths for economic and financial variables such as inflation, interest rates and equity returns. Their clients purchase these models to generate scenarios of their own. This means that not only their economic forecasting accuracy is of great importance, but also the models interpretability.

In Chapter 2, it is shown that the DFM of OF is based on an advanced version of the DFM of Stock and Watson [82, 83] that uses Principal Component factors based on a large number of macroeconomic and financial variables. Using frequency filtered time series, the Ortec Finance model places these Principal Component factors in a Vector Autoregressive (VAR) model for the short-, medium- and long-term dynamics of economic variables.

In the same chapter, simplified representative versions of these models are built for structure and performance evaluation. This way, subjective adaptations of the final DFMs are excluded. It is shown that, although these models demonstrate satisfactory forecasting behavior, their structure prevents for a structured way to incorporate economic laws and principles. This is in line with OF's own findings [77, 79], who classify this as an essential limitation to further increase the forecasting capabilities of their models. Engineers classify these models as black-box models for which incorporation of a-priori information is known to be a difficult matter [93]. This chapter concludes by formulation of the specific limitations based on the representative recreated models.

1-2 Part 2: Macroeconomic Theory and the Economic-Engineering Approach

This thesis introduces an entirely different approach that results in a grey-box model. By using economic-engineering [55], economic principles and laws are leveraged to model economic systems, similar to how engineers use Newtonian or analytical mechanics to model physical systems [88]. A big advantage of this approach is that it offers additional tools to build and analyze interpretable models and explicitly capture specific economic relations.

Economic-engineering models have been successfully applied for economic modelling. These engineering models offer relatively low complexity while they are powerful in terms of outputs: even low order economic-engineering models can represent the same dynamics as highly complex econometric models. For example, see the theses of Vos [94] and Huisman [31] that use fourth-order models to mimic the dynamical behavior of econometric models of (inter)national economic systems. This insight proves that using an economic engineering approach is a promising option to develop grey-box models of economic systems.

In Chapter 3, the macroeconomic theory is provided that will function as the backbone of this thesis' grey-box model. This chapter also describes the limitations of this theory that need to be overcome to develop a dynamic model. The most limiting one is the general equilibrium assumption which states that the theory only holds in equilibrium.

In Chapter 4, the economic-engineering analogy is made specific to macroeconomics. In economics, there is no clear method to derive the equations of motions like for physical models. To model physical systems, Newtonian or analytical mechanics are used [88]. Economic systems do not have such methods to derive the equations of motion, but they do obey economic laws [65, 92]. In the field of economic engineering, these economic laws are reformulated as mathematical definitions to specify the dynamics of economic systems [55].

This thesis is a first effort to apply the economic-engineering theory to model an entire open economy. As a test case, this thesis considers the United States economy. This requires a different approach than previously used within economic-engineering. In Chapter 4, an approach is formulated to model the complex macroeconomic system using national accounting theory. First, the system is broken up into eight subsystems. For each of the subsystems energy storing elements are formulated. By introducing these energy storing elements, the equilibrium limitation of the macroeconomic theory [52] is overcome. A distinction is formulated between the financial domain, analogues to the rotational mechanical energy domain, and the economic domain, analogues to the translational mechanical energy domain. This results in a modelling approach that is also in line with macroeconomic theory such as the quantity theory of money [65].

This part is concluded by the formulation of an economic-engineering approach that incorporates economic theory and overcomes the limitations of national accounting.

1-3 Part 3: The Fundamental Grey-Box Model and its Transfer Function

In Chapter 5, the proposed modelling approach of the previous chapter is used to develop a grey-box model of the entire U.S. economy. Bond graphs are used to structurally model economic relations. A bond graph [38] is a tool frequently used within economic-engineering to model interconnected economic systems in an intuitive and graphical manner while obeying physical/economic laws. A bond-graph model is developed for each of the eight subsystems and subsequently aggregated for the total economic system. From the bond graph model, the state-space model is derived where each of the parameters and signals are given an economic interpretation, making the model interpretable to both economists as engineers.

The state-space model is not able to directly model the equity returns and the yields for government bonds with different maturities. These variables involve the discounting of future cash flows [20, 89] which are inherent to the frequency domain [90]. In Chapter 6, this is solved by taking the Laplace transform of signals from the time domain model to compute a nation's equity value, government bond value and labor value. This way, these variables are expressed as a function of a complex discount rate.

The chapter not only demonstrates the potential of the engineering frequency-domain but also facilitates the use of frequency domain analysis tools for macroeconomic scenario analysis. In Systems and Control (S&C) engineering, the Laplace transform is used to solve differential equations algebraically [28] and to analyze system in the frequency domain. This is a particularly interesting property because this thesis aims to perform scenario analysis. This chapter therefore gives an economic interpretation to Bode plots and transfer function. By doing so S&C frequency domain concepts and tools, can be used for macroeconomic scenario analysis in the frequency domain. The chapter introduces the notions of liquidity and efficiency as the economic interpretation to transfer functions. It is shown that by using Bode plots, the coincidence and the effectiveness of an input-output relation can be visualized as a function of frequency. Finally it is shown that the transfer functions can be used to obtain the long-run steady-state value of a response by means of the final value theorem. A new and useful property for economic scenario analysis.

1-4 Part 4: Model Validation and Scenario Analysis in the Time- and Frequency Domain

In Chapter 7, the developed state-space model and the transfer functions are used to perform macroeconomic scenario analysis. Firstly, the models behavior is qualitatively validated (using manually assigned parameters) by comparing economic theory to unit-step responses of the model. Then, the capabilities of the grey-box model are demonstrated by applying existing systems and control engineering tools such as Bode plots, pole-zero maps, transient responses, and root-locus analysis. It is demonstrated that the grey-box model allows for extensive macroeconomic scenario analysis in both the time and frequency domain. This part concludes that the economic-engineering grey-box model developed in this thesis is a promising competitor of the current black-box models.

Part 1

Chapter 2

Black-Box Approach to Current Macroeconomic Scenario Models

2-1 Introduction

In this chapter, I present a recreated simplified version of Ortec Finance's macroeconomic scenario models to formulate the specific interpretability limitations they have. The recreated simplified versions exclude the (subjective) fine-tuning the currently used models have undergone and allows for a more objective evaluation.

A more elaborate explanation of the modelling methodology of Ortec Finance (OF) is included in Appendix C. Finally, the performance validation results and quantitative evaluation of the recreated simplified version of OF's models are included in Appendix D.

I conducted both theoretical and practical research. Economic and econometric literature was reviewed to acquire the theoretical knowledge and a practical basis was set by means of an internship at OF at the Scenarios & Asset Valuation department. I define the goal of this chapter as follows.

Chapter goal:

1. Evaluate the Ortec Finance's black-box modelling approach to formulate its specific limitations.

First, in Section 2-2, I explain what economic scenario models are and what their relevance is. I give concepts related to economic scenario models an engineering interpretation. In Section 2-3, I explain OF's modelling approach. In section Section 2-4, I present a recreated simplified version of Ortec Finance's (OF) economic scenario models. Based on the simplified model, I formulate the interpretability limitations. Finally, in Section 2-5, I draw the conclusions from this chapter.

2-2 Macroeconomic Scenario Models

Because this is a thesis on macroeconomic [1] scenario models, it is important to understand what they are and their relevance.

2-2-1 Economic Scenarios for Optimal Policy Decisions

Economic scenarios are an important component of Asset & Liability Management (ALM) models. ALM models are used to support strategic policy decisions [77]. I adopt the definition for economic scenario's of Bunn and Salo [15]:

Definition 2-2.1: Economic Scenario

A scenario is a possible future scenarios which can be validated with a clear set of assumptions.

A model that is able to more accurately describe the underlying system is able to better forecast scenario's. A more accurate model decreases model uncertainty. A controller designed for a more accurate model is able to compute more efficient control actions and hence a more optimal strategic policy decision. The forecasting behavior of these models are validated with so-called stylized facts.

2-2-2 Stylized Facts as Economic Laws

Ortec Finance uses a set stylized facts to compare the scenario forecast results to. Stylized facts are defined as Sewell [69]:

Definition 2-2.2: Stylized Fact

A term used in economics to refer to empirical findings that are so consistent (for example, across a wide range of instruments, markets and time periods) that they are accepted as truth. Due to their generality, they are often qualitative.

These stylized-facts can thus be interpret as empirical economic laws that the economic scenario's responses must be able to exhibit. The stylized facts used by OF concern the term structure of risk and return, business cycles, time-varying volatility, tail-risk, non-normal distributions and yield curves. An example, the term structure of risk and return is a stylized fact which states that risk and return in terms of volatilities, correlations and distributions can be different depending on the investment horizon. Another example is the existence of business cycles [78].

[1] Macroeconomics is the branch of economics concerned with large-scale or general economic factors, such as interest rates and national productivity

2-2-3 Economic Cycles as Frequencies

Each post-19th-century economic school of thought has investigated the existence of cyclic behaviour in macroeconomics. From Burns and Mitchell [16] the definition of a business cycle is stated to be:

Definition 2-2.3: Business Cycle

A type of fluctuations found in the aggregate economic activity of nations that organize their work mainly in business enterprises: in duration business cycles vary from more than on year to ten or twelve years.

Some examples of identified economic cycles are: The Kitchin 'inventory' cycle named after Kitchin [44] with a pattern of fluctuation in economic growth rates with a length of three to five years. This cycle seems to be caused by constant over- and undersupply in business inventories compared to the required levels. Another example is the Juglar 'investment' cycle [37] with a length of seven to eleven years which appears to be driven by over- and undersupply in investments in fixed assets such as plants and machinery.

The existence of theses cycles lay the foundation of the frequency-domain approach used by OF [77] and why it is intuitive to use this approach. In Section 4-3-2, I show that these economic cycles will be integrated in the engineering grey-box model by using combinations of I- and C-elements. In Section 7-3, I use transfer functions and Bode plots to analyze the frequency behavior of this thesis' grey-box macroeconomic scenario model.

2-3 Dynamic Scenario Generator Modelling Approach

The Dynamic Scenario Generator (DSG) is a tool OF has developed to generate economic scenarios. Here, I present the general approach used by OF to develop their macroeconomic scenario models, i.e. their DSG.

OF's modelling approach can be described the three steps of the pyramid setup shown in Figure 2-1. The lower two steps involve the modelling simulation, validation and back-testing. The top layer involves the tuning of the model to include more information than the data describes.

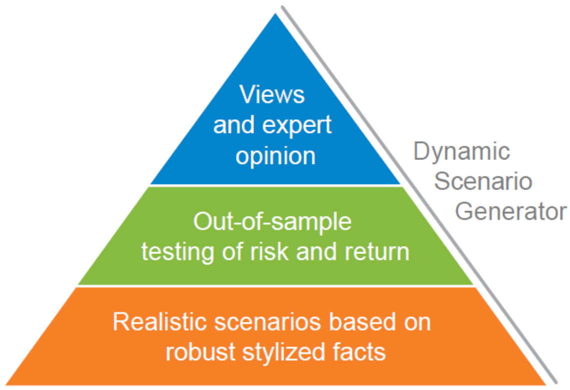


Figure 2-1: Ortec Finance's DSG modelling methodology [24]

Compared to similar models in industry, the DSG primarily distinguishes itself by using a frequency domain approach. A frequency domain approach refers to two things. Firstly, it refers to the way the time series data are processed; The time series data are transformed to the frequency domain using the Fourier transform and are subsequently band pass filtered using an in-house designed frequency filter, the Zero Phase Shift filter [47] [77]. From the filtered series three separate models are developed. Secondly, frequency domain tools such as spectral analysis are used to verify if the models incorporates certain stylized facts.

I will evaluate OF's models without the inclusion of expert views and opinions. Expert views and opinions depend on knowledge of the model maker and can be subjective. However, the potential of implementing them is important. A model that consists of signals and parameters that have an economic interpretation, simplifies the implementation of expert views and opinions. For example, to apply an impulse responses or shock to a specific economic variable using the models of OF requires the computation of a specific combination of initial states and inputs. In Section 7-2, I demonstrate this is evident for the grey-box model developed in this thesis.

The interpretability issue also influences the incorporating stylized facts in the models. A grey-box approach allows for structured incorporation of new and different stylized facts. Since these stylized facts represent economic laws, their implementation should increase the scenario model's forecasting performance. In Chapter 4, I show how the economic-engineering theory incorporates economic laws in the models.

2-4 Recreating the Dynamic Scenario Generator for Evaluation

To evaluate the economic scenario models of OF without the inclusion of expert views and opinions, I recreated the trend and business cycle DSG models. This allows for objective comparison to this thesis's grey-box model. The out-of-sample performance results of the simplified models give an impression of what the current models are capable of in performance. This gives an indication of what the grey-box model should be able to match. I have included the model identification steps and performance results in Appendix D.

OF has defined five variables that need be modelled for this thesis:

1. year-on-year Gross Domestic Product (GDP);
2. year-on-year Consumer Price Index (CPI);
3. year-on-year Equity Total Returns (EQTR);
4. Nominal Government Bond with 10 years maturity yield rate (NGLR);
5. Nominal Government Bill with 3 months maturity yield rate (TBSR)

EQTR refers to the total return on stocks including dividends. It is equal to the Morgan Stanley Capital International Index (MSCI) of US equity returns.

This thesis is a first effort to develop a grey-box macroeconomic scenario model. As a test case, I will only consider the United States economy. The United States economy is the largest in terms of GDP, one of the better understood, transparent, and frequently studied economies of the world. As a result there is sufficient data available for validation and comparison.

2-4-1 The Black-Box Model Structure

The modelling approach of OF is a large scale Dynamic Factor Model that uses Principal Component Analysis (PCA) factors based on a large number of macroeconomic and financial variables. Separate models are developed for three separate frequency bands. A more detailed and elaborate explanation of OF's modelling approach are included in Appendix C.

At the basis of the models is Eq. (2-1) [7]. In state-space form these equations form the system of Eq. (2-2).

$$
\begin{aligned}
Y(k) &= \Lambda F(k) + \Phi Y(k-1) + c_y + \epsilon(k) \\
F(k) &= \Gamma F(k-1) + c_f + \eta(k)
\end{aligned} \tag{2-1}
$$

$$
\begin{aligned}
\underbrace{\begin{bmatrix} F(k+1) \\ Y(k+1) \end{bmatrix}}_{x(k+1)} &= \underbrace{\begin{bmatrix} \Gamma & 0 \\ \Lambda\Gamma & \Phi \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} F(k) \\ Y(k) \end{bmatrix}}_{x(k)} + \begin{bmatrix} 1 & 0 \\ B & -1 \end{bmatrix} \begin{bmatrix} c_f \\ c_y \end{bmatrix} + \underbrace{\begin{bmatrix} I & 0 \\ \Lambda & I \end{bmatrix}}_{E} \underbrace{\begin{bmatrix} \eta(k+1) \\ \epsilon(k+1) \end{bmatrix}}_{\zeta(k)} \\
Y_{out}(k) &= \underbrace{\begin{bmatrix} 0 & I \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} F(k) \\ Y(k) \end{bmatrix}}_{x(k)} + \begin{bmatrix} c_\mu \end{bmatrix}
\end{aligned} \tag{2-2}
$$

where,

$$\zeta(k) \overset{i.i.d.}{\sim} N\left(0, \begin{bmatrix} \Sigma_\eta & \Sigma_\eta \Lambda^T \\ \Lambda \Sigma_\eta & \Lambda \Sigma_\eta \Lambda^T + \Sigma_\epsilon \end{bmatrix}\right) \tag{2-3}$$

and, $Y(k) \in \mathbb{R}^{(309\times1)}$, $F(k) \in \mathbb{R}^{(N\times1)}$, $\Gamma \in \mathbb{R}^{(N\times N)}$, $\Lambda \in \mathbb{R}^{(309\times N)}$, and $\Phi \in \mathbb{R}^{(309\times309)}$. $Y(k)$ is the discrete time set of filtered time series on a yearly time-scale for the business cycle model and on a 8-yearly time-scale for the trend model. N corresponds to the number of factors. The number 309 refers to the number of time series data used.

The model has no economically interpretable inputs because the modelling method only considers time series output data to obtain the principal components. The PCA procedure causes loss of economic interpretation of the factors/states, $F(k)$. Finally, the parameters are estimated directly from the data and therefore have no economic interpretation. The combination of these three aspects cause the interpretability issues.

In engineering these models are considered to be black-box models as the structure and parameters are determined from experimental modelling solely based on measurement data[59]. In the next section, I state the specific limitations of these models based on this model structure and the modelling approach.

2-4-2 The Interpretability Limitations

The previous chapter showed why incorporating of economic theory and a-priori information of the system is challenging or not possible at all. Here, I state the specific limitations that result from the lack of interpretability:

1. The combination of latent states, the lack of measurable inputs, and the economically meaningless parameters, makes simulation of certain shocks and responses challenging;
2. Incorporation of economic principles or a-priori system information is unintuitive or not possible;
3. The lack of interpretable states and measured inputs makes controller design and closed loop scenario analysis impossible.

In addition, OF has clients that purchase their economic scenario models. Since these clients want to generate scenario's of their own, not only the forecasting accuracy, but also the interpretability of the models are a highly desired property.

A grey-box model [2] is able to resolve these issues. Using economic-engineering, a grey-box model can be developed that uses economic principles at the basis to derive the equations of motion (see Chapter 4). As a result, a grey-box model directly capture these economic principles and expresses them in economic meaningful parameters, signals, and variables.

In Chapter 4, I formulate analogies between engineering and the macroeconomic concepts of Chapter 3. Using both theories, I develop a structured method and grey-box model in Chapter 5.

[2]see Section 4-2 for an engineering definition of a grey-box and black-box model

2-5 Conclusions and Next Steps

OF develops economic scenario models using a black-box modelling approach. The loss of economic interpretability is caused by the combination of 1) only considering output time series data, 2) using Principal Component Analysis to obtain the states, and 3) the use of statistic parameters estimation methods,

The evaluation performed in this chapter results in the identifications of three main limitations of the current black-box approach. 1) The incorporation of economic principles and a-priori system information is often not possible. 2) Simulating specific economic shocks is a time-consuming process. 3) The model does not allow for interpretable control scenario's. Additionally, OF desires a more interpretable model because their clients prefer this. This motivates the development of a grey-box model. This chapter concludes Part 1 of this thesis. Part 2, 3 and 4 of this thesis will consists of the following steps.

Next Steps Towards Engineering Macroeconomic Scenario Analysis

2. **Macroeconomic Theory and Economic-Engineering Approach**
   - (a) Macroeconomic theory as the basis for Grey-Box Modelling (Chapter 3)
   - (b) Economic-Engineering Analogy for Macroeconomic Scenario Modelling (Chapter 4)
3. **The Fundamental Grey-Box Model and its Transfer Function**
   - (a) Development of a Time Domain Model of the United States Economy (Chapter 5)
   - (b) The Engineering Frequency Domain for Valuation and Liquidity (Chapter 6)
4. **Model Validation and Scenario Analysis in the Time- and Frequency Domain**
   - (a) Time and Frequency Domain Analysis and Validation (Chapter 7)

Part 2

Chapter 3

Macroeconomic Theory as a Basis for Grey-Box Modelling

3-1 Introduction

In this chapter, I provide macroeconomic and national accounting theory. This is important, since this theory functions as a basis for this thesis' grey-box scenario model. Because the modelling approach used by Ortec Finance (OF) is an entirely different approach, I can not use it in a similar way to develop a grey-box model. The goal of this chapter is as follows:

Chapter goal:

1. Introduce macroeconomic concepts and theory that will be used to build a grey-box macroeconomic scenario model.

In Section 3-2, I introduce important general aspects of macroeconomic theory. In Section 3-3, I present national accounting and the circular flow theory of an economy which will function as the backbone of my economic-engineering model. Finally, I state this chapter's conclusions in Section 3-4.

3-2 Macroeconomic Theory

Economics today is divided into two major subfields, microeconomics and macroeconomics. For this thesis's models, I most prominently use macroeconomic theory. However, because macroeconomic behavior is the result of aggregating many microeconomic decisions, both are essentially captured by the theory. This is similar to how classical mechanics is used to describe the motion of macroscopic objects which is the result of the dynamics of many moving particles. Besides these two major subfields, there exist several theories of how economies behave which are described by the different economic schools of thought.

3-2-1 Macroeconomic Schools of Thought

I use the term 'economic first principles' to describe laws that are based on economic theory. There are several macroeconomic theories within modern macroeconomics which are formulated by different economic schools of thought. An economic school of thought can be defined as a group of economic thinkers who share or shared a common perspective on the way economies work [1]. Hence, contrary to the natural sciences there is more than one set of laws to model macroeconomic systems.

Since I use macroeconomic underlying theory to model an economy, i.e. the United States, the outcome is dependant on the theory used. This thesis's macroeconomic models are based on mainstream economics. It is not a school of economic thought itself, rather uses a combination of some modern schools of thought. These obey both micro- and macroeconomic laws. Some essential assumptions are market clearing, budget constraints, price as the sum of cost, rational behavior of firms and households and business cycle behavior among others. How these assumptions and laws are incorporated in the economic-engineering theory is explained in Section 4-3-2.

3-2-2 The Short, Medium and Long Run

Macroeconomic literature [6, 52, 65, 95] distinguish between the short, medium and long run macroeconomic theory and their models. This is in line with the approach taken by OF. The different runs are subdivided as follows:

The Short Run This part examines the behavior of the economy when prices are inflexible or 'sticky'. Price stickiness is the resistance of market prices to change quickly despite shifts in demand or supply [64]. This means that in economics it is assumed that markets do not clear in the short run. The time scale is from month to month or from year to year,

The Medium Run This parts examines the behavior of the economy in the classical sense where prices are flexible or non-sticky. In other words, the key assumption is that markets clear continuously and prices adjust instantly to the changes in supply and demand. It is best suited for analyzing a time horizon for at least several years like business cycles [52]. This horizon involves most of the national accounting from Section 3-3.

The Long Run The long run part examines structural growth of the economy in capital stock, the labor force, and technological knowledge. Again, the assumption of flexible or non-sticky prices holds.

In Section 7-4, I show how pole-zero maps can be used to locate the different cycles of the system using the grey-box scenario model of Chapter 5.

3-3 National Income Accounts as the Model Backbone

The national accounts relate the different markets and economic actors in a clear and mathematical way. It is therefore a good candidate to use as a basis for the modelling of a whole economy

National income accounting is a double-entry bookkeeping system used by governments that measures the level of a country's economic activity in a given time period [65]. This economic producing activity is called the Gross Domestic Product (GDP). It is the market value of all final goods and services produced within an economy in a given period of time and is measured in $/year.

There are two approaches to compute GDP, the expenditure and income approach. The two approaches are two sides of the same coin, both yield the same results by the rules of accounting, the expenditure of buyers on products is the income to the sellers of those products.

3-3-1 Expenditure Approach

The most common used approach to computing GDP is the expenditure approach [65]. It measures the flow of the total number of final goods and services bought or expended. The GDP can be computed with the expenditure approach using the relation of Eq. (3-1).

$$\mathrm{GDP} = \mathrm{C} + \mathrm{I} + \mathrm{G} + \mathrm{NX} \tag{3-1}$$

Here, gross domestic product (GDP) is the sum of the dollar values of consumption (C), gross investment (I), government purchases of goods and services (G), and net exports (NX) produced within a nation during a given year.

This equation is an identity - an equation that must hold because of the way the variables are defined [52]. All variables are measured in $\$/year$. In this thesis, I will leverage this equation to build an economic-engineering model. I provide the definition as stated in Mankiw [52] for each of the components.

- *Consumer Spending (C).* Household expenditures on durable and nondurable goods and services.
- *Gross Investment (I).* Items bought for future use. They are divided into three subcategories: business fixed investment, residential fixed investment.
- *Government Spending (G).* The goods and services bought by federal, state, and local governments.
- *Net Export (NX).* The value of goods and services sold to other countries (exports) minus the value of goods and services that foreigners sell us (imports).

3-3-2 Income Approach

The other approach to compute GDP is the income approach. I will combine the relations of both the income as the expenditure approach for the grey-box model basis.

Income refers to the flow of wages, interest payments, dividends and other things of value accruing during a period of time [65]. National Income (NI) is the aggregate of all income in an economy. The income approach measures the NI received by economic agents contributing to production. Following Mankiw [52], the five categories are:

- *Compensation of employees.* The wages, salaries and fringe benefits earned by workers.
- *Proprietors' income.* The income of noncorporate businesses or self-employed firm owners.
- *Rental income.* The income that landlords receive, including the imputed rent that homeowners "pay" to themselves, less expenses, such as depreciation. Rental income includes the 'rent' paid for land to extract natural resources.
- *Corporate profits.* The income of corporations after payments to their workers and creditors
- *Net interest.* The interest domestic businesses pay minus the interest they receive.

Within the United States, the two most contributing factors to NI are compensations of employees and corporate profits (approximately 80 % [52]). Furthermore, the relation between NI and GDP is:

$$\text{GDP} = \text{NI} + \text{SD} + \text{IBT} + \text{Dep - NFFI} \tag{3-2}$$

where the components are [52]:

- *Depreciation (Dep).* The consumption of fixed capital or durable goods.
- *Net Foreign Factor Income (NFFI).* The payments to domestically owned factors of income located abroad minus payments to foreign factors located domestically.
- *Indirect Business Tax (IBT).* Certain taxes on businesses, such as sales taxes, less offsetting business subsidies. These taxes place a wedge between the price that consumers pay for a good and the price that firms receive and are a type of income of the government.
- *Statistical Discrepancy (SD).* Discrepancy arising from different data sources that may not be completely consistent.

3-3-3 Factors of Production

The incomes of previous sections are the payments for the outputs generated by the factors of production. The definition for factors of production as given by Samuelson and Nordhaus [65] is:

Definition 3-3.1: Factor of Production

Input used to produce goods and services.

Economist divide the factors of production into four categories as defined by [65]:

- *Land.* The gift of nature to our societies. This includes natural resources. It consists of the land used for farming or for underpinning houses, factories, and roads; the energy resources; and the nonenergy resources like copper and iron ore and sand.
- *Labor.* The human time spent in production.
- *Capital.* Assets of an economy, produced in order to produce yet other goods. Capital goods include machines, roads, computers, software etc.
- *Entrepreneurship.* The ability to combine the other three factors of production to innovate and earn a profit.

Land, labor and capital are considered to be the classic the factors of production. The last, entrepreneurship, is a more modern concept [67].

All five categories of NI as stated in Section 3-3-2, are the payments for the outputs produced by the factors of production. Wage or salary is the price of labor, interest the price of capital, rent the price of land and profit and proprietors' income the price of entrepreneurship.

3-3-4 Circular Flow Theory

To connect and visualize the mathematical accounting relations of previous sections, macroeconomist use the circular flow theory [52].

In an economy, several economic decision makers or economic actors trade goods and services or factors of production in exchange for a price. These result in flows of income and expenditure flowing from one decision maker to the other. How these income flows propagate through an economic system is referred to as the circular flow of an economy. The circular flow theory follows the accounting relations of previous sections. Figure 3-1 [52] displays a circular flow diagram for a simplified two-sector economy. The two sectors are the firms and the households. Each cash flow has a flow of inputs or outputs going in opposite direction.

The circular flow diagrams provide a clear graphical basis for my model while incorporating the national accounting relations. In Chapter 5, I use an extended version of this circular flow diagram to build a bond graph model of the United States economy.

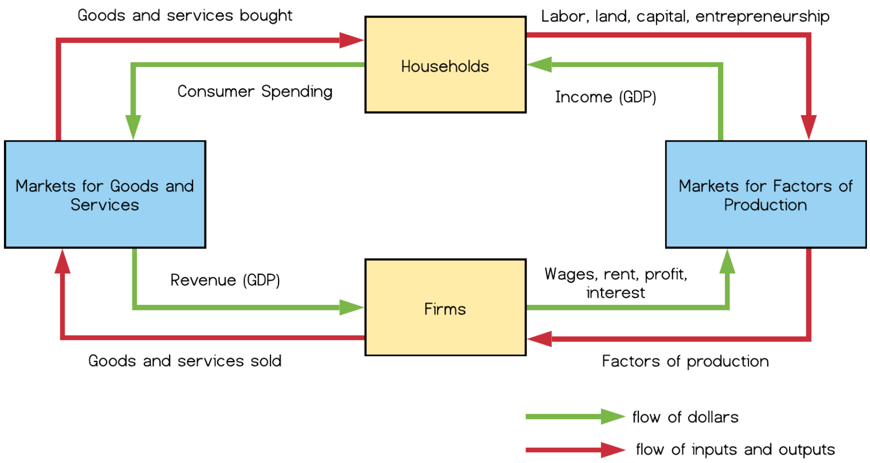


Figure 3-1: Circular flow diagram of a simplified economy with two actors and two types of markets

3-3-5 The General Equilibrium Assumption as Limitation

The circular flow theory and its corresponding national income accounts assume a situation of macroeconomic general equilibrium, i.e. aggregate demand and aggregate supply are in balance and all markets clear [65]. This means that GDP does not change. The assumption limits the usefulness of the macroeconomic theory for modelling dynamic behavior.

With the use of a structured economic-engineering system theory approach, I am able to overcome this limitation. The elements in bond graphs store and release energy in disequilibrium (see Section 4-3). The macroeconomic interpretation of these elements is not always available in macroeconomic theory and needs to be defined (see Section 4-4).

The equilibrium assumption in the national accounts theory for a four-sector economy leads to the Saving-Investment identity:

Saving-Investment identity for four-sector economy:

$$\underbrace{\text{S + (M-X)}}_{\text{Supply of financial capital}} = \underbrace{\text{I + (G-T)}}_{\text{Demand for financial capital}} \tag{3-3}$$

Saving (S), imports (M), and taxation (T) are referred to as withdrawals or leakages and investment (I), government expenditure (G), and exports (X) are known as injections to the system [42].

In this thesis I will assume that this equality must only hold in equilibrium. Within the context of the circular flow diagram, this is achieved if the amount of leakage or withdrawal is equal to the amount of injection. If leakages > injections, GDP contracts and inversely, if injections < leakages, GDP expands. Or similarly, if more energy is dissipated than enters the system, the energy of the closed system decreases. If more energy enters the system than is dissipated, the total energy of the closed system increases.

Because these variables are considered to represent either leakages or injections to economic system, I will use them as inputs to the system to simulate dynamic scenarios in Section 7-2.

3-3-6 Quantity Theory of Money for Domain Equivalence

Within macroeconomics, money supply plays a key role in determining aggregate demand [65]. The quantity theory of money is considered to be the leading explanation of how money affects the economy in the long run [52].

The money circulating in an economy is assumed to move with a certain velocity [65]. It is assumed that during periods of rapid inflation, money circulates quickly. The concept of velocity of money is defined as the rate at which money circulates through and economy Samuelson and Nordhaus [65]. The quantity equation of exchange that introduces the velocity of money is given by [65]:

$$\begin{array}{rcrcrcr} \text{Money} & \times & \text{Velocity} & = & \text{Price} & \times & \text{Output} \\ \text{M} & \times & \text{V} & = & \text{P} & \times & \text{Y} \end{array} \tag{3-4}$$

Here, money is measured in \$, velocity is measured in $\frac{1}{\text{yr}}$, price in $\frac{\$}{\#}$, and output in $\frac{\#}{\text{yr}}$.

Velocity is expressed as a rate, which I interpret to be an analogue of angular velocity. In Section 4-3-2, I present an alternative interpretation of this theory where the identity is the equivalent of the conservation of energy between rotational and translational kinetic energy. I use this concept to distinguish between the financial and economic domain analogues to the rotational mechanical and translational mechanical domain, respectively.

3-4 Conclusions

Macroeconomic theory is a concatenation of several theories from different economic schools of thought. It is often divided into the short, medium and long run [6, 52, 65]. In this thesis, I use concepts of several economic schools of thought gathered under mainstream economics to develop one modelling methodology for all time horizons using a economic-engineering approach.

The combination of the national accounting theory and the circular flow theory provide a clear mathematical and visual relationship between the cash flows in an economy. Because of its clear mathematical accounting definitions, general agreement within macroeconomic theory, and clear visualization, I will use this theory as a basis to model an economy.

The theory has its limitation. For example, the bookkeeping identities only hold for an economy in equilibrium, the theory is vague on dynamics within the markets and of the actors, and many of the presented interconnections are too blunt to make a realistic model. In addition, not all variables I want to model are described by the national accounting theory.

In the next chapter I will translate the macroeconomic theory of this chapter to engineering concepts using economic-engineering. This structured approach also enables me to overcome the limitations of the theory. In Section 4-4, I present my structured engineering approach to model an economy based on the theory presented in this chapter.

Chapter 4

Economic-Engineering Analogy for Macroeconomic Scenario Modelling

4-1 Introduction

In this chapter, first I give a definition for the different type of models. This way, it is clear for both economist as engineers what I mean when I use the terms grey-box and black-box model. Then, I present the economic-engineering theory reformulated to macroeconomics so that it can be used for the development a grey-box model. It is also shown how the bond graph formalism incorporates (macro)economic laws to verify that the modelling approach adheres to economic principles. Finally, I formulate the proposal of how I intend to model the economic system using economic-engineering and macroeconomic theory. I state the goal of this chapter as follows:

Chapter goals:

1. Introduce economic-engineering theory specific to macroeconomics and show how it incorporates economic laws and principles
2. Develop a preliminary modelling methodology using economic-engineering and macroeconomic theory from the previous chapters to build a grey-box model

First, I state the different types of models within systems engineering in Section 4-2. Subsequently, I present the economic-engineering analogue theory specific to macroeconomic scenario models and explain how they incorporate economic laws in Section 4-3. In Section 4-4, I postulate the beginning of the economic-engineering approach to macroeconomic scenario model development. Finally, I conclude and state the next steps in Section 4-5.

4-2 White-box, Grey-box and Black-box Models in Engineering

Because the modelling methodologies used by me and Ortec Finance (OF) are different in structure, I want to clarify which types of models exist. This way, the reader should be able to clearly distinguish between the economic scenario models of OF and this thesis.

There is not an unambiguous definition of these classifications, but in the field of engineering, most sources will refer to somewhat the same definitions. For this thesis, I follow the definitions given by Nelles [59] for the types of models:

1. *White-Box Models* are fully derived using first principles, i.e. physical, economic etc. laws. All parameters can be derived using theoretical modelling. These models do not depend on data and their parameters possess a direct interpretation in first-principles
2. *Black-Box Models* are based solely on measurement data. Both model structure and parameters are determined from experimental modelling. No or very little prior knowledge is exploited.
3. *Grey-Box Models* are a compromise or combination of the above two classes. Typically, the determination of the model structure relies on prior knowledge while the model parameters are determined mainly by measurement data.

In this thesis, I build a grey-box model using first principles from the economic-engineering analogue theory which includes the macroeconomic theory from Chapter 3.

The essence of this grey-box modelling methodology is that the structure of the model carries a great deal of the dynamical behavior of the system. The systems are designed using analogue economic principles derived from macroeconomic theory. Consequently, the parameters and variables have underlying economic meaning which increases interpretability of the model. In contrast, OF starts with the measurement data and tends to correct the model using stylized facts and expert opinions during or after the main system identification steps.

4-3 Economic-Engineering for Macroeconomics

An economy can be simplified to a system of interconnected markets and agents that interact through both endogenous and exogenous signals. The signals can be analyzed or controlled in the time and frequency domain. Economic-Engineering aims to apply this knowledge to economic systems.

4-3-1 Analogues of Newtonian Mechanics for Macroeconomic Modelling

To build a grey-box model, an engineer uses first principles expressed in physical domain variables to model a physical system. Within the field of engineering there exist several domains that can be modelled using the same technique due the similar properties of the different domains [35, 38]. The similar properties of the different domains are captured by the formulation of analogues.

Analogues are pairs of variables of different domains that show identical behavior. They are used to model connected systems such as mechantronic systems, where electrical and mechanical subsystems are interconnected. These mechatronic systems can be modelled using the same analogue first principles.

Mendel [55] has extended the analogue theory to economic. Table 4-1 displays list of economic-engineering analogues specific to macroeconomic scenario modelling. The finance domain variables are direct analogues to rotational mechanics whereas the analogues to economics can be viewed as the direct analogues to translational mechanics.

Table 4-1: Analogues in economic and finance domain

General	Economics		Finance	
	Variable	Unit	Variable	Unit
Momentum (p)	Price(p)	$\frac{\$}{\#}$	Financial Capital (L)	\$
Displacement (q)	Asset(q)	#	Interest (a)	%
Effort (e)	Want ($\dot{p}$)	$\frac{\$}{\#\text{-yr}}$	Loanable Funds (LF)	$\frac{\$}{\text{yr}}$
Flow (f)	Asset Flow($\dot{q}$)	$\frac{\#}{\text{yr}}$	Interest Rate (r)	$\frac{\%}{\text{yr}}$
Power (P)	Growth(G)	$\frac{\$}{\text{yr}^2}$	Growth(G)	$\frac{\$}{\text{yr}^2}$
Energy (E)	Income (E)	$\frac{\$}{\text{yr}}$	Cash Flow (CF)	$\frac{\$}{\text{yr}}$
Action (S)	Wealth	\$	Wealth	\$

The behavior of physical systems is determined by a system's power or energy flows. In general, there are energy storing elements which store momentum and position and energy dissipating elements which dissipate momentum or a position. Position and momentum are therefore known as *energy variables*. Power is the time derivative of the energy and is the product of the effort and flow. Efforts and flows are therefore known as *power variables* [35, 38].

The (macro)economic analogues give intuitive economic meaning that underline the interpretability I aim to introduce into the scenario models.

4-3-2 Bond Graphs for Macroeconomic Modelling and Economic Laws

A bond graph is a graphical representation of a dynamical system citetbond2006. The use of bond graphs for macroeconomic systems is advantageous for the following reasons:

1. *Domain Neutrality*: Bond graphs are domain neutral. Economic-engineering theory exploits this property using analogues to model systems. Each of the markets within macroeconomics operate in different domains and are connected into one system.
2. *Laws Integration*: The physical and thus analogues economic laws are built into the design of bond graphs. This allows for modelling economic systems without violating economic principles.
3. *Graphical Modelling*: Bond graphs model systems in a graphical manner. Although the underlying principles can be complicated, its graphical properties make the modelling process more intuitive.

4. *System Derivation*: Bond graphs allow for direct state space or transfer function derivation so that systems and control theory can be applied.

1-Ports and Multi-Ports as Economic Laws

There are two types of 1-port elements. The first type is active ports. These are effort and flow source and sink elements and provide or withhold the system of some effort or flow and correspond to the modelled inputs and outputs of a system and are the analogue of the injections and withdrawals from Section 3-3-5. An overview of the 1-ports and their economic analogues are displayed in Table 4-2.

Table 4-2: Analogues of 1-port elements in electrodynamics and mechanics

General	Mechanics	Economic
S_e (Effort Source)	External Force (F)	Marketer
S_f (Flow source)	External displacement ($\dot{x}$)	Producer
C (Compliance)	Spring (k)	Inventory
I (Inertance)	Mass (m)	Agent
R (Resistance)	Damper (b)	Dissipator

The second type is inactive ports and correspond to the C-,I- and R-elements. Each of the three has a specific transformation effect on the flows and efforts of the system. The 1-port elements are listed in Table 4-2. Their functions are as follows:

- *I-elements* store a price, analogues to momentum in mechanics. Here, a net cost is integrated or accumulated which gives the reservation price. Consequently, the price is transformed to a flow of goods using the price elasticity m^{-1}. The element obeys Newtons second law, equivalent to Marshall's principle as shown below.

$$f(t) = \frac{1}{m} \int^t e(t)dt \tag{4-1}$$

- *C-elements* store asset position, analogues to distance in mechanics. Here, a net flow of goods is integrated or accumulated resulting in the scarcity or excess of, for example, an ideal inventory level. Consequently, this asset position is transformed to the holding cost or convenience yield by the accounting liquidity k. The element obeys Hooke's law, best described by the economic equivalent Hotelling's law [55] as shown below.

$$e(t) = k \int^t f(t)dt \tag{4-2}$$

- *R-elements* do not store anything. Instead, they dissipate energy or income. It can be used to represent the unmodelled degrees of freedom of a system that diminish a cash flow either by reducing the spread between two flows, consumption, or the spread between two yields, depreciation, amortizaton, and depletion.

$$e(t) = Rf(t) \tag{4-3}$$

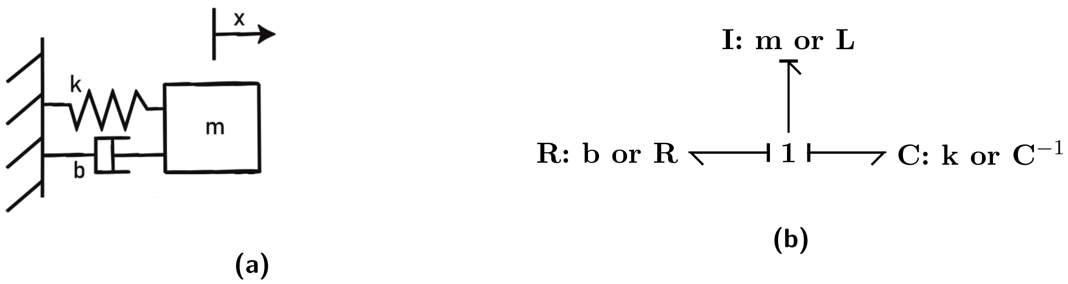


Figure 4-1: The bond graph (4-1a) and mass-spring-damper system (4-1b) are the same

This makes the bond graph and the mass spring damper systems of Figure 4-1 equivalent.

It is the combination of an I- and C-element, known as the harmonic oscillator, that cause the oscillations or trading cycles. The economic interpretation of a harmonic oscillator is a trader that buys and sells stock as is shown in Figure 4-2. The mechanism is juxtaposed to how economic agents trade on each of the connected markets. Their actions create different trading cycles for each of the markets. This way business cycles are included in the modelling methodology.

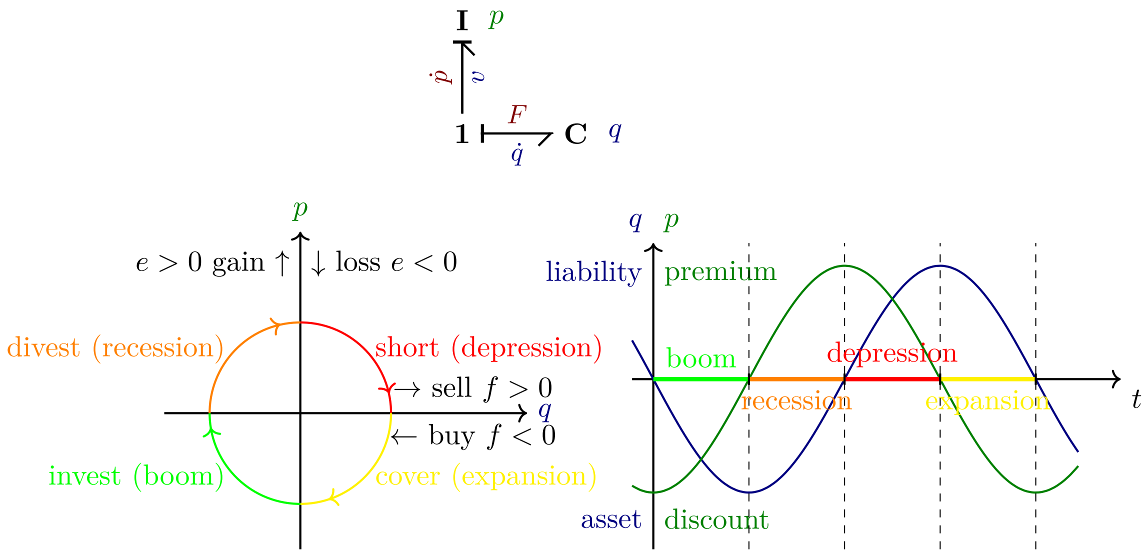


Figure 4-2: The analogy between a harmonic oscillator and an ideal trader. Bond graph and phase portrait of the trader. Stock levels and price movements during the various phases of the trade cycle [55]

Multi-ports enable the flow of energy between the various elements. There are two kinds of multi-ports, also known as junctions. The 0-junction and the 1-junction. On the junctions the net power equals zero, hence they conserve energy and are reversible.

- *0-junctions* can be viewed as resource allocation mechanisms. The flows (of goods) are balanced out and the costs/benefits or efforts coincide. In mechanics these are the kinematic constraints and the economic equivalent is the Edgeworth box of an exchange economy. The goods clear and the budget is constraint as displayed in Eq. (4-4).

$$\begin{aligned} e_1 = e_2 = ...e_n \\ \sum f_{in} = \sum f_{out} \end{aligned} \tag{4-4}$$

- *1-junctions* can be viewed as a price determination mechanism. The 'wants' or efforts are balanced out and the flows (of goods) coincide. In mechanics this is described by Newton's third law which states that 'action=reaction' which is the analogue of market clearing where 'demand=supply'. The relation is displayed in Eq. (4-5).

$$\begin{aligned} f_1 = f_2 = ...f_n \\ \sum e_{in} = \sum e_{out} \end{aligned} \tag{4-5}$$

Table 4-3 provides an overview of the junctions of the junction and their interpretation in the mechanical, electrical and economic domains.

Table 4-3: Analogues of 3-port elements in electrodynamics, mechanics and economics

Junction	Mechanics	Electrodynamics	Economics
0	Kinematic Constraint	Kirgoff 1^{st} law	Coincidence of Wants
1	d'Alambert's Principle	Kirgoff 2^{nd} law	Say's Law

2-port Elements for Agent-Market Interconnections

2-ports change specific attributes of the effort and flow while conserving power. They can transform the domain, power direction, or effort-flow ratio. There are two types of 2- ports. The gyrator maps flow variables to effort variables and effort variables to flow variables. The transformer maps flow variables to flow variables and effort variables to effort variables [38].

I use 2-port elements to interconnect markets and economic agents. Both the firms and the households are connected to multiple markets: The markets for factors of production (capital, land, labour, entrepreneurship), the market for consumer goods and services and the market for loanable funds, as described in Section 4-4.

The Quantity Theory of Money in Bond Graph Formalism

The first law of thermodynamics states that energy can neither be created nor destroyed, but can change form [57]. This is analogues to stating that cash flows can neither be created, nor destroyed but can change form. I interpret the quantity theory of money from Section 3-3-6 to be a statement that obeys this description.

I apply and substitute the analogues defined in this chapter to the quantity theory of money to demonstrate that the theory, Eq. (4-6), can be viewed as the equivalent of the law of conservation of energy, Eq. (4-7).

$$M \times V = \quad P \times Y \tag{4-6}$$

$$\frac{1}{2}I \times \omega^2 \quad = \quad \frac{1}{2}m \times v^2 \tag{4-7}$$

Here, M is the money measured in \$, V is the velocity in 1/yr, P the price in \$/yr and Y the output in #/yr from the quantity theory of money presented in Section 3-3-6.The variables that have not been covered in this chapter are: I which is the moment of inertia, analogues to the dollar duration in \$yr, and ω which is the angular velocity, analogues to the interest rate r in %/yr.

The cash flow is conserved, but changes form. It is transformed from the domain of finance to the domain of economics or in mechanics, from the rotational to translational mechanics domain. In this regard, this approach is in line economists' idea of money [41].

The 2-port elements do exactly this. They transform from one energy domain to the other while conserving both power as energy. The bond graph formalism thus builds in the quantity theory of money.

Energy Domain Tetrahedrons of State

Figure 4-3a shows the tetrahedron of state that is used to model the market for consumer goods and services and the market of factors of production. Note that assets measured in #, could also indicate land in Square Foot (SF), labor in Full-Time Equivalents (FTE), capital assets in units (#) or consumer goods and services in quantity (#).

The financial market does not use quantities of goods but rather expresses a cash flow in \$/yr as the product of an interest rate measured in %/yr and financial capital in \$ [65].

Figure 4-3b shows the tetrahedron of state in angular momentum-angle coordinates, analogue to the conjugate pair financial capital-interest. In physics, the C-elements conventionally store the angle variable and I-elements store the (angular) momentum [38]. Consequently, directly juxtaposed to macroeconomics this would mean that financial capital measured in \$ behaves as the price of a flow of interest, the interest rate. This is in contrast to macroeconomic literature which state that the interest rates are the price of lending or borrowing money [52, 65, 95].

This is a fundamental difference, however in practice does not cause difficulties for the modelling approach. It can be solved with the application of a gyrator instead of a transformer.

Finally, van Ardenne [90] proposes the use of action-angle variables to express the capital asset market. Although the units are equivalent, these are two different energy domains. Angular momentum is a vector quantity- it has a magnitude and a direction. Which energy domain to use is a choice. I choose to apply the angular momentum-angle energy domain analogue because of its direct application to the quantity theory of money and the economic intuition of cash flows circulating or rotating within an economy.

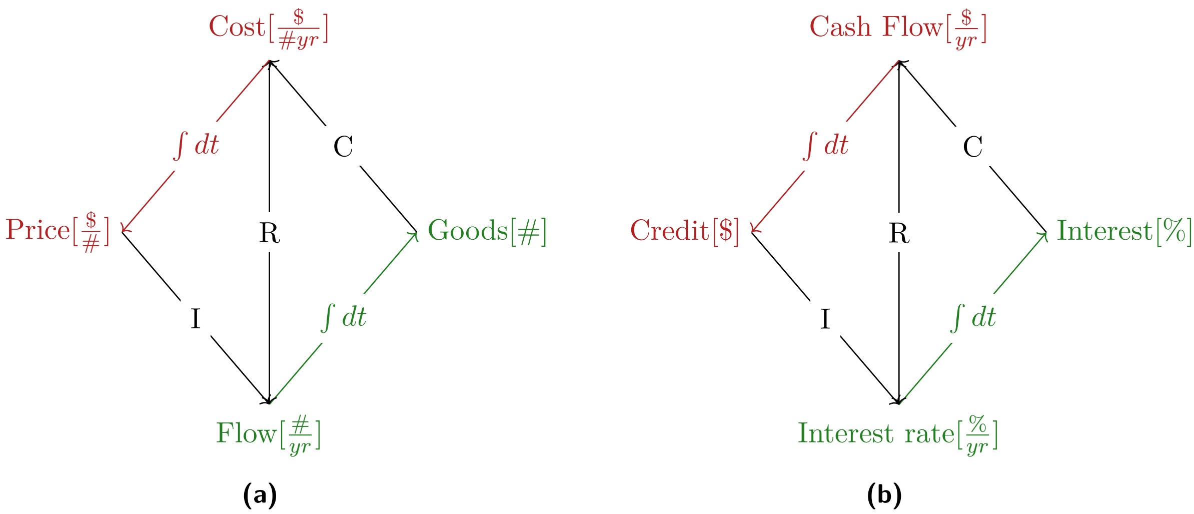


Figure 4-3: Economic Tetrahedra of state in two coordinate systems: momentum-position coordinates 4-3a, angular momentum-angle 4-3b

4-4 Towards an Economic-Engineering Approach to Macroeconomic Modelling

To model a complex system such as a total economy, it is usually necessary first to break it up into smaller parts. Each part can be modelled and perhaps studied experimentally and subsequently assembled in to the total system model [38]. I break up the system into two actors, the firms and households, and six markets. I refer to these eight major parts of the system as the subsystems. Here, I motivate and explain the adjustments I make compared to macroeconomic theory to model each. In Chapter 5, I use the approach formulated here to develop a bond graph and state-space model.

4-4-1 A New Circular Flow Diagram as the System's Structure

The existing circular flow diagrams [52] provide a good basis to model a whole economy. However, there are reasons why I can not readily use this diagram as the structure for my model. They are all the result of oversimplification of the system. Here, I name the most limiting ones.

Firstly, the existing diagram distinguishes between the physical capital and the financial capital market, whereas the theory often does not [52, 65]. Secondly, the market for entrepreneurship (the profit market) is only vaguely defined and does not express the units of entrepreneurship for which profits are received. Thirdly, there are multiple theories that describe the dynamics of the financial market [52, 65]. Fourthly, the households and firms are considered to only redirect cash flows and not store them, whereas theory does suggest storing of capital and cash [65]. Finally, the government is considered to be an integral part of the closed economy [52], whereas I assume it is a controller and should therefore be disconnected from the system.

Based on these limitations, I propose a different circular flow diagram to function as the interconnecting structure of my model. The new flow diagram is shown in Figure 4-4. The diagram uses the disposable income [1] of households instead of the national income and distinguished between the subsystems in the rotational mechanical and the translational mechanical domain. This configuration represents a more realistic version of an economy and incorporates economic-engineering theory.

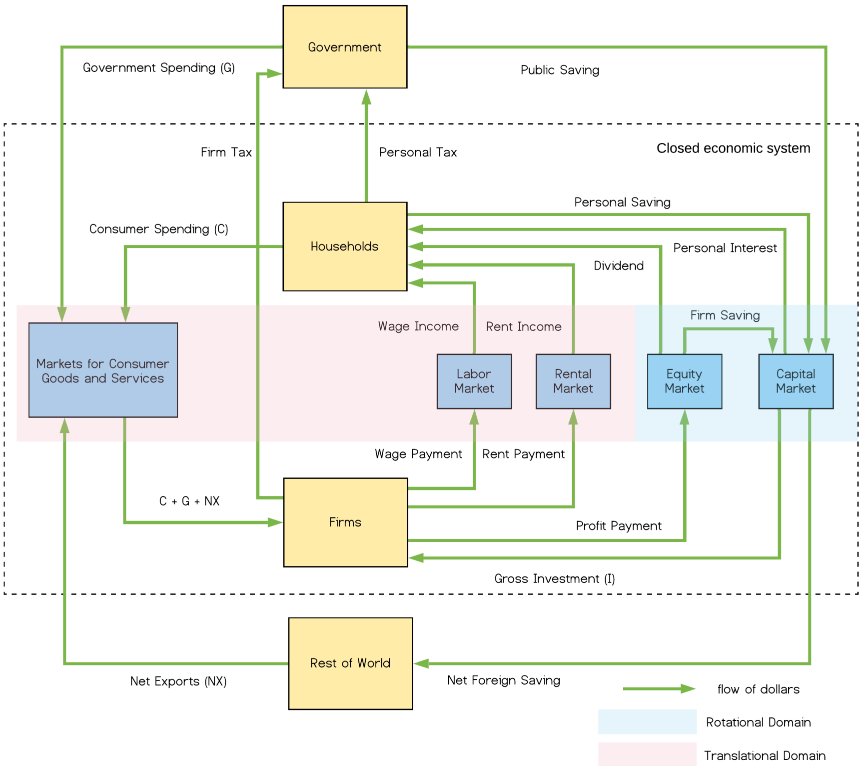


Figure 4-4: Circular flow diagram of a more realistic economy using disposable income instead of national income where the translational and rotational domains are distinguished. The diagram only displays the cash flows.

4-4-2 Energy Storing Elements as Thesis Contribution

The most limiting part of the national accounting theory, is the fact that the relations only hold in equilibrium [52]. I want to develop a dynamic model that is also able to simulate situations of disequilibrium to perform scenario analysis. To overcome this obstacle, I introduce energy storing elements for each of the subsystems. This way, energy is stored or released in situations of disequilibrium and in equilibrium the accounting relations of Section 3-3 hold.

The theory introduced in Section 2-2-3 attributes the cause of economic cycles to oversupply and undersupply. As the economic-engineering literature points out [55], over and under only exist due to the existence of energy storing elements. However, in the national accounts it is not explicitly mentioned but for one exception. That is the storage of consumer and capital

[1] Disposable income is the number of dollars per year the households have to spend [52]

goods which are counted as inventory investment [52, 65]. For all other markets, the effect of storage is overlooked.

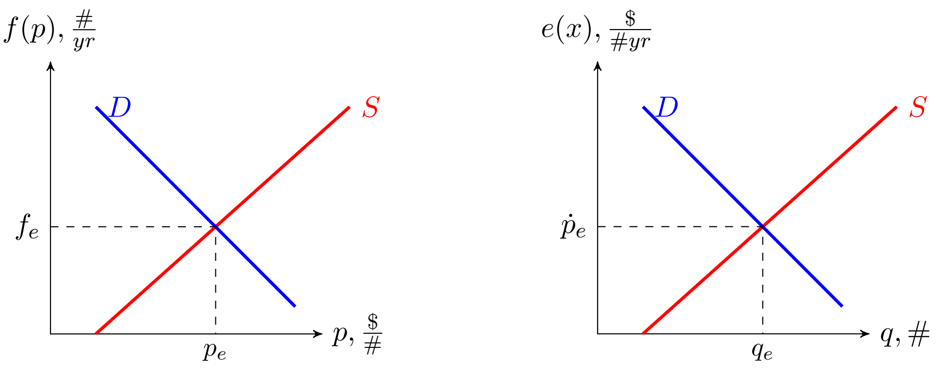
By using an economic-engineering approach, I define cash flow storing elements for all market and actors. This means that for some of the subsystems, I have to define energy storing elements that are not straightforwardly obtained from macroeconomic literature. In the next sections, I show in which variables each of the subsystems are expressed. In Chapter 5, I develop a model for each of the subsystems and define the energy storing elements.

4-4-3 Subsystems in the Translational Mechanics Domain

I define a total of three subsystems in the translational mechanical domain. These are the labor market, the rental market and the market for consumer goods and services. For each subsystem a combination of C- and I-elements are determined. The general dynamics of these elements are described by Figure 4-5b. Because each subsystems is expressed in different units, I include Table 4-4 to give an overview complementing Figure 4-5b. In the next subsections, I briefly explain how I intend to model each of the subsystem. In Chapter 5, I will develop the grey-box model and specify the meaning of each C- and I- element for each subsystem.

Table 4-4: The units for each of the subsystems in the translational mechanical domain

Variables	Consumer Goods & Services Market	Labor Market	Rental Market
Price (p)	Price basket of goods [$/#]	Wage [$/FTE]	Rental price [$/SF]
Asset (q)	Basket of goods [#]	Labor [FTE]	Rental [SF]
Effort (e)	Want [$/(# yr)]	Want[$/(FTE yr)]	Want [$/(SF yr)]
Flow (f)	Flow of basket of goods [#/yr]	Flow of labor [FTE/yr]	Flow of rentals [SF/yr]



(a) Price storage in the I-element **(b)** Asset storage in the C-element

Figure 4-5: The dynamics of the C- and I-elements in the translational mechanical domain. An overview of the units for each of the markets is given in Table 4-4

Market for Consumer Goods and Services as a Subsystem

The market for consumer goods and services is where governments, households and foreign countries purchase goods to satisfy their personal wants and needs. Britannica [13] defines it as:

Definition 4-4.1: Consumer Goods and Services

Commodities produced and subsequently purchased to satisfy the current wants and perceived needs of the buyer

They are divided into durable goods: durable goods such as automobiles, nondurable goods such as food, and services such as medical care [65]. In an economy there exist a wide range of different goods. Together they are summarized as one product: a fixed basket of goods and services measured in units (#).

Market for Labor as a Subsystem

The market for labor is where firms (demanders) interact with households (suppliers) by employing them to perform labor. I adopt the definition from [52]:

Definition 4-4.2: Labor

Human time spent in production

Firms that employ people compensate them for their labor with salaries, wages or benefits. In reality the labor market trades in various types of labor dependant on their education and skill sets [80]. In this thesis, I assume a homogeneous labor force that receives an average nominal wage per full-time equivalent (FTE).

Market for Land as Rental Market Subsystem

The market for land is where firms (demanders) interact with households (suppliers) by renting land, housing, capital or natural resources in exchange for rent. Land is a collective term that includes housing, non-residential properties and royalties [60]. The price paid for land is called rent and the income derived from rent is called rental income.

The bulk of rental income originates from the provision of housing services by households and non-profit institutions (75%) and its contribution to national income is approximately 3% [60]. The largest share of the rental income originates from imputed rent of owner occupied housing. Imputed rent is an estimate of the rent a house owner would be willing to pay to live in his or her own house [65].

Because of the prominence of housing properties for rental income, I assume that housing and office properties represent this market for the United States (and exclude non housing properties). I will therefore use the term 'rental market'. I assume imputed rent is paid by the firms and received by the households. I adopt the definition from [52] for rental:

Definition 4-4.3: Rental

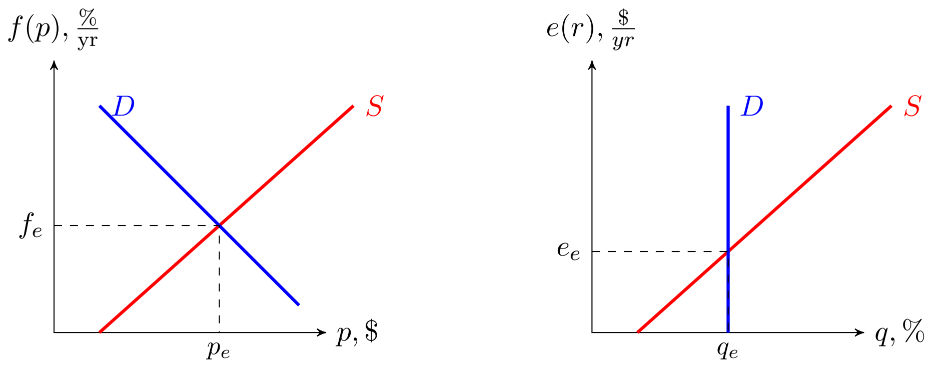
A piece of property available for renting

4-4-4 Subsystems in the Rotational Mechanics Domain

I define a total of five subsystems in the translational mechanical domain. These are the households, the firms, the equity market, the loanable funds market and the government bond market. For each subsystem a combination of C- and I-elements are determined. The general dynamics of these elements are described by Figure 4-5b. Because each subsystems is expressed in different units, I include Table 4-4 to give an overview complementing Figure 4-5b. In the next subsections, I briefly explain how I intend to model each of the subsystem. In Chapter 5, I will develop the grey-box model and specify the meaning of each of the C- and I- elements for each subsystem.

Table 4-5: The units for each of the subsystems in the rotational mechanical domain

Variables	Loanable Funds Market	Government Bond Market	Equity
Price (p)	Financial capital [$]	Par value [$]	Cash [$]
Asset (q)	Interest [%]	Accumulated yield [%]	Return on investment [%]
Effort (e)	Loanable funds [$/yr]	Flow of bonds [$/yr]	Cash flow [$/yr]
Flow (f)	Interest rate [%/yr]	Yield [%/yr]	Rate of return on investment [%/yr]
	Households	**Firms**	
Price (p)	Income [$]	Expenditure [$]	
Asset (q)	Growth [%]	Growth [%]	
Effort (e)	Flow of income [$/yr]	Flow of expenditure [$/yr]	
Flow (f)	Growth Rate [%/yr]	Growth Rate [%/yr]	



(a) Financial capital storage in the I-element **(b)** Interest storage in the C-element

Figure 4-6: The dynamics of the C- and I-elements in the rotational mechanical domain. An overview of the units for each of the markets is given in Table 4-4

Households and the Firms as Separate Subsystems

In the national accounting theory, households and firms are assumed to only redirect cash flows, most likely because the theory only models equilibria (Section 3-3-5).

I have defined energy storing elements to store flows and efforts in periods of disequilibrium. These storage elements are not considered within macroeconomic theory although their economic definition does exist. I model both the household as the firms in a similar manner.

Both the households and the firms are modelled using one I-element and one C-element. In Figure 5-4 and Figure 5-5, I motivate and the use of each and model them.

Market for Capital as Loanable Funds Market Subsystem

Saving and investment can be interpret in terms of supply and demand. Saving is the supply of loanable funds- households lend their saving to investors or deposit their savings in a bank that subsequently loans the funds out. Loanable funds are defined as:

Definition 4-4.4: Loanable Fund

Any form of credit, such as loans, bonds or saving deposits.

Note that I use saving, investment and loanable funds as the flow variables in $/yr and savings, investments and financial capital as the stock variables in $ (see Section 4-3-1).

Gross investment and Gross foreign investment constitute the demand for loanable funds. Domestic and foreign investors borrow from the public directly by selling bonds or indirectly by borrowing from banks [52]. The process of transferring funds from savers to borrowers is called financial intermediation. In this thesis, I model the commercial banking system as the ones to carry out this financial intermediation.

Although I treat the interest as the good and the financial capital as the price of interest, my design of the financial market follows macroeconomic literature: Flows of supply and demand for loanable funds enter the financial market which ultimately result in interest [65].

Market for Entrepreneurship as Equity Market Subsystem

Entrepreneurship is considered to be the fourth factor of production [67, 68]. Its exact form remains abstract in literature. For example, in what kind of variable can entrepreneurship be measured or expressed? How do the dynamics of such a market look like? In this thesis, I will therefore use a different approach.

I model the market for entrepreneurship as a type of equity market but not the stock market itself. The purchase of stock represents a transfer of ownership shares from one person to another and does not provide new funds for investment projects and therefore does not directly contribute to national production [52]. Therefore, most stock market activities do not fit within the national accounting theory. Instead, I model the market for entrepreneurship as a market mechanism where free cash flows of the firms are either paid out as dividends, kept in the form of cash or used to payoff debt. This depends on the demand of the shareholders/households and the supply of the firms. The free cash flow is defined as [65]:

Definition 4-4.5: Free Cash Flow

The cash flow available for distribution to a company's equity-holders

The variable I want to model is the year-on-year total equity returns as explained in Section 2-4. These are the returns due to increase or decrease of stock prices in addition to dividend yields. This design of the market does not directly yield the equity returns. Equity returns are based on their value which involves discounting future cash flows [20]. Discounting is inherent to the frequency domain [90]. In Section 6-4-1, I take the Laplace transform of signals from the time domain model to determine the value and returns of a nation's equity in the frequency domain.

Market for Government Bonds as a Subsystem

Government bonds are borrowing instruments by the government. The total dollar value of accumulated government borrowings is known as the government debt. If more cash goes into buying government bonds, less is directly invested in capital and therefore increases of growth of output decline [65]. I adopt this construction where the government bond market is connected to the market for loanable funds and takes up a portion of the investment.

Loans differ in their term or maturity -the length of the time until they must be paid off, the coupon rate- the amount that the issuer/borrower agrees to repay the bondholder/lender- and the principal or face value -the amount that the issuer/borrower agrees to repay the bondholder/ lender [89]. Loans also vary in risk. The safest assets in the world are securities of the U.S. [89]. This thesis only deals with U.S. government bonds, which are assumed to be the safest kind [65], and therefore no risk modelling is included.

I model the bond market as one general market for government bonds with all maturities, as if there only exist one maturity bond.

The prices of bonds and equity are related to their value. To compute the value of bonds and equity involves discounting future cash flows and coupon payments [65, 89]. Discounting is inherent to the frequency domain [90]. In Section 6-4-3, I demonstrate this and use the Laplace transform to model government bond yields of one specific maturity.

4-4-5 Government as a Controller

At the basis of (macro)economic lies the rational choice theory. This theory states that individuals use rational calculations to make rational choices and achieve outcomes that are aligned with their own personal objectives [73]. Households are constantly trying to maximize their utility and firms their profits [23].

A government has many functions. There are the obvious ones like upholding the rule of law and providing safety to its citizens etc., but a government also exercises economic policy. It does so by levying taxes, implementing regulations and spending money [65].

By applying economic policy, a government does not act out of self-interest, but out of the interest of its citizens, the households and firms. I interpret the economic policy of a government as the set of control inputs to the economic system. Because the government acts according to different principles than the rest of the economy, I decouple the government subsystem from the rest of the closed economy and model its influence either as input signals or dissipative elements.

4-5 Conclusions and Next Steps

The national accounting and circular flow theory are specifically useful for the goal of this thesis as they provide clear mathematical relations between economic markets and actors. However they come with a big drawback: The relations only hold for economic systems in equilibrium.

The economic-engineering theory functions as the bridge between the macroeconomic and the mechanical domain. The theory allows for the development of a macroeconomic grey-box model using an engineering modelling approach. By using bond graphs, a whole range of economic laws and principles are integrated into the model.

The drawback of the national accounting and circular flow theory is overcome by introducing energy storing elements for each of the subsystems. This allows the theory to be used for the development of a dynamic grey-box model. The complex system is broken up into eight subsystems which will be modelled separately and subsequently interconnected. A distinction is made between the economic domain -expressed in prices and quantities of assets- and the financial domain -expressed in financial capital and rates to follow macroeconomic theory.

By formulation of an economic-engineering approach to macroeconomic modelling, Part 2 of this thesis is completed. This formulated approach is put to use in the next part of this thesis where I develop a method to develop a time-domain model. This model will be used to perform national valuation and analyze economic scenarios in the time and frequency domain. Parts 3 and 4 will consist of the following steps:

Next Steps Towards Engineering Macroeconomic Scenario Analysis

3. **The Fundamental Grey-Box Model and its Transfer Function**
   (a) Development of a Time Domain Model of the United States Economy (Chapter 5)
   (b) The Engineering Frequency Domain for Valuation and Liquidity (Chapter 6)
4. **Model Validation and Scenario Analysis in the Time- and Frequency Domain**
   (a) Time and Frequency Domain Analysis and Validation (Chapter 7)

Part 3

Chapter 5

Development of a Time-Domain Model of the United States Economy

5-1 Introduction

In this chapter, I develop a bond-graph model for the open United States economy. From this model I derive the state-space model so that economic scenario analysis can be performed. The system is broken up the system into subsystems and each of them are modelled separately.

This modelling effort consists of four steps: 1) Subsystems' behavior is derived from the theory as presented in Chapter 3, 2) bond graph representations are given by using the analogues of Chapter 4, 3) all signals and elements are given an economic interpretation so that the model is intuitive to both engineers and economists, and 4) the state-space/time domain model is derived. The goals of this chapter are as follows:

Chapter goals:

1. Development of a macroeconomic time-domain model of the United States economy to perform economic scenario analysis

First in Section 5-2, I state fundamental modelling assumptions. I then directly present the total model of the aggregated subsystems in Section 5-3. From this I continue to model each subsystem in separate sections. In Section 5-4, I present the bond graph and state-space model of the firms. Subsequently, I do the same for the households in Section 5-5, the consumer goods and services market in Section 5-6, the market for loanable funds in Section 5-10, the labor market in Section 5-7, the equity market in Section 5-9 and for the government bond market in Section 5-11. Finally, I conclude and state my contributions in Section 5-12.

5-2 Fundamental Assumptions

I make some practical assumptions to model this complex macroeconomic system. I state the most fundamental ones here:

Linearity

All elements in the model are assumed to be linear. This assumption is not in line with reality. For example, demand and supply curves are known to be slightly skewed [92]. The fact is that there are also no real physical systems that are truly linear [38, 43]. It is often assumed that in and around linearization points these systems behave linearly. For linear systems, there exist an abundance of analytical tools for obtaining exact analytical solutions to the linear equations and for extracting detailed information about the response of the system. I model the linearized system directly using bond graph theory. The parameter identification with data should take care of which linearization points are used. This does mean that parameter estimation has to be repeated.

Homogeneous Assets within Markets

Within each market, I assume the assets are homogeneous. With assets I mean labor, land, products and financial capital. In reality, there exist many different types products, labor and even land as was explained in Section 4-4. This assumption is in line with macroeconomic theory. In the consumer goods and services market, baskets of goods and their basket prices are evaluated. The labor market assumes an average wage for one unit of average labor. Finally, within the market for land I assume there is one product corresponding to one average rental price.

Government and Federal Reserve as Controllers

I assume that a government and Federal Reserve act as controllers of the system. The government applies economic policy and the federal reserve monetary policy. These policies do not abide the same underlying principle of self-interest. The interconnections of the closed-economy with the government and federal reserve are modelled as effort sources, flow sources or R-elements. This is not in line with macroeconomic or econometric theory, where the government is assumed to be an integral part of the economy.

Rest of the World as Exogenous Actor

The rest of the world is modelled as an actor exogenous to the closed economic system. In reality the world exist of multiple interconnected economies. In this thesis, I only model the United States economy. Net exports net foreign saving or net capital outflow are modelled as flow- and effort sources. Influences of the rest of the world such as competition and opportunity costs are modelled using R-elements.

National Accounting Relations in Equilibrium

The relations as stated by the national accounts in Chapter 3 only hold in equilibrium. This was discussed in Section 3-3-5. In this thesis, I model situations of disequilibrium and the transients that correspond to them. If energy (analogues to cash flows) and power (analogues to growth) are conserved, they can only hold in equilibrium if initial cash flow conditions are equal. Since I make use of R-elements to account for unmodelled dissipative parts of the economy, energy is not conserved for the closed economic system.

Financial Capital and Capital are Interchangeable

I model financial capital to fundamentally express the same behavior as (physical) capital. Therefore, I will not model the capital market as a separate market but as an integral part of the firms. In reality, there exist a capital market where firms exchange capital and financial capital acquired from investment. In this thesis, I assume that investment directly results in capital accumulation within the firms.

Non-shifting Demand or Supply Curves

The markets assume a situation where the demand curves shift, and the supply curve do not. In disequilibrium, only movements along the supply curve occur until a state of equilibrium is reached. If prices increase or decrease, so do the flows that result from them. Shifts of both curves occur in reality. They represent economic growth caused either by production or demand [62]. Incorporation of both phenomena involves additional complexity.

5-3 The Aggregated Total Macroeconomic System

The bond graph system of the total market is obtained by interconnecting the eight subsystems. Together they represent the United States economy. Figure 5-1 gives the bond graph of the total interconnected macroeconomic system and Table 5-1 provides an overview of the elements and their economical meaning. The parameters values have been assigned manually. Their values are included in Appendix F.

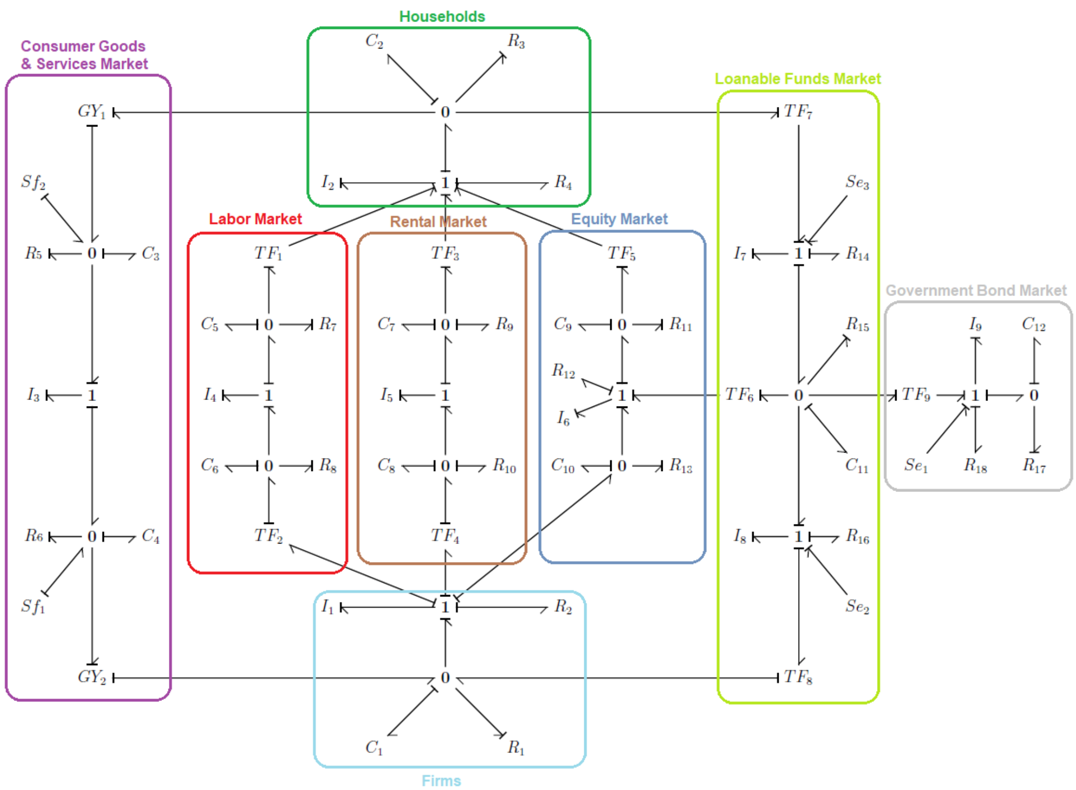


Figure 5-1: Bond graph model of the total system. The dynamics are described by twenty-one states and five inputs.

he linear state-space representation allows the application of the systems and control toolbox for macroeconomic analysis. The analysis of the behavior is performed in Chapter 7.

The six markets and the two economic actors mimic the behavior of the U.S. economy and carries the same general structure as described the macroeconomic theory from Chapter 3 with the engineering adaptations from Chapter 4. In the next sections, I model each of the eight subsystems separately.

Table 5-1: Overview of element and their economic meaning in the bond graph for the U.S. economy.

	Labor Market		**Rental Market**		**Consumer Goods & Services Market**
C_5	Unemployed	C_7	Household-owned Rentals	C_3	Backlog
C_6	Human Capital	C_8	Firm-owned Rentals	C_4	Inventory Level
I_4	Nominal Wage	I_5	Rent Price Level	I_3	Consumer Price Level
R_7	Inactive Labor	R_9	Depreciation	R_5	Competition
R_8	Amortization	R_{10}	Depreciation	R_6	Inventory Write Down
	Households		**Firms**		**Government Bonds Market**
C_2	Standard of living	C_1	Technology level	C_{12}	Accumulated yield
I_2	Currency	I_1	Capital	I_9	Value of outstanding bonds
R_3	Amortization	R_1	Government	R_{17}	Treasury auction
R_4	Informal economy	R_2	Impairment	R_{18}	Government
	Loanable Funds Market		**Equity Market**		
C_{11}	Interest	C_9	Risk discount		
I_7	Bank Deposits	C_{10}	Economic value added		
I_8	Firms' Debt	I_6	Cash holdings		
R_{14}	Commercial Bank	R_{11}	Government		
R_{15}	Financial Instruments	R_{12}	Cash Equivalents		
R_{16}	Commercial Bank	R_{13}	Rest of World		

5-4 Firms as a Subsystem

All the firms of an economy combined are modelled as one subsystem. This subsystem is connected to five different markets. It has five inputs and returns two different outputs to the markets. Table 5-2 provides and overview of the inputs, outputs and states of the firms as a subsystem.

Table 5-2: Inputs, outputs and states of the firm

Inputs	**Outputs**	**States**
Labor cost [$/yr]	GDP [$/yr]	Capital [$]
Rental cost[$/yr]	Economic growth rate [%/yr]	Technology level [%]
Free cash flow (FCF) [$/yr]		
Growth of sales [%/yr]		
Growth of investment [%/yr]		

Figure 5-2 depicts the firms connected to the five markets. The blue arrows represent the flow variables in %/yr and the red arrows the effort variables $/yr. Their product is the power or change of the cash flow in $\$/yr^2$. The direction of the arrows indicates causality and not the direction of positive flows.

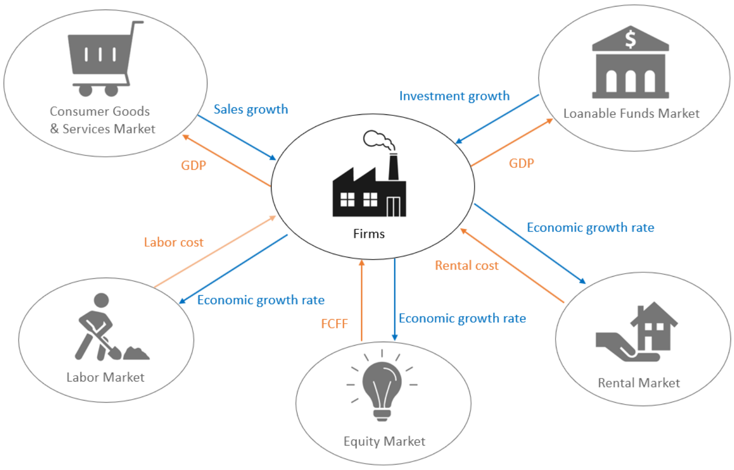


Figure 5-2: The firm as subsystem connected to five different markets. The three markets for factors of production impose costs and receive an effort. The markets for consumer goods and services and the market for loanable funds receive a growth rate and returns Gross Domestic Product (GDP).

5-4-1 Bond Graph of the Firm

Figure 5-3 displays the bond graph of the firms as a subsystem. The consumer goods & services market and loanable funds market give a growth rate of revenue for the firms and receive a GDP effort. In the lower end of the bond graph, the markets of factors of production receive a flow economic growth rate, and return different types of costs to the firms. Table 5-3 presents the elements, signals and their economic meaning.

Technology Level

If the sum of the growth rates of sales and investment is greater than the cost incurred by the economies of scale variable, it increases the technology level. The technology level, sometimes also referred to as a form of factor productivity, increase the output (GDP) per input [52]. The logic behind it is that if firms expect an increase of revenue they are likely to invest in innovation. This subsequently leads to a higher technology level and results in a higher GDP. However, the government reduces this difference by levying indirect taxes. This R-element could be exchanged for an effort sink if the government is modelled.

Capital

Business accounting states that the difference between net income and free cash flow are capital expenditures (CAPEX) and depreciation. This is what I have incorporated into the model. If capital expenditures increase, so does the accumulated capital which results in economies of scale; a cost advantage because of the growing firm [65].

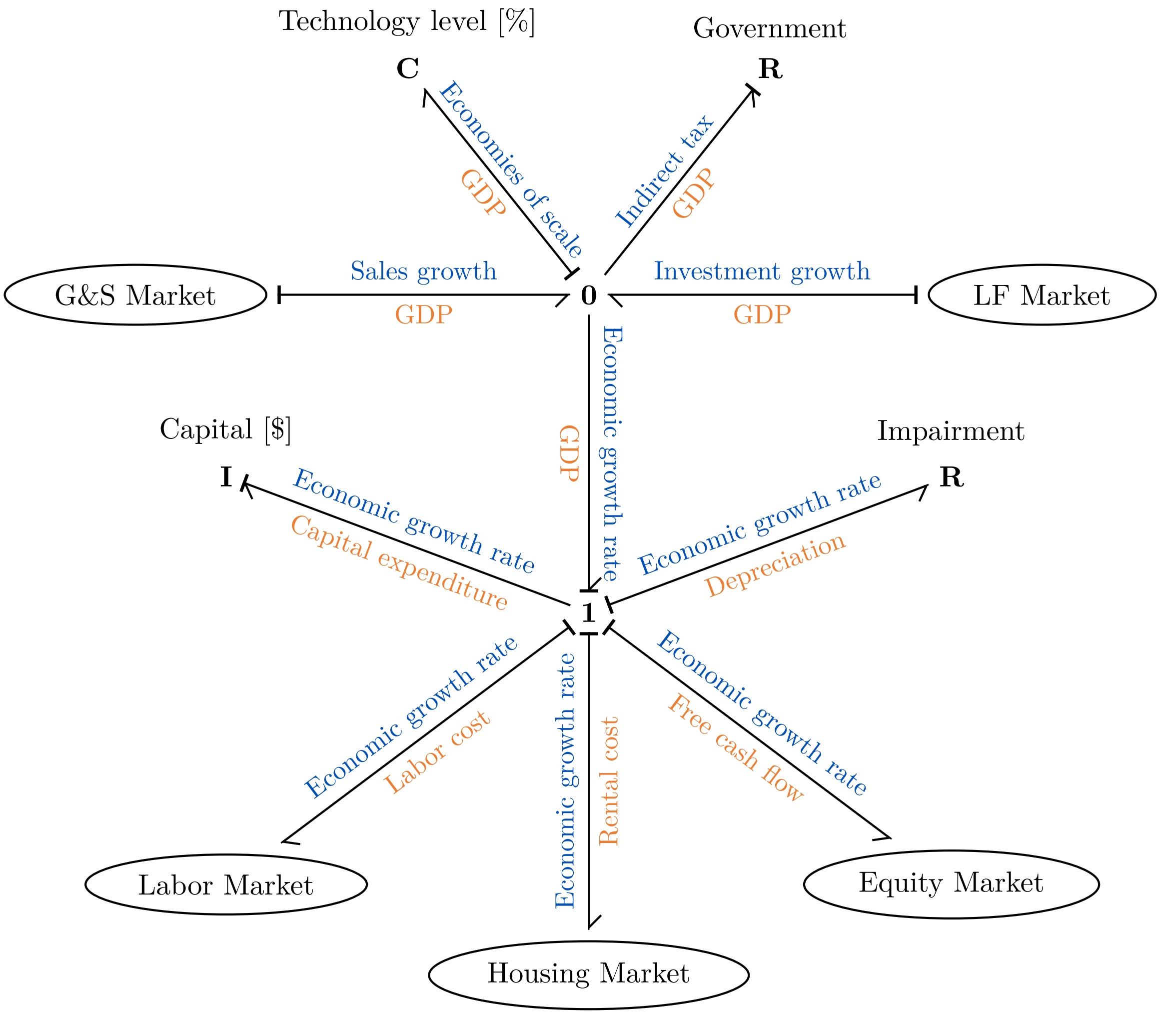


Figure 5-3: Bond graph of the firms. GDP expressed in $/yr is determined by the growth potential of the firm. Economies of scale in %/yr is determined by the net investments or capital accumulation. Accumulated capital leads to relative decrease of the cost incurred by the firms, also known as economies of scale.

5-4-2 State-space Representation of the Firms

From the bond graph of Figure 5-3 the state-space derivation is derived [38]. The state vector is defined as $x(t) = [p_1\ q_1]$', where p_1 is capital, and q_1 the technology level. The input vector $u(t) = [u_1\ u_2\ u_3\ u_4\ u_5]$' represent sales growth, investment growth, labor cost, rental cost and free cash flow, respectively. Finally, the output vector $y(t) = [y_1\ y_2\ y_3\ y_4\ y_5]$' are two times GDP and three times economic growth rate depending on which market they are connected to. This leads to the following state-space system:

Table 5-3: Definitions of elements and signals in the bond graph of the firm subsystem

	Definition	Interpretation	Units
Elements	Capital	An asset that are used as productive inputs for further production [52].	$
	Technology level	The level of the discovery of new and improved methods of producing goods [14].	%
	Impairment	Diminishing of long-term asset value [49].	
	Government	The political system by which a country or community is administered and regulated [52].	
Efforts	GDP	The market value of all final goods and services produced within an economy in a given period of time [52].	$/yr
	Net investment	The amount of investment after the replacement of depreciated capital; the change in the capital stock [52].	$/yr
	Labor cost	The sum of all wages paid to employees, as well as the cost of employee benefits and payroll taxes paid by an employer [92].	$/yr
	Rental cost	The cost incurred by a business to utilize a property or location for an office, retail space, factory, or storage space [52].	$/yr
	Depreciation	The reduction in the capital stock that occurs over time because of aging and use [52].	$/yr
	Free cash flow	The cash a company generates after accounting for cash outflows to support operations and maintain its capital assets [20].	$/yr
Flows	Growth of sales	The percent change in the net sales of a business from one period to another.	%/yr
	Growth of investment	The percent change in investment of a business from one period to another.	%/yr
	Economic growth rate	The percentage change in the value of all of the goods and services produced in a nation during a specific period of time [52].	%/yr
	Indirect tax	Sales and excise taxes paid by businesses [95].	%/yr
	Economies of scale	When the average cost of producing each individual unit declines as total output increases [46].	%/yr

State-space Model of the Firm Dynamics

$$\begin{aligned}\dot{x}(t) &= \underbrace{\begin{bmatrix} -\frac{R_2}{I_1} & \frac{1}{C_1} \\ -\frac{1}{I_1} & -\frac{1}{C_1 R_1} \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 & 0 & -1 & -1 & -1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}}_{B} u(t) \\ y(t) &= \underbrace{\begin{bmatrix} 0 & \frac{1}{C_1} \\ 0 & \frac{1}{C_1} \\ \frac{1}{I_1} & 0 \\ \frac{1}{I_1} & 0 \\ \frac{1}{I_1} & 0 \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(t)\end{aligned} \tag{5-1}$$

5-5 Households as a Subsystem

The households of an economy are modelled as one economic actor or subsystem. This subsystem is connected to the same five markets as the firms. As for the firms, this subsystem has two states, five different inputs and two different outputs. Table 5-4 provides an overview of the inputs, outputs and states of the firms as a subsystem.

Table 5-4: Inputs, outputs and states of the households

Inputs	Outputs	States
Labor income [$/yr]	Disposable income (DI) [$/yr]	Standard of living [%]
Rental income [$/yr]	Disposable income growth [%/yr]	Currency [$]
Equity income [$/yr]		
MPC rate [%/yr]		
MPS rate [%/yr]		

Figure 5-4 depicts the households as a subsystem connected to the five markets. The blue arrows represent the flow variables in %/yr and the red arrows the effort variables in $/yr. Their product is the power or change of the cash flow in $/yr^2. The direction of the arrows indicates causality and not the direction of positive flows.

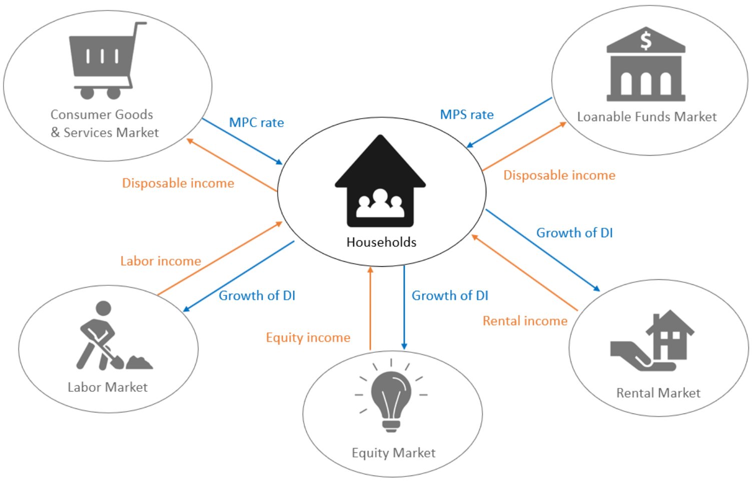


Figure 5-4: The households as subsystem is connected to five different markets. The households receive different forms of income and give back a flow. The markets for consumer goods and services and the market for loanable funds give a growth rate and return Disposable Income (DI).

5-5-1 Bond Graph of the Households

Figure 5-5 displays the bond graph of the households as a subsystem. The consumer goods & services market and loanable funds market receive a growth rate of expenditure of the firms and return DI. In the lower end of the bond graph, the market of the factors of production give forms of income and receive back the DI growth rate. Table 5-5 presents the elements, signals and their economic meaning.

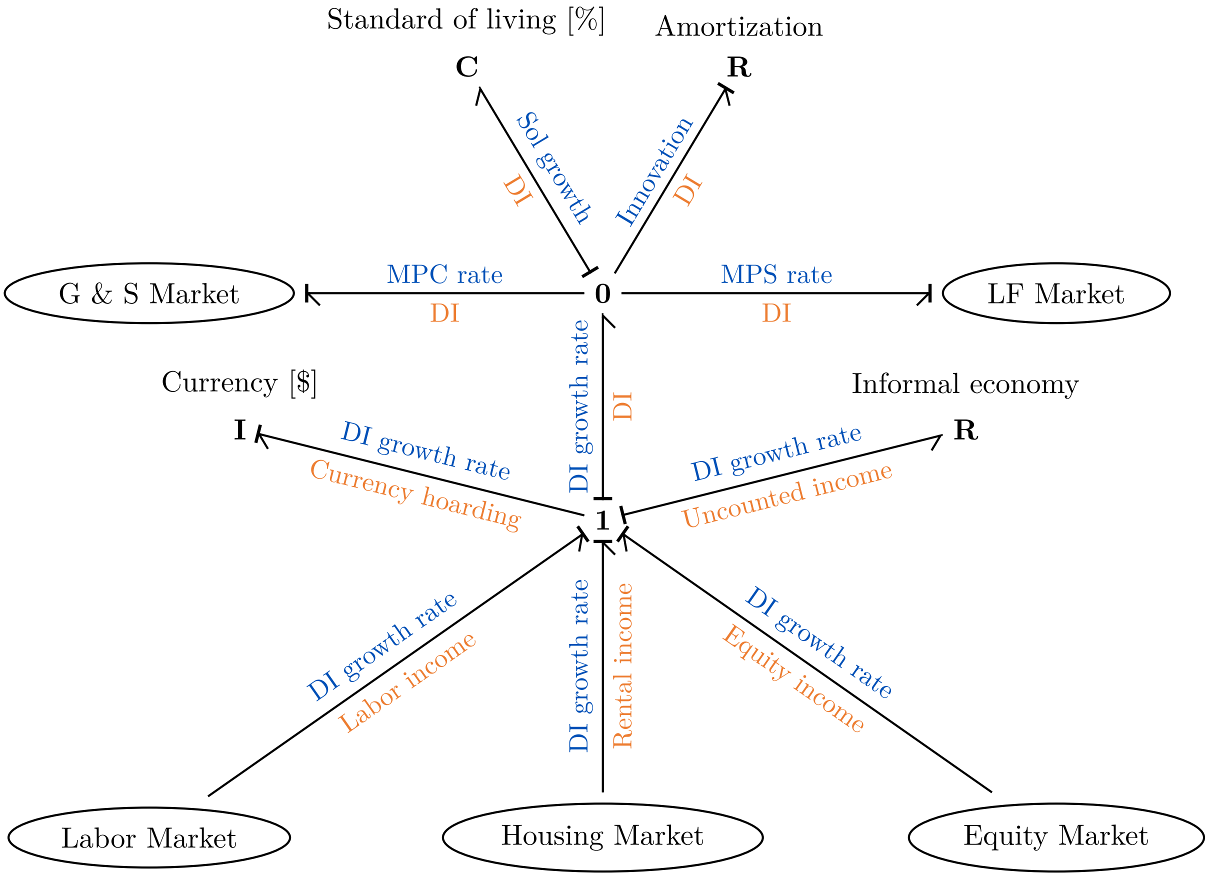


Figure 5-5: Bond graph of the households. Disposable income (DI) [%/yr] is determined by the standard of living. The growth of this disposible income [%/yr] is determined by currency or cash the households hold.

Standard of Living

If the growth rate of DI is greater than the household saving and consumption rates, it is stored in the form of standard of living which returns a new DI level. The logic behind it is that if households expect an increase of income, they are likely to increase their consumer spending and saving behavior accordingly. This subsequently leads to a higher standard of living and requires a higher need for disposable income. Due to constant technical advancements within an economy the standard of living of today, is different to that of a decade ago [65]. Therefore, I also assume the standard of living amortizes.

Table 5-5: Definitions of elements and signals in the bond graph of the households subsystem

	Definition	Interpretation	Units
Elements	Currency	Coins and paper money held outside the banking system [65].	$
	Standard of living	The level of income, necessities, luxury, and other goods and services that are generally readily available to a designated population [33].	%
	Informal economy	Economic activity that falls outside the regulated economy and tax system [33].	
	Amortization	Lowering of value of intangibles over time [33].	
Efforts	Disposable income (DI)	Take-home pay, or that part of the total national income that is available to households for consumption or saving [65].	$/yr
	Currency hoarding	Holding currency in quantities greater than needed for immediate use [33].	$/yr
	Labor income	The income earned from labor [52].	$/yr
	Rental income	The income earned from rentals [52].	$/yr
	Uncounted income	The income of an economy that goes uncounted as it goes to the informal economy.	
	Equity income	Money earned from stock dividends [20].	$/yr
Flows	MPC rate	The rate of change of the amount by which consumption changes when disposable income increases by one dollar [52].	%/yr
	MPS rate	The rate of change of the amount by which saving changes when disposable income increases by one dollar [52].	%/yr
	DI growth	The percentage change of disposable income in a period of time [52].	%/yr
	Innovation	Sales and excise taxes paid by businesses [95].	%/yr
	Standard of living (Sol) growth	The rate of change of the standard of living.	%/yr

Currency

If the incomes from labor, rental and equity are greater than the disposable income measured in macroeconomics, the income is stored in the form of physical currency. These are coins and paper money held outside the banking system [65]. The access to more currency provides liquidity to households, i.e. there is no need to go to a bank to withdraw currency or sell assets to acquire money. Therefore, if more currency is stored it provides a possibility to higher growth of disposable income once used. Because economies deal with informal parts within the economy, i.e. illegal traffic or trade in officially controlled or scarce commodities, cash can be dissipated out the system or introduced back into the system (negative dissipation).

5-5-2 State-space Representation of the Households

From the bond graph of Figure 5-5, the state-space is derived [38]. The state vector is defined as $x(t) = [p_2\ q_2]'$, where p_2 is currency, q_2 standard of living. The input vector $u(t) = [u_1\ u_2\ u_3\ u_4\ u_5]'$ represent the Marginal Propensity to Consume (MPC) rate, Marginal Propensity to Save (MPS) rate, labor income, rental income and equity income, respectively. Finally, the output vector $y(t) = [y_1\ y_2\ y_3\ y_4\ y_5]'$ are two times DI and three times the growth rate of DI depending on which market they are connected to. This leads to the following state-space system:

State-space Model of the Household Dynamics

$$\begin{aligned}
\dot{x}(t) &= \underbrace{\begin{bmatrix} -\frac{R_4}{I_2} & -\frac{1}{C_2} \\ \frac{1}{I_2} & -\frac{1}{C_2 R_3} \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 & 0 \end{bmatrix}}_{B} u(t) \\
y(t) &= \underbrace{\begin{bmatrix} 0 & \frac{1}{C_1} \\ 0 & \frac{1}{C_1} \\ \frac{1}{I_1} & 0 \\ \frac{1}{I_1} & 0 \\ \frac{1}{I_1} & 0 \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(t)
\end{aligned} \tag{5-2}$$

5-6 Consumer Goods and Services Market as a Subsystem

The goods and services market is connected to the households and firms from which it receives an effort and returns a flow. Additionally, the market receives a flow from the government and rest of the world. Table 5-6 provides an overview of the inputs, outputs and states of the goods and services market.

Table 5-6: Inputs, outputs and states of the consumer goods and services market

Inputs	Outputs	States
GDP [$/yr]	Sales growth [%/yr]	Inventory [#]
Disposable income [$/yr]	MPC rate [%/yr]	backlog [#]
Export [#/yr]		Consumer price level [$/#]
Import [#/yr]		
Government consumption [#/yr]		

Figure 5-6 depicts the goods and services market connected to both the households and the firms. The orange and red arrows represent the effort variables measured in $/yr and $\frac{\$}{\#\ \mathrm{yr}}$, respectively. The blue arrows measured in %/yr and the green arrows in #/yr represent the flow variables. Gyrators are used to go from the rotational mechanical energy domain (orange and blue) to the translational mechanical energy domain (red and green). Net exports and government consumption are displayed as one external flow source in Figure 5-6. In reality these are two separate flow sources from which the flows are addable. The direction of the arrows indicates causality and not the direction of positive flows.

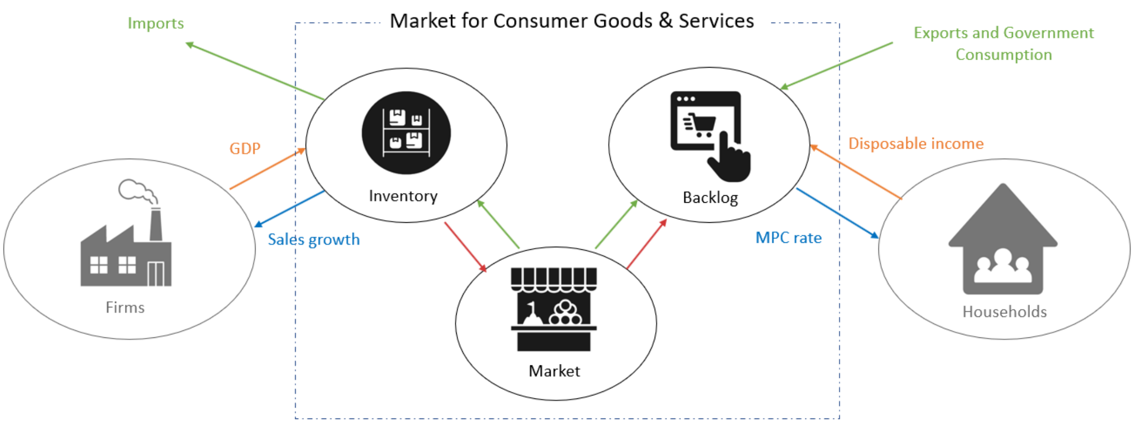


Figure 5-6: The market for goods and services is interconnected to the firms and households. The external modelled inputs are net exports and government consumption. The red and orange arrows represent flows, the green and blue arrows represent the efforts.

5-6-1 Bond Graph of the Consumer Goods and Services Market

Figure 5-7 displays the bond graph of the consumer goods & services market as a subsystem connected to both the households and the firms. Two gyrators are used to move from the rotational mechanical domain to the translational mechanical domain. GDP and DI in $/yr are linearly gyrated to goods produced and goods ordered in #/yr, respectively. MPC rate and Growth of Sales in %/yr are gyrated to opportunity cost of goods and order yield in $\frac{\$}{\#\ \text{yr}}$, respectively.

Demand and supply are collected and stored in the combination of the zero junction and C-elements. Storage exists in the form of inventory/surplus and backlog/scarcity. The C-elements return efforts that represents the convenience (yield) or disconvenience (cost). These then act as the forces of supply and demand which results in a net force and a movement along the supply curve.

Table 5-7 presents the elements, signals and their economic meaning. The underlying

Table 5-7: Definitions of elements and signals in the bond graph of the consumer goods and services market subsystem

	Definition	Interpretation	Units
Elements	Inventory level	The number of items kept in stock to process or sell [52].	#
	Consumer price level	Measure of the overall level of prices that shows the cost of a fixed basket of consumer goods [52].	$/#
	Backlog	Buildup of orders that needs completing [33].	#
	Innovation	Improvements of existing products or creation of new products [33].	
	Government	System or method that controls the country [65].	
	Rest of the world	All countries outside the United States.	
Efforts	Opportunity cost of goods	Potential benefits the customer misses out on when choosing the firm's product [92].	$\frac{\$}{\#\ \text{yr}}$
	Free market force	Unobservable market force that helps the demand and supply of the product reach equilibrium [41].	$\frac{\$}{\#\ \text{yr}}$
	Convenience yield	The benefit or premium associated with holding an underlying product or physical good [66].	$\frac{\$}{\#\ \text{yr}}$
Flows	Domestic consumption	The domestic spending on consumer goods and services [52].	#/yr
	Government consumption	The spending of the government on consumer goods and services [52].	#/yr
	Exports	Goods and services sold to other countries [52].	#/yr
	Imports	Goods and services bought from other countries [52].	#/yr
	Product life cycle decline	Decrease in product demand due to its ending life cycle [33].	#/yr
	Shortage	The flow demand exceeding the flow of supply [92].	#/yr
	Sales	The total amount of goods and services purchased or sold.	#/yr
	Surplus	The flow of supply exceeding the flow of demand [62].	#/yr
	Goods produced	The flow of goods leaving producing factories.	#/yr

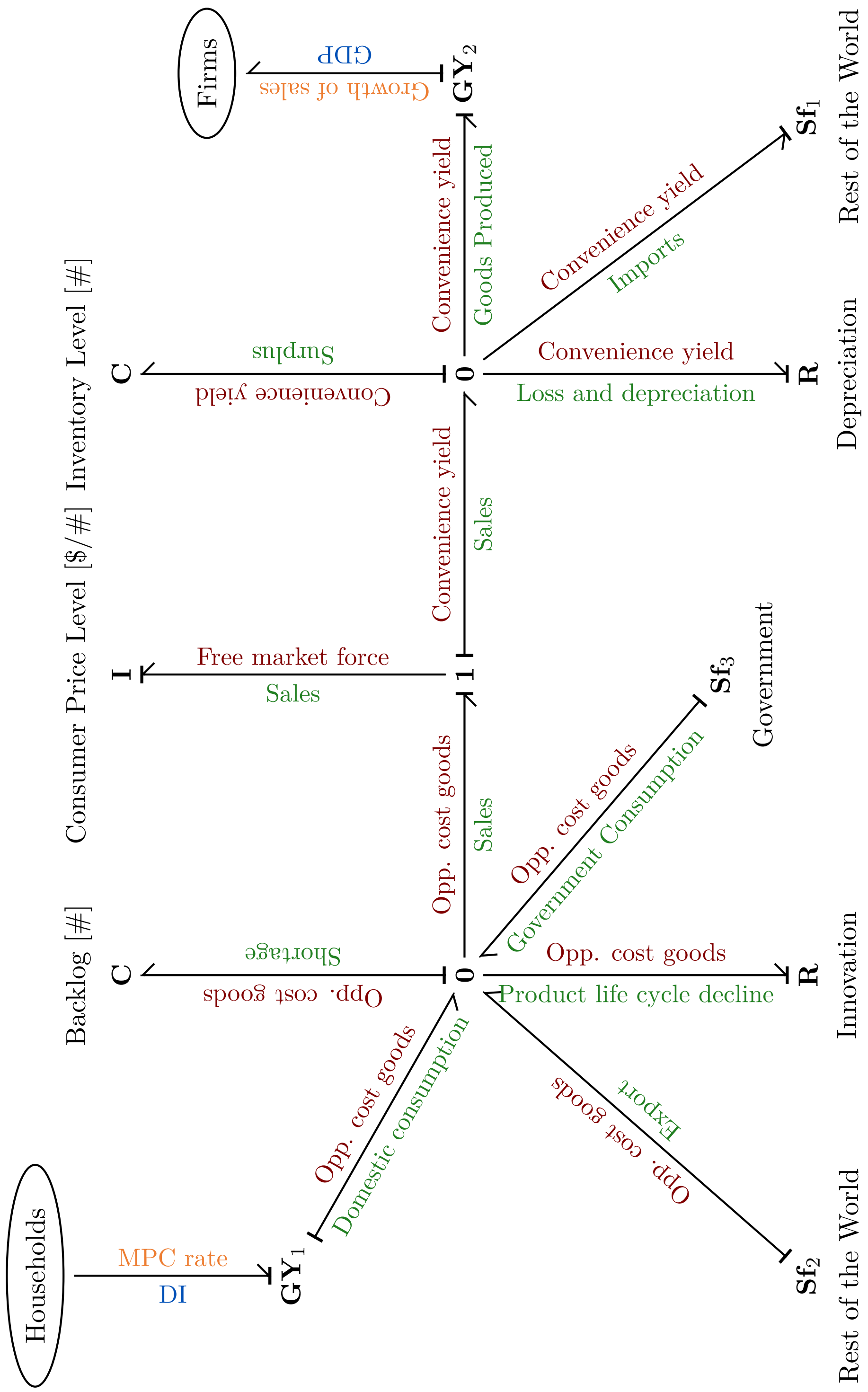


Figure 5-7: Bond graph of the market for consumer goods and services. The households and firms interact with the markets by giving a flow of goods and receiving a cost or benefit. The goods are stored in C-elements and the price is stored in the I-element.

5-6-2 State-Space Representation of the Consumer Goods and Services Market

From the bond graph of Figure 5-7, the state-space is derived [38]. The state vector is defined as $x(t) = [p_3\ q_3\ q_4]$', where p_3 is consumer price level, q_3 the backlog, and q_4 the inventory level. The input vector $u(t) = [u_1\ u_2\ u_3\ u_4\ u_4\ ]$' represent the imports, exports, government consumption, disposable income, and GDP respectively. Finally, the output vector $y(t) = [y_1\ y_2]$' consists of the MPC rate and growth rate of sales. This leads to the following state-space system:

State-Space Model of the Consumer Goods and Services Market Dynamics

$$
\begin{aligned}
\dot{x}(t) &= \underbrace{\begin{bmatrix} 0 & \frac{1}{C_3} & -\frac{1}{C_4} \\ -\frac{1}{I_3} & -\frac{1}{C_3 R_5} & 0 \\ \frac{1}{I_3} & 0 & -\frac{1}{C_4 R_6} \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & \frac{1}{GY_1} & 0 \\ -1 & 0 & 0 & 0 & -\frac{1}{GY_2} \end{bmatrix}}_{B} u(t) \\
y(t) &= \underbrace{\begin{bmatrix} 0 & \frac{1}{C_3 GY_1} & 0 \\ 0 & 0 & \frac{1}{C_4 GY_2} \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(t)
\end{aligned}
\tag{5-3}
$$

5-7 Labor Market as a Subsystem

The labor market is connected to the households and firms. It receives the cost of capital from the firms and growth of disposable income from the households and returns the labor cost to the firms and the labor income to the households. Table 5-8 provides an overview of the inputs, outputs and states of the labor market. It consists of three parts: Human capital, the market itself and the number of employed.

Table 5-8: Inputs, outputs and states of the labor market

Inputs	Outputs	States
Economic growth rate [%/yr]	Labor cost [$/yr]	Human capital [FTE]
Disposable income growth [%/yr]	Labor income [$/yr]	Unemployed [FTE]
		Nominal wage [$/FTE]

Figure 5-8 depicts the labor market connected to both the households and the firms. The orange arrows measured in $/yr and red arrows in $\frac{\$}{\text{FTE yr}}$ represent the effort variables. The blue arrows measured in %/yr and green arrows in FTE/yr represent the flow variables. The direction of the arrows indicates causality and not the direction of positive flows.

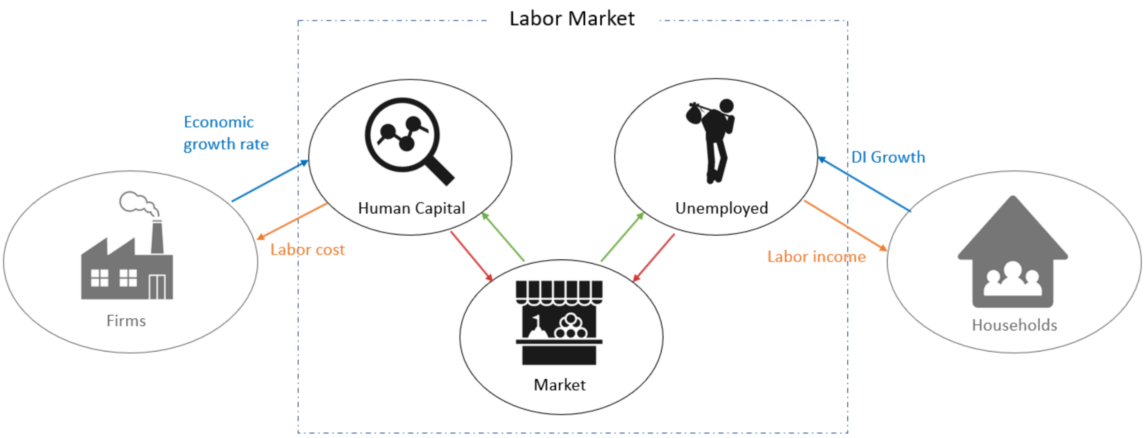


Figure 5-8: The labor market is interconnected to the firms and households. There are no modelled external sources. The red and orange arrows represent flows, the green and blue arrows represent the efforts.

5-7-1 Bond Graph of the Labor Market

Figure 5-9 displays the bond graph of the labor market as a subsystem connected to both the households and the firms. Two transformers are used to move from the rotational mechanical domain to the translational mechanical domain. Cost of capital and growth of disposable income in %/yr are linearly transformed to direct labor and employment in FTE/yr, respectively. Labor income and labor cost in \$/yr are transformed to opportunity cost of labor and labor productivity in $\frac{\$}{\text{FTE yr}}$, respectively.

Demand and supply are collected and stored in the combination of the zero junction and C-elements. Storage of flows of labor exists in the form of human capital and the number of unemployed in the labor force. The C-elements return efforts that represent the productivity or opportunity cost. These subsequently act as the forces of supply and demand which results in a net force and a movement along the supply curve of labor.

Table 5-9 presents the elements, signals and their economic meaning. The underlying

Table 5-9: Definitions of elements and signals in the bond graph of the labor market subsystem

	Definition	**Interpretation**	**Units**
Elements	Human capital	The knowledge and skills that workers acquire through education [52].	FTE
	Unemployed	The number of those in the labor force who do not have jobs [52].	FTE
	Nominal wage	Amount of money paid for working one full time equivalent (FTE) [92].	\$/FTE
	Amortization	Lowering of value of intangibles over time [33].	
	Inactive labor	Persons outside the labor force [52].	
Efforts	Opportunity cost of labor	Potential benefits the laborer misses out on when choosing to work [92].	$\frac{\$}{\#yr}$
	Free market force	Unobservable market force that helps the demand and supply of the product reach equilibrium [41].	$\frac{\$}{\#yr}$
	Labor productivity	The output per worker [52].	$\frac{\$}{\#yr}$
Flows	Employment	Those in the labor force who have jobs per time period [52].	FTE/yr
	Unemployment	Those in the labor force who do not have jobs per time period [52].	FTE/yr
	Hired labor	Number of persons moving in or out of the firm's employment [33].	FTE/yr
	Amortized intangibles	Value decrease of intangibles over time [33].	FTE/yr
	Indirect labor	Work done on activities other than production (Proces optimization, marketing, R&D) [49].	FTE/yr
	Direct labor	Work done directly on producing products [49].	FTE/yr
	Incapacitation	Persons that are permanently unable to work [33].	FTE/yr

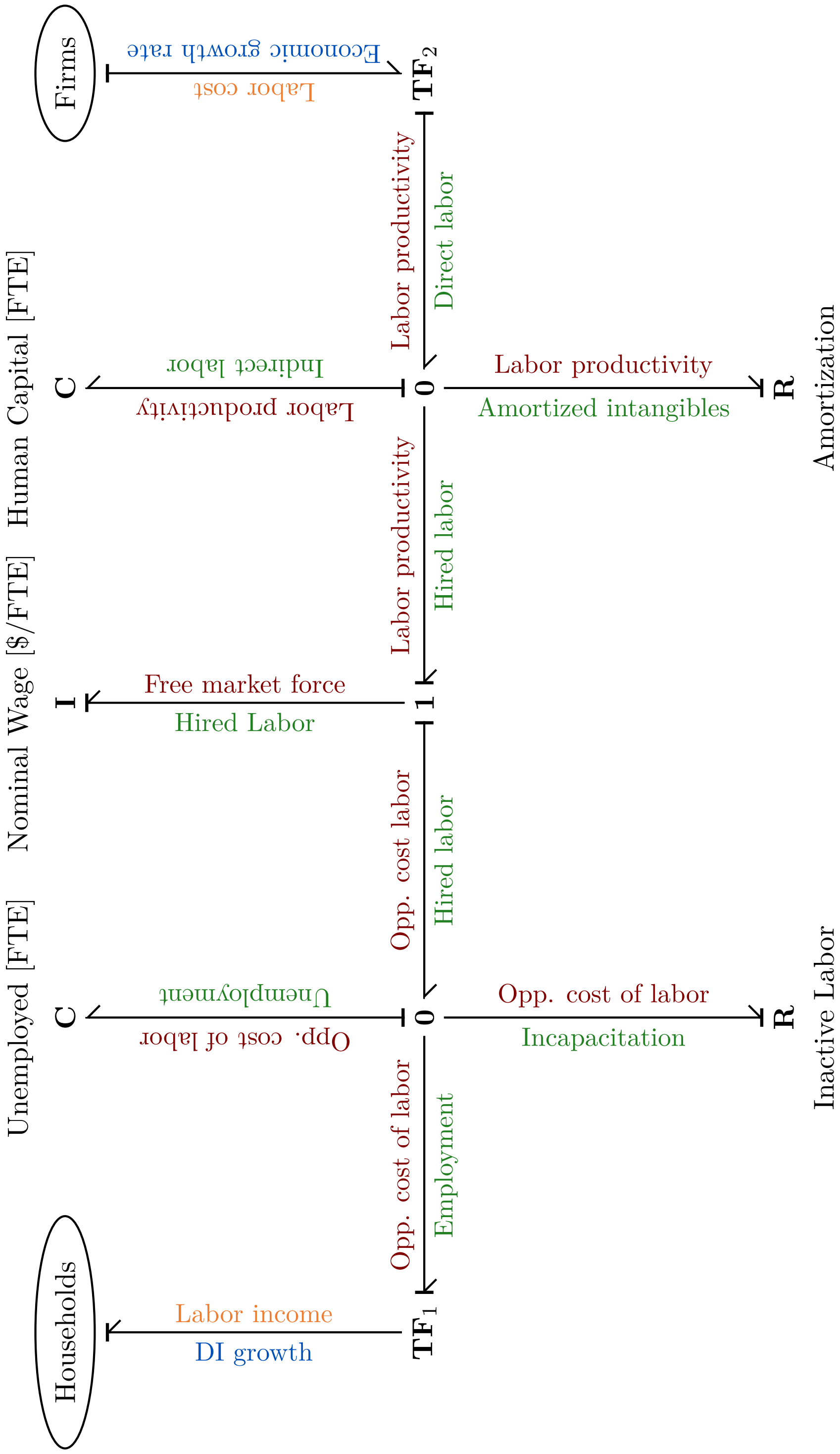


Figure 5-9: Bond graph of the labor market. The households and firms interact with the markets by giving a flow of of full-time equivalent (FTE) and receiving a cost or benefit. Units of labor expressed in FTE are stored in C-elements and the price of labor is stored in the I-element.

5-7-2 State-Space Representation of the Labor Market

From the bond graph of Figure 5-9, the state-space is derived [38]. The state vector is defined as $x(t) = [p_4\ q_5\ q_6]$', where p_4 is the nominal wage, q_5 the unemployed, and q_6 human capital. The input vector $u(t) = [u_1\ u_2]$' represent the disposable income growth and the economic growth rate. Finally, the output vector $y(t) = [y_1\ y_2]$' consists of the labor income and labor cost. This leads to the following state-space system:

State-Space Model of the Labor Market Dynamics

$$
\begin{aligned}
\dot{x}(t) &= \underbrace{\begin{bmatrix} 0 & -\frac{1}{C_5} & \frac{1}{C_6} \\ \frac{1}{I_4} & -\frac{1}{C_5 R_7} & 0 \\ -\frac{1}{I_4} & 0 & -\frac{1}{C_6 R_8} \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ -\frac{1}{TF_1} & 0 \\ 0 & TF_2 \end{bmatrix}}_{B} u(t) \\
y(t) &= \underbrace{\begin{bmatrix} 0 & \frac{1}{C_5 TF_1} & 0 \\ 0 & 0 & \frac{TF_2}{C_6} \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(t)
\end{aligned}
\tag{5-4}
$$

5-8 Rental Market as a Subsystem

The rental market represents the macroeconomic market for land as explained in Section 4-4-3. It is not to be confused with the real estate market where houses are bought as a form of residential investment [52]. It is connected to the households and firms. The firms are assumed to be the renters and the households the renters. In reality, this is not necessarily the situation but it follow macroeconomic theory convention.

The market receives the economic growth rate in %/yr from the firms and the growth rate of disposable income in %/yr from the households. The market returns the rental cost in $/yr to the firms and the rental income in $/yr to the households. Table 5-10 provides an overview of the inputs, outputs and states of the rental market. It consists of three parts: Household owned rentals, the market itself and the firm owned rentals. The unit of rentals are expressed in square foot (SF), a common metric for property rental pricing.

Table 5-10: Inputs, outputs and states of the rental market

Inputs	Outputs	States
Economic growth rate [%/yr]	Rental cost [$/yr]	Household-owned rentals [SF]
Disposable income growth [%/yr]	Rental income [$/yr]	Firm-owned rentals [SF]
		Rent price level [$/SF]

Figure 5-10 depicts the rental market connected to both the households and the firms. The orange arrows in $/yr and red arrows in $\frac{\$}{\text{SF yr}}$ represent the effort variable. The blue arrows in %/yr and the green arrows in SF/yr represent the flow variables. The direction of the arrows indicates causality and not the direction of positive flows.

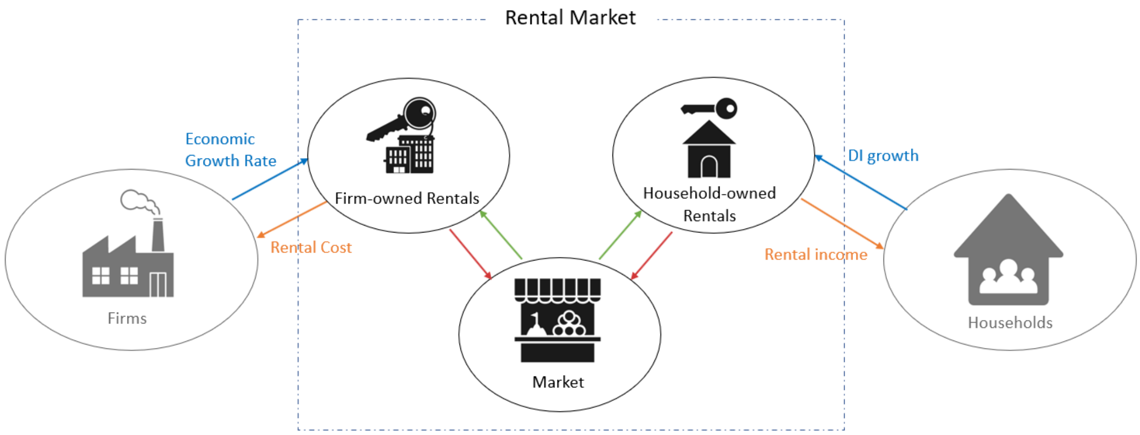


Figure 5-10: The rental market is interconnected to the firms and households. There are no modelled external sources. The red and orange arrows represent flows, the green and blue arrows represent the efforts.

5-8-1 Bond Graph of the Rental Market

Figure 5-11 displays the bond graph of the rental market as a subsystem connected to both the households and the firms. Two transformers are used to move from the rotational mechanical domain to the translational mechanical domain. The economic growth rate and growth rate disposable income in %/yr are linearly transformed to renting and renting out in SF/yr, respectively. Rental income and rental cost in $/yr are transformed to opportunity cost of rentals and imputed rent in $\frac{\$}{\text{SF yr}}$, respectively.

Demand and supply are collected and stored in the combination of the zero junction and C-elements. Storage of flows of rentals exists in the form of firm-owned- and household-owned properties. The C-elements return efforts that represent the imputed rent or opportunity cost. These subsequently act as the forces of supply and demand which results in a net force and a movement along the supply curve of rentals.

Table 5-11 presents the elements, signals and their economic meaning. The underlying

Table 5-11: Definitions of elements and signals in the bond graph of the rental market subsystem

	Definition	**Interpretation**	**Units**
Elements	Rent price level	Measure of the overall level of prices that shows the cost of a square foot of rentals [52].	$/SF
	Firm-owned properties	Rentals owned by the firms.	SF
	Household-owned properties	Rentals owned by the households.	SF
	Depreciation	Value decrease or consumption of fixed capital [52].	
Efforts	Opportunity cost of rental	Potential benefits the renter misses out on when choosing to rent [92].	$\frac{\$}{\#yr}$
	Free market force	Unobservable market force that helps the demand and supply of the product reach equilibrium [41].	$\frac{\$}{\#yr}$
	Imputed rent	The rental value that an owner would get from renting the home they occupy at market rates [52].	$\frac{\$}{\#yr}$
Flows	Renting out	The act of paying for the use of rentals.	FTE/yr
	Renting	The act of receiving payment for the lending of rentals.	FTE/yr
	Owner-occupied renting	Form of housing tenure in which a person, called the owner-occupier, owns the home in which they live [52].	FTE/yr
	Tenant-occupied renting	Form of housing tenure in which a person, called the tenant, rents the property [60].	FTE/yr
	Capital consumption adjustments	Measures of depreciation that are based on historical-cost accounting [60].	FTE/yr

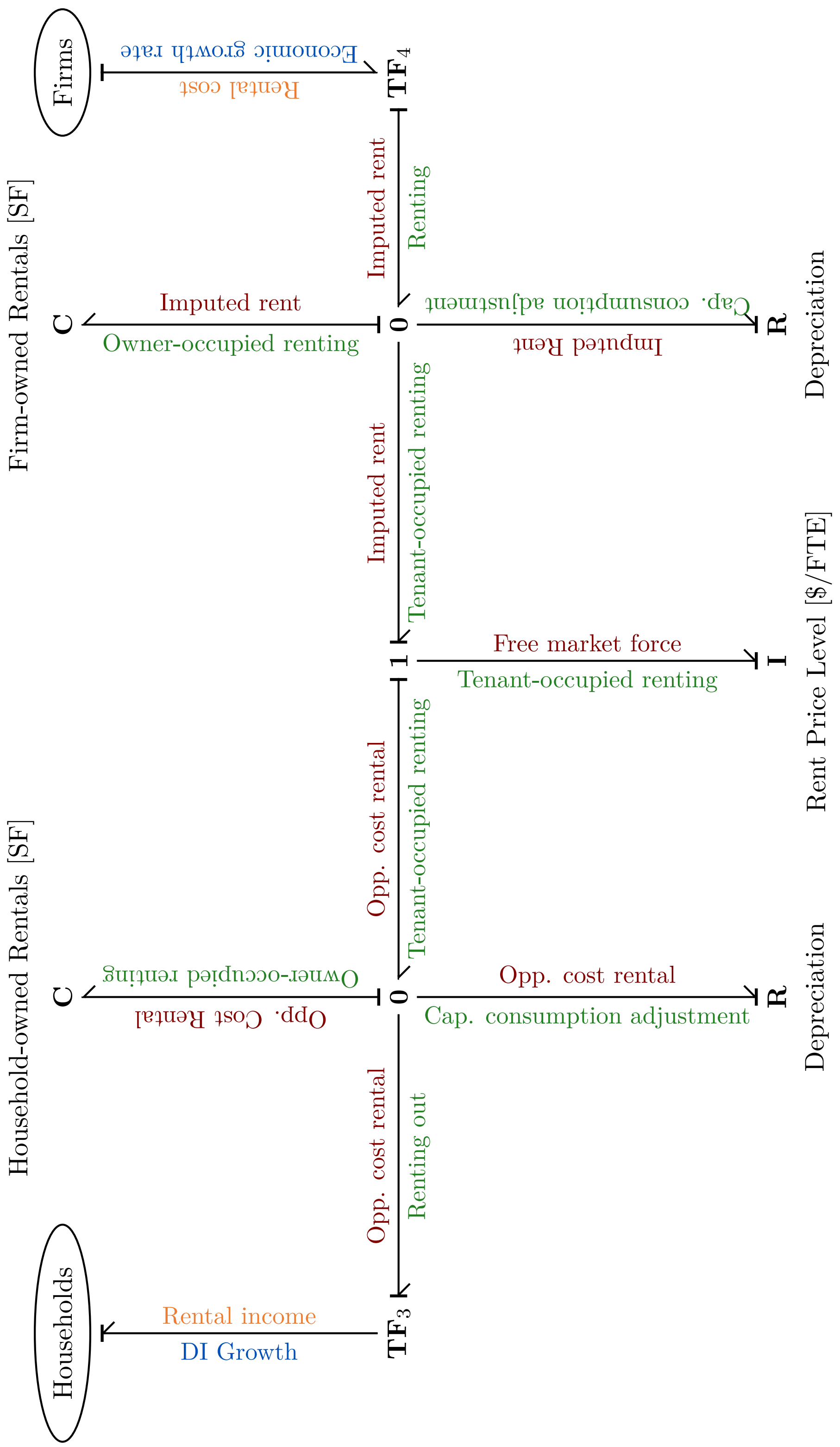


Figure 5-11: Bond graph of the rental market. The households and firms interact with the markets by giving a flow of square foot (SF) and receiving a cost or benefit. Units of rentals expressed in square foot are stored in C-elements and the rent price level is stored in the I-element.

5-8-2 State-Space Representation of the Rental Market

From the bond graph of Figure 5-11, the state-space is derived [38]. The state vector is defined as $x(t) = [p_5\ q_7\ q_8]$', where p_5 is the rent price level, q_7 household-owned rentals, and q_8 firm-owned rentals. The input vector $u(t) = [u_1\ u_2]$' represent the disposable income growth and economic growth rate. Finally, the output vector $y(t) = [y_1\ y_2]$' consists of the rental income and rental cost. This leads to the following state-space system:

State-Space Model of the Rental Market Dynamics

$$
\begin{aligned}
\dot{x}(t) &= \underbrace{\begin{bmatrix} 0 & -\frac{1}{C_7} & \frac{1}{C_8} \\ \frac{1}{I_5} & -\frac{1}{C_7 R_9} & 0 \\ -\frac{1}{I_5} & 0 & -\frac{1}{C_8 R_{10}} \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ -\frac{1}{TF_3} & 0 \\ 0 & TF_4 \end{bmatrix}}_{B} u(t) \\
y(t) &= \underbrace{\begin{bmatrix} 0 & \frac{1}{C_7 TF_3} & 0 \\ 0 & 0 & \frac{TF_4}{C_8} \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D}
\end{aligned}
\tag{5-5}
$$

5-9 Equity Market as a Subsystem

The equity market represents the macroeconomic market for profits and entrepreneurship. I view the profit market as the equity market as explained in Section 4-4-4.

The market is connected to the households, firms and market for loanable funds. The firms distribute profits to the households in the form of dividends or stores them in the form of cash. The market receives the economic growth rate from the firms and growth rate of disposable income from the households and returns Free Cash Flow to Firm (FCFF) and Free Cash Flow to Equity (FCFE) to the firms and households, respectively. Finally, financing cost are made which refer to the interest payments on loans. Table 5-12 provides an overview of the inputs, outputs and states of the equity market.

Table 5-12: Inputs, outputs and states of the equity market

Inputs	Outputs	States
Economic growth rate [%/yr]	FCFF [$/yr]	Cash holdings [$]
Disposable income growth [%/yr]	Equity income [$/yr]	Economic value added [%]
Investment [$/yr]	Financing Cost [%/yr]	Risk discount [%]

Figure 5-12 depicts the equity market and its interconnections. The orange arrows represent the effort variables in $/yr and the blue arrows represent the flow variables in %/yr. The direction of the arrows indicates causality and not the direction of positive flows.

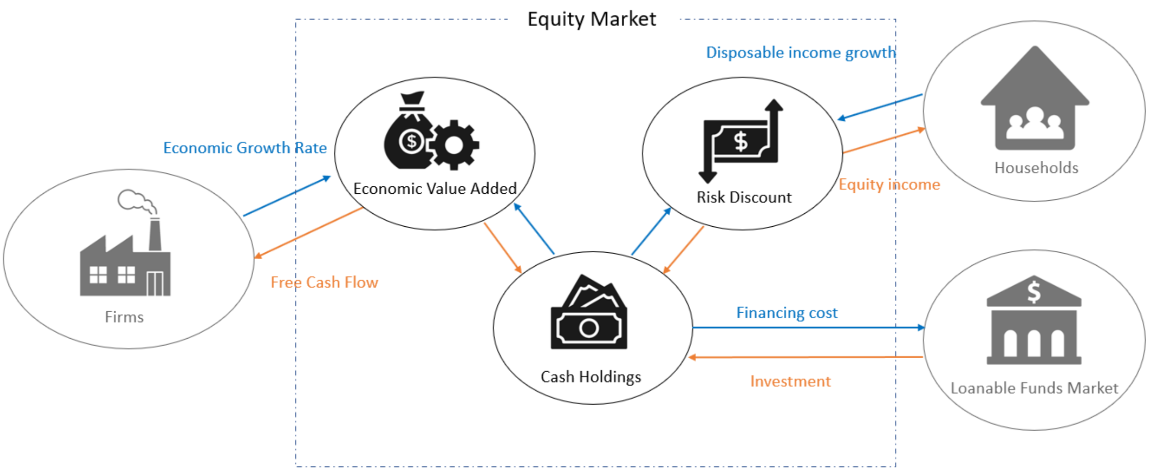


Figure 5-12: The rental market is interconnected to the firms and households. There are no modelled external sources. The red and orange arrows represent flows, the green and blue arrows represent the efforts.

5-9-1 Bond Graph of the Equity Market

Figure 5-13 displays the bond graph of the equity market as a subsystem connected to the households, firms and market for loanable funds.

The market consists of three parts: the risk discount, cash holdings and the economic value added. The amount of cash holdings is defined by weighing of the difference between FCFF and FCFE, i.e., if more cash flows enters than exits the market, it is stored in the form of cash holdings. The C-elements store the differences between the demanded and supplied rates of return. If the rate of return demanded by the household is lower than the rate of return realized by the firms, it is considered an attractive investment [86]. This either increases the economic value added or risk discount associated with an investment.

This market does not model the stock market directly. Instead, it deals with how free cash flows of the firms are distributed. To model the market value of a firm and hence its shares, requires a net present value computation using future cash flows. In Section 6-4-1, I will use the variables from this time domain model to estimate the value and the change of the value of equity. Table 5-13 presents the elements, signals and their economic meaning.

Two transformers are used to transform from different subsystems or markets within the rotational mechanical domain. They represent a ratio between the different types of investment and growth rates. The growth rate of disposable income and cost of capital are linearly transformed to cost of equity and financing cost, respectively. FCFE and investment are transformed to equity income and net borrowing, respectively.

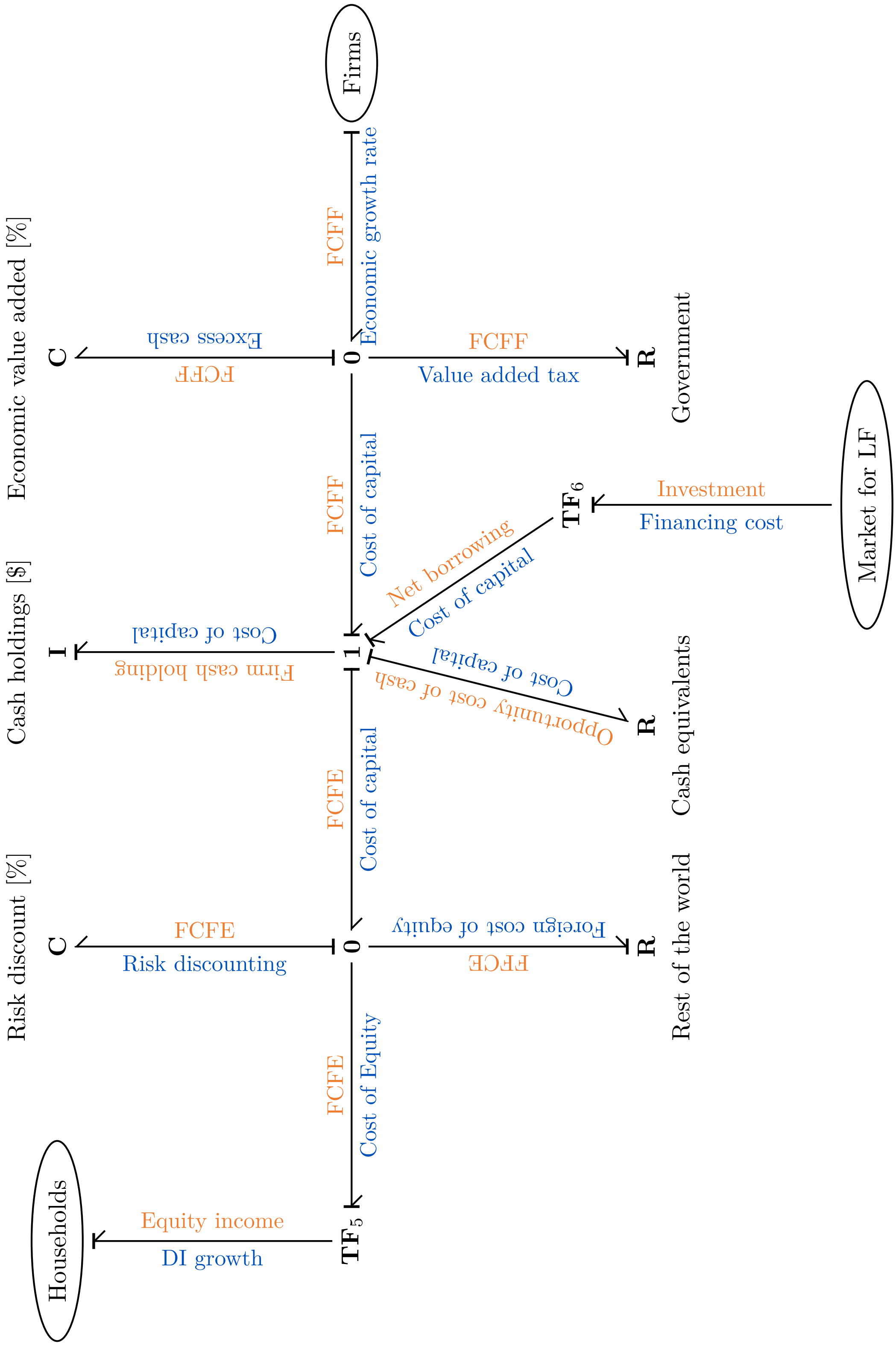


Figure 5-13: Bond graph of the equity market. There are three parts of the market that determine FCFF, FCFE and the Financing Cost. They are the discount rate, the cash holdings and the economic value added

Table 5-13: Definitions of elements and signals in the bond graph of the equity market subsystem

	Definition	Interpretation	Units
Elements	Cash holdings	The assets that you hold in ready cash [20].	\$
	Risk discount	Situation in which an investor is willing to accept a lower expected return in exchange for lower risk or volatility [32].	%
	Economic value added	A measure of a company's financial performance based on the residual wealth [86].	%
	Rest of the World	All countries outside the United States	
	Cash alternatives	investments securities that are meant for short-term investing [86].	
Efforts	Free cash flow to the firm (FCFF)	The free cash flow at the business enterprise level, used to value the firm or, indirectly, the firm's equity [86].	\$/yr
	Free cash flow to equity (FCFE)	The free cash flow a company can afford to pay out in dividends [86].	\$/yr
	Firm cash holding	The flow of assets firms hold in ready cash [20].	\$/yr
	Net borrowing	Debt issued less debt repaid over the period [86].	\$/yr
	Loan amortization	The process of paying off debt [86].	\$/yr
Flows	Cost of capital	The required return necessary to make a capital budgeting project worthwhile, the opportunity cost for its investors [86].	%/yr
	Cost of equity	Returns demanded by investors who are part of the company's ownership structure [86].	%/yr
	Financing cost	The interest and other costs incurred by the company while borrowing funds [86].	%/yr
	Excess cash	The spread between the return on the capital and the cost of the capital [48].	%/yr
	Foreign cost of equity	Returns demanded by foreign investors who are part of the company's ownership structure [86].	%/yr
	Value added tax	Tax levied upon a firm as a percentage of its value added [65].	%/yr

5-9-2 State-Space Representation of the Equity Market

From the bond graph of Figure 5-13, the state-space is derived. The state vector is defined as $x(t) = [p_6\ q_9\ q_{10}]$ which represents the cash holdings, risk discount and economic value added. The input vector $u(t) = [u_1\ u_2\ u_3]$' represents the disposable income growth rate, economic growth rate and investment. Finally, the output vector $y(t) = [y_1\ y_2\ y_3]$' represents the equity income, free cash flow to the firm and financing cost. This leads to the following state-space system:

State-Space Model of the Equity Market Dynamics

$$\dot{x}(t) = \underbrace{\begin{bmatrix} -\frac{R_{12}}{I_6} & -\frac{1}{C_9} & \frac{1}{C_{10}} \\ \frac{1}{I_6} & -\frac{1}{R_{11}C_9} & 0 \\ -\frac{1}{I_6} & 0 & -\frac{1}{R_{13}C_{10}} \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 & 0 & \frac{1}{TF_6} \\ -\frac{1}{TF_5} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{B} u(t)$$

$$y(t) = \underbrace{\begin{bmatrix} 0 & \frac{1}{C_9 TF_5} & 0 \\ 0 & 0 & \frac{1}{C_{10}} \\ \frac{1}{I_6 TF_6} & 0 & 0 \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(t) \tag{5-6}$$

5-10 Market for Loanable Funds as a Subsystem

The subsystem is connected to the firms, households, government bond market and equity market. Each influence the interest rate by demanding and supplying loanable funds. The market takes disposable income, GDP, financing cost, and cost of bonds from the households, firms, equity market and bond markets as inputs, respectively. In turn, it returns investment growth rate, MPC rate, and two times investment to the households, firms, equity market and bond markets, respectively.

Table 5-14 provides an overview of the inputs, outputs and states of the market for loanable funds. It consists of three parts: The bank deposits or personal savings, the market itself and the firms' debt.

Table 5-14: Inputs, outputs and states of the loanable funds market

Inputs	Outputs	States
GDP [$/yr]	Investment growth rate [%/yr]	Bank deposits [$]
Disposable income [$/yr]	MPC rate [%/yr]	Firm debt [$]
Net foreign saving		Interest [%]
Net foreign investment		

Figure 5-14 depicts the market for loanable funds connected to the households, firms, equity market and government bond markets. The blue and orange arrows represent the flow variables in %/yr and effort variables in $/yr, respectively. The direction of the arrows indicates causality and not the direction of positive flows.

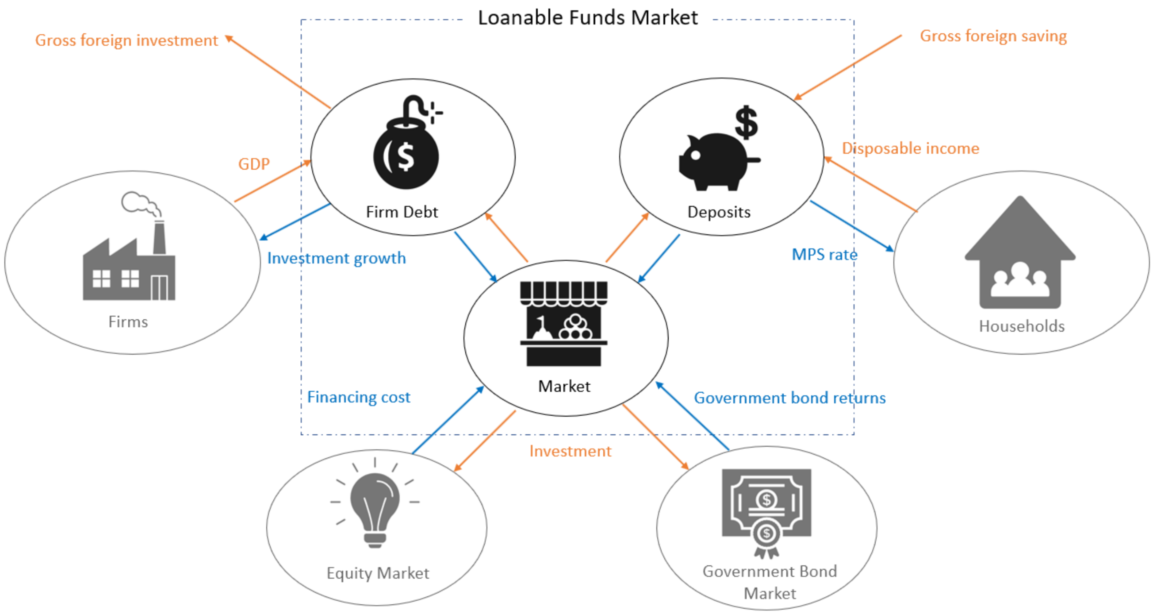


Figure 5-14: The loanable funds market is interconnected to the firms, households, equity market and bond markets. The external source is net foreign saving. The orange arrows represent flows, the blue arrows the efforts.

5-10-1 Bond Graph of the Loanable Funds Market

Figure 5-15 displays the bond graph of the loanable funds market as a subsystem connected to the households, firms, equity market and government bond markets. In reality, the market for loanable funds is a complex market with many submarkets for each of the different forms of financial investments. I replace the unmodeled financial investments with R-elements. In addition, I use R-elements to mimic the unmodeled dissipating behavior of the intermediary banks that require fees for intermediating financial capital.

Contrary to the other markets, the C-element functions as a market element where return on saving is weighed off against the returns on several types of investment. The market determines one average interest rate from them.

The I-elements weigh of the supply and demand for loanable funds. The difference is stored either as deposits on the supply side or as debt on the demand side. If more investment is supplied than needed by the firms, the firm goes into higher debt to finance its activities. Likewise, if the funds provided by the households are greater than the ones used for investment, they are stored in a bank account in the form of deposits.

Two transformers are used to transform from different subsystems within the rotational mechanical domain. The return on saving and return on investment in %/yr are linearly transformed to the MPS rate and the investment growth rate, respectively. GDP and disposable income $/yr are transformed personal saving and gross investment, respectively. Table 5-15 presents the elements, signals and their economic meaning.

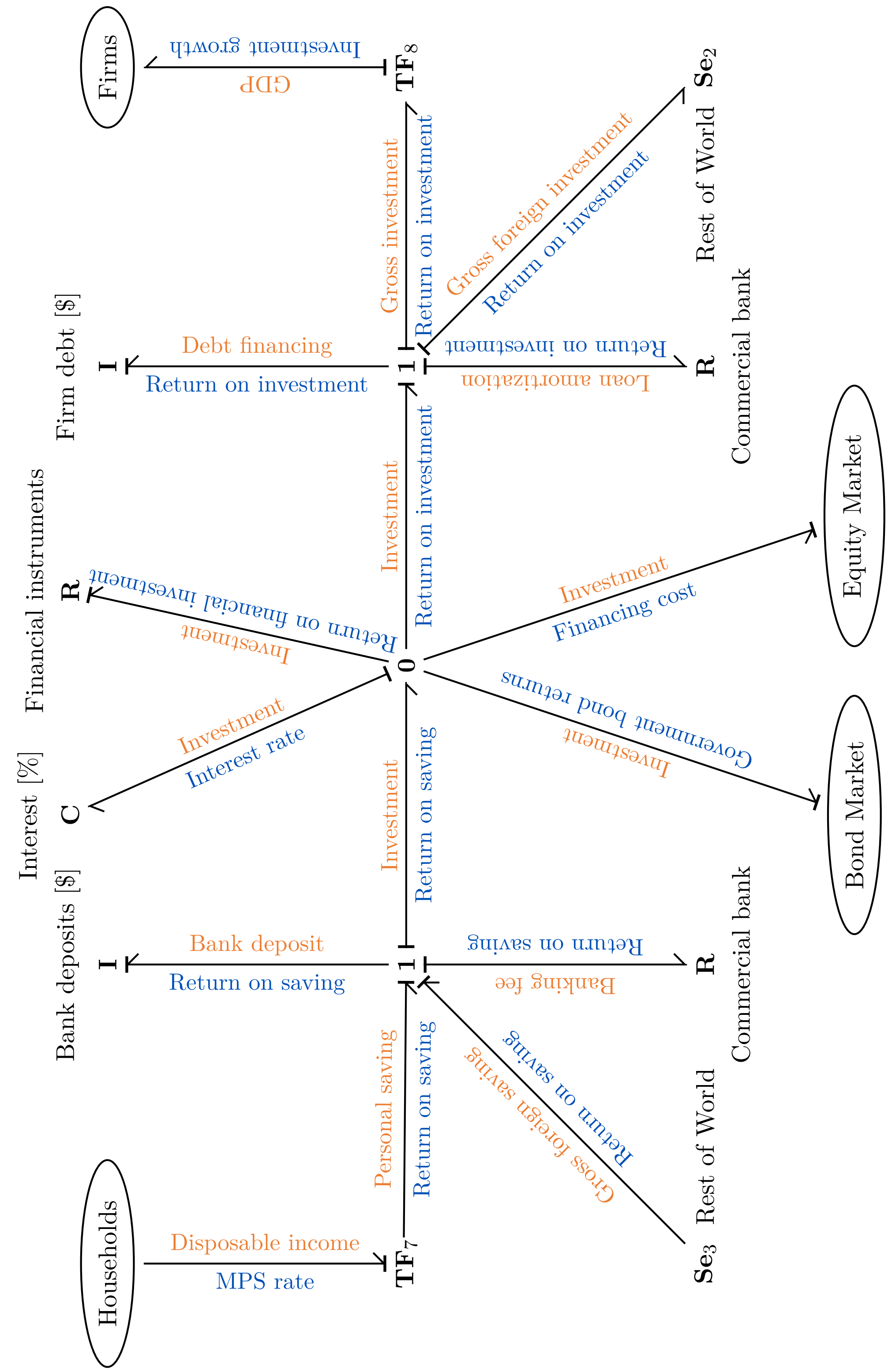


Figure 5-15: Bond graph of the loanable funds market. There are three parts of the market that determine investment growth and MPS rate: deposits, interests and firms' debt. Cash is lost through unmodelled financial instruments and the interest and fees commercial banks withhold.

Table 5-15: Definitions of elements and signals in the bond graph of the loanable funds market subsystem

	Definition	Interpretation	Units
Elements	Bank deposits	Money placed into banking institutions for safekeeping. These deposits are made to deposit accounts such as savings accounts, checking accounts, and money market accounts [53].	$
	Firm debt	The amount of money which needs to be paid back by the firms to lenders.	$
	Interest	Accumulated dollar return per year per dollar invested [65].	%
	Commercial bank	Financial institution that grants loans, accepts deposits, and offers basic financial products such as savings accounts and certificates of deposit to individuals and businesses. It makes money primarily by providing different types of loans to customers and charging interest [33].	
	Rest of the world	All countries outside the United States.	
	Financial instruments	A monetary contract between two parties [52].	
Efforts	Investment	Production of durable capital goods [65]. Here I use the sum of real investment and financial investment.	$/yr
	Gross investment	Investment including all investment goods produced [65].	$/yr
	Debt financing	Money raised by a firm by selling debt instruments, most commonly in the form of bank loans or bonds [33].	$/yr
	Loan amortization	The process of paying off debt [86].	$/yr
	Personal saving	The part of disposable income that is not consumed [65].	$/yr
	Bank deposit	The flow of money placed into banking institutions for safekeeping [53].	$/yr
	Banking fee	Charges imposed by financial institutions on their personal and business customers for account set-up, maintenance, and minor transactional services [34].	$/yr
	Gross foreign investment	The amount of investment from US citizens abroad [22].	$/yr
	Gross foreign saving	The amount of foreign saving in the US [22].	$/yr
Flows	Return on saving	The net dollar return per year for every dollar saved in a savings account [65].	%/yr
	Interest rate	A dollar return per year per dollar invested [65].	%/yr
	Government bond returns	The dollar return per year per dollar invested in government bond investment as a percentage of the total investment. Here I use the average for government bonds of all maturities.	%/yr
	Financing cost	The interest and other costs incurred by the company while borrowing funds [86].	%/yr
	Return on financial investment	The dollar return per year per dollar invested in unmodelled financial instruments.	%/yr
	Return on investment	The net dollar return per year for every dollar of invested financial capital [65].	%/yr

5-10-2 State-Space Representation of the Loanable Funds Market

From the bond graph of Figure 5-15, the state-space is derived [38]. The state vector is defined as $x(t) = [p_7\ p_8\ q_{11}]$ which represents the bank deposits, interest and firms' debt. The input vector $u(t) = [u_1\ u_2\ u_3\ u_4\ u_5\ u_6]$' represent the gross foreign saving, gross foreign investment, disposable income, government bond returns, financing cost and GDP. Finally, the output vector $y(t) = [y_1\ y_2\ y_3\ y_4]$' represent the MPC rate, investment growth rate and two times investment. This leads to the following state-space system:

State-Space Model of the Loanable Funds Dynamics

$$\dot{x}(t) = \underbrace{\begin{bmatrix} \frac{R_{14}}{I_7} & 0 & -\frac{1}{C_{11}} \\ 0 & -\frac{R_{16}}{I_8} & \frac{1}{C_{11}} \\ \frac{1}{I_7} & -\frac{1}{I_8} & -\frac{1}{R_{15}C_{11}} \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 1 & 0 & \frac{1}{TF_7} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -TF_8 \\ 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}}_{B} u(t)$$

$$y(t) = \underbrace{\begin{bmatrix} \frac{1}{I_7 TF_7} & 0 & 0 \\ 0 & \frac{TF_8}{I_8} & 0 \\ 0 & 0 & \frac{1}{C_{11}} \\ 0 & 0 & \frac{1}{C_{11}} \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(t) \tag{5-7}$$

5-11 Market for Government Bonds as a Subsystem

The market for government bonds is where governments demand for loans in exchange for an agreed rate of interest, the coupon rate. The suppliers of these loans are the households, who supply through saving in the market for loanable funds. There are two inputs to the system: Investment and bond purchases by the Federal Reserve.

Table 5-16 provides an overview of the inputs, outputs and states of the government bond market. It consists of three parts: The personal savings or deposits, the market for loanable funds and the scarcity of loanable funds or firms' debts.

Table 5-16: Inputs, outputs and states of the government bond market

Inputs	Outputs	States
Bond Buyback [$/yr]	Bond Return [%/yr]	Value of Bonds Outstanding [$]
Investment [$/yr]		Accumulated Yield [%]

Figure 5-16 depicts the market for government bonds connected to the market for loanable funds. The blue and orange arrows represent the flow variables in units of %/yr and effort variables in units of $/yr, respectively. The direction of the arrows indicates causality and not the direction of positive flows.

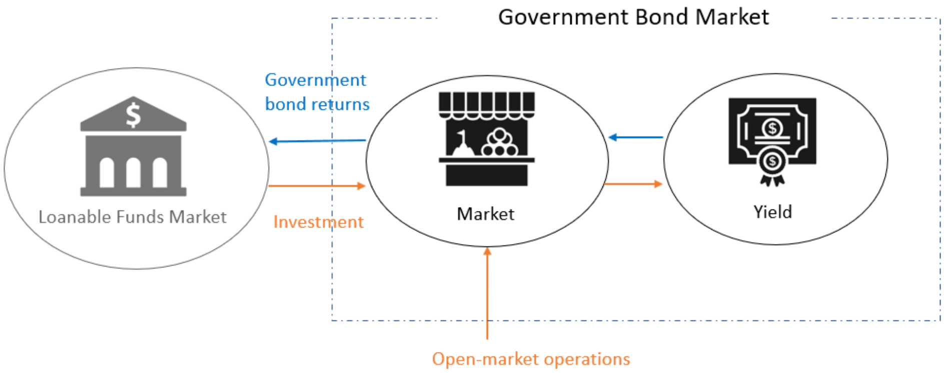


Figure 5-16: The government bonds market is interconnected to the loanable funds market. The external source is the purchasing of bonds by the Federal Reserve. The orange arrows represent flows, the blue arrows the efforts.

5-11-1 Bond Graph of the Government Bond Market

Figure 5-17 displays the bond graph of the government bond market as a subsystem connected to the loanable funds market and the Federal Reserve. Two inputs determine values of the two states: the outstanding dollar value of the bonds and the accumulated bond yield. The subsystem outputs the bond price changes to the loanable funds market.

The I-element weighs of the supply and demand for government bonds. If the demand exceeds the supply, the total dollar value of the outstanding bonds increases [89]. This causes an increase in the percentage price change of the bonds. In the C-element, the sum of the agreed coupon rates and the percentage price change are integrated and stored. The sum of the percentage price change and the original coupon rate gives the current yield.

The government and treasury auction are modelled with an R-element. The Federal Reserve applies monetary policy by either buying back treasury bills (monetary expansion) or selling treasury bills (monetary tightening) [52]. Treasury bills are government bonds with a short maturity. By doing so, it controls the money supply and therewith the interest rate of an economy. One transformer is used as a market interconnection between this market and the loanable funds market. The transformer represents a ratio between the total investment and the part of investment in government bonds and between the bond return rate to the bond percentage price change.

Because bonds are loans which pay interest, the value of a bond is a net present value calculation, similar to the net present value calculation for the market value of a firm [65]. In Section 6-4-3, I will perform such a computation using a Laplace transformation.

Table 5-17 presents the elements, signals and their economic meaning.

Table 5-17: Definitions of elements and signals in the bond graph of the government bond market subsystem

	Definition	**Interpretation**	**Units**
Elements	Value of outstanding bonds	The current oustanding par values of all government bonds combined.	$
	Accumulated Yield	The return an investor realizes on a bond [89].	%
	Treasury Auction	Auction where the government sells marketable securities (bills, notes, bonds, FRNs, and TIPS), by which the rate, yield, or discount margin of these securities are determined [89].	
	Government	System or method that controls the country [65].	
	Federal Reserve	The central bank of the United States [65].	
Efforts	Bond supply	The total supply of governments bonds.	$/yr
	Open-market operations (OMO)	The activity of a central bank in buying or selling government bonds to influence bank reserves, the money supply, and interest rates [65].	$/yr
	Traded bonds	The bonds traded on the secondary market.	$/yr
	Principal repayment	The repayment of the original bond value to the bond holder [89].	$/yr
Flows	Price Change	The rate of change of the bonds market value due to the effect of supply and demand [89].	%/yr
	Current Yield	The annual coupon payment divided by the bond's current market value [89].	%/yr
	Coupon Rate	The rate of interest it pays annually over the bonds par value [89].	%/yr

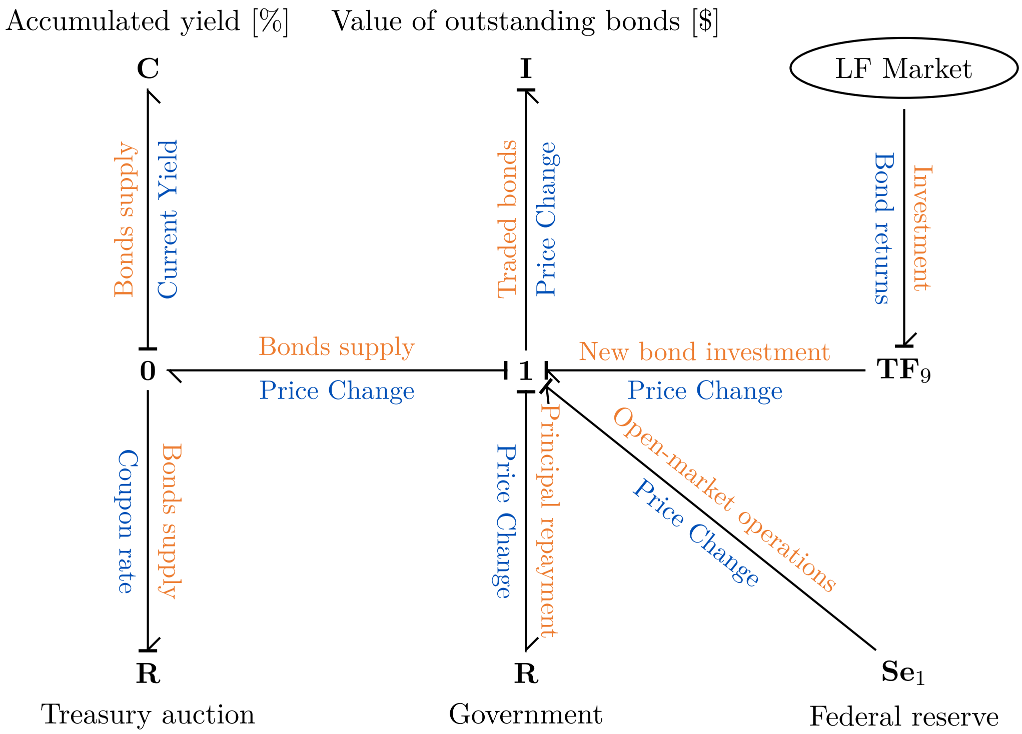


Figure 5-17: Bond graph of the government bond market. There are two parts of the market that determine the bond returns: value of the outstanding government bonds and accumulated yield.

5-11-2 State-Space Representation of the Government Bond Market

From the bond graph of Figure 5-17, the state-space is derived [38]. The state vector is defined as $x(t) = [p_9 \; q_{12}]$ which represents the value of bonds outstanding and the accumulated bond yield. The input vector $u(t) = [u_1 \; u_2]$' represent investment and the open-market operations of the Federal Reserve. Finally, the output $y(t) = y_1$ represent the bond return rate. This leads to the following state-space system:

State-Space Model of the Government Bond Dynamics

$$
\begin{aligned}
\dot{x}(t) &= \underbrace{\begin{bmatrix} -\frac{R_{18}}{I_9} & \frac{1}{C_{12}} \\ -\frac{1}{I_9} & -\frac{1}{R_{17}C_{12}} \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} \frac{1}{TF_9} & 1 \\ 0 & 0 \end{bmatrix}}_{B} u(t) \\
y(t) &= \underbrace{\begin{bmatrix} \frac{1}{I_9 TF_9} & 0 \end{bmatrix}}_{C} x(t) + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(t)
\end{aligned}
\qquad (5\text{-}8)
$$

5-12 Conclusions and Contributions

The economic system is modelled using a system engineering approach. The complex interconnected economic system is subdivided in eight subsystems: households, firms, market for consumer goods and services, labor market, rental market, equity market, loanable funds market, and the government bond market. The labor market, rental market and market for goods and services are expressed in the translational mechanical energy domain, the remainder in the rotational mechanical domain.

The bond-graph and corresponding state-space model is entirely based on macroeconomic theory, in contrast to current economic scenario models. The model consists of a total of 21 states and 50 different parameters. All states and elements have a direct economic interpretation which makes them understandable for both economists and engineers. This introduces the interpretability desired in industry for such models (see Chapter 2). The interpretable linear state-space representation allows for macroeconomic scenario analysis using Systems and Control (S&C) tools.

The model uses an ex-post approach to compute the values of the states, i.e. the variables are stored by integrating flows and efforts from the past to the present. This is not how economists compute the market value of bonds and equity. Instead, future cash flows are discounted to compute the present market value. Variables from this model will be used in Chapter 6 to compute the present value of equity and bonds using the Laplace transformation.

Contributions:

1. Development of a fundamental grey-box model of the United States economy consisting of the households, firms, labor market, rental market, consumer goods and services market, equity market, loanable funds market and government bond market.

2. Formulation of economic interpretation for the states, variables and inputs for economic interpretability of the model.

Chapter 6

The Engineering Frequency Domain for Valuation and Liquidity Analysis

6-1 Introduction

In this chapter, I use the Laplace transform to be able to model total equity returns and government bonds yields with a specific maturity. I show that these variables are inherent to the frequency domain. Therefore, the time domain model of previous chapter is not able to compute these variables directly. It is shown that Bode plots are a powerful tool to perform macroeconomic scenario analysis in the frequency domain. To be able to use Bode plots, transfer functions and Bode pots are given an economic interpretation. I define the chapter goals as follow:

Chapter goals:

1. Application of the Laplace transform to model total equity returns, government bond yields and the value of labor.
2. Introduce frequency domain tools and formulate their economic interpretation to perform macroeconomic scenario modelling in the frequency domain.

In Section 6-2, I map the economic definition of net present value to the engineering frequency domain and explain and I show that the Laplace transform of future cash flows gives the ex-ante measure for the cash-flow equivalent. In Section 6-3, I give an economic interpretation to the complex variable of the Laplace transform and the Internal Rate of Return (IRR). In Section 6-4, I show how the Laplace transform of the signals over bonds of the model from Chapter 5 can be used to determine the net present value (NPV). Subsequently, I determine the value and cash-flow-equivalents of equity, labor and government bonds with a specific maturity. In Section 6-5, I show how transfer functions can be used to determine the liquidity and efficiency in macroeconomics and how the final value theorem can be used to obtain the long-run steady-state value of a response. Additionally in this section, I give Bode plots a macroeconomic interpretation. Finally, in Section 6-6 I conclude that the use of the frequency domain for macroeconomic modelling and analysis is a contributing application.

6-2 The Laplace Transform for Net Present Value

The five variables that Ortec Finance (OF) proposed to model were stated in Section 2-4. These included year-on-year total equity returns and bond yields for two different maturities. The computation of the value of these variables involve discounting future cash flows. This makes these variable inherently frequency domain variables [90]. The time domain model of Chapter 5 is therefore not able to directly compute these variables.

The time domain model of the previous chapter determines cash flows as what macroeconomists call, an ex-post quantity [58]. This means that the quantity is measured after the event. Value of equity and bonds are determined by discounting future cash flows [65]. The cash flows are an ex-ante quantity, which means that the quantity is predicted beforehand based on the past [58].

The present value is the dollar value today of a stream of future income. It is measured by calculating how much money invested today would be needed, at the going interest rate, to generate the asset's future stream of receipts [65]. In economics the Net-Present Value (NPV) is usually computed using a discrete time relation [20, 65]:

$$\mathrm{NPV}(i) = \sum_{k=0}^{\infty} \frac{C(k)}{(1+i)^k} - c(0) \tag{6-1}$$

If transformed to continuous time, the relation becomes very similar to the Laplace transform:

$$\mathrm{NPV}(i) = \int_0^{\infty} C(t) e^{-\ln(1+i)t} - c(0) \tag{6-2}$$

Here, $C(t)$ is the cash flow in \$/yr at time step t in years, i is a discount rate or return that could be earned in %/yr, and $c(0)$ the value of initial investment in \$.

The NPV gives the value of a series of cash flows, while accounting for the time value of money: A dollar to be paid out tomorrow is worth less than a dollar paid out today. The decrease (or increase) in present value of future cash flows is based on a discount rate [86]. By introducing the Laplace transform, the computation is performed using a complex discount rate [29]. In this case, NPV becomes:

$$\mathrm{NPV}(s) = \int_0^{\infty} C(t) e^{-st} \tag{6-3}$$

Here, s is a complex variable ($s = \sigma + i\omega$) as opposed to a real-valued variable used in economic valuation theory [20, 89].

Engineers use the Laplace transform to express differential equations in the time domain as algebraic equations in the frequency domain. This simplifies the solving of differential equations and permits frequency domain model evaluation, which is particularly useful here.

By directly applying the Laplace transform, the NPV becomes a frequency dependent variable and therefore a frequency domain concept. The use of the Laplace transform has several benefits. Firstly, it introduces the fully explored set of possible solutions [76] compared to the handful of currently known analytical solutions to the NPV problem. Secondly, it allows for cyclical discounting by the introduction of a complex discounting factor[17]. Finally, if the cash flow is expressed as a differential equation, solving it is relatively simple.

I will now work towards Eq. (6-6) which I use throughout this chapter. If we define the ex-post credit or financial capital dollar value as $c(t)$ in \$, the cash flow as $\dot{c}(t)$ in \$/yr and the changes of the cash flow as $\ddot{c}(t)$ in $\frac{\$}{\mathrm{yr}^2}$ in \$/yr^2, the NPV of a project can be determined from the cash flow using:

$$\begin{aligned} C(s) &= \int_0^\infty c(t)e^{-st} \\ \mathrm{NPV}(s) &= \int_0^\infty \dot{c}(t)e^{-st} = sC(s) - c(0) \end{aligned} \tag{6-4}$$

Similarly, evaluated using the power- the change of the cash flow w.r.t. time- yields:

$$K(s) \quad = \quad \int_0^\infty \ddot{c}(t)e^{-st} = s^2C(s) - s\dot{c}(0) - c(0) \tag{6-5}$$

I define $K(s)$ as the cash-flow-equivalent. $K(s)$ is expressed in \$/yr but contrary to the original cash flow it is a frequency dependent ex-ante variable. The cash-flow equivalent is the variable of particular interest because it determines the change of the value in the frequency domain. From the change of the value of government bonds, the yield can be obtained and from the change of the value of equity, the total equity returns.

If one assumes the initial conditions are zero, NPV and the cash-flow-equivalent $K(s)$ are related through:

$$\mathrm{NPV}(s) \quad = \quad \frac{K(s)}{s} \tag{6-6}$$

With this insight, valuation of equity, bonds and other economic variables can be performed using the power variables of the bonds from the time domain model of Chapter 5.

6-3 The Internal Rate of Return as Frequency Variable

The IRR can be defined as the discount rate at which the present value of all future cash flows (or monetized expected hypothetical benefits) is equal to the initial investment, that is, the rate at which an investment breaks even [63]. Rearranging Eq. (6-5), this condition is satisfied if:

$$\mathrm{NPV}(s) = 0 \qquad sC(s) = c(0) \tag{6-7}$$

where,

$$s = \sigma + i\omega \tag{6-8}$$

In engineering, the real part represents the dissipative trending behavior, and the imaginary part the oscillating behavior. Juxtaposed to economic valuation, I interpret the real part as some trending exponential discount rate related to the time value of money, and the imaginary part as some economic cycle.

Economists currently only use real-valued discount rates for valuation [20, 65]. The complex discount rate is something new by the use of this approach. The imaginary part of the IRR explains where in a cycle the real value of the IRR is when computed. It introduces a variability.

6-4 Application of Frequency Domain Valuation to Time Domain Variables

Here, I will use the variables of the time domain model from Chapter 5 to compute their value by taking the Laplace transform. This way, the scenario generating capabilities of the model in the time domain is extended to valuation in the frequency domain.

For subsystems in the rotational mechanical domain, I propose two different methods to compute the NPV. Figure 6-1 functions as a supportive figure to this explanation.

The first option is to directly consider the cash flow effort variable $e(t)$ and take the Laplace transform as in Eq. (6-4).

The second option is to take the Laplace transform of the power -the derivative of the cash flow over time- over a bond. The relation defined in Eq. (6-6) can subsequently be used to obtain the NPV.

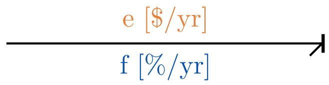


Figure 6-1: The product of the effort and flow over a bond gives the power. In the rotational mechanical domain (financial domain) both the power, as the cash flow (the effort) can be used to compute the NPV

Alternatively, it is also possible to compute the NPV of variables from bonds in the translational mechanical domain (see Figure 6-2). In the translational domain, the effort, $e(t)$, represents a cost in $\frac{\$}{\#\,\mathrm{yr}}$, and the flow, $f(t)$, a flow of goods measured in #/yr. The only possibility is therefore to use the power variable to compute the cash flow equivalent and relate it to NPV(s) using Eq. (6-6).

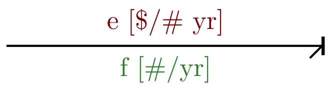


Figure 6-2: The product of the effort and flow over a bond gives the power. In the translational mechanical domain (economic domain) only the power variable can be used to compute the NPV

6-4-1 The Laplace Transform for Equity Valuation

I apply two different equity valuation methods by using the Laplace transform: the Free Cash Flow to Equity (FCFE) discount model and the dividend discount model [20]. The FCFE discount model applies to both dividend as non dividend paying firms [86]. The dividend discount model only applies to dividend paying firms.

Valuation of stocks is complex because much of the market movements of stock prices are influenced by short-term psychological effects of people trading on the stock market [65]. Because the theory on firm foundations is defined in a structural manner and market psychology is not, I will only follow the firm foundation approach which holds that assets should be valued on the basis of their intrinsic value. I assume that over time, the stock prices should approach their present value.

Free Cash Flow to Equity Discount Model as Frequency Domain Concept

The free cash flow to equity model discounts the FCFE by cost of equity [20, 86]. FCFE is the cash flow remaining for equity holders after all other claims have been satisfied [86]. The time dependant function for FCFE appears as one of the signals in the time domain model of Section 5-9-1. Figure 6-3 redisplays the bond of the time domain model of the equity market from this section. FCFE acts as an effort along the bond.

Figure 6-3: Bond connecting the equity market and the households. FCFE is the effort variable in $/yr and the Cost of Equity in %/yr the flow variable.

I use $\mathrm{EV}_1(s)$ to define the present equity value. Reformulating the FCFE model from [86] with the Laplace transformation gives:

$$\mathrm{EV}_1(s) \quad = \quad \int_0^\infty \mathrm{FCFE}(t) e^{-st} dt \tag{6-9}$$

To obtain the change of equity value over time, a differentiation in the frequency domain is used. This corresponds to the multiplication of $1/s$ [76]. The product of Eq. (6-9) and $1/s$ therefore represents the change of the equity value as a frequency domain variable. I define this variable as the Derivative Equity Value (DEV_1):

$$\mathrm{DEV}(s) \quad = s\mathrm{EV}(s) \quad = \int_0^\infty \frac{\mathrm{FCFE}(t)}{dt} e^{-st} dt \tag{6-10}$$

This concludes the computation of a nation's equity value and the change of that value using a engineering version of the FCFE discount model. The value and its changes are posed as a frequency domain concept and as a function of a complex discount rate.

Dividend Discount Model as Frequency Domain Concept

Using the Laplace transform to compute the value of equity using the dividend discount model [86] results in the following equation:

$$\mathrm{EV}_2(s) \quad = \quad \int_0^{\infty} \mathrm{Div}(t) e^{-st} dt \tag{6-11}$$

Here, $\mathrm{Div}(t)$ is the time dependant function of the dividend incomes. Its time derivative, $\frac{d\mathrm{Div}(t)}{dt}$, is the product of $\mathrm{FCFE}(t)$ and $\mathrm{COE}(t)$ from Figure 6-3.

I define the Laplace transform of the Cost of Equity (COE) separately as the Equity-Cost-Equivalent $\mathrm{ECE}(s)$:

$$\mathrm{ECE}(s) \quad = \quad \int_0^{\infty} \mathrm{COE}(t) e^{-st} dt \tag{6-12}$$

The Laplace transform of a product of two time domain functions equals the convolution of the frequency domain functions [76]. Convolution expresses the amount of overlap of a function as it is shifted over another function. In economics convolutions are used to aggregate different dimensions into a single-dimensional sufficient statistic [56].

Hence, the change of the dividend equity value measured in \$/yr equals the convolution of Eq. (6-9) and Eq. (6-12). The change of the dividend equity value then becomes:

$$\mathrm{DEV}_2(s) = \mathrm{EV}_1(s) * \mathrm{ECE}(s) \tag{6-13}$$

Finally, the dividend equity value $\mathrm{EV}_2(s)$ is obtained using Eq. (6-13) and the relation described by Eq. (6-6) as follows:

$$\mathrm{EV}_2(s) = \frac{\mathrm{EV}_1(s) * \mathrm{ECE}(s)}{s} \tag{6-14}$$

This shows that that year-over-year total equity returns can be computed using a frequency domain approach. This solves the limitation of the time domain model of Chapter 5.

6-4-2 The Laplace Transform for the Labor Theory of Value

To show the versatility of this valuation method, I will also use the Laplace transform to compute the ex-ante value of labor. Comparison of the IRR and the inflation rate gives and indication of the change of the purchasing power of labor income in an economy.

The labor theory of value is a major pillar of traditional Marxian economics. The theory's basic claim is: the value of a commodity can be objectively measured by the average number of labor hours required to produce that commodity. I use an adjusted notion of labor value from Adam Smith [74], value-in-exchange: The relative proportion with which a commodity exchanges for another commodity. Here, I will use the price of labor: the wage.

The Laplace transform of labor income gives the equivalent value of labor as frequency domain variable. I use the term labor present value (LPV) to define it.

$$\mathrm{LPV}(s) = \int_0^\infty I(t)e^{-st}dt \tag{6-15}$$

where $I(t)$ in \$/yr is the labor income and s represents the complex labor discount rate, or amortization rate of labor. The variables are obtained from the bond connected to the I-element in the labor market model of Section 5-7-1. Figure 6-4 redisplays how this element appears:

Figure 6-4: Bond of the I-element of the labor market subsystem. The free market force, which represents the time derivative of the nominal wage in $\frac{\$}{\text{yr}\ \#}$, is the effort variable. Hired labor in FTE/yr is the flow variable.

The Free Market Force (FMF) is the time dependant effort variable and the Hired Labor (HL) is the time dependant flow variable. I define the Laplace transform of the free market force, $\mathrm{FMF}(t)$, as the unit labor value $V(s)$:

$$\mathrm{V}(s) = \int_0^\infty \mathrm{FMF}(t)e^{-st}dt \tag{6-16}$$

The labor value is the total value of sales -here the total value of labor- divided by the sum of the quantities - the hired laborers [54]. Free market force measured in $\frac{\$}{\mathrm{FTE\ yr}}$, is discounted for unit labor value similarly to how cash flows are discounted for NPV. The discount rate could, for example, represent a complex inflation rate.

Similarly the flow variable, $\mathrm{HL}(t)$, can be discounted with, for example, the population growth rate. The hiring rate can be compared to the discount rate to conclude if an economy is hiring faster than the population is growing. I define the Laplace transform of the hired labor as the Labor-Equivalent $\mathrm{LE}(s)$:

$$\mathrm{LE}(s) = \int_0^\infty \mathrm{HL}(t)e^{-st}dt \tag{6-17}$$

The Labor Present Value (LPV) is obtained by the convolution of Eq. (6-16) and Eq. (6-17) and subsequently applying the relation ofEq. (6-6). The equation for the LPV becomes:

$$\mathrm{LPV}(s) = \frac{\mathrm{V}(s) * \mathrm{L}(s)}{s} \tag{6-18}$$

Discounting the $\mathrm{LPV}(s)$ with a combination of population growth and the inflation rate and comparing it to the wage growth rate gives a measure of the net purchasing power expressed in \$. Likewise, the IRR of Eq. (6-18) provides a minimal required wage growth rate in %/yr.

With this equation, the versatility of this approach is shown as it can be used to other variables than financial variables to compute value.

6-4-3 The Laplace Transform for Government Bond Valuation

In the previous two sections, I showed how to compute a nations equity and labor value using the properties of the Laplace transform. Unlike for these two variables, the signals for a government bond of a specific maturity exist only for a finite time interval. I solve this issue by using the Laplace transform of delayed unit-step responses and the multiplicative properties of the frequency.

For bonds with a fixed annual coupon interest rate, economists calculate the value of bonds using [89]:

$$\mathrm{V}(i) = \sum_{k=1}^{M} \frac{C(k)}{(1+i)^k} + \frac{F(M)}{(1+i)^M} \tag{6-19}$$

Here, $C(k)$ in \$/yr are the coupon payments , i in %/yr is the real-valued yield-to-maturity or discount rate, $F(M)$ in \$ is the face value at maturity, and M measured in years represents the maturity. Note that contrary to NPV, this is not a 'net' value calculation, i.e. the initial investment is not subtracted from the value but instead its discounted face value F is added.

Eq. (6-19) is a discrete time discounting function. This can be motivated by the fact that for government bonds of a specific maturity, the coupon payments are made in pre-determined time periods (usually semi-annually or annually) until maturity.

This thesis considers all government bonds of an economy, and not one single type of government bond. I assume that government bonds are constantly issued and therefore the coupon rates are constantly adjusted and so is the total amortized yearly face value. Therefore I will use continuous time discounting. To compute the present value of government bonds with a specific maturity using a continuous discounting function, I use:

$$\mathrm{V}(s) = \int_{1}^{M} C(t)e^{-st}dt + F(M)e^{-Ms} \tag{6-20}$$

Here M is the bonds maturity measured in years, $C(t)$ are the coupons payments in \$/yr and $F(M)$ is the returned face value at maturity. This equation uses a finite time interval, $1 \leq t \leq M$. At $t = 0$ the government bond is purchased and hence no coupon payments are made. $C(0)$ is therefore equal to 0. At $t = M$ the bond reaches maturity, and the face value is paid out to the bond owner.

Like in the previous sections, elements of the bond graph model are used to determine the value of government bonds for a specific maturity. The bonds from Section 5-11-1 are redisplayed here:

Figure 6-5: Bond connecting the government bond market and the treasury auction. Bonds supply is the effort variable in \$/yr and the coupon rate is the flow variable in %/yr.

Figure 6-6: Bond connecting the government bond market and the government. Principal Repayment is the effort variable in \$/yr and the price change is the flow variable in %/yr.

I determine the ex-ante values of the effort and flow of Figure 6-5 to acquire the Coupon-Equivalent $\mathrm{CE}(s)$ in % and the total issued Bond Value $\mathrm{BV}(s)$ in \$:

$$\begin{aligned} \mathrm{CE}(s) &= \int_0^\infty \mathrm{CR}(t)e^{-st}dt \\ \mathrm{BV}(s) &= \int_0^\infty \mathrm{BS}(t)e^{-st}dt \end{aligned} \tag{6-21}$$

Here, $\mathrm{CR}(t)$ is the Coupon Rate in %/yr from Figure 6-5 and $\mathrm{BS}(t)$ is the Bond Supply from Figure 6-5.

To ensure the frequency domain variables are defined within the time interval $1 \leq t \leq M$, I apply two delayed unit step operations and a subtraction. The delayed unit step response $u(t-\tau)$ is defined as follow:

$$u(t-\tau) = \begin{cases} 0, & t < \tau \\ 1 & t \geq \tau \end{cases} \tag{6-22}$$

The first operation is to multiply the time domain functions by $u(t-1)$ and subsequently subtract the time domain function by the product of itself and $u(t-M)$. I provide an example for $\mathrm{CE}(t)$:

$$\mathrm{CR}_i(t) = u(t-1)\mathrm{CR}(t) - u(t-M)\mathrm{CR}(t) \tag{6-23}$$

$$\mathrm{CR}_i(t) = \begin{cases} 0, & t < 1 \\ \mathrm{CR}(t), & 1 \leq t \leq M \\ 0, & t > M \end{cases} \tag{6-24}$$

Transforming this variable into the frequency domain using the Laplace transform properties [76] gives:

$$\mathcal{L}\{\mathrm{CR}_i(t)\} = \frac{\mathrm{CE}(s)e^{-s} - \mathrm{CE}(s)e^{-Ms}}{s} \tag{6-25}$$

Applying the same operations as above to $\mathrm{BS}(t)$ to obtain $\mathrm{BS}_i(t)$ and taking the Laplace transform of the product of $\mathrm{BS}_i(t)$ and $\mathrm{CR}_i(t)$ gives:

$$\mathcal{L}\{\mathrm{CR}_i(t)\mathrm{BS}_i(t)\} = \frac{\mathrm{CE}(s) * \mathrm{BV}(s)e^{-2s} - 2\mathrm{CE}(s) * \mathrm{BV}(s)e^{-(M+1)s} + \mathrm{CE}(s) * \mathrm{BV}(s)e^{-2Ms}}{s^2} \tag{6-26}$$

Finally, I need to add the face value repayment at time M. I use the product of the effort variable, Principal Repayment (PR), and the flow variable, Price Change (PC), from Figure 6-6 at time M. I define the face value equivalent as:

$$\mathrm{FE}(s) = \mathrm{PC}(M) \cdot \mathrm{PR}(M)e^{-2Ms} \tag{6-27}$$

Addition of Eq. (6-27) to Eq. (6-26) gives the change of the value of government bonds as a function of a complex discount rate s. I define this value as the Derivative Value (DV). DV(s) is measured in units of \$/yr and is:

$$\mathrm{DV}(s) = \frac{\mathrm{CE}(s) * \mathrm{BV}(s)e^{-2s} - 2\mathrm{CE}(s) * \mathrm{BV}(s)e^{-(M+1)s} + \mathrm{CE}(s) * \mathrm{BV}(s)e^{-2Ms}}{s^2} + \mathrm{PC}(M) \cdot \mathrm{PR}(M)e^{-2Ms} \tag{6-28}$$

The product of Eq. (6-28) and $\frac{1}{s}$, which is an integration operator in the frequency domain, gives the value of a nation's government bonds with a specific maturity.

With this equation, I have shown that year-over-year government bond returns can be computed from the time domain signals of Chapter 5 using the Laplace transform. The use of the frequency domain solves the limitations the time domain models has with respect to valuation.

6-5 Frequency-Domain Macroeconomic Scenario Modelling using Transfer Functions

In engineering, the frequency domain is often used for both controller design as system analysis [25]. Because I want to perform scenario and system analysis, I introduce the concept of transfer functions and give them an economic interpretation. This way, existing frequency domain analysis tools can also be applied to economic models.

6-5-1 Transfer Functions for Liquidity, Illiquidity and Efficiency

The transfer function can be formally defined as follows: The function $H(s)$, which is the transfer gain from $Q(s)$ to $P(s)$— input to output— is called the transfer function of the system. It is the ratio of the Laplace transform of the output of the system to the Laplace transform of the input [25]. The transfer function of the state space model of Chapter 5 is obtained using Eq. (6-29) [26] and has the form shown in Eq. (6-30).

$$H(s) = C(sI - A)^{-1}B + D \tag{6-29}$$

$$H(s) = \frac{Q(s)}{P(s)} \tag{6-30}$$

The transfer function is a function of a complex discount rate $s = \sigma + i\omega$. In an economic application, σ represents the value-decrease of money over time, and ω gives the frequency of some economic cycle. The rationale behind these interpretations are explained in Section 6-3.

In Section 6-4-1 - Section 6-4-2, I used several different economic interpretations for $P(s)$ and $Q(s)$ without the notion of a transfer function $H(s)$. For a macroeconomic system, I distinguish between the rotational mechanical and translational mechanical domain following the argument of Section 4-3-2. Here, I first present the economic interpretation of $Q(s)$ and $P(s)$ for each domain individually. Then, I give a general economic interpretation of $Q(s)$ and $P(s)$ to be used for the formulation of the general economic interpretation of transfer functions.

I define $Q(s)$ as the Laplace transform of some flow $f(t)$:

$$Q(s) = \int_0^\infty f(t)e^{-st}dt \tag{6-31}$$

I define $P(s)$ as the Laplace transform of some effort $e(t)$:

$$P(s) = \int_0^\infty e(t)e^{-st}dt \tag{6-32}$$

Note that $e(t)$ is the effort variable and e^{-st} an exponent.

Translational Mechanical Domain Interpretation

In the translational mechanical domain, the flow $f(t)$ measured in #/yr is the time derivative of the quantity considered. These quantities can be labor measured in full-time equivalent (FTE), rentals measured in SF, and consumer goods measured in #. The discounting factor e^{st} performs exponential depreciation, depletion or amortization on the product flows over time.

The effort $e(t)$ measured in $\frac{\$}{\#\ \mathrm{yr}}$ is some economic force which is the time derivative of the price of the before mentioned goods. Prices can thus refer to wages in \$/FTE, rentals prices in \$/SF, and consumer goods prices in \$/#. The discounting factor e^{st} performs exponential discounting or depreciation on economic forces over time.

I define $Q(s)$ as the Economic Output (EO) for consumer goods and services, and as the Economic Input (EI) for labor and rentals. I define $P(s)$ as the economic value (EV) for all three of the above.

Rotational Mechanical Domain Interpretation

In the rotational mechanical domain, the flow $f(t)$ is the time derivative of the performance or interest. The interest or performance are measured in %. The discounting factor e^{st} performs exponential devaluation, depreciation, amortization on the performance rate over time.

In the rotational mechanical domain, the effort $e(t)$ in \$/yr is the time derivative of the financial capital or cash (hence, the cash flow). The discount factor e^{st} performs exponential devaluation, depreciation or amortization on the cash flow over time.

I define $Q(s)$ as the as the Performance Equivalent (FE) for interest and performance measures. I define $P(s)$ as the present value (PV) for financial capital and cash.

Generalization of the Interpretation

Henceforth, I will use the term Output for $Q(s)$ to indicate either of the three possibilities from above. Output is the result of an economic process that uses inputs to produce a product or service that is available for sale [65]. Inputs are the commodities or services used by firms in their production processes; also called factors of production [65]. Finally, performance is a measure of issues dealing with the amount and value of money, wealth, debt, and investment. The economic prefix refers to the ex-ante property, like used for economic value.

I define $P(s)$ as economic value for consumer goods and services, labor and rentals, and as present value for cash flows. Economic value is the benefit that will be derived from a good or service and changes due to economic forces [73]. Similar to business value, economic value is discounted over time [20]. Henceforth, I will use the term value for $P(s)$ to indicate either of the two possibilities.

Transfer Function Mappings in Macroeconomics

Using these generalized definition for output $Q(s)$ and value $P(s)$ macroeconomic transfer functions can describe three different type of linear mappings [90]: The first is from $P(s)$ to $P(s)$ and from $Q(s)$ to $Q(s)$. The second is from $P(s)$ to $Q(s)$, and the third from $Q(s)$ to $P(s)$.

The first mapping, from output $Q(s)$ to output $Q(s)$, or value $P(s)$ to value $P(s)$, can be interpret as the economic efficiency, or the liquidity of exchange. Economist refers to economic efficiency as the relationship between aggregate benefits and costs to the individuals concerned [71]. A widely used criterion to define efficiency between different goods is Pareto Optimality [71]. Another example of efficiency in economics is productivity: the ratio of outputs to inputs [65]. A highly efficient transfer $H(s)$ mesured in %, indicates that the input-output relation has high value transfer in the case of $P(s)$, and high output transfer in the case of $Q(s)$.

The second mapping, from value $P(s)$ to product $Q(s)$, can be interpret as the liquidity. Economists classify assets as liquid if it can be converted into cash quickly for close to their current value [65]. Liquidity in macroeconomic is often linked to the function of money in the financial system and therefore the function of the Federal Reserve and monetary policy [42, 52, 65]. The field of electromechanics better explains this type of transfer than mechanics and calls it the impedance: The ratio of the complex representation of the sinusoidal voltage between its terminals, to the complex representation of the current flowing through it [18]. High liquidity or high value for $H(s)$ measured in $\#^2/\$$ means that output $Q(s)$ increases much upon an increase of value $P(s)$ which is in accordance to the definition in economics as stated above.

The third mapping, from output $Q(s)$ to value $P(s)$, can be interpret as the illequidity or inverse liquidity. In electromechanics, this is type of transfer is called the admittance. The admittance describes the current response to a voltage stimulus and is the inverse of the impedance [18]. High illequidity $H^{-1}(s)$ measured in $\$/\#^2$ is a situation in which a large increase of value results from an increase in the number of assets.

With these definitions, all signals of the time-domain macroeconomic model of Chapter 5 are given economic interpretation in the frequency-domain. Transfer functions give the input-output relation of the signals as a function of complex discount rates and can be used for frequency domain analysis of macroeconomic scenario's as is performed in Section 7-3.

6-5-2 Bode Plots for Frequency Response Visualization

In control theory, a Bode plot is graph of the frequency response of a transfer function [25]. A Bode plot consists of a magnitude plot and a phase plot. Both of the plots are a function of the frequency ω expressed in rad/s. The magnitude plot gives the gain or amplification in decibels (dB) of the input signal as a function of the frequency. The phase plot gives the phase shift of the output with respect to the input expressed in degrees as a function of the frequency [25].

I interpret the frequency ω as the trading frequency or economic cycle of the input of the system. When the purchasing of government bonds by the Federal Reserve (FED) is considered, ω corresponds to the trading frequency. When considering the flow of foreign demand for consumer goods, ω corresponds to the economic cycles of the rest of the world.

I interpret the magnitude $|H(s = i\omega)|$ of the frequency response of a transfer function $H(s)$ as the input-output effectiveness. In macroeconomics, the term effectiveness is used to indicate the effect of monetary or economic policy on variables like Gross Domestic Product (GDP), unemployment, and price levels [65]. The phase plots show the gain amplification between the input and output of the economic system. These two notions coincide.

I interpret the phase $\angle H(s = i\omega)$ of the frequency response of a transfer function $H(s)$ as the input-output coincidence. Economist use coincidence to indicate that the variable neither lags nor leads [95]. The phase plots show the phase lead or lag between the input and output of the economic system. The two notions coincide.

With these macroeconomic interpretations of the variables of Bode plots, macroeconomic scenario analysis can be performed in the frequency domain. In Section 7-3, I first demonstrate the similarities between Bode plots and the liquidity preference-money supply (LM) curve. Then I use Bode plots to perform macroeconomic scenario analysis. Additionally, I demonstrate that Bode plots are particularly useful to identify optimal trading frequencies and liquidity trap frequencies.

6-5-3 Final Value Theorem for Long-Run Steady-State

Similar to engineering systems, macroeconomist define the steady state as a condition in which key variables are not changing [52]. The steady-state value is of special interest in macroeconomic growth theory as it gives the long-run value of macroeconomic variables [65].

A particularly useful property of the Laplace transform in control is known as the final value theorem which allows us to compute the steady-state value a time domain model given its Laplace transform. Only if all poles of $sY(s)$ are in the left half of the $s-plane$, then [25]:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sH(s)U(s) \tag{6-33}$$

Here $Y(s)$ is the Laplace transform of the output of a system, $H(s)$ is the transfer function, and $U(s)$ is the Laplace transform of the input signal.

Macroeconomic theory suggest the ever present business cycles in economic systems [52, 65]. I view an economy as a stable underdamped system that dies out and reaches a steady-state over time. The business-cycles are caused by the constant re-excitation of the system of its participants. The pole zero maps of OF's models also verify this (see Appendix D). This makes the final value theorem a new and exceptionally useful tool for macroeconomic systems.

In Section 7-5, I apply the final value theorem to one of the outputs of the time domain model from Chapter 5 to compute the long-run steady state value due to a certain shock. Additionally, I give the macroeconomic interpretation of poles and zeros and their influence to the different economic runs described in Section 7-4.

6-6 Conclusion and Contributions

It is shown that equity returns and government bond value are inherently frequency domain concepts. Contrary to the modelling strategy of the previous chapter, the Laplace transform models variables as ex-ante or forward looking quantities which is in-line with the approach of current practices.

The first contribution of this chapter is the application of the Laplace transform for valuation of equity, government bonds and even labor using the signals of the bond graph model of Chapter 5.

The second contribution of this chapter is the formulation of key analogues between frequency domain concepts and tools and macroeconomics. By formulating these, frequency domain analysis tools such as Bode plots can be used to perform macroeconomic scenario modelling using engineering formalism. Because this thesis aims to perform scenario analysis in an economically interpretable way, this is an important result. In the next chapter, the findings of this chapter are used to perform scenario analysis in the frequency domain.

Contributions:

1. Given an economic interpretation to frequency domain concepts and tools so that scenario analysis can be performed in the frequency domain.
2. The use of frequency domain tools to compute total equity returns and government bond yields for bonds with a specific maturity.

Part 4

Chapter 7

Time and Frequency Domain Analysis

7-1 Introduction

The application of an engineering grey-box modelling approach presents the opportunity to use existing systems and control engineering tools to perform system analysis in the frequency and time domain. In this chapter, I use these tools to perform macroeconomic system and scenario analysis of the model developed in Section 5. The model has not been identified with economic data. I have assigned the values for the parameters manually to qualitatively validate the model and be able to use it for analysis. I define the goal of this chapter as:

Chapter goals:

1. Validate the model by comparing unit-step responses to economic theory.
2. Give Systems and Control (S&C) tools an economic interpretation and use them for macroeconomic system and scenario analysis

In Section 7-2, I qualitatively validate the aggregated total model and one of the subsystems by comparing the unit-step input responses of different inputs to (macro)economic theory. In Section 7-3, I use Bode plots as an alternative to IS-LM models and use them to perform macroeconomic scenario analysis in the frequency domain. In Section 7-4, I analyze pole-zero maps of the model to determine the cycles, trends and stability of the system. In Section 7-5, use the interpretations of this chapter to perform open-loop scenario analysis in the time-domain. In Section 7-6, I use concepts defined in this chapter to perform a closed-loop macroeconomic scenario analysis using the root-locus method. Finally, I state the conclusions and contributions in Section 7-7.

7-2 Step Inputs for Model Validation Using Scenarios

In this section, I perform a qualitative validation of the time domain model by analyzing unit-step transient behavior of different economic scenarios. I compare the transient responses to how economic theory describes the behavior of comparable scenario's. The validation is done using unit-step input responses instead of shocks/unit-impulse responses. The step responses result in new steady-state values which are more straightforward to analyze.

I first do this for the total system -the total economy- and subsequently for one of the subsystems -the consumer goods and services market. The parameters have been assigned values manually. They are included in Appendix F.

The time-domain model has a total of 21 states and consists of 8 subsystems. To keep the analysis compact, I will only show the scenario responses of the system as a whole for the following variables: Gross Domestic Product (GDP), consumer price level, nominal wage level, rental price level, unemployed, Free Cash Flow to Firm (FCFF), the interest and current government bond yield. The reason is that macroeconomic theory is clear on the responses these variables in specific scenario's [65, 95].

7-2-1 Consumer Goods and Services Demand Flow Step Input Response

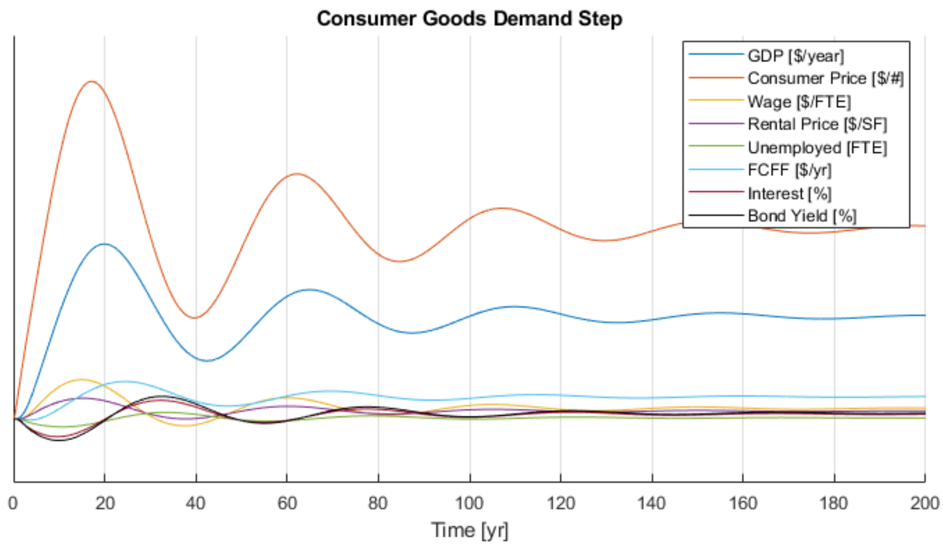


Figure 7-1: The time domain step response of the total model shows that increased demand for consumer goods and services lead to a economic upswing. The overshoot indicates that the economic system cant directly absorb the increased demand which causes extra costs or benefits.

Figure 7-1 shows a scenario where the demand flow for consumer goods & services instantly increases to a new constant level by for example an increased consumer demand from abroad. The transient response is a unit-step input response of the source flow Sf_2 at $t = 0s$.

The demand step causes an economic upswing resulting in increased GDP, inflation and subsequently, an increase in all forms of income. This is in line with macroeconomic theory described by [52] which states that: "*Over time, the high level of aggregate demand pulls up* *wages and prices. As the price level rises, the quantity of output demanded declines, and the economy gradually approaches the natural level of production. But during the transition to the higher price level, the economy's output is higher than its natural level".* This is exactly what can be observed from the figure. Additionally, the interest rates and hence the interest decrease because firms initially require less investment, however at higher levels of economic growth, the demand for investment increase and push the interest levels back to a higher steady-state value.

7-2-2 Consumer Goods and Services Supply Flow Step Input Response

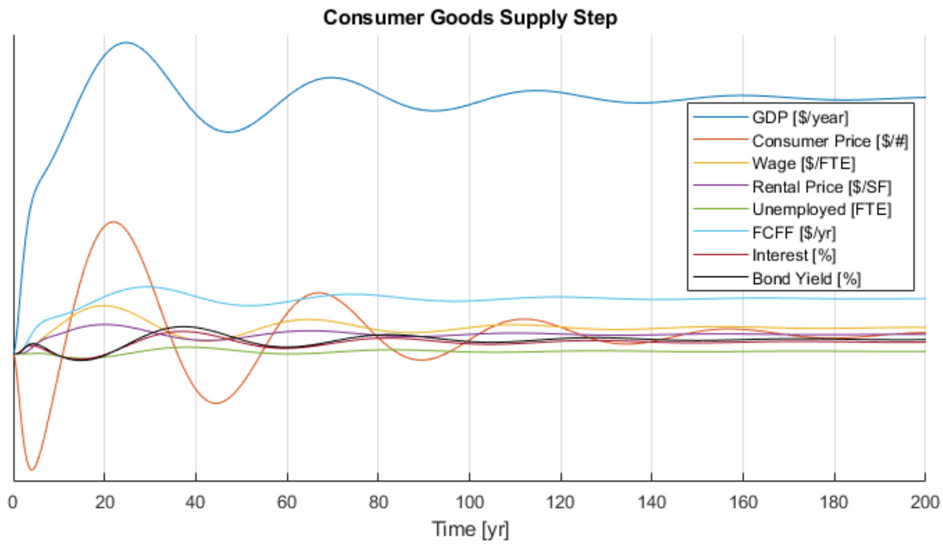


Figure 7-2: The time domain step response of the total model shows that increased supply of consumer goods and services lead to a economic upswing.

Figure 7-2 shows a scenario of a production flow step input of consumer goods & services. I call this production flow input the supply input. At $t = 0$, the supply input of consumer goods is increased to a new constant value measured in #/yr. This means that the supplier decides to supply an increased constant flow of goods. The scenario is a more theoretical one in macroeconomics, but they do occur in the oil market [65]. The transient response shows a unit step input response of the supply input Sf_3 at $t = 0s$.

The consumer price level displays non-minimum phase behavior. The response can be viewed as a price cutting strategy on macro level: to increase final sales, the prices initially must be cut. The momentum created by the increase of demand due to lower prices causes an economic upswing. Again, this is in line macroeconomic theory which state that a beneficial supply shock leads to increase of nominal GDP and (initial) decrease of inflation [65].

7-2-3 Saving Supply Flow Step Input Response

Figure 7-3 shows a scenario where the supply of the flow of saving instantly increases to a new level expressed in units $/yr. The transient response shows a unit step response of the supply of saving input Se_3 at $t = 0s$.

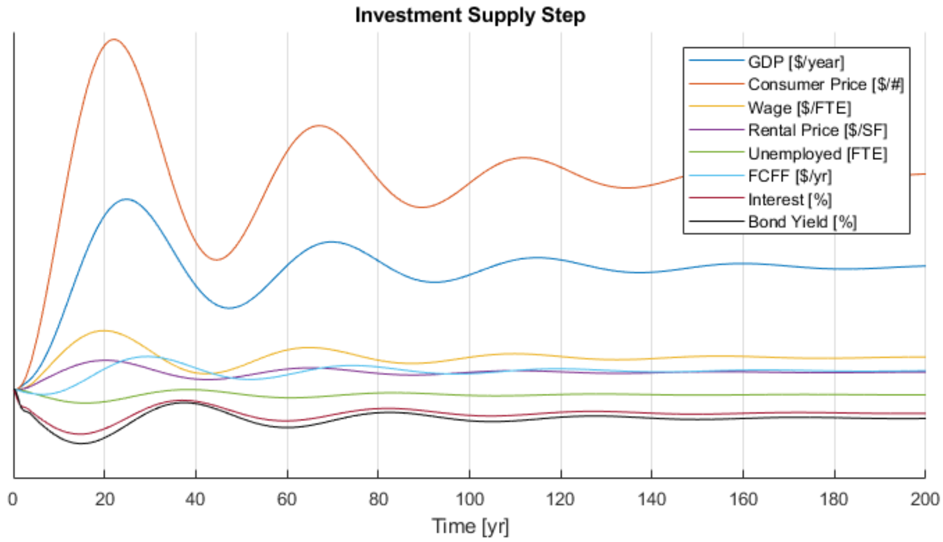


Figure 7-3: The time domain step response of the total model shows that increased supply of saving cause an economic upswing. The overshoot indicates that the economic system cant directly absorb the increased demand which causes extra costs or benefits.

The increase of saving supply leads to lower interest rates which in turn lead to a greater amount of investment. GDP relies directly on the amount of investment (Eq. (3-1)). The scenario response is comparable to the Federal Reserve's monetary policy. When the Federal Reserve loosens monetary policy, the money supply is said to be increased [65]. An increase the money supply leads to lower interest rates which in turn lead to more investment and subsequently economic growth [52]. The scenario is comparable, however the supply of investment could, for example, also be caused by increased government investment or increased foreign investment. Additionally, this is in line with the findings of the ECB for U.S. economic shocks [72].

7-2-4 FED Open Market Operations Effort Step Input Response

Figure 7-4 shows a scenario where the the Federal Reserve tightens monetary policy by selling extra government bonds while holder other things constant. At $t = 0$ the Federal Reserve decides to sell an increased constant flow of bonds measured in $/yr. The transient response shows a unit step response of the supply of government bond input Se_1 at $t = 0s$.

A decrease of the money supply (monetary tightening) leads to decreased consumer spending, hence lower inflation and reduced economic growth [52]. The transient response shows exactly this. Additionally, the response is in line with the findings of the model used by the European Central Bank [72] tot analyze shocks of the United States.

With these insights I conclude that the transient responses for the total model behave in accordance with macroeconomic theory.

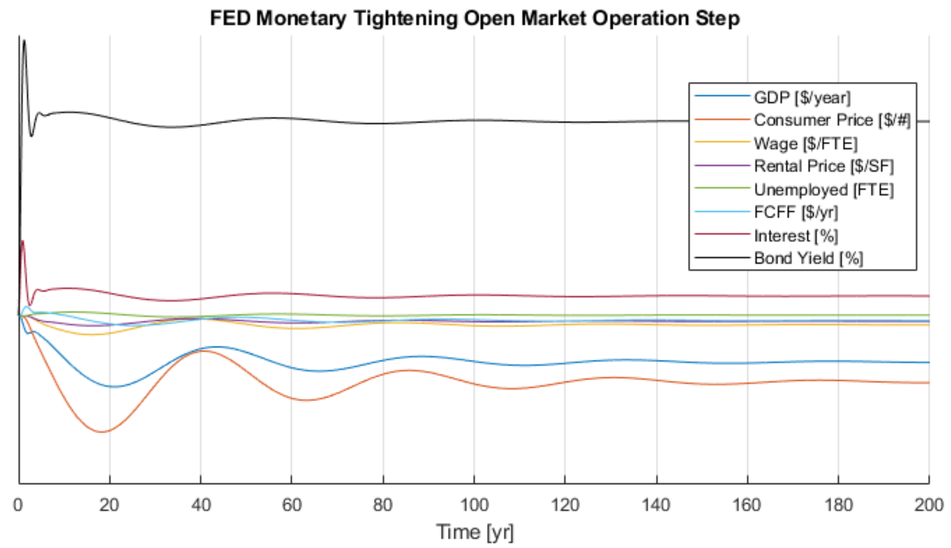


Figure 7-4: The time domain step response of the total model shows that increased monetary tightening by open-market operations causes the economy to slow down

7-2-5 Step Inputs for Validation of Consumer Goods and Services Market

I use the model of the consumer goods and services market to qualitatively validate one of the subsystems by comparing it to economic theory. This way, not only the total system is validated, but also the subsystems.

The subsystems dynamics are based on both macroeconomic theory as microeconomic theory. I validate the unit-step responses by comparing them with economic theory.

Figure 7-5 shows a demand unit-step response. At time $t = 4$, the demand for consumer goods and services is increased to a new constant flow measured in #/yr. I named this demand input the consumption.

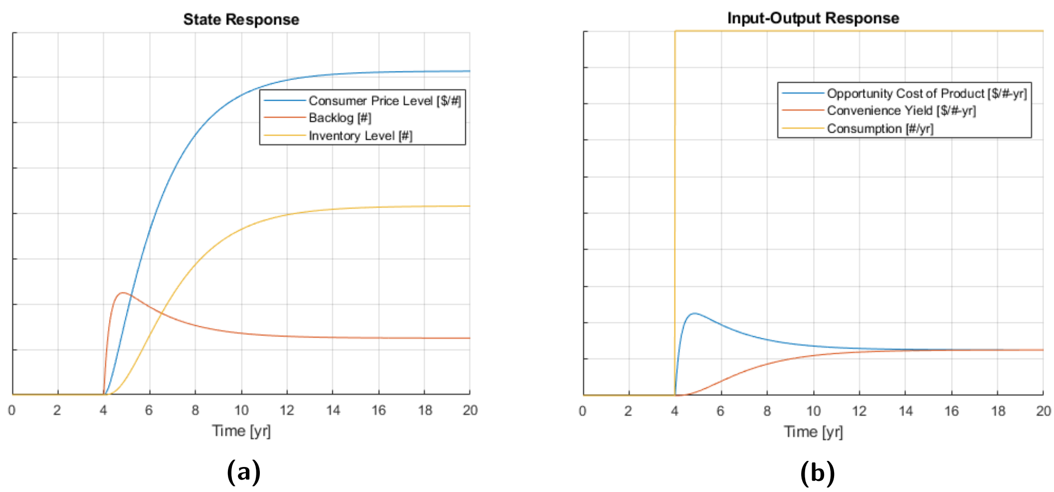


Figure 7-5: Unit-Step Consumption Flow Response of the Consumer Goods and Services Market

An increase in the demand flow for consumer goods causes an increase of the backlog [19]. The backlog is a measure of scarcity. A good is scarce if the choice of one alternative requires that another be given up [61]. An increase in scarcity lead to an increases of the opportunity cost of these goods [61]. The opportunity cost of the product are the potential benefits the customer misses out on when choosing the firm's product [92]. The increase in opportunity cost due to the increase in demand results in an increase of the consumer price level. The market prices measure the opportunity cost of those factors [92]. The fact that an increase in demand leads to an increase in the price level is in agreement with the law of demand [92]. The increase of opportunity cost result in an increase in price and an increase of the goods supplied. This corresponds to a shift along the supply curve [92]. The increase of the goods supplied lead to an increase of the inventory levels i.e. a firm is expecting to sell more and therefore requires an increase of the expected inventory level. The increased inventory levels correspond to an increase of the convenience yield [66]. The convenience yield is the opportunity cost of holding inventory [75]. The consumer price level reaches an equilibrium when the opportunity cost of the goods and the convenience yield are balanced.

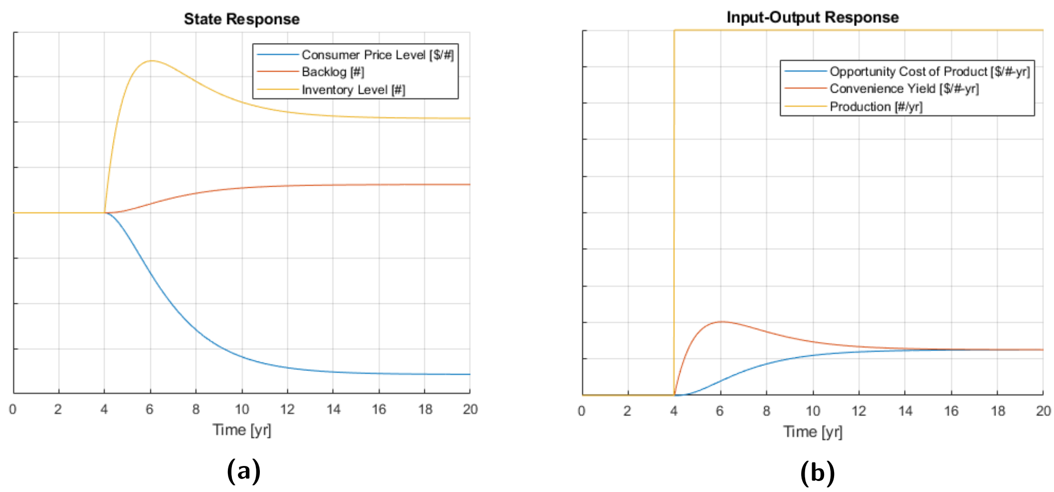


Figure 7-6: Unit-Step Production Flow Response of the Consumer Goods and Services Market

Figure 7-6 shows the unit-step production flow response of the consumer goods and services market. It is similar to the response of Figure 7-5, but the input is now acting on the opposite side of the market. The increase of the flow of goods produced leads to an increase of the inventory levels and therefore the convenience yield/cost. The increased convenience yield acts as an increased force of supply [74] on the market of consumer goods and services. This causes a decrease of the price level, which is in agreement with the law of supply [92]. The decrease of the consumer price results in an increase of the opportunity cost of the products leading and an increase of the backlog.

With these insights, I conclude that the input-output and state response of the consumer goods and services market are in line with economic literature.

7-3 The Use of Bode Plots for Economic Scenario Analysis in the Frequency Domain

In macroeconomics, the IS-LM model is a two-dimensional macroeconomic tool that shows the relationship between interest rates or price levels and aggregate output or GDP. IS stands for investment and saving and LM for the liquidity preference-money supply. The theory is still used as an educative tool for aggregate macroeconomic analysis, but its practical use has fallen out of favor due to its lack of precision [1, 39, 52]. Here, I use Bode plots as an alternative method to model the relations between inputs and outputs of the aggregated macroeconomic system, but now as a function of frequency.

I will first consider the Bode plot of the transfer function mapping the input Se_1, the open-market operations (OMO) of the Federal Reserve (FED), to the output GDP. The transfer function represents the economic efficiency as it maps a quantity of value to another quantity of value. Figure 7-7 shows this Bode plot.

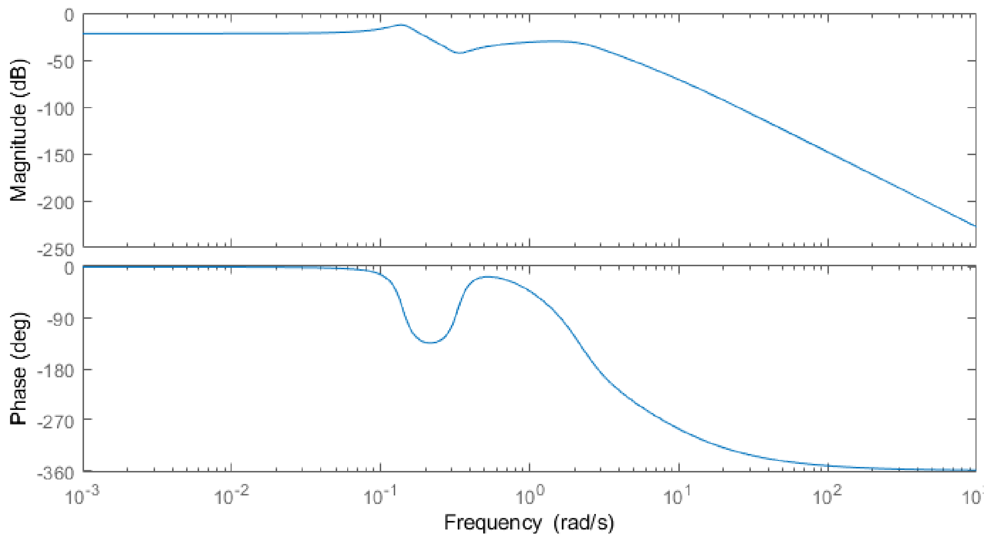


Figure 7-7: Bode plot transfer function between the input open market operation Se_1 and output GDP. The plot shows a resonance peak at around 1.4 rad/s of which the magnitude is still below 0 dB. At this frequency the effectiveness of open market operation on GDP is the highest.

The magnitude plot of Figure 7-7 displays the effectiveness of OMOs on GDP. The plot displays a resonance peak at a frequency $\omega \approx 0.14$ rad/s. This peak corresponds to the presence of a complex pole pair [26] (see Section 7-4). Since the peak causes the effectiveness of the FED OMO's to become exceptionally high, I interpret this to be the optimal trading cycle. I give a general economic interpretation of resonance peaks:

Resonance Peak

The resonance peaks shows the optimal trading cycles

At a frequency $\omega \approx 0.33$ rad/s the opposite happens. Here the figure shows an anti-resonance peak, corresponding to the presence of complex zero pair [26] (see Section 7-4). I interpret the anti-resonance peak as a liquidity trap. A liquidity trap is a situation in which the nominal interest rate has fallen to its lower bound of zero, calling into question the efficacy of monetary policy to further stimulate the economy [52]. I give a general economic interpretation of anti-resonance peaks:

Anti-Resonance Peak

Anti-resonance peaks show the liquidity trap cycles

The magnitude plot shows a roll-off at a frequency $\omega \approx 2$ rad/s, also known as the corner frequency in control theory [26]. The roll-off shows that trading at higher frequencies will decrease the effectiveness of monetary policy further. I interpret the corner frequency to be the liquidity risk frequency. Higher frequency will only further decrease the effectiveness of monetary policy on GDP [65]. I give a general macroeconomic interpretation of the corner frequency:

Corner Frequency

The liquidity risk frequency

Finally, the magnitude plot shows that for this model, OMOs are not a very effective tool to increase GDP as the magnitude remains low over the full frequency range of the Bode plot.

The phase plot shows the coincidence of OMOs and GDP. This is important information for the Federal Reserve to anticipate when the OMOs' effects will take place. At lower frequencies, the two coincide, i.e. there is no phase lag or lead. At very high frequencies the effects of the OMO's and GDP again coincide (360 degrees lag or lead is the same as 0). The optimal trading frequency comes with a drawback, a phase lag of a quarter cycle.

The second Bode plot I consider, relates the five input variables to GDP. I will use the following transfer functions to define them:

1. $H_1(s)$ is the transfer function between the flow of supply for consumer goods, Sf_1 #/yr, and GDP.
2. $H_2(s)$ is the transfer function between the flow of demand of consumer goods, Sf_2 in #/yr, and GDP.
3. $H_3(s)$ is the transfer function between the accommodative monetary policy of OMOs, Se_1 in \$/yr, and GDP.
4. $H_4(s)$ is the transfer function between the investment demand Se_2 in \$/yr, and GDP.
5. $H_5(s)$ is the transfer function between the saving supply Se_3 in \$/yr, and GDP.

The Laplace of the unit-impulse is one [76]. Therefore the Bode plot of the above transfer functions directly corresponds to unit-impulse responses of the system in the frequency domain. Figure 7-8 shows the Bode plots of the five transfer functions. The transfer functions are a measure of liquidity, mapping an economic quantity to value, and efficiency, mapping value to value.

The Bode plot shows the frequency responses relating each of the five defined inputs of the total aggregated open economic system to GDP. The effectiveness and coincidence can be analyzed for each of the economic scenario's corresponding to the different cycles/frequencies

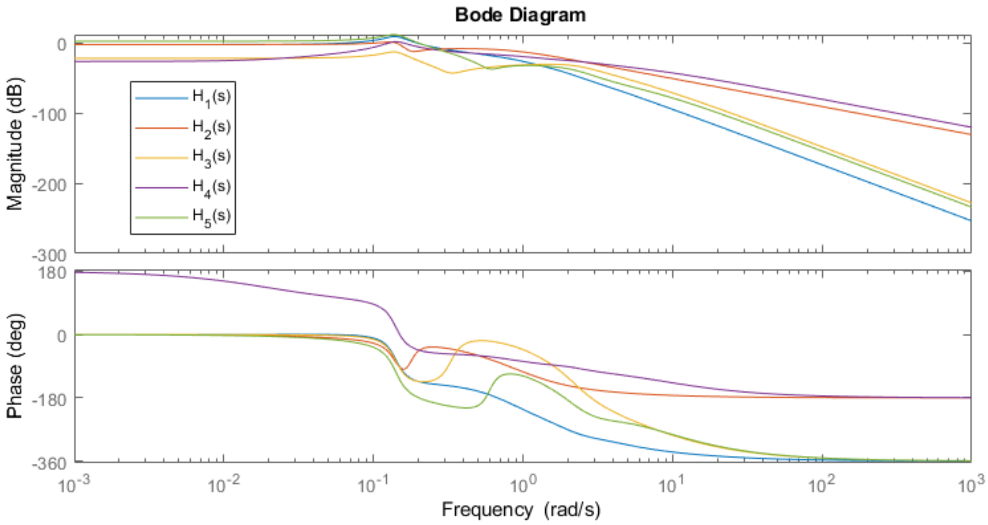


Figure 7-8: Bode plot of the five transfer functions relating each of the five modelled inputs to output GDP. Each transfer function displays a range of scenario's for each input frequency.

of the inputs. It can for example be concluded that for each of the inputs, the optimal trading cycle is approximately 0.14 rad/s. However, some attain a lag or lead at this trading cycle.

Besides the analysis of the phase and gain of transfer functions, Bode plots also allow for visual inspection of the phase margin (PM) and the gain margin (GM) [26]. The PM and GM are the stability criteria indicating the amount of gain and phase, respectively, that can be added to the loop before the closed-loop system becomes unstable [28]. This would allow for analysis of the robustness of closed loop stability behavior. I will not further treat this in this thesis, but in Section 7-6 I will use the root-locus method to perform closed loop macroeconomic stability and scenario analysis.

7-4 Poles and Zeros for Cycles, Trends and Stability

Pole-zero maps offer a visual tool to analyze the stability and the damping of the transient response of a system [25]. Because I aim to perform macroeconomic scenario analysis using engineering tools, I will give poles and zeros an economic interpretation. This way, the stability and damping of the transient response of macroeconomic scenario's can be determined.

The poles and zeros of a system correspond to the roots of the numerator and denominator, respectively, of the transfer function of a system [25].

Figure 7-9 displays the pole-zero maps of two of the eight subsystem. The pole-zero maps of the other six subsystems are included in Appendix G. Figure 7-10 displays the pole-zero map of the aggregated total system, the United States economy.

All poles are located in the left-half of the s-plane, which ensures stability. This is in line with macroeconomic theory which defines markets to be stable in general [52].

I interpret the real-valued poles in the left-hand of the s-plane as some economic factor that causes depreciation or amortization of the macroeconomic system. In engineering, poles in

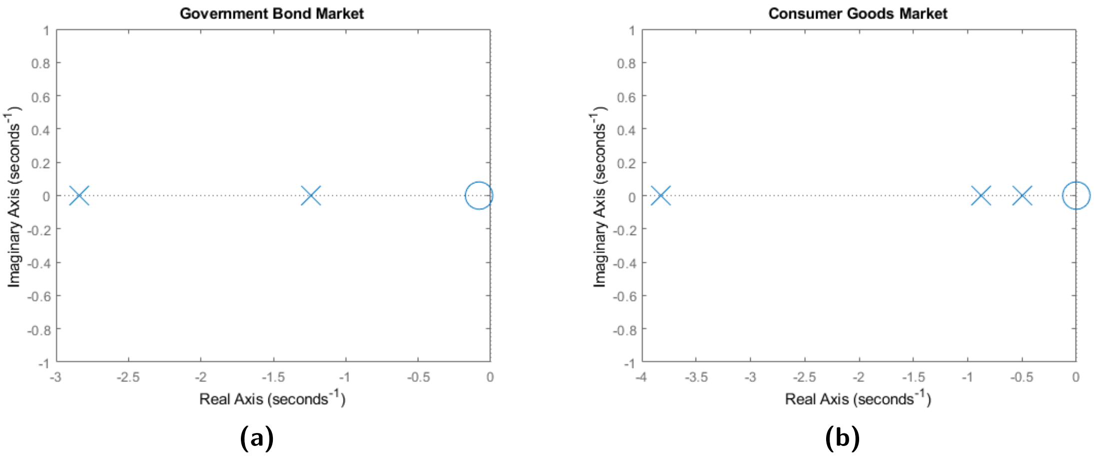


Figure 7-9: The pole-zero maps of the equity market and the consumer goods and services market. The circles represent the location of the zeros, the crosses the location of the poles. The imaginary part of the poles correspond to the economic cycles. The real part of the poles correspond to economic factors. All poles are in the left half s-plane. The two subsystems are stable.

the left half of the s-plane correspond to a exponential decay [25]. In (macro)economics, depreciation is often modelled using exponential decaying functions [30].

I interpret the real-valued poles in the right-half s-plane as an economic factor causing unstable growth. In engineering, poles in the right half of the s-plane lead to exponential growth. In macroeconomics, the population growth is an example of a exponentially growing entity, only bound by the limited food supply [65]. I give a general economic interpretation of the real-valued part of poles:

Real-valued part of Poles

The real-valued part of the poles indicate the presence of economic factors causing either economic dissipation or unstable growth. Poles in the left-hand plane correspond to depreciation, amortization or depletion, poles in the right-hand plane indicate appreciation.

The imaginary part of poles and zeros define the frequency of the response [25]. The importance of business cycles and their frequency interpretation was explained in Section 2-2-3. I give the following definition of a general economic interpretation to the imaginary part of poles and zeros:

Imaginary Part of Poles

The imaginary part of poles indicate the presence of economic cycles

In engineering, the zeros correspond to the signal transmission-blocking properties of the system and are also called the transmission zeros of the system [25]. Transmission zeros are associated with modes of behavior wherein the input and states of a system are nonzero, yet the output equals zero. I interpret a zero as a transmission blocking economic factors. Examples include laws and policies that block certain activities in an economy. In macroeconomics, the term transmission is used for describing the effect of interest rate policies [8]. I give a general economic interpretation of the zero's of a system:

Zeros

Zeros indicate the presence some transmission blocking economic factor or cycle.

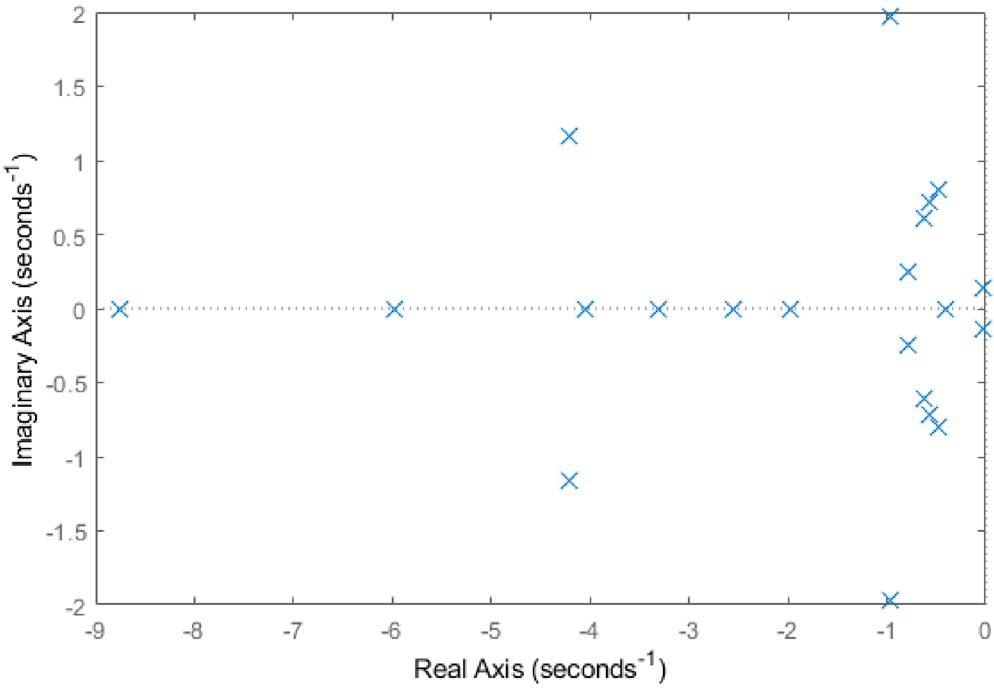


Figure 7-10: The pole-zero map of the aggregated total open economy. The circles represent the location of the zeros, the crosses the location of the poles. All poles and zeros are in the left half of the s-plane, the system is stable.

The economic interpretation of poles and zero's allow for a straightforward evaluation of economic models. By inspecting the pole zero maps, the present cycles and factors of an economic system can be obtained.

7-5 Open-Loop Macroeconomic Scenario Analysis using Engineering Tools

With the formulated economic analogues of previous sections, it is possible to perform macroeconomic transient response analysis using engineering tools and concepts. Here, I first analyze a transfer function from the model, and subsequently create a scenario.

For this analysis, I take a single-input single-output transfer function $H_6(s)$ which maps Sf_1 to the Free cash flow to the firms (FCFF). This transfer function describes illiquidity as it maps a total output to value. The pole-zero map of this transfer function is given in Figure 7-11.

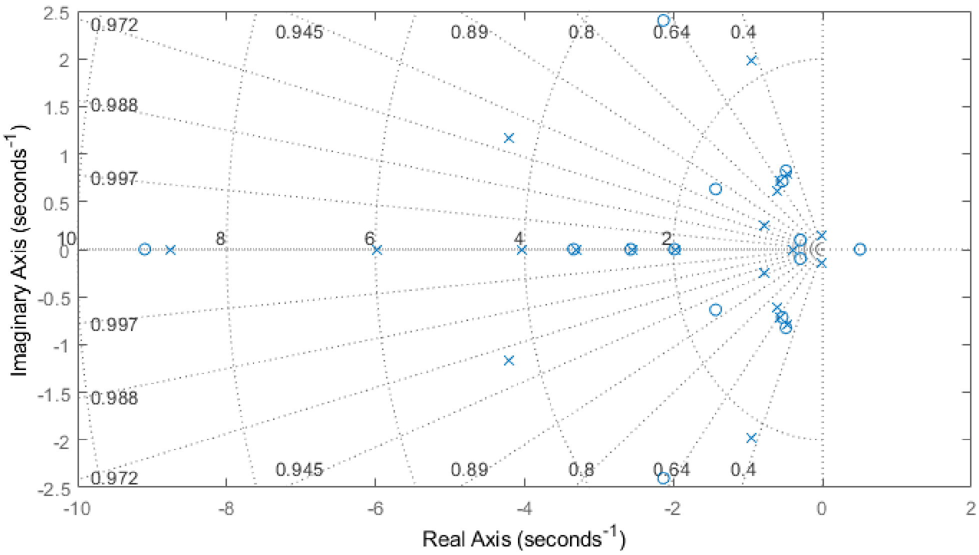


Figure 7-11: The pole-zero map of transfer function between Sf_1, consumer goods supply, and FCFF. The system is stable and has a zero in the right-half plane causing non-minimum phase behavior. The grid is added to observe the natural frequency and the damping ratio

From the map, it can be observed that the system is non-minimum phase because of the right half plane zero. This means that the transient response initially moves in opposite direction [40]. An economic interpretation is that by increasing production, the consumer price level initially decreases due to which less profits are made. Then due to the lower prices, more goods are sold and profits eventually increase. This phenomenon is similar to the physical example of an airplane pitching the nose down to generate lift.

Additionally, the system has complex zeros and poles with a low damping ratio. The system response is underdamped behavior causing the output to oscillate.

Since the system is underdamped, the long-run steady state value can be computed even before the transient response is shown (see Section 6-5-3). Using the Matlab command `dcgain` gives the long run steady state value of 0.1731 \$/yr.

All of the above mentioned is verified by the transient response shown in Figure 7-12.

Based on the notion of the optimal frequency from Section 7-3, I create a scenario where the production of consumer goods of the firms (accidentally) reach this frequency. Since here a transfer of illiquidity is considered, this frequency corresponds to the non-optimal trading frequency or a liquidity trap. From the Bode plot in Figure 7-13, the liquidity trap frequency $\omega_{lt} \approx 0.14$ rad/s. Input $u = 1+\sin(0.14*t)$ results in the time domain response of Figure 7-14.

Figure 7-14 shows that if consumer goods are supplied at the liquidity trap frequency, the effectiveness or amplification is high compared to that of Figure 7-12. This response can be disadvantageous to the firms because their profits or free cash flows oscillate a lot throughout time. A liquidity trap hence leads to big oscillations in profits. This is in accordance with

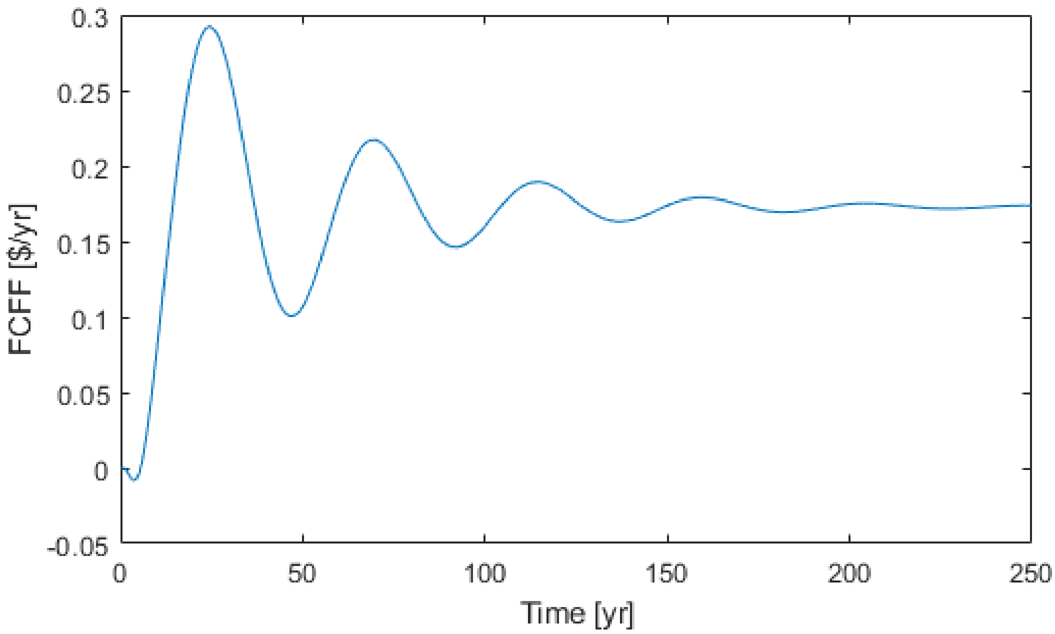


Figure 7-12: Transient response of FCFF by applying a unit-step to supply of consumer goods, Sf_1. The response is non-minimum phase and oscillatory. The response has an overshoot.

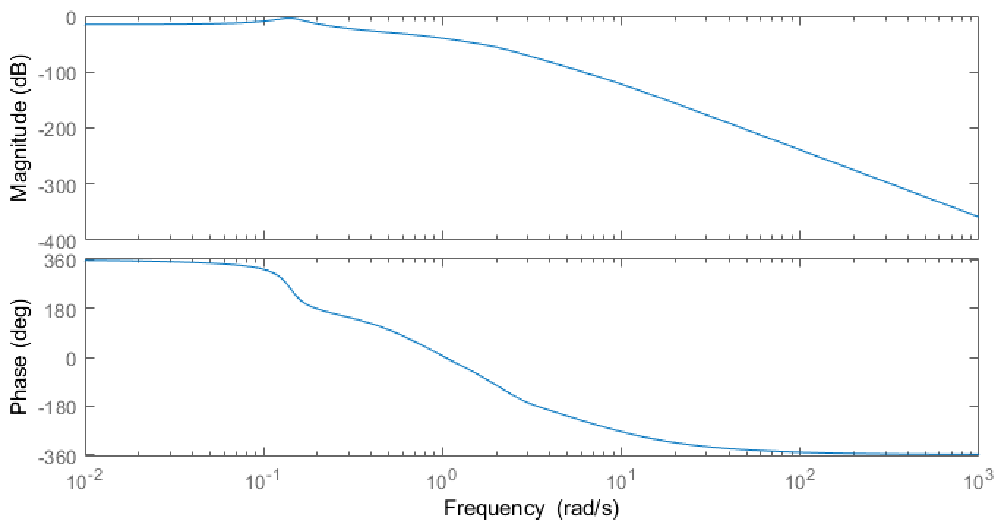


Figure 7-13: Bode plot of from Sf_1 to FCFF. The plot shows an optimal trading frequency at 0.14 rad/s.

economic theory which states that low liquidity is a situation where large changes in prices and therefore profits are caused by relatively small changes in traded assets [2, 87].

The two components by which the poles and zeros, and hence the behavior, of a system are described are the damping ratio and natural frequency [25]. They therefore form the basis

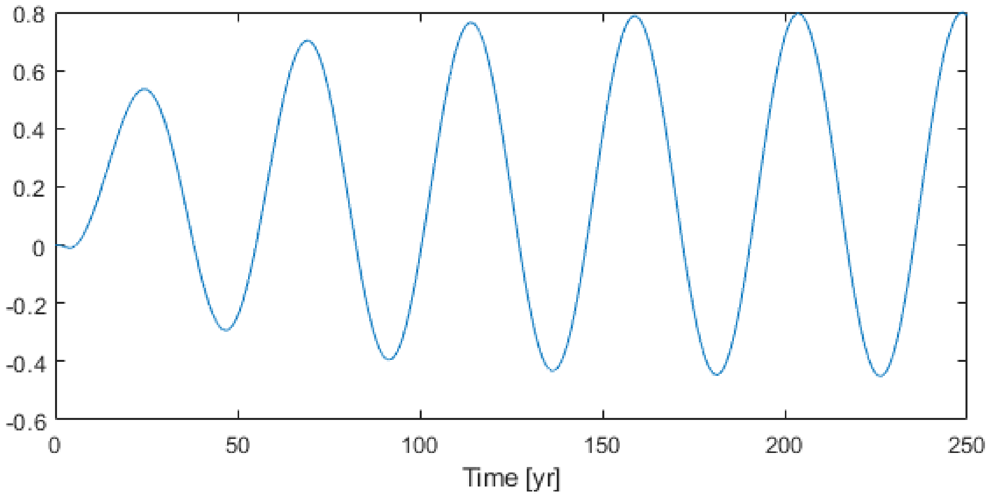


Figure 7-14: Transient response of $H_6(s)$ trading at the optimal trading frequency of 0.14 rad/s

of transient response analysis. By giving them an economic interpretation, economic systems may be analyzed in the same way physical systems are analyzed.

I interpret the natural frequency, ω_n, as the real business cycle of an economic system measured in 1/yr. Economist define a real business cycle as an economic cycle that can be explained by real changes in the economy (such as changes in technology) and without the role of nominal variables (such as the money supply) [52]. I interpret this explanation of real business cycles to relate to the change of parameters of the system instead of external inputs affecting the systems behavior. I give a general macroeconomic interpretation of the natural frequency:

Natural Frequency

The natural frequency is the real business cycle of an economy

I interpret the damping ratio as the rate of depreciation or appreciation per real business cycle. This can be motivated with by the expression for the damping ratio:

$$\zeta = \frac{\sigma}{\omega_n} \tag{7-1}$$

Where σ is the real-valued part of the pole analogues to some economic factor causing exponential depreciation or appreciation (see Section 7-4) depending on whether the system is stable or not. An overdamped economic system is therefore a system where the rate of depreciation is larger than the real business cycle frequency and therefore shows no oscillatory behavior and takes a longer time to reach the long-run steady state compared to a critically damped system. A critically damped system is one where the exponential depreciation rate is equal to the frequency of the business cycle and an underdamped system is one where the depreciation rate is smaller than the real business cycle frequency and therefore shows an oscillatory response and overshoot. I give a general macroeconomic interpretation of the damping ratio:

Damping Ratio

The damping ratio of a system is the depreciation ratio describing the depreciation rate per business cycle.

With these insights, I conclude that transient response analysis can be viewed as a real business cycle and depreciation ratio analysis. This is another result of using a grey-box model where each state and each element has an economic interpretation.

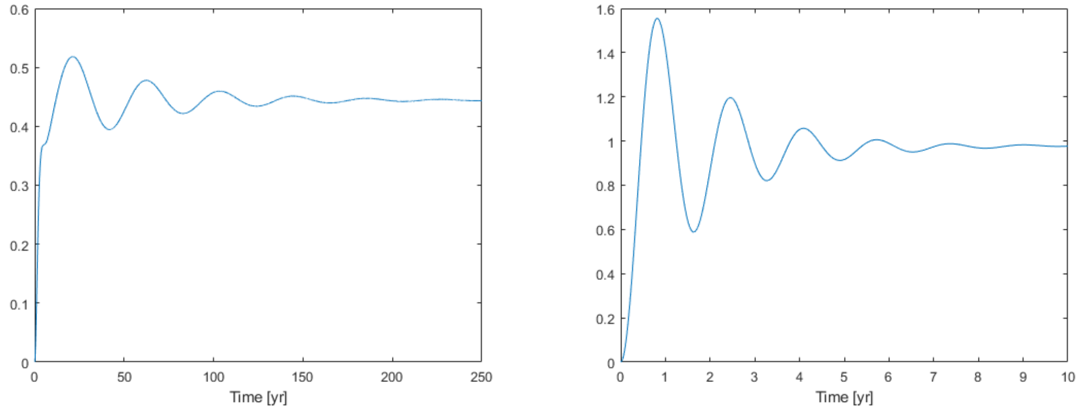
In Section 7-6, I combine the concepts of this section and of Section 7-4 to design a closed-loop scenario using the root-locus method.

7-6 Root-Locus Methods for Closed-Loop Fiscal and Monetary Policy Analysis

In this section, I apply the root-locus method to analyze closed-loop scenarios. The two scenario's considered are: 1) the increase of fiscal policy by increasing the government spending and 2) expanding monetary policy by increasing the purchasing of government bonds by the Federal Reserve.

The root locus gives the location of the roots of the characteristic equation of the closed-loop system in the complex plane as a parameter (here the loop gain) changes between zero and infinity [40]. The main goals of monetary and fiscal policy are to increase the growth of GDP, increase employment while stabilizing prices [65]. By using the root-locus method, it can be determined if and when the system becomes unstable by varying the gain of the closed-loop system from either of the two inputs from $0 \rightarrow \infty$. Besides the stability, also the dynamic properties can be determined approximately from the location of the poles and zeros.

I use two different transfer functions. The first maps Sf_2 to GDP, H_2 from Section 7-3. This transfer functions resembles the illiquidity relation between fiscal policy and GDP. The second transfer function maps Se_1 to GDP, H_3 from Section 7-3. This transfer function resembles the efficiency of monetary policy with respect to GDP.



(a) **(b)**

Figure 7-15: Closed-loop unit-step response of transfer function H_2 with a static gain of 1 (7-15a) and a static gain of 48.7 (7-15a)

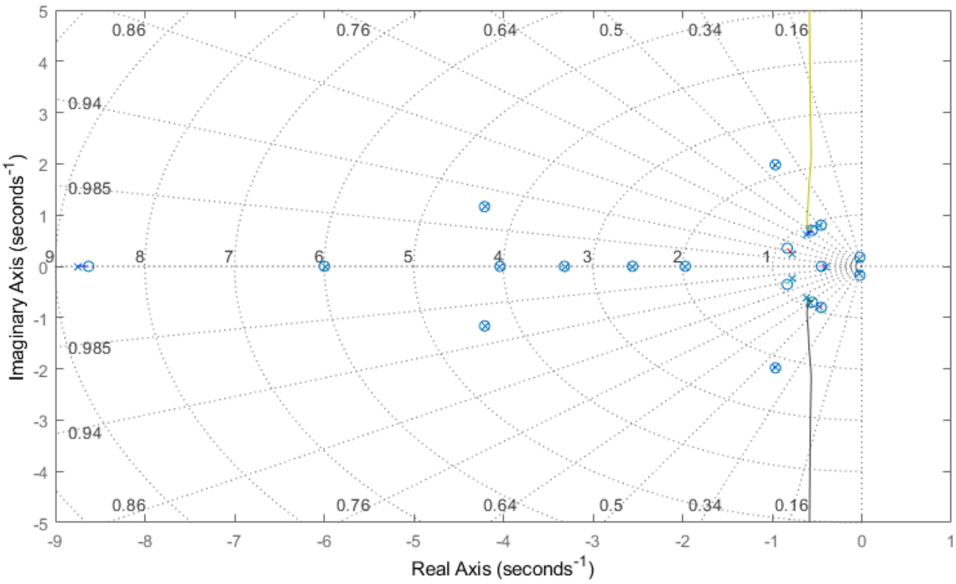
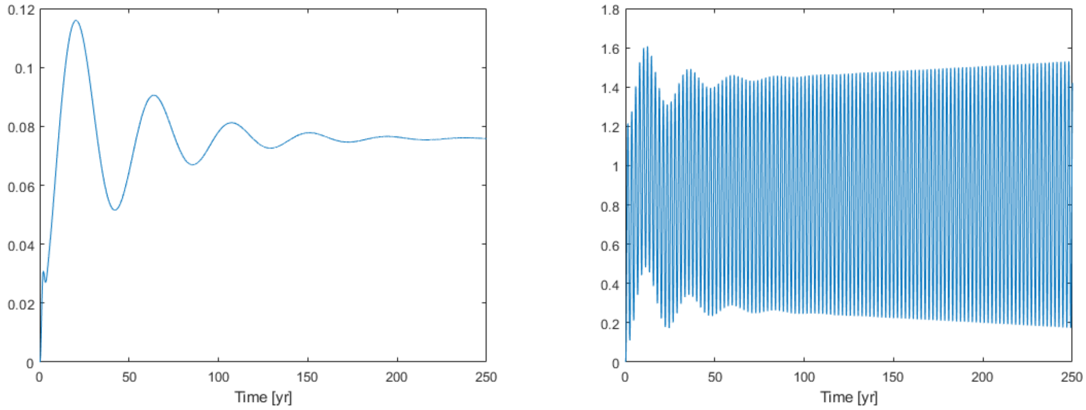


Figure 7-16: The root-locus plot of the transfer function mapping government consumption to GDP. The root-locus resembles a situation where the government keeps increasing fiscal policy in a closed-loop control setting. The system does not become unstable.



(a) **(b)**

Figure 7-17: Closed-loop unit-step response of transfer function H_3 with a static gain of 1 (7-17a) and a static gain of 70.5 (7-17b). Too much monetary policy destabilizes the economy

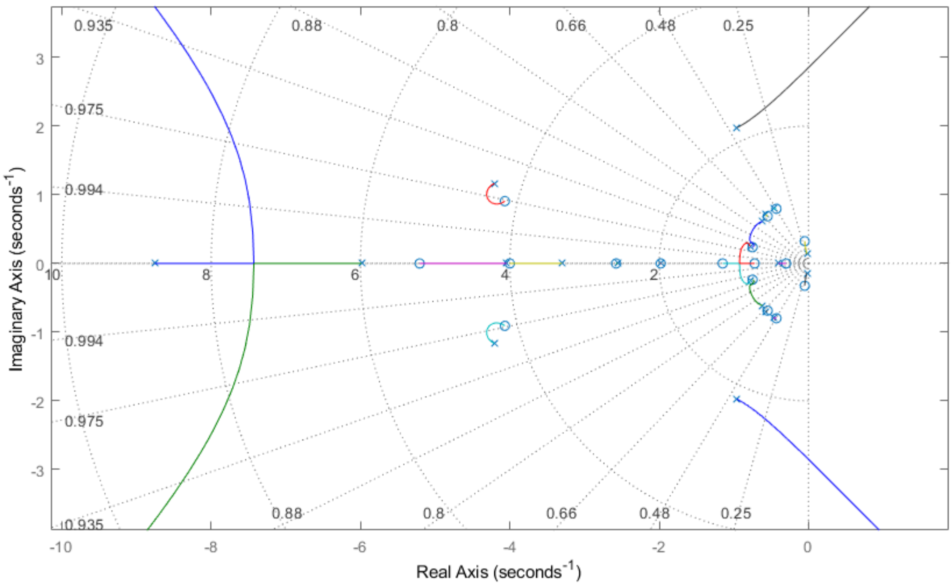


Figure 7-18: The root-locus plot of the transfer function mapping purchasing of government bonds by the Federal Reserve to GDP. The root-locus resembles a situation where the Federal Reserve keeps expanding monetary policy in a closed-loop control setting. It can be seen that monetary policy can destabilize the system

Figure 7-16 shows the root-locus plot of the transfer function H_2 describing the effect of the government's fiscal policy on GDP. The root locus plot shows that the open loop poles move towards the open loop zeros. The system remains stable, even for high gains. Increasing the static controller gain increases the effect of fiscal policy, but also causes additional (unwanted) cyclical behavior. The unit-step transient responses of Figure 7-15 demonstrate this. Figure 7-15 also shows that the steady state error between the input and output decreases and the rise time significantly decreases corresponding the effect of the proportional action [91]. The economic interpretation is a government that significantly increases their spending behavior by which GDP is steered into their reference goal more aggressively.

Figure 7-18 shows the root-locus plot of the transfer function H_3 describing the effect of the Federal Reserve's monetary expansion policy on GDP. The root-locus plot shows that the open-loop poles move towards the open-loop zeros. Since the system has a total of twenty one poles, I will only discuss the movement of one complex pole pair that causes the system to become unstable. This concerns the complex pole pair on the right side of the figure moving to the right-half of the s plane.

For the pole pair on the right the real business cycle frequency and the depreciation ratio initially increase until the closed-loop system becomes unstable. From this it is concluded that monetary policy can destabilize an economy. The closed-loop unit-step transient responses of these two scenario's are shown in Figure 7-17. This result is in line with economic literature which acknowledges the possibility of monetary policy destabilizing an economy [52, 65].

7-7 Conclusions and Contributions

The existence of a macroeconomic grey-box model allows the use of powerful S&C tools for macroeconomic scenario analysis. Because the grey-box model consists of signals, parameters and variables with an economic interpretation, a whole new set of possibilities exist to perform macroeconomic scenario analysis in an economically interpretable way.

This chapter has demonstrated the usefulness of just a few of many existing analysis tools of S&C theory new to the field of economics and econometrics. Key to being able to use these tools is the formulation of economic interpretations to engineering concepts. This does not only extend the existing economic analysis methods of responses in the time-domain, but also introduces new methods to analyze scenario's in the frequency domain using Bode plots, transfer functions and pole-zero maps. Finally, the root-locus method is used to perform closed-loop scenario analysis of monetary and fiscal policies. This is also an entirely new scenario analysis method in the field of economics and econometrics. By demonstrating how the tools can be used the door is opened to the application of more existing S&C tools that have not been mentioned or used in this thesis.

The contributions of this chapter are based on the proposition of economic interpretations of S&C engineering concepts. These are:

Table 7-1: Propositions of (macro)economic interpretations to system and control engineering concepts

Systems and Control	**Macroeconomics**
Resonance peak	Optimal trading frequency
Anti-resonance peak	Liquidity trap frequency
Corner frequency	Liquidity risk frequency
Real-valued pole	Economic factor
Imaginary-valued pole	Economic cycle
Zeros	Transmission block
Natural frequency	Real business cycle
Damping ratio	Depreciation ratio

Chapter 8

Conclusions and Recommendations

The purpose of this thesis is to develop a macroeconomic scenario model using and engineering grey-box approach. For this purpose, two main contributions have been made.

The first main contribution is the development of a method, a bond graph model and a grey-box state-space model of the entire open United States economy. By using an economic-engineering approach, a method is developed to build a dynamic model of an entire open economy leveraging the national accounting and circular flow theory. Although governments use the national accounting theory to compute a country's Gross Domestic Product (GDP), the theory in the strict sense only holds for systems in equilibrium. This obstacle is overcome by first breaking the system up into eight subsystems- the markets and economic actors- and subsequently introducing economically interpretable energy storing elements for each. This way, the model adheres to the national accounting relations in equilibrium situations, but is able to dynamically forecast economic scenarios for disequilibria. By applying the economic-engineering framework the model incorporates a whole set of additional economic laws and theories besides the national accounting theory. Examples are the laws of supply and demand, economic cycle theory, the quantity theory of money, and liquidity theory among others. The macroeconomic grey-box model is the first of its kind and opens the doors to a whole new way of performing economic scenario analysis.

The second main contribution is the application of Systems and Control (S&C) engineering concepts and analysis tools to the macroeconomic grey-box model. A key step to achieve this is the defining of key analogues between S&C engineering and the field of macroeconomics. This not only makes the model economically interpretable, but also allows the use of existing analysis tools from the field of S&C engineering, new to economics and econometrics. The use of these tools yields a number of results: 1) equity returns and government bond yields with a specific maturity are computed in the frequency domain using the Laplace transform 2) Bode plots were used to perform macroeconomic scenario analysis in the frequency domain 3) Economic transient response analysis is performed using control system performance specifications, and 4) closed-loop scenario analysis is performed using the root-locus method.

In this thesis, the behavior of the model is qualitatively validated by comparing transient responses to economic theory. Using the manually assigned parameters, scenario analysis is performed. The last step I recommend towards practical quantitative application is system identification using economic data. This way the tools presented in this thesis can be used to quantitatively analyse macroeconomic scenarios. This thesis demonstrates the potential and the benefits the grey-box economic scenario has over the existing black-box models. With the notions of this thesis, others tools than the ones mentioned here can be used to perform macroeconomic scenario analysis using engineering formalism.

Appendix A

Dynamic Factor Models

Dynamic Factor Models (DFM) are considered to be a parsimonious representations of time series in the field of econometrics. Factor analysis is a dimension reduction technique summarizing the sources of variation among variables. Most of the results and developments on Dynamic Factor models are summarized in surveys by Bai and Ng [4], Stock and Watson [81, 82, 85] and Lütkepohl [50]. The premise of DFM is that there are a number of possibly unobserved common factors $f(t)$, that drive the dynamics of observed economic time series of interest $y(t)$.

A-0-1 Types

A total of three types of DFM's can be distinguished [21]:

1. *Macroeconomic Factor Models*: The factors are observed, the loadings are unknown parameters. Some of the macroeconomic variables typically used as factors are inflation, the percentage change in industrial production, the excess return to long-term government bonds, and the realized return premium of low-grade corporate bonds relative to high-grade bonds. An example of these models is the Fama-French factor model. These are asset pricing models where the size of firms, book-to-market values and excess return on the market among others are used as factors.

2. *Fundamental Factor Models*: Factors are unknown parameters, loadings are observed. Observed asset-specific fundamentals, such as industry, market cap, book value, etc. are used as the factor loadings. An example is the Nelson-Siegel model which is used to forecast yield curves. Factors are computed using cross-sectional Weighted Least Squares (WLS) regression as apposed to time-series regression.

3. *Statistical Factor models*: Both factors and loadings are unknown parameters. Uses various maximum-likelihood or Principal Component Analysis (PCA) based analysis to compute the factors and regression techniques to obtain the factor loadings. This requires sufficient data.

A-0-2 Formulation

Following Bai and Ng [4], Lütkepohl [50], Stock and Watson [82], the dynamic factor model for a stationary Linear Time-Invariant (LTI) system as a function of lag operator L and step size k can be described as:

$$\begin{aligned} f(k) &= A(L)f(k-1) + \eta(k) \\ e(k) &= \Phi(L)e(k-1) + \epsilon(k) \\ y(k) &= C(L)f(k) + e(k) \\ \eta(k) \overset{i.i.d.}{\sim} N(0,\Sigma_\eta) & \qquad \epsilon(t) \overset{i.i.d.}{\sim} N(0,\Sigma_\epsilon) \end{aligned} \tag{A-1}$$

$$\begin{aligned} A(L) &= A_0 + A_1 L + ... + A_q L^q \\ C(L) &= C_0 + C_1 L + ... + C_p L^p \\ \Phi(L) &= \Phi_0 + \Phi_1 L + ... + \Phi_r L^r \end{aligned} \tag{A-2}$$

Where $y, e, \epsilon \in \mathbb{R}^N$, $f, \eta \in \mathbb{R}^K$, $A_q \in \mathbb{R}^{K\times K}$, $C_p \in \mathbb{R}^{N\times K}$ and $\Phi \in \mathbb{R}^{N\times N}$. The term $e(k)$ is a referred to as an idiosyncratic disturbance that consists of a measurement error $\epsilon(k)$ and of special features that are specific to an individual series. The term $\eta(k)$ and possible added constant vectors are also referred to as idiosyncratic components where $E[e(k)\eta(k)^T] = 0$ holds. Both $\eta(k)$ and $\epsilon(k)$ are assumed to be independent identically distributed and hence a type of white noise. Here, K is the number of selected factors, equivalent to the model's order, and N is the number of measured economic time series Bai and Ng [4].

Appendix B

Factor-augmented Vector Autoregression Models

Factor-augmented Vector Auto Regression (FAVAR) models combine the properties of VAR models and DFM. It was originally developed by Bernanke et al. [5]. The main idea is to be able to work with a Dynamic Factor Model (DFM) in which the idiosyncratic errors are serially uncorrelated [85]. To obtain this, Eq. (B-1) has been left multiplied by $(1 - \Phi(L)L)$ which results in Eq. (B-2).

$$y(k) = C(L)f(k) + e(k) \tag{B-1}$$
$$y(k) = \tilde{C}(L)f(k) + \Phi(L)y(k-1) + \epsilon(k), \quad \text{where} \quad \tilde{C} = (1 - \Phi(L)L)C(L) \tag{B-2}$$
$$f(k) = A(L)f(k-1) + \eta(k) \tag{B-3}$$

where,

$$\tilde{C}(L) = \begin{bmatrix} \tilde{c}_1(L) \\ \vdots \\ \tilde{c}_N(L) \end{bmatrix}, \Phi(L) = \begin{bmatrix} \phi_1(L) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \phi_N(L) \end{bmatrix}, e(k) = \begin{bmatrix} e_1(k) \\ \vdots \\ e_N(k) \end{bmatrix}$$

After converting Eq. (B-1) - (B-2) into the static form, seen in Eq. (B-4), the FAVAR system can be rewritten in the state-space formulation as shown in Equation B-5 [84]. The dynamics, $\Phi(L)$, of the idiosyncratic error $e(k)$ are implicitly encaptured by the lag of $y(k)$ and in the term $\Lambda(L)$.

$$\begin{aligned} Y(k) &= \Lambda F(k) + \Phi Y(k-1) + \epsilon(k) \\ F(k) &= \Gamma F(k-1) + G\eta(k) \end{aligned} \tag{B-4}$$

$$\begin{aligned}\underbrace{\begin{bmatrix}F(k+1)\\Y(k+1)\end{bmatrix}}_{x(k+1)} &= \underbrace{\begin{bmatrix}\Gamma & 0\\\Lambda\Gamma & \Phi\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}F(k)\\Y(k)\end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix}G & 0\\\Lambda G & I\end{bmatrix}}_{E}\underbrace{\begin{bmatrix}\eta(k+1)\\\epsilon(k+1)\end{bmatrix}}_{\zeta(k)} \\ Y_{out}(k) &= \underbrace{\begin{bmatrix}0 & I\end{bmatrix}}_{C}\underbrace{\begin{bmatrix}F(k)\\Y(k)\end{bmatrix}}_{x(k)}\end{aligned} \tag{B-5}$$

where,

$$\zeta(k) \overset{i.i.d.}{\sim} N\left(0, \begin{bmatrix}G\Sigma_\eta G^T & G\Sigma_\eta G^T\Lambda^T\\\Lambda G\Sigma_\eta G^T & \Lambda G\Sigma_\eta G^T\Lambda^T + \Sigma_\epsilon\end{bmatrix}\right) \tag{B-6}$$

Appendix C

Ortec Finance Modelling Approach

This appendix includes a summary of the approach and methods used by Ortec Finance (OF). As this is interdisciplinary research effort, it serves the purpose of familiarizing the engineering reader with the current modelling approach and methodology.

C-1 Dynamic Scenario Generator Modelling Methodology

The four main system identification and validation steps of Ortec Finance's (OF) modelling methodology are: 1. filtering the time series; 2. system identification using Dynamic Factor Models (DFMs) with Principal Component Analysis (PCA); 3. out-of-sample validation and; 4. scenario generation. I will elaborate on each of the steps as some differ from how engineers would apply them.

Filtering and Pre-Processing The time series data are transformed to the frequency domain using the Discrete Fourier Transform (DFT). The filtering decomposes the frequency series into the three separate frequency bands of interest, namely:

1. The trend component, for which: 16 years $\leq$ period
2. The business cycle component, for which: 2 years $<$ period $\leq$ 16 years
3. The monthly cycle component, for whic:h 2 months $\leq$ period $\leq$ 2 year

An example of what this would look like is given in Figure C-1. After the series have been filtered into the separate components, they are inversely transformed to time series. All time series are standardized and the data that are non-stationary are transformed to a natural logarithmic-return scale (units of ln(%)).

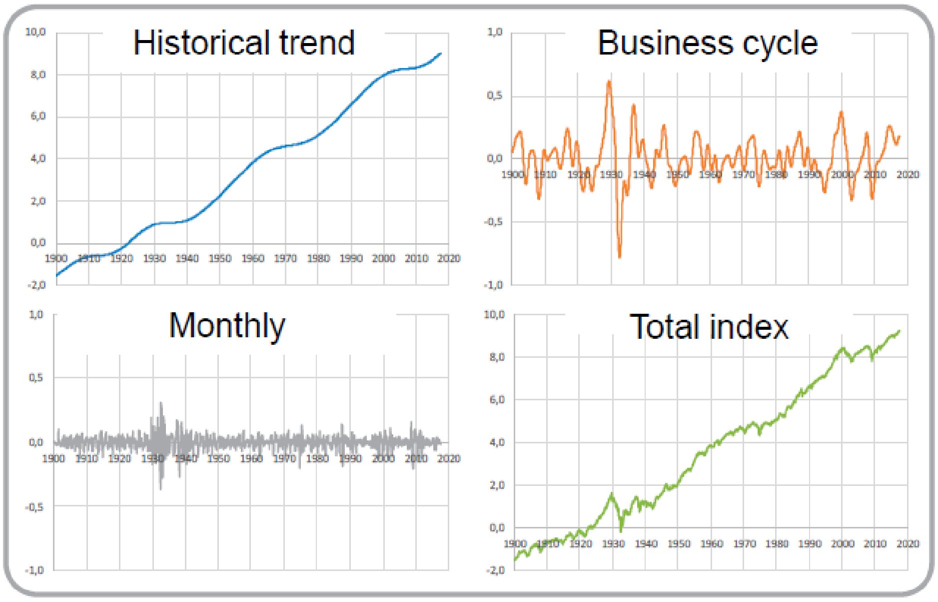


Figure C-1: A graphical example of Ortec Finance's decomposed time series [24]

Dynamic Factor Models In general, a factor model is a model in which the dynamics of the variables of interest are driven by another set of underlying (possibly unobserved) other variables called factors [3, 51, 70, 83, 85].

OF uses an adapted version of DFMs named Factor-augmented Vector Auto Regression (FAVAR) models. Apart from factors, these models include autoregressions of output time series. They can be considered a combination of DFMs and Vector Autoregressive (VAR) models. A more detailed description of DFMs and FAVAR are included in chapter A and chapter B, respectively.

There are several methods to obtain the factors. For the monthly and business cycle models, OF uses Principal Component Analysis (PCA) on the output time-series data. The trend model is built using observable factors [45]. Here, a total of 13 time series such as world interest rates, world growth and world inflation are used as factors. The result is a state-space model with factors that have an economic interpretation. This is a step towards the goal which OF strives for: More interpretability of the models. This also means that implementing stylized facts and expert views and opinions becomes a more straightforward process.

Principal Component Analysis The central idea of PCA is to reduce the dimensionality of a data set consisting of a large number of correlated variables while retaining as much as possible of the variation present in the dataset. This is achieved by transforming to a new set of variables, the principal components (PCs), which are uncorrelated, and which are ordered so that the first few retain most of the variation present in all of the original variables [36]. The initial formulation of the problem is as shown in (C-1)

$$y_i(k) = \lambda_i' f(k) + \epsilon_i(k), \quad i = 1...N, \;\; k = 1...T \tag{C-1}$$

where $\lambda_i' = [\lambda_{i1}...\lambda_{ir}]$ are the loadings, $f(k) = [f_1(k)...f_r(k)]'$ are the factors, $y(k)$ are the economic time series variables, r represent the number of factors and i the number of output time series. The objective is to obtain factors $f_j \in \mathbb{R}^K$ where $K < N$ to compress the system while finding the best approximation for y_i. The factors and their loadings are obtained by solving the nonlinear least-squares optimization problem in matrix form seen in Eq. (C-2) which equals the minimization of the (co)variance matrix of the idiosyncratic error $\epsilon(k)$.

$$\begin{aligned} \min_{\Lambda,F} \frac{1}{TN} \sum_{i=1}^{N} \sum_{k=1}^{T} (y_i(k) - \lambda_i' f(k))^2 &= \min_{\Lambda,F} ||Y - \Lambda' F||^2 \\ s.t. \qquad & \Lambda\Lambda' = I_K \end{aligned} \tag{C-2}$$

Here, $Y \in \mathbb{R}^{N \times T}$, $\Lambda' \in \mathbb{R}^{N \times K}$, and $F \in \mathbb{K}^{K \times T}$. Without further derivation, the solution is obtained if Λ comprises of the normalized eigenvector scaled by the square roots of the K largest eigenvalues of $Y'Y$ [36].

Pseudo-out-of-sample Testing Model validation is performed with a type of out-of-sample or cross-validation testing. Two methods of out-of-sample testing are distinguished: Iterative and direct forecasting. Iterative is also referred to as pseudo-out-of-sample testing.

Direct forecasting is most comparable to how engineers would apply cross-validation testing. A sample set is split up into a modelling set and a validation set. The modelling set, from $f(0) \rightarrow f(k)$, is used to estimate the model and its parameters. A validation set, ranging from $f(k) \rightarrow f(\text{end})$, is then used to directly measure the performance by measuring the error between the response of the model and the validation set.

Iterative or pseudo-out-of-sample testing uses the estimated model to predict the one-step-ahead output, $f(k+1)$. This uses the last data point of the model set, $f(k)$, and assumes that the input noises and disturbances equal zero (since $E[\eta(k)] = 0$). The error between the one-step-ahead forecasting output of the model and the validation set is then measured. The model is re-estimated including the new data point at $f(k+1)$. OF uses an expanding window iterative forecasting approach to validate the models. This refers to validation where the model set is expanded by the inclusion of the new data-point. These steps are repeated until the whole validation set has been forecasted.

Scenario Generation The final models forecast scenarios using Monte Carlos simulation. The forecasts describe a set of possible scenarios and their confidence bands. The inputs used to excite the model are the the zero mean i.i.d. idiosyncratic disturbances as presented in chapter B. An example of what these scenarios look like is shown in Figure C-2.

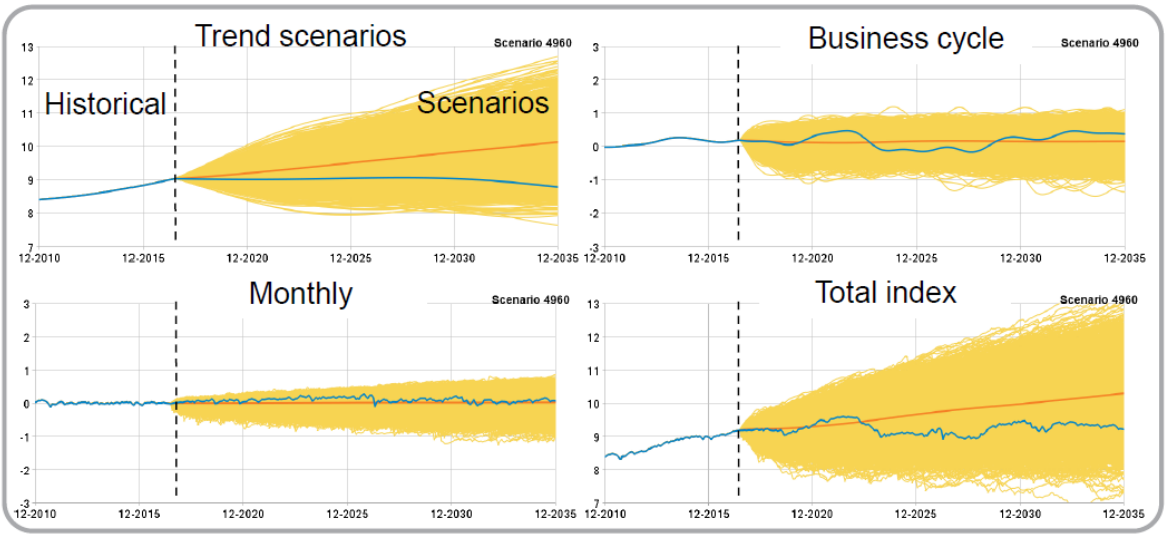


Figure C-2: A graphical example of the Monte Carlo scenarios generated by the DSG [24]

Appendix D

Dynamic Scenario Generator Performance Evaluation

This appendix presents the performance results of the representative simplified Dynamic Scenario Generator (DSG) models I built. The results give an impression of what the grey-box model from Chapter 5 should be able to match in performance.

D-1 Variable and Parameter Estimation Steps

At the basis of the DSG models are the two equations of Eq. (D-1).

$$\begin{aligned} Y(k) &= \Lambda F(k) + \Phi Y(k-1) + c_y + \epsilon(k) \\ F(k) &= \Gamma F(k-1) + c_f + \eta(k) \end{aligned} \tag{D-1}$$

where, $Y(k) \in \mathbb{R}^{(309\times 1)}$, $F(k) \in \mathbb{R}^{(N\times 1)}$, $\Gamma \in \mathbb{R}^{(N\times N)}$, $\Lambda \in \mathbb{R}^{(309\times N)}$, and $\Phi \in \mathbb{R}^{(309\times 309)}$. $Y(k)$ is the discrete time set of filtered time series on a yearly time-scale for the business cycle model and on a 8-yearly time-scale for the trend model. N corresponds to the number of factors/states.

I have only built a trend- and business cycle model. Gross Domestic Product (GDP) and Consumer Price Index (CPI) are measured half-yearly or yearly and therefore do not include higher frequency behaviour. For the other variables the time-series data do contain these higher frequencies. I am solely interested in modelling the fundamental behavior of these variables and not the high frequency noise. Therefore, I assume that higher frequency behavior is caused by short term market volatility [27] and do not model these frequencies.

A total of 309 filtered and transformed time series are used. These consist of several economic and financial variables such as for example gross domestic products, consumer price indices, exchange rates, commodities etc. of 21 different countries.

Using these two equations the identification process excluding the filtering of the time series is as follows:

1. The time series data is standardized to 0 mean and unit standard deviation;
2. The factors, $F(k)$, are computed using Principal Component Analysis (PCA) decomposition of the filtered time series data, $Y(t)$. Nine factors for the business cycle model and four factors for the trend model;
3. The loadings, Λ, are computed using a Elastic Net regression [96];
4. Ordinary Least Squares (OLS) regression is used to compute the autoregression matrix Φ. For the business cycle model autoregression are applied to Nominal Government Bond with 10 years maturity yield rate (NGLR) and for the trend model to Equity Total Returns (EQTR);
5. The constant c_y is computed using the mean of the difference between the output time series and the fitted series: $Y(k) - \Lambda F(k) - \Phi Y(k-1)$;
6. Yule-Walker (YW) equations are used to compute the dynamics matrix, Γ, and the constant c_f. The result is a system with a set of stable eigenvalues;
7. The subtracted mean in step 1 gives the constant c_μ

D-2 Measures for Performance Evaluation

The performance of the models are assessed with a Variance Accounted For (VAF) score [93] and a Root Mean Square Error (RMSE) value and relative difference. The VAF score is a percentage that reflects the how much of the variance of the data is encaptured by the model's response and is computed with Eq. (D-2) [93]. It is also referred to as the R squared error.

$$\text{VAF} = \left(1 - \frac{\text{var}(y(k) - \hat{y}(k))}{\text{var}(y(k)}\right) \cdot 100\% \tag{D-2}$$

where $y(k)$ are the observations and $\hat{y}(k)$ are the predictions or estimates. The RMSE measures the standard deviation of the difference between the predicted values and the observed values. It is computed using Eq. (D-3). The units are the same as the variable for which it is computed. The higher the value the worse the performance. As opposed to VAF it also takes into account a bias.

$$\text{RMSE} = \sqrt{\frac{\sum_{k=1}^{T}(\hat{y}(k) - y(k))^2}{T}} \tag{D-3}$$

D-3 Business Cycle Model Performance Results

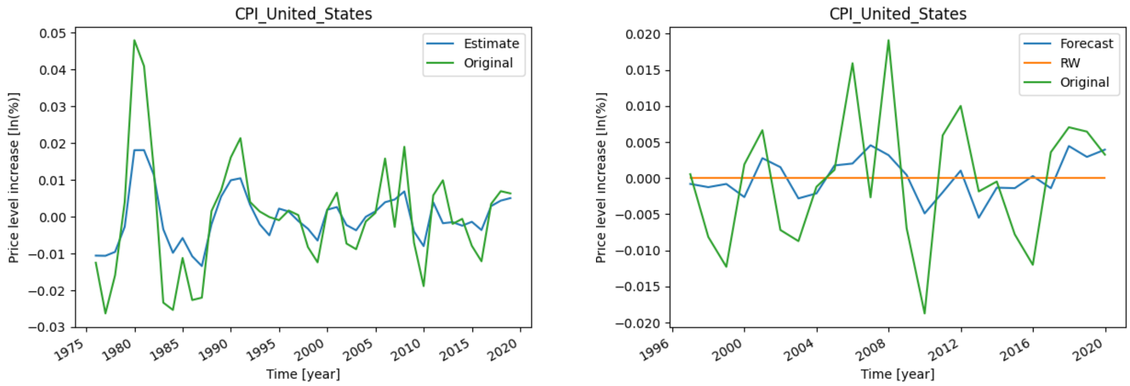
The pseudo-out-of-sample performance is validated and compared to a benchmark model by means of RMSEs. The benchmark model used is a static value estimation of the outputs. For the detrended business cycle data with mean 0, this translated to a constant value of 0. Ortec Finance (OF) refers to this as a static Random Walk (RW) model. Table D-1 shows the in-sample and out-of-sample performance results. The initial in-sample data set ranges from the end 1975 to end 1995. The set expands with 1 year steps following the pseudo-out of sampling procedure. A total of 24 years were forecasted, ranging from end 1996 to end 2019. Only the United States is considered. The model is however also able to predict the same variables among others for several other countries. The in-sample and pseudo-out-of-sample average VAF for all countries used and variables are 50.63% and 12.09%, respectively. Higher in-sample VAF resulted in overfitting and worse out-of-sample performance.

	in-sample		out-of-sample		relative to RW
Variables	VAF [%]	RMSE [-]	VAF [%]	RMSE [-]	decrease RMSE [%]
CPI	63.63	0.0092	22.04	0.0078	11.65
EQTR	52.06	0.0939	4.90	0.1450	2.55
GDP	47.15	0.0131	19.01	0.0129	10.50
NGLR	55.77	0.0066	0.00	0.0055	-2.38
TBSR	63.91	0.0097	41.31	0.0099	23.39

Table D-1: In- and out-of-sample results of the business cycle model for the United States

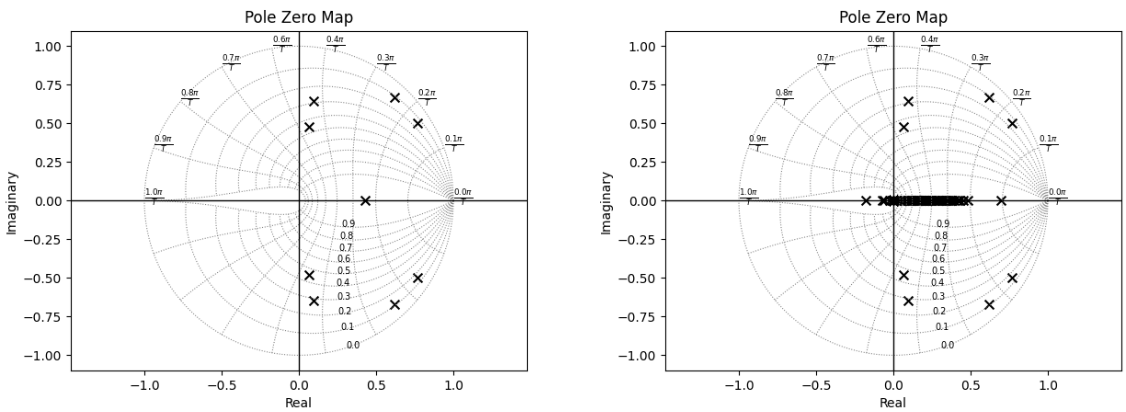
The model outperforms the static benchmark for every variable except for NGLR. In advance, I expected NGLR and Nominal Government Bill with 3 months maturity yield rate (TBSR) to perform worse since both variables are highly dependant on the actions taken by the Federal Reserve, comparable to control actions. In the actual DSG both these variables depend highly on expert views and opinions. This applies to TBSR in particular that performs strongly here and is an unexpected outcome.

A graphical example of the in-sample and pseudo-out-of-sample performance of CPI are shown in Figure D-1a and Figure D-1b, respectively. Figure D-2 shows the poles and zeros of the business cycle model with and without the autoregressive states. The system has no zeros and a combination of stable complex and purely imaginary poles. The in- and out-of-sample results of the other four variables are included in section D-6 - D-9.



(a) (b)

Figure D-1: In-sample estimation (D-1a) and pseudo-out-of-sample forecast versus static random walk benchmark (D-1b) of the business cycle model for CPI



(a) (b)

Figure D-2: Pole-zero plot of the business cycle model without (D-2a) and with (D-2b) autoregressive states. All poles are in the unit circle, the system is stable

D-4 Trend Model Performance Results

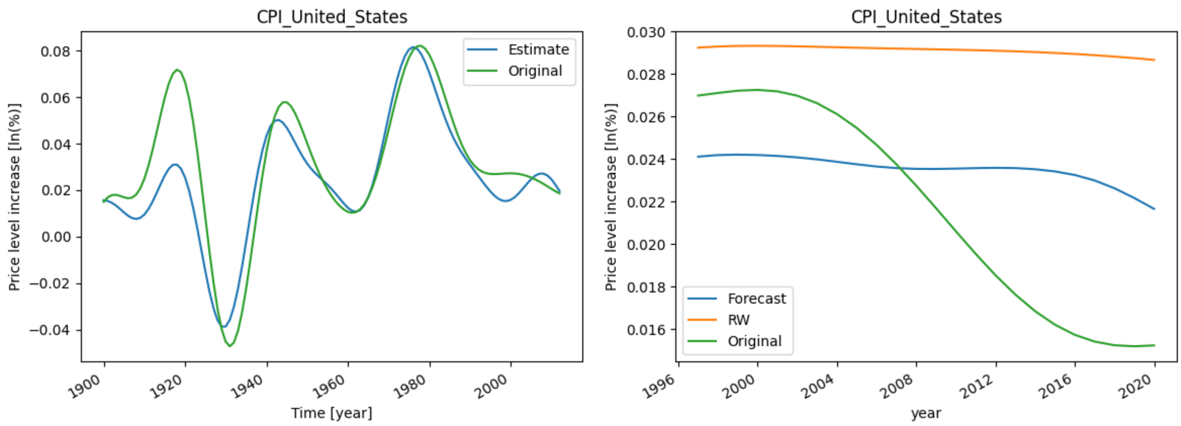
Like for the business cycle model, the pseudo-out-of-sample performance is validated and compared to a benchmark model by means of RMSEs. For the trend data, OF uses long-term mean estimates computed from separate models. Instead, I use the mean of the time-series from the data ranging from 1950 to the the beginning of the validation set, which starts at 1988 and expands in steps of 8 year along with the pseudo-out-of-sample testing procedure as described in Section 2-3. The reason for this is because I consider the use of the long-term mean estimates as a form of implementation of expert views. This can disturb an objective comparison of the models. The initial in-sample data set ranges from the end 1899 to end 1988. The set expands with 8 year steps following the pseudo-out of sampling procedure. A total of 24 years were forecasted, ranging from end 1996 to end 2019. Al variables are given for only for the United States but the model can be used for forecasts of other included regions as well. The in-sample and pseudo-out-of-sample average VAF for all countries and variables used are 70.29% and 12.25%, respectively.

The model outperforms the benchmark on all variables. The pseudo-out-of sample VAF of GDP is 0 %, yet outperforms the benchmark model. The cause of this is a weakly performing benchmark model. In reality OF uses long-term mean estimates from a different model as the benchmark which performs better, though, these are also incorporated in the model itself. The 0 % out-of-sample VAF for GDP is, in itself, an unexpected result because GDP is considered to be outcome structural economic behaviour. The low in-sample VAF could cause this.

A graphical representation of the in-sample and pseudo-out-of-sample performance of CPI are shown in Figure D-3a and Figure D-3b, respectively. Figure D-4 shows the poles and zeros of the trend model with and without the autoregressive states. The system has no zeros and a combination of stable complex and purely imaginary poles. The in- and out-of-sample results of the other four variables are included in section D-6 - D-9.

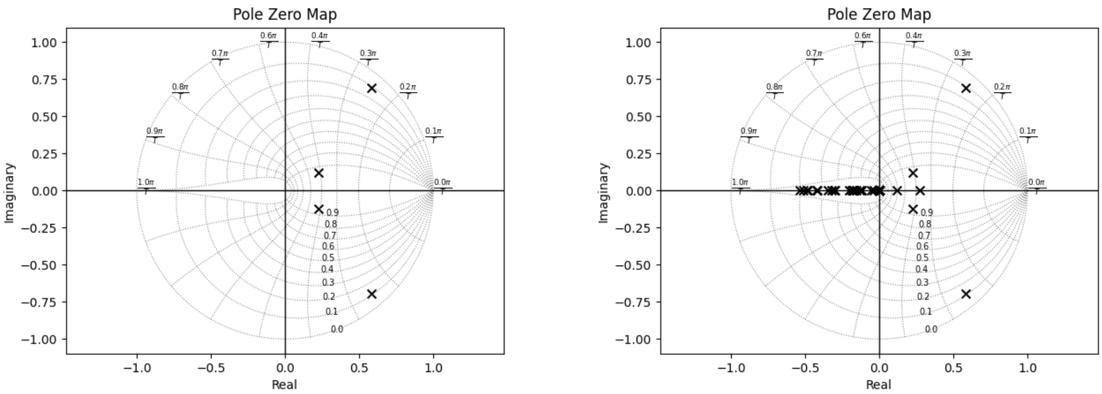
	in-sample		out-of-sample		relative to RW
Variables	VAF [%]	RMSE [-]	VAF [%]	RMSE [-]	decrease RMSE [%]
CPI	78.43	0.0144	19.68	0.0046	47.02
EQTR	85.82	0.0240	6.28	0.0419	10.06
GDP	41.56	0.0153	0.00	0.0123	9.79
NGLR	95.13	0.0055	14.84	0.0135	21.05
TBSR	76.91	0.0119	11.68	0.0179	25.19

Table D-2: In- and out-of-sample results of the trend model for the United States



(a) **(b)**

Figure D-3: In-sample estimation (D-3a) and pseudo-out-of-sample forecast versus static random walk benchmark (D-3b) of the trend model for CPI



(a) **(b)**

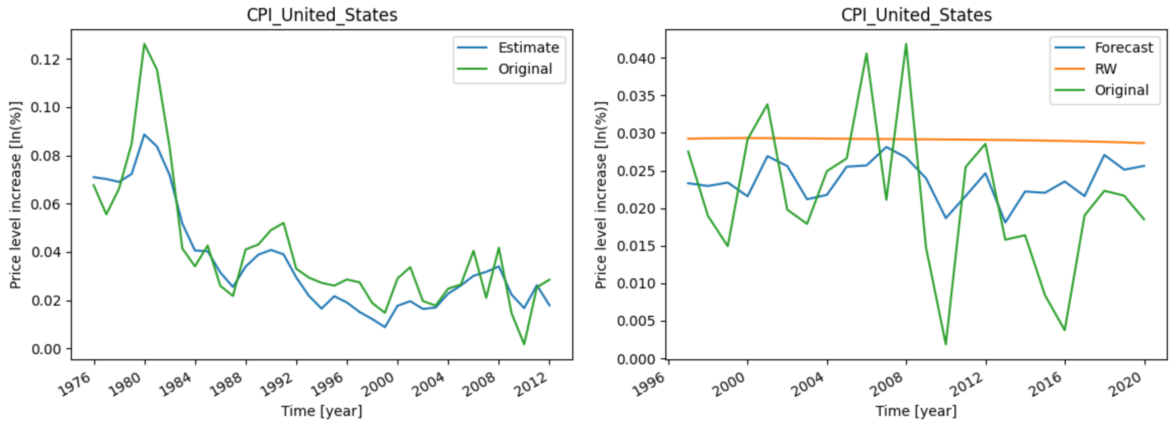
Figure D-4: Pole-zero plot of the trend model without (D-4a) and with (D-4b) autoregressive states. All poles are in the unit circle, the system is stable

D-5 Combined Performance Results

The results are combined by summing the output estimates and forecasts of the trend and business cycle models. The combined performance results are listed in Table D-3. A graphical representation of the combined in-sample and pseudo-out-of-sample results for CPI are shown in Figure D-5a and Figure D-5b, respectively. The recreated DSG model outperforms the static RW benchmark models for all listed variables of the United States.

	in-sample		out-of-sample		relative to RW
Variables	VAF [%]	RMSE [-]	VAF [%]	RMSE [-]	decrease RMSE [%]
CPI	83.55	0.0116	19.73	0.0088	27.76
EQTR	54.79	0.1044	4.65	0.1569	2.84
GDP	38.14	0.0161	13.45	0.0178	7.25
NGLR	88.82	0.0099	11.04	0.0145	18.43
TBSR	83.21	0.0137	25.11	0.0202	24.63

Table D-3: In- and out-of-sample results of the combined model for the United States



(a)

(b)

Figure D-5: In-sample estimation (D-1a) and pseudo-out-of-sample forecast versus static random walk benchmark (D-1b) of the combined model for CPI

Below are the figures of the in-sample and out-of-sample simulation results. The results for EQTR in shown in Figure D-6, GDP in Figure D-7, TBSR in Figure D-8, and NGLR in Figure D-9.

D-6 EQTR Simulation Results

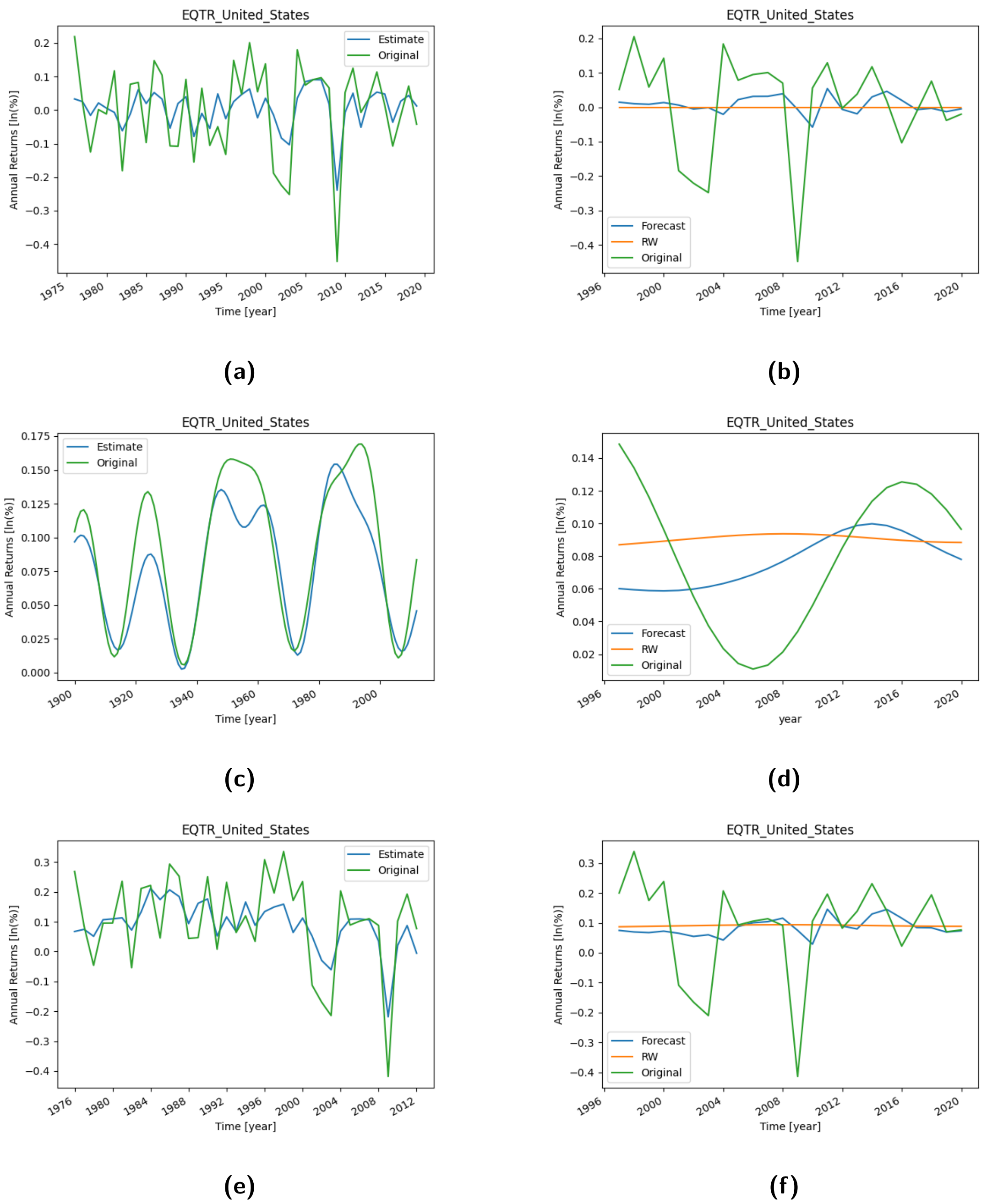
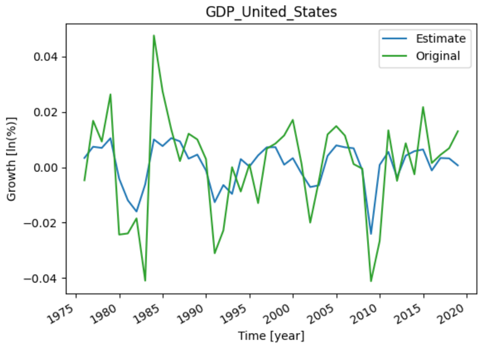
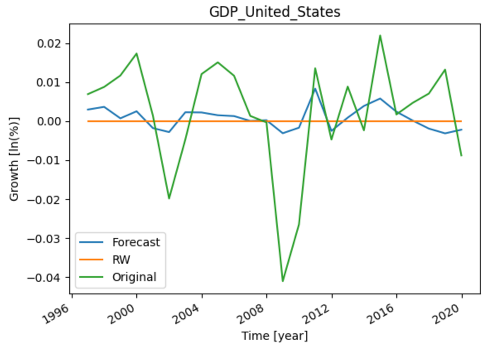


Figure D-6: In-sample estimation (D-6a) and pseudo-out-of-sample forecast versus static random walk benchmark (D-6b) of the business cycle model for EQTR. In-sample estimation (D-6c) and pseudo-out-of-sample forecast versus static random walk benchmark (D-6d) of the trend model for EQTR and in-sample estimation (D-6e) and pseudo-out-of-sample forecast versus static random walk benchmark (D-6f) of the combined model for EQTR

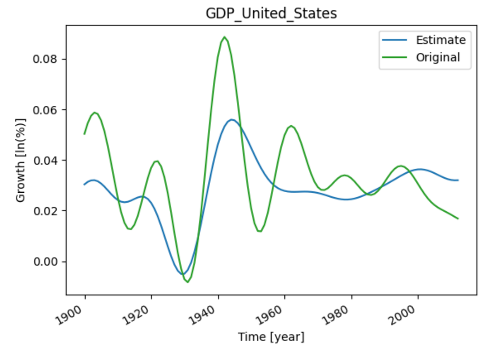
D-7 GDP Simulation Results



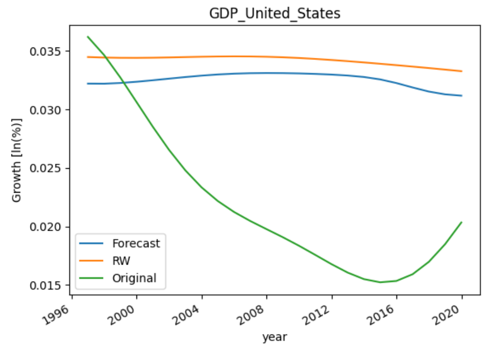
(a)



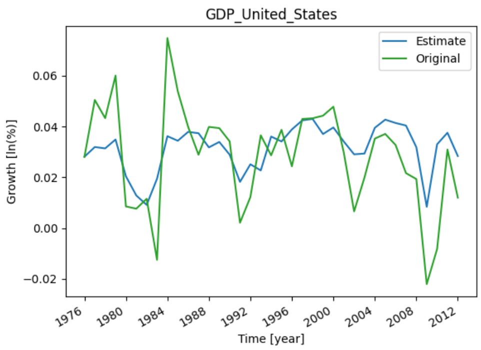
(b)



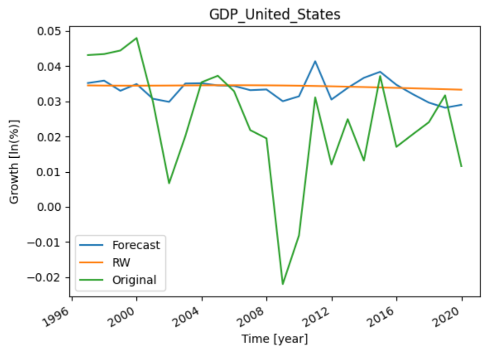
(c)



(d)



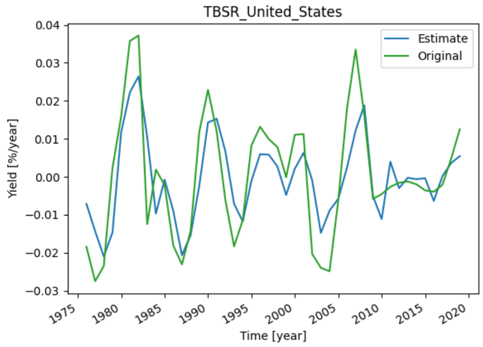
(e)



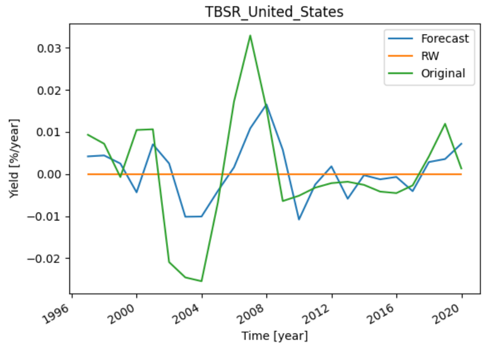
(f)

Figure D-7: In-sample estimation (D-7a) and pseudo-out-of-sample forecast versus static random walk benchmark (D-7b) of the business cycle model for GDP. In-sample estimation (D-7c) and pseudo-out-of-sample forecast versus static random walk benchmark (D-7d) of the trend model for GDP and in-sample estimation (D-7e) and pseudo-out-of-sample forecast versus static random walk benchmark (D-7f) of the combined model for GDP

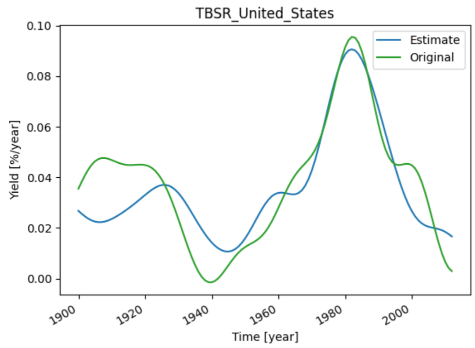
D-8 TBSR Simulation Results



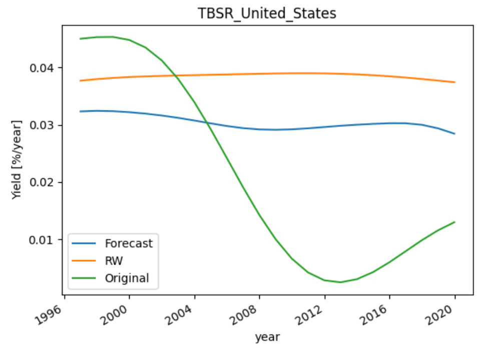
(a)



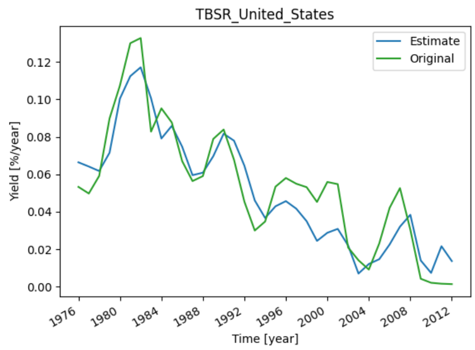
(b)



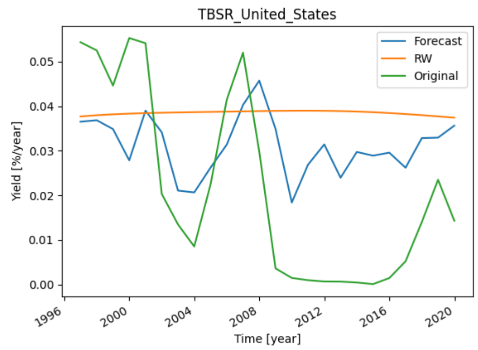
(c)



(d)



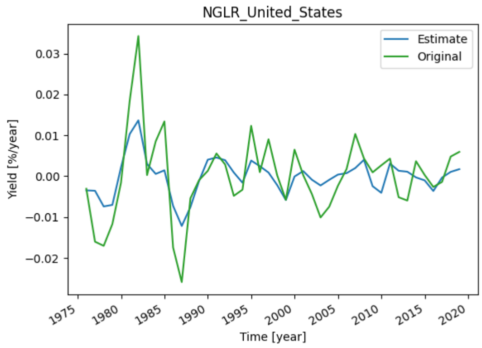
(e)



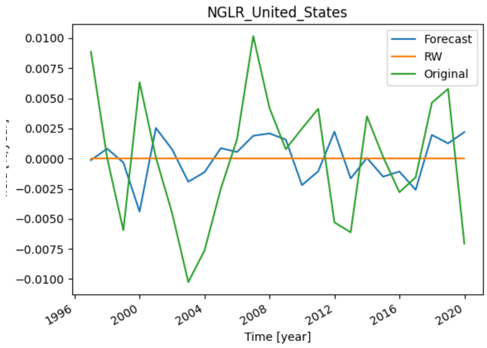
(f)

Figure D-8: In-sample estimation (D-8a) and pseudo-out-of-sample forecast versus static random walk benchmark (D-8b) of the business cycle model for TBSR. In-sample estimation (D-8c) and pseudo-out-of-sample forecast versus static random walk benchmark (D-8d) of the trend model for TBSR and in-sample estimation (D-8e) and pseudo-out-of-sample forecast versus static random walk benchmark (D-8f) of the combined model for TBSR

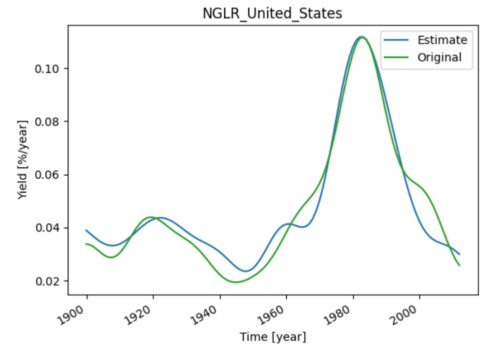
D-9 NGLR Simulation Results



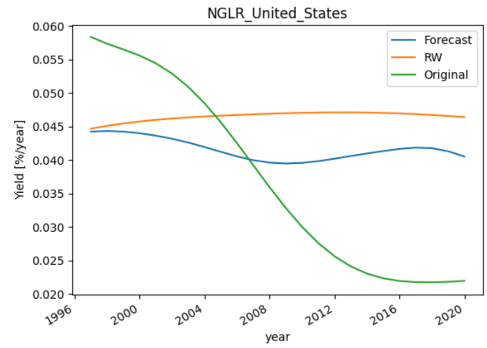
(a)



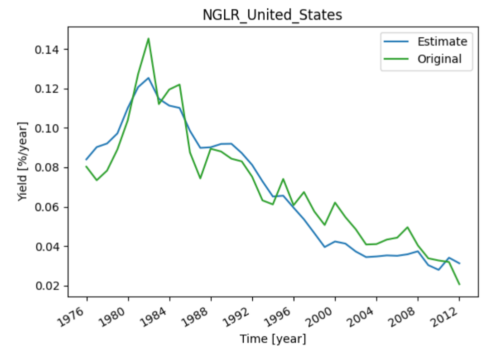
(b)



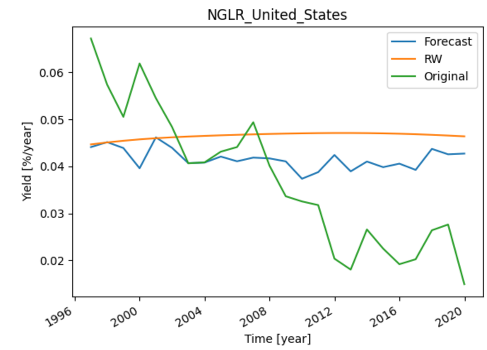
(c)



(d)



(e)



(f)

Figure D-9: In-sample estimation (D-9a) and pseudo-out-of-sample forecast versus static random walk benchmark (D-9b) of the business cycle model for NGLR. In-sample estimation (D-9c) and pseudo-out-of-sample forecast versus static random walk benchmark (D-9d) of the trend model for NGLR and in-sample estimation (D-9e) and pseudo-out-of-sample forecast versus static random walk benchmark (D-9f) of the combined model for NGLR

Appendix E

Yule-Walker Equations

In this appendix the Yule-Walker estimation method is described to estimate A, c and Σ from $f(0), ..., f(T-1)$. The Yule-Walker equations are based on the assumption that the process is stationary, i.e. the mean and covariance of the series are stationary.

Take $f(0), ..., f(T-1) \in \mathbb{R}^m$ for the system:

$$\begin{aligned} f(k+1) &= Af(k) + c + \eta(k) \quad \text{(E-1)} \\ \eta(k) &\overset{i.i.d.}{\sim} N(0, \Sigma_\eta) \quad \text{(E-2)} \end{aligned}$$

And assume the following relations hold where $+i$ and $-i$ yield the same result:

$$\mu = \lim_{k\to\infty} \mathbb{E}f(k), \qquad \Gamma(i) = \lim_{k\to\infty} \mathbb{E}(f(k)-\mu)(f(k-i)-\mu)' \quad \text{(E-3)}$$

The response of the system from time instant j to time instant $k+j$ is given by:

$$\begin{aligned} f(k+j) &= A^k f(j) + \sum_{i=0}^{k-1} A^{k-i-1}(c + \eta(i+j)) \quad \text{(E-4)} \\ k &= 1, ..., T \end{aligned}$$

Following the stationary assumption, taking the mean of the process results in:

$$\begin{aligned} \mathbb{E}f(k+j) &= \mathbb{E}A^k f(j) + \mathbb{E}(c + \eta(i+j)) \quad \text{(E-5)} \\ \mu_f &= A^k \mu_f + c \quad \text{(E-6)} \end{aligned}$$

Then (E-1) can be rewritten as:

$$(f(k+j) - \mu_f) = A^k(f(j) - \mu_f) + \sum_{i=0}^{k-1} A^{k-i-1}(c + \eta(i+j)) \quad \text{(E-7)}$$

(E-8)

Right multiplication with $f(j)-\mu_f)'$ and taking the expectation gives:

$$\begin{aligned}\mathbb{E}(f(k+j)-\mu_f)(f(j)-\mu_f)' &= A^k\mathbb{E}(f(j)-\mu_f)(f(j)-\mu_f)' \\ \Gamma(k)_f &= A^k\Gamma(0)\end{aligned} \tag{E-9}$$

Another way of obtaining the parameters is by multiplying the left-hand side and the right-hand side of (E-1) and taking the expected values:

$$\mathbb{E}(f(k+j)-\mu_f)(f(k+j)-\mu_f)' =$$
$$\mathbb{E}(A^k f(j)+\sum_{i=0}^{k-1}A^{k-i-1}(c+\eta(i+j)))(A^k f(j)+\sum_{i=0}^{k-1}A^{k-i-1}(c+\eta(i+j)))'$$
$$\Gamma(0) = A^k\Gamma(0)(A^k)' + \sum_{i=0}^{k-1}A^{k-i-1}(\Sigma_\eta + c^2)(A^{k-i-1})' \tag{E-10}$$

Assuming you start at an arbitrary j but look one step ahead, $k=1$, we obtain:

$$\Gamma(0) = A\Gamma(0)A' + \Sigma_\eta + c^2 \tag{E-11}$$

From this the A, c and Σ_η can be derived using simple matrix algebra.

Appendix F

Model Parameter Values

Table F-1: The parameter values of the total model

Labor Market		Rental Market		Consumer Goods & Services Market	
C_5	0.83	C_7	1.00	C_3	1.00
C_6	3.33	C_8	3.33	C_4	3.33
I_4	1.00	I_5	0.67	I_3	1.43
R_7	0.50	R_9	0.33	R_5	0.25
R_8	0.50	R_{10}	1.00	R_6	0.25
TF_1	1.00	TF_3	1.00	GY_1	1.00
TF_2	1.00	TF_4	1.00	GY_2	1.00
Households		**Firms**		**Government Bonds Market**	
C_2	1.11	C_1	1.00	C_{12}	0.33
I_2	5.00	I_1	5.00	I_9	0.25
R_3	7.14	R_1	20.00	R_{17}	10.00
R_4	5.00	R_2	6.00	R_{18}	1.00
				TF_9	2.00
Loanable Funds Market		**Equity Market**			
C_{11}	0.50	C_9	0.33		
I_7	1.00	C_{10}	1.00		
I_8	1.00	I_6	2.00		
R_{14}	5.00	R_{11}	1.00		
R_{15}	1.25	R_{12}	10.00		
R_{16}	9.00	R_{13}	2.00		
TF_6	1.00	TF_5	1.00		
TF_7	1.00				
TF_8	1.00				

Appendix G

Pole-zero Maps Subsystems

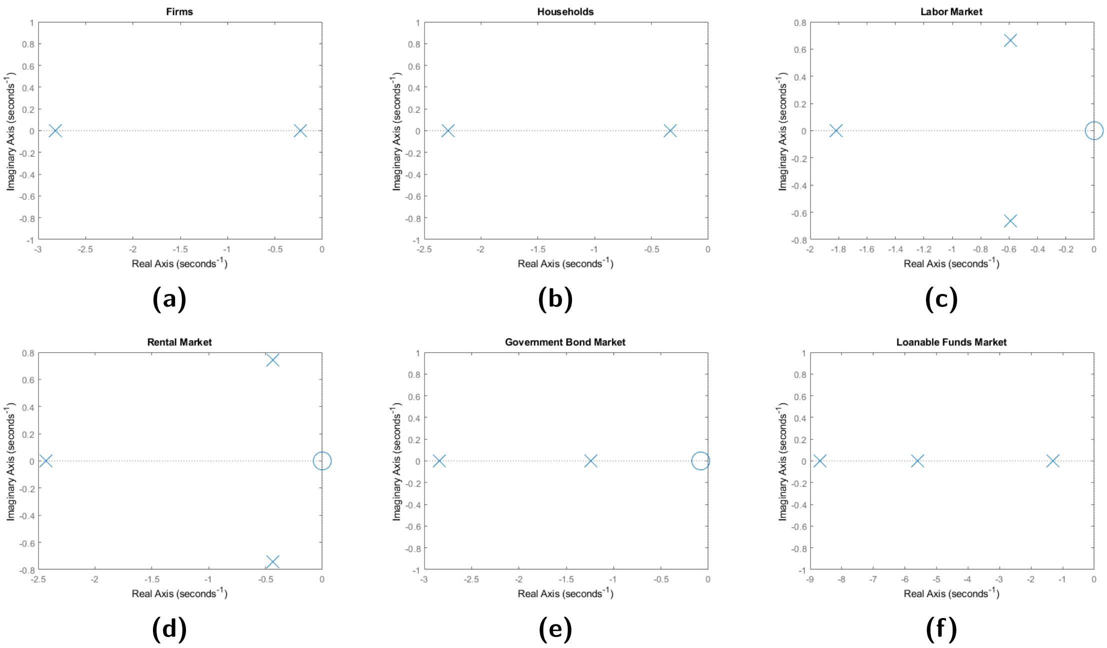


Figure G-1: The pole-zero maps for the six other subsystems. All poles are in the left half of the s-plane. All subsystems are stable

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Glossary

List of Acronyms

OF	Ortec Finance
RW	Random Walk
PCA	Principal Component Analysis
DFM	Dynamic Factor Model
DSG	Dynamic Scenario Generator
VAR	Vector Autoregressive
OLS	Ordinary Least Squares
WLS	Weighted Least Squares
YW	Yule-Walker
FAVAR	Factor-augmented Vector Auto Regression
VAF	Variance Accounted For
GDP	Gross Domestic Product
CPI	Consumer Price Index
EQTR	Equity Total Returns
DFT	Discrete Fourier Transform
LTI	Linear Time-Invariant
RMSE	Root Mean Square Error
ALM	Asset & Liability Management
NGLR	Nominal Government Bond with 10 years maturity yield rate
TBSR	Nominal Government Bill with 3 months maturity yield rate
MSCI	Morgan Stanley Capital International Index
NI	National Income
NFFI	Net Foreign Factor Income
IBT	Indirect Business Tax

SD	Statistical Discrepancy
S&C	Systems and Control
FTE	full-time equivalent
DI	Disposable Income
MPS	Marginal Propensity to Save
MPC	Marginal Propensity to Consume
FCFE	Free Cash Flow to Equity
FCFF	Free Cash Flow to Firm
NPV	Net-Present Value
IRR	Internal Rate of Return