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DOI
10.1080/15472450.2024.2408024

Publication date
2024

Document Version
Final published version

Published in
Journal of Intelligent Transportation Systems: technology, planning, and operations

JOURNAL OF INTELLIGENT TRANSPORTATION SYSTEMS
https://doi.org/10.1080/15472450.2024.2408024

Optimizing dedicated lanes and tolling schemes for connected and autonomous vehicles to address bottleneck congestion considering morning commuter departure choices

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ABSTRACT
The introduction of connected and autonomous vehicles (CAVs) provides a significant opportunity to address the persistently increasing problem of urban traffic congestion. By virtue of their connectivity and automation features, CAVs can reduce vehicle headways, thereby increasing road capacity and enhancing throughput. It has been hypothesized that CAV-infrastructure design policies can influence traveler behavior in ways that could reduce congestion. This research focuses on the potential of using CAV-dedicated lanes (CAVL) to alleviate traffic congestion in a bottleneck corridor that serves both human-driven vehicles (HDVs) and CAVs. We delve into investigating the impacts of CAVLs on the departure time and lane choices of morning commuters. The study first expresses traffic equilibrium conditions as a linear program with complementarity constraints. Then, a system-optimal commute congestion management design is formulated to minimize the overall system cost, which consists of queuing delays and early and late arrival costs. The results of the computational experiments suggest that: (i) the CAV technological advancements can significantly reduce traffic congestion under CAVL deployment with an almost similar effect as a tolling policy; and (ii) the lower value of time for CAV commuters leads them to depart closer to their desired arrival time without a tolling policy, which could significantly increase the bottleneck traffic congestion that commuters experience, particularly HDVs.

ARTICLE HISTORY
Received 26 September 2023
Revised 18 September 2024
Accepted 19 September 2024

KEYWORDS
CAV-dedicated lane; commuter departure time; highway bottleneck; system-optimal; tolling

Introduction

Background

Over the past few years, connected and autonomous vehicles (CAVs) have emerged as a promising technology with opportunities for improving the transportation system, particularly in terms of traffic congestion. In the United States, commuters experienced double the hours of delay in 2019 compared to 1982 (Schrank et al., 2021). Vehicle automation and connectivity enable vehicles to travel with reduced headways and thereby significantly increase road capacity (Kummetha et al., 2024; Levin & Boyles, 2015; Milakis et al., 2017; Shladover, 2016) to as much as three times that of human-driven vehicles (Tientrakool et al., 2011). To earn such prospective benefits of automation and connectivity, infrastructure owners and operators (IOOs) need to modify the transportation infrastructure to facilitate the operations of CAVs without jeopardizing the travel time and safety of human-driven vehicles (HDVs). These modifications may include the provision of separate lanes for autonomous vehicles during the transition horizon (defined as the decades during which both CAVs and HDVs will co-exist in a traffic stream). For this reason, the concept of dedicated lanes for CAVs (CAVL) continues to receive growing attention (Chen et al., 2016; Ghiasi et al., 2017; Lu et al., 2019; Ngoduy et al., 2024). To address the traffic congestion at all times and particularly during the morning peak period, IOOs could deploy dedicated CAV lanes at specified urban road links during the CAV transition horizon. The road links of interest in this article are corridors that have a bottleneck effect on traffic from a city's suburbs to the downtown area.

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Literature review

CAVL deployment

The existing literature on the impacts of CAVL on traffic network performance can be classified into two groups: The first group deals with the long-term impacts of CAVLs and investigates network-wide equilibrium states in the road network (Chen et al., 2016; Liu & Song, 2019; Pourgholamali et al., 2023; Seilabi, 2022; Seilabi et al., 2022; Seilabi et al., 2020, 2023a). These studies mainly identify optimal lane deployment strategies in terms of the number of lanes that minimize total travel time over a given analysis period. Chen et al. (2016) developed CAVL strategies using a multi-year framework. Defining the CAV market penetration as the percent share of CAV travel demand across all travelers, the authors use a diffusion model to capture the CAV market penetration by comparing the net benefits of CAVs in terms of travel time savings and safety. Ye and Wang (2018) investigate the synergetic effect of deploying CAVL with a tolling policy to minimize total travel time. Liu and Song (2019) propose a lane management scheme where HDVs pay a toll if they wish to use CAVLs. Madadi et al. (2019) develop a framework to minimize total travel time costs and prepare a CAV-ready road subnetwork through various road work including retrofitting. Subsequently, Madadi et al. (2021) combine the idea of a CAV-ready subnetwork with CAVLs to provide more flexibility for an IOO in accommodating CAVs during the transition horizon. Instead of using a fixed road capacity, Movaghar et al. (2020) capture link capacity as a function of CAV proportion when deploying CAVLs.

The second group of studies addressed the short-term performance impacts of CAVL deployment (Ghiasi et al., 2017, 2020; Ji et al., 2024; Seilabi et al., 2023b). This group considers a single highway corridor over a short evaluation period (a few hours). Ghiasi et al. (2017) develop an analytical formulation using Markov chain modeling to identify the optimal number of CAVLs to maximize traffic throughput under different CAV market penetration and CAV demand levels. Subsequently, Ghiasi et al. (2020) relaxed the assumption of fixed lane width to incorporate the possibility of having narrower lanes in the optimal solution. Table 1 summarizes the literature on CAVL deployment.

Bottleneck models

A highway segment with a localized disruption of vehicular traffic is referred to as a bottleneck. One of the earliest bottleneck models was developed by Vickrey (1969). In the Vickrey model, commuters make individual departure time choices in such a manner that they minimize their travel cost which consists of travel time and schedule delay costs. At equilibrium, commuters cannot further minimize their travel costs by unilaterally changing their departure times. Arnott et al. (1990) applied the travel demand management strategy, i.e., tolling, to determine the system-optimal departure rates to minimize the total cost (schedule delay and travel time).

Several researchers later expanded the studies by Vickrey (1969) and Arnott et al. (1990) by relaxing assumptions such as the homogeneity of commuters in terms of schedule delay penalties. The schedule delay penalty includes early and late arrival penalties for commuters. These studies can be categorized into two classes. The first class determines the commuters' departure rates under user equilibrium and system-optimal conditions using the continuous-time model. For example, Vickrey (1973) proposed the tolling policy in the context of managing morning commute congestion, where commuter homogeneity is relaxed by assuming the special case of heterogeneity (i.e., fixed ratios of schedule delay penalties to the value of time). van den Berg and Verhoef (2011) derived the impact of tolling on managing morning commute

Table 1. Summary of literature.

Network/corridor level	Study	Objective	Travel decisions	Other congestion management strategy
Network	Chen et al. (2016)	Costs of safety and total travel time	Route/lane choice	None
	Ye and Wang (2018)	Total travel time	Route/lane choice	Lane-specific tolling policy
	Liu and Song (2019)	Total travel time	Route/lane choice	Lane-specific tolling policy
	Madadi et al. (2020)	Costs of network adjustment for CAVs and total travel time	Route/lane choice	None
	Wu et al. (2020)	Total travel time and distance	Route/lane choice	Cordon-based tolling policy
	Movaghar et al. (2020)	Total travel time	Route/lane choice	None
	Madadi et al. (2021)	Costs of network adjustment for CAVs and total travel time	Route/lane choice	None
Corridor	Ghiasi et al. (2017)	Highway throughput	Lane choice	None
	Ye and Yamamoto (2018)	Highway throughput	Lane choice	None
	Ghiasi et al. (2020)	Highway throughput	Lane choice	None
	Our study	Total travel cost (travel time and schedule delay)	Departure time/lane choice	Lane-specific time-varying tolling policy

congestion under the assumption that commuters have a continuous distribution of the value of time and a schedule delay penalty with an identical desired arrival time. Other studies on bottleneck models in a continuous time setting used similar assumptions to examine the impact of tolling policy on the value of time and schedule delay penalties. In the context of CAVs, Liu (2018) explored the equilibrium conditions for departure time and parking location choices of commuters with a fully CAV fleet. After passing a bottleneck, CAV commuters will be dropped off at the workplace, and then CAVs will drive themselves to parking locations. The system-optimal design of the tolling policy and parking fees is determined to minimize the total system cost. Zhang et al. (2022) investigated the impact of a lower value of time for CAV commuters compared to HDV commuters to understand commuters' departure time choices. This is because CAV commuters can spend their in-vehicle time on various activities, such as work or entertainment (Kolarova et al., 2018; Steck et al., 2018).

Another class of studies uses a mathematical program in the context of a discrete time setting to analyze morning commute congestion. These studies divide the morning peak period into several time intervals and determine the departure rates for each time interval. These studies consider commuter heterogeneity in terms of schedule delay penalty, the value of time, and desired arrival time. Ramadurai et al. (2010) formulated the single bottleneck model as a linear complementarity problem (LCP) to determine the departure rates of morning commuters under equilibrium conditions. Doan et al. (2011) extended the LCP to capture the impact of tolling on departure rates during the morning peak period. They proved that, under system-optimal conditions, the travel time of commuters is equal to zero, which implies that the total system cost consists of commuters' schedule delay costs only. Miralinaghi et al. (Miralinaghi, 2018, 2019; Miralinaghi & Peeta, 2016) used a tradable credit scheme concept to manage morning commute congestion by considering the loss aversion of commuters toward purchasing credits. The present article falls into the second class of studies that analyze the morning commute congestion in a highway bottleneck with CAVLs during the transition horizon with a mixed fleet of CAVs and HDVs.

Problem statement

Several studies have investigated the impacts of CAVL on traffic congestion at corridor or network levels. These deal mainly with the lane and route choices of commuters. However, there is also a need to understand the departure time choices of commuters, particularly during the morning peak period. In particular, there is a need to examine morning commutes during the CAV transition horizon where there is a mixed flow of CAVs and HDVs on the same road corridor that has a bottleneck. Commuters traverse the highway bottleneck during the morning peak period. There are two types of commuters: (i) CAV commuters and (ii) HDV commuters. Commuters travel either using CAVs or HDVs. Commuters are identical in terms of schedule delay penalty and desired arrival time. CAV commuters have a lower value of time compared to HDV commuters. Two types of lanes exist in the bottleneck: (i) CAVLs and (ii) GPLs. The capacity of CAVLs is assumed to be higher than that of GPLs. This is due to the Cooperative Adaptive Cruise Control, which increases the CAVL capacity by decreasing driving time headway (Chen et al., 2016; Madadi et al., 2020; Shladover, 2016). For computational simplicity, the capacity of GPLs is assumed to be independent of the proportion of CAVs and HDVs. CAV travelers can choose between CAVLs and GPLs, while HDV travelers are restricted to using only GPLs. Each lane on the highway is treated as a separate bottleneck, and the lane-changing behavior of commuters in the bottleneck is not considered. The IOO implements a lane-specific tolling policy under which commuters are charged a toll based on the lanes they use.

Research contributions

The contributions of this research are threefold. First, the study develops a framework for managing morning commute congestion in a highway bottleneck during the transition horizon with a mixed fleet of CAVs and HDVs, considering the departure time choices of commuters. To the best of our knowledge, this is the first study that analyzes the synergetic impact of CAV lanes and tolling schemes on managing morning commute congestion during the CAV transition horizon. In this context, the study develops a linear complementarity problem to determine commuters' equilibrium departure rates under the CAV-lane and tolling schemes. This helps shed light on the synergetic impact of CAVLs and tolling schemes on commuters' departure rates. Also, the existence of a solution (in terms of departure rates) has been proven. For example, computational experiments show that CAV technological advancement, which could further increase CAVL capacity, can significantly reduce traffic congestion with an almost similar effect as a tolling policy.

Secondly, this study investigates the lane and departure time choices of CAV commuters and their impacts on their travel costs and travel times for CAVLs and GPLs. It is shown that the CAVL queuing delay is less than or equal to that of GPL in any time interval. Further, CAV commuters use GPLs in any time interval only if they use CAVLs in that time interval. This implies that the equilibrium cost of CAV commuters is always less than that of HDV commuters. In practice, this could have social inequity implications because it is expected that CAVs will be affordable to high-income commuters only, particularly during the early part of the transition horizon where CAV volumes are low and scale economies are yet to kick in. Thirdly, the system-optimal design model as a linear problem is developed to determine the optimal tolling policy, which can also be used to identify the optimal number of lanes in terms of minimal overall travel cost during the morning peak period.

The remaining sections of this article are as follows: The next section introduces the preliminary notions. Then, user equilibrium conditions are presented. Next, we investigate the solution's existence and properties under equilibrium conditions. Then, the system-optimal condition using the tolling policy is formulated. Next, computational experiments are conducted. Finally, concluding remarks are provided.

Preliminaries

This section presents the preliminary steps to investigate the morning bottleneck in a discrete time setting. In this context, commuters travel on a highway from their residence to their workplace during the morning peak period, which is divided into Γ time intervals. Let T denote the set of time intervals. The highway bottleneck section has multiple lanes of two types: CAVL and GPL (Figure 1). Let L denote the set of lanes with two subsets of L_{CAVL} and L_{GPL} that denote CAVL and general-purpose lane (GPL), respectively. The numbers of CAVLs and GPLs are equal to $|L_{CAVL}|$ and $|L_{GPL}|$, where $|X|$ denotes the cardinality of set X. Each lane l has a deterministic capacity, denoted by s_l. Upon reaching a bottleneck, commuters are served in a first-in-first-out order, and it is assumed that the number of lane changes are negligible.

Based on their choice of vehicle type (HDV vs. CAV), two groups of commuters are considered, denoted by G: (i) connected and autonomous vehicles

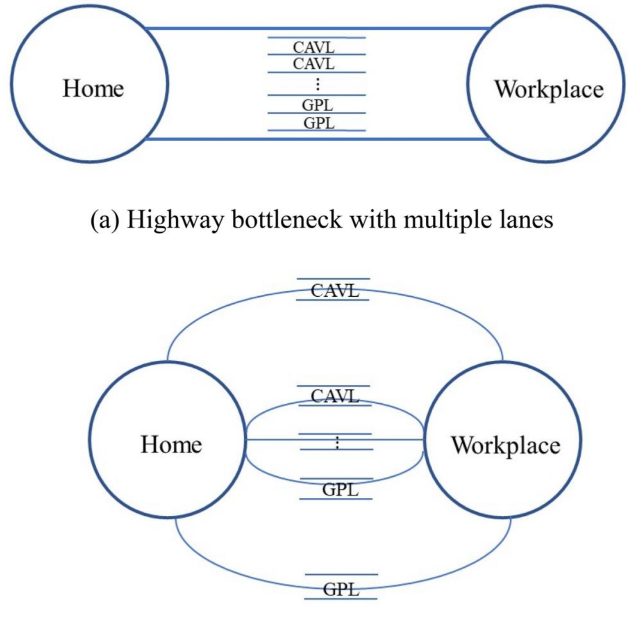


Figure 1. Highway bottleneck with CAVL and transformed network.

($g = 1$) and (ii) human-driven vehicles ($g = 2$). The detailed notation list is provided in Appendix A. Commuters are identical in terms of schedule delay penalty, i.e., early arrival penalty β_g and late arrival penalty γ_g, which are expressed in \$/(time interval). The CAV and HDV commuters have the same desired arrival time (t^*). However, each group of commuters has a different value of time, expressed in \$/(time interval). CAV commuters have a strictly lower value of time compared to HDV commuters (that is, $\alpha_1 < \alpha_2$). Further, based on the empirical studies, the early arrival penalty is assumed to be lower than the value of time for each group (i.e., $\beta_g \leq \alpha_g$) (Doan et al., 2011; Ramadurai et al., 2010; Small, 1982).

Due to the options available to them, CAV commuters make choices of both departure time and lane type (i.e., CAVL vs. GPL); on the other hand, HDV commuters make choices only of their departure time. These decisions are based on the total travel cost (which consists of schedule delay, queuing delay, and time-varying lane-specific toll). Commuters are unable to reduce their travel costs by unilaterally changing their departure times (and in the case of CAV commuters, both lanes and departure times). By modifying the function proposed by Ramadurai et al. (2010), the queuing delay of commuters can be formulated as follows:

$$\tau_{0,l} = \max\left(0, \frac{\sum_g r_{g,0,l} - s_l}{s_l}\right) \quad \forall l \in L \tag{1}$$

$$\tau_{t,l} = \max\left(0, \tau_{t-1,l} + \frac{\sum_g r_{g,t,l} - s_l}{s_l}\right) \quad \forall t > 0, \forall l \in L \tag{2}$$

Queuing delays of commuters using lane l departing at time interval t can be calculated using Eqs. (1) and (2). These equations state that a queue is generated at bottleneck l if bottleneck capacity is less than total departure rates of commuters. The early arrival duration of commuters departing in time interval t using lane l can be derived as follows:

$$e_{t,l} = \max(0, t^* - t - \tau_{t,l}) \quad \forall t \in T, \forall l \in L \tag{3}$$

Constraint (3) states that if commuters using lane l arrive later than the desired arrival time, early arrival duration is equal to zero and they experience late arrival cost. Finally, the travel cost of commuters of group g departing at time t using lane l $(\sigma_{g,t,l})$ can be formulated as follows:

$$\sigma_{g,t,l} = \beta \cdot e_{t,l} + \alpha_g \cdot \tau_{t,l} + \gamma \cdot (e_{t,l} - (t^* - t - \tau_{t,l})) \quad \forall t \in T, \forall l \in L \tag{4}$$

User equilibrium under CAVL and tolling policies

This section presents the user equilibrium conditions for managing morning commute congestion under integrated policies of CAVL and tolling. The travel cost of commuters under the integrated policies can be formulated as follows:

$$\sigma_{g,t,l} = \beta \cdot e_{t,l} + \alpha_g \cdot \tau_{t,l} + \gamma \cdot (e_{t,l} - (t^* - t - \tau_{t,l})) + p_{t,l} \quad \forall t \in T, \forall l \in L \tag{5}$$

where $p_{t,l}$ denotes the tolls charged to commuters departing at time interval t using lane l using lane l. Under equilibrium conditions, (i) CAV commuters cannot reduce their travel costs further by unilaterally changing their departure times and lanes, and (ii) HDV commuters cannot reduce their travel costs further by unilaterally changing their departure times. The equilibrium condition can be formulated as a mixed-linear complementarity problem (MLCP) as follows:

$$0 \le r_{g,t,l} \perp \alpha_g \cdot \tau_{t,l} + \beta \cdot e_{t,l} + \gamma \cdot (e_{t,l} - (t^* - t - \tau_{t,l})) + p_{t,l} - \mu_g \ge 0 \quad \forall t \in T, \forall l \in L, g = 1 \tag{6}$$

$$0 \le r_{g,t,l} \perp \alpha_g \cdot \tau_{t,l} + \beta \cdot e_{t,l} + \gamma \cdot (e_{t,l} - (t^* - t - \tau_{t,l})) + p_{t,l} - \mu_g \ge 0 \quad \forall t \in T, \forall l \in L_{GPL}, g = 2 \tag{7}$$

$$r_{g,t,l} = 0 \quad \forall t \in T, \forall l \in L_{CAVL}, g = 2 \tag{8}$$

$$0 \le \tau_{0,l} \perp \tau_{0,l} - \frac{\sum_g r_{g,t,l} - s_l}{s_l} \ge 0 \quad \forall l \in L \tag{9}$$

$$0 \le \tau_{t,l} \perp \tau_{t,l} - \left(\tau_{t-1,l} + \frac{\sum_g r_{g,t,l} - s_l}{s_l}\right) \ge 0 \quad \forall t \in T \backslash 0, \forall l \in L \tag{10}$$

$$0 \le e_{t,l} \perp e_{t,l} - (t^* - t - \tau_{t,l}) \ge 0 \quad \forall t \in T, \forall l \in L \tag{11}$$

$$\sum_l \sum_t r_{g,t,l} - N_g = 0 \quad \forall g \in G \tag{12}$$

where μ_g is the equilibrium travel cost of commuters of group g. The mathematical operator "$\perp$" means that vectors $z \perp d$ if and only if $z^T d = 0$. Complementarity constraints (6) and (7) are the user equilibrium conditions which state that commuters of group g depart at time interval t using lane l only if their travel costs, including queuing delay, schedule delay, and tolls are equal to the minimum travel cost of that group. Constraints (8) ensure that HDV commuters do not travel on CAV lanes. Complementarity constraints (9) and (10) calculate the queueing delay for lane l at time interval 0 and $t > 0$, respectively. Complementarity constraints (11) determine the early arrival duration for commuters using lane l departing at time interval t. Constraints (12) satisfy the travel demand of commuters of group g.

Linear complementarity problems (LCPs) are a type of mathematical optimization problem with widespread applications in various fields, such as engineering, economics, and game theory. Although several examples of the linear complementarity problem may be traced back to writings as early as 1940, focused research into the LCP began in the mid-1960s. The basic form of LCP involves finding a vector $\boldsymbol{x} \in R^n$ that satisfies the following conditions:

$$Mx + \boldsymbol{q} \ge 0$$

$$\boldsymbol{x} \ge 0$$

$$\boldsymbol{x}^T(Mx + \boldsymbol{q}) = 0$$

here, $\boldsymbol{M}$ is an $n \times n$ matrix ($\boldsymbol{M} \in R^{n \times n}$) and $\boldsymbol{q}$ is an n-dimensional vector ($\boldsymbol{q} \in R^n$). The first condition, $Mx + \boldsymbol{q} \ge 0$ ensures that each component of the vector $Mx + \boldsymbol{q}$ is non-negative. The second condition, $\boldsymbol{x} \ge 0$, ensures that each component of the vector $\boldsymbol{x}$ is also non-negative. The third condition, $\boldsymbol{x}^T(Mx + \boldsymbol{q}) = 0$, is known as the complementarity condition. It ensures that for each i, at least one of $\boldsymbol{x}_i$ or $(Mx + \boldsymbol{q})_i$ must be zero (Cottle et al., 1992).

To apply the existing theorems in the context of linear complementarity problems (LCP) for investigating the solution existence, the MLCP (6)–(12) needs to be reformulated as the equivalent LCP as follows:

$$0 \le r_{g,t,l} \perp \alpha_g \cdot \tau_{t,l} + \beta \cdot e_{t,l} + \gamma \cdot (e_{t,l} - (t^* - t - \tau_{t,l})) + \varphi_{g,t,l} + p_{t,l} - \mu_g \ge 0 \qquad \forall t \in T, \forall l \in L, g \in G \tag{13}$$

$$0 \le \mu_g \perp \textstyle\sum_l \sum_t r_{g,t,l} - N_g \ge 0 \quad \forall g \in G \tag{14}$$

(9)–(11)

Let $\varphi_{g,t,l}$ denote the extra pseudo-cost incurred by the commuters due to HDV travel restrictions on CAVLs. $\varphi_{2,t,CAVL}$ is a sufficiently large positive value to ensure that HDV commuters are not using CAVL. As CAVs are allowed to use CAVL, $\varphi_{1,t,l_{CAV}}$ is zero. As there is no restriction on using GPL for CAV and HDV commuters, $\varphi_{g,t,l}$ is equal to zero for GPLs. The equivalence between MLCP and LCP can be established using the following theorem:

Theorem 1. MLCP is equivalent to LCP. This means that every solution to LCP can solve MLCP and vice versa.

Proof. Appendix B presents the proof for this proposition.

Solution existence and uniqueness

To facilitate proof that a solution exists, the right-hand sides of the complementarity Eqs. (13) and (14) are divided by $(\alpha_g + \gamma)$ and those of Eqs. (9) and (10) are multiplied by S. Then, the model can be described as a general linear complementarity form of:

$$0 \le \boldsymbol{v} \perp A v + \boldsymbol{b} \ge 0$$

in which the $\boldsymbol{v}$ is the variable vector $\boldsymbol{v} \equiv \begin{pmatrix} \boldsymbol{r} \\ \boldsymbol{\tau} \\ \boldsymbol{e} \\ \boldsymbol{\mu} \end{pmatrix}$:

where $\boldsymbol{r} \equiv (r_{g,t,l})_{(g,t,l) \in G \times T \times L}$, $\boldsymbol{\tau} \equiv (\tau_{t,l})_{(t,l) \in T \times L}$, $\boldsymbol{e} \equiv (e_{t,l})_{(t,l) \in T \times L}$, and $\boldsymbol{\mu} \equiv (\mu_g)_{g \in G}$. $\mathbf{b}$ is the constant vector $\boldsymbol{b} \equiv \begin{pmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \\ \mathbf{b}_4 \end{pmatrix}$ and matrix A is as defined as $\boldsymbol{A} \equiv \begin{pmatrix} 0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & -\boldsymbol{A}_3 \\ -\boldsymbol{A}_1^T & \boldsymbol{S} & 0 & 0 \\ 0 & \boldsymbol{A}_4 & \boldsymbol{A}_5 & 0 \\ \boldsymbol{A}_3^T & 0 & 0 & 0 \end{pmatrix}$. The introduced vectors and matrices are shown in more detail in Appendix C.

The solution of $LCP(\boldsymbol{b}, \boldsymbol{A})$, which is vector $\boldsymbol{v}$, is denoted by $SOL(\boldsymbol{b}, \boldsymbol{A})$. Cottle et al. (1992) proved that $SOL(\boldsymbol{b}, \boldsymbol{A}) \ne \emptyset$ if the following conditions hold:

Condition (i): $\boldsymbol{v} = 0$ is the only solution to $LCP(\boldsymbol{b} = 0, \boldsymbol{A})$ (for $\boldsymbol{A} \in \Re^{n \times n}$). In this case, matrix $\boldsymbol{A}$ belongs to a specific class of matrices called $\boldsymbol{R}_0$-matrix.

Condition (ii): $\boldsymbol{A}$ is copositive. In this case, $\boldsymbol{v}^T \boldsymbol{A} \boldsymbol{v} \ge 0$ for every $\boldsymbol{v} \ge 0$.

Therefore, if matrix $\boldsymbol{A}$ of the proposed model satisfies the above conditions, the proposed complementarity model has solutions. In the following, it is proved that matrix $\boldsymbol{A}$ satisfies both mentioned conditions.

Proof. Based on the elements of matrix $\mathbf{A}$, it can be decomposed into a positive semi-definite matrix $\hat{\boldsymbol{A}}$ and a non-negative matrix $\overline{\boldsymbol{A}}$ (i.e., $\boldsymbol{A} = \hat{\boldsymbol{A}} + \overline{\boldsymbol{A}}$)

$$\hat{\boldsymbol{A}} \equiv \begin{pmatrix} 0 & \boldsymbol{A}_1 & 0 & -\boldsymbol{A}_3 \\ -\boldsymbol{A}_1^T & \boldsymbol{S} & 0 & 0 \\ 0 & 0 & \boldsymbol{A}_5 & 0 \\ \boldsymbol{A}_3^T & 0 & 0 & 0 \end{pmatrix}$$

$$\overline{\boldsymbol{A}} \equiv \begin{pmatrix} 0 & 0 & \boldsymbol{A}_2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{A}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Condition (i):

Clearly, $\boldsymbol{v} = 0$ yields in $\boldsymbol{A}\boldsymbol{v} \ge 0, \boldsymbol{v} \ge 0$ and finally $\boldsymbol{v}^T \boldsymbol{A} \boldsymbol{v} = 0$. So, $sol(\boldsymbol{b} = 0, \boldsymbol{A}) = \{\boldsymbol{v} = 0\}$.

Then, we need to show that if there exist a $\boldsymbol{v} \ge 0$ such that $\boldsymbol{v}^T \boldsymbol{A} \boldsymbol{v} = 0$, then $\boldsymbol{v} = 0$.

Based on the decomposed form of $\boldsymbol{A}$, $\boldsymbol{v}^T \boldsymbol{A} \boldsymbol{v} = \boldsymbol{v}^T \hat{\boldsymbol{A}} \boldsymbol{v} + \boldsymbol{v}^T \overline{\boldsymbol{A}} \boldsymbol{v} = 0$. Therefore, $\boldsymbol{\tau} \boldsymbol{S} \boldsymbol{\tau} + \boldsymbol{e} \boldsymbol{A}_4 \boldsymbol{\tau} + \boldsymbol{r} \boldsymbol{A}_2 \boldsymbol{e} + \boldsymbol{e} \boldsymbol{A}_5 \boldsymbol{e} = 0$. As $\boldsymbol{S}, \boldsymbol{A}_2, \boldsymbol{A}_4$, and $\boldsymbol{A}_5$ are positive matrices, therefore $\boldsymbol{r}, \boldsymbol{e}$, and $\boldsymbol{\tau}$ are zero matrices. As $\boldsymbol{r} = 0$, there is no traffic congestion and delay in the network which implies $\mu = 0$ and thus $\boldsymbol{v} = 0$. Therefore, $\boldsymbol{v} = 0$ is the only solution to $LCP(0, A)$ and $\boldsymbol{A}$ is a $\boldsymbol{R}_0$ matrix.

Condition (ii):

$$\boldsymbol{v}^T \boldsymbol{A} \boldsymbol{v} = \boldsymbol{v}^T \hat{\boldsymbol{A}} \boldsymbol{v} + \boldsymbol{v}^T \overline{\boldsymbol{A}} \boldsymbol{v}$$

As $\hat{\boldsymbol{A}}$ and $\overline{\boldsymbol{A}}$ are a positive semi-definite matrix and non-negative matrix, respectively, it is concluded that

$$\boldsymbol{v}^T \hat{\boldsymbol{A}} \boldsymbol{v} + \boldsymbol{v}^T \overline{\boldsymbol{A}} \boldsymbol{v} \ge 0$$

Therefore, $\boldsymbol{A}$ is a copositive matrix. The conditions (i) and (ii) hold, and the proof is complete. ■

Doan et al. (2011) investigated the uniqueness of equilibrium departure rates in the context of a single bottleneck model. It is shown that the uniqueness of departure rates depends on the ratios of $\frac{\beta}{\alpha}$ and $\frac{\gamma}{\alpha}$ which are different across groups in our study. However, this does not guarantee uniqueness in the context of multiple bottleneck model of our study. Since there could exist multiple CAVLs and GPLs that can be used by CAVs and HDVs, they can switch lanes if it does not impact their travel costs. For example, assume the demand of HDVs is equal to zero. If the demand of CAVs is a small positive constant (e.g., ε) which is less than the capacity of CAVL and GPL, they depart at t^* using any lanes with zero travel cost. Hence, the equilibrium departure rates can be non-unique. However, if switching lanes results in higher travel costs for travelers (that is, unique equilibrium departure rates), then it reduces to the multiple single bottleneck model, and since, $\frac{\beta}{\alpha}$ and $\frac{\gamma}{\alpha}$ are different across groups, the equilibrium departure rates are unique. In this condition, it can also be inferred that the equilibrium travel cost is unique, following the proof shown by Ramadurai et al. (2010).

User equilibrium solution properties

In this section, we investigate the relationship between CAVL and GPL queuing delays and the departure rates of commuters under user equilibrium without tolling.

Proposition 1. Under the equilibrium condition, the queuing delay of CAVL for any time interval t is less than or equal to the one for GPL in that time interval (that is, $\tau_{t,l} \leq \tau_{t,l'} \forall t$ where $l \in L_{CAVL}$ and $l' \in L_{GPL}$).

Proof. Appendix D presents the proof for this proposition.

This proposition shows that the queuing delay of CAVL is less than or equal to the GPL in every time interval. This is because if the queuing delay of CAVL is higher than that of GPL in any time interval, then CAV commuters can change their lane choice in that time interval to reduce their travel costs. This continues until the queuing delay for both lanes in that time interval becomes equal. Hence, HDV commuters experience higher queuing delays compared to CAV commuters at every time interval, which is socially inequitable. This leads to Proposition 2, which shows the relationship between the equilibrium travel costs of CAV and HDV commuters.

Proposition 2. Under user equilibrium, the travel cost of CAV commuters is always less than the travel cost of HDV commuters.

Proof. Since CAV commuters can experience lower or equal queuing delays in any time interval compared to HDV commuters (who are restricted to GPL only), and given the lower value of time for CAVs, it results in CAV commuters having a lower equilibrium travel cost compared to HDV commuters.

This proposition shows that the flexibility of CAV commuters in using both CAVL and GPL enables them to experience lower travel costs compared to HDV commuters. This is amplified by the lower value of time for CAV commuters. In practice, this has important equity implications. In the early stages of the CAV transition period, they will be affordable only to higher-income commuters. They can experience lower travel costs compared to lower-income commuters who cannot afford to purchase CAVs. To reduce inequity, Seilabi et al. (2020) propose a Pareto-optimal tradable credit scheme that enables all travelers to experience lower travel costs. Next, the lane choice behavior of CAV commuters during the morning peak period is analyzed.

Proposition 3. If departure rates are unique, CAV commuters use GPLs in time interval t only if there exists at least one CAV commuter who uses CAVL in that time interval.

Proof. Appendix E presents the proof for this proposition. This proposition shows that CAVL always has priority for CAV commuters because of the lesser or equal queueing delay compared to GPL. They choose to only use GPL if it allows them to reduce their queuing delays. This occurs only when GPL has significantly less flow compared to CAVL. Otherwise, due to the higher capacity of CAVL, queueing delays are always higher for GPLs at a comparable level of flow.

Proposition 4. The departure rates of CAV and HDV commuters that use GPL do not overlap in two or more consecutive time intervals.

Proof. Appendix F presents the proof for this proposition. This proposition implies that departure rates of CAV and HDV commuters that use GPL overlap in less than two consecutive time intervals in discrete time setting. This indicates that if this analysis is extended to the continuous time setting $(\Delta t \rightarrow 0)$, the departure rates of CAV and HDV commuters that use GPL do not overlap during the morning peak period.

System-optimal design of CAVL and tolling strategies

This section develops the system-optimal design of CAVL and tolling strategies using a linear model. The goal is to determine the optimal lane-specific toll amount and the number of CAVLs to deploy to achieve the minimum system cost (which consists of total queueing and schedule delays). To develop a system-optimal tolling strategy for a single highway bottleneck, Doan et al. (2011) proved that travelers experience zero queueing delays under system-optimal conditions. The same proof can be applied to the multiple highway bottlenecks, which implies that queueing delays are equal to zero. This property enables us to develop a system-optimal tolling strategy. First, the method to determine the optimal tolling policy given the number of CAVLs is shown and then generalized to calculate both the number of CAVLs and the optimal tolling policy. Under a given number of CAVLs and a zero-queuing delay property, the system-optimal model that yields the optimal departure rates and tolling strategy can be expressed as the following mathematical model with complementarity constraints (MPCC):

$$0 \le r_{g,t,l} \perp \beta \cdot e_{t,l} + \gamma \cdot \left(e_{t,l} - (t^* - t)\right) + p_{t,l} + \varphi_{g,t,l} - \mu_g \ge 0 \quad \forall t \in T, \forall l \in L, g \in G \tag{15}$$

$$0 \le e_{t,l} \perp e_{t,l} - (t^* - t) \ge 0 \quad \forall t \in T, \forall l \in L \tag{16}$$

$$\textstyle\sum_l \sum_t r_{g,t,l} - N_g = 0 \quad \forall g \in G \tag{17}$$

By inserting zero queueing delays, Constraints (15)–(17) satisfy user equilibrium constraints (6)–(12), respectively. The MPCC consists of linear complementarity constraints, which makes it difficult to solve. Hence, it is necessary to develop a mathematical program that can be easily solved. The MPCC can be formulated as the following linear program (LP):

$$\min_p Z = \sum_{(g,t,l)} r_{g,t,l} u_t \tag{18}$$

$$\textstyle\sum_g \sum_t r_{g,t,l} \le s_l \quad \forall l \in L \tag{19}$$

$$r_{2,t,l} = 0 \quad \forall t \in T, \forall l \in L_{CAVL} \tag{20}$$

$$\textstyle\sum_l \sum_t r_{g,t,l} - N_g = 0 \quad \forall g \in G \tag{21}$$

$$r_{g,t,l} \ge 0 \quad \forall t \in T, \forall g \in G, \forall l \in L \tag{22}$$

where $u_t = \begin{cases} \beta(t^* - t) \\ \gamma(t - t^*) \end{cases}$ denotes the schedule delay of commuters departing in time interval t. The objective function Z also denotes the travel cost, which only consists of schedule delay under the system-optimal condition. The objective function (18) is to minimize the total cost of commuters. Constraint (19) states the total departure rates of commuters using lane l should not exceed the capacity of that lane. Constraints (20) and (22) are identical to constraints (8) and (12), respectively. Using the first-order conditions, it is straightforward to demonstrate that the solution of LP (18)–(22) is also a solution to MPCC (15)–(17) where the Lagrangian multiplier for constraints (21) is the optimal time-varying lane-specific tolling policy.

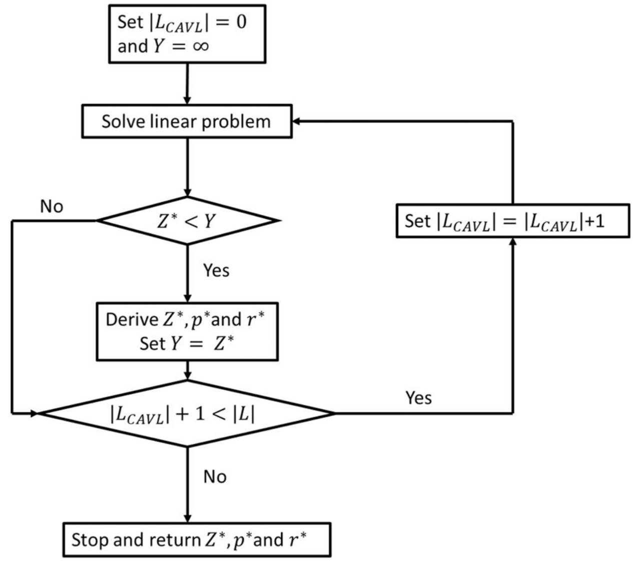


Figure 2. Solution algorithm to determine the optimal CAVL and tolling strategies.

Proposition 5. If there is any time interval in which CAVs use both CAVL and GPL, then, in that time interval, the toll charged for the CAVL is equal to that for the GPL.

Proof. If CAVs using CAVL and GPL experience equal travel cost, and zero travel time in time interval t, they should be charged equal tolls on both types of lanes in that time interval. Hence, CAVs and HDVs departing in time interval t pay the same toll amount.

The main assumption, used to develop the LP (18)–(22), is that the number of CAVLs is constant. However, identifying the optimal number of CAVLs can be another policy that the IOO could use to further minimize the total travel cost. To develop the system-optimal CAVL and tolling policy, it is necessary to solve LP (18)–(22) using the enumeration technique for the available number of lanes to allocate to CAVs. For example, if there are four lanes on a highway, the IOO can allocate up to three lanes to CAVs because it is necessary to have at least one lane open for HDV use on this highway. Finally, the framework for deriving the optimal CAVL and tolling strategy is formulated in Figure 2.

Computational experiments

This section aims to analyze the impacts of the number of CAVLs and toll fees under different CAV market penetration rates on total system cost, and lane and departure time choices of commuters. During the morning peak period, the CAV and HDV commuters travel along a highway with four lanes per direction, and the total travel demand of commuters is equal to 1,000. The morning peak period is divided into 100 intervals, and commuters desire to arrive by the 70th interval. The capacities of CAVL and GPL are assumed to be equal to 30 and 10 vehicles per lane per time interval, respectively. The early and late arrival penalties for CAV and HDV commuters are assumed to be equal to 0.8 and 4 \$/(time interval), respectively. The CAV and HDV values of time are equal to 1 and 2 \$/(time interval), respectively. Commercial solvers embedded in GAMS (Rosenthal, 2015) are used to solve MLCP (6)–(12) and LP (18)–(22).

First, we analyze the total system cost under different numbers of CAVLs and CAV penetration rates without a toll. Figure 3 presents the total system cost under user equilibrium conditions for different CAV market penetrations and the number of CAVLs. For zero CAVLs, the total system cost initially decreases as the CAV market penetration rate increases. This is mainly due to the lower value of commuters' time for CAVs relative to HDVs. After achieving a minimum of around 45% CAV market penetration, total travel costs rise as the CAV market penetration rate increases. It is because CAV commuters have a lower value of time compared to HDV commuters. Hence, as the CAV market penetration rate increases, more commuters travel before the desired arrival time, despite incurring higher queuing delays due to the higher traffic congestion, to avoid a late arrival penalty (Figure 4). Figure 4 also illustrates that the lesser value of time for CAVs can make commuters indifferent toward travel time and hence depart closer to their desired arrival time, which can significantly increase bottleneck congestion, especially endured by HDV commuters. Consequently, the system cost increases as commuters experience higher queueing delays. A similar pattern can be observed for one, two, and three CAVLs, where total system cost initially reduces and then increases at different CAV market penetration rates. To determine the optimal number of CAVLs, Figure 3 is divided into four areas with blue circles. In each area, the total queuing delays of different CAV market penetrations are the minimum for either 0, 1, 2, or 3 CAVLs. In other words, the IOO deploys 0, 1, 2, and 3 CAVLs for areas labeled as 1, 2, 3, and 4, respectively.

The optimal CAVL deployment plan without a tolling policy can be determined using Figure 3. Hereafter, this policy is referred to as "CAVL only." Under this policy, the equilibrium travel costs of HDV and CAV commuters are presented for different CAV market penetrations in Figure 5. The equilibrium travel cost of CAVs is less than that of HDVs, which is consistent with Proposition 2. When the CAV market penetration is lower than 25%, the total system cost under zero CAV-dedicated lane is less than the other cases (Area 1 of Figure 3). When the CAV market penetration increases to 25%, the travel cost of CAV commuters increases until the IOO deploys additional CAVL in the system given the lesser total travel cost under one CAVL (Area 2 of Figure 3). After increasing the CAV market penetration to 45% and 75%, IOO deploys the second and third CAV-dedicated lanes, respectively (areas 3 and 4 of Figure 3). Interestingly, this also leads to a

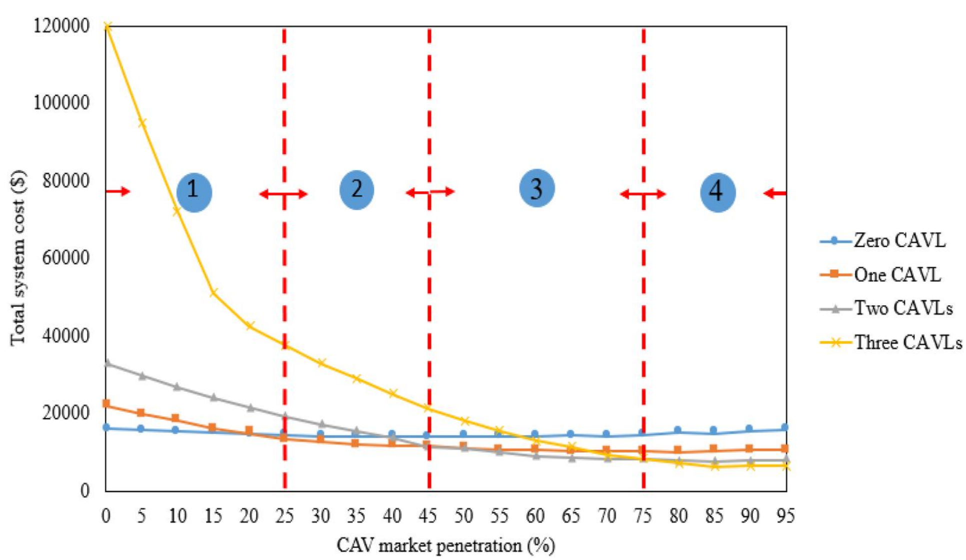
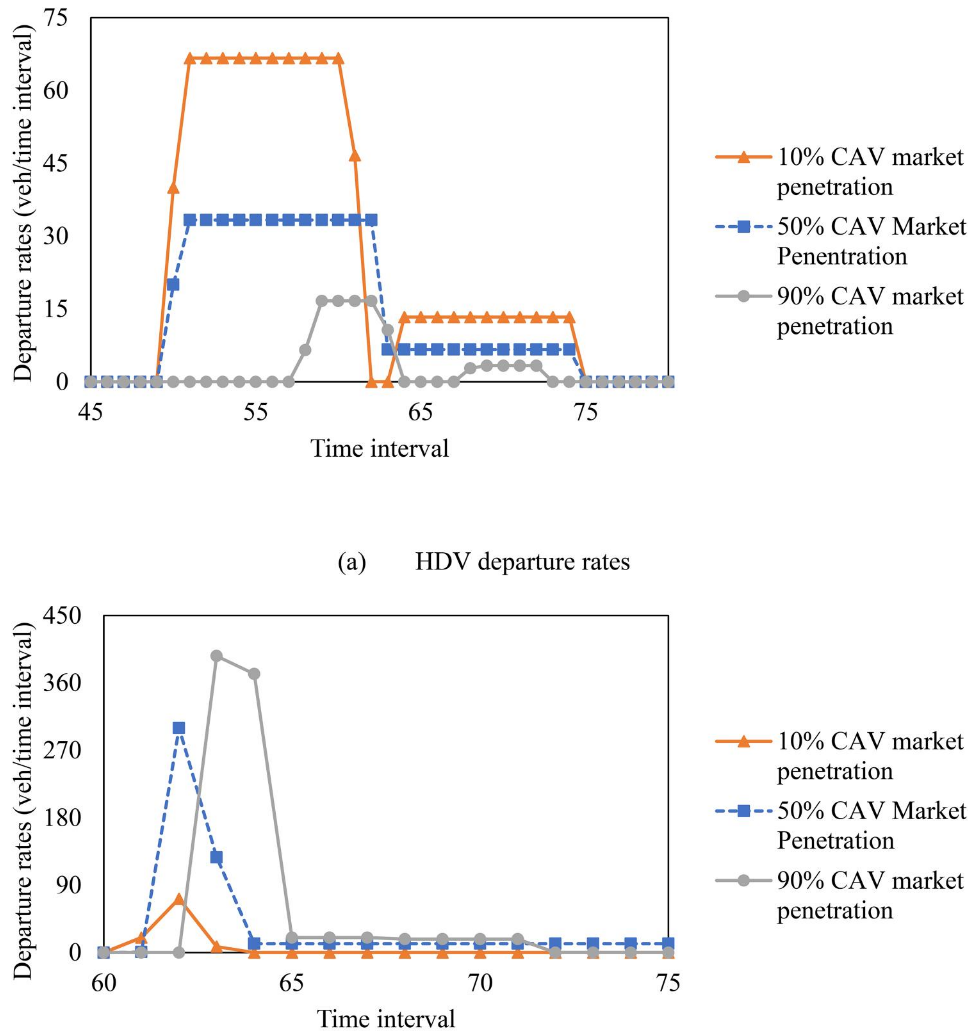


Figure 3. Total system cost under different CAVLs and CAV market penetrations without tolling policy.



(a) HDV departure rates

(b) CAV departure rates

Figure 4. Aggregate HDV and CAV departure rates under zero CAVLs and without tolling policy.

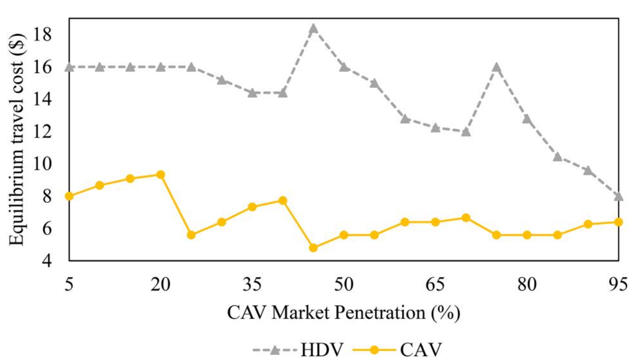


Figure 5. Equilibrium travel costs under different CAV market penetration.

reduction in HDV travel costs. This is because although CAVL causes a reduction in road capacity for HDVs, it can also lead to an increase in available capacity as CAV prefers to use CAVL with higher capacity. This reduces traffic congestion for the system users and leads to a reduction in travel costs for HDVs as CAV market penetration increases. Hence, there is less social inequity between CAV and HDV commuters in terms of travel costs as CAV market penetration increases.

Next, Figure 6 shows the total system cost under different optimal strategies, in terms of CAVLs and a tolling policy (OCAVLT), to achieve the minimum system cost. When there exist zero CAVL, system cost remains unchanged as CAV market penetration increases under optimal tolling policy. It is because the queueing delay is zero under the SO condition. Hence, the total system travel cost only includes the schedule delay costs of travelers (i.e., early and late arrival costs) and consequently, the total system cost does not change irrespective of the share of CAVs in the mixed-traffic flow.

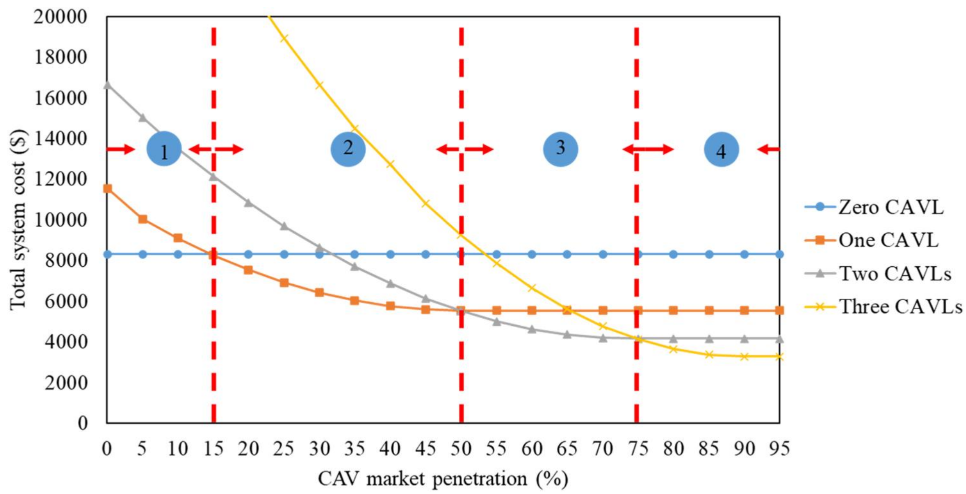


Figure 6. The optimal total system cost under OCAVLT for different CAV market penetrations.

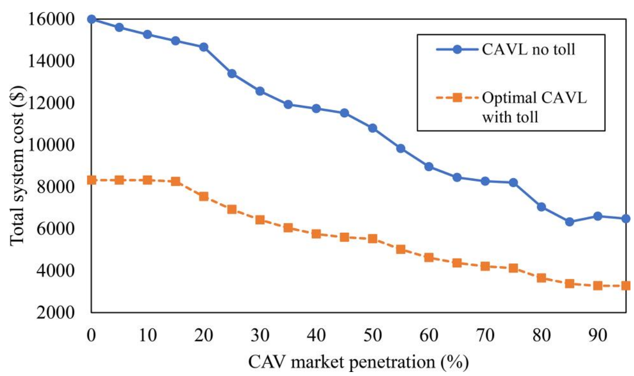


Figure 7. Comparison of the optimal total system cost under CAVL without toll and OCAVLT for different CAV market penetrations.

On the other hand, the total system cost decreases for one, two, and three CAVLs. This is because CAVL leads to an increase in bottleneck capacity, but this increased capacity is only available to CAV commuters. Hence, as the CAV market penetration increases, the total system cost is reduced. It is interesting to note that the tolling policy leads to deploying the first CAVL under a lesser CAV market penetration (that is, 15%) compared to the case without the tolling policy in Figure 3 (that is, 25%). That happens because the tolling policy impacts commuter departure time choices, enabling the system to leverage the higher capacity of CAVL to reduce total system cost. Figure 7 compares the system-optimal condition under the optimal CAVL only and OCAVLT policies. It is observed that the total system cost under OCAVLT is approximately half of that under optimal CAVL only. Further, as CAV market penetration increases, the system cost difference between these policies reduces, which highlights the advantage of deploying CAVL.

Finally, we investigate the impact of the CAVL capacity increase on the total system cost under optimal CAVL only and OCAVLT policies. So far, it has been assumed that CAVL capacity is three times that of GPL capacity (Tientrakool et al., 2011). That is, the capacities of CAVL and GPL are assumed to be equal to 30 and 10 vehicles per time unit, respectively. That is, the CAVL capacity coefficient is equal to 3. Figure 8 illustrates the impacts of CAVL capacity increase coefficients on total system cost under CAVL only and OCAVLT. As the CAVL capacity coefficient increases, the difference between the total system costs under CAVL only and OCAVLT decreases. This shows the importance of technological advancements in CAVs, which reduce the necessity of implementing a tolling policy. For example, when the capacity of CAVL is 15 vehicles per time unit, the optimal system cost with the tolling policy is almost identical to the one under CAVL only when the CAVL capacity is 30 vehicles per time unit.

Concluding remarks

This study proposes an analytical framework to alleviate traffic congestion in a highway corridor during the transition era with a mixed fleet of CAVs and HDVs using CAVL and tolling policies. First, the user equilibrium condition is formulated as LCP to understand the impact of CAVL on traffic congestion under different CAV market penetrations. Further, the solution's existence is investigated in terms of departure rates and travel costs. Finally, the system-optimal condition is determined to achieve the minimum system cost by deriving the optimal number of CAVL and tolling policy.

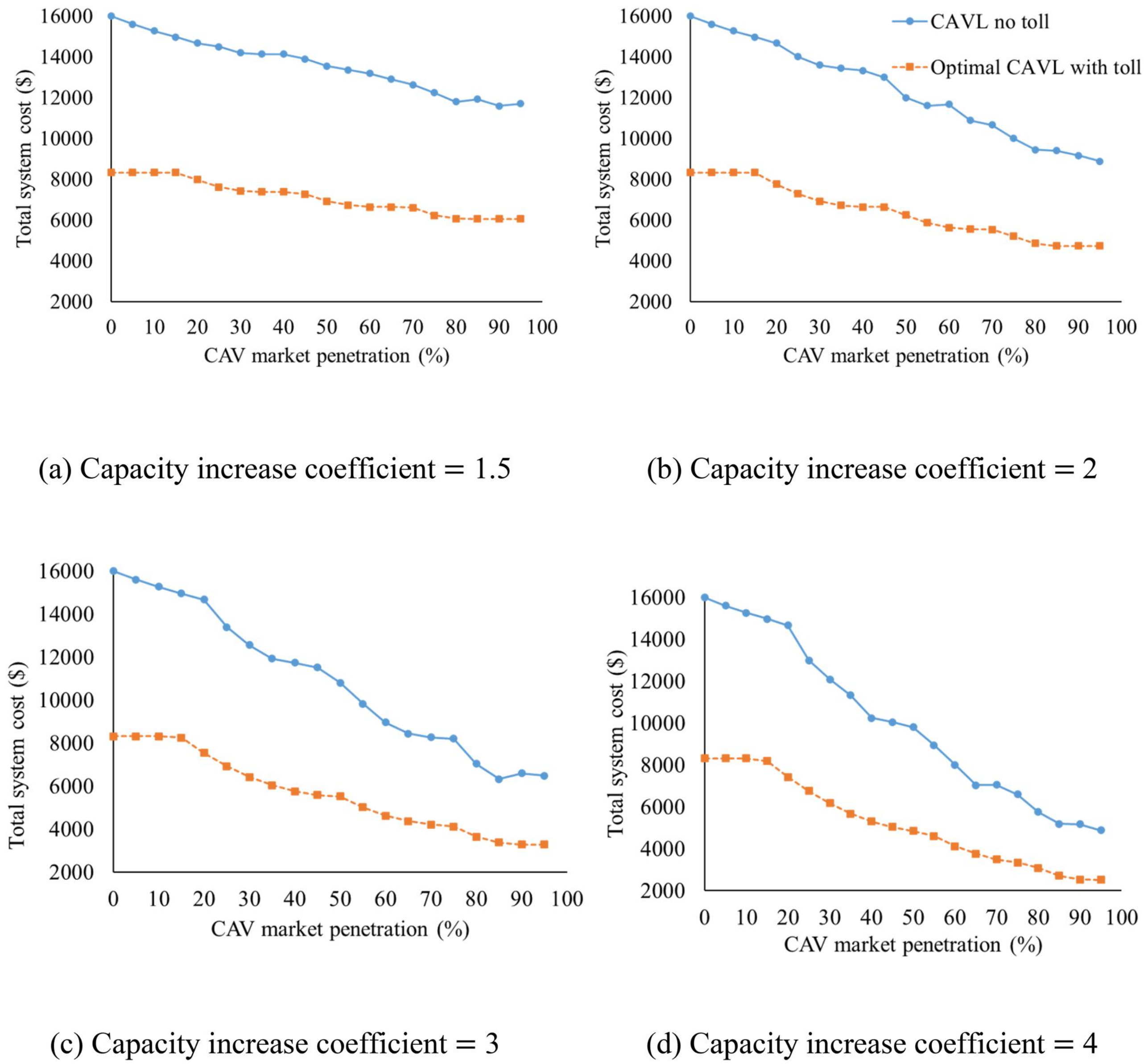


Figure 8. Total system cost under CAVL only and OCAVLT under different CAVL capacity increase coefficients.

Computational experiments are conducted to understand the impacts of different parameters, such as CAVL capacity and CAV market penetration, on the total system cost and departure rates. First, it is shown that HDV commuters' travel costs reduce as CAV market penetration increases by deploying CAVL. In the case of bottleneck experiment of this research, HDV travel costs can be reduced by 50% as CAV market penetration increases by deploying CAVL. Second, it is shown that the difference between CAV and HDV travel costs reduces as CAV market penetration increases. This leads to less social inequity in terms of the travel cost difference between HDVs and CAVs. In the case of the bottleneck experiment of this research, the difference between HDV and CAV travel costs can be reduced by 80% as CAV market penetration increases by deploying CAVL. Third, CAVs depart closer to their desired arrival time because of the lesser value of time, which can significantly increase bottleneck traffic congestion, especially endured by HDV commuters. Fourth, it is illustrated that under a system-optimal tolling policy, CAV commuters utilize CAVLs more than they would without a tolling policy. The framework suggests the allocation of CAVLs at 15, 50, and 75% of CAV market penetration to achieve optimal total system cost.

In future efforts to implement this model in the real world, there are several considerations that need to be addressed. The first consideration is the existence of multiple highway bottlenecks instead of one bottleneck as used in the current study, their configurations (in series or parallel), and their interactions. One of the best-known examples is New York City, where there are multiple highway bridges connecting Manhattan to the rest of the city. In this context, travelers have the option of choosing their departure times in addition to the paths that need to be considered in practice. The second consideration to enhance real-world application is related to the dependency of GPL capacity on the CAV percentage of the GPL traffic stream. For example, it has been shown in other studies (Liu & Song, 2019) that the GPL capacity can increase through an increase in the percentage of the CAV fleet. In other words, the higher share of CAVs in the traffic flow can result in higher road capacity for GPLs and consequently, reduced total travel costs. This can be a decisive factor in determining the optimal number of CAVLs, especially under high CAV market penetration. However, considering the GPL capacity as a function of shares of CAVs and HDVs turns the model (6)–(12) into a nonlinear complementarity problem instead of a linear complementarity problem, which significantly increases its complexity. Hence, there is a need for another study that holistically considers road capacity changes based on the share of CAVs in traffic flow. The third consideration is related to the characteristics of HDV and CAV travelers. This study assumes that CAV and HDV have identical desired arrival times and early and late arrival penalties. However, in reality, commuters are heterogeneous in terms of desired arrival time and early and late arrival penalties. This can affect the departure time choices of commuters and, consequently, the state of traffic congestion in the bottleneck. Our framework can be made more realistic to capture the heterogeneity of commuters.

The development of autonomous technology may have a variety of effects on dedicated lane management. First, the advancements in automation will enhance the car-following and lane-changing performance of the CAVs and increase traffic capacity in a mixed-stream (Ahmed et al., 2022; Olia et al., 2018). Second, the road agency can dynamically reassign dedicated lanes or their direction in short time intervals, such as peak and off-peak hours, as CAVs and HDVs equip themselves with more advanced sensing and communication technologies. Third, other autonomous vehicle types, such as autonomous buses and autonomous modular vehicles, could be deployed. We expect these advanced transit systems to operate in the future as part of the public transit network. As a result, the autonomous transit fleet has the potential to use the dedicated lanes for autonomous vehicles to further improve public transit service.

Acknowledgments

The contents of this article reflect the view of the authors, who are solely responsible for the facts and accuracy of the data presented and do not necessarily reflect the official views or policies of the sponsoring organizations. These contents do not constitute a standard, specification, or regulation.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This study is based on research supported by the Center for Connected and Automated Transportation (CCAT), Region V University Transportation Center, funded by the US Department of Transportation, Award #69A3551747105. It is also supported by metaCCAZE (flexibly adapted MetaInnovations, use cases, collaborative business, and governance models to accelerate the deployment of smart and shared zero-emission mobility for passengers and freight) funded by Horizon Europe.

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Appendix A. Summary of notations

Sets	
L	Set of lanes
T	Set of time intervals

Parameter	
s_l	Capacity of lane l
N_g	Travel demand of group g
α_g	Value of time of group g
β_g	Early arrival penalty of group g
γ_g	Late arrival penalty of group g
t^*	Desired arrival time

Variables	
$r_{g,t,l}$	Departure rates of commuters of group g using lane l in time interval t
$\tau_{t,l}$	Queuing delay of commuters using lane l in time interval t
$e_{t,l}$	Early arrival duration of commuters departing in time interval t using lane l
$\sigma_{g,t,l}$	Travel cost of group g departing in time interval t using lane l
$p_{t,l}$	Toll of lane l for commuters departing at time interval t
μ_g	Equilibrium travel cost of commuters of group g

Appendix B. Proof of Theorem 1

To prove this theorem, $r_{2,t,l}$ should be proved to be equal to zero for any $l \in L_{CAVL}$ and μ_g has a strictly positive value. First, it is proved that $r_{2,t,l}$ is equal to zero for any $l \in L_{CAVL}$. $\varphi_{g,t,l}$ of HDV commuters for using CAVLs is set to be a sufficiently large positive constant. Therefore, the right-hand side of constraint (13) is always greater than zero for $l \in L_{CAV}$. This results in $r_{g,t,l} = 0$ for HDV commuters that use CAVLs, which is equivalent to Eq. (8). Moreover, as $\varphi_{g,t,l}$ is assumed to be zero for other cases, Eq. (13) is equivalent to Eqs. (6) and (7) for all CAV commuters and HDV commuters that use GP lanes. Ultimately, Eq. (13) of LCP is equivalent to Eqs. (6)–(8) of MLCP.

Next, it is proved that μ_g has a strictly positive value. According to Eq. (14), $\sum_l \sum_t r_{g,t,l} \geq N_g$ which means that some of $r_{g,t,l}$, for both CAV and HDV commuters, are greater than zero. Therefore, the right-hand side of Eq. (13), for both CAV and HDV commuters and under some (t, l), are zeros. As the right-hand side of Eq. (13) is a linear combination of non-negative variables and it is assumed that congestion exists (travel demand exceeds capacity), μ_g is restrictively positive. Therefore, $\sum_l \sum_t r_{g,t,l} - N_g = 0$ in Eq. (14) which is consistent with the Eq. (12) of MCLP. This concludes the proof. ∎

Appendix C. Detailed vectors of solution existence

The introduced MLCP model is presented in a generalized form of

$$0 \le \boldsymbol{v} \perp A\boldsymbol{v} + \boldsymbol{b} \ge 0$$

$\boldsymbol{v}$ is the variable vector $\boldsymbol{v} \equiv \begin{pmatrix} \boldsymbol{r} \\ \boldsymbol{\tau} \\ \boldsymbol{e} \\ \overline{\boldsymbol{\mu}} \end{pmatrix}$:

where $\boldsymbol{r} \equiv (r_{g,t,l})_{(g,t,l)\in G\times T\times L}$, $\boldsymbol{\tau} \equiv (\tau_{t,l})_{(t,l)\in T\times L}$, $\boldsymbol{e} \equiv (e_{t,l})_{(t,l)\in T\times L}$, and $\overline{\boldsymbol{\mu}} \equiv (\overline{\mu})_{g\in G}$. **b** is the constant vector $\boldsymbol{b} \equiv \begin{pmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \\ \mathbf{b}_4 \end{pmatrix}$

where

$$\boldsymbol{b}_1 = \begin{pmatrix} \frac{1}{\alpha_1+\gamma}(-\gamma(t^*-t) + (\varphi_{1,t,l}) + (p_{t,l}))_{t\in T, l\in L} \\ \frac{1}{\alpha_2+\gamma}(-\gamma(t^*-t) + (\varphi_{2,t,l}) + (p_{t,l}))_{t\in T, l\in L} \\ \vdots \\ \frac{1}{\alpha_g+\gamma}(-\gamma(t^*-t) + (\varphi_{g,t,l}) + (p_{t,l}))_{t\in T, l\in L} \end{pmatrix} \in \Re^{|G|\times|T|\times|L|},$$

$$\boldsymbol{b}_2 = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_l \\ s_1 \\ \vdots \\ s_l \\ \vdots \\ s_l \end{pmatrix} \in \Re^{|T|\times|L|},$$

$$\boldsymbol{b}_3 = \left(-(t^* - t - \tau_{t,l})_{t\in T, l\in L} \right) \in \Re^{|T|\times|L|},$$

$$b_4 = \begin{pmatrix} \frac{1}{\alpha_1+\gamma}N_1 \\ \frac{1}{\alpha_2+\gamma}N_2 \\ \vdots \\ \frac{1}{\alpha_g+\gamma}N_g \end{pmatrix} \in \Re^{|G|},$$

Also, Matrix A is as follows:

$$\boldsymbol{A} \equiv \begin{pmatrix} 0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & -\boldsymbol{A}_3 \\ -\boldsymbol{A}_1^T & \boldsymbol{S} & 0 & 0 \\ 0 & \boldsymbol{A}_4 & \boldsymbol{A}_5 & 0 \\ \boldsymbol{A}_3^T & 0 & 0 & 0 \end{pmatrix},$$

where

$$\boldsymbol{A}_1 = \begin{pmatrix} \boldsymbol{I} \\ \boldsymbol{I} \\ \vdots \\ \boldsymbol{I} \end{pmatrix} \in \Re^{(|G|\times|T|\times|L|)} \times \Re^{(|T|\times|L|)},$$

$\boldsymbol{I}$ is an identity matrix: $\boldsymbol{I} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} \in \Re^{(|T|\times|L|)} \times \Re^{(|T|\times|L|)}$,

$$\boldsymbol{A}_2 = \begin{pmatrix} \frac{\beta+\gamma}{\alpha_1+\gamma} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & \frac{\beta+\gamma}{g+\gamma} \end{pmatrix} \in \Re^{(|G|\times|T|\times|L|)} \times \Re^{(|G|\times|T|\times|L|)},$$

$$\boldsymbol{A}_3 = \begin{pmatrix} \frac{1}{\alpha_1+\gamma} & 0 & \ldots & 0 \\ \frac{1}{\alpha_1+\gamma} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & 0 \\ 0 & \frac{1}{\alpha_2+\gamma} & \ldots & 0 \\ \vdots & \vdots & \vdots & 0 \\ 0 & 0 & 0 & \frac{1}{\alpha_g+\gamma} \end{pmatrix} \in \Re^{(|G|\times|T|\times|L|)} \times \Re^{(|G|)},$$

$$\boldsymbol{A}_4 = \begin{pmatrix} \frac{1}{\alpha_1+\gamma}\boldsymbol{I} \\ \frac{1}{\alpha_2+\gamma}\boldsymbol{I} \\ \vdots \\ \frac{1}{\alpha_g+\gamma}\boldsymbol{I} \end{pmatrix} \in \Re^{(|G|\times|T|\times|L|)} \times \Re^{(|T|\times|L|)},$$

$$\boldsymbol{A}_5 = \begin{pmatrix} \frac{1}{\alpha_1+\gamma} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & \frac{1}{\alpha_g+\gamma} \end{pmatrix} \in \Re^{(|G|\times|T|\times|L|)} \times \Re^{(|G|\times|T|\times|L|)},$$

$$\boldsymbol{S} = \begin{pmatrix} s_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_l \end{pmatrix} \in \Re^{(|T|\times|L|)} \times \Re^{(|T|\times|L|)}$$

Appendix D. Proof of Proposition 1

Proposition 1 is proved using contradiction. Under equilibrium conditions, assume that

$$\tau_{t,l} > \tau_{t,l'} \quad \forall t, l \in L_{CAVL}, l' \in L_{GPL} \tag{23}$$

To conduct this proof, the peak period is divided into three parts, as follows:

Part 1. $(t + \tau_{t,l} \le t^*$ for $l \in L_{CAVL})$

In this part, both HDV and CAV commuters incur early arrival costs. The relationship between the travel costs of CAV commuters using GPL and CAVL can be formulated as follows:

$$\alpha_1 \cdot \tau_{t,l'} + \beta \cdot \left(t^* - t - \tau_{t,l'}\right) < \alpha_1 \cdot \tau_{t,l} + \beta \cdot (t^* - t - \tau_{t,l}) \quad \forall t \in T, l \in L_{CAVL}, l' \in L_{GPL} \tag{24}$$

This implies that

$$\sigma_{1,t,l'} < \sigma_{1,t,l} \quad \forall t \in T, l \in L_{CAVL}, l' \in L_{GPL} \tag{25}$$

There are four cases regarding departure rates of CAV commuters as follows:

1. $r_{1,t,l'} > 0, r_{1,t,l} > 0$
2. $r_{1,t,l'} = 0, r_{1,t,l} > 0$
3. $r_{1,t,l'} > 0, r_{1,t,l} = 0$
4. $r_{1,t,l'} = 0, r_{1,t,l} = 0$

Given the lesser travel costs of CAV commuters under GPL compared to CAVL, cases 1 and 2 cannot occur under the equilibrium conditions since CAV commuters can change lanes from CAVL to GPL to reduce their travel costs. The relationship between the travel times of CAV commuters using CAVL and GPL under case 3 can be formulated as follows:

$$\tau_{t-1,l'} + \frac{\sum_g r_{g,t,l'} - s_{l'}}{s_{l'}} < \tau_{t-1,l} \quad \forall t \in T, l \in L_{CAVL}, l' \in L_{GPL} \tag{26}$$

For case 4,

$$\tau_{t-1,l'} + \frac{r_{g,t,l'} - s_{l'}}{s_{l'}} < \tau_{t-1,l} \quad \forall t \in T, l \in L_{CAVL}, l' \in L_{GPL} \tag{27}$$

From inequalities (26) and (27), it follows that $\tau_{t-1,l'}$ is less than $\tau_{t-1,l}$. Similar to inequalities (24) and (25), it infers that $\sigma_{1,t-1,l'}$ is less than $\sigma_{2,t-1,l}$. Following the same pattern, it results that

$$\tau_{t',l'} < \tau_{t',l} \quad \forall t' \in T, l \tag{28}$$

This implies that $\sigma_{1,t',l'}$ is strictly less than $\sigma_{2,t',l}$ for any $t' \le t$. Given the higher cost of using CAVL, CAV commuters do not use CAVL and travel using GPL in any time interval $t' \le t$. Since CAV commuters do not use CAVL, its queueing delay should be equal to zero ($\tau_{t,l} = 0, \forall t \in T, \forall l \in L_{CAVL}$). This means that the queuing delay of GPL is strictly less than zero, which is not possible. This completes the proof for part 1. ■

Part 2. ($t + \tau_{t,l} \ge t^*$ and $t + \tau_{t,l'} \le t^*$ for $l \in L_{CAVL}, l' \in L_{GPL}$)

In this part, CAV commuters using CAVL and GPL, incur early and late arrival delays, respectively. Let $\tilde{t}$ denote the greatest time interval in which departing commuters incur early arrival cost (that is, $\tilde{t} + \tau_{\tilde{t},l} \le t^*$ and $\tilde{t} + 1 + \tau_{\tilde{t}+1,l} \ge t^*$ for $l \in L_{CAVL}$). Based on the part 1, it can be inferred that $\tau_{\tilde{t},l} \le \tau_{\tilde{t},l'}$ and $\sigma_{1,t,l} \le \sigma_{1,t,l'}$. If $t^* \le \tilde{t} + 1 + \tau_{\tilde{t}+1,l'}$, then the proof can be done using part 3. Hence, for part 2, it is assumed that $t^* \ge \tilde{t} + 1 + \tau_{\tilde{t}+1,l'}$. We need to prove that $\tau_{\tilde{t}+1,l}$ is less than or equal to $\tau_{\tilde{t}+1,l'}$. To prove by contradiction, we need to prove $\tau_{\tilde{t}+1,l} > \tau_{\tilde{t}+1,l'}$. Then,

$$\tau_{\tilde{t},l'} + \frac{\sum_g r_{g,\tilde{t}+1,l'} - s_{l'}}{s_{l'}} < \tau_{\tilde{t},l} + \frac{r_{1,\tilde{t}+1,l} - s_l}{s_l} \quad l \in L_{CAVL}, l' \in L_{GPL} \tag{29}$$

Since $\tau_{\tilde{t},l'} \ge \tau_{\tilde{t},l}$ and $s_l \le s_{l'}$, it follows that $r_{1,\tilde{t}+1,l} \ge \sum_g r_{g,\tilde{t}+1,l'}$ which implies that $\sigma_{\tilde{t}+1,1,l} \le \sigma_{\tilde{t}+1,1,l'}$. Then,

$$\alpha_1 \cdot \tau_{\tilde{t}+1,l} + \gamma \cdot \left(\tilde{t} + 1 + \tau_{\tilde{t}+1,l} - t^*\right) < \alpha_1 \cdot \tau_{\tilde{t}+1,l'} + \beta \cdot \left(t^* - (\tilde{t}+1) - \tau_{\tilde{t}+1,l'}\right) l \in L_{CAVL}, l' \in L_{GPL} \tag{30}$$

Inequality (30) can be reformulated as follows:

$$(\alpha_1 + \gamma) \cdot \tau_{\tilde{t}+1,l} - \gamma \cdot (t^* - (\tilde{t}+1)) < (\alpha_1 - \beta) \cdot \tau_{\tilde{t}+1,l'} + \beta \cdot \left(t^* - (\tilde{t}+1)\right) l \in L_{CAVL}, l' \in L_{GPL} \tag{31}$$

By reformulating inequality (31), it follows:

$$(\alpha_1 + \gamma) \cdot \tau_{\tilde{t}+1,l} - (\alpha_1 - \beta) \cdot \tau_{\tilde{t}+1,l'} < (\gamma + \beta) \cdot \left(t^* - (\tilde{t}+1)\right) \ l \in L_{CAVL}, l' \in L_{GPL} \tag{32}$$

Since $\tau_{\tilde{t}+1,l} \ge t^* - (\tilde{t}+1)$, it follows

$$(\alpha_1 + \gamma) \cdot \tau_{\tilde{t}+1,l} - (\alpha_1 - \beta) \cdot \tau_{\tilde{t}+1,l'} < (\gamma + \beta) \cdot \tau_{\tilde{t}+1,l} \quad l \in L_{CAVL}, l' \in L_{GPL} \tag{33}$$

This means that $\tau_{\tilde{t}+1,l} < \tau_{\tilde{t}+1,l'}$ which contradicts the original assumption of $\tau_{\tilde{t}+1,l} > \tau_{\tilde{t}+1,l'}$. This completes the proof for part 2. ■

Part 3. $t + \tau_{t,l} \ge t^*$ and $t + \tau_{t,l'} \ge t^*$ for $l \in L_{CAVL}, l' \in L_{GPL}$

Similar to part 1, it can be shown that if $\exists t \ge t^*$ such that $\tau_{t,l} > \tau_{t,l'}$, then $\tau_{t-1,l} > \tau_{t-1,l'}$. This continues until $\tau_{\tilde{t}+1,l} > \tau_{\tilde{t}+1,l'}$ which contradicts the finding in part 2. This completes the proof. ■

Appendix E. Proof of Proposition 3

To prove by contradiction, it is assumed that there exists a time interval t in which CAV commuters depart using GPL without using CAVL in that time period. Since departure rates are unique, travelers cannot switch lanes without changing their travel cost which means that the equilibrium travel cost of CAVs using GPL is strictly less than that of the CAVL. Given Proposition 1, the following three scenarios are possible for CAV travel costs using CAVL and GPL:

$$\alpha_1 \cdot \tau_{t,l'} + \beta \cdot \left(t^* - -\tau_{t,l'}\right) < \alpha_1 \cdot \tau_{t,l} + \beta \cdot (t^* - t - \tau_{t,l}) \quad \forall t, l \in L_{CAVL}, l' \in L_{GPL} \tag{34}$$

$$\alpha_1 \cdot \tau_{t,l'} + \gamma \cdot \left(t + \tau_{t,l'} - t^*\right) < \alpha_1 \cdot \tau_{t,l} + \beta \cdot (t^* - t - \tau_{t,l}) \quad \forall t, l \in L_{CAVL}, l' \in L_{GPL} \tag{35}$$

$$\alpha_1 \cdot \tau_{t,l'} + \gamma \cdot \left(t + \tau_{t,l'} - t^*\right) < \alpha_1 \cdot \tau_{t,l} + \gamma \cdot (t + \tau_{t,l} - t^*) \quad \forall t, l \in L_{CAVL}, l' \in L_{GPL} \tag{36}$$

Scenarios 1 and 3 indicate when CAV commuters arrive either early or late, respectively. Scenario 2 corresponds to the case that CAV commuters using CAVL arrive earlier, while CAV commuters using GPL arrive later than the desired arrival time based on Proposition 1. Under all scenarios, it results in the queuing delay of CAVL being higher than GPL, which contradicts Proposition 1. It completes the proof. ■

Appendix F. Proof of Proposition 4

To prove by contradiction, it is assumed that CAV and HDV commuters depart using GPL in time intervals $t-1$ and t. Then, it follows:

$$\alpha_1 \cdot \tau_{t-1,l'} + \beta \cdot e_{t-1,l'} + \gamma \cdot \left(e_{t-1,l'} - \left(t^* - (t-1) - \tau_{t-1,l}\right)\right) = \alpha_1 \cdot \tau_{t,l'} + \beta \cdot e_{t,l'} + \gamma \cdot \left(e_{t,l'} - (t^* - t - \tau_{t,l})\right) \forall t, \forall l' \in L_{GPL} \tag{37}$$

$$\alpha_2 \cdot \tau_{t-1,l'} + \beta \cdot e_{t-1,l'} + \gamma \cdot \left(e_{t-1,l'} - \left(t^* - (t-1) - \tau_{t-1,l}\right)\right) = \alpha_2 \cdot \tau_{t,l'} + \beta \cdot e_{t,l'} + \gamma \cdot \left(e_{t,l'} - (t^* - t - \tau_{t,l})\right) \forall t, \forall l' \in L_{GPL} \tag{38}$$

By subtracting constraints (37) and (38), it follows that $\tau_{t-1,l'} = \tau_{t,l'}$. By substituting the equality condition of queueing delays in both time intervals $t-1$ and t into Eqs. (37) and (38), it results:

$$(\beta + \gamma) \cdot e_{t-1,l'} + \gamma = (\beta + \gamma) \cdot e_{t,l'} \quad \forall t, \forall l' \in L_{GPL} \tag{39}$$

If CAV commuters departing in time intervals $t-1$ and t arrive later than desired arrival time, $e_{t-1,l'}$ and $e_{t,l'}$ are equal to zero which implies that Eq. (39) is infeasible. If CAV commuters arrive earlier than desired arrival time, then it follows:

$$(\beta + \gamma) \cdot (t^* - (t-1) - \tau_{t-1,l'}) + \gamma = (\beta + \gamma) \cdot (t^* - t - \tau_{t,l'}) \forall l' \in L_{GPL} \tag{40}$$

Equation (40) is also infeasible as β and γ are strictly greater than zero. Finally, if CAV commuters depart in time intervals $t-1$ and t arrive earlier and later than desired arrival time, then it follows:

$$(\beta + \gamma) \cdot e_{t-1,l'} + \gamma = 0 \quad \forall l' \in L_{GPL} \tag{41}$$

This equation is also infeasible as β, γ and $e_{t-1,l'}$ are positive. Hence, both CAV and HDV commuters can't depart at two consecutive intervals using the GPL. The same proof can be applied to more than two time intervals. This completes the proof. ■