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Efficient anisoplanatic aberration correction in digital holography via a single-step Zernike–Fourier approach

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Abstract. We demonstrate an efficient approach for correcting spatially varying (anisoplanatic) aberrations in digital holographic imaging by leveraging a Zernike-Fourier domain representation. The imaging operator was modelled in a matrix form as a combination of Fourier basis functions and Zernike decomposed field-dependent wavefront aberrations. The single-step matrix multiplication greatly reduces computational complexity compared to traditionally relying on explicit matrix inversion or point-wise convolution.

1 Introduction

Spatially-varying aberrations, commonly referred to as anisoplanatic aberrations, greatly degrade image quality in optical imaging systems in a nonlinear way. Traditional correction approaches rely on heavy computational procedures [1], which strongly limit the feasibility of practical applications.

In this study, we show efficient anisoplanatic field aberration correction in digital holography (DH). Our method is based on the orthogonality and unitarity of the Fourier basis combined with Zernike phase decomposition. These two ingredients are combined in a single aberration matrix to allow for single-step anisoplanatic aberration correction.

2 Methodology

The field on the Fourier plane in an imaging system can be described as the linear superposition of the spectra of all individual point sources over the imaged field of view [2]. Based on the linearity of Fourier transform, the vectorized aberrated spectrum in a discrete form can be expressed as.

$$\mathbf{G}[f_k] = \frac{1}{\mathrm{N}^2}\sum_k \mathbf{o}[k]e^{-i2\pi\boldsymbol{\Phi}_k[f_k]} \odot e^{i2\pi\mathrm{Z}^m\mathbf{c}_k[f_k]} \odot \mathbf{P}[f_k]\ , \tag{1}$$

where $\odot$ is the Hadamard product operator. Here the total measured Fourier domain field $\mathbf{G}[f_k]$ is a superposition of planewaves from all spatial samples $\mathbf{o}[k]$, each modulated both by the standard shifted phase $e^{-i2\pi\boldsymbol{\Phi}_k[f_k]}$ and the local aberration phase $e^{i2\pi\mathrm{Z}^m\mathbf{c}_k[f_k]}$ which

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is decomposed using an m-mode Zernike projection matrix Z^{m} and a coefficient vector $\mathbf{c}_{\boldsymbol{k}}$. Finally constrained by a pupil function $\mathbf{P}[f_k]$. A compact form can be further expressed as

$$\mathbf{G} = \mathrm{T}\mathbf{o}\ . \tag{2}$$

Given the orthogonality of the Fourier basis and the continuous nature of wave propagation, T^{-1} can be approximated by the Hermitian transpose of T, denoted as T^{H}. As a result, an aberration corrected image can be efficiently reconstructed by a single matrix multiplication.

$$\mathbf{o} \cong \mathrm{T}^{\mathrm{H}}\mathbf{G}\ . \tag{3}$$

3 Results

Spatially varying aberrated wavefronts—simulated and parametrized for an imperfect 4f imaging system in Zemax—are applied to a sharp, noisy resolution target (101×101 pixels) to generate an aberrated image. Visual and quantitative comparisons of the correction methods are shown in Fig. 1.

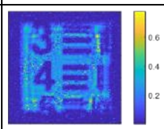
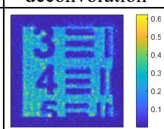
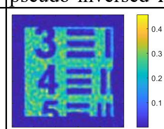
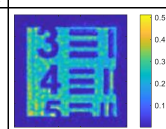
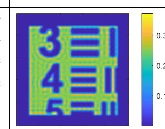
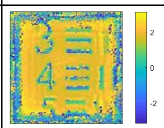
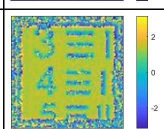
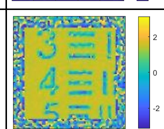
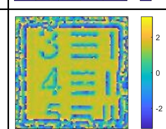
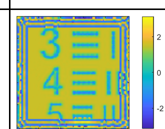
	Aberrated Image	pixel-sliding deconvolution	Correction by pseudo-inversed T	Correction by T^{H}	Diffraction limited
Intensity					
Phase					
SSIM	0.715	0.916	0.958	0.924	1.000
Correction time	-	14.562 s	0.063 s	0.063 s	-
T calibration time	-	-	0.346 s	0.346 s	-
T^{-1}computing time	-	-	91.325 s	0.312 s	-

Fig. 1 Comparison of spatially-varying aberrated images and the reconstruction results. The SSIM value was calculated between the intensity of input matrix and the diffraction-limited image.

4 Conclusion

In this study, we demonstrate an efficient method for correcting anisoplanatic aberrations. Compared to pixel-sliding deconvolution, correction by matrix multiplication significantly reduces the time cost—a benefit that becomes even more apparent as the image size increases. Although the method saves computational time relative to traditional approaches, it requires a lot of memory during computation. In the future, to achieve calibration-independent correction, the parametric model and efficient correction method make spatially varying blind deconvolution feasible.

References

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