Proposal for a tool to design masonry double-curved shells

Appendices

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structural
d e s i g n *lab*
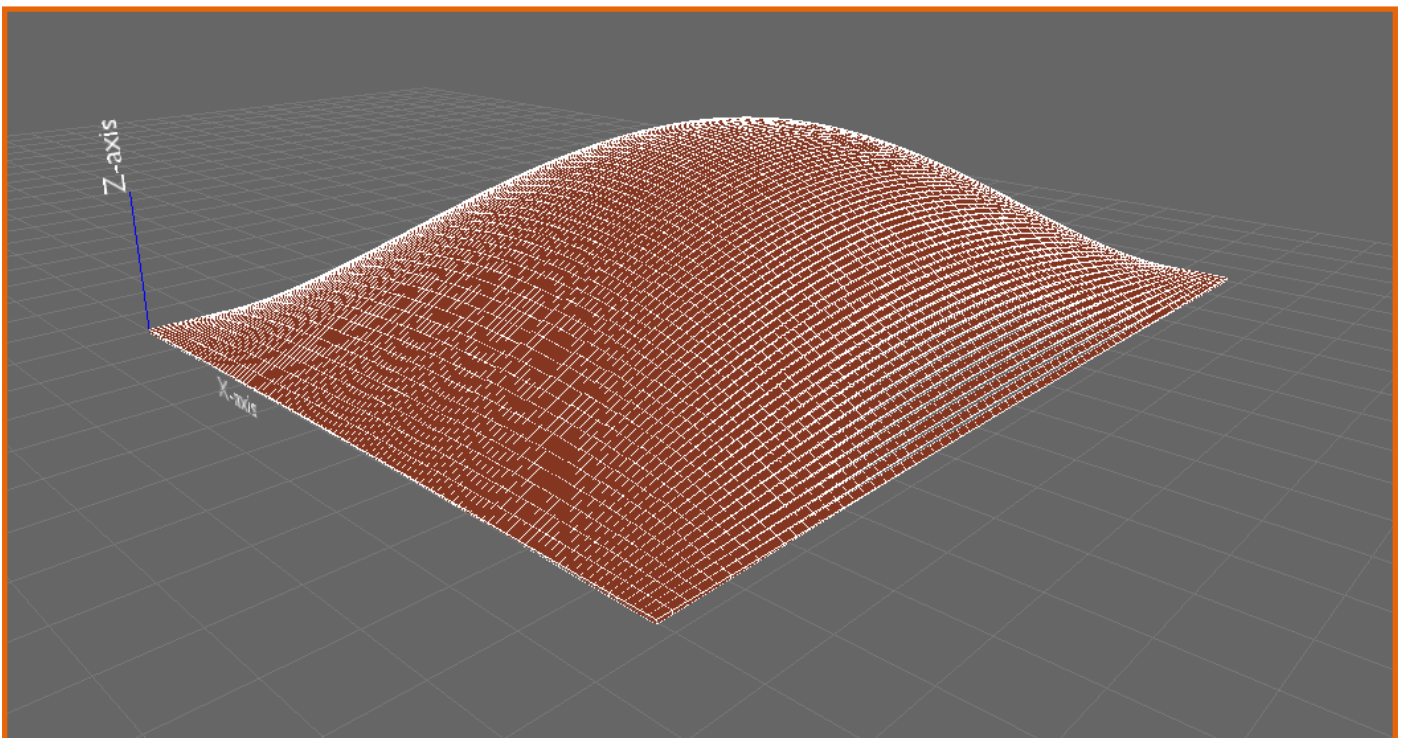
Part 2 - Appendices

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Specialization Structural Design

PREFACE

This is the second report of the Master Thesis research '*Proposal for a tool to design masonry double-curved shells*' and consists of the appendices and the user manual.
It contains five appendices and the user manual, which has been added as an extra appendix.

The five appendices are:
-   Appendix A   Reciprocal relationship;
-   Appendix B   Method of Gauss-Jordan (Pivotting);
-   Appendix C   The Simplex method;
-   Appendix D   Personal data student;
-   Appendix E   Data of graduation committee.

And the user manual is presented in:
-   Appendix F   The user manual.

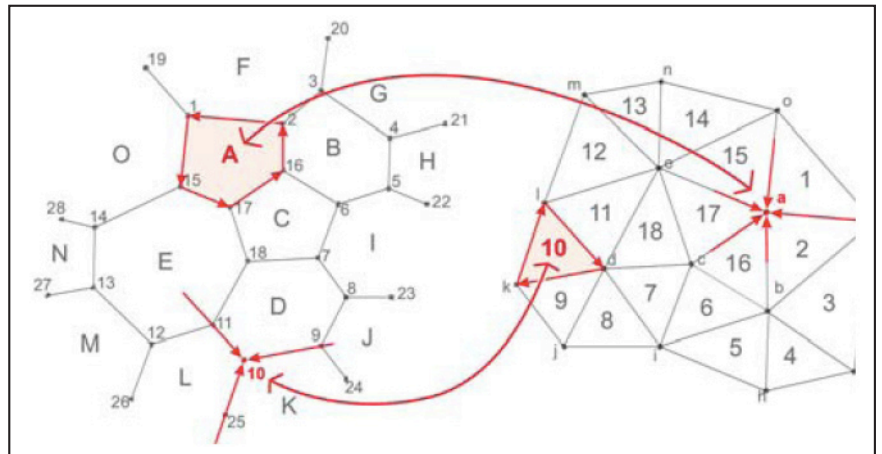Tom van Swinderen, August 2009

APPENDICES

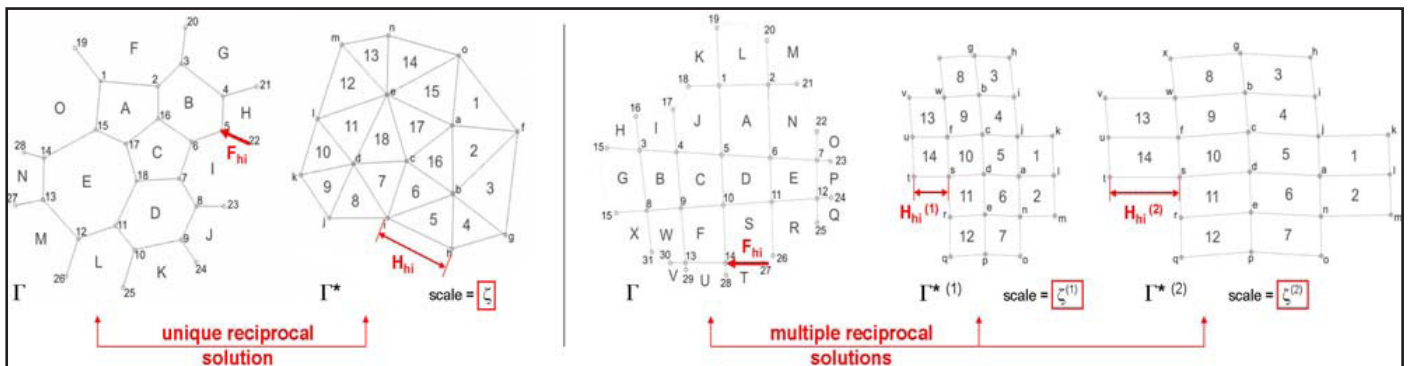**Figure A-1** - *The two-way relation between primal and dual grid*

Figure A-2 - *Explanation of valency and the influence of the scalefactor*

Appendix A   Reciprocal relationship between the Primal and Dual grid

Description of important elements

First the descriptions of the Primal and Dual grid and the reciprocal relationship are mentioned:

Primal grid

The planar projection of the force network of a 3D model. In other words: the grid consists of the horizontal reflections of the network beams on the ground plane (X-Y plane).

Dual grid

The dual grid is the *reciprocal figure* of the primal grid. The lines connected to a node in the primal grid, form a polygon of forces in the dual grid (Figure A-1). A definition of *polygon of forces* is: *"The sides of a force polygon represent in magnitude and direction a system of forces in equilibrium"* [8].

Reciprocal relationship

These two grids are related with a *reciprocal relationship*. A description of a *reciprocal figure* is defined as: *'Two plane figures are reciprocal when they consist of an equal number of lines, so that corresponding lines in the two figures are parallel and corresponding lines which converge to a point in one figure form a closed polygon in the other.'* [10].

The angle of the network lines are equal in both grids. This is one of the main characteristics to use in switching between the primal and dual grid. One more important observation is the relation of the lengths of the lines in the grids, which depends on the force polygon.

Scalefactor ζ

A node is connected with lines to a certain number of other nodes. The amount of neighbouring nodes, and so number of lines, is referred to with the *valency* of that node and grid.

If a primal grid has valency 3, it is said to be determinate. As a result the Dual grid has only got one unique solution (situation at the left in Figure A-2).

When the valency is higher than three, several solutions are possible for the Dual grids, all representing different possible distributions of the internal forces in the system. A solution is determined with the scalefactor ζ (situation at the right in Figure A-2).
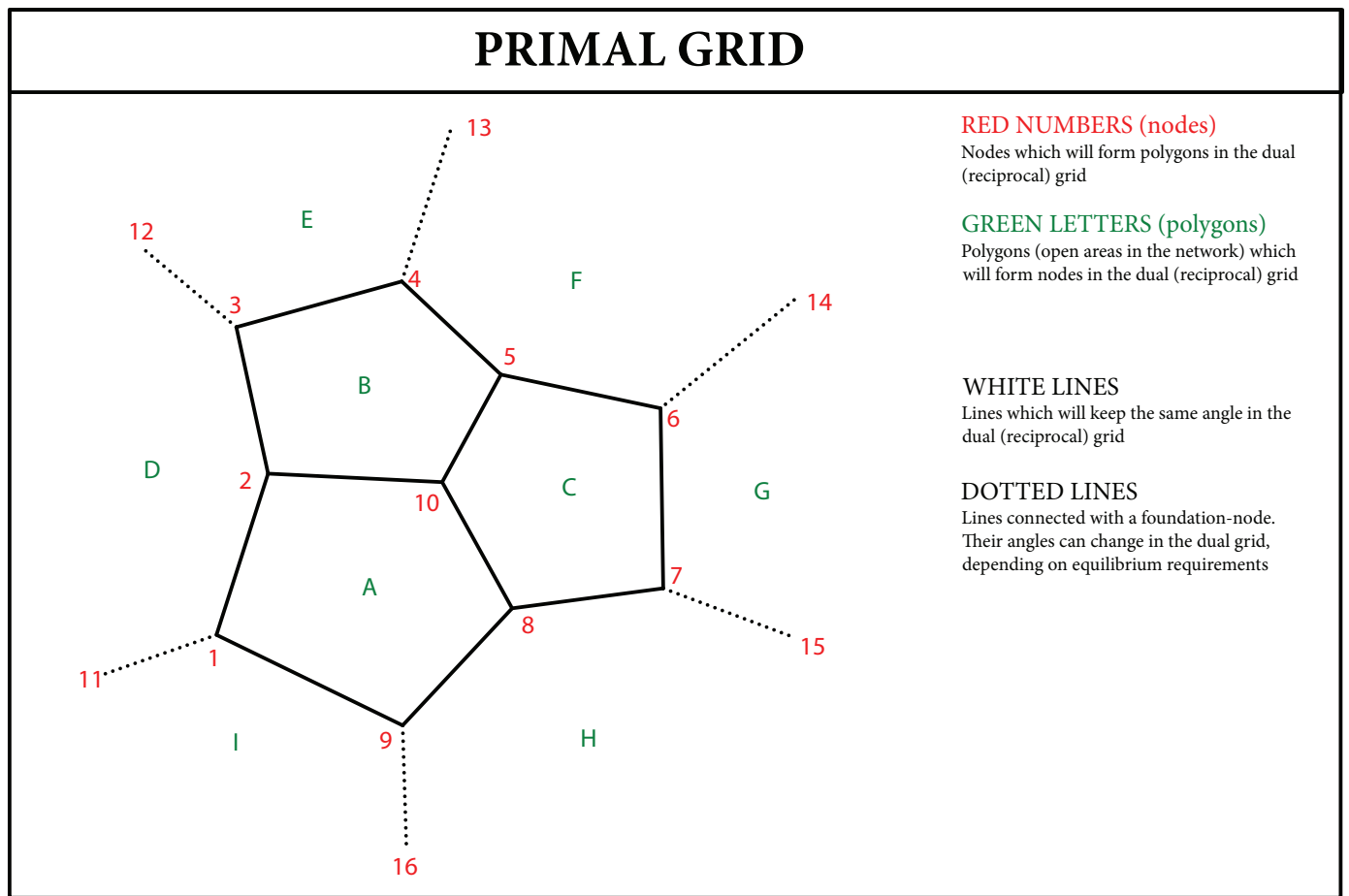
PRIMAL GRID

Figure A-3 - Primal grid, which is used as example to show the steps

5 Choose any other node related to an earlier inspected node (as long as it is NO foundation node)   In this case: node 5

α 1 = 137°
α 2 = 241°
α 3 = 348°

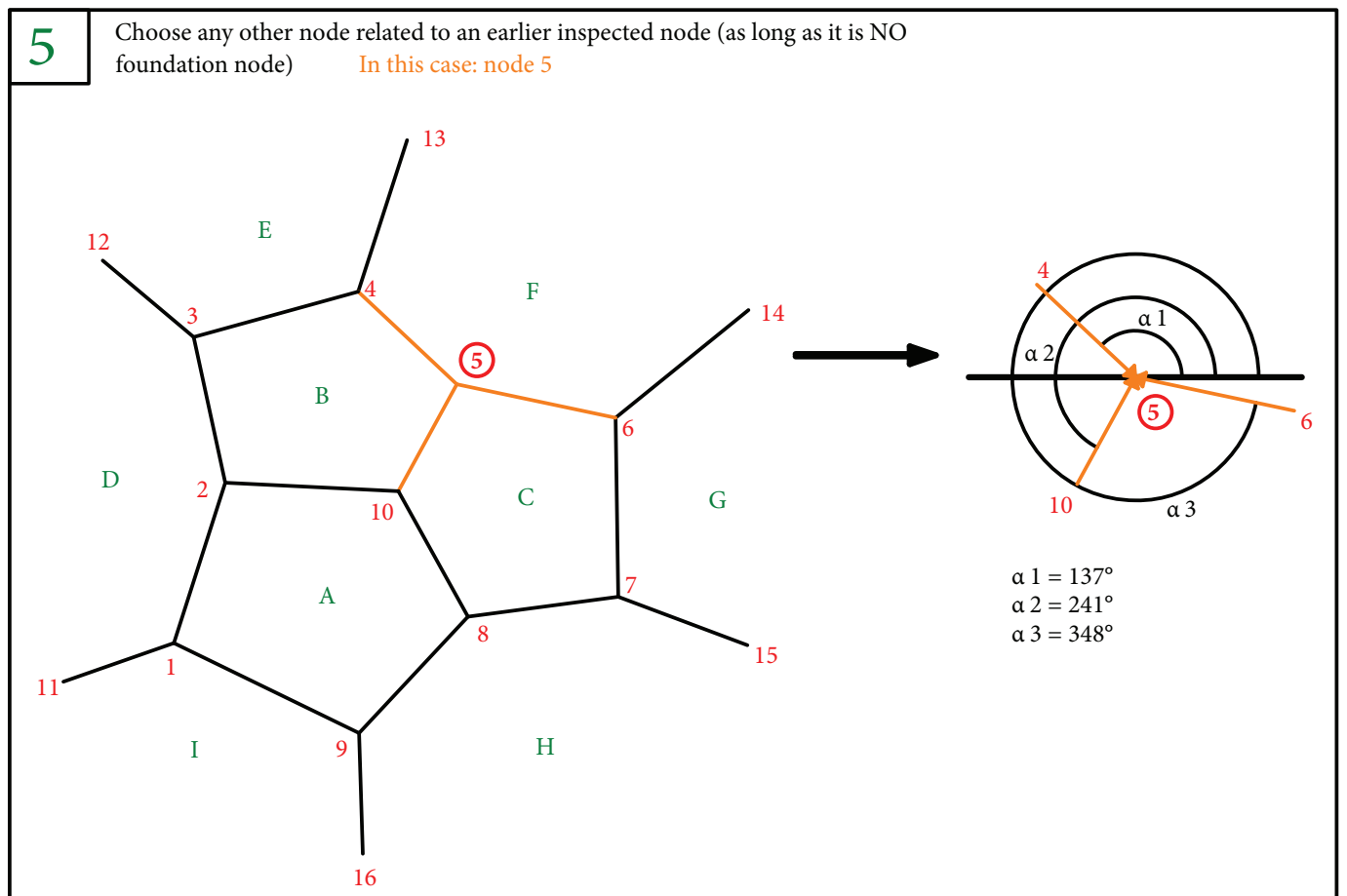*Figure A-4 - Find lines related to a node. In the case: of node 5*

The process behind the reciprocal relationship

General remarks

An important aspect is the dependency of all inner lines – lines which are not connected to a foundation node – to each other. Every line is connected to two nodes, and so to two force polygons. The force related to that line must be the same in both polygons. Therefore the force polygons of the inner nodes are all dependent on each other.

The consequence of this is:

- Nodes with valency of three will form a triangular polygon in the dual grid. A triangle is always a fixed shape. As a result the dual grid has only got one solution and can not be scaled with the scalefactor;

- Nodes with higher valency will form a polygon of at least four sides. This means the shape is not fixed, and so the dual grid can have several solutions.

Foundation nodes and lines connected to them

As mentioned before, foundation nodes are different than normal nodes. This is caused by certain limits imposed on the foundation nodes.

The closed force polygons of the nodes without lines connected to foundation nodes are restricted to change position and angle, because they are dependent on other force polygons. Though the lines connected to the foundation nodes, and the foundation nodes themselves are not bounded and dependent. This will be explained and shown in step 5 of the process.

The consequence of this is that the force polygons of the nodes, which have a line connected to a foundation node, are not closed yet and have a range of possible solutions. Therefore the process has to be started with another node that has not got any line connected to a foundation node.

Where to start

It does not matter where to start with creating the dual grid for 3D models with nodes of valency three. The same result is obtained always. Though for models with higher valency nodes, the result can be influenced by the used scale. Nevertheless it also does not matter where to start with creating the 3D model, since still the dependency of nodes is valid and active.

The steps of the process

1.   For every single node (excluding the foundation nodes): find the lines connected to the node in the primal grid

2.   Make a polygon of these lines, by adding one to the end of the other. Start with any line and add the next, as it is the next in clockwise order around the node in the primal grid

3.   If any of the lines of the regarded node is not connected to a foundation node: close the force polygon, by finding the intersection point of the two lines that are not connected yet.

4.   Position and scale all polygons

5.   Determine the range of possible angles for the foundation lines and set it at the midpoint of this range

Explanation of steps

The steps are best explained using an example. A random primal grid has been created (Figure A-3) and the dual grid will be formed using the following steps:

1.   Find the lines connected to the node in the primal grid

For each node, check which lines are connected to it. Remember these lines and their direction and lengths for step 2 (Figure A-4).
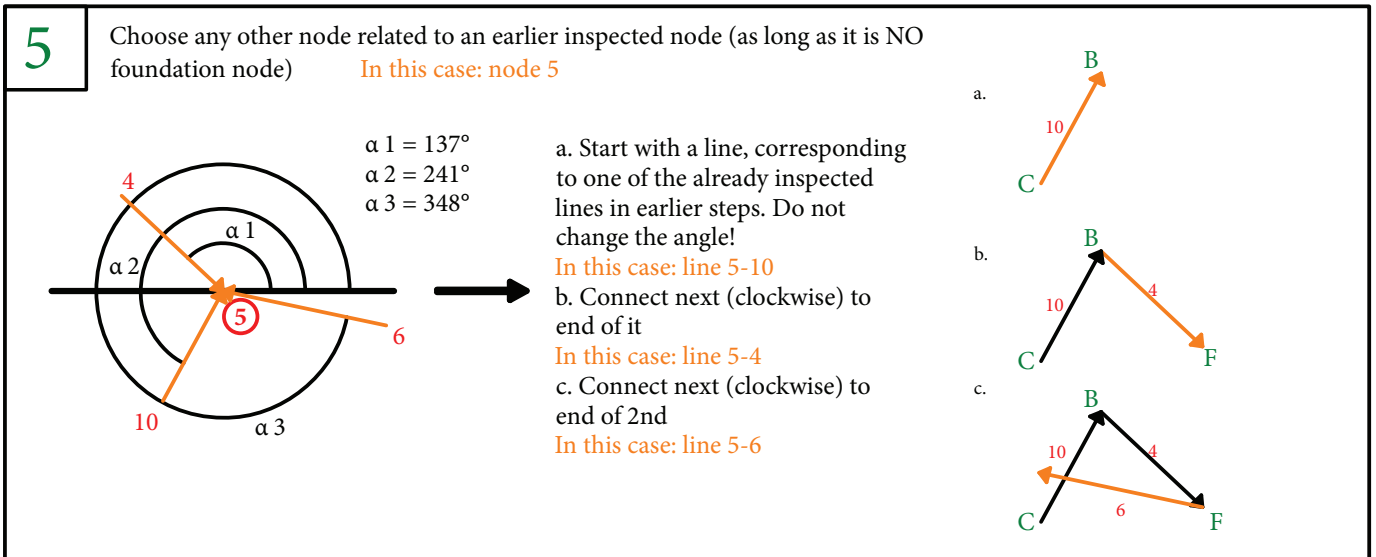
5 Choose any other node related to an earlier inspected node (as long as it is NO foundation node)   In this case: node 5

α 1 = 137°
α 2 = 241°
α 3 = 348°

a. Start with a line, corresponding to one of the already inspected lines in earlier steps. Do not change the angle!
In this case: line 5-10
b. Connect next (clockwise) to end of it
In this case: line 5-4
c. Connect next (clockwise) to end of 2nd
In this case: line 5-6

Figure A-5 - Form polygon by placing the lines in clockwise order

5 Choose any other node related to an earlier inspected node (as long as it is NO foundation node)   In this case: node 5

α 1 = 137°
α 2 = 241°
α 3 = 348°

Find intersection between 1st and last line, and calculate scaling factor of the lines to form the polygon (to use in the other steps of the process)

RESULT
Force polygon.
Angles are the same and have to stay the same!
Length proportion is known, BUT can still change.
This will be seen when regarding the other nodes

NODE 5
Length 4   :   Length 6   :   Length 10
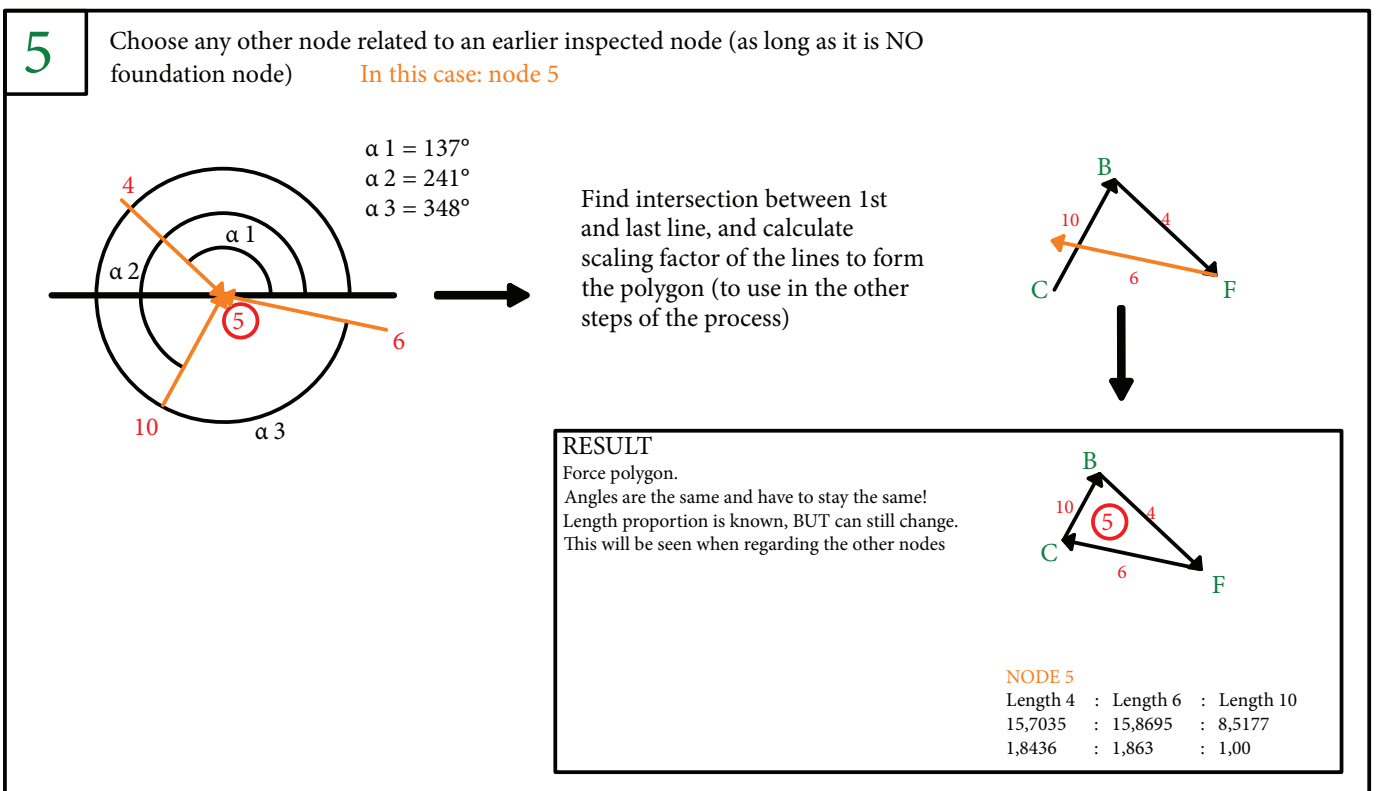15,7035   :   15,8695   :   8,5177
1,8436   :   1,863   :   1,00

Figure A-6 - Close the force polygon by finding the intersection point

2.   Make a polygon of these lines

Create a force polygon for each node. Use the lines obtained in step 1. Start with any of the lines. Use its length and direction. Connect the next line at the end of the first line. One thing is important: the order of the lines to form the polygon, has to be the clockwise order of lines around the original node in the primal grid (Figure A-5).

3.   Close the polygon

The force polygon of step 2 will very likely not be closed. It is a requirement though to have equilibrium of forces in the node. Therefore the polygon has to be closed, which will be done in this step.

The intersection of the two lines is calculated, and the lines will be extended towards this point. As a result, the force polygon is closed and complete (Figure A-6). The ratio of lengths of the lines is in fact the ratio of size of the forces.
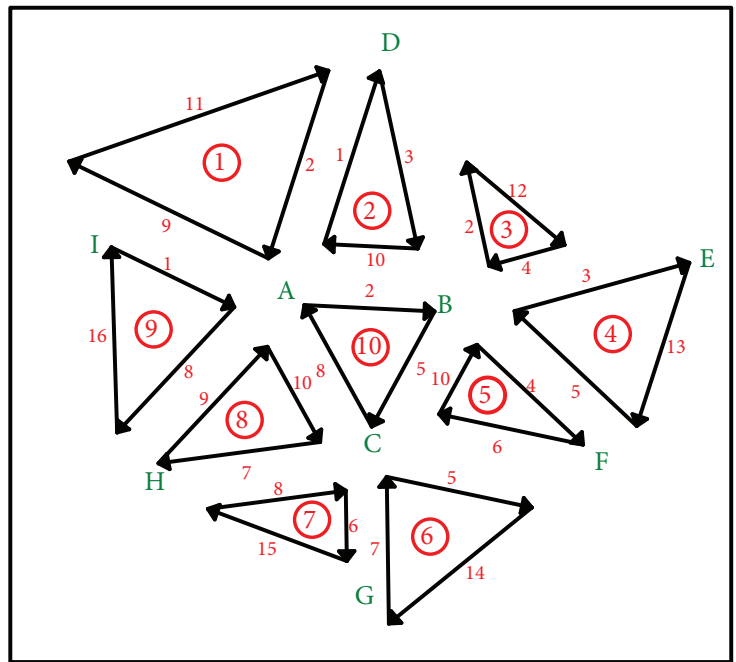

Figure A-7 - Reposition the polygons, so that matching lines are linked

4.   Position and scale all polygons

The force polygons consist of lines that are also present in other polygons. Both lines correspond with the same line-element in the structure though. Therefore the force has to be the same. This is accomplished by positioning the force polygons in such a way, that two lines corresponding to the same network-element are placed next to each other (Figure A-7).

Next step is to make make the lines the same length, by scaling one of the two polygons. This has to be done for every pair of lines, until all pairs are the same length (Figure A-8).

The resulting grid consists of all closed polygons and of several open polygons at the borders, which will be closed in step 5.

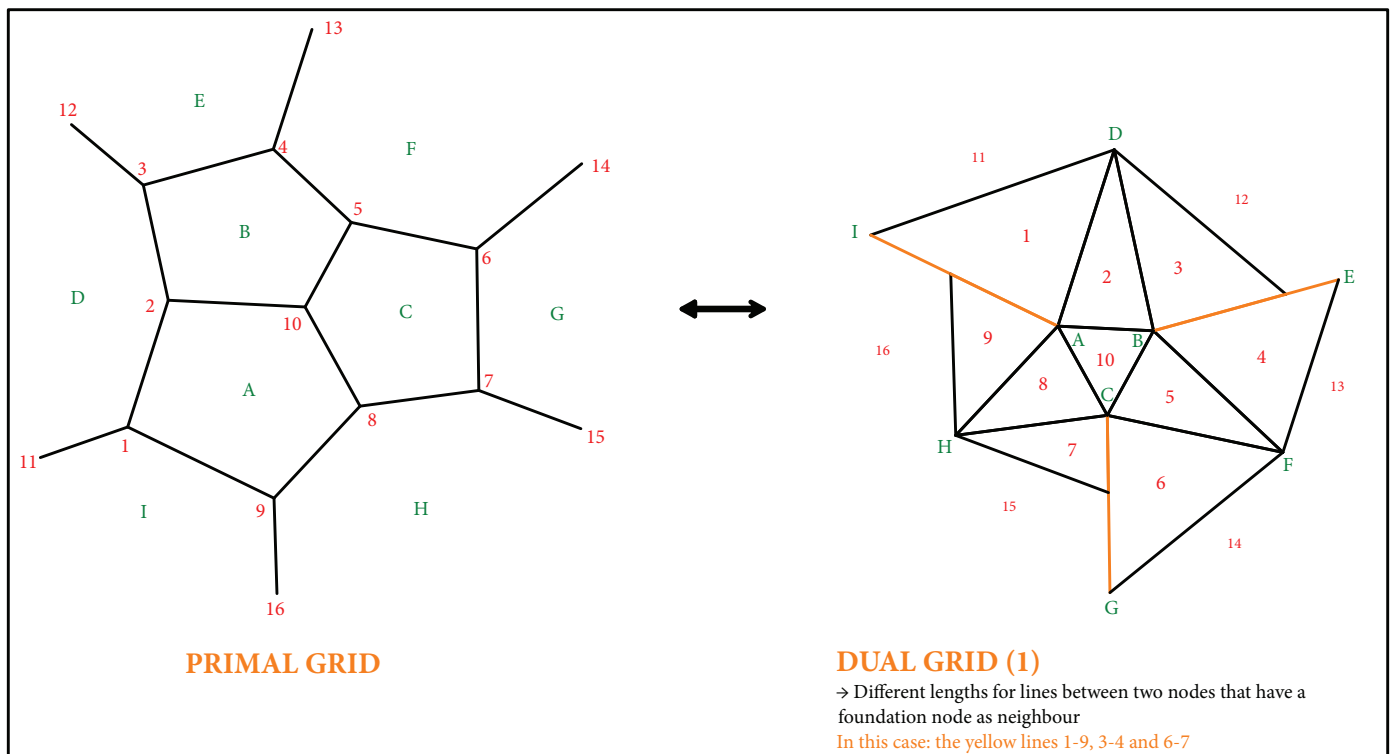**PRIMAL GRID**

DUAL GRID (1)
→ Different lengths for lines between two nodes that have a foundation node as neighbour
In this case: the yellow lines 1-9, 3-4 and 6-7

Figure A-8 - Scale the polygons to match the lenght of the matching lines

1. Ignore angles and lengths of foundation lines
In this case: Lines DE, EF, FG, GH, HI and ID

2. Draw lines with the angle of the lines with unknown lenght -
mentioned in the first dual grid (see above).
In this case: The purple lines with angle of lines 1-9, 3-4 and 6-7

*1+2. Purple working lines - on
which the intersection has to lie -
and the original dual lines, of which
the angle can be changed.*

3. Now we can define a range of possible solutions by adapting the
angle of the foundation lines. This range is restricted by two aspects:
a. The intersection of them must lie on the working line given by the
angle of the line connected to a closed polygon in the primal grid.
b. The length of this line and of the foundation lines may not exceed
a certain value, given by the maximum allowable force in the line
(given by the maximum allowable stress in the brick).

In this case: The yellow lines are the corresponding new angles of
the foundation lines in the primal grid. The length of the purple lines
from starting point till intersection corresponds with the force.

*3b. Maximum lengths
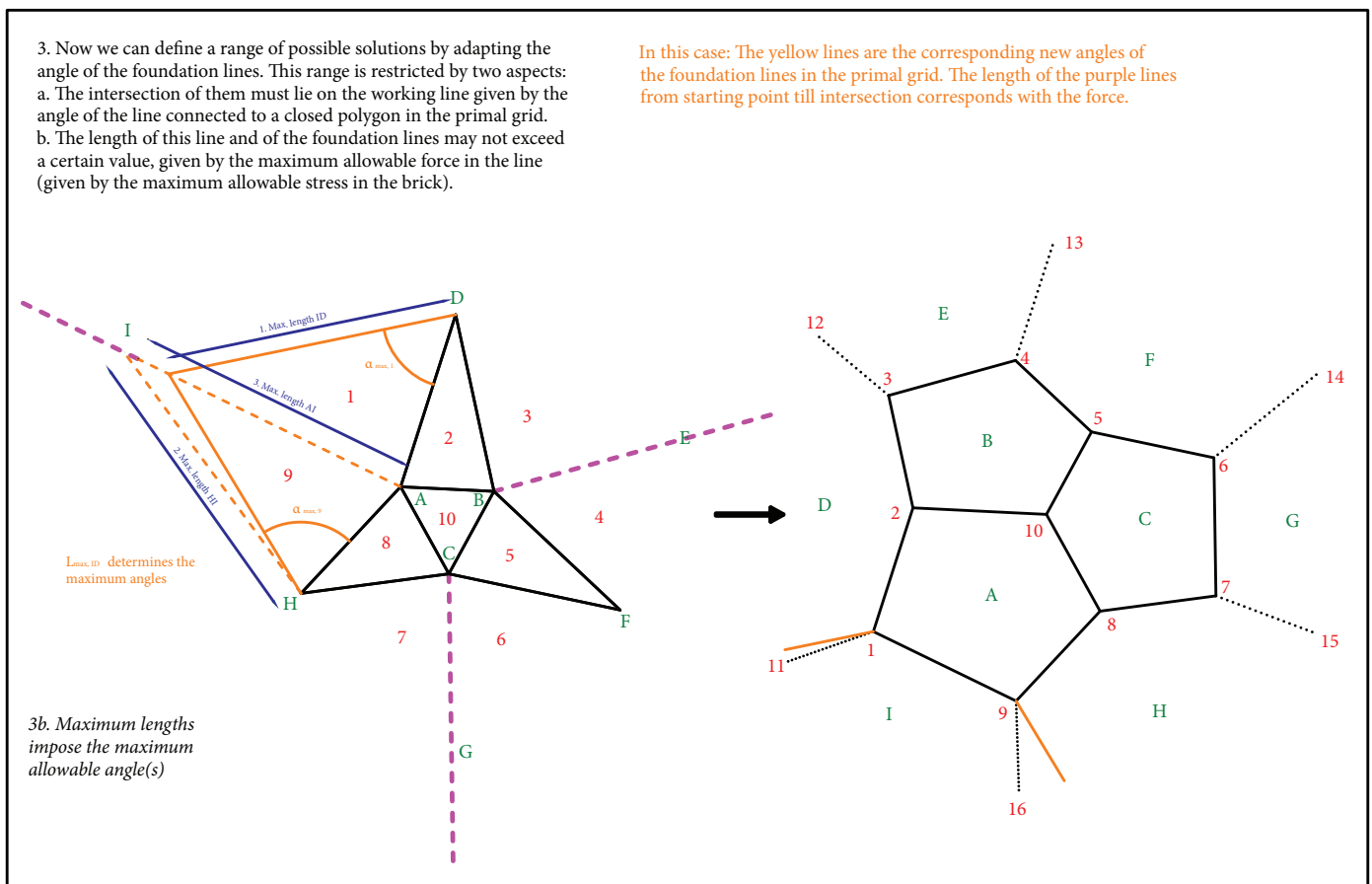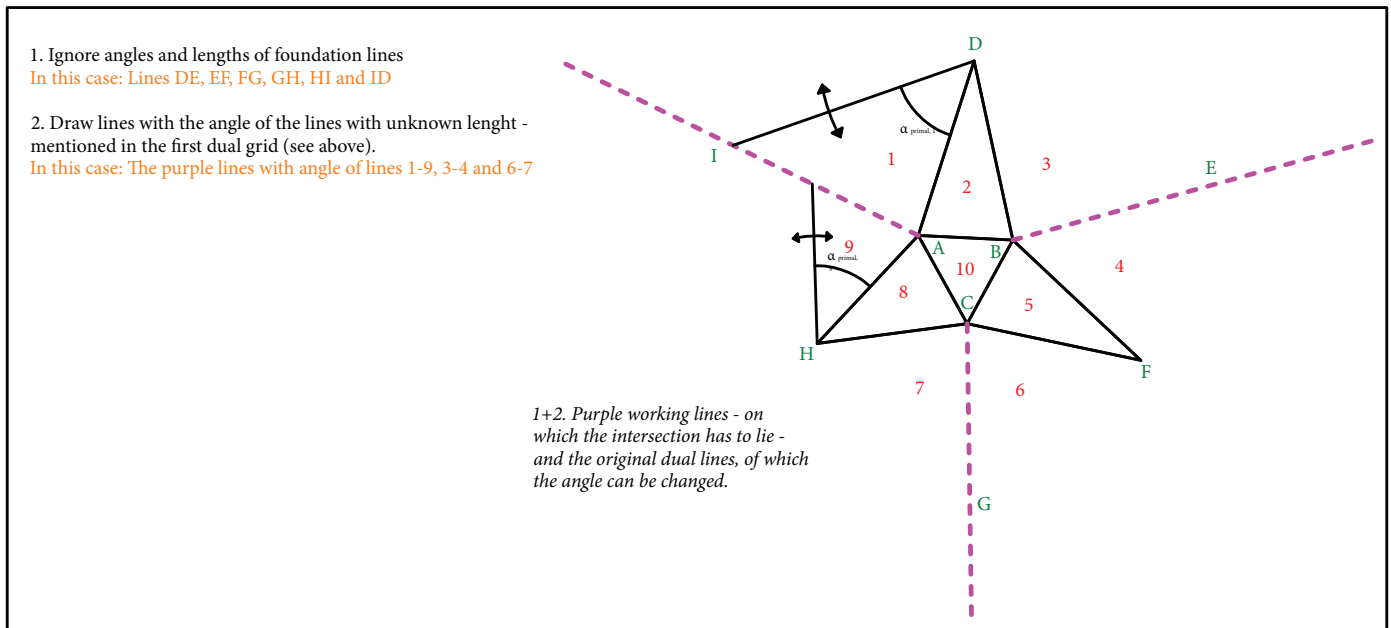impose the maximum
allowable angle(s)*

***Figure A-9a & b** - Show possible range for the foundation lines*

5.   Set foundation lines and nodes

For all nodes that have no closed polygon yet, determine what the maximum angles for the final line can be. This line is in fact the line that is connected to the foundation node in the primal grid. So by changing the angle of the line, the position of the foundation node is changed (Figure A-9). The application places the lines in the midpoint of the possible angle-range.

Final result: the dual grid
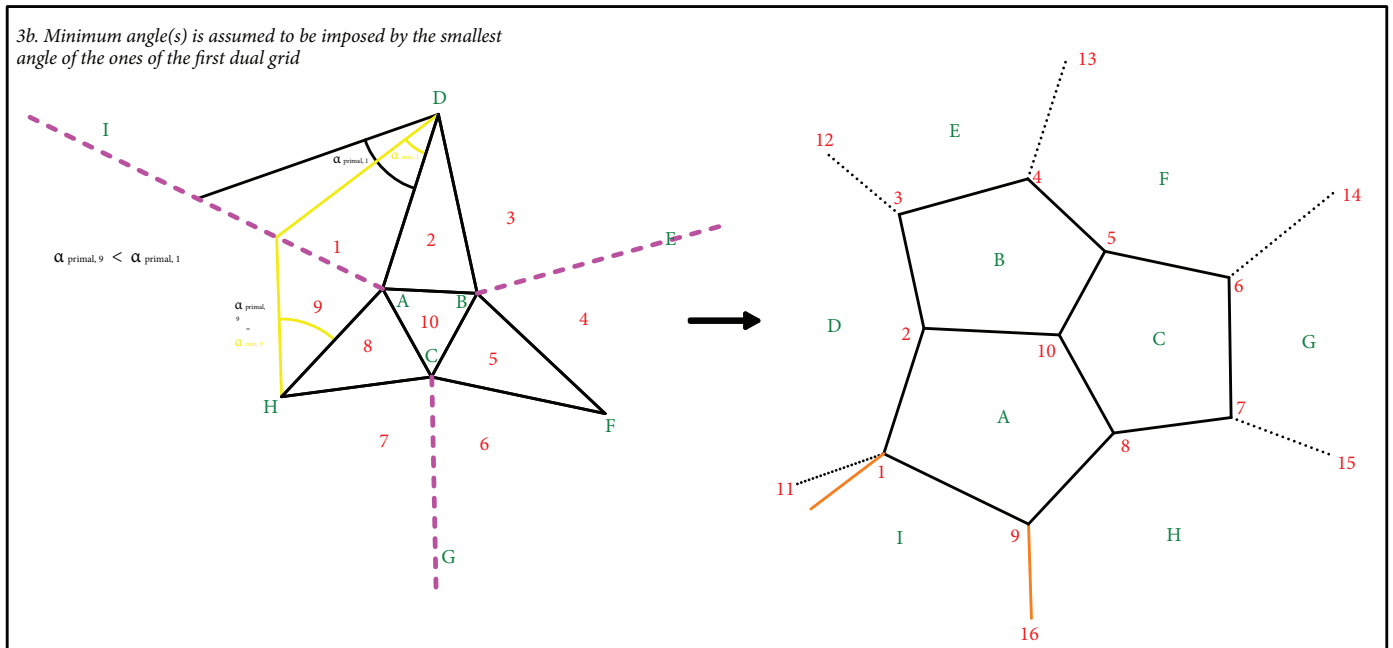
The final result is the dual grid (figure A-10).

3b. Minimum angle(s) is assumed to be imposed by the smallest angle of the ones of the first dual grid

$\alpha_{primal, 9} < \alpha_{primal, 1}$

Figure A-9 c- *Show possible range for the foundation lines*

Intersection has to lie somewhere on this line

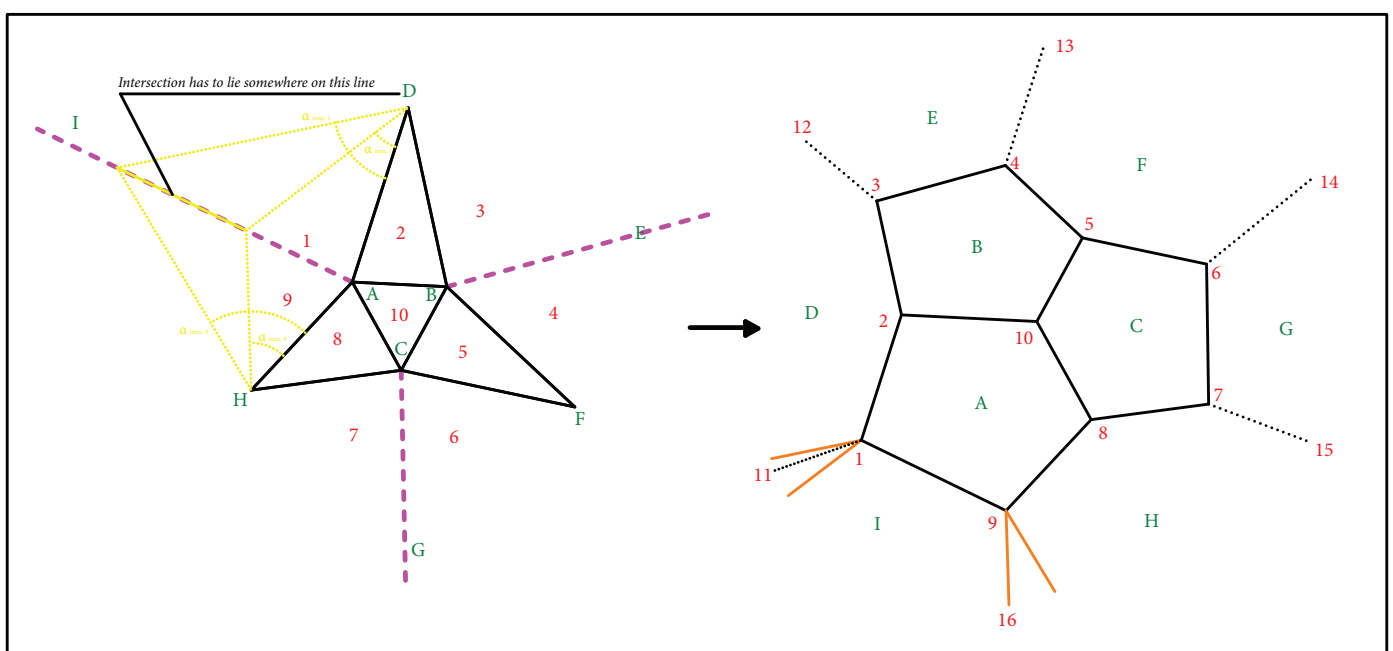**Figure A-10** - *The dual grid*

The starting matrix to be solved:

$$\begin{array}{cccc} \mathbf{x} & \mathbf{y} & \mathbf{z} & \mathbf{rhs} \end{array}$$
$$\begin{bmatrix} 3 & 2 & -4 & 3 \\ 2 & 3 & 3 & 15 \\ 5 & -3 & 1 & 14 \end{bmatrix}$$

The element in row and column 1 is chosen as pivot:

$$\begin{array}{cccc} \mathbf{x} & \mathbf{y} & \mathbf{z} & \mathbf{rhs} \end{array}$$
$$\begin{bmatrix} *3 & 2 & -4 & 3 \\ 2 & 3 & 3 & 15 \\ 5 & -3 & 1 & 14 \end{bmatrix}$$

Process to get all other column-elements zero:

$$\begin{array}{cccc} \mathbf{x} & \mathbf{y} & \mathbf{z} & \mathbf{rhs} \end{array}$$
$$\begin{bmatrix} *3 & 2 & -4 & 3 \\ 2 & 3 & 3 & 15 \\ 5 & -3 & 1 & 14 \end{bmatrix}$$

replace the 3 in R_2C_2, you would take 3(3) - 2(2) = 9 - 4 = 5.
replace the 3 in R_2C_3, you would take 3(3) - 2(-4) = 9 +8 = 17.
replace the 15 in R_2C_4, you would take 3(15) - 2(3) = 45 - 6 = 39.
replace the -3 in R_3C_2, you would take 3(-3) - 5(2) = -9 - 10 = -19.
replace the 1 in R_3C_3, you would take 3(1) - 5(-4) = 3 + 20 = 23
replace the 14 in R_3C_4, you would take 3(14) - 5(3) = 42 - 15 = 27.

The result after the whole process for the 1st pivot:

$$\begin{array}{cccc} \mathbf{x} & \mathbf{y} & \mathbf{z} & \mathbf{rhs} \end{array}$$
$$\begin{bmatrix} 3 & 2 & -4 & 3 \\ 0 & 5 & 17 & 39 \\ 0 & -19 & 23 & 27 \end{bmatrix}$$

After repeating this proces two more times, for the remaining two rows and columns, the final result is:

$$\begin{array}{cccc} \mathbf{x} & \mathbf{y} & \mathbf{z} & \mathbf{rhs} \end{array}$$
$$\begin{bmatrix} 5 & 0 & 0 & 15 \\ 0 & 5 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Figure B-1 - Example of pivoting operations in the Gauss-Jordan theory

Appendix B   Theory of Gauss-Jordan regarding Pivoting

The goal when solving a system of equations is to place the augmented matrix into reduced row-echelon form, if possible. There are three elementary row operations that you may use to accomplish placing a matrix into reduced row-echelon form. Each of the requirements of a reduced row-echelon matrix can satisfied using the elementary row operations.

<u>Pivoting</u>

Pivoting uses row operations (known as Gauss-Jordan row operations) to change one matrix entry (the Pivot) to "1", and to change all other entries in the pivot's column into zero-values.

Select the pivot:

- Pick the column with the most zeros in it;
- Use a row or column only once;
- Pivot on a one if possible;
- Pivot on the main diagonal;
- Never pivot on a zero;
- Never pivot on the right hand side.

Once a pivot is chosen, the row operations of pivoting must be as follows:

Step 1: Make the pivot "1" by dividing the pivot's row by the pivot number;

Step 2: Make the remainder of the pivot's columns into zero-values by adding to each row a suitable multiple of the Pivot Row.

Note

The number changing to "1" is called a *Pivot* and is usually encircled and cannot be zero. If it is zero, then interchange this row with a row below it with a non-zero element in that column (if there is none, then the conversion is impossible).

<u>Row operations</u>

- If there is a row of all zeros, then it is at the bottom of the matrix:
  Interchange two rows of a matrix to move the row of all zeros to the bottom.

- The first non-zero element of any row is a one. That element is called the leading one:
  Multiply (divide) the row by a non-zero constant to make the first non-zero element into a one.

- The leading one of any row is to the right of the leading one of the previous row:
  Multiply a row by a non-zero constant and add it to another row, replacing that row. The point of this elementary row operation is to make numbers into zeros. By making the numbers under the leading ones into zero, it forces the first non-zero element of any row to be to the right of the leading one of the previous row.

- All elements above and below a leading one are zero:
  Multiply a row by a non-zero constant and add it to another row, replacing that row. The point of this elementary row operation is to make numbers into zero. The difference here is that you're clearing (making zero) the elements above the leading one instead of just below the leading one.

<u>Example</u>

On the left page an example of the pivoting operations is given (Figure B-1).
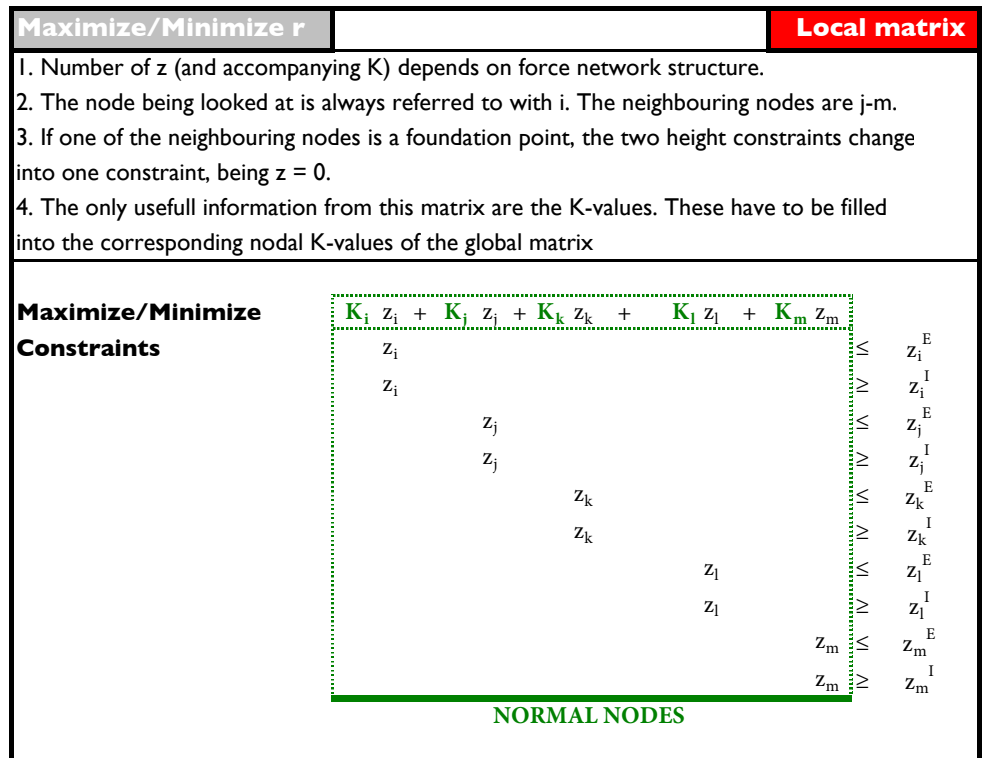
Maximize/Minimize r | **Local matrix**

1. Number of z (and accompanying K) depends on force network structure.

2. The node being looked at is always referred to with i. The neighbouring nodes are j-m.

3. If one of the neighbouring nodes is a foundation point, the two height constraints change into one constraint, being z = 0.

4. The only usefull information from this matrix are the K-values. These have to be filled into the corresponding nodal K-values of the global matrix

Maximize/Minimize Constraints

$$K_i\,z_i + K_i\,z_j + K_k\,z_k + K_l\,z_l + K_m\,z_m$$

$$z_i \leq z_i^E$$
$$z_i \geq z_i^I$$
$$z_j \leq z_j^E$$
$$z_j \geq z_j^I$$
$$z_k \leq z_k^E$$
$$z_k \geq z_k^I$$
$$z_l \leq z_l^E$$
$$z_l \geq z_l^I$$
$$z_m \leq z_m^E$$
$$z_m \geq z_m^I$$

NORMAL NODES

Figure C-1 - Local matrix to solve with the Simplex Method

Maximize/Minimize r | **Global matrix**

1. The K-values are the combined node-corresponding K-values from all local matrices.

…

…

…

Maximize/Minimize Constraints

$$K_1\,z_1 + K_2\,z_2 + \ldots + K_{n-1}\,z_{n-1} + K_n\,z_n \quad + \quad K_{f1}\,z_{f1} + K_{f2}\,z_{f2} + \ldots + K_{fn}\,z_{fn}$$

$$z_1 \leq z_1^E$$
$$z_1 \geq z_1^I$$
$$z_2 \leq z_2^E$$
$$z_2 \geq z_2^I$$
$$\ldots \leq \ldots$$
$$\ldots \geq \ldots$$
$$z_{n-1} \leq z_{n-1}^E$$
$$z_{n-1} \geq z_{n-1}^I$$
$$z_n \leq z_n^E$$
$$z_n \geq z_n^I$$
$$z_{f1} = 0$$
$$z_{f2} = 0$$
$$\ldots = 0$$
$$z_{fn} = 0$$

NORMAL NODES   **FOUNDATION NODES**
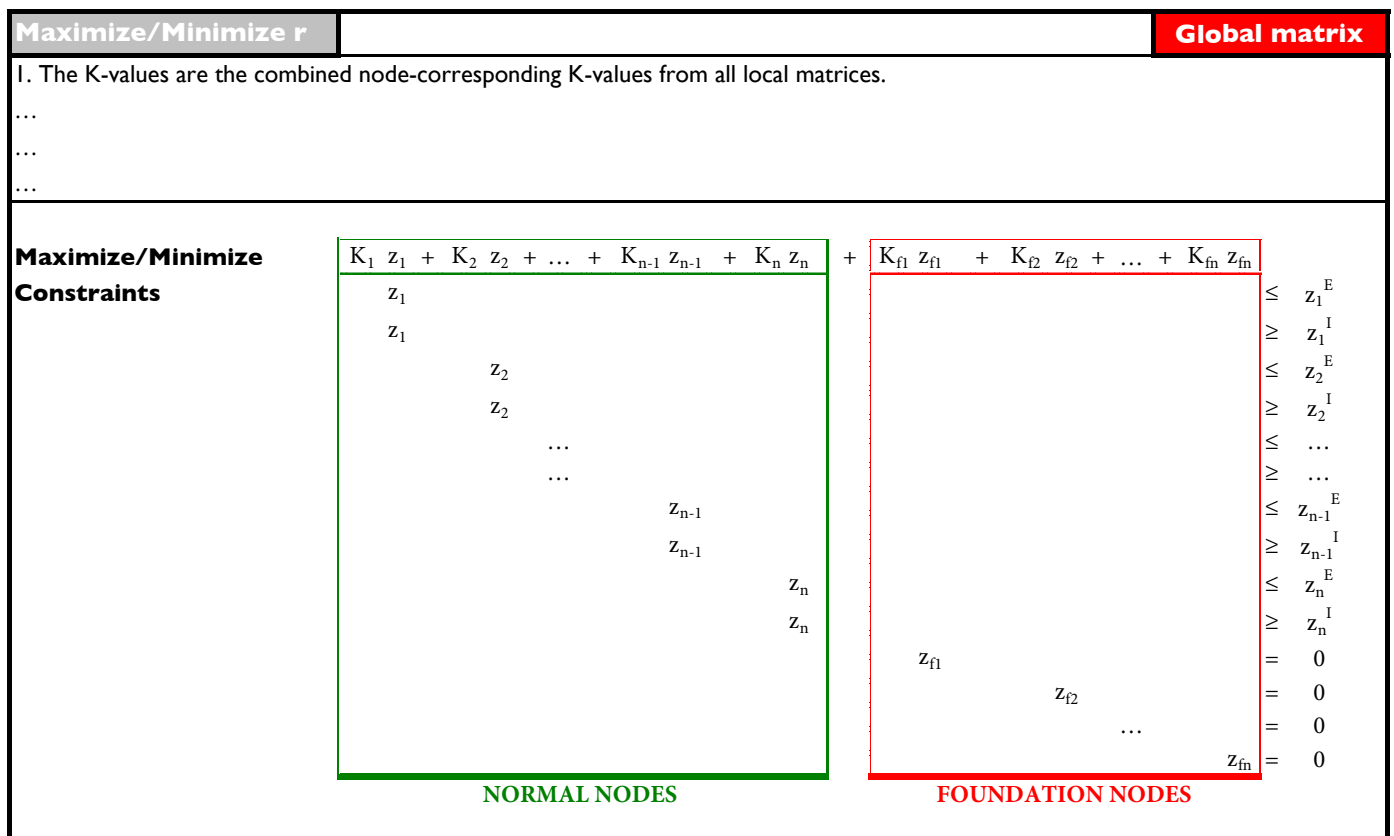
Figure C-2 - Global matrix to solve with the Simplex Method

Appendix C   The Simplex Method

In this appendix is explained how the Simplex method is applied in this research and which steps are taken to solve a matrix with this method. The steps are supported with pictures from Excel.

The following process describes all steps involved in applying the Simplex solution algorithm:
1.   Convert the LP to the a the form that is suitable to use the Simplex method;
2.   Convert all ≤ constraints to equalities by adding a different slack variable for each one of them;
3.   Construct the initial simplex tableau with all slack variables in the BVS. The last row in the table contains the coefficient of the objective function (row Cj);
4.   Determine whether the current tableau is optimal;
5.   If the current BVS is not optimal, determine which nonbasic variable should become a basic variable and which basic variable should become a nonbasic variable. Find the new BVS with the better objective function value.

1.   Convert the LP to the following form
-   Convert the minimization problem into a maximization one (by multiplying the objective function by -1)
-   All variables must be non-negative
-   All RHS values must be non-negative (multiply both sides by -1, if needed)
-   All constraints must be in ≤ form (except the non-negativity conditions). No strictly equality or ≥ constraints are allowed
Since all nodes will be located above the ground, it is assumed that the Z-values of the boundary conditions are always positive.
To obtain this form the following steps are taken:
1.   Start with the matrix (Figure C-1) in which the objective function and boundary conditions are presented
2.   Add variables y to convert ≥ constraints into the ≤ form (Figure C-3).

2.   Convert all ≤ constraints to equalities
Add a different slack variable for each boundary constraint (Figure C-4).

3.   Construct the initial simplex tableau
The matrix that has be transformed into the simplex tableau is the matrix as obtained in the end of step 2 (Figure C-5).
Now make sure that all slack variables are in the BVS. The last row in the table must contain the coefficient of the objective function (row Cj) (Figure C-6).

1. Minimize y_0
2. Maximize r **2**

$$y_0 \qquad\qquad\qquad - y_1 - y_2 - y_3 - y_4 = 0$$
$$r - K_i z_i - K_j z_j - K_k z_k - K_l z_l \qquad\qquad\qquad\qquad = 0$$
$$z_i \qquad\qquad\qquad\qquad\qquad\qquad\qquad \le z_i^E$$
$$z_i \qquad\qquad\qquad\qquad + y_1 \qquad\qquad\qquad \le z_i^I$$
$$z_j \qquad\qquad\qquad\qquad\qquad\qquad\qquad \le z_j^E$$
$$z_j \qquad\qquad\qquad\qquad\qquad + y_2 \qquad\qquad \le z_j^I$$
$$z_k \qquad\qquad\qquad\qquad\qquad\qquad\qquad \le z_k^E$$
$$z_k \qquad\qquad\qquad\qquad\qquad\qquad + y_3 \qquad \le z_k^I$$
$$z_l \qquad\qquad\qquad\qquad\qquad\qquad\qquad \le z_l^E$$
$$z_l \qquad\qquad\qquad\qquad\qquad\qquad + y_4 \le z_l^I$$
$$y_i \ge 0 \quad , i=1\ldots4$$

Figure C-3 - *Step1: convert the $\ge$ constraints into the $\le$ form*

1. Minimize y_0
2. Maximize r **2**

$$y_0 \qquad\qquad\qquad - y_1 - y_2 - y_3 - y_4 = 0$$
$$r - K_i z_i - K_j z_j - K_k z_k - K_l z_l \qquad\qquad = 0$$
$$z_i \qquad\qquad\qquad + x_1 \qquad\qquad\qquad = z_i^E$$
$$z_i \qquad\qquad\qquad - x_2 \qquad\qquad + y_1 \qquad = z_i^I$$
$$z_j \qquad\qquad + x_3 \qquad\qquad\qquad = z_j^E$$
$$z_j \qquad\qquad - x_4 \qquad\qquad\qquad + y_2 \qquad = z_j^I$$
$$z_k \qquad + x_5 \qquad\qquad\qquad = z_k^E$$
$$z_k \qquad - x_6 \qquad\qquad\qquad + y_3 \qquad = z_k^I$$
$$z_l \qquad + x_7 \qquad\qquad = z_l^E$$
$$z_l \qquad - x_8 \qquad\qquad + y_4 = z_l^I$$
$$x_i \ge 0 \quad , i=1\ldots8$$
$$y_i \ge 0 \quad , i=1\ldots4$$

Figure C-4 - *Result after step 2: convert the boundary conditions into equalities by adding extra variables x*

Maximize r **8a**

$$r \qquad - K_i x_2 \qquad - K_j x_4 \qquad - K_k x_6 \qquad - K_l x_8 = K_i z_i^I + K_j z_j^I + K_k z_k^I + K_l z_l^I$$
$$x_1 + x_2 \qquad\qquad\qquad\qquad = z_i^E - z_i^I$$
$$z_i \qquad - x_2 \qquad\qquad\qquad\qquad = z_i^I$$
$$x_3 + x_4 \qquad\qquad\qquad = z_j^E - z_j^I$$
$$z_j \qquad - x_4 \qquad\qquad\qquad = z_j^I$$
$$x_5 + x_6 \qquad\qquad = z_k^E - z_k^I$$
$$z_k \qquad - x_6 \qquad\qquad = z_k^I$$
$$x_7 + x_8 = z_l^E - z_l^I$$
$$z_l \qquad - x_8 = z_l^I$$
$$x_i \ge 0 \quad , i=1\ldots8$$

Figure C-5 - *Result after optimizing the matrix of Figure C-4. The y-values are all zero and so have been deleted*

Simplex Tableau   **6**   **8b**

rij	basis	z_i	z_j	z_k	z_l	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
1	r						$-K_i$		$-K_j$		$-K_k$		$-K_l$	$K_i z_i^I + K_j z_j^I + K_k z_k^I + K_l z_l^I$
2	x_1					1	1							$z_i^E - z_i^I$
3	z_i	1					-1							z_i^I
4	x_3							1	1					$z_j^E - z_j^I$
5	z_j		1						-1					z_j^I
6	x_5									1	1			$z_k^E - z_k^I$
7	z_k			1							-1			z_k^I
8	x_7											1	1	$z_l^E - z_l^I$
9	z_l				1								-1	z_l^I

Figure C-6 - *The Simplex tableau of figure C-5 (step3)*

4. Determine whether the current tableau is optimal

To do so, check that:

- Are all RHS values are non-negative (called, the feasibility condition)?

- Are all elements of the last row, that is C_j row, non-positive (called, the optimality condition)?

- If the answer to both of these two questions is Yes, than stop. The current tableau contains an optimal solution (Figure C-8).

→ If not and the tableau is not optimal: continue with step 5.

Maximize   r   **8a**

$$r \qquad - K_i\, x_2 \qquad - K_j\, x_4 \qquad - K_k\, x_6 \qquad - K_l\, x_8 = K_i\, z_i^I + K_j\, z_j^I + K_k\, z_k^I + K_l\, z_l^I$$

$$x_1 + \boxed{x_2} = z_i^E - z_i^I$$

$$z_i \qquad - x_2 = z_i^I$$

$$x_3 + x_4 = z_j^E - z_j^I$$

$$z_j \qquad - x_4 = z_j^I$$

$$x_5 + x_6 = z_k^E - z_k^I$$

$$z_k \qquad - x_6 = z_k^I$$

$$x_7 + x_8 = z_l^E - z_l^I$$

$$z_l \qquad - x_8 = z_l^I$$

$$x_i \geq 0 , i=1...8$$

Simplex Tableau   6   **8b**

rij	basis	z_i	z_j	z_k	z_l	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
1	r						- K_i		- K_j		- K_k		- K_l	$K_i z_i^I + K_j z_j^I + K_k z_k^I + K_l z_l^I$
2	x_1					1	1							z_i^E - z_i^I
3	z_i	1					- 1							z_i^I
4	x_3							1	1					z_j^E - z_j^I
5	z_j		1						- 1					z_j^I
6	x_5									1	1			z_i^E - z_k^I
7	z_k			1							- 1			z_k^I
8	x_7											1	1	z_l^E - z_l^I
9	z_l				1								- 1	z_l^I

(a) - Before one optimizing step

Maximize   r   **9a**

$$r \qquad + K_i\, x_1 \qquad - K_j\, x_4 \qquad - K_k\, x_6 \qquad - K_l\, x_8 = K_i\, z_i^E + K_j\, z_j^I + K_k\, z_k^I + K_l\, z_l^I$$

$$x_1 + x_2 = z_i^E - z_i^I$$

$$z_i \qquad x_1 = z_i^E$$

$$x_3 + x_4 = z_j^E - z_j^I$$

$$z_j \qquad - x_4 = z_j^I$$

$$x_5 + x_6 = z_k^E - z_k^I$$

$$z_k \qquad - x_6 = z_k^I$$

$$x_7 + x_8 = z_l^E - z_l^I$$

$$z_l \qquad - x_8 = z_l^I$$

$$x_i \geq 0 , i=1...8$$

Simplex Tableau   7   **9b**

rij	basis	z_i	z_j	z_k	z_l	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
1	r					K_i			- K_j		- K_k		- K_l	$K_i z_i^E + K_j z_j^I + K_k z_k^I + K_l z_l^I$
2	x_2					1	1							z_i^E - z_i^I
3	z_i	1				1								z_i^E
4	x_3							1	1					z_j^E - z_j^I
5	z_j		1						- 1					z_j^I
6	x_5									1	1			z_i^E - z_k^I
7	z_k			1							- 1			z_k^I
8	x_7											1	1	z_l^E - z_l^I
9	z_l				1								- 1	z_l^I

(b) - After one optimizing step

Figure C-7 - *Optimizing the Simplex tableau of figure C-6*

Figure C-8 - *Final result: the optimal Simplex tableau and corresponding values for the variables*

Maximize r						10a
r	=	$K_i \, z_i^E + K_j \, z_j^I + K_k \, z_k^I + K_l \, z_l^I$				
z_i	=	z_i^E				
z_j	=	z_j^I				
z_k	=	z_k^I				
z_l	=	z_l^I				

$\rightarrow$ r   = $\rightarrow$ ζ =

Simplex Tableau   8   **10b**

rij		basis	z_i	z_j	z_k	z_l	
1		r					$K_i \, z_i^E + K_j \, z_j^I + K_k \, z_k^I + K_l \, z_l^I$
2		z_i	**1**				z_i^E
3		z_j		**1**			z_j^I
4		z_k			**1**		z_k^I
5		z_l				**1**	z_l^I

5.   Determine which nonbasic-variable becomes the new basic-variable

If the current BVS is not optimal, determine, which nonbasic variable should become a basic variable and, which basic variable should become a nonbasic variable. To find the new BVS with the better objective function value, perform the following tasks:

-   Identify the entering variable: The entering variable is the one with the largest positive C_j value (In case of a tie, we select the variable that corresponds to the leftmost of the columns).

-   Identify the outgoing variable: The outgoing variable is the one with smallest non-negative column ratio (to find the column ratios, divide the RHS column by the entering variable column, wherever possible). In case of a tie we select the variable that corresponds to the upmost of the tied rows.

-   Generate the new tableau: Perform the Gauss-Jordan pivoting operation to convert the entering column to an identity column vector (including the element in the C_j row).

-   RESULT: the chosen z has become a basis-variable (so exists only once in the tableau) and an y has become a non-basis variable (Figure C-7).

→ Return to step 4

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1. SETUP 3D MODEL	2. CONTROL OVER THE (NETWORK) MODEL	3. BRICK PATTERN	4. EXPORT THE 3D MODEL

IMPORT AN .OBJ FILE
CREATE A PARAMETER MODEL

RECTANGULAR GROUNDPLAN
SPHERICAL GROUNDPLAN

CREATE 3D MODEL MANUAL (RECTANGULAR BASEPLAN)   -

10.77   WIDTH SET TO 10.771429 M

15.11   DEPTH SET TO 15.114286 M

3.00   HEIGHT SET TO 3.0 M

COLUMNS (PARALLEL TO Y-AXIS): 6

ROWS (PARALLEL TO X-AXIS): 5

Z-axis

Appendix F   User Manual
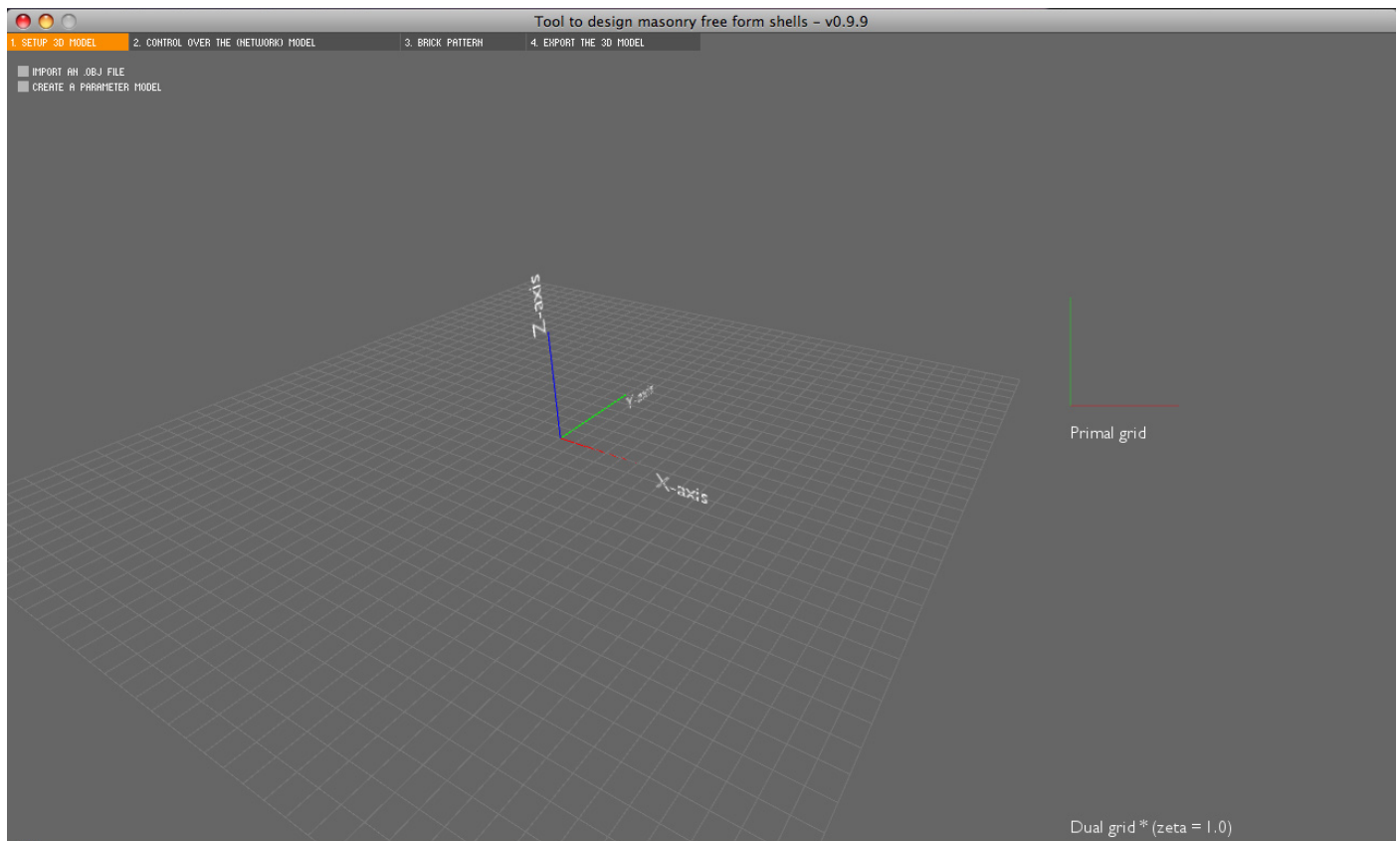
Figure F-0 *- Start screen*

INTRODUCTION

When the application is started, the screen is empty (Figure F-0). In the top of the screen four tabs are found, and several texts and an axis-system are shown.

<u>THE TABS</u>

The four tabs represent the steps to take when you use the application. By default Tab 1. 'Setup 3D model' is activated at start up. It shows two options to create the starting model (Chapter 1). First a model – which is going to be analyzed in the application – has to be created.

After a model has been created, several drawings appear:

- the 3D model in the middle of the screen;
- the corresponding primal grid, and;
- the corresponding dual grid.

All aspects concerning the 3D model and how it is shown are found in Tab 2. 'Control over the network model'. (Chapter 2)

Another option in the application is to change the force network model. For instance the position of nodes is adaptable and lines can be removed or added. (Chapter 3)

When the force network model has been analyzed and optimized and the result is satisfying, a brick pattern is created. This is coordinated in Tab 3. 'Brick pattern'. (Chapter 4)

After this the design elements of this application are finished; all steps to create, analyze and optimize a model and corresponding brick pattern are completed. The last step is to export specific layers of the model, to be able to use them in the continuation of the design process. In Tab 4. 'Export the 3D model' the options are presented for exporting the model. (Chapter 5)

! Pay attention !

Several important remarks:

- After changing any aspect regarding the model, the model is updated immediately and the analysis will be done automatically. The new results are presented. There is no option to return to the old situation. Therefore be careful when changing and adapting element;
- When you want to import a model, the .OBJ file has to be placed in the sketchbook of the application.
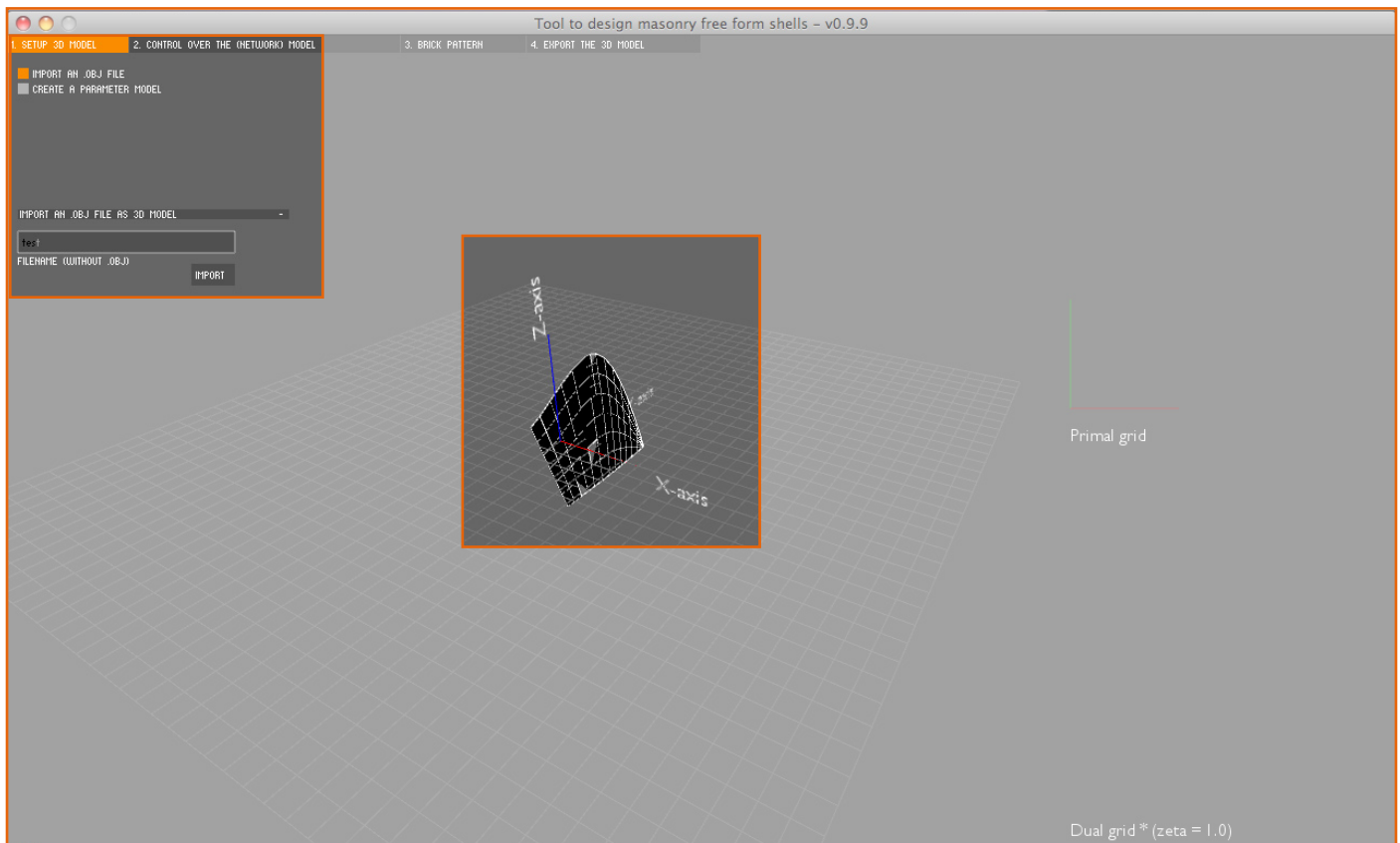
Figure F-1 - Tab 1: Setup model - Option 1: import a model. The imported model is shown with white lines and black polygon surface

1. CREATE A STARTING MODEL

All options to create a starting model, as presented below, are located in Tab 1. 'Setup 3D model'. (Figure F-0)

IMPORT A MODEL

Steps

1. Make a nice free form design in a 3D modelling program, such as Autodesk Maya or Rhinoceros. Since it has to be exported as .OBJ file to be able to import it in the application, make sure the final model is in polygons. If not, transform it into polygon format and save it. After saving the 3D model and exporting it as .OBJ file, it can be imported into the Processing application.

2. Switch the toggle in Tab 1 to 'Import a model'. A textfield and button appear. Input the filename in the textfield and press the button 'Import'.

The model can be shown as line-model (press l) or as polygon model (showing polygon surfaces) (press p) (Figure F-1)

The .OBJ file consists of vertices (v) (x, y and z coordinate), vertice-textures (vt) and vertice-normals (vn). In addition the polygons are added (f) by using v, vt and vn for every corner-node.

CREATE A PARAMETER MODEL

Two different types of models can be chosen from with the second couple of radio buttons in Tab 1:

1. Rectangular grid and rectangular groundplan;

2. Spherical grid and elliptical groundplan.

Both models are based on five parameters, of which three are the same for both and two parameters are model specific. The parameters have a preset value, but can be adapted to the values specified by the user with the sliders and knobs (Figure F-2). When a switch is made from one type of model to the other, the three common parameters will remain the same. This assures the ability to see the difference of shape while having the same shape dimension parameters.

The three common parameters are:

1. Width of the base;

2. Depth of the base;

3. Height of the model.

The two rectangular network model specific parameters are:

4. Number of networklines in X direction;

5. Number of networklines in Y direction.

The two spherical network model specific parameters are:

4. Number of (horizontal) rings;

5. Number of (vertical) slices/bays.

IMPORTED AND PARAMETER MODEL

It is possible to work with a imported model and parameter model at the same time. This is intended to approximate the imported shape with a parameter model. In the situation where both an import model and parameter model are loaded, a third radiobutton group appears in Tab 1. (Figure F-3)
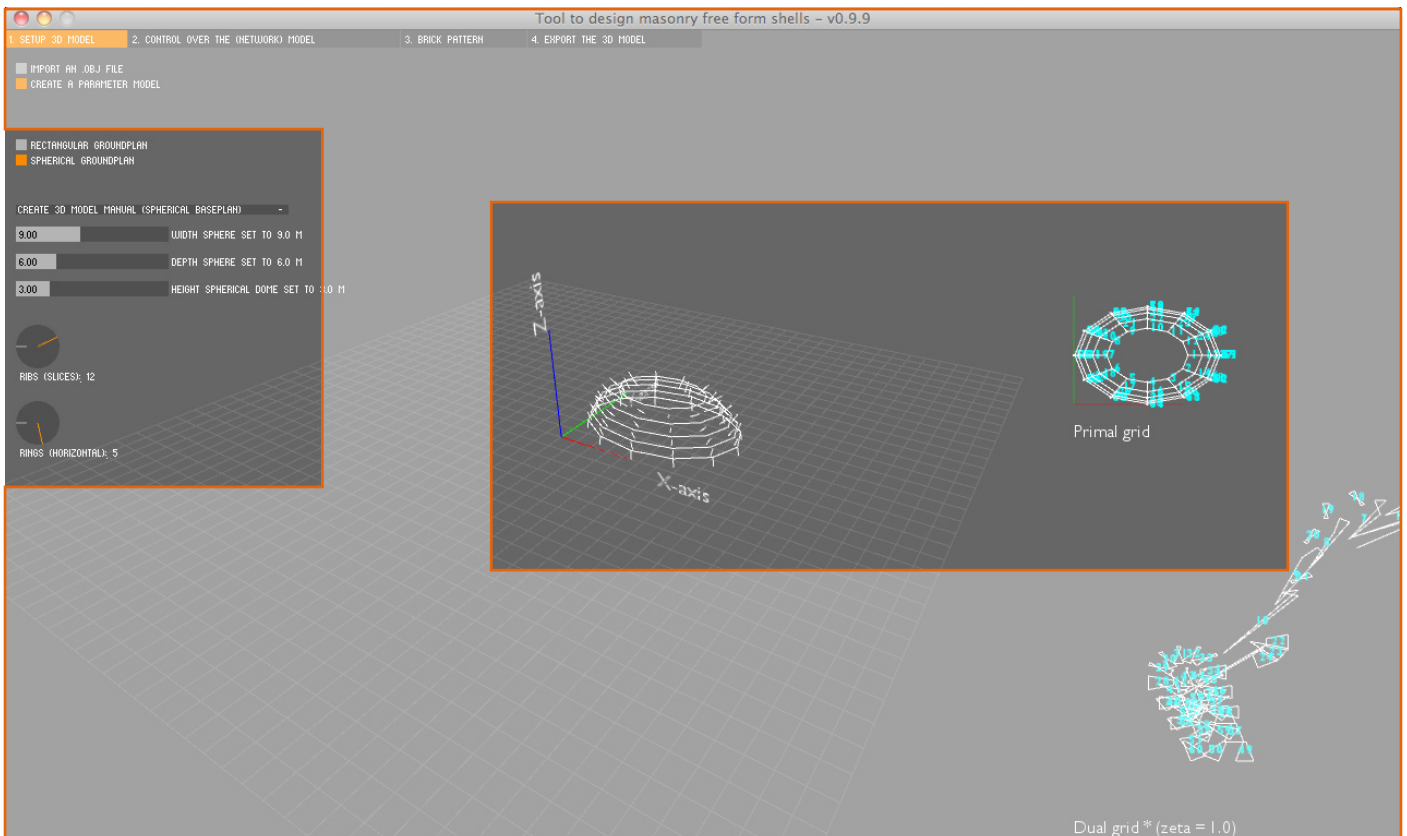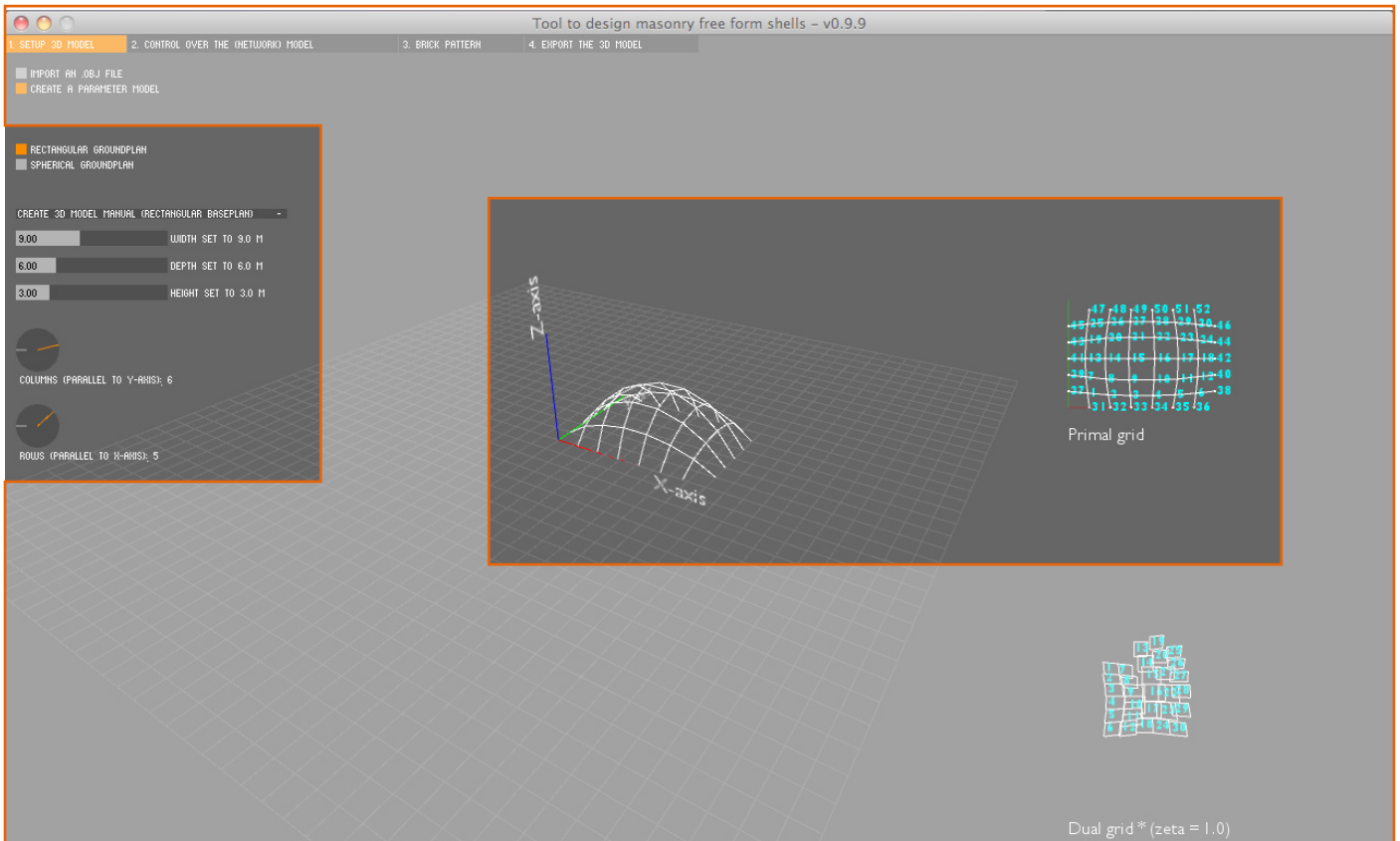
Figure F-2 - Tab 1: Setup model - Option 2: Create a parameter model. Either a rectangular grid (top picture) or a spherical grid (bottom picture)

With it you can select which model you want to see:

- Only the imported model;

- Only the parameter model;

- Both models.

! Pay attention !

Every time any of these five parameters is changed or the type of model is switched, the model will be reset according to the updated five parameters. As a result any changes made to the current model by hand (for instance adapting loading or relocate nodes) are lost.
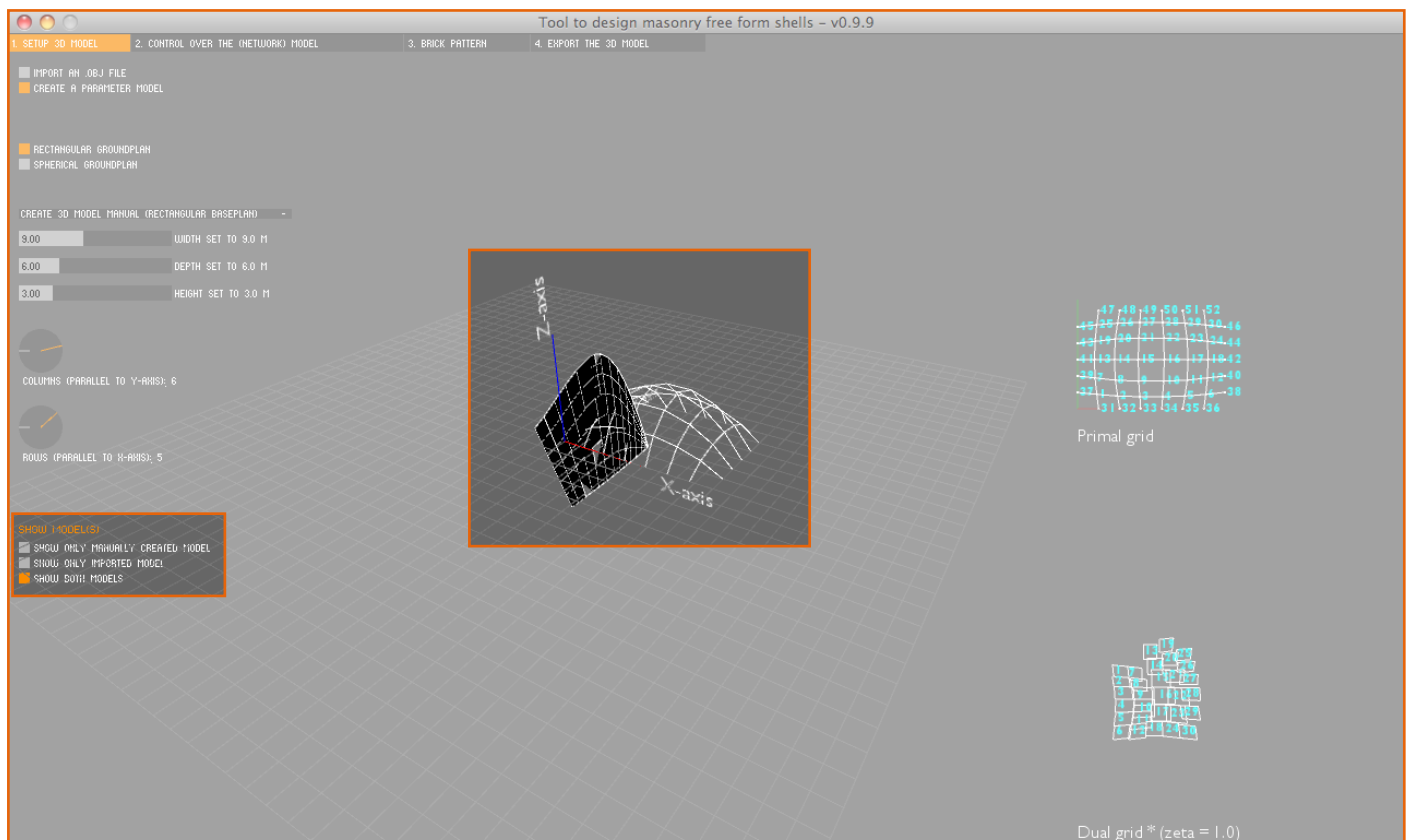
Figure F-3 - *Tab 1: Setup model - Choose which model is shown: the importmodel, the parameter, or both (in the displayed situation both are shown)*
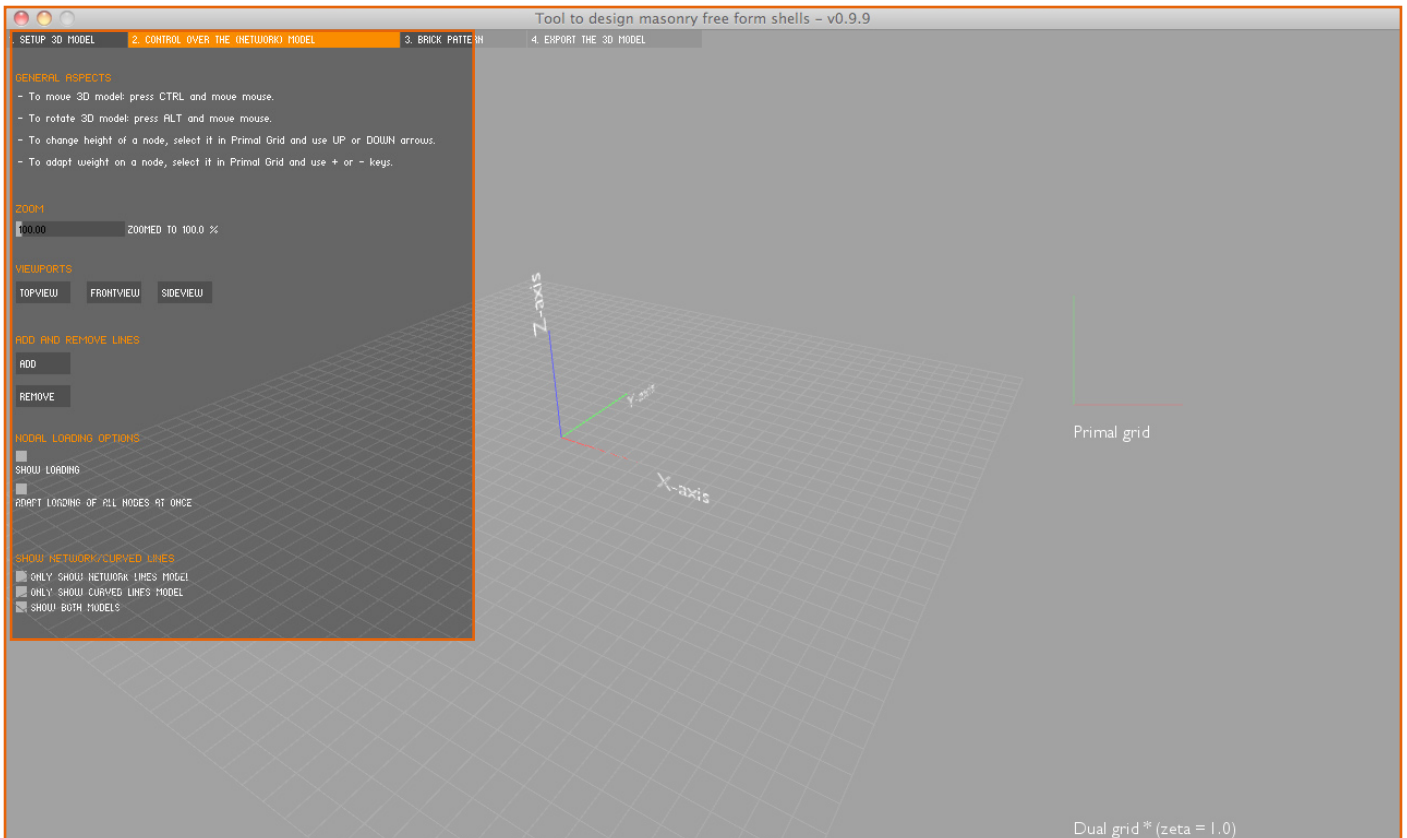
Figure F-4 - *Tab 2: adapt the appearance of the 3D model*
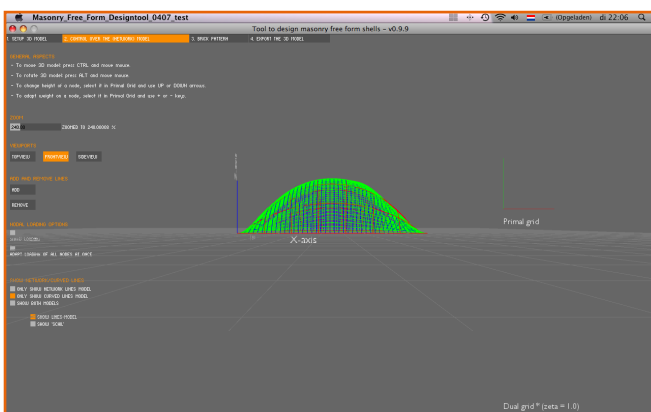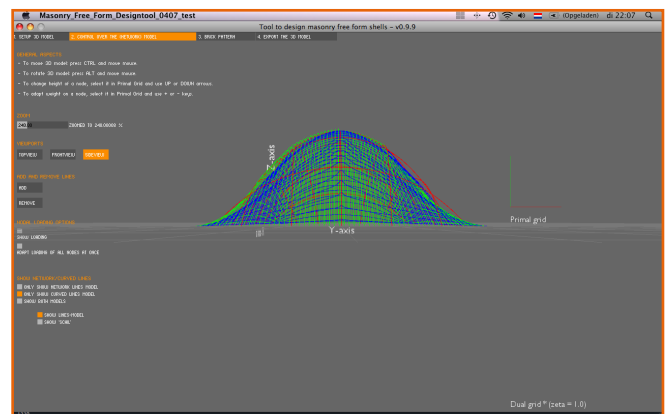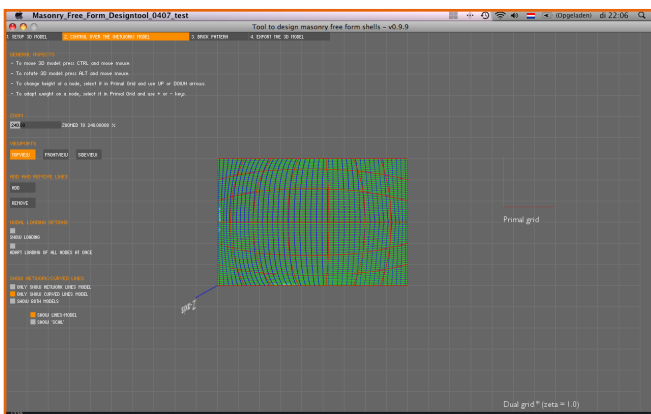
Figure F-5 - *Tab 2: Three viewports - topview (topleft), frontview (bottomleft) and sideview (right)*

2.   CHANGE THE VIEW OF THE 3D MODEL

Most of the options to change the view of the 3D model, as presented below, are located in Tab 2. 'Control Network Model'. (Figure F-4)

PAN

This refers to the ability to relocate both the origin and the model, as is possible in AutoCAD and other 3D modeling software. This is achieved by clicking the left mouse button (LMB) while holding the SHIFT-key. At that moment the angle of the 3D model is frozen while the origin, together with the model, can be relocated by dragging the mouse.

ROTATE

This refers to the ability to rotate both the origin and the model, so that the 3D model can be looked at from every point of view that is wanted. The same is possible in most 3D modeling software like AutoCAD. It is achieved by clicking the LMB, while holding the CTRL-key. When the mouse is now dragged, the model rotates.

ZOOM

This refers to the ability to zoom in and out on the 3D model. There are two options to zoom:

Using a slider

1. Go to Tab 2. 'Control Network Model'.
2. Under the heading 'Zoom': move the slider until it reached the percentage you want to zoom the 3D model to. The zoom-range has been set from 50% - 1000%.

Using the mouse

Hold down the LMB in combination with the Z-key. When the mouse is moved downwards, the model will be zoomed in, and the the camera will zoom out the model when the mouse if moved in upward direction.

VIEWPORTS

This refers to the ability to select three predefined views for the 3D model (Figure F-5).

1. Go to Tab 2. 'Control Network Model';
2. Under the heading 'Viewports': click on of the three buttons Topview, Frontview or Sideview to show respectively the top, front or side view of the 3D model.

VISIBILITY LOADING

The loading is shown by arrows. The longer the arrow, the higher the loading (Figure F-6).

1. Go to Tab 2. 'Control Network Model';
2. Switch the toggle-button 'Show/Hide Loading'.

DISPLAY NETWORK OR SURFACE

1. Go to Tab 2. 'Control Network Model';
2. With the radiobuttons, choose whether to show (Figure F-7):
    -   The network model;
    -   The surface model;
    -   Both the network and surface model.
3. If you choose either option 2 or 3 (to show the surface model) a new set of radiobuttons appears:
    -   Show a curves model (surface made out of curves placed close to each other without filling);
    -   Show polygons (surface made out of black polygons and white borders).
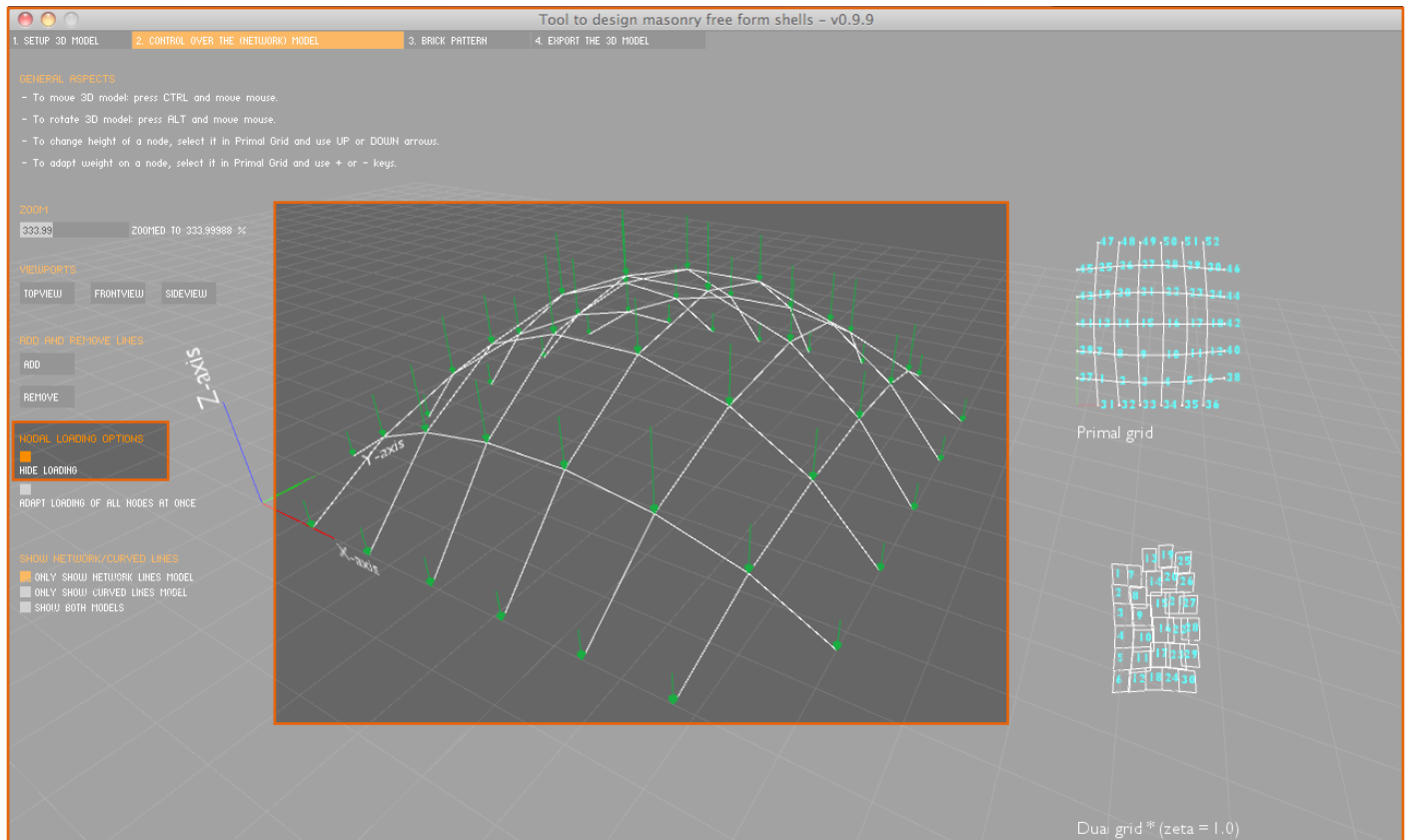
Figure F-6 - *Tab 2: display or hide the loading on the model (green arrows)*
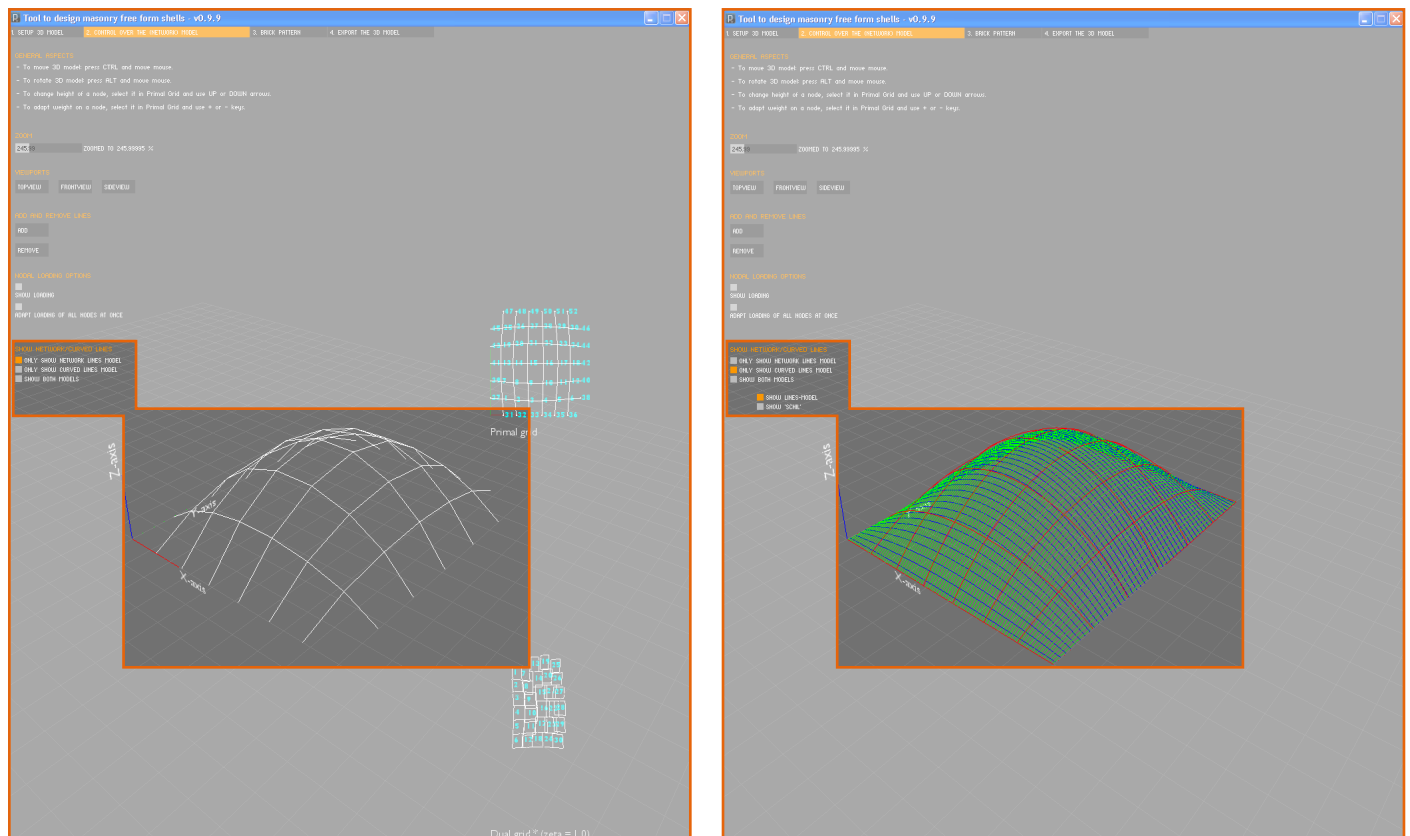

Figure F-7 - *Tab 2: display the force network model (left), the curves-surface model (middle) or the polygon surface model (right)*

There are two more options to change the way the model is shown in the screen:

- In Tab 1: the visibility of the models (in the situation that a model has been imported and a parameter model is active);

- In Tab 3: show or hide the brick pattern.

VISIBILITY MODELS

This refers to either show the imported model, or the parameter-model or both. This last function might be useful if an approximation of the imported model is created with the parameter-model. (Figure F-3)

1. Go to Tab 1. 'Setup 3D model';

2. Under the heading 'Visibility models' select one of the three radio-button options:

    - Only show parameter-model;

    - Only show imported model;

    - Show both models.

! Pay attention !

This option is only available when a model has been imported ánd a parameter model has been created.

VISIBILITY BRICKS

1. Go to Tab 3. 'Brick Patterns';

2. Switch the toggle-button 'Show/Hide Brick'. (located under the four sliders to set the dimension of the brick); (Figure F-10)

3. If you choose to show the brick pattern a new set of radiobuttons appears:

    - Show the lines pattern;

    - Show the shell pattern.
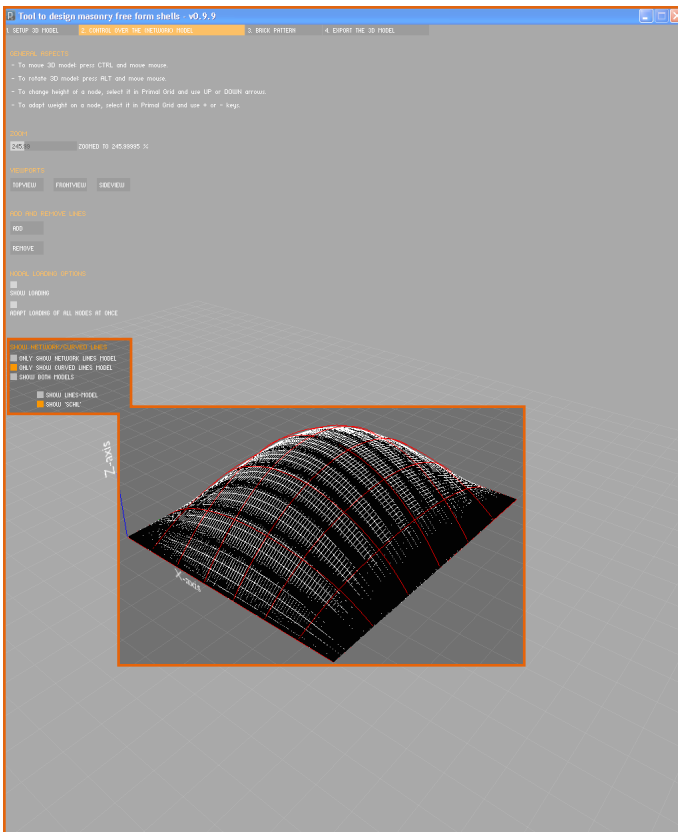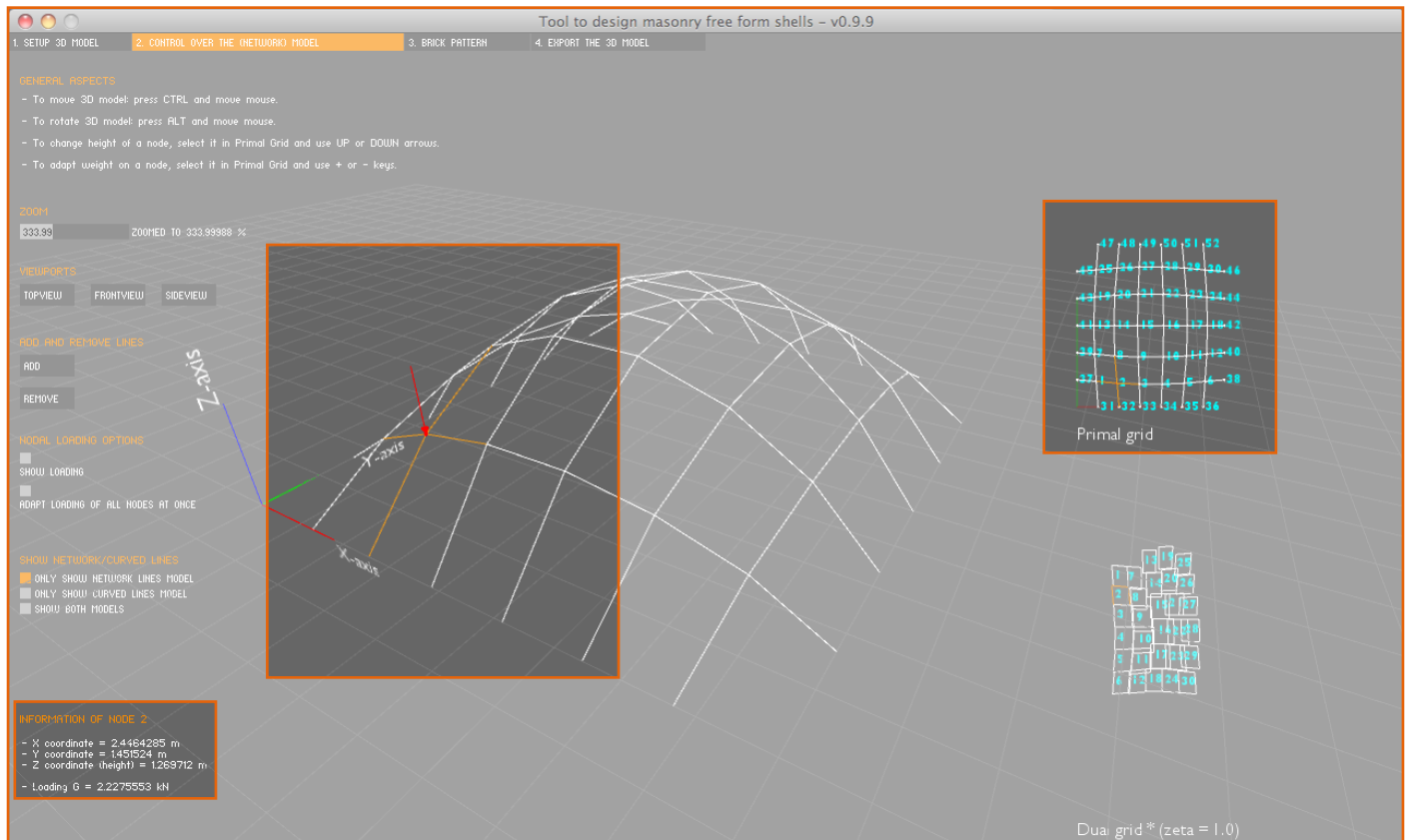
Figure F-8 - *Tab 2 - Control the model and network - Coloured node and lines when a node is selected. At the same time the Node Information Box is shown in the left bottom of the screen*
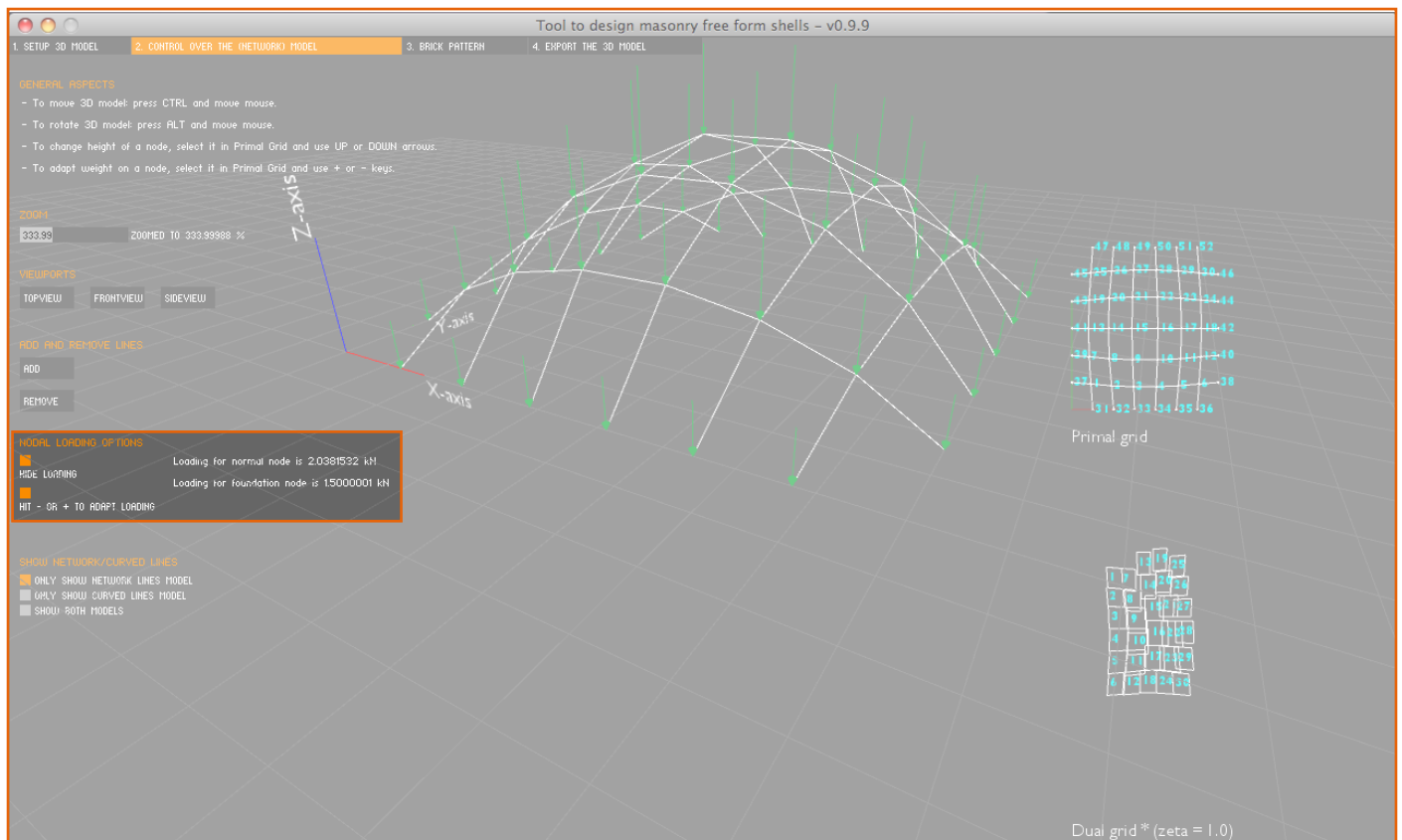

Figure F-9 - *Tab 2 - Control the model and network - adapt loading of all nodes*

3. ADAPT THE NETWORK MODEL

SELECT NODES – The Node Information Box

When a node is selected in the Primal Grid, the Node Information Box (NIB) is shown in Tab 2. 'Control Network Model', left in the bottom of the screen (Figure F-8). This NIB contains:
-   The node number, corresponding with the Primal Grid node numbering;
-   The X, Y and Z coordinates;
-   The loading G.

The selected node will be colored red - both in the Primal Grid and the 3D model. The lines connected to this node will be colored orange, and in the Dual Grid the corresponding polygon is also highlighted in orange (Figure F-8).

MOVE NODES

In X and Y direction by dragging a selected node (in the Primal Grid)
1. Select the node of which you want to change position;
2. Drag the node towards the new position;
3. Release the mouse. The node is still selected, but it will not move anymore.

In Z direction by pressing UP and DOWN keys
1. Select the node of which you want to change height;
2. Increase or decrease the height by pressing respectively the UP or DOWN key. The result is shown immediately in the Node Information box and in the 3D model.

ADAPT LOADING
1. *Adapt loading of all nodes at once* (Figure F-9).
This can be used in the situation when a live load, such as snow, is added or adapted.
a. Switch the toggle 'Adapt loading of all nodes' to 'on'.
b. Use the + and – key increase or decrease the loading of all nodes.
c. When finished, switch the toggle back to 'off'.

2. *Adapt loading of one specific node*
This can be used in the situation when a local load, such as a person or more own weight, is added or adapted.
a. Select the node of which you want to change the loading (Figure F-8);
b. Increase or decrease the loading by pressing respectively the + or – key.

<u>ADD AND REMOVE LINES</u>

<u>Add</u>

This refers to the option to add a line to the model, by selecting two nodes.

1. Go to Tab 2. 'Control Network Model';

2. Under heading 'Add and remove lines': press the button 'Add' once. The text 'Select the first node' is displayed;

3. In the Primal Grid: select the (first) node which is the first node of the to be added line. The text 'Select the second node' is displayed;

4. In the Primal Grid: select the (second) node which is the second node of the to be added line. The new line is shown. The text 'New line: node <nodenumber1> - <nodenumber2>' is displayed.

<u>Remove</u>

This refers to the option to remove a line.

1. Go to Tab 2. 'Control Network Model';

2. Select the line that has to be removed;

3. Under heading 'Add and remove lines': press the button 'Remove' once. The text 'Line <nodenumber1> - <nodenumber2> deleted' is displayed.

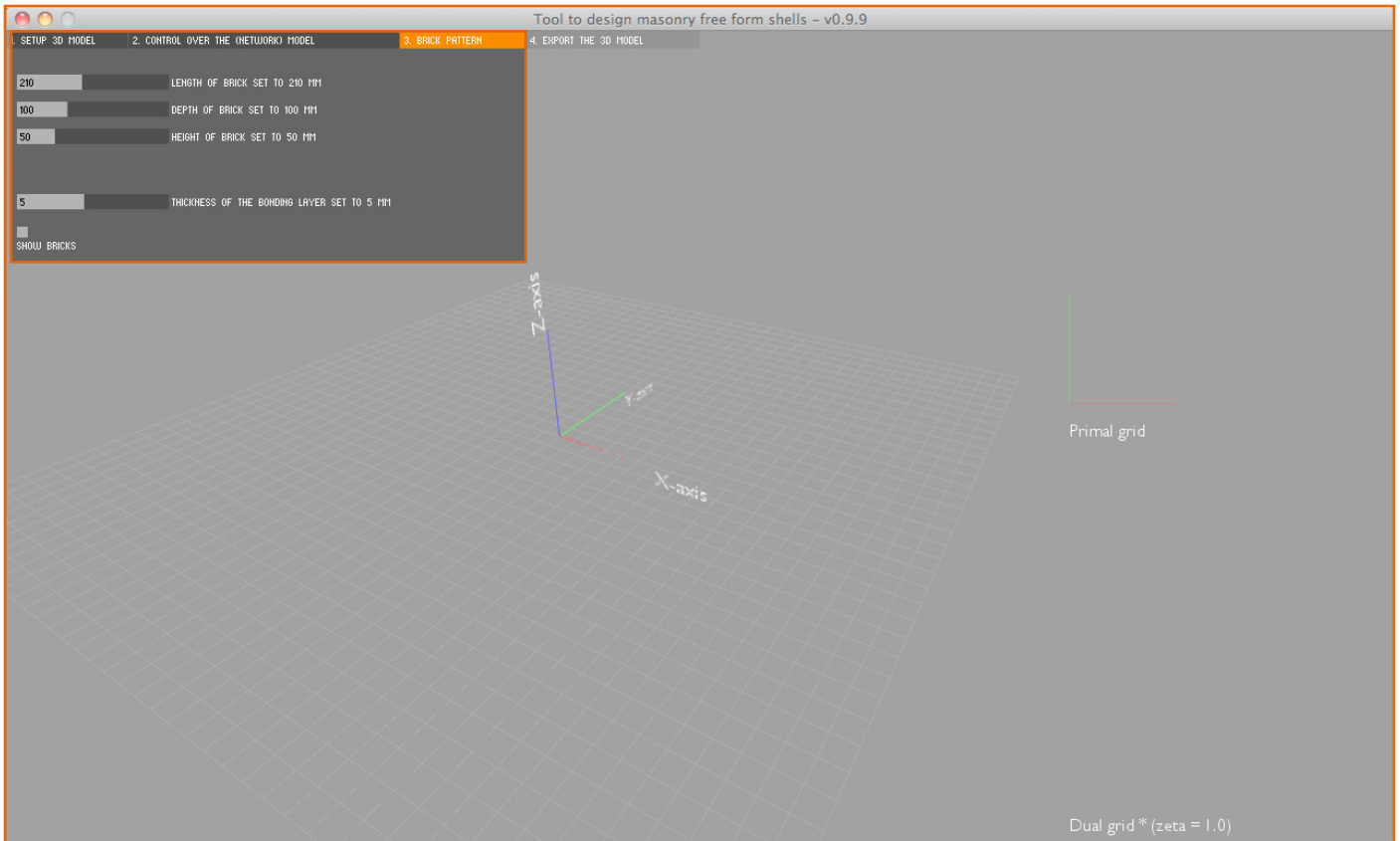   If no line is selected yet, the text 'No line selected yet'.

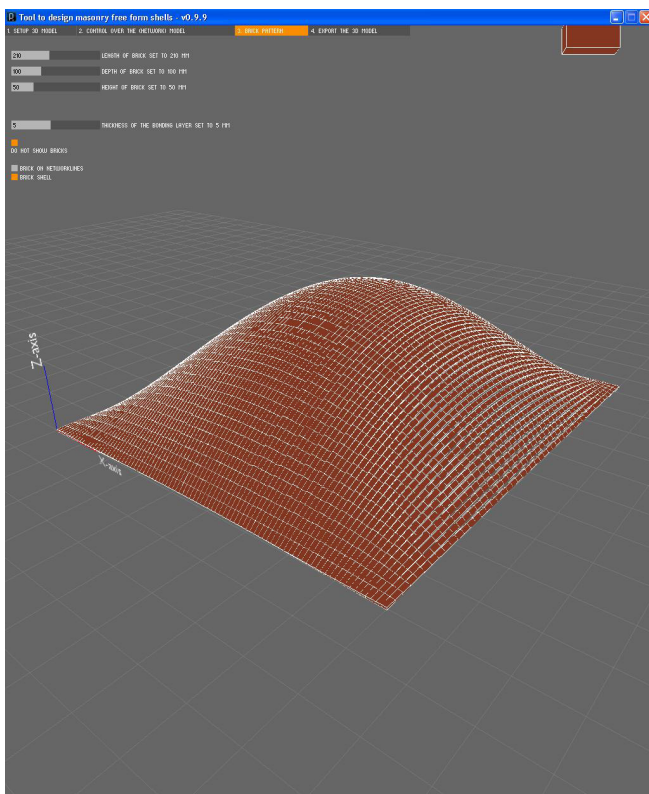**Figure F-10** - *Tab 3: Brick pattern, with the option to show or hide the brick.*

Figure F-11 - *Tab 3: Example of a generated brick pattern*

4. CREATE BRICK PATTERN

All options regarding the masonry and brick and patterns, as presented below, are located in Tab 3. 'Brick pattern' (Figure F-10).

DIMENSIONS OF THE USED BRICK

The dimensions of the brick to build the shell with are adaptable. An example of the brick and its dimensions is shown in the right upper corner.

1. Go to Tab 3. 'Brick pattern';
2. Slide the corresponding slider to adapt the length, depth or height of the standard brick.

! Pay attention !

When one of the values is changed, the pattern is changed according to the new brick size automatically. Earlier made adaptations to the pattern will be lost.

GENERATE PATTERN

There are two options available:

1. Create a model where only a pattern is made for the network;
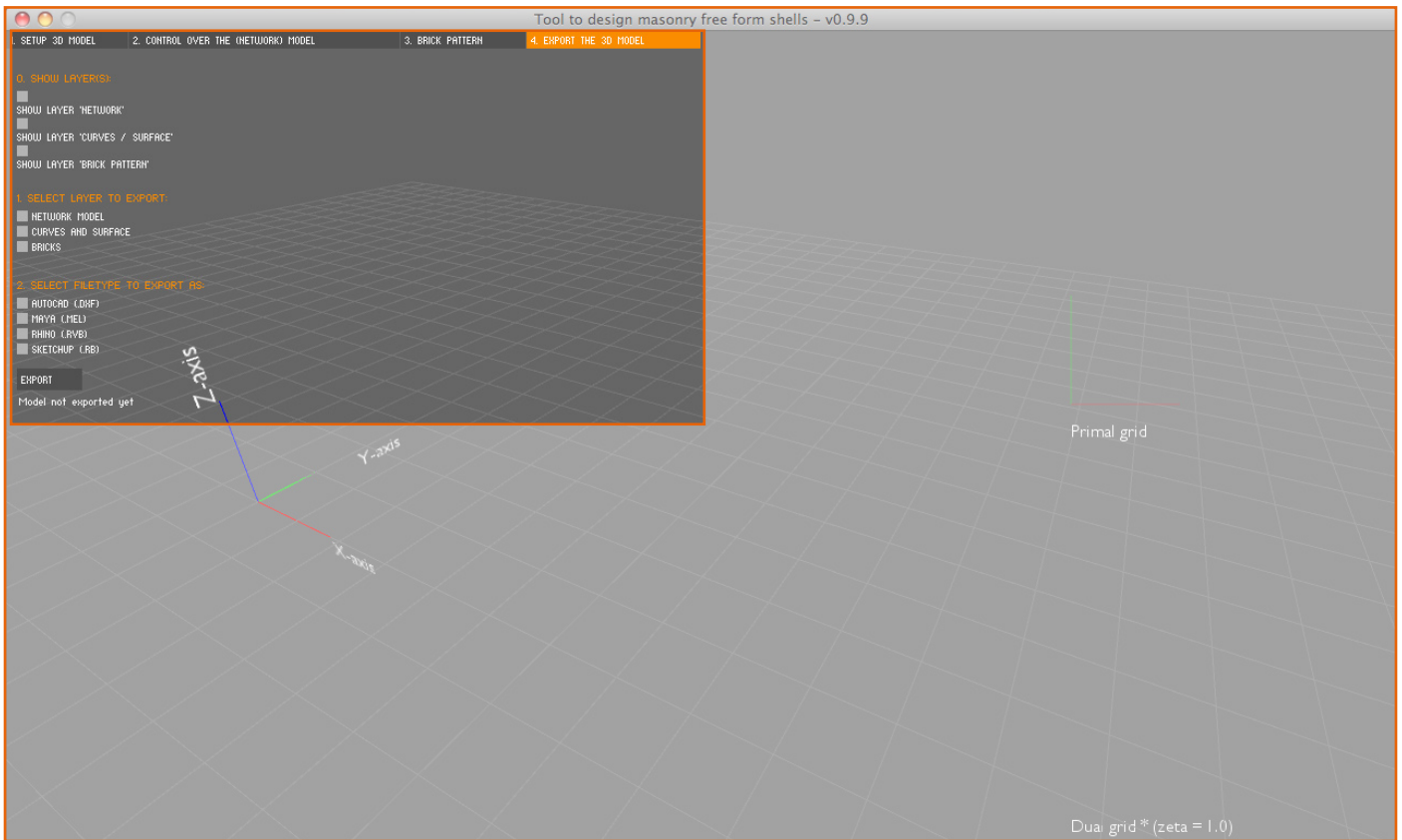2. Create a full shell (Figure F-11).

Figure F-12 - *Tab 4: Export 3D model and brick pattern,. Possible in four different file formats*
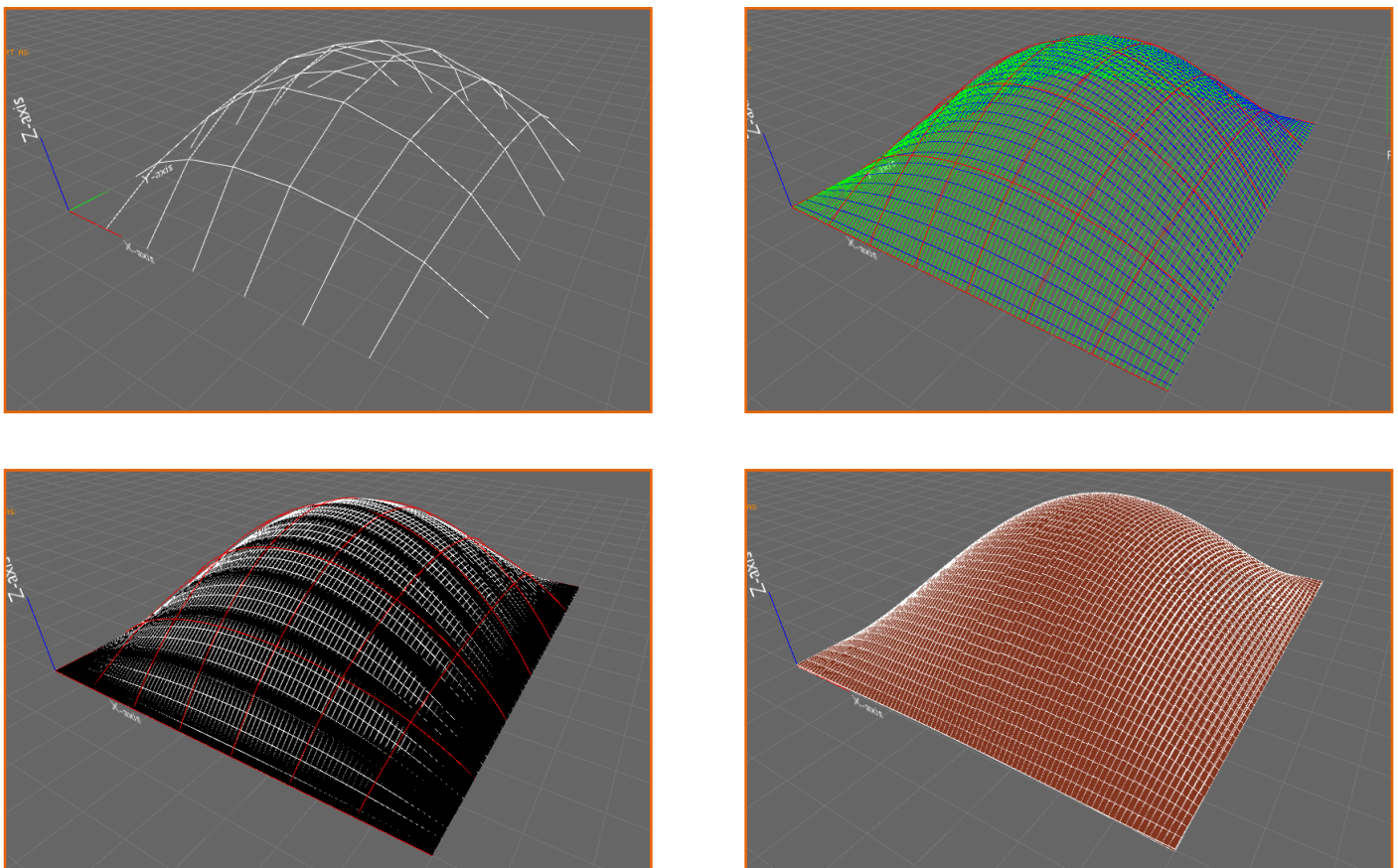

Figure F-13 - *Tab 4: Three layers to export: force network, shell surface/curves and the brick pattern*

5. EXPORT MODEL

All options to export a 3D model, as presented below, are located in Tab 4. 'Export the 3D model'. (Figure F-12)

There are two variables to select before to export the model.

1. The layer to export;
2. The file format to export as.

1.   THE LAYER TO EXPORT

Three layers are interesting to export (Figure F-13):

-   The force network model   (nodes and lines)
-   The surface shape   (either curves or surfaces)
-   The brick pattern   (blocks representing all bricks)

2.   THE FILE FORMAT TO EXPORT AS

The 3D model can be exported as one of the following four file formats:

-   AutoCAD   .DXF file
-   Autodesk Maya   .MEL file
-   Rhinoceros   .RVB file
-   SketchUP   .RB file

AutoCAD

File format	.DXF

When opening the file directly in AutoCad, it is opened as 2D file; therefore it is recommended to take the following steps:

Steps to take   1. Start AutoCad;

2. Open a .dwt file (drawing template) called: acad3D;

3. Drag the exported .DXF file into the .DWT file opened in step 2.

Autodesk Maya

File format	.MEL

Steps to take   1. Start Maya;

2. Open the Script Editor in Maya;

3. In the Script Editor: Load the exported .MEL file;

4. Press CTRL + ENTER and the model is shown.

Rhinoceros

File format	.RVB

Steps to take   1. Start Rhinoceros;

2. Open the Visual Basic Script Editor;

3. Open/load the .rvb file and the model is shown.
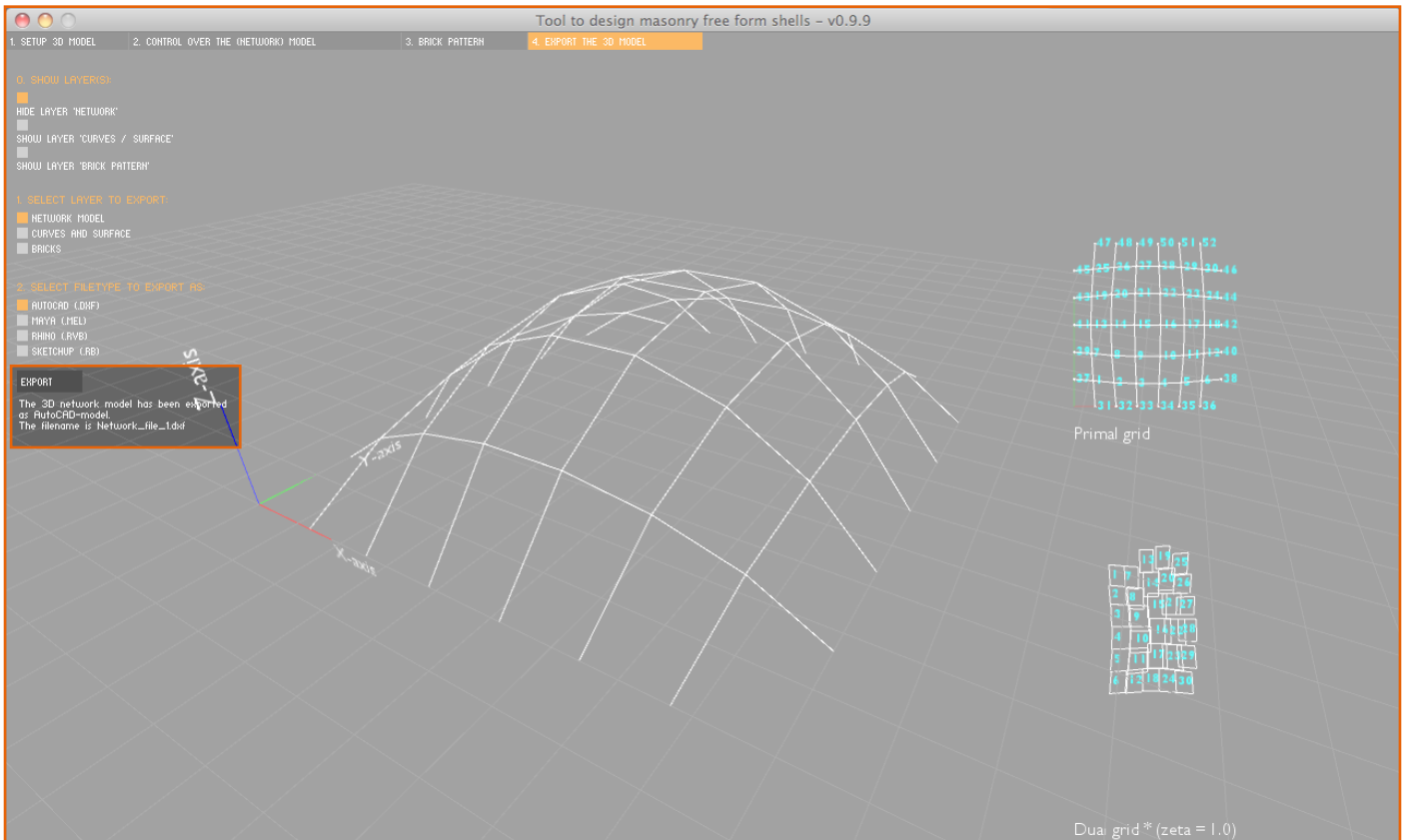
SketchUP

Figure F-14 - *Tab 4: Filename convention when exporting a file*

File Format   .RB file

Steps to take   1. Place the .rb file in the Plugins folder of SketchUP.

                    This folder is in most cases located at:

                                For Windows: Depending on where the folder SketchUP is located during installation. Default is:

                                                    “ C:\Program Files\Google\Google SketchUp 7\Plugins ”

                                For Mac OSX: “ Library / Application Support / Google Sketch Up 7 / SketchUp / plugins ”

                    2. Start SketchUP. The file is loaded automatically.

                    or:

                    2. Start SketchUp

                    3. Open the Ruby Console (located in the tab “Window”).

                    4. Type: load ‘<filename>.rb’ (for example: filename is ‘test’, than type: load ‘test.rb’)

3.   THE FILE NAME

When a file has been exported, the file name is shown below the button ‘Export’.

This name consists of:

- The layer that is being exported;
- The number showing how many files have already been exported;
- The extension of the file format used to export.

As a result the file name will be such as: <exported layer>_<file number>.<file format>

Example (Figure F-14):   Network_file_1.dxf

APPLIED LIBRARIES

- Saito OBJECT loader   to import an .obj file   (http://code.google.com/p/saitoobjloader/)
- P5   to create the User Interface   (http://www.sojamo.de/libraries/controlP5/)
- matrixMath   to do calculations with matrices   (http://matrixmath.fadarch.com/)
- SuperCAD   to export the model   (http://labelle.spacekit.ca/supercad/)

RECOMMENDATIONS FOR FURTHER RESEARCH

Chapter 1 - IMPORT MODEL

Use the actual imported model as network model. The nodes from the .obj file can already be read, though the lines still have to be transformed into the Line-elements that are used in the application.

One remark and warning regarding this possibility: the network that is imported has to make sense. When a surface from Maya is transformed into polygons and than imported in the application, the network is no *network*, but a representation of the surface with rectangular shapes. This is not the same as a force network, and as a result can also not be used as one.