Reliability-based Dynamic Network Design with Stochastic Networks

Hao Li

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Hao Li (李浩)
Master of Science in Transportation and Planning, Delft University of Technology
geboren te Zhongning, Ningxia Province, P.R. China

Dit proefschrift is goedgekeurd door de promotoren:

Prof. dr. ir. P.H.L. Bovy
Prof. dr. H. J. van Zuylen

Samenstelling promotiecommissie:

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Prof. dr. ir. P.H.L. Bovy	Technische Universiteit Delft, promotor
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P.O. Box 5017
2600 GA Delft
The Netherlands
T: +31 (0) 15 278 6046
F: +31 (0) 15 278 4333
E: info@rsTRAIL.nl

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To my parents: AiYing & ZhanSheng

Preface

Since October 2003, I have been living in the Netherlands. These past few years were spent on my master study and my PhD research at the Transportation and Planning Department, Delft University of Technology. The PhD life in Delft is certainly an unforgettable, wonderful and precious experience. It involved independent research, scientific supervision, free discussions, all kinds of opportunities for high level education, training and visiting interesting conferences, ample chances to meet experts in our filed, etc. In addition, the working environment, research atmosphere, supervisors, and the group of research fellows surrounding me are extraordinary. I have enriched myself during the PhD study, not only in the capability of conducting a scientific research, but also in other skills, for which I feel very grateful. Through this dissertation I would like to express my gratitude to the many people who played an important role in my PhD life here in Delft.

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I would like to thank my family and Huizhao's family. I am grateful to have a very happy family and family in-law. Thank you very much for all the time supporting me and taking care of each other when we are aboard. My great mum, thank you for supporting, understanding, and hard working for your four kids by yourself. Thank you, my dear sisters and lovely brother, for being so considerate and nice to me. I am writing this thesis in remembrance of my beloved father.

Last but not least, special thanks go to my dear husband, Huizhao. Thank you so much for your caring, loving and understanding, which always make me feel cherished and protected. Thank you for always cheering me up and staying with me. Without your supports and love I would not have reached what I have achieved so far.

Hao Li (李浩)
Delft, May 2009

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Notations

Variables

$u_p^{od}(t)$	Individual's utility of making a trip on route p between OD pair (o, d) when departing at t
$c_p^{od}(t)$	Individual's travel cost on route p between OD pair (o, d) when departing at t
$\tau_p^{od}(t)$	Individual's travel time on route p between OD pair (o, d) when departing at t in the deterministic case
$\tilde{\tau}_p^{od}(t)$	Experienced stochastic travel times on route p between OD pair (o, d) when departing at t
$\tilde{c}_p^{od}(t)$	Experienced stochastic travel costs on route p between OD pair (o, d) when departing at t
$E\left[\tilde{c}_p^{od}(t)\right]$	Expected travel cost on route p between OD pair (o, d) for travelers departing at t in the stochastic capacity case
$E\left(\tilde{\tau}_p^{od}(t)\right)$	Expected travel time on route p between OD pair (o, d) for travelers departing at t in the stochastic capacity case
$Std\left[\tilde{c}_p^{od}(t)\right]$	Standard deviation of travelers' experienced travel costs on route p between OD pair (o, d) when departing at t
$Std\left(\tilde{\tau}_p^{od}(t)\right)$	Standard deviation of travelers' experienced travel time on route p between OD pair (o, d) when departing at t
$\tilde{x}(t)$	Time dependent random variable
$\lvert\tilde{x}(t)\rvert$	Absolute values of random variable $\tilde{x}(t)$
$\mu(t)$	Time dependent mean of random variable $\tilde{x}(t)$
$\sigma(t)$	Time dependent standard deviation of random variable $\tilde{x}(t)$
$Q(t)$	Queue length at time instant t in the deterministic case

$D_1(t)$	Cumulative early departures at time instant t in the deterministic case
$D_2(t)$	Cumulative late departures at time instant t in the deterministic case
$r_1(t)$	Early departure rate at time instant t in the deterministic case
$r_2(t)$	Late departure rate at time instant t in the deterministic case
C	Deterministic bottleneck capacity
t_0	Start time of the first departure in the deterministic capacity case
t_t	Time instant at which travelers departing before t_t will arrive early, while those departing after t_t will arrive late in deterministic capacity case
t_e	Time instant at which the last user departs, or the end of the peak period in the deterministic capacity case
N	Number of travelers using the bottleneck
N_{late}	Number of travelers arriving late at destination
N_{early}	Number of travelers arriving early at destination
$C_{\max}$	Upper bound of the capacity distribution
$C_{\min}$	Lower bound of the capacity distribution
$\tilde{C}$	Stochastic capacity
$f(t)$	Departure flow rate at time instant t
t_0^*	Start time for the first departure in the long term equilibrium
t_t^*	Time instant at which travelers departing before t_t^* will arrive early, while those departing after t_t^* will arrive late based on the expected travel time at the long term equilibrium
t_e^*	Time instant at which the last user departs, or the end of the peak period in the long term equilibrium
t_c^*	Time instant at which the capacity distribution becomes time-dependent
$D_1^*(t)$	Cumulative early departure flow in the long term equilibrium
$D_2^*(t)$	Cumulative late departure flow in the long term equilibrium
$D_3^*(t)$	Cumulative late departure flow in the long term equilibrium with time-dependent capacity distribution
$D_T^*(t)$	Total cumulative departure flow in the long term equilibrium, $D_T^*(t) = D_1^*(t_t^*) + D_2^*(t_c^*) + D_3^*(t)$
$r_1^*(t)$	Early departure rate at time instant t in the long term equilibrium with stochastic capacities
$r_2^*(t)$	Late departure rate at time instant t in the long term equilibrium with stochastic capacities
$r_3^*(t)$	Late departure rate at time instant t in the long term equilibrium with stochastic capacities
$\tilde{v}(t)$	Stochastic outflow rates for time instant t
$p_u(t)$	Probability of non-zero travel times

$f_p^{od}(t)$	Departure flow rate at time instant t on route p between OD pair (o, d)
W^*	Cumulative outflow from the bottleneck at long term equilibrium
q^{od}	Total travel demand from origin o to destination d (constant)
$\bar{c}_p^{od}(k)$	Equilibrium travel cost when departing at time interval k on route p between OD pair (o, d)
$c_p^{od}(k)$	Deterministic travel cost when departing at time interval k on route p between OD pair (o, d)
$\hat{c}_p^{od}(k)$	Perceived travel cost when departing at time interval k on route p between OD pair (o, d)
$\bar{f}_p^{od}(k)$	Equilibrium flow for time interval k on route p between OD pair (o, d)
$\varepsilon_p^{od}(k)$	Random component accounting for travelers' perception errors on the travel cost at time interval k on route p between OD pair (o, d)
$\hat{f}_p^{od}(k)$	Intermediate departure flow for time interval k on route p between OD pair (o, d)
$f_p^{od(i)}(k)$	Departure flow for time interval k on route p between OD pair (o, d) at iteration i
$c_p^{od(i)}(k)$	Deterministic travel cost when departing at time interval k on route p between OD pair (o, d) at iteration i
$\tilde{\tau}_a(k)$	Stochastic link travel time on link a at time interval k
F	Cumulative distribution function of standard normal distribution
Δ	Cholesky triangle of the variance-covariance matrix of link capacities in a network
M_a	Number of routes in the network traversing link a
$G^{(i)}$	Duality gap value at iteration i
L_a	Number of lanes on link a
$\bar{\boldsymbol{L}}$	Optimal solution, a vector of number of lanes for designed links
I	Construction costs
B	Given budget for network constructions
d_a	Length of link a
$x(L_a, d_a)$	Average construction cost per kilometer per lane, firstly decreases and then increases with the number of lanes
d_p^{od}	Length of route p between (o, d)
Z	Objective function

Parameters

θ	Fraction factor of the minimum capacity to the maximum capacity, $0 < \theta < 1$
χ_C	Capacity-dependent parameter

n	Number of Halton draws
ϑ	Proportion of network efficiency in the design objective function, $0 \le \vartheta \le 1$
β_{PS}	Path Size parameter to capture the route correlation due to spatial overlap
p_m	Mutation probability in GA
$G_{\max}$	Maximum number of generations in GA

Behavioral parameters

PAT	Preferred arrival time
α	Value of travel time
β	Value of travel time reliability
γ_1	Value of being early at work
γ_2	Value of being late at work
λ_t	A constant parameter of schedule delay cost given a travel time distribution at departure time instant *t*, $\lambda_t = 1$ or 0
δ_t	A constant parameter of reliability cost given a travel time distribution at departure time instant *t*, $0 \le \delta_t \le 1/2$
ξ_t	A constant parameter of travel time reliability given a travel time distribution at departure time instant *t*, $0 \le \xi_t \le 1/2 \cdot (\gamma_1 + \gamma_2)$
β_t	Attribute dependent parameter of travel time reliability
φ_t	A constant parameter of quadratic schedule delay cost given a travel time distribution at departure time instant *t*, $\varphi_t = 0$ or 1
ϕ_t	A constant parameter of reliability cost given a travel time distribution at departure time instant *t* in quadratic schedule delay cost function, $0 \le \phi_t \le 1$
ρ_t	A constant parameter of quadratic schedule delay costs
ς_t	A constant parameter of travel time reliability in quadratic schedule delay cost function
ω	Scale parameter in MNL choice model

Functions

$g\left(\tilde{x}(t)\right)$	$\max\left\{0, \tilde{x}(t)\right\}$, i.e. $\left(\tilde{x}(t)\right)^+$

Sets

$\boldsymbol{O}$	Set of origins in a network
$\boldsymbol{D}$	Set of destinations in a network
$\boldsymbol{P}^{od}$	Route set between OD pair (o, d)

$\boldsymbol{A}$	Set of links in a network
$\boldsymbol{K}$	Set of departure time intervals
$\boldsymbol{T}$	Set of time steps for dynamic network loading
$\boldsymbol{\Lambda}$	Solution space of numbers of lanes for all designed links, i.e. a set of points in a multidimensional space where each point represents a lane design with axes of links and numbers of lanes
$\boldsymbol{\Omega}_{\mathbf{d}}$	Set of feasible time dependent route flow matrix $\mathbf{f}$ in case of dynamic traffic assignment with joint departure time/route choice, the row indicates a route in the network, the column indicates the time interval
$\boldsymbol{\Omega}_s$	Set of feasible route flow vectors $\mathbf{f}$ in case of static traffic assignment
Γ_p^{od}	Set of links on route p between OD pair (o, d)

Indices

*	Equilibrium state in Bottleneck model
~	Stochastic variable
‾	Equilibrium state in general network modeling
a	Link
p	Route
o	Origin
d	Destination
k	Time interval
t	Time instant
i	Iteration index
g	Index of generation in GA

Matrix or vectors

$\mathbf{X}$	Vector of Halton draws
$\mathbf{C}_a$	Vector of stochastic link capacities drawn based on Halton draws to estimate the capacity distribution for link a
$\mathbf{C}_n$	Matrix of stochastic link capacities for all links in a network
$\mathbf{f}$	Vector of route flows in case of static assignment or matrix of dynamic route flows in case of dynamic assignment
$\tilde{\mathbf{C}}$	Vector of a certain combination of link capacities for all links in a network
$\boldsymbol{\Sigma}$	Variance-covariance matrix of link capacities in a network
$\mathbf{L}$	Vector of numbers of lanes on all the designed links

Abbreviations

SN-DDDUEV	Stochastic Network and Departure time and Dynamic Deterministic User Equilibrium with Vertical Queues
SN-DDPUEH	Stochastic Network and Departure time and Dynamic Probabilistic User Equilibrium with Horizontal Queues
DN-DDDUEV	Deterministic Network and Departure time and Dynamic Deterministic User Equilibrium with Vertical Queues
SN-SPUE	Stochastic Network and Static Probabilistic User Equilibrium
PAT	Preferred Arrival Time
FIFO	First-In-First-Out
MSA	Method of Successive Averages
CNDP	Continuous Network Design Problem
DNDP	Discrete Network Design Problem
MNDP	Mixed Network Design Problem
RDDNDP	Reliability-based Dynamic Discrete Network Design Problem
MPEC	Mathematical Program with Equilibrium Constraints
MPRDEC	Mathematical Program with Reliability-based Dynamic Equilibrium Constraints
RSDNDP	Reliability-based Static Discrete Network Design Problem
TNT	Total Network Travel time
TNC	Total Network Travel Cost
CoV	Coefficient of Variation
VOT	Value of Time
VOR	Value of Reliability

Part I: Introduction

1 Introduction

1.1 Introduction to the Research Topic

Road networks are dynamic and stochastic systems. It is dynamic in a sense that the traffic flows and traffic conditions are changing on the networks over space and time dimensions. It is stochastic in a way that the link capacities and travel demand are stochastic over time. Attibuted to the dynamism and the stochasticity, the traffic condition on the network at any location at any time is uncertain. Many sources contribute to the stochasticity in capacity and demand, ranging from irregular and random incidents such as earthquakes, floods, adverse weather, traffic accidents, breakdowns, signal failures, and road works to regular fluctuations of travel demand in times of day, days of the week, and seasons of the year (Taylor 1999). All these sources lead to variations in network capacities or fluctuations in travel demand, which cause travel times and travel costs for travelers to vary on the same network in the same time period from day to day and to vary from time to time within a day (Emam and Al-Deek 2005; Tu *et al.* 2007b; Tu 2008).

Travel time reliability has been defined and widely adopted as a concept reflecting the properties of the travel time stochasticity. We refer to (Asakura and Kashiwadani 1991; Cassir *et al.* 2000; Chen *et al.* 2002b; Tu 2008) for categories of reliability definitions and measures of travel time reliability for different purposes. On the one hand, travel time reliability might be adopted for purpose of modeling travelers' choice behavior in response to stochastic and uncertain travel times. On the other hand, it is also applicable as a measure of the service quality that a road network provides to its users. Therefore the travel time reliability related research could be generally grouped into reliability-based choice behavior modeling under uncertainty and reliable network performance assessment or network designs.

Modeling travelers' choice behavior under uncertainty, for instance route choice and departure time choice behavior, has gained increasing attention (Abdel-Aty *et al.* 1996; Bogers and van Zuylen 2004; Van Amelsfort 2009). Especially in the context of uncertainty, modeling departure time adaptation is fairly important since travelers attempt to arrive on time at their destination with a high probability and to reduce the schedule delay late by departing earlier (Li *et al.* 2008b; Li *et al.* 2008d) (Schedule delay refers a difference between the actual arrival time and the preferred arrival time. If a traveler arrives at his/her destination later than his/her preferred arrival time, then the schedule delay is the schedule delay late). At the same time, this time adaptation appears to be one of the major responses of travelers to changing congestion conditions on their trips. How to model travelers' choice behavior under uncertainty and whether to model departure time choice behavior under uncertainty will have significant impacts on the traffic assignment and thus lead to different network performance evaluations and different network designs.

When facing stochastic network conditions, road authorities or designers aim to design a network which is required to provide efficient and reliable services to road users and to ensure smooth travels under normal traffic flow fluctuations with stochastic networks. Since the link capacities and the travel demand are not deterministic and network conditions are not static, it is desired to design and assess road networks considering the stochastic and dynamic nature of the road network systems and taking into account travelers' major choice behavior under uncertainty (Li *et al.* 2008c; Li *et al.* 2009b; Li *et al.* 2009a).

Currently, in most road network design studies where stochastic networks or travel demand are considered, choice behavior under uncertainty are not modeled (Lou *et al.* 2009; Ng and Waller 2009; Sharma *et al.* 2009). In most cases with stochastic networks or travel demand, static network design problems are proposed and formulated in which the lower level traffic assignment cannot capture the dynamics, flow propagations and spillback effects. Departure time choice modeling is in most cases neglected, which is however a very important aspect of travel behavior that needs to be modeled in the context of uncertainty (Van Amelsfort 2009). In several dynamic network design studies, which are nework designs based on dynamic traffic assignment, for instance (Heydecker 2002), deterministic networks and travel demand are considered.

Therefore this dissertation, other that earlier network design studies as aforementioned, focuses on road network designs with stochastic capacities based on dynamic traffic assignment (we call it dynamic road network design) with modeling travelers' route/departure time choice behavior under uncertainty. Travel time reliability is explicitly taken into account on both the network design level and the individual behavior. The structure of the network and numbers of lanes on a set of potential links are aimed to be designed to provide efficient and reliable services to the road users, since the sensitivity of the network is strongly dependent on the physical structure and physical characteristics of the network. Different network structures, having different capability to handle the variability in the demand and in flows caused by stochastic capacities, exhibit different reactions to the same variability level of capacity or demand. The less sensitive, the more reliable the network can be. The goal of this research is to develop a comprehensive dynamic network design approach with stochastic capacities, which explicitly models travelers' departure time/route choice behavior under uncertainty and models travel time reliability at the network design level. The approach aims to find a fairly good and even the optimal network structure given the spatial land use pattern, which could best handle the traffic flows and their variations caused by stochastic capacities.

With regard to road networks, the operational conditions may be normal and abnormal. Because natural resources are limited, performance of road networks under abnormal conditions cannot be required to be the same as that under normal conditions. The condition modeled in this dissertation will concentrate on the normal daily traffic conditions. Extreme conditions, for instance natural disasters, are not considered.

This chapter will firstly specify the main research questions and objective which need to be solved or achieved in this dissertation. The approaches to solve the research questions and to achieve the research objectives in this dissertation will be presented briefly. Then the research scope will be described. After that, the scientific contributions and the practical relevance of the research in this dissertation will be introduced. Finally the outline of this dissertation will be given.

1.2 Research Questions and Objectives

This dissertation deals with the road network design problem with stochastic networks. Stochastic networks denote networks in which the link capacities are stochastic and varying between days. In general, a network design problem is a bi-level problem and a Mathematical Problem with Equilibrium Constraints (MPEC). In general, a bi-level problem is an optimization problem which is constrained to another optimization problem. Bi-level problems are special cases of multi-level optimization problems where only two levels are involved and mutually interrelated. In the network design problem, on the upper level road authorities aim to optimize certain network performances while on the lower level traveler's choice is modeled to reach a certain equilibrium at which individual traveler's travel utility is optimized. Therefore, the network design problem is an optimization problem constrained to lower level user equilibrium, which is a typical MPEC. Dealing with network design problems requires a realistic modeling of travelers' choice behavior, based on which the network performance can be evaluated and the networks can be designed.

As aforementioned in most studies tackling road network design problems with stochastic capacities or travel demand, the travelers' choice behavior under uncertainty is not modeled. This is not plausible since travel time/cost uncertainty significantly influences travelers' choice behavior. In addition, a static road network design approach is followed in most studies with only route choice modeling. The dynamics in flow propagation, spillback effects, etc., are not captured. As previously discussed, departure time choice adaptation under uncertainty appears to be one of the major responses of travelers to changing congestion conditions on their trips, which is however neglected in most research on network modeling under uncertainty and network designs under uncertainty. This dissertation describes a dynamic network design approach where travelers' route and departure time choice behavior under uncertainty is explicitly modeled with stochastic capacities. Travel time reliability plays a role in both the network design level and the individual choice behavior level. The proposed dynamic network design approach is an equilibrium network design approach, implying that a long term dynamic user equilibrium under stochastic capacities is modeled. It is assumed that travelers based on their past accumulative experiences are aware of the distributive properties of the travel time distributions at any departure time instant t on all available routes. A long term user equilibrium will emerge with which no traveler can reduce his long term travel cost by unilaterally changing routes or departure times (for detailed discussions, see Chapter 3).

Generally, this dissertation consists of four distinguishable but related parts:

1. Investigation of the influence of the travel time/cost uncertainty on travelers' choice behavior and modeling travelers' departure time/route choice behavior under uncertainty (Chapter 2);

2. Theoretical investigations on the long term dynamic user equilibrium with stochastic capacities (Chapter 3);

3. Establishment of network assignment procedures that adequately capture the stochasticities in the capacities and capture the influence of this on the travelers' choice behaviors for general road networks (Chapter 4);

4. Establishment of a reliability-based dynamic discrete road network design methodology with stochastic networks and its applications. Travel time reliability is considered on both the network design level and the individual choice behavior level. Numbers of lanes on a set of potential links in the network are the design variables (Chapter 5-7).

The research questions/objectives involved in each of the four parts will be presented sequentially.

Part 1: Modeling travelers' departure time/route choice behavior under uncertainty

Dealing with network design problems requires realistic modeling of travelers' choice behavior. In this dissertation, stochastic networks (i.e. stochastic link capacities) are modeled, which lead to stochastic travel times and travel costs experienced by travelers between days. This part of research focuses on how to model the influence of the travel time/cost uncertainty on travelers' departure time and route choices. Several approaches have been proposed in the literature to model travelers' choice behavior under uncertainty, for instance the scheduling approach (Noland and Small 1995) and the mean-variance approach (Jackson and Jucker 1981). Briefly, the scheduling approach assumes that travelers make their departure time/route choice decisions in order to minimize their individual disutility, which is composed of the expectation of the travel time and the expectation of the schedule delay early and late. While the mean-variance approach assumes that travelers try to minimize their individual disutility which consists of the expectation of the travel time and the variance of the travel time (or standard deviation of travel time). Both approaches are based on different hypothese. The research questions/objectives involved in this part are:

Research objective 1: Clarify the relationships among the different behavioral modeling approaches currently available for incorporating travel time uncertainty.

Research objective 2: Develop an appropriate way to model travelers' route/departure time choice behavior under uncertainty and derive the disutility components that should be included in the utility function.

For modeling traveler's choice behavior, the term of utility is used. This dissertation will deal with the choice modeling and traffic assignment. Therefore, in Chapter 2 in terms of choice behavior modeling, the term of utility function is used. Since in the utility function we defined, all the components are disutilities (i.e. costs), in the traffic assignment part, travel cost function is then used. By achieving the first research objective on the relationships among the currently available approaches for modeling travelers' choice behavior under uncertainty, the second research objective aims to derive a utility function which is appropriate for modeling travelers' route/departure time choice behavior under uncertainty. This derived utility function will be used to model the long term dynamic user equilibrium, based on which the road network is designed.

Part 2: Theoretical investigations on the long term dynamic user equilibrium under uncertainty

With the derived utility function for modeling travelers' departure time/route choice behavior under uncertainty, the long term dynamic user equilibrium is defined and investigated analytically. It puts forward a few research objectives/questions:

Research objective 3: Prove the existence of the long term dynamic user equilibrium with stochastic capacities.

If the long term dynamic user equilibrium indeed exists, then the following research question comes up:

Research question 4: Are there significant differences in departure patterns resulting from adopting the reliability-based and non-reliability based utility functions?

The reliability-based utility function is derived by the second objective. The non-reliability based utility function is the traditional utility function in the deterministic capacity case, where travel time is deterministic. With answers from question 4, a decision will be made on which user equilibrium type will be modeled for the network design problem under uncertainty. In the process of investigating the departure patterns with reliability-based utility function, a few sub-questions are solved, for instance the question of the relationship between travel time reliability and the expected travel times.

Part 3: Establishment of network assignment procedures for general stochastic network modeling

Theoretical analysis of the long term dynamic user equilibrium is quite complicated for general road networks. A comprehensive approach for modeling the long term dynamic user equilibrium is eagerly needed for general network modeling, which should be able to capture the stochastic natures in capacities between days and to capture the consideration of this by travelers. It puts forward a few research objectives in this part, among which:

Research objective 5: Establish a network assignment procedure to derive the long term dynamic user equilibrium with stochastic networks, which is able to model stochastic and correlated link capacities and to incorporate travel time reliability in the utility function in which simultaneous departure time and route choices are modeled.

The established network assignment procedure from achieving the long term dynamic user equilibrium under stochastic capacities will be utilized to model the equilibrium constraint for the network design problem.

Part 4: Establishment of a reliability-based dynamic network design methodology with stochastic networks

A network design approach is established, with which travelers' departure time and route choice behavior under uncertainty is modeled on the demand level and network reliability is considered on the network design level. Numbers of lanes on a set of potential links are the design variables. The research objectives involved in this part are:

Research objective 6: Formulate the reliability-based dynamic network design approach with discrete design variables and a construction budget constraint.

For comparison purpose of the traditional static network design approach and the proposed dynamic network design approach, the following research objective comes up:

Research objective 7: Formulate the reliability-based static network design approach with discrete design variables and a construction budget constraint.

Research objective 8: Establish a network assignment procedure to derive the long term static user equilibrium with stochastic networks.

For solving the proposed reliability-based dynamic network design problem with discrete design variables, the properties of the design problem need to be clarified. Appropriate and efficient approaches need to be developed to solve the design problem. The research objetives involved in this part are:

Research objective 9: Develop a network-oriented approach to solve the proposed dynamic discrete network design problem.

The proposed discrete network design problem is a combinatorial problem. Many combinations of the possible numbers of lanes on the potential links are not feasible, or unconnected, or illogical (see the definitions in Chapter 6). Evaluating these lane designs loses the efficiency and costs a lot of computation times. The following research objective therefore needs to be solved:

Research objective 10: Develop a systematic approach to filter the solution space by eliminating the infeasible, unconnected, and illogical lane designs prior to the design step.

The proposed dynamic network design approach will be applied to a specific road network. To assess the merits of a dynamic design approach, a static network design approach will be applied as well. The related research objective thus is:

Research objective 11: Investigate the differences in the design solutions from static and dynamic network design approaches.

Research objective 11 aims to find out whether the design solutions from the proposed dynamic network design approach are significantly different from and better than that achieved with static network design approach, and whether the application of a dynamic design approach really matters for the design decisions. The deficiency and the advantages of the dynamic and static network design approaches need to be explored.

Research objective 12: Investigate the value and the impacts of departure time choice modeling on the network performance evaluations and network designs.

Objective 12 aims to find out whether modeling departure time choice is of importance and necessary in terms of network designs.

Two different network design objective functions are proposed, one of which takes into account travel time reliability implicitly in the network performances and the other defines network reliability explicitly in the objective function. The related research question is:

Research question 13: Do different network design objectives lead to significantly different design solutions?

In order to investigate the influence of including travel time reliability in travelers' choice behavior on the network performance evaluations and network designs, different levels of capacity variations are modeled and compared. In case that the capacity variations equal zero, the reliability-based dynamic network design approach boils down to a dynamic network design approach with deterministic capacities, giving rise to the following research objective:

Research objective 14: Investigate the influence of including travel time reliability in travelers' choice behavior on network designs and differences in the design solutions from stochastic and deterministic dynamic network design approaches.

Below we will describe with which methodologies we intend to tackle the various research questions.

1.3 Research Approach

The research approach for solving the research questions and for achieving the research objectives will be presented for the four groups respectively and sequentially.

1.3.1 Research approach for questions/objectives in part 1: modeling travelers' choice behavior under uncertainty

For achieving the research objectives 1-2, a theoretical approach is adopted to investigate the relationship between currently available approaches for modeling travelers' choice behavior under uncertainty. Based on the theoretical analysis, the generalized utility function will be derived with appropriate utility components for modeling travelers' route/departure time choice behavior under uncertainty.

1.3.2 Research approach for questions/objectives in part 2: dynamic user equilibrium under uncertainty

For achieving the research objective 3 and solving the research question 4, again a theoretical approach is adopted with continuous time and continuous flows by extending Vickrey's deterministic bottleneck model (Vickrey 1969) in a twofold way: incorporating the stochastic capacities and modeling travel time reliability in the (dis)utility function. Due to the complexity of the mathematical formulations for solving the long term dynamic user equilibrium caused by the reliability term and stochastic capacities, a closed solution is not possible. A numerical solution approach is applied to solve the systematic mathematical equations to derive the equilibrium departure patterns with stochastic capacities.

1.3.3 Research approach for questions/objectives in part 3: dynamic network assignment procedures for stochastic networks

A simulation-based approach is developed to model stochastic link capacities and to derive the travel time reliability on any specific route at any departure time interval. An iterative method is applied to gradually achieve the long term dynamic user equilibrium (research objective 5). Stochastic link capacities are modeled by the quasi-Monte Carlo approach with Halton draws while Cholesky decomposition is utilized to generate correlated link capacities by decomposing the link capacity covariance matrix.

Dynamic network loading (DNL) modeling approaches are established for both vertical and horizontal queues for different comparative purposes.

1.3.4 Research approach for questions/objectives in part 4: a reliability-based dynamic network design methodology

The reliability-based dynamic discrete network design problem is formulated mathematically as a bi-level problem and MPEC (research objective 6). So is the static network design problem (research objective 7). A similar simulation-based approach is adopted to derive the long term static user equilibrium as for dynamic long term user equilibrium (research objective 8).

A combined road network-oriented genetic and set evaluation approach is developed to solve the reliability-based dynamic discrete network design problem (research objective 9). For a detailed description, see Chapter 6.

A new systematic approach is developed to eliminate the infeasible, unconnected, and illogical lane designs (for the definitions, see Chapter 6) and to reduce the solution space prior to the design step, thus to save computation time (research objective 10).

To achieve the research objectives 11-14, a case study is carried out by applying the proposed dynamic network design approach and the static network design approach on a hypothetical network with different levels of capacity variations.

1.4 Research Scope

Given the challenging research questions, we have to limit the research scope in this dissertation in various ways. Firstly, the road networks we are focusing on in this dissertation are limited to freeways. The developed network design approach and the network modeling procedure are applicable to urban road networks as well. However the simulation program needs extensions with intersections.

The population of travelers is assumed to be homogeneous having the same (dis)utility function and an identical preferred arrival time.

It is assumed that the travel demand is deterministic and constant from day to day. Stochastic travel times and travel costs are mainly caused by stochastic link capacities. The reason why fluctuating travel demand is not considered is that a more refined definition of the user equilibrium and the flow conservation constraint with stochastic travel demand need to be carefully established in case the departure time choice is modeled (see the detailed discussions in Section 4.1). Once the definition of the user equilibrium with stochastic travel demand and departure time choice is clearly established, stochastic travel demand can be easily integrated into our framework of the reliability-based dynamic network design approach.

The impacts of the limitations listed above on the general validities of the conclusions drawn from this research will be discussed in the dissertation on occasions the issues are encountered (see discussions with Figure 3.9 in Section 3.3.5 for heterogeneous travelers and Section 4.1 for the discussions on the demand fluctuations) and in the conclusion Chapter 8.

1.5 Main Research Contributions

The contributions of the research in this dissertation will be elaborated from a scientific and a practical perspective respectively.

1.5.1 Scientific contributions

The scientific contributions of this dissertation can be summarized for the four groups as follows:

Group 1: modeling travelers' departure time/route choice behavior under uncertainty

- The principal equivalence of the mean-variance approach and the scheduling approach conditional to departure times are proven analytically for linear schedule delay cost functions without any assumptions on the travel time distributions.

- The expectation of the schedule delay costs for any given departure time t in case of a linear schedule delay cost function can be decomposed into a linear function of the schedule delay cost based on the expectation of travel time and the standard deviation of travel times. It is analytically proven that the scheduling approach indeed partly captures risk aversion attitudes.

- A new generalized utility function is proposed for modeling travelers' route/departure time choice behavior under uncertainty, of which the mean-variance and the scheduling approaches are special cases.

- A new concept and understanding of the disutility parameters in the utility function is established, which states that the (dis)utility parameters contain two kinds of information: heterogeneity of travelers (different reactions to the same situation) and attribute dependence (the same traveler may have different reactions to different traffic situations or to different values of the attribute values).

Group 2: dynamic user equilibrium under uncertainty

- A dynamic long term user equilibrium with joint departure time and route choices is specified for stochastic networks.

- Theoretical foundations are established on the departure patterns under uncertainty by extending Vickrey's bottleneck model toward the stochastic capacity case with reliability-based utility function.

- The non-linear relationship between the standard deviation of travel time and the expectation of travel time is established analytically.

Group 3: dynamic network assignment procedures for stochastic networks

- A simulation-based network assignment approach is established, which is able to model stochastic and correlated link capacities and reliability-based departure time/route choice behaviors. This approach is very flexible for achieving both dynamic and static user equilibria with stochastic capacities.

Group 4: a reliability-based dynamic network design methodology

- A dynamic road network design approach is established with stochastic networks, where travel time reliability is considered on both the network design level and individual departure time/route choice behavior.

- Departure time choice under uncertainty is modeled explicitly, which has significant impacts on the network performance evaluations and network designs.

- Instead of the traditional continuous network design approach, a discrete network design problem is established in which network structures and numbers of lanes on a set of potential links in a network are the design variables.

- A combined road network-oriented genetic and set evaluation approach is developed to solve the dynamic network design problem.

- The concepts of a universal set and a master set of lane designs are established, using the notions of infeasible, unconnected, and illogical lane designs. A systematic approach is developed to reduce the solution space by eliminating the infeasible, unconnected, and illogical network solutions.

- A comparison of the reliability-based dynamic network design approach and static network design approach has been performed, which shows the deficiency of the traditional static network design approach.

1.5.2 Practical relevance

Some achievements listed above, besides their scientific contributions, have their potential practical relevance as well.

- The derived utility function for modeling travelers' departure time and route choice behavior can be used in general network modeling with stochastic networks and fluctuating travel demand.

- The established simulation-based network assignment approach can be directly used to model network equilibrium with stochastic networks, which can be used to evaluate dynamic traffic management measures in stochastic networks, for instance dynamic speed limit, peak hour lanes, etc.

- The proposed reliability-based dynamic network design approach with stochastic networks is capable of: 1) providing a completely new network design (a kind of network structure design since a link can have zero lanes) given the spatial land use pattern, 2) an improvement design on an existing network structure.

- The proposed dynamic network design approach can be applied to dynamic lane design problems. For instance the peak hour lane, a dynamic lane configuration to meet the dynamic travel demand, can be designed with the proposed design approach in this dissertation. The only modification is that the design variables are changed to a matrix of numbers of lanes on a set of potential links in different time intervals.

- The proposed dynamic network design approach can be utilized to dynamic speed limit design problems in which stochastic capacities are taken into account. The only difference is that the design variables are changed to the speed limits on a set of potential links in the network in different time intervals.

- The proposed dynamic network design approach can be adapted and then used for static or dynamic toll design problems on a set of potential links in the network with stochastic networks. The utility function needs modifications by including the levies as a disutility component. The design objective function could be adapted according to the design purposes. The design variables are a matrix of toll levels on a set of potential links in different time intervals. For an example on dynamic toll designs, see (Joksimovic 2007) with deterministic road networks.

- The systematic approach developed to eliminate the infeasible, unconnected, and illogical lane designs can be directly used for general discrete network design problems to reduce the solution space and to save computation time.

1.6 Thesis Outline

This thesis is gradually built up from behavioral modeling under uncertainty, network modeling under uncertainty, to constrained reliability-based dynamic network design problems and applications. The thesis is mainly divided into five parts: 1) introduction; 2) travelers' choice behavior modeling under travel time uncertainty; 3) theoretical and comprehensive network modeling under uncertainty; 4) reliability-based dynamic network design problem including applications; 5) conclusions.

Figure 1.1 shows the setting up of the chapters in this dissertation. Chapter 2 focuses on the travelers' choice behavior under uncertainty. An analytical investigation on the principal equivalence of the scheduling approach and the mean-variance approach will be performed for given departure time instant t. A new generalized utility function for modeling travelers' departure time/route choice behavior is proposed which is more behaviorally sound (i.e. have face validity) and can be used flexibly for different purposes and in different contexts of adopted approaches.

Based on the generalized utility function, an analytical investigation of the departure pattern under stochastic capacities is performed in Chapter 3 with continuous time and continuous flows, as an extension of Vickrey's bottleneck model towards stochastic capacities with reliability-based utility function. The travel time reliability in relation to the expectation of stochastic travel times is explored as well.

Due to the complexity of theoretical derivations of departure time equilibrium together with stochastic capacities and reliability, a comprehensive simulation-based network assignment approach is developed for more general network modeling under uncertainty, given in Chapter 4. Different models and their assumptions, purposes are presented in detail in Chapter 4. The simulation-based approach with SN-DDDUEV (Stochastic Network and Departure time and Dynamic Deterministic User Equilibrium with Vertical queues) model is applied to the single bottleneck as well to validate the theoretical findings from Chapter 3.

The findings from Chapters 3 and 4 motivate our network approach. Chapter 5 defines and formulates a reliability-based dynamic discrete network design approach with stochastic networks, with which departure time choice under uncertainty is explicitly modeled. Numbers of lanes for all the potential links are the design variables.

Approaches for solving the proposed network design problem are elaborated in Chapter 6. A combined road network-oriented Genetic and set evaluation approach is proposed to solve the dynamic network design problem. A new systematic approach is proposed to eliminate the infeasible, unconnected and illogical lane designs in order to reduce the solution space and to save computation time.

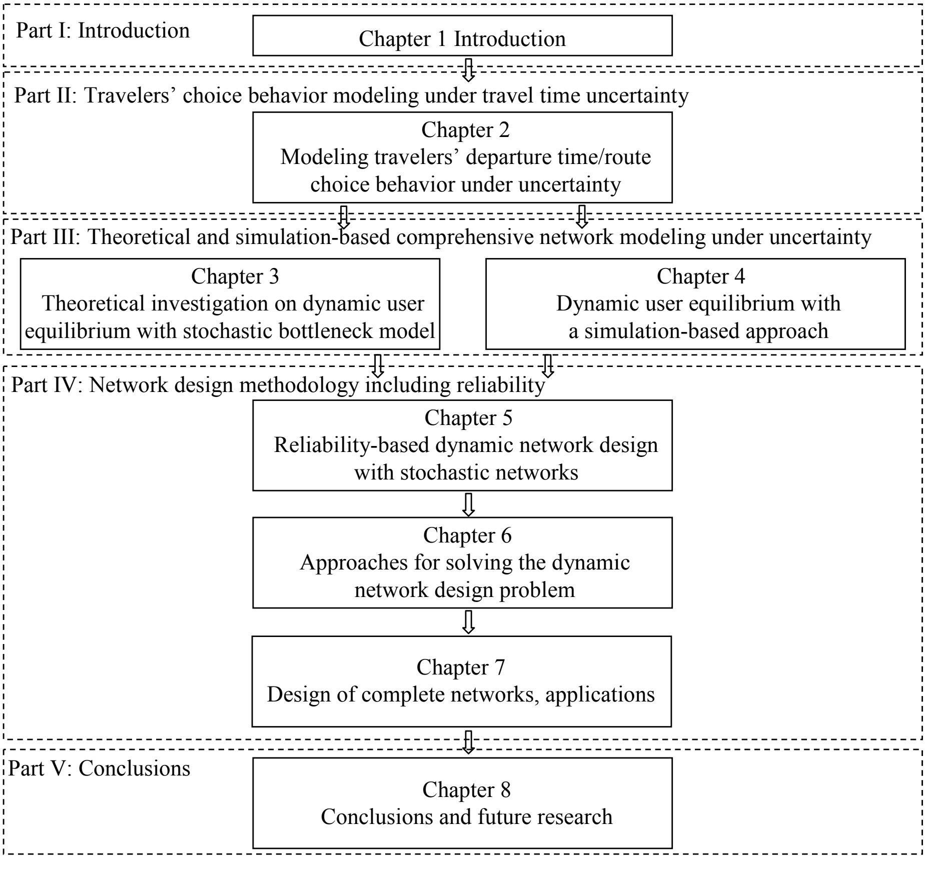


Figure 1.1: Outline of the dissertation

Applications of the proposed dynamic network design approach on a hypothetical network for a complete new network design will be presented in Chapter 7. The proposed reliability-based dynamic network approach will be compared with the traditional reliability-based static network design approach. Different capacity variation levels are modeled and compared to investigate the impacts of modeling travel time reliability in travelers' choice behavior on network designs.

Finally conclusions drawn and future research will be discussed in Chapter 8.

Part II: Traveler's Choice Behavior Modeling under Travel Time Uncertainty

2 Modeling Traveler's Departure Time/Route Choice Behavior under Uncertainty

2.1 Introduction

Continuous fluctuations in the roadway environment cause travel times to vary on the same facility in the same time period from day to day (Emam and Al-Deek 2005; Tu 2008). Stochastic travel times within-day and between-days can be attributed to two main sources: variations in road capacities and fluctuations in travel demand within-day and over-days. Of course, uncertainty and diversity in travelers' behaviors (for instance driving behavior) lead to variations and unpredictability in travel times as well, thus in travel costs. Traveler's choice behavior, for instance route choice, departure time choice and mode choice, under uncertainty has gained increasing attention. Especially in the context of uncertainty, departure time adaptation appears even more significant than route choice adaptation for the sake of attempting to arrive on time at work and to reduce the schedule delay, and minimizing the disutility by departing earlier. Thus, the impacts of travel time unreliability on traveler's choice behavior, especially on departure time choice behavior, are badly needed to be investigated. Modeling travelers' departure time/route choice behaviors is crucial in network modeling, network evaluation and network designs under uncertainty. Especially a main task in the modeling is to determine which components should be included in the utility function and what the parameter value is for each component.

Currently, many different theories are applicable to describe and model human choice behavior under uncertainty, such as expected utility theory, prospect theory, cumulative prospect theory, extended prospect theory (Van de Kaa 2008), etc. The difference of expected utility theory from other theories is that it does not account for the cognitive tasks in traveler's decision making. However, in general the expected utility theory is sufficiently widely accepted as useful to deal with choices made under uncertainty. In addition, the expected utility theory can be straightforwardly applied to the transport situation and traffic assignment models. Therefore, this research works in the framework of expected utility theory. For research on the choice behavior under uncertainty using prospect theory, we refer to for example (Avineri and Prashker 2006; Van de Kaa 2008). For a review of different theories for modeling traveler's choice behavior under uncertainty, we refer to for example (Fujii and Kitamura 2004).

This chapter will firstly address how travel time unreliability may influence traveler's departure time/route choice behavior and how travelers might cope with the uncertainty. Then several existing approaches and potential hypotheses are formulated, representing travelers' perceived utilities and their mental reasoning on the choice decisions. After that, the utility functions based on the behavioral hypotheses are investigated and compared. The equivalence among different utility function specifications, especially between the mean-variance approach and the scheduling approach, and related parameter values is analytically explored. Based on this equivalence, a generalized utility function is derived, which is more behaviorally sound and can be used flexibly due to different purposes and in different contexts of adopted approaches. The underlying meaning of the parameters in the utility function is discussed and the non-linearity in scheduling costs is analyzed as well. Finally some conclusions are drawn.

2.2 Behavioral Investigation of Uncertainty Considerations

In transport networks, due to the stochastic supply and fluctuating travel demand, travel times and travel costs experienced by traveler's within-day and between-days are stochastic. The influence of travel time variability on travelers' choice behavior under uncertainty has gained increasing attention (Bates *et al.* 2001; De Palma and Picard 2005; Batley 2007; Liu and Polak 2007). A lot of studies (Van Amelsfort *et al.* 2008) have been carried out analyzing the impacts of travel time variability on travelers' departure time/route choice behavior and on how to model their choice behaviors under uncertainty. Several empirical studies (e.g., (Abdel-Aty *et al.* 1996; Lam 2000; Ghosh 2001; Brownstone *et al.* 2003; Bogers and van Zuylen 2004; Brownstone and Small 2005; Bovy *et al.* 2008)) suggest that travelers are interested not only in travel time savings but also reduction of travel time variability. Most travelers prefer reliable travel times. The reason why the travel time variability is so important can be explained at least from two aspects: the anxiety or stress caused by uncertainty, additional cognitive burden associated with planning activities, and sensitivity to the consequences of the uncertainty, for instance late arrivals, etc (Bates *et al.* 2001).

A hypothesis of a 'safety margin' being selected by travelers has once been specified by (Gaver 1968; Knight 1974), which assumed that travelers make their choice decisions by considering the expected travel time and adding an extra time budget, the so-called safety margin, to cope with the uncertainties. Travelers might also trade off their expected travel time and travel time variability based on their past experiences. However with these hypotheses, the impacts of travel time variability on scheduling costs and travelers' reaction on the scheduling cost uncertainty are not explicitly modeled, which turns out to be very important in representing travelers' choice behavior under uncertainty (Small 1982).

A mean-variance approach has been proposed (Jackson and Jucker 1981) (see Equation (2.1)), in which the individual utility function is composed of expected travel time and travel time variance (or standard deviation of travel time). It hypothesizes that travelers are interested not only in travel time savings but also a reduction of travel time variability. Variability of travel time is usually measured by the standard deviation of travel time (Small *et al.* 1999). The mean-variance approach does not explicitly model the effect of travel time variability on scheduling decisions.[1]

$$u_p^{od}(t) = \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \beta \cdot Std\left[\tilde{\tau}_p^{od}(t)\right], \ \forall (o,d), p, t, \tag{2.1}$$

where $u_p^{od}(t)$, $\tilde{\tau}_p^{od}(t)$ denote the individual's (dis)utility and stochastic travel times on route p between OD pair (o, d) departing at t under uncertainty respectively. In this dissertation all stochastic variables are denoted with a tilde. $E\left[\tilde{\tau}_p^{od}(t)\right]$ and $Std\left[\tilde{\tau}_p^{od}(t)\right]$ denote the expectation of travelers' experienced travel times and the standard deviation of travel times on route p between OD pair (o, d) departing at t respectively. Parameters α and β are value of time and value of reliability respectively.

Due to the stochastic properties of travel times, a traveler departing everyday at the same time instant t may arrive early, late or on time as preferred, largely relying on the traffic situations on that specific day. Travel time variability directly leads to uncertainty in arrival times at the destination. Departure time shifts are very important reactions of travelers to travel time unreliability, since travel time unreliability causes uncertainty of their arrival time, which cause punishment for delays. Travelers attempt to depart earlier in order to arrive at the destination on time and reduce the schedule delay costs. Several empirical studies show that scheduling costs play a major role in the timing of departures (Small 1982).

A scheduling approach based on the expected utility theory (Polak 1987) has originally been specified (Noland and Small 1995), which hypothesizes that scheduling delay cost plays a very important role in the timing of departures under uncertainty. A related (dis)utility function may be formulated as:

$$\begin{aligned} u_p^{od}(t) = \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot E\left[\left(PAT - \left(t + \tilde{\tau}_p^{od}(t)\right)\right)^+\right] \\ + \gamma_2 \cdot E\left[\left(t + \tilde{\tau}_p^{od}(t) - PAT\right)^+\right], \ \forall (o,d), p, t, \end{aligned} \tag{2.2}$$

PAT denotes the preferred arrival time. The second and the third component are the expected schedule delay costs of being early and late respectively. Function $(x)^+$ is equivalent to $\max\{0, \boldsymbol{x}\}$, since there is either early schedule delay or late schedule delay on a specific day. It can never occur that travelers experience both delay costs at the same time. Parameters α, γ_1, and γ_2 represent value of time, value of early schedule delay and late schedule delay respectively. It has been shown that scheduling costs explain the aversion to uncertain travel times (Small *et al.* 1995). They conclude that in models with a fully specified set of scheduling costs, it is unnecessary to add an additional disutility for unreliability of travel time. Expected scheduling costs account for all the aversion to travel time uncertainty.

[1] There are other travel costs which might be considered by travelers, for instance fuel cost, tolls, etc. In our research, we will not consider these disutility components.

However it is found that scheduling delay costs cannot capture the travel time unreliability completely (Noland and Polak 2002; Van Amelsfort *et al.* 2008). Besides a scheduling effect, travel time unreliability appears to be a separate source of travel disutility. Travelers may not only consider the expected travel time (i.e. duration of their trip), the expected schedule delay costs, but also the variability in travel times as an indicator representing travelers' perceived uncertainty. Therefore an alternative utility function might be expressed as:

$$u_p^{od}(t) = \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot E\left[\left(PAT - \left(t + \tilde{\tau}_p^{od}(t)\right)\right)^+\right] + \gamma_2 \cdot E\left[\left(t + \tilde{\tau}_p^{od}(t) - PAT\right)^+\right] + \beta \cdot Std\left[\tilde{\tau}_p^{od}(t)\right], \ \forall (o,d), p, t, \tag{2.3}$$

Due to the complexity of human decision making, travelers make their choice decisions based on different reasoning about the traffic situation and perceive different utilities. With regard to travelers' mental representations, different measures, different indicators and different disutility components can be chosen, thus different models based on the hypotheses can be adopted to represent travelers' choice behavior under uncertainty.

Based on the past cumulative experiences under uncertainty, travelers might have an image of the distributive properties of their perceived travel time distributions at a departure time instant t. We hypothesize that travelers attempt to minimize a weighted function of the expected travel time, the schedule delay costs, which is derived based on their perceived expected travel time, and the variability in travel time, and schedule delays, which can be formulated as:

$$u_p^{od}(t) = \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot \left(PAT - \left(t + E\left[\tilde{\tau}_p^{od}(t)\right]\right)\right)^+ + \gamma_2 \cdot \left(t + E\left[\tilde{\tau}_p^{od}(t)\right] - PAT\right)^+ + \beta \cdot Std\left[\tilde{\tau}_p^{od}(t)\right], \ \forall (o,d), p, t, \tag{2.4}$$

Travelers make a trade-off between all the disutility components.

The models we have discussed are mainly for purpose of our research to investigate the relationship between the typical mean-variance approach and the scheduling approach. There are many other models based on different hypotheses on travelers' departure time/route choice behavior under uncertainty. The full overview of these models is out of the scope of this dissertation.

2.3 A Generalized Model for Traveler's Departure Time/Route Choice Behavior Under Uncertainty

Based on above models representing travelers' choice behavior under uncertainty, some questions come up, for instance which utility function performs best, which disutility components should be included, what are the relationships among different utility functions, is there any better utility function and forth alike. Already in 1994, the relationship between the expected utility approach and the mean-standard deviation approach has been analyzed (Senna 1994), but in his work, the utility does not account for any scheduling costs. Some analytical investigations on the utility function (2.2) has been performed by (Fosgerau and

Karlstrom 2008), which showed that in case the travel time distribution is independent of departure times, the expected scheduling cost is linear in mean and standard deviation of travel time by assuming normally distributed travel time. The equivalence between the travel time budget approach and the scheduling approach has been demonstrated by (Siu and Lo 2007) for a given travel time distribution. The travel time budget approach is proposed by (Lo *et al.* 2006), which is defined based on the probability requirement of arrivals within the travel time budget. This approach shares a similar form as the mean-variance approach, but carries an entirely different interpretation. In case the travel time distribution is known, the parameter of the standard deviation in the travel time budget function can be derived given a probability that a trip completes within the travel time budget.

In this section, we analyze the relationship between the expected utility approach including scheduling costs (the so-called scheduling approach) and the mean-variance approach without specific assumptions on the travel time distribution and we prove the principal equivalence of the mean-variance approach and the scheduling approach conditional on departure times, which is the same conclusion as from (Fosgerau and Karlstrom 2008), but for general travel time distributions. Based on these results, a new generalized utility function is proposed for modeling travelers' departure time and route choice behavior under uncertainty.

2.3.1 Equivalence of the mean-variance approach and the scheduling approach

In this section, we will prove the equivalence of the mean-variance approach and the scheduling approach conditional on departure times.

Let's start with the scheduling approach based on Equation (2.2). Its utility function assumes linearity of schedule delays, which means that travelers have constant values for early and late schedule delays. This assumption will be relaxed later in Section 2.5. With this utility function, it is assumed that travelers have perfect knowledge on the distributive properties of their future travel times and the schedule delay early and late due to their past experiences, for all departure time instants and all available routes.

In the following, we will analytically prove the equivalence of the mean-variance approach and the scheduling approach from a totally different approach without any assumption on the travel time distribution.

Theorem 2.1: *If schedule delay cost is a linear function, then for any departure time t, the expected schedule delay cost on route p between OD pair* (o, d) *can be decomposed into a linear function of schedule delay cost with expected travel time on route p between OD pair* (o, d) *when departing at t and the standard deviation of travel time on route p between OD pair* (o, d) *when departing at t.*

This theorem can be expressed by Formula (2.5), where ξ_t is a constant for any departure time instant t. In case the travel time distribution $\tilde{\tau}_p^{od}(t)$ at any departure time instant t follows a normal distribution, ξ_t can be derived directly and expressed mathematically.

$$\gamma_1 \cdot E\left[\left(PAT-\left(t+\tilde{\tau}_p^{od}(t)\right)\right)^+\right]+\gamma_2 \cdot E\left[\left(t+\tilde{\tau}_p^{od}(t)-PAT\right)^+\right]$$
$$=$$
$$\begin{cases} -\gamma_1 \cdot E\left[\tilde{\tau}_p^{od}(t)\right]+\xi_t \cdot Std\left[\tilde{\tau}_p^{od}(t)\right]+\gamma_1 \cdot (PAT-t), & \left(t+\tilde{\tau}_p^{od}(t)\right)\le PAT \\ \gamma_2 \cdot E\left[\tilde{\tau}_p^{od}(t)\right]+\xi_t \cdot Std\left[\tilde{\tau}_p^{od}(t)\right]+\gamma_2 \cdot (t-PAT), & \left(t+\tilde{\tau}_p^{od}(t)\right)> PAT \end{cases}, \forall (o,d), p, t, \tag{2.5}$$

For any given departure time instant t, PAT and t are constants.

Proof: Let's assume a function $g\left(\tilde{x}(t)\right)=\max\left\{0,\tilde{x}(t)\right\}=\left(\tilde{x}(t)\right)^+$, where $\tilde{x}(t)$ is a time dependent random variable that follows a general distribution expressed as $\tilde{x}(t)\sim P\left(\mu(t),\sigma^2(t)\right)$ with mean $\mu(t)$ and standard deviation $\sigma(t)$. We aim to investigate the relationship between $E\left[g\left(\tilde{x}(t)\right)\right]$ and $E\left[\tilde{x}(t)\right]$.

The function $g\left(\tilde{x}\right)$ can be transformed into:

$$g\left(\tilde{x}(t)\right)=\frac{\tilde{x}(t)+\left|\tilde{x}(t)\right|}{2} \tag{2.6}$$

Therefore,

$$E\left[g\left(\tilde{x}(t)\right)\right]=E\left[\frac{\tilde{x}(t)+\left|\tilde{x}(t)\right|}{2}\right]=\frac{1}{2}E\left[\tilde{x}(t)\right]+\frac{1}{2}E\left[\left|\tilde{x}(t)\right|\right] \tag{2.7}$$

It holds that:

$$E\left[\tilde{x}(t)\right]\le E\left[\left|\tilde{x}(t)\right|\right]\le\sqrt{E\left[\left|\tilde{x}(t)\right|^2\right]}=\sqrt{E\left[\tilde{x}(t)^2\right]}=\sqrt{\mu^2(t)+\sigma^2(t)} \tag{2.8}$$

Substituting expression (2.8) into Formula (2.7), we get:

$$\begin{aligned} &\frac{1}{2}\mu(t)+\frac{1}{2}\mu(t)\le\frac{1}{2}E\left[\tilde{x}(t)\right]+\frac{1}{2}E\left[\left|\tilde{x}(t)\right|\right]\le\frac{1}{2}\mu(t)+\frac{1}{2}\sqrt{\mu^2(t)+\sigma^2(t)} \\ &\Rightarrow \mu(t)\le E\left[g\left(\tilde{x}(t)\right)\right]\le\frac{1}{2}\mu(t)+\frac{1}{2}\sqrt{\mu^2(t)+\sigma^2(t)} \end{aligned} \tag{2.9}$$

It can be seen that $E\left[g\left(\tilde{x}(t)\right)\right]$ is always larger than or equal to $E\left[\tilde{x}(t)\right]$.
Due to the fact that:

$$\sqrt{\mu^2(t)+\sigma^2(t)}\le\left|\mu(t)\right|+\sigma(t) \tag{2.10}$$

we further get that:

$$
\begin{aligned}
&\mu(t) \le E\left[g\left(\tilde{x}(t)\right)\right] \le \frac{1}{2}\mu(t) + \frac{1}{2}\left(\left|\mu(t)\right| + \sigma(t)\right) \\
&if \quad \mu(t) \ge 0, \;\; \mu(t) \le E\left[g\left(\tilde{x}(t)\right)\right] \le \mu(t) + \frac{1}{2}\sigma(t) \\
&\qquad\qquad 0 \le E\left[g\left(\tilde{x}(t)\right)\right] - \mu(t) \le \frac{1}{2}\sigma(t) \\
&if \quad \mu(t) < 0, \;\; 0 \le E\left[g\left(\tilde{x}(t)\right)\right] \le \frac{1}{2}\sigma(t)
\end{aligned}
\tag{2.11}
$$

Therefore, $E\left[g\left(\tilde{x}(t)\right)\right]$ for any departure time instant *t* can be expressed by a linear function of $\mu(t)$ and $\sigma(t)$ at departure time instant *t* such as:

$$
E\left[g\left(\tilde{x}(t)\right)\right] = \lambda_t \cdot \mu(t) + \delta_t \cdot \sigma(t), \quad with \;\; 0 \le \delta_t \le \frac{1}{2}, \quad \text{and } \lambda_t = \begin{cases} 1, & \text{if } \mu(t) \ge 0 \\ 0, & \text{if } \mu(t) < 0 \end{cases}
\tag{2.12}
$$

where λ_t and δ_t are constants for any combination of $\mu(t)$ and $\sigma(t)$ at departure time instant *t*. $E\left[g\left(\tilde{x}(t)\right)\right]$ is a linear function of $\mu(t)$ and $\sigma(t)$ for any departure time instant *t*.

As can be seen in Formula (2.12), λ_t and δ_t are constants for any travel time distribution with a combination of $\mu(t)$ and $\sigma(t)$ at departure time instant *t*, but are different for different travel time distributions. Therefore, these parameters are attribute dependent (attributes denote the distulity components in the utility function). Formula (2.12) has the very important implication that the expectation of schedule delay costs for a given travel time distribution with a given combination of $\mu(t)$ and $\sigma(t)$ can be expressed as a linear function of expectation of $\tilde{x}(t)$ and standard deviation of $\tilde{x}(t)$ at time instant *t*. In our case, $\tilde{x}(t)$ represents the schedule delay cost with $\tilde{x}(t) = PAT - \left(t + \tilde{\tau}_p^{od}(t)\right)$ or $\tilde{x}(t) = t + \tilde{\tau}_p^{od}(t) - PAT$. Since the standard deviation of the schedule delay is the same as the standard deviation of travel time with $Var\left(PAT - \left(t + \tilde{\tau}_p^{od}(t)\right)\right) = Var\left(t + \tilde{\tau}_p^{od}(t) - PAT\right) = Var\left(\tilde{\tau}_p^{od}(t)\right)$, the expectation of the (dis)utility can be expressed as:

$$
\begin{aligned}
u_p^{od}(t) &= \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot E\left[\left(PAT - \left(t + \tilde{\tau}_p^{od}(t)\right)\right)^+\right] + \gamma_2 \cdot E\left[\left(t + \tilde{\tau}_p^{od}(t) - PAT\right)^+\right] \\
&= \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot \left(\lambda_{t1} \cdot \left(PAT - \left(t + E\left[\tilde{\tau}_p^{od}(t)\right]\right)\right) + \delta_{t1} \cdot Std\left[\tilde{\tau}_p^{od}(t)\right]\right) \\
&\qquad + \gamma_2 \cdot \left(\lambda_{t2} \cdot \left(t + E\left[\tilde{\tau}_p^{od}(t)\right] - PAT\right) + \delta_{t2} \cdot Std\left[\tilde{\tau}_p^{od}(t)\right]\right) \\
&\text{with } \; 0 \le \delta_{t1}, \delta_{t2} \le \frac{1}{2}, \;\; \text{and } \begin{cases} \lambda_{t1}=1, \; \lambda_{t2} = 0, \; \text{if } \; E\left[\tilde{\tau}_p^{od}(t)\right] \le PAT - t \\ \lambda_{t1}=0, \; \lambda_{t2} = 1, \; \text{otherwise} \end{cases} \\
&\forall (o,d), p, t,
\end{aligned}
\tag{2.13}
$$

Formula (2.13) can be simplified to:

$$
\begin{aligned}
u_p^{od}(t) &= \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot E\left[\left(PAT - \left(t + \tilde{\tau}_p^{od}(t)\right)\right)^+\right] + \gamma_2 \cdot E\left[\left(t + \tilde{\tau}_p^{od}(t) - PAT\right)^+\right] \\
&= \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot \left(PAT - \left(t + E\left[\tilde{\tau}_p^{od}(t)\right]\right)\right)^+ + \gamma_2 \cdot \left(t + E\left[\tilde{\tau}_p^{od}(t)\right] - PAT\right)^+ \\
&\qquad + \xi_t \cdot Std\left[\tilde{\tau}_p^{od}(t)\right] \\
&= \begin{cases} (\alpha - \gamma_1) \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \xi_t \cdot Std\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot (PAT - t), & \left(t + \tilde{\tau}_p^{od}(t)\right) \le PAT \\ (\alpha + \gamma_2) \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \xi_t \cdot Std\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_2 \cdot (t - PAT), & \left(t + \tilde{\tau}_p^{od}(t)\right) > PAT \end{cases} \\
&\textit{with } \ \xi_t = (\gamma_1 \delta_{t1} + \gamma_2 \delta_{t2}), \quad 0 \le \delta_{t1}, \delta_{t2} \le \frac{1}{2}, \ \forall (o,d), p, t,
\end{aligned} \tag{2.14}
$$

where ξ_t is a constant for any travel time distribution with a combination of $\mu(t)$ and $\sigma(t)$ at departure time instant t, but is different for different travel time distributions with combinations of $\mu(t)$ and $\sigma(t)$. ξ_t has a range of:

$$0 \le \xi_t \le 1/2 \cdot (\gamma_1 + \gamma_2) \tag{2.15}$$

Formula (2.14), except for the first disutility component of $\alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right]$, appears to be identical to Formula (2.5). ■

2.3.2 A new generalized utility function

During the process of the proof in previous section, better understanding of the schedule delay costs is gained, which inspires an idea of a generalized utility function for modeling travelers' departure time/route choice behavior under uncertainty. We propose a new generalized (dis)utility function, formulated as:

$$
\begin{aligned}
u_p^{od}(t) &= \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot \left(PAT - \left(t + E\left[\tilde{\tau}_p^{od}(t)\right]\right)\right)^+ + \gamma_2 \cdot \left(t + E\left[\tilde{\tau}_p^{od}(t)\right] - PAT\right)^+ \\
&\qquad + \beta_t \cdot Std\left[\tilde{\tau}_p^{od}(t)\right]
\end{aligned} \tag{2.16}
$$

where β_t is a parameter of the standard deviation, instead of a constant. The generalized (dis)utility function (2.16), from the mathematical formulations, is different from the scheduling approach. The schedule delay cost in the generalized utility function is calculated based on the expectation of travel time, rather than the expected schedule delay cost in the scheduling approach. There is a separate disutility component of the standard deviation of travel time. The generalized utility function is also different from the mean-variance approach, since it explicitly includes the schedule delay early and late disutility.

2.3.3 Conclusions

This section discusses the advantages and disadvantages of the different approaches of modeling traveler's choice behavior under uncertainty and draws some conclusions.

Conclusion 1: We have proven analytically the principal equivalence of the scheduling approach and the mean-variance approach in traveler's choice behavior modeling for any arbitrary departure time given a combination of $\mu(t)$ and $\sigma(t)$, without assuming any type of the travel time distributions. The uncertainty is predominantly captured by travel time variability since the uncertainty on the arrival time at destination is caused by the uncertainty in travel time. The variability of arrival time is strongly correlated to variability in travel time. Thus in the utility function traveler's perceived uncertainty can be reflected by the variability in travel time.

Conclusion 2: A new generalized utility function (2.16) is proposed. Although the new generalized utility function (2.16) is consistent with the scheduling approach (see utility function (2.2)) when β_t is well chosen to be the same as ξ_t, their behavioral explanations are quite different. With the scheduling approach, travelers consider the expected schedule delay costs, where the average values of both schedule delay early and late are positive. It means that travelers will experience both schedule delay early and late on the same trip and will never expect to be on time, which is not logical. However, with the generalized utility function, travelers consider the schedule delay costs based on the expectation of travel time. This is more plausible and behaviorally sound since travelers based on their expected travel time anticipate that they will be either early or late or on time. Travelers are assumed to be able to know the expected travel time based on their past accumulative experiences. Then they infer when they will arrive at the destination.

Conclusion 3: Scheduling costs account for the aversion to travel time unreliability as found by (Small *et al.* 1999; Hollander 2006; Van Amelsfort *et al.* 2008). From Formula (2.14), the expectation of schedule delay cost already includes travel time variability, which proves their empirical findings analytically. The scheduling approach indeed represents travelers' risk aversions since based on our derivations the expected utility is larger than the utility of expected values:

$$\begin{aligned} &E\left[\left(PAT-\left(t+\tilde{\tau}_p^{od}(t)\right)\right)^+\right] \geq \left(PAT-\left(t+E\left[\tilde{\tau}_p^{od}(t)\right]\right)\right)^+, \ \forall(o,d),p,t, \\ &E\left[\left(t+\tilde{\tau}_p^{od}(t)-PAT\right)^+\right] \geq \left(t+E\left[\tilde{\tau}_p^{od}(t)\right]-PAT\right)^+, \quad \forall(o,d),p,t, \end{aligned} \tag{2.17}$$

Adding a separate component of variability in travel time as shown in Equation (2.3) together with expected schedule delay cost seems unnecessary and may overestimate the impacts of uncertainty on traveler's departure time/route choice behavior.

Figure 2.1 shows the curves of schedule delay early and late respectively for departure time instant *t*. The horizontal axis denotes the stochastic travel times when departing at time instant *t*. Both curves are convex, according to Jensen's inequality, inequality (2.17) holds. The expected schedule delays and the schedule delays based on expected travel time are illustrated by dots in Figure 2.1 to show the relative relationships.

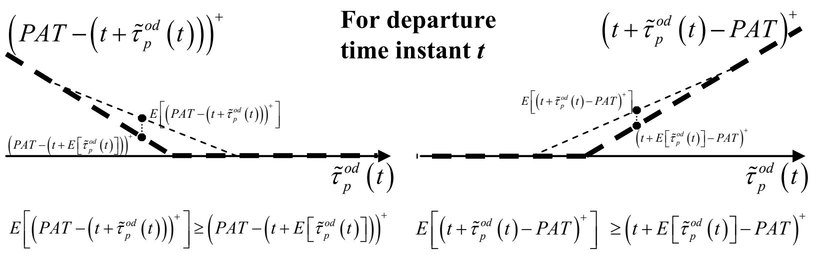


Figure 2.1: Illustration of the schedule delay early and late curves for departure time instant *t*

Conclusion 4: It is concluded from (Noland and Polak 2002) based on empirical studies that there is a fundamental link between the travel time variability and scheduling costs. However in some limited circumstances there appear to be some residual effects of reliability that are not completely subsumed by scheduling considerations. Similar results are also found by (Van Amelsfort *et al.* 2008) showing that travel time unreliability cannot be captured completely by scheduling delay costs, and besides a scheduling effect, travel time reliability is a separate source of travel disutility. The new generalized utility function (2.16) is able to overcome above shortcoming of the scheduling approach since the disutility parameter of travel time variability β_t can be very flexible. The scheduling approach is a special case of the generalized utility function (2.16).

Conclusion 5: Compared to the mean-variance approach, the generalized utility function (2.16) explicitly accounts for the scheduling delay early and late. Thus the mean-variance approach is also a special case of the generalized utility function (2.16). The generalized utility function (2.16) is more general for modeling travelers' departure time/route choice behavior under uncertainty.

The proposed generalized utility function needs calibration by conducting empirical studies.

2.4 Two-dimensional Parameters in the Utility Functions

This section discusses the underlying two dimensional meanings of ξ_t in the Formula (2.14).

As discussed for the equivalence that ξ_t is a constant for a given travel time distribution with a combination of $\mu(t)$ and $\sigma(t)$ at departure time instant *t*. For different travel time distributions, ξ_t will be different (we call it attribute dependent), but bounded. This attribute dependency of the parameters is different from varied parameter values representing heterogeneity of travelers.

Normally we assume that travelers' parameters of the attributes follow distributions representing the taste variations among travelers in a population. It is mostly neglected that the parameters also should depend on the attribute values for instance travel time, schedule delay and travel time variability. So, modeling travelers' behavior should take into account: heterogeneity of travelers and time dependent traffic situations (travel time and travel time variability). The parameter of each (dis)utility component should contain two forms of information, namely stemming from heterogeneity of travelers (different reactions to the same situation) and attribute dependence (same travelers may have different reactions to different traffic situations or to different values of the attribute over time). Therefore the parameters have a two-dimensional distribution, one is crossing the population and the other is a dimension in attribute values. The second one is helpful in capturing the non-linearities of different (dis)utility components. For instance, the parameter of schedule delay should be schedule delay dependent. This might hold for many other (dis)utility attributes like travel time or travel time variability. If the travel time variability is very small, travelers might not care that much (variability will be insignificant in the utility function). If the variability is very high, travel time variability might be overwhelming in the utility function and appears fairly significant. All these behavioral thinking can be explained by the attribute dependent parameters, such as the parameters δ_t and ξ_t in Formulae (2.13) and (2.14). Another example is that a lot of surveys investigate the percentage of risk neutral, risk aversion and risk prone travelers. It is hypothesized here that the results may depend on the degree of risks. With different degrees of risk, a traveler can be risk neutral, risk aversion and risk prone, which means that the way travelers treat risk or uncertainty depends on the degree (i.e. attribute value, like variability, travel time, schedule delay, etc). It helps generalize the utility function with attribute dependent parameters especially when modeling departure time choice with scheduling costs.

However it will make the modeling problem even more complicated. Depending on the purposes of the research, appropriate parameter values will be adopted.

2.5 Non-linearity in Schedule Delay Costs

This section relaxes the assumption on the linearity in the schedule delay costs made in Section 2.3.

A lot of empirical surveys indicate the non-linearity in the scheduling cost (Small *et al.* 1999). Different approaches are available to capture the non-linearity. One option is assigning attribute dependent parameter value distributions to schedule delay early or late depending on the schedule delay, as discussed in the previous section. Another option is assuming a non-linear function in scheduling delay costs. It is found that a quadratic rather than linear function in scheduling cost will improve the models (Small *et al.* 1999), especially for modeling schedule delay early, which suggests a utility function such as:

$$u_p^{od}(t) = \alpha \cdot E\left[\tilde{\tau}_p^{od}(t)\right] + \gamma_1 \cdot E\left[\left(\left(PAT - \left(t + \tilde{\tau}_p^{od}(t)\right)\right)^+\right)^2\right] + \gamma_2 \cdot E\left[\left(\left(t + \tilde{\tau}_p^{od}(t) - PAT\right)^+\right)^2\right] \tag{2.18}$$

Equation (2.18) is an improved scheduling approach with quadratic scheduling cost function. Although it is suggested that specifying a quadratic term in schedule delay late is inferior to allowing for an additional penalty for lateness beyond a particular point (Small *et al.* 1999), we just employ quadratic terms in both schedule delay early and late to capture the non-linearity in schedule delay costs for the sake of simplicity.

This section mainly aims to investigate the relationship between the improved scheduling approach and a transformed mean-variance approach in case of quadratic scheduling costs.

Theorem 2.2: *For any departure time t, the expectation of the quadratic schedule delay cost on route p between OD pair* (*o*, *d*) *can be decomposed into a quadratic function of schedule delay cost with the expected travel time on route p between OD pair* (*o*, *d*) *when departing at t and of the standard deviation of travel time on route p between OD pair* (*o*, *d*) *when departing at t.*

This theorem can be expressed by Equation (2.19). ρ_t and ς_t are constants for a given travel time distribution at departure time instant *t*:

$$\begin{aligned}&\alpha\cdot E\left[\tilde{\tau}_p^{od}(t)\right]+\gamma_1\cdot E\left[\left(\left(PAT-\left(t+\tilde{\tau}_p^{od}(t)\right)\right)^+\right)^2\right]+\gamma_2\cdot E\left[\left(\left(t+\tilde{\tau}_p^{od}(t)-PAT\right)^+\right)^2\right]\\&=\\&\alpha\cdot E\left[\tilde{\tau}_p^{od}(t)\right]+\rho_t\cdot\left(PAT-\left(t+E\left[\tilde{\tau}_p^{od}(t)\right]\right)\right)^2+\varsigma_t\cdot Var\left[\tilde{\tau}_p^{od}(t)\right]\end{aligned}\tag{2.19}$$

Proof: We will explore the relationship between $E\left[g^2\left(\tilde{x}(t)\right)\right]$ and $E\left[\tilde{x}(t)\right]$, where $g\left(\tilde{x}(t)\right)=\max\left\{0,\tilde{x}(t)\right\}=\left(\tilde{x}(t)\right)^+$.

$$E\left[g^2\left(\tilde{x}(t)\right)\right]=E\left[\frac{\tilde{x}(t)+\left|\tilde{x}(t)\right|}{2}\right]^2=\frac{1}{4}E\left[\tilde{x}(t)+\left|\tilde{x}(t)\right|\right]^2\tag{2.20}$$

It holds that:

$$E^2\left[\tilde{x}(t)+\left|\tilde{x}(t)\right|\right]\le E\left[\tilde{x}(t)+\left|\tilde{x}(t)\right|\right]^2\le E\left[\left|\tilde{x}(t)\right|+\left|\tilde{x}(t)\right|\right]^2=4E\left[\tilde{x}(t)\right]^2\tag{2.21}$$

According to Formula (2.11):

$$\begin{aligned}&E\left[\tilde{x}(t)+\left|\tilde{x}(t)\right|\right]\ge 2\mu(t),\ \textit{if}\ \mu(t)\ge 0,\\&E\left[\tilde{x}(t)+\left|\tilde{x}(t)\right|\right]\ge 0,\qquad \textit{if}\ \mu(t)<0,\\\text{Therefore}\quad&E^2\left[\tilde{x}(t)+\left|\tilde{x}(t)\right|\right]\ge 4\mu^2(t),\ \textit{if}\ \mu(t)\ge 0,\\&E^2\left[\tilde{x}(t)+\left|\tilde{x}(t)\right|\right]\ge 0,\qquad \textit{if}\ \mu(t)<0,\end{aligned}\tag{2.22}$$

Combining inequations (2.21) and (2.22), it is derived that:

$$\begin{aligned}&4\mu^2(t)\le E\left[\tilde{x}(t)+\left|\tilde{x}(t)\right|\right]^2\le 4\mu^2(t)+4\sigma^2(\mathrm{t}),\ \textit{if}\ \mu(t)\ge 0,\\&0\le E\left[\tilde{x}(t)+\left|\tilde{x}(t)\right|\right]^2\le 4\mu^2(t)+4\sigma^2(\mathrm{t}),\qquad \textit{if}\ \mu(t)<0,\end{aligned}\tag{2.23}$$

With Equation (2.20), we derive:

$$\begin{aligned}&\mu^2(t)\le E\left[g^2\left(\tilde{x}(t)\right)\right]\le \mu^2(t)+\sigma^2(\mathrm{t}),\ if\ \ \mu(t)\ge 0,\\ &0\le E\left[g^2\left(\tilde{x}(t)\right)\right]\le \mu^2(t)+\sigma^2(\mathrm{t}),\qquad if\ \ \mu(t)<0,\end{aligned} \tag{2.24}$$

Thus $E\left[g^2\left(\tilde{x}(t)\right)\right]$ can be expressed as a quadratic function of $\mu(t)$ and $\sigma(t)$ with constants φ_t and ϕ_t for departure time instant *t* such as:

$$\begin{aligned}&E\left[g^2\left(\tilde{x}(t)\right)\right]=\varphi_t\cdot\mu^2(t)+\phi_t\cdot\sigma^2(t),\\ &\text{with }\begin{cases}\varphi_t=1, & \text{if } \mu(t)\ge 0\\ 0\le\varphi_t<1, & \text{if } \mu(t)<0\end{cases}\quad \text{and } 0\le\phi_t\le 1.\end{aligned} \tag{2.25}$$

Again $\tilde{x}(t)=PAT-\left(t+\tilde{\tau}_p^{od}(t)\right)$ or $\tilde{x}(t)=t+\tilde{\tau}_p^{od}(t)-PAT$. Thus the expectation and the variance can be derived as $\mu(t)=PAT-\left(t+E\left[\tilde{\tau}_p^{od}(t)\right]\right)$ or $\mu(t)=t+E\left[\tilde{\tau}_p^{od}(t)\right]-PAT$ and $\sigma(t)=Std\left[\tilde{\tau}_p^{od}(t)\right]$.

Therefore, the improved scheduling approach (2.18) can be expressed as:

$$\begin{aligned}&u_p^{od}(t)=\alpha\cdot E\left[\tilde{\tau}_p^{od}(t)\right]+\gamma_1\cdot E\left[\left(\left(PAT-\left(t+\tilde{\tau}_p^{od}(t)\right)\right)^+\right)^2\right]+\gamma_2\cdot E\left[\left(\left(t+\tilde{\tau}_p^{od}(t)-PAT\right)^+\right)^2\right]\\ &=\alpha\cdot E\left[\tilde{\tau}_p^{od}(t)\right]+\gamma_1\varphi_{t1}\cdot\left(PAT-\left(t+E\left[\tilde{\tau}_p^{od}(t)\right]\right)\right)^2+\gamma_1\phi_{t1}\cdot Var\left[\tilde{\tau}_p^{od}(t)\right]\\ &+\gamma_2\varphi_{t2}\cdot\left(t+E\left[\tilde{\tau}_p^{od}(t)\right]-PAT\right)^2+\gamma_2\phi_{t2}\cdot Var\left[\tilde{\tau}_p^{od}(t)\right]\\ &\text{with }\begin{cases}\varphi_{t1}=1,\ 0\le\varphi_{t2}<1,\ 0\le\phi_{t1},\phi_{t2}\le 1,\ \text{if } E\left[\tilde{\tau}_p^{od}(t)\right]\le PAT-t\\ 0\le\varphi_{t1}<1,\ \ \varphi_{t2}=1,\ 0\le\phi_{t1},\phi_{t2}\le 1,\ \text{otherwise}\end{cases}\end{aligned} \tag{2.26}$$

Expression (2.26) can be simplified as:

$$\begin{aligned}&u_p^{od}(t)=\alpha\cdot E\left[\tilde{\tau}_p^{od}(t)\right]+\rho_t\cdot\left(PAT-\left(t+E\left[\tilde{\tau}_p^{od}(t)\right]\right)\right)^2+\varsigma_t\cdot Var\left[\tilde{\tau}_p^{od}(t)\right]\\ &\text{with } \rho_t=\gamma_1\varphi_{t1}+\gamma_2\varphi_{t2},\ \varsigma_t=\gamma_1\phi_{t1}+\gamma_2\phi_{t2},\ \text{for } \forall(o,d),p,t,\end{aligned} \tag{2.27}$$

where $0\le\varsigma_t\le\gamma_1+\gamma_2$ and is dependent on the travel time distribution at time instant *t*. ■

In case the quadratic schedule delay cost function is considered, the schedule delay costs can still be expressed as a function of the expected travel time and the standard deviation of travel time.

2.6 Conclusions

This chapter mainly investigates the relationship between the expectation of schedule delay costs and the expectation of travel time and standard deviation of travel time. The principal equivalence of the scheduling approach and the mean-variance approach in choice modeling under uncertainty has been proven analytically for any travel time distribution at departure time instant t.

The expectation of schedule delay costs at departure time instant t can be decomposed into a linear function of the expected travel time and the standard deviation of travel time at departure time instant t in case assuming linearity in schedule delay costs.

Scheduling costs already account for the aversion to travel time unreliability. A new generalized utility function, composed of expected travel time, schedule delays based on the expected travel time and travel time variability, is proposed and formulated. This utility function, compared to the mean-variance approach, explicitly accounts for the schedule delay costs. Compared to the scheduling approach, the new generalized utility function is more flexible in terms of capturing risk aversions caused by travel time uncertainty and is more behaviorally sound. The scheduling approach and the mean-variance approach are special cases of the proposed generalized utility function.

A special case of the non-linear schedule delay cost function, i.e. a quadratic function, is considered. It turns out that the improved scheduling approach is equivalent to a transformed mean-variance approach since the quadratic schedule delay costs can be expressed as a quadratic function of the expected travel time and the standard deviation of travel time.

The new generalized utility function will be adopted to model travelers' departure time/route choice behavior under uncertainty. The departure pattern with stochastic networks will be investigated analytically for a single bottleneck and with a simulation-based approach for larger networks in the upcoming two chapters respectively. With the modeled departure pattern under uncertainty, the network performance will be evaluated and road networks will be designed.

Part III: Theoretical and Simulation-based Comprehensive Network Modeling under Uncertainty

3 Theoretical Investigation of Dynamic User Equilibrium in a Stochastic Bottleneck Model

3.1 Introduction

As discussed in Chapter 2, scheduling costs play a very important role in modeling traveler's choice behavior under uncertainty. Due to the variability in travel times, a crucial decision travelers have to make is their departure time, which together with travel time and travel time reliability will directly determine their scheduling costs. Modeling departure time choice and traveler's departure adaptation under uncertainty is very important. The real life traffic situation can only be represented realistically when the dynamics in travelers' departures are captured. A lot of empirical investigations have been carried out to obtain the departure patterns under uncertainty directly from observations (Van Amelsfort 2009). However based on our best knowledge, in network modeling no attention has been paid and no research has been carried out on the analysis of departure time patterns under uncertainty. Most research on network modeling related to uncertainty focus on route choice modeling and static assignment (Bell 2000; Yin and Ieda 2001; Lo and Tung 2003; Yin *et al.* 2005; Shao *et al.* 2006; Sumalee and Kurauchi 2006; Dimitriou *et al.* 2008; Zhang and Levinson 2008). These studies mostly model travel time reliability in the choice behavior by introducing a reliability index or using exponential or quadratic forms of the travel time function. Reliability is only indirectly encapsulated in the utility function.

Analytical research on dynamic network equilibrium with departure time choice under uncertainty has been quite limited. Therefore this chapter will mainly investigate the departure patterns with modeling travel time reliability in travelers' choice behavior under uncertainty analytically. It is assumed that travel time variability is caused by stochastic capacities representing between-days dynamics. Within-day peak-hour capacity is assumed constant. It is assumed that travelers are aware of the stochastic properties of their experienced travel time and schedule delays at any departure time t and attempt to adapt their departure times in order to minimize their future long trip disutility as expressed in Formula (2.16) in Chapter 2. The research questions in this chapter are: How does the departure pattern with including travel time reliability in the choice behavior differ from that without including travel time reliability? Does a long term temporal equilibrium exist? How does the travel time reliability develop along with departure time during the peak hour?

In order to gain insights into the fundamental relationships and solve aforementioned questions, an analytical investigation is performed by extending Vickrey's bottleneck model towards the stochastic capacity case. The challenge of this chapter is to develop a theoretical analysis of departure patterns including travel time reliability in the travel cost function in stochastic networks.

This chapter firstly presents a brief introduction into Vickrey's bottleneck model with deterministic capacity and some basic notions and formulations. Then the theoretical analyses of departure pattern with stochastic bottleneck model are given with the mathematical derivations and analytical results. A comparison will be made between the departure patterns with and without an inclusion of travel time reliability. Finally some conclusions will be drawn. Since the utility function proposed in Chapter 2 is only composed of travel costs, travel costs and travel cost function will be used in the rest of the dissertation.

3.2 Dynamic User Equilibrium with a Deterministic Bottleneck Model

A widely used model to study the departure pattern is Vickrey's bottleneck model (Vickrey 1969). This model assumes a single bottleneck with constant capacity with a given constant total demand larger than capacity during a limited time period, the usual peak, and known identical preferred arrival times of all travelers with the same origin and destination. The model predicts the within-day equilibrium temporal distribution of demand based on the behavioral hypothesis that all travelers choose their departure times so as to minimize their total individual travel costs consisting of travel (queuing) time and schedule delay costs of arriving early or late than preferred. Due to a single bottleneck, there is only departure time choice and no route choice. Thus in this chapter all notations in the Formulae are simply time specific without any index on route and OD. It is assumed that travelers have perfect information about the traffic conditions they may encounter at all potential departure times. Vickrey's travel cost function, assuming linearity in the schedule delay costs, is given as:

$$c(t)=\alpha\cdot\tau(t)+\gamma_1\left(PAT-\left(t+\tau(t)\right)\right)^{+}+\gamma_2\left(t+\tau(t)-PAT\right)^{+} \tag{3.1}$$

where $c(t)$ denotes the travel cost at departure time instant *t*. $\tau(t)$ denotes the queuing travel time (i.e. delay in the bottleneck) at departure time instant *t*. PAT denotes the preferred arrival time. Again the function $\left(x(t)\right)^{+}=\max\left\{0,x(t)\right\}$. Parameters α, γ_1 and γ_2 denote value of travel time, values of schedule delay early and late, respectively. Homogeneous travelers are assumed with an identical PAT and the same parameter values. This travel cost function assumes linearity in schedule delay costs which simplifies the reality.

Based on the travel cost function (3.1), there is a transition point, defined as t_t, at which both schedule delay early and late are equal to zero, mathematically expressed as:

$$PAT-\left(t_t+\tau(t_t)\right)=t_t+\tau(t_t)-PAT=0 \tag{3.2}$$

Travelers departing at t_t will arrive exactly on time at their destination. Travelers departing before t_t will arrive early at their destination, while those who depart after t_t will arrive late at the destination relative to their PAT.

The original Vickrey model assumes a vertical queue at the head of the bottleneck without spillback. Thus the travel time for travelers departing at time t simply equals the queue length at time t divided by the bottleneck capacity (see (Vickrey 1969)):

$$\begin{aligned} \tau(t) &= \frac{Q(t)}{C} = \frac{D_1(t)}{C} - (t - t_0), & t_0 \le t \le t_t \\ \tau(t) &= \frac{Q(t)}{C} = \frac{D_1(t_t)}{C} + \frac{D_2(t)}{C} - (t - t_0), & t_t \le t \le t_e \end{aligned} \tag{3.3}$$

where $Q(t)$ denotes the queue length at time instant t. C is the deterministic constant capacity rate of the bottleneck. $D_1(t)$ denotes cumulative early departures since all travelers departing before t_t will arrive early at destination, $D_2(t)$ denotes the cumulative late departures, which are expressed as:

$$\begin{aligned} D_1(t) &= \int_{t_0}^{t} r_1(x)\,dx, \;\; t_0 \le t \le t_t, \\ D_2(t) &= \int_{t_t}^{t} r_2(x)\,dx, \;\; t_t \le t \le t_e. \end{aligned} \tag{3.4}$$

$r_1(t)$ and $r_2(t)$ denote early departure rate and late departure rate respectively. t_0 denotes the start time for the first departure at which queueing starts. t_e denotes the time of the last departure experiencing delays. This travel time function is really simple. Of course, other travel time functions can be adopted.

The dynamic user equilibrium for departure time choice results when no traveler can reduce his/her travel cost by unilaterally altering its departure time (Mahmassani and Herman 1984), which can be expressed mathematically as $dc/dt = 0$ for all time instants t with non-zero departures. From the first order derivative of Formula (3.1), the early and late departure rates in the deterministic case are derived as (Vickrey 1969):

$$\begin{cases} r_1(t) = C\left(1 + \dfrac{\gamma_1}{\alpha - \gamma_1}\right), & t_0 \le t \le t_t \\ r_2(t) = C\left(1 - \dfrac{\gamma_2}{\alpha + \gamma_2}\right), & t_t \le t \le t_e \end{cases} \tag{3.5}$$

It can be seen that the early and late departure rates are constant, which is a consequence of the linearity assumption in the schedule delay costs and the simple travel time function.
Assume an inelastic total demand N uses the bottleneck with

$$\int_{t_0}^{t_t} r_1(t)\,dt + \int_{t_t}^{t_e} r_2(t)\,dt = N \tag{3.6}$$

and assume the continuity of the travel costs at the critical time points, then t_0, t_t, and t_e can be derived. For details, see (Vickrey 1969). Figure 3.1 shows the within-day equilibrium departure pattern for the case with total demand N and deterministic capacity C. Homogeneous travelers are assumed with an identical PAT. The critical time points are indicated. Travelers departing at t_t have the longest travel time but will arrive exactly on time. Travelers departing before t_t will arrive earlier than desired, while the travelers departing after t_t will arrive late at the destination. Values of 1.0, 0.8, 1.2 have been chosen for parameters α, γ_1 and γ_2 respectively.

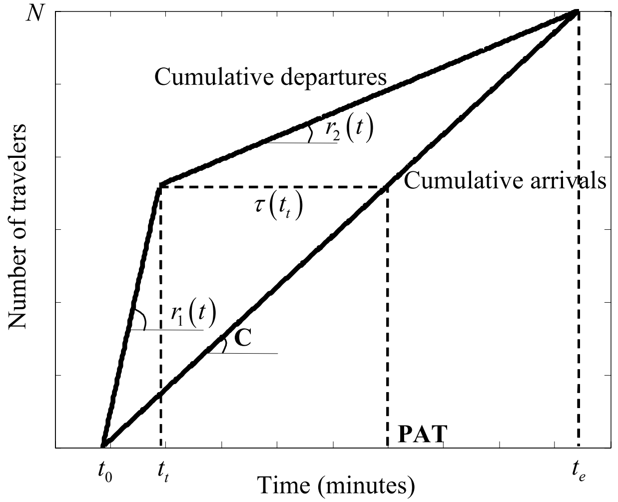


Figure 3.1: Cumulative departures and arrivals in the deterministic case with constant demand *N* and capacity *C*

Figure 3.2 presents the cumulative departures and arrivals for different capacities relative to the capacity C presented in Figure 3.1 with the same constant travel demand N and the same PAT.

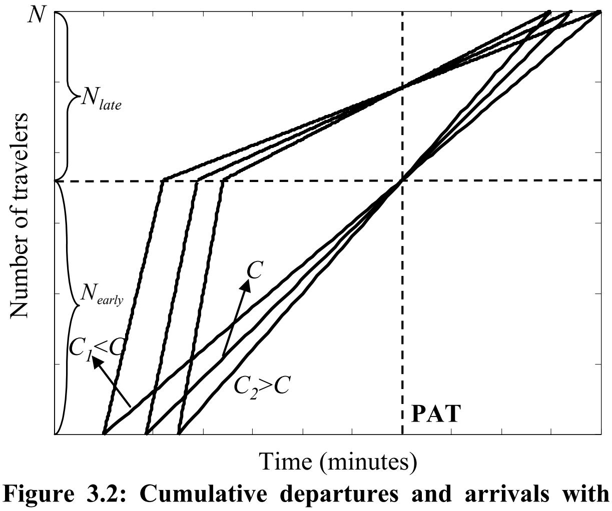


Figure 3.2: Cumulative departures and arrivals with different capacity rates C and constant demand N

As expected, earlier departure rates increase with increasing capacity *C*, while the peak duration decreases with increasing *C*. With larger *C*, the starting time for departures shifts to the right with late departure. All the cumulative curves from different *C* values cross the same two points at PAT, which implies that the number of travelers arriving early or late than preferred is constant and independent of capacities. The total travel time lost in queuing decreases with increasing *C*. This can be proven theoretically since it can be derived that:

$$\frac{N_{early}}{N_{late}} = \frac{\gamma_2}{\gamma_1} \tag{3.7}$$

where N_{early} and N_{late} respectively denote the number of travelers arriving earlier and later than the preferred arrival time.

This bottleneck model, further elaborated by (Fargier 1983), has been extended by many others. Hendrickson and Kocur explored the bottleneck model for several cases: lateness is not allowed; system optimal with tolls; PAT follows a distribution instead of being identical and fixed (Hendrickson and Kocur 1981). The analyses of the bottleneck model with a uniform distribution of PAT has also been presented by (Small 1992). The bottleneck model has been extended by (Mahmassani and Herman 1984) with an additional dimension of choice by considering joint departure time and route choices. However only one origin and one destination with parallel routes is considered and travelers are assumed to pass only one bottleneck to their destination. The departure time pattern for a network with two origins and one destination and for a network with two consecutive bottlenecks has been studied by (Kuwahara 1990). The departure patterns for a network with overlapping routes as an extension of the single bottleneck model have been analyzed by (Zhang *et al.* 2008). The departure pattern for non-identical travelers with different cost functions (i.e. different parameter values for the cost components) has been studied analytically (Newell 1987), while a numerical approach has been adopted to obtain the equilibrium for finite groups of heterogeneous travelers with elastic and fixed demand (Van der Zijpp and Koolstra 2002). Various toll schemes with the bottleneck model have been investigated (Vickrey 1969; Arnott *et al.* 1989). The bottleneck model was used to analyze ramp metering (Arnott *et al.* 1993a). Extensions of the bottleneck model for heterogeneous travelers with the same relative cost of late to early arrival have been performed by (Arnott *et al.* 1994; Arnott *et al.* 1998). They also did analyses of bottleneck model for travelers with different parameters for the late arrivals and for discrete PAT's among travelers. The bottleneck model has been extended with predictable and unpredictable fluctuations in capacity and demand (Arnott *et al.* 1997), where the effects of information on participation and departure time choices were analyzed. A review on the extensions of the Vickrey's bottleneck model has been provided by (Arnott *et al.* 1998), including modeling of stochastic demand, stochastic capacity, multiple routes and OD pairs, toll schemes, ramp metering, system optimum, heterogeneous travelers, discrete PAT and PAT distributions. In this chapter, we will add two extensions of this deterministic bottleneck model by incorporating stochastic capacities over days and modeling travel time reliability in the travel cost function (Li *et al.* 2008b; Li *et al.* 2008d).

3.3 Dynamic User Equilibrium with a Stochastic Bottleneck Model

Day-to-day capacity and demand fluctuations lead to travel time variations over days. The travel times experienced by the travelers departing at the same time instant vary over days. The travel time for a specific departure time instant follows some probability distribution. In Chapter 2 a generalized (dis)utility function has been proposed to model travelers' choice behavior under uncertainty. This section will adopt the derived (dis)utility function and analytically investigate traveler's departure time choice and the resulting long term departure pattern by extending Vickrey's bottleneck model. The travel cost defined by the generalized mean-variance scheduling approach is named long term future trip cost, which is based on traveler's past cumulative experiences. Since the added reliability component (i.e. travel time standard deviation) is a function of a series of within-day traffic situations, which also depends on the reliability in the (dis)utility function, an analytical analysis of the departure pattern with including reliability in the choice behavior is quite challenging. We restrict ourselves to day-to-day fluctuations in capacity and assume that the total daily travel demand using the bottleneck is fixed. For elastic demand, see (Arnott *et al.* 1993b; Van der Zijpp and Koolstra 2002). A long term equilibrium departure pattern is derived by assuming that travelers make their strategic departure time choice based on their past experiences. A deterministic departure pattern will result in which no traveler can reduce his/her long term future trip costs by unilaterally changing their departure time. With this deterministic long term departure pattern, the number of travelers suffering bottleneck delays however may vary from day to day and is a random variable due to the stochastic capacities on specific days. The within-day traffic situation is however not necessarily an equilibrium due to the varied capacities from day to day. It is assumed that travel time variability is only caused by stochastic capacities representing day-to-day dynamics, while within-day capacity is assumed constant.

3.3.1 Behavioral assumptions

We assume a bottleneck with random fluctuations of the capacity from day-to-day and a fixed number of travelers N using that bottleneck, constant from day-to day, such as a fixed number of daily commuters. Due to the past accumulative experiences in using the bottleneck, travelers are assumed to be aware of the stochastic properties of the waiting time (i.e. travel time) distributions at all departure times (i.e. expectations and variances) that emerge from day to day due to the random capacities, without being able to predict the daily traffic situation before starting its trip. As discussed in Chapter 2, it is assumed that travelers make their strategic departure time choice so as to minimize their long term future trip cost $c(t)$, which is defined and formulated as (3.8) according to the generalized mean-variance scheduling approach (see Formula (2.16)), including travel cost components such as expected waiting time, schedule delays based on the expected waiting time, and standard deviation of waiting times. Based on their accumulative experienced travel costs, it is assumed that travelers are aware of their future trip costs at any departure time and make their strategic departure time choice accordingly.

$$c(t)=\alpha\cdot E\left[\tilde{\tau}(t)\right]+\gamma_1\cdot\left(PAT-\left(t+E\left[\tilde{\tau}(t)\right]\right)\right)^{+}+\gamma_2\cdot\left(t+E\left[\tilde{\tau}(t)\right]-PAT\right)^{+}+\beta\cdot Std\left[\tilde{\tau}(t)\right] \tag{3.8}$$

For the sake of simplicity, we still assume a fixed number of N homogeneous travelers, all having the same travel cost function and identical preferred arrival time. A fixed parameter β is assumed for the underlying analytical analysis instead of the attribute dependent parameter β_t in Formula (2.16). Not all of these travelers will suffer delays on arbitrary days. The number of travelers experiencing delays on a specific day depends on the capacity on that day, so does the delay at departure time t.

3.3.2 Existence of a long term equilibrium

We assume that each of the travelers chooses his/her departure time t in order to minimize its long term future trip cost. We will show that a kind of long term equilibrium however emerges from day-to-day capacity variations and travel time variations. Continuous time and continuous flows are dealt with in this chapter. Notice that the equilibrium may be different for continuous flows and discrete flows. $dc^*(t)/dt = 0$ ($c^*(t)$ denotes equilibrium costs at t for the bottleneck) holds at the long term departure time equilibrium for the time instants with positive departure flow rates. For the formulation of the single bottleneck in this chapter, the equilibrium variables are denoted by a superscript *. For the general network equilibrium formulation dealt with in the next chapter, the equilibrium variables are denoted by a bar.

The long term equilibrium for a single bottleneck model can also be formulated as an infinite-dimensional continuous-time Variational Inequality Problem (VIP) aimed to determine the vector of the equilibrium departure flow rates $\mathbf{f}^*$ for all t ($\mathbf{f}^* \equiv \left[f^*(t) \right]$, $\forall t$, where $f^*(t)$ denotes the departure flow rate at time instant t at equilibrium):

$$\int c\left(\mathbf{f}^*, t\right)\left(f(t) - f^*(t)\right) dt \geq 0, \quad \forall \mathbf{f} \in \mathbf{\Omega_d} \tag{3.9}$$

where $c\left(\mathbf{f}^*, t\right)$ denotes the equilibrium cost at time instant t as a function of the vector $\mathbf{f}^*$ representing the departure flow rates at all t. Especially the departure flow rates before t play important roles in $c\left(\mathbf{f}^*, t\right)$ since the travel cost at t is mainly determined by the cumulative departure flows from previous time instants. Ω denotes a bounded infinite feasible set of departure flow rates satisfying flow conservation (3.10) and nonnegativity constraints (3.11), respectively:

$$\int f(t) dt = N, \tag{3.10}$$

$$f(t) \geq 0, \quad \forall t \tag{3.11}$$

Since this chapter only deals with the single-bottleneck model, there is no route choice. The objective of this VI formulation is to find the optimal departure pattern $\mathbf{f}^* \in \mathbf{\Omega_d}$ with which no traveler would be better off by changing his/her departure time. The formulation for a general network equilibrium with continuous time and joint departure time/route choices will be slightly different by incorporating routes and OD pairs (discrete variables) in above formulations. Ω defined by expressions (3.10) and (3.11) is a nonempty and closed convex set.

Under stochastic bottleneck capacities, the experienced travel times over days are stochastic for any departure time t and follow a general distribution. In order to derive the long term future trip costs, we firstly start with the general formulation of the stochastic travel time (stochastic travel times at equilibrium are a special case of Formula (3.12) with equilibrium departure flow rates $f^*(t)$ instead of $f(t)$), which is formulated as:

$$\tilde{\tau}(t) = \frac{\int_{t_0^*}^{t} f(x)\,dx}{\tilde{C}} - \left(t - t_0^*\right), \tag{3.12}$$

where t_0^* denotes the deterministic time instant for the first departures at the long term equilibrium. $\tilde{C}$ denotes the stochastic capacities. $\tilde{\tau}(t)$ is a continuous function of $\mathbf{f}$.

The cost function (3.8) is a function of $\tilde{\tau}(t)$ and thus a continuous function of the departure flow rates $\mathbf{f}$. Together with the condition that Ω is a nonempty and closed convex set, the VIP (3.9) is equivalent to a fixed point problem. With the additional property that Ω is also bounded defined by (3.10), the fixed point problem has at least one solution according to Brouwer's fixed point theorem. See Theorem 3.1 taken from (Chen 1999) and the detailed proof. Thus a long term equilibrium exists and will emerge from day-to-day capacity variations. Because of its long-term orientation, this assumed departure time choice behavior is called strategic.

Theorem 3.1 (Existence of solutions to the VIP) (Chen 1999) If Ω is a compact convex set and $c(\mathbf{f})$ is continuous on Ω, then the VIP admits at least one solution $\overline{\mathbf{f}}$.

The uniqueness of the equilibrium is difficult to prove in general, while as seen later in Chapter 4 with simulations, it always converges to a unique equilibrium state at which the long term future trip costs for travelers departing at any used time t are equal and minimal. With this long term equilibrium departure pattern, a within-day user's departure time equilibrium is not necessarily achieved as travelers are not able to know the traffic situation they will encounter and cannot predict what will happen without any information before making a trip.

3.3.3 Random capacities

It is assumed in this chapter that travel time uncertainty is caused by day-to-day capacity variations. Day-to-day capacity variations are the major factor leading to stochastic travel time variations over days (Tu *et al.* 2007b). We assume a certain random distribution in the capacity (see (Chen *et al.* 2002a)) due to all kinds of causes such as weather volatility, relatively minor incidents, traffic composition, and so on. Extreme cases caused by accidents, disasters, etc. are not intended to be modeled.

To model daily disturbances, one must consider various levels of capacity degradation. In other words, roadway capacities are continuous quantities subject to routine degradation due to physical and operational factors. A reasonable way to capture these variations and their impact on the network performance is to model roadway capacities using probability distributions (Chen *et al.* 2002a). Current research applies different assumptions of the distribution of stochastic link capacity, such as uniform (Li *et al.* 2008b; Li *et al.* 2008d), normal (Li *et al.* 2007a; Li *et al.* 2008a; Li *et al.* 2009b), exponential (Lo and Tung 2003), gamma (Brilon and Zurlinden 2003), weibull (Brilon 2005), etc. Depending on characteristics of road facilities in a specific area, different link capacity distributions might be chosen. It is found that the weibull distribution, apart from its theoretical attractiveness, is the function that best fitted the capacity observations (Brilon 2005).

For the sake of simplicity, the stochastic capacity (denoted as $\tilde{C}$) is assumed uniformly distributed with an upper bound $C_{\max}$ and a lower bound $C_{\min}$. Of course more realistic distributions like weibull can be assumed. For the impact of different classes of capacity distributions on the departure patterns under uncertainty, further research needs to be carried out. Without loss of generality, we assume that the minimum capacity is proportional to the maximum capacity with a fraction factor θ, (i.e., $C_{\min} = \theta C_{\max}, 0 < \theta < 1$). A larger θ value indicates a smaller capacity variation. The following properties then hold in general:

$$E\left(\frac{1}{\tilde{C}}\right) = \int_{\theta C_{\max}}^{C_{\max}} \frac{1}{\tilde{C}} \cdot \frac{1}{C_{\max} - \theta C_{\max}} d\tilde{C} = \frac{1}{C_{\max}(1-\theta)} \ln\frac{1}{\theta} = \frac{1}{f_\theta \cdot C_{\max}} \tag{3.13}$$

with a capacity distribution parameter $f_\theta = (1-\theta)/\ln\frac{1}{\theta}, \quad 0 < f_\theta < 1$.

Since

$$\lim_{\theta\to 1} f_\theta = \lim_{\theta\to 1}\left((1-\theta)/\ln\frac{1}{\theta}\right) = 1, \tag{3.14}$$

the deterministic capacity case is a special case of the random capacity case.

$$E\left(\frac{1}{\tilde{C}^2}\right) = \int_{\theta C_{\max}}^{C_{\max}} \frac{1}{\tilde{C}^2} \cdot \frac{1}{C_{\max} - \theta C_{\max}} d\tilde{C} = \frac{1}{C_{\max}(1-\theta)}\left(\frac{1}{\theta C_{\max}} - \frac{1}{C_{\max}}\right) = \frac{1}{\theta C_{\max}^2} \tag{3.15}$$

These properties are needed to derive the stochastic temporal travel time distributions.

3.3.4 Formulation and analytical solutions to the dynamic user equilibrium

In this subsection, we will derive the long term equilibrium temporal demand distribution (departure time pattern) that emerges in the stochastic capacity case.

A three-regime approach will be adopted, which differs from the two-regime approach used in the deterministic bottleneck model. The analysis will be performed for the three possible regimes ($t_0^* \le t \le t_t^*$, $t_t^* \le t \le t_c^*$, and $t_c^* \le t \le t_e^*$) separately.

Again let's define the transition time point at the long term equilibrium t_t^*, at which it holds that the schedule delay based on the expected travel time is equal to zero:

$$PAT - \left(t_t^* + E\left[\tilde{\tau}\left(t_t^*\right)\right]\right) = t_t^* + E\left[\tilde{\tau}\left(t_t^*\right)\right] - PAT = 0 \tag{3.16}$$

which is equivalent to saying that the summed schedule delay early and late based on the expected travel time is zero if departing at t_t^*.

Travelers departing at t_t^* are expected to arrive exactly on time (PAT) at the destination, while travelers departing before t_t^* will arrive early at their destination, and after t_t^* will arrive late based on the expected travel time. Thus the division of the first two-regimes is determined by different schedule delay functions.

We define t_c^* as the time instant at which the maximum capacity line intersects with the long term cumulative departures at equilibrium, expressed as:

$$\left(t_c^* - t_0^*\right) C_{\max} = D_1^*(t_t^*) + D_2^*(t_c^*) \tag{3.17}$$

where $D_1^*(t)$ denotes cumulative early departures (based on the expected travel time) at the long term equilibrium state during time period $t_0^* \le t \le t_t^*$, while $D_2^*(t)$ denotes the cumulative late departures at long term equilibrium state in the second regime $t_t^* \le t \le t_c^*$, expressed respectively as:

$$\begin{aligned} D_1^*(t) &= \int_{t_0^*}^{t} r_1^*(x)\,dx \\ D_2^*(t) &= \int_{t_t^*}^{t} r_2^*(x)\,dx \end{aligned} \tag{3.18}$$

$r_1^*(t)$ and $r_2^*(t)$ denote the equilibrium early departure flow rate and late departure flow rate during $t_0^* \le t \le t_t^*$ and $t_t^* \le t \le t_c^*$ respectively. Of course the existence of the intersection between the cumulative departures and the maximum capacity line (i.e. existence of t_c^*) strongly depends on the capacity distributions. If capacity factor θ is smaller than a threshold value, implying very large variations in the capacities, then there is no intersection between the cumulative departures and the maximum capacity line. Since we model daily fluctuations in the capacities caused by the weather and minor incidences, θ will be larger than the threshold value. So, t_c^* will always exist. A detailed discussion will be given later in this chapter.

Departing after time instant t_c^*, it may occur that travelers experience no queues on some days with large capacities, implying that the outflow rate equals the departure flow rate rather than the capacity. Travel times are directly influenced by the outflow rate on the bottleneck instead of the given capacities. Therefore the stochastic travel times will be calculated differently due to the varying outflow rates. So does the long term future travel cost. Thus the third regime division, divided by t_c^*, is due to the changed outflow rate distribution. Before t_c^*, the outflow rate distribution is equivalent to the capacity distribution, uniformly distributed with an upper bound $C_{\max}$ and a lower bound $C_{\min}$. After t_c^*, the outflow rate distribution becomes time-dependent, partly following a uniform distribution with a time-dependent upper bound $C_b(t)$ and partly equal to the departure rate. Therefore t_c^* is defined to distinguish the outflow rate distributions. In the deterministic case, the outflow rate always equals the bottleneck capacity. The definition of $C_b(t)$ and other critical time points are illustrated in Figure 3.3:

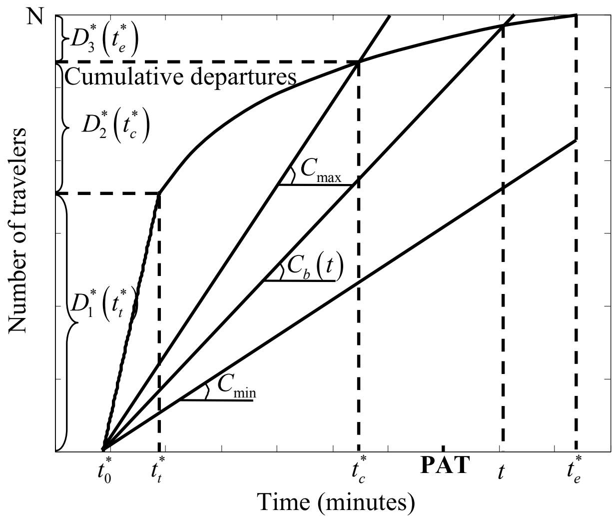


Figure 3.3: Illustration of critical time instant t_c^* and time-dependent upper boundary capacity $C_b(\mathrm{t})$

The time-dependent boundary capacity $C_b(t)$, with which travelers departing at *t* experience exactly zero delays, is expressed as:

$$C_b(\mathrm{t}) = \frac{D_1^*(t_t^*) + D_2^*(t_c^*) + D_3^*(t)}{t - t_0^*} = \frac{D_T^*(t)}{t - t_0^*}, \text{ with } D_T^*(t) = D_1^*(t_t^*) + D_2^*(t_c^*) + D_3^*(t) \quad (3.19)$$

$D_3^*(t)$ denotes the cumulative late departures at long term equilibrium state during $t_c^* \le t \le t_e^*$, expressed as:

$$D_3^*(t) = \int_{t_c^*}^{t} r_3^*(x)\,dx \quad (3.20)$$

$r_3^*(t)$ denotes the equilibrium late departure rate during $t_c^* \le t \le t_e^*$. On days with capacities larger than $C_b(t)$ and smaller than $C_{\max}$, travelers departing at *t* will experience no delays.

Let's define $\tilde{v}(t)$ as the stochastic outflow rate for time instant *t*, then the properties of $\tilde{v}(t)$ in each of the three regimes can be derived. As aforementioned, before t_c^* the outflow rate distribution is equivalent to the capacity distribution since the outflow rate is always equal to the capacity on any specific day, thus we have:

$$E(\tilde{v}(t)) = E(\tilde{C}) = \frac{C_{\max}(1+\theta)}{2}, \quad t_0^* \le t \le t_c^* \quad (3.21)$$

$$E\left(\frac{1}{\tilde{v}(t)}\right) = E\left(\frac{1}{\tilde{C}}\right) = \frac{1}{f_\theta \cdot C_{\max}}, \quad t_0^* \le t \le t_c^* \quad (3.22)$$

$$E\left(\frac{1}{\tilde{v}^2(t)}\right)=E\left(\frac{1}{\tilde{C}^2}\right)=\frac{1}{\theta C_{\max}^2},\quad t_0^*\le t\le t_c^* \tag{3.23}$$

After t_c^*, the outflow rate $\tilde{v}(t)$ will partly follow a uniform distribution with a time-dependent upper bound $C_b(t)$, denoted as $\tilde{v}_u(t)$, and partly be equal to departure rate. Therefore the expectation of the outflow rate at time instant *t* can be calculated from the two parts as:

$$\begin{aligned}E(\tilde{v}(t)) &= p_u(t)\cdot E(\tilde{v}_u(t))+(1-p_u(t))\cdot r_3^*(t),\ t_c^*\le t\le t_e^*\\ &= p_u(t)\cdot\frac{C_{\min}+C_b(t)}{2}+(1-p_u(t))\cdot r_3^*(t)\\ &= \frac{C_b(t)-C_{\min}}{C_{\max}(1-\theta)}\cdot\frac{C_{\min}+C_b(t)}{2}+\frac{C_{\max}-C_b(t)}{C_{\max}(1-\theta)}\cdot r_3^*(t)\\ &= \frac{C_b^{\,2}(t)-\theta^2C_{\max}{}^2}{2C_{\max}(1-\theta)}+\frac{C_{\max}-C_b(t)}{C_{\max}(1-\theta)}\cdot r_3^*(t)\\ &= \frac{\left(\dfrac{D_T^*(t)}{t-t_0^*}\right)^2-\theta^2C_{\max}{}^2+(2C_{\max}-2C_b(t))\cdot r_3^*(t)}{2C_{\max}(1-\theta)}\end{aligned} \tag{3.24}$$

where $\tilde{v}_u(t)$ represents the part of the outflow rate following a time-dependent uniform distribution in the range of $C_{\min}\le\tilde{v}_u(t)\le C_b(t)$. $p_u(t)$ is the probability of the outflow rate $\tilde{v}_u(t)$ following an uniform distribution, which is equivalent to the probability of the non-zero travel times that will be experienced by travelers, expressed as:

$$p_u(t)=\frac{C_b(t)-C_{\min}}{C_{\max}(1-\theta)},\quad t_c^*\le t\le t_e^* \tag{3.25}$$

Since we mainly need to derive the non-zero experienced travel times, we will focus on the properties of $\tilde{v}_u(t)$ after t_c^* as following:

$$E\left(\frac{1}{\tilde{v}_u(t)}\right)=\int_{C_{\min}}^{C_b(t)}\frac{1}{\tilde{v}_u(t)}\cdot\frac{1}{C_b(t)-C_{\min}}d\tilde{v}_u=\frac{1}{C_b(t)-C_{\min}}\ln\frac{C_b(t)}{C_{\min}},\quad t_c^*\le t\le t_e^* \tag{3.26}$$

$$\begin{aligned}E\left(\frac{1}{\tilde{v}_u^2(t)}\right)&=\int_{C_{\min}}^{C_b(t)}\frac{1}{\tilde{v}_u^2(t)}\cdot\frac{1}{C_b(t)-C_{\min}}d\tilde{v}_u=\frac{1}{C_b(t)-C_{\min}}\left(\frac{1}{C_{\min}}-\frac{1}{C_b(t)}\right)\\ &=\frac{1}{C_{\min}C_b(t)},\quad t_c^*\le t\le t_e^*\end{aligned} \tag{3.27}$$

Now let's derive the cost components for the long term future travel cost given in Formula (3.8). The expectation of travel time at departure time *t* for each regime is derived respectively as:

$$E\left(\tilde{\tau}(t)\right)=E\left(\frac{D_1(t)}{\tilde{v}(t)}-\left(t-t_0^*\right)\right),\quad t_0^*\le t\le t_t^* \tag{3.28}$$

and
$$E\left(\tilde{\tau}(t)\right)=E\left(\frac{D_1\left(t_t^*\right)}{\tilde{v}(t)}+\frac{D_2(t)}{\tilde{v}(t)}-\left(t-t_0^*\right)\right),\quad t_t^*\le t\le t_c^* \tag{3.29}$$

We define $\tilde{\tau}_u(t)$ as the stochastic non-zero travel times experienced by travelers departing at t, caused by the stochastic $\tilde{v}_u(t)$. Then the expectation of the travel time for the third regime can be computed as:

$$\begin{aligned}E\left(\tilde{\tau}(t)\right)&=\left(1-p_u(t)\right)\cdot 0+p_u(t)\cdot E\left(\tilde{\tau}_u(t)\right),\quad C_{\min}\le\tilde{v}_u(t)\le C_b(t),\quad t_c^*\le t\le t_e^*\\&=p_u(t)\cdot E\left(\frac{D_1\left(t_t^*\right)+D_2\left(t_c^*\right)+D_3(t)}{\tilde{v}_u(t)}-\left(t-t_0^*\right)\right)\end{aligned} \tag{3.30}$$

On an arbitrary day, queue transition and ending times vary according to the capacity on that day. Given a stable long term equilibrium departure pattern exists, of which the temporal departure flow pattern is deterministic/constant from day-to-day, implying that

$$E\left(D(t)\right)=D^*(t) \tag{3.31}$$

where $D^*(t)$ denotes cumulative departures at long term equilibrium state.

Then the expected travel time (i.e. the long-term equilibrium travel time) for each regime can be expressed as:

$$E\left(\tilde{\tau}(t)\right)=D_1^*(t)E\left(\frac{1}{\tilde{v}(t)}\right)-\left(t-t_0^*\right),\quad t_0^*\le t\le t_t^* \tag{3.32}$$

$$E\left(\tilde{\tau}(t)\right)=\left(D_1^*\left(t_t^*\right)+D_2^*(t)\right)E\left(\frac{1}{\tilde{v}(t)}\right)-\left(t-t_0^*\right),\quad t_t^*\le t\le t_c^* \tag{3.33}$$

$$E\left(\tilde{\tau}(t)\right)=p_u(t)\cdot\left[D_T^*(t)\cdot E\left(\frac{1}{\tilde{v}_u(t)}\right)-\left(t-t_0^*\right)\right],\ C_{\min}\le\tilde{v}_u(t)\le C_b(t),\ t_c^*\le t\le t_e^* \tag{3.34}$$

Once we substitute the properties of the stochastic outflow rates, we get:

$$E\left(\tilde{\tau}(t)\right)=\frac{D_1^*(t)}{f_\theta\cdot C_{\max}}-\left(t-t_0^*\right),\quad t_0^*\le t\le t_t^* \tag{3.35}$$

$$E\left(\tilde{\tau}(t)\right)=\frac{\left(D_1^*\left(t_t^*\right)+D_2^*(t)\right)}{f_\theta\cdot C_{\max}}-\left(t-t_0^*\right),\quad t_t^*\le t\le t_c^* \tag{3.36}$$

$$\begin{aligned}E\left(\tilde{\tau}(t)\right)&=p_u(t)\cdot\left[\frac{D_T^*(t)}{C_b(t)-C_{\min}}\ln\frac{C_b(t)}{C_{\min}}-\left(t-t_0^*\right)\right]\\&=\frac{C_b(t)-C_{\min}}{C_{\max}(1-\theta)}\left[\frac{D_T^*(t)}{C_b(t)-C_{\min}}\ln\frac{C_b(t)}{C_{\min}}-\left(t-t_0^*\right)\right]\\&=\frac{D_T^*(t)}{C_{\max}(1-\theta)}\ln\frac{C_b(t)}{C_{\min}}-\frac{C_b(t)-C_{\min}}{C_{\max}(1-\theta)}\left(t-t_0^*\right)\\&=\frac{D_T^*(t)}{C_{\max}(1-\theta)}\ln\frac{D_T^*(t)}{C_{\min}\left(t-t_0^*\right)}-\frac{D_T^*(t)-C_{\min}\left(t-t_0^*\right)}{C_{\max}(1-\theta)},\quad C_{\min}\le\tilde{v}_u(t)\le C_b(t),\ t_c^*\le t\le t_e^*\end{aligned} \tag{3.37}$$

The variance and the standard deviation of the travel times at departure time t can then be derived for the three regimes respectively as:

1) Regime I, for time period $t_0^*\le t\le t_t^*$:

$$\begin{aligned}Var\left(\tilde{\tau}(t)\right)&=E\left(\tilde{\tau}^2(t)\right)-E^2\left(\tilde{\tau}(t)\right)\\&=E\left(\frac{D_1^{*2}(t)}{\tilde{v}_u^{\,2}(t)}-2\left(t-t_0^*\right)\frac{D_1^*(t)}{\tilde{v}_u(t)}+\left(t-t_0^*\right)^2\right)-\left(D_1^*(t)\cdot E\left(\frac{1}{\tilde{v}_u(t)}\right)-\left(t-t_0^*\right)\right)^2\\&=D_1^{*2}(t)\cdot E\left(\frac{1}{\tilde{v}_u^{\,2}(t)}\right)-2\left(t-t_0^*\right)\cdot D_1^*(t)\cdot E\left(\frac{1}{\tilde{v}_u(t)}\right)+\left(t-t_0^*\right)^2\\&\quad-D_1^{*2}(t)\cdot E^2\left(\frac{1}{\tilde{v}_u(t)}\right)-\left(t-t_0^*\right)^2+2D_1^*(t)E\left(\frac{1}{\tilde{v}_u(t)}\right)\left(t-t_0^*\right)\\&=D_1^{*2}(t)\cdot E\left(\frac{1}{\tilde{v}_u^{\,2}(t)}\right)-D_1^{*2}(t)\cdot E^2\left(\frac{1}{\tilde{v}_u(t)}\right)\\&=D_1^{*2}(t)\cdot\frac{1}{C_{\max}^2}\left(\frac{1}{\theta}-\frac{1}{(1-\theta)^2}\ln^2\frac{1}{\theta}\right)\\&=D_1^{*2}(t)\cdot\frac{1}{C_{\max}^2}\left(\frac{1}{\theta}-\frac{1}{f_\theta^2}\right),\qquad t_0^*\le t\le t_t^*\end{aligned} \tag{3.38}$$

$$Std\left(\tilde{\tau}(t)\right)=\chi_C\cdot D_1^*(t),\quad t_0^*\le t\le t_t^*,\ \text{with}\ \chi_C=\sqrt{\frac{1}{C_{\max}^2}\left(\frac{1}{\theta}-\frac{1}{(1-\theta)^2}\ln^2\frac{1}{\theta}\right)} \tag{3.39}$$

2) Regime II, for time period $t_t^*\le t\le t_c^*$:

$$
\begin{aligned}
&Var\left(\tilde{\tau}(t)\right)=E\left(\tilde{\tau}^2(t)\right)-E^2\left(\tilde{\tau}(t)\right)\\
&=E\left(\left(\frac{D_1^*\left(t_t^*\right)}{\tilde{v}_u(t)}+\frac{D_2^*(t)}{\tilde{v}_u(t)}-\left(t-t_0^*\right)\right)^2\right)-\left(D_2^*(t)\cdot E\left(\frac{1}{\tilde{v}_u(t)}\right)+\frac{D_1^*\left(t_t^*\right)}{\tilde{v}_u(t)}-\left(t-t_0^*\right)\right)^2\\
&=\left(E\left(\frac{1}{\tilde{v}_u^{\,2}(t)}\right)-E^2\left(\frac{1}{\tilde{v}_u(t)}\right)\right)\cdot\left(D_1^{*2}\left(t_t^*\right)+D_2^{*2}(t)+2D_1^*\left(t_t^*\right)D_2^*(t)\right)\\
&=\left(D_1^*\left(t_t^*\right)+D_2^*(t)\right)^2\cdot\frac{1}{C_{\max}^2}\left(\frac{1}{\theta}-\frac{1}{(1-\theta)^2}\ln^2\frac{1}{\theta}\right)\\
&=\left(D_1^*\left(t_t^*\right)+D_2^*(t)\right)^2\cdot\frac{1}{C_{\max}^2}\left(\frac{1}{\theta}-\frac{1}{f_\theta^2}\right),\qquad t_t^*\le t\le t_c^*
\end{aligned}
\tag{3.40}
$$

$$
Std\left(\tilde{\tau}(t)\right)=\chi_C\cdot\left(D_1^*\left(t_t^*\right)+D_2^*(t)\right),\qquad t_t^*\le t\le t_c^* \tag{3.41}
$$

3) Regime III, for time period $t_c^*\le t\le t_e^*$:

$$
\begin{aligned}
Var\left(\tilde{\tau}(t)\right)&=p_u\cdot E\left(\tilde{\tau}_u^2(t)\right)-E^2\left(\tilde{\tau}_u(t)\right)\\
&=p_u\cdot E\left(\tilde{\tau}_u^2(t)\right)-p_u^{\,2}\cdot E^2\left(\tilde{\tau}_u(t)\right)\\
&=\frac{\left(t-t_0^*\right)\left(D_T^*(t)-C_{\min}\left(t-t_0^*\right)\right)}{C_{\max}(1-\theta)}-\frac{\left(D_T^*(t)-C_{\min}\left(t-t_0^*\right)\right)^2}{C_{\max}^{\,2}(1-\theta)^2}-\frac{2D_T^*(t)\left(t-t_0^*\right)}{C_{\max}(1-\theta)}\ln\frac{D_T^*(t)}{C_{\min}\left(t-t_0^*\right)}\\
&\quad+\frac{2D_T^{*2}(t)}{\left(C_{\max}(1-\theta)\right)^2}\ln\frac{D_T^*(t)}{C_{\min}\left(t-t_0^*\right)}-\frac{2\theta D_T^*(t)\left(t-t_0^*\right)}{C_{\max}(1-\theta)^2}\ln\frac{D_T^*(t)}{C_{\min}\left(t-t_0^*\right)}\\
&\quad+D_T^{*2}(t)\left[\frac{1}{C_{\max}^{\,2}\theta(1-\theta)}-\frac{\left(t-t_0^*\right)}{C_{\max}(1-\theta)D_T^*(t)}-\frac{1}{C_{\max}^{\,2}(1-\theta)^2}\ln^2\frac{D_T^*(t)}{C_{\min}\left(t-t_0^*\right)}\right],\ t_c^*\le t\le t_e^*
\end{aligned}
\tag{3.42}
$$

All the cost components in the long term future trip cost function have been analytically established. According to the principle of the dynamic user equilibrium for departure time choice, it holds $dc^*/dt=0$, which means that travel costs are equal at all used departure times with non-zero flows. No traveler can reduce his/her long term future trip cost by unilaterally changing his/her departure time. With the first order differential equation, we can derive the early and late departure flow rates at the long term equilibrium state. For the detailed derivations, we refer to Appendix A. For the first two regimes, the departure flow rates can be directly derived as:

$$
r_1^*(t)=\frac{\alpha\cdot C_{\max}}{\dfrac{\alpha-\gamma_1}{f_\theta}+\beta\sqrt{\left(\dfrac{1}{\theta}-\dfrac{1}{f_\theta^2}\right)}},\qquad t_0^*\le t\le t_t^* \tag{3.43}
$$

$$r_2^*(t) = \frac{\alpha \cdot C_{\max}}{\dfrac{\alpha+\gamma_2}{f_\theta} + \beta\sqrt{\left(\dfrac{1}{\theta} - \dfrac{1}{f_\theta^2}\right)}}, \qquad t_t^* \le t \le t_c^* \tag{3.44}$$

It can be seen from Formulae (3.43) and (3.44) that the departure flow rates for the first two regimes are constant, which is a consequence of the linearity assumption in the schedule delay cost function, the simple travel time function and the assumption of the independence of the capacity from the cumulative departures. The departure flow rates strongly depend on the capacity factor θ. With higher capacities, departure rates in the first two periods also will increase. For given θ, with increasing $C_{\max}$ and increasing capacity range, departure rates go up. For given $C_{\max}$, departure rates rise along with the increase of θ. In general, the higher the capacities, the higher the departure rates. These two departure flow rates appear independent of the demand *N*.

There is no closed-form expression for the late departure rate $r_3^*(t)$ during $t_c^* \le t \le t_e^*$. The finite difference method is used to obtain the numerical solution for the departure pattern at the third time period through the differential equation derived from $dc^*/dt = 0$. The differential of the cumulative departure $D_3^*(t)$ over time t is approximated by the finite difference given as:

$$\frac{dD_3^*(t)}{dt} = \frac{D_3^*(t+\Delta) - D_3^*(t)}{\Delta} \tag{3.45}$$

The differential of the travel cost over t for the third regime is given as:

$$\frac{dc^*(t)}{dt} = (\alpha+\gamma_2)\frac{dE(\tilde{\tau}(t))}{dt} + \beta\frac{dStd(\tilde{\tau}(t))}{dt} + \gamma_2 = 0, \;\; t_c^* \le t \le t_e^* \tag{3.46}$$

Substitute Formula (3.45) into Formula (3.46), the relationship between $D_3^*(t+\Delta)$ and $D_3^*(t)$ can be derived.

Since we have the initial value at t_c^* that $D_3^*(t_c^*) = D_2^*(t_c^*)$ and ending condition that $D_3^*(t_e^*) = N$, the numerical solution of the departure pattern for time period $t_c^* \le t \le t_e^*$ can be derived. A very small numerical step Δ of 0.01 is adopted to derive the departure pattern with the boundary conditions. The smaller the step is chosen, the more accurate the results will be.

However there are still four unknown variables t_0^*, t_t^*, t_c^*, t_e^*.

Several boundary conditions (see Formulae (3.47), (3.48), (3.49)) can be used to derive the relationship between t_0^* and t_t^*, t_c^*, t_e^*.

$$c^*(t_0^*) = c^*(t_e^*) \tag{3.47}$$

$$c^*(t_0^*) = c^*(t_c^*) \tag{3.48}$$

$$c^*(t_0^*) = c^*(t_t^*) \tag{3.49}$$

Because there is no-closed-form solution for $r_3^*(t)$, we firstly initialize one of the variables (we choose t_0^*) satisfying Formula (3.50):

$$N = \int_{t_0^*}^{t_t^*} r_1^*(x)\,dx + \int_{t_t^*}^{t_c^*} r_2^*(x)\,dx + \int_{t_c^*}^{t_e^*} r_3^*(x)\,dx \tag{3.50}$$

and select the value of t_0^* with which the equilibrium cost is the minimum expressed as :

$$t_0^* = \arg\min_{t_0^* \le t^*} c^*\left(t_0^*\right) \tag{3.51}$$

Demand N directly influences the value of t_0^*. Having established t_0^*, the other critical time points t_t^*, t_c^*, t_e^* can be derived sequentially. So does the equilibrium departure pattern. With the long term equilibrium departure pattern, the real experienced travel times and schedule delays vary over days according to the stochastic capacities. Appendix A provides the detailed derivations and information on the numerical method.

3.3.5 Graphical results on the departure pattern at long term equilibrium

This subsection presents the graphical results for the long term equilibrium departure flow patterns under degradable capacity for a bottleneck model with a demand N, a uniformly distributed stochastic capacity with $C_{\max}$ and $\theta = 2/3$, and an identical PAT. The mean capacity is equal to the deterministic capacity in the deterministic bottleneck case in Section 3.2. The capacity distribution satisfies that $std(\tilde{C}) \approx 0.1E(\tilde{C})$, meaning that the standard deviation of the capacity distribution is about 10% of the mean capacity. Standard deviation and the expectation of the uniform capacity distribution are expressed as $std(\tilde{C}) = \left(\left(C_{\max}^2 + C_{\max}^2 + C_{\max}C_{\min}\right)/3 - E^2\left(\tilde{C}\right)\right)^{1/2}$ and $E(\tilde{C}) = \left(C_{\max} + C_{\min}\right)/2$. Values of 1.0, 1.2, 0.8 and 1.2 are adopted for parameters α, β, γ_1, γ_2 respectively.

Figure 3.4 shows the equilibrium departure pattern of the stochastic capacity case including travel time reliability in traveler's choice behavior and the deterministic capacity case (equaling the mean capacity) with a constant travel demand N. In the picture, all the critical times and moments are indicated. It can be seen that the departure patterns are significantly different with and without including travel time reliability in the travel cost function. The larger the capacity variation is, the more significant the departure patterns are different. Departure flows are more spread over a longer time period ($\left(t_e^* - t_0^*\right)$ increases). Travelers depart earlier after a long term adaptation when they consider travel time reliability as part of the travel cost under uncertainty. The number of earlier arrivals based on the expected travel time increases due to travelers attempting to depart earlier. With inclusion of travel time reliability in the travel cost function, the congestion onset at the bottleneck starts earlier and its end is also earlier for the same PAT, compared with the deterministic case.

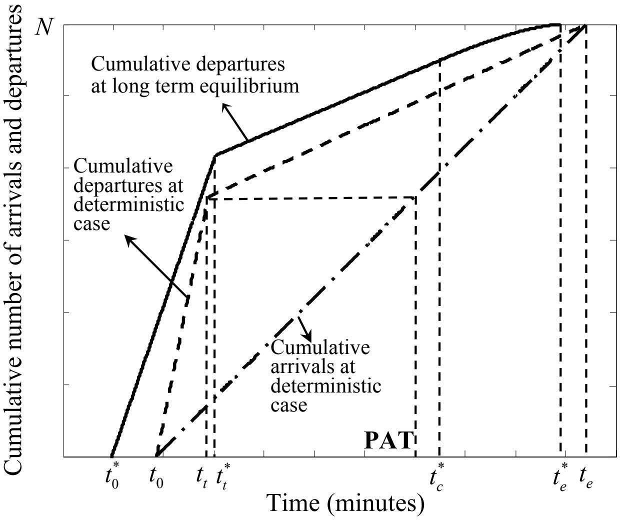


Figure 3.4: Cumulative departures with and without including travel time reliability in the travel cost function

Figure 3.4 didn't present the capacity line/outflow rate line at the long term equilibrium simply because the cumulative capacity line/outflow rate line (denoted as W^*) does not exist at all at equilibrium. For some purposes, we define two categories of cumulative outflow lines at long term equilibrium. One refers to the average cumulative outflow from the bottleneck (i.e. $E\left(\tilde{W}(t)\right)$, named category I) and another refers to the cumulative outflow with which $W^*(t+E(\tilde{\tau}(t)) = D^*(t)$ holds (named category II).

Cumulative outflow category I at long term equilibrium:

On a single day, the cumulative outflow can be calculated as:

$$\tilde{W}(t) = \int_{t_0^*}^{t} \tilde{v}(t)dt \tag{3.52}$$

Then the expectation of the cumulative outflow is expressed as:

$$E\left(\tilde{W}(t)\right) = E\int_{t_0^*}^{t} \tilde{v}(t)dt \tag{3.53}$$

Specifically for each regime, the expectation of the cumulative outflow is formulated as:

$$W_{1,2}^*(t) = E\left(\tilde{W}(t)\right) = E\left(\tilde{v}(t)\right)\left(t-t_0^*\right) = \frac{C_{\max}(1+\theta)}{2}\left(t-t_0^*\right) \text{ with } \tilde{v}(t)=\tilde{C},\ t_0^* \le t \le t_c^* \tag{3.54}$$

$$W_3^*(t) = E\left(\tilde{W}(t)\right) = \frac{C_{\max}(1+\theta)}{2}\left(t_c^*-t_0^*\right) + \int_{t_c^*}^{t} E\left(\tilde{v}(t)\right)dt \ , \ t_c^* \le t \le t_e^* \tag{3.55}$$

Cumulative outflow category II at long term equilibrium:

Another capacity line is defined in order to be able to see the expected travel time for any time instant *t* directly, which is equivalent to saying that $W^*(t+E(\hat{\tau}(t))=D^*(t)$.

If we define $v^*(t)$ as the equilibrium outflow rate, with which it holds that:

$$W^*(t)=\int_{t_0^*}^{t} v^*(t)dt \tag{3.56}$$

then for the first two regimes we have:

$$v_{1,2}^*(t)=\frac{1}{E\left(\tilde{C}^{-1}\right)}, \quad t_0^*\le t\le t_c^* \tag{3.57}$$

For the third regime $t_c^*\le t\le t_e^*$, $v_3^*(t)$ can be derived from the following equation:

$$1/E\left(\frac{1}{\tilde{C}}\right)\cdot\left(t_c^*+E\left(\tilde{\tau}\left(t_c^*\right)\right)-t_0^*\right)+\int_{t_c^*}^{t} v_3^*(t)dt=D_1^*\left(t_t^*\right)+D_2^*\left(t_c^*\right)+D_3^*(t), \quad t_c^*\le t\le t_e^* \tag{3.58}$$

Figure 3.5 shows the cumulative departures and the defined two categories of cumulative outflow lines at the long term equilibrium.[2] Since small capacity variation is defined, the two categories of the outflow lines are very close to each other except in the third regime. If larger capacity variations are chosen, these two lines are significantly different, see (Li *et al.* 2008b; Li *et al.* 2008d).

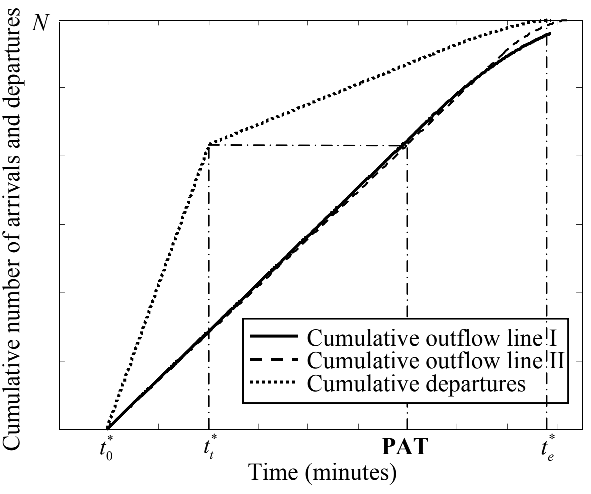


Figure 3.5: Cumulative departures and two categories of cumulative outflow lines at the long term equilibrium

[2] The graphical results given in Chapter 3 show the long term equilibrium departure patterns and travel costs for general stochastic bottleneck models. Therefore, no scales are provided in the figures. Appendix B provides the graphical results with scales for a comparison with the simulation-based approach.

Figure 3.6 shows the expected travel times at the long term equilibrium with stochastic capacities and the travel times at deterministic capacity case. On average, travel times are shorter in the stochastic case, because the departure flows are more spread over a longer time period. In the third regime, the expected travel time is non-linear and convex. The decreasing speed of the expected travel time gradually declines.

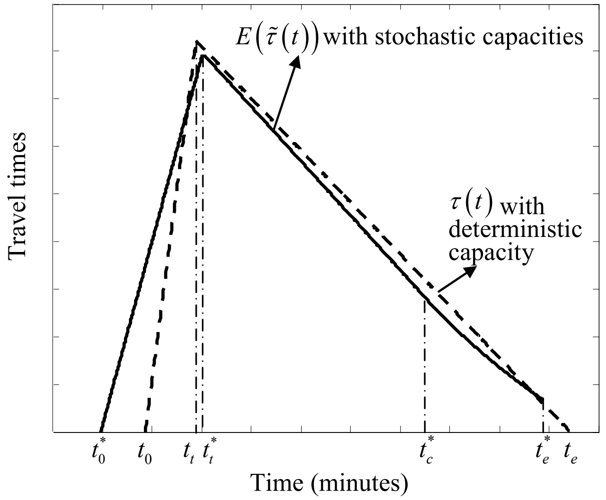


Figure 3.6: Expected travel times with stochastic capacities and travel times with deterministic capacity

Figure 3.7 presents the long term future trip costs at equilibrium (namely the long term equilibrium costs) by component.

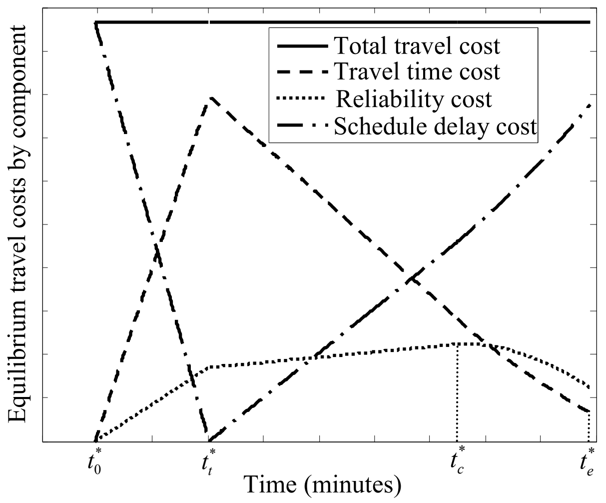


Figure 3.7: Long term equilibrium costs by component

It can be seen that at the long term equilibrium, the travel costs for any traveler departing at any time t are equal. This result satisfies the user departure time equilibrium principle and validates the analytical analysis. It also can be seen that the schedule delays based on the expected travel time decrease to zero while approaching t_t^*. Travel time reliability in relation to the departure time and to the expected travel time can be seen as well. Travel time variability in the experienced travel time firstly increases and then decreases, but not to zero. It is noticed that in the third regime after t_c^*, all the cost components are non-linear.

An important finding is that the standard deviation of travel time is not proportional to the expectation of travel time, as many assume to be, see (Daganzo and Sheffi 1977; Bovy 1990). Although the travel time in the bottleneck model is waiting time at a bottleneck, while (Daganzo and Sheffi 1977; Bovy 1990) refer to the section travel times. Given a fixed number of travelers, the first travelers departing at t_0^* will never experience queues and always have free flow travel time, thus there is no experienced travel time variability for them. The later a traveler departs, the larger the variability he/she might encounter between days. That is also one reason why travelers attempt to depart early to reduce their travel time variability and risks in case of stochastic capacities. Of course it is constrained to the case that all the travelers have an identical preferred arrival time. The findings are specifically for the peak hour within a time window around the PAT. If a traveler departs very late, which is out of the scope of our analysis period, the variability of travel time might be very low due to the absence of congestion.

Previous pictures all present the results at the long term equilibrium. It should be born in mind that the resulting departure pattern at long term equilibrium is deterministic. However the experienced day-to-day travel times and the number of travelers experiencing queues change according to varied capacities over days. Figure 3.8 presents the daily experienced traffic situations with maximum and minimum capacities. The maximum and minimum experienced travel times for travelers departing at t can be derived. For instance, travelers departing at t_c^* will encounter zero delays on the days with the maximum capacity (see the dashed line), while a large delay on the days with the minimum capacity (see dash-dotted line). Travelers departing at t_t^* may encounter the maximum delays on the days with the minimum capacity. On the days with maximum capacity, about 92% (out of a total demand N) travelers will experience delays. The rest of the travelers departing after t_c^* will not encounter any delays. Their experienced travel times are equal to free flow travel time on those days.

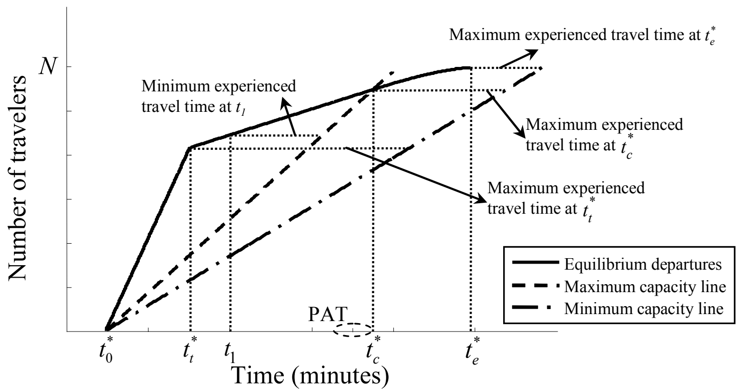


Figure 3.8: Range of experienced day-to-day travel times

Figure 3.9 presents the expected travel times and the experienced travel times with maximum capacity and with minimum capacity. Travelers departing at t_0^* always experience zero delays and t_0^* is fixed point. This is the limitation of the simple Vickrey's bottleneck model. A lot of assumptions are not realistic, for instance homogeneous travelers with an identical PAT and the same travel cost functions. The analysis can be made more realistic by relaxing the assumptions and modeling heterogeneous travelers with a distribution of PAT and different travel cost functions. Taste diversity and PAT distribution through the population will lead to much wider departure patterns than derived in above analysis. Then the queue starting point will vary over days as well.

It is clearly seen from Figure 3.9 that the travel times are zero after t_c^* on days with maximum capacity, which reduces the travel time variations dramatically around the expectation of travel time, thus leads to decreased travel time variability after t_c^*. It also leads to the non-linearity in the expected travel time and in other cost components in the third regime. The maximum experienced travel time after t_c^* is also non-linear due to the non-linear departure pattern. Before t_c^* the range of travel time variations increases around the expectation of travel time, leading to rising travel time variability, which further proves the findings from Figure 3.7 on the travel time reliability in relation to expected travel time and departure time.

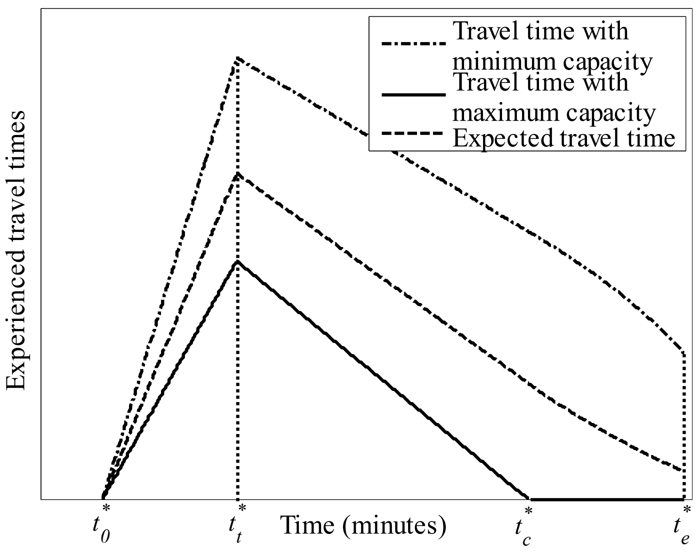


Figure 3.9: Extreme experienced travel times with minimum and maximum capacities and expected travel times

As aforementioned, it may happen in Figure 3.8 that the maximum capacity value is larger than the early departure rate and that there is no intersection between the cumulative departures and the maximum capacity line. Then all travelers will have free flow travel time on that day. It should be clarified that the position of the resulting cumulative departure curve relative to the extreme positions of the capacity lines is fully depending on the capacity fraction factor θ. We have that $r_1^*(t) > r_2^*(t) > r_3^*(t)$ (Appendix A shows why this holds in general), thus we only need to investigate the relationships between $r_1^*(t)$ and $C_{\max}$, and $r_2^*(t)$ and $C_{\min}$.

Table 3.1 presents the relative slope of $r_1^*(t)$ and $r_2^*(t)$ to the maximum capacity and minimum capacity respectively in relation to different θ values (according to Formula (3.44), $r_1^*(t)/C_{\max}$ and $r_2^*(t)/C_{\min}$ are simply a function of θ, a larger θ value indicates a smaller capacity variation).

Table 3.1: Departure rates in relation to the capacity variation factor θ

θ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$r_1^*(t)/C_{\max}$	0.36	0.64	0.93	1.25	1.63	2.07	2.60	3.23	4.01
$r_2^*(t)/C_{\min}$	1.27	0.89	0.74	0.65	0.59	0.55	0.52	0.49	0.47

It is found that in the case that θ satisfies $1/3 \le \theta < 1$, then the early departure rate $r_1^*(t)$ will be always larger than the maximum capacity, which means that the equilibrium departure pattern always starts to the left of the maximum capacity line so that there is always an intersection between the cumulative departure curve and the maximum capacity line. With this condition that $1/3 \le \theta < 1$, it holds as well that $r_2^*(t)$ will be smaller than the minimum capacity line, thus $r_3^*(t) < C_{\min}$ holds as well. There is always an intersection between the cumulative departure line and the minimum capacity line, which implies that on the worst days with minimum capacity the queues will ultimately resolve. We think that $\theta = 1/3$ already represents a very large variation in real-world road capacities. $1/3 \le \theta < 1$ is a realistic range capturing the day-to-day capacity variations caused by minor incidents, weather volatility, etc. In a previous analysis, we had $\theta = 2/3$. For extreme case with $\theta = 1/3$, see (Li *et al.* 2008d). Of course, this conclusion depend on the parameter values α, β, γ_1, γ_2. For different parameter values α, β, γ_1, γ_2, different ranges of θ values will be derived.

Figure 3.10 and Figure 3.11 present a comparison of the equilibrium departure patterns, category II outflow lines and costs for different capacity variations with the same mean capacity and different θ values respectively. The same demand *N* is modeled. In both figures, the solid lines represent the departure pattern and all the costs for capacities with $C_{\max 1}$ and $\theta_1 \approx 0.56$. The dashed lines represent the departure pattern and the costs for capacities with $C_{\max 2}$ and $\theta_2 \approx 0.67$. We have that $E(\tilde{C}_1) = E(\tilde{C}_2)$. It can be concluded that given the same mean capacity, the larger the θ is, the larger the early departure rate is and the shorter the time period is (i.e. $t_e^* - t_0^*$). With smaller θ and larger capacity variations, travelers depart much earlier due to the larger travel time variability as can be seen in Figure 3.11 that the travel time unreliability is much higher with capacities $\tilde{C}_1$. It is quite plausible that travelers depart earlier in case of dealing with high risks with larger travel time uncertainty. Comparing the costs from the capacity distributions, only the travel time reliability costs are significantly different, whereas the other cost components are very similar. The larger the θ is, the smaller the equilibrium cost is.

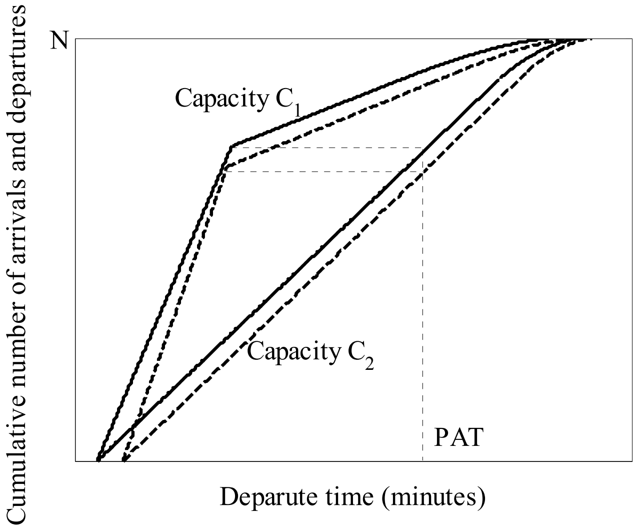


Figure 3.10: Departure patterns comparison with different capacity distributions with the same mean capacity

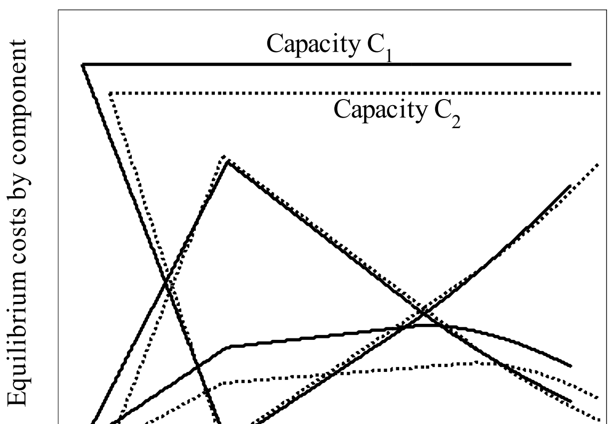


Figure 3.11: Equilibrium travel costs comparison with different capacity distributions with the same mean capacity

Figure 3.12 and Figure 3.13 show the equilibrium departure patterns and the equilibrium costs for different capacity distributions with the same maximum capacity with the same demand *N*. The solid lines present the departure pattern, category II outflow line and the equilibrium costs with a uniformly distributed capacities with a $C_{\max 1}$ and $\theta_1 = 1/3$. The dashed lines instead show the case with a $C_{\max 2}$ and $\theta_2 = 2/3$. We have $C_{\max 1} = C_{\max 2}$ and $E\left(\tilde{C}_1\right) < E\left(\tilde{C}_2\right)$. While the previous comparison showed the impacts of θ, this comparison shows the joint impacts of θ and overall capacity (i.e. mean capacity). It can be seen that with a large capacity factor θ (i.e. a smaller capacity variation) and a smaller mean capacity, travelers depart much earlier and the travel costs especially the travel time reliability costs are remarkably higher. The departure pattern is fully dependent on the capacity variations.

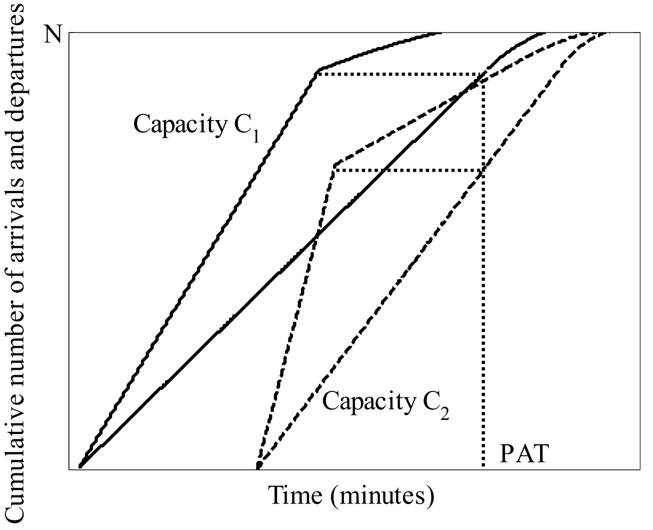


Figure 3.12: Departure patterns comparison with different capacity distributions with the same maximum capacity

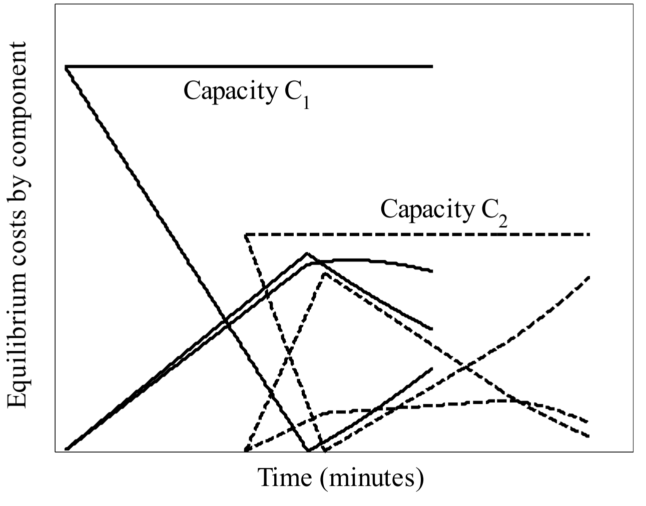


Figure 3.13: Equilibrium travel costs comparison with different capacity distributions with the same maximum capacity

3.4 Conclusions and Discussions

Vickrey's bottleneck model has been extended twofold toward the stochastic capacities and including travel time reliability in the travel cost function. Based on the generalized mean-variance scheduling approach for modeling traveler's departure time/route choice behavior under uncertainty, this chapter analytically investigated the departure pattern under stochastic networks. A long term equilibrium pattern emerges from day-to-day adaptation. A deterministic departure time and dynamic user equilibrium with stochastic network and vertical queues (SN-DDDUEV) is achieved.

It appears that consideration of random capacity and travel time reliability leads to significantly different departure time patterns compared to the deterministic capacity case, mainly yielding shifts towards earlier departure times. Travel time unreliability appears to increase firstly with departure time and then decrease to a certain nonzero value. The later a traveler departs, the larger travel time variability he/she might encounter. This is also a reason why travelers depart earlier in order to reduce travel time variability.

In reality, the queue starting point will change everyday according to the network capacity since the departure pattern in reality is more spread over a longer time period. Travelers are heterogeneous in PAT, in travel cost function, etc, which is more complicated than what we modeled. Our bottleneck model is a very simple model, a lot of assumptions are not realistic. We assumed homogeneous travelers with an identical PAT and the same travel cost functions. Linear schedule delay function and simple travel time functions are assumed. Therefore in our analysis the queue onset point is fixed over days (which will be proven also by the simulation-based approach in next chapter). The analysis can be improved more realistically by relaxing the assumptions and modeling heterogeneous travelers with a distribution of PAT and different travel cost functions. Taste diversity and PAT distribution through the population will lead much wider departure patterns than derived in our analysis. Then the queue starting point will vary over days as well.

Due to the complexity of the theoretical derivations capturing dynamics within-day and over days considering stochastic capacity, the theoretical analysis is only carried out for a single-link bottleneck with homogeneous travelers. The next chapter will adopt and introduce a simulation-based approach capable of analysis on larger and more complex networks with joint departure time/route choices. The simulation-based approach with the vertical queues assumption and horizontal queues are presented for different purposes. The departure pattern will be analyzed with a simulation-based approach as well in the next chapter.

4 Dynamic User Equilibrium and a Simulation-based Solution Approach

4.1 Introduction

In Chapter 3 we have established analytical analyses of the reliability-based departure time user equilibrium on a single bottleneck under stochastic capacity with continuous time and continuous flows. Already this simple case appears to be fairly complex. Aiming at analyzing large and realistic networks and to be useful for real applications in network evaluation or network designs, we will develop a general reliability-based network modeling framework and establish a simulation-based approach dealing with discrete time and continuous flows to solve the general problem. Another purpose of developing the simulation-based approach is to validate the analytical analysis and its findings presented in Chapter 3.

There are two major sources leading to travel time unreliability over days, namely day-to-day capacity variations and demand fluctuations. With regard to demand variation modeling, it is normally assumed by most of the researchers (Shao *et al.* 2006) that the OD demand follows normal/lognormal distributions approximated by multivariate normal/lognormal link flow distributions. Static user equilibrium is achieved with a flow conservation constraint for the mean route flows and mean OD demand. However, with dynamic traffic assignment with departure time choice, modeling stochastic demand will be much more complicated than with static assignment. Based on our best knowledge, no research has been done on the dynamic network modeling with stochastic travel demand. From our opinion, the queue onset points, ending points, the time period, route flows for any k will be all stochastic variables for a given PAT. A more refined definition of the long term equilibrium with stochastic demand should be established, especially the flow conservation at the long term equilibrium needs a clear definition and formulation. Based on our work (Li *et al.* 2007a) on network reliability-based optimal toll designs considering simultaneous variations in travel demand and capacities, it may be expected that if the demand fluctuation is considered, the travel time variation experienced by travelers will be significantly larger than only modeling capacity variations.

Given the same PAT, travelers might depart even earlier in order to reduce the probability of arriving late at destinations. The departure time point t_0^* and t_e^* will be distributions as well, which will be left out for future research. Only stochastic capacity is modeled as the major factor causing unreliable travel times. Although day-to-day demand fluctuation is not considered, the amount of travelers experiencing delays is a stochastic variable over days due to capacity variations.

It is assumed throughout this chapter that travelers are homogeneous with the same long term travel cost function and an identical PAT.

This chapter will formulate several user equilibrium models meant for different purposes such as validating the simulation-based approach, validating our analytical analysis on the stochastic bottleneck model, more realistic modeling for general stochastic networks, and facilitating the investigations on the influence of reliability and dynamic modeling on the network designs. The models are distinguished by whether deterministic or stochastic capacities, deterministic or probabilistic user equilibrium, static or dynamic flows, and vertical or horizontal queues are modeled. Four models will be depicted, which are SN-DDDUEV (Stochastic Network and Departure time and Dynamic Deterministic User Equilibrium with Vertical queues), SN-DDPUEH (Stochastic Network and Departure time and Dynamic Probabilistic User Equilibrium with Horizontal queues), DN-DDDUEV (Deterministic Network and Departure time and Dynamic Deterministic User Equilibrium with Vertical queues), and SN-SPUE (Stochastic Network and Static Probabilistic User Equilibrium). Table 4.1 summarizes the basic properties of the four models.

Table 4.1: Properties of different network models

		SN-DDDUEV	SN-DDPUEH	DN-DDDUEV	SN-SPUE
Demand	Constant and Deterministic	×	×	×	×
	Elastic and Stochastic				
Capacity	Deterministic			×	
	Stochastic	×	×		×
Equilibrium	Deterministic	×		×	
	Probabilistic		×		×
Queue modeling	Vertical	×		×	
	Horizontal		×		
Network flows	Static				×
	Dynamic	×	×	×	

This chapter will firstly introduce these different models with their different assumptions. Then a general framework of a simulation-based approach solving the different user equilibria will be illustrated. The model specifications of joint departure time/route choice, the dynamic network loading, correlated link capacity randomization and convergence criteria will be provided for each model. The simulation-based approach with vertical queues is applied to the single bottleneck model for both the deterministic and stochastic capacity cases. The results will be presented and compared to the analytical findings from Chapter 3. Finally some conclusions are drawn.

4.2 Stochastic Network and Departure time and Dynamic Deterministic User Equilibrium with Vertical Queues

The long term deterministic user equilibrium with stochastic capacity and deterministic departure time choice was formulated in Chapter 3 for a single bottleneck model in continuous time space, which belongs to the class of SN-DDDUEV (Stochastic Network and Departure time and Dynamic Deterministic User Equilibrium with Vertical queues, see (Chen *et al.* 2002a)). For the sake of validating the theoretical analysis given in Chapter 3, a more general formulation of the model SN-DDDUEV will be presented in this subsection for a general network for which a simulation-based approach will be developed. Due to this simulation-based approach, time will be discretized into time intervals $k \in K$ and flow propagations will be processed interval by interval. The model formulation will be presented in a discrete time version instead of in a continuous-time version. In case of continuous time formulations, departure flow rate at any time instant *t* is aimed to be derived, while in case of discrete time formulations, departure flows are derived for each time interval *k*.

In this model, stochastic networks are modeled with stochastic link capacities. Travel time unreliability is caused by the day-to-day capacity variations. Within-day capacity is assumed constant. This model assumes that travelers are rational and have perfect knowledge of the distributive properties of the travel times at any departure time *t* and all routes and make their long term departure time and route choices so as to minimize their long term future trip cost $c_p^{od}(k)$. According to the derivation in Chapter 2 (see Formula (2.16)), the deterministic long term future trip cost for general networks in discrete-time is formulated in (4.1) by assuming linearity in schedule delay cost, which is an approximation to the reality:

$$\begin{aligned} c_p^{od}(k) = \alpha \cdot E\left[\tilde{\tau}_p^{od}(k)\right] + \gamma_1 \cdot \left(PAT - \left(k + E\left[\tilde{\tau}_p^{od}(k)\right]\right)\right)^+ + \gamma_2 \cdot \left(k + E\left[\tilde{\tau}_p^{od}(k)\right] - PAT\right)^+ \\ + \beta \cdot Std\left[\tilde{\tau}_p^{od}(k)\right] \end{aligned} \tag{4.1}$$

The long term future travel cost $c_p^{od}(k)$ on route *p* between (*o*, *d*) when departing at time interval *k* is deterministic. Homogeneous travelers with an identical PAT and the same travel cost function are modeled. Again $(x(k))^+ = \max\{0, x(k)\}$. $\tilde{\tau}_p^{od}(k)$ denotes the day-to-day stochastic route travel times when departing at time interval *k*. In case of working with continuous time *t*, the stochastic travel time $\tilde{\tau}_p^{od}(t)$ given a randomized combination of link capacities is unique. Whereas in case of working with discrete times, the stochastic travel time $\tilde{\tau}_p^{od}(k)$ for departure time interval *k* is an average of the travel times from all the time steps lying in this departure time interval.

4.2.1 Formulation of SN-DDDUEV

The dynamic user equilibrium with joint departure time/route choices under daily travel cost uncertainty can be formulated as a finite-dimensional variational inequality problem for discrete time:

$$\sum_{o,d}\sum_{p}\sum_{k}\overline{c}_p^{od}(k|\overline{\mathbf{f}})\left(f_p^{od}(k)-\overline{f}_p^{od}(k)\right)\geq 0, \quad \forall \mathbf{f}\in\mathbf{\Omega_d}. \tag{4.2}$$

This VI formulation simply states that given user equilibrium costs $\overline{c}_p^{od}(k|\overline{\mathbf{f}}), \forall(o,d), p, k,$ any deviations from the dynamic user equilibrium flow matrix $\overline{\mathbf{f}}$ cannot reduce total costs. In other words, no traveler would be tempted to reduce his/her long term travel cost by changing his/her departure time or route, as no alterative route or departure time would offer a lower long term cost. Still, continuous flows are assumed instead of discrete flows (the equilibrium may be different for continuous flows and discrete flows). $\overline{f}_p^{od}(k)$ denotes the deterministic equilibrium flows on route p between (o, d) departing at time interval k. $\overline{c}_p^{od}(k|\overline{\mathbf{f}})$ is a function of the dynamic equilibrium flow matrix $\overline{\mathbf{f}}$. The travel costs at k is mainly determined by the cumulative departure flows from previous time intervals. It should be noticed that with discrete time, $\overline{f}_p^{od}(k)$ denotes the flows for a time interval k while with continuous time $\overline{f}_p^{od}(t)$ denotes flow rate at time instant t. $\mathbf{\Omega_d}$ is the set of feasible dynamic route flow matrix $\mathbf{f}$ (dimension in route by dimension in time interval), defined by the flow conservation (4.3) and non-negativity (4.4) constraints, respectively:

$$\sum_{p\in P^{od}}\sum_{k} f_p^{od}(k)=q^{od}, \quad \forall(o,d), \tag{4.3}$$

$$f_p^{od}(k)\geq 0, \quad \forall(o,d), p, k. \tag{4.4}$$

Due to the constant travel demand, the flow conservation (4.3) holds at the long term equilibrium. If stochastic travel demand is modeled, a new formulation of the flow conservation needs to be defined. The objective of this VI formulation is to find the optimal departure pattern $\overline{\mathbf{f}}\in\mathbf{\Omega_d}$ with which no traveler could reduce his/her long term trip cost by changing his/her departure time or the route. The long term trip costs of the used departure time intervals on all the used routes are minimum and equal within each OD pair.

4.2.2 Existence of SN-DDDUEV

For purpose of validating the analytical analysis in Chapter 3, the same travel time function is used as in stochastic Vickrey's bottleneck model, which is simply computed by the queue length over the bottleneck capacity. As shown in Chapter 3 in a continuous time version the long term trip cost at any time instant is a continuous function of the departure flow rates $\mathbf{f}$. In a discrete time version, the stochastic travel time at time interval k, which is an average of the travel times from several time steps, is a continuous function of the departure flows at time interval k. The long term trip cost when departing at time interval k then is also a continuous function of the departure flows $\mathbf{f}$. $\mathbf{\Omega_d}$ is a nonempty and closed convex set. According to the theorem 3.1 from (Chen 1999), the VIP has at least one solution. A long term deterministic equilibrium will emerge.

4.2.3 Simultaneous departure time/route choices

Simultaneous departure time and route choices are modeled instead of sequential modeling. Deterministic equilibrium assignment is performed.

4.2.4 DNL with vertical queues

In order to capture the dynamics of the traffic and propagation of the traffic flows, a dynamic network loading (DNL) model is needed, which simulates the dynamic propagation of route flows over the links in the network and yields time-dependent link travel times $\tilde{\tau}_a\left(k\middle|\tilde{\mathbf{C}}\right)$, $\forall a \in A$ with a specific capacity vector $\tilde{\mathbf{C}} = [\tilde{C}_a]$, $\forall a \in A$. The input of DNL at iteration i is the derived departure flows $f_p^{od(i)}(k)$.

A DNL model simulating vertical queues without spillback is modeled for comparison purpose with the theoretical analysis performed in Chapter 3. A simple link travel time function consistent with Vickrey's bottleneck model is applied, which is simply computed by the queue length on link a divided by the link capacity, given as:

$$\tilde{\tau}_a\left(k\middle|\tilde{\mathbf{C}}\right) = \frac{Q_a(k)}{\tilde{C}_a}, \ \forall a \in A \tag{4.5}$$

where $Q_a(k)$ denotes the queue length on link a at time interval k.

In contrast to the horizontal queue modeling, vertical queue modeling does not control the link inflows by assuming that each link does not restrict to the physical capability and can accommodate infinite number of vehicles without spillback to the upstream link and without influencing the inflow capacity into this link and the outflow rate of the upstream link. The link outflow rate is determined by the node model from (Bliemer 2007). The node model relates the inflows into and outflows out of each node, which takes into account the inflow capacities into the outgoing links and the potential outflow rates coming from the incoming links. The outcomes of the node model are the dynamic link outflow flows rates. With modeling vertical queue, the inflow capacity of the downstream link always equals its capacity. The potential outflow rate of the link may equal its inflow rate, its capacity, or the inflow capacity of the downstream link.

4.2.5 Modeling stochastic capacities

Link capacities are assumed to follow a distribution representing the day-to-day variations. Let $\mathbf{X}$ denote a vector of random draws with $|\mathbf{X}| = n$. For the purpose of comparing the simulation-based approach with the theoretical bottleneck analysis given in Chapter 3, a uniformly distributed link capacity is modeled. A vector of uniformly distributed link capacities can be drawn by:

$$\mathbf{C}_a = C_{a\min} + \left(C_{a\max} - C_{a\min}\right) \cdot \mathbf{X} \tag{4.6}$$

In order to well represent the capacity distribution, a quasi-Monte Carlo approach using Halton draws (Halton 1960) is adopted to model these stochastic capacities. In this thesis, $\mathbf{X}$ is a vector of Halton draws (also called Halton sequences). Halton sequences are designed to give fairly even coverage over the domain of the distribution. These sequences are deterministic, but of low discrepancy. They were first introduced in 1960 and are an example of a quasi-random number sequence. The Halton sequences are defined with a number, for instance with number 2, 3, or 5.[3]

4.3 Stochastic Network and Departure Time and Dynamic Probabilistic User Equilibrium with Horizontal Queues

This subsection presents the SN-DDPUEH model (Stochastic Network and Departure time and Dynamic Probabilistic User Equilibrium with Horizontal queues). Compared to the SN-DDDUEV, probabilistic user equilibrium is achieved and horizontal queue is modeled to consider the physical capacity of the link and to capture the spill back effects in the network. More accurate and realistic travel times can be derived with modeling horizontal queues instead of vertical queues. This model is used for general stochastic network modeling and network design problem dealt with in Chapter 5. Still stochastic networks are modeled with stochastic link capacities. Due to the errors that may stem from perception variations on the cost component, the component-specific parameters, the type of cost function and incorrect or missing cost components, modelers are not sure how exactly travelers make their choices, therefore by adding an error term in the travel cost function to capture the uncertainties. Travelers are assumed to make their strategic departure time and route choices so as to minimize their perceived long term trip costs $\hat{c}_p^{od}(k)$ which includes a random component. The perceived long term travel cost, which is a stochastic variable (different from previously defined stochastic travel cost $\tilde{c}_p^{od}(k)$, $\tilde{c}_p^{od}(k)$ is caused by stochastic capacities, while $\hat{c}_p^{od}(k)$ is due to the uncertainty in modeling from the modelers), is defined as:

$$\hat{c}_p^{od}\left(k\right) = c_p^{od}\left(k\right) + \tilde{\varepsilon}_p^{od}\left(k\right) \tag{4.7}$$

where $c_p^{od}(k)$ denotes the deterministic travel cost as defined in Formula (4.1). $\tilde{\varepsilon}_p^{od}\left(k\right)$ is the random error component used by the modelers.

4.3.1 Formulation of SN-DDPUEH

The SN-DDPUEH model can be formulated as a finite-dimensional variational inequality problem (see e.g. (He 1997)) for discrete time:

$$\sum_{o,d} \sum_{p \in P^{od}} \sum_{k} \overline{F}_p^{od}(k \big| \overline{\mathbf{f}}) \left(f_p^{od}(k) - \overline{f}_p^{od}(k) \right) \geq 0, \quad \forall \mathbf{f} \in \mathbf{\Omega_d}. \tag{4.8}$$

where

3 Monte Carlo, also called pseudo-Monte Carlo, is based on sequences of pseudorandom numbers, while Quasi-monte Carlo is based on a subsequence of a low-discrepancy sequence. For illustrations, a Halton sequence with a number 3 is constructed as follows (Train, K. 1999). Take the unit interval (0, 1) and divide it into 3 parts. The dividing points become the first two elements of the Halton sequence: 1/3 and 2/3. Now take each of the three parts and divide them into 3 parts. The dividing points constitute the next elements in the Halton sequence: 1/9, 4/9 (which is 1/9 above 1/3), 7/9 (which is 1/9 above 2/3), and 2/9, 5/9 (which is 2/9 above 2/3), and 8/9 (which is 2/9 above 2/3). The unit interval has now been divided into nine parts. Divide each of these nine parts into thirds. The dividing points are 1/27, 10/27, 19/27, 4/27, 13/27, 22/27, 7/27, 16/27, 25/27 and 2/27, 11/27, 20/27, 5/27, 14/27, 23/27, 8/27, 17/27, 26/27. Each of the 27 spaces are then divided into three parts, and so on for as many numbers as needed in the sequence. Similar sequences are defined for other numbers, such as 2 (1/2, 1/4, 3/4, 1/8, 5/8, 3/8, ...) and 5 (1/5, 2/5, 3/5, 4/5, 1/25, 6/25, 11/25, ...).

$$\bar{F}_p^{od}(k) = \left(\bar{f}_p^{od}(k) - \hat{f}_p^{od}(k)\right)\frac{\partial \bar{c}_p^{od}(k)}{\partial f_p^{od}(k)}, \tag{4.9}$$

where $\bar{f}_p^{rs}(k)$ is the equilibrium flow departing at time interval *k* on route *p* between OD pair (*o*, *d*). $\hat{f}_p^{rs}(k)$ is the intermediate departure flow, calculated by the OD demand for (*o*, *d*) multiplied by the proportion of travelers choosing route *p* among all route alternatives for an OD pair (*o*, *d*) and departure time interval *k*. The proportion can be calculated by any probabilistic model such as Path-Size logit model, C-logit model, Path Size Correction Logit (PSC-Logit) (Bovy *et al.* 2008), which account for the spatial overlap of routes. See Section 4.3.3 for the formulations of $\hat{f}_p^{rs}(k)$. $\bar{c}_p^{od}(k)$ denotes the equilibrium cost on when departing at time interval *k* on route *p* between OD pair (*o*, *d*).

$\mathbf{\Omega_d}$ is the same as defined in the model SN-DDDUEV. The long term equilibrium is also subject to the flow propagation constraint.

4.3.2 Existence of SN-DDPUEH

Dissimilar to the SN-DDDUEV with modeling vertical queues and a simple queue travel time function, the travel times and travel costs are much more complicated to be derived with horizontal queues than simply analytically derivable. We assume that any departure flow on route *p* corresponds to at least one and only one travel cost. Then the long term trip cost at any departure time interval is a continuous function of the departure flows $\mathbf{f}$. $\mathbf{\Omega_d}$ is a nonempty and closed convex set. According to the theorem 3.1 from (Chen 1999), the VIP has at least one solution. A long term probabilistic user equilibrium will emerge after day-to-day adaptation of departure time and route choices.

4.3.3 Simultaneous departure time/route choices

Since a probabilistic user equilibrium is modeled, a probabilistic equilibrium approach is applied. Depending on the probability distribution of the random component included in the perceived travel cost, different models such as logit model, path-size logit model, probit model may be applied for calculating the flow proportions of travelers taking route *p* and departure time interval *k*.

In case that $\tilde{\varepsilon}_p^{od}(k)$ follow independent and identical Gumbel distributions. Then the intermediate departure flow on route *p* at time interval *k* at iteration *i*, denoted as $\hat{f}_p^{od}(k)$ may be computed by a multinomial logit model (MNL):

$$\hat{f}_p^{od}(k) = q^{od} \cdot \frac{\exp\left(-\omega c_p^{od(i)}(k)\right)}{\sum_{p\in P^{od}}\sum_{k\in K}\exp\left(-\omega c_p^{od(i)}(k)\right)}, \quad \forall (o,d), p, k, \tag{4.10}$$

where ω is a scale parameter. In case a large scale parameter is chosen, then a deterministic user equilibrium can be achieved as well with above model. $c_p^{od(i)}(k)$ denotes the deterministic travel cost on route *p* departing at time interval *k* between OD pair (*o*, *d*) at iteration *i*. However, MNL is the simplest model which cannot capture the correlations among routes and departure time intervals. With modeling departure time choices, the correlations between adjacent departure time intervals are expected significant. Therefore a more sophisticated model is needed to calculate the intermediate departure flows.

A Path Size Correction Logit (PSC-Logit) model proposed by (Bovy *et al.* 2008) is adopted to compute the intermediate departure flows. PSC-Logit is mathematically derivable and easy to be applied compared to other complex models such as PCL, CNL, Logit-Kernel and Probit. It also offers a more natural interpretation of the role of the correlation due to spatial overlap. The performance of PSC-logit is as good as or even better than the classical PS-Logit (Bovy *et al.* 2008). However this model only captures the correlations among routes. Correlations among different departure time intervals are not accounted for. No research has been done so far on the modeling of correlations among different departure time intervals. We assume an extra factor M_{TI} (time interval factor), similar to the path size factor, to capture the correlations among different departure time intervals. The model is given in Formula (4.11):

$$\hat{f}_p^{od}(k) = q^{od} \cdot \frac{\exp\left(-\omega\left(c_p^{od(i)}(k) + \beta_{PS}\frac{1}{d_p^{od}}\sum_{a\in\Gamma_p^{od}} d_a \ln M_a + \beta_{TI} M_{TI}\right)\right)}{\sum_{p\in P^{od}}\sum_{k\in K}\exp\left(-\omega\left(c_p^{od(i)}(k) + \beta_{PS}\frac{1}{d_p^{od}}\sum_{a\in\Gamma_p^{od}} d_a \ln M_a + \beta_{TI} M_{TI}\right)\right)}, \quad \forall (o,d), p \tag{4.11}$$

where d_p^{od} is the length of the route p between (o, d). Γ_p^{od} is a set of all links on route p between (o, d). M_a is the number of routes in the network traversing link a. β_{PS} and β_{TI} are the path size parameter and time interval parameter. If the correlations among all routes in a real network may cancel out, the path size factor does not influence of modeling results very much. We think the adjacent departure time intervals within a certain time period are highly correlated and outside of that time period they are weakly correlated or non-correlated. Since empirical analyses are needed to calibrate and validate the model with the time interval factor to capture the correlations among departure time intervals, we assume the correlations among departure time intervals cancel out in the end.

4.3.4 DNL with horizontal queues

As mentioned, DNL with modeling horizontal queues takes into account the physical capability of the links and captures the spillback effects, which simulates reality better and can provide more realistic travel times than vertical queue modeling does. The applied DNL model version simulating horizontal queues and spillback is taken from (Bliemer 2007). The reason to choose this DNL model is that it does not rely on the unrealistic restrictions made by other models. For instance many DNL models (Astarita 1996; Wu *et al.* 1998; Chabini 2001; Bliemer *et al.* 2004) propagate the traffic flow using link travel time functions which assumes that the travel times are fixed while traversing the link and cannot deal with specific phenomena such as spillback and DTM measures with capacity changes. Also the delays are predicted inside the bottleneck instead of upstream of the bottleneck. The model proposed by (Ran and Boyce 1996) deals with queues and spillback by splitting a link into a free-moving part followed by a queuing part, but assuming the queue length and outflow capacity will not change while traversing the link which is not realistic. Variable queue length is considered by (He 1997; Roels and Perakis 2004), but a constant outflow capacity is assumed. The model taken from (Bliemer 2007) does not rely on travel time functions that look forward in time, but it only uses queuing and exit functions that look backward in time. This model leads to a completely time-responsive dynamic queuing modeling without any assumptions on stationary flows, queue lengths, or outflow capacities. This DNL model consists of a link model and a node model. The link model describes the flow propagation through each link and assumes a dynamic horizontal queue. The node model relates the inflows into and outflows out of each node according to the dynamic route flow proportions, taking into account possible restricted flow capacities due to queue spillback. The formulation correctly deals with time-varying link attributes, such as inflow and outflow capacities and maximum speeds. Assuming first-in-first-out (FIFO) within each user class, the link travel times can easily be determined from the cumulative inflows $U_a\left(k\middle|\tilde{\mathbf{C}}\right)$ and cumulative outflows $V_a\left(k\middle|\tilde{\mathbf{C}}\right)$ with a specific capacity vector as follows:

$$\tau_a\left(k\middle|\tilde{\mathbf{C}}\right) = V_a^{-1}\left(U_a\left(k\middle|\tilde{\mathbf{C}}\right)\middle|\tilde{\mathbf{C}}\right) - k, \quad \forall a \in A \tag{4.12}$$

Formula (4.12) is illustrated in Figure 4.1 showing the derivation of the link travel times.

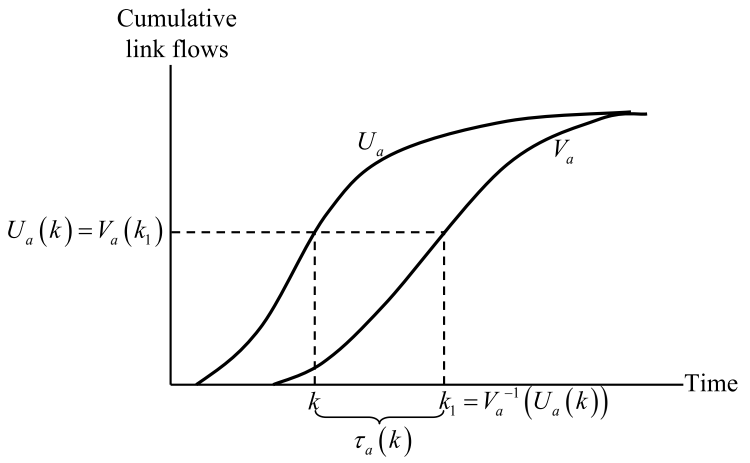


Figure 4.1: Derivation of link travel times

Afterwards, route travel times given a specific capacity vector $\tau_p^{od}\left(k\middle|\tilde{\mathbf{C}}\right)$ can be calculated. Since the capacity vector $\tilde{\mathbf{C}}$, representing a realization of stochastic link capacities for all links, is stochastic, the link/route travel times are stochastic as well.

4.3.5 Capacity modeling

As has been discussed in Section 3.3.3, different capacity distributions can be assumed. Due to the extreme value properties and the non-negativity of link capacities, weibull and gamma distributions are better to fit the capacity distributions. However, we assume that the capacity distribution can be sufficiently well represented by a normal distribution by controlling the possible negative values. Another reason to choose normal distribution is that the apparent correlations among link capacities can be modeled due to the mathematical properties of normal distributions. Therefore the link capacities $\tilde{C}_a$ in a general network is assumed to follow a normal distribution with a mean μ_a and a standard deviation σ_a :

$$\tilde{C}_a \sim N\left(\mu_a,\ \sigma_a^2\right), \tag{4.13}$$

Let F denote the cumulative distribution function of the standard normal distribution, then link capacities can be drawn with a vector of random draws $\mathbf{X}$:

$$\mathbf{C}_a = \mu_a + \sigma_a \cdot F^{-1}\left(\mathbf{X}\right), \tag{4.14}$$

Based on empirical analyses (AVV Transport Research Center 1999), the standard deviation of the link capacity is about 10% of the mean value. When generating random capacities, the maximum absolute value drawn from the standard normal distribution is about 3.5. Then the stochastic capacity $\tilde{C}_a$ is seldom negative due to $\tilde{C}_a \geq \mu_a + (-3.47)\cdot 10\% \cdot \mu_a \approx 60\% \cdot \mu_a > 0.$ When dealing with large capacity variations, we can control the random draws to avoid the negative link capacities. In case a negative capacity is drawn, the capacity is set to zero.

Adjacent link capacities in a network naturally are correlated. In case of bad weather conditions, link capacities are prone to be positively correlated. In order to capture the correlations among link capacities during the quasi-Monte Carlo simulations, Cholesky decomposition of the covariance matrix of link capacities distribution is adopted to facilitate the correlated capacities draws.

In a network, all the link capacities (a matrix, denoted as $\mathbf{C}_n$) follow normal distributions $\mathbf{C}_n \sim N\left(\boldsymbol{\mu},\ \boldsymbol{\Sigma}\right)$ with a symmetric positive definite covariance matrix $\boldsymbol{\Sigma}$ as:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_m^2 \end{bmatrix} \tag{4.15}$$

where m is the total number of links in the network. A symmetric positive-definite matrix can be decomposed into a lower triangular matrix and the transpose of the lower triangular matrix. The matrix of link capacities can therefore be expressed as:

$$\begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \vdots \\ \mathbf{C}_m \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix} + \Delta \cdot F^{-1} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_m \end{bmatrix} \tag{4.16}$$

where $\mathbf{X}_{1\cdots m}$ are independent vectors of Halton draws. $\mu_{1\cdots m}$ denote the mean of link capacities for m^{th} link. Δ is a the lower triangular matrix such that the product of $\boldsymbol{\Delta}$ and its transpose $\boldsymbol{\Delta}^T$ equals matrix $\boldsymbol{\Sigma}$:

$$\mathbf{\Delta}\mathbf{\Delta}^T = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_m^2 \end{bmatrix} = \mathbf{\Sigma} \tag{4.17}$$

which is called Cholesky decomposition of $\mathbf{\Sigma}$. $\mathbf{\Delta}$ is called the Cholesky triangle. The Cholesky decomposition is commonly used in the Monte Carlo method for simulating systems with multiple correlated variables.

Independent distribution among all link capacities is a special case of Formula (4.17) with:

$$\mathbf{\Delta} = \mathbf{\Delta}^T = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_m \end{bmatrix} \tag{4.18}$$

and

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_m^2 \end{bmatrix} \tag{4.19}$$

In the simulation, the variance-covariance matrix of link capacities needs to be firstly determined. Then the Cholesky decomposition is conducted with the variance-covariance matrix, after which the correlated or uncorrelated random capacities for all links in the network can be drawn simultaneously using Formula (4.16). We use a quasi-Monte Carlo approach using Halton draws **X**. For different types of capacity distributions, Monte Carlo simulation is able to generate random draws for correlated variables by derivation of the joint CDF (Cumulative Density Function). A procedure of Monte Carlo simulation being able to preserve the correlations among variables for different types of distributions and for a mixture of different distributions has been demonstrated by (Chang *et al.* 1994).

4.4 Deterministic Network and Departure Time and Dynamic Deterministic User Equilibrium with Vertical Queues

This section briefly presents the DN-DDDUEV model (Deterministic Network and Departure time and Dynamic Deterministic User Equilibrium with Vertical queues). Compared to the SN-DDDUEV (Section 4.2), a deterministic network is modeled without considering stochasticity in capacities and travel times. Therefore there is no travel time reliability involved in the travel cost function and a deterministic dynamic user equilibrium is achieved. The purpose of establishing this model is to validate our simulation-based approach by comparing the analytical results from Vickrey's bottleneck model with deterministic capacities with the results from simulation-based approach. Therefore vertical queues are modeled. A consistent travel cost function as proposed by Vickrey is used to modeling traveler's departure time/route choice behaviors, given as in a discrete time version:

$$c_p^{od}(k) = \alpha \cdot \tau_p^{od}(k) + \gamma_1 \cdot \left(PAT - \left(k + \tau_p^{od}(k)\right)\right)^+ + \gamma_2 \cdot \left(k + \tau_p^{od}(k) - PAT\right)^+ \tag{4.20}$$

The formulation of the DN-DDDUEV model is the same as that of SN-DDDUEV given in subsection 4.2.1. The same DNL model and same deterministic assignment approach are applied as from SN-DDDUEV.

4.5 Static Probabilistic User Equilibrium with Stochastic Networks

This section presents the SN-SPUE model (Stochastic Network and Static Probabilistic User Equilibrium). This model assumes stochastic networks, while static traffic assignment is performed. A long term static probabilistic user equilibrium will emerge at which no traveler can reduce his/her long term route costs by unilaterally changing routes. The perceived route travel cost function $\hat{c}_p^{od}$, adopted in SN-SPUE, is given as:

$$\hat{c}_p^{od} = \alpha \cdot E\left[\tilde{\tau}_p^{od}\right] + \beta \cdot Std\left[\tilde{\tau}_p^{od}\right] + \tilde{\varepsilon}_p^{od} \tag{4.21}$$

where $\tilde{\tau}_p^{od}$ and $\tilde{\varepsilon}_p^{od}$ denote the stochastic route travel time and route-specific error term. There is no time dimension involved. SN-SPUE can be formulated as a VI problem, which is similar to the VI formulation of SN-DDPUEH (see Section 4.3.1) except that there is no time dimension. The VI formulation simply states that given user equilibrium route costs, any deviations from the user equilibrium route flows cannot reduce individual travel costs (Bell and Iida 1997).

With static assignment, a travel time function is needed to estimate the travel time on route p. In this research, the BPR function is adopted to calculate the travel time on each of the routes. Static traffic assignment is not realistic, which can't capture the propagation of flows and assumes the route flows are over all links on the same route. The capacity in the travel time function is normally in veh/hour, therefore the OD demand and route flows need to be transformed into a one-hour period to compute the route costs. Or the capacity should be transformed corresponding to the demand interval to make the travel time calculation realistic. In some cases, the time period is subdivided into small time intervals where for each time interval a static traffic assignment is performed with the dynamic OD demand matrix, which, although better than carrying out one static assignment for the whole period, is still not realistic and not comparable to dynamic traffic assignment model. For more discussions, see Chapter 7.

The same assignment approach is applied as for SN-DDPUEH without departure time choice. Normally distributed link capacities are modeled using the same technique illustrated in subsection 4.3.5. This model will be used for static network design problem, which will be compared to the dynamic network design problem.

4.6 Convergence Criteria

The method of successive averages (MSA), which is a special case of the Frank-Wolfe algorithm (Ortúzar and Willumsen 2000), is adopted to solve the equilibrium problem and to derive the deterministic equilibrium departure pattern:

$$f_p^{od(i)}(k) = \left(1 - \frac{1}{i}\right) \cdot f_p^{od(i-1)}(k) + \frac{1}{i} \cdot \hat{f}_p^{od}(k), \quad \forall (o,d), p, k, \tag{4.22}$$

where $f_p^{od(i)}(k)$ denotes the departure flow on route p for time interval k between OD pair (o, d) at iteration i. Although MSA is a type of heuristic solution method, which does not guarantee descent for instance in the convergence criteria at every iteration, still it provides satisfactory convergence to the equilibrium with sufficient iterations.

In order to check if the equilibrium is achieved or sufficiently achieved, some convergence criteria or measures are needed. Several measures are available to judge the convergence of the iterative simulations. Three measures are elaborated in this section: 1) gap function; 2) the relative changes in dynamic route flows; 3) duality gap.

4.6.1 Gap function

Since the VI problem can be transformed into an equivalent gap function minimization problem, many studies focus on the reformulation of the user equilibrium problem to an optimization problem with which many solution techniques are available. A function G is a gap function for the VI formulation if:

$$\begin{aligned} &G(\bar{\mathbf{f}}) = 0, \ \bar{\mathbf{f}} \in \mathbf{\Omega_d}, \\ &G(\mathbf{f}) \geq 0, \ \mathbf{f} \in \mathbf{\Omega_d}. \end{aligned} \tag{4.23}$$

The global minima of the gap function G coincide with the solutions of the VI problem. Therefore the gap function G provides a measure of convergence of the VI formulation at $\bar{\mathbf{f}}$. By minimizing G over $\mathbf{f}$, equilibrium solutions $\bar{\mathbf{f}}$ are obtained. The VI formulation is equivalent to the minimization problem:

$$\min G(\mathbf{f}), \ \mathbf{f} \in \mathbf{\Omega_d} \tag{4.24}$$

Different gap functions have been defined by other researchers. See some application for static assignment in (Facchinei and Soares 1995; Lo and Chen 2000) and for dynamic assignment (Smith 1993; Lu *et al.* 2009).

4.6.2 Relative flow changes

The measure of relative flow changes is defined as:

$$G^{(i)} = \frac{\sum_{o,d} \sum_{p \in P^{od}} \sum_{k \in K} \left| f_p^{od(i)}(k) - f_p^{od(i-1)}(k) \right|}{\sum_{o,d} q^{od}}, \tag{4.25}$$

where $\left| f_p^{od(i)}(k) - f_p^{od(i-1)}(k) \right|$ indicates the absolute value of flow changes on route p between (o, d) at time interval k relative to the previous iteration (i-1). The relative flow change is such a measure to indicate how much flow is transferred to other departure time interval or other route at each iteration. Relative flow change is a good indicator of convergence in case of dealing with probabilistic user equilibrium, where travelers take into account the perceived travel costs. Although the duality gap will not converge to zero in case of SUE, the flow patterns should converge to a stable state and the relative flow change should go to zero.

4.6.3 Duality gap

The duality gap for the overall network convergence is defined instead of explicitly for each departure time interval since the departure time choice is modeled. The equilibrium should be reached among all the departure time intervals instead of within each departure time interval. The duality gap is a good indicator of the convergence when we model deterministic user equilibrium with deterministic travel costs. Travelers choose the departure time and route with the actual minimum travel costs. Define π^{od} as the minimum route travel cost for users traveling from o to d among all departure time intervals $k \in K$,

$$\pi^{od} = \min_{p \in P^{od},\, k \in K} \left\{ c_p^{od}(k) \right\}, \ \forall (o,\, d), \tag{4.26}$$

At each iteration i, the duality gap is computed as follows:

$$G^{(i)} = \frac{\sum_{o,d} \sum_{p \in P^{od}} \sum_{k \in K} \left(c_p^{od(i)}(k) - \pi^{od(i)} \right) f_p^{od(i)}(k)}{\sum_{o,d} \pi^{od(i)} q^{od}}, \tag{4.27}$$

where q^{od} is the travel demand between (o, d). The duality gap should finally converge to zero in case of a SN-DDDUEV, and DN-DDDUEV. In case of SN-DDSUEH, DN-DDSUEH and SN-SSUE however, the duality gap won't go to zero since travellers consider the perceived travel cost which involves perception errors in the travel cost. The duality gap would decrease and stabilize at a certain positive value.

The numerator of the duality gap is an example of the gap function. In this thesis, the duality gap is adopted to check the convergence of the equilibrium.

From the simulation experiences, it is found that departure time choice equilibrium is much harder to be achieved than network equilibrium with route choice only. Normally with a dynamic OD matrix, modeling route choice only converges faster since the route costs are relatively independent of each other and mainly determined by the route flows. Route cost is a monotone increasing function of the route flow. Modifying the route flows according to the route travel costs leads to the convergence gradually to the equilibrium flow patterns, while with departure time choices, an increase or a decrease in the departure flows for a certain departure interval $f_p^{od}(k)$ does not mean a corresponding increase or decease in the travel cost $c_p^{od}(k)$ as we initially expected. The travel cost $c_p^{od}(k)$ largely depends on the cumulative departure pattern from previous time intervals and correlates with the departure flows from previous time intervals significantly. Using traditional algorithms, as done for route choice user equilibrium, by adjusting the departure flows based on the travel cost for that time interval is not very efficient leading to the equilibrium. Thus the departure time choice equilibrium takes much longer time and much more iterations to be achieved, especially for larger networks.

Several simulation trials were performed to speed up the convergence. Dynamic scale parameters are used to converge better. For instance, start with a small scale parameter leading to more spread assignment among routes and departure time intervals. After a certain number of iterations, large scale parameters could be utilized, which indeed helps speed up the convergence. In order to reach SN-DDDUEV and DN-DDDUEV with a logit model being used, it is better to use route cost difference than route cost since a larger scale parameter can be used to converge better. It should be noticed that most departures occur before the PAT. The modeled time period and PAT should be set in such a way that the time period is best utilized instead of a time waste for useless calculations and sufficient departure time options are provided to travelers.

4.7 Development of a Simulation-based Approach

In this section, a simulation-based approach is developed and a framework will be established for solving all the elaborated models presented in previous subsections. In contrast to the analytical approach applied in Chapter 3, the simulation-based approach discretizes the time period into time intervals and time steps and traffic flow propagation is modeled per time step. The smaller the time step, the more accurate the travel time estimation, however the longer the computation time consumption. The results from the simulation-based approach are approximations to those from the analytical approach. Due to the complexity of the analytical approach, the simulation-based approach takes an advantage of the capability of modeling larger and complex stochastic networks.

A general framework of the simulation-based approach is depicted in Figure 4.2, which is able to model stochastic/deterministic networks, dynamic/static user equilibria, with/without departure time choices.

The inputs of the model are static OD demand, the travel cost function with parameter values and PAT, and the infrastructure network with link capacities. Firstly route set generation is performed to generate a predefined route set. Then the route costs are initialized for the first iteration based on the free flow travel times and the initial travel cost is simply composed of only travel time and schedule delays, so without an initial reliability cost. The outputs will be the equilibrium travel time reliability, departure pattern and equilibrium travel costs for all p and k. Of course there are other unreliabilities, however we only consider travel time reliability as the major explanatory factor of travelers choice behavior under uncertainty. There are two loops in the simulation-based approach:

1) an inner loop with DNL (Dynamic Network Loading), which propagates the traffic flows through the network and models the within-day dynamics. Each iteration loop represents a hypothetical day with a randomly drawn link capacity vector by repeating the within-day dynamics with varied link capacity vectors (representing over-day variations in capacities), the experienced travel time distributions and their properties can be obtained. Then the travel cost will be derived.

2) an outer loop with simultaneous departure time/route choices. This is a typical process with simulation-based approaches. The departure flow on route p at k is derived iteratively with MSA since new travel costs will be obtained with predetermined flow patterns after each iteration and a new flow pattern will result. Until a certain defined convergence criterion is satisfied, the equilibrium is considered to be sufficiently achieved. The initial solution has remarkable impacts on the convergence. Some initial solutions appear not to lead to a convergence to the equilibrium with the heuristic approach MSA. From the simulation experiences, a unique equilibrium will be achieved.

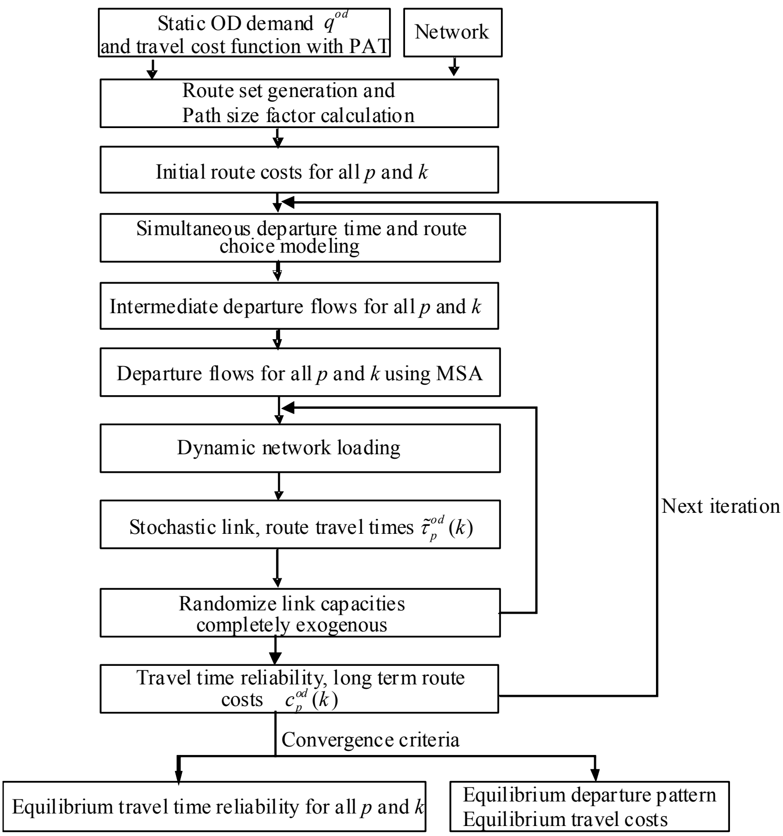


Figure 4.2: Framework of the iterative simulation-based approach

4.8 Application to a Single Bottleneck Model with Deterministic Capacities and Vertical Queues

The developed simulation-based approach has been programmed in Matlab for different models presented in previous sections. This section presents the application of the DN-DDDUEV model on the single bottleneck to validate our simulation-based approach. The same demand *N* and bottleneck capacity are modeled as done in Section 3.2 on the Vickrey's bottleneck model. The same travel cost function is applied with the same PAT.

Figure 4.3 presents the equilibrium cumulative departures and arrivals for the single bottleneck with deterministic capacities from the simulation-based approach. Compared to the analytical results (see Figure 3.1 in section 3.2, both figures are illustrated in the same dimensions), consistent equilibrium departure patterns are derived from the simulation-based approach.

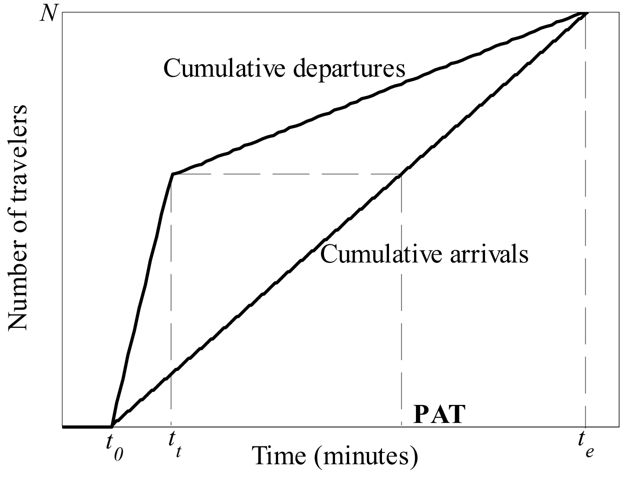


Figure 4.3: Cumulative departures and arrivals from the simulation-based approach for the deterministic single bottleneck case

Figure 4.4 shows the equilibrium travel costs by components for the deterministic single bottleneck. It can be seen that the equilibrium has reached with the simulation-based approach, at which all the travel costs at the used departure times are equal and minimal. A unique equilibrium exists based on the simulation-based approach.

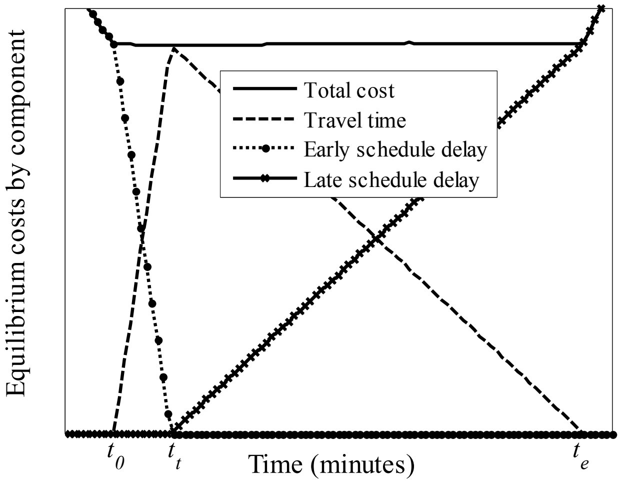


Figure 4.4: Equilibrium costs by component from the simulation-based approach for the deterministic single bottleneck case

This application to the single bottleneck with deterministic capacity validates our simulation-based approach since the results from the simulation-based approach are very much consistent with the theoretical solutions.

4.9 Application to a Single Bottleneck Model with Stochastic Capacities and Vertical Queues

To test the theoretical findings from Chapter 3, the validated simulation-based approach is applied to the SN-DDDUEV on the single bottleneck model. The same demand N and capacity distributions are modeled for the sake of consistency as presented in subsection 3.3.4. The same parameter values are adopted in the travel cost function with the same PAT. 20 halton draws (n=20) are used to approximate the link capacity distribution.

Figure 4.5 and Figure 4.6 present the cumulative departures at the long term equilibrium and the equilibrium costs by component respectively from the simulation-based approach for the single bottleneck with stochastic capacities. Figure 4.5 and Figure 4.6 are plotted in the same dimensions as Figure 3.5 and Figure 3.7 respectively in Chapter 3 and are totally comparable. Appendix B presents Figure 3.5 and Figure 3.7 with scales to facilitate the comparisons. In Figure 4.6 the equilibrium cost is not a straight line since the convergence criterion didn't reach exactly zero. Due to the computation time, we think the equilibrium is sufficiently achieved. From the comparison, the solution from the simulation-based approach is highly consistent with the theoretical findings, which validates our theoretical analyses on the single bottleneck under stochastic capacities.

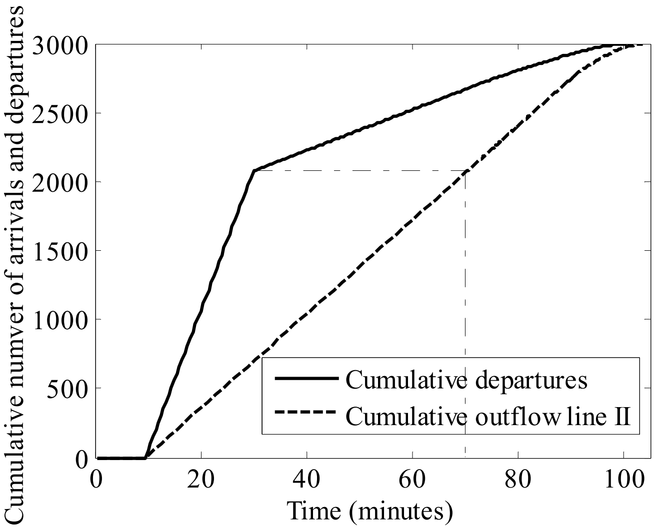


Figure 4.5: Cumulative departures and outflow line II from the simulation-based approach for the stochastic single bottleneck case

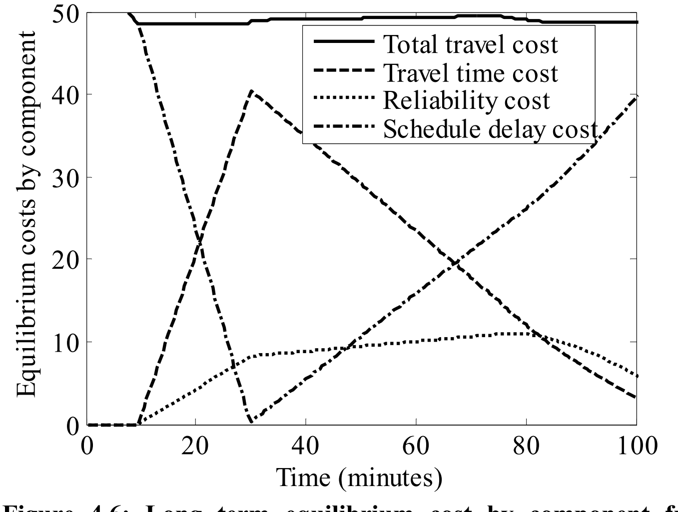


Figure 4.6: Long term equilibrium cost by component from the simulation-based approach for the stochastic single bottleneck case

The simulation-based approach appears to give very accurate results as from the theoretical approach in modeling the departure time choice equilibrium under deterministic/stochastic capacities. Further it is easy to be applied for larger and more complicated networks. Thus the simulation-based approach rather than theoretical analysis with modeling horizontal queues will be used for large network analysis under stochastic capacities.

Another advantage of the simulation-based approach is its flexibility in employing different reliability measures which are difficult to be derived mathematically, for instance percentile travel times.

4.10 Conclusions

This chapter has formulated the long term dynamic user equilibrium under stochastic capacities with discrete time (SN-DDDUE and SN-DDSUE) and presented the framework of a simulation-based approach to solve the equilibrium problem with joint departure time/route choices. The detailed model specifications of simultaneous departure time/route choices, DNL models with horizontal queues and vertical queues, randomization of correlated link capacity draws, and the convergence criteria are depicted. The simulation-based approach with vertical queues has been applied to the single bottleneck case for comparison purpose with the analytical analyses performed in Chapter 3. It shows that highly consistent results are obtained from the simulation-based approach and the analytical analyses. The simulation-based approach is able to derive very accurate results in modeling the long term dynamic user equilibrium and is very flexible and easy to be applied into more general networks. Thus the simulation-based approach with horizontal queues modeled in this chapter, capable of evaluating network performance considering travel time reliability in traveler's departure time/route choice behaviors under uncertainty, will be adopted for road network designs, which will be the topic in the next chapter.

Part IV: Network Design Methodology including Reliability

5 Reliability-based Dynamic Network Design with Stochastic Networks

5.1 Introduction

As has been concluded in Chapters 3 and 4, the departure pattern considering travel time reliability in traveler's choice behavior will be significantly different from that without including travel time reliability in the choice behavior. As discussed in Chapter 2, modeling travel time reliability in the travel cost function better reflects traveler's choice behavior under uncertainty. Without considering travel time reliability in traveler's choice behavior under uncertainty, the network performance will be evaluated inaccurately, which will directly influence network assessment and network designs. Furthermore, the potential applications presented in subsection 1.5.2 (reliability-based dynamic toll design, dynamic peak hour lane design, and dynamic speed limit design) together motivate our reliability-based dynamic road network design approach, in which a road network is designed constrained to reliability-based SN-DDSUE under uncertainty as formulated in Chapter 4.

The network design problem (NDP) has been recognized as one of the most difficult and challenging problems in the transport field. From different perspectives, the network design problem can be categorized in different ways. Depending on the question whether an equilibrium is achieved or not on the lower level, the network design problem can generally be divided into equilibrium network design and disequilibrium network design (Friesz and Shah 2001). Traditional network design problems are network designs constrained to lower level user equilibrium. Friesz and Shah (2001) argued that the traffic on a network is not necessarily in equilibrium and that capacity changes to the network must induce transient phenomena not captured by invocation of the static version of Wardrop's first principle (user equilibrium). They modeled the day-to-day adjustments of flows and costs which guides the network from one disequilibrium state to another. Eventually the network settles down to a conventional steady static equilibrium for any given time t, which is a long term equilibrium for any given time t without modeling departure time choice, although we may believe that in reality, equilibrium does not exist. Especially no perfect information can be provided, travelers cannot predict what is going to happen, travelers may reroute according to the real time situation. Equilibrium or disequilibrium is just a hypothesis which we need to make to model the complicated reality.

Within equilibrium or disequilibrium network design problems, a static network design problem and dynamic network design problem can be distinguished relying on whether a static or dynamic traffic assignment is performed on the lower level. In most literature, the static network design problem is defined and solved. Heydecker (2002) presented a dynamic equilibrium network design in which the design problem is based on the joint departure time and route choice user equilibrium. He indicated that a static assignment limits to flows which are modeled without reference to their variation over time and can't represent the real traffic situation. He concluded that departure time choice and arrival time-specific costs play a very important role and contribute to a more appropriate evaluation and ultimately design of road networks.

With regard to the design variables, network design problems can be grouped into continuous, discrete and mixed network design problems. The Continuous Network Design Problem (CNDP) treats the design variables as continuous, for instance the capacity improvement of a subset of existing network links, which is particularly sensible for a road network for which specifically signalization and ramp metering are considered (Yang and Bell 1998). The Discrete Network Design Problem (DNDP) deals with discrete design variables for instance number of lanes for a set of virtual or existing links, determining where to add lanes or to delete lanes and how many lanes needed. The Mixed Network Design Problem (MNDP) copes with both discrete and continuous decision variables simultaneously.

As known, road networks are dynamic systems in which the variations in travel demand and supply (i.e. capacities) lead to stochasticity in travel conditions, travel costs, etc. Travelers in general dislike uncertainty in travel times. Road authorities and road designers pay increasing attention to the reliability of the services that transport networks can provide to the users. Road networks are now required to have a high degree of reliability to ensure road users smooth travel under normal traffic flow fluctuations and to avoid serious unexpected delays. Depending on whether the stochastic travel demand or network is considered or not, the network design problem can be categorized as a stochastic network design problem or a deterministic network design problem. Nowadays, many studies have been performed on reliability related network design problems. Some research has been done on reliable network design problems (Yin *et al.* 2005; Lou *et al.* 2009; Ng and Waller 2009; Sharma *et al.* 2009), (Rakha and Zhang 2005; Sumalee *et al.* 2006; Dimitriou *et al.* 2008; Zhang and Levinson 2008) or on network reliability modeling (Clark and Watling 2005), however without specifically considering the impact of travel time reliability on traveler's choice behavior. Few research work is available on network design problem in which travel behavior under uncertainty is modeled (Yin and Ieda 2002). However all these network design studies are restricted to static traffic assignment with only route choice modeling. The network dynamics, spillback, and departure time choice cannot be captured at all, which is actually very important since a better understanding of route/departure time choice and the factors that influence the routes chosen are fundamental in developing the strategies to better utilize the network capacity and to improve the network performance. Reliability is modeled either on the network level or on the individual level. Based on our best knowledge, no research tackles the reliable network design problem considering travel time reliability in the choice behavior explicitly using dynamic assignment and modeling departure time choice.

This chapter proposes a reliability-based dynamic network design problem with joint departure time and route choice modeling, aiming to capture the dynamics in the network with stochastic capacities and to evaluate the network performance more reasonably. The design problem is a bi-level problem. Reliability will be modeled on both the network design level and individual travel choice level.

A network design problem is an optimization problem which typically consists of three parts: 1) the objective function specifies the goal or objective. Sometimes a multi-objective design problem may be formulated depending on the purpose; 2) the design variables represent the choices to be made; 3) the constraints restrict the choices of the decision variables to those that are possible or acceptable or even constrain the objective function. Therefore this chapter will be organized in the following order: firstly a definition of the reliability-based network design problem is presented. Then the objectives of the design problem will be illustrated, followed by a description of the design variables. Afterwards, the constraints and the formulation of the design problem will be given.

5.2 Definition of Reliability-based Dynamic Network Design Problem

Our research focuses on the stochastic and dynamic network design problem. As aforementioned, network conditions can be divided into normal and abnormal conditions. In our research, we aim to design a road network considering normal daily traffic conditions under stochastic capacities. We assume that the temporal travel demand is inelastic and deterministic. Extreme traffic conditions, for instance incidents, an earthquake, a flood, etc which cause serious delays due to a big drop in capacities or even non-connectivity, will not be explicitly accounted for. Stochasticity in traffic situations, travel times, and travel costs is caused by day-to-day variations in link capacities.

This chapter proposes an integrative reliable network design problem with two main actors involved, namely road authorities/designers on the network level and individual travelers on the lower level. In the circumstance with uncertainties, travelers make their travel decisions considering travel costs and travel time reliability. Road authorities/designers also pay increasing attentions to the reliability of the services provided to road users. Thus the network design problem is a typical bi-level optimization problem. Reliability will be modeled on both network level and individual level, which is depicted in Figure 5.1.

On the upper level, road authorities/designers design a road network aiming to optimize network performance, for instance network reliability aiming to provide reliable service to road users, total network travel time and total network costs aiming to provide efficient service to road users, by designing network structure and road characteristics like the numbers of lanes and so forth. The decision they make on the network design determines the sensitivity of the network to variations in link capacities, thus determines the travel time reliability under stochastic capacities and the travel costs. Travelers make their choice decisions not only taking into account travel times, schedule delays, but also travel time reliability. The design strategy influences the choice behavior of travelers on the lower level, such as departure time choice, route choice, mode choice, and in the long term, destination choice. Reversely, the changes in travelers' choice behavior affect the network performance which the road authority aims to optimize. The use of a network is determined by the costs and the costs are determined by the use of the network. This is a classical equilibrium problem.

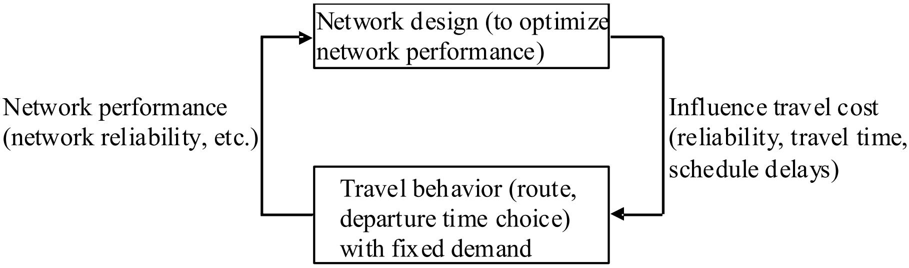


Figure 5.1: The role of reliability in a bi-level network design problem

As a follow up from Chapters 3 and 4 on the long term user equilibrium under stochastic network, a network design problem constrained to a long term dynamic user equilibrium including travel time reliability in a joint departure time/route choices is proposed in this chapter. We aim to help road authorities to optimize their network design such that network efficiency, network reliability and the utilization of the investment are optimized. At the same time, appropriate travel choice behavior in the view of travel time reliability is modeled.

5.3 Design Objectives

The difference between social benefits and costs of the transportation system might be chosen as the objective function to be maximized. The benefit is the contribution of the spatial structure to the well-being of the society. Due to the difficulty of defining, measuring and evaluating the social benefits, we assume the benefits to be the fact that people can travel and do travel between the different places. Since we assume temporally constant travel demand without thinking about the induced travel demand after constructing new road in the network, the social benefits may be treated as constant (Steenbrink 1974). Thus a network design problem with benefit maximization is transformed to an equivalent cost minimization problem.

Generally the cost of a transport network includes: construction and maintenance costs, travel costs, incident costs, environment costs, operational costs, etc. The construction costs will refer to the use of the land, raw materials and labor for the construction and maintenance, which predicts long-run expenditures on infrastructure and outputs (distance traveled by passenger vehicle, single unit truck, and combination truck). For different types of roads and intersections the construction costs are significantly different. A detailed distinction of the construction costs is made by (Steenbrink 1974) for different road types. Environmental costs are difficult to define and quantify in a social evaluation, which covers the pollution of air and soil, noise, disturbance of the ecological balance, damage to the beauty of the landscape, etc. Accident costs depend on the accident type, number of accidents of a certain type driven kilometer. A detailed analysis of accident cost estimations has been given by (Steenbrink 1974). We will not further elaborate on this important aspect of road networks. The travel costs specifically refer to the costs experienced by the travelers, for instance travel time, toll charges, delays, etc. The traditional road network design model is able to prescribe optimal improvement schemes that minimize the total travel time. On top of that, we attempt to improve network reliability as a part of travel costs and optimize network efficiency with a given construction budget.

With regard to road networks, the operational conditions could be normal and abnormal. Because natural resources are limited, performance of road networks under abnormal conditions cannot be required to be the same as that under normal conditions. In the case of normal daily operations, road networks should ensure that people and goods overcome the friction of geographic space efficiently and reliably. Network efficiency can be represented by a measure of total network travel time, total travel costs, congestion durations, total queue length, etc.

Improving network reliability can be achieved by two ways: one is to define the network reliability in the objective function which needs to be optimized (a direct approach); the other is to define certain reliability criteria as constraints in the design problem (an indirect approach). The design problem proposed in this chapter is a budget constraint design problem where the network reliability is directly incorporated in the objective function. The construction costs will be defined in the constraints. The environment costs, accident costs, although form an important part of the total social costs, due to the difficulty of quantification and social evaluation, are not incorporated in the objective function. Therefore the objective function in our research will focus on the experienced travel costs.

Depending on the policy objectives of road authorities, different design objective function can be defined. In the following subsections, some possible and acceptable objectives are discussed, such as optimizing network efficiency and network reliability from road authority's point of view and minimizing the total network travel costs for travelers. These different objective functions will be compared in terms of design solutions in the applications in Chapter 7.

5.3.1 Optimize network efficiency and network reliability

Given the long term equilibrium departure pattern, the daily total network travel time is a stochastic variable, following a distribution. Road authorities, aiming to provide more reliable services to the road users, may not only consider network efficiency, which accounts for the average performance of the network, i.e. average total network travel time, but also the network reliability which is of importance as well to be explicitly incorporated in the design problem.

Reliability is generally defined as the ability of a system or process to perform a required function under given environmental and operational conditions according to defined quality standards and for a stated period of time (Hoyland and Rausand 1994). In the case of road networks, the required function can be described in terms of performance variables like travel time, throughput, accessibility, equity, and so forth. Different functions may lead to different reliability measures (Cassir *et al.* 2000).

Generally there are three categories of network reliability definitions: connectivity reliability, capacity reliability and travel time reliability. Connectivity reliability, defined as the probability that network nodes are connected (Iida and Wakabayashi 1989), is a measure in the case of major exceptional events (abnormal conditions), such as serious natural disasters and huge accidents. Since we consider normal daily conditions, connectivity reliability is not considered. We focus on the service reliability, which could be represented by capacity and travel time reliability. Capacity reliability is defined as the probability that the network can accommodate a certain volume of traffic demand at a required service level (Chen *et al.* 2002b). It is a measure to evaluate the performance of a degradable road network. Travel time reliability is defined as the probability that a trip between an OD pair can be successfully made within a specified time and a specified level-of-service (Asakura and Kashiwadani 1991; Bell and C.Cassir 1999). It is useful to evaluate network performance under normal daily demand fluctuations. A network reliability measure will somehow describe the stochastic properties of an outcome (e.g. total travel time, network capacity, congestion durations, etc). The network performance reliability was defined by (Bell 2000) as the expected total trip cost when travelers are extremely pessimistic about the state of the network. The trip cost was obtained in a mix-strategy Nash equilibrium. A network reliability indicator was proposed by (Clark and Watling 2005) as the probability of total network travel time does not exceed a certain threshold. The network effectiveness and network reliability under demand uncertainty were analyzed by (Yin *et al.* 2005). They used the standard deviation of total network travel time as the indicator of network reliability. A route flow weighted route travel time reliability was defined by (Li *et al.* 2008a) as an indicator for network reliability.

We choose some properties of the daily total network travel time as the indicators of network efficiency and network reliability. A relative weight of these two measures is sufficient instead of evaluating them in some units. We postulate that network efficiency (average total network travel time) and network reliability (the variation in total network travel time) are both crucial involved in decision making on the design strategies. Percentile total network travel time could be adopted, which in some senses includes the expectation of total travel time and a range representing the variations (for instance 90 percentile minus 50 percentile). Different distributions, with different expectations and different variations, may have the same certain percentile travel times (for instance 90 percentile). Thus a single percentile total network travel time cannot capture the relative importance of network efficiency and network reliability. A trade-off between network efficiency and network reliability will be made. We adopt the standard deviation and percentile total travel time difference as indicators of network reliability. Therefore, our objective functions that explicitly account for network efficiency and network reliability are formulated as follows:

$$Z = \left(\overbrace{\vartheta \cdot E\left(\sum_{(o,d)} \sum_{p} \sum_{k} \tilde{\tau}_p^{od}\left(k \middle| \bar{\mathbf{f}}\right) \bar{f}_p^{od}(k) \right)}^{network\ efficiency} + (1-\vartheta) \cdot \overbrace{\left(\sum_{(o,d)} \sum_{p} \sum_{k} \tilde{\tau}_p^{od}\left(k \middle| \bar{\mathbf{f}}\right) \bar{f}_p^{od}(k) \right)^{90th-50th}}^{network\ unreliability} \right), \quad (5.1)$$

or:

$$Z = \left(\vartheta \cdot \overbrace{E\left(\sum_{(o,d)} \sum_{p} \sum_{k} \tilde{\tau}_p^{od}\left(k \middle| \overline{\mathbf{f}}\right) \overline{f}_p^{od}(k) \right)}^{network\ efficiency} + (1-\vartheta) \cdot \overbrace{Std\left(\sum_{(o,d)} \sum_{p} \sum_{k} \tilde{\tau}_p^{od}\left(k \middle| \overline{\mathbf{f}}\right) \overline{f}_p^{od}(k) \right)}^{network\ unreliability} \right), \quad (5.2)$$

where $\tilde{\tau}_p^{od}\left(k \middle| \overline{\mathbf{f}}\right)$ denotes the stochastic route travel times when departing in time interval k given the long term equilibrium departure patterns. $\overline{\mathbf{f}}$ is a matrix of departure flows on all routes at all departure time intervals. The multiplication of the stochastic travel time $\tilde{\tau}_p^{od}\left(k \middle| \overline{\mathbf{f}}\right)$ with the equilibrium flows $\overline{f}_p^{od}(k)$ and summing them over all OD pairs, routes, and departure times yields the stochastic total network travel time experienced by all travelers. The first part in Formulae (5.1) or (5.2) indicates the expectation of total network travel time at the long term equilibrium. The second part indicates network reliability which is calculated by the 90th percentile minus 50th percentile in Formula (5.1), and by standard deviation of the total network travel time in Formula (5.2). Proportions ϑ and $(1-\vartheta)$ are the weights of network efficiency and network reliability in the design objective function. ϑ is a percentage between 0 to 1, showing the importance of network efficiency and network reliability that the road authorities consider. Different road authorities with different design purposes may apply different ϑ values. In case $\vartheta = 0,$ the road authority only aims to optimize network reliability, while in case setting $\vartheta = 1,$ the road authority only attempts to optimize network efficiency defined by average total network travel time.

5.3.2 Minimizing total network cost for travelers

Given constant travel demand and social benefits, a most common objective function for road authorities or road designers is to minimize the total network travel costs, as formulated in expression (5.3):

$$Z = \sum_{(o,d)} \sum_{p} \sum_{k} \overline{c}_p^{od}(k) \overline{f}_p^{od}(k), \quad (5.3)$$

where $\overline{c}_p^{od}(k)$ and $\overline{f}_p^{od}(k)$ denote the long term equilibrium travel costs and equilibrium departure flows on route p between (o, d) when departing in time interval k as discussed in Chapter 4. The equilibrium cost $\overline{c}_p^{od}(k)$ consists of travel time, schedule delays and travel time reliability. Thus this objective function implicitly accounts for the overall travel time costs, schedule delay costs and travel time unreliability.

5.4 Design Variables

It is argued that the construction or improvement of transport infrastructure is one of the most important instruments for the implementation of a regional economic development policy. The physical transport road network is characterized by its network attributes, normally described by capacities, structures, lengths of roads, maximum speed, etc and is a part of the service network.

In this research we will not deal with the graphical design problem for optimal physical planning. Instead we aim to find the optimal capacity allocations for a given spatial distribution of population, employment and land-use patterns. Decision variables may be classified into integer variables, for instance numbers of lanes, and continuous variables, for instance link capacities. The traditional network design problem is treated as a continuous network design problem (CNDP), which treats link capacities as continuous variables and aiming to determine a continuous capacity expansion plan for an existing transportation network to optimize a certain system objective. We design the dimensions of the links, i.e. the numbers of lanes for a subset or a full set of road segments in a potential road network, denoted as $\mathbf{L}=[L_a]$, where $\mathbf{L}\in\Lambda$, $\Lambda=\{\mathbf{L}\,|\,L_a=0,1,2,\ldots,L_a^{\max}\}$. L_a represents the number of lanes for link *a*, which is an integer value. The dimension zero is also included, so that in fact a network structure is also chosen from a set of possible structures.

Given the number of lanes for a link in the potential network, the link capacity $\tilde{C}_a$ is assumed a stochastic continuous variable, following a distribution, expressed as:

$$\tilde{C}_a(L_a)\sim P\Big(\big(\mu_a(L_a)\big),\big(\sigma_a^2(L_a)\big)\Big), \tag{5.4}$$

where $\mu_a(L_a)$ and $\sigma_a(L_a)$ are the expectation and standard deviation of the link capacity distribution. Clearly the characteristics of the link capacity distribution are functions of the numbers of lanes. The standard deviation of the link capacities is a function of the mean capacity depending on the number of lanes, expressed as:

$$\sigma_a=f\big(\mu_a(L_a)\big), \tag{5.5}$$

The larger the number of lanes of a link, the smaller its relative capacity variation. For instance, with a one-lane road, the standard deviation of its capacity is about 10% of the mean capacity (AVV Transport Research Center 1999), while with a two-lane road, the standard deviation is 8% of the corresponding mean capacity.

5.5 Formulation of the Reliable Dynamic Network Design Problem

A reliable road network design problem constrained to reliability-based dynamic user equilibrium with joint departure time/route choice modeling (i.e. SN-DDSUEH) is formulated as following, which is a Mathematical Program with Reliability-based Dynamic Equilibrium Constraints (MPRDEC). As indicated by (Luo *et al.* 1996; Lawphonegpanich and Hearn 2004), MPEC (Mathematical Program with Equilibrium Constraints) is a special case of a bi-level programming problem.

$$\overline{\mathbf{L}}=\arg\min_{\mathbf{L}\in\Lambda}\left(\begin{array}{l}\vartheta\cdot E\left(\sum\limits_{(o,d)}\sum\limits_{p}\sum\limits_{k}\tilde{\tau}_p^{od}\left(k\middle|\mathbf{L},\overline{\mathbf{f}}\right)\overline{f}_p^{od}\left(k\middle|\mathbf{L}\right)\right)\\ \qquad\qquad+(1-\vartheta)\cdot std\left(\sum\limits_{(o,d)}\sum\limits_{p}\sum\limits_{k}\tilde{\tau}_p^{od}\left(k\middle|\mathbf{L},\overline{\mathbf{f}}\right)\overline{f}_p^{od}\left(k\middle|\mathbf{L}\right)\right)\end{array}\right) \tag{5.6}$$

or

$$\bar{\mathbf{L}} = \arg\min_{\mathbf{L}\in\Lambda} \sum_{(o,d)} \sum_{p} \sum_{k} \bar{c}_p^{od}(k)\bar{f}_p^{od}(k), \tag{5.7}$$

subject to reliability-based dynamic long term equilibrium constraints under stochastic capacities, the budget constraint, and integer solution space limitations:

$$\sum_{o,d} \sum_{p\in P^{od}} \sum_{k} \bar{F}_p^{od}(k\left|\bar{\mathbf{f}}\right.,\mathbf{L})\left(f_p^{od}(k)-\bar{f}_p^{od}(k)\right) \geq 0, \quad \forall \mathbf{f} \in \mathbf{\Omega_d}. \tag{5.8}$$

$$I\left(\mathbf{L}\right) \leq \mathbf{B} \tag{5.9}$$

$$\mathbf{L} \in \Lambda, \qquad \Lambda = \{\mathbf{L} \mid L_a = 0,1,2,\ldots,L_a^{\max}\} \tag{5.10}$$

where $\bar{F}_p^{od}(k\left|\bar{\mathbf{f}}\right.,\mathbf{L})$ has been defined in Chapter 4 in Equation (4.9). The objective function (5.6) aims to derive the optimal lane strategies $\bar{\mathbf{L}}$ to minimize a weighted sum of the average total network travel time (TNT) and network travel time unreliability (for short, objective mean+std of TNT). The total network travel time distribution at long term equilibrium varies according to the given design strategy $\mathbf{L}$. The objective function (5.7) aims to find the optimal lane design $\bar{\mathbf{L}}$ to minimize the total network costs at equilibrium (for short, objective TNC).

Expression (5.8) is the finite-dimensional VI formulation of the long term probabilistic user equilibrium with stochastic networks in a discrete-time version (see Expression (4.8) in Chapter 4). The equilibrium travel costs and equilibrium departure flows are dependent on the design strategies. $\mathbf{\Omega_d}$ is the set of feasible dynamic route flow matrix $\mathbf{f}$, defined by the flow conservation (5.11) and non-negativity (5.12) constraints, respectively:

$$\sum_{p\in P^{od}} \sum_{k} f_p^{od}(k) = q^{od}, \quad \forall\left(o,d\right), \tag{5.11}$$

$$f_p^{od}(k) \geq 0, \quad \forall\left(o,d\right), p, k. \tag{5.12}$$

where q^{od} is the total travel demand between (o, d).
Expression (5.9) gives the budget inequality constraints. $\boldsymbol{B}$ is the available budget for the construction. The construction cost, denoted by $I(\mathbf{L})$, is a function of lane design $\mathbf{L}$ and the lengths corresponding to $\mathbf{L}$. The construction cost is computed as a sum of the construction costs over all the links. The construction cost on link a, denoted by I_a, can be formulated as a function of L_a and the length of link a, denoted as d_a:

$$I_a = x\left(L_a, d_a\right) \cdot L_a \cdot d_a \tag{5.13}$$

where $x\left(L_a, d_a\right)$ is the average construction cost per kilometer per lane, which firstly decreases and then increases with the number of lanes. It is shown that the starting investments (up to 2×2 lanes) are generally rather high; expansion to 2×3 and 2×4 lanes is assumed to be comparatively cheap, while with further expansion 2×5 and 2×6 lanes again high investments are involved (Steenbrink 1974).

Expression (5.10) is called the domain constraint with a restriction on the maximum number of lanes $L_a^{\max}$ for each link. Note that zero-lane is also an option, which means not building a certain road segment. The problem must be solved for the values of the variables L_a that satisfy the restrictions and meanwhile minimize the objective function. A vector $\mathbf{L} \in \Lambda$ satisfying all the constraints is called a feasible solution to the optimization problem. The collection of all feasible solutions forms a feasible region.

The formulated network design problem is a constrained combinatorial optimization problem capable of providing a completely new network design (kind of network structure design since a link can have zero lanes) and of an improvement design on an existing network.

Table 5.1 summarizes the properties of the proposed and formulated network design problem.

Table 5.1: The classification of the proposed network design problem

Uncertainty		Demand		Capacity	
Yes	No	Fixed	Stochastic	Constant	Stochastic
×		×			×
Design property		Lower assignment		Choice modeling	
Discrete	Continuous	Static	Dynamic	Route choice	Departure time choice
×			×	×	×
Long term dimension		Within-day dimension		Reliability modeling	
Equilibrium	Disequilibrium	Equilibrium	Disequilibrium	Choice behavior	Network level
×			×	×	×

The network reliability is related to the sensitivity of a road network to the variations in the capacities and travel demand. The sensitivity of the network actually depends on the physical structure and physical characteristics of the network. If the network can easily handle the variations in capacities and in travel demand without severe delays, the network is not sensitive. The less sensitive, the more reliable the network is considered to be. The proposed network design problem is in fact designing the network structure and network capacities in order to establish an insensitive and reliable road network under stochastic capacities.

5.6 Formulation of the Static Reliable Network Design Problem

The dynamic network design problem requires much longer computation time and more efforts compared to the static network design problem. Although static assignment is not realistic, it demands less computation time. In order to investigate and compare the optimal solutions from static and dynamic network designs, a reliable static network design problem is formulated as well, which is a special case of the formulated dynamic network design problem.

The same network design objective function is adopted to facilitate the comparison (see Section 5.5 in Formulae (5.6) and (5.7). The difference from the previously formulated design problem is that there is no departure time choice. Capacity is still the source leading to between-day travel time variations. Travelers, based on their past experiences, are aware of the distributive properties of the route travel time and make their route choices so as to minimize their long term future route costs (see the formulation in Section 4.5).

Table 5.2 presents the experiments we aim to carry out in Chapter 7 for comparison purpose between static and dynamic network designs, dynamic network designs with different objective functions, and dynamic network designs with different levels of travel time reliability.

Table 5.2: Experimental design

<table>
<tr><th colspan="3"></th><th>A 7-link hypothetical network (Chapter 7)</th></tr>
<tr><td rowspan="4">Dynamic network design with stochastic capacity (subject to SN-DDPUEH)</td><td rowspan="2">High level capacity variation</td><td>Total network costs as design objective</td><td>×</td></tr>
<tr><td>Network reliability and network travel time as design objective</td><td>×</td></tr>
<tr><td rowspan="2">Low level capacity variation</td><td>Total network costs as design objective</td><td>×</td></tr>
<tr><td>Network reliability and network travel time as design objective</td><td>×</td></tr>
<tr><td rowspan="4">Static network design with stochastic capacity</td><td rowspan="2">High level capacity variation</td><td>Total network costs as design objective</td><td>×</td></tr>
<tr><td>Network reliability and network travel time as design objective</td><td>×</td></tr>
<tr><td rowspan="2">Low level capacity variation</td><td>Total network costs as design objective</td><td>×</td></tr>
<tr><td>Network reliability and network travel time as design objective</td><td>×</td></tr>
</table>

5.7 Summary

As the real travel environment is never deterministic, a network design problem should be able to capture the stochasticity and its influence in choice behavior, especially the departure time choice behavior. Network reliability is nowadays also a big issue attracting increasing policy attention. This chapter proposed and formulated a reliable network design approach constrained to reliability-based dynamic user equilibrium with joint departure time and route choice modeling under stochastic networks. Reliability is incorporated in both the network design level and individual travel choice level in order to design an efficient and reliable road network with a given budget taking into account the influence of travel time reliability in traveler's departure time and route choice behavior. The proposed design problem deals with discrete design variables, i.e. numbers of lanes for a subset or a full set of links in the potential network.

In order to gain insights into the importance of dynamic network modeling involved in network design problem, especially departure time choice modeling, a static reliable network design approach is formulated as well. The static reliable network design approach, dealing with stochastic networks, considers reliability on both network and individual level, while the dynamics in the flow propagation and departure time choices are neglected. The design solutions from different approaches will be compared in the case study in Chapter 7.

The design approaches have been formulated already. The next chapter will be devoted to the algorithms to solve the proposed network design problem.

6 Solution Approaches of the Dynamic Network Design Problem

6.1 Introduction

The proposed reliability-based dynamic discrete network design problem is a typical combinatorial optimization problem. A number of algorithms are available to solve such a combinatorial optimization problem. This chapter will firstly discuss the properties and typical problems of the dynamic network design problem. Then a road network-oriented genetic algorithm (GA) is selected to solve this design problem. Specifically for the road network design problem, infeasible, unconnected and illogical designs massively exist. These will be defined and a systematic approach to eliminate the infeasible, unconnected and illogical designs is developed. The same types of infeasible, unconnected and illogical lane designs might be generated during GA operations which requires a special treatment of the adopted GA. Due to the long computation time of GA, a combined GA and set evaluation approach is proposed to derive the optimal or a fairly good design solution. Finally a summary of this chapter is presented.

6.2 Properties of the Design Problem

The formulated network design problem in Chapter 5 is a bi-level problem and a combinatorial optimization problem, whose decision variables (numbers of lanes on a set of potential links, zero-lane is allowed as well) are discrete. Combinatorial optimization problems are characterized by a finite number of feasible solutions. The goal is to find an optimal arrangement, grouping, ordering or selection of discrete objects usually finite in numbers (Lawler 1976).

In a road network, a combination of lane numbers for a particular set of links, called a Lane Design, has its physical meaning. A typical problem of a lane design is that it may lead to an unconnected network structure, which is attributed to the zero lanes leading to disconnectivity between any Origin-Destination (OD) in the network. Besides, with a certain lane design, a certain number of constructed links may never be used by any travelers. All these lane designs, specifically for road networks, are unrealistic and illogical.

As already clarified, the discrete decision version of the design problem is a NP-complete problem (Johnson *et al.* 1978). The class NP stands for "Nondeterministic Polynomial" time. A problem is *NP-complete* if it can be used to efficiently solve any other problem in NP. In essence, any NP-complete problem is as hard as any problem in NP, see (Evans and Minieka 1992). In the work of (Jeroslaw 1985), it is already mentioned that bi-level problems are in general NP-hard, which implies that the proposed dynamic network design problem is NP-hard and cannot be solved with any existing algorithm in polynomial time (which means the time for solving the problem is not a polynomial function of the number of the decision variables). We challenge this NP-hard problem by tackling an even more difficult problem in which on the lower level departure time choice is modeled under stochastic capacities instead of traditional network design problems which are constrained to a static assignment with route choice modeling only.

An algorithm is a sequence of steps designed to solve a problem. Well known algorithms for solving the optimization problem can be classified as either exact or approximate algorithms. An exact algorithm is the one which always produces an optimal solution if it exists. For finding the exact optimal solution for the formulated dynamic network design problem in Chapter 5, a systematic search in the solution space is needed, for instance the exhaustive search and full enumeration of all possible designs. However such algorithm involves exponential asymptotical computational complexity. For large networks, it is inefficient and infeasible to apply this exhaustive search algorithm. To increase efficiency, all modern exact methods use pruning rules to discard parts of the search space in which the (optimal) solution cannot be found. These approaches are doing an implicit enumeration of the search space (Stutzle 1998). For optimization problems the best known examples are branch & bound algorithms, and dynamic programming.

Therefore approximate or heuristic algorithms, which sacrifice the guarantee of finding the optimal solution and produce a feasible and hopefully very good solution in polynomial-time, are frequently adopted for finding a sufficiently good solution (Evans and Minieka 1992). Promising feasible heuristic algorithms to solve the combinatorial problem are among others: local search (LS), simulated annealing (SA) (Kirkpatrick *et al.* 1983), genetic algorithm (GA) (Obitko 1998; Gen and Cheng 2000), ant colony optimization (ACO) (Colorni *et al.* 1991; Dorigo *et al.* 1996), tabu search (TS) (Glover 1989; Glover 1990), particle swarm optimization (PSO) (Kennedy and Eberhart 1995; Eberhart and Shi 2001), etc. SA, GA, ACO and PSO are naturally inspired metaheuristics.

Often one uses heuristics when an exact algorithm is not available or when it is computationally impractical. Due to the NP-hardness of the proposed network design problem and since many decision variables needed to be designed simultaneously, a heuristic algorithm, GA, is adopted in this thesis to solve the network design problem. GA is characterized by the flexibility in dealing with the constraint of a multi-variable design problem and is a global search algorithm instead of local search algorithm.

6.3 Generation of the Master Set of Lane Designs

Prior to the application of the Genetic Algorithm for solving the dynamic network design problem, the set of lane designs firstly is filtered. Such a filtering is especially useful in a transport network design context where trips between origin and destination have to be served. A universal set of lane designs is defined, which contains all possible combinations of the numbers of lanes on all the potential links in the network satisfying the domain constraint with $L_a^{\max}$ for each link. The size of the universal set can be easily computed as a product of the possibility for link *a* over all links, expressed as: $\prod_a \left(\boldsymbol{L}_{a,\max}+1\right)$. Since zero-lane is also a possibility, the possible numbers of lanes for link *a* therefore is $\boldsymbol{L}_{a,\max}+1$. Each lane design in this universal set can be represented as a point in a multidimensional space where the links are the axes and the numbers of lanes are the values on these axes.

The master set is defined as the subset of the universal set (i.e. a subset of points in the multidimensional space), which contains all feasible, connected, and logical lane designs. A design satisfying the budget constraint is defined as a feasible design. An unconnected design is defined as a lane vector **L**, which disables the connectivity between any OD pair in the network. Illogical designs are defined by lane vectors with which the roads constructed will never be used. The size of the master set is strongly dependent on the network structure.

An example grid network shown in Figure 6.1 with the link numbers is adopted to illustrate the concepts of the universal set, the master set and their sizes. In addition, the demonstration of the infeasible, unconnected, and illogical lane designs will be given with this network. In this network, a lane design is a vector of lane numbers on all 12 links in a sequence as [L_1 L_2 L_3 L_4 L_5 L_6 L_7 L_8 L_9 L_{10} L_{11} L_{12}].

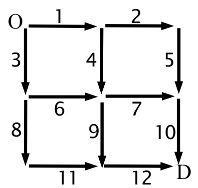


Figure 6.1: An example grid network

A maximum $L_a^{\max}=2,\ \forall a\in A$ is defined for all the 12 links in the network. Since all the links are equal in length, a construction budget of 8 lanes is defined instead of the monetary costs. The size of the universal set equals $3^{12}=531,441$. The size of the subset of feasible designs satisfying the construction budget however only is 57,720. Figure 6.2 illustrates the infeasible designs, unconnected designs and illogical designs with examples. The first example design consists of 10 lanes, which exceeds the budget constraint, thus is an infeasible design. The second example design does not provide any connectivity from the origin to the destination, thus is an unconnected design. With the third example design, link 1 will never be used. The lane designs [2 0 1 0 0 1 0 1 1 0 1 1] and [1 0 1 0 0 1 0 1 1 0 1 1] are equivalent to [0 0 1 0 0 1 0 1 1 0 1 1] in terms of the network performance evaluation. Therefore [2 0 1 0 0 1 0 1 1 0 1 1] and [1 0 1 0 0 1 0 1 1 0 1 1] are illogical designs and should be eliminated from the design set for evaluations.

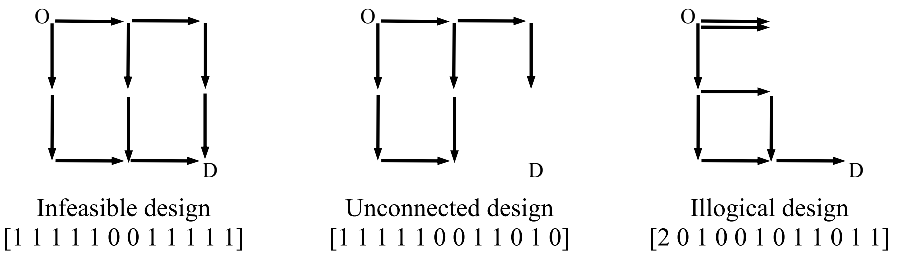


Figure 6.2: Illustrations of infeasible, unconnected, and illogical lane designs

A new systematic approach of generating the master set is developed to eliminate the infeasible, unconnected, and illogical lane designs, to reduce the solution space to a considerable extent, thus to save computation time.

Figure 6.3 presents the systematic approach to eliminate the infeasible, unconnected and illogical designs and to derive the master set. Due to the construction budget constraint, a finite number of feasible designs can be derived from the universal set. As shown in the example network in Figure 6.1, 57,720 feasible designs are derived from the universal set with a size of 531,441.

The unconnected designs, which disable the connectivity between any OD pairs, are distinguished and deleted by checking the number of OD pairs as a result of the designs. A given potential network has a predefined OD matrix and a certain number of OD pairs. An OD-Route-Link matrix consists of information on all OD indices, route indices, and a series of links on each of the route in sequence. With each feasible design, the zero-lane links are extracted and the routes traversing these zero-lane links are deleted from the OD-Route-Link matrix, after which a new OD-Route-Link matrix is derived for this specific feasible design. The number of OD pairs in the new OD-Route-Link matrix can be derived. If the number of OD pairs in the new OD-Route-Link matrix is smaller than the predefined number of OD pairs in the potential network, implying that some OD pairs are disconnected due to the zero-lane links, then this feasible design is an unconnected design and deleted. By eliminating the unconnected designs from the feasible design set, a complete set of feasible and connected designs is created.

With the example network in Figure 6.1, 7,588 connected designs are derived from 57,720 feasible lane designs, i.e. the size of the feasible and connected design set is 7,588.

For each design in the feasible and connected subset, an OD-Route-Link-lanes matrix can be generated, which differs from the OD-Route-Link matrix in such a way that the numbers of lanes on the links for each route instead of link numbers are included in the matrix. The feasible and connected design set can be grouped into different subsets. Within each subset, all the designs lead to the same OD-Route-Link-lanes matrix, i.e. the same physical network. Due to the existence of zero-lane links, some links on the routes traversing these zero-lane links might not be used due to disconnectivity. Therefore different numbers of lanes on these unused links do not change the physical network and will not influence the network performance. These designs are illogical since some roads constructed will never be used. A design with the minimum construction costs from each subset is the design with which all the unused links have zero-lanes, which is called the logical design in this subset. The remaining designs in this subset are illogical designs. By collecting the logical designs from all the subsets, the master set is generated, which is an exhaustive set of all feasible, connected and logical designs, i.e. the master set.

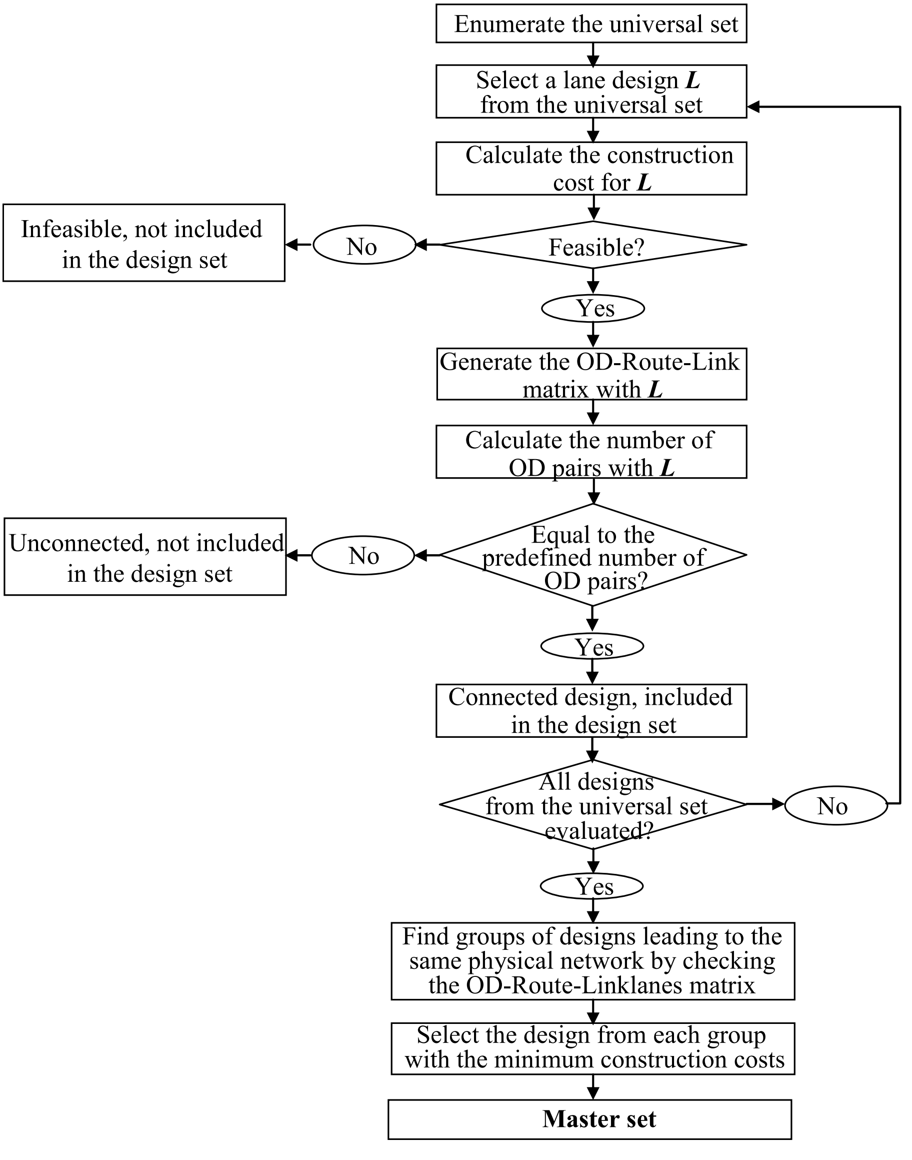


Figure 6.3: A systematic diagram of deriving the master set of lane designs

Again with the example network in Figure 6.1, by eliminating the illogical designs from the feasible and connected design set, a master set with a size of 654 is derived from 7,588 feasible and connected designs. This example network shows that the solution space can be reduced dramatically from 531,441 to 654 (i.e. the size of the master set is only 0.12% of the size of the universal set, and 1.13% of the size of the feasible and connected design set), which can reduce the computation time for the optimization problem considerably. With this example network, it only takes a few minutes to generate the master set.

Of course, the size of the master set depends on the construction budget and the network structures. For large and complex networks with many overlaps among routes, eliminating unconnected and illogical designs enables to decrease dramatically the solution space for the design problem and saves considerable computation time, as shown with the example grid network.

6.4 A Road Network-oriented Genetic Algorithm

Upon the derived master set, a Genetic Algorithm (GA) is applied to find a fairly good or even the optimal design. A Genetic algorithm is firstly developed by Holland in 1975 (Holland 1975). In the past decades, GA has been extensively dealt with and widely applied in many fields (Obitko 1998; Gen and Cheng 2000), therefore this section will not elaborate on GA in general, but rather only describe the intelligence and parameter settings of the approach used in this thesis. The framework of solving the dynamic discrete network design problem with GA is illustrated in Figure 6.4.

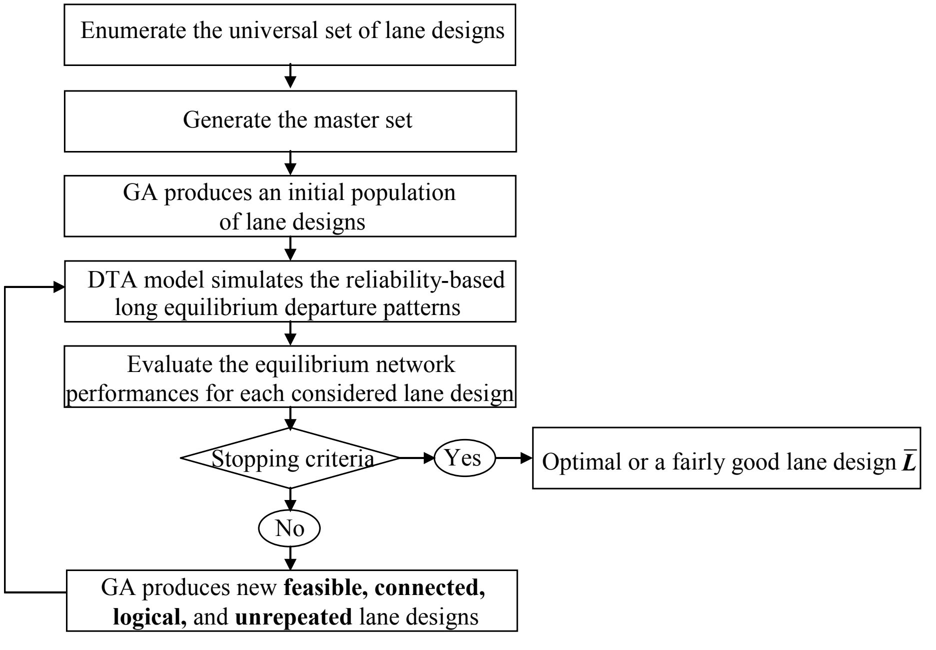


Figure 6.4: Framework of solving the network design problem with GA

With the master set derived from the universal set, GA is used to produce and evolve the lane designs from the master set toward a fairly good or the optimal solution for the network design problem. The fitness of each lane design is evaluated based on the user's long term equilibrium considering travel time reliability in departure time and route choices, which is simulated by the DTA model. GA iteratively evolves the lane designs until reaching the predefined stopping criteria.

An initial set of lane designs, namely a population, is randomly selected from the master set. All the designs are different in the population. The size of the population is predefined according to the number of design variables.

There are three GA operators: crossover, mutation, and reproduction, with which the new lane designs (so-called offspring) are created from the designs from the previous generation (so-called parents). The behavior of GA is characterized by a balance between exploitation and exploration in the search space. The balance is strongly affected by several parameters such as size of the design set at each generation, coefficient of reproduction selectivity (a parameter involved in the fitness function), probability of crossover, probability of mutation, and maximum generations. Fixed parameters are used in most applications of genetic algorithms. Since a genetic algorithm is an intrinsically dynamic and adaptive process, the use of constant parameters is thus in contrast to the general evolutionary spirit. Therefore, it is natural to try to modify the values of the parameters during the run of the algorithm. There are three principal categories of adaptation: deterministic, adaptive and self-adaptive (Gen and Cheng 2000). A deterministic adaption is taken in our study. The mutation probability p_m is decreased gradually along with the elapse of generations to facilitate the exploration in the whole solution space at the beginning and convergence at the end using the following equation (similar function, see (Gen and Cheng 2000)):

$$p_m = 0.5 - 0.4\frac{g}{G_{\max}} \tag{6.1}$$

where g denotes the current generation number and $G_{\max}$ denotes the maximum number of generations. The mutation probability will decrease from 0.5 to 0.1 as the number of generations increases to $G_{\max}$.

At each generation, the best two designs from the previous generation (the so-called parents) are kept in the set for current generation (the so-called offspring). Each design is characterized by its fitness derived from the simulation with the DTA model. The fitness of each design $\boldsymbol{L}$ is defined as the inverse of the objective function in the optimization problem:

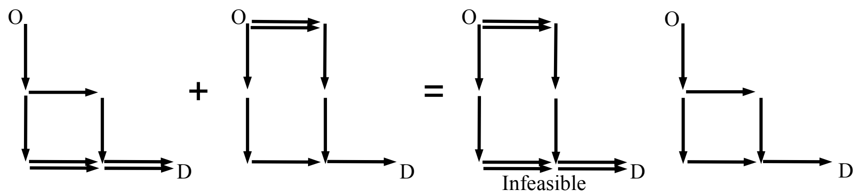
$$v(\mathbf{L}) = \frac{1}{Z(\mathbf{L})} \tag{6.2}$$

where $Z(\mathbf{L})$ has been defined by either Formula (5.2) or (5.3) in Chapter 5. The fitness function in some literature is calculated as a negative exponential transformation of the objective function. According to the fitness, each design has a probability of being selected to generate new designs. The higher the fitness, the higher the possibility it will be selected. Whether to perform crossover or not and the selection of designs are random processes. The selection process used in this study is the so-called ‘roulette wheel’ (Obitko 1998). Crossover point and mutation position are all randomly chosen.

All the evaluated designs after each generation are stored in a pool with the fitness, all the network cost components (for instance, average total network travel time at equilibrium, standard deviation of total network travel time, total schedule delay early and late, construction costs, etc) and the duality gap. At each generation, the newly generated designs are checked before the evaluation to make sure that whether it has already been evaluated in previous generations or not. If so, the fitness can be directly retrieved from the pool to avoid repeated evaluations.

Dealing with generated infeasible, unconnected and illogical designs

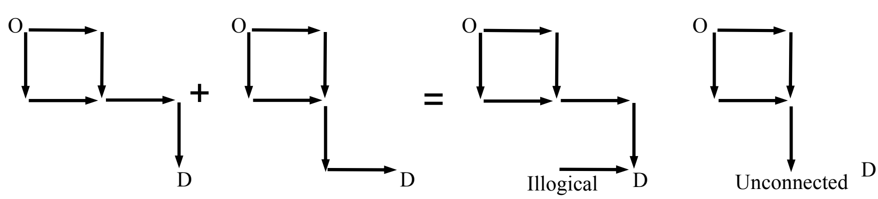
A problem encountered in applying GA to the constraint optimization problem is that the genetic operators may yield infeasible, unconnected or illogical designs, already excluded in the master set generation step. The example network given in Figure 6.1 is again used to demonstrate the process of crossover, mutation and the generation of infeasible, unconnected, and illogical designs. Figure 6.5 shows a crossover between two lane designs at a randomly chosen crossover point at link 7 and the generated lane designs, one of which is infeasible.



$$\begin{matrix}[0\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 2\ 2] \\ [2\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 1]\end{matrix} \Rightarrow \begin{matrix}[2\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 2\ 2] \\ [0\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 1]\end{matrix}$$

Figure 6.5: Generation of infeasible design after crossover

Figure 6.6 shows an example of generating unconnected and illogical lane designs after the crossover with a crossover point at link 11.



$$\begin{matrix}[1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 0] \\ [1\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 1]\end{matrix} \Rightarrow \begin{matrix}[1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1] \\ [1\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0]\end{matrix}$$

Figure 6.6: Generation of unconnected and illogical designs after crossover

After crossover, a mutation is performed on the generated lane designs with a mutation probability. The mutation points (link numbers) are randomly chosen for randomly selected lane designs with randomly generated integer numbers of lanes on that link. With this mutation, infeasible, unconnected, and illogical designs may be generated as well. Figure 6.7 illustrates the process of mutation and the generated infeasible, unconnected, and illogical designs. However it is also possible that the infeasible, unconnected, and illogical designs generated after crossover may become designs in the master set after the mutation. For instance with the generated illogical design after crossover presented in Figure 6.6, if a mutation point at link 12 is selected and the number of lanes on link 12 is changed to zero, then this illogical design becomes logical. The unconnected design in Figure 6.6 will become a connected design after the mutation takes place at link 12 to one or two lanes.

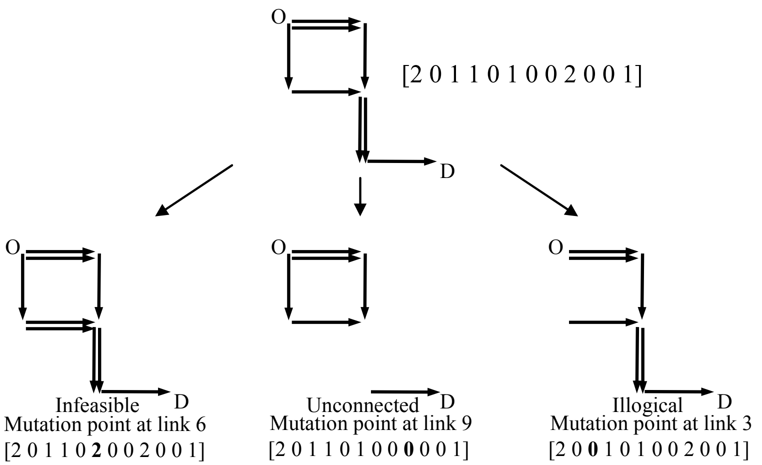


Figure 6.7: Generation of infeasible, unconnected, and illogical lane designs after mutation

Existing techniques to handle this problem can be roughly classified as follows: 1) rejecting methods, 2) repairing methods, and 3) penalty methods (Gen and Cheng 2000).

A rejecting method discards all infeasible, unconnected and illogical designs created throughout the evolutionary process.

Repairing methods involve taking an infeasible design and generating a feasible solution through a repair procedure.

The penalty technique is used to keep a certain number of infeasible solutions in each generation so as to enforce the genetic search toward an optimal solution from both sides of feasible and infeasible regions and mixture of both regions. Some infeasible solutions may provide more useful information about the optimal solutions than some feasible solutions do.

The rejecting method is the simplest way to handle the problem and is adopted in this thesis. At each generation, the generated designs are checked before the reproduction of a new population. All previously evaluated designs are recorded in a pool so that in case that a generated design is infeasible, or unconnected or illogical, a new design from the set (master set – pool) is randomly selected to take place in order to avoid the repeated selection. This step is specific for transport network designs.

The encoding of the designs can be evaluated by several principles: Nonredundancy, Legality, Completeness, Lamarckian property and Causality (Gen and Cheng 2000). In our study, a real value encoding is adopted which satisfies all the properties. Therefore the GA can be called a real-coded genetic algorithm.

The convergence of GA is considered to be achieved when the average of the set performance will not be significantly improved for *M* successive generations, or a maximum number of *N* generations has been performed.

During the applications, some practical problems are encountered. Different $L_a^{\max}$ could be defined for different links. Then in case of mutation, the randomly generated number of lanes for different links should be defined differently depending on the $L_a^{\max}$.

Intermediate flow patterns from previously evaluated designs at equilibrium can be used as the initial solution for the convergence to save computation time. However, different designs lead to different network structures, so flow patterns among routes need to be adjusted. For instance the flows on the deleted routes are added to another route of the same OD.

Attention must be paid to convergence which possibly influences the optimal solution generation. An ideal case is that all the evaluated designs converge to the same duality gap value, since some designs are very close in terms of the network performance at equilibrium. Different convergence may lead to different evaluations, ordering of the designs and the selection of the best strategy. However the computation time is also enormous if a same small duality gap value is required for all the designs. Computation time and convergence should be traded off. In our study, a maximum number of iterations is defined to terminate the simulation. If the maximum iterations are performed, the equilibrium is considered achieved. A sufficiently large number for the maximum iterations is defined to ensure the convergence and to reduce the influence of the convergence on the network performance evaluations. Although the equilibria from different designs still do not converge to the same degree, the influence of the convergence can be neglected.

It is also tried to provide a good initial design set. Since a static assignment costs much less computation time, a complete evaluation of the designs in the master set is practically feasible which will be discussed in Chapter 7. Several good solutions from the static network design approach could be included in the initial design set for solving the dynamic network design problem. However it turns out that including good designs in the initial design set is not a good solution as expected, since the good designs are too dominating which hinders the generation of better designs and explorations in the solution space.

6.5 A Combined Genetic Algorithm and Set Evaluation Algorithm

In order to find a fairly good solution or the optimal solution, more generations are needed with GA. However it takes very long computation time due to the time consuming simulations to reach the dynamic user equilibrium with randomized capacities. Therefore a set evaluation approach is proposed and combined with GA. This means that firstly a full evaluation of all the designs in the master set is performed with a reliability-based static assignment with stochastic capacities. Then, a number of the top designs in terms of network performance are selected and evaluated by the reliability-based dynamic traffic assignment model. The best design is derived with the set evaluation. The GA is also applied. If the solution derived from GA is found in the top design set from static network design approach, then we think the set is sufficiently large and the set evaluation derives a better solution than GA. Set evaluation is quite efficient in terms of computation time and enables a better solution for the dynamic network design problem.

It will be discussed in Chapter 7 that static assignment is deficient from an empirical point of view. Good solutions based on static network evaluation might be week solutions based on the dynamic network evaluation. However if the set selected from the static network design approach is sufficiently large, a good or even the optimal solution could be found within this set for the dynamic network design problem. Combined with GA, it is believed that the set evaluation results in a better solution than GA. It can be explained that due to the good solutions and the optimal solution for both static and dynamic network design problems lie in a region which more or less make the maximum utilization of the construction budget. A very bad solution from the static assignment evaluation must be a design with low construction costs, which cannot be a good solution for the dynamic design problem neither.

6.6 Summary

This chapter focused on the approaches and algorithms adopted to solve the reliability-based dynamic network design problem. A universal set and a master set of lane designs have been defined. A new systematic approach is proposed to establish a master set by a priori eliminating the infeasible, unconnected and illogical designs to reduce the solution space and to save computation time. An example grid network has been applied to demonstrate the definitions of the infeasible, unconnected and illogical designs and the process of the crossover and mutations. It showed that the solution space can be reduced dramatically in this way. With the example network the derived master set size is only 0.12% and 1.13% of the sizes of the universal set and the feasible, connected design set respectively. The crucial parameters for the GA operators are presented. Some critical issues influencing the design solution are emphasized, for instance the influence of the equilibrium convergence on the network performance evaluations.

A combined genetic and set evaluation algorithm is proposed to solve the dynamic network design problem. A set of top designs is selected based on the reliability-based static network evaluation for a complete master set. This set is evaluated by the reliability-based dynamic assignment model and the best design within this set is derived. GA is applied as well. If the solution from GA is found in the set, then the set evaluation gives a better solution than GA for the dynamic network design problem. The combined GA-Set evaluation approach is efficient in terms of the computation time and the quality of the solution.

The GA and the set evaluation approach will be applied to the reliability-based dynamic network design problem on a hypothetical network in Chapter 7. The solution from static and dynamic network design approaches will be compared with different reliability degrees and different objective functions.

7 Design of Complete Networks

7.1 Introduction

This chapter will present a series of applications of the proposed dynamic network design approach to a hypothetical network for a complete design. All the potential links in the network are designed with the possibility to have zero lanes. The dynamic and static network design approaches formulated in Chapter 5 will be applied to the network. The purpose of this chapter is to show the feasibility and the merits of the proposed reliability-based dynamic network design approach. Furthermore this chapter aims to compare the designs from:

1) static reliability-based network design approach and dynamic reliability-based network design approach,

2) dynamic network design approach with difference levels of capacity variations and travel time reliability,

3) reliability-based dynamic network design approach with different objective functions,

in order to investigate the impacts of the travel time reliability and departure time choice modeling on the network designs. The GA and the set evaluation approach as discussed in Chapter 6 are adopted to solve the design problem. The lower level long term equilibrium with stochastic networks is modeled by the simulation-based approach.

This chapter will firstly start with the impacts of link independence on the network performance evaluations. Then the hypothetical network for the design application will be described. After that the design solutions from the static and dynamic network design approaches with two levels of capacity variations will be elaborated respectively. Comparisons among different design approaches will be used to draw conclusions.

7.2 Impacts of Link Independences

In case of modeling stochastic link capacities, it should be born in mind that link divisions in a network have impacts on the network performance evaluations, which differs from the deterministic capacity case. An example is presented to explain the influences of link divisions. Figure 7.1 shows a three-link network with link numbers and link lengths in the brackets in kilometers.

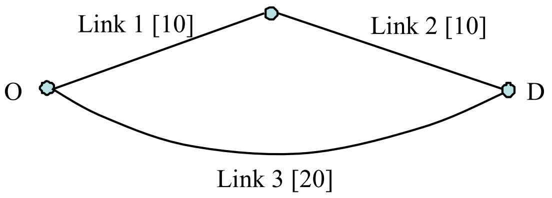


Figure 7.1: Three-link network example

At first thought, strategies [2 2 0] and [0 0 2] should be equivalent. However it is only true in the deterministic capacity case. With stochastic capacities, it is found that design strategy [0 0 2] performs better than [2 2 0] with independent link capacities given the same demand and the same travel cost functions with the same PAT. It is due to the bottleneck capacity of a single route consisting of two links in series is determined by the minimum of the stochastic capacities of these two links. Thus in general the outflow capacity is lower than the route with [0 0 2], which leads to longer travel times and thus higher total travel costs. Dividing one link into two links will lead to different network equilibria and network performance in case of modelling stochastic capacities.

In the deterministic capacity case, the design strategies of [1 1 1], [2 1 1] and [1 2 1] will give exactly the same equilibrium given the same demand and the same travel cost functions with the same PAT, thus the same total network travel time and total travel costs. It is quite easy to understand because the route performance is determined by the bottleneck on the route. With all the three designs, the bottleneck on the upper route is always the one-lane link. However, in the stochastic capacity case, all these strategies will not reach exactly the same equilibrium. Since with design strategy [1 1 1], the upper route performance is determined by the minimum of the capacities on the two links. While with strategies of [2 1 1] and [1 2 1], the upper route performance is determined by the one-lane link. Thus the stochastic travel times will be different. So are the expected travel time, travel reliability, schedule delays and equilibrium travel costs.

This can be explained with the result from extreme value theory that the expectation of the minimum of a set of distributions is always smaller or equal to the minimum of the expectation of each distribution (Bovy 1984; Li *et al.* 2007a; Li *et al.* 2007b). In stochastic networks, the expectation of the bottleneck capacity on a route is smaller or equal to the minimum of the mean capacities of all links on this route.

Since capacity is a local characteristic of a road, normally we define link capacity by assuming a homogeneous road section with the same capacity at all locations. Link divisions should be carefully determined in a network in case of modeling stochastic capacities, depending on the road geometry, speed limits, etc. Of course, positive link capacity correlations between the upper two links in case of the strategies [2 2 0] will reduce the difference between the designs [2 2 0] and [0 0 2]. An extreme case would be that the capacities of the upper two links are completely positively correlated. In this case designs [2 2

0] and [0 0 2] are totally equivalent in case stochastic capacities are considered. Therefore, when working with stochastic link capacities, a capacity covariance matrix is very important to define the link independences, as we will do in Section 7.4.

7.3 Network Description

The proposed reliability-based dynamic discrete network design approach will be applied to a hypothetical network with 7 potential links, as depicted in Figure 7.2, given the land use patterns.

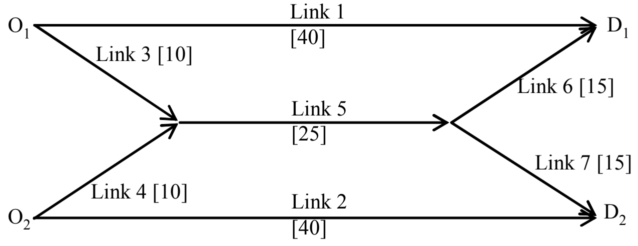


Figure 7.2: Network description

The length of each link is given in square brackets with a unit of kilometer. In total 6 routes are potentially available between the two origins and destinations. Routes are numbered from 1 to 6 with the route-link table given as:

Table 7.1: Link incidence matrix for the potential complete network

		Link 1	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7
(O_1, D_1)	**Route 1**	1	0	0	0	0	0	0
(O_1, D_1)	**Route 2**	0	0	1	0	1	1	0
(O_1, D_2)	**Route 3**	0	0	1	0	1	0	1
(O_2, D_1)	**Route 4**	0	0	0	1	1	1	0
(O_2, D_2)	**Route 5**	0	0	0	1	1	0	1
(O_2, D_2)	**Route 6**	0	1	0	0	0	0	0

The total travel demand for each OD pair are given in Table 7.2 in vehicles.

Table 7.2: OD Demand (unit in vehicles)

	D_1	D_2
O_1	5000	1000
O_2	2000	5000

Each link capacity is assumed to follow a normal distribution. The correlation among link capacities and the impacts of the covariance matrix will be analyzed in the next section.

Fifty Halton draws are used to replicate the link capacity distribution in the quasi-Monte Carlo process. The domain constraint is defined as $L_{\max} = 3$, restricting a maximum number of lanes for each potential link to three.

Two levels of capacity variations are modeled to investigate the influence of reliability on the network design problem. One level assumes that the standard deviation is equal or less than 10% of the mean capacity, depending on the number of lanes. The mean capacities for one-lane, two-lane, and three-lane are 2150, 4300 and 6900pcu/hour, and the standard deviations are 215, 350, and 500 respectively (for short, level 10%). The second level assumes that the standard deviation is equal or less than 20% of the mean capacity, depending on the number of lanes. The standard deviations for one-lane, two-lane, and three-lane are 430, 700, 1000 respectively (for short, level 20%).

The path-size factor depends on the network structure and the route sets in this potential network. Different designs will result in different network structures and route sets, thus changed path-size factors. If all the potential links are built, the path-size factors are computed for all the 6 routes sequentially as [0, 1.0397, 1.0397, 1.0397, 1.0397, 0]. For general networks, path-size factors need to be recomputed for different designs.

According to the definition of unconnected designs given in chapter 6, links 3, 4, 5, 6, 7 cannot have zero lanes to preserve the connectivity between OD pairs (O_1, D_2) and (O_2, D_1) in the network. Illogical routes do not exist for this hypothetical network. Therefore the path size factors will remain the same regardless of the network structures and designs for this hypothetical network. For this hypothetical network, PSC-logit is applied for the traffic assignment.

A value of time of 10 euros/hour has been adopted. Relative parameters of 1.2, 0.8 and 1.2 are utilized in the travel cost function for travel time reliability, schedule delay early and schedule delay late respectively. The impact of the different parameter settings on the design solution will be investigated in future work. A time period of 90 minutes is modeled with PAT = 80 [min]. The maximum speed on each link is 100km/hour. A time step of 30 seconds is adopted in the dynamic network loading, while the minimum link travel time in the network is 6 minutes. A departure time period of 1 minute is used in modeling the departure flows.

A construction cost of about 210,000 euros/km/lane/year is taken from (Meeuwissen 2003). Lack of cost information forced us to use only rough cost factors. Since we work on a constrained network design, for this hypothetical network the construction cost calculation does not influence the designs since the budget and the construction costs are scalable and not a component in the objective function. The demand modelled in this case is assumed 20% of the daily total demand, 5 days a week and 52 weeks per year are accounted for. Thus the total travel cost for a year is computed by multiplying the total travel cost derived from the simulation for 90 minutes by 1300.

With the genetic algorithm, a decreasing mutation probability from 0.5 to 0.2 is taken. The convergence of GA is considered to be achieved with M=10 and N=50.

7.4 Impacts of Covariance Matrix on The Evaluations

Depending on the network structure and the modeling situations, a different covariance matrix $\mathbf{\Sigma}$ of the link capacities (see section 4.3.5) in this network may be defined. For instance when it is rainy, it is found that all the link capacity decreases at the same time by a 10-20%, which means that all the link capacities are completely positively correlated (Brilon and Ponzlet 1996; Maze *et al.* 2006; Tu *et al.* 2007c; Tu *et al.* 2007a). Since link capacity is determined by many factors such as the weather conditions, the traffic composition, driving behavior, etc, link capacity covariance matrix may change over time. For purpose to show the impacts of the covariance matrix on the network performance evaluation, four link capacity covariance matrices are defined for this hypothetical network, given as:

Table 7.3: Covariance matrices (unit in vehicles)

$$\mathbf{\Sigma}_1 = \begin{array}{c} 1\\2\\3\\4\\5\\6\\7 \end{array} \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_6^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_7^2 \end{bmatrix} \qquad \mathbf{\Sigma}_2 = \begin{array}{c} 1\\2\\3\\4\\5\\6\\7 \end{array} \begin{bmatrix} \sigma_1^2 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & \sigma_2^2 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & \sigma_3^2 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & \sigma_4^2 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & \sigma_5^2 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & \sigma_6^2 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & \sigma_7^2 \end{bmatrix}$$

$$\mathbf{\Sigma}_3 = \begin{array}{c} 1\\2\\3\\4\\5\\6\\7 \end{array} \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 100 & 0 & 0 \\ 0 & 0 & 100 & 100 & \sigma_5^2 & 100 & 100 \\ 0 & 0 & 0 & 0 & 100 & \sigma_6^2 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 & \sigma_7^2 \end{bmatrix} \qquad \mathbf{\Sigma}_4 = \begin{array}{c} 1\\2\\3\\4\\5\\6\\7 \end{array} \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 5 & 0 & 0 \\ 0 & 0 & 5 & 5 & \sigma_5^2 & 5 & 5 \\ 0 & 0 & 0 & 0 & 5 & \sigma_6^2 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 & \sigma_7^2 \end{bmatrix}$$

Covariance matrices $\mathbf{\Sigma}_1$, $\mathbf{\Sigma}_2$, $\mathbf{\Sigma}_3$, $\mathbf{\Sigma}_4$ assume independent link capacities, completely positively correlated link capacities, and partly correlated link capacities to different degrees respectively. The reliability-based dynamic traffic assignment model with joint departure time and route choices are performed for a lane design [2 2 1 2 3 2 1] with each of these covariance matrices. The network performances at equilibrium are compared with different covariance matrices.

Table 7.4 presents the distributive properties of the total network travel time and total network travel costs by components for the four different link capacity covariance matrices. A value of $\vartheta = 0.6$ is adopted in the objective function (see Formula (5.6)).

It is noticed that the most remarkable case is the network with covariance matrix $\mathbf{\Sigma}_2$ assuming all links are positively correlated to model for instance the bad weather. The network with $\mathbf{\Sigma}_2$ outperforms the rest cases with the other three covariance matrices, with which the total network travel cost is much less and the total network travel time is on average much smaller and more reliable than that with the other three covariance matrices. With covariance matrix $\mathbf{\Sigma}_2$, the relative differences and variations among the link capacities will be less due to the positive correlations, thus more reliable and more efficient. The difference among the independent link capacity case with $\mathbf{\Sigma}_1$ and few correlated link cases with $\mathbf{\Sigma}_3$ and $\mathbf{\Sigma}_4$ are not significant.

Table 7.4: Impacts of covariance matrices

	$\mathbf{\Sigma}_1$	$\mathbf{\Sigma}_2$	$\mathbf{\Sigma}_3$	$\mathbf{\Sigma}_4$
Mean+std of TNT (hours)	8346017.4	**8066295.7**	8347828.1	8332811.2
Mean of TNT (hours)	13551742.7	13095250.4	13556175.5	13532446.5
Std of TNT (hours)	537429.4	**522863.7**	535307.1	533358.2
Total travel costs (euros)	188994415.6	**183820854.6**	189223152.2	188971867.2
Total travel time costs (euros)	135517427.1	130952503.6	135561754.8	135324464.8
Total reliability costs (euros)	9403592.4	9209375.3	9393415.0	9370989.6
Total schedule delay early costs (euros)	29190189.5	30078294.9	29091356.8	29127604.4
Total schedule delay late costs (euros)	14883206.6	13580680.8	15176625.6	15148808.5

Therefore, considering capacity correlations have impacts on the network performances and network evaluations. Different covariance matrices exhibit different impacts. In this example the difference is about 5% at most. There is no unique link capacity covariance matrix for a given network over time and the influence of a specific covariance matrix depends on the network structures. For the design problem, all different designs will be evaluated with the same covariance matrices. We assume that a specific covariance matrix will have the same influences on the network performances of all different designs. The consequence of this assumption is that the selection of the covariance matrices does influence the network performance for a specific design, while it does not influence the relative goodness of the different designs and thus the optimal solutions to the network design problem. Any reasonable covariance matrix may be chosen. In following design applications, an independent covariance matrix $\mathbf{\Sigma}_1$ is selected to model the stochastic link capacities.

7.5 Design Solution in Case of SN-SPUE

This section starts with the static network design problem with stochastic capacities (see the formulation of the static network design problem in Section 5.6 and the travel cost function in (4.21)). Parameters of 0.15 and 4 are chosen for the BPR function to estimate the route travel time.

An exhaustive set of feasible lane designs (master set) contains 2964 possibilities, satisfying the construction budget constraint and excluding the unconnected and illogical designs. Since the static assignment is enormously fast to reach equilibrium and a full evaluation of the 2964 possibilities only takes few minutes for this hypothetical network. Therefore all the 2964 designs are evaluated and the optimal solution is found.

The two levels of capacity variations are modeled and the two different design objective functions are considered. Table 7.5 illustrates the top 10 lane designs in terms of the objective function of mean+std of TNT in hours, which turns out to have similar construction costs. An optimal design $\bar{\mathbf{L}}_s$ (denotes the optimal solution from static network design) of [2 3 1 1 2 2 1] is found with which the weighted network efficiency and network reliability is optimized. The expectation and the standard deviation of the total network travel time at equilibrium are presented as well. It is noticed that the variation of the total network travel time is relatively small, which is on average only 2.2% of the mean total network travel time (i.e. the coefficient of variation of TNT is about 2.2%).

Table 7.5: Design evaluations with level 10% capacity variations on a yearly basis

Lane designs	Weighted mean+std of TNT (hours/year)	Total travel costs (Millions of Euros/year)	Mean of TNT (Hours/ year)	Std of TNT (Hours/year)	Total travel time costs (Millions of Euros/year)	Total reliability costs (Millions of Euros/year)	Construction costs (Millions of Euros/year)
[2 3 1 1 2 2 1]	5041101	85.77	8278582	184878.5	82.79	2.98	65.63
[3 2 2 2 2 1 1]	5044122	85.87	8288688	177274.3	82.89	2.98	66.68
[3 2 1 3 2 1 1]	5046498	85.92	8289389	182162.3	82.89	3.02	66.68
[3 2 1 2 2 1 1]	5049941	85.96	8291679	187334.8	82.92	3.04	64.60
[3 2 1 1 2 2 1]	5097991	86.96	8354704	212922.3	83.55	3.4	65.63
[3 2 1 1 2 1 2]	5117852	87.36	8391531	207333.1	83.92	3.45	65.63
[2 3 2 2 2 1 1]	5118624	87.43	8386537	216755.3	83.87	3.58	66.68
[3 2 3 1 2 1 1]	5123687	87.44	8399110	210553.1	83.99	3.45	66.68
[2 3 1 3 2 1 1]	5123973	87.54	8391606	222524.6	83.92	3.63	66.68
[3 2 2 1 2 1 1]	5124443	87.50	8397241	215245.5	83.97	3.53	64.60

Therefore explicitly including the network travel time unreliability (as in objective of mean+std of TNT) does not contribute to network reliability unless a very high weight is chosen for the network unreliability.

As can be seen that optimizing the weighted mean+std of TNT leads to the same optimal design and similar sequence of the 10 designs (except for the three designs which are not in the correct order, see the bolded mean of TNT) as optimizing the mean of TNT since network reliability is relatively too small. However if only network reliability is optimized (with $\vartheta = 0$), the design [3 2 2 2 2 1 1] leads to the most reliable network.

It is found that the standard deviation is not a monotone function of the expectation of the total network travel time. Large expectation of the total network travel time does not necessarily lead to large variations for the same network. With the same degree of capacity variations, the standard deviation of the total network travel time strongly depends on the network structure as a result of the designs.

It is also found that the two different objective functions in the stochastic and static network design problem lead to the same optimal solution and the same order of the 10 designs. Due to the small scale of this hypothetical network and low demand, the network performance differences among different designs are not large. For larger networks and realistic networks, the network performance among different designs may be very significant. The value of choosing an optimal design for a specific potential network becomes more remarkable.

Table 7.6 compares the equilibrium flow patterns and route costs for the optimal design $\bar{\mathbf{L}}_s$ and the most reliable design [3 2 2 2 2 1 1] to investigate the sensitivity of the network structure and its influence on the network reliability. With the optimal design $\bar{\mathbf{L}}_s$, there are two bottlenecks on both route 3 and 5 (although route 5 is not used by travelers) with one-lane links, therefore the variability of the route outflow capacity is larger than that on the same routes with the most reliable design [3 2 2 2 2 1 1]. The remaining routes in both designs are similar. Therefore the design [3 2 2 2 2 1 1] would be more reliable than the optimal design $\bar{\mathbf{L}}_s$.

Table 7.6: Route flows and route costs at equilibrium with level 10% capacity variations

	[2 3 1 1 2 2 1]		[3 2 2 2 2 1 1]	
	Route Flows	Route Costs	Route Flows	Route Costs
Route 1	4545.5	32.36	5000	26.67
Route 2	454.5	33.07	0	34.15
Route 3	1000	32.98	1000	32.97
Route 4	2000	33.71	2000	34.26
Route 5	0	33.63	454.5	33.07
Route 6	5000	26.67	4545.5	32.22

Larger capacity variations are modeled to investigate the influence of reliability on network design. Table 7.7 presents the mean and standard deviation of the total network travel time and the total network travel costs by component with 20% variations in link capacities.

An optimal design of [3 2 2 2 2 1 1] is derived, which is different from that with 10% variations in link capacities, implying that modeling travel time reliability in traveler's choice behavior has significant influence on the network equilibrium and network designs.

Table 7.7: Design evaluations with level 20% capacity variations on a yearly basis

Lane designs	Weighted mean+std of TNT (hours/year)	Total travel costs (Millions of Euros/year)	Mean of TNT (Hours/ year)	Std of TNT (Hours/year)	Total travel time costs (Millions of Euros/year)	Total reliability costs (Millions of Euros/year)	Construction costs (Millions of Euros/year)
[3 2 2 2 2 1 1]	5252511	93.07	8453662	450784.2	84.54	8.53	66.68
[2 2 1 1 2 2 1]	5273378	93.69	8449840	508685.3	84.50	9.19	65.63
[3 2 1 3 2 1 1]	5285039	94.15	8468919	509219.5	84.69	9.47	66.68
[3 2 1 2 2 1 1]	5296127	94.30	8470636	534363.2	84.71	9.59	64.59
[3 1 1 3 3 1 2]	5341332	94.71	8686654	**323350.3**	86.87	7.84	66.68
[2 2 2 2 3 2 1]	5349559	95.19	8635028	421355.4	86.35	8.84	66.68
[2 2 1 3 3 2 1]	5411885	97.35	8687630	498267.8	86.88	10.48	66.68
[2 2 1 2 3 2 1]	5425970	97..65	8704423	508289.2	87.04	10.60	64.59
[3 2 1 1 2 2 1]	5430382	97.86	8604016	669930.3	86.04	11.82	65.63
[2 3 2 2 2 1 1]	5436732	97.71	8632186	643551.2	86.32	11.39	66.68

Compared to Table 7.5, it is noticed that the top 10 designs are different from different capacity variations. Again the two objective functions lead to the same optimal solution and similar sequences of the 10 designs.

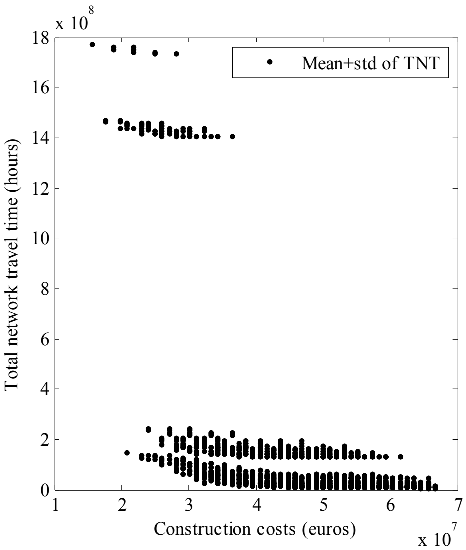
The standard deviation of the total network travel time is relatively small, for which a coefficient of variation of TNT is about 5.3%. If only the network reliability is aimed to be optimized, the most reliable design will be [3 1 1 3 3 1 2]. The reliability of a network depends on the sensitivity of the network to the variations in link capacities. The less sensitive, the more reliable the network is. The sensitivity of the network depends on the network structure.

Table 7.8 compares the equilibrium flow patterns and route costs of the optimal design and the most reliable design respectively.

Table 7.8: Route flows and route costs at equilibrium with level 20% capacity variations

	[3 2 2 2 2 1 1]		[3 1 1 3 3 1 2]	
	Route Flows	Route Costs	Route Flows	Route Costs
Route 1	5000	27.80	5000	27.79
Route 2	0	37.46	0	37.93
Route 3	1000	35.88	1000	36.71
Route 4	2000	37.66	2000	38.17
Route 5	714.3	36.10	2903.2	36.97
Route 6	4285.7	35.83	2096.8	37.10

Figure 7.3 and Figure 7.4 plot the distributive properties of the total network travel time and total network travel costs for all the feasible, connected and logical lane configurations (all the designs in the master set) in relation to the construction costs with 20% capacity variations. Mean+std of total network travel time has a similar pattern as total network travel costs in relation to construction costs.



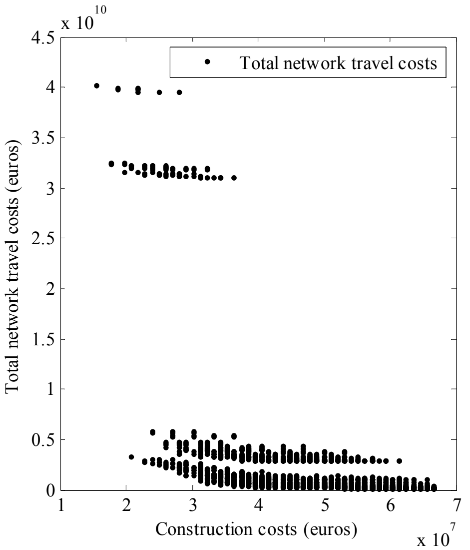


Figure 7.3: Network performances in relation to construction costs with 20% capacity variations

It can be seen that when the investment is small, extra investment on the construction improves dramatically the network performance, which is to say that the marginal benefit of investment is very large. However when the investment reaches a certain amount, the marginal benefit of the investment on the network performance is rather limited.

The same construction costs may lead to different designs, thus highly different network performances. Larger investment may lead to worse network performance than smaller amount of investment, depending on the network structures. Therefore the optimal capacity allocation over space should be determined with which the investment is optimally and maximally utilized.

Figure 7.4 shows the means and standard deviations of total network travel time of all the feasible, connected and logical networks in the master set in relation to construction costs. It can be seen that the standard deviation of total network travel time of all the designs have larger variability than the expectations of total network travel time, since std of TNT can be very large or very small as seen in the right figure. Therefore the reliable designs and the reliable network structures can be derived with which the network reliability is high.

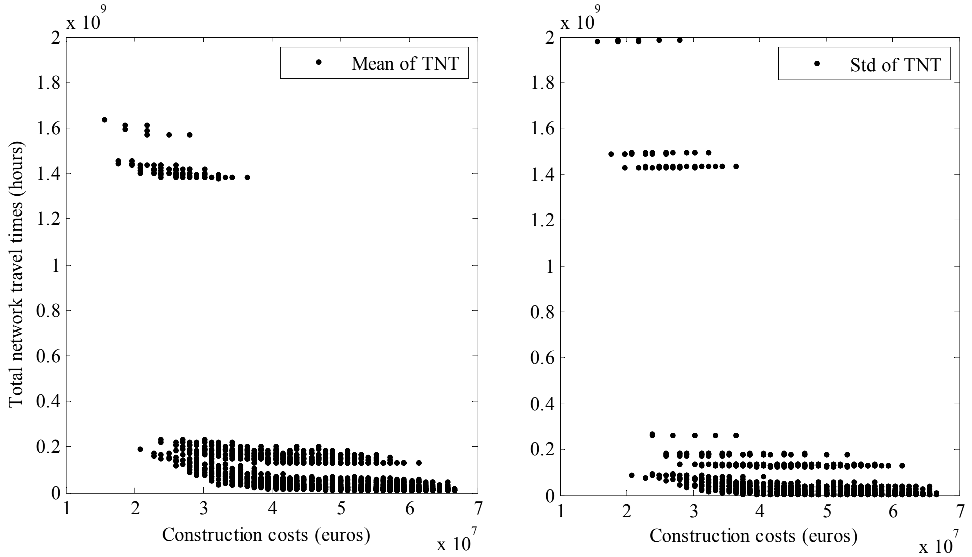


Figure 7.4: Expectations and standard deviations of total network travel times in relation to construction costs

Among the 2964 designs in the master set, more than 13000 pairs of designs are found which have only one-lane difference in the physical network. In order to investigate the network performance with adding an extra lane in the network at any location, Figure 7.5 presents the frequency of the network performance differences of the 13491 pairs of designs.

A negative difference in the network performance means that adding one extra lane in the network improves the network performances, vice versa. It is found that most pairs have differences in performance which are close to zero from both sides, implying that adding one extra lane does not change the network performance efficiently. Some differences are positive, meaning that adding one extra lane in the network may deteriorate the network performances. This shows that the so-called Braess paradox appears in reliability-based static networks.

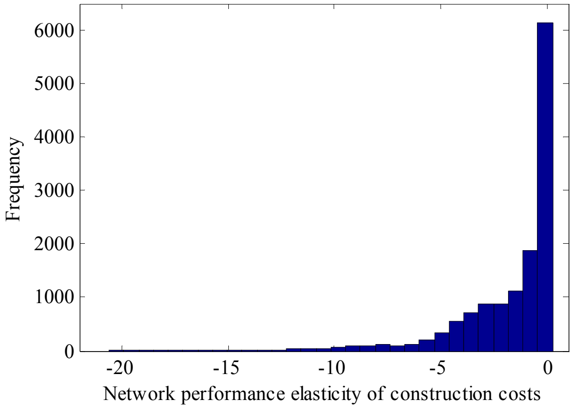


Figure 7.5: Network performance changes with adding one extra lane in the network

7.6 Design Solution in Case of SN-DDPUEH with Low Capacity Variations

In contrast to the previous section on a static network design, this section presents the design solution from the reliability-based dynamic road network design approach with 10% capacity variations. The purpose is to investigate the effects of dynamic network modeling and departure time choice on network designs and to determine whether a dynamic network design approach indeed outperforms the static network design approach.

The GA approach has been tested with static network designs showing that GA is able to find good solutions which however are not necessarily the optimal solution, while each run with GA may generate different solutions. If the optimal is desired, the GA is required to be run many times and the solutions need to be compared. For practical purpose, we adopt the combined GA and set evaluation approach to derive a fairly good design or even the optimal design with dynamic networks.

A sufficient set of the top 100 designs from static designs with 10% capacity variations is evaluated with dynamic traffic assignment model. The GA is also applied to the reliability-based dynamic network design problem. The solution given by GA appears to be one of the designs in the 100 designs, which means that the evaluation of the set of designs will lead to a better solution than the GA algorithm.

Table 7.9 presents the mean+std of total network travel time and total network travel costs by components of the top 10 designs. Both objective functions lead to the same design solution $\bar{\mathbf{L}}_d$ (denotes the design solution from dynamic network design) of [3 2 2 2 2 1 1], which is different from the solution $\bar{\mathbf{L}}_s$ from static network design with 10% capacity variations.

It appears that the static solution $\bar{\mathbf{L}}_s$ [2 3 1 1 2 2 1] from the static network design with 10% capacity variation not even is a good solution in the top 10 best designs from dynamic network design approach. Because of its poor travel time estimations, a static traffic assignment may lead to incorrect approximation of the network performance, thus to poor network designs. Again, the two objective functions lead to the same design solution, but different order of the 10 design in terms of network performance.

Compared to Table 7.5, in general the estimated total network travel times from dynamic traffic assignment expectedly are much higher than from static assignment since the dynamics and spillbacks are captured. It is also noticed that larger variations of total network travel time (the coefficient of variation of TNT is about 3-4%) appear than in the static network design case (about 2.2%) with 10% capacity variations. Static traffic assignment significantly underestimates the total network travel time and the variability of total network travel time, thus also underestimates the value of the investments.

Table 7.9: Dynamic design evaluations with level 10% capacity variations on a yearly basis

Lane designs	Weighted mean+std of TNT (Hours/year)	Total travel costs (Millions of Euros/year)	Mean of TNT (Hours/year)	Std of TNT (Hours/ year)	Total travel time costs (Millions of Euros/year)	Total reliability costs (Millions of Euros/year)	Construction cost (Millions of Euros/year)
[3 2 2 2 2 1 1]	7040815	158,10	11489484	367812	114,89	**7,72**	66.68
[3 2 1 2 2 1 1]	7116073	160.94	11615972	**366226**	116,16	8,21	64.59
[2 2 3 2 2 2 1]	7238355	170,79	11736390	491302	117,36	8,86	63.55
[2 2 3 3 2 2 1]	7238411	170,49	11735793	492339	117,36	8,91	65.63
[2 2 2 3 2 3 1]	7288140	171,65	11816261	495960	118,16	8,92	66.68
[2 2 2 3 2 2 2]	7293935	171,69	11827147	494116	118,27	8,94	66.68
[2 2 1 3 2 2 2]	7295356	171,56	11829379	494322	118,29	8,96	64.59
[2 2 2 2 2 1 2]	7295545	172,85	11848845	465595	118,49	9,25	61.47
[2 2 2 2 2 1 3]	7302294	172,93	11857264	469840	118,57	9,29	64.59
[2 2 3 2 2 3 1]	7314904	171,98	11857998	500262	118,58	8,92	66.68

Travelers make a trade-off between the route travel times, route travel time reliability, schedule delay costs at all available departure time intervals and make their route and departure time choice accordingly. For comparison, Figure 7.6 and Figure 7.7 present the equilibrium departure patterns on all routes in the network with the dynamic design $\bar{\mathbf{L}}_d$ and the static design $\bar{\mathbf{L}}_s$ (see Section 7.5).

Different equilibrium departure patterns over time appear for both designs, which directly influence the network performance. With designs for instance $\bar{\mathbf{L}}_s$, departures from different OD pairs start and finish within almost the same time region whereas with designs for instance $\bar{\mathbf{L}}_d$, the departures from different OD pairs appear relatively separate over time. As seen in Figure 7.6, with the dynamic design solution $\bar{\mathbf{L}}_d$ travel demand between OD pairs (O_2, D_1) depart much earlier than the demand departing from (O_2, D_2), followed by that from (O_1, D_2) and (O_1, D_1). The departures from different OD pairs avoid each other over time. The dynamic design solution $\bar{\mathbf{L}}_d$ leading to relatively separate departures among the 4 OD pairs, explains the best network performance. A wider distribution of departures over time among different OD pairs will generate lower total network travel time and total network travel costs.

The distribution of departures is a result of the designs. Therefore the design solution, leading to the best network performance, makes the maximum utilization not only of the physical capacity itself, but also the efficiency over time.

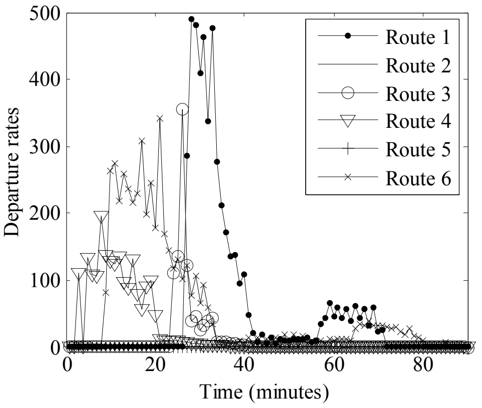


Figure 7.6: Dynamic equilibrium departure patterns with $\overline{\mathbf{L}}_d$ [3 2 2 2 2 1 1]

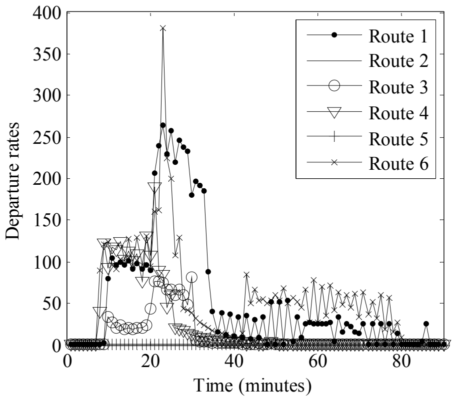


Figure 7.7: Dynamic equilibrium departure patterns with $\overline{\mathbf{L}}_s$ [2 3 1 1 2 2 1]

A static assignment model cannot capture the dynamics over time, even not mention the departure dynamics over time which has significant influence on network performance. A static network design approach only attempts to utilize the physical capacity, not the capacity over time. Utilizing network capacity over the time dimension can reduce congestions, thus result in a better network performance. That's the reason why some research has suggested that a wider distribution of PAT from the employer may help alleviate the congestions (Arnott *et al.* 2005). A wider distribution of PAT will lead to relatively separate departures from different OD pairs, which has been shown to lead to lower network costs and better network performances.

For comparison, Figure 7.8 and Figure 7.9 present the total network travel time distributions based on the dynamic assignment caused by the stochastic link capacities with the design solutions $\overline{\mathbf{L}}_d$ and $\overline{\mathbf{L}}_s$. It is found that with the dynamic solution $\overline{\mathbf{L}}_d$, the total network travel time at long term equilibrium is narrowly distributed, while with static solution $\overline{\mathbf{L}}_s$ the total network travel time exhibit much larger variability. It means the design $\overline{\mathbf{L}}_d$ is much more reliable than $\overline{\mathbf{L}}_s$. In this sense also a static network design approach may lead to a poor network design.

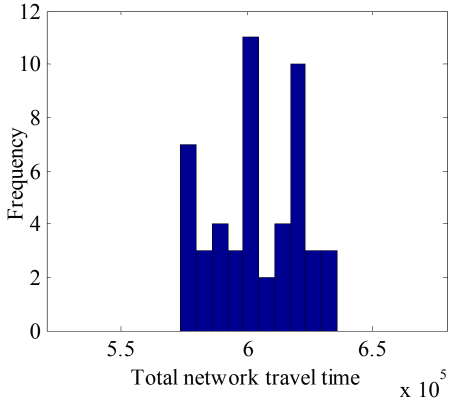


Figure 7.8: Total network travel time distribution with the dynamic solution $\bar{\mathbf{L}}_d$ based on the dynamic assignment

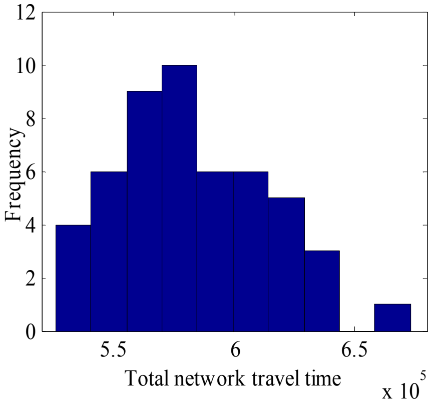


Figure 7.9: Total network travel time distribution with the static solution $\bar{\mathbf{L}}_s$ based on the dynamic assignment

Figure 7.10 presents the network performances in relation to construction costs with dynamic networks based on the evaluation of the 100 designs. Since these designs are the best designs from static network design approach, they have similar construction costs. The spots are widely scattered. The same construction cost may lead to very different network performance in terms of mean+std of TNT and total network travel costs. Higher construction cost may lead to worse network designs than with lower construction costs.

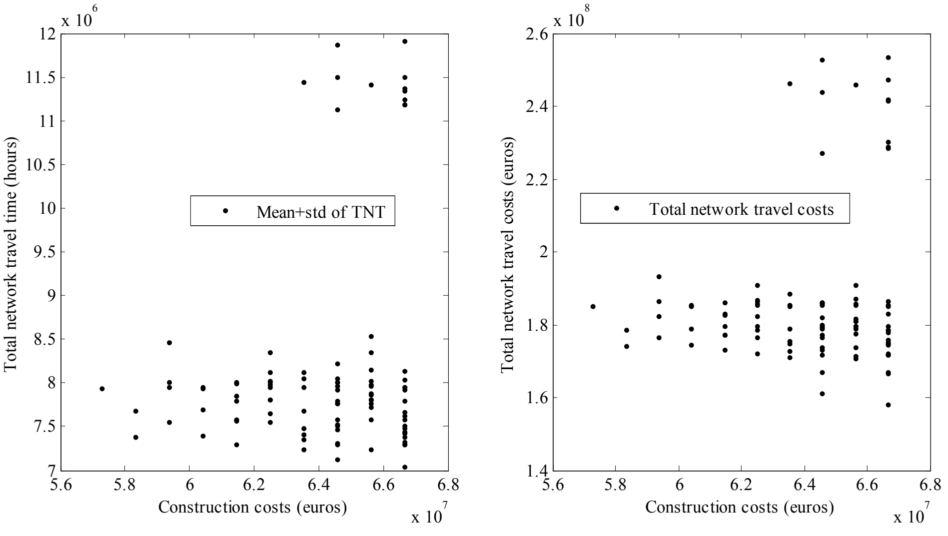


Figure 7.10: Dynamic network performance in relation to construction costs with a set size of 100

From the evaluated 100 designs, 183 pairs of designs are found having only one-lane difference in the network structure at any location. Figure 7.11 shows the network performance differences of the 183 pairs.

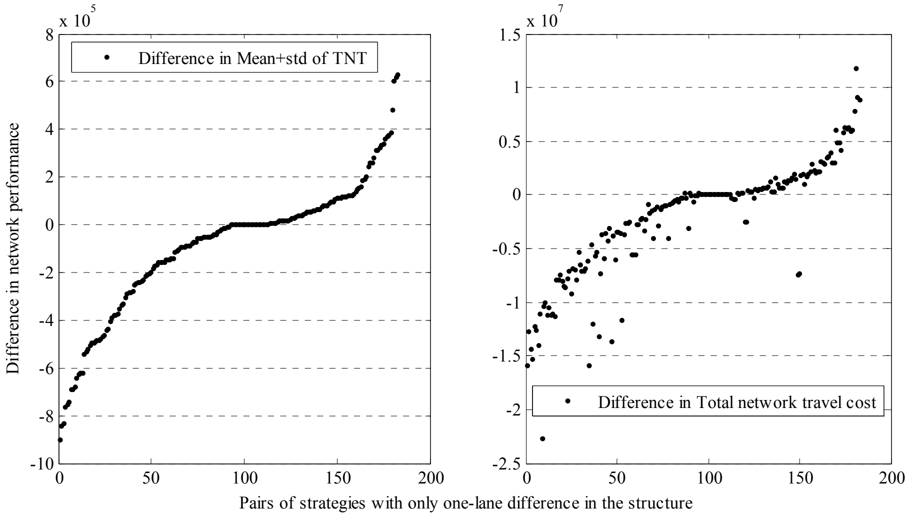


Figure 7.11: Network performance differences with adding one extra lane

Negative differences in network performance in Figure 7.11 denote that adding one extra lane improves the network performances. About 63% of the pairs have negative differences in total network costs while 61% in mean+std of the total network travel time. About 37% and 39% of the pairs have positive differences in the total network travel costs and mean+std of total network travel time respectively, showing that network performance is worse by adding one extra lane in the network. This shows that the Breass paradox thus also appears in the reliability-based travel cost function in dynamic networks.

7.7 Design Solution in Case of SN-DDSUEH with High Capacity Variations

This subsection presents the design solution from the proposed reliability-based dynamic road network design approach with level 20% capacity variations, compared with 10% variations in previous section.

Again 100 top designs from the static design approach with 20% capacity variations are selected and evaluated by the reliability-based dynamic assignment. Table 7.10 presents the network travel time and network travel costs by components of the top 10 designs.

A different design solution $\overline{\mathbf{L}}_d$ of [3 2 1 3 2 1 1] is derived from that with dynamic network design with 10% capacity variations.

Still the two objective functions lead to the same design solution. Compared to the evaluations with 10% capacity variations (see Table 7.9), the expectations of the total network travel times are similar. While the standard deviations of total network travel time increases with a Coefficient of Variation (CoV) of about 7-8%.

Table 7.10: Dynamic design evaluations with level 20% capacity variations on a yearly basis

Lane designs	Weighted mean+std of TNT (Hours/year)	Total travel costs (Millions of Euros/year)	Mean of TNT (Hours/ year)	Std of TNT (Hours/ year)	Total travel time costs (Millions of Euros/year)	Total reliability costs (Millions of Euros/year)	Construction cost (Millions of Euros/year)
[3 2 1 3 2 1 1]	7176719	171.79	11432486	793067	114.32	16.43	66.68
[2 3 2 2 2 1 1]	7368276	174.99	11748856	797406	117.49	16.33	66.68
[2 3 1 3 2 1 1]	7508894	178.11	11986206	**792927**	119.86	16.25	66.68
[3 2 2 2 2 1 1]	7587431	177.74	12060172	878320	120.60	16.84	66.68
[2 3 1 2 2 1 1]	7698630	180.80	12270011	841559	122.70	17.32	64.59
[3 2 1 2 2 1 1]	7701104	181.13	12238948	894339	122.39	18.65	64.59
[3 2 1 1 2 1 2]	7730374	182.89	12201332	1023937	122.01	19.29	65.63
[2 2 1 1 3 3 1]	7867659	196.16	12237819	1312418	122.38	22.29	65.63
[3 2 3 1 2 1 1]	7933741	188.56	12360740	1293243	123.61	22.89	66.68
[2 2 1 1 3 2 2]	7944800	197.56	12389640	1277539	123.90	21.86	65.63

It can be seen that the mean of the total network travel time is equal to the sum of all flow weighted mean of all route travel times at all departure time intervals (due to the mean of TNT equals the total travel time costs divided by VOT=10euros/hour). However the variance of the total network travel time is not simply a sum of all flow weighted route travel time variances at all departure time intervals, which could be a consequence of the correlations of travel times among routes and time intervals.

Table 7.11 collects the design solutions and the most reliable designs from static and dynamic network design approaches with difference capacity variations.

Table 7.11: Overview of the design solutions and the most reliable designs from different design approaches

Static network design approach	10% capacity variations	Design solution	[2 2 2 2 3 2 1]
		Most reliable design	[2 2 2 2 3 2 1]
	20% capacity variations	Design solution	[2 2 2 2 3 2 1]
		Most reliable design	[2 2 2 2 3 2 1]
Dynamic network design approach	10% capacity variations	Design solution	[3 2 2 2 2 1 1]
		Most reliable design	[3 2 1 2 2 1 1]
	20% capacity variations	Design solution	[3 2 1 3 2 1 1]
		Most reliable design	[2 3 1 3 2 1 1]

Based on the comparisons, different design solutions are derived from the static and dynamic network design approaches. Different capacity variations also lead to different design solutions, which implies that modeling travel time reliability in traveler’s choice behavior has significant impacts on the departure pattern and the network performance evaluations. The design solution is very sensitive to the approach adopted. The knowledge of the capacity distributions is very important for network designs.

Figure 7.12 presents the departure pattern with the design solution from the 10% capacity variation case [3 2 2 2 2 1 1], in case of 20% capacity variations are modeled. Compared to Figure 7.6, it can be seen that with larger capacity variations and larger travel time unreliability, travelers attempt to depart much earlier than that with 10% capacity variations.

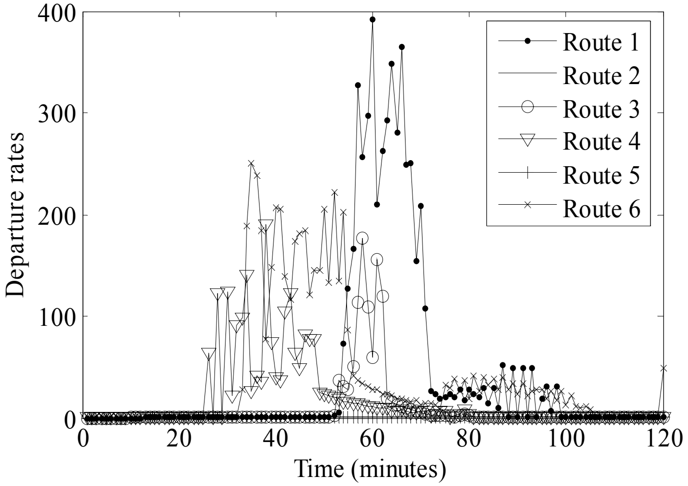


Figure 7.12: Equilibrium departure patterns with design [3 2 2 2 2 1 1] with PAT= 110

Again it is concluded that modeling travel time reliability in traveler's choice behavior has a significant influence on the departure pattern and network performance evaluations, thus the network designs.

Figure 7.13 presents the dynamic network performances in relation to the construction costs in the case of 20% capacity variations for all the 100 designs.

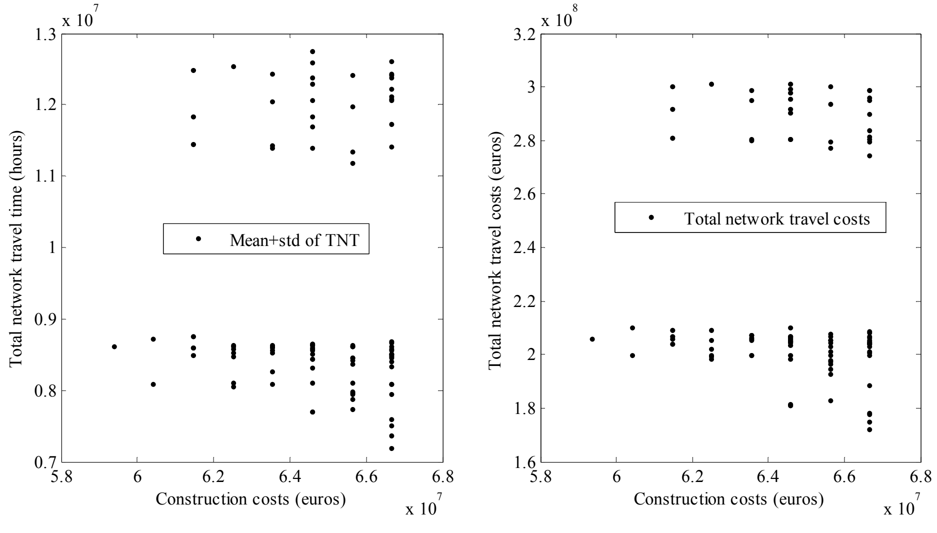


Figure 7.13: Dynamic network performances in relation to construction costs

Due to the fact that spots are widely scattered, the same construction cost may lead to different network performances. In addition, higher construction cost does not necessarily mean an improvement in the network performance. The appropriate allocation of the construction cost and capacity distribution over space will make the maximum utilization of the infrastructure, even over time dimension.

Figure 7.14 presents the frequency distribution of the network performance changes by adding one extra lane in the network at any location, based on the evaluations of the top 100 designs selected from the static network design with 20% capacity variations. Negative difference means that adding one lane decreases costs, i.e. improves network performance. It can be seen that both negative and positive network performance changes are observed. Adding lanes in the network sometimes deteriorates the network performances. This again is an instance of Braess paradox in case of reliability-based cost functions in a dynamic network.

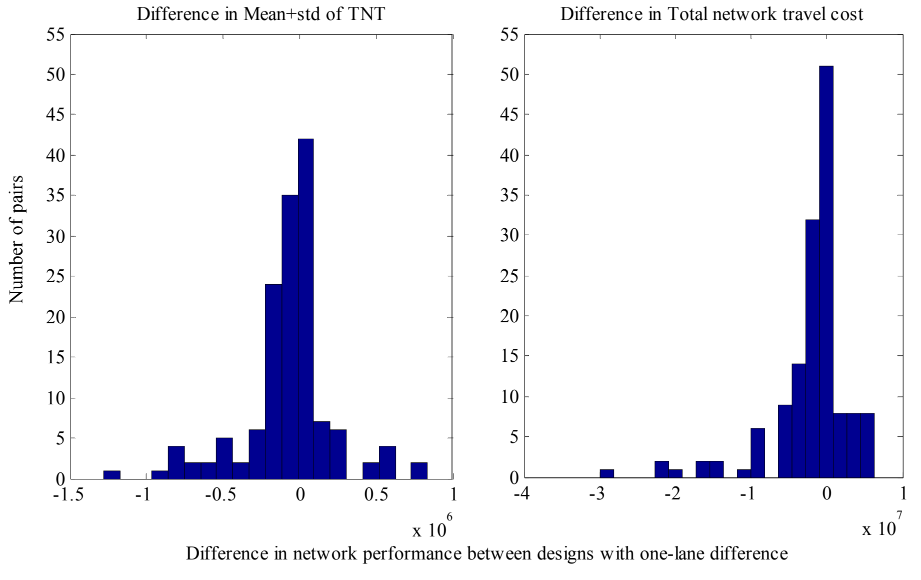


Figure 7.14: Frequency distribution of network performance changes by adding one extra lane at any location in the network

Figure 7.15 presents the network performance elasticity of construction costs computed from the pairs of designs with one lane difference in the network structure. Large absolute values of elasticity imply that a change in the construction cost leads to significant changes in the network performances. Elasticity around zero means the investment is not efficient, which from the figure occurs with a large frequency. In most cases the investment is not efficient.

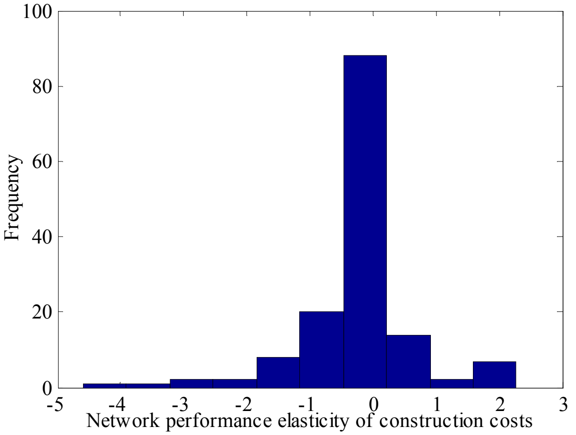


Figure 7.15: Network performance elasticity of construction costs

7.8 Conclusions

This chapter applied the proposed reliability-based dynamic network design approach to a hypothetical network with 7 potential links. Different capacity covariance matrices are defined and tested. It is found that covariance of disturbances have significant influence on the network performance evaluation for a specific network design, while it is assumed that it does not influence the network design solution in case the same covariance matrix is adopted for the evaluation of all different designs.

The design solutions from a static network design approach with two levels of capacity variations are investigated and compared with that from dynamic network design approach with two levels of capacity variations.

It is found that a static network design may lead to a poor approximation of the network performances and to poor network designs. A static network design only aims to make the best utilization of the physical capacity, regardless of the utilization over time dimension based on unrealistic assumptions. The static assignment underestimates the total network travel time and the variability of total network travel time, thus underestimates the effectiveness of the investment.

Modeling departure time choice is crucial when dealing with the network design problem. A dynamic network design approach aims to derive a fairly good allocation of the capacity over space, at the same time making the best utilization of the network over time. Relatively separate departures among different OD pairs, as a result of the designs, lead to better network performances. This effect cannot be captured by static network design approach.

It is found that the two objective functions (minimize mean+std of total network travel time and minimize total network travel cost) lead to the same design solution with this small network. Different level of capacity variations leads to different design solutions, implying that modeling travel time reliability in traveler’s choice behavior has significant influence on the network flow patterns over space and time, thus on the network performance evaluations and the network design. The knowledge of the capacity distribution is important in network designs.

The experimental results also showed that the appearance of Braess paradoxes when applying reliability-based traffic flow assignments.

In conclusion, network designs should be based on the dynamic network modeling with considering travel time reliability in traveler’s choice behavior, especially departure time choice behavior. Such an approach offers in addition ample apportunities in designing dynamic link properties as part of a dynamic traffic management policy, such as dynamic lane configuration, dynamic speed limits during peak hours, dynamic toll design, etc.

Part V: Conclusions

8 Conclusions and Future Research

8.1 Introduction

This concluding chapter, as the epilogue of this dissertation, summarizes the research that has been performed, the approaches adopted to solve the research questions given in Chapter 1, and the main achievements. The findings and conclusions having their scientific and practical implications will be presented. Some recommendations and potential values of our research to the practitioners and decision makers are given. This chapter will discuss the general validity of our developed approaches and a few potential research subjects to enhance our methodology. Finally a few research topics which are of highly scientific interest will be given for future research.

8.2 Summary

Transportation systems are stochastic and dynamic systems. The road capacities and the travel demand are fluctuating from time to time within a day and at the same time from day to day. We focus on the two main stakeholders involved in the stochastic transportation systems: the road authorities or the designers, and the road users. For road users, the travel time and travel costs experienced over time and space are stochastic, thus desire reliable travel times and low travel costs by altering routes or departure times. Road authorities and designers aim to design road networks such that they provide efficient and reliable services to the road users. Traveler's choice behavior under stochastic travel times/costs and the design of the road networks are mutually affecting each other. While on the one hand travelers' choice behavior determines the network traffic situations and thus the network performances, the design of the road network influences the choice behaviors of travelers on the other hand.

Traveler's choice behavior under stochastic travel times, especially route/departure time choice behaviors has attracted increasing attentions in the last decade. However, in most studies on network design problems with stochastic capacity or travel demand, traveler's choice behavior, especially their departure time choice behavior under stochastic travel times are not considered. Instead, static traffic assignment is followed in most network design studies with stochastic travel times, which from our point of view is not realistic in terms of network performance assessments. This motivated our research direction. We aim to establish a network design approach with which dynamic traffic assignment is followed to capture the dynamics in flow propagations and spillback effects. Traveler's departure time/route choice behavior under stochastic travel times is explicitly modeled. Travel time reliability is taken into account on both the individual choice behavior and network performance evaluation with stochastic networks.

We started the research with modeling traveler's departure time and route choice behavior under uncertainty. In general two approaches are available to model traveler's choice behavior under uncertainty, namely the mean-variance and the scheduling approach. We theoretically investigate the relationship between these two approaches. We proved the principal equivalence of the mean-variance and the scheduling approach conditional to departure times. Through the mathematical derivations based on these two approaches, we propose a *generalized travel cost function* for modeling traveler's departure time/route choice behavior under uncertainty. The generalized travel cost function, compared to the scheduling approach, is more behaviorally sound (has face validity) and plausible. It is also more flexible to capture more complete risk aversion attitude. Compared to the mean-variance approach, the generalized travel cost function explicitly accounts for the schedule delay cost, which is very important in the timing of departures for travelers.

Compared to the dynamic user equilibrium without capturing traveler's choice behavior under uncertainty, we defined in the second part a type of long term dynamic user equilibrium with stochastic networks, at which no traveler can reduce his long term accumulated trip cost by unilaterally altering routes and departure times. The long term trip cost is formulated as the derived generalized travel cost function. We analytically derive the long term dynamic user equilibrium by extending Vickrey's bottleneck model to the stochastic capacity case and including travel time reliability in the travel cost function, given uniformly distributed link capacities with continuous time and continuous flows. Compared to the two-regime division with Vickrey's deterministic bottleneck model, our analyses with stochastic capacities adopt a three-regime division in the analyzed time period. We compared the equilibrium departure patterns with and without modeling traveler's choice behavior under uncertainty. The long term equilibrium travel costs at all departure time instants t by component are derived analytically and presented graphically.

Due to the mathematical complexity inherent of the theoretical approach of deriving the long term dynamic user equilibrium with stochastic networks, we established a comprehensive dynamic network assignment procedure with discrete time and continuous flows. This procedure is able among other matters to capture the correlated stochastic link capacities and traveler's departure time/route choice behavior under uncertainty. We established a new method to generate correlated link capacities realistically by using the Cholesky decomposition with predefined link capacity covariance matrix.

We defined and formulated four different user equilibrium models with their assumptions and specifications for different comparative purposes. These models are distinguished by whether deterministic or stochastic capacities, deterministic or probabilistic user equilibrium, static or dynamic flows, and vertical or horizontal queues are modeled. The established dynamic network assignment procedure is capable of deriving the four different user equilibria with stochastic capacities.

In the third part we established a reliability-based dynamic discrete network design methodology under stochastic capacities. Its purpose is to design completely new (from scratch) or redesign existing networks given a policy objective and policy constraints. Travel time reliability is considered on both the network design level and individual choice behavior. Dynamic network modeling with joint departure time/route choice is adopted. The design variables are the network structure and numbers of lanes on a set of potential links in the network. The design problem aims to minimize a weighted network inefficiency and network unreliability.

We developed a combined network-oriented genetic and set evaluation approach to solve the reliability-based dynamic discrete network design problem. A new concept of the universal set and the master set of lane designs is defined. We established a new systematic approach to derive the master set by eliminating the infeasible, unconnected and illogical lane designs in order to reduce the solution space and to save computation time. We adopted an example grid network to demonstrate the notions of the infeasible, unconnected and illogical lane designs and the process of crossover and mutation. The example network gives a general impression on the relative sizes of the universal set and the master set.

We investigated the feasibility and the merits of the established dynamic network design approach by a case study on a 7-link hypothetical road network. We compared the design solutions from the reliability-based dynamic and static network design approaches with different capacity variation levels. The influence of two different objective functions on the design solutions has been investigated. We tested the network performances with different link capacity covariance matrices to get insight into the influence of link correlations on network performances.

8.3 Findings and Conclusions

After the summary of the research and the main methodological achievements given in the previous section, this section summarizes the important findings and main conclusions which can be useful for other researchers and meaningful in both practical and scientific aspects.

This section is organized following the structure of the dissertation and presents the findings and conclusions for each group sequentially.

8.3.1 Modeling traveler's departure time/route choice behavior under uncertainty

From the theoretical derivations, we established that the scheduling approach and the mean-variance approach are equivalent, conditional to departure times. We showed that the expectation of the schedule delay costs at any departure time instant t can be decomposed into a linear function of the expected travel time and standard deviation of the travel time at departure time instant t in case of assuming linearity in schedule delay costs. We proved analytically that the scheduling approach indeed already represents traveler's risk aversion attitudes.

The proposed generalized travel cost function, from its mathematical implications, is more behaviorally sound (has face validity) and more plausible compared to the scheduling approach. The reason is that with the scheduling approach, the expectations of schedule delay early and late are both positive, implying that travelers would experience schedule delay early and late at the same trip and will not arrive at destination on time, which is not realistic. Whereas with the generalized travel cost function, travelers will expect to be either early, late or on time based on the expected travel times.

Since the scheduling approach only partly captures the risk aversion attitudes based on others' research, the generalized travel cost function is more flexible in terms of capturing a more complete risk aversion attitude caused by travel time uncertainty.

The generalized travel cost function, compared to the mean-variance approach, accounts for the schedule delay costs explicitly. The scheduling approach and the mean-variance approach are special cases of our generalized mean-variance scheduling approach.

With regard to quadratic schedule delay cost functions, we showed that the expectation of the quadratic schedule delay cost when departing at any time instant t can be decomposed into a quadratic function of the schedule delay cost based on the expected travel time when departing at time instant t and the standard deviation of the travel time when departing at time instant t.

We claim that the cost parameters in this cost function have a two dimensional meaning, where one dimension reflects the heterogeneity of travelers in terms of taste variations, while the other depends on the prevailing attribute values.

8.3.2 Dynamic user equilibrium under uncertainty

We showed that the equilibrium departure pattern under uncertainty is significantly different from the deterministic capacity case. The larger the capacity variation is, the more significant the difference will be from the departure pattern in the deterministic capacity case. Consideration of random capacities and travel time reliability yields systematic shifts towards earlier departures. The later a traveler departs in the congested period, the larger the travel time variability he/she might encounter between days. This finding may be interpreted as a reason why travelers attempt to depart early to reduce their travel time variability in case of stochastic travel costs.

We established analytically that the standard deviation of travel time under congested conditions is not proportional to the expectation of travel time, as most sources assume to be the case.

8.3.3 Dynamic network assignment procedures for stochastic networks

By applying the simulation-based dynamic network modeling approach to a single bottleneck with deterministic capacity, we showed that the results (i.e. equilibrium departure pattern and equilibrium travel costs by component) from our established simulation-based approach are consistent with the analytical results from Vickrey's deterministic bottleneck model, which validates our established simulation-based dynamic network modeling approach.

By applying the simulation-based dynamic network modeling approach to a single bottleneck with stochastic capacities, we showed that results from our established simulation-based approach for stochastic capacities are consistent to that from our analytical derivations with stochastic bottleneck model, which validates our theoretical findings on the dynamic user equilibrium with stochastic capacities.

8.3.4 A reliability-based dynamic network design methodology

The developed combined genetic and set evaluation approach for solving dynamic network design problem appears to be a feasible approach. It is applicable for general networks.

The application of the developed systematic approach for deriving the master set of lane designs on an example grid network showed that the solution space for a network design problem can be reduced considerably by eliminating the infeasible, unconnected and illogical lane designs.

The established reliability-based dynamic network design methodology is applied to a hypothetical road network and appears to be feasible. By comparing our dynamic network design approach to the static network design approach, it is found that static traffic assignment significantly underestimates the total network travel time and the variability of total network travel time compared to the dynamic traffic assignment, thus also underestimates the value of the investments in reducing these network costs. The static and dynamic network design approaches lead to significantly different design solutions. Since static traffic assignment cannot capture the dynamics in flow propagation and spillback effects, a dynamic traffic assignment is more realistic. So is the dynamic network design approach. We found that the static network design approach may lead to poor designs compared to the dynamic network design approach.

It is found with the small network that minimizing total network travel cost leads to the same design solutions as from minimizing mean plus standard deviation of total network travel times. It is expected to be the case in general, since the network travel time cost is dominating to other network costs.

We showed that the same construction cost budget may lead to significantly different design solutions with highly different network performances. Larger construction cost does not mean better network performances. It is shown that adding one extra lane sometimes deteriorates the network performances. This is a new evidence of the Braess paradox appearing in reliability-based static and dynamic networks. The marginal benefit of the investment is very small when the investment reaches a certain amount, which is quite common.

The results from the hypothetical network exercise showed that modeling departure time choice plays a very crucial role in network designs. Different designs lead to different departure patterns, which have significant impacts on the network performances. Our dynamic network design approach with departure time choice modeling results in a lane design which gives the best allocation of the capacity distribution over space given the construction budget. In addition, it gives the maximum utilization of the link capacities over time. Static network design approach however cannot capture these dynamics in departures, nor can make use of the time dimension.

By comparing the design solutions from our dynamic network design approach with different capacity variation levels, we conclude that modeling traveler's choice behavior under uncertainty has significant impacts on the departure patterns and network performance evaluations, thus on the network designs.

We applied the Cholesky decomposition to generate realistically correlated link capacities with different link capacity covariance matrices in the same network. We showed that considering link capacity correlations significantly influences the network performance evaluations. A fully positive correlated link capacity covariance matrix outperforms other kinds of matrices by having less total network travel cost and less total network travel time on average.

8.4 Recommendations and values to practical applications

The research in this dissertation and its findings and conclusions has potential values to practitioners and decision makers.

Based on the research, dynamic network design approach is recommended to the road authority/designers instead of current static network design approach, since static traffic assignment leads to poor evaluations of the network performance, thus to poor network designs.

Modeling traveler's choices under uncertainty has significant influences on the network equilibrium performances. It is recommended that network designs should be based on reliability-based choice modeling, especially departure time choices taking into account the impact of the stochastic nature of the transport systems.

The generalized travel cost function for modeling traveler's departure time and route choices under uncertainty can be used in general network modeling with stochastic networks and fluctuating travel demand.

The established dynamic network modeling procedure, which captures the stochastic properties of the road networks and the consideration of this by travelers, can be directly used to assess road network performances and dynamic traffic management measures, for instance dynamic speed limit, dynamic lane configurations, etc.

The established reliability-based dynamic road network design approach can help road authorities to optimize their network designs such that network efficiency, network reliability and the utilization of the investment are optimized with realistic travelers' choice modeling under uncertainty.

The dynamic network design approach proposed in this dissertation offers in addition ample apportunities in designing dynamic link properties as part of a dynamic traffic management policy, such as dynamic lane configuration design, dynamic speed limit design during peak hours, dynamic toll design, etc., considering stochastic networks.

The systematic approach of filtering the design solution space by eliminating the infeasible, unconnected, and illogical designs can be directly used by the road designers to save the computation time for solving the design problem, irrespective to the design approaches (static or dynamic).

8.5 Discussion

Our established reliability-based dynamic discrete network design methodology is a comprehensive and realistic network design approach compared to other existing network design methodologies in the literatures. It takes into account stochastic natures of the road networks and explicitly models traveler's departure time/route choice behavior under uncertainty. Correlated link capacities can be modeled quite flexibly according to the link capacity covariance matrix. Travel time reliability is considered on both the network design level and individual choice behavior. Our methodology is capable of designing network structures and numbers of lanes on a set of potential links by making the best utilizations of the construction cost budget and the network capacities over time dimension. It also offers ample opportunities in the context of dynamic traffic management such as with respect to dynamic lane configuration, dynamic speed limits, and dynamic toll designs.

However, given a number of assumptions adopted, our methodology might be enhanced and be more complete by performing more research in several ways. Firstly our generalized travel cost function for modeling travelers' choice behavior under uncertainty so far has been derived based on mathematical derivations. Since the scheduling approach and the mean-variance approach are widely used and empirical studies have been carried out to calibrate and validate these two approaches, and since these two approaches are special cases of our generalized travel cost function, it is logical that our generalized travel cost function represents travelers' choice behavior under uncertainty. To be more convincing, empirical analyses are required to testify the generalized travel cost function.

Secondly, in our dissertation the cost parameter values in the travel cost function are selected based on the empirical studies in the literature which however show different values. Adopting such cost parameters will influence the equilibrium departure patterns and route flow patterns, thus the network designs. Performing sensitivity analyses of the cost parameter values helps generalize our network design methodology. The analytical approach can be adoped to study the influence of the parameters α, β, γ on the equilibrium departure patterns and travel costs.

Thirdly, we assumed homogeneous travelers in our research having the same travel cost functions and an identical preferred arrival time. To be more realistic, heterogeneous travelers should be modeled. Taste diversity and preferred arrival time distribution in the population will lead to much wider departure patterns than derived in the dissertation. Modeling heterogeneous travelers will lead to different network equilibrium, or to more spread departure patterns over the time period, which influences the network performance evaluations and thus the network design solutions. Our methodology allows to be extended by modeling heterogeneous travelers.

Fourthly, the number of Halton draws for representing the random capacity distributions might influence the equilibrium flow patterns. It is necessary to investigate the influence of the number of Halton draws on the network designs and to determine the appropriate number for the required Halton draws.

With our established network assignment procedure for stochastic networks, it is very flexible and feasible to adopt different reliability measures in both choice behavioral modeling and network design objectives. Some analyses can be carried out with different reliability measures to investigate the influence of adopted reliability measures on the equilibrium flow patterns and network designs.

The case study in our dissertation is performed given a certain link covariance matrix. The influence of different link capacity covariance matrices on the network design solutions can be investigated further.

8.6 Future Research

We deliberately confined our research to deterministic travel demand with only stochastic capacity variation as a source for travel time/cost uncertainty. Next to capacity, demand fluctuation however is another major factor leading to stochastic travel times. As discussed in the dissertation, modeling demand fluctuations together with stochastic network capacities will lead to larger travel time variability, which will lead to even earlier departures. It is expected that different network design solutions will be derived. It is very important to establish a more refined definition of the long term dynamic user equilibrium with departure time choice in case of stochastic demand as well. We need the formulation of the dynamic user equilibrium with departure time choice for stochastic capacity and stochastic demand.

The convergence of the dynamic traffic assignment is paramount in dynamic network design problems. Faster convergence saves considerable computation time for solving the dynamic network design problems and enlarges the scalability of the road networks to be designed. It will contribute substantially to develop new and efficient algorithms for the convergence of the dynamic traffic assignment, with departure time choice.

Our established network design methodology so far has been only applied to a hypothetical network. It is very important and meaningful to apply our methodology to real-sized realistic road networks in order to show the feasibility of our network design methodology for real and practical uses.

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Appendix A: Detailed Derivations of the Analytical Analyses in Stochastic Bottleneck Model

The analytical analyses in stochastic bottleneck model are quite complicated. In the dissertation, the detailed derivations are not presented. Therefore, this appendix provides more detailed derivations for readers who are interested.

In Chapter 3, Formulae (3.35)-(3.42) provide the cost components for the travel cost function (3.8) for all three regimes.

We start with the derivations of departure flow rates in the first and second regimes. The long term travel costs for regimes I and II are given as:

$$\begin{aligned} c(t) = {} & \alpha E\left(\frac{1}{\tilde{v}(t)}\right) D_1^*(t) - \alpha\left(t - t_0^*\right) + \beta \cdot \chi_C \cdot D_1^*(t) \\ & + \gamma_1 \cdot \left(PAT - \left(t + E\left(\frac{1}{\tilde{v}(t)}\right) D_1^*(t) - \left(t - t_0^*\right)\right)\right), \quad t_0^* \le t \le t_t^* \end{aligned} \tag{A.1}$$

$$\begin{aligned} c(t) = {} & \alpha E\left(\frac{1}{\tilde{v}(t)}\right) D_2^*(t) + \alpha E\left(\frac{1}{\tilde{v}(t)}\right) D_1^*\left(t_t^*\right) - \alpha\left(t - t_0^*\right) + \beta \cdot \chi_C \cdot \left(D_1^*\left(t_t^*\right) + D_2^*(t)\right) \\ & + \gamma_2 \left(\left(t + E\left(\frac{1}{\tilde{v}(t)}\right) D_2^*(t) + E\left(\frac{1}{\tilde{v}(t)}\right) D_1^*\left(t_t^*\right) - \left(t - t_0^*\right)\right) - PAT\right), \quad t_t^* \le t \le t_c^* \end{aligned} \tag{A.2}$$

According to the definition of user equilibrium, we have $dc^* / dt = 0$. By calculating the differential equations with the travel cost functions given in Equation (A.1) and (A.2), we derive:

$$\frac{\alpha}{C_{\max} f_\theta}\frac{dD_1^*}{dt}-\alpha+\beta\cdot\chi_C\cdot\frac{dD_1^*}{dt}+\gamma_1\cdot\left(-1-\frac{1}{C_{\max} f_\theta}\frac{dD_1^*}{dt}+1\right)=0,\ \ t_0^*\le t\le t_t^* \tag{A.3}$$

$$\frac{\alpha}{C_{\max} f_\theta}\frac{dD_2^*}{dt}-\alpha+\beta\cdot\chi_C\cdot\frac{dD_2^*}{dt}+\gamma_2\left(1+\frac{1}{C_{\max} f_\theta}\frac{dD_2^*}{dt}-1\right)=0,\ t_t^*\le t\le t_c^* \tag{A.4}$$

where $\chi_C=\frac{1}{C_{\max}}\sqrt{\left(\frac{1}{\theta}-\frac{1}{f_\theta^2}\right)}$.

Due to

$$\frac{dD_1^*}{dt}=r_1^*(t)\quad\text{and}\quad\frac{dD_2^*}{dt}=r_2^*(t), \tag{A.5}$$

From Equations (A.3)-(A.5), we can derive the departure flow rates for regimes I and II, as given in Equations (3.43) and (3.44) in Chapter 3.

Now we discuss the derivation of the departure flow rate $r_3^*(t)$ in regime III. The travel cost function in regime III is given as:

$$\begin{aligned}
c(t)&=\alpha E\left(\tilde\tau(t)\right)+\beta Std\left(\tilde\tau(t)\right)+\gamma_2\left(t+E\left(\tilde\tau(t)\right)-PAT\right),\qquad t_c^*\le t\le t_e^*\\
&=\left(\alpha+\gamma_2\right)E\left(\tilde\tau(t)\right)+\beta Std\left(\tilde\tau(t)\right)+\gamma_2 t-\gamma_2\cdot PAT\\
&=\left(\alpha+\gamma_2\right)\left(\frac{D_T^*(t)}{C_{\max}(1-\theta)}\ln\frac{D_T^*(t)}{C_{\min}\left(t-t_0^*\right)}-\frac{D_T^*(t)-C_{\min}\left(t-t_0^*\right)}{C_{\max}(1-\theta)}\right)+\gamma_2 t-\gamma_2\cdot PAT\\
&\quad+\beta\sqrt{\begin{aligned}&\left[\frac{\left(D_T^*(t)-C_{\min}\left(t-t_0^*\right)\right)\left(C_{\max}(1-\theta)\left(t-t_0^*\right)-\left(D_T^*(t)-C_{\min}\left(t-t_0^*\right)\right)\right)}{\left(C_{\max}(1-\theta)\left(t-t_0^*\right)\right)^2}\right]\\
&\cdot\left[\left(t-t_0^*\right)^2-2D_T^*(t)\frac{\left(t-t_0^*\right)^2}{D_T^*(t)-C_{\min}\left(t-t_0^*\right)}\ln\frac{D_T^*(t)}{C_{\min}\left(t-t_0^*\right)}\right]\\
&+D_T^{*2}(t)\left[\begin{aligned}&\frac{D_T^*(t)-C_{\min}\left(t-t_0^*\right)}{C_{\max}(1-\theta)}\frac{1}{C_{\min}D_T^*(t)}\\&-\left(\frac{D_T^*(t)-C_{\min}\left(t-t_0^*\right)}{C_{\max}(1-\theta)\left(t-t_0^*\right)}\right)^2\frac{1}{\left(C_b(t)-C_{\min}\right)^2}\ln^2\frac{C_b(t)}{C_{\min}}\end{aligned}\right]\end{aligned}}
\end{aligned} \tag{A.6}$$

where $C_b(t)$ and $D_T^*(t)$ are defined in Equation (3.19) in Chapter 3 with $D_T^*(t)=D_1^*\left(t_t^*\right)+D_2^*\left(t_c^*\right)+D_3^*(t)$.

The differential of the Expression (A.6) is very complicated. There is no closed form solution for $r_3^*(t)$ by solving $dc^*/dt=0$ for regime III. Therefore, we adopted the finite difference method to obtain the numerical solution for this stochastic bottleneck model. We define:

$$f(t)=D_3^*(t) \tag{A.7}$$

$$\frac{dD_3^*}{dt}=\frac{f(t+\Delta)-f(t)}{\Delta} \tag{A.8}$$

$$A=D_1^*(t_t^*)+D_2^*(t_c^*), \tag{A.9}$$

The differential of the travel cost in regime III is given as:

$$\begin{aligned}
\frac{dc^*(t)}{dt}&=(\alpha+\gamma_2)\frac{dE(\tilde{\tau}(t))}{dt}+\beta\frac{dStd(\tilde{\tau}(t))}{dt}+\gamma_2, \qquad t_c^*\le t\le t_e^*\\
&=\frac{\alpha+\gamma_2}{C_{\max}(1-\theta)}\left(\frac{f(t+\Delta)-f(t)}{\Delta}\cdot\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}-\frac{(A+f(t))}{(t-t_0^*)}\right)+\frac{(\alpha+\gamma_2)\theta}{(1-\theta)}+\gamma_2\\
&+0.5\cdot\beta\cdot x/\sqrt{\begin{aligned}&\left[\frac{(t-t_0^*)(A+f(t))}{C_{\max}(1-\theta)}-\frac{\left(A+f(t)-C_{\min}(t-t_0^*)\right)^2}{\left(C_{\max}(1-\theta)\right)^2}-\frac{2(A+f(t))(t-t_0^*)}{C_{\max}(1-\theta)}\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}\right.\\
&\left.-\frac{\theta(t-t_0^*)^2}{(1-\theta)}+\frac{2(A+f(t))^2}{\left(C_{\max}(1-\theta)\right)^2}\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}-\frac{2(A+f(t))\theta(t-t_0^*)}{C_{\max}(1-\theta)^2}\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}\right]\\
&+\frac{(A+f(t))^2}{C_{\max}{}^2\theta(1-\theta)}-\frac{(t-t_0^*)(A+f(t))}{C_{\max}(1-\theta)}-\frac{(A+f(t))^2}{C^2{}_{\max}(1-\theta)^2}\ln^2\frac{A+f(t)}{C_{\min}(t-t_0^*)}\end{aligned}}
\end{aligned} \tag{A.10}$$

where

$$\begin{aligned}
x=&\frac{2(1+\theta)(A+f(t))}{C_{\max}(1-\theta)^2}-\frac{2\theta(t-t_0^*)}{(1-\theta)^2}-\frac{2(t-t_0^*)}{C_{\max}(1-\theta)}\frac{f(t+\Delta)-f(t)}{\Delta}-\frac{2(t-t_0^*)}{C_{\max}(1-\theta)^2}\frac{f(t+\Delta)-f(t)}{\Delta}\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}\\
&-\frac{2(A+f(t))}{C_{\max}(1-\theta)}\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}-\frac{2\theta(A+f(t))}{C_{\max}(1-\theta)^2}\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}+\frac{4(A+f(t))}{\left(C_{\max}(1-\theta)\right)^2}\frac{f(t+\Delta)-f(t)}{\Delta}\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}\\
&-\frac{2(A+f(t))}{C_{\max}{}^2(1-\theta)^2}\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}\frac{f(t+\Delta)-f(t)}{\Delta}-\frac{2(A+f(t))}{C_{\max}{}^2(1-\theta)^2}\frac{f(t+\Delta)-f(t)}{\Delta}\ln^2\frac{A+f(t)}{C_{\min}(t-t_0^*)}\\
&+\frac{2(A+f(t))}{C_{\max}{}^2\theta(1-\theta)}\frac{f(t+\Delta)-f(t)}{\Delta}-\frac{2(A+f(t))^2}{\left(C_{\max}(1-\theta)\right)^2(t-t_0^*)}+\frac{2(A+f(t))^2}{C_{\max}{}^2(1-\theta)^2(t-t_0^*)}\ln\frac{A+f(t)}{C_{\min}(t-t_0^*)}
\end{aligned} \tag{A.11}$$

Solving the equation $dc^*/dt=0$, we derive $f(t+\Delta)$ as a function of $f(t)$:

$$f(t+\Delta)=g(f(t)), \tag{A.12}$$

We firstly initialize t_0^*, then we can derive t_t^*, t_c^* and A in Equations (A.10) and (A.11). The point at (t_c^*, A) is the initial point of the $D_3^*(t)$ curve. We choose a small Δ=0.01. The smaller the Δ value is, the more accurate the result will be. With the derived Equation (A.12) and the initial point, we can gradually get the numerical coordinates of the points on the $D_3^*(t)$ curve. The calculation will stop when the accumulative number of departures reaches N, from which t_e^* is derived. However, most of the initial values of t_0^* cannot lead to a solution which satisfies the Equation (3.50) in Chapter 3. Since the $D_3^*(t)$ curve derived from the differential equation in regime III is concave, which means the departure flow rate $r_3^*(t)$ decreases with the departure times. Therefore $r_1^*(t) > r_2^*(t) > r_3^*(t)$. The maximum values of $D_3^*(t)$ with different initial values of t_0^* sometimes cannot reach N. Only a few values of t_0^* satisfies the Equation (3.50) in Chapter 3. Then we choose the value of t_0^* which gives the minimum long term equilibrium costs. The stochastic bottleneck problem is solved.

Appendix B: Analytical Results on Stochastic Bottleneck Model with Scales

Since the analytical results presented in Chapter 3 hold for general stochastic bottleneck models, the graphical results are given without scales. However, for a comparison of the analytical and the simulation-based results, an example stochastic bottleneck is adopted as seen in Chapter 4. It is important to have scales in the graphical presentations. Therefore, in this appendix we provide graphical results of the equilibrium departures and equilibrium travel costs with scales from the analytical approaches, which facilitates the comparisons of Figures 3.5, 3.7 in Chapter 3 and Figures 4.5, 4.6 in Chapter 4.

Figure B.1 and Figure B.2 present the cumulative departures and long term equilibrium costs respectively for the stochastic bottleneck from the analytical approach with scales.

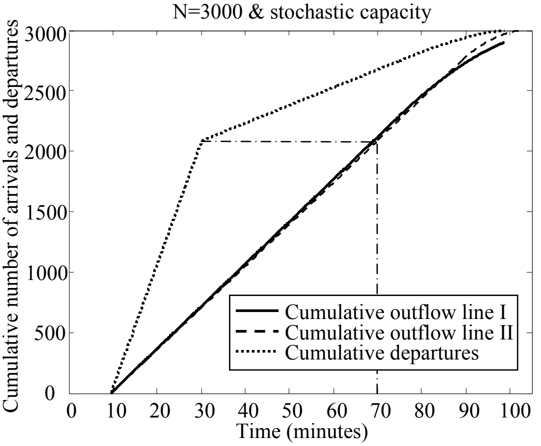

Figure B.1: Cumulative departures and two categories of cumulative outflow lines at the long term equilibrium from the analytical approach

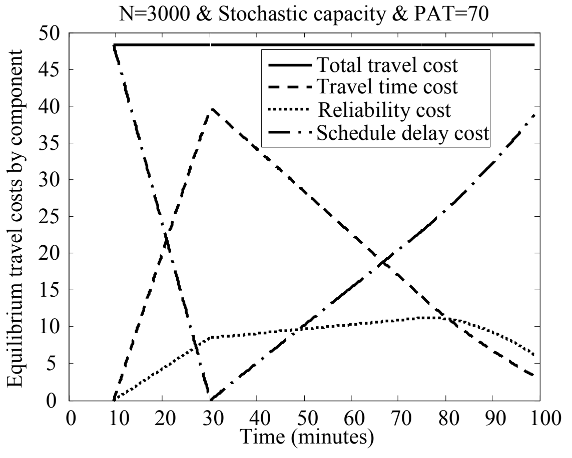

Figure B.2: Long term equilibrium costs by component from the analytical approach

Summary

Transportation systems are stochastic systems where the road capacities and the travel demand are fluctuating within a day and between days. The traffic situations experienced by travelers are stochastic as well. It is known that the economy of a nation or region depends heavily upon an efficient and reliable transportation system to provide accessibility and to promote the safe and efficient movement of people and goods. In the context of varying capacities and travel demand, road authorities/designers need a network design approach which is able to capture the stochastic nature of the transport system, and the consideration of this by its users.

Despite the important role of uncertainty in travel decision making, in most studies of road network design problems considering stochastic capacities or travel demand, traveler's choice behavior under uncertainty, especially their departure time choices, are not modeled, which is however a very important aspect of travel choices that need to be modeled in the context of uncertainty. In addition, static traffic assignment is followed in most network design studies with stochastic travel times, which is not appropriate in terms of network performance assessments.

We established a comprehensive network design approach in which dynamic traffic assignment is adopted (named as dynamic network design approach) to capture the dynamics in departure time choice and in flow propagations and spillback effects. Stochastic capacities are modeled, with which traveler's departure time/route choices under stochastic travel times are explicitly modeled.

We started developing the dynamic network design approach with modeling traveler's departure time and route choices under uncertainty. Generally, two approaches are available, namely the mean-variance and the scheduling approaches. We analytically proved the principal equivalence of the mean-variance and the scheduling approaches conditional on the departure times. We showed that the expectation of the schedule delay costs at any departure time instant t can be decomposed into a linear function of the expected travel time and the

standard deviation of the travel time at departure time instant t in case of assuming linearity in schedule delay costs. Based on this result we proposed a *generalized* (*dis*)*utility function* for modeling traveler's departure time/route choices under uncertainty. The generalized utility function, from its mathematical implications, is more behaviorally sound (has face validity) compared to the scheduling approach. It is also more flexible to capture the risk aversion attitude. Compared to the mean-variance approach, the generalized utility function explicitly accounts for the schedule delay cost, which is very important in the timing of departures for travelers. The scheduling approach and the mean-variance approach are special cases of our generalized mean-variance scheduling approach.

A long term dynamic user equilibrium is defined by assuming that travelers make their strategic departure time and route choices in order to minimize their accumulated travel costs over a longer period which are defined by our generalized utility function.

We analytically derived the long term dynamic user equilibrium by extending Vickrey's bottleneck model to the stochastic capacity case and including travel time reliability in the travel cost function (i.e. disutility function), given uniformly distributed link capacities with continuous time and continuous flows. The analyses with stochastic capacities adopted a three-regime division in the analyzed time period due to different schedule delay cost functions and capacity distribution changes. We showed that the equilibrium departure pattern with modeling traveler's choices under uncertainty is significantly different from the deterministic capacity case. The larger the capacity variation is, the more significant the difference will be. Consideration of random capacities and travel time reliability in traveler's choices yields systematic shifts of trips towards earlier departures. The later a traveler departs in the congested period, the larger the travel time variability he/she might encounter between days. That's the reason why travelers attempt to depart early to reduce their travel time variability. We found that the standard deviation of travel time under congested conditions is not proportional to the expectation of travel time, as most sources assume to be.

We established a dynamic network assignment procedure with discrete time and continuous flows for modeling different user equilibria under stochastic capacities, for instance dynamic or static user equilibrium, deterministic or probabilistic user equilibrium, with or without reliability in the choices, dynamic user equilibrium with vertical or horizontal queues. This procedure is able to capture the correlated stochastic link capacities and traveler's departure time/route choice behavior under uncertainty. A simulation-based solution approach was applied to the deterministic bottleneck and the stochastic bottleneck, from which we found that the simulation-based approach gives results consistent with the theoretical approach. It validates our theoretical findings of the long term departure patterns with stochastic capacities.

A reliability-based dynamic discrete network design methodology under stochastic capacities has been developed, which is an optimization problem constrained to the long term dynamic user equilibrium as we defined. Dynamic network modeling with joint departure time/route choice is adopted. Travel time reliability is considered on both the network design and individual choice level. The design variables are the network structure and numbers of lanes on a set of potential links in the network.

To solve the reliability-based dynamic discrete network design problem, we developed a combined network-oriented genetic and set evaluation approach. New concepts of the universal set and the master set of lane designs are defined. We developed a new systematic approach to derive the master set by eliminating the infeasible, unconnected and illogical lane designs in order to reduce the solution space and to save computation time. An example grid network was adopted to demonstrate the notions of the infeasible, unconnected and illogical lane designs and the process of crossover and mutation. With the example network, we showed that the solution space for a network design problem can be reduced substantially by eliminating the infeasible, unconnected and illogical lane designs.

We applied the dynamic network design approach to a hypothetical road network to investigate the feasibility and the merits of our proposed design approach. The combined genetic and set evaluation approach for solving the dynamic network design problem appears to be a feasible approach. We compared the design solutions from the reliability-based dynamic and static network design approaches. We found that static traffic assignment significantly underestimates the total network travel time and the variability of total network travel time, compared to the dynamic traffic assignment. The static and dynamic network design approaches lead to significantly different design solutions. Since static traffic assignment cannot capture the dynamics in flow propagation and spillback effects, the static network design approach may lead to poor designs. A dynamic traffic assignment is more realistic, thus the dynamic network design approach is recommended. We tested the network performances with different link capacity covariance matrices, from which we found that considering link capacity correlations significantly influences the network performance evaluations. We showed a new instance of the Braess paradox, namely that adding one extra lane in reliability-based static and dynamic networks sometimes deteriorates the network performances. We found that modeling departure time choice plays a very crucial role in network designs. The dynamic network design approach with modeling departure time choices gives a solution which maximizes the utilization of the link capacities over space and time. A static network design approach however cannot capture these dynamics in departures, nor can make use of the time dimension. We compared the design solutions with different capacity variation levels, form which we conclude that modeling traveler’s choices under uncertainty has significant impacts on the departure patterns and network performance evaluations, thus on the network designs.

It is recommended to the road authority/designers that the dynamic network design approach should replace the current static network design approach. Traveler’s choices under uncertainty, especially departure time choices, should be explicitly modeled in the network design problems. The established reliability-based dynamic road network design approach can support road authorities to design a completely new network or to improve the existing networks with considering the stochastic networks and consideration of this by the travelers. Our dynamic network design approach offers in addition ample opportunities in designing dynamic link properties as part of a dynamic traffic management policy, such as dynamic lane configuration design, dynamic speed limit design during peak hours, dynamic toll design, etc., considering stochastic networks.

Samenvatting

Transport systemen zijn stochastisch van aard. Niet alleen de capaciteit van wegen maar ook de verkeersvraag fluctueert binnen een dag en tussen dagen. De verkeerssituaties waarmee reizigers geconfronteerd worden zijn eveneens stochastisch. Het is bekend dat de economie van een land of regio sterk afhankelijk is van een efficiënt en betrouwbaar transport systeem om in de toegankelijkheid van het gebied te voorzien en veilige en efficiënte stromen van goederen en mensen te faciliteren. In de context van variërende wegcapaciteiten en een fluctuerende verkeersvraag, hebben wegbeheerders/ ontwerpers een netwerkontwerp benadering nodig, die rekening houdt met de stochastische aard van een transportsysteem en de manier waarop de gebruikers van het systeem hiermee om gaan.

Ondanks de belangrijke rol van onzekerheid in het maken van verplaatsingskeuzes wordt in de meeste studies over het ontwerpen van wegennetwerken met stochastische capaciteiten of een stochastische verkeersvraag het keuzegedrag van reizigers bij onzekerheid en in het bijzonder de keuze van de vertrektijd niet gemodelleerd. Dit is echter een zeer belangrijke component van de keuzes die een reiziger moet maken. Deze component moet derhalve worden gemodelleerd in de eerder genoemde context van onzekerheid. Daarnaast wordt in de meeste studies over het ontwerpen van netwerken met stochastische reistijden gebruik gemaakt van statische verkeerstoedelingen, hetgeen niet geschikt is wanneer de prestaties van een netwerk moeten worden beoordeeld.

We hebben een veelomvattende benadering voor het ontwerpen van netwerken ontwikkeld, die gebruik maakt van dynamische verkeerstoedeling (dynamische netwerkontwerp benadering genoemd) om rekening te kunnen houden met de dynamiek in vertrektijdkeuzes, in verkeersstromen en met terugslageffecten. Stochastische wegcapaciteiten worden gemodelleerd, waarbij de route- en vertrektijdstipkeuzes van reizigers in het geval van stochastische reistijden expliciet worden gemodelleerd.

Bij het ontwikkelen van deze dynamische netwerkontwerp benadering is begonnen met het modelleren van het route- en vertrektijdstipkeuzegedrag in het geval van onzekerheid. Grof gesteld zijn er hierbij twee benaderingen mogelijk, namelijk de zog. variantie-gemiddelde en de zog. tijdstipaanpassingsbenadering. We hebben analytisch aangetoond dat de variantie-gemiddelde benadering en de tijdstipaanpassings benadering in principe equivalent zijn bij gegeven vertrektijden. We hebben aangetoond dat de verwachting van de vertragingskosten op elk vertrektijdstip *t* opgedeeld kan worden in een lineaire functie van de verwachte reistijd en de standaarddeviatie van de reistijd op vertrektijdstip *t* onder de aanname van lineaire vertragingskosten. Hierop gebaseerd hebben we een *gegeneraliseerde reiskostenfunctie* voor het modelleren van de gecombineerde vertrektijd- en routekeuze onder onzekerheid opgesteld. Deze gegeneraliseerde kostenfunctie is, door zijn mathematische implicaties, meer valide in vergelijking met de tijdaanpassingsbenadering. De functie is ook flexibeler in het omgaan met een risicomijdende houding van de reizigers. In tegenstelling tot de variantie-gemiddelde benadering houdt de gegeneraliseerde kostenfunctie expliciet rekening met de vertragingskosten, die zeer belangrijk zijn voor de tijden waarop reizigers vertrekken. De tijdaanpassingsbenadering en de variantie-gemiddelde benadering zijn speciale gevallen van onze gegeneraliseerde variantie-gemiddelde tijdaanpassingsbenadering.

Een lange termijn dynamisch gebruikersevenwicht is vervolgens gedefinieerd door aan te nemen dat reizigers hun strategische vertrektijd en route kiezen met het doel om hun geaccumuleerde reiskosten, die door onze gegeneraliseerde kosten functie gedefinieerd zijn, over een langere periode te minimaliseren.

We hebben het lange termijn dynamische gebruikersevenwicht analytisch afgeleid door het bottleneckmodel van Vickrey uit te breiden naar de situatie met stochastische capaciteiten en het toevoegen van reistijdbetrouwbaarheid in de reiskostenfunctie, gegeven uniform verdeelde wegcapaciteiten, waarbij de tijd en verkeersstroom als continu-grootheden zijn verondersteld. De analyses met stochastische capaciteiten maken gebruik van een verdeling van te analyseren tijdsperiode in drie regimes met verschillende vertragingskostenfuncties en veranderingen in de verdeling van capaciteiten. We hebben aangetoond dat het evenwichtsvertrekpatroon waarbij de keuzes van reizigers onder onzekerheid worden gemodelleerd significant afwijkt van de situatie met deterministische capaciteiten. Hoe groter de variantie in capaciteiten, des te significanter zal het verschil zijn. Het incorporeren van variërende capaciteiten en reistijdbetrouwbaarheid in de keuzes van reizigers resulteert in systematische verschuivingen van reizen naar vroegere vertrektijden. Hoe later een reiziger vertrekt in de periode met congestie, des te groter de dag-tot-dag variatie van reistijden waarmee hij/zij geconfronteerd wordt. Om die reden proberen reizigers vroeger te vertrekken om zo de reistijdvariatie te verminderen. We hebben onder meer gevonden dat de standaarddeviatie van de reistijd tijdens congestie niet proportioneel is met de verwachte reistijd, zoals de meeste bronnen aannemen.

We hebben een dynamische netwerktoedelingsprocedure ontwikkeld met discrete tijden en continue stromen voor het modelleren van verschillende gebruikersevenwichten in het geval van stochastische capaciteiten, zoals onder meer met een dynamisch of een statisch gebruikersevenwicht, met een deterministisch of een probabilistisch gebruikersevenwicht, met of zonder betrouwbaarheid in de keuzes, en een dynamisch gebruikersevenwicht met verticale of horizontale wachtrijen. Deze procedure houdt rekening met correlaties tussen stochastische wegcapaciteiten en de vertrektijd/ routekeuze onder onzekerheid. Een op simulatie gebaseerde oplossingsbenadering is toegepast op een deterministische en een stochastische bottleneck situatie. De resultaten tonen aan dat de op simulatie gebaseerde benadering consistent is met de theoretische benadering. Dit valideert onze theoretische bevindingen over de lange termijn vertrekpatronen in het geval van stochastische capaciteiten.

Gegeven deze nieuwe toedelingsmethode is een op betrouwbaarheid gebaseerde dynamische discrete netwerkontwerpmethode ontwikkeld die rekening houdt met stochastische capaciteiten. Dit is geformuleerd als een optimaliseringsprobleem met als randvoorwaarde het door ons gedefinieerde lange termijn dynamisch gebruikersevenwicht. Een dynamische netwerkmodellering met een gecombineerde vertrektijd/ routekeuze is toegepast. De reistijd onbetrouwbaarheid is zowel op het netwerkontwerpniveau als op het individuele keuzeniveau meegenomen. De ontwerpvariabelen zijn de structuur van het netwerk en het aantal rijstroken op een set van potentiële links binnen het netwerk.

Om het op betrouwbaarheid gebaseerde dynamische discrete netwerkontwerpprobleem op te lossen, hebben we een gecombineerde netwerkgeoriënteerde genetische en set evaluatie benadering ontwikkeld. Nieuwe concepten voor de universele set en de feasible set van strookontwerpen zijn gedefinieerd. We hebben een nieuwe systematische benadering ontwikkeld voor het afleiden van de feasible set door het elimineren van onmogelijke, niet-verbonden en onlogische strookontwerpen met als doel om de oplossingsruimte te beperken en de rekentijd te verkorten. Een voorbeeld roosternetwerk is gebruikt om de noties van onmogelijke, niet-verbonden en onlogische strookontwerpen en het proces van overgang en mutatie te illustreren. Met dit voorbeeldnetwerk laten we zien dat de oplossingsruimte voor een netwerkontwerpprobleem substantieel ingeperkt kan worden door het elimineren van onmogelijke, niet-verbonden en onlogische strookontwerpen.

We hebben de dynamische netwerkontwerpbenadering toegepast op een hypothetisch wegennetwerk om de haalbaarheid en de verdiensten van de door ons voorgestelde ontwerp benadering te onderzoeken. Het blijkt dat de gecombineerde genetische en set evaluatie benadering voor het oplossen van dynamische netwerkontwerpproblemen haalbaar is. De ontwerpoplossingen voor de op betrouwbaarheid gebaseerde dynamische en statische netwerk ontwerpbenaderingen zijn vergeleken en er is gevonden dat een statische verkeerstoedeling de totale netwerkreistijd en de variabiliteit van de totale netwerkreistijd significant onderschat in vergelijking met dynamische verkeerstoedeling. De statische en dynamische netwerkontwerp benaderingen resulteren in significant verschillende ontwerpoplossingen. Omdat een statische verkeerstoedeling geen rekening houdt met de dynamiek van verkeersstromen en terugslag effecten, leidt de statische netwerkontwerpbenadering tot slechte ontwerpen. Dynamische verkeerstoedeling is realistischer, vandaar dat een dynamisch netwerkontwerp aanbevolen wordt.

We hebben de prestaties van het netwerk getest met verschillende wegvakcapaciteit covariantie matrices. Deze analyses tonen aan dat het meenemen van correlaties tussen wegvakcapaciteiten de evaluatie van de netwerkprestaties significant beïnvloedt. Ook een nieuw voorbeeld van de zog. Braess paradox aangetoond, dat wil zeggen dat toevoeging van een strook in op betrouwbaarheid gebaseerde statische en dynamische netwerken de prestatie van een netwerk soms drastisch vermindert. We hebben aangetoond dat het modelleren van de vertrektijdstipkeuze een cruciale rol speelt in het netwerkontwerp. De dynamische netwerk ontwerpbenadering waarin vertrektijdstipkeuzes gemodelleerd worden resulteert in een oplossing waarbij het gebruik van de wegcapaciteiten over ruimte en tijd gemaximaliseerd wordt. Een statische netwerkontwerpbenadering kan daarentegen niet omgaan met de dynamiek in de vertrektijdstipkeuzes en de tijdsdimensie. We hebben de ontwerpoplossingen vergeleken voor verschillende niveaus van capaciteitsvariatie. Hieruit valt te concluderen dat het modelleren van de keuzes van reizigers onder onzekerheid significante effecten heeft op vertrekpatronen en netwerk prestatie evaluaties, dus op de netwerkontwerpen.

Wegbeheerders en wegontwerpers wordt aanbevolen om de dynamische netwerkontwerp benadering te gebruiken in plaats van de huidige statische benadering. De verplaatsingskeuzes bij onzekerheid, in het bijzonder de vertrektijdkeuzes, moeten expliciet gemodelleerd worden in netwerkontwerpstudies. De ontwikkelde op betrouwbaarheid gebaseerde dynamische wegennetwerk ontwerpbenadering kan wegbeheerders steunen in het ontwerpen van een geheel nieuw netwerk of in het verbeteren van bestaande netwerken waarbij rekening wordt gehouden met stochastische netwerken en de wijze waarop reizigers daarmee om gaan. Onze dynamische netwerkontwerpbenadering biedt daarnaast een grote hoeveelheid aan toepassingsmogelijkheden bij het ontwerpen van dynamische kenmerken van wegvakken als onderdeel van een dynamisch verkeersmanagement beleid, zoals het ontwerp van dynamische rijbaanconfiguraties, van dynamische snelheidslimieten gedurende de spits, van dynamische tolheffing, enz., met in acht neming van stochastische netwerken.

(Dutch translation by Saskia Ossen)

概述(Summary in Chinese)

交通运输系统所具有的随机特性主要是指道路通行能力和交通需求随时间变化所表征的变异性和不确定性。因此道路使用者经历的交通状况也是随机的。一个国家或地区的经济状况很大程度上依赖于有效、可靠的交通运输系统来提供可达性，并促进人和货物安全有效的运转。由此可见，道路管理部门和规划设计人员需要一种基于可靠度的双层次（bi-level）设计方法。一方面，这种方法需要能捕捉体现路网的随机性；另一方面这种方法也需要考虑道路使用者的路径和出行时间选择行为对路网的的影响。

尽管随机交通状况在道路使用者的出行抉择行为中起着重要的作用，但是大部分针对路网设计的研究都没有考虑交通状况随机特性对此抉择行为的影响，尤其是出行时间选择的影响。然而在面对路网的随机特性，这种出行时间选择对路网性能的影响是不能忽略不计的。而且，大部分关于路网设计的研究是基于静态交通分配模型的。静态交通分配模型有很多不切实际的假设：假设任意路径上流量在时间和空间上是没有变化的、交通拥挤往往发生在瓶颈内部，而不是瓶颈的上游路段、不能模拟交通拥挤（水平）排队及其回溢效应 (spillback effects)，等等，直接导致了不准确的路网性能评估及不精确的路网设计结果。

本文建立了一个全面的、系统的、基于动态交通分配模型的路网设计方法（称之为动态路网设计方法）。 动态交通分配模型可以模拟道路使用者的出行时间选择，以及交通流在路网上的传播 (propagation)和回溢效应。除此之外，本文提出的动态路网设计方法也模拟了道路通行能力的变异性。由于道路通行能力的变异性直接导致了行程时间的不可靠性，因此模拟行程时间可靠度对道路使用者的出行抉择行为的影响是该动态路网设计方法的关键所在。

本文首先着手于道路使用者在基于路网随机性的路径和出行时间选择行为研究。通常有两种方法可以用来模拟在随机交通状况下道路使用者的出行抉择行为。一种是均值方差法(the mean-variance approach)，另一种是预达时间延误法 (the scheduling approach)。 本文通过理论方法证明了这两种方法在给定出行时间的等同性，并得出在

一定出行时刻中，预达时间延误的均值可以分解成行程时间均值和行程时间方差的线性方程。基于此结论，本文提出了一种新的广义的效用方程来模拟道路使用者路径及出行时间的选择行为。从数学含义来看，新的广义效用方程比预达时间延误法更符合道路使用者的行为逻辑，而且这个效用方程更具备模拟道路使用者对不可靠度不选择偏好的灵活性。与均值方差法相比较，新的广义效用方程明确模拟了预达时间延误费用在道路使用者出行时间选择行为中的重要性。因此，均值方差法或预达时间延误法是新的广义效用方程的特殊情况之一。

在每天随机变化的交通状况和行程时间情况下，本文假设一种长期用户平衡会逐渐形成。道路使用者基于自身经历，做出路径和出行时间的决策性选择，以达到他们的长期费用最小化的目的。在一定出行时间和路径上，新的广义效用方程所定义的长期费用包括几个元素：行程时间的均值、行程时间的标准方差、以及基于行程时间均值的预达时间早到费用和预达时间晚到费用。

为了通过理论方法研究长期出行时间用户平衡的特性，本文使用了 Vickrey 瓶颈模型。传统的 Vickrey 瓶颈模型是基于确定性的道路通行能力，因此没有行程时间的不可靠度和随机特性的问题。该论文从两个方面拓展并延伸了 Vickrey 瓶颈模型：引入道路通行能力的变异性以及考虑道路使用者在随机性路网的长期费用，包括行程时间可靠度。与传统的 Vickrey 瓶颈模型相比，本文提出的随机 Vickrey 瓶颈模型将出行时段划分为三个区域。这些时间段的划分主要是取决于不同的预达时间延误方程以及道路通行能力分布。本文通过以上理论推断得出以下结论：不考虑行程时间可靠度对道路使用者出行选择行为影响下所得出的出行分布完全不同于考虑行程时间可靠度所得出的出行分布。道路通行能力分布的变异性越大，出行分布的差别越显著。道路出行者会出发的更早，以达到降低行程时间的不可靠性的目的。在高峰期间，越晚出发，越有可能经历更不可靠的行程时间。同时也发现行程时间的方差并不是与目前很多研究人员所假设的随行程时间的均值而线性增长或降低的规律相符合。

本文建立的基于可靠度的动态路网分配模型模拟了连续形交通流和离散的时间段。所建立的路网分配模型可以灵活运用到不同的路网模型，比如基于可靠度的静态及动态路网分配、确定性或者概率形的用户平衡、考虑可靠度或者不考虑可靠度的选择行为模拟、基于垂直排队(vertical queues)或者水平排队(horizontal queues)的动态分配模型。动态路网分配模型也可以捕捉相关联的路段通行能力，以及基于可靠度的路径和出行时间的选择行为。本文利用仿真的方法模拟了传统的 Vickrey 瓶颈模型和新的随机 Vickrey 瓶颈模型。比较仿真结果与理论结果显示本文提出的仿真模型得出了与理论方法完全一致的结果，与此同时也证明了随机 Vickrey 瓶颈模型的理论分析的正确性。

本文提出了基于可靠度的动态离散路网设计方法来模拟道路通行能力的变异性。该路网设计方法是一种基于用户平衡约束条件的优化问题、是对道路使用者在随机交通状况下的路径和出行时间选择行为的模拟。可靠度在路网设计目标及出行选择行为的模拟中都得到了充分的体现。路网设计的变量是路网结构及所有路段的车道数量即路段通行能力。

为了解决所提出的动态路网设计问题，本文设计了针对路网的遗传算法及集评估的组合算法、提出了关于车道分布设计总集(the universal set of lane designs)和优化集(the master set of lane designs)的概念。在此基础上，提出并建立了一种系统性的、产生车道

分布设计优化集的方法。该方法通过剔除不可行的、不连接的、以及不符合逻辑的车道分布设计来得到优化集。通过这种方法，路网设计的解空间可以得到很大程度的缩减。这样不仅可以提高解决路网设计问题的效率而且大大节省计算时间。该方法运用到一个小型路网上来演示不可行、不连接、不符合逻辑的车道分布设计的定义，以及演示遗传算法中交叉和突变的过程。

本文将基于可靠度的动态路网设计方法运用在一个假设的路网之上。其目的在于测试该设计方法的可行性及突出该方法的优点。所提出的遗传算法和集评价法的组合算法显示了其有效性和可行性。然后，对传统的基于静态分配模型的路网设计方法和本文提出的动态路网设计方法进行了对比。结果表明基于静态分配模型的路网评价远远低估了实际的路网费用，从而严重影响了投资效益的评估。从对比分析中可以看出，静态路网设计方法和动态路网设计方法得出完全不同的最优路网设计方案。由于静态分配模型是基于很多不合实际的假设基础上，因此静态路网设计方法可能会得出很差的路网设计方案。由此，本文推荐使用动态路网设计方法。此外，分析表明路段通行能力协方差矩阵不同性对路网性能评价产生了很大的影响。值得提出的是 Braess paradox 同样出现在基于可靠度的动态路网当中。在某些路网中添加路段，可能会导致整体路网的服务水平下降。研究进一步表明，模拟道路使用者的出行时间选择行为对路网设计至关重要。基于出行时间选择的动态路网设计方法不仅可以优化路网在空间上的分配，而且可以充分利用路网在时间上的可使用性。这也正是静态分配模型所不能模拟的。本文还针对不同的道路通行能力分布的变异性等级(capacity variation levels)对路网设计的影响进行了分析。结果表明模拟可靠度及交通状况随机性对道路使用者的出行选择行为的影响和对于路网评估和设计都是至关重要的。

综上所述，本文建议道路设计者使用基于动态分配模型的路网设计方法，从而取代当前广泛使用的静态路网设计方法，以克服静态分配模型对路网造成的错误评估以至于得出很差的路网设计方案。在随机性路网设计时，对道路使用者的出行选择行为，尤其是出行时间选择行为的模拟是至关重要的。本文所建立基于可靠度的动态路网设计方法可以用来设计全新的、更符合道路使用者逻辑的路网结构，同时也可以用来改善现有的路网结构。此外，对该方法进行稍加修改便可用于设计其他的动态路段特性：比如动态的高峰车道分布设计、动态限速设计、动态拥挤收费费用设计等等，以达到支持动态交通管理决策的目的。

(Summary in Chinese has been checked by dr. Huizhao Tu: 涂辉招)

About the Author

Hao Li (李浩) was born in NingXia Province, China, on 1st January, 1982. She did her bachelor study from 1999 ~ 2003 at the Faculty of Civil Engineering of Tongji University, Shanghai. She received the first prize of the student scholarship for three consecutive academic years and other scholarships for her academic results. She received her Bachelor degree with honor in June 2003 and was admitted for the master program at the Faculty of Transportation Engineering in Tongji University.

After her graduation in 2003, she continued her master study in Transportation and Planning, at the Faculty of Civil Engineering and Geosciences, Delft University of Technology, in the Netherlands. She received her master degree with honor in August, 2005, with her thesis entitled "Influence of Road Pricing on Network Reliability" under the supervision of Professor P.H.L. Bovy and Dr. M.C.J. Bliemer.

In September 2005 she started her PhD study at the same department, with research on reliability-based dynamic network design with stochastic networks, under the supervision of Professor P.H.L. Bovy, Professor H.J. van Zuylen, and Dr. M.C.J. Bliemer. During her PhD study, she published several journal papers and conference papers (see a list of author's publication). She, together with Professor P.H.L. Bovy and Dr. M.C.J. Bliemer, won the first prize of the best scientific paper at the 14th ITS world congress, in Beijing, China, 2007.

As part of her PhD education, she completed a study into the probabilistic properties of stochastic route set generation for use in network assignment studies.

Since February, 2009, she has been participating in a Transumo project on Personal Rapid Transit (PRT) at Rotterdam Port aiming to improve the Public Transport Services to Rotterdam Port, for which her main responsibility is the development of SP experimental design and discrete choice models.

Author’s Publications

Books

1. Li, H., P.H.L. Bovy, G. Hooghiemstra (2007) *Probabilistic properties of stochastic choice set generation*. Delft University of Technology. TRAIL Studies in Transportation Science. No. S2007/1.

2. Li, H., M.C.J. Bliemer, P.H.L. Bovy (2009) An integrated dynamic road network design appraoch with stochastic networks. In: Rudy R. Negenborn, Zofia Lukszo, Hans Hellendoorn (Eds.) *Intelligent Infrastructures*. Forthcoming.

Journals

3. Li, H., M.C.J. Bliemer and P.H.L. Bovy (2008) Network reliability-based optimal toll design. *Journal of Advanced Transportation* **42**(3): 311-332.

4. Li, H., P.H.L. Bovy and M.C.J. Bliemer (2008) Departure time distribution in the stochastic bottleneck model. *International Journal of ITS Research* **6**(2): 79-86.

5. Li, H., M.C.J. Bliemer and P.H.L. Bovy (2009) Modeling departure time choice with stochastic networks involved in network design. *Transportation Research Record*. In press.

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6. Li, H., M.C.J. Bliemer and P.H.L. Bovy (2009) Reliability-based Dynamic Discrete Network Design with Stochastic Networks. In *Proceedings: ISTTT18*, Hong Kong. In press.

7. Li, H., M.C.J. Bliemer and P.H.L. Bovy (2009) Modeling departure time choice with stochastic networks involved in network design. In *Proceedings: 88th Transportation Research Board*, Washington DC.

8. Li, H., M.C.J. Bliemer and P.H.L. Bovy (2008) Strategic Departure Time Choice in a Bottleneck with Stochastic Capacities. In *Proceedings: 87th Transportation Research Board*, Washington DC.

9. Li, H., G. Hooghiemstra, P.H.L. Bovy, M.C.J. Bliemer (2008) Selection properties of stochastic route choice set generation. In *Proceedings*: *87th Transportation Research Board*, Washington DC.

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11. Tu, H., H.J. van Zuylen, H. Li (2008) Application of a travel time reliability model in dynamic traffic assignment. In *Proceedings: 2nd International Symposium on Dynamic Traffic Assignment*, Katholieke Universiteit, Leuven, Belgium.

12. Li, H., M.C.J. Bliemer and P.H.L. Bovy (2007) Optimal toll design from a network reliability perspective. In *Proceedings: TRISTAN VI*, Phuket, Thailand.

13. Li, H., P.H.L. Bovy, and M.C.J. Bliemer (2007) Departure Time Distributions in the Stochastic Bottleneck Model. In *Proceedings:* 14th world congress on ITS, Beijing, China (**First prize of the best scientific paper**).

14. Bliemer, M.C.J., P.H.L. Bovy, and H. Li, (2007) Some properties and implications of stochastically Generated Route Choice Sets. In *proceedings: TRISTAN VI.* Phuket, Thailand.

15. Li, H., M.C.J. Bliemer, and P.H.L. Bovy (2007) Network reliability-based optimal toll design. In *Proceedings: 3rd International Symposium on Transport Network Reliability*, Volume II, The Hague, The Netherlands.

16. Li, H., M.C.J. Bliemer and P.H.L. Bovy (2006) Influence of Road pricing on network reliability. In *Proceedings*: *6th International conference on transport and traffic studies (ICTTS)*, pp. 379-391, Xi'an, China.

17. Li, H., M.C.J. Bliemer and P.H.L. Bovy (2006) Impacts of road pricing on network travel time reliability: a preliminary analysis. In: *Seminar Infrastructure reliability TU* Delft, Delft, The Netherlands.

18. Li, H., M.C.J. Bliemer and P.H.L. Bovy (2006) Network reliability-based optimal toll design. In *Proceedings: 9th International TRAIL Congress*, Rotterdam, The Netherlands, TRAIL Research School.

19. Tu, H., L. Sun, and H. Li (2003) Feasibility Study on Implementing Electronic Road Pricing System in Shanghai. In *Proceedings: 8th Conference of Hong Kong Society for Transportation Studies.* Hong Kong, P.R. China.

Reports

20. Li, H. (2005) *Calibration and Validation of MARPLE*. Technical report, AVV Transport Research Center. Rotterdam, The Netherlands.

21. Li, H. (2005) *Influence of road pricing on network reliability*. Master thesis. Faculty of Civil Engineering and Geosciences, Transportation and Planning Section, Delft University of Technology. The Netherlands.

TRAIL Thesis Series

The TRAIL Thesis Series is a series of the Netherlands TRAIL Research School on transport, infrastructure and logistics.

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