Reliability Analysis And Safety Assessments of Structural Wall with Nonlinear Finite Element Analyses

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RELIABILITY ANALYSIS AND SAFETY ASSESSMENTS OF STRUCTURAL WALL WITH NONLINEAR FINITE ELEMENT ANALYSES

by

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in partial fulfillment of the requirements for the degree of

Master of Science in Structural Engineering

at the Delft University of Technology

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PREFACE

This report is written as a final part of Additional Graduation Work (CIE5050-09). The additional graduation work is carried out as a partial fulfillment of of the requirements for the degree of Master of Science in Structural Engineering at Delft University of Technology.

This report discusses the results of reliability analysis of safety assessment of a structural wall. The results are based on a particular case study. Reliability assessments are performed based on self developed script in MATLAB and Python used in conjunction with results from DIANA. The inspiration to perform a study in this topic came from discussion with Dr. ir. M. A. N. Hendriks, Associate Professor, TU Delft.

I would like to express my sincere gratitude to Dr. Hendriks for his continuous support and enthusiasm during the project. I would also like to thank Dr. ir. Yuguang Yang, TU Delft for being in the evaluation committee. Last two months have provided me a wonderful opportunity to understand the concepts learned in lectures and apply them in tackling the research problems. I hope to use the skills I have acquired in this project in being coherent, critical and focused during my graduation project. I would also like to thank my colleagues at TU Delft, friends and family back in Nepal for their continuous support and motivation throughout.

Suman Bhattarai Delft, October 2017

ABSTRACT

Inclusion of safety formats method in nonlinear finite element analysis of a structure is beneficial to realize the safe and real behavior of structures. In addition, reliability analysis in combination with NLFEA results and safety assessments provide an insight into consistency of the safety assessments recommendations related to targeted safety. However, various complications arise regarding the appropriate constitutive model to be used in analysis, estimation and inclusion of modelling uncertainty in results from nonlinear finite element analysis, unavailability of a suitable function that defines the response of a structure when subjected to a certain load etc.

In this additional thesis, a case study is performed to study the combination of structural safety assessment and reliability analysis using nonlinear finite element analysis. A wall specimen experimented by Lefas et. al is selected for the study [1]. Solution strategy is adopted based on the recommendations of Dutch guidelines with some deviations [2]. The nonlinear finite element analyses are performed in DIANA 10.1. Since only one solution strategy was used during the analysis of a single wall specimen, the modelling uncertainty parameters were estimated using results from previous studies[3]. The modelling uncertainty parameters were estimated to be, $\theta_m = 1.21$ and $V_{\theta} = 6.88\%$. These parameters infer that the results obtained from different solution strategies were close to each other but still had a considerable bias with respect to the structural capacity obtained from the experimental values. The NLFEA underestimated the capacity as compared to experimental results. Only 79.3 % of the experimental capacity was realized from NLFEA.

Three different safety format methods suggested in [2] and a new safety format method by Schlune et al. [4] are used in NLFEA to estimate the design load capacity. The effect of inclusion of estimated model uncertainty parameters on design load capacity is observed too. The results of safety assessments indicated that the design load estimated by using ECOV and Schlune et al. safety format are comparatively higher than using Partial Safety Factor and Global Resistance Factor, but still conservative with respect to the experimental results.

The reliability of the design load estimated by using different safety assessment methods is checked using response surface method and first order reliability methods. A response surface method together with FORM is used to obtain a quadratic limit state function that closely represents the response of the structural wall subjected to the design load value. The limit state function thus obtained is then used for FORM analysis, Importance sampling and/or Monte Carlo simulations to observe if the intended safety level is achieved. The study of acheived reliability index indicated that the design load values estimated by using three safety formats recommended in [2] are conservative, whereas the one estimated by using Schlune et al. format was found to be non-conservative.

The additional thesis was carried out with various approximations and limitations. All limitations and approximations are explained in detail and justified where necessary and possible. The results and discussions of this additional thesis are dependent only on a single solution strategy adopted in a single wall specimen and in a single failure mode. Therefore, the results should instead be interpreted as an indication of need of further and extensive research and studies required in the field of reliability analysis and safety assessment using nonlinear finite element analysis.

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1

INTRODUCTION

Maintaining a balance between the safety and optimum use of capacity have always been a point of discussion in assessing the design load capacity of a structure. Safety has to be of paramount priority as the major purpose of any creation, be it structure or a new technology is to make human lives convenient in a safe and efficient manner. However, the issue of discussion is to not make optimum use of structure for what it is actually designed for. The structure has to be safe to use and must comply the serviceability requirements. But on the same time, the true capacity of structure has to be utilized. It would not be wise and economical to keep a huge buffer between the capacity of a structure and its utilization. Therefore, the need is to have a realistic and safe estimation of capacity of structures.

Several provisions and recommendations are formulated to assess safe design load estimation of structure. These recommended safety methods are used to calculate the design load capacity after reducing the material parameters or by reducing the ultimate load capacity by a specified value of safety factor. In Partial Safety Factor method, the material parameters that are used in NLFEA are heavily reduced by using material safety factor. Similarly, in Global Resistance Factor method the material parameters are reduced by a certain factor and again the load assessed by NLFEA is reduced using global safety factor. Since the material parameters and also the load assessed by NLFEA are reduced in safety assessments, it is sometime doubted if estimation of safety formats methods are indeed reliable and not too conservative. A reliability analysis of the predicted design load capacity will be useful to observe if intended target reliability was met sufficiently or there are rooms for improving the methods to obtain more realistic estimation.

Finite element analyses have been evolving throughout in analyzing and assessing the structural responses. Nonlinear finite element analysis (NLFEA) is generally used to replicate the realistic response of a structure. The possibilities to include nonlinear behavior such as cracking of concrete, yielding of reinforcement, change in the position and cross section area etc. aids to obtain real response of a structure. Use of representative material parameters and material models is necessary to acquire realistic characteristics of a structure. Therefore, new studies are done to use the safety assessment methods in conjunction with NLFEA. The incorporation of safety formats in NLFEA may benefit in obtaining a realistic but also safe estimation of structural response.

However, the possibility to include various parameters to replicate true response of structure has aided in bringing variability in the use of NLFEA in structural capacity assessments. As various uncertainties related to material parameters, material models and interaction, and cross-section are to be incorporated, it would not be understatement to state that NLFEA deals with uncertain uncertainties. It infers that the result of NLFEA depends a lot in the adopted material parameters, models, solution method and finite element software in the analysis. Therefore, this aspect of NLFEA must be taken into account too. The mean modelling uncertainty is generally defined as the ratio of experimental capacity and numerical capacity assessed by NLFEA. The parameters defining this uncertainty should be included in the realistic assessment of the structural capacity of structure.

This thesis aims to study the above mentioned factors in some details. A structural wall specimen which has been tested experimentally by Lefas [1], will be used as a case study. The thesis aims to study the use of NLFEA in structural safety assessment of the chosen wall specimen. First, a NLFEA will be carried out to assess the ultimate load capacity of the structure after making a justifiable selection of the material parameters, constitutive model and solution procedure. Then after, different safety assessment methods recommended

in Dutch Guidelines, [2] will be adopted in NLFEA. The effect of inclusion of modelling parameters in design load estimation will be studied too. And at the end, reliability analysis will be carried out to observe the safety level achieved in using certain design load values.

Few approximations and limitations are made during the analyses to make the study doable and complete in stipulated amount of time. These approximations are about the choice of independent random variables in assessing structural response of structure, constitutive model adopted in NLFEA etc. All approximations and limitations are described in detail and justified where possible.

2

RELIABILITY ANALYSIS & SAFETY ASSESSMENT IN NLFEA

A realistic structural response of structure can be obtained in a finite element analysis only when non-linearities are included in the analysis. The sources of these non-linearities are materials, geometry and boundary conditions. Realistic stress redistribution and capacity beyond the elastic stage can be assessed only through NLFEA. Thus, in order to predict realistic structural response, NLFEA are carried out with mean material parameters. Different safety formats are applied in the material parameters and load assessed by NLFEA to obtain realistic but also safe design load capacity. The reliability analysis of these NLFEA assessments is then carried out using different probabilistic methods. The uncertainties and safety formats that are used in NLFEA in this thesis for assessing design capacity of the structure are discussed in this chapter.

2.1. UNCERTAINTIES

Uncertainties are very important parameters that affect the structural safety assessment of a structure. In order to obtain realistic structural safety assessment of a structure, uncertainties that arise from numerous factors need to be properly characterized, evaluated and incorporated while adopting the basic variables representing resistance and load of the structure.

Uncertainties in mechanical material parameters and geometry are usually taken into consideration when material parameters, loading and geometry are modeled as random variables following a certain probability distribution. Besides these uncertainties, modelling uncertainty is generally always present in a finite element analysis. However, accurately quantifying it and adopting it in safety assessment is still an ongoing research in structural reliability analysis.

Modelling uncertainty is actually a result of various factors, such as: simplification of the real physical structure, the FEA softwared used, solution procedure, user judgment, physical and geometrical uncertainties etc. The modelling uncertainty is characterized by two statistical parameters θ_m and V_{θ} . These parameters represent the mean ratio of experimental load capacity to ultimate load capacity estimated by NLFEA, and coefficient of variation of modelling. Parameters θ_m and V_{θ} are based on combinations of experimental results and NLFEA results. Low value of V_{θ} , and $\theta_m \approx 1$ implies a well validated finite element model. Estimation of modelling uncertainty parameters and effect of including them in safety assessments of structure is described later in this thesis.

2.2. SAFETY FORMAT METHODS IN NLFEA

The ultimate load capacity estimated by NLFEA using mean material parameters has to incorporate a suitable safety factor to estimate the design load capacity. As discussed earlier, there are number of variables affecting the resistance of a structure. All these uncertain variables have to be handled appropriately to predict a safe and realistic design load capacity. Four different safety formats are described below and later used in this thesis. Three of these safety formats viz. Partial safety method, global resistance method and estimation of coefficient of variation of resistance method are also included in the Dutch guidelines[2]. The recommendations of Dutch Guidelines are followed in using these safety formats to estimate a design load capacity fulfilling the required level of safety. Besides these three safety formats, the new format proposed by Schlune et al. in 2012 is also used in this thesis^[4]. It must be noted that in discussions below, *r* represents the assessment of load capacity by nonlinear analysis with the specified material parameters^[5].

2.2.1. PARTIAL SAFETY FACTOR METHOD

In partial safety method (PSF), deterministic values of random variables are used in NLFEA to estimate the design load capacity. Design values are assigned to random variables determining the structural resistance. These design values are already incorporated with uncertainties and variabilities arising from different sources[2]. The values are determined using concrete partial safety coefficient $\gamma_c = 1.5$ and steel partial safety coefficient $\gamma_s = 1.15[2]$. Since the material parameters are significantly reduced using partial safety coefficient, the actual response of structural analysis might not be replicated by NLFEA using this method. The ultimate load capacity obtained from NLFEA using this method is the design load capacity itself. Mathematically,

$$R_d = r(f_d, \ldots) \tag{2.1}$$

2.2.2. GLOBAL RESISTANCE FACTOR METHOD

According to global resistance factor method (GRF), the global resistance of the structure is a random variable[2]. The mean material parameters to be used in NLFEA according to global resistance factor method are, $f_{cm} = 0.85 f_{ck}$ and $f_{ym} = 1.1 f_{yk}$. Other concrete and steel parameters to be used in the analysis are then derived from f_{cm} and f_{ym} using standard relations recommended in DG[2]. The design load capacity is obtained by dividing the NLFEA ultimate load capacity by a global resistance safety factor $\gamma_{GL} = \gamma_R \cdot \gamma_{Rd} = 1.27$, where $\gamma_R = 1.2$ is partial factor of resistance and $\gamma_{Rd} = 1.06$ is the modeling uncertainty factor. Mathematically,

$$R_d = \frac{r(f_{mGRF}...)}{\gamma_{GL}} \tag{2.2}$$

It needs to be noted that the ration 1.27 : 0.85 equals the concrete partial safety coefficient and the ration 1.27 : 1.1 equals the steel partial safety coefficient[2]. Thus, this method is actually an extension of PSF method.

2.2.3. ESTIMATION OF COEFFICIENT OF VARIATION OF RESISTANCE METHOD

In estimation of coefficient of variation of resistance method (ECOV), the probability distribution of resistance of reinforced concrete structure is assumed to be a lognormal distribution described by two parameters: the mean resistance, $r(f_{cm},...) = R_{um}$ and the coefficient of variation V_R [2]. Two NLFEA are carried out to estimate the coefficient of variation V_R . One analysis is run using mean properties of materials and the second one using characteristic properties of materials.

$$V_R = \frac{1}{1.65} \ln \left(\frac{r(f_{cm}, ...)}{r(f_{ck}, ...)} \right)$$
(2.3)

Then the design resistance is calculated using:

$$R_d = \frac{r(f_{cm}\ldots)}{\gamma_R \gamma_{Rd}} \tag{2.4}$$

where $\gamma_{Rd} = 1.06$ is the adopted value of model uncertainty safety factor for ECOV and GRF in this thesis, and

$$\gamma_R = e^{\alpha_R \beta V_R} \tag{2.5}$$

with $\alpha_R = 0.8$ and $\beta = 3.8[2]$. The modeling uncertainty factor $\gamma_{Rd} = 1.06$ should be increased if the model is not properly validated.

2.2.4. SAFETY FORMAT METHOD BY SCHLUNE ET AL.

Schlune et al. proposed a new safety format method in 2012 which allows direct inclusion of modeling uncertainty parameters (θ_m and V_{θ}) in estimating the design load capacity[4]. This method also assumes the resistance to follow a lognormal distribution. The method can be interpreted as an improved form of ECOV[5].

In this thesis, the in-situ value for the concrete compressive strength is obtained using $f_{cm,is} = 0.85 f_{cm}$. The coefficient of variation of the resistance is assessed using all three coefficients of uncertainties viz., coefficient of variation (cov) of material uncertainties (V_f), cov of geometrical uncertainties (V_g) and cov of modeling uncertainties (V_{θ}). The coefficient of variation of material, (V_f) is evaluated using two additional NLFEA with reduction of two basic material parameters ($f_{cm,is}$ and f_{ym}) related to failure by one at a time[5]. A step size parameter, c = 2.15 is used as recommended by Schlune et al. based on results from experimental experiences. [4, 5]

$$\Delta f_i \approx f_i (1 - e^{-cV_{f_i}}) \tag{2.6}$$

$$f_{\Delta i} \approx f_{im} - \Delta f_i \tag{2.7}$$

$$V_{f} \approx \frac{1}{r(f_{cm,is}, f_{ym})} \cdot \sqrt{\left(\frac{r(f_{cm,is}, f_{ym}) - r(f_{cm,is} - \Delta f_{c}, f_{ym})}{\Delta f_{c}}\right)^{2} \sigma_{f_{c}}^{2} + \left(\frac{r(f_{cm,is}, f_{ym}) - r(f_{cm}, f_{ym} - \Delta f_{y})}{\Delta f_{y}}\right)^{2} \sigma_{f_{y}}^{2}} \tag{2.8}$$

The design load capacity according to this method is calculated using:

$$R_d = \frac{r(f_{cm,is}, f_{ym})}{\gamma_R} \tag{2.9}$$

$$\gamma_R = \frac{e^{\alpha_R \beta V_R}}{\theta_m} \tag{2.10}$$

$$V_R = \sqrt{V_{\theta}^2 + V_g^2 + V_f^2}$$
(2.11)

2.3. Reliability Analysis

The other scope of the thesis was to analyze the structural reliability using different probabilistic methods. It was tested if the design load satisfied the required safety level. The desired safety level is $\beta \ge 3.8$, where 3.8 is the 50-year reliability index. The approach to the reliability analysis starts with defining a limit state function (LSF), usually represented by G = R - S where R is the resistance and S is the action effect. Once the LSF is obtained, probability of failure can be obtained using $P(G \le 0)$. If G follows a standard normal distribution or is transformed to a standard normal distribution with mean μ_G and standard deviation σ_G , then reliability index β and failure probability P_f are determined using Equations 2.12 and 2.13.

$$\beta = \frac{\mu_G}{\sigma_G} \tag{2.12}$$

$$P_f = \phi(-\beta) \tag{2.13}$$

The reliability methods used in this thesis are discussed below.

2.3.1. FIRST ORDER RELIABILITY METHOD

In first order reliability method (FORM), the reliability index β is as suggested by Hasofer/Lind; the shortest length between the linearized limit state surface and the origin in standard normal space[6].FORM is used in conjunction with response surface method in this thesis. The second order LSF obtained from NLFEA is linearized in standard normal space and then used to compute the reliability index. The outline of procedure of FORM analysis, inclusion of FORM in response surface method, transformation of random variables into standard normal space and linearization based on first oder Taylor series are discussed in Section 6.3.

2.3.2. Response Surface Method

A suitable LSF that distinguishes failure and safe domain is very much necessary to conduct reliability analysis of a structure. However, in most of the practical situations, it is quite difficult to obtain a closed form mechanical model of the response. The response surface method (RSM) approximates limit state function by a polynomial function depending on the realizations of random variables used in NLFEA[7]. The response surface is constructed based on method suggested by Bucher and Bourgund[8]. The outline of RSM procedure used for finding the LSF in this thesis is discussed in Section 6.2

2.3.3. MONTE CARLO SIMULATIONS & IMPORTANCE SAMPLING

Monte Carlo method can be used with the response surface generated by RSM method or directly coupled with NLFEA. In this method, random number x_u is generated from a uniform distribution between zero and one, which then represents the cumulative probability for $F_X(x)$ of a random variable. The realization x corresponding to the cumulative probability x_u is then[6]

$$x = F_X^{-1}(x_u) \tag{2.14}$$

If coupled directly with NLFEA, these realizations of random variables are used as material parameters to assess the structural safety of structure. And, if used with the response surface, these realizations of random variables are used in the function to assess if it falls in safe or failure domain. After number of such random samples, the probability of failure is assessed using:

$$P_f = \frac{N_f}{N} \tag{2.15}$$

where N_f is the number of failures and N is total number of simulations. Since the structural safety is assessed with reliability index $\beta \ge 3.8$, the failure probability is, $P_f \approx 10^{-6}$. It means more than 10^6 simulations are required thus demanding huge computation cost and time. Therefore, due to significant computation cost requirement, Monte Carlo simulations are not carried out in coupled with NLFEA directly in this thesis.

The number of simulations required can be minimized if more realizations of the random variable result in unsafe domain. This can be achieved by using a new sampling function $f_s(x)$ such that its maximum is located in the domain that contributes most to P_f . This method is known as "importance sampling". Detailed procedure and description can be found in [6]. The number of samples required for reliability analysis decreases significantly using this method. Both Monte carlo simulations and importance sampling are used later in this thesis.

3

SOLUTION STRATEGY

The term solution strategy as defined by Engen represents choices regarding finite element type and integration scheme, material parameters and constitutive model, and solution procedure[5, 9]. Uniformity and validation of solution strategy is very important as the result of a finite element analysis is strongly dependent on the solution strategy. As number of material and geometrical parameters that varies during the analysis can be modelled during NLFEA, adopting a verified solution strategy could help making reliable and uniform use of NLFEA. This in turn will help to reduce the moelling uncertainty and get results that are more stable and also in compliance with the real experimental scenario[5]. Therefore, a solution strategy based on recommendations of Dutch guidelines for nonlinear finite element analysis of existing structures is used in this thesis. The guidelines are validated for use on structural elements like beams, girders and slabs, reinforced or prestressed[2].

Structural walls are often constructed to resist the lateral forces such as in earthquake or severe wind. As these walls mainly carry in-plane shear forces and axial loads, they can be idealized as plane stress problem. Also two dimensional finite element model will help to simplify the structure and requires less computation effort. Therefore, two dimensional plane stress element is used to define a base model with solution strategy based on recommendations of DG[2]. The NLFEA of the base model assesses the ultimate load capacity and structural behavior of the wall.

3.1. EXPERIMENT BY LEFAS ET AL.

Experiments on thirteen structural walls subjected to monotonically increasing horizontal load at top edge of the wall was conducted by Lefas et al.[1]. Two types of walls (Type I and Type II) with varying cross-section and material properties, and an additional load were tested in the experiment. The capacity of wall, deformations and stress state, and failure mode are discussed in the experimental report[1]. This experimental report is used as a reference case through out the thesis to assess load capacity, failure mode and modeling uncertainty.

Due to time constraints, only one of thirteen structural walls that were used in the experiment is used as a case study for this thesis. The wall specimen (SW-21) used for this thesis is Type II wall with height to width ratio two and loaded only with the horizontal force at the top edge of wall. The wall is not subjected to any additional axial load. The experimental setup, geometry and reinforcement layout is presented in Figure 3.1.

3.1.1. RESULTS FROM EXPERIMENTS

The experimental report indicates that wall specimens failed in a ductile manner. The vertical propagation of crack reduced the depth of the compressive zone ultimately leading to collapse of specimens due to failure of the compressive zone. Flexural cracks initiated near the bottom third of the tensile edge at about 15% of the ultimate horizontal load capacity. The results also indicated that contrary to general belief, horizontal web re-inforcement did not have significant effect on shear capacity. Two wall specimens with half of recommended horizontal web reinforcement also failed in ductile manner after reaching the shear capacity.

The experimental report has discussed about the possible cause of high shear resistance of the concrete compressive zone. It has been concluded that the triaxial compressive stress was developed in the compressive zone subjected to combinations of high compressive and shear forces. The triaxial compressive stress state is developed as a result of volume dilation of the compressive zone. This resulted in higher resistance



Figure 3.1: (a) Experimental setup: elevation and plan view (b) Dimension and reinforcement layout of Type II wall specimen[1]

of the concrete compressive toe. As failure under such stress condition is caused by cracking in direction of maximum principal compressive stress, it justifies the vertical splitting of the wall's compressive zone.

3.2. PREVIOUS STUDIES

The experiment is validated and are later referred to in a number of studies[5]. All wall specimens were analyzed numerically by Vecchio and it was observed that the results from NLFEA and experiments were in good correlation[5, 10]. A master's thesis [5] at Norwegian University of Science and Technology by Nilsen-Nygaard was also based on six of experimentally tested thirteen wall specimens. The aim of the thesis was to study the significance of modelling uncertainty and perform structural safety assessment with NLFEA. The load capacity assessed by using NLFEA in DIANA was lower than the experimental value. The lack of volumetric concrete expansion modelling, post-peak concrete compressive behavior and the poor structural stiffness prediction were indicated to be the main limitation of adopted solution strategy in DIANA 9.6. This study is also used as another reference case for the present study. With some modifications in the solution strategy adopted by Nilsen-Nygaard, NLFEA including safety format assessment and reliability analyses are carried out.

3.3. Solution Strategy Based on Recommendations of Dutch Guide-Lines

The solution strategy used for NLFEA of the structural wall is based on recommendations of DG and the solution strategy adopted by Nilsen-Nygaard in [5]. The adopted solution strategy however deviates a bit from the recommendations. Deviations are noted in remarks and will be discussed later. The NLFEA for ultimate load capacity estimation, structural safety assessments and reliability analysis are done in DIANA software, version 10.1. The displacement control loading method is used during all the analyses.

A linear elastic analysis of a simplified structure was performed prior to the nonlinear analysis of the structural wall model. The linear elastic analysis was performed to validate the model regarding elastic stiffness, units and determination of suitable finite element size. The simplification, modeling and result of linear fi-



Figure 3.2: Finite element model of wall SW 21 representing mesh, boundary conditions and loading

nite element analysis are presented in Appendix A. After this preliminary analysis, it was determined to adopt a finite element size of 130 mm for the analysis. The selection of finite element size also confers with the element size adopted by Nilsen-Nygaard in [5].

The mechanical model used in this thesis is slightly different than the physical model. The lower beam was omitted in the mechanical model, and the base of the wall was restrained in both directions. Restraining both nodal displacements along the base were considered to be sufficient without inclusion of lower beam. This approximation can be justified as it is stated in the experimental report by Lefas et al. that "The lower beam was utilized to clamp down the specimens to the floor, simulating a rigid foundation"[1]. In [5] a study of the influence of lower beam effect was performed and results indicated that the approximation had a little significance on the overall analysis. The upper beam was modelled with linear elastic concrete to avoid damage at high concentrated load, and also to serve the beam only as a load transferring structures. This approximation should be acceptable as the thickness of the upper beam was more than three times the thickness of wall, and also the beam was heavily reinforced[1, 5]. The structural model that was modelled in DIANA and used for all analyses in this thesis is presented in Figure 3.2 The figure clearly represents the geometry, loading and boundary conditions.

The solution procedure adopted in this thesis is summarized through Table 3.1. The material parameters for concrete and steel were calculated on basis of mean concrete compressive strength f_{cm} and mean yield strength f_{ym} respectively. The relation between the material parameters and mean strength is recommended in [2]. The mean cylinder strength of concrete was calculated using $f_{cm} = f'_c = 0.85 f_{cu}$, where f_{cu} is the concrete cube strength given in the experimental report[1]. Similarly, the mean yield strength for reinforcement was calculated using $f_{ym} = f_{yk} + 10$, where f_{yk} is characteristic yield strength given in the experimental report for three different reinforcement bar diameters.

Table 3.1: Adopted Solution Strategy

	Concrete	
Finite Element		
Element Type	Plane Stress Element CQ16M	
Interpolation Scheme	Quadratic	
Integration Scheme	Full (2x2 point Gauss)	
Constitutive Modelling		
Model	Total strain based fixed crack model	
Crack Bandwidth	Rots	
Shear Behavior	Constant	Deviation
Shear Retention	0.1 Frances are attached for the second	
Compressive Behavior	Exponencial Soliening Parabolic	
Reduction of Compressive	Falabolic	
Strength due to lateral cracking	Yes	
Reduction Model	Vecchio & Collins 1993	
Lower Bound Reduction Curve	0.4	
Stress Confinement Model	Selby & Vecchio	
Poisson's ratio Reduction Model	Damage Based	
Material Parameters		
Mean Compressive Strength	fcm	36.38 MPa
Mean Tensile Strength	$f_{ct} = 0.3(f_{cm} - 8)^{\frac{2}{3}}$	2.79 MPa
Fracture Energy	$G_F = 0.073 f_{cm}^0.18$	0.14 N/mm
Compressive Fracture Energy	$G_c = 250G_F$	34.85 N/mm
Linear Material Parameters		
Young's Modulus of Elasticity	E _c	27549 MPa
Poisson's Ratio	ν	0.15
	Reinforcement Steel	
Finite Element		
Embeded Reinforcement	Yes	
Interpolation Scheme	Quadratic	
Integration Scheme	Reduced	
Constitutive Modelling		
Model	Elasto Plastic Material Mode	$E_{har} = 0.02E_s$
Tension Stiffening	Neglected	Deviation
Material Parameters		
Reinforcement Steel diameter (mm)	Mean Yield Strength, $f_{ym} = f_{ck} + 10$	Ultimate Strength, f_{uk}
4	430 MPa	490 MPa
6.25	530 MPa	610 MPa
8	480 MPa	565 MPa
Linear Material Parameters		
Young's Modulus of Elasticity	Es	200000 MPa
Loading, I	teration and Convergence Criterion	
Loading	Displacement Controlled	
Load Steps	by load steps of 0.5 mm until failure	Deviation
Equilibrium Iteration	Regular Newton Raphson	
Maximum Number of Iterations	50	
Line Searches per Iteration	10	
Force Norm	0.01	
Energy Norm	0.001	Deviation
No Convergence	Continue	

3.3.1. DEVIATION FROM DUTCH GUIDELINES

As noted in Table 3.1, there are few deviations in solution strategy than what is recommended in DG. These deviations are chosen based on some analyses and approximations. The first deviation in solution strategy from recommended guidelines is the use of constant shear retention factor. The Dutch guidelines suggest the use of variable shear retention model in conjunction with fixed crack model.

Three additional analyses were carried out to see the effect of this choice. The first additional analysis was done using rotating crack model, second one using damage based shear retention model and third one using constant shear retention factor 0.01. These three analyses resulted in ultimate load capacity 1% higher, 19% lower and 12% lower than using constant shear retention factor of 0.1. It was also found that during the analysis the principal stress field is not rotated significantly until the failure starts. Thus, it explains the small difference in result of rotating crack model and adopted shear retention factor. Also, it justifies the use of fixed crack model over rotating model since there is not much rotation of the principal directions. The results of damage based model and lower shear retention factor seem to be more conservative as they resulted in early failure. This suggests less shear stress was transferred to the concrete toe when damage based model and lower shear retention factor were ussed. This also justifies the use of adopted model. Therefore, for realistic stress transfer, total strain fixed crack model was used in conjunction with constant shear retention factor 0.1[5, 9].

The second deviation was regarding the omission of tension stiffening effect. In order to correctly model the redistribution of tensile stresses between concrete cracks and reinforcement bars, guidelines suggests the use of increased fracture energy G_F if element size is smaller than the estimated average crack spacing[2]. Nilsen-Nygaard in [5] performed a calculation of average crack spacing and evaluated that the fracture energy should be increased to $1.5G_F$. Performing an additional NLFEA using the increased fracture energy resulted in 0.23% difference in ultimate load capacity compared with the result of adopted solution strategy. Since the difference was not significant enough, the tension stiffening effect was neglected and unmodified tensile fracture energy G_F is used in the solution strategy.

The guidelines suggested to adopt arch-length procedure in load incrementation. However load steps were defined manually. Five initial load steps are of 0.1 mm which is approximately one third of displacement at initiation of flexural cracking according to experimental report[1]. Smaller load step factor is adopted to correctly capture the initial cracking phenomenon during the analysis. Later the load step factor is increased to reduce the computation effort.

The final deviation from the guidelines was regarding the energy norm. It is suggested to use an energy norm of 10^{-4} , but adopting this norm resulted in non convergence before the failure occurs. Peforming a post-analysis check it was evident that the non convergence was due to the adopted norm instead of structural failure. Therefore, reduced energy norm of 10^{-3} is adopted in this thesis which is in agreement with analyses performed by Engen and Nilsen-Nygaard[5, 9]. Since the solution procedure might result in non convergence before actual failure, it was chosen to allow analysis to proceed beyond non converged load step. Then, post analysis check was done to make an educated decision about the failure behavior.

4

RESULTS & DISCUSSION OF NLFEA

A finite element model of the wall specimen, SW-21 was analyzed using solution strategy discussed in Section 3.3. The aim of the analysis is to assess the ultimate load capacity of the structure and compare it with the experimental load capacity, $R_{exp} = 127 \text{ kN}[1]$. The assessment of load capacity also forms a reference case for structural safety assessments and reliability analysis of the wall specimen. The results from NLFEA, and its comparison with experimental and previous studies results are presented in sections below.

4.1. RESULTS OF ANALYSIS

The force-displacement response obtained from NLFEA is presented in Figure 4.1. The response of the structure from NLFEA is presented only upto reaching the ultimate strength of the reinforcement bar. Similarly, for the experimental results, it is only presented upto the failur load obtained from the experimental study. Initiation of flexural crack, yielding of reinforcement and peak load are clearly marked in the figure for both NLFEA and experimental results. In addition, first non-converged load step and reaching of ultimate strength (f_{uk}) in reinforcement are also marked in the figure for NLFEA results. The non-convergence occurs at 28th load step, and then the convergence recovers again. From Figure 4.1, the load can be seen to be again increasing after the convergence is recovered. From the analysis it was found that the peak load was obtained at load step 25. The peak load is used as the ultimate load capacity which was assessed to be $R_{um} = 100.8$ kN at 10.5 mm displacement. The NLFEA estimation is 79.3% of the experimental capacity. The experimental capacity was $R_{exp} = 127$ kN at 20.61 mm displacement[1].

Convergence behavior before reaching the failure load was observed, and it was found that on average each load step reached convergence after approximately 6 iterations. This indicates that the results are stable. First load step that required more than one iteration was load step 3 when the flexural crack is initiated. Similarly, the load step that needed maximum number of iterations to reach convergence was load step 10 when the yielding of vertical reinforcement took place. Out of 25 load steps, only three load steps satisfied both force and energy norm, and rest 22 steps satisfied only energy norm. However, the out of balance force was still in the order of 10^{-2}

4.1.1. FAILURE CHARACTERISTICS

In finite element analysis, the first crack was observed at load step 3, 0.3 mm displacement and 13.9 kN force. Similarly, the yielding of vertical reinforcement was observed at 5.0 mm displacement and 68.68 kN force. The experimental values for initiation of cracking and yielding of reinforcement were 0.32 mm displacement with 10 kN force and 5.81 mm with 80 kN force respectively.

The cracking pattern observed in the model during progressive load steps is presented in Figure 4.2. The figure shows absolute deformation with scale factor 2. Four representative load steps are chosen: load step 3 when flexural cracking initiates, load step 14 when vertical reinforcement yielding occur, load step 24 just before the peak load level and load step 28, the first non converged load step. Uniform color range is used in all crack plots. It can be seen that after the peak load level, there is a vertical crack propagation in compressive zone implying the initiation of splitting failure. Immediately after the peak load level, there is a shift of critical compressive zone. The initiation of crack, its propagation and ultimately the failure are in good compliance with results discussed in the report of experiments on thirteen wall specimens.



Figure 4.1: Load deflection response obtained from experiment and NLFEA

The initiation of yielding of reinforcement occurred on the tensile edge whereas the ultimate strength was reached on reinforcement under compression. The results of NLFEA showed that the horizontal reinforcement and shear stirrups do not reach yielding till the load when vertical reinforcement reaches its ultimate strength. This result is also in compliance with the experimental results. This particular finding will be used later in structural safety assessments and reliability analysis to simplify the problem model. The stresses in vertical reinforcement bars in tension and compression are represented in Figure 4.3. The stress in integration points at bottom of outermost reinforcement bars at different load steps is also presented in the figure. It is to be noted that the figure represents stresses developed in vertical bars upto the failure load only.

An additional NLFEA was performed with no hardening characteristics in reinforcement steel. The result is presented in Figure 4.4. It can be seen that not only the peak load decreases but also beyond the post-peak drop, the force increases very less. This infers that with the adopted solution strategy the vertical reinforcement acts under compression and thus helps to increase the ultimate load capacity of the structure.

In order to study the shift of compressive zone in more detail, principal compressive stresses at the outermost element in the wall base at compressive edge was observed too. Stresses at two outermost integration points of the element are analyzed, and then represented in Figure 4.5. As shown in the figure, outermost integration points in the element are marked P1 and P2. It can be observed that at peak load level, there is significant change of stresses in above mentioned two integration points. At point P1, the stress is seen to be decreasing rapidly and simultaneously, the stress at point P2 is increasing.

As already discussed in Section 3.1.1, one of the main conclusion of experimental report by Leflas et al. was that the development of triaxial compressive state at compressive zone resulted in high resistance of the compressive zone against failure. The triaxial stress state was result of volume dilation of concrete compressive zone. Thus, to view if similar process occurred in finite element analysis, volumetric strain was assessed at two points identified in Figure 4.5 (P1 and P2). The volumetric strain at two points is presented in Figure 4.6, and it can be observed that volumetric expansion effects did not occur in finite element analysis of the model.

4.2. ESTIMATION OF MODELLING UNCERTAINTY

The modelling uncertainty measures the deviation of NLFEA from experimental results. Since one of the aim of the thesis is to study the effect of including modelling uncertainty in structural safety assessments, two pa-



Figure 4.2: Crack propagation in the model during different load steps



Figure 4.3: (a) Stress profile in vertical reinforcement in Load step 25 (b) No yielding of Horizontal reinforcement at Load step 25 and (c) Stress in bottom integration points of two outermost vertical bars at different load steps



Figure 4.4: Load deflection response for hardening and non hardening reinforcement bar



Figure 4.5: Variation of compressive stress at two integration points of outermost element during progressive load steps



Figure 4.6: Volumetric strain at two integration points of outermost element during progressive load steps

Solution Method	R _{um}	$\theta_{m,i}$
Present Study	100.8	1.26
Nilsen-Nygaard 2D Model	97.7	1.30
Nilsen-Nygaard 3D Model	108.0	1.18
Engen Model	113.99	1.11
Mean modelling uncertain Coefficient of Variation, V	nty, θ _m ' _θ (%)	1.21 6.88

Table 4.1: Estimation of Modelling Uncertainty Parameters [3, 5]

rameters describing the uncertainty are estimated. Since only one wall specimen was used as the case study in this thesis, and also the analysis was done based on a single solution strategy it was difficult to estimate parameters accurately. Engen in [11] has described various methods suggested in literatures to estimate the modelling uncertainty parameters. The method that is used in this thesis is based on one experimental result compared to NLFEA results obtained using different solution strategies. The experimental result is obtained from [1], and NLFEA results are obtained from solution strategies adopted in this thesis, in [5] and in [3]. The results are presented in Table 4.1.

The estimated values of mean modelling uncertainty θ_m and coefficient of variation of modelling V_{θ} are used in estimating the design capacity by Schlune et al. safety format. The effect of modelling uncertainty parameters on design load estimation including sensitivity analyses are discussed in Section 5.3.

4.3. SUMMARY OF NLFEA RESULTS

The adopted solution strategy replicated the structural behavior and failure characteristics quite impressively. The initiation of crack, initiation of yielding of reinforcement in tension and less significance of horizontal web reinforcement and shear stirrups in ultimate load capacity were found to be in good correlation with the experimental report by Lefas[1]. Vertical reinforcement in compressive edge takes a significant role in assessing the failure load. The effect of perfectly plastic steel and hardening steel can be seen in Figure 4.4.

However, as compared to the experimental result, the ultimate load capacity assessed by NLFEA was conservative. The ultimate load capacity assessed by NLFEA was only found to be 79.3% of the experimental load capacity. The major difference obtained between behavior of finite element model and experimented wall specimen is the lack of volumetric expansion in two dimensional finite element model. As already discussed in Section 3.1.1, volumetric expansion in the compressive zone was regarded to be the contributing factor for high resistance of the structure during the experiment. The lack of such characteristics in finite element analysis is predicted to be the major limitation of the adopted solution strategy. The results using three dimensional finite element model could have been higher than the one assessed by using two dimensional model. Nilsen-Nygaard performed NLFEA on both 2D and 3D model. The results were found to be 76.9% and 85.0% of the experimental load capacity for 2D and 3D models respectively. The result obtained from adopted solution strategy in this thesis is quite similar with results obtained in previous studies too. Nilsen-Nygaard has mentioned in [5] that results obtained by Vecchio in [10] yielded $\theta_m = 1.14$ and $V_{\theta} = 7.3\%$ for models without concrete expansion, and $\theta_m = 1.02$ and $V_{\theta} = 6.5\%$ for models with concrete expansion. It can be observed that the values of θ_m and V_{θ} estimated in Section 4.2 is similar with results obtained by Vecchio in models without concrete expansion.

5 NLFEA USING DIFFERENT SAFETY FORMATS

In this chapter the inclusion of different safety formats in NLFEA of the wall is discussed. The design load capacity R_d was predicted by incorporating different safety formats. The safety factors were applied to the mean material parameters used to assess the ultimate load capacity R_{um} using NLFEA and/or to the assessed load as recommended. The model uncertainty that was calculated in Section 4.2 is included in estimating the design load according to Schlune et al. safety format. The sensitivity analyses of effect of those parameters to the estimation of design load capacity according to Schlune et al. safety format is studied too. The same solution strategy that was used in assessing the ultimate load capacity of SW 21 wall specimen is used in these analyses as well.

5.1. MATERIAL PARAMETERS

The material parameters that are used in the structural safety assessments using PSF, GRF and ECOV were calculated by formulas in Chapter 2 and DG unless mentioned and discussed[2]. As mentioned in Section 3.3, the characteristic yield stress of all three steel bars were used directly from the experimental data. *Mean DG* implies the mean material parameters as recommended by Dutch Guidelines and used in the previous NLFEA.

In addition to the NLFEA using mean DG material paremeters, two additional nonlinear analyses is suggested by Schlune et al. to calculate the material uncertainty $V_f[4]$. However, since there are three different reinforcement bars being used in the model, a study was done to see how to incorporate this issue into the calculation of material uncertainty. It has already been discussed in Section 4.1.1 that yielding does not occur in the horizontal reinforcement and shear stirrups prior to the peak load level. Therefore, it should be a reasonable approximation to only consider changing the material parameters of vertical reinforcement.

Two nonlinear analyses were carried out to study this claim; one with changed material parameters for all reinforcement bars and another with changed material parameters only for the vertical reinforcement. It was observed that there was no difference in these two non linear analyses up to the failure point. Thus only vertical reinforcement and concrete material parameters were changed for calculation of material uncertainty. This study also serves to validate the approximation of using only two independent random variables f_c and f_γ in FORM and RSM analysis to be discussed in Chapter 6.

The reduced concrete compressive strength, $f_{\Delta c}$ and reduced steel yield strength, $f_{\Delta y}$ were calculated by Equation 2.7, and are presented in Table 5.1 along with material parameters of other safety formats. The random variables f_{cm} and f_{ym} representing the wall resistance were assumed to be lognormally distributed with coefficient of variation $V_{fc} = 4\%$ and $V_{fy} = 15\%$ respectively. The choice of distribution and coefficient of variation adopted for material parameters are discussed in more detail in Section 6.1. Using the results of NLFEA, and Equation 2.8, the final material uncertainty to be used in Equation 2.11 to calculate V_R was calculated to be $V_f = 4.70\%$.

Similarly, $V_g = 5\%$ was used as the coefficient of variation of geometrical uncertainty for both concrete and steel dimension. The estimated $V_{\theta} = 6.88\%$ from Section 4.2 was used as the coefficient of variation for model uncertainty. These values resulted to $V_R = 9.72\%$. For both Schlune et al. and ECOV, same values for

Concrete Material Parameters							
	IInit	PSF	Schlune et al.		CRE	ECOV	
	omt		Mean insitu	Reduced, $f_{\Delta c}$	UIU	Mean	Charcteristic
$\overline{f_c}$	MPa	18.92	30.92	22.40	24.12	36.38	28.38
f_t	MPa	1.30	2.96	2.38	2.50	2.79	1.95
\mathbf{G}_{f}^{I}	N/mm	0.12	0.14	0.13	0.13	0.14	0.13
$\mathbf{G}_{c}^{'}$	N/mm	30.98	33.85	31.94	32.37	34.85	33.33
\mathbf{E}_{c}	MPa	22642.16	26237.72	23818.15	24354.04	27548.66	25570.86
ν		0.15	0.15	0.15	0.15	0.15	0.15

Table 5.1: Material parameters used in NLFEA for different safety formats

Reinforcement Material Parameters						
Diameter of bar	PSF	Schlune et al.		GRF		ECOV
	101	Mean	Reduced, $f_{\Delta y}$	GIU	Mean	Charcteristic
4 mm	365.22	430.00	430.00	462.00	430.00	420.00
6.25 mm	452.17	530.00	530.00	572.00	530.00	520.00
8 mm	408.70	480.00	440.45	517.00	480.00	470.00

Table 5.2: Comparison of design load assessed by using different safety formats with R_{um} = 100.8 kN

Safety Format	Design Load Capacity, \mathbf{R}_d (kN)	$\frac{R_d}{R_{um}}$
PSF	80.97	1.24
GRF	77.04	1.31
ECOV	85.88	1.17
Schlune et al.	88.47	1.14

the reliability index (β = 3.8) and sensitivity factor(α_R = 0.8) was used[2].

5.2. RESULTS OF SAFETY FORMATS INCLUDED NLFEA

The design load capacity obtained from NLFEA using the material parameters suggested by different safety formats is presented in Table 5.2. The reduction factor relative to the ultimate load capacity predicted by NLFEA using mean DG material parameters is also presented in the table. Similar study was done by Nilsen-Nygaard[5], and the results are comparable in most of the cases. Some differences are to be expected due to the difference in adopted solution strategy. The major difference is in the design capacity predicted using PSF. This is mainly because, in[5], in addition to material safety factor (γ_c and γ_s), safety factor $\gamma_{Rd} = 1.06$ is also included in assessing the design material parameters. However, in this thesis, only material safety factor (γ_c and γ_s) are included in estimating material parameters according to PSF as recommended by DG[2].

Though GRF and ECOV also included model uncertainty safety factor, $\gamma_{Rd} = 1.06$ in assessment of design load capacity, direct inclusion of model uncertainty parameters (θ_m and V_θ) was done only in Schlune et al. safety format. As *f ib* allows to use higher value of γ_{Rd} in ECOV, model uncertainty parameters could be used to calculate a new value of γ_{Rd} using the formulas provided by Kadlec and Cervenka[12, 13]. However, the lognormal formula provided by Kadlec and Cervenka resulted in a value less than 1.0. The result was expected as the result of the NLFEA was conservative with respect to the experimental result. Thus the value of γ_{Rd} would be less than 1 to scale the result closer to the experimental result. Adopting this value was not appropriate as it was less than 1.06 and also gave the design load higher than the the ultimate load capacity, R_{um} . It was not clear how to adopt the formula for conservative analysis results. Due to lack of time, no further study was done regarding it and the specified value $\gamma_{Rd} = 1.06$ was used in ECOV safety format as well.

For constant $\theta_m = 1.21$				For C	onstant	$\mathbf{v}_{\theta} = 6.88\%$				
	$\overline{\mathbf{v}_{ heta}}$	\mathbf{V}_R	γ_R	\mathbf{R}_d		$\overline{\theta_m}$	γ_R	\mathbf{R}_d		
	0	0.069	0.978	100.477		1.3	1.034	95.049		
	5	0.085	1.027	95.624		1.25	1.075	91.393		
	10	0.121	1.148	85.614		1.2	1.120	87.737		
	15	0.165	1.310	74.970		1.1	1.222	80.426		
	20	0.211	1.509	65.089		1	1.344	73.114		
	30	0.308	2.023	48.569		0.9	1.493	65.803		
15	10	15	20 25	30	- 28 - 0.0 -	0.9 0		105 11 115	12 125	-
- •		Coefficient of Var	riation, V _g					Modelling Uncertainty, θ _m		

Table 5.3: Effect of varying θ_m and V_{θ} on design load estimated by Schlune et al. safety format

Figure 5.1: Variation of R_d with change in (a) Coefficient of Variation V_{θ} and (b) Mean modelling uncertainty θ_m

5.3. EFFECT OF MODEL UNCERTAINTY IN DESIGN LOAD CAPACITY

This section aims to describe the effect of model uncertainty in estimating the design load capacity. Schlune et al. safety format is the only safety format that allows direct inclusion of model uncertainty parameters (V_{θ} and θ_m) in assessing the design load capacity[4, 5]. The design load capacity presented in Table 5.2 according to Schlune et al. safety format was based on V_{θ} and θ_m estimated in Section 4.2. A small sensitivity study was carried out to see how the design capacity is affected by varying model uncertainty parameters independently. It must be noted that this study only describes the effect of model uncertainty parameters on result of a particular NLFEA. The results are presented in Table 5.3 and Figure 5.1.

It can be observed that for a particular NLFEA result, the design load capacity decreases with increment of θ_m and decrease of V_{θ}. It was an expected trend, also suggested by:

$$R_d = \frac{R_{um}\theta_m}{e^{\alpha\beta V_R}} \tag{5.1}$$

Ideally, a lower value of θ_m would imply less variation from the experimental value and thus would give larger ultimate and design load capacity. In that case even R_{um} value would change and a different result will be obtained. But as mentioned above this result should only be applied on a particular NLFEA result and varying model uncertainty parameters.

5.4. SUMMARY OF NLFEA WITH DIFFERENT SAFETY FORMATS

The result obtained from NLFEA incorporating different safety formats can be summarized through Figure 5.2. The figure represents the reduction factor for design load capacities estimated by using different safety formats relative to the experimental load capacity of the structural wall. Six different bars can be observed representing experimental load capacity, ultimate load capacity estimated by using DG and design load capacity using PSF, GRF, ECOV and Schlune et al. respectively. This bar chart will be updated later in Section 6.6 after obtaining results of FORM, Monte Carlo simulations and importance sampling.

The lowest ultimate load capacity was obtained using PSF method. It was an expected outcome as material parameters are heavily reduced using material safety factors γ_c and γ_s in PSF method. But, in PSF method the design load capacity was estimated to be equal to the ultimate load capacity assessed using reduced material parameters. The ultimate load capacity assessed by GRF was approximately 17 kN higher than PSF



Figure 5.2: Comparison of design load capacity assessed using different method with experimental capacity

method, but owing to global safety factor γ_{GL} = 1.27 the design load capacity of GRF was close to 4 kN less than the PSF method.

Results of PSF and GRF were found to be close enough to result obtained using ($\theta_m = 1$ and $V_{\theta} = 5\%$) in Schlune et al. format. This helps to give an idea about the level of uncertainty used in PSF and GRF method. Using the lognormal formula by Kadlec, it can be observed that the specified value $\gamma_{Rd} = 1.06$ implies a value of $V_{\theta} = 4.9\%$ for finite element analysis results having mean modeling uncertainty unity[5, 13].

The higher design load capacity is provided by ECOV and Schlune et al. safety formats. The similarities in both these formats is inclusion of material uncertainty V_f using one and two additional non linear analyses respectively. In addition, the inclusion of bias $\theta_m = 1.21$ makes the Schlune et al. format more non conservative than other safety formats.

The design load capacity for the structural wall using strut and tie method was calculated by Nilsen-Nygaard in [5]. The analytical design load was found to be 62.2 kN. The design load capacity assessed using all four safety formats are higher than the analytical design load but still conservative with respect to the experimental load capacity. It confers with the result obtained by Nilsen-Nygaard as well[5].

All the NLFEA provided ultimate load capacity between 26 to 28 load steps. And also the failure mode for each analyses was similar to the one discussed in Section 4.1.1.

6

RELIABILITY ANALYSIS OF THE STRUCTURAL WALL

The simplest way to understand the reliability analysis of a structure and to verify its safety is to express that resistance action is larger than the load action.

$$Z = R - S > 0 \tag{6.1}$$

However, the reliability analysis of a practical structure is not a simple task and involves various complexes. The limitation arises from the unavailability of a explicit limit state equation. Therefore, conducting a reliability analysis by analytical evaluation of integration describing probability of failure is generally not possible. Further, there is possibility of having a structural failure by combination of different failure mode. As failure domain and failure probability are different for different combination, the failure mode also must be observed with caution.

In this thesis, the reliability analysis of the structural wall was carried out using several methods. First of all an implicit Limit State equation was evaluated based on number of NLFEA results and fitting those results to a response surface. The LSF thus obtained was used to carry out FORM (First Order Reliability Method), Monte Carlo simulations and Importance Sampling. The procedure and results of the analysis are presented in sections below.

6.1. RANDOM VARIABLES

Resistance and load in the limit state equation are usually random variables. These random variables follow a certain distribution and are characterized by specific parameters. In this thesis, the resistance is assumed to be a random variable whereas a deterministic value is used for the load.

Random variables representing the wall's resistance are concrete compressive strength f_c and vertical reinforcement bar yield stress f_y . Other concrete material parameters like f_{ct} , E_c , G_c , G_F^I are assumed to be completely dependent on the random variable f_c . This can be assumed to be a realistic approximation as the correlation of these variable with f_c is very high. Literature reviews suggest that the correlation is not exactly 1 but sufficiently high ($\approx 0.8 - 0.9$) to make the approximation justifiable.

Similarly, only vertical reinforcement yield stress is considered to be the random variables. Two NLFEA carried out to observed the effect of this approximation was explained in Section 5.1. The random variables f_c and f_y are assumed to be independent to each other. All these approximations help to simplify the situation. Then, it is only necessary to deal with two independent random variables, f_c and f_y to obtain an implicit limit state equation by fitting NLFEA results to a response surface and thus carry out reliability analyses.

The random variables f_c and f_y are assumed to follow lognormal distribution. Generally, it is preferred to use normal distribution as a representative distribution for random variables. However, since normal distribution has negative values and negative strength does not make sense, lognormal distribution is preferred to normal distribution. The parameters to describe the lognormal distribution are its mean and standard deviation. The mean of random variables f_c and f_y are assumed to be equal to mean DG values i.e. 36.38 MPa and 480 Mpa respectively. The coefficient of variation V_{fc} and V_{fy} are approximated to be equal to 15.0% and 4.0% respectively. This approximation was made based on number of literature studies.[4, 5, 7, 14]

6.2. RESPONSE SURFACE

The set of results obtained from number of NLFEA with different deterministic input values for the basic variables is fitted to a suitable function. The fitted function represents a limit state function and thus can be used to distinguish the failure domain.

Since only two random variables f_c and f_y were representing the wall resistance and the load was deterministic, the second order response surface consists of 6 terms.

$$g(x) = b_1 + b_2 f_c + b_3 f_y + b_4 f_c^2 + b_5 f_y^2 + b_6 f_c f_y$$
(6.2)

The regression coefficients b_1 to b_6 were unknown and the task was to determine them by using the iterative process suggested by Bucher & Bourgund[8]. Equation 6.2 when represented using matrix notation becomes

$$\mathbf{g}(\mathbf{x}) = \mathbf{A}\mathbf{b} \tag{6.3}$$

where, **A** is a matrix containing the input values for the basic variables. $[1, f_{c1}, f_{y1}, f_{c1}^2, f_{y1}^2, f_{c1}f_{y1}]$ is the first row of **A**. **b** is a column vector of undetermined coefficients and $g(\mathbf{x})$ is a column vector of responses. The cross-term $b_6 f_c f_v$ is included to improve the accuracy of the response surface.

The response G(X) was obtained by subtracting load from the NLFEA response.

$$G(\mathbf{X}) = R(\mathbf{X}) - L \tag{6.4}$$

where, $R(\mathbf{X})$ is the resistance obtained from NLFEA as function of basic variables (f_c and f_y), and L is the load value. Two deterministic load values were used in the reliability analysis. Analytical load capacity was chosen as one representative design load at the beginning. But, FORM analysis of first response surface obtained using the analytical load gave a reliability index of $\beta \approx 12$. No further iteration was done for that load then because the aim was to find a load value close to limit state.

The design load values for which the reliability analysis were performed were two-third of the experimental capacity and the design load estimated by using Schlune et al. safety format. The two third experimental capacity is higher than the design load value estimated by PSF and GRF method, and only 1.27 kN smaller than the design load estimated by ECOV safety format. Thus performing a reliability analysis on design load value equal to two third of experimental capacity would give an insight about the results of safety format method recommended in [2]. Similarly, the other design load value that was used as a deterministic load was equal to the design load estimated by using Schlune et al. format. Since the design load estimated by using this method resulted in higher value compared to other it was chosen to perform the reliability analysis in this load value too. These load values gave some insights to compare the reliability level of estimated design load capacities.

For the first iteration (response surface) the sampling points x_i were chosen to be the mean values $\bar{x_i}$ and $x_i = \bar{x_i} + f_i \sigma_i$, where f_i is an arbitrary factor and σ_i is standard deviation of the random variable respectively. Nine NLFEA results were used to get a response surface. Since the number of responses were higher than the number of variables, a least squares approach was used to find **b** by Equation 6.5 [7].

$$\mathbf{b} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{g}$$
(6.5)

This method was used together with FORM to update the response surface by forming an iterative process. New design points, and reliability index β were calculated using FORM analysis with the function obtained. More detailed information about the procedure of FORM analysis is described in Section 6.3.

The β value calculated from FORM analysis was used to check the convergence.

$$\epsilon = \frac{\beta_{i+1} - \beta_i}{\beta_{i+1}} \tag{6.6}$$

The convergence criterion was set to be 0.001. If the convergence was achieved, the RSM iteration was stopped and the function thus obtained was used as the final LSF. The LSF then was used for Monte Carlo simulations and Importance Sampling. However, if the convergence was not achieved, updated design points from FORM analysis was used as the new center in generating new response surface.

For the first iteration in response surface method, the value of $f = \pm 3$ was used in this thesis. Zhao and Qiu have mentioned in [15] that the value of f = 3 is recommended by several scholars. For the remaining iterations, f = 1,2 were used. First it was attempted to use only $f = \pm 1$. But, the result obtained in second



Figure 6.1: Quadratic LSFs obtained for $R_d = \frac{2R_{exp}}{3}$ during iterations 2,3 while using $f = \pm 1$

and third iterations were very absurd. The design point after third iteration was found to be $f_c = 51.62$ and $f_y = 464.35$ whereas the first iteration and also other iterations using f = 1,2 gave design points close to $f_c = 21$ and $f_y = 420$. The possible reason for this might be due to low concrete compressive strength obtained while using f = -1. Since stable results were obtained using f = 1,2, it was continued to use the mentioned values in all iterations after the first one. The graph of function obtained after second and third iteration while using $f = \pm 1$ is presented in Figure 6.1.

The convergence was achieved in the fifth iteration while using load equal to two third of the experimental load capacity and in second iteration while using load equal to the design load capacity estimated by Schlune et al. format. The convergence for second load value was achieved sooner because the convergence criterion used in this case was equal to 0.01.

6.3. FORM ANALYSIS

The second order LSF obtained from RSM iteration was used to carry out First order reliability method (FORM). Since only two random variables were used in this thesis, the linearized LSF will then be in the form

$$G = a_0 + a_1 U_c + a_2 U_y \tag{6.7}$$

where a_0 , a_1 , a_2 are coefficients to be determined and U_c , U_y are: random variables f_c , f_y transformed into standard normal space. First, the random variables in quadratic LSF obtained from RSM were transformed into the standard normal space. The transformation between lognormal distribution and standard normal space as suggested by Hasofer/Lind are shown in Equations 6.8 and 6.9[7]

$$U(x) = \frac{\ln(x) - \mu_{ln}}{\sigma_{ln}} \tag{6.8}$$

$$X(u) = \exp(u\sigma_{ln} + \mu_{ln}) \tag{6.9}$$

where the lognormal mean, μ_{ln} , and lognormal standard distribution, σ_{ln} are given as

$$\mu_{ln} = \ln\left(\mu_x^2 \sqrt{\frac{1}{\sigma_x^2 + \mu_x^2}}\right),$$
(6.10)

$$\sigma_{ln} = \sqrt{\ln(V_x^2 + 1)} \tag{6.11}$$

After transforming the random variables into standard normal space, the LSF is now linearized using only the linear terms in Taylor series expansion around a certain design point x_i . After finding the linearized LSF, the mean and standard deviation of linearized LSF were calculated. These values were used to obtain



Figure 6.2: (a)Quadratic LSF in real space (b)Nonlinear and linearized LSF in standard normal space (c) Response Surface in real space (d)Response Surface in standard normal space for $R_d = \frac{2R_{exp}}{3}$

sensitivity factors α_i , the reliability index β and the next design point x_i^* . After finding the design point, convergence was checked using

$$\epsilon_{FORM} = \sqrt{\sum_{i=1}^{n} (x_{ij} - x_{i,j-1})^2}$$
 (6.12)

where j is the iteration number. The convergence criterion used in this thesis was 10^{-5} . The probability of failure $P_f = \phi(-\beta) = \phi(-\frac{\mu}{\sigma})$ was calculated on satisfying the convergence criterion, else new iteration was carried out with the new design point. Detailed procedure for FORM analysis can be found in [6].

6.4. RESULTS OF RSM & FORM ANALYSIS

The results of iterations of RSM and FORM are presented in Figures 6.2, 6.3, 6.4 and 6.5. Graphs of response surfaces, limit state functions and linearized LSF are presented only for the final iteration of RSM as only these results will be used for further analysis. Also LSF obtained after each iteration of RSM is presented and it can be observed that results have similar pattern through out all iterations. The design points obtained in each iteration are marked too. All design points are close to each other near $f_c = 20$ and $f_y = 420$, which indicates stable design points were obtained after each iterations. The resulting reliability index, sensitivity factors of each material strength and corresponding failure probability for both load values obtained by FORM are presented in Table 6.1. Also, the values of basic variables used in NLFEA, response values used for RSM, residues after obtaining response surface, and number of iterations required in FORM for last iteration of both design loads are presented in Table 6.2. Residue is defined as the difference of response obtained from NLFEA results and value predicted from the obtained response surface using input values of basic variables. Similar results for all iterations of RSM for both design load values are presented in Appendix D

6.5. MONTE CARLO & IMPORTANCE SAMPLING

The LSF obtained after final iteration of RSM were also used for calculating failure probability P_f by using Monte-Carlo simulations and Importance Sampling. The failure probability for LSF obtained using design load equal to two third of experimental load capacity, was in the order of 10^{-6} . Therefore more than 10^7 samples of random variables f_c and f_y were to be generated. As it was computationally time consuming, Importance sampling was used too to calculate the failure probability.



Figure 6.3: Quadratic LSFs obtained for $R_d = \frac{2R_{exp}}{3}$ during iterations 1-5



Figure 6.4: (a)Quadratic LSF in real space (b)Nonlinear and linearized LSF in standard normal space (c) Response Surface in real space (d)Response Surface in standard normal space for $R_d = R_{d,Schluneetal}$.

Design Load	Reliability Index	Sensit	ivity Factor	Failure Probability	Legend used
R _d (k N)	β	α_c	α_y	\mathbf{P}_{f}	for Figure 6.6
84.67	4.75	0.678	0.735	$1.01 \ge 10^{-6}$	FORM1
88.47	3.44	0.7	0.714	$2.87 \text{ x} 10^{-4}$	FORM2

Table 6.1: Summary of results of FORM analysis



Figure 6.5: Quadtratic LSFs obtained for $R_d = \frac{2R_{exp}}{3}$ during Iteration 1 and 2

Table 6.2: Results of Final Iteration of RSM for (a) $R_d = \frac{2R_{exp}}{3}$ and (b) $R_d = R_{Schluneetal}$.

(a)					
f_y (MPa)	Response (N)	Residue (N)			
421.61	246.99	102.62			
440.81	2398.66	-212.42			
460.01	5049.59	106.96			
421.61	3364.38	-51.58			
440.81	6154.81	109.11			
460.01	8480.02	-60.25			
421.61	6156.46	-53.77			
440.81	9103.60	100.59			
460.01	11611.19	-49.43			
Sum of Squ	ares (RSS)				
due_i^2		317.40 N			
of FORM It	erations	7			
value, f _{cd}		22.25 MPa			
value, f _{vd}		417.11 MPa			
ity Index, β		4.751			
ity factor α_c		0.678			
ity factor α_1	,	0.735			
	f_y (MPa) 421.61 440.81 460.01 421.61 440.81 460.01 421.61 440.81 460.01 421.61 440.81 460.01 $\frac{1}{duc_i^2}$ or FORM It value, f_{yd} ity Index, β ity factor a_c ity factor a_c	(a) f_y (MPa) Response (N) 421.61 246.99 440.81 2398.66 460.01 5049.59 421.61 3364.38 440.81 6154.81 460.01 8480.02 421.61 6156.46 440.81 9103.60 460.01 11611.19 Sum of Squares (RSS) due_i^2 or of DRM Iterations $ralue, f_{rd}$ tiy Index, β liy factor a_v			

		(b)			
<i>f_c</i> (MPa)	f_y (MPa)	Response (N)	Residue (N)		
25.35	434.11	-1.81	67.66		
25.35	453.31	2448.03	178.72		
25.35	414.91	-2159.16	-245.24		
30.81	434.11	3431.54	-111.28		
30.81	453.31	6136.40	-49.74		
30.81	414.91	482.34	162.18		
19.89	434.11	-3207.86	44.75		
19.89	453.31	-679.92	-127.78		
19.89	414.91	-5512.01	84.13		
Residue $\sqrt{\Sigma Resi}$	Residue Sum of Squares (RSS) $\sqrt{\Sigma Residue_i^2}$				
Number	of FORM It	erations	8		
Design v	alue, f _{cd}		25.11 MPa		
Design v	434.71 MPa				
Reliabili	3.444				
Sensitiv	0.700				
Sensitiv	ity factor a_y	,	0.714		

Method Used	Number of Random	Design Load	Reliability Index	Legend used
	Samples Used	\mathbf{R}_d (kN)	β	for Figure 6.6
Importance Sampling Importance Sampling Monte Carlo	$10^5 \\ 10^3 \\ 10^5$	84.67 88.47 88.47	4.75 3.45 3.37	IS 1 IS 2 MC

Table 6.3: Summary of Importance Sampling and Monte Carlo simulations

In order to achieve the goal of calculating failure probability using less randomly generated samples of variables, a new sampling function was chosen. The mean value of f_c and f_y for new sampling function were chosen to be equal to the design point used for final RSM iteration. Dtailed procedure of Importance Sampling can be found it [6].

The failure probability for load equal to the design load estimated by using Schlune et al. format was higher. Therefore, both Monte Carlo simulations and Importance sampling were carried out for the LSF obtained using design load estimated by using Schlune et al. format. The number of samples of random variables used to obtain the indicated reliability index for each load level are presented in Table 6.3.

The entire procedure of RSM, FORM, Monte Carlo simulations and Importance Sampling were carried out by running self developed script in Python and MATLAB. Since nine NLFEA were to be carried out for each iteration of finding a new response surface, a python script was developed to carryout the nine NLFEA automatically. Similarly after getting the responses from NLFEA, self developed MATLAB script was used to evaluate a new response surface and then carryout FORM, Monte Carlo and Importance Sampling analysis. The scripts are attached in Appendix B and C. Automated use of DIANA results in response surface was not tried to use the results only after performing a post analysis check.

6.6. SUMMARY OF THE RESULTS OF RELIABILITY ANALYSIS

The figure 5.2 presented in Section 5.4 is updated with the inclusion of results from FORM, Monte Carlo and importance sampling analysis. The updated results are presented in Figure 6.6. It can be seen from results of FORM analysis that target reliability of β = 3.8 was safely achieved for all design load estimated by all safety formats except Schlune et al. format.

Two third of the experimental load capacity is higher than the design load estimated by PSF and GRF, and only 1.27 kN lower than ECOV estimation. The reliability index obtained for design load, $L = \frac{2R_{exp}}{3}$ using FORM and importance sampling is approximately 4.75. As all safety formats used $\beta = 3.8$, it indicates that the design load estimated by PSF, GRF and ECOV are conservative in this case.

However, while using design load equal to the one estimated by Schlune et al. format, it did not meet the intended safety level. The reliability index obtained using Schlune et al. format was $\beta \approx 2.76$, inferring that Schlune et al. format provided a non conservative design load capacity.

FORM, Monte Carlo simulations and importance sampling provided failure probability using the LSF obtained from RSM. Though FORM was already used in conjunction with RSM to fit a stable response surface, Monte Carlo simulations and importance sampling were used too in order to get more insight into different methods of reliability analysis. Since the reliability index obtained from all analysis are comparable, it can be concluded that for this case FORM is a good approximation and computationally expensive Monte Carlo simulations and importance sampling are not really necessary. However, it must also be noted that the LSF obtained is an approximate one based on FORM and RSM. The conclusion might be different for an analytical LSF.

Different RSM results obtained while using $f = \pm 1$ showed that the response surface is sensitive to adopted values of f_i . Choosing f = -1 resulted in lower concrete grade that affected the response surface estimation. However, using the value of f_i that resulted in higher concrete compressive strength gave a stable design point and response surface in every iterations. This study infers that the failure mode might be different for lower concrete grades. It was discussed in Section 4.1.1 that the vertical reinforcement acts under compression in left edge. For lower concrete grades the compressive action in hardening steel might be higher than usual and might result in a different failure mode. As choosing $f = \pm 1$ includes both higher and lower concrete grade, the failure mode might be transitioning and thus affect the response surface estimation. Further analysis about this problem was not done and f = 1, 2 were adopted for stable response surface estimation.

Sensitivity factor α_i of concrete was found to be smaller than that of steel from FORM analysis of final RSF



Figure 6.6: Comparison of design load capacity assessed using different safety formats and reliability analyses with experimental capacity

for both design load values. This suggests that concrete compressive strength had less impact on assessing the ultimate load and thus the design load capacity of the structure. This result is in compliance with the discussion in the experimental report that the concrete compressive strength had less influence in the overall strength of the structural wall [1]. The result indicated that the variability of concrete compressive strength did not significantly affect the strength of the wall.

7

CONCLUSION AND RECOMMENDATIONS

Allowing the inclusion of various parameters in modelling during non-linear finite element analysis benefits to replicate the true characteristics of a structure. However, it also creates a dilemma in choosing the appropriate constitutive model to assess a realistic and safe response of the structure. Only one solution strategy was adopted in this thesis, but the results obtained from using different solution strategies in different studies were used to estimate the modelling uncertainty. The difference in result of the analyses indicate the variability that arise in using NLFEA. It infers the need to validate and regulate a uniform and stable solution strategy for using NLFEA in assessing the capacity of a structure.

The modelling uncertainty estimated in this thesis was $\theta_m = 1.21$ and $V_{\theta} = 6.88\%$. The coefficient of variation suggested relatively stable results obtained from using different solution strategies in finite element analysis. However, high value of mean modelling uncertainty reveals the lack of reproducing the realistic response of the structure. Therefore, it indicates the need of a well validated method and model to be used in non-linear finite element analysis. However, the study in this thesis was only based on a single solution strategy and analysis of a single model. The adopted solutions strategy could be used for other wall specimens and then measure the modelling uncertainty parameters by comparing with corresponding experimental results. Another way to assess the modelling uncertainty is to modify the solution strategy based on the limitations experienced in this thesis and thus calculate uncertainty parameters by comparing experimental result with results obtained from different modifications of solution strategy.

Major failure characteristics for the wall specimen SW 21 were closely replicated by using finite element analysis. However, the ultimate load capacity assessed for wall specimen was found to be conservative with respect to the experimental results. The report indicated the important effect of volumetric expansion and triaxial stress state in attributing high resistance of the wall specimen. Since this particular characteristics was not obtained in finite element analysis, the use of two dimension plane stress element is assumed to be one of the limiting factor in obtaining realistic results.

Inclusion of different safety formats recommended in Dutch guidelines in NLFEA of the structure yielded higher design load capacity than the analytical load capacity. The study of effect of varying modelling uncertainty parameters in design load capacity by new safety format indicated the importance of including these parameters in the assessment. The design load estimated by using ECOV and Schlune et.al safety format were relatively higher than the one estimated by other safety formats. Same reliability index ($\beta = 3.8$) and sensitivity factor ($\alpha_R = 0.8$) were used in ECOV and Schlune et al. safety assessment. However, since the modelling uncertainty obtained from analysis was not directly included in ECOV, comparable difference was noted between the design load estimated by these two methods. *fib* allows to use higher value of modelling uncertainty safety factor (γ_{Rd}) for poorly validated model. However for analysis giving conservative results, using higher values of γ_{Rd} would result in more conservative estimation thus making the use of provision less advantageous.

Performing reliability analyses with combination of FORM and RSM (response surface method), it was found that the design load estimated by Schlune et al. method achieved the reliability index of 3.44. Similarly, the reliability level obtained while using design load equal to one-third of the experimental capacity was approximately 4.75. The design load estimated by ECOV was only 1.2 kN higher than the two third of experimental capacity. This concludes that all safety assessment method recommended in the guidelines [2] have reached the intended safety level. However, it is important to note that the safety assessments used in this

thesis are based on reliability level, $\beta = 3.8$. But, the reliability analysis resulted in reliability level, $\beta = 4.75$ while using the design load approximately equal to ECOV estimation which is the highest among the methods recommended in [2]. This indicates that the recommended safety assessment methods provide more conservative results. However, these finding regarding usual safety assessment methods and Schlune et al. recommendation was based on limited studies and analyses. Thus, this finding should instead be regarded as an indication of need to study and analyze the methods adopted in more detail.

During reliability analysis study, the choice of f' seem to be very important in obtaining an accurate response surface from results of finite element analyses. Depending on choice of f and material parameters values, there might be change in failure mode which affects the reliability analysis calculation. Adopting the values of $f = \pm 1$ during response surface method led to results that was difficult to interpret and describe.

All the results presented in this thesis are based on similar failure mode. Thus, it would be necessary to study the combination of different failure modes and its effect in safety level assessment. The redistribution of forces and interaction between components would then be more realistic and would probably lead to values that can aid more in making a well validated conclusion about the safety assessments methods used in NLFEA.

BIBLIOGRAPHY

- [1] Lefas and N. N. Ioannis D and Kotsovos, Michael D and Ambraseys, *Behavior of reinforced concrete structural walls: strength, deformation characteristics, and failure mechanism,* Structural Journal **87**, 23 (1990).
- [2] M. A. Hendriks, A. de Boer, and Belletti Beatrice, *Guidelines for Nonlinear Finite Element Analysis of Concrete Structures*, Rijkswaterstaat Centre for Infrastructures, Report RTD:1016-1:2017.
- [3] M. Engen, M. A. Hendriks, J. Köhler, J. A. Øverli, and E. Åldstedt, *A quantification of the modelling uncertainty of non-linear finite element analyses of large concrete structures*, Structural Safety 64, 1 (2017).
- [4] H. Schlune, M. Plos, and K. Gylltoft, *Safety formats for non-linear analysis of concrete structures*, Magazine of Concrete Research **64**, 563 (2012).
- [5] I. Nilsen-Nygaard, Structural Safety Assessment of Reinforced Concrete Structures with Nonlinear Finite Element Analyses and the Significance of the Modelling Uncertainty, Master's thesis, Norwegian University of Science and Technology (2015).
- [6] S. Jonkman, R. Steenbergen, O. Morales-Nápoles, A. Vrouwenvelder, and J. Vrijling, *Probabilistic design: Risk and reliability analysis in civil engineering*, Lecture Notes CIE4130. Delft University of Technology (2015).
- [7] H. Eklund, A. Skorve, and A. Strand, *Reliability assessments of concrete structures using non-linear finite element analyses*, Master's thesis, Norwegian)University)of)Science)and)Technology (2017).
- [8] C. G. Bucher and U. Bourgund, *A fast and efficient response surface approach for structural reliability problems*, Structural safety **7**, 57 (1990).
- [9] M. Engen, M. A. N. Hendriks, J. A. Øverli, and E. Åldstedt, *Solution strategy for non-linear finite element analyses of large reinforced concrete structures*, Structural Concrete 16, 389 (2015).
- [10] F. J. Vecchio, *Finite element modeling of concrete expansion and confinement*, Journal of Structural Engineering 118, 2390 (1992).
- [11] M. Engen, Aspects of design of reinforced concrete structures using non-linear finite element analyses: Solution strategy, modelling uncertainty and material uncertainty, Phd thesis, Norwegian University of Science and Technology (2017).
- [12] Code Model special activity group, *fib Model Code for Concrete Structures 2010*, September (2013) pp. 1–402.
- [13] L. Kadlec and V. Cervenka, Uncertainty of numerical models for punching resistance of RC slabs, in Concrete - Innovation and Design, fib Symposium (Copenhagen, 2015) pp. 1–13.
- [14] P. Evangeliou, *Probabilistic Nonlinear Finite Element Analysis of Reinforced Concrete Beams without Shear Reinforcement*, Master's thesis, Delft University of Technology (2016).
- [15] W. Zhao and Z. Qiu, *An efficient response surface method and its application to structural reliability and reliability-based optimization*, Finite Elements in Analysis and Design **67**, 34 (2013).

A

PRELIMINARY ANALYSIS

In order to make a choice about the finite element size and confirm the boundary and loading conditions, a preliminary analysis was performed in DIANA 10.1. Only the wall structure without upper and lower beams were chosen for the analysis. The wall was modelled as a non-reinforced concrete wall with linear elastic isotropic properties. The wall was clamped at the base by restraining motion in both horizontal and vertical direction. It is subjected to a concentrated load of 127 kN at the top as shown in FigureA.1. The linear material properties along with the cross-section properties used in the model is presented in Table A.1. The wall was modelled using eight noded quadratic elements. The aim of the analysis is to observe the horizontal deflection of the beam and compare it with analytical results. A linear analysis was performed.



Figure A.1: Simplified model of the wall specimen SW 21 used in preliminary analysis

Cross-section and Material Properties				
Width, b (mm)	650			
Height, h (mm)	1300			
Thickness, t (mm)	65			
Material Class	Concrete and Masonry			
Material Model	Linear Elastic Isotropic			
Young's modulus, E (MPa)	36000			
Shear modulus, G (MPa)	$\frac{E}{2(1+\gamma)}$			
Poisson's ratio	0.15			

Table A.1: Input in FEA

Table A.2: Summary of preliminary analysis

Element Size, mm x mm	Horizontal Deflection, δ_{FEA} (mm)	Error = $\frac{\delta - \delta_{FEA}}{\delta} 100\%$
65 x 65	1.9834	2.58
130 x 130	1.9876	2.38
325 x 325	1.9825	2.63

In order to compare and verify the results of finite element analysis, analytical solution was dervied as well. The analytical results for the maximum deflection is found by deriving an expression based on virtual work theory for beams. This theory is based on principle of conservation of energy. Contributions of both shear and bending on the deformation is included in this result. The expression used for deriving the formula of deflection is:

$$\delta = \int_0^L \frac{mM}{EI} dx + \int_0^L \frac{\nu V}{GA_s} dx \tag{A.1}$$

where δ is the deflection to be calculated, M and V are moment and shear force due to external load (here F = 127 kN) at point where deflection is to be determined, and m and v are moment and shear force due to virtual work at the point where deflection is to be measured. Similarly, E, I, G and A_s are modulus of elasticity, moment of inertia of cross section, shear modulus and cross section area effective in shear respectively. The shear area is calculated as $A_s = \frac{1}{1.2}bh$. The analytical calculation yielded deflection, $\delta = 2.036mm$.

The results of Linear finite element analyses using 2D regular plane stress element of different size are presented in Table A.2. Three different size were adopted during the analysis: 65 mm, 130 mm and 325 mm.

It can be observed that all three sizes provided reasonably close estimation to the analytical calculation. Adopting the size of 130 mm would be reasonable owing to computation efforts and requirement of Dutch guidelines. For the adopted solution strategy, the guidelines recommend to have maximum element edge length approximately half of the equivalent length (h_{eq}). Where,

$$h_{eq} < \frac{EG_F}{f_t^2} \tag{A.2}$$



Figure A.2: Contour plot of vertical Stress, $S_{\gamma\gamma}$ (N/mm²) for (a) element size 325 mm (b) element size 130 mm

For the mean material parameters adopted in 5.1, the element edge length should then be less than 247 mm. Thus, 130 mm element size seem to be more appropriate. Also it can be observed in Figure A.2 that for 130 mm element size the stress distribution is smoother as compared to the element size 325 mm. The element size of 65 mm is not chosen mainly because of the computation efforts. The computation memory increases approximately by 4 times on reducing the size of element to half.

B

PYTHON SCRIPT

Attached is the script that was used to run automated analysis in DIANA during Response Surface Method:

```
for i in range(1,10):
2
3
           #Defining the central values of concrete compressive strength
4
           cm = 27.61
5
           ym = 446.21
6
7
           #Opening the file to modify it and save it as a new file
8
           openProject( ".....( Project Directory) ......" )
9
10
           #Saving the file as a new one
11
           saveProjectAs( ".....(Project Directory)...../ "+str(i)+".dpf")
12
13
           #Changing the parameters
14
15
           #For analysis named 2 and 3 the concrete values are central values
16
           if i == 1 or i == 2 or i == 3:
17
                    c = cm
18
                                 # c is the new compressive strength
                                                                                  # ft is the
                    ft = (0.3*pow((c-8),(2/3)))
19
                        mean tensile strength
                    gft = 0.073 * pow(c, 0.18)
                                                                                  # gft is the
20
                          tensile fracture energy
                                                                                           #
                    gfc = 250*gft
21
                         gfc is the compressive fracture energy
                    E = 0.85 * 22000 * pow((c/10), 0.3)
                                                                         # E is the reduced
22
                        modulus of elasticity
23
                    setParameter ( "MATERIAL" , "CONCRETE" , "COMPRS/COMSTR" , c )
24
                    setParameter ( "MATERIAL", "CONCRETE", "TENSIL/TENSTR", ft )
25
                    setParameter ( "MATERIAL", "CONCRETE", "TENSIL/GF1", gft )
26
                    setParameter ( "MATERIAL", "CONCRETE", "COMPRS/GC", gfc )
setParameter ( "MATERIAL", "CONCRETE", "LINEAR/ELASTI/YOUNG", E )
27
28
                    setParameter ( "MATERIAL", "CONLIN", "LINEAR/ELASTI/YOUNG", E )
29
30
           #For analysis named 4, 5 or 6 the concrete values are with f = 1
31
           if i == 4 or i == 5 or i == 6:
32
```

c = cm + 0.15 * 36.38# c is the new compressive strength ft = (0.3 * pow((c-8), (2/3)))# ft is the mean tensile strength gft = 0.073 * pow(c, 0.18)# gft is the tensile fracture energy gfc = 250 * gft# gfc is the compressive fracture energy # E is the reduced E = 0.85 * 22000 * pow((c/10), 0.3)modulus of elasticity setParameter ("MATERIAL", "CONCRETE", "COMPRS/COMSTR", c) setParameter ("MATERIAL", "CONCRETE", "TENSIL/TENSTR", ft) setParameter ("MATERIAL", "CONCRETE", "TENSIL/GF1", gft) setParameter ("MATERIAL", "CONCRETE", "COMPRS/GC", gfc) setParameter ("MATERIAL", "CONCRETE", "LINEAR/ELASTI/YOUNG", E) setParameter ("MATERIAL", "CONLIN", "LINEAR/ELASTI/YOUNG", E) #For analysis named 7, 8 or 9 the concrete values are with f = 2if i == 7 or i == 8 or i == 9: c = cm - 0.15 * 36.38# c is the new compressive strength ft = (0.3*pow((c-8),(2/3)))# ft is the mean tensile strength # gft is the gft = 0.073 * pow(c, 0.18)tensile fracture energy gfc = 250 * gft# gfc is the compressive fracture energy E = 0.85 * 22000 * pow((c/10), 0.3)# E is the reduced modulus of elasticity setParameter ("MATERIAL", "CONCRETE", "COMPRS/COMSIR", c) setParameter ("MATERIAL", "CONCRETE", "TENSIL/TENSTR", ft) setParameter ("MATERIAL", "CONCRETE", "TENSIL/GF1", gft) setParameter ("MATERIAL", "CONCRETE", "COMPRS/GC", gfc) setParameter ("MATERIAL", "CONCRETE", "LINEAR/ELASTI/YOUNG", E) setParameter ("MATERIAL", "CONLIN", "LINEAR/ELASTI/YOUNG", E) #Changing the steel properties #For analysis named 2 or 4 or 7 the yield strength of steel is with f = 1if i == 2 or i == 5 or i == 8: y = ym + 0.04 * 480# y is the new yield strength vpm = 4082 * 0.05 + v# calculated based on h(har) = 4082setParameter ("MATERIAL", "RE8H", "PLASTI/HARDI2/KAPSIG", [0, y, 0.05, ypm]) #For analysis named 3 or 4 or 7 the yield strength of steel is with f = 2if i == 3 or i == 6 or i == 9: y = ym - 0.04 * 480# y is the new yield strength ypm = 4082 * 0.05 + y# calculated based on h(har) = 4082setParameter("MATERIAL", "RE8H", "PLASTI/HARDI2/KAPSIG", [0, y,

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54

55 56

61 62

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```
0.05, ypm ] )
#For analysis named 2 or 4 or 7 the yield strength of steel is the mean
   value
if i == 1 or i == 4 or i == 7:
        y = ym
                                                                         # y
           is the new yield strength
       ypm = 4082 * 0.05 + y
                                                                 # calculated
            based on h(har) = 4082
        setParameter ( "MATERIAL", "RE8H", "PLASTI/HARDI2/KAPSIG", [ 0, y,
            0.05, ypm ] )
#Renaming the analysis to the name of the file
renameAnalysis( "Nonlinear", str(i))
saveProject()
#Running the analysis
runSolver( str(i) )
#Saving the result in vcf file
analysis = str(i)
                            # i is my analysis name
Output = "Output"
                            # My output is saved as Output in Structural
   Nonlinear Analysis
                      # a new counter j because I have 46 load steps...
for j in range(0,46):
   started with zero because of the way its stored in Python
        # results process
                  = resultCases ( analysis , Output )
        rCase
        rCase
                  = rCase[j]
                                                                 # load steps
        rLabel
                  = 'Reaction Forces'
                            # Looking at reaction force
        rComp
                  = 'FBX'
                                            # Looking at FBX component
        rNodes
                  = [226]
                                            # this is the node id in which we
             are looking for reaction
                  = [ analysis, Output, rCase, rLabel, rComp ] # this is
        rTab
            the "resultTable" please see DIANA Manual for the syntax
        FBXtable = resultData( rTab, rNodes )
                    # stores the value of FBX for node number rNodes
        outputDir = " ...... (Project Directory) ......"
        name_csv = "Result_" + str(i)
        with open( outputDir + name_csv +'.csv', 'a' ) as file:
                file.writelines(str(FBXtable)+ '\n')
```

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87 88 89

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109 110

111

112 113 114

115#Saving the project116saveProject ()

C

MATLAB SCRIPT

The following matlab script was written and used to obtain the response surface and then perform FORM in conjunction with DIANA results. Direct linking of DIANA results and the script was not done for post analysis check and observe the result of NLFEA before using for response surface calculation.

```
1
   clc
2
3
   syms fc fy Uc Uy;
4
  %for finding coefficients of the expressions
5
  % c is the mean concrete compressive strength value
6
   c = 21.53;
7
  %y is the mean yield strength value of vertical reinforcement
8
  y = 421.61;
9
  % different points used are based on f = 1, 2
10
   c1 = c + 0.15 * 36.38;
11
   c2 = c + 2*0.15*36.38;
12
  y1 = y + 0.04 * 480;
13
  y_2 = y + 2 * 0.04 * 480;
14
  %nine different combinations/points/experiments/results are used
15
  A = [1 c y c^2 y^2 c^*y; 1 c y 1 c^2 y^1 c^2 c^*y]; 1 c y^2 c^2 y^2 c^*y^2; 1 c y c^2 y^2
16
       cl*y; 1 cl y1 cl^2 y1^2 cl*y1; 1 cl y2 cl^2 y2^2 cl*y2; 1 c2 y c2^2 y^2 c2*y; 1
       c2 y1 c2^2 y1^2 c2*y1; 1 c2 y2 c2^2 y2^2 c2*y2];
  %g is obtained from NLFEA result - the STM capacity
17
  g = [246.9926368]
18
   2398.662432
19
   5049.591832
20
   3364.376826
21
   6154.814438
22
   8480.023746
23
   6156.457321
24
   9103.600696
25
   11611.18818];
26
  % b is the coefficient array
27
  b = inv(A'*A)*A'*g;
28
29
  %For FORM ANALYSIS
30
  %g1 is LSF
31
  g1 = (b(1) + b(2)*fc + b(3)*fy + b(4)*fc^2 + b(5)*fy^2 + b(6)*fc*fy);
32
33
  %s1 is the Response Surface in Real Space co ordinate system
34
```

```
s1 = @(fc1, fy1) (b(1) + b(2)*fc1 + b(3)*fy1 + b(4)*fc1^2 + b(5)*fy1^2 + b(6)*fc1*fy1
35
                       );
36
         %Value of normal parameters of concrete compressive strength
 37
         mc = \log(36.38^2/((0.15*36.38)^2+36.38^2)^{0.5});
38
          sdc = (log(0.15^{2}+1))^{0.5};
39
         %value of normal parameters of yield strength of vertical reinforcment
 40
         my = \log (480^{2}/((0.04*480)^{2}+480^{2})^{0.5});
41
          sdy = (log(0.04^{2}+1))^{0.5};
 42
 43
         %Ucy is the LSF transformed to standard normal varaiables Uc and Uy
 44
         Ucy = expand((b(1) + b(2)) exp(Uc*sdc + mc) + b(3)) exp(Uv*sdv + mv) + b(4) exp(Uc*sdv + mc) + b(4) 
 45
                       sdc + mc) ^{2} + b(5) * (exp(Uy*sdy + my))^{2} + b(6) * exp(Uc*sdc + mc) * exp(Uy*sdy + my)
                       ));
          Ucy = simplify(Ucy);
 46
 47
         %s3 is the Response Surface in Standard Normal Space
 48
          s_3 = @(Uc1, Uy1) ((b(1) + b(2) * exp(Uc1 * sdc + mc) + b(3) * exp(Uy1 * sdy + my) + b(4) * (exp(Uy1 * sdy + my) * (exp(Uy1 * sdy + my) + b(4) * (exp(Uy1 * sdy + my) * (exp(
 49
                       (Uc1*sdc + mc))^2 + b(5)*(exp(Uy1*sdy + my))^2 + b(6)*exp(Uc1*sdc + mc)*exp(Uy1*sdc + mc))^2 + b(6)*exp(Uy1*sdc + mc)*exp(Uy1*sdc + mc))^2 + b(6)*exp(Uy1*sdc + mc))^2 +
                       sdy + my)));
 50
         % is for error, co is for design point of c and yo is for design point of
51
         %y. here co and yo are 0 because they have been transformed to standard
 52
         %normal already..... required is the number of iterations required
 53
          e = 1; co = 0; yo = 0; fcd = c; fyd = y; required = 0;
 54
 55
         %checking if the error is less than 0.0001
 56
 57
          while e > 0.00001
 58
                       %lin is the linearization of LSF at design point (co, yo)
 59
                        lin = (subs(Ucy, [Uc, Uy], [co, yo]) + (Uc - co)*subs(diff(Ucy, Uc, 1), [Uc, Uy])
 60
                                      ], [co, yo]) + (Uy - yo)*subs(diff(Ucy, Uy, 1), [Uc, Uy], [co, yo]));
                       %meanZ is mean of Linearized LSF and sdz is the sd of lineariszed LSF
 61
                       meanZ = (subs(Ucy, [Uc, Uy], [co, yo]) + (-co)*subs(diff(Ucy, Uc, 1), [Uc, Uy],
 62
                                       [co, yo]) + (-yo) * subs(diff(Ucy, Uy, 1), [Uc, Uy], [co, yo]));
                        sdZ = sqrt((subs(diff(Ucy, Uc, 1), [Uc, Uy], [co, yo]))^2 + (subs(diff(Ucy, Uy, Uy, Uy, Uz, Uy))^2)^2
 63
                                      1), [Uc, Uy], [co, yo]))^2);
 64
                       %beta is reliability index and a1, a2 are sensitivity factors
 65
                        beta = meanZ/sdZ;
 66
                        a1 = (subs(diff(Ucy, Uc, 1), [Uc, Uy], [co, yo]))/sdZ;
 67
                        a2 = (subs(diff(Ucv, Uv, 1), [Uc, Uv], [co, vo]))/sdZ;
 68
                       %now new design point in standard normal space is evaluated
 69
                        co = -a1 * beta;
 70
                       yo = -a2*beta;
 71
                       %saving old design point in real space to compare with new one to find
 72
                       %an error
 73
                         fco = fcd; fyo = fyd;
 74
                       %new desing point in real space is evaluated
 75
                         fcd = exp(co*sdc + mc);
 76
                         fyd = exp(yo*sdy + my);
 77
                        e = sqrt((fcd - fco)^{2} + (fyd - fyo)^{2});
 78
                         required = (required + 1);
 79
          end
 80
 81
        %Extracting/Displaying results of the FORM Analysis
```

```
g1;
                                        %The LSF in Real Space
83
   Ucy;
                                        %Transformed LSF in Standard Normal Space
84
                                        %The linear LSF in Standard Normal Space
   lin;
85
                                        %The value of Reliability Index
   beta
86
                                        %Number of iterations required to get stable design
   required;
87
         point
                                        %Tolerance level of successive design points
   e:
88
                                        %Substituting the value of design point to get the
   subs(g1, [fc, fy], [fcd, fyd]);
89
        response
                                        %Concrete compressive strength in new design point
   fcd
90
                                        %Steel yield strength in new design point
   fvd
91
                                        %Concrete compressive strength in Standard Normal
   co:
92
        Space
                                        %Steel yield strength in Standard Normal Space
   yo;
93
                                        %sensitivity factor for concrete, defined earlier
   a1
94
   a2
                                        %sensitivity factor for steel, defined earlier
95
96
   %PLOTTING GRAPHS/FIGURES/RESPONSE SURFACES
97
98
   %The first one is the Quadratic LSF in Real Space Co ordinate System
99
   figure
100
   plot1 = ezplot(g1, [20 50 300 600]);
101
   set(plot1, 'linewidth',2)
102
   ax = gca;
103
   ax.YAxisLocation = 'origin';
104
   ax.XAxisLocation = 'origin';
105
   set(gca, 'box', 'off', 'Fontsize', 24)
106
   xlabel('f_c (N/mm^2)', 'Fontsize', 30);
ylabel('f_y (N/mm^2)', 'Fontsize', 30);
107
108
   zlabel('Response, Z (N)');
109
110
   %The second one is the Quadratic LSF and Linearised LSF in Standard Normal Space Co
111
        ordinate System
   figure
112
                                                %This sketches the linearised LSF
   ez1 = ezplot(lin, [-5, 5, -5, 5]);
113
   hold on
114
                                                %This sketches the Quadratic LSF
   ez2 = ezplot(Ucy, [-5, 5, -5, 5]);
115
   legend('Linearized LSF', 'Nonlinear LSF')
116
   ax = gca;
117
   ax.YAxisLocation = 'origin';
118
   ax.XAxisLocation = 'origin';
119
   set(gca, 'box', 'off', 'Fontsize', 24)
120
   set(ez1, 'linewidth',2, 'color', 'r')
121
   set(ez2, 'linewidth',2, 'color', 'b')
122
   xlabel('U_c (N/mm^2)', 'Fontsize', 30);
ylabel('U_y (N/mm^2)', 'Fontsize', 30);
123
124
   zlabel('Response, G (N)');
125
126
   %The third one is the Response Surface in Real Space
127
   figure
128
                                       %This is the Limit State Surface i.e. Z = 0
   s2 = @(fc1, fy1) 0;
129
   fsurf(s1, [20 50 300 600])
130
   hold on
131
   fsurf(s2,[20 50 300 600],'r')
132
   %box on
133
   legend('Response Surface', 'Limit State i.e. Z = 0')
134
```

```
%title('Limit State Surface in Real Coordinate Space')
135
   set(gca, 'box', 'off', 'Fontsize', 24)
136
   xlabel('fc (N/mm^2)', 'Fontsize', 30);
137
   vlabel('fv (N/mm^2)', 'Fontsize', 30);
138
   zlabel('Response, Z (N)');
139
140
   %The fourth one is the Response Surface in Standard Normal Space
141
   figure
142
   s4 = @(Uc1, Uy1) 0;
                                      %This is the Limit State Surface i.e. G = 0
143
   fsurf(s3, [-5 \ 0 \ -5 \ 0])
144
   hold on
145
   fsurf(s4, [-5 \ 0 \ -5 \ 0], 'g')
146
   legend('Response Surface','Limit State i.e. G = 0')
147
   %title('Limit State Surface in Standard Normal Space')
148
   set(gca, 'box', 'off', 'Fontsize', 24)
149
   xlabel('Uc (N/mm^2)', 'Fontsize', 30);
150
   ylabel('Uy (N/mm^2)', 'Fontsize', 30);
151
   zlabel('Response, G (N)', 'Fontsize', 30);
152
153
   % %The fifth one is the joint pdf of Uc and Uy. As both follow Standard
154
   % %Normal distribution, the pdf is the product of two standard normal pdf
155
   % figure
156
   \% s5 = @(Uc2, Uv2) 1/(2) * exp(-(Uc2^2 + Uv2^2)/2);
157
   % fsurf(s5,[-3 3 -3 3], 'ShowContours', 'on')
158
   % title ('Joint Pdf of G')
159
   \% xlabel('Uc [N/mm^2]');
160
   % ylabel('Uy [N/mm^2]');
161
   % zlabel('g(Uc, Uy)');
162
   % box on
163
```

After obtaining the response surface, following script was used to perform Importance Sampling. The part of response surface calculation in not included. And the similar symbols used in previous script is used here too.

```
%Value of normal parameters of concrete compressive strength based on new
2
  %design point
3
  mcs = \log(c^2/((0.15*c)^2+c^2)^{0.5});
  %value of normal parameters of yield strength of vertical reinforcment
5
  %based on new design point
  mys = \log(\frac{v^2}{(0.04 * v)^2 + v^2})^{0.5};
8
9
                                            %This is the variable to count number of
  n = 0;
10
      failures
  i = 0;
11
12
  for j = 1:(100000)
13
                                             %A random number is generated
      p1 = rand;
14
                                             %Another random number is generated as conc
      p2 = rand;
15
           comp strength and yield strength are independent parameteres
                                             %Finding a value of inverse cdf of
      fcm = logninv(p1, mcs, sdc);
16
          lognormal distribution with probability = p1 for conc comp strength
      fym = logninv(p2, mys, sdy);
                                             %Finding a value of inverse cdf of
17
          lognormal distribution with probability = p2 for yield strength
       e = subs(g1, [fc, fy], [fcm, fym]); %Substituting the value of design point to
18
           get the response
```

```
19
       if e < 0
                                            %Counting the failure if response is
20
           negative
           n = (n + 1);
21
           i = (i + (lognpdf(fcm, mc, sdc)*lognpdf(fym, my, sdy))/(lognpdf(fcm, mcs,
22
               sdc)*lognpdf(fym, mys, sdy)));
       end
23
   end
24
25
  %Extracting/Displaying results of the FORM Analysis
26
                                      %The LSF in Real Space
   g1;
27
   pf = i/j
                                      %Probability of Failure
28
                                      %Number of failure responses
  n
29
   beta = -norminv(pf, 0, 1)
30
```

D

Response Surface Method Results

Five iterations were required to satisfy the convergence criterion in Response Surface Method for $R_d = \frac{2R_{exp}}{3}$. Similarly, two iterations were required for $R_d = R_{Schluneet.al}$. Nine NLFEA results were used to obtain the response surface in each iteration. The values of responses to be used in curve fitting is obtained by finding the difference between results of NLFEA and the design load adopted. Following tables represent the values of basic variables used in NLFEA, response used in RSM and the residues for the obtained response surface. The table also lists the residual sum of squares, number of FORM iterations required to reach convergence, design points, sensitivity factors and reliability index obtained after the final FORM iteration.

Table D.1: Results of RSM Iterations for $R_d = \frac{2R_{exp}}{3}$

	1	teration 1	
f _c (MPa)	f_y (MPa)	Response (N)	Residue (N)
36.38	480.00	16202.98	-92.58
36.38	537.60	23899.13	282.14
36.38	422.40	8047.86	-189.57
52.75	480.00	24280.98	-5.33
52.75	537.60	31983.48	-115.26
52.75	422.40	15857.79	120.59
20.01	480.00	6569.10	97.91
20.01	537.60	13134.76	-166.88
20.01	422.40	-1026.97	68.97
Residue Sum of Squares (RSS) $\sqrt{\Sigma Residue^2}$			440.63 N
Number of FORM Iterations			9
Design value, f _{cd}			22.86 MPa
Design value, f_{yd}			417.67 MPa
Reliability Index, β			4.605
Sensitivity factor α_c			0.660
Sensitivity factor α_{γ}			0.751

	,		
	1	teration 1	
fc (MPa)	f_y (MPa)	Response (N)	Residue (N)
21.89	418.72	-109.98	-75.52
21.89	437.92	2231.36	83.37
21.89	457.12	4939.48	-8.15
27.35	418.72	3207.41	166.54
27.35	437.92	5217.15	-198.94
27.35	457.12	8440.89	32.41
32.81	418.72	5725.36	-91.32
32.81	437.92	8500.24	115.58
32.81	457.12	11545.86	-23.95
Residue	Sum of Squ	ares (RSS)	
$\sqrt{\Sigma Resi}$	due ²		321.49 N
V Z_{i} V Z_{i} V Z_{i} V Z_{i} Z_{i			20
Design value. f			21.32 MPa
Design value, f_{yd}			422.46 MPa
Reliability Index. β			4.730
Sensitivity factor α_c			0.742
Sensitivity factor α_{y}			0.671

	It	eration 2	
f_c (MPa)	f_y (MPa)	Response (N)	Residue (N
22.48	415.92	-129.33	5.24
22.48	435.12	2396.69	23.40
22.48	454.32	5037.17	-24.47
27.93	415.92	3016.70	7.61
27.93	435.12	5546.03	-71.33
27.93	454.32	8473.39	67.28
33.39	415.92	5624.60	-9.42
33.39	435.12	8394.19	51.50
33.39	454.32	11192.71	-39.13
Residue S	um of Squa	res (RSS)	
$\sqrt{\Sigma Residue_i^2}$			122.96 N
Number of FORM Iterations			10
Design value, f _{cd}			22.02 MPa
Design value, f_{yd}			419.26 MPa
Reliability Index, β			4.706
Sensitivity factor α_c			0.699
Sensitivity factor α_{y}			0.715

	Ite	eration 2	
f_c (MPa)	f_y (MPa)	Response (N)	Residue (N)
20.92	420.92	-447.70	-68.47
20.92	440.12	1925.31	26.54
20.92	459.32	4498.44	45.01
26.37	420.92	3053.55	162.59
26.37	440.12	5304.16	-96.10
26.37	459.32	8122.09	-64.13
31.83	420.92	5589.04	-91.88
31.83	440.12	8493.43	71.92
31.83	459.32	11460.37	21.60
Residue S $\sqrt{\Sigma Resid}$	247.55 N		
Number of FORM Iterations			10
Design value, fcd			21.45 MPa
Design value, f_{yd}			421.29 MPa
Reliability Index, β			4.748
Sensitivity factor α_c			0.730
Sensitivity factor α_y			0.683

Iteration 1			
f_c (MPa)	f_y (MPa)	Response (N)	Residue (N)
36.38	480.00	12401.24	92.57
36.38	537.60	20097.39	-282.14
36.38	422.40	4246.13	189.57
52.75	480.00	20479.24	5.33
52.75	537.60	28181.74	115.26
52.75	422.40	12056.06	-120.59
20.01	480.00	2767.37	-97.91
20.01	537.60	9333.02	166.88
20.01	422.40	-4828.71	-68.97
Residue Sum of Squares (RSS) $\sqrt{\Sigma Residue_i^2}$			440.63 N
V Number of FORM Iterations			7
Design value, f_{cd}			25.37 MPa
Design value, f_{yd}			434.18 MPa
Reliability Index, β			3.418
Sensitivity factor α_c			0.685
Sensitivity factor α_v			0.728

Table D.2: Results of RSM Iterations for $R_d = R_{Schluneet.al}$	

	It	eration 2	
f_c (MPa)	f_y (MPa)	Response (N)	Residue (N)
25.35	434.11	-1.81	67.66
25.35	453.31	2448.03	178.72
25.35	414.91	-2159.16	-245.24
30.81	434.11	3431.54	-111.28
30.81	453.31	6136.40	-49.74
30.81	414.91	482.34	162.18
19.89	434.11	-3207.86	44.75
19.89	453.31	-679.92	-127.78
19.89	414.91	-5512.01	84.13
Residue Sum of Squares (RSS) $\sqrt{\Sigma Residue_i^2}$			404.02 N
Number of FORM Iterations			8
Design value, f_{cd}			25.11 MPa
Design value, f_{yd}			434.71 MPa
Reliability Index, β			3.444
Sensitivity factor α_c			0.700
Sensitivity factor α_{y}			0.714