A two-parameter method for e^N transition prediction

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A correction to the e^N method for transition prediction, that at present is in use at Delft University, is developed and tested. The existing method is based on solutions of the Falkner-Skan similar boundary layer equation without suction or blowing. The correction is based on additional solutions of this equation with suction and blowing. For the development of the method an Orr-Sommerfeld solver is used for the calculation of the stability diagrams for several series of Falkner-Skan flows with suction or blowing for fixed shape factor (*H*). This results in differences in the velocity profile curvature near the wall. A correction based on only one additional parameter has been devised using these results. A comparison between the corrected e^N method and direct stability calculations for several non-similar flows with and without suction shows good agreement.

Nomenclature

a	wave amplitude	Re_{θ}	Reynolds nr. with respect to θ
a_0	wave amplitude at $x = x_0$	$(Re_{\theta})_{crit}$	critical Reynolds nr. with
C_q	suction flow coefficient		respect to θ
c	airfoil chord	T	amplification of unstable
c	$\omega/lpha$		disturbances
c^*	c/U	T_{max}	maximum value of T at
curve1	$\frac{\partial^2 u^*}{\partial y^{*2}}$ at $y^* = 1$		specific Re_{θ}
dc1	curve1 of actual minus $curve1$	T_{maxmax}	global maximum of T at "top"
	of Arnal profiles for same H		for specific $(Re_{\theta})_{crit}$
f	non-dimensional streamfunction	Tu	turbulence level [%]
F	$\omega \nu / U^2$	U	boundary layer edge velocity,
F_{∞}	$\omega \nu / U_{\infty}^2$		parallel to surface
H	shape factor δ^*/θ	U_{∞}	freestream velocity
l	$ au_0 heta/\mu U$	U^*	U/U_{∞}
M	$\frac{x}{U}\frac{dU}{dx}$	u	tangential velocity in
m_T	$\left(\frac{\partial^2 u^*}{\partial u^{*2}}\right)_0$		boundary layer
n	amplification factor	u^*	u/U
N	amplification factor,	v	normal velocity in
	envelope of all n		boundary layer
r	$^{10}log(Re_{\theta}) - ^{10}log(Re_{\theta})_{crit}$	v_0	normal velocity at wall
$r^*,ar{r}$	r/r_{top}		(negative for suction)
r_{top}	r at T_{maxmax}	x	distance along wall
Re_c	Reynolds nr. with respect to chord	x^*	x/c

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x_0	wall coordinate at which	θ	boundary layer momentum
	a discrete frequency disturbance		loss thickness
	becomes unstable in the spatial	μ	dynamic viscosity
	mode	ν	kinematic viscosity
y	distance normal to surface	ρ	(air) density
y^*	y/ heta	au	shear stress
		$ au_0$	wall shear stress
Greek symbols		ψ	streamfunction
		ω	frequency
α	$\alpha_r + i\alpha_i$		
$-\alpha_i$	spatial growth rate of	Subscripts	
	disturbances		
α_r	wave number	0	at wall
β	Hartree/Falkner-Skan	∞	freestream
	pressure gradient parameter	axis	position in ω or F for T_{max}
δ	boundary layer thickness	c	chord
δ^*	boundary layer displacement	crit	instability point
	thickness	max	local maximum at specific Re_{θ}
ϕ	amplitude function in	maxmax	global maximum
	Orr-Sommerfeld equation	scale	frequency difference expressed
η	non-dimensional y coordinate		in $\omega \theta / U$ between upper neutral
	in boundary layer = $\frac{y}{x}\sqrt{\frac{Ux}{\nu}}$		curve and global maximum
		ω_{scaled}	$((\omega\theta/U) - (\omega\theta/U)_{axis})/scale$
		top	position of global maximum

I. Introduction

The e^{N} method for transition prediction was introduced in 1956 by Smith and Gamberoni¹ and Van Ingen.² This method has enjoyed much success in the past half century and is still very useful, especially for design purposes. In 2006 Van Ingen published a CD-ROM with a historical review of work at TU Delft on this method. The CD-ROM³ includes his publications on the subject during that 50 year period. A shorter summary was published in 2008 in an AIAA paper.⁴ In addition a new database method was included that was based on a one-parameter family of 15 stability diagrams by Arnal.⁵ These diagrams were calculated for 11 attached and 4 separated solutions of the Falkner-Skan equation without suction or blowing. Table 1 gives an overview of these 15 Arnal profiles. In the present paper values of H for members of this profile family will be indicated by $H1, H2, \ldots, H15$. In most $e^{\mathbf{N}}$ database methods the velocity profile shape factor (H) is used as the single parameter characterizing all stability diagrams for arbitrary pressure gradient and suction or blowing. Van Ingen³ analyzed a series of velocity profiles on a flat plate with (strong) suction and blowing. It was concluded that for these profiles the non-dimensional velocity, shear and curvature profiles as a function of $y^* = y/\theta$ were very similar to the Falkner-Skan profile with pressure gradient but zero suction or blowing at the same value of H. Based on this observation the new method continued the tradition of using H as the single parameter to characterize all possible stability diagrams. The Arnal series was then used as a basis and applied to flows with suction and blowing as well. Another interesting observation was that the Arnal stability diagrams for favorable pressure gradient were very similar and even collapsed on one diagram after proper scaling and shifting. Fig.1 shows the idea of scaling by using the scale indicated in the figure to define:

$$\omega_{scaled} = \frac{\frac{\omega\theta}{U} - \left(\frac{\omega\theta}{U}\right)_{axis}}{scale} \tag{1}$$

Shifting is done by introducing

$$r = {}^{10} \log(Re_{\theta}) - {}^{10} \log(Re_{\theta}crit)$$
⁽²⁾

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Figure 1. Definition of "scale", "axis" and "top" for a stability diagram.

Table 1. Overview of the Falkner-Skan velocity profiles without suction or blowing, analyzed for stability by Arnal.⁵ Note that for the asymptotic suction profile according to Hughes & Reid, H = 2 and $(U\delta^*/\nu)crit = 46270$.

number	β	Н	$Re_{\delta^*}crit$
1	1.00	2.216	12510
2	0.50	2.297	7750
3	0.20	2.411	2860
4	0.10	2.481	1390
5	0.05	2.529	872
6	0.00	2.591	520
7	-0.05	2.676	315
8	-0.10	2.802	198
9	-0.15	3.023	126
10	-0.185	3.378	89
11	-0.1988	4.029	67
12	-0.16	6.752	46.3
13	-0.12	10.056	40.5
14	-0.08	16.467	36.5
15	-0.04	35.944	33.0

The axis used in Eq.1 is the collection of points where in the cross sections at constant Reynolds number the maximum amplification rate T_{max} is reached. The global maximum amplification rate T_{maxmax} is reached at the top. As an example Fig. 2 shows the striking similarity for the scaled and shifted Arnal diagrams for the cases 1 through 6 with favourable pressure gradient. Figure 3 shows the non-dimensional amplification rate $T^* = T_{max}/T_{maxmax}$ along the axis of the diagram for all 15 Arnal cases. A nearly perfect analytical approximation is :

$$\bar{T} = \bar{r}e^{(1-\bar{r})} \tag{3}$$

where $\bar{r}=r/r_{top}$

Using these similarities a scaled and shifted database was developed in which interpolation between the 15 Arnal diagrams was easy and accurate. The present paper will not discuss the existing method in detail. References² and³ provide the necessary background.

The observed similarity was also used to advantage in developing the new two-parameter database method.



Figure 2. Scaled and shifted stability diagram for the Arnal profiles with favorable pressure gradient (nr 1:6).

II. Methodology

Recently, the present authors calculated solutions of the Falkner-Skan equation for various combinations of pressure gradient parameter β and the suction or blowing parameter f_{wall} where:

$$\psi_{wall} = \int_0^x -v_0 dx = \sqrt{\nu x U} f_{wall} \tag{4}$$

These combinations were chosen in such a way that for each of the 11 Arnal values of the shape factor H (H1:H11) for attached flow, a whole series of velocity-, shear- and curvature-profiles was obtained with that same value of H. The earlier observed similarity also seems to hold for this extended series of Falkner-Skan solutions.

It was found that for all members of each of the series the velocity and shear profiles showed only minor differences. Near the wall the curvature profiles showed larger differences however. Stability diagrams for a number of these flows were calculated using the Orr-Sommerfeld solver to be discussed in section III.



Figure 3. Scaled amplification rate along the axis for all 15 Arnal diagrams.

Results show that the observable differences near the wall led to differences in critical Reynolds number and maximum amplification rate in the order of 10 % compared to the Arnal cases of no suction or blowing for the same value of H. The effect of these differences on the parameters used to scale the stability diagrams was calculated for the 11 series using the Orr-Sommerfeld solver. These parameters are ${}^{10}log(\text{Re}_{\theta}crit)$, the global maximum amplification rate T_{maxmax} , the scale and the position of the top (Fig.1) expressed in:

$${}^{10}logF = {}^{10} log(\frac{\omega\nu}{U^2}) = {}^{10} log(\frac{\omega\theta}{U}/Re_{\theta})$$
(5)

Based on this analysis a correction to the one-parameter database method was developed, to be discussed in section IV.

III. Basics of the e^N method and the Orr-Sommerfeld solver

In the present paper we will discuss the basics of linear stability theory for two-dimensional incompressible flow only as far as is needed to explain the idea of the e^N method. For a more detailed account see Van Ingen.^{3,4} Disturbances in the boundary layer are described by a stream function :

$$\psi(y) = \phi(y)e^{i(\alpha x - \omega t)} \tag{6}$$

Various quantities will be non-dimensionalized using the velocity at the edge of the boundary layer (U) and the momentum loss thickness (θ) as scales. Boundary conditions for ϕ follow from u' and v' = 0 at the wall and for $y \to \infty$:

$$\phi(0) = \phi'(0) = \phi(\infty) = \phi'(\infty) = 0 \tag{7}$$

The Orr-Sommerfeld code used in the present investigation is based on the global solution of the spatial stability problem using an expansion in Chebychev polynomials. For the treatment of the non-linearity of the eigenvalue the companion matrix technique is used as proposed by Bridges & Morris.⁶ More specifically the Orr-Sommerfeld equation can be written as

$$\left[\left(\frac{d^2}{dy^2} - \alpha^2\right)^2 - iRe((\alpha U - \omega)(\frac{d^2}{dy^2} - \alpha^2) - \alpha U'')\right]\phi = 0$$
(8)

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Equation 8 can be integrated indefinetely four times giving

$$\phi - (i\omega Re - 2\alpha^2) \iint \phi - i\alpha Re \int \int U\phi + 2i\alpha Re \iiint U'\phi + (\alpha^4 - i\alpha^2\omega Re)t\phi + i\alpha^3 Re \iiint U\phi + \frac{b_1y^3}{6} + \frac{b_2y^2}{2} + b_3y + b_4 = 0$$
(9)

The eigenfunction ϕ can be expressed in a series of Chebychev polynomials as:

$$\phi(y) = \frac{\alpha_0}{2} + \sum_{n=1}^{N} \alpha_n T_n(y)$$
(10)

Equation 10 is substituted into Equation 9 along with the formula for Chebychev multiplication and integration. Finally the system is solved by a standard eigensolver to obtain the global spectrum of eigenvalues (α) for a given combination of frequency (ω) and Reynolds number (Re).

Note that the velocity profile and its curvature play a prominent role in the Orr-Sommerfeld equation. Since the curvature is influenced by pressure gradient, suction/blowing, heating/cooling at the wall, etc. these factors have a strong influence on the solutions, and therefore on boundary layer stability. Furthermore the Reynolds number and the frequency of the imposed disturbances are found to be very important. In the present paper we will only discuss the effects of pressure gradient and suction/blowing at the wall. Note that the curvature $(u^*)''$ in Eq.8 is multiplied by ϕ and because of the boundary conditions Eq.7 this term disappears near the wall and at the edge of the boundary layer. We will return to this observation later when we will discuss in more detail the effect of the curvature term. In the spatial mode of the stability analysis we take the circular frequency ω to be real and the wave number α to be complex. Also ϕ and ψ are complex but in the present paper we will only need to specify $\alpha = \alpha_r + i\alpha_i$.

Introducing α into Eq.6 leads to:

$$\psi(y) = \phi(y)e^{(-\alpha_i x)}e^{i(\alpha_r x - \omega t)}$$
(11)

It follows from Eq.11 that disturbances grow, remain constant or decrease with x for $\alpha_i < 0$, = 0 and > 0 respectively, meaning that the given flow is unstable, neutral or stable against the given disturbance. Which case occurs depends on the shape of the velocity profile, the frequency and the Reynolds number Re_{θ} . Below the critical Reynolds number $(Re_{\theta}crit)$ the boundary layer is stable to small disturbances of all frequencies. At higher Reynolds numbers there is a range of frequencies for which instability occurs. As can be seen from Eq.11 the rate of amplification or damping is determined by $-\alpha_i$.

The amplitude a of the disturbance can be computed as a function of x using Eq.11. The ratio of the amplitudes a and a + da at x and x + dx is given by:

$$\frac{a+da}{a} = \frac{e^{-\alpha_i(x+dx)}}{e^{-\alpha_i x}} = e^{-\alpha_i dx}$$
(12)

or:

$$ln(a+da) - ln(a) = d(ln(a)) = -\alpha_i dx$$
(13)

and after integration:

$$n = ln(\frac{a}{a_0}) = \int_{x_0}^x -\alpha_i dx \tag{14}$$

where x_0 is the station where the disturbance with frequency ω and amplitude a_0 first becomes unstable. The quantity *n* will be denoted as the "amplification factor" while $-\alpha_i$ is the "amplification rate". Then e^n gives the "amplification ratio". In applications we will write Eq.14 as follows:

$$n(x,\omega) = (10^{-6}U_{\infty}\frac{c}{\nu}) \int_{x_0/c}^{x/c} 10^6 \frac{-\alpha_i \theta}{Re_{\theta}} \frac{U}{U_{\infty}} d(\frac{x}{c})$$
(15)

or by denoting $T = 10^6 (\frac{-\alpha_i \theta}{Re_{\theta}})$

$$n(x,\omega) = \left(\frac{10^{-6}U_{\infty}c}{\nu}\right) \int_{x_0/c}^{x/c} T(\frac{U}{U_{\infty}}) d(\frac{x}{c})$$
(16)

The factors 10^6 and 10^{-6} have been introduced for convenience. If we calculate n as a function of x for a range of frequencies $(F_{\infty} = \frac{\omega\nu}{U_{\infty}^2})$ we get a set of *n*-curves; the envelope of these curves gives the maximum amplification factor N as a function of x. The e^N method assumes that transition occurs when N has reached a critical value (of the order of 9) but dependent (as determined by experiment) on the disturbance environment in the flow such as turbulence level.

IV. Correction parameter selection

The observed differences in the stability data for the same H led to the idea of taking some measure of the curvature near the wall as a second parameter in addition to H to account for the effect of non Falkner-Skan similarity and suction or blowing on the stability characteristics of the flow. Because existing e^N methods using only H as a single parameter have been reasonably successful, it was expected that a relatively simple correction to the one-parameter database method could be developed in this way. For the time being the development of the correction has been restricted to the 11 values H1 through H11 for attached flow.

An obvious parameter would seem to be the curvature of the velocity profile at the wall as expressed by:^a

$$m_T = \left(\frac{\partial^2 u^*}{\partial y^{*2}}\right)_0 \tag{17}$$

Fig.4 shows a plot of the non-dimensional wall shear stress parameter $l = \frac{\tau_0 \theta}{\mu U} = (\frac{\partial u^*}{\partial y^*})_0$ for a series of characteristic laminar boundary layers as a function of m_T . It is obvious that the Hartree solutions of the Falkner-Skan equation for zero suction or blowing are situated at one end of the spectrum. Of the boundary layers calculated by Tani for $U = 1 - (x^*)^n$ the one for n = 8 is at the other end of the spectrum. The parameter m_T shows appreciable differences for these cases.



Figure 4. Non-dimensional wall shear stress (l) vs. non-dimensional curvature at the wall (m_T) for a number of classical boundary layers.

For each of the 11 series (each at one single value of H) m_T was plotted as function of the suction or blowing parameter f_{wall} . Note that also the Falkner-Skan pressure gradient parameter β had to be changed with f_{wall} in order to maintain a constant H. As an example Fig.5 and Fig.6 respectively show the results for the Blasius value H6 = 2.5911. Analysis of all results showed that for some values of H different combinations of β and f_{wall} could lead to the same value of m_T . An example is shown in Fig.7 for H5 = 2.526. This multiplicity excludes m_T as the second parameter. Another argument to exclude m_T as a parameter is that

^aThe subscript T stands for Thwaites, who seems to have been the first to make extensive use of this parameter

in the Orr-Sommerfeld equation the velocity profile curvature is multiplied by the disturbance amplitude function ϕ . Since ϕ and its first derivative are zero at the wall (because of the zero values for the u and vfluctuations) this term is small very near to the wall and hence m_T cannot be expected to have much direct influence on the solution.



Figure 5. Curvature at the wall (m_T) and at $y^* = 1$ ("curve 1") for the profiles with H = 2.5911 (Blasius) as a function of f_{wall} .

After excluding m_T as second parameter, the only remaining possibility seemed to be the utilization of the curvature at some small but finite distance from the wall. Fig.5 and Fig.7 also show the curvature at y^* = 1 (indicated by curve1) for the profile series with H6 =2.5911 and H5=2.526. The new parameter shows a monotonic variation with f_{wall} for these cases , while m_T does not. It turned out that curvature at y^* = 1 could be used for the range 2.48 < H < 4.03. (That is $H4 \leq H \leq H11$). However for H < H4the new parameter also started to show a non-monotonic behaviour. As an ad-hoc solution, not supported by sufficient computational evidence, the various splines used in the correction procedure were extrapolated linearly from ${}^{10}log(H5)$ and ${}^{10}log(H4)$ down to ${}^{10}log(H1)$.

The present paper will not discuss the development of the correction method in detail. The main items are:

- Relative changes (with respect to the Arnal values for the same H) in some parameters that are used in the original database method have been plotted as functions of dc1, the difference in curvature (curve1) of the profile at $y^* = 1$ compared to the Arnal profiles
- For each of these parameters y_p a second degree polynomial was fitted of the form

$$y_p = a \cdot dc1^2 + b \cdot dc1 \tag{18}$$

There is no constant term in Eq.18 because by definition all y_p 's are zero for the Arnal profiles (dc1 = 0). From the examples to be discussed in section V it can be concluded that in several cases the linear term may be sufficient for practical purposes.

• For each parameter y_p the coefficients a and b were fitted by splines in ${}^{10}log(H)$ in the interval ${}^{10}log(H4) <= {}^{10}log(H1)$.



Figure 6. β as function of f_{wall} for the profiles with H = 2.5911 (Blasius).



Figure 7. Curvature at the wall (m_T) and at $y^* = 1$ ("curve 1") for the profiles with H=2.526 (Arnal nr. 5) as a function of f_{wall} .

- Because some plots for y_p vs dc1 in the interval ${}^{10}log(H1) <= {}^{10}log(H) <= {}^{10}log(H4)$ showed a nonmonotonic behaviour they could not be included in the splines. Therefore a rather bold step was taken by linear extrapolation of the splines below ${}^{10}log(H4)$. Using one of the examples to be discussed in section V (Iglisch flow), an improved extrapolation will be defined.
- It is expected that in practical applications the splines will only have to be used for H < H4 at very high Reynolds numbers.
- Note that the correction is based on a detailed knowledge of the curvature of the velocity profile. Using an integral relation method for the boundary layer calculation will (in general) not provide this information. In case the integral relations would be based on the Falkner-Skan profiles without suction or blowing the corrections will even turn out to be exactly zero by definition.

The authors are currently working on a further investigation for the smaller value of H. Based on the results of this the correction procedure will be published in full detail. For information on the progress of this work the reader is referred to the website that is mentioned in reference.³ The Arnal series contains 4 stability diagrams for separated flow. The correction for these flows is thought to be less important for the practical applications envisaged at TU Delft that are intended to avoid separation. This issue will also be addressed at a later stage. The original one-parameter database method employed the strong similarity that is present between the Arnal stability diagrams (see work by van Ingen^{3,4} and Figs.2 and 3). A further investigation has indicated that such a similarity is also present within a series with the same H at the various combinations of β and f_{wall} .

Further work on the consequences of the observed similarity is ongoing. It is thought that this may result in a much simplified version of the present e^N database method.

V. Examples

The present section will present some results of the new two-parameter method. The flows to be discussed are:

- The Tani flow $(U^* = 1 (x^*)^8)$ without suction.⁷
- The Tani flow $(U^* = 1 (x^*)^8)$ with such a suction distribution that the boundary layer first approaches separation and then returns to a nearly flat plate value of H.
- An airfoil designed by Boermans (⁸) with a constant value of U^* (1.6) over the first 32 % of the chord and an adverse pressure gradient over the downstream part. A special suction distribution over the downstream part was designed to keep the value of H constant at the value for a flat plate without suction (H6 = 2.5911).
- The Iglisch flow for a flat plate with constant suction velocity.⁹

A. Tani flow without suction

The Tani flow $(U^* = 1 - (x^*)^8)$ without suction shows a long initial stretch of nearly constant U^* (Fig.8) followed by an adverse pressure gradient leading to separation at x/c=0.63 indicated by the large increase of H (Fig.9). To obtain interesting values of the N-factors the Reynolds number Re_c was set at 3×10^6 . The correction parameter dc1 is shown as a function of x and H in Figs.10 and 11 respectively. The N-factors according to the Orr-Sommerfeld solution, the one-parameter database method and the corrected database method (linear and quadratic) are presented in Fig.12

B. Tani flow with suction

The same Tani flow pressure distribution $(U^* = 1 - (x^*)^8)$ with suction at the same Re_c was investigated next. In this case suction was applied downstream of x/c = 0.30 in an attempt to avoid or at least delay separation. A solution was found for which the flow tends towards separation first, but then returns to the flat plate value for H. (Fig.9). The correction parameter dc1 follows from Figs.10 and 11. Note that although the shape factor H returns to the flat plate value, dc1 does not. Hence the curvature of the profiles with the same value of H is different for stations at beginning and end of the suction region. The N-factors are shown in Fig.13.



Figure 8. U^* for the Tani flow with and without suction.



Figure 9. H for the Tani flow with and without suction.



Figure 10. dc1 as a function of x^* for the Tani flow with and without suction.



Figure 11. dc1 as a function of H for the Tani flow with and without suction.



Figure 12. Amplification factor for the Tani flow without suction.



Figure 13. Amplification factor for the Tani flow with suction.

C. Boermans airfoil

After our experience with the Tani flow with suction, we developed a version of our finite difference method for solving the boundary layer equations, for which the distribution of H with x can be prescribed. This method was applied to an airfoil designed by Boermans at TU Delft (reference 7). The pressure distribution for this airfoil at the chosen angle of attack is such that U^* is constant (1.6) over the first 32 % chord followed by an adverse pressure gradient. (Fig.14). In our finite difference boundary layer solver the pressure gradient is expressed as:

$$M = \frac{x}{U}\frac{dU}{dx} \tag{19}$$

For Falkner-Skan pressure distributions M is constant. For the present airfoil M shows a linear dependence on x in the adverse pressure gradient region. A suction distribution was designed such that H remains constant at the flat plate value (H6 = 2.5911) all the way down to 100 % chord. The original design was made by Boermans for a sailplane at a relatively low value of Re_c (a little below 1×10^6). To obtain higher N-factors, more relevant for the present study, we used $Re_c = 5 \times 10^6$. The correction parameter dc1 is shown as a function of x in Fig.16. The N-factors follow from Fig.17.

D. Iglisch flow with suction

For the boundary layer on an infinitely long flat plate with constant suction velocity an exact solution has been given by Iglisch.⁹ In this solution a new independent variable x^* is introduced by:

$$x^* = \left(\frac{-v_0}{U}\right)^2 \frac{Ux}{\nu} \tag{20}$$

This implies that a "reference length" c is defined by:

$$c = U\left(\frac{\nu}{-v_0}\right)^2 \tag{21}$$

In what follows we will use the suction coefficients $c_q = -v_0/U$ and $(c_q)_{red} = 10^4 c_q$.

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Figure 14. $U^*(x^*)$ for the suction airfoil.



Figure 15. $M(x^*)$ for the suction airfoil.



Figure 16. $dc1(x^*)$ for the suction airfoil.



Figure 17. *N*-factor vs. x^* for the suction airfoil.

If for the reference speed U_{∞} the constant mainstream velocity U is used it follows that: $Re_c = (c_q)^{-2}$. From Iglisch's solution it is known that at $x^* = 0$ the boundary layer starts as the Blasius flat plate without suction (H = 2.5911) and that for $x \to \infty$ the asymptotic suction boundary layer is obtained (H = 2). It is also known that near $x^* = 0$ a regular behaviour is obtained if $\sqrt{x^*}$ is used as independent variable. Interesting values of c_q are of the order of 10^{-4} and hence the Reynolds number based on c is of the order of 10^8 .

Fig.18 shows H vs $\sqrt{x^*}$, the small circles indicate the points where the first 6 Arnal values (H6 to H1) are reached (H6 is the Blasius value at the leading edge). The N-factors using the database method without correction for 12 values of $(c_q)_{red}$ from 0.4 to 1.5 in steps of 0.1 are shown in Fig.19. For $(c_q)_{red} = 1.5$ complete stability is reached. For just keeping the boundary layer laminar the maximum N-factor may be allowed to grow until about 9. This reduces the required suction velocity to about 1/3 of the value for complete stabilization. Since the maximum N-factor is only reached locally a further reduction in suction quantity would follow from taking a non-constant suction velocity, adjusted to the stability characteristics of the boundary layer.



Figure 18. H vs. $\sqrt{x^*}$ for the Iglisch flow.

To test the new correction method we recalculated the boundary layer and its stability characteristics for $(c_q)_{red} = 0.5$. Figs.20 and 21 show the correction parameter dc1 as a function of $\sqrt{x^*}$ and H respectively. Fig.22 shows the *N*-factors for the Orr-Sommerfeld solution and the database method with and without correction. It appears that the correction is rather large. The figure also shows the positions where the first 6 Arnal values for H are reached. It should be stressed that the database correction beyond the position of H4 is based on an ad-hoc linear extrapolation of the correction splines. An improved linear correction was developed using the Orr-Sommerfeld solutions for the stations where H3, H2 and H1 are reached. The resulting N factors are also shown in Fig.22. Additional flows will be investigated in the future.

VI. Conclusions

- A correction to the one-parameter e^N method that was used for many years at TU Delft was developed. The resulting two-parameter method gives good results in comparison with direct stability calculations with the Orr-Sommerfeld equation.
- The correction function contains a linear and a quadratic term. In many cases the linear term seems to be sufficient.
- There is an interesting similarity in the stability diagrams for similar Falkner-skan flows with pressure gradient and suction/blowing. Further research is in progress to fully exploit this similarity.
- The correction requires a detailed knowledge of the velocity profile curvature. Hence the correction may not be applicable to boundary layers calculated with a simple integral relation method.



Figure 19. N-factor vs. $\sqrt{x^*}$ for the Iglisch flow with various suction coefficients.



Figure 20. dc1 vs. $\sqrt{x^*}$ for the Iglisch flow.



Figure 21. dc1 vs. H for the Iglisch flow.



Figure 22. N-factor for the Iglisch flow.

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