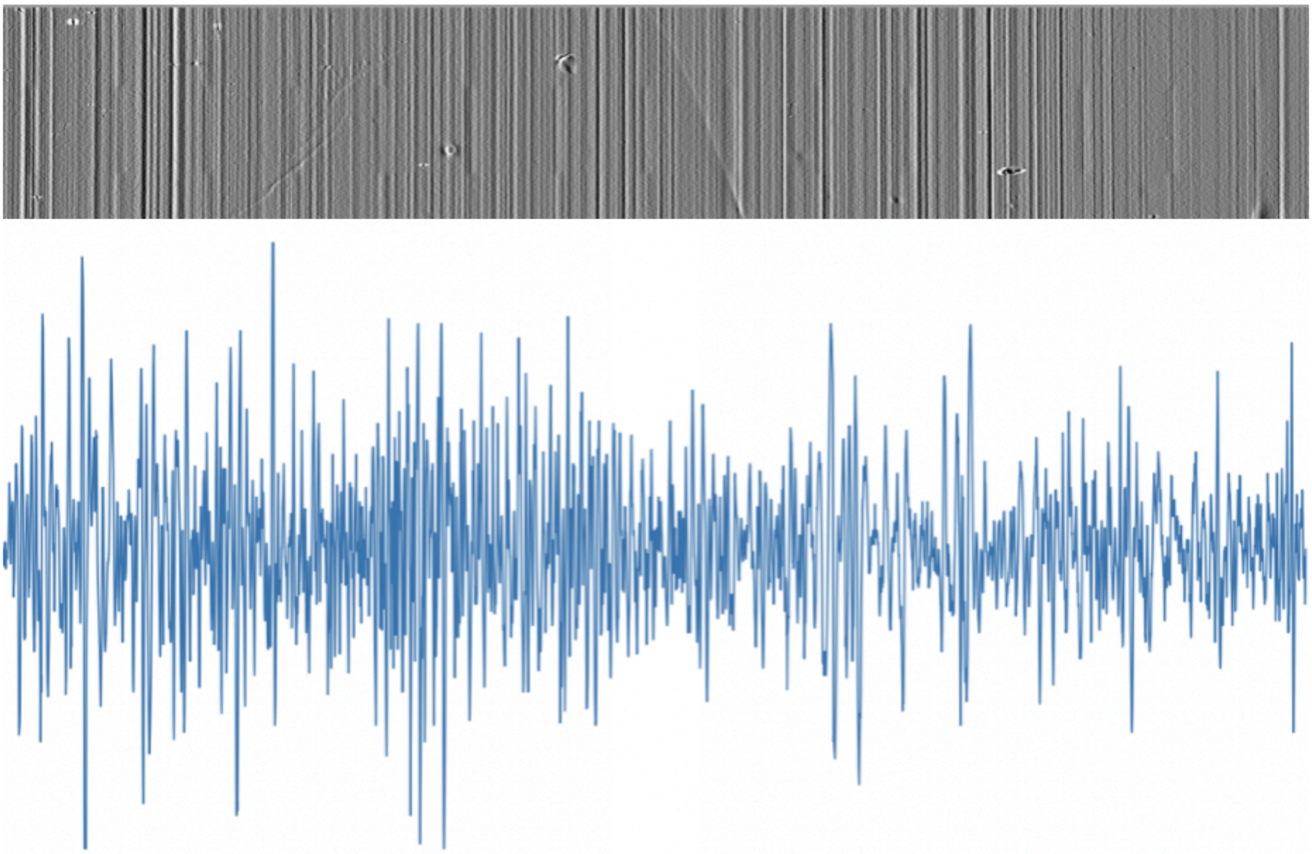
Automating the Comparison of Knife Striation Patterns

Master Thesis

Nina Serdar



TU Delft

Netherlands Forensic Institute
Ministry of Justice and Security

Automating the Comparison of Knife Striation Patterns

by

Nina Serdar

in partial fulfillment of the requirements for the degree of
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Nina Serdar
Delft, May 2026

Abstract

When a knife cuts into a material, it creates in it a set of lines, i.e. a striation pattern. Since the striation pattern is individual to each knife, by looking at it, it can be determined (with more or less uncertainty) whether a cut was created by a given knife. This is particularly useful in forensic science for stabbing cases, where it is of interest to know which knife was used for the stabbing. However, the comparison of striation patterns is quite challenging, as they appear different depending on the angle of attack under which they were created, which is generally unknown for the stabbing cases. This project provides a model to compare striation patterns created at different angles of attack, using Dynamic Time Warping. This model was tested with striation patterns created by 6 different knives, and it showed promising results, however some improvements should still be made, which are discussed in the research. Finally, this model provides a solid base to calculate likelihood ratios, which are used to express the certainty that a cut was created by a given knife, as opposed to by another unknown knife.

Contents

1
Introduction

In the last years, since 2020, the number of murders in the Netherlands has been roughly constant, in fact each year between 120 and 125 people were killed. However, the number of deaths by stabbing has been increasing. Namely the number of people who were killed by stabbing or by a battle weapon was 46 in 2020, 58 in 2023, and 60 in 2024. In particular, in 2024 the murders by stabbing or a battle weapon represented exactly half of the total number of homicides [21]. This concerning increase of cases of stabbings makes it crucial to be able to understand how these cases need to be analyzed and investigated.

One of the main questions that is asked while investigating a stabbing case is what was the knife that was used for the stabbing. This question is explored in the following way: a "suspect knife" is found/selected. This could be, for example, a knife that is found on the crime scene. Then, some test cuts are made with this suspect knife in a forensic laboratory. These cuts are then compared to the cuts in the wounds of the victim that was stabbed [9]. The comparison between the two sets of cuts is made by looking at the striation patterns present in them and the individual characteristics that they present [8]. In fact, the striation patterns are the fine lines that the knife creates while cutting through the material (a picture is shown in Figure 1.1) and each knife, while cutting, leaves the same pattern, which is individual to that specific knife. These patterns can be seen better in harder materials, as they deform less while being cut. Thus, typically, the victim's wounds that are considered for the comparison are the ones in their bones [20] and cartilage [25] [17] (preferably the cartilage [12]), and the test cuts made in the forensic laboratory are made in a material which resembles the human cartilage in its physical properties. If the test cuts present the same striation pattern as the wounds of the victim, it is strong evidence that the knife used for the stabbing is indeed the same, i.e. the suspect knife.

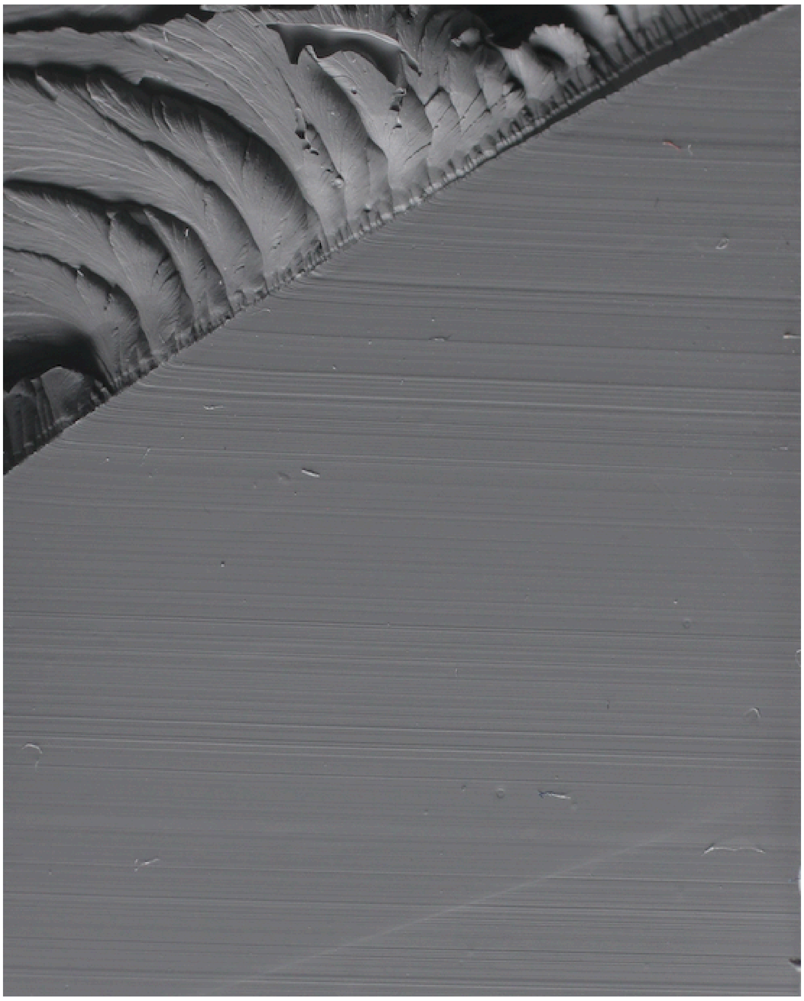


Figure 1.1: A cast of the striation pattern made by a knife

This approach to find out whether the suspect knife is the knife that was used in the stabbing case presents many challenges and issues. Namely:

- The comparison is currently done by hand
  As of now, the test cuts made in the forensic laboratory are compared to the wounds of the victim by a forensic scientist, without the help of a software or an automated system. In fact, the forensic scientists compares the two sets of cuts by looking at them under a microscope (to better see the patterns) and trying to see if the patterns of the two sets of cuts match. The success of this process, thus, depends heavily on the abilities of the forensic scientists working on it.
- The comparison is very time consuming
  As previously explained, a given knife will produce the same striation pattern while cutting into a material, i.e. it will create the same lines in the material, as long as the cutting edge is not altered. However, the distance between each of the lines that it will create depends on the angle under which the material was cut. So two cuts made by the same knife under two different angles will present the same striation pattern, i.e. the same set of lines, however in one striation pattern the lines will be closer together and in the other one they will be further apart from each other. This complicates the comparison between the striation patterns in the test cuts and in the victim's wounds. In fact, it is generally unknown to the forensic scientist what was the angle of attack for the cuts in the victim's wounds. Thus, it is impossible to recreate some test cuts under that same angle to then compare the two. What is currently done by the forensic scientists is to make multiple test cuts from different angles and investigate if any match the striation patterns in the victim's wounds. As this process requires a lot of trial and error, it is very time consuming.

- It is difficult to evaluate the strength of the evidence
  Normally, if a match is found between the striation patterns in the test cuts and the striation patterns in the victim's wounds, the forensic laboratory is then asked to calculate a likelihood ratio [11] [15]. The likelihood ratio is calculated with the following formula [2]:

  $$LR = \frac{P(E|H_1)}{P(E|H_2)}$$

  where E is the evidence, i.e. that there is a correspondence between the striation patterns of the test cuts and the victim's wounds, H_1 is the hypothesis that the knife used in the stabbing incident is the suspect knife, and H_2 is the hypothesis that the knife used in the stabbing incident is an unknown knife. The higher the likelihood ratio, the higher the evidence supports H_1, and the lower the likelihood ratio, the higher the evidence supports H_2. As of now, it is challenging to estimate $P(E|H_2)$, i.e. the probability that the correspondence/match that was found was a random match. This is because we are interested in knowing how likely it is to find a correspondence between the striation pattern in the victim's wounds and the striation pattern that a random knife, unrelated to the crime, would produce. In order to answer this question, a comparison between the striation pattern in the victim's wounds and the striation pattern caused by multiple different knives would need to take place. However, this process would take a very long time and thus it is unfeasible. So currently $P(E|H_2)$ is determined by the forensic laboratory by making an estimate, and not by making multiple other comparisons with different knives.

The aim of this research is to build a software that automatizes the comparison of the striation pattern of the test cuts and the striation pattern of the victim's wounds. More specifically, the goal is to build a software that takes as input two different pictures of striation patterns created by either the same knife or by two different knives, and helps to determine whether the two striation patterns are the same (and thus created by the same knife) or if they are different. This software can then be used to automatizes the comparison of the striation pattern of the test cuts and the victim's wounds by giving as input a picture of the striation pattern made by the suspect knife in the forensic laboratory and a picture of the striation pattern on the victim's wounds. The software will then be able to determine whether the striation pattern created by the suspect knife is the same striation pattern found in the victim's wounds, and thus provide an insight on whether the suspect knife was used for the stabbing attack.

This research is relevant as its results will significantly improve the current method that is used to find out whether the suspect knife is the knife that was used in the stabbing case. In fact, it will solve the three main problems that the current method has. Namely, firstly it will automatize the comparison process, so that the outcome of the comparison process will no longer depend on the abilities of the specific forensic scientist working on it, but rather on a method that is consistently used each time. Secondly, it will be more time effective, as it will be able to compare larger amounts of striation patterns in a much shorter time compared to a human. Finally, it will be able to provide a useful insight on how to find $P(E|H_2)$ to calculate the likelihood ratio. Namely, if the comparison process can be executed fast, the striation pattern in the victim's wounds can be compared to the striation patterns created by multiple other (unrelated) knives. This would help determining how likely it is to find a match with another (unrelated) knife as well.

The rest of this project is structured as follows: Chapter 2 provides some background information as to how the knife striation patterns are created and analyzed and Chapter 3 illustrates the previous work that was done for this research. In Chapter 4 the sub-research questions for this research are defined and in Chapter 5 some mathematical tools that could be useful to answer these sub-research questions are explained. Chapter 6 presents the first approach that was used for this research, which, however, was not successful. The final model that was created is explained in Chapter 7, and Chapter 8 describes how this model was tested and with what data. The results of these tests are shown in Chapter 9, and they are discussed in detail in Chapter 10. This Section also provides insights on how to further improve the model proposed, and a comparison with the model that was previously developed. Finally, Chapter 11 presents the conclusions and questions for further research.

2

Background Information

2.1. Making the Test Cuts

When a suspect knife needs to be analyzed, some test cuts are made with it in order to see what the striation pattern created by that knife looks like, so that it can be compared to the striation pattern found in the victim's wounds. The test cuts are made using the machine shown in Figure 2.1.

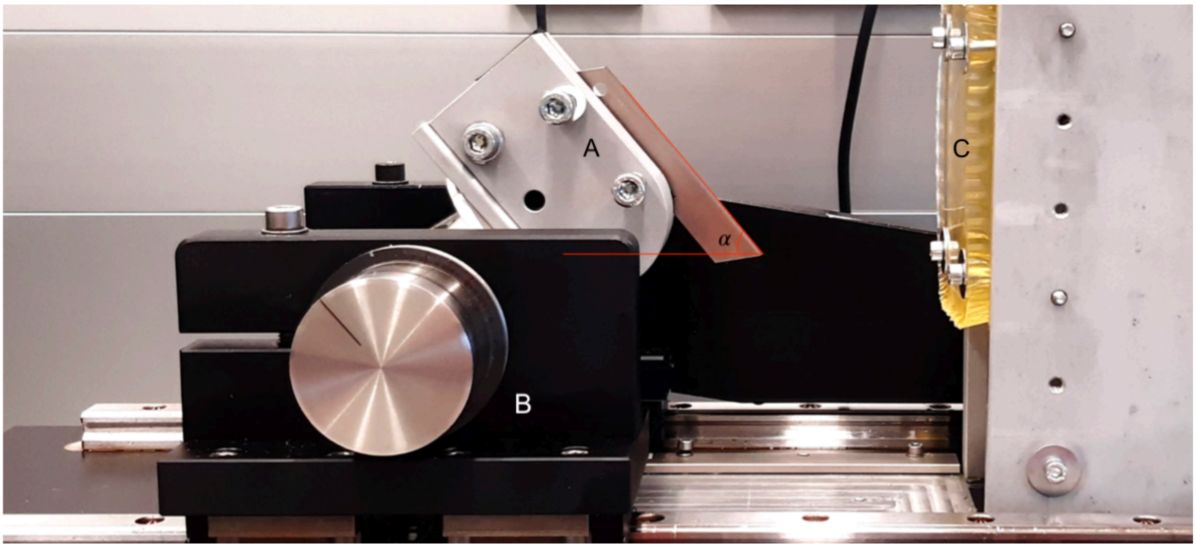


Figure 2.1: The machine used to create the test cuts [14]

This machine works as follows: the knife is put and fixed in the position where the paper cutter is in the figure (position A). The angle of the blade α, which can be seen in the figure, needs to be regulated depending on the angle of attack that needs to be investigated. Regulating the angle of attack can be done by putting a level box on the knife and thus checking this angle. Once the knife is fixed and its inclination is regulated, the machine is activated so that the black part of the machine (part B in the picture) starts sliding to the right, together with the knife as it is attached to it. With this movement, the knife reaches the block on the right (position C in the picture) and it cuts into it. This block is made of a material called ”Dip-Pak” [19] [6] which is a material that resembles the physical properties of the human cartilage and it is similar to wax. It is one of the best materials that can be used to make the lab-created cuts [3] as it is soft enough so that a knife can cut through it, however it is also stiff enough so that it does not deform much while being cut, and in this way the striation pattern created by the knife remains visible in it.

The block of Dip-Pak is then taken out of the machine, and it is carefully opened along the side where the cut was made, so that the striation pattern created by the knife is shown. As the Dip-Pak has a transparent-yellow color, it is difficult to see the striation pattern well in it, and so a cast of the striation pattern is made in a gray material, so that the lines of the striation pattern can be seen more easily. Figure 1.1 shows a picture of a cast of a striation pattern.

2.2. Processing the Test Cuts

After creating the physical cut in the Dip-Pak and making a cast of the striation pattern it created, this data is processed so that it can be read easily by a computer. More specifically, it is of interest to know precisely the distance between each of the lines in the striation pattern, as well as their physical height on the cast, and their width. To retrieve this information, the cast of the striation pattern is scanned with a 3D scanner. This machine, in fact, is able to accurately detect the differences in heights of the multiple lines on the surface of the cast creating the striation pattern, and it uses this information to produce a 3D model of the cast. This 3D model is then further processed with a software called "Scratch" [4] which turns the 3D model into a two-dimensional picture. This is done by converting the height dimension in the 3D model (i.e. the height of each of the lines in the striation pattern) into a gray-scale, applying a filtering for noise and shape reduction, and keeping the two other original dimensions are they are. So, in other words, the lines of the striation pattern are shown in the two-dimensional picture with different colors, depending on their original height on the cast. More specifically, the lighter the color of the lines, the higher their original height in the cast was. Figure 2.2 shows an example of a striation pattern after it has been processed with Scratch.

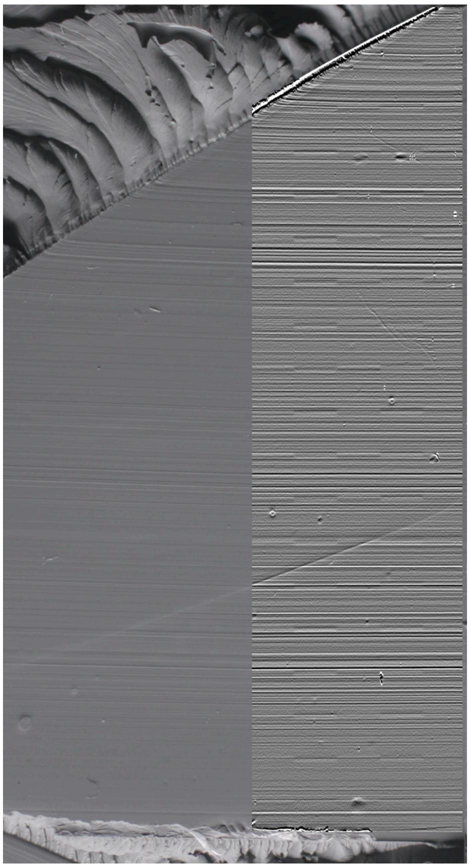


Figure 2.2: A cast of a striation pattern and its image processed in Scratch

Figure 2.2 shows on the left a cast of a striation pattern created by a knife, and on the right the two-dimensional image of that striation pattern created by Scratch. As it can be seen, the lines of striation patterns on the left and on the right perfectly coincide, however their differences in height is much easier to spot on the image on the right as it has been turned into a gray-scale.

2.3. Challenges in Comparing Striation Patterns

Once the striation pattern has been processed with Scratch, it can be compared to another striation pattern that needs to be investigated, for example the striation pattern found in the victim's cartilage. This step, however, presents two main difficulties. The first one is that it is not generally known what part of the knife created the striation pattern on the victim's cartilage and so it is difficult to know which lines of the striation pattern of the knife (if any) they are. In other words, the striation pattern observed on the cartilage is only a small part of the total striation pattern that that knife can produce. This is because only a fraction of the blade of the knife cuts through the cartilage leaving a striation pattern, and not the whole blade. Thus, to match the striation pattern on the cartilage, one has to look at the whole striation pattern produced by a given knife and see if the smaller striation pattern from the cartilage can be found in it at some point, without knowing where exactly it should appear.

The second difficulty in comparing the striation patterns is that the angle of attack for the striation pattern in the cartilage is unknown. Thus, it is very hard to create a test cut with the knife that needs to be investigated with that same angle of attack. This is a problem since it is only meaningful to compare two different striation patterns if they are made with the same angle of attack. To understand why this is the case, it is useful to consider what the striation pattern of a knife looks like for different angles of attack.

The lines in the striation pattern of a knife get closer together or further apart, depending on the angle of attack for the cut. Moreover, this stretching or shrinking of the lines in the striation pattern depending on the angle of attack is not linear. This is because each line in the striation pattern is created by a different point on the blade of the knife and the distances between the lines in the striation pattern depend on the projection of the blade onto the surface that is cut. These distances change non-linearly when the blade is rotated, as the blade is generally curved. As a consequence, the striation patterns undergo a stretching or a shrinking of the lines which is not linear. A visual explanation of this is shown in the following figure.

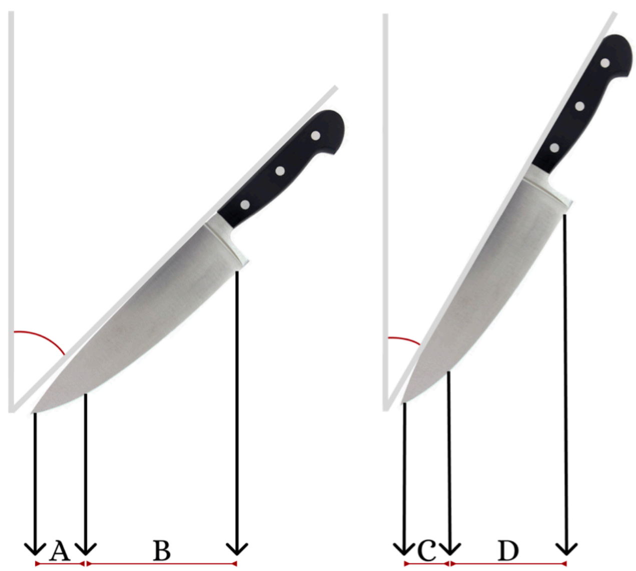


Figure 2.3: Projections of the same points of the blade for two different angles [14]

In Figure 2.3, the same three points of the blade are projected onto the surface that is cut. The distance A is larger than the distance C and the distance B is larger than the distance D. Thus, in the striation pattern created by the knife on the left, the lines will be further apart compared to the striation pattern created by the knife on the right. Thus, the striation pattern for the knife on the left will be a stretching of the striation pattern for the knife on the right. Moreover, the amount by which the distances A and B are larger than the distances C and D respectively is not linear. Thus, the stretching of the striation pattern for the knife on the left is not linear.

Therefore, since the distance between the lines in a striation pattern depends non-linearly on the angle of attack, it is very difficult to visually compare two striation patterns made at different angles of attack. For this reason, it is much more meaningful and preferred to compare striation patterns made with the same angle of attack. However, in real life stabbing incidents, it is almost always unknown what the angle of attack was, and so comparing the resulting striation pattern with some test cuts becomes challenging.

3

Previous Work

3.1. Attempt to Automatize the Comparison of Striation Patterns

In 2023, the problem of automatizing the comparison of knife striation patterns was researched by Pim Meulensteen, while he was working on his Bachelor Thesis [14] as an intern at the NFI. The main idea of the method that he wanted to use can be summarized with the following steps:

1. Make two test cuts with the knife that needs to be investigated at two different known angles, namely α_1 and α_2
2. Get a processed image of these striation patterns that were made
3. Turn these striation patterns into signals
4. Create virtual striation patterns for all possible angles of attack based on these striation patterns and the shape of the blade of the knife
5. Compare each of these striation patterns with the striation pattern that needs to be investigated

The first two steps of this method are clear, as they were already explained in the previous section. The other steps will be explained in detail in the next subsections.

3.2. Turning the Striation Patterns into a Signal

The images of the striation patterns for α_1 and α_2 are turned into signals so that they can be analyzed by a computer. Each of these images is a 2D list of pixels and it is processed in the following way: for each row, the median pixel value is calculated and stored, so that the image is summarized into a 1D list. These values are then normalized between 1 and -1 and they are the values that the signal takes. The signal is then plotted on an x-y plane, normalizing the x-axis between 0 and 1. An example of this process is shown in the following figures.

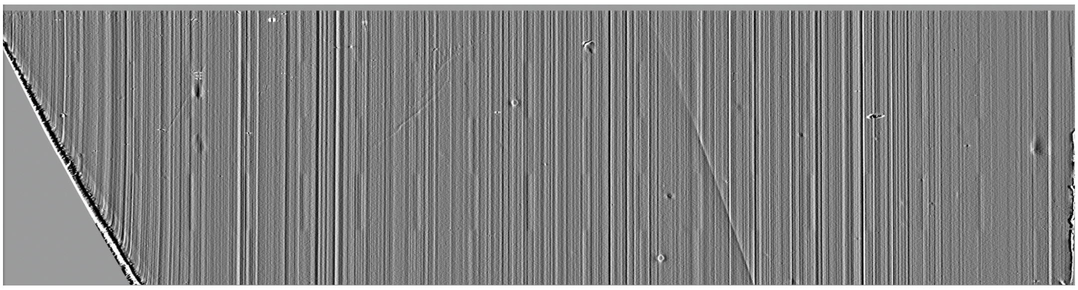


Figure 3.1: The striation pattern after it has been processed in Scratch

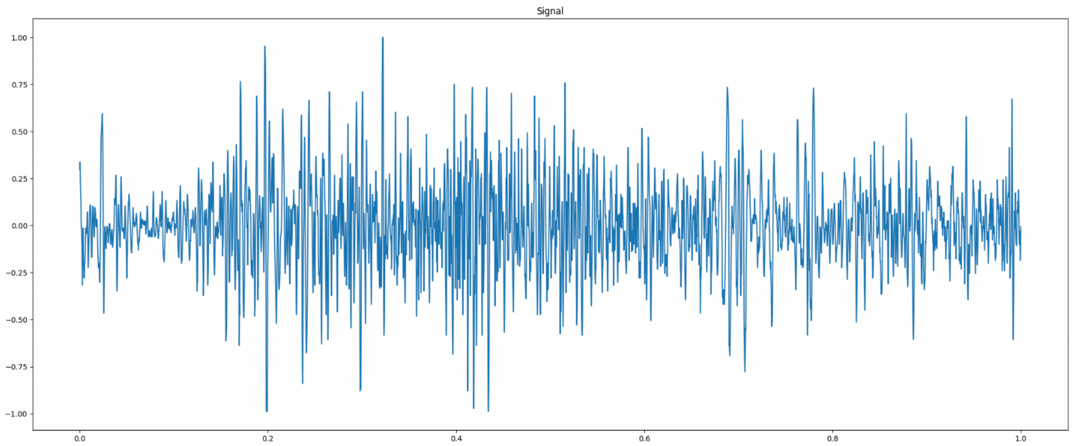


Figure 3.2: The signal resulting from this striation pattern

3.3. Creating a Virtual Knife

Once the striation patterns for α_1 and α_2 are turned into signals, they are used to predict what the striation patterns of that knife will look like for different angles of attack. The main idea, in fact, is that there exists a transformation such that if it is applied to the signal for α_1, it gives the signal for α_2. This is because the striation patterns for α_1 and α_2 are the same, however one is the non-linear compression/shrinking of the other. Thus, once this transformation is known, the striation patterns for any angle of attack can also be known.

In the striation pattern for the angle α_1, each line is created by a different point on the blade of the knife. If it was known which point exactly on the blade created exactly which line, it would be easy to know what striation pattern that knife would create under any angle of attack. In fact, it would be sufficient to scale the distances between the lines in the striation pattern for α_1 based on how the blade of the knife is projected back on the cutting surface, for any given angle. However, it is unknown what points on the blade created exactly which lines in the striation pattern for α_1, so another method is used.

Namely, the part of the blade of the knife that created the striation pattern for α_1 is guessed. Then, based on this estimate, a signal for the striation pattern at α_2 is generated. So, in other words, the signal for α_1 is stretched or compressed depending on the shape of the part of the blade from which the signal is guessed to be from. This generated signal for α_2 is then compared to the known signal for α_2. If the two signals (partially) match, it means that the original guess of which part of the blade created the striation pattern for α_1 is correct, and thus the striation pattern for any other angle of attack can be generated. If the two signals do not match, this process is repeated with another initial guess for which part of the blade created the striation pattern for α_1.

This method was implemented in the following way:

1. A picture of the blade of the investigated knife is taken
2. The shape of the blade of the knife is detected via Canny edge detection
3. A polynomial is fitted to the detected blade of the knife (see Figure 3.3 [14])
4. The part of the blade from which the striation pattern for α_1 comes from is guessed by selecting the interval (begin1, end1) on the blade, where begin1 $\in [x1_{min}, x1_{max}]$ and end1 $\in [y1_{min}, y1_{max}]$
5. For each interval (begin1, end1), a signal for the striation pattern at α_2 is generated
6. An interval (begin2, end2) is chosen, where begin2 $\in [x2_{min}, x2_{max}]$ and end2 $\in [y2_{min}, y2_{max}]$. This interval is a guess of which part of the blade the known striation pattern for α_2 comes from

7. For each (begin2, end2), the generated signal for α_2 and the known signal for α_2, rescaled between (begin2, end2), are compared. This is done by calculating their Pearson product-moment correlation coefficient
8. The values for begin1, end1, begin2, and end2 that lead to the highest Pearson product-moment correlation coefficient are stored
9. Steps 4-8 are repeated up to n other times, each time changing the values for begin1, end1, begin2 and end2 such that they are centered around their respective values found in step 8, $\pm$ a small ε

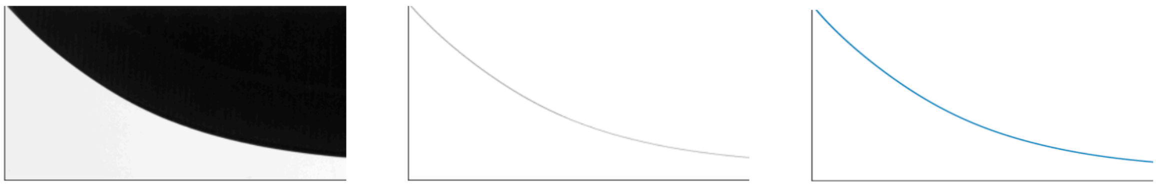


Figure 3.3: Left: Picture of the blade of the knife, Center: Shape of the blade after Canny edge detection, Right: Polynomial fitted to the blade of the knife

In theory, this method should be able to identify which part of the blade the striation pattern for α_1 comes from, although it might take many iterations with many initial guesses for (begin1, end1) and (begin2, end2), and then this information could be used to generate the signal of the striation pattern for any other angle of attack. However, before getting to Step 5 in Section 3.1, i.e. comparing the generated signals of the striations patterns under any angle of attack to the investigated striation pattern, this method was tested and evaluated. This process is explained in detail in the next subsection.

3.4. Evaluation of this Attempt

In 2025, Gerard Vermaas, another intern at the NFI, wrote his Bachelor Thesis [24] on testing and evaluating the method just described. Ideally, in fact, this method should be able to generate a virtual signal of the striation pattern for α_2 and match it to the known signal for α_2 so that their Pearson product-moment correlation coefficient is 1. Thus, in order for this method to be evaluated, it was run with different data and the resulting Pearson product-moment correlation coefficient was analyzed.

More specifically, 6 different random knives were selected. For each knife, 5 different cuts were made and processed, as explained in Chapter 2. These 5 different cuts were made at the angles of attack of 0°, 10°, 20°, 30° and 40°, respectively. Then, the following were computed:

- **Approach 1:** For each knife, the code explained in Section 3.1-Section 3.3 was run with, as inputs, the striation pattern at $\alpha_1 = 40°$ and the striation pattern at $\alpha_2 = 30°$ for that given knife. So, in other words, for each knife, the code generated multiple virtual signals for the striation pattern at 30°, based on the striation pattern at 40° and where on the blade it presumably came from. Then, it compared each of these generated signals to the known signal of the striation pattern at 30°, which is the second input in the code, and gave the maximum cross-correlation coefficient that was found, and thus which generated signal was the best. The interval (begin1, end1) on the blade of the knife that was used to generate this best signal was also then stored. The results of this approach are shown in the following table.

Knife	Maximum Cross-Correlation Coefficient
Knife 1	0.14
Knife 2	0.50
Knife 3	0.49
Knife 4	0.49
Knife 5	0.33
Knife 6	0.32

As it can be seen from the numbers in the table, the maximum cross-correlation coefficient that was found by the code between the generated signals for 30° and the known signals for 30° is generally quite low, especially for Knives 1, 5 and 6, and thus not very meaningful.

However, since it was now "known" which part of the blade each striation pattern for α_1 came from, i.e. the interval (begin1, end1), other signals could be generated for any other angle of attack. This was done next.

- **Approach 2:** For each knife, the signal of a striation pattern for $\alpha_2 = 20°$ was generated, based on the striation pattern for $\alpha_1 = 40°$ and the interval (begin1, end1) previously found and stored. Then, this generated signal was compared to the known signal of the striation pattern for 20° for that knife, as well as to the known signals of the striation patterns for 20° for all the other knives. The cross-correlation coefficients obtained from these comparisons are shown in the following table.

	Physical Trace for 20°					
Generated Trace for 20°	Knife 1	Knife 2	Knife 3	Knife 4	Knife 5	Knife 6
Knife 1	0.10	0.11	0.14	0.05	0.09	0.16
Knife 2	0.15	0.18	0.15	0.12	0.12	0.22
Knife 3	0.07	0.16	0.08	0.06	0.09	0.19
Knife 4	0.16	0.17	0.13	0.18	0.17	0.18
Knife 5	0.12	0.16	0.13	0.08	0.15	0.21
Knife 6	0.11	0.16	0.13	0.05	0.11	0.32

In the table above, the entries in the diagonal are the results for the Known Matches, whereas all the other ones are the results for the Known-Non-Matches. In other words, the entries in the diagonal represent the cross-correlation coefficient between two signals that should be the same, whereas the other entries represent the cross-correlation coefficient between two signals that are known to be different. In theory, thus, we would expect the entries in the diagonal to be much higher than the other ones.

From the results in the table, however, it can be seen that there is not a large difference between the entries in the diagonal and the other entries. Moreover, in many cases, a higher cross-correlation was found between signals for different knives than from the same knife. For example, the generated signal for 20° of Knife 1 resulted in a higher cross-correlation with the known signals for 20° of Knives 2, 3 and 6 rather than with the known signal for 20° of Knife 1.

Comparing this table to the previous one, it can be seen that the values in the previous table are in general higher than the values for the random matches, however this difference is not meaningful because, especially for Knives 1, 5 and 6 it is not very large. Moreover, the values in the previous table are much higher than the values in the diagonal in the second table, i.e. the Known Matches. This suggests that the interval (begin1, end1) that was found is not correct, and thus a generated signal for α_2, based on the signal for $\alpha_1 = 40°$ and (begin1, end1), becomes less and less accurate as the difference between α_1 and α_2 increases.

- **Approach 3:** The first approach (i.e. for each knife, starting from $\alpha_1 = 40°$, generate multiple signals for $\alpha_2 = 30°$ and compare them to the known signal at 30° to find the best one) was repeated, however the input pictures of the striation patterns for α_1 and α_2 were cropped, such that they were roughly half the length of the original ones. In this way, the signals in which they were turned into were simpler. This was done to test whether these simpler signals were easier to compare and thus lead to better cross-correlation coefficients. The results of this approach are shown in the following table.

Knife	Maximum Cross-Correlation Coefficient
Knife 1	0.31
Knife 2	0.77
Knife 3	0.82
Knife 4	0.45
Knife 5	0.53
Knife 6	0.72

Comparing these results to the ones in the first table, it can be seen that the cross-correlation found this time was (much) higher for all knives, except for Knife 4. This suggests that indeed simpler signals are easier to compare and match, leading to higher cross-correlation coefficients. Moreover, for each knife, a new interval for (begin1, end1) was found, called (begin1crop, end2crop). These intervals were all different from the ones previously found.

- **Approach 4:** The second approach was also repeated, taking as inputs, for each knife, the cropped picture of the striation pattern for $\alpha_1 = 40°$ and the interval (begin1crop, end2crop). The generated signal for $\alpha_2 = 20°$ was then compared to the signals of the cropped striation patterns for $\alpha_2 = 20°$ for all knives. The results are shown in the following table.

	Physical Trace for 20°					
Generated Trace for 20°	Knife 1	Knife 2	Knife 3	Knife 4	Knife 5	Knife 6
Knife 1	0.16	0.20	0.19	0.07	0.15	0.19
Knife 2	0.16	0.24	0.24	0.09	0.18	0.18
Knife 3	0.11	0.16	0.12	0.06	0.12	0.21
Knife 4	0.25	0.18	0.21	0.24	0.16	0.20
Knife 5	0.20	0.18	0.22	0.08	0.30	0.20
Knife 6	0.15	0.17	0.12	0.05	0.15	0.28

Comparing this table to the second one, it can be seen that all the values for the Known Matches increased, except for Knife 6. However, also the values for the random matches have increased. For example, all the values of the first row of this table are higher compared to the values of the first row of the previous one. Moreover, it is still the case that some random matches produce higher cross-correlation coefficients than the Known Matches. For example, the generated signal for 20° of Knife 1 has a higher cross-correlation coefficient with the known signals for 20° of Knives 2, 3 and 6 than with the known signal for 20° of Knife 1.

- **Approach 5:** The third approach was repeated, however this time α_2 was 20°. So, for each knife, starting from a cropped picture of the striation pattern for 40°, multiple signals for $\alpha_2 = 20°$ were generated and then compared to the known cropped striation pattern at 20°. The maximum cross-correlation coefficients resulting from these comparisons are shown in the following table.

Knife	Maximum Cross-Correlation Coefficient
Knife 1	0.81
Knife 2	0.71
Knife 3	0.51
Knife 4	0.25
Knife 5	0.41
Knife 6	0.43

Comparing these results to the ones in the third table, it can be seen that the maximum cross-correlation coefficient is lower for all knives, except for Knife 1. This suggests, once again, that the larger the difference between α_1 and α_2, the more inaccurate the generated signals for α_2 will be. In fact, the results in the third table are for a rotation to $\alpha_2 = 30°$ starting from a cropped image of the striation pattern for $\alpha_1 = 40°$, whereas for this table, the rotation is to $\alpha_2 = 20°$, starting from the same cropped image.

Moreover, the intervals (begin1, end1) that were found for each knife with this approach are different than the intervals (begin1crop, end1crop) that were found with the third approach. However, in theory, they should have been the same, since they are an estimate of where on the blade the same cropped striation pattern for 40° comes from. This, thus, suggests that this code is not robust.

- **Approach 6:** The second approach was repeated, taking as inputs, for each knife, the cropped picture of the striation pattern for $\alpha_1 = 40°$ and the interval (begin1, end2) found in Approach 5. This was used to generate a signal for $\alpha_2 = 30°$, which was then compared to the signals of the cropped striation patterns for $\alpha_2 = 30°$ for all knives. The results are shown in the following table.

	Physical Trace for 30°					
Generated Trace for 30°	Knife 1	Knife 2	Knife 3	Knife 4	Knife 5	Knife 6
Knife 1	0.56	0.18	0.23	0.20	0.18	0.20
Knife 2	0.18	0.70	0.21	0.12	0.16	0.19
Knife 3	0.13	0.13	0.62	0.11	0.13	0.16
Knife 4	0.26	0.20	0.16	0.46	0.20	0.24
Knife 5	0.27	0.20	0.18	0.20	0.53	0.18
Knife 6	0.17	0.18	0.18	0.12	0.18	0.62

In this table, it can be seen that there is a large difference between the entries in the diagonal, i.e. the Known Matches, and the other entries, i.e. the Known-Non-Matches. This is surprising as, in general, the maximum cross-correlations obtained in the fifth table are lower than the ones obtained in the third table, which suggests that setting (begin1crop, end1crop) as an input would lead to better results. However, from the results in the table, this seems not to be the case.

It is thus clear that the code used to generate and compare the different striation patterns needs some improvements, as it produces inconsistent and conflicting results.

3.5. Observations

In [24], a section is dedicated to explore the possible reasons as to why the code is performing so poorly. This is done by making some useful observations on the problem. In particular, the following two observations seem to be especially relevant.

- **Observation 1:** As previously explained, in the code, given an interval on the blade for the striation pattern at α_1, the signal of the striation pattern for α_2 is generated, by rotating the blade of the knife to α_2 and projecting it back onto the cutting surface. In general, the resulting signal is a non-linear compression/stretch of the signal for α_1, since the blade of the knife is generally curved. However, in the case in which the blade of the knife is straight, this compression/stretch is linear.

  To test whether this model accurately represents the reality, a knife with a straight blade was considered. This is Knife 6 in the previous analysis and it is shown in Figure 3.4 [24].

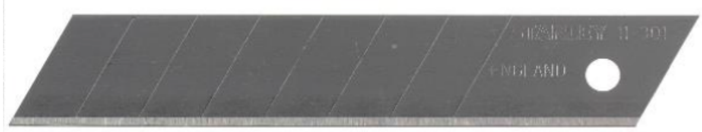


Figure 3.4: Knife with a straight blade (Knife 6)

  The image of the striation pattern for this knife at 30° was enlarged (linearly) so that the distances between the lines in the striation pattern increased linearly. Then, it was compared to the striation pattern for this knife at 40°, as shown in Figure 3.5 [24].

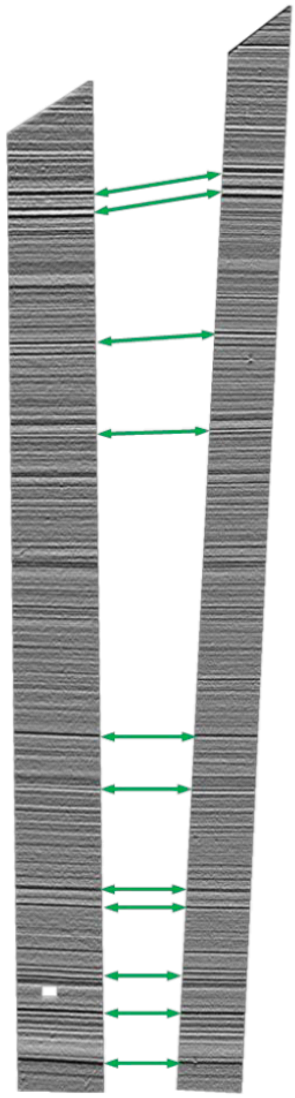


Figure 3.5: On the left , the picture of the striation pattern at 30° enlarged, on the right the picture of the striation pattern at 40°

From this picture, it can clearly be seen that the lines in the bottom of the striation pattern line up, however the higher lines do not. This suggests that the stretch of the striation pattern for 40° is not exactly linear. This might be caused by the fact that the material in which the cut is made deforms a little while being cut. Thus, there is a discrepancy between the theoretical model and the physical data.

Moreover, this probably explains why better results were obtained in the previous analysis by taking a cropped image of the striation pattern, as opposed to the whole image. In fact, if only the bottom part of the striation pattern is taken, it is possible to generate a signal for another angle without causing a misalignment in the lines, as shown in Figure 3.5.

Since this problems arises when a knife with a straight blade is considered, it is likely that this also happens when knives with curved blade are considered, since in general these cases are more complex.

- **Observation 2:** It was observed that the maximum cross-correlation coefficients found in the previous analysis (in Approaches 1, 3 and 5) are highly dependent on the initial inputs for (begin1, end1). In other words, the code produces very different results based on the initial guess of the user as to where on the blade the striation pattern for α_1 comes from. This is expected, however not to this extent. In fact, the code is very sensitive to small variations in this interval, creating very different results. This suggests that this method is not robust.

  Moreover, due to this high sensitivity of the code on the initial input (beign1, end1), in order to find the maximum cross-correlation coefficients for the analysis, multiple trials were needed. This made this task very time consuming. However, this process could have been accelerated by considering the known length of the striation patterns, instead of guessing it.

The code, and more specifically the way in which the signals for α_2 are generated, could be improved based on these observations. In fact, the current method used to rotate and project the blade on the knife onto the cutting surface should be modified, to avoid the problem of misalignment of lines in the striation patterns, as in Figure 3.5. Also, the method used to determine which part of the blade the striation pattern for α_1 comes from should be refined, so that it can be more robust and less sensitive to the initial input of the user.

4
Problem Definition

As explained in Chapter 3, this first attempt to automate the comparison of striation patterns presents multiple issues and does not produce consistent results. Thus, the aim of this research is to improve this method by solving the problems it currently has such that it will be able to produce better results. More specifically, this will be done by answering some research sub-questions concerning the problematic parts of the method that is currently used.

The sub-questions that will be answered in this research are the following.

1. **Is there a more robust way of creating a Virtual Knife?**

   As previously explained, the current method used to create a virtual knife does not produce consistent results. In fact, for example, for the same image of the striation pattern at α_1, two different intervals (begin1, end1) can be found, depending on α_2. Two of the reasons as to why this current method is failing are the following:

   - The values for (begin1, end1) and (begin2, end2) need to be estimated by the user and this method is highly sensitive to the initial input
   - The way in which the signal for α_1 is rotated to create the virtual signal for α_2 does not accurately represent how the signal α_2 looks like in real life, as shown in Figure 3.5

   The aim of this research sub-question is, thus, to understand whether the two issues found in this method can be solved or, alternatively, if there is a new other method that can be used in order to model what the striation patterns for all angles will look like for a given knife.

2. **Is there a more robust way of comparing the signals representing the striation patterns?**

   With the current method, the signals representing the striation patterns are compared via a Pearson product-moment correlation coefficient. In particular, the signals are compared in order to:

   - Create the Virtual Knife (i.e. comparing the generated signals for α_2 to the known signal for α_2 to determine (beign1, end1))
   - Determine how well a signal matches with a Known Match as opposed to a know-non-match

   As it was explained, however, the creation of the Virtual Knife has some issues, and often the generated signals for α_2 have a higher cross-correlation coefficient with Known-Non-Matches rather than with the Known Matches. This might be due to the fact that the Pearson product-moment correlation coefficient is not a good metric to compare signals.

Thus, the aim of this research sub-question is to determine whether some other techniques can be used to compare the signals for the striation patterns, so that they can produce more consistent and robust results. This will therefore improve the creation of the Virtual Knife, as well as give better insights on how well the generated signals match with their Known Matches and Known-Non-Matches.

3. **Can the process to compare (virtual) striation patterns be made less time consuming?**

   The current method to create the virtual knife is quite time consuming, as it requires the user to:

   - Provide a picture of the blade of the knife
   - Provide two pictures of the striation patterns for α_1 and α_2 that have been processed as explained in Chapter 2
   - Guess the intervals (begin1, end1) and (begin2, end2) via trial and error

   The aim of this research sub-question is to find out whether the method used to create the Virtual Knife can be improved, such that it would need less inputs from the user and thus become more time-efficient. This could be done, for example, by requiring less pictures as the input for the code, and/or by creating a robust method to determine the intervals (begin1, end1) and (begin2, end2) without guessing, e.g. using the information of the physical length of the cuts.

Finding the answer to these sub-questions will bring insights on how to improve the current method. Moreover, if the research is successful in building a robust way to create a Virtual Knife and compare its signals to other ones, the following (bonus) research question should be explored:

Can this method also be used to compare striation marks from the crime scenes?

This question is worth asking since often times, in stabbing incidents, there is not a single angle of attack, but rather multiple, since the knife slightly rotates while cutting the person. Thus, in the resulting striation pattern, only a small fraction matches with a specific angle of attack (used in the aggression). However, in the method that is currently being researched, only striation patterns resulting from a single angle of attack can be compared. Thus, it will be needed to adapt the method such that it can also work in cases where a single striation pattern is the result of multiple angles of attack.

5

Potential Approaches

In order to find answers to the research questions previously mentioned, and thus to improve the current method used to generate and compare striation patterns, it is important to first know what mathematical tools might be relevant and useful. This section is dedicated to explore four mathematical techniques which might be particularly fitting for this problem. For each of these techniques, a description is provided, as well as a brief reason as to why it could be useful for this research.

5.1. Fast Fourier Transform

A Fast Fourier Transform [10] [18] is an algorithm to compute the Discrete Fourier Transform of a signal in a much faster way. The Discrete Fourier Transform takes a sequence $(x_n) \in \mathbb{C}$ of N numbers and it transforms it into another sequence $(X_k) \in \mathbb{C}$ of N numbers where

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2i\pi kn/N}$$

for $k = 0, 1, ..., N-1$. The complexity of this method is $O(N^2)$.

The algorithm for the Fast Fourier Transform reduces the complexity of this method to $O(N \log N)$. This is done by splitting X_k into two components of length $N/2$ each, namely one for the x_n with n even and one for the x_n with n odd. For simplicity, we can assume that N is even.

We thus have

$$\begin{aligned} X_k &= \sum_{m=0}^{N/2-1} x_{2m} e^{-2i\pi k(2m)/N} + \sum_{m=0}^{N/2-1} x_{2m+1} e^{-2i\pi k(2m+1)/N} \\ &= \sum_{m=0}^{N/2-1} x_{2m} e^{-2i\pi k(2m)/N} + e^{-2i\pi k/N} \sum_{m=0}^{N/2-1} x_{2m+1} e^{-2i\pi k(2m)/N} \\ &= E_k + W_N^k O_k \end{aligned}$$

where $E_k := \sum_{m=0}^{N/2-1} x_{2m} e^{-2i\pi k(2m)/N}$, $W_N := e^{-2i\pi/N}$ and $O_k := \sum_{m=0}^{N/2-1} x_{2m+1} e^{-2i\pi k(2m)/N}$.

The first $N/2$ terms of the sequence (X_k) are then computed with the formula $X_k = E_k + W_N^k O_k$ for $k = 0, 1, ..., N/2-1$.

The elements of the second half of the sequence are of the form $X_{k+N/2}$ for $k = 0, 1, ..., N/2 - 1$, so we have

$$\begin{aligned} X_{k+N/2} &= \sum_{m=0}^{N/2-1} x_{2m} e^{-2i\pi(k+N/2)(2m)/N} + e^{-2i\pi(k+N/2)/N} \sum_{m=0}^{N/2-1} x_{2m+1} e^{-2i\pi(k+N/2)(2m)/N} \\ &= \sum_{m=0}^{N/2-1} x_{2m} e^{-2i\pi(k)(2m)/N} e^{-2i\pi m} + e^{-2i\pi(k)/N} e^{-i\pi} \sum_{m=0}^{N/2-1} x_{2m+1} e^{-2i\pi(k)(2m)/N} e^{-2i\pi m} \\ &= \sum_{m=0}^{N/2-1} x_{2m} e^{-2i\pi(k)(2m)/N} - e^{-2i\pi(k)/N} \sum_{m=0}^{N/2-1} x_{2m+1} e^{-2i\pi(k)(2m)/N} \\ &= E_k - W_N^k O_k \end{aligned}$$

So computing E_k and $W_N^k O_k$ for $k = 0, 1, ..., N/2 - 1$ easily gives the whole sequence (X_k). Applying this recursively reduces significantly the complexity of the problem.

As it will be explained more in detail in the next sub-section, one of the ways to compare signals is via a Matched filter. This method can be implemented more efficiently using a Discrete Fourier Transform, so being able to compute it fast with a Fast Fourier Transform is crucial.

5.2. Matched Filter

A common way used to compare signals is the Matched Filter [22] [23]. More specifically, this method is used in order to determine whether a known signal is present in another unknown signal. This is done by choosing a suitable filter and convoluting it with the unknown signal to determine their highest correlation. This method is described in detail in what follows. For this sub-section, to indicate the n^{th} element of a signal x, the notation $x[n]$ will be used, instead of x_n.

Suppose we have a known signal s and an unknown signal $x = s + w$, where w is some noise. We want to find a matched filter h (i.e. a finite sequence) such that the output

$$y[n] = (x * h)[n]$$

maximizes the signal-to-noise power ratio for some shift n, say n_{max}. So, in other words, the signal s is detected in the signal x at the shift n_{max}, and their correlation at this point is given by $y[n_{max}]$.

In general, the choice of the matched filter h needed to maximize the signal-to-noise power ratio in the output depends on the type of noise w present in the unknown signal x, as well as on the known signal s. The reason for this is that, in order to maximize the signal-to-noise power ratio in the output, the matched filter needs to be "as parallel as possible" to the signal s and "as orthogonal as possible" to the noise w.

The general formula for the optimal matched filter h is

$$h = \mathbb{E}(ww^H)^{-1} s$$

where w^H is the Hermitian Transpose of w. This result can be derived by writing the output as $y[n] = ((s + w) * h)[n]$, so that it can be split into two components, namely y_s and y_w, representing the output due to the signal and the output due to the noise, respectively. Then, the signal-to-noise power ratio of the output is given by $\frac{|y_s|^2}{\mathbb{E}(|y_w|^2)}$. This ratio can be re-written and bounded using the Cauchy–Schwarz inequality, producing the desired result.

Once the matched filter h is found, the convolution $y[n] = (x * h)[n]$ needs to be computed for all n in order to find which n (i.e. which shift) gives the maximum output, as well as what this output is (i.e. the correlation between the signals). This is very computationally expensive, as the complexity of this method is, once again, $O(N^2)$. However, using the Convolution Theorem and the Fast Fourier Transform method previously explained, this process can be executed much faster, having a complexity of $O(N \log N)$.

In fact, the Convolution Theorem states that the Fourier Transform of a convolution $f * g$ of two functions f and g is the product of the Fourier Transforms for f and g. Equivalently, this applies for Fast Fourier Transforms. So we have that

$$y[n] = (x * h)[n]$$

$$\begin{aligned} \Rightarrow FFT(y[n]) &= FFT((x * h)[n]) \\ &= FFT(x[n])FFT(h[n]) \\ &= X_k \cdot H_k \end{aligned}$$

$$\Rightarrow y[n] = IFFT(X_k \cdot H_k)$$

where X_k and H_k are the Fast Fourier Tranforms of x and h respectively, and $X_k \cdot H_k$ is a point-wise multiplication. In this way, the whole sequence y_n can be computed fast, so that both the correlation of the signals, as well as their shift can be found quickly.

This method can be useful in this research as it provides a robust way to detect whether a signal is present in another signal. More specifically, it could be used to compare the generated signal for α_2 to the known signal for α_2, to find the interval (begin1, end1) on the blade of the knife. Also, it could be used to determine how well the generated signals for α_2 match with their Known Matches and their Known-Non-Matches.

5.3. POD

The Proper Orthogonal Decomposition Method [13] [7], or in short the POD Method, is a method that was initially introduced in the field of fluid dynamics in order to find the coherent structures hidden inside turbulent flows. However, it can also be used in other fields to find the most dominant structures in a given data set. The main idea of this method is to find a function (or multiple) which is best correlated with the data that needs to be investigated, e.g. the turbulent flow. The details of this method are described in what follows.

Suppose we have an $m \times n$ matrix X where each column represents a snapshot of the same spatial domain for a different time n. We are interested in finding a function ϕ which is best correlated on average with X. This is equivalent to finding a function ϕ such that the projection

$$\langle x, \phi \rangle^2$$

is maximized for all the columns of X, and $||\phi|| = 1$. So we want to find

$$\max_{||\phi||=1} \sum_{k=1}^{n} \langle x_k, \phi \rangle^2$$

where x_k is the k^{th} column of X.

It can be shown that this maximization problem is equivalent to finding the largest eigenvalue of the problem

$$R\phi = \lambda\phi$$

where $R = XX^T$, and finding the corresponding eigenvector ϕ.

To find the eigenvalues of $R = XX^T$, it is useful to first notice that they are the same eigenvalues of X^TX. In fact, suppose that λ is an eigenvalue of XX^T, then

$$XX^Tv = \lambda v \Rightarrow X^TXX^Tv = \lambda X^Tv \Rightarrow X^TX(X^Tv) = \lambda(X^Tv)$$

so λ is also an eigenvalue of X^TX. Vice versa, suppose that λ is an eigenvalue of X^TX, then

$$X^TXv = \lambda v \Rightarrow XX^TXv = \lambda Xv \Rightarrow XX^T(Xv) = \lambda(Xv)$$

so λ is also an eigenvalue of XX^T.

Moreover, let the Singular Value Decomposition (SVD) of X be given by $X = U\Sigma V^T$, where the elements σ_i on the diagonal of Σ are given by $\sigma_i = \sqrt{\lambda_i}$, where $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$ are the eigenvalues of X^TX, and thus also of XX^T. We have that

$$\begin{aligned} XX^T &= U\Sigma V^T(U\Sigma V^T)^T \\ &= U\Sigma V^TV\Sigma^TU^T \\ &= U\Sigma^2U^T \end{aligned}$$

$$\begin{aligned} \Rightarrow XX^TU &= U\Sigma^2U^TU \\ &= U\Sigma^2 \\ &= \Sigma^2U \end{aligned}$$

This implies that the columns of U are the eigenvectors of XX^T.

Thus, finding the SVD of $X = U\Sigma V^T$ gives the (square root of) the eigenvalues of XX^T on the diagonal of Σ, as well as their corresponding eigenvectors as the columns of U. The eigenvector corresponding to the highest eigenvalue represents the most correlated function ϕ to X, and, in general, the higher the eigenvalue, the higher the corelation between its associated eigenvector/function ϕ and X is.

This method would be useful in this research to create a new method to compare striation patterns. Namely, it could be investigated whether there is a strong underlying structure between the striation patterns created by a given knife and the striation pattern that needs to be investigated, using the POD Method. In this way, the presence of a strong underlying structure would suggest that the considered striation patterns come from the same knife.

5.4. Dynamic Time Warping

A very versatile and commonly used way to compare signals is via Dynamic Time Warping [5]. This method, in fact, is used to determine whether two signals have the same underlying shape, even if they appear different and present some slight variations. This could be the case, for example, for two signals representing the same musical piece played by two different interpreters, causing difference in tempo, and local accelerations/decelerations of the music, or two signals representing two people saying the same word (in fact this method is also wildly for speech recognition [1]). The main idea of Dynamic Time Warping is to find an alignment between the two signals which allows for non-linear local shifts, to accommodate for the local changes in velocities of the two signals. This method is described in detail in what follows.

Suppose we have two signals $X := (x_1, x_2, ..., x_N)$ and $Y := (y_1, y_2, ..., y_M)$ and we want to find their optimal alignment. This is equivalent to defining an "alignment cost" between the two signals and minimizing it. To do this, we define $c(i, j) := |x_i - y_j|$ which is the local cost of aligning x_i to y_j, for $i \in 1, 2, .., N$ and $j \in 1, 2, .., M$.

Then, we definite a cumulative cost, i.e. the cost of aligning $(x_1, x_2, ..., x_i)$ to $(y_1, y_2, ..., y_j)$. The minimum cumulative cost of aligning $(x_1, x_2, ..., x_i)$ to $(y_1, y_2, ..., y_j)$ is given by

$$D(i,j) = c(i,j) + \min \begin{cases} D(i-1,j) \\ D(i,j-1) \\ D(i-1,j-1) \end{cases}$$

In this formula, the terms $D(i-1, j)$, $D(i, j-1)$ and $D(i-1, j-1)$ represent the three possible paths that lead to the alignment of the terms x_i and y_j. Namely, to align x_i and y_j we can either:

- Have advanced in the sequence X from x_{i-1} to x_i and not advanced in the sequence Y. In this case the cost is $D(i-1, j)$
- Have advanced in the sequence Y from y_{i-1} to y_i and not advanced in the sequence X. In this case the cost is $D(i, j-1)$
- Have advanced in both sequences X and Y from x_{i-1} to x_i and from y_{i-1} to y_i respectively. In this case the cost is $D(i-1, j-1)$

If the minimum between these three costs is $D(i-1, j-1)$, the signals are already aligned for x_i and y_i. Otherwise, the cost metrics $D(i-1, j)$ and $D(i, j-1)$ can accommodate for the local non-linear stretching/shrinking of the two signals.

The minimum cumulative cost $D(i, j)$ is defined recursively, with the initial condition that $D(1, 1) = c(1, 1)$. This is 0 if the signals have the same starting point, otherwise it should be adjusted based on the delay of the signals. The Dynamic Time Warping between the signals X and Y is given by the cumulative cost of aligning $(x_1, x_2, ..., x_N)$ and $(y_1, y_2, ..., y_M)$, which is $D(N, M)$. The smaller the value of $D(N, M)$ is, the similar the signals X and Y are.

This method could be used in this research to create a new method of comparing signals. Namely, as previously explained, the lines in the striation patterns for a given knife are all the same, however the distances between them are non-linearly shrunk/extended depending on the angle of attack. Thus, instead of comparing all the possible (virtual) striation patterns for a given knife to the striation pattern that needs to be investigated, it could just be investigated whether one single striation pattern of a given knife can be non-linearly deformed into the striation pattern that needs to be investigated, via Dynamic Time Warping.

6

First Approach

As explained in Chapter 4, the sub-research questions that we want to answer are:

1. Is there a more robust way of creating a Virtual Knife?
2. Is there a more robust way of comparing the signals representing the striation patterns?
3. Can the process to compare (virtual) striation patterns be made less time consuming?

These research question could be answered either by trying to improve the current method used to generate and compare striation patterns or by creating a new one. As explained in Chapter 5, the current method could be improved using a Matched Filter to compare the generated signal for α_2 to the known signal for α_2, to find the interval (begin1, end1) on the blade of the knife and thus create a more robust Virtual Knife. Moreover, the Matched Filter could also then be used to compare the virtual striation patterns generated by the Virtual Knife to the striation patterns of other knives/the striation pattern that needs to be investigated to see if they match.

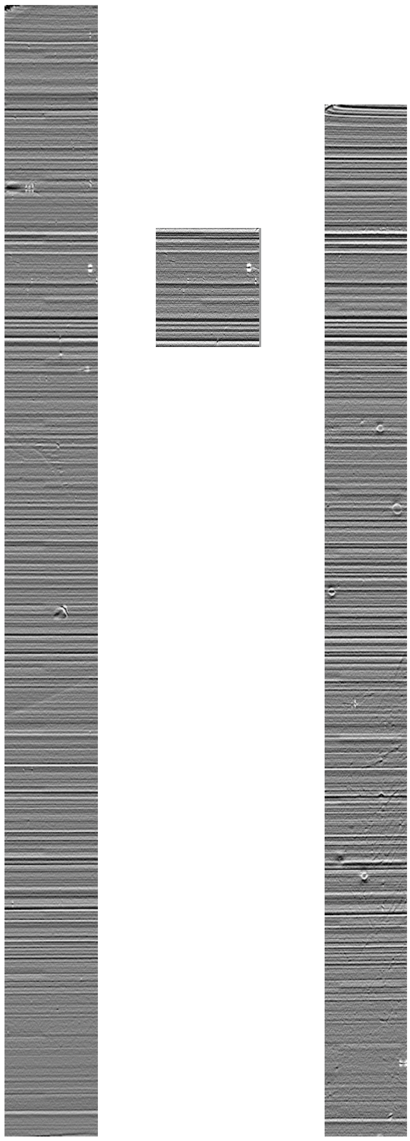
This method should, thus, help answer the first two research questions, however it would not make the process less time consuming as the code would still need, as inputs, a picture of the blade of the knife, two processed images of the striation patterns for each knife at different angles and the user would still need to guess the values for (begin1, end1), which is a very time consuming process.

Nevertheless, a Matched Filter was implemented to investigate whether this tool could be useful to improve the current method. Namely, suppose that we have the striation pattern created at the known angle α_1 and we want to use it to virtually generate the striation patterns for that knife for all the possible angles of attack. So, following steps 1-5 of the algorithm described in Section 3.3, we guess an interval (begin1, end1) on the blade of the knife where we think the striation pattern that we have was created from, and, based on this interval, we generate a virtual striation pattern for the angle of attack α_2.

Then, we compare this generated striation pattern for α_2 to a lab-created striation pattern at α_2, to see if they match and thus if the guess for (begin1, end1) was correct. However, instead of guessing the physical size of the lab-created striation pattern for α_2 as in Step 6 of the algorithm described in Section 3.3, we calculate it knowing that each pixel in the picture of the striation pattern corresponds to 1.763 micro meters. Then, instead of finding the correlation coefficient between the generated signal for α_2 and the rescaled known signal for α_2, as in Step 7 of the algorithm in Section 3.3, we compare them using a Matched Filter. In this way, assuming the initial guess for (begin1, end1) was correct, (a part of) the generated signal for α_2 will be found in (a part of) the (rescaled) lab-created signal for α_2. On the contrary, if the initial guess for (begin1, end1) was incorrect, there will be no significant overlap between (parts of) the generated and the (rescaled) lab-created signals for α_2.

This was tested in the following way: two striation patterns, Mark1 and Mark2, were considered, both created at an angle of attack of 20° with the same knife. In particular, Mark1 represented the generated striation pattern for $\alpha_2 = 20°$, assuming the inital guess for (begin1, end1) was correct, and Mark2 represented the lab-created striation pattern for α_2. It is important to notice that these two signals had different amplitudes. This mimics the fact that, in general, the generated signal and the lab-created signal for α_2 will have different amplitudes, even if the initial guess for (begin1, end1) is correct. In fact, the generated signal for α_2 is created from the striation pattern for α_1, and it is likely that the gray-scales for the original lab-created striation patterns for α_1 and α_2 were not calibrated exactly equally. Thus, the two signals for the striation patterns for α_2 are likely to have different amplitudes.

A part of the striation pattern for Mark1 was cropped out, and a Match Filter was used to "find it" inside the full striation pattern of Mark2, to simulate the procedure previously described. The small part of the striation pattern of Mark1 was correctly identified with the Matched Filter in the striation pattern of Mark2, as it can be see from the figures below.



(a) Striation pattern of Mark1 **(b)** Cropped part **(c)** Striation pattern of Mark2

Figure 6.1

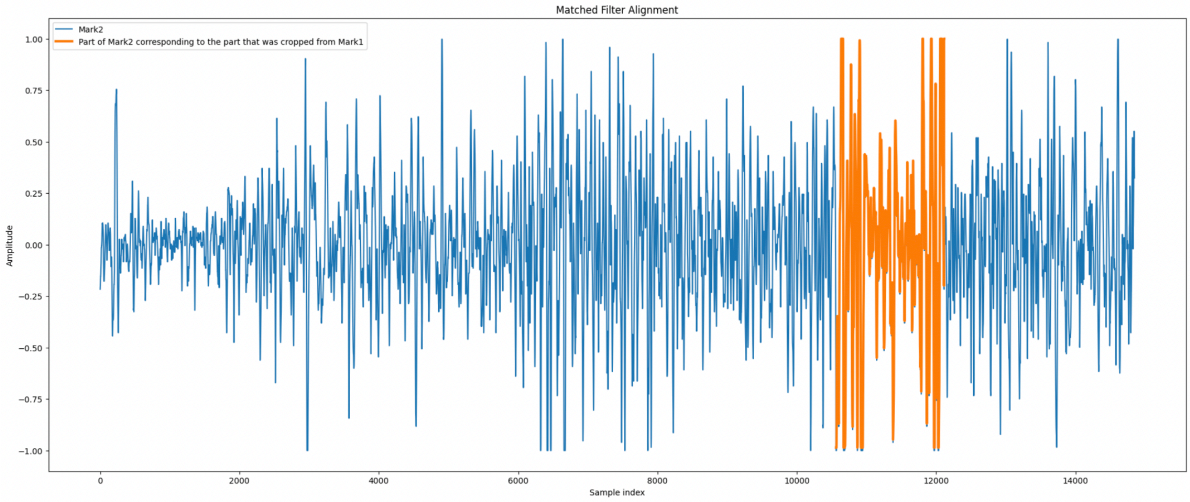


Figure 6.2: Where the cropped part of Mark1 was supposed to be found in Mark2

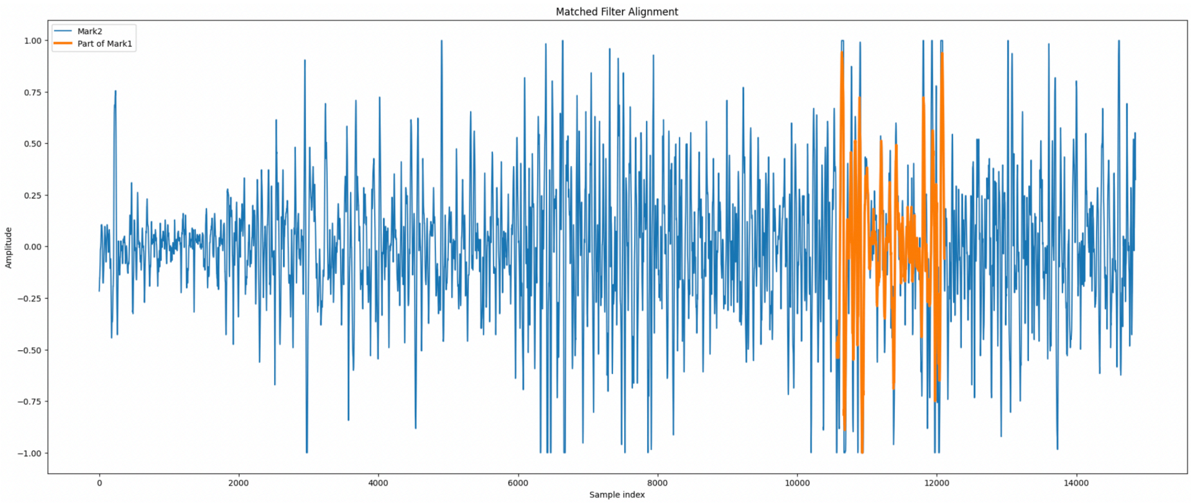
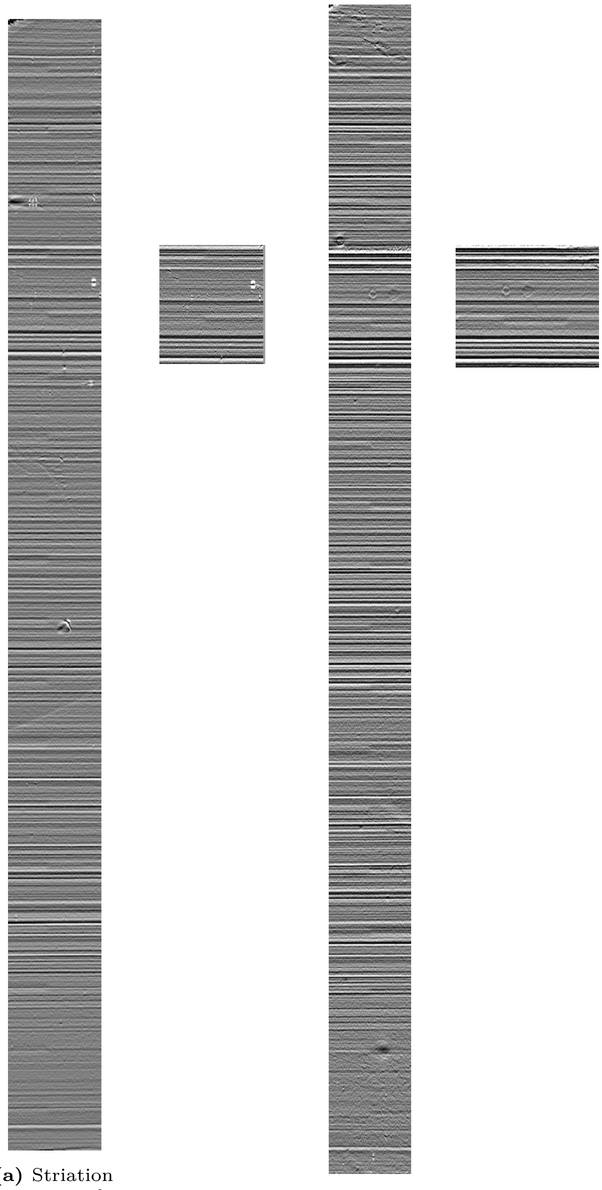


Figure 6.3: Where the cropped part of Mark1 was found in Mark2 with the Matched Filter

This seems to suggest that, provided that the initial guess for (begin1, end1) is correct, the generated and the lab-created striation patterns for α_2 can be compared using a Matched Filter. However, as it was previously explained in Section 3.5, the generated signal for α_2 can never exactly look like the lab-created signal for α_2, even if the initial guess for (begin1, end1) is correct, due to the slight deformation of the material while begin cut, that are not taken into consideration which generating the virtual striation pattern for α_2. Therefore, in practice, the Matched Filter would need to be able to find the overlap between the generated and the lab-created striation patterns for α_2 even if one is slightly stretched compared to the other.

Thus, in order to assess this, the previous test was repeated, however this time instead of Mark1, another cut (Mark3) was considered, which was created at an angle of attack of 22° with the same knife. The same part of Mark1 that was previously considered was cropped out of Mark3, and the Matched Filter was used to ”find it” in the longer striation pattern for Mark2. Therefore, the test was essentially the same as the previous one, however this time the lines in the smaller striation pattern were slightly further apart from each other, as Mark3 was created at an angle of attack of 22°, and not 20° as Mark1. This was done to mimic the slight difference that would come up in real life between the generated and the lab-created striation patterns for α_2 due to small deformations of the material that is being cut.

The data used, as well as the results of these tests are shown in the following figures.



(a) Striation pattern of Mark1 (previous test)

(b) Cropped part (previous test)

(c) Striation pattern of Mark3

(d) Cropped part from Mark3

Figure 6.4

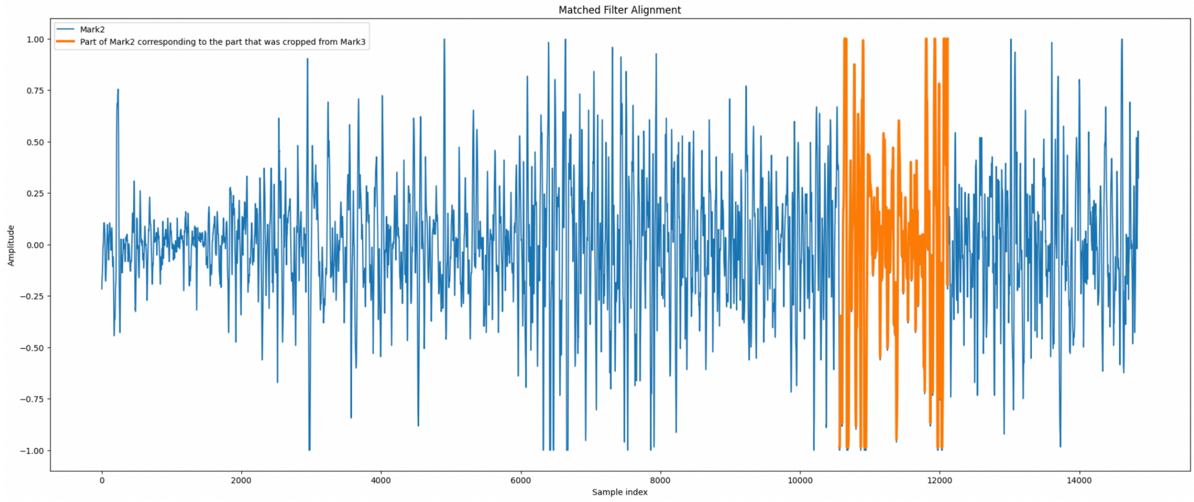


Figure 6.5: Where the cropped part of Mark3 was supposed to be found in Mark2

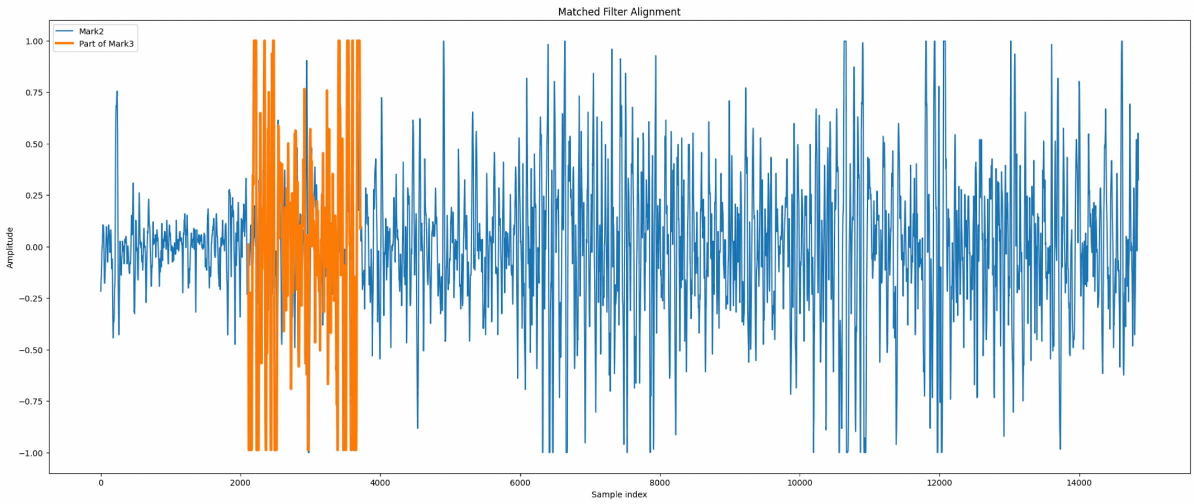


Figure 6.6: Where the cropped part of Mark3 was found in Mark2 with the Matched Filter

As we can see from the figure above, the part of the signal for Mark3 that was considered was not correctly identified in the longer signal for Mark2. As explained above, this test mimics a real life situation, where (parts of) the generated and the lab-created striation patterns for α_2 appear stretched/compressed, compared to each other, due to deformation of the material that is begin cut. From the results of the test, we can see that the Matched Filter cannot be used to improve the current method as it cannot be used to compare signals that appear slightly stretched/compressed relative to each other.

7
Final Model

As explained in Chapter 5, another mathematical tool that could be used in order to answer the sub-research questions mentioned in Chapter 4 is the Dynamic Time Warping. This mathematical tool, in fact, could be used to create a new method to compare the signals, as it can help determine whether two (parts of) signals can be stretched/shrunk non-linearly one into the other, and thus if they correspond to striation patterns created by the same knife or not.

More specifically, as explained in Chapter 5, the Dynamic Time Warping is a mathematical method that allows to find the best alignment between two sequences, or equivalently, between two signals, by locally stretching non-linearly either of the signals so that it can match the other one. As previously explained, the striation patterns created by the same knife are always the same, however the distances between the lines in the striation pattern will change non-linearly depending on the angle of attack under which a specific cut was created with that knife. So, in other words, the signals representing the striation patterns created by a given knife will all look the same, however they will all be a non-linearly stretched/shrunk version of each other, depending on the angle of attack under which they were created. As a consequence, if two (parts of) these signals can be well aligned by stretching one into the other, we can conclude that they were created by the same knife. Therefore, the Dynamic Time Warping can be used to determine the best alignment between two (parts of) signals, and, depending on "how good" this alignment is, it can be determined whether these signals correspond to striation patterns created by the same knife or not.

For this method, thus, the only inputs that are needed are the two processed images of the two (partial) striation patterns that need to be compared. It is therefore not needed to create a Virtual Knife, and so this method should be significantly less time consuming and easy to implement compared to the current one, previously explained. For this reason, this method was constructed and the following sections explain how this was done in detail.

7.1. Method

As explained in Chapter 5, Dynamic Time Warping is a mathematical method used to find the best alignment between two sequences or signals X and Y, by defining an "alignment cost" between the two signals and minimizing it. This is done accordingly to the formula in Section 5.4, and the sequence of pairs of points in X and Y that are aligned to each other is called the "warping path". In general, either one of the signals X and Y can be locally stretched in order to match the other one. So, in other words, to align $x_i \in X$ and $y_j \in Y$ one can either:

- Have advanced in X from x_{i-1} to x_i and have not advanced in Y. In this case both x_{i-1} and x_i will be aligned to the same point in Y, namely y_j. Thus, in the warping path we will have $...(x_{i-1}, y_j), (x_i, y_j)...$, which corresponds to a local stretching of Y relative to X, since the point y_j is repeated.
- Have advanced in Y from y_{j-1} to y_j and have not advanced in X. In this case both y_{j-1} and y_j will be aligned to the same point in X, namely x_i. Thus, in the warping path we will have $...(x_i, y_{j-1}), (x_i, y_j)...$, which corresponds to a local stretching of X relative to Y, since the point x_i is repeated.
- Have advanced in both X and Y from x_{i-1} to x_i and from y_{j-1} to y_j respectively. In this case x_{i-1} will be aligned to y_{j-1} and x_i will be aligned to y_j so neither or the signals will be stretched relative to the other

In the context of the research, we want to use Dynamic Time Warping to align (parts of) different signals corresponding to different striation patterns to determine whether they were created by the same knife. So suppose that two signals $X = (x_1, x_2, .., x_N)$ and $Y = (y_1, y_2, ..., y_M)$ correspond to two different striation patterns created by the same knife at two different angles of attack. Without loss of generality, we can assume that the signal X was created at a higher angle of attack compared to the signal Y, and thus it will look stretched compared to the signal Y, since the higher the angle of attack, the higher the distances between the lines in the striation pattern. Moreover, at no point will the lines in the striation pattern for Y be further apart than the lines in the striation pattern of X, since X was created at a higher angle of attack. Therefore, to align the two signals via Dynamic Time Warping we should only allow the signal Y to stretch. Therefore, to align $x_i \in X$ and $y_j \in Y$ one can either:

- Have advanced in X from x_{i-1} to x_i and have not advanced in Y.
- Have advanced in both X and Y from x_{i-1} to x_i and from y_{j-1} to y_j respectively.

Therefore, in this setting, the formula for the minimum cumulative cost of aligning $(x_1, x_2, ..., x_i)$ to $(y_1, y_2, ..., y_j)$ is given by

$$D(i,j) = |x_i - y_j| + \min \begin{cases} D(i-1, j) \\ D(i-1, j-1) \end{cases}$$

As it is generally unknown where in X the signal Y should be aligned to, the best alignment between X and Y is found for each starting point $x_i \in X$. So, in other words, multiple alignments between (a part of) X and Y are computed, each with a different starting point, i.e. initially aligning y_1 to x_1, or to x_2, etc. The alignment that produces the smallest matching cost $D(i, M)$ is the best alignment between Y and (a part of) X, and in case two alignments produce the same smallest cost $D(i, M)$, the one with the shortest path is chosen. Then, the correlation coefficient between the two aligned signals is computed (considering only the part of X to which Y is aligned to).

This method assumes that the whole signal Y will be able to be matched to (part of) the signal X. This assumption is realistic since in a real life case, the signal Y will represent the striation pattern found in the victim's wounds and the signal X will represent the striation pattern that was created in the lab with the suspect knife. So, if the cuts created in the lab are long enough, the signal X will represent a large part of the striation pattern of the suspect knife, if not all of it, and thus it will contain the part of the striation pattern that is found in the victim's wounds (assuming that the suspect knife was indeed used in the stabbing incident). Moreover, since it is assumed that the signal Y was created at a smaller angle of attack compared to the signal X, the cuts created in the lab with the suspect knife will have to be made at a high angle of attack. In fact, although it is unknown at what angle of attack the cuts in the victim's wounds were created, it can be assumed that they were created at an angle of attack of at most 40°, based on the previous cases. Thus, creating the test cuts in the lab at an angle of attack of 40° should ensure that the striation pattern represented by the signal Y was indeed created at a lower angle of attack compared to the striation pattern represented by the signal X.

7.2. Calibration

In the method described in Section 7.1 there are no set constrains as to how much the signal Y can be stretched in order to match locally the signal X, or, in other words, there is not a maximum amount of times that the same point y_j can be repeated in the warping path to match multiple points x_i, x_{i+1}, x_{i+2} etc. This is because, in general, the angle of attack under which Y was created is unknown, and therefore it is unknown how much it should be stretched in order to match X. For example, if Y was created at an angle of attack of 20°, it will have to be stretched more than if it were created at 30°, since it needs to be aligned to the signal X which is known to be created at 40°.

However, this is not a problem in the case in which both the signals for X and Y represent striation patterns created with the same knife. In fact, this method will stretch the signal Y exactly by the amount that it needs to be stretched to match X, as this is the only way that will ensure the lowest possible matching cost. Therefore, there is no need to set a maximum amount of times that the same point in Y can be repeated in the warping path, as this is implicitly regulated by the method.

However, in the case when the signals for X and Y represent striation patterns created with different knives, the situation is different. In fact, for these cases, the alignment that produces the lowest matching cost might require Y to be stretched by a very large amount. This is because there is no real match between the signals X and Y, and therefore what might decrease the total matching cost is to repeat a point in Y multiple times until the signals show a similar shape (and thus produce a low matching cost). As a consequence, the two aligned signals X and Y might produce a high correlation coefficient, however the required stretch of the signal Y is unrealistic since if X and Y were indeed created by the same knife, no amount of difference in the angle of attack between X and Y could justify such a long stretch of the signal Y.

In a real life case, it is unknown whether the signals X and Y represent striation patterns created by the same knife. Therefore, in order to limit the cases described above where X and Y are not a match, a constraint on the maximum amount of times that the same point in Y can be repeated in the warping path should be set. More specifically, the maximum amount of times that the same point in Y can be repeated should be set to be the smallest amount that allows the signal Y to be matched to the signal X when they are a match. So, in other words, this constraint should be such that it is just large enough to allow the signal Y to be aligned to the signal X (when they are a match), regardless of the difference in angle of attack between the signals X and Y.

It is not obvious to see how large this constraint should be, however it is expected that when X and Y are a match, the larger the difference in angle of attack between X and Y is, the higher the amount of times that the same point in Y will have to be repeated in the warping path to match X. Moreover, it is important to notice that if the constraint that is chosen is slightly too strict, in the cases in which there is a large difference in angle of attack between X and Y and they are a match, the correlation coefficient between the two aligned signals will be lower than it would have otherwise been if no constraints were put, however a match between the two signals will still be found. To analyze this further, some tests were run. Namely, three different knives were selected and for each one the striation pattern created at 40° was considered (signal X). Then, it was matched to (parts of) the striation pattern for that knife created at 0°, 10°, 20° and 30°, however the maximum amount of times that the same point in Y could be repeated in the warping path was chosen to be either 1, 2, 3, 4, 5, or 6 times for each comparison. The correlation coefficients between the aligned signals were then computed for each test and they are shown in the figures below.

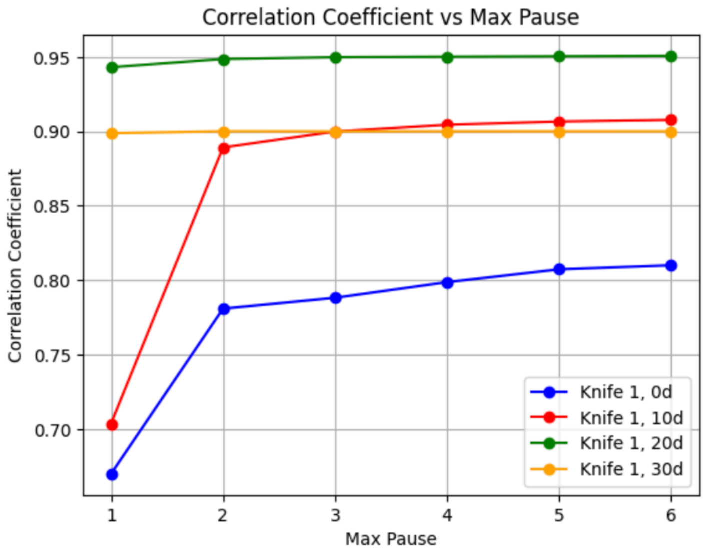


Figure 7.1: How the correlation coefficient between the aligned signals X and Y changes depending on the maximum stretch of Y allowed, for multiple differences in angle of attack between X and Y. The cuts are made with Knife 1, the angle of attack under which X was created is 40°, whereas for Y it is 0°, 10°, 20°, 30°.

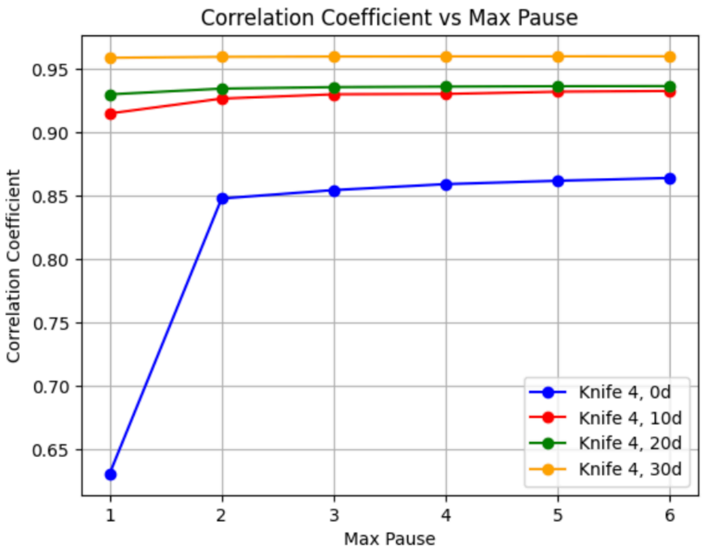


Figure 7.2: How the correlation coefficient between the aligned signals X and Y changes depending on the maximum stretch of Y allowed, for multiple differences in angle of attack between X and Y. The cuts are made with Knife 4, the angle of attack under which X was created is 40°, whereas for Y it is 0°, 10°, 20°, 30°.

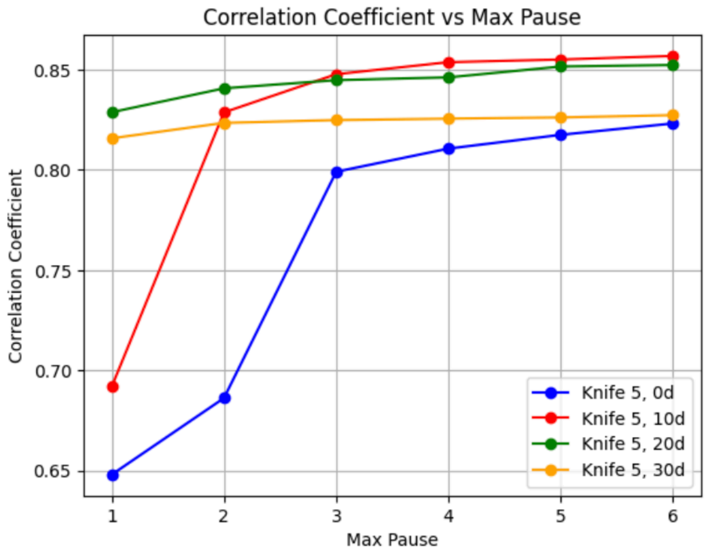


Figure 7.3: How the correlation coefficient between the aligned signals X and Y changes depending on the maximum stretch of Y allowed, for multiple differences in angle of attack between X and Y. The cuts are made with Knife 5, the angle of attack under which X was created is 40°, whereas for Y it is 0°, 10°, 20°, 30°.

As it can be seen from the graphs above, when the difference in angle of attack between X and Y is 10° or 20° (so for the data in yellow and green), it suffices to allow each point of Y to be repeated at most 2 times in order to obtain the (almost) optimal alignment between X and Y. In fact, for these cases, the correlation coefficient between the aligned signals does not change almost at all when the amount of times that each point in Y can be repeated increases. When the difference in angle of attack between X and Y is increased to 30° or 40° (so for the data in red and blue), to obtain a close to optimal alignment between X and Y, each point of Y needs to be allowed to be repeated at least 3 times. This is expected since the larger the difference in angle of attack between X and Y is, the higher the amount of times that the same point in Y will have to be repeated in the warping path to match X.

Therefore, for this project, the maximum amount of times that each point in Y can be repeated in the warping path in order to match X was set to 3. In this way, all the matches with a difference in angle of attack between X and Y of at most 20° can be found with an almost optimal alignment, while the matches with a difference in angle of attack between X and Y greater than 20° can still be found, with a slightly worse alignment. Moreover, with this constraint, the correlation coefficients produced by the alignment of the non-matches are lower compared to what they would have otherwise been without it.

7.3. Comparison with the Matched Filter

The method just described in Section 7.1 and Section 7.2 uses Dynamic Time Warping to stretch the signal Y to match (a part of) the signal X, with the constraint that each point y_j can be repeated at most 3 times in the warping path. The alignment between Y and (a part of) X that is found is the one producing the smallest matching cost, and in case multiple alignments produce the same matching cost, the one with the shortest path is selected. Then, once the two signals have been aligned with this method, their correlation coefficient is computed. Therefore, the best alignment between the (stretched) signal Y and (a part of) the signal X is not found based on the correlation coefficient that this alignment produces.

It is therefore important to investigate whether the alignment found between the stretched signal Y and (a part of) the signal X is indeed the one resulting in the highest correlation coefficient. In order to test this, the method just described should be used only to produce a stretched version of the signal Y via Dynamic Time Warping, and then the Matched Filter should be used to determine where in the larger signal X the stretched signal Y best aligns to, i.e. produces the highest correlation coefficient. If this alignment is the same as the one found in the method described, then indeed this method finds the alignment between the stretched signal Y and (part of) the signal X which produces the largest correlation coefficient.

This test was carried out with more than 30 different pairs of signals X and Y. In all cases, the alignment that was found between the stretched signal Y and (part of) the signal X using the Matched Filter was the same as the alignment that was found originally using the method described in Section 7.1 and Section 7.2. An example of such a test can be seen in the figures below, which show the alignments that were found between a longer signal X representing the striation pattern of a knife created at 40° and a shorter signal Y representing the striation pattern for the same knife created at 0°.

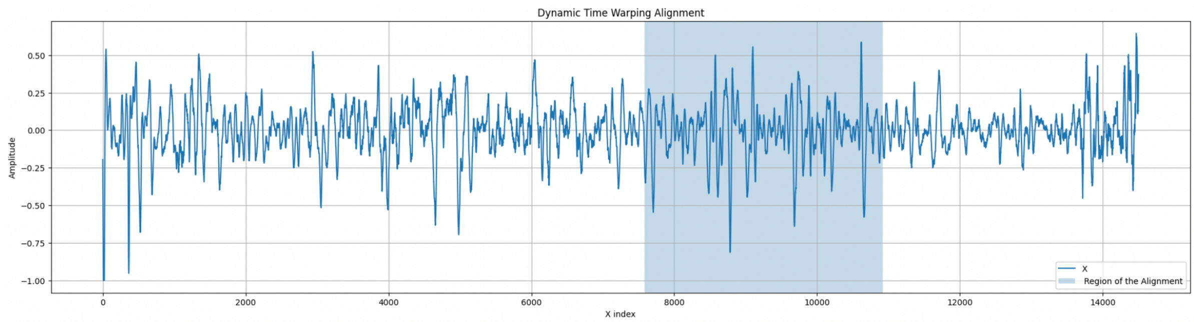


Figure 7.4: Where in X the signal Y was aligned to, using Dynamic Time Warping

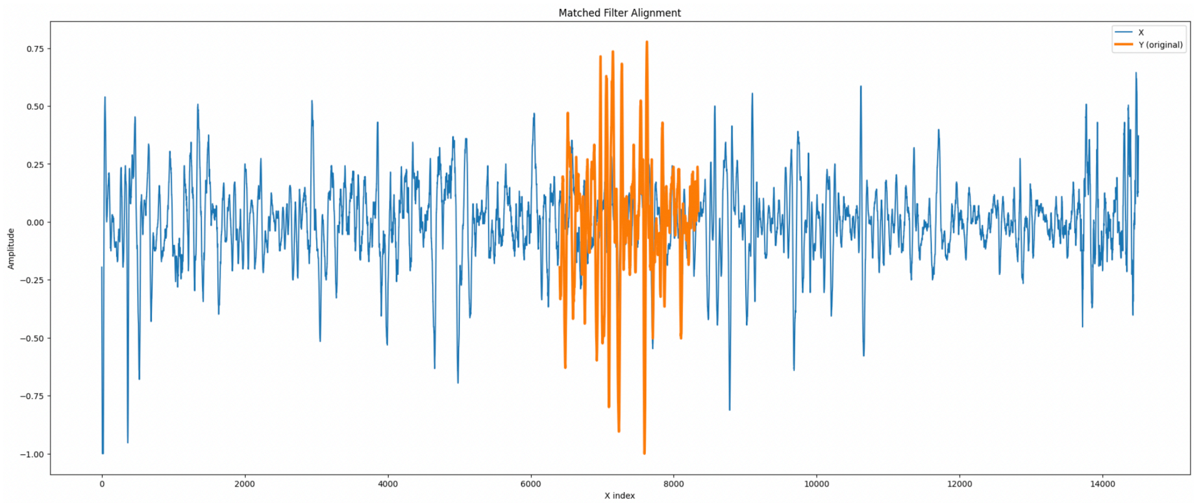


Figure 7.5: Where in X the original signal Y was aligned to using the Matched Filter

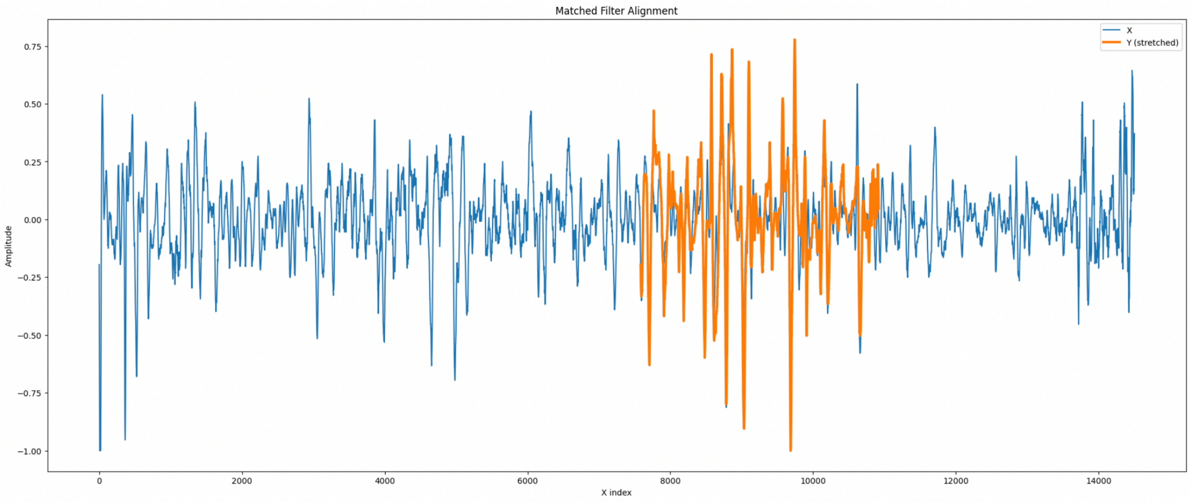


Figure 7.6: Where in X the signal Y was aligned to using the Matched Filter, after it was stretched using Dynamic Time Warping

The first of the figures above shows the alignment that was found between the (stretched) signal Y and the signal X using the method described in Section 7.1 and Section 7.2, the second one shows the alignment that was found between the signal X and the signal Y using the Matched Filter only, so without stretching the signal Y, and finally the third figure shows the alignment that was found between the signal X and the signal Y using the Matched Filter, after the signal Y was stretched as in the first figure, using Dynamic Time Warping. As we can see, the matching region in the first and in the third figure is the same, and therefore the alignment found via Dynamic Time Warping is indeed the alignment between the signal X and the stretched signal Y which results in the highest correlation coefficient.

From the results of these tests, we can infer the method described in Section 7.1 and Section 7.2 finds an alignment between the signals X and Y which is also the one resulting in the highest correlation coefficient between the (stretched) signal Y and the (partial) signal X. Furthermore, this method can also just be considered a tool to stretch the signal Y, which can then be aligned to a part of the signal X using the Matched Filter.

8 Testing the Model

As previously explained, the ultimate goal of the model described in Chapter 7, is to determine whether two (partial) striation patterns were created by the same knife. For this reason, it was tested to determine how well it could perform this task.

8.1. Data used

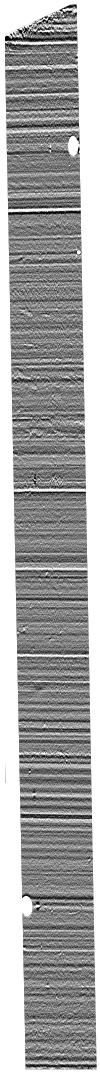
The data that was used to run these tests were the test cuts created by the previous student working on this problem at the NFI, [24]. As previously explained, in fact, he selected 6 different knives and created 5 different cuts for each of them. More specifically, the cuts that he created for each knife were at the following angles of attack: 0°, 10°, 20°, 30° and 40°. These cuts were then processed as explained in Chapter 2, in order to obtain, for each of them, a picture of the striation pattern in a gray-scale.

For these tests, the pictures of the striation patterns were considered. For each knife, in fact, the pictures of the striation patterns created at 0°, 10°, 20° and 30° were cropped so that they would all only show the same (small) part of the striation pattern. This was done by visually selecting the same (small) part of the striation pattern in each picture of these striation patterns. An example of this can be see in the figures below (Figure 8.1 and Figure 8.2), as they represent the full striation patterns at 20° and 30° respectively for Knife 1 on the left, and the same small part of the striation pattern that was considered after cropping these pictures, on the right.

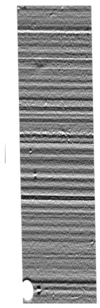
The pictures of the striation patterns created at an angle of attack of 40° for the 6 knives were duplicated. Then, one copy of the pictures was cropped so that it would only show the same small part of the striation pattern as the cropped pictures for the striation patterns at 0°, 10°, 20° and 30° for the same knife. The other copy of the picture, on the contrary, was left untouched.

Therefore, in the end, the data used to test the model consisted of, for each of the 6 knives:

- a picture of the same small part of the striation pattern, created at the angles of attack of 0°, 10°, 20°, 30° and 40° (cuts Y)
- a picture of the whole striation pattern created at 40° (cut X)

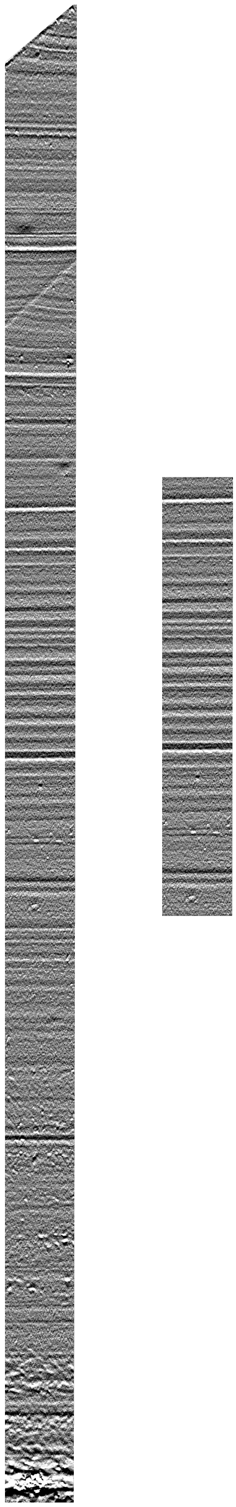


(a) Striation pattern at 20°



(b) Part selected, measuring 6.2 mm

Figure 8.1



(a) Striation pattern at 30° **(b)** Part selected, measuring 8.3 mm

Figure 8.2

8.2. Remarks on the Data used

One of the potential limitations of testing the model using the data previously mentioned is that it was created in the lab, and thus it might differ from the data that would be found in a real life case, e.g. the striation pattern that would be found in a bone or in a cartilage tissue in the case of a stabbing incident. As a consequence, it could be the case that the model might perform well with the lab-created data, but perform worse with the real life data.

The following remarks explain how the real life data might be different from the lab-created data, and how these differences compare.

- **Length of the cuts Y**
  In a real life case, the part of the striation pattern found in a bone or in a cartilage tissue is generally small. This is because the physical length of the striation pattern found can be no longer than the depth and the height of the tissue that it is cutting through, and as the bones and the cartilage tissue are generally quite small, the striation pattern on them is also rather short.

  As a consequences, the real life traces of the striation patterns created at higher angles of attack are less informative. This is because in the striation patterns created at higher angles of attack the lines are further apart from each other. Thus, if a trace of a striation pattern created at a higher angle of attack is very small, it might not be informative as it will contain a limited amount of lines.

  For this research it was assumed that in real life cases the striation patterns found for higher angles of attack are at least physically long enough to be informative. In other words, it is assumed that the real life traces for striation patterns created at higher angles of attack are physically longer.

  Thus, the cuts Y were cropped from the longer striation patterns created at 0°, 10°, 20°, 30° and 40° so that their length would be similar to that of a striation pattern found in a real life case for the ones created at a lower angle of attack, and possibly slightly higher for the ones created at higher angles of attack. This is done by considering the same part of the striation pattern for all the cuts created with the same knife. Since the distance between the lines in a striation pattern is higher for cuts created at higher angles of attack, the cuts Y that were cropped from striation patterns created are higher angles of attack are longer.
- **Quality of the striation patterns**
  In a real life case, the quality of the striation pattern found is not always perfect. This could be due to multiple factors, and it might affect how clear the lines in the striation pattern appear to be.

  The data that is used to test the model described in Chapter 7 reflects this. In fact, as in can be seen from the figures in Section 8.1, for some of the considered striation patterns the lines appear clearer than for others. This is due to the fact that the gray-scale for each of the cuts in the data set was calibrated differently, based on the highest and lowest points of that cut. Therefore, the same part of a striation pattern is set to a slightly different gray-scale, depending on the cut of which is it part of, and thus shows a different degree of clarity for its lines accordingly.

  Therefore, testing the model with this data set provides an insight on how the model will behave with data that (partially) lacks clarity and quality, although perhaps not as much as the real life data does.
- **Knives used**
  In real life cases, multiple types of knives are used to create the cuts that will be analyzed. As they can have different blade shapes, the way in which their striation patterns will change for different angles of attack will be different, as explained in Section 2.3. Thus, in order to account for this, the data set used to test the model was created using 6 different knives, with very different blade shapes.

The following figures show the 6 knives that were used to create the data set [24].

Knife 1 Knife 2

Knife 3 Knife 4

Knife 5 Knife 6

Figure 8.5

Finally, it is important to remark that the cuts for Knife 2 at an angle of attack of 10°, and for Knife 6 at an angle of attack of 0° were not included in the data set, as their quality was very compromised.

8.3. Design of the Test

In order to check whether the model described in Chapter 7 can be used to determine if two striation patterns X and Y were created by the same knife, it is tested for the two following scenarios:

- *Case 1:* The two striation patterns for the signals X and Y come from the same knife, and they are possibly created at two different angles of attack
- *Case 2:* The two striation patterns for the signals X and Y come from two different knives

As explained in Chapter 7, the model takes as input two images representing two striation patterns X and Y, finds the best alignment of Y to (a part of) X while allowing for a non-linear stretching, and outputs the alignment that was found, as well as the correlation coefficient between the two aligned signals (for the part of X to which Y is aligned to). Therefore, the expected outcomes for the two cases previously mentioned are:

- *Case 1:* The striation pattern Y is correctly aligned to its corresponding part in the striation pattern of X, and the correlation coefficient between the two aligned signals is rather high
- *Case 2:* The correlation coefficient between the two aligned signals X and Y (for the part of X to which Y is aligned to) is rather low

The test is therefore conducted as follows. A small cut $Y_{i,\alpha}$ created with Knife i (for $i \in \{1, 2, 3, 4, 5, 6\}$) at the angle of attack α (for $\alpha \in \{0°, 10°, 20°, 30°\}$ is selected. Then, the model is used to determine the best alignment between this small cut $Y_{i,\alpha}$ and (a part of) the longer cuts X created at 40° for the other 5 knives, and the resulting correlation coefficients are stored. So, for example, if the short cut $Y_{1,0°}$ created at 0° with Knife 1 is selected, it is then compared to the longer cuts X created at 40° with Knives 2, 3, 4, 5, and 6, producing five different correlation coefficients for the five different alignments found. These tests correspond to *Case 2*, as mentioned above.

Then, the selected shorter cut $Y_{i,\alpha}$ is compared to the longer cut X created at 40° with Knife i. This test corresponds to *Case 1* mentioned above, and thus in this case we are not only interested in the resulting correlation coefficient but also in whether the shorter cut $Y_{i,\alpha}$ was correctly aligned to its corresponding part in the striation pattern created at 40°. However, this is generally not obvious to see, and thus the following comparison is considered as well. Namely, the shorter cut $Y_{i,40°}$ created at 40° with Knife i is compared to the longer cut X also created at 40° with Knife i. In this way, it will be aligned perfectly to a part of the longer cut X created at 40°, and the resulting correlation coefficient will be 1.

Since all the shorter cuts $Y_{i,\alpha}$ were originally cropped so that they all represent the same part of the striation pattern for Knife i, we know that the considered shorter cut $Y_{i,\alpha}$ created at angle α for Knife i will be correctly aligned to its corresponding part in the striation pattern X created at 40° for Knife i if it will be aligned to to same part as to where the shorter cut $Y_{i,40°}$ created at 40° with Knife i was aligned to. In this way, thus, it can be checked whether the alignment between the shorter cut $Y_{i,\alpha}$ for Knife i and the longer cut X created at 40° for Knife i is correct, as in, in the correct region.

An example of this is shown in the figures below. In fact, suppose that we want to compare the shorter cut $Y_{1,30°}$ created at 30° with Knife 1 (Figure 8.2) to the longer cut X created at 40° also with Knife 1. From their alignment, we get a correlation coefficient of 0.90 and the region of the alignment is shown in the following figure.

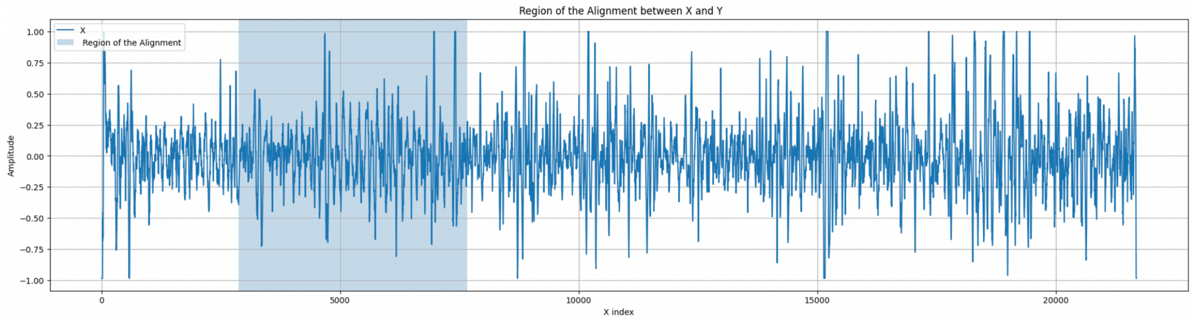


Figure 8.6: Region of the alignment between the signals X (created at 40°) and Y (created at 30°) for Knife 1

However, as previously explained, we are not only interested in the resulting correlation coefficient but also in whether the shorter cut $Y_{1,30°}$ was correctly aligned to its corresponding part in the striation pattern created at 40°. Thus, in order to know this, we also compare the shorter cut $Y_{1,40°}$ created at 40° with Knife 1 to the longer cut X created at 40° with Knife 1. For this comparison, we get a correlation coefficient of 1, and the region of this alignment is shown in the following figure.

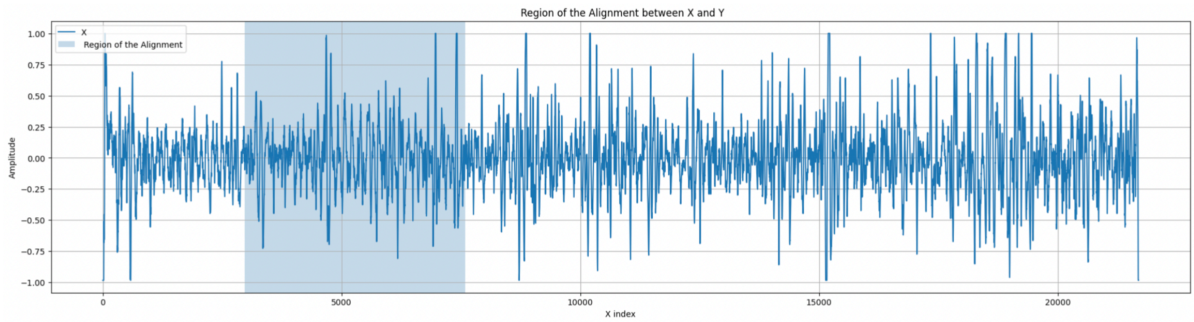


Figure 8.7: Region of the alignment between the signals X (created at 40°) and Y (created at 40°) for Knife 1

Since both of the short cuts $Y_{1,30°}$ and $Y_{1,40°}$ (created at 30° and 40°) are aligned to the same part of the longer cut X created at 40°, we know that the alignment found in Figure 8.6 is correct. This is because both of the shorter cuts $Y_{1,30°}$ and $Y_{1,40°}$ were cropped so that they would show the same part of the striation pattern of Knife 1, and the alignment between the shorter cut $Y_{1,40°}$ created at 40° to the longer cut X also created at 40° has to be correct.

The selected shorter cut $Y_{i,\alpha}$ is compared to the longer cut X created at 40° with Knife i, and the resulting correlation coefficient is stored, as well as whether the shorter cut $Y_{i,\alpha}$ was aligned correctly to its corresponding part in X. This procedure is then repeated for all $i \in \{1, 2, 3, 4, 5, 6\}$ and for all $\alpha \in \{0°, 10°, 20°, 30°\}$.

9

Results

9.1. Confusion Matrix for the Tests

The following table shows the correlation coefficients obtained when comparing the small cuts Y made at either 0°, 10°, 20° or 30° to the larger cuts X made at 40°, for all the knives.

	Longer cut X at 40°					
Shorter cut Y	Knife 1	Knife 2	Knife 3	Knife 4	Knife 5	Knife 6
Knife 1, 0°	0.79	0.58	0.65	0.74	0.64	0.74
Knife 1, 10°	0.90	0.51	0.61	0.73	0.60	0.72
Knife 1, 20°	0.95	0.41	0.53	0.73	0.63	0.68
Knife 1, 30°	0.90	0.41	0.58	0.66	0.52	0.55
Knife 2, 0°	0.75	0.66	0.71	0.70	0.72	0.64
Knife 2, 20°	0.67	0.90	0.79	0.66	0.65	0.74
Knife 2, 30°	0.69	0.95	0.70	0.69	0.62	0.74
Knife 3, 0°	0.77	0.71	0.75	0.71	0.66	0.79
Knife 3, 10°	0.76	0.73	0.73	0.73	0.67	0.74
Knife 3, 20°	0.68	0.47	0.69	0.71	0.62	0.65
Knife 3, 30°	0.73	0.65	0.94	0.68	0.66	0.74
Knife 4, 0°	0.63	0.54	0.66	0.85	0.63	0.67
Knife 4, 10°	0.72	0.55	0.59	0.93	0.61	0.61
Knife 4, 20°	0.67	0.40	0.54	0.94	0.60	0.55
Knife 4, 30°	0.59	0.39	0.42	0.96	0.57	0.69
Knife 5, 0°	0.74	0.51	0.60	0.68	0.80	0.55
Knife 5, 10°	0.77	0.54	0.63	0.66	0.85	0.64
Knife 5, 20°	0.67	0.42	0.50	0.76	0.84	0.63
Knife 5, 30°	0.73	0.50	0.52	0.65	0.82	0.73
Knife 6, 10°	0.46	0.56	0.30	0.55	0.58	0.29
Knife 6, 20°	0.63	0.49	0.60	0.69	0.61	0.76
Knife 6, 30°	0.72	0.40	0.64	0.68	0.61	0.92

Table 9.1

The colored diagonal blocks of this table represent the "Known Matches", i.e. the comparison of two cuts made at different angles but with the same knife. The other entries of the table represent the "Known-Non-Matches", i.e. the comparisons of two cuts that we know were not made with the same knife. Thus, ideally, the correlation coefficients in the diagonal blocks of the table should be 1, whereas the correlation coefficients in all the other entries of the table should be as low as possible.

Moreover, in the table, the entries in green represent the Known Matches that were correctly identified, whereas the entries in orange represent the Known Matches that were not identified. In other words, for the entries in green, given a small cut Y, the code was able to correctly identify its corresponding part in the striation pattern created at 40°, whereas this was not the case for the entries in orange. Thus, the correlation coefficients in orange in the table have the same significance as random matches.

9.2. Distributions

The two following histograms show the distributions of the correlation coefficients obtained in the tests for the Known-Non-Matches and the Known Matches respectively. The entries in orange from the table above are considered as Known-Non-Matches, as these correlation coefficients are practically the results of random matches, i.e. the alignment of two signals that are known not to be representing the same (part of the) striation pattern.

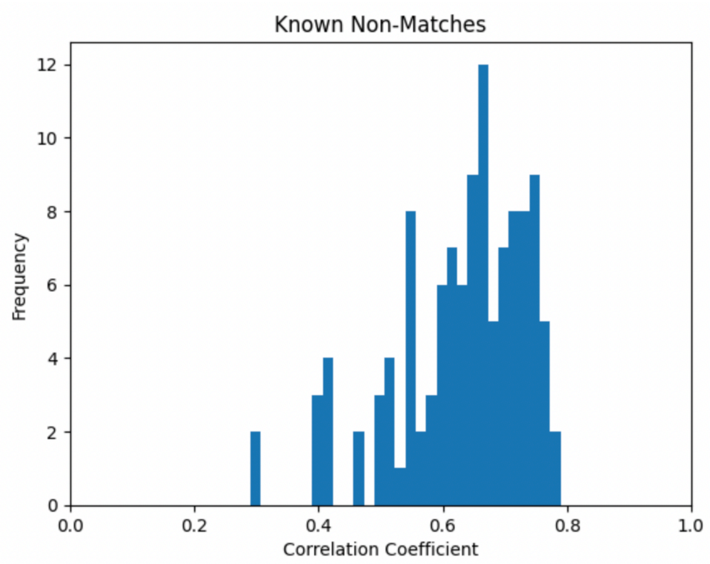


Figure 9.1: Distribution of the correlation coefficients for the Known-Non-Matches

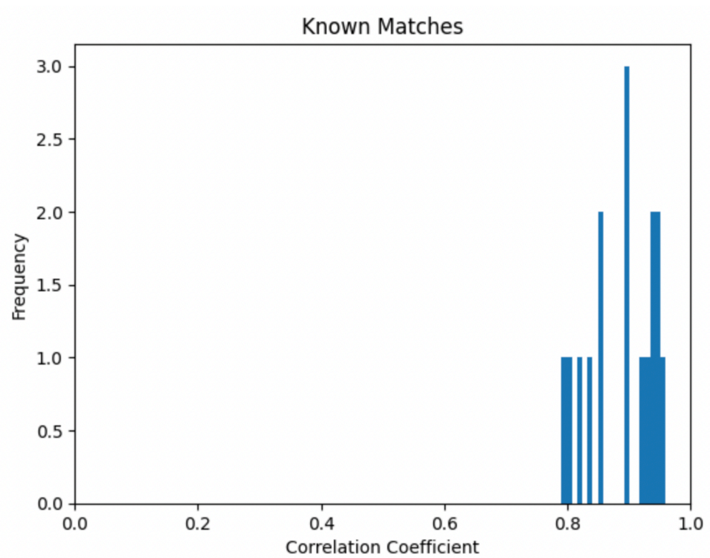


Figure 9.2: Distribution of the correlation coefficients for the Known Matches

9.3. Remarks on the Results

From the table shown in Section 9.1, we can see that a total 132 tests were run, out of which 22 were the comparisons of Known Matches. The model was able to correctly identify 16 out of these 22 Known Matches. When the Known Matches were not correctly identified, the shorter cut Y was created at an angle of attack of either 0°, 10° or 20°. On the contrary, all the Known Matches of the shorter cuts Y created at 30° were correctly identified.

From Figure 9.1 and Figure 9.2, we can see that there is very little overlap between the two distributions for the correlation coefficients for the Known Matches and the Known-Non-Matches. This suggests that this model can be used to produce meaningful likelihood ratios. In fact, as explained in Chapter 1, we are also interested in knowing the likelihood ratio

$$LR = \frac{P(E|H_1)}{P(E|H_2)}$$

where E is the evidence, i.e. the correlation coefficient produced by the code, H_1 is the hypothesis that the two cuts X and Y were created by the same knife, and H_2 is the hypothesis that the short cut Y was created by an unknown knife. Thus, roughly speaking, given the correlation coefficient produced by the model, $P(E|H_1)$ is the probability of finding this correlation coefficient in the distribution of the Known Matches, whereas $P(E|H_2)$ is the probability of finding this correlation coefficient in the distribution of the Known-Non-Matches. Therefore, the less overlap there is between these two distributions, the larger the strength of the likelihood ratio will be.

10

Discussion

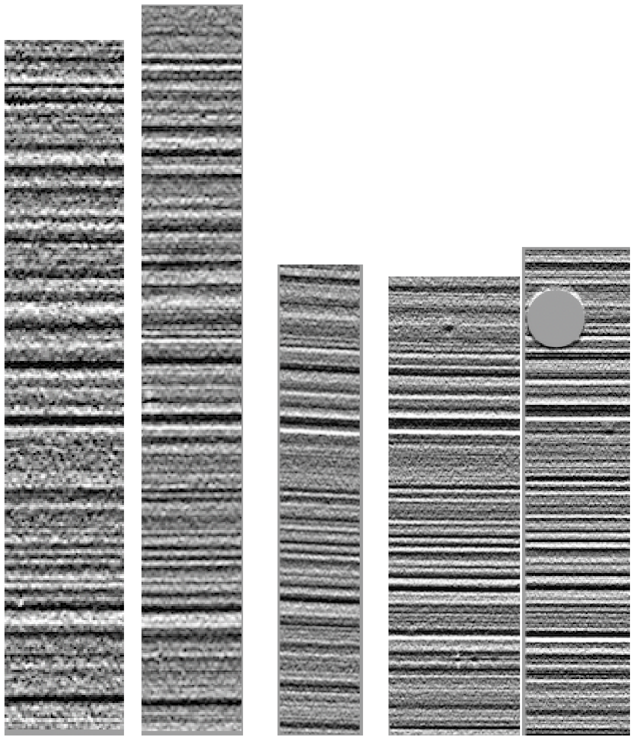
10.1. Why certain Known Matches were not Identified

As mentioned in Section 9.3, all the Known Matches where the shorter cut Y was created at an angle of attack of 30° were correctly identified, and for the Known Matches that were not correctly identified, the shorter cut Y was created at an angle of attack of either 0°, 10° or 20°. As the shorter cuts Y were compared to longer cuts X created at an angle of attack of 40°, this suggests that the model is less likely to perform well when the difference in angle of attack between the cuts X and Y increases.

At first, it might seem that this is caused by the fact that in this model each point is allowed to be repeated at most 3 times. In fact, the higher the difference in angle of attack between X and Y, the more the signal Y has to be stretched to match X, and if the difference in angle of attack between X and Y is increased too much, it could be the case that the said maximum stretching allowed is not enough to align Y to X. As a consequence, the model will not be able to correctly identify the Known Matches of the shorter cuts Y created at smaller angles, like 0°, 10° and 20°.

However, increasing the number of times that the same point can be repeated, or in other words, allowing Y to stretch more in order to match X, the model still cannot correctly identify the Known Matches for the short cuts Y created at said lower angles. This, in fact, was tested by setting the maximum number of times that the same point y_j could be repeated up to 20. Thus, the reason why the Known Matches for these shorter cuts Y cannot be correctly identified by the code is not because they cannot be stretched enough to match the longer cuts X.

To understand why the model has difficulties in identifying the Known Matches for the shorter cuts Y created at smaller angles, it is useful to look at the physical striation patterns for these shorter cuts Y, and compare them to the (partial) physical striation pattern created at an angle of attack of 40°. The following figures show these for Knife 3, which was one of the knives for which the Known Matches for the shorter cuts Y created at 0°, 10° and 20° were not correctly identified.



(a) 0° **(b)** 10° **(c)** 20° **(d)** 30° **(e)** 40°

Figure 10.1: The images of the striation patterns created at lower angles of attack are linearly enlarged to visually match the striation pattern created at 40° on the right

In the figures above it is easy to identify the same lines in the striation pattern for the different cuts. However, if we were to compare directly the striation pattern created at 0° to the striation pattern created at 40°, it would be more difficult to identify the same lines in the striation pattern. This is because in the cuts made at higher angles of attack, more lines of the striation pattern are visible and distinguishable, whereas in the cuts made at lower angles of attack some lines are blurred or even non-existent. Thus, without having the ”intermediate angle” cuts, it is challenging to identify the same lines in the striation pattern for the cut at 0° and the cut at 40°.

The reason why the cuts made at lower angles of attack present less distinguishable lines in the striation pattern is not because their scans/gray-scales were made with a lower resolution, but rather because of physical reasons. In fact, when a material is cut with a lower angle of attack, the lines its striation pattern will be closer together, however, depending on the physical properties of the material, they might get so close together that they are blended into one line only. Therefore, the striation pattern for such a cut made with a smaller angle of attack might appear to have physically less lines.

This is the reason why the model fails to identify some of the Known Matches for the shorter cuts Y created at smaller angles of attack. In fact, when the striation patterns for these cuts are turned into signals, they look different from the signals for the striation patterns created at 40°, since they originally contained less lines and in general less information. Thus, they can potentially match more regions in the longer signal X, and, as a consequence, the model is more likely to mismatch them.

Furthermore, it can be noticed that for the Known Matches that were correctly identified, the shorter cut Y created at 0° always produces a lower correlation coefficient compared to the shorter cuts made at 10°, 20° and 30° with that knife. For example, for Knife 4, the correlation coefficients of the Known Matches for the shorter cuts Y created at 0°, 10°, 20° and 30° are 0.85, 0.93, 0.94 and 0.96 respectively, so the shorter cut Y created at 0° produces the lowest correlation coefficient.

This is another consequence of the fact that the cuts made at smaller angles of attack present less physical lines in their striation pattern. In fact, even when the model correctly identifies a Known Match for a shorter cut Y created at a small angle of attack, the correlation coefficient of this match will be lower compared to the correlation coefficient for a Known Match for a shorter cut Y created at a higher angle of attack, because the signal Y will be physically be missing some information compared to the signal X, and thus their correlation coefficient will be lower.

10.2. Decreasing the Difference in Angle of Attack between X and Y

In order to avoid the problem described in Section 10.1, and thus to ensure that the model can correctly identify the Known Matches also for shorter cuts Y created at lower angles of attack, another test was performed. This time, however, the shorter cuts Y were compared to a longer cut X created at 20°, instead of 40°. Since the model only allows Y to be stretched to match X, the shorter cuts Y that were considered had to be made at an angle of attack had to be <20°, and so only the ones created at 0° and 10° were considered.

The hypothesis for this test was that, as the physical striation patterns created at a lower angle of attack are more similar to the physical striation pattern created at 20°, this time the model should be able to correctly identify all the Known Matches. Moreover, for the same reason, the correlation coefficients for the Known Matches that were correctly identified should be higher than the ones obtained for the same shorter cuts Y when they were compared to the long cut X created at 40°. For example, the shorter cut Y created at 0° for Knife 1 resulted in a correlation coefficient of 0.79 for its Known Match with the longer cut X created at 40° with Knife 1. Thus, from the comparison of the same shorter cut to the longer cut X created at 20° with Knife 1, we expect a correlation coefficient greater than 0.79.

The following table shows the correlation coefficients obtained when comparing the small cuts Y made at 0° and 10° to the larger cuts X made at 20°, for all the knives.

	Longer cut X at 20°					
Shorter cut Y	Knife 1	Knife 2	Knife 3	Knife 4	Knife 5	Knife 6
Knife 1, 0°	0.94	0.56	0.71	0.68	0.60	0.58
Knife 1, 10°	0.97	0.58	0.68	0.76	0.57	0.54
Knife 2, 0°	0.71	0.83	0.74	0.77	0.54	0.57
Knife 3, 0°	0.81	0.73	0.84	0.75	0.67	0.68
Knife 3, 10°	0.82	0.70	0.82	0.69	0.66	0.74
Knife 4, 0°	0.64	0.59	0.77	0.90	0.55	0.49
Knife 4, 10°	0.70	0.54	0.74	0.93	0.53	0.56
Knife 5, 0°	0.64	0.59	0.64	0.69	0.83	0.51
Knife 5, 10°	0.73	0.64	0.66	0.69	0.86	0.65
Knife 6, 10°	0.52	0.35	0.60	0.74	0.40	0.73

Table 10.1

This time a total of 60 tests were run, out of which 10 were comparisons of Known Matches. The model was able to correctly identify all 10 of them. Moreover, the correlation coefficients for all the Known Matches are higher than the correlation coefficients previously obtained for the same comparisons, i.e. when the larger cut X was created at an angle of attack of 40° (except for one case in which the correlation coefficient remained the same), as expected.

Furthermore, the distributions of the correlation coefficients for the Known-Non-Matches and the Known Matches respectively are shown in the histograms below.

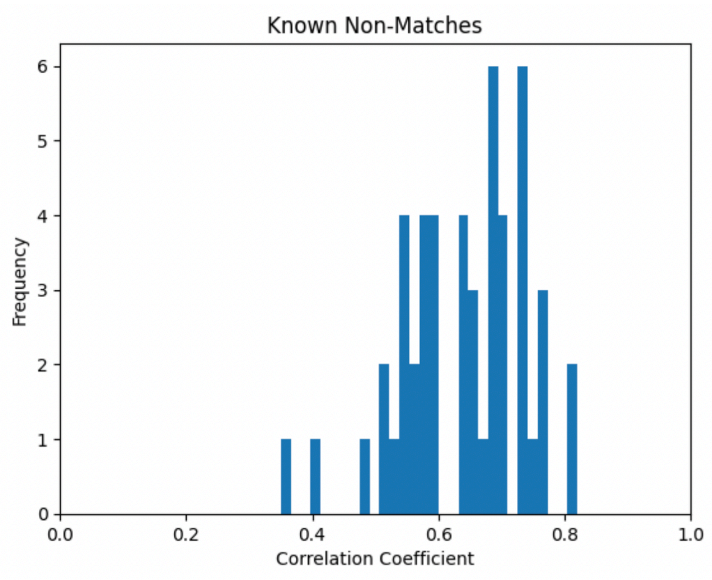


Figure 10.2: Distribution of the correlation coefficients for the Known-Non-Matches (X created at 20°)

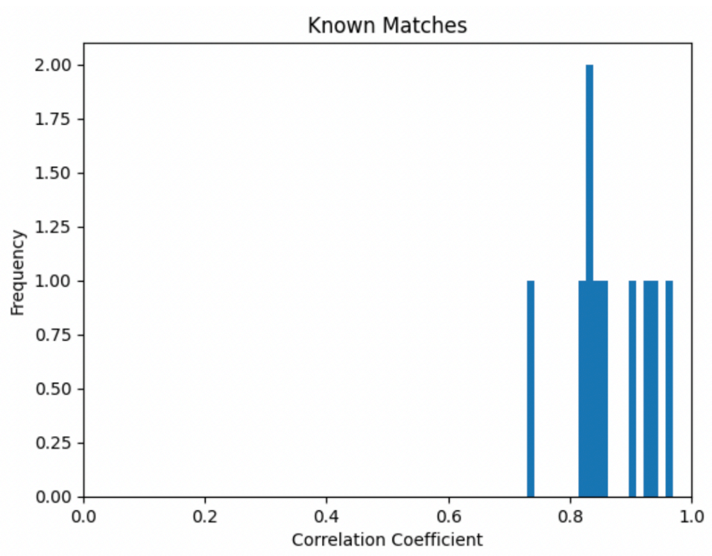
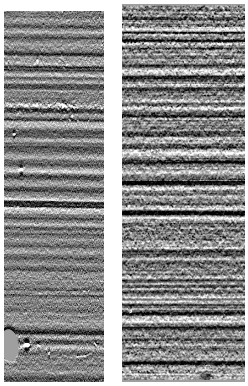


Figure 10.3: Distribution of the correlation coefficients for the Known Matches (X created at 20°)

As explained in Section 9.3, the less overlap there is between these two distributions, the more meaningful the likelihood ratio for the evidence of a given correlation coefficient will be. Compared to the previous tests (in Chapter 9), the distributions in Figure 10.3 and Figure 10.2 present more overlap, and, as a result, given a correlation coefficient it can be determined with less certainty whether it comes from the distribution of the Known Matches or the Known-Non-Matches. More specifically, this overlap consists of two of the Known-Non-Matches that present correlation coefficients >0.80 and one of the Known Matches that presents a correlation coefficient <0.8.

The cases in which this happens are for the comparisons of the shorter cuts Y of Knife 3, 0°, Knife 3, 10° and Knife 6, 10°. These cases are analyzed in detail in what follows:

- **Knife 3, 0°**
  The correlation coefficient for the Known-Non-Match with Knife 1 is 0.81. The following images show (the small part of) the striation pattern of Knife 3, 0° that was used for the tests, and the part of the longer striation pattern of Knife 1, 20° that it was matched to.

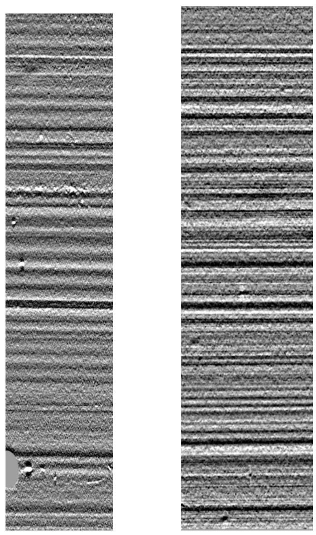


Knife 1 Knife 3, 0°

Figure 10.4

  From the images above (not to scale), we can observe that these two (parts of) striation patterns do look similar, and with a non-linear stretching of the striation pattern of Knife 3 at 0°, they could be aligned quite well. Therefore, the fact that their matching produces a high correlation coefficient is not caused by an issue in the model but rather by the physical nature of the data.

- **Knife 3, 10°**
  The correlation coefficient for the Known-Non-Match with Knife 1 is 0.82. The following images show (the small part of) the striation pattern of Knife 3, 10° that was used for the tests, and the part of the longer striation pattern of Knife 1, 20° that it was matched to.

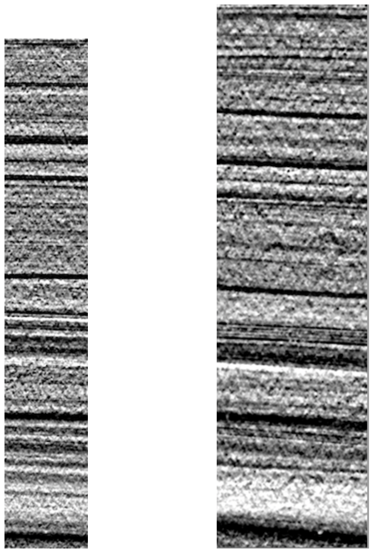


Knife 1 Knife 3, 10°

Figure 10.5

The part of the striation pattern of Knife 3 that is shown in the picture above created at 10° is the same as the one in the previous case, created at 0°. Moreover, they both are matched to the same part in the striation pattern of Knife 1 (created at 20°). Therefore this case is analogous to the previous one. Namely, these (part of) striation patterns for Knife 1 and Knife 3 look similar, and therefore produce a high correlation coefficient even though they are not produced by the same knife.

- **Knife 6, 10°** The correlation coefficient for the Known Match with Knife 6 is 0.73. The following images show (the small part of) the striation pattern of Knife 6, 10° that was used for the tests, and the part of the longer striation pattern of Knife 6, 20° that it was correctly matched to.



Knife 6, 20° Knife 6, 10°

Figure 10.6

The model was able to correctly match the shorter cut Y created at 10° to its corresponding part in the striation pattern created at 20°, both with Knife 6. However, it produced a rather low correlation coefficient for this match. From the images above, we can see that, although the striation patterns look similar, the striation pattern created at 20° presents more distinguishable lines than the one created at 10°. This is not surprising, for the reasons explained in Section 10.1. For this reason, the two striation patterns can be identified to match, however their correlation coefficient will be low as they present these differences.

10.3. Reducing the Maximum Stretch Allowed

From the results presented in Section 10.2, we can see that decreasing the difference in angle of attack between X and Y, and in particular comparing the shorter cuts Y created at 0° and 10° to the longer cuts X created at 20° instead of 40°, improves the outcomes of the comparisons. More specifically, all the Known Matches were correctly identified (and this was not the case for the results in Section 9.1) and their correlation coefficients were in general higher than the ones obtained for the same comparisons in Section 9.1.

Thus, these results suggest that in order to determine whether a shorter cut Y was created by a given knife, it would be better to create at least two longer cuts X with that knife at two different angles of attack, e.g. 20° and 40°, and compare the shorter cut Y to both of these. This is the case since the angle of attack for Y is generally unknown, however, assuming that it will not be greater than 40°, if we compare it to longer cuts X created at 20° and 40°, we know that the difference in angle of attack between Y and X will be at most 20° for each of the X. Therefore, assuming that the model can correctly identify the Known Matches with a difference in angle of attack less than or equal to 20°, this will guarantee that the Known Matches will be identified. More generally, creating even more cuts X

at different angles of attack will reduce the maximum difference in angle of attack between Y and (on of the) X even further, leading to even better results.

In this setting, the maximum difference in angle of attack between Y and (on the cuts) X will be lower, and thus for the Known Matches also the maximum stretching that Y will have to undergo in order to match X will be lower. So, in other words, the maximum amount of times that the same point in Y will be repeated for the matching will be lower. Therefore, the model should be adjusted accordingly, i.e. the maximum amount that the same point in Y can be repeated should be lowered, so that the Known Matches will still be correctly identified, however the Know-Non-Matches will produce lower correlation coefficients as they will have less freedom in how to be matched to the signal X. This will therefore create less overlap between the distribution for the correlation coefficient of the Known Matches and the Known-Non-Matches, leading to more certainty in the interpretation fo the results.

To verify this hypothesis, another test was run, setting the maximum amount of times that the same point in Y could be repeated for the stretching to 1, instead of 3. The data that was used for this test was the same as the one used in Section 9.1, however only the shorter cuts Y created at 20° and 30° were considered. So the maximum potential difference in angle of attack between X and Y was 20°, and more specifically the longer cuts X were created at 40°, whereas the shorter cuts Y were created at at 30° and 20°, for each knife.

As mentioned above, the expected results were to obtain less overlap between the distribution for the correlation coefficient of the Known Matches and the Known-Non-Matches, while still correctly identifying the Known-Matches. Moreover, some of the correlation coefficient for the Known Matches were expected to be slightly lower than the ones obtained in Section 9.1 for the same comparisons. This is to to the fact that reducing the maximum stretch allowed for the shorter cut Y might result in slightly worse alignment of the signals Y and X for the Known Matches, especially when there is a higher difference in angle of attack between X and Y, leading to slightly lower correlation coefficients.

The following table shows the correlation coefficients obtained when comparing the small cuts Y made at 20° and 30° to the larger cuts X made at 40°, for all the knives.

	Longer cut X at 40°					
Shorter cut Y	Knife 1	Knife 2	Knife 3	Knife 4	Knife 5	Knife 6
Knife 1, 20°	0.94	0.40	0.52	0.65	0.55	0.67
Knife 1, 30°	0.90	0.32	0.47	0.58	0.51	0.41
Knife 2, 20°	0.62	0.88	0.62	0.62	0.60	0.66
Knife 2, 30°	0.59	0.95	0.68	0.54	0.64	0.59
Knife 3, 20°	0.69	0.39	0.64	0.66	0.56	0.63
Knife 3, 30°	0.68	0.57	0.94	0.52	0.62	0.66
Knife 4, 20°	0.63	0.37	0.50	0.93	0.57	0.50
Knife 4, 30°	0.57	0.36	0.42	0.96	0.56	0.65
Knife 5, 20°	0.53	0.33	0.46	0.58	0.83	0.52
Knife 5, 30°	0.60	0.49	0.48	0.54	0.82	0.51
Knife 6, 20°	0.61	0.48	0.58	0.57	0.56	0.48
Knife 6, 30°	0.69	0.37	0.52	0.58	0.52	0.91

Table 10.2

This time a total of 72 tests were run, out of which 12 were Known Matches. The model correctly identified 10 out of 12 of them, and the two Known Matches that were not correctly identified (i.e. the ones in orange in the table) were also not correctly identified in the results in Section 9.1, i.e. when the maximum repetition allowed for each point in Y was 3 instead of 1. Furthermore, all the correlation coefficients obtained for the shorter cuts Y created at 30° were the same as the ones obtained in Section 9.1 for the same comparisons, except for Knife 6, where the correlation coefficient obtained is slightly lower than the one obtained in Section 9.1. Also, all the shorter cuts Y created at 20° resulted into slightly lower correlation coefficients compared to the ones obtained for the same comparisons in Section 9.1, as expected.

The following histograms show the the distributions of the correlation coefficients for the Known-Non-Matches and the Known Matches respectively, for these test. Once again, the Known Matches that were not identified, i.e. the ones in orange in the table above, were considered as Known-Non-Matches.

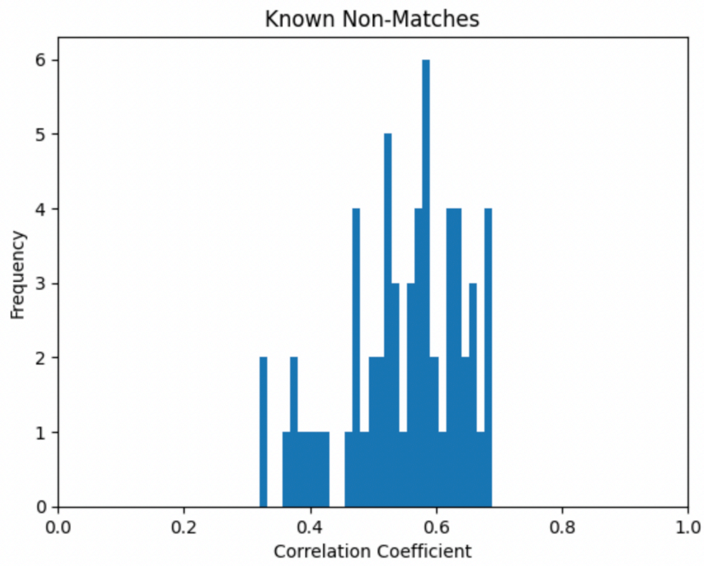


Figure 10.7: Distribution of the correlation coefficients for the Known-Non-Matches (maximum repetition of each point in Y set to 1)

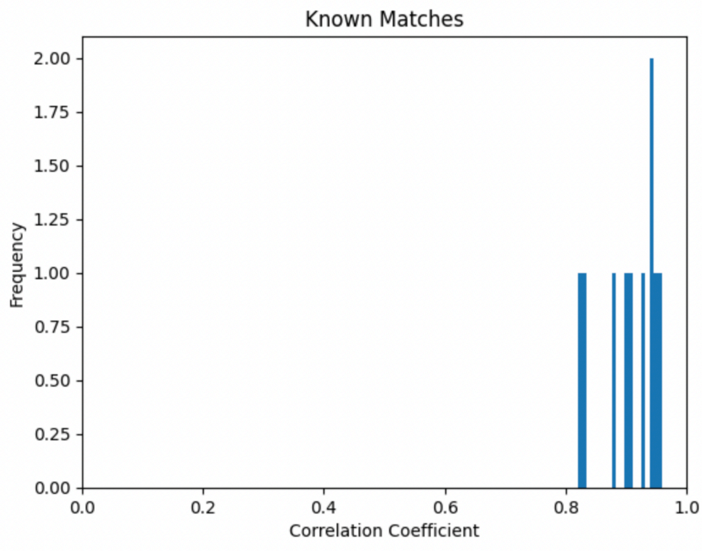


Figure 10.8: Distribution of the correlation coefficients for the Known Matches (maximum repetition of each point in Y set to 1)

For comparison, the distributions of the correlation coefficients for the Known-Non-Matches and the Known Matches for the results for the same comparisons in Section 9.1 are also shown. For these distributions, thus, out of the results shown in Section 9.1, only the ones for the shorter cuts Y created at 20° and 30° and their comparisons are considered. The Known Matches that were not correctly identified are counted as Known-Non-Matches.

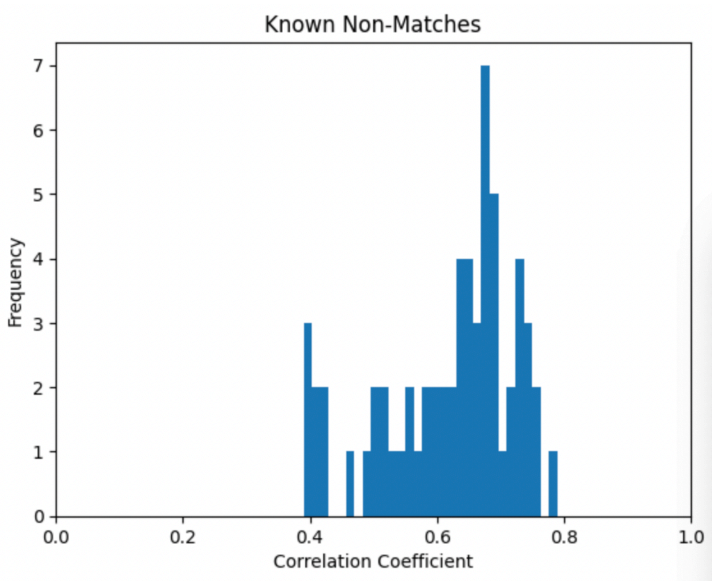


Figure 10.9: Distribution of the correlation coefficients for the Known-Non-Matches (maximum repetition of each point in Y set to 3)

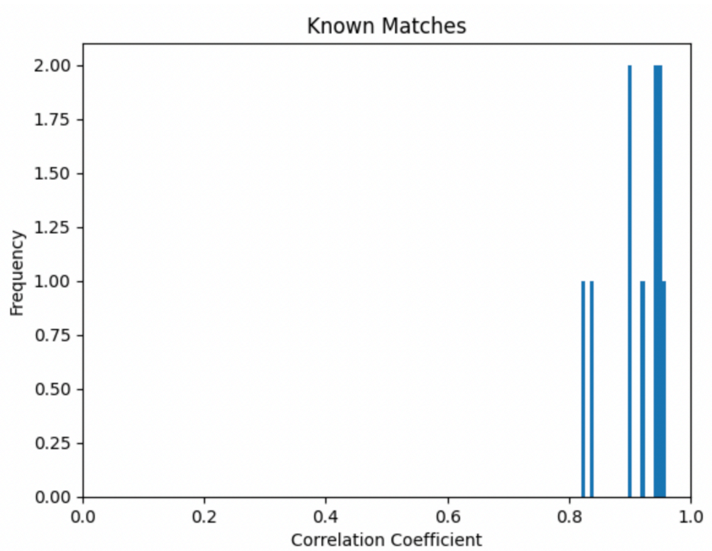


Figure 10.10: Distribution of the correlation coefficients for the Known Matches (maximum repetition of each point in Y set to 3)

From the comparison of Figure 10.8 and Figure 10.10, we can see that the distributions for the Known Matches look very similar. Therefore, as mentioned above, setting the maximum repetition of each point in Y to 1 or 3 makes no meaningful difference in whether the Known Matches are correctly identified and in the values of the correlation coefficients for their comparisons.

Comparing Figure 10.7 to Figure 10.9, however, we can see that the distributions for the correlation coefficients of the Known-Non-Matches change significantly depending on what the maximum repetition of each point in Y is set to. More specifically, when the maximum repetition of each point in Y is set to 1, the correlation coefficients for the Known-Non-Matches are significantly lower than the ones for the maximum repetition of each point in Y set to 3, as expected. In fact, in the first case, the correlation coefficients are in a range between [0.32, 0.69], whereas in the second case they are in a range between [0.39, 0.79].

Finally, we can notice that there is no overlap in the distributions of the correlation coefficients for the Know-Non-Matches and the Know Matches for both choices of the maximum repetition of each point in Y. However, setting the maximum repetition of each point in Y to 1 is preferable, as it leads to clearer results. In fact, in this setting, the distributions of the correlation coefficients for the Know-Non-Matches and the Know Matches are further apart from each other, and so given a correlation coefficient, it can be established with more certainty whether it comes from a distribution or the other. This, therefore, reduces the risk of wrongly interpret a random match as a real one, or a real one as a random one.

10.4. Comparison to the Previous Method

The method proposed provides multiple improvements compared to the method that was previously developed which was explained in Chapter 3. These improvements regard the following:

- **Results**
  The previous method did not produce meaningful results for the first two sets of tests that were run (i.e. Approach 2 and Approach 4 in Section 3.4), in the sense that there was not a (significant) difference between the correlation coefficients obtained for the Known Matches and the Known-Non-Matches. Approach 6 in Section 3.4 did produce more meaningful results in this sense, however these results were surprising and conflicting because the Virtual Knife that was used to generate them was not well calibrated, as we can see from the table in Approach 5 in Section 3.4.

  The results obtain with the new method, on the contrary, show that there is very little overlap in the distributions for the correlation coefficients of the Known Matches and the Known-Non-Matches, as we can see in Section 9.2. Moreover, making the adjustments proposed in the discussions in Section 10.2 and Section 10.3 (i.e. comparing the shorter cuts Y to longer cuts X created also at 20°, and not just at 40°, and decreasing the maximum amount of times that the same point in Y can be repeated), the results produced by this method further improve. Namely, it can correctly identify 20 out of the 22 Known Matches and the distance between the distribution of the correlation coefficients for the Known Matches and Known-Non-Matches can be significantly increased.

- **Robustness**
  In the previous method, a striation pattern created at a known angle α_1 with a given knife was used to generate a Virtual Knife, i.e. the striation patterns for all the other angles of attack for that knife. However, the creation and calibration of the Virtual Knife was significantly dependent on the initial guess of where on the blade the striation pattern created at α_1 presumably came from. In other words, locating the striation pattern created at α_1 in two slightly different positions of the blade resulted into very different generated striation patterns for the other angles of attack. For this reason, the model was not robust.

  The new method does not present this problem since it does not depend on any initial condition.

- **Time consumption**
  In the previous method, to investigate whether a small cut was created by a given knife, the following inputs were needed from the user:

  1. The small cut to investigate
  2. A striation pattern of the given knife at a known angle α_1
  3. A striation pattern of the given knife at another known angle α_2
  4. A picture of the blade of the knife

  Moreover, as explained above, for the calibration of the Virtual Knife, the part of the blade from which the striation pattern at α_1 presumably came from had to be guessed. This process required a lot of trial and error and thus was very time consuming.

  The new method is significantly less time consuming since no parameters have to be estimated through a process of trial and error. Moreover, it only requires items 1-3 of the list above as inputs (for example letting α_1=40° and α_2=20°), and it does not need a picture of the blade of the knife.

- **Accounting for the physical nature of the problem**
  As explained in Section 3.5, there is a discrepancy between the theoretical model that describes how the striation pattern of a given knife will look like for different angles of attack (used for the previous method) and the physical data. In fact, the model predicts what a given striation pattern will look like for a different angle of attack based on the shape of the blade of the knife. However, the distances between the lines in the striation pattern will change non-linearly also depending on whether and how the material deforms while being cut. For this reason, the striation patterns for different angles of attack generated by the previous method will not exactly look like what they look like in real life, and more specifically the distances between some lines in the striation pattern will be different. Therefore the correlation coefficient resulting from the (rigid) matching on the two will be lower.

  In the new method, the signals Y for the striation patterns created at lower angles of attack are stretched to match the striation patterns X created at higher angles of attack. Thus, even if the distance between some lines in the striation pattern of X are due to the deformation of the material that was being cut, the two signals Y and X can still be matched (assuming that they were created by the same knife), as the stretching of Y is local and non-linear. So, in other words, the stretching of the signal Y can accommodate for the effects of the deformation of the material on the striation pattern for X.

10.5. Further Improvements

From the discussion presented, the following improvements could be made to the model in order to have better results. More specifically, these improvements will increase the number of Known-Matches correctly identify, reduce the overlap between the distributions of the correlation coefficients for the Known Matches and the Known-Non-Matches, and improve the interpretation of the results.

- **Comparing Y to multiple X at different angles of attack**
  As discussed in Section 10.2, the model can better identify the Known Matches when the difference in angle of attack between Y and X is lower. More specifically, from the tests run, it seems that the Known Matches can be identified when the difference in angle of attack between Y and X is up to 20°, however this is not always the case when this difference in angle of attack increases.

  For this reason, it would be beneficial to compare the shorter cuts Y to at least two longer cuts X, created at 20° and 40°. Moreover, it would be interesting to determine whether comparing the shorter cuts to only two longer cuts X is enough to ensure that all the Known-Matches can be correctly identified. In fact, from the results shown in Section 9.1, we can see that for two knives the Known Match created at 20° could not be identified, when compared to a longer cut

X created at 40°. These cases could, thus, suggest that these shorter cuts should be compared to a longer cut X created at a smaller angle, e.g. 30°, in order to ensure that their Known Match will be identified, and thus that, in general, comparing the shorter cuts Y to only two longer cuts X is not enough.

Finally, in general, comparing the shorter cuts Y to multiple longer cuts X will increase the correlation coefficients for the Known Matches, as explained in Section 10.2, while the correlation coefficients for the Known-Non-Matches will roughly remain the same. Therefore, comparing the shorter cuts Y to multiple cuts X created at different angles of attack will also produce results easier to interpret.

- **Decreasing the maximum stretch allowed**
  As shown and discussed in Section 10.3, if more longer cuts X are created and used for the comparisons, and thus the maximum difference in angle of attack between Y and (on the cuts) X is lower, it is beneficial to decrease the maximum stretch that Y can undergo to match X. In fact, this leads to lower correlation coefficients for the Known-Non-Matches, without significantly altering the correlation coefficients for the Known Matches.

  It is therefore advised to study what should be the maximum amount of times that the same point in Y can be repeated, in order to minimize the overlap in the distributions of the correlation coefficients for the Known Matches and the Known-Non-Matches, while still correctly identifying the Known Matches.

  In Section 10.3, it was shown that this number is 1, when the shorter cuts Y created at 30° and 20° are compared to a longer cut X created at 40°. However, it might be the case that for the comparisons of shorter cuts Y created at 0° and 10° and a longer cut X created at 20° this number is too low and it should be increased, as some preliminary tests that were run suggest.
- **Common Source vs. Specific Source**
  Suppose a shorter cut Y is compared against a longer cut X, and it produces a correlation coefficient for their match. In order to interpret the correlation coefficient produced by the comparison of these two (partial) striation patterns, and thus to determine the probability that it is a match or a random match, there are two possible approaches. [16]

  Namely, the first one is the ”common source” approach, in which the probability that it is a match or a random match is calculated based off the distributions of the correlation coefficients for the Known Matches and the Known-Non-Matches, usually via a likelihood ratio. In this method, therefore, it is questioned what is the probability that the given correlation coefficient comes from either distribution. This is the approach used in this project.

  The second approach is called the ”specific source” approach, and to determine whether the correlation coefficient refers to a match or a random match, the distributions of the correlation coefficients for the Known Matches and the Known-Non-Matches are not considered. In fact, the correlation coefficient in question is compared only against the correlation coefficient obtained from the comparison of that shorter cut Y to other longer cuts X for different knives. The higher the correlation coefficient is compared to the ones obtained from the comparisons with other knives, the higher the probability that it is a match.

  One of the reasons why this approach can be preferable compared to the common source approach is that if a shorter cut Y contains very little information, it will produce high correlation coefficients when compared to the longer cuts X of all knives. However, in the common source approach all these matches could be mistaken for real matches when considered separately, whereas in the specific source approach it will be clear that the results are inconclusive.

The results obtained in Section 9.1, Section 10.2 and Section 10.3 suggest that it would be beneficial to interpret the data also via the specific source approach, as when a Known-Match is found it presents a significantly higher correlation coefficient compared to the correlation coefficients for the matching of the same shorter cut Y to the longer cuts X for other knives. Moreover, in the cases in which the Known Match was not correctly identified, it can be seen that it produces a correlation coefficient similar to the ones for the matching of the same shorter cut Y to the longer cuts X for other knives.

It would, thus, be helpful to model a specific source approach for this problem, so that the results could be interpreted better.

11

Conclusion

The aim of this research project was to produce a method to automate the comparison of striation patterns created by knives. More specifically, the goal was to create a model to determine the likelihood ratio in favor of the hypothesis that a (partial) striation pattern was created by a given knife, as opposed to by an unknown different knife. This problem was at first approached by trying to improve an already existing model, which was previously developed for this purpose. Namely, in this model, the creation of the "Virtual Knife" was particularly challenging and time consuming, and so in this project, a Matched Filter was introduced to improve this. However, this did not result in any significant improvement of the model, since the Matched Filter could only match the striation patterns generated with the "Virtual Knife" to the lab created one if they were precisely for the same angle of attack, and even with a small difference in angle of attack between the two (e.g. 2°), a match could no longer be found.

The second approach that was used for this research was to create a new model to compare the signals representing the knife striation patterns. More specifically, the idea of this model was to non-linearly stretch via Dynamic Time Warping the signal for the (small) striation pattern of interest to (partially) match the striation pattern created with a given knife. Then, once an alignment between these signals was found, their correlation coefficient was calculated. For this model, it was assumed that the striation pattern of interest was created at a lower angle of attack compared to the striation pattern for the given knife, and that there was a limit as to how much the striation pattern of interest could be stretched locally for the matching. This model was tested with the data for 6 different knives, comparing the small striation patterns created at angles of attack of 0°, 10°, 20° and 30° for each knife to the longer striation patterns created at an angle of attack of 40°, for each knife. The results showed almost no overlap between the distributions of the correlation coefficients for the Known Matches and the Known-Non-Matches, however only 16 out of the 22 Known Matches were correctly identified.

To increase the rate of Known Matches that were able to be identified by the model, another test was run, however this time only the small striation patterns created at angles of attack of 0° and 10° were considered for each knife, and they were compared to the longer striation patterns created at an angle of attack of 20°, for each knife, instead of 40°. This time 10 out of 10 of the Known Matches were able to be correctly identified. Additionally, to further decrease the overlap between the distributions of the correlation coefficients for the Known Matches and the Known-Non-Matches, another test was run. Namely, the small striation patterns created at angles of attack of 20° and 30° for each knife were compared to the longer striation patterns created at an angle of attack of 40°, for each knife, however the local maximum stretched allowed for the small striation patterns was decreased. This resulted in a drop of the correlation coefficients for the Known-Non-Matches (from [0.39, 0.79] to [0.32, 0.69]), whereas the correlation coefficients for the Known Matches remained almost unchanged, leading to a further separation between the distributions of the correlation coefficients for the Known Matches and the Known-Non-Matches.

Overall, the results obtained in this research seem promising and the model proposed is a clear improvement compared to the previous one, in terms of robustness, time consumption and results. Moreover, the running time for a striation pattern comparison with this model is about 2-4 minutes, making it feasible also for real-life purposes, where hundreds of comparisons need to be made in a couple of weeks. Finally, as previously explained, this model can still be further improved. Namely, the (short) striation patterns could be compared to more longer cuts created at different angles of attack (e.g. 20° and 40°), to ensure that more Known Matches can be correctly identified, although this would slow down the creation of the striation patterns database, as for each knife at least two cuts would need to be created instead of one. Also, the maximum local stretch allowed for the (short) striation pattern should be reduced, in order to ensure an even larger separation between the distributions of the correlation coefficients for the Known Matches and the Known-Non-Matches.

11.1. Further Research

The following research questions should still be investigated in order to further improve the model proposed:

- **How many long striation patterns created at different angles of attack are needed for each knife? And under which angles of attack should they be created?**
  These questions are worth asking, since, as previously explained, the results show that the short striation patterns should be compared to at least two different long striation patterns, created at angles of attack of 20° and 40°. However, it should be investigated whether comparing the short striation pattern to only these two longer striation patterns in enough to ensure that all the Known Matches can be correctly identified, and if the choice of these two angles of attack is a good one.
- **What should be the local maximum stretch allowed for the small striation pattern?**
  As previously explained, the results show that reducing the local maximum stretch allowed for the small striation pattern results in a further separation between the distributions of the correlation coefficients for the Known Matches and the Known-Non-Matches. However, the tests conducted only investigated what the maximum stretch allowed for the small striation pattern should be in cases where the small striation patterns created at angles of attack of 20° and 30° were compared to the longer striation patterns created at an angle of attack of 40°. Thus, it is important to investigate also how the maximum stretch allowed for the small striation pattern should change when small striation patterns created at 0° and 10° are compared to longer striation patterns created at 20°, for example. The goal would be to find the lowest maximum stretch for the small striation pattern which still ensures that all the Known Matches can be correctly identified.
- **How should the results be interpreted if the short striation patterns are compared to multiple long striation patterns created at different angles of attack?**
  This questions is worth asking since comparing the short striation patterns to multiple long striation patterns created at different angles of attack might produce results that are not obvious to interpret. This might be the case, for example, if a short striation pattern results in a high correlation coefficient when compared to a longer striation pattern created at 20°, however it produces a low correlation coefficient when compared to a longer striation pattern created at 40°. In this case, thus, it is not immediately obvious to determine how these results should be interpreted. Moreover, in a case like this, it is not obvious to determine what data should be considered to calculate a likelihood ratio. In fact, to calculate a likelihood ratio, this short striation pattern would need to be compared to longer striation patterns created by other knives. However, it is not obvious to determine *a priori* whether only the longer striation patterns created at 20° should be considered for these comparisons, or if also the longer striation patterns created at 40° are needed.

- **How does this model perform with real life data?**
  One of the limitation of this research was that it was not possible to test the model with real life crime scene data and so it was tested with lab created data. However, the lab created data presents some differences compared to the real life data. Namely, the for lab created data, the lines in a striation pattern are all straight, since the are created under a specific angle of attack which remains constant while cutting, while in real life data, the lines in a striation pattern are curved, since the victim or the knife were moving while creating the cut, and the angle of attack was not constant. Moreover, the quality of the lab created striation patterns is higher than the one of the real life striation patterns, since the material in which the lab created cuts are made captures better the striation pattern, compared to the human cartilage. For this reason, it should be further researched how this model would perform with real life data. More specifically, in order to do so, it would need to be investigated how the striation patterns with curved lines can be turned into signals, and test the model with striation patterns found in crime scenes, or created in a lab in a material that closely resembles the human cartilage.

Finally, in Chapter 5, another potential model was briefly explained. Namely, the idea of this model was to determine the likelihood that a given striation pattern was created by a given knife, by investigating whether there is a strong underlying structure between the given striation pattern and the striation patterns created by that given knife. This would be done using the POD method. This model was not implemented in this research as it would require creating multiple cuts at different angles of attack for each knife that needs to be investigated, making it very time consuming, and thus unfeasible. However, if in the future the creation of multiple cuts at different angles of attack for each knife will be a feasible task, or if it will be possible to virtually generate the striation patterns for a given knife under any angle of attack, this method should be reconsidered and possibly implemented.

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