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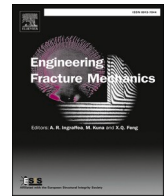
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# A proposal for similitude in characterizing fatigue delamination behavior with fibre bridging of carbon-fibre reinforced polymer composites

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## ABSTRACT

Methods based on fracture mechanics have been widely used in fatigue delamination growth (FDG) characterization of composite laminates. These methods are based on the similitude hypothesis. It is therefore important to have appropriate parameters to well represent the similitude, which is useful for fatigue delamination test standard development aimed by Technical Committee 4 of the European Structural Integrity Society (ESIS TC4) and the ISO/TC61/SC13. In the present study, discussions on similitude parameters for fibre-bridged fatigue delamination interpretation have been conducted via fatigue data with fibre bridging at different  $R$ -ratios. The results clearly demonstrate that the strain energy release rate ( $SERR$ ) indeed applied around the crack front, rather than the total applied  $SERR$ , should be employed to represent the similitude for FDG interpretation with large-scale fibre bridging. Particularly, the use of  $\Delta\sqrt{G_{tip}}$  can well determine fibre-bridged delamination behavior of a given  $R$ -ratio, but it is not valid for FDG at different  $R$ -ratios in accordance with the similitude principles. A new similitude parameter, in terms of both  $\Delta\sqrt{G_{tip}}$  and the maximum  $SERR$   $G_{max\_tip}$ , was therefore proposed to appropriately represent FDG behavior with fibre bridging at different  $R$ -ratios. This study can not only provide database, but also give important insights for the development of mode I fatigue delamination test standard of composite laminates.

## 1. Introduction

Carbon-fibre reinforced polymer composites have been widely used in aerospace engineering and other engineering disciplines, due to their excellent mechanical performance with weight-saving potential. The weight percentage of composites in modern aviation aircrafts, such as Boeing 787 and Airbus A350XWB, is about 50–53 % [1]. However, these materials are vulnerable to FDG [2,3], a unique damage evolution frequently observed between neighboring layers under cyclic loading. Understanding and describing this phenomenon has been a big challenge for the application of advanced composites in structural design [3–5].

Furthermore, the change from *no-growth* to *slow-crack growth* in the certification of composites and adhesively bonded structures,

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recommended by the US Federal Aviation Administration (FAA) since 2009, has focused the attentions to appropriately representing delamination growth behavior under cyclic loading [6,7]. In the last decade, both ESIS TC4 and Subcommittee D30.06 of the American Society for Testing and Materials (ASTM D30.06) have conducted separated round-robin test programs for establishing a protocol for mode I FDG in composite laminates [8–10].

For these purposes, generally methods based on fracture mechanics have been frequently used to characterize and determine FDG behavior [4], in which fatigue crack growth rate  $da/dN$  is correlated to the *SERR*  $G$  (usually in terms of the maximum *SERR*  $G_{\max}$  or the *SERR* range  $\Delta\sqrt{G}$ ). The Paris relation

$$\frac{da}{dN} = C(G_{\max})^n \text{ or } \frac{da}{dN} = C(\Delta\sqrt{G})^n = C\left[\left(\sqrt{G_{\max}} - \sqrt{G_{\min}}\right)^2\right]^n \quad (1)$$

is the most frequently adopted formulation for this correlation. Variations on Eq. (1) have been proposed to represent fatigue delamination behavior considering the effects of stress ratio, mixed-mode opening and other factors [2–4,9,11]. The Paris relation and its variants are indeed based on the similitude hypothesis [7,12–14], which form the backbone in fatigue delamination characterization and prediction. This hypothesis is formulated as [7,12,13]:

*When fatigue delamination behavior in composites is characterized with the SERR  $G$ , then the same material with different crack lengths has the same delamination growth, provided that the applied SERR range  $\Delta\sqrt{G}$  and  $G_{\max}$  are the same.*

According to this hypothesis, both  $\Delta\sqrt{G}$  and  $G_{\max}$  should be employed to determine the similitude in fatigue delamination, because neither  $\Delta\sqrt{G}$  nor  $G_{\max}$  is an appropriate similitude parameter in itself. Intuitively, one can explain this with that a fatigue loading cycle cannot be uniquely determined with either the range or the maximum load [2,4]. This in essence is the subject in debates on fatigue data interpretation, particularly with respect to the *R*-ratio dependence [4]. Accordingly, several researchers [15–21] have proposed fatigue models involving both  $\Delta\sqrt{G}$  and  $G_{\max}$  in the similitude. Furthermore, it has been demonstrated that both  $\Delta\sqrt{G}$  and  $G_{\max}$  distinctively relate to microscopic failure features observed on fatigue fracture surfaces [19,20], physically supporting the proposed formulations using both  $\Delta\sqrt{G}$  and  $G_{\max}$ . Hence, a Paris-type relation Eq.(2) employing both  $\Delta\sqrt{G}$  and  $G_{\max}$  to represent the similitude has been proposed recently to successfully interpret FDG behavior at different *R*-ratios [21]. This formulation can appropriately account for *R*-ratio effects, demonstrating its validity in characterizing fatigue delamination behavior (i.e. obeying the similitude hypothesis).

$$\frac{da}{dN} = C \left[ \Delta\sqrt{G} \left[ 1 - \left( \frac{G_{\max}}{G_C} \right)^\gamma \right] G_{\max} \left( \frac{G_{\max}}{G_C} \right)^\gamma \right]^n \quad (2)$$

where  $G_{\max}$  and  $G_{\min}$  respectively represent the maximum and minimum *SERR* of a fatigue cycle;  $G_C$  is the material fracture toughness;  $C$ ,  $n$  and  $\gamma$  are curve-fitting parameters.

Fibre bridging, visible in Fig. 1, is an important and unique shielding mechanism frequently reported in fatigue delamination studies of composite laminates [9,22–24]. The presence of these bridging fibres at the behind of the crack front can cause fatigue delamination behavior crack length dependence using Eq.(1) in data reduction [22], which violates the similitude hypothesis and makes Eq. (1) essentially incomplete. This underlines the importance of appropriately addressing fibre-bridging in FDG study towards fatigue test standard development [9].

For delamination with fibre bridging, in accordance with the *J*-integral concept Eq.(3) [24–27], the total *SERR*  $G_{\text{total}}$  in the system (i.e. specimen) can be divided as the *SERR* applied to the crack front  $G_{\text{tip}}$  and the *SERR* stored in bridging fibres  $G_{\text{br}}$ .

$$G_{\text{total}} = G_{\text{tip}} + G_{\text{br}} = G_{\text{tip}} + \int_0^{\delta_c} \sigma_{\text{br}}(\delta) d\delta \quad (3)$$

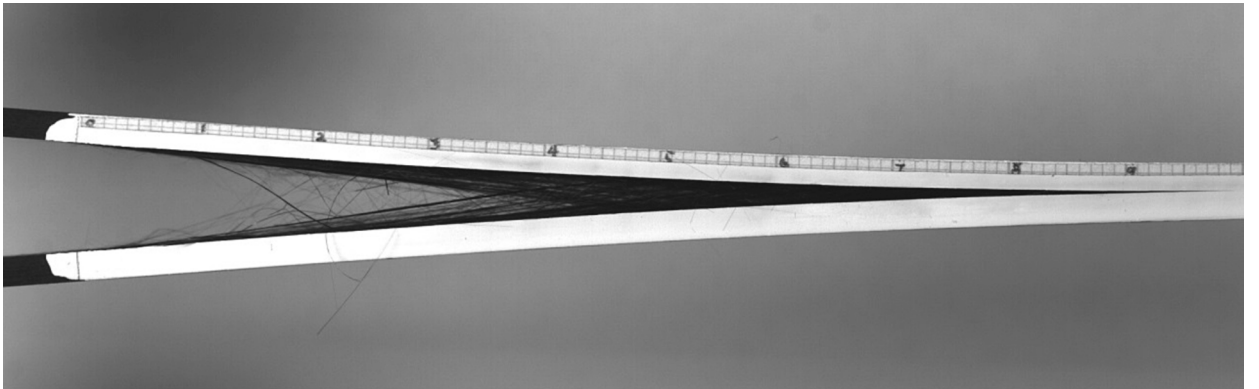


Fig. 1. Obvious fibre bridging in FDG.

where  $\sigma_{br}(\delta)$  represents the bridging closure stress; and  $\delta$  is the crack opening displacement.

It has been reported several times that the use of  $G_{total}$  as similitude parameter in the Paris relation cannot appropriately represent fibre-bridged FDG behavior, violating the similitude principles [22,27,28]. Gregory et al., [29] conducted an investigation on fibre-bridged mode I FDG in composites at different temperatures. It was found that the use of  $G_{tip}$ , rather than  $G_{total}$ , can well represent temperature effects on FDG behavior with fibre bridging. Donough et al., [30] also recommended using  $G_{tip}$  in the formation of a new similitude parameter to determine FDG behavior with fibre bridging at different  $R$ -ratios. And in other studies completed by Farmand-Ashtiani et al., [24] and Yao et al., [31], strong correlation between  $da/dN$  and  $G_{tip}$  has been reported in FDG with fibre bridging in composite laminates. Furthermore, it has been reported that fatigue damage evolution around crack front remains the same for FDG with fibre bridging [22,31]. Particularly, both dominant fibre/matrix interface debonding and localized matrix brittle failure were identified on the delamination fracture surfaces. As a result, it seems reasonable to deduce that there is similarity in fibre-bridged FDG at the crack front in the perspective of damage evolution, i.e. there is self-similar fatigue crack propagation around the delamination front regardless of fibre bridging development. To the authors' opinion, this can play the physical background for the success of using  $G_{tip}$  in fibre-bridged FDG characterization reported in the above mentioned studies [24,29–31].

According to above discussions, the objective of the present study is therefore to have thorough discussions on the validity of using different similitude parameters (i.e.  $\Delta\sqrt{G}$  and  $\Delta\sqrt{G_{tip}}$ ) in fibre-bridged FDG interpretations. And a new similitude parameter, based on the authors' previous study [21], was proposed to appropriately characterize FDG behavior with fibre bridging at different  $R$ -ratios in accordance with the similitude principles.

## 2. Material, fatigue experiments and data reduction

For the convenience of the readers, this section will briefly repeat the description of the specimen preparation, fatigue experimental set-up, fatigue experimental procedure, and data reduction method, as previously introduced in [22,28,31,32].

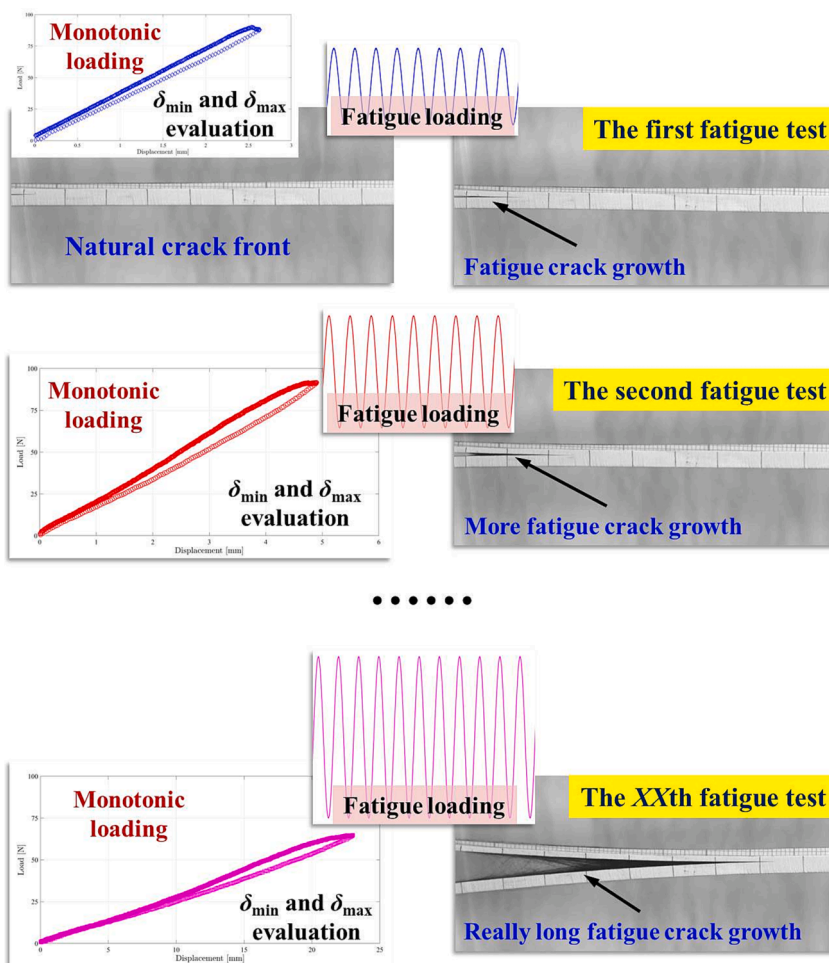


Fig. 2. A schematic illustration of the test procedure for fatigue delamination with different amounts of fibre bridging.

## 2.1. Material and specimen preparation

Unidirectional double cantilever beam (DCB) specimens were manufactured and tested to investigate fatigue delamination behavior with fibre bridging. The composite laminates were produced by hand-lay-up of thermosetting carbon/epoxy prepreg M30SC/DT120 with stacking sequence  $[0_{16}/0_{16}]$ . A 12.7  $\mu\text{m}$  Teflon film was inserted in the middle plane of these laminates during the hand-lay-up process to act as an initial delamination typically around  $a_0 = 60.0$  mm. The laminates were cured in vacuum in an autoclave at a pressure of 6 bars and curing temperature of 120°C for 90 min. The nominal thickness of the composite laminates after curing is 5.0 mm. These laminates were C-scanned to detect potential imperfections. They were subsequently cut with a diamond saw into 25.0 mm width beams with 200.0 mm length. Only those samples were tested where the C-scan did not reveal any obvious imperfection. A pair of aluminum loading blocks, 25.0 mm width by 20.0 mm length with 6.0 mm thickness, was adhesively bonded onto the specimen at the side of the Teflon insert for load introduction. And the distance from the initial crack front to the loading line is equal to  $a_0$  subtracting 15.5 mm (i.e. the distance between the loading line and the pre-cracked DCB end).

One side of the DCB specimen was coated with a thin layer of typewriter correction fluid to enhance visibility of the delamination front during the fatigue test. A strip of grid paper was pasted on the coated side of the specimen to aid in measuring the delamination propagation length  $a-a_0$ .

## 2.2. Experimental procedure and fatigue data reduction

All fatigue experiments were conducted on a 10kN MTS servo-hydraulic test machine under displacement control at a frequency of 5 Hz. Images of fatigue crack propagation were automatically recorded at the maximum displacement during the test with pre-defined intervals via a computer controlled digital camera system with high resolution. The corresponding information of force, displacement and fatigue cycle number was automatically stored in an Excel file enabling data reduction after the test.

To have fatigue delamination with different amounts of fibre bridging, DCB specimens were repeatedly tested for several times with increased displacements keeping the  $R$ -ratio constant. This specific test procedure is schematically illustrated in Fig. 2. And both ESIS TC4 and ISO/TC61/SC13 have paid keen attention to this test procedure proposed by the first author in developing mode I fatigue delamination test standard for composite laminates. Typically, DCB specimen was first quasi-statically loaded to generate a really short natural crack front (2–3 mm), as well as to determine the minimum and maximum displacements used in the subsequent fatigue delamination test. One should note that FDG rate  $da/dN$  gradually decreases with decreasing  $SERR$  in displacement controlled tests. Each test was therefore manually terminated in case of crack retardation. Subsequently, a monotonic loading–unloading cycle was performed on the tested specimen until the load–displacement curve becomes slightly nonlinear (i.e. avoiding new fibre bridging generation in quasi-static delamination) to evaluate the minimum and maximum displacements applied in the subsequent fatigue test sequence. This sequence was repeated multiple times until the maximum displacement capacity of the test machine was reached. With this test procedure, multiple delamination resistance curves were obtained, with each one representing delamination resistance equivalent to a specific fatigue pre-crack length, i.e. delamination length at which that particular fatigue test was initiated.

To evaluate the  $R$ -ratio effects on FDG behavior, incorporating with the FDG data of  $R = 0.1$  and  $0.5$  that have been reported in our previous studies [28,31], another group of fatigue experiments were performed at a different  $R$ -ratio  $0.3$  in this study. A summary of all these fatigue experiments are provided in Table 1.

The Modified Compliance Calibration (MCC) method Eq.(4), recommended in the ASTM D5528 standard, was employed to calculate the  $SERR$   $G$  with the load, displacement and crack propagation length information recorded in fatigue delamination tests. And the 7-point Incremental Polynomial Method, recommended in the ASTM E647 standard, was used to fatigue crack growth rate  $da/dN$  calculation.

$$G = \frac{3P^2 C^{(2/3)}}{2A_1 Bh} \quad (4)$$

where  $P$  is the load;  $C$  is the compliance of the DCB specimen;  $A_1$  is the slope of the curve in the graph where  $a/h$  is plotted against  $C^{1/3}$ .

**Table 1**  
Fatigue delamination test matrix.

Specimen	$R$ -ratio	Fatigue pre-crack length [mm]
Spe-1	0.1	4.26; 15.52; 28.48; 40.49; 54.02; 68.64
Spe-2	0.1	5.80; 17.02; 29.77; 44.24; 60.95; 80.25; 100.85
Spe-3	0.3	3.96; 16.54; 29.77; 40.59; 55.90; 65.90; 85.28;
Spe-4	0.3	2.62; 14.97; 27.19; 39.66; 48.90; 62.25; 80.31
Spe-5	0.5	3.83; 12.14; 19.80; 27.07; 37.51; 47.86; 60.24; 68.69
Spe-6	0.5	4.09; 13.16; 29.26; 40.08; 51.92; 64.79; 81.33

### 3. Results and discussions

#### 3.1. Using the total SERR $\Delta\sqrt{G}$ as similitude parameter in FDG interpretations

All fatigue data were first interpreted via the Paris relation Eq.(1) and expressed in terms of  $da/dN$  against  $\Delta\sqrt{G}$ . People who have interesting to these data interpretations with  $R$ -ratios 0.1 and 0.5 are referred to literature [28,31]. Fig. 3 provides a summary of these data interpretations for  $R = 0.3$ . It is clearly that the presence of fibre bridging can cause significant retardation effects on FDG behavior. Particularly, these Paris resistance curves can downwards shift from left to right in the graph, and finally tend to converge into a narrow band region with delamination propagation (i.e. relevant to bridging development and saturation).

The presence of fibre bridging, as illustrated in Fig. 1 at the behind of crack front, is the main reason for this crack growth retardation, as a large number of SERR could be periodically stored and released in these bridging fibres with delamination propagation (i.e. alleviate stress concentration around the crack front). The amount of bridging fibres generated in FDG can be phenomenologically represented by crack propagation length  $a-a_0$ . If one takes close examinations on these results, it can be found that FDG behavior with the same pre-crack length  $a-a_0$  (i.e. the same amount of bridging fibres) remains the same. This can also provide indirect evidence to support above discussion of fibre bridging retardation effects on FDG behavior.

According to the requirements of the similitude hypothesis, one can make a statement, with the resistance curve distribution illustrated in Fig. 3, that the use of  $\Delta\sqrt{G}$  cannot well represent the similitude in FDG with fibre bridging, as delamination cannot be uniquely characterized by  $\Delta\sqrt{G}$  but depends on crack length.

The use of energy principles in fibre-bridged FDG has demonstrated that the presence of fibre bridging indeed has little contribution to permanent energy dissipation, but periodically store and release strain energy in fatigue cycles [32]. Most damage evolution in fibre-bridged FDG indeed is still concentrated around the crack front, regardless of fibre bridging. Furthermore, it has been reported [22,31] that the microscopic damage features located on fracture surfaces remain the same in crack propagation with fibre bridging. All these indicate that damage mechanisms in fibre-bridged FDG around the crack front remain the same or similar in physics, i.e. there is self-similar crack evolution in fibre-bridged FDG at the crack front. As a result, similitude still exists in fatigue delamination with fibre bridging. Thus, the left issue is that: *how to employ an appropriate parameter to well represent this similitude?*

Referring to the J-integral Eq.(3), it seems reasonable to attempt the SERR applied to the crack front as similitude parameter to represent FDG behavior with fibre bridging, as this parameter closely related to the self-similar damage propagation around the crack front. Thus, the use of  $\Delta\sqrt{G_{tip}}$  as the similitude parameter in fibre-bridged FDG interpretations will be given and discussed in what follows.

#### 3.2. Using $\Delta\sqrt{G_{tip}}$ as similitude parameter in FDG interpretations

A modified Paris relation Eq.(5), employing  $\Delta\sqrt{G_{tip}}$  as the similitude parameter, has been proposed and verified by the authors in FDG interpretations in the previous studies [31,32]. The determination of  $\Delta\sqrt{G_{tip}}$  in this correlation is indeed based on an empirical method introduced in literature [30].

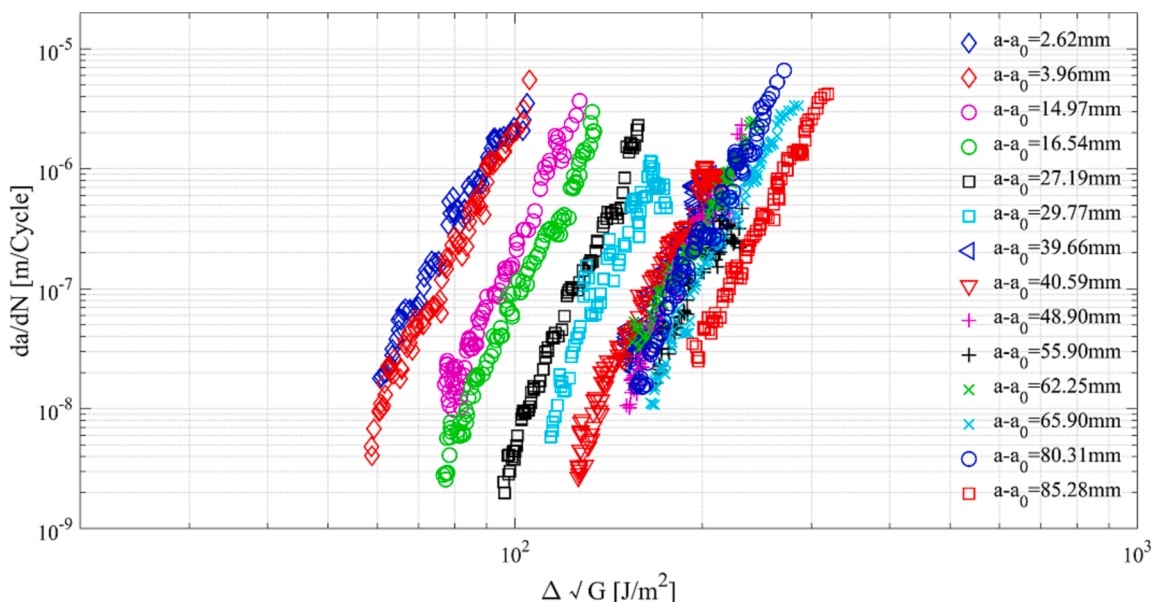


Fig. 3. Fatigue data interpreted via the similitude parameter  $\Delta\sqrt{G}$  ( $R = 0.3$ ).

$$\frac{da}{dN} = C \Delta \sqrt{G_{tip}}^n = C \left[ \frac{G_{IC0}}{G_{IC}(a-a_0)} \Delta \sqrt{G} \right]^n \quad (5)$$

where  $G_{IC}(a-a_0)$  represents fatigue delamination fracture toughness increase because of fibre bridging. This fatigue resistance curve (i. e. fatigue R-curve) can be conveniently determined with the monotonic loading–unloading cycle. And  $G_{IC0}$  is the fatigue fracture toughness excluding fibre bridging contribution.

Fig. 4 summarizes the fatigue R-curves of different R-ratios. And the second-order polynomial function was employed to appropriately fit these results. It is clear that delamination resistance can increase with crack propagation because of fibre bridging. And the magnitude of these resistance curves seems to be R-ratio dependent. Particularly, the R-curve for  $R = 0.5$  is slightly higher than the other stress ratios, indicating more bridging fibres present in FDG of  $R = 0.5$ . In the studies conducted by Yao et al., [33] and Jensen et al., [23,34], it has been also reported that more fibre bridging can be present in fatigue delamination with high *SERR*, as compared to that with low *SERR*.

One should keep in mind that the amount of bridging fibres created in fatigue delamination is different from that in quasi-static. As a result, one cannot use the quasi-statically determined R-curve in fatigue data reduction. The fatigue R-curve derived from the specific test procedure proposed by the first author should be employed in fatigue data analysis.

All fatigue data of different stress ratios were interpreted via Eq.(5) to verify the validity of using  $\Delta \sqrt{G_{tip}}$  in appropriately representing FDG behavior with fibre bridging. Figs. 5–7 summarize all these interpretations in terms of  $da/dN$  against  $\Delta \sqrt{G_{tip}}$ . As a comparison, the results illustrated in Figs. 3 and 6 for  $R = 0.3$  clearly demonstrate that FDG behavior with different amounts of bridging fibres tend to collapse into a narrow band region once using  $\Delta \sqrt{G_{tip}}$  as the similitude parameter. In other words, fatigue delamination behavior can be uniquely characterized via  $\Delta \sqrt{G_{tip}}$  with fibre bridging development keeping the R-ratio constant, which agrees well with the similitude principles. The same conclusion can be also made with the fatigue data interpretations summarized in Figs. 5 and 7 for  $R = 0.1$  and  $0.5$ . Accordingly, one can make an important conclusion that  $\Delta \sqrt{G_{tip}}$  is a reasonable similitude parameter in fibre-bridged fatigue delamination interpretations at a given R-ratio.

All above discussions clearly demonstrate the great importance of employing an appropriate similitude parameter in fibre-bridged FDG behavior interpretations. One may reasonably ask a question that: *Does  $\Delta \sqrt{G_{tip}}$  still work for fatigue delamination with fibre bridging at different R-ratios?*

To answer this question, a comparison study was conducted on these fatigue data interpretations, as illustrated in Fig. 8. It is clear that there is obvious R-ratio dependence on fatigue delamination behavior. Particularly, fatigue crack growth can accelerate with increased R-ratio. This dependence indeed violates the requirement of the similitude hypothesis. As a result, one can make a conclusion that  $\Delta \sqrt{G_{tip}}$  can only be valid in determining fibre-bridged FDG behavior of the same R-ratio. However, it cannot appropriately represent fatigue delamination behavior of different R-ratios in the perspective of similitude.

Thus, the new question arises here as *how could one appropriately determine fatigue delamination behavior with fibre bridging at different R-ratios in accordance with the requirements of the similitude principles.* With consideration of fully characterizing a cyclic loading, it seems reasonable to use both  $\Delta \sqrt{G_{tip}}$  and  $G_{max,tip}$  around the crack front to represent FDG behavior with different amounts of bridging fibres at various R-ratios. And detailed discussions on the application of using both  $\Delta \sqrt{G_{tip}}$  and  $G_{max,tip}$  as similitude parameter in fibre-bridged FDG interpretations will be provided as follows.

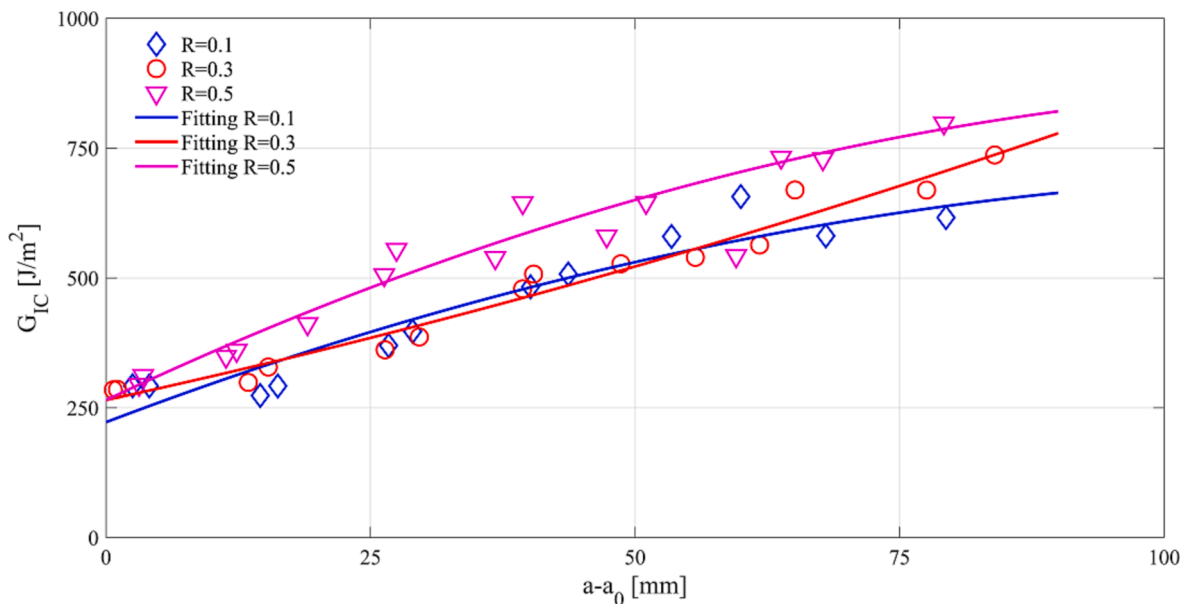


Fig. 4. Fatigue delamination resistance curves of different R-ratios.



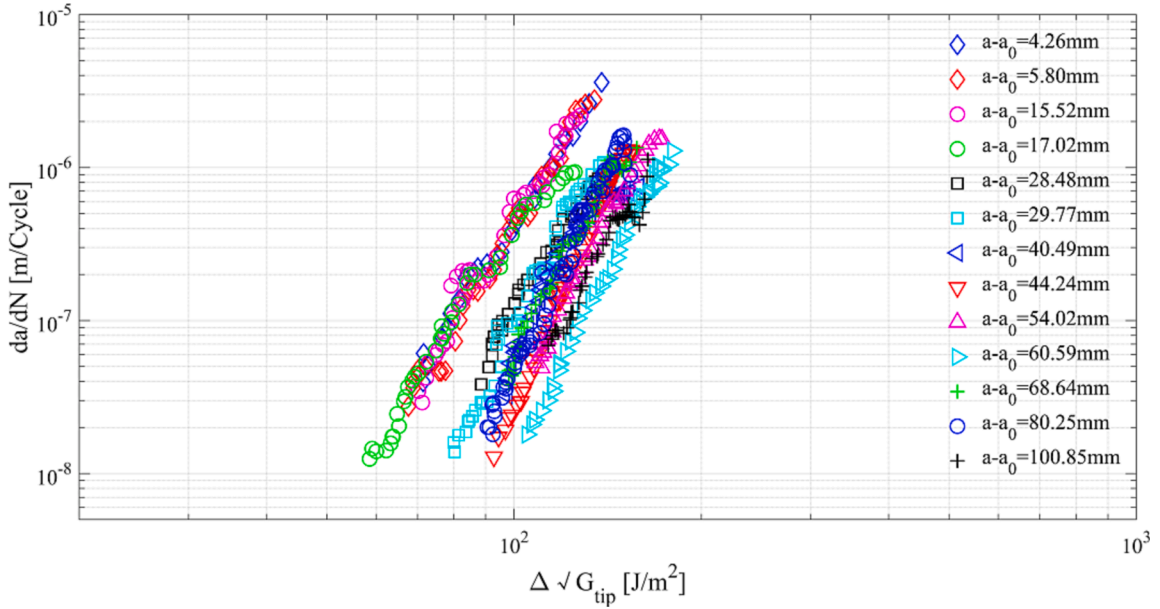


Fig. 5. Fatigue data of  $R = 0.1$  interpreted via the similitude parameter  $\Delta\sqrt{G_{tip}}$ .

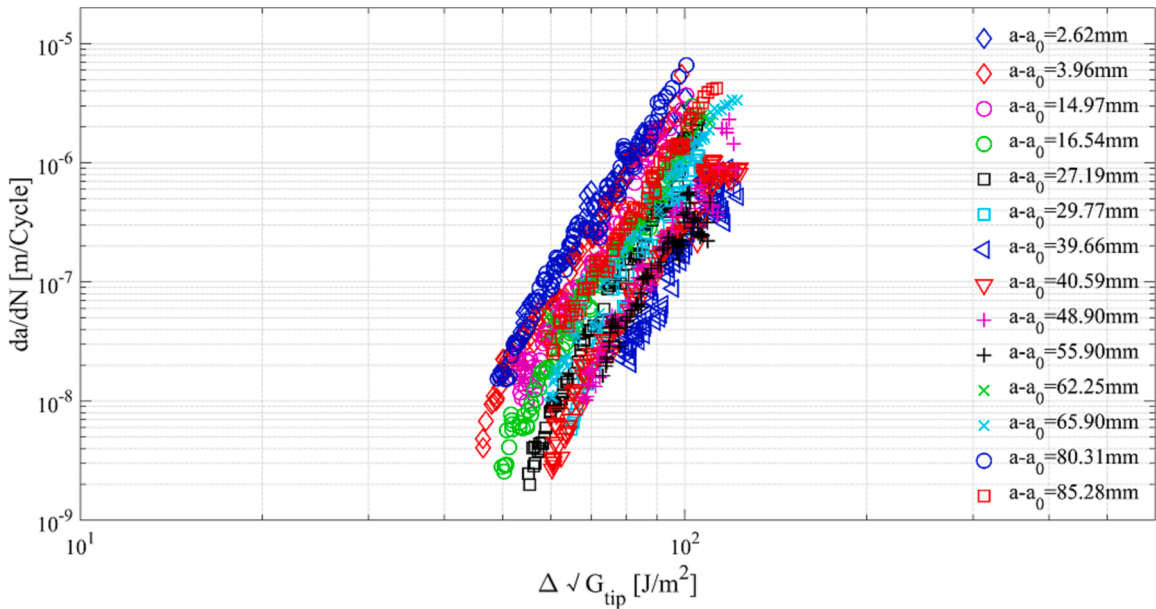


Fig. 6. Fatigue data of  $R = 0.3$  interpreted via the similitude parameter  $\Delta\sqrt{G_{tip}}$ .

### 3.3. Using both $\Delta\sqrt{G_{tip}}$ and $G_{max\_tip}$ as similitude parameter in FDG interpretations

According to a recent study completed by the authors [21], the Paris-type correlation, employing both  $\Delta\sqrt{G_{tip}}$  and  $G_{max\_tip}$  to represent the similitude in fatigue delamination with fibre bridging at different  $R$ -ratios, can be expressed as

$$\frac{da}{dN} = C\Delta\sqrt{G_{eff}}^n = C \left[ \Delta\sqrt{G_{tip}} \left[ 1 - \left( \frac{G_{max\_tip}}{G_{IC0}} \right)^\gamma \right] G_{max\_tip} \left( \frac{G_{max\_tip}}{G_{IC0}} \right)^\gamma \right]^n \quad (6)$$

where  $\gamma$  is a curve-fitting parameter; and  $G_{max\_tip}$  can be conveniently calculated via

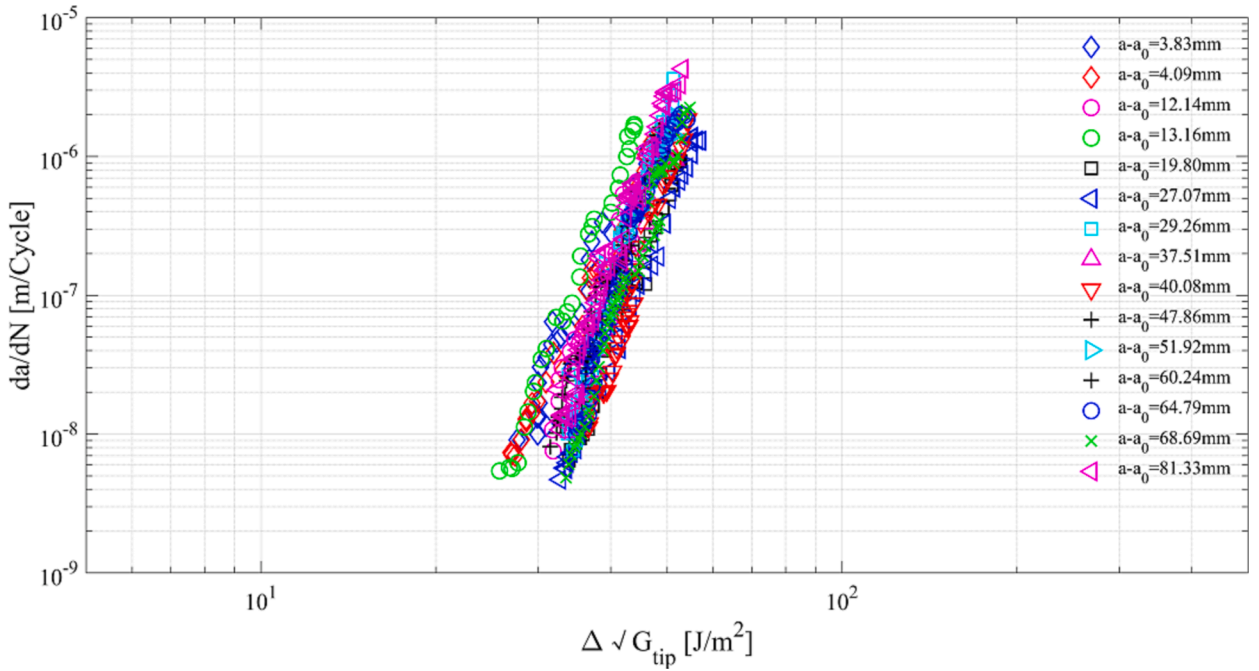


Fig. 7. Fatigue data of  $R = 0.5$  interpreted via the similitude parameter  $\Delta\sqrt{G_{tip}}$ .

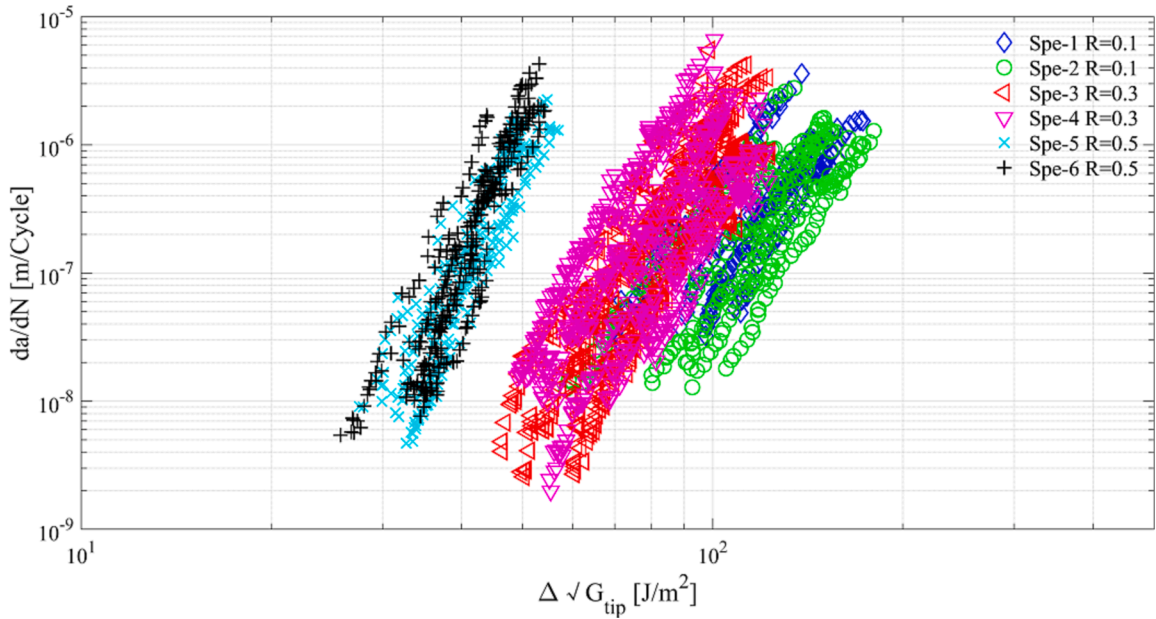


Fig. 8.  $R$ -ratio dependence of FDG behavior interpretations using  $\Delta\sqrt{G_{tip}}$ .

$$G_{max\_tip} = \frac{\Delta\sqrt{G_{tip}}}{(1 - R)^2} \tag{7}$$

Fig. 9 provides a summary of all fatigue data of various  $R$ -ratios interpreted via Eq.(6). It is clear that all these data with different amounts of bridging fibres at various  $R$ -ratios can converge into a band region. Despite some data scatter, a master resistance curve can be calibrated to represent fatigue delamination behavior, obeying well with the similitude hypothesis. Incorporating with the results illustrated in Fig. 8, it is therefore recommended to use the similitude parameter  $\Delta\sqrt{G_{eff}}$ , in terms of both  $\Delta\sqrt{G_{tip}}$  and  $G_{max\_tip}$ , for fibre-bridged FDG interpretations at different stress ratios in the perspective of obeying the similitude principles.

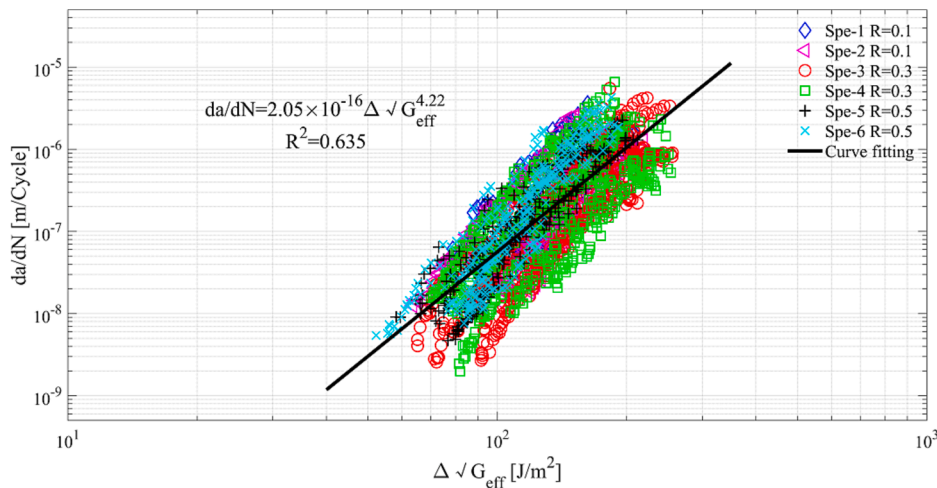


Fig. 9. Fibre-bridged fatigue delamination data interpreted via  $\Delta\sqrt{G_{\text{eff}}}$ .

Scatter indeed is an important and inevitable issue frequently reported in fatigue delamination study. Researchers [35,36] indeed have provided some critical discussions on the sources of scatter, i.e. intrinsic and extrinsic scatter, and gave suggestions for reducing scatter. Particularly, both automated manufacturing process and digital technology were recently recommended to scatter reduction [36]. In the perspective of fatigue data interpretation, Murri [9] reported that the application of normalization, whereby the total applied *SERR* is normalized by delamination resistance, can significantly reduce fatigue delamination data scatter. Furthermore, in a recent study conducted by Jones et al., [37], an alternative normalization approach, i.e. dividing by the threshold, has been proposed to reduce scatter in the important “near-threshold” region with consideration of using no crack growth philosophy in composite structural design.

To the authors’ opinion, the present study can provide evidence that the importance of using appropriate similitude parameters in reducing scatter, as clearly illustrated in Figs. 3 and 6 for  $R = 0.3$ , Figs. 8 and 9 for different  $R$ -ratios. This study indeed provides a clue that people can follow this pattern to reduce fatigue scatter in future study: *First, one should employ a reasonable parameter to appropriately represent the similitude in fatigue delamination. Based on data interpretations with this appropriate similitude parameter, other methods, such as normalisation and so on, could be subsequently applied for reducing scatter further.*

#### 4. Concluding remarks

Similitude principles play an important role in fatigue delamination data interpretation. The present study highlights the importance of obeying similitude in appropriate fatigue data interpretation. The results clearly demonstrate that the *SERR* applied on the crack front should be employed to represent fatigue delamination behavior with large-scale fibre bridging. Particularly, the use of  $\Delta\sqrt{G_{\text{tip}}}$  as similitude parameter can well determine fatigue delamination behavior with fibre bridging keeping the  $R$ -ratio constant. However, there is still obvious  $R$ -ratio dependence using this parameter in fibre-bridged fatigue data interpretations, violating the similitude principles. As a result, the use of  $\Delta\sqrt{G_{\text{tip}}}$  cannot fully represent the similitude in fatigue delamination with fibre bridging at different  $R$ -ratios.

According to our previous study on stress ratio effects on FDG [21], a new similitude parameter  $\Delta\sqrt{G_{\text{eff}}}$ , in terms of both  $\Delta\sqrt{G_{\text{tip}}}$  and  $G_{\text{max\_tip}}$ , has been proposed for FDG with fibre bridging at different  $R$ -ratios. The use of this parameter can well represent fibre-bridged fatigue delamination behavior, which agrees well with the requirements of the similitude principles. Particularly, all fatigue data with different amounts of fibre bridging at different  $R$ -ratios can converge into a band region, contributing to a master resistance curve in determining FDG behavior.

This study can provide solid evidence on the importance of using an appropriate similitude parameter in fibre-bridged fatigue delamination characterization, which is important for the development of mode I fatigue delamination test standard of composite laminates aimed by ESIS TC4 and ISO/TC61/SC13.

#### CRedit authorship contribution statement

**Liaojun Yao:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Mingyue Chuai:** Visualization, Validation, Software, Methodology, Formal analysis, Data curation. **Zhangming Lyu:** Resources, Methodology, Investigation. **Xiangming Chen:** Resources, Methodology, Investigation. **Licheng Guo:** Resources, Methodology. **R.C. Alderliesten:** Writing – review & editing, Methodology, Data curation.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.

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