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Enhancing Bandgap Depth in Locally Resonant Metastructures via Notch-filtered Piezoelectric Actuation

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***Abstract*—This paper proposes an effective approach to enhance bandgap depth in metastructures with high damping ratios, utilizing piezoelectric actuators coupled with notch filters for improved vibration isolation. The active control strategy focuses on dynamically attenuating specific resonant frequencies through the application of notch filters within the actuator control loops. AI algorithm, in particular Reinforcement Learning, is employed to optimize the notch filter parameters, thereby fine-tuning the system's response. Numerical validation reveals that this approach not only maintains system stability but also significantly deepens the bandgap. The results highlight that utilizing notched piezo-actuation achieves a more pronounced bandgap depth in overdamped systems compared to traditional piezo-actuated resonators, marking a substantial advancement in vibration control technologies.**

***Index Terms*—Metastructures, Notch Filters, Piezoelectric Actuators, Bandgap Enhancement.**

I. INTRODUCTION

In the realm of engineering, the quest for efficient vibration control in mechanical structures and metastructures is a perennial challenge that has significant implications for the longevity, safety, and performance of various systems. The ability to manipulate and manage vibrational energy via bandgaps—frequency ranges where wave propagation is inhibited—has emerged as a potent strategy in the design of such structures. However, the presence of high damping ratios within these systems has historically presented a considerable obstacle, as it tends to diminish the effectiveness of the bandgap and thus, the overall vibration isolation capabilities.

The complexity of integrating active vibration control mechanisms that can adapt and respond to varying operational conditions further accentuates the need for innovative solutions. It is within this context that the current research endeavors to bridge the gap, proposing a novel approach that leverages the sophistication of active control strategies to enhance the bandgap depth even in highly damped metastructures.

This paper is structured as follows: Section two provides a literature review and background, setting the stage for the current study by discussing previous work in the field. Section three succinctly describes the methodology used, employing advanced artificial intelligence algorithms to optimize the parameters of notch filters in piezo-actuated systems for improved vibration isolation. Section four presents the results and discussion, delving into the efficacy of the proposed approach through a comparative analysis with traditional methods. Finally, section five offers a conclusion that encapsulates the contributions and insights gleaned from this research and proposes avenues for future work that could expand on the findings presented herein.

II. LITERATURE REVIEW AND BACKGROUND

The literature on vibration isolation in metastructures has extensively covered passive and active control strategies. Passive methods, such as the incorporation of resonators, have been widely researched for their natural bandgap properties. Soukoulis et al. [1] outlined the basic principles of bandgap creation through periodic structuring, while the work of Johnson and Rifaie et al. [2] expanded on the impact of intrinsic material damping on these bandgaps

Active control strategies, including the use of piezoelectric actuators, were explored by Wang and Inman [3] as a means to adaptively tune vibration characteristics. However, these methods often fall short in systems with high damping ratios, as noted by Van Spengen [4], where the active components can introduce additional complexity without significantly improving isolation.

The application of notch filters within the control loop of piezoelectric actuators has been less documented, with pioneering work by Song et al. [5] suggesting potential improvements in bandgap depth. The use of AI algorithms for system optimization is a relatively new approach in this field, with Huang et al. [6] demonstrating the feasibility of machine learning methods for parameter tuning in complex systems.

This study builds on the foundation laid by previous research while addressing the noted gap. By employing an active control strategy using notch filters, the research provides a novel solution to the challenges posed by high damping ratios in metastructures, a solution that is both robust and adaptable.

The existing research gap is identified in the inadequate performance of current vibration isolation approaches in highly damped metastructures, where traditional methods fall short in fully utilizing the bandgap effect. This study primarily aims to explore whether implementing an active control approach with notch filters can enhance the manipulation of bandgap properties in overdamped systems more efficiently than typical passive or active methods. The research centers on assessing how well these custom-designed notch filters can replicate the vibration isolation capabilities of undamped systems, thereby addressing the difficulties posed by high damping levels.

III. Enhancing Metastructure Bandgap Depth with Notch Filters

In the Method section of your academic article, the dynamics of a metastructure with integrated piezoelectric actuators and resonators are explored through a mathematical model. The model is represented by a set of differential equations that describe the motion of the beam and resonators, their interactions, and the role of the piezoelectric actuators in controlling vibration.

The primary beam's motion is captured by (1), which includes the effects of the flexural rigidity and mass distribution, along with the interactions with attached resonators. The resonators' dynamics are detailed in (2), which accounts for their mass, damping, stiffness, and the influence of the piezoelectric actuators. (3) represents the electrical dynamics of the piezoelectric actuator, linking its voltage to the resonator's motion. The governing equations are as follows [7]:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + C\frac{\partial w(x,t)}{\partial t} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} - \sum_{r=1}^{N_r}\left(k_r z_r(t) + c_r\frac{\partial z_r(t)}{\partial t}\right)\delta\left(x - x_r\right) = \mathcal{F}_{b_m}(x,t) \tag{1}$$

$$m_r\frac{\partial^2 z_r(t)}{\partial t^2} + c_r\frac{\partial z_r(t)}{\partial t} + k_r z_r(t) + m_r\frac{\partial^2 w\left(x_r,t\right)}{\partial t^2} - \vartheta_{p,r} v_{p,r}(t) = \mathcal{F}_{b_r}(t) \tag{2}$$

$$C_{p,r}^*\frac{\partial v_{p,r}(t)}{\partial t} + \vartheta_{p,r} z_r(t) = 0 \tag{3}$$

Parameters EI, ρA, and C represent the beam's flexural rigidity, mass per unit length, and damping coefficient, respectively. Terms $\vartheta_{p,r}, C_{p,r}^*$, and $\mathcal{F}_{b_m}(x,t)$ denote the piezoelectric properties: electromechanical coupling, capacitance, and external force on the main beam. Functions $w(x,t)$ and $z_r(t)$ describe the primary structure's transverse vibrations and the resonators' movements. The Kronecker delta function $\delta(x - x_r)$ positions the resonators on the beam. N_r is the number of resonators, m_r their mass, k_r their stiffness, and c_r their damping coefficient.

Orthogonality conditions are employed to simplify the complex dynamics into a more manageable form, leading to a modal decomposition of the beam's deflection in (4). By substituting this modal expansion into the governing equations and applying orthogonality conditions, a reduced set of equations is obtained, capturing the interactions between the structure's modes and the resonators.

$$w(x,t) = \sum_{m=1}^{N_m}\phi_m(x) z_m(t), \tag{4}$$

Where $\phi_m(x)$ represents the spatial configuration of the m-th mode shape, and $z_m(t)$ corresponds to its time-dependent amplitude. This approach simplifies the intricate dynamics of a flexible structure with embedded resonators into a set of more comprehensible modal elements.

The application of Laplace transforms to these equations, assuming no initial conditions, provides a linear set of equations in the Laplace domain, which are used to derive the transfer function for the resonator's displacement, as shown in (5). This equation demonstrates the effect of applying voltage to the piezoelectric actuator on the dynamics of the resonator.

The influence of the resonators' mass distribution on the structure is taken into account, leading to the approximation of the mass distribution across the structure in terms of a mass ratio. This mass ratio ensures that the resonator masses are synchronized with the structural dynamics.

The transfer function for the displacement of the resonator is derived from the aforementioned definitions and mathematical rearrangement, as follows:

$$Z_r(s) = \frac{-s^2 w_b - s^2\sum_{m=1}^{N_m} Z_m(s)\phi_m\left(x_r\right)}{s^2 + 2\zeta_r\omega_r s + \omega_r^2 + \frac{v_a\omega_r^2}{s}}, \quad r = 1,2,\ldots,N_r \tag{5}$$

Here, the active voltage applied to the piezo, v_a, equals κ_e^* and assumed that κ_e^* is defined as $\kappa_e^* = \frac{\vartheta_{p,r}^2}{C_{p,r}^* k_r}$. When considering the voltage source as an input in piezoelectric as an actuator, we assume that the effective coupling stiffness, κ_e^*, can be represented by the $\kappa_e^* = \alpha\omega_r v_0 = v_a$. In this equation, ω_r denotes the resonator's natural frequency, v_0 is the voltage applied to the piezoelectric element, and α is an empirical coefficient with units of Farads per Coulomb (F/C). Equation (5) represents the Laplace transform of the displacement of the r-th resonator, $Z_r(s)$, in a dynamic system. It incorporates Z_m, the modal displacement for each of the N_m number of modes of the system, and w_b, the base excitation displacement. The term $\phi(x_r)$ indicates the mode shape at the location of the r-th resonator.

Further mathematical manipulation brings to light the interaction between structural modes and resonators in the Laplace domain, as demonstrated in equation (6). This equation incorporates $Z_m(s)$, representing the Laplace-transformed displacement of the m-th structural mode, and $\mathcal{Q}_{b_m}(s)$ for the transformed external disturbances affecting the system. Here, ζ_m and ζ_r denote the modal and resonator damping ratios,

$$\left(s^2+2\zeta_m\omega_m s+\omega_m^2\right)Z_m(s)+\frac{s^2\left(\omega_r^2+2\zeta_r\omega_r s\right)\sum_{r=1}^{N_r}m_r\phi_m(x_r)\sum_{p=1}^{N_m}\phi_p(x_r)Z_p(s)}{s^2+2\zeta_r\omega_r s+\omega_r^2\left(1+\frac{v_a}{s}\right)}=\mathcal{Q}_{b_m}(s),\quad m=1,2,\ldots,N_m \tag{6}$$

respectively. The equation thus underscores the resonators' dynamic response to applied voltage and delineates the effect of piezoelectric actuation on the system, taking into account the damping characteristics of both the modes and the resonators.

The masses of the resonators, represented by (m_r), are calculated based on the structure's mass distribution at the points where resonators are installed. This calculation uses a mass ratio, denoted as (μ), to define the resonator mass in relation to the total mass of the primary structure. The formula $m_r = \mu m\left(x_r\right)dx_r$ links resonator masses to the structure's mass distribution, ensuring their behavior aligns with the structural dynamics. In systems with many resonators, their distribution can be approximated by $\sum_{r=1}^{N_r} m\left(x_r\right)\phi_m\left(x_r\right)\phi_p\left(x_r\right)dx_r \approx \int_0^L m(x)\phi_m(x)\phi_p(x)dx = \delta_{mp}$. This approximation considers the cumulative impact of resonators over the entire structure, in line with the orthogonality expressed by the Kronecker delta function δ_{mp}.

Enhancing the bandgap depth with notch filters introduces an innovative approach to mitigate the challenges posed by high damping ratios in mechanical systems that affect the performance of the bandgap, especially its depth. The utilization of notch filters is proposed as a means to fine-tune the system, deepening the bandgap and improving vibration isolation.

The method involves integrating a notch filter, represented by (7), into the piezoelectric actuator circuit to refine the performance and stabilize the system. The notch filter's parameters, such as the quality factor Q, the adjusted quality factor Q_β, and the gain k, are key to its function. These parameters allow the filter to selectively attenuate specific frequencies, which corresponds to the resonant frequencies that contribute to excess damping.

$$H_{no} = k\frac{s^2+\frac{\omega_{no}}{Q_\beta}s+\omega_{no}^2}{s^2+\frac{\omega_{no}}{Q}s+\omega_{no}^2} \tag{7}$$

Where H_{no} describes the filter's response, with k as its gain, and ω_{no} as the notch frequency. The quality factor Q determines the filter's selectivity and bandwidth width at ω_{no}, with a higher Q leading to narrower bandwidth and sharper resonance. In contrast, Q_β is an adjusted quality factor that controls the depth of the notch and the filter's response outside the notch frequency. This differentiation between Q and Q_β allows for precise control over the filter's frequency response, with changes in Q_β enabling modulation of attenuation at the notch frequency and altering the filter's behavior in specific ways. The use of Q_β varies based on the filter design and the engineer's objectives.

The integration of the notch filter into the system is evaluated through the derived (8), which now accounts for the notch filter's effects. This revised equation facilitates the analysis of the system's performance, illustrating the modulation of the bandgap via active control, made possible by the inclusion of the notch filter.

Equation (8) incorporates the dynamics of a notch filter, enabling the analysis of its influence on the system's performance (eliminating modal damping for simplicity). This includes examining how the filter affects the depth of the system's bandgap through active control using piezoelectric actuators. This equation provides a framework for understanding how the notch filter's parameters interact with the system dynamics, especially in terms of modifying bandgap characteristics.

To deepen the bandgap, Reinforcement Learning (RL) with an actor-critic method is utilized, focusing on optimizing notch filter parameters. This RL approach refines control parameters by iteratively adjusting actions based on the environmental state and corresponding rewards. The optimization leverages the Deep Deterministic Policy Gradient (DDPG) algorithm to target optimal values for parameters k, Q, and Q_β, which are critical for enhancing the bandgap depth and achieving superior vibration isolation.

IV. Results and Discussion

The parameters of this study are detailed in Table I, that presents the geometric and material properties of the rectangular aluminum beam.

TABLE I
Geometric and material properties of the studied rectangular aluminum beam

Parameter	Value	Parameter	Value
L_m	300 mm	w_r	87 Hz
w_m	40 mm	ω_m	$18, 114, 319, ..$ Hz
h_m	2 mm	ζ_r	0.08
ρ_m	2700 kg/m^3	N_m	8
E_m	69.5 GPa	N_r	8
ζ_m	0.02	v_0	1 V
μ	0.66	α	0.098 F/C

Fig. 1 illustrates the sensitivity of the notch filter with varying parameters Q, Q_β, and k. The graph demonstrates the filter's frequency response, highlighting the attenuation levels across a range of frequencies. The shaded areas represent the variation in attenuation due to changes in the notch filter parameters, providing insight into the filter's effectiveness in suppressing specific frequency bands. The notch depths and bandwidths at various frequencies show how the system's response can be fine-tuned for enhanced vibration isolation. A comparison of the optimized parameters obtained through the Reinforcement Learning algorithm demonstrates their impact on the system's performance. The sensitivity analysis confirms that precise adjustments of Q, Q_β, and k are crucial for achieving the desired bandgap depth, validating the RL approach's efficacy in optimizing the metastructure's dynamic behavior.

$$\frac{Z_m(s)}{Q_{b_m}(s)} = \frac{1}{s^2\left(1+\frac{\mu(2\zeta_r\omega_r s+\omega_r^2)}{s^2+2\zeta_r\omega_r s+\omega_r^2\left(1+\frac{kv_a}{s}\frac{s^2+\frac{\omega_{no}}{Q_\beta}s+\omega_{no}^2}{s^2+\frac{\omega_{no}}{Q}s+\omega_{no}^2}\right)}\right)+2\zeta_m\omega_m s+\omega_m^2} \tag{8}$$

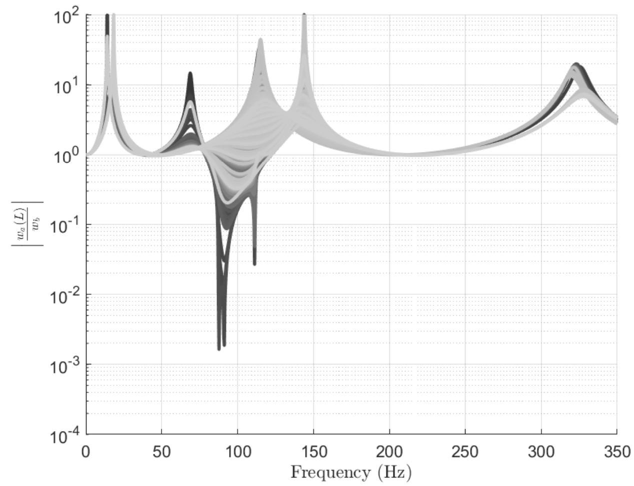

Fig. 1. Sensitivity of Transmittance from the Base to the Tip of the Aluminum Beam Metastructure, Illustrating Attenuation Variability with Parameter Adjustments of Q, Q_β, and k.

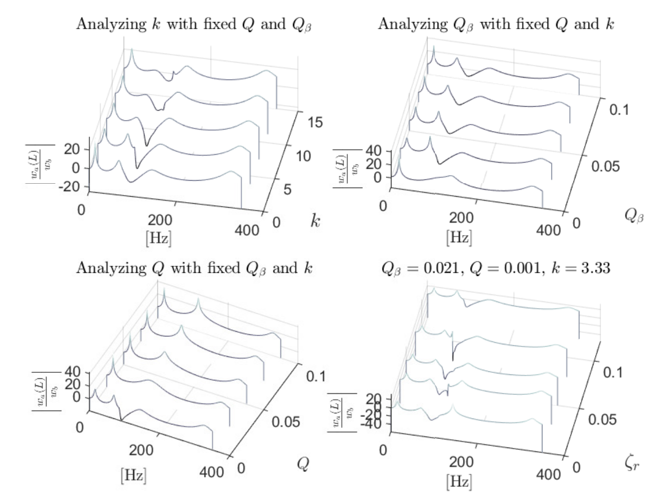

Fig. 2. Parameter Sensitivity Analysis of Notch Filter Performance on Metastructure Transmittance, Displaying the Effects of Variations in Gain (k), Quality Factor (Q), and Adjusted Quality Factor (Q_β) Across Frequency Bands.

Fig. 2 illustrates the impact of individual notch filter parameters on the vibration transmittance in a beam metastructure, through a series of contour plots. The top left plot illustrates the variation of gain k while Q and Q_β are held constant. Different levels of attenuation over the frequency spectrum can be observed as k changes, indicating the filter's sensitivity to gain alterations. The top right plot seems to focus on varying Q_β with fixed values for Q and k. This plot would be particularly useful for understanding how the depth and sharpness of the notch in the filter's response are affected by Q_β. The bottom left plot probably shows the effect of altering Q with Q_β and k remaining constant. Adjusting Q affects the bandwidth of the notch, and this visualization helps in finding the balance between selectivity and attenuation efficiency. The bottom right plot could be a specific case where Q_β, Q, and k are set to optimal values determined by prior optimization algorithms. This plot would exemplify the achieved balance between attenuation depth and bandwidth for effective vibration suppression.

For optimizing the notch filter parameters Q, Q_β, and k, Artificial Intelligence algorithms were utilized. The Reinforcement Learning (RL) approach pinpointed an optimal parameter set, achieving a structural damping ratio ζ_m of 0.03 and a resonator damping ratio ζ_r of 0.08. The parameters were determined to be $Q_{\beta_{\text{opt}}}$ at 0.021, Q_{opt} at 0.001, and k_{opt} at 33.3. These values are indicative of the RL algorithm's capability to finely adjust the system, enhancing its performance and demonstrating a similar yet uniquely effective solution compared to other algorithms for influencing the metastructure's transmittance and bandgap properties.

Fig. 3 compares transmittance across a range of frequencies for different damping treatments and system configurations. It includes four scenarios: an undamped metastructure, a damped system, a conventional piezo-actuated system with constant voltage, and a notch piezo-actuated system with optimized parameters. The optimized notch parameters, obtained through a Reinforcement Learning algorithm to reduce structural damping to $\zeta_m = 0.03$, illustrate the notch filter's effectiveness in decreasing transmittance at resonant frequencies, thereby deepening the bandgap and enhancing vibration isolation. This comparison underscores the enhanced vibration suppression of the notch piezo-actuated system compared to other configurations. The results highlight that in an overdamped metastructure, a piezo-actuator with constant voltage is less effective. However, integrating a notch filter significantly improves bandgap characteristics, even surpassing an undamped system's performance. This finding is pivotal in addressing the research objective of augmenting bandgap depth in highly damped metastructures, demonstrating that an

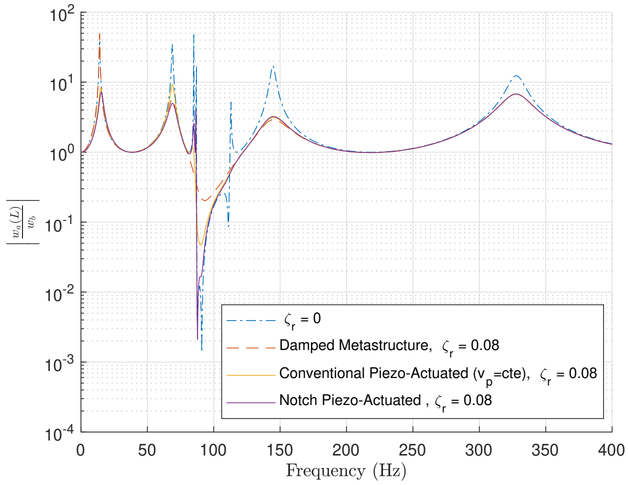

Fig. 3. Performance Comparison of Transmittance in High Damping Metastructures, Demonstrating the Effectiveness of Notch Filter Application in Piezo-Actuated Systems Versus Constant Voltage Application, with parameters: $\zeta_m = 0.02$, $\zeta_r = 0.08$, $v_0 = 1$ V, $Q_{\beta_{opt}} = 0.021$, $Q_{opt} = 0.001$, $k_{opt} = 33.3$. Achieving Bandgap Characteristics Comparable to Conventional and Undamped System.

actively controlled piezo-actuated system with a notch filter can achieve favorable bandgap properties in an overdamped system.

V. Conclusion

This research has made significant contributions to the field of vibration control in mechanical metastructures. Through the integration of notch filters in piezo-actuated systems, the study has demonstrated an innovative approach to enhancing bandgap characteristics in systems with high damping ratios. The use of Reinforcement Learning algorithms to optimize the parameters of the notch filter represents a noteworthy advance, enabling the precise tuning of the system's dynamic response. Results have shown that the application of a notch filter in an overdamped metastructure can effectively emulate the bandgap features of an undamped system, thereby achieving superior vibration isolation.

The insights gained from this study underscore the potential of smart materials and control strategies in engineering applications where vibration suppression is crucial. It affirms the viability of using advanced AI algorithms for system optimization, setting a precedent for their application in more complex dynamic systems.

Looking forward, future research could explore the scalability of this approach to larger structures or those with varying damping characteristics. Further investigation into the long-term stability and robustness of the control system under different operational conditions would also be valuable. Additionally, the integration of this approach with other smart material technologies could lead to the development of even more sophisticated vibration control systems, broadening the scope of practical applications in industries ranging from aerospace to civil engineering.

The principal contribution of this paper is the introduction of an active control strategy using notch filters to enhance the bandgap depth within highly damped metastructures. This approach has proven to be more effective than traditional piezo-actuated methods that employ constant voltage. The study demonstrates that by carefully adjusting the notch filter parameters, it is possible to achieve bandgap characteristics similar to those in undamped systems, thus providing a significant improvement in vibration isolation. This methodology represents a substantial advancement in metastructural design, offering a sophisticated tool for engineers to optimize dynamic responses in a variety of practical applications.

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