# Pin-pin resonance as a reference in determining ballasted railway track vibration behaviour

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**Pin-pin resonance is one of the most significant preferred vibration modes of beams, which are supported at equal distances, such as rails at sleepers in railway track structures do. Pin-pin resonance is a vibration that appears in one basic (first) mode and several higher modes, however the basic mode will have the highest amplitudes. In operational conditions of railways, pin-pin resonance only partly influences wheel-rail contact of the train while the speed dependent sleeper-passing frequency is more important. Among other track resonances, pin-pin resonance plays an important role in noise and vibration radiation of the rails and can be used as a meaningful instrument in track system dynamics recognition and optimization. However, existing simple analytic approximations are not sufficiently reliable to perform these recognitions. This will be shown in this paper by means of new tests and simulations that are based on improved models and methods but restricted to ballasted track structures.**

*Key words: Railwaytrack, Dynamics, Vibration modes, Simulation*

## 1 Introduction into pin-pin resonance

Present railway tracks are characterised by large distances of continuous welded rails and accurately positioned rails and sleepers. These things make the mechanical modelling of railway track rather simple: two straight parallel beams at equal distances supported by sleepers (or sometimes by blocks). As a consequence of discretely supporting beams like rails, the rails in the track "framework" will obtain vibration modes related to this type of supporting. The most important vibration mode resembles a kind of bending between discrete points or pins. This mode is schematically shown in (Fig.1). With some simplifying assumptions, this pin-pin vibration resonance occurs at a specific frequency ($f_{pp}$), which can be calculated by:

$$f_{pp} = \frac{\pi}{2l^2}\sqrt{\frac{EI}{m}} \tag{1}$$

Where:
$l$ : distance between two supports [m]
$EI$: bending stiffness of the rail (static) [Nm$^2$]
$m$ : mass of the rail per unit length [kg/m']

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*Fig. 1. Pin-pin vibration mode (first mode).*

Equation (1) is a general one, based upon an infinitely long beam and perfectly supported at small but rigid points. The assumption of these small and rigid points is in railway practice not a realistic one: the rail is at an elastically supported sleeper in ballast (or at a block in an elastic boot) with an elastic railpad in-between (Fig. 2). These and other elements may influence pin-pin resonance.
By consequence these elements introduce new significant vertical resonances, like rail on railpad (at frequency $f_r$) and sleeper on ballast (at frequency $f_s$). For most track structures these three vertical resonances are dominant and lie between 30 and 2000 Hz.

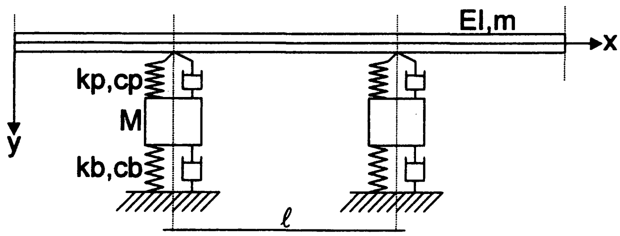


*Fig. 2. Track model with elastic discrete supports.*

## 2 Former work

In order to predict track vibration resonances in vertical direction, several analytical calculation methods have been presented by others. These methods differ in number of activated parameters and come up with several approximation formulas for the frequencies of the three resonances $f_{pp}$, $f_r$ and $f_s$.

In [1] it is outlined that the effect of introducing elastic layers or non-rigid supports underneath the bending beam on the pin-pin frequency can be satisfied by changing the mass per unit length of equation (1) into an equivalent value. For wooden sleepers with low mass, the compensation will be higher than for concrete sleepers as the preference of lighter wooden sleepers to move together with rails is higher than for heavier ones.

In [2] a discrepancy between analytical model results, based on an infinite Euler beam on discrete supports, and measured data for pin-pin resonance and some other vertical track resonances above 400 Hz is found. Compensation is found in a 25% reduction factor of the rail's vertical bending moment of inertia parameter I. By introducing reduction factors on mass and on bending stiffness, the results of field measurements can be approached, see [1], [2]. These measurements had shown that the really occurring pin-pin frequencies lied 10-300 Hz lower than the analytically approximated ones according to equation (1).

In [2] it is already stated that a Timoshenko beam modelling will make the application of reduction factors redundant. In [3] it is shown that the application of Timoshenko beam properties in an analytical model using Green's function is fairly simple. Then the frequency response function of track structures will be more reliable in frequencies above 500 Hz, where the pin-pin resonance occurs. Nevertheless a quick method for a reliable estimate of the three frequencies is still not presented.

# 3 New measurements and modelling

In recent measurements performed at railway structures in the Netherlands, the same differences occurred: pin-pin frequencies estimated by (1) were much higher than those really recorded. In stead of developing another analytical model, a finite element program called RAIL [4] was applied for the simulation of track behaviour and the performance of sensitivity tests on track behaviour.

RAIL uses Timoshenko beam elements for rails, which means that shear and bending of elements are taken into account as well as translational and rotational inertia, only in the 2D plane. The restriction to 2D plane automatically implies block supports in stead of sleepers as only one rail can be analysed. Another important feature of modelling in finite elements is that support length of the rail at the block (or half sleeper) and the support length of the block itself is taken into consideration. Finally different behaviour of the elastic layers on tension and compression could be modelled, which is however not done yet.

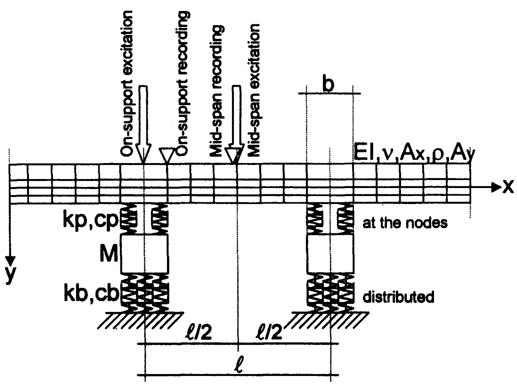


*Fig. 3. Model as used in RAIL for excitation calculations.*

As finite element modelling is restricted to finite lengths, an evaluation per model should make clear if the modelled length is sufficient and not influencing the results. Generally 20 m to both ends turns out to be sufficient for vertical point excitation simulations. For each span between two supports it suffices to use 6 elements of equal length for dynamic analyses in which the pin-pin resonance occurs.

# 4 Some examples from former work

***Example 1: British Rail track structure***

The following track data, which were used in [1], describe a ballasted track structure with concrete sleepers and 133-FB rails. Over 70 meters of this track have been modelled in finite elements in RAIL.

Table 1. Track data applied in [1], * adapted for simulation.

| Symbol | Value | Description |
|---|---|---|
| $E$ | $210\ 10^{9}$ N/m$^2$ | Young's modulus of rail material |
| $I$ | $2314\ 10^{-8}$ m$^4$ | Rail moment of inertia close to UIC54 |
| $\nu$ | 0.3 | Poisson's ratio of rail material |
| $A_x$ | $71.3\ 10^{-4}$ m$^2$ | Rail cross section area close to UIC54 |
| $\rho$ | 7850 kg/m$^3$ | Rail material mass density |
| $A_y$* | $28.5\ 10^{-4}$ m$^2$ | Rail shear cross section area<br>(rail shear coefficient $K = 0.4$) |
| $l$ | 0.698 m | Sleeper spacing |
| $b$* | 0.068 m | Rail support length per sleeper |
| $K_p$ | $280\ 10^{6}$ N/m | Railpad stiffness |
| $C_p$ | $82\ 10^{3}$ Ns/m | Railpad viscous damping |
| $M$ | 110 kg | Half sleeper (block) mass |
| $K_b$ | $180\ 10^{6}$ N/m | Ballast stiffness per half sleeper (block) |
| $C_b$ | $63\ 10^{3}$ Ns/m | Ballast viscous damping per half sleeper (block) |

According to equation (1) and omitting any contribution of the sleepers, the pin-pin frequency is 950 Hz, while the observed value is 770 Hz. Detailed calculations with Timoshenko analytical solutions in [1] have set 766 Hz to be the resonance.

A FEM in RAIL approximately comes to the same solution in this case: $f_{pp}$ is 775 Hz (Fig. 6). In this case the rail shear coefficient $K$ is 0.4 in stead of 0.34 in [1]. Both other vertical resonances ($f_s$ and $f_r$) are at 140 Hz and 473 Hz respectively.

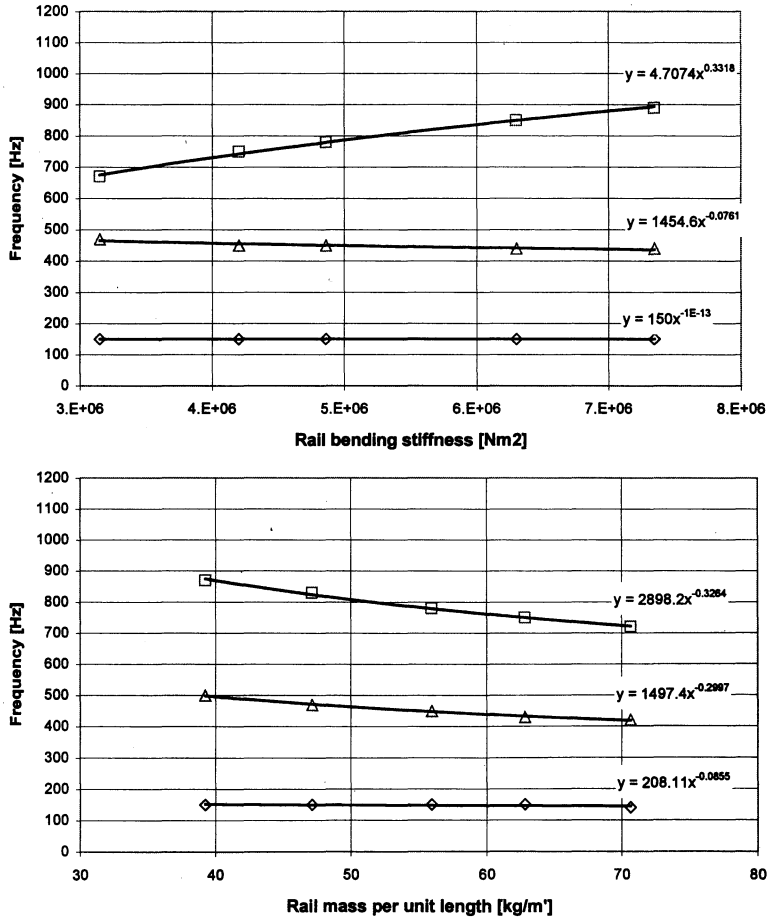


*Fig. 4a,b Sensitivity of rail bending stiffness (EI) and rail mass per unit length (m) to 3 resonant frequencies: $f_{pp}$ (□), $f_s$ (◊) and $f_r$ (Δ).*

Which parameters are most important for pin-pin resonance? Referring to equation (1), $EI$ influences $f_{pp}$ proportionally while $m$ and $l$ do the same inversely. Support stiffnesses $K_p$ and $K_b$ (to a smaller extent), support mass $M$ and support length $b$ may influence $f_{pp}$, $f_r$ or $f_s$. The graphs of Fig. 4 and Fig. 5 show the sensitivities of $EI$, $m$, $l$ and $M$ on $f_{pp}$ (box line) and on the two other resonances $f_s$ (diamond line) and $f_r$ (triangle line). Please note that $m$ is composed from a varying $A_x$ multiplied by $r$ and that $A_y$ is derived from the same $A_x$ by multiplying with the rail shear coefficient $K = 0.4$.

With these and other graphs for *Kb*, *Kp*, and *b*, sensitivities come clear. We assume that the influences can be described by the depicted power formulae. If the absolute value of the power of a specific parameter is smaller than 0.1, that parameter is generally considered as not sufficiently important. Then the following sensitive parameters remain:

For $f_{pp}$: *EI*, *m* and *l*

For $f_s$: $K_b$, *l* and *M*

For $f_r$: *M*, *m* and $K_p$

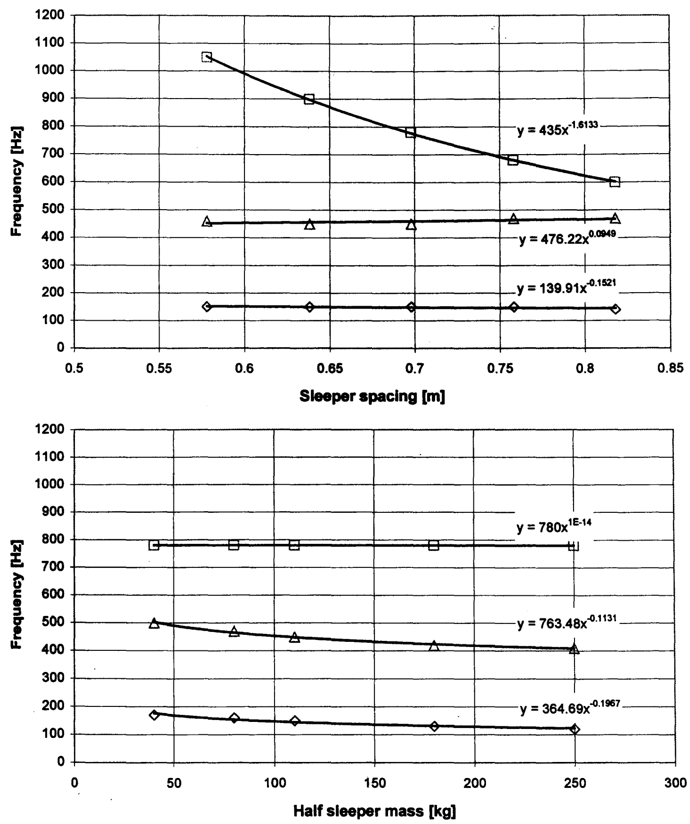


*Fig. 5a,b Sensitivity of sleeper spacing (l) and half sleeper mass (M) to 3 resonant frequencies:* $f_{pp}$ (□), $f_s$ (◊) *and* $f_r$ (Δ).

The values for $A_y$ have not been varied independently from $A_x$. The conventional shapes of rails approach the I-shape and the effective shear cross-section area is about 40% of the cross-section area ($K = 0.4$). Non-conventional rail shapes may have higher effective areas on shear up to about 70% for square cross-sections.

With help of a multiple power approximation, the following equations have been composed for mid-span excitation and recording. It should be stressed at this point that these can be closely related to the modelled type of track (which is ballasted track). In future papers the resemblance with other types of track such as block supported track will be investigated.

$$f_{pp} = n1.l^{-1.61} \cdot EI^{0.33} \cdot m^{-0.33} \quad (2)$$

$$f_s = n2.l^{-0.15} \cdot EI^{-0.20} \cdot K_b^{\ 0.50} \quad (3)$$

$$f_r = n3.m^{-0.30} \cdot M^{-0.11} \cdot K_p^{\ 0.59} \quad (4)$$

The constants in these equations are calculated ($n1 = 10.2$, $n2 = 0.0253$ and $n3 = 0.0275$) by filling the supplied track data in Table 1 and the following resonant frequencies: $f_{pp} = 775$ Hz, $f_s = 140$ Hz and $f_r = 473$ Hz. A frequency response function (FRF) on the given track data is shown in Fig. 6.
It depicts track receptance of mid-span excitation and recording and the phase lag relative to the force. It clearly shows the three resonances.

The simulated response function shows good resemblance with experimentally obtained response functions, see Fig. 10 in [1]. Resonant frequencies were determined there at $f_s = 140$ Hz, $f_r = 470$ Hz and finally at $f_{pp} = 770$ Hz.

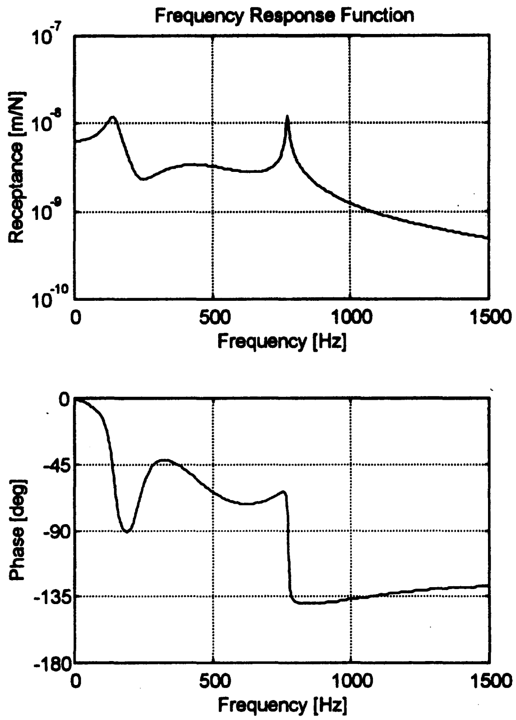


*Fig. 6. FRF of ballasted track data in Table 1, calculated with* RAIL.

*Example 2: Swedish Railways track structure*

In [2] a different ballasted track structure has been simulated and tested in the laboratory. It comprises concrete sleepers and UIC50 rails. Apart from vertical vibration modes of the rail and the supports, also flexural modes of the sleeper have been considered in [2]. This is however beyond the capabilities of the RAIL program.

Table 2. Track data applied in [2], * adapted for simulation.

| Symbol | Value | Description |
|---|---|---|
| $E$ | $210\ 10^9$ N/m$^2$ | Young's modulus of rail material |
| $I$ | $2060\ 10^{-8}$ m$^4$ | Rail moment of inertia of UIC50 |
| ν | 0.3 | Poisson's ratio of rail material |
| $A_x$ | $66.3\ 10^{-4}$ m$^2$ | Rail cross section area of UIC50 |
| ρ | 7850 kg/m$^3$ | Rail material mass density |
| $A_y$* | $26.5\ 10^{-4}$ m$^2$ | Rail shear cross section area ($K = 0.4$) |
| $l$ | 0.65 m | Sleeper spacing |
| $b$* | 0.05 m | Rail support length per sleeper |
| $K_p$ | $500\ 10^6$ N/m | Railpad stiffness |
| $C_p$* | $19.5\ 10^3$ Ns/m | Rail viscous damping (approx.) |
| $M$ | 125 kg | Half sleeper (block) mass |
| $K_b$* | $21.25\ 10^6$ N/m | Ballast stiffness per half sleeper (block) |
| $C_b$* | $27.5\ 10^3$ Ns/m | Ballast viscous damping per half sleeper (block) |

Based on the equations for mid-span excitation and recording (2) to (4), the following frequencies are calculated: $f_{pp} = 858$ Hz, $f_s = 47$ Hz and $f_r = 670$ Hz. A simulation with the above data entered in RAIL shows the following resonant frequencies (also see the FRF in Fig. 7): $f_{pp} = 858$ Hz, $f_s = 50$ Hz and $f_r = 615$ Hz.

Tests on a laboratory specimen of 8.5 m ballasted track have shown resonances at 50, 320, 600, 800, 930 and 1150 Hz; $f_{pp}$ is identified at 930 Hz, $f_s$ at 50 Hz and $f_r$ at 800 Hz according to [2]. The differences between these values, the approximations via (2) to (4) and RAIL results are due to:

- tested and modelled influences of sleeper bending, while simulation in RAIL is restricted to block supports;
- some adaptations on the available data for modelling in RAIL;
- mid-span excitation tests in laboratory with close to <u>next</u> mid-span recording, while excitation and recording in RAIL are at the same point;

- limited length of the laboratory specimen, which causes additional resonances of reflecting waves.

In order to overcome the last two effects, Fig. 8 shows the results of the modelled specimen of 8.5 m calculated with RAIL. The shifts of resonant frequencies are very small.

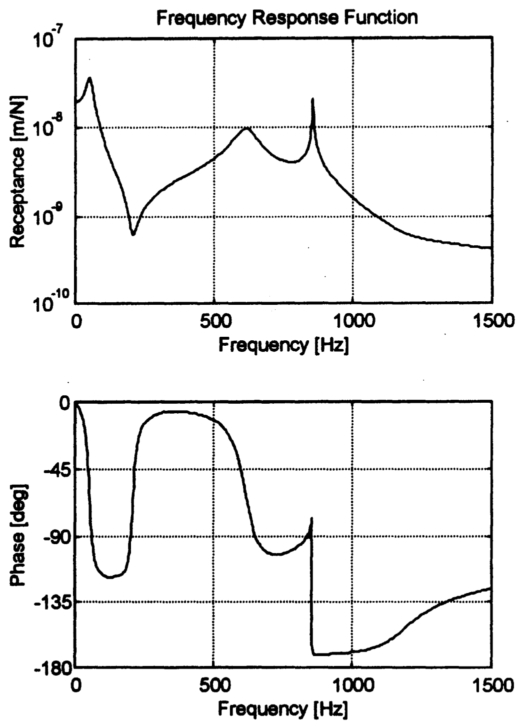


*Fig. 7. FRF of ballasted track data in Table 2, calculated with* RAIL.

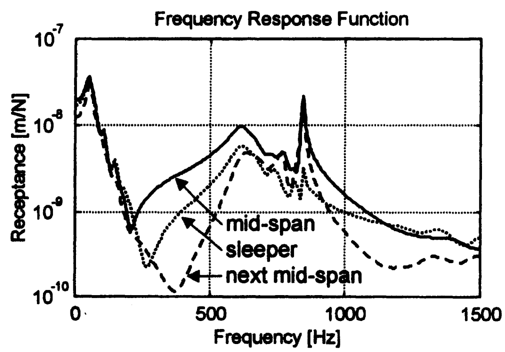


*Fig. 8. FRF of ballasted track sample of 8.5 m, calculated with* RAIL.

Nevertheless the estimate of $f_{pp}$ of 858 Hz, and calculated by RAIL at 845 Hz is closer to the stated 930 Hz than 1068 Hz according to (1).

***Example 3: German Railways track structure***

Finally in [3] a third ballasted track structure has been simulated, consisting of heavy concrete sleepers and UIC60 rails. This is a typical German track structure.

Table 3. Track data applied in [3], * adapted for simulation.

| Symbol | Value | Description |
|---|---|---|
| $E$ | $210\ 10^9$ N/m$^2$ | Young's modulus of rail material |
| $I$* | $3055\ 10^{-8}$ m$^4$ | Rail moment of inertia of UIC60 |
| $\nu$ | 0.3 | Poisson's ratio of rail material |
| $A_x$* | $76.9\ 10^{-4}$ m$^2$ | Rail cross section area of UIC60 |
| $\rho$* | 7850 kg/m$^3$ | Rail material mass density |
| $A_y$* | $30.8\ 10^{-4}$ m$^2$ | Rail shear cross section area ($K$ = 0.4) |
| $l$ | 0.60 m | Sleeper spacing |
| $b$* | 0.06 m | Rail support length per sleeper |
| $K_p$ | $300\ 10^6$ N/m | Railpad stiffness |
| $C_p$* | $10.4\ 10^3$ Ns/m | Rail viscous damping (approx.) |
| $M$ | 162 kg | Half sleeper (block) mass |
| $K_b$* | $75\ 10^6$ N/m | Ballast stiffness per half sleeper (block) |
| $C_b$* | $48.8\ 10^3$ Ns/m | Ballast viscous damping per half sleeper (block) |

In Fig. 9 the FRF is shown with the following resonances: $f_{pp}$ = 1038 Hz, $f_s$ = 85 Hz and $f_r$ = 480 Hz. There is a clear anti-resonance at 190 Hz, representing a vibration of the support and only slight vibration in the rail. This anti-resonance can be distinguished in other FRFs as well.

For this track structure, equations (2) to (4) give the following estimates for the three resonant frequencies: $f_{pp}$ = 1058 Hz, $f_s$ = 86 Hz and $f_r$ = 461 Hz. The initial estimate for pin-pin resonance using equation (1) gives $f_{pp}$ = 1423 Hz.

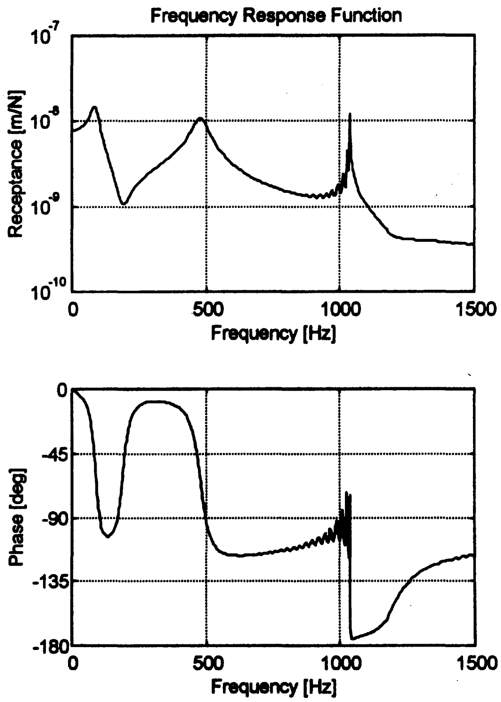


*Fig. 9. FRF of ballasted track data in Table 3, calculated with* RAIL.

Compared to the results of analytical calculation presented in [3], there are some differences with the RAIL simulation results. These are mainly due to some adaptations or conversions on the available data for modelling in RAIL. Especially the rail shear cross-section ($A_y$) is not stated explicitly, which directly affects the $f_{pp}$ value. Both stiffness and damping parameters ($K_p$, $K_b$, $C_p$ and $C_b$) are defined frequency-dependent, which has apparently some consequences on the $f_s$ and $f_r$ values. The resonances listed in [3] are: $f_{pp}$ = 1230 Hz, $f_s$ = 85 Hz and $f_r$ = 500 Hz.

## 5 Two track structures in the Netherlands

In order to investigate the appropriate application of "prediction"-equation (2) for pin-pin resonance in particular, some ballasted tracks in the Netherlands have been investigated and presented here.

First a ballasted track structure in the Circle Line of the Amsterdam Metro has been examined. This track was built in 1995 of S49 rails and concrete monoblock sleepers, which are about 20% lighter than the standard NS prestressed monoblock sleepers NS90.

Table 4. Ballasted track data of Circle Line of Amsterdam Metro.

| Symbol | Value | Description |
|---|---|---|
| $E$ | $210\ 10^9\ \mathrm{N/m^2}$ | Young's modulus of rail material |
| $I$ | $1819\ 10^{-8}\ \mathrm{m^4}$ | Rail moment of inertia of S49 |
| $\nu$ | 0.3 | Poisson's ratio of rail material |
| $A_x$ | $63.4\ 10^{-4}\ \mathrm{m^2}$ | Rail cross section area of S49 |
| $\rho$ | $7850\ \mathrm{kg/m^3}$ | Rail material mass density |
| $A_y$ | $25.4\ 10^{-4}\ \mathrm{m^2}$ | Rail shear cross section area ($K = 0.40$) |
| $l$ | 0.71 m | Sleeper spacing |

For mid-span excitation and recording, the pin-pin resonant frequency is calculated with equation (2): $f_{pp} = 725$ Hz.

Field tests with excitation hammer equipment have shown resonances at 150, 530 and 740 Hz, as can be seen in the average transfer function on receptance (Fig. 10). The 740 Hz resonance is interpreted as the pin-pin resonance. The prediction of $f_{pp}$ of 725 Hz is very close to the stated 740 Hz compared to 867 Hz according to (1). The frequency values are based on mid-span excitation and recording.

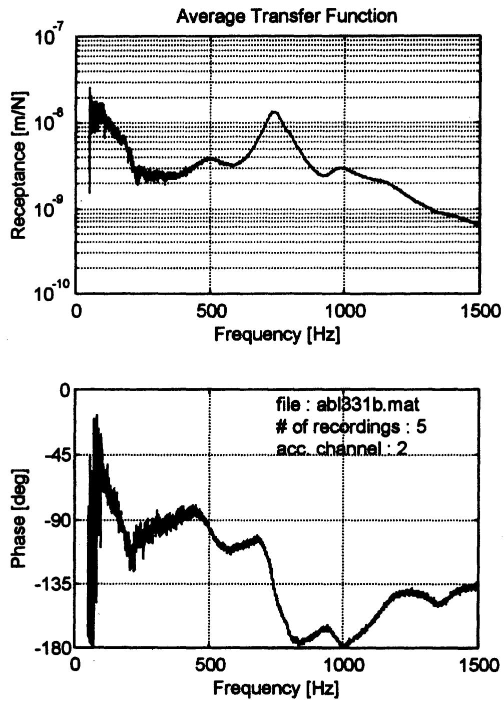


*Fig. 10. ATF recorded on the rail of ballasted track on the Circle line of Amsterdam Metro.*

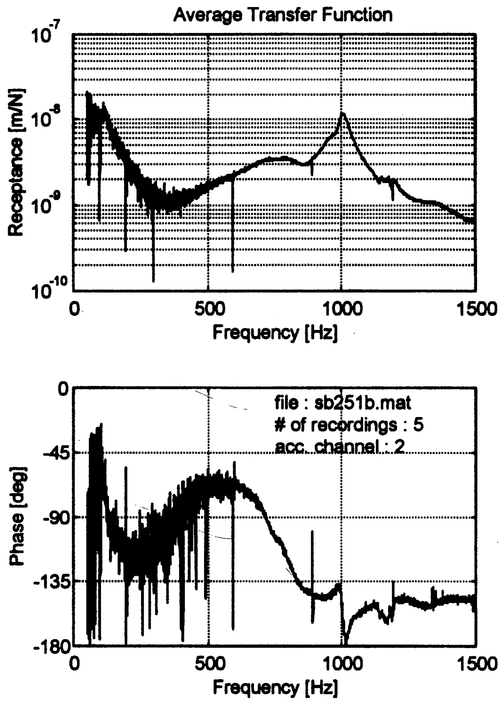


*Fig. 11. ATF recorded on the rail of ballasted track on the Brabant-route of NS.*

Second a ballasted track structure in the national network of Netherlands Railways (NS) on a heavily used line (Brabant-route) is presented. In Table 5 track data are listed.

Based on the equations for mid-span excitation and recording, pin-pin resonant frequency is calculated with equation (2): $f_{pp}$ = 1004 Hz.

Table 5. Ballasted track data of Brabant-route of NS.

| Symbol | Value | Description |
|---|---|---|
| $E$ | $210\ 10^9\ \mathrm{N/m^2}$ | Young's modulus of rail material |
| $I$ | $2346\ 10^{-8}\ \mathrm{m^4}$ | Rail moment of inertia of UIC54 |
| $\nu$ | 0.3 | Poisson's ratio of rail material |
| $A_x$ | $69.3\ 10^{-4}\ \mathrm{m^2}$ | Rail cross section area of UIC54 |
| $\rho$ | $7850\ \mathrm{kg/m^3}$ | Rail material mass density |
| $A_y$ | $27.7\ 10^{-4}\ \mathrm{m^2}$ | Rail shear cross section area ($K = 0.40$) |
| $l$ | 0.60 m | Sleeper spacing |

In this particular case, tests have shown resonances at 120, 790 and 1005 Hz (Fig. 11). The 1005 Hz resonance is interpreted as the pin-pin resonance. The prediction of $f_{pp}$ of 1004 Hz is almost equal to the measured 1005 Hz, while equation (1) would have given 1313 Hz.

In both ballasted track structures, the estimates for $f_s$ and $f_r$ have not been checked. In stead of checking afterwards, the opportunity is there to use the estimate functions (3) and (4) to determine values for $K_b$ and $K_p$ by transforming them into:

$$K_b = \left(\frac{f_s}{l^{-0.15} \cdot M^{-0.20} \cdot n2}\right)^{\frac{1}{0.50}} \tag{5}$$

$$K_p = \left(\frac{f_r}{m^{-0.30} \cdot M^{-(0.11)} \cdot n3}\right)^{\frac{1}{0.59}} \tag{6}$$

These equations give quite realistic stiffness parameter values, which are listed in Table 6. The tendency that railpad stiffness of the NS track structure is higher than the metro structure agrees with the different railpad properties: NS railpads are thinner and older than those applied at Amsterdam Metro.

Table 6. Stiffness values determined with resonance estimate equations.

| Track structure and company | $M$ [kg] | $f_s$ [Hz] | $K_b$ [$10^6$N/m] | $f_r$ [Hz] | $K_p$ [$10^6$/m] |
|---|---|---|---|---|---|
| Circle line Amsterdam Metro | 100 | 150 | 200 | 530 | 315 |
| Brabant-route NS | 105 | 120 | 125 | 790 | 655 |

## 6 Conclusions

Pin-pin and other vertical track resonant frequencies are clear functions of stiffness and damping properties, bending and shear stiffness, support spacing and masses of track structure components. This general conclusion is evident and supported by others. Analytical models, recordings from structures in the field and test specimen in laboratory have proven this.

The finite element program RAIL, especially developed for track related mechanical calculations, has shown its value in analysing and predicting track resonances and – more general – in predicting track vibration behaviour. The introduction of FEM comprising Timoshenko beam elements turned out to be essential for higher frequencies than approximately 400 Hz, where pin-pin resonances are occurring definitely. With FEM, sensitivity studies of specific track parameters have been performed and the results have been combined in improved estimation equations. For the frequently applied ballasted tracks, where still useful analytical estimation equations are missing and reduction factors have had to be introduced, these more accurate estimation equations can be helpful. It should be accepted however that present FEM calculation accuracy ($\Delta f = 10$ Hz) is the major reason for the little errors in estimations. Computational restrictions partly forced this.

The derived equations for ballasted structures seemed to be applicable on other structures. In further papers, ballastless as well as embedded track structures will be analysed and compared to field recordings.

## 7 Acknowledgements

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## 8 References

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