CoBaPreM

Component-Based Predictive Maintenance to reduce unscheduled occurrences.

A new strategy to improve reliability practices

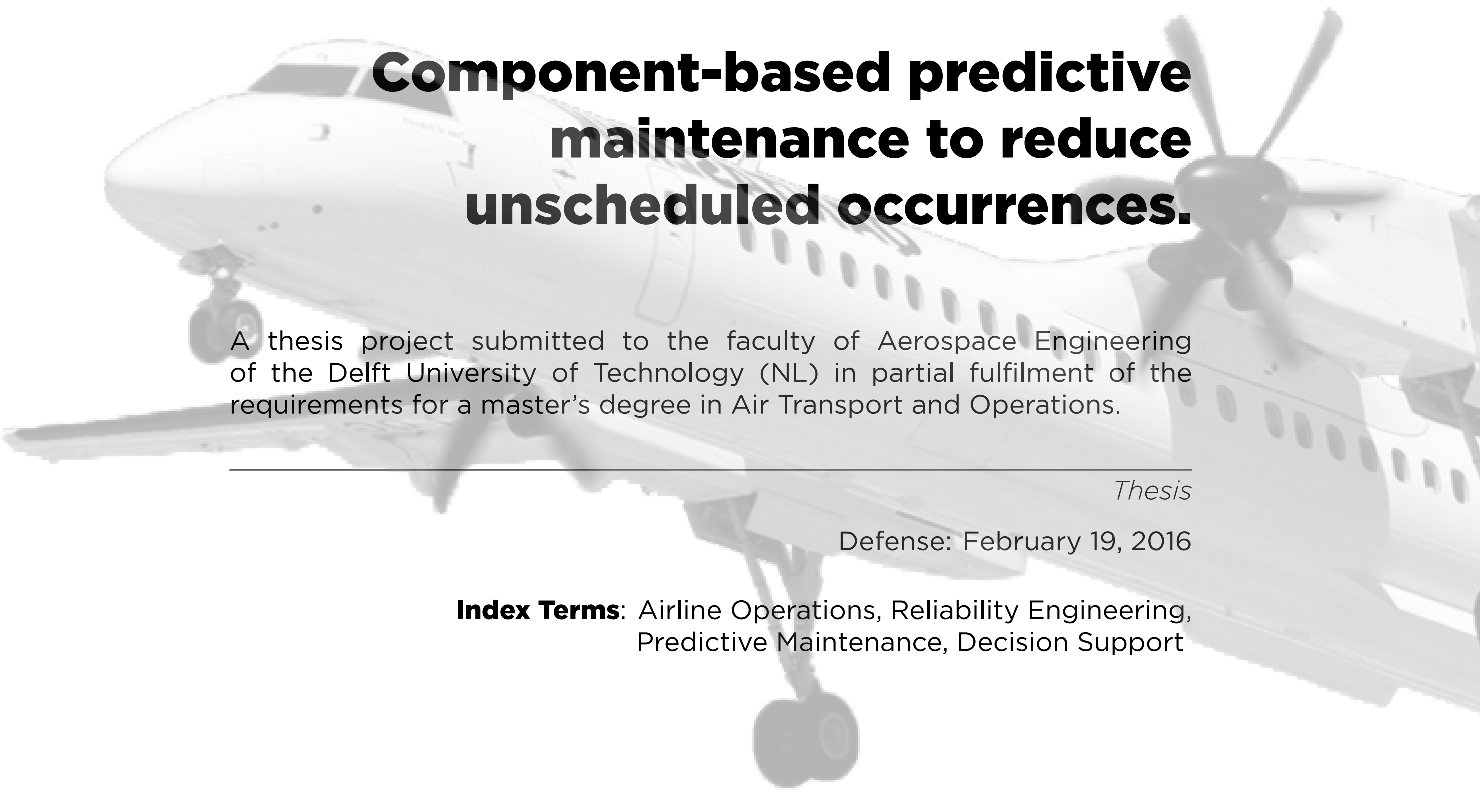
L.W.M. de Boer/4012682

Air Transport and Operations Thesis

Defense: 10:00-11:00 February 19, 2016
Location: LR-CZ C

TU Delft Delft University of Technology

QANTASLINK



Component-based predictive maintenance to reduce unscheduled occurrences.

A thesis project submitted to the faculty of Aerospace Engineering of the Delft University of Technology (NL) in partial fulfilment of the requirements for a master's degree in Air Transport and Operations.

Thesis

Defense: February 19, 2016

Index Terms: Airline Operations, Reliability Engineering, Predictive Maintenance, Decision Support

Lennaert W.M. de Boer (4012682)
deboer.lwm@gmail.com
CO: Air Transport and Operations
Faculty of Aerospace Engineering
Delft University of Technology

Abstract

Costs associated to unscheduled and excessively prudent maintenance can contribute significantly to an airline's expenditure. Due to the complex nature of aviation related operations, reliability practices have been limited. In an attempt to improve component reliability, i.e. reduce maintenance-related costs, a new strategy was proposed which pursuits new avenues w.r.t. reliability modelling. The strategy focusses on identifying operational factors affecting component reliability and assessing whether these can be used to reduce the number of unscheduled occurrences (i.e. failures).

Currently, reliability models are limited to exponential distributions, which assume hazard rates remain constant throughout the component's operational life. Studies have shown that time-independent Proportional Hazard Models (PHMs) could improve overall reliability, however, due to poor data, this was not verified. In this solution, the benefits of both time-independent and -dependent PHMs are assessed. Furthermore, underlying hazard functions were supplemented by introducing: normal, log-normal, logistic, exponential, Weibull, and gamma distributions. To address repairables, restoration events were simulated using Kijima type II General Repair Processes (GRPs).

This project would not have been possible without the cooperation of QantasLink that supplied the data required. Results obtained from analysing historical data of the top ten components w.r.t. unscheduled removals indicated that adopting new maintenance schedules, derived from the proposed reliability models, could reduce the number of unscheduled occurrences by approximately 37% while limiting the increase in Mean Time Till (next) Repair (MTTRep). The variables identified by the solution were validated using literature, however, due to variable reduction and data limitations, the exact nature of component failures is yet to be established.

Undeniably the potential benefits of adopting the proposed strategy are extensive. Nonetheless, numerous assumptions were introduced to overcome challenges imposed by the complex nature of the data. To overcome these challenges, recommendations and suggestions were proposed for the future development of airline reliability practices.

Words: 299

Acknowledgement

Firstly, I would like to express my sincere gratitude to my supervisor, Dr. Ir. Wim Verhagen, for his unprecedented support during the thesis project and related research. Above all, his guidance and knowledge helped me during the development of the solution and the writing of my literature review and thesis. No one was better equipped to assist me when facing various challenges and struggles.

Besides my supervisor, I would like to thank my manager at QantasLink, performance and reliability engineer Reece Griffiths, for supporting this project from day one. This project would not have been realised without his knowledge on maintenance related operations and his connections at QantasLink.

My gratitude also goes out to Arlene Vera, Manager of Flight Data, who patiently sat beside and assisted me as I extracted over five years of Flight Data from QantasLink's servers.

Finally I would like to thank my colleagues and associates at QantasLink, the Delft University of Technology, and the University of Technology Sydney, as well as relatives working with Leiden University Medical Centre and Consultants for Development Programmes for their insightful comments and encouragement.

Table of Contents

List of Abbreviations and Symbols

Abbreviations

AC	Aircraft
AD	Anderson Darling
BLUE	Best Linear Unbiased Estimator
CDF	Cumulative Distribution Functions
CI	Confidence Interval
CM	Condition Monitoring
CS	Cramer-von Mises Smirnov
dCOX	Dependent COX
DIR	Dispatch Reliability Rates
EHM	Engine Health Monitoring
EVA	Extreme Value Analysis
FDR	Flight Data Recorders
FH	Flight Hours
FRACAS	Failure Reporting, Analysis, and Corrective Action System
GOF	Goodness-of-Fit
GRP	General Renewal Process
HPP	Homogeneous Poison Processes
HT	Hard Time
indCOX	Independent COX
KS	Kolmogorov Smirnov
LHS	Left Hand Side
MDA	Maximum Difference Analysis
MEL	Minimum Equipment List
MLG	Main Landing Gear
MMEL	Master MEL
MS	Mission Success
MTBF	Mean Time Between Failures
MTTF	Mean Time Till Failure
MTTRep	Mean Time Till (next) Repair
NHPP	Non-Homogenous Poisson Process
NLG	Nose Landing Gear
NRR	Nikulin-Rao-Robson Test
OC	On-Condition
OEM	Original Equipment Manufacturer
PDF	Probability Density Function
PHM	Proportional Hazard Models
PN	Part Number
RAMS	Reliability, Availability, Maintainability and Supportability
RHS	Right Hand Side
RP	Renewal Processes
SDR	Significant Defect Reports
SN	Serial Number
SoFa	State of Failure
SoFu	State of Functioning
UR	Unscheduled Removal
URR	UR Rates
US	Unscheduled
VAR	Variable

Symbols

Symbol	Description	Unit
β	Vector of covariate coefficients	-
δ	State or type of X (0 - censored, 1 - failed)	-
$\lambda(t)$	Hazard function	-
$\Lambda(t)$	Cummulative hazard function	-
$\mathcal{L}$	Log Likelihood	-
μ	Mean	-
∇	Partial derivative	-
ρ	Density	kg/m^3
σ	Standard deviation	-
Θ	Vector of unkown parameters	-
D	Direction	-
f(t)	Probability density function	-
F(t)	Cummulative density function	-
g	Group	-
H	Hessian	-
H_0	Null hypothesis	-
L	Likelihood	-
M_i	Set of flights related to maintanance event i	-
n	number of observations	-
N	Set of maintenance events	-
q	Dynamic pressure	Pa
R	Residual sum of error	-
R(t)	Reliability function	-
S_{AD}	Anderson Darling test	-
S_K	Kolmogorov test (with Bolshev's correction)	-
S_{NRR}	Nikulin-Rao-Robson test	-
S_W	Cramer-von Mises-Smirnov test	-
S(t)	See R(t)	-
t	Time, Inter-arrival time	Cycles
T	Component real age	Cycles
V	Velocity, Set of Variables, Virtual Age	m/s, -, Cycles
X	Independent variable	-
Y	Dependent variable	-
z	Z-score	-
Z	Vector of covariates	-

Subscripts and Superscripts

$\bar{x}$	Mean of x
$\hat{x}$	Estimate of x
A	All
C	Censored
d	Death (Failure)
f	Factor f, Final
F	Failed
i	Interval censored
l	Left censored
r	Right censored
s	Start
v	Variable

Executive Summary

In recent years the number of Low Cost Carriers has grown tremendously and with it the level of competitiveness in the industry (OAG 2012). To compete, airlines have explored new avenues in marketing, fare classes, services, and in overall quality. However, due to strict regulations and complexity, the preventative nature of aviation related maintenance has remained unchanged consisting primarily of Hard-Time, On-Condition and Condition Monitoring tasks.

Actual maintenance data shows that, due to the diverse operational conditions, the number of unscheduled removals can differ significantly per operator. As an example, see Figure A, which compares the number of unscheduled removals of QantasLink's top nine [1] components, with respect to costs induced by failures, and the world-wide fleet. The data implies that the preventative maintenance conditions proposed by aircraft component manufacturers can be inefficient.

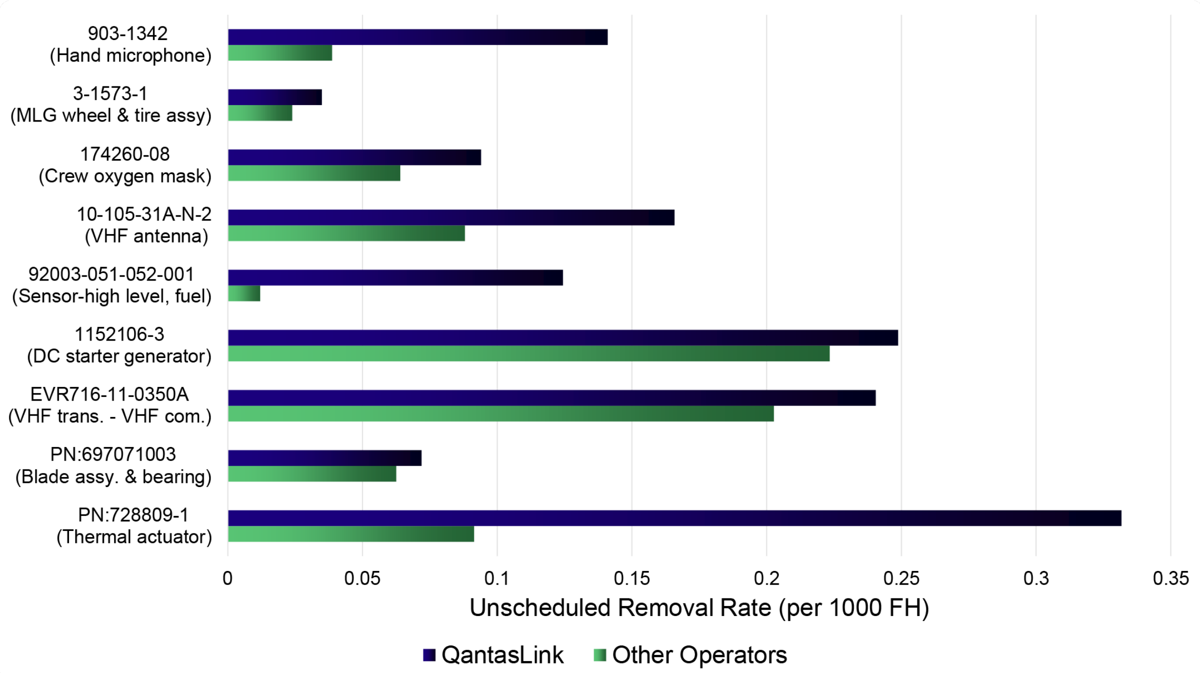


Figure A: Comparing QantasLink's top nine URR components [1] with other operators (Bombardier 2014*a*, Bombardier 2014*b*).

At the hope of reducing the costs associated to unscheduled maintenance events, well-known aircraft manufactures are designing systems that use real-time flight data to generate notifications such that operators can plan maintenance events prior to an aircraft's arrival. Theoretically these systems could reduce costs significantly for long-haul airlines operating at larger airports. However, these systems are ineffective for regional airlines, such as QantasLink, operating at smaller rural airports with limited maintenance capabilities.

To tackle this problem, other industries have implemented reliability models that use current and historic operational data to predict when a component will fail. Research has shown that, currently, reliability practices in the aviation industry are restrictive. This is due to the complex nature of aviation-related data and the quantity of data available. However, since the start of this century there has been a growing trend to monitor and collect data. This paper focuses on investigating whether this data can be used to identify factors related to component failures and determining whether these can be used to reduce the number of unscheduled maintenance occurrences.

A similar study was conducted before. However, this study was limited in terms of reliability models and the number of components. Furthermore, due to poor quality input data, the potential contribution of adopting more complex reliability models in the industry could not be quantified (Schotte 2015).

Research Objective and Proposed Solution

Following the aforementioned analysis the research question was formulated as follows:

> *"Can operational factors recorded by on-board Flight Data Recorders (FDRs) be used in a predictive nature to optimise maintenance schedules such that Unscheduled Removal Rates (URRs) are reduced?"*

The solution proposed in this paper consists of the consecutive application of five modules: Data acquisition and preparation, data analysis, reliability modelling, and decision support. The following section will briefly describe each module, such that the objectives and methodology used are better understood.

Data acquisition and preparation The focus of this module is to extract and prepare data from the on-board flight data recorders, TRAX maintenance management systems, and Engine Health Monitoring (EHM) Systems. The aforementioned data consists of over 320 million observations, hence the programming language used in this paper is R, which was designed for large datasets (Muenchen 2015).

Besides extracting the data, this module also prepares the data for Module 2, Data Analysis. Data preparation consists of the removal and completion of errors and missing/incomplete entries. In this module several assumptions had to be introduced to deal with the limitations of FDR and TRAX data, specifically with respect to the time of maintenance events and flights (Sec. 1). The final step in this module is the scaling of operational factors such that these can be interpreted directly during data analysis and reliability modelling.

As for most data processing solutions, data preparation and analysis needs to be designed explicitly for the format and structure of the datasets in question. Information regarding the nature and quality of the datasets is given in App. A and the literature review (de Boer 2015).

Data analysis The literature review stated that the focus of 'Data Analysis' was to identify the factors affecting component reliability using multivariate techniques, such as cluster analysis and discriminant analysis (de Boer 2015). However, due to the preventative nature of aviation related maintenance activities, the failure data consisted primarily of scheduled (censored) events (>90%), making the aforementioned analysis techniques inaccurate. As a result, the focus of this module shifted from factor identification to variable reduction, with as primary goal to reduce the 1531 input variables for reliability modelling.

To reduce the number of input variables the modules use four techniques: linear correlation, semi-parametric COX modelling, Extreme Value Analysis (EVA), and Maximum Difference Analysis (MDA). The implementation of linear correlation and semi-parametric COX models is well-explained in academic papers, EVA and MDA on the other hand were designed explicitly for this project. The focus of EVA and MDA is to identify which operational factors were significantly different between failure and non-failure related flights.

EVA focusses on identifying these operational factors in flights most recent to the failure. In essence, this technique determines whether a factor, at one instance of time, is responsible for the component's failure. EVA was designed as a quadratic optimisation problem, however, due to R's limitations with respect to quadratic optimisation, the technique had to be redesigned as a linear optimisation problem.

MDA focusses on identifying the operational factors that were significantly different during all flights related to a component's operational life-cycle by comparing the mean and standard deviation to that of non-failure related flights.

Reliability modelling In this module the failure data from Module 1, 'Data Acquisition and Preparation', and operational factors analysed in Module 2, 'Data Analysis', are used in a forward selection procedure to identify which operational factors most affect the component's reliability, up to a certain significance level, computed using a diverse selection of reliability models and underlying distributions.

Up to this date the most complex reliability models used in aviation related studies were limited to time-independent Proportional Hazard Models (PHMs) with a limited selection of distributions (Schotte 2015). To extend its scope the diversity of underlying distributions was extended significantly to include: normal, log-normal, logistic, exponential, Weibull, and gamma functions. This solution further distinguishes itself by attempting to improve reliability modelling of repairable components by utilising Kijima's (GRP) type II General Repair Processes (GRPs), which, in studies, proved to perform very well (Kijima & Sumita 1986). Finally, to account for operational factors varying over time, this solution also implements time-dependent PHMs to aviation-related failure data for the first time.

Models are compared using modified Kolmogorov Smirnov (KS), Carmer-von Mises-Smirnov (CS), Anderson-Darling (AD), and Nikulin-Rao-Robson (NRR) Goodness-of-Fit (GOF) tests (Sec. Background Information: *Goodness-of-Fit Tests*). In addition, the figures and tables provided in the decision support module can assist in the validation of underlying assumptions and selection of most effective models.

Forecasting In order to predict failures using time-independent and -dependent PHMs operational factors need to be forecast. The nature of aviation related operational factors can make forecasting very complex. To limit the scope, however, in this assignment operational factors are assumed to be related to seasonal and annual trends only. This accounts for changes in the atmosphere and aircraft loading due to seasonal changes.

To test the model, the reliability of various components was computed using operational factor forecasts up to 250 cycles in advance (approximately one month). The tests revealed that, within this interval, the computed error was well below 5%. At 100 cycles, the minimum number of cycles required by QantasLink, the extrapolated error is below 3% and 1% for time-independent and -dependent PHMs respectively.

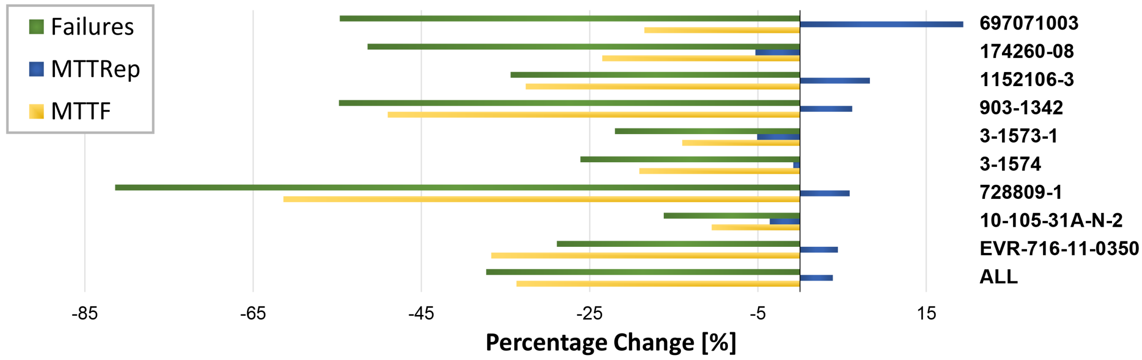
Decision Support The aim of this module is to assist the user in the identification of operational factors affecting component reliability and the selection of an appropriate model and threshold. To do so, the user is supplied with tables containing each model's GOF test scores and the percentage of unscheduled (scheduled) maintenance events prevented (postponed), computed using historical data. Furthermore, for graphical inspection the reliability and hazard functions of each model are presented.

For the convenience of the user a decision logic diagram was created, to be used in a step-wise manner with the aforementioned tables and figures in order to select the appropriate model and threshold for future maintenance activities.

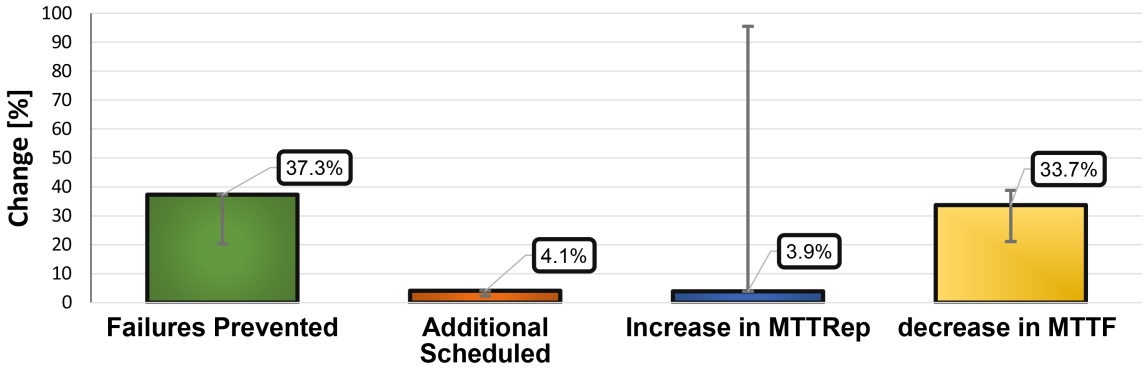
Results

The solution proposed in this paper was assessed using historical data obtained from the top 10 components [1] , with respect to URRs. Results showed that the cost of errors, obtained from estimating model parameters, could be minimised if failures were predicted assuming worst-case scenarios and by increasing the reliability threshold. Figure B shows that, of the previously recorded failures, between 16 to 81% could have been prevented, depending on the components criticality, with respect to failure costs, and the reliability level used. This corresponds to a total reduction of 37% of the failures (263 failures). Additional costs could have been saved from postponing current maintenance events, however, since the data was highly censored (over 90%), the increase in Mean Time Till (next) Repair (MTTRep) was limited such that the probability of unforeseen failures was minimised.
Overall, the results suggest that the implementation of the proposed strategy can reduce the number of unscheduled maintenance events significantly. Once the solution has been tested in real-life the models can be adjusted to allow for more (or less) time between repairs based on the operators demands.

In many of the reliability models between four and ten operational factors could be identified at a significance level of 99%. Of these, generally two (or three) affected component reliability significantly. The results show that the operational factors identified are often indirectly related to the true cause of the failure.



(i) Top 10 components.



(ii) Summary of top 10 components.

Figure B: Summary of potential outcome using historical data (Same as Fig. DC1).

As an example, consider pitch angle, vertical velocity, and torque during climb. Logically, all three could be correlated, however in some cases, only one is identified. As such, it remains a challenge to identify the true cause of a component's failure. Nonetheless, the factors identified in this solution can be used with logical reasoning to understand the nature of a components failure and hence its origin.

Implementation

The solution can be implemented in a variety of ways. As an example, consider the following non-invasive and invasive implementations:

Non-invasive implementation During this procedure, maintenance tasks are carried out and scheduled as usual. Using a reliability model obtained from this solution, every week the reliability of all components is predicted two weeks in advance (100 cycles). If it is 'almost' certain that a component will fail, e.g. reliability is extremely low ($\mathbf{0 \leq R_L \leq 15}$), a maintenance task is scheduled such that the component is removed/repaired. This non-invasive implementation guarantees that additional maintenance tasks are only performed once a component's failure is close to certain. This method will not accumulate any additional costs associated to over-maintenance.

Invasive implementation During this procedure all maintenance tasks are carried out and scheduled according to the solution proposed in this paper. Initially, the Mean Time Till (next) Repair (MTTRep) is set for all components according to a reliability model produced by this solution and using an averaged set of covariates. Every week the model is utilised, in combination with the component's age and operational history, to predict its reliability within the coming two weeks (100 cycles). If the component's reliability drops below the predetermined threshold, a maintenance task is planned. If not, no task are planned for the removal of that component within the coming weeks. Components with a predicted reliability level close to the threshold, i.e. $\mathbf{R_L \leq R_{comp} \leq R_L + 0.05}$, are put on to a watch list. The operator can use the watch list to determine whether multiple components can be removed/repaired during maintenance events.

Conclusion and Recommendations

The results obtained from evaluating historical data suggest that component reliability can be improved significantly. Factors associated to component failures could be identified, however it remains uncertain whether these were directly or indirectly related. Nonetheless the data suggests that, if current reliability thresholds and Mean Time Till (next) Repair (MTTRep) are maintained, over 37% of previously recorded failures can be prevented. In the solution, models and reliability thresholds are assigned such that changes in MTTRep are minimised. This guarantees that costs associated to errors are minimised.

Overall, the results imply that, depending on the nature of the component, both time-independent and -dependent Proportional Hazard Models, in combination with Kijima's Type II General Repair Processes, can significantly improve overall reliability. In addition, the identified operational factors can be used to explore new avenues to prevent failures, such as implementing/adapting operational procedures and (or) improving structural integrity.

In terms of future development, this paper has identified three area of significant improvement to reliability practices in the aviation industry:

1. In terms of data analysis: One of the major differences between failure data in aviation and that in other industries is the censoring level. Censored data does contains some information, however, with a significantly lower amount of failure data, approximately 10% of the observations, the accuracy of the reliability models is reduced. In general, by increasing the amount of data available, all reliability models can be improved. As an example, the quality and quantity of data can be increased by airline collaboration.

2. In terms of reliability modelling: The reliability models used in this solution are only valid given that the underlying assumption of one-failure mode is not violated. Often, when two or more failure-modes exists, the reliability models are inaccurate and incapable to identify the operational factors affecting component reliability. Up to this date, there are no numerical techniques to determine whether two (or more) failure-modes exists. As a consequence, in this solution the user is required to validate the assumption graphically. To improve this solution significantly, future research should focus on the development of a numerical algorithm that can identify multiple failure-modes. The algorithm can then be used to cluster failure data into multiple groups, one corresponding to each failure mode.

3. In terms of forecasting: In this paper it was assumed that a model incorporating annual and seasonal changes could accurately forecast operational factors. However, the facts remains that aviation related operational factors are stochastic and complex. Tests showed that the overall error produced using the model was highly dependent on the operational factors used. In the future, airlines would like to be able to forecast component-reliability beyond 100 cycles in advance. This can be done by researching the complex relationships between the operational factors and utilising external sources, such as weather forecasts and loading history, to forecast operational factors more effectively.

Notes:

[1] Analysis was performed on QantasLink's top ten components, w.r.t. unscheduled removal rates. Due to insufficient data, high-level fuel sensors (PN: **92003-051-052-001**) could not be analysed/modelled (see Sec. 3.7).

Introduction

With the aviation industry continuously growing and faced with an increased number of low-cost carriers, airlines are looking for new strategies to reduce costs while maintaining the high levels of security and safety imposed on the industry (IATA 2015). Up to this day, despite research showing promising results for (more) complex models, component-based reliability modelling has been limited to exponential hazard functions (Schotte 2015).

In collaboration with QantasLink, a regional airline operating in Australia, research was proposed to examine a variety of analysis and modelling techniques to establish whether operational factors causing component failures can be identified and whether this insight can assist in the forecasting of failures, such that maintenance schedules can be optimised and costs induced by component-failures reduced.

The problem

Component failures contribute to delays, maintenance fees, operational costs, and overall safety issues. To reduce the number of Unscheduled Removals (URs) the industry is focussed on preventive maintenance tasks such as: Hard Time (HT), On-Condition (OC), and Condition Monitoring (CM). Although these activities have reduced the number of URs, most operators still struggle with high Unscheduled Removal Rates (URRs) and the costs associated with these.

Figure I1 shows QantasLink's top nine components [1] that show significantly different Unscheduled Removal Rates (URRs) than experienced by other operators (Bombardier 2014*a*, Bombardier 2014*b*). In the aviation industry each operator is subject to unique operational conditions which vary annually, seasonally, and even daily, and as such, for each operator, this list may vary.

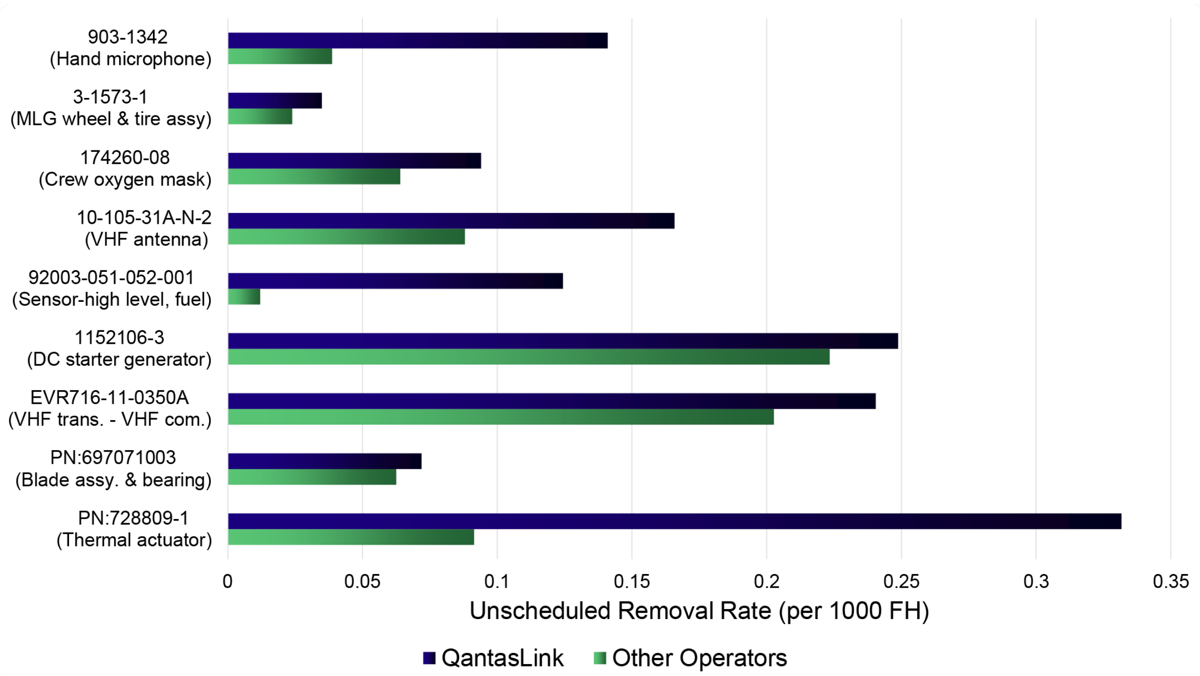


Figure I1: Comparing QantasLink's top nine URR components [1] with other operators (Bombardier 2014*a*, Bombardier 2014*b*).

The concept of reliability modelling has been around for quite some years, but its application and significance to the aviation industry have been limited due to the scarce availability of data and its accessibility. Studies indicate that complex reliability models show potential in increasing the reliability of components in the industry, but have failed to quantify this due to "poor" quality input data (Schotte 2015).

The aforementioned information suggests that there is growing demand for new strategies that can assist in the identification of failure-related operational factors and improve overall component-reliability. For this reason the main research question was defined as followed:

> **Research Question:**
> Can operational factors recorded by on-board Flight Data Recorders (FDRs) be used in a predictive nature to optimise maintenance schedules such that Unscheduled Removal Rates (URRs) are reduced?

Research proposal

Nowadays most civil aircraft are equipped with Flight Data Recorders (FDRs) and most operators are digitally logging their maintenance activities since the start of the 21^{st} century. With the collaboration of QantasLink the problem stated in the aforementioned research question was resolved and it was possible to make an attempt at quantifying the potential benefits of component-based reliability models.

In this paper a solution will be proposed in the form of a structured approach to identifying potential operational factors related to component failures and assessing their ability in improving component reliability. Referring back to my previous statement, reliability models currently used in the industry are limited in terms of functionality; due to the assumed constant hazard rate (Schotte 2015). This paper will not only consider time-based reliability models with restoration coefficients, but also time (in)dependent Proportional Hazard Models (PHMs) subject to General Repair Processes (GRP) with normal, log-normal, logistic, exponential, gamma, and Weibull underlying distributions. As such, in addition to improving component reliability at QantasLink, we hope to contribute to the current state of the art by providing insights into the possible benefits of using advanced reliability models in the industry.

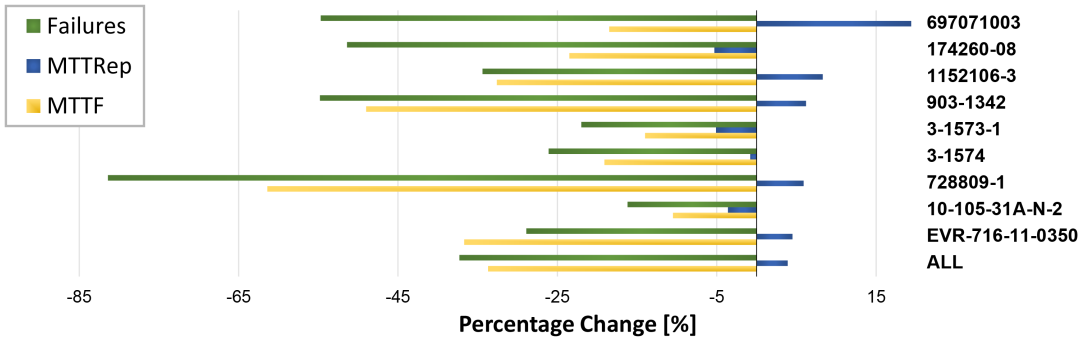
The program consists of four modules (Sec. 2): data acquisition, data analysis, reliability modelling, and predictions. As generally the case with software development projects, verification and validation are an ongoing process to verify the program's results analytically and validate it logically.

The solution

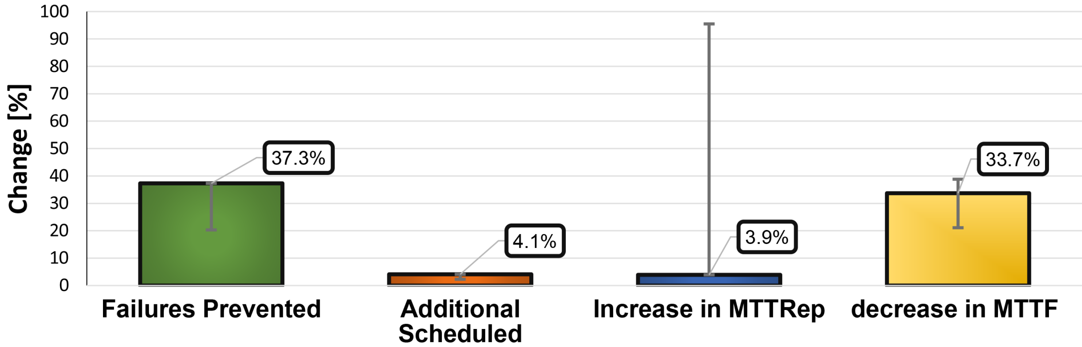
In order to obtain the final solution, numerous decisions and assumptions were made to overcome the challenges associated with analysing large-scale datasets and modelling highly censored failure data. An elaborate list of the assumptions made is given in Sec. 1. However, in short, to perform analysis the input data was processed to identify potential flights related to component failures and to reduce the number of operational factors. To perform reliability modelling, it was assumed that only one failure mode was present in the datasets. This assumption could be verified using graphical analysis, however, in a few cases it was evident that poor results were related to the 'potential' violation of the assumption. Finally, to limit the scope, forecasting was limited to 100 cycles and to annual (linear) and seasonal (sinusoidal) trend models.

At a first glance, the results obtained from analysing the top ten components [1] with respect to URRs indicated that reliability models currently used in the industry (exponential distributions) lack the proficiency to model failures accurately. In fact, graphical analysis and Goodness-of-Fit tests showed that, in most cases, significant improvements could be made if alternative time-based models were used.

To assess more complex time-independent and -dependent Proportional Hazard Models (PHMs), the results obtained from evaluating historical data of the top ten components with respect to URRs were used. In an ideal study, the input data can be split into two sets: the first to be used during analysis and reliability modelling, and the second to be used for evaluating the obtained models. In this paper the initial datasets were already limited in size (number of failures) and hence could not be split into two. Generally, the next best option would be to acquire a newer (or different) set of data. However, due to security regulations, obtaining a second set was not feasible. As a consequence, the results described below were computed using the same data used for analysis and modelling, which imposes its own set of limitations.



(a) Top 10 components.



(b) Summary of top 10 components.

Figure I2: Summary of potential outcome using historical data (Same as Fig. DC1).

Using historical data, Table DC1 in Sec. Discussion and Conclusion summarises the number of failures of the top ten components prevented with respect to URRs when maintenance schedules were adjusted using the strategy proposed in this paper. The results, shown graphically in Fig. I2, suggest that considerable costs could have been saved in the past by preventing a total of 263 (37.3%) of the recorded failures. Since component reliability is strongly correlated with age, we also observe a decrease in the overall Mean Time Till Failure (MTTF) of 33%.

These figures were computed using reliability models and thresholds that focused on limiting the increase in Mean Time Till (next) Repair (MTTRep). By limiting the increase, failure prevention is maximised while probability of unforeseen failures occurring from postponing scheduled events is minimised.

The reliability threshold used to compute these values was derived from current, component-specific, reliability levels which can be estimated using actual MTTRep and the reliability function of the empirical data (Kaplan Meier). For more information on the selection of an appropriate reliability model and threshold read Sections 3 and 4.

Time-independent and -dependent modelling showed that generally the covariates identified by each underlying distribution were the same. Nonetheless, the user should always be conscious of relationships among the operational factors and limitations imposed by data reduction. As an example, assume that dynamic pressure ($q_{inc.} = \frac{1}{2}\rho V^2$) was identified. Component reliability could indeed be directly related to dynamic pressure, but also, assuming incompressible fluid dynamics, to the fluid density or velocity.

To understand the true cause of failures, operators could run tests in collaboration with manufacturers. However, with respect to failure prevention, as long as an operational factor is a measure of the component's reliability, it does not matter whether the failure was a direct or an indirect cause.

To conclude, in this paper the objective was to establish whether operational factors affecting component reliability could be identified, and, if so, whether these could be used to reduce the number of unscheduled occurrences. It is evident from the results that operational factors directly and (or) indirectly affecting component reliability could be identified. However, in cases of multiple failure modes, the results were inconclusive. Generally this can be solved by splitting the dataset into two (or more) groups and by finding more effective ways, other than graphical techniques, to address failure mode identification.

Results derived from analysing historical data (Tab. DC1 in Sec. 4) show that up to 37% of the failures can be prevented whilst limiting increases in MTTRep. Although these results suggest that a significant amount of cost can be saved, almost 90% of the input data was censored. As a consequence, costs induced by increasing the MTTRep cannot be quantified. Nonetheless, by preserving current reliability levels, or increasing them, we managed to minimise the impact of unforeseen 'new' failures.

Report Structure

As an aid to the reader, below is an ordered list of the elements discussed in this paper:

Background Info.	To assist the reader, the concepts of data analysis, reliability modelling, and goodness-of-fit tests are introduced. This section also briefly describes the various datasets and presents the challenges associated with censored data.
1. Assumptions	A complete list of the assumptions made is given in this section. The motivation behind each assumption is further discussed in Sec. 2.
2. Methodology	This section contains a comprehensive description of the methodology used to obtain thhe final solution. This includes: data acquisition, data analysis, reliability modelling, and forecasting.
3. Results	In this section the results obtained from analysing and modelling the top ten components, w.r.t. URRs, are presented.
4. Solution Decision Logic	In order to assist the future user, this section introduces a decision logic diagram that can be used, in combination with the results generated by the program, to select a reliability model and threshold.
Discussion & Conclusion	In this section the results, described in Sec. 3, are discussed and final conclusions regarding the solution and results are presented.
Recommendations and Suggestions	Based on the discussion and conclusion, bottlenecks and areas of interest are identified for future studies and presented in this section.
Implementation	As a side note: a brief description on the implementation of this strategy is outlined.
References	Lists the references cited.
Appendix	Supplemental information, tables, and figures.

Notes:

[1] Analysis was performed on QantasLink's top ten components, w.r.t. unscheduled removal rates. Due to insufficient data, high-level fuel sensors (PN: **92003-051-052-001**) could not be analysed/modelled (see Sec. 3.7).

Background information

To present a justification of the content and nomenclature used in this paper the following section introduces the data made available by QantasLink, the concept of data analysis, and the theory behind reliability modelling. Due to the preventative nature of maintenance in the aviation industry reliability data is often censored. Censoring has its own implications for data analysis and reliability modelling, hence, in addition to the aforementioned topics, this chapter will also describe censoring as well as the modifications performed to statistical goodness-of-fit tests to adjust the database. This section is concluded with a brief description of R, a programming language used frequently for large scale data analysis.

This section is based on the findings of the literature review performed as part of this thesis project (de Boer 2015). For further information regarding data analysis techniques, specifically in the multivariate domain, the reader is referred to "Multivariate data analysis" (Jr., Black, Babin & Anderson 2009). More information regarding reliability models can be found in an online portal for reliability engineers and professional (ReliaSoft 2015).

Data sets (QantasLink)

Data provided by QantasLink forms the foundation of this thesis project. As in the case with any other statistical project, it is essential to understand the nature of the input variables. The following section describes the relevant sources that provide input data for this thesis. To be adequate, additional detail on each source (acquisition, type, and quality) is provided in Appendix A.

The information provided in the following section and in Appendix A is almost equivalent to the Sections presented in the literature study (de Boer 2015).

Aircraft Maintenance Management System (TRAX) TRAX is an airline Maintenance Management Suite (TRAX Systems 2015*b*) aimed at assisting airlines in tracking, monitoring, and planning maintenance and engineering activities. TRAX allows its users to input schedules, defects, incidents, component utilisation and shop data, Minimum Equipment Lists (MELs), technical defect and supplementary maintenance data, and Significant Defect Reports (SDR). By doing so the airline can monitor its inventory, schedule maintenance events, and identify trends.

TRAX contains a significant amount of information on daily operations and has been in use since 2004. It is important to note that all data collected by TRAX is acquired by human input and thus is subject to error.

More detail in Appendix A.1.

Flight Data Recorders (FDR) Flight Data Recorders are used to monitor in-flight conditions such as flight crew inputs and outputs measured by sensors on-board the aircraft. FDRs are not subject to human interaction prior to data retrieval and hence are only subject to systematic errors. The software/server interfacing with the FDRs pre-filters all the data and identifies outliers, missing values, and errors prior to retrieval.

More detail in Appendix A.2.

Engine Health Monitoring (EHM) Engine Health Monitoring is an initiative of QantasLink to assess the health of an engine. The method monitors an engine's performance based on five different aspects. Once each component has been assessed the engine is given a grade defining how often it will be inspected and, in case it has to be removed, the maximum amount of Flight Hours (FH) before removal has to take place.

Data returned from EHM is categorical and may vary depending on the observer.

More detail in Appendix A.3.

Original Equipment Manufacturer (OEM) technical information services Bombardier has the responsibility to provide the design, parts, assemblies, and technical services to its customers. Bombardier provides many services, one of which are quarterly and semi-annual Failure Reporting, Analysis, and Corrective Action System (FRACAS) reports (Bombardier 2014*b*, Bombardier 2014*a*). The FRACAS reports provide information regarding aircraft statistics, utilization, system & component statistics, and subsystem/component Dispatch Reliability Rates (DIR) & Cancellation Rates.

FRACAS reports provide insights on current maintenance operations w.r.t. other airlines operating Bombardier Dash 8s. In addition FRACAS reports also contain the latest test results of components on-board the aircraft. The reports have the potential to assist in the identification of weaknesses in current maintenance practices.

More detail in Appendix A.4.

AC technical delay report AC delay reports track the delays that occur during the day. For each delay/defect, it is logged when the issue was raised (in terms of time, flight, aircraft, and station), why the delay occurred (e.g. component or system failure, warning indicators, failed check-ups, etc.), and when & how the event was resolved (e.g. part was replaced with component B).

The reports are primarily used to identify events with large repercussions. Once an event has been identified procedures are adjusted such that their impact on operations in the future is reduced.

More detail in Appendix A.5.

Data analysis

In recent years, the rapid growth of informational technology has enabled users to record, monitor, and store data in a multitude of forms. With this trend, the demand for large-scale multivariate analysis techniques has increased drastically. Data analysis is the process of filtering, transforming, and modelling of data for a multitude of objectives, such as: hypothesis testing, forecasting & predicting, sorting, reducing, & filtering, and dependence analysis (Johnson & Wichern 2007, Jr. et al. 2009).

The following section will first present the theory behind data analysis followed by a brief introduction of the concepts of multivariate regression and cluster analysis. The section is concluded with an overview of the Z-test, which was used in combination with linear optimisation to identify operational factors during the analysis phase.

What is data analysis?

In general, there are two major types of data: non-metric (qualitative) and metric (quantitative), and four types of scales: nominal, ordinal, interval, and ratio. Modern day data analysis techniques are subdivided into six categories: descriptive, exploratory, inferential, predictive, causal, and mechanistic. The *type* of data and *research objective* are the primary factors influencing the data analysis techniques used. In general terms, data analysis consists of the following steps:

Processing The process of combining the data (in rows, columns, matrices) in preparation of cleaning and modelling.

Cleaning The process of removing outliers, finding errors, and missing values.

Preparing The process of standardising, normalising, rescaling, or weighting data prior to analysis.

Reduction The process of reducing the number of variables using any multivariate technique (e.g. cluster analysis).

Analysis The process of performing descriptive, exploratory, inferential, predictive, causal, and (or) mechanistic analysis on the data.

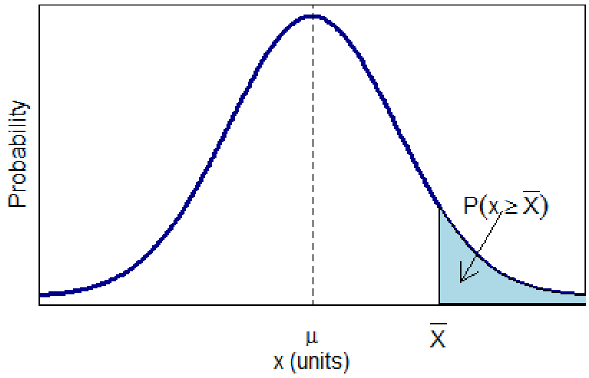


Figure BG1: Overview of Z-distribution and statistic.

Multivariate regression and cluster analysis

Motivated by the type of input data (Sec.) and research objective: "*identify which operational factors (if any) affect component reliability*", discrete multivariate regression (or multiple discriminant analysis) and cluster analysis were identified as potential candidates for data analysis in the literature review (de Boer 2015).

The goal of discrete multivariate regression is to approximate a model that can predict the changes in one or more (metric or non-metric) dependent variables in response to known changes in one or more independent variables (BG1).

$$\text{Discrete Multivariate Regression} \quad \overset{\text{(metric, non-metric)}}{\boldsymbol{Y_1, Y_2, \cdots, Y_n}} = \overset{\text{(metric, non-metric)}}{\boldsymbol{X_1, X_2, \cdots, X_n}} \tag{BG1}$$

Discrete multivariate regression is an extension of well-known multivariate regression to address dichotomous, polytomous, ordinal, and count dependent variables (Orme & Combs-Orme 2009, Jr. et al. 2009). The technique assumes the relationships among the dependent and independent variables are linear, and that errors are equally distributed, independent, and normal. Despite the lack of statistical tests to test the assumptions, there are enough tests to verify the validity of the results.

Cluster analysis is an independence technique used to identify groups ("clusters") within datasets. In general terms the technique often consists of three steps: determine number of clusters, allocate each observation into its respective group (clustering process), and finally to inspect the groups to identify features that distinguish them. Cluster analysis is a quantitative method, hence does not rely on any statistical assumptions.

The Z-test

After the two aforementioned analysis techniques it became evident, due to the preventative nature of aviation data (censored data), that discrete multivariate regression and cluster analysis were not reliable methods to identify operational factors (Sec. 2.2.5). To address this issue Z-tests were implemented in combination with linear optimisation (Sec. 2.2.3.1).

$$\begin{array}{lll} \text{Null hypothesis} & \text{where,} & \\ \boldsymbol{H_0 = X_f \sim Norm(\mu_f, \sigma_f)} & \boldsymbol{f} & \boldsymbol{= 1, 2, \ldots, F} \\ & \boldsymbol{F} & \sim \text{Number of operational Factors} \\ & \boldsymbol{X_f} & \sim \text{Observations of factor } \boldsymbol{f} \\ & \boldsymbol{\mu_f} & \sim \text{Mean factor } \boldsymbol{f} \\ & \boldsymbol{\sigma_f} & \sim \text{Standard deviation factor } \boldsymbol{f} \end{array} \tag{BG2}$$

The Z-test is a statistical test in which it is assumed that the distribution of the test statistic under the null hypothesis (BG2) is normal. Due to the central limit theorem, this assumption is specifically true for large samples. In practice the test is most often used to determine whether an observation X belongs to the normal distribution with mean μ and variance σ^2.

Reliability modelling

Failures are caused by various failure mechanisms and in the domain of reliability engineering, it is of interest to determine how long a product will remain functional (reliability function), how long the maintenance task will last (maintainability function), and how long it will take to support the maintenance task (supportability function) (Verhagen 2014). Environmental factors affect both reliability and maintainability of a component. In addition, reliability can also be affected by operational factors and maintainability can be affected by personal factors. Supportability is affected by location and organisational factors. Availability of a product is dependent on the reliability, maintainability and supportability functions.

The focus of this thesis is to identify what operational factors affect component-reliability and to determine whether these factors can be used to improve component-reliability by means of reliability models. The following section will cover the underlying concepts of reliability engineering, as well as the principles behind the modelling of non-repairables and repairables using time-based, time-independent and time-dependent Proportional Hazard Models (PHM).

For a recap on Reliability, Availability, Maintainability and Supportability (RAMS) the reader might want to read the literature review (de Boer 2015), or use the online reliability portal for professionals (ReliaSoft 2015).

Reliability

In this report functionality is defined as "the inherit characteristics of a product related to its ability to perform a specified function according to the specified requirements under the specified operating conditions" (Verhagen 2014). A product (or component) is said to be in a State of Functioning (SoFu), when it is functional (as defined above), and a State of Failure (SoFa) if it fails to meet the specified requirements. Components that can be restored to SoFu from SoFa are referred to as restorables (repairables), those that cannot are non-restorables (non-repairables).

The reliability function, $R(t)$, also referred to as the survivability function, $S(t)$, gives the probability of a component surviving up to time t (BG3).

$$R(t) = Pr(T > t), \qquad F(t) = 1 - R(t) = Pr(T \leq t) \tag{BG3}$$

The complement of the reliability function, often labelled $F(t)$, computes the probability that a component will fail before time t and is referred to as the lifetime distribution function. Its differential, denoted $f(t)$, gives the rate of failures per unit time (event density).

In practice engineers are often interested in computing the probability of a component surviving between the interval $[t_s, t_f]$. This probability is referred to as Mission Success (MS) and can readily be found using $R(t)$ (BG4) (Ghobbar 2011).

$$\text{MS}(t_s, t_f) = P(T > t_f | T > t_s) = \frac{R(t_s) - R(t_f)}{R(t_s)} \tag{BG4}$$

Quite often, especially for non-repairables it is useful to compute the Mean Time Between Failures (MTBF) or Mean Time Till Failure (MTTF) (BG5). In literature, MTBF and MTTF are the terms used to describe the expected time till failure for repairable and non-repairable systems respectively (ReliaSoft 2015). For the remainder of this paper MTBF will be referred to as MTTF which will describe the mean time till failure for both repairable and non-repairable systems.

$$\begin{aligned} \text{Event density} \quad & f(t) = \frac{dF(t)}{dt} \\ \text{MTBF/MTTF} \quad & E(t) = \int_0^\infty t f(t)\, dt = \int_0^\infty R(t)\, dt \end{aligned} \tag{BG5}$$

The hazard function, denoted $\lambda(t)$, is used to understand the failure properties of a system (component) and is defined as the instantaneous event rate at time t or more specifically:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{Pr(t \leq T < t + \Delta t)}{\Delta t \cdot Pr(T > t)} = \frac{f(t)}{R(t)} \tag{BG6}$$

Figure BG2 presents the relationship between hazard rate and various failure properties.

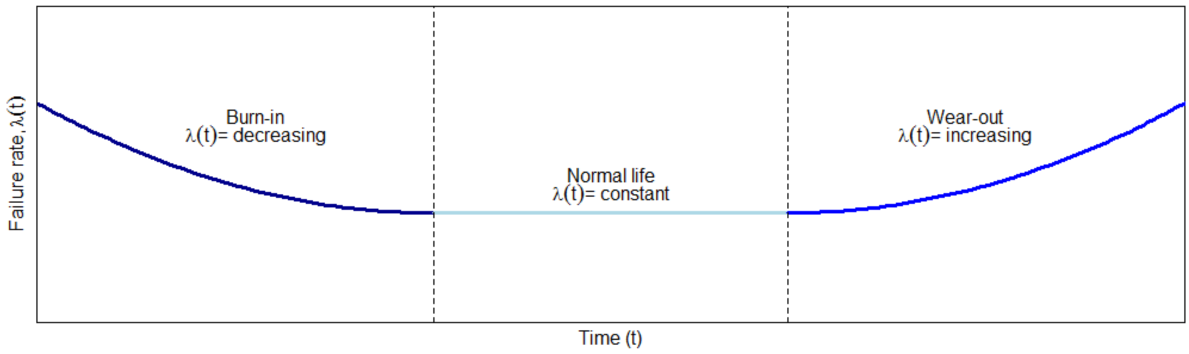


Figure BG2: Using the hazard function to understand a component's failure properties.

The hazard function and reliability function can both be used to compute the cumulative hazard function $\Lambda(t)$, a function used to predict the expected number of failures up to time t or between interval $[t_0 \; t_1]$ (BG7).

$$\begin{aligned} \Lambda(t) &= -\log(R(t)) = \int_0^t \lambda(t)\, dt \\ \text{or equivalently} \quad R(t) &= e^{-\Lambda(t)} \end{aligned} \tag{BG7}$$

Modelling

The aforementioned section covered the basic principles related to reliability modelling and engineering. The following section will briefly outline the various models used in the modelling process. Derivations and other research specific tasks are outlined more explicitly in Sec. 2.

Standard nomenclature It is common in reliability practices to present data in the form of (t_i, δ_i), where t_i denotes the failure (or censoring) time and δ_i equals 0 if observation i is censored and 1 otherwise (BG8).

$$\begin{aligned} X &\sim \{X_1, X_2, \ldots, X_n\} \\ X &\sim \{(t_1, \delta_1), (t_2, \delta_2), \ldots, (t_n, \delta_n)\} \end{aligned} \tag{BG8}$$

In the remainder of this paper, X will denote the set of observations ($X = X_1, X_2, \ldots, X_n$) where each observation defines a failure or censoring time t_i and the corresponding δ_i.

Time-based models Time-based reliability models use component age (time) to model reliability. In its simplest form its lifetime distribution function ($F(t)$) and event density ($f(t)$) are based on common statistical distribution functions (e.g. exponential, normal, log-normal). For more complex components, such as repairables, the statistical models can be reformulated to include a renewal parameter (also known as a Repair Process). Research has shown that type II General Renewal Processes (GRP-II) generally provide better estimates than [Non] Homogeneous Poison Processes ([N]HPPs) and Renewal Processes (RPs) (Kijima & Sumita 1986, Kaminskiy & Krivtsov 2006, Kaminskiy & Krivtsov 2010). In GRP models the i^{th} failure is formulated using the previous failure time ($(i-1)^{th}$ failure) and with a renewal function, derived from the renewal parameter. More information on the theory and application of GRP-II models with various underlying statistical distributions is presented in Sec. 2.3.

Proportional Hazard Models (PHMs) Proportional Hazard Models, also known as COX models, are an extension to time-based models by introducing covariates. The standard (time-based) statistical hazard function is reformulated to introduce the covariates and corresponding parameters β (BG9).

$$\lambda(t, Z|\theta, \beta) = \lambda_0(t|\theta)e^{\beta^T Z} \tag{BG9}$$

In (BG9) Z are the covariates corresponding to failure t, $\lambda_0(t)$ is the underlying hazard function (e.g. normal distribution), θ (θ_0, θ_1,..., θ_p) denotes the unknown parameters of the underlying distribution function, and β (β_0, β_1,..., β_c) denotes the unknown parameters corresponding to each covariate.

Equation (BG9) can readily be reformulated to incorporate time dependent covariates $Z(t)$. The drawback of using time dependent covariates is that the computation of the unknown parameters θ and β becomes more complex and resource demanding.

Censoring

In statistics an observation is considered censored if its value is only partially known. As an example lets identify two cases: in case one, a component is operated till failure, and in case two, the component is operated till it has successfully completed 300 Flight Hours (FH). In the first case, the exact time of failure is known, however in the second it is only known that the component has survived beyond 300 FH. The aforementioned example describes Type I right censoring, a censoring type which occurs when the experiment (operation) is stopped at a predetermined time.

Censoring is common in the aviation industry as component maintenance is preventative (e.g. hard time, on-condition) orientated. The theory and application of other censoring types (e.g. left, interval, type II) is readily available, however in context to this paper only right censoring will be considered. To account for censored data, the Maximum Likelihood Estimator, used in the computation of model coefficients can be reformulated to (Stevens 2013, Zhang 2005, Ionescu & Limnios 1999):

$$\text{Likelihood,} \quad L = \prod_{d \in D}^{n_d} f(t_d) \prod_{l \in L}^{n_l} (1 - S(t_l)) \prod_{r \in R}^{n_r} S(t_r) \prod_{i \in I}^{n_i} (S(U_i) - S(V_i)) \tag{BG10}$$

In (BG10) subscript d (l, r, i) denotes the d^{th} (l^{th}, r^{th}, i^{th}) observation corresponding to the group of failure truncated (left, right, interval censored) observations D (L, R, I). n_g denotes the total number of observations belonging to observation group $g = \{D, L, R, I\}$.

Goodness-of-fit tests

The testing of models derived from censored data has been an ongoing issue in reliability engineering (D'Agostino 1986). The following section will briefly outline how common tests such as Kolmogorov, Cramer-von Mises-Smirnov and Anderson-Darling were modified to account for right censored data. Study has shown that the modified tests depend highly on the distribution of censoring times and perform poorly when the data is truly randomly censored (Nikulin, Lemeshko, Chimitova & Tsivinskaya 2010). The study further assessed the Nikulin-Rao-Robson (NRR) χ^2 test, which is less subjective to variations in the censoring distribution times, however is limited to smaller sample sizes (Balakrishnan, Chimitova & Vedernikova 2014). In the absence of (or with limited) censoring, several studies suggest transforming the censored models into complete samples, such that classical Kolmogorov, Cramer-von Mises-Smirnov and Anderson-Darling tests can be performed (Chimitova, Tsivinskaya & Vedernikova 2010).

Modified test statistics The following section will briefly introduce the modified test statistics as defined in the research of (Chimitova et al. 2010, Nikulin et al. 2010, Balakrishnan et al. 2014). The reader is expected to be familiar with the well establish Kaplan-Meier estimator for Cumulative Distribution Functions (CDF) $\hat{F}_n(t)$. Further we define the models

CDF as $F(t|\theta)$. The modified Kolmogorov test statistic with Bolshev's correction can then be found using:

$$S_K = \frac{6nD_n + 1}{6\sqrt{n}} \tag{BG11a}$$

where,

$$D_n = \sup_{t<\infty} |\hat{F}_n(t) - F(t|\theta)| \tag{BG11b}$$
$$n \sim \text{Number of observations}$$

The modified Cramer-von Mises-Smirnov test statistic can be computed using:

$$S_W = \int_{-\infty}^{\infty} \left(\hat{F}_n(t) - F(t|\theta)\right)^2 dF(t|\theta) \tag{BG12a}$$

or using empirical data,

$$\begin{aligned} S_W = & \frac{n_r}{3} + n_r \sum_{i\in\delta_i=1} \Big[\hat{F}_n^2(t_i)\big(F(t_{i+1}|\theta) - F(t_i|\theta)\big) \\ & - \hat{F}_n(t_i)\big(F^2(t_{i+1}|\theta) - F^2(t_i|\theta)\big)\Big] \end{aligned} \tag{BG12b}$$

Finally the modified Anderson Darling test statistic can be derived from:

$$S_{AD} = \int_{-\infty}^{\infty} \left(\hat{F}_n(t) - F(t|\theta)\right)^2 \frac{dF(t|\theta)}{F(t|\theta)(1 - F(t|\theta))} \tag{BG13a}$$

and computed using:

$$\begin{aligned} S_{AD} = & - n_r + n_r \sum_{i\in\delta_i=1} \Big[\big(\hat{F}_n^2(t_{i-1}) - \hat{F}_n^2(t_i)\big) \log\left(F(t_i|\theta)\right) \\ & - \left(\big(1 - \hat{F}_n(t_{i-1})\big)^2 - \big(1 - \hat{F}_n(t_i)\big)^2\right) \log\left(1 - F(t_i|\theta)\right)\Big] \end{aligned} \tag{BG13b}$$

In the above formulae n_r denotes the number of uncensored observations.

Due to the limitations imposed on randomly distributed censoring the Nikulin-Rao-Robson (NRR) χ^2 test was also introduced. The test statistic has proven to be more effective with randomly distributed but has limited power with large datasets. Prior to computing the test the data must first be grouped into k intervals. Nikulin suggested selecting the intervals such that the expected number of failures in each group is approximately equal (Nikulin et al. 2010).

The paper suggests its own method for allocating group interval times a_j, however in this paper groups were allocated by computing the total number of expected failures (E_k) and the approximate expected number of failures per interval j (e_a):

$$E_k = \sum_{i=1}^{n} \Lambda(t_i|\theta), \qquad e_a = \frac{E_k}{k} \tag{BG14a}$$

To ease the allocation of intervals, the function $E(t_0, t_1)$ is specified (BG14b) such that the number of expected failures in the interval $[t_0, t_1]$ is returned.

$$E(t_0, t_1) = \sum_{i=1}^{n_1} \Lambda(t_i|\theta) - \sum_{i=1}^{n_0} \Lambda(t_i|\theta) = \sum_{i=n_0}^{n_1} \Lambda(t_i|\theta) \tag{BG14b}$$

Then by arranging each observation $X_i = (t_i, \delta_i)$ in occurrence of time such that $0 \le t_1 \le t_2 \le \cdots \le t_n$ and iterating through i we can find each interval time a_j where $a_0 = 0 \le a_1 \le a_2 \le \cdots \le a_k = t_n$(BG15).

$$\begin{aligned} a_0 &= 0 \\ a_1 &= t_j, \quad \text{where} \quad E(a_0, t_j) \le e_a \le E(a_0, t_{j+1}) \\ a_2 &= t_j, \quad \text{where} \quad E(a_1, t_j) \le e_a \le E(a_1, t_{j+1}) \\ &\vdots \\ a_{k-1} &= t_j, \quad \text{where} \quad E(a_{k-2}, t_j) \le e_a \le E(a_{k-2}, t_{j+1}) \\ a_k &= t_n \end{aligned} \tag{BG15}$$

Now that each interval j is specified, $I_j = (a_{j-1}, a_j | a_0 = 0, a_k = t_n)$ for $j = 1, 2, \ldots, k$, the NRR χ^2 test statistic S_{NRR} can be computed.

$$S_{NRR} = Z^T \hat{V}^{-1} Z \tag{BG16a}$$

where Z can be found using

$$\begin{gathered} Z_j = \frac{1}{\sqrt{n}}(U_j - e_j) \\ U_j = \sum_{i:X_i \in I_j} \delta_i, \qquad e_j = E(a_{j-1}, a_j) \end{gathered} \tag{BG16b}$$

and $\hat{V}^{-1}$ is computed using

$$\begin{gathered} \hat{V}^{-1} = \hat{A}^{-1}\hat{C}^T\hat{G}^{-1}\hat{C}\hat{A}^{-1} \\ \textbf{diag}(A_j) = \frac{U_j}{n}, \qquad \hat{G}_{ll'} = \hat{i}_{ll'} - \sum_{j=1}^{k} \hat{C}_{lj}\hat{C}_{l'j}\hat{A}_j^{-1} \qquad \text{for} \quad (l, l') = 1, 2, \ldots, m \\ \hat{i}_{ll'} = \frac{1}{n}\sum_{i=1}^{n} \delta_i \frac{\delta\, \textbf{ln}(\lambda(t_i|\theta))}{\delta\theta_l} \frac{\delta\, \textbf{ln}(\lambda(t_i|\theta))}{\delta\theta_{l'}} \\ \hat{C}_{lj} = \frac{1}{n} \sum_{i:X_i \in I_j} \delta_i \frac{\delta}{\delta\theta_l}\, \textbf{ln}(t_i, \theta) \end{gathered} \tag{BG16c}$$

In the above formulae m denotes the number of model parameters θ estimated during modelling. The NNR χ^2 test statistic requires the computation of the partial-derivatives of the log hazard function. As this function depends on the restoration factor q and on the covariate coefficients β in time-(in)dependent PHMs it can become rather complex. Hence to compute the partial derivatives a numerical approach was adopted based on the findings of K. Kopecky (Kopecky 2007, Miranda & Fackler 2004, Nocedal & Wright 2006).

The limit distribution of the statistic S_{NRR} is χ^2_r with $r = \textbf{rank}(V^{-1})$ degrees of freedom (Nikulin et al. 2010, Chimitova et al. 2010).

The aforementioned tests, Kolmogorov, Cramer-von Mises-Smirnov, Anderson-Darling, and NRR χ^2, have shown to yield poor results for highly censored datasets. Despite this drawback, in cases of high censoring, the tests can still be used for comparing multiple models.

Programming language

In the literature study Python was identified as the programming language of choice, due to its well-established modules in terms of data analysis, and its similarities with Matlab, a programming language familiar to the author of this paper. After implementing and testing two similar modules used for data fetching and processing, results indicated that R was significantly quicker at processing the FDR data due to Rs well established 'ff' package (Adler, Gläser, Nenadic, Oehlschlägel & Zucchini 2015).

For those unfamiliar with R, this is a programming language for statistical computing and graphics. In terms of statistics R is the most used language with the number of users still growing (Muenchen 2015). To cope with excessive resource utilisation, generative functions were implemented and in hopes of increasing computational speed, the usage of loops was minimised and procedures were written/tested in parallel.

In context of this thesis the following packages were utilised:

ff/ffbase	Used in the acquisition and processing of flight data.
MASS	Support functions for the application of modern statistical solutions.
fpc/mclust	Used during clustering analysis.
mvnormtest	Used in the testing of normality in multivariate variables.
lpSolve	Linear optimisation module used for the identification of extreme values in failed flights.
numDeriv	Compute numerical derivatives.
Reliability/ flexsurv	Adds additional survival functions to R to assist in survival analysis.
goftest	Goodness-of-fit tests for complete datasets (not censored).
jpeg & tiff	Used to convert images produced by program.

To compute the reports, this program relies on the command-shell 'xelatex' command available in the open-source project MiKTeX (MiKTeX 2016). In the case that this resource is not available, the user has to find an alternative way to compile the .tex files produced in the output folder.

1 Assumptions

The following section will briefly summarise the assumptions relevant to the final solution. The assumptions were derived from the challenges identified during the production phase hence Sec. 2 elaborates on the motive(s) behind each assumption. To aid the reader the assumptions are categorised per module:

A. Data acquisition and processing

A1 All maintenance events are logged in TRAX.
A2 Failure events are listen as 'Unscheduled'.
A3 Incomplete/censored events are listen as 'Scheduled'.
A4 Maintenance events occurring between 00:00 and 05:00 AM relate to failures occurring the day before.
A5 Maintenance events with zero operational cycles are assumed to be non existent.
A6 All flights occurring on day of maintenance event are 'potentially' related to the event.
A7 All flights are listed in FDR data.
A8 Flights listed per day are ordered in time of occurrence.
A9 Operational factors with more than 80% of the measurements missing are irrelevant.
A10 Operational factors with more than 80% of identical measurements are irrelevant.
A11 Flights with all operational factors missing are assumed to be 'normal' (averaged).

B. Data analysis

B1 Operational factors are normally distributed in accordance with the central limit theorem.
B2 Extreme value analysis and maximum difference analysis (Sections 2.2.3.1 and 2.2.3.2) are effective techniques for preliminary operational factor identification.
B3 During Extreme Value Analysis, Sec. 2.2.3.1, flight variables with a probability of 60% or larger for being significantly different from the norm are relevant.
B4 During Maximum Difference Analysis, Sec. 2.2.3.2, flight variables with a probability of 80% or larger for being significantly different from the norm are relevant.
B5 During variable reduction (linear correlation), Sec. 2.2.3.3, operational factors with linear correlation $\geq \mathbf{0.95}$ are considered related.
B6 Correlated operational factors can be represented using the variables showing the largest difference from the norm.

C. Reliability modelling

C1 Components can be modelled using time-based, time-independent Proportional Hazard (PH), and (or) time-dependent PH reliability models with underlying normal, log-normal, logistic, exponential, Weibull, and (or) gamma distributions adjusted for General Repair Processes (GRPs).
C2 Component failures are related to one failure-mode.
C3 Nelder-Maed numerical optimisation algorithms are most effective, w.r.t. time and accuracy, in estimating model coefficients using Maximum Likelihood Estimation (MLE).
C4 Forward-selection variable identification techniques are most effective, w.r.t. time and accuracy, in identifying factors related to component reliability.
C5 Forward-selection with a significance level threshold of $\geq \mathbf{0.95}$% will identify relevant operational factors.
C6 Forward-selection assumes variables are uncorrelated (independent).
C7 Numerical differentiation is an accurate estimator of a function's differential at x.
C8 The Fisher information matrix is approximately equal to the Hessian of the MLE function.
C9 The Delta method is effective in estimating the confidence intervals of the reliability and (or) hazard function.

D. Future predictions

D1 Trend-line fitting using historical data is effective at forecasting operational factors.
D2 Linear models capture annual changes.
D3 Sinusoidal models with a period of 365.25 capture seasonal changes.

2

Methodology

The aim of this thesis was to identify and investigate the effects of operational factors on component reliability. The following section outlines the methodology used to successfully address the issue of factor identification and reliability modelling. It is assumed that the reader is familiar with the concepts of data analysis and reliability modelling mentioned in Sec. *Background information*.

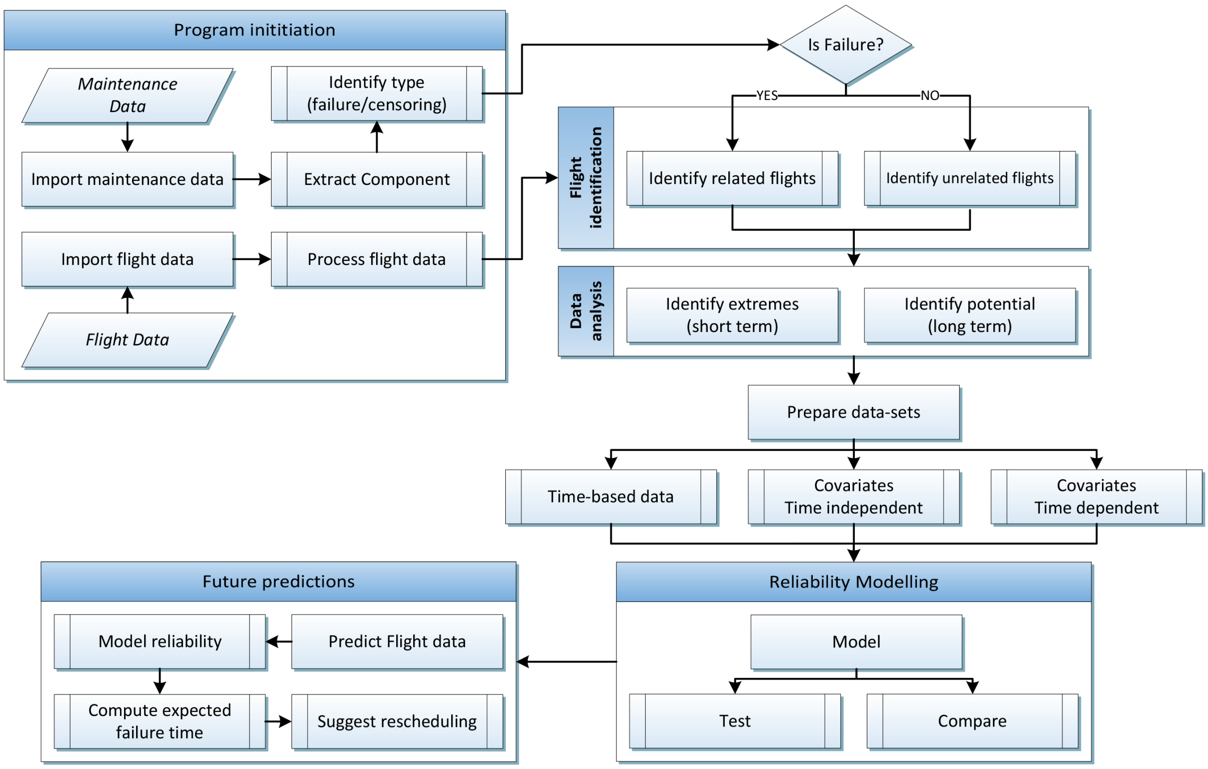


Figure 2.0.0.1: Overview of data flow in program.

Figure 2.0.0.1 presents an overview of the modules associated with the solution to this problem. Each module (highlighted in blue) will be described in more detail. Program initiation and flight identification will be presented in the section labelled Data acquisition and processing (Sec. 2.1). The following section will outline the methods used during data acquisition and processing (Sec. 2.1), data analysis (Sec. 2.2), reliability modelling (Sec. 2.3), and future predictions.

For the readers convenience each section/module was structured systematically:

Objective	Description of module's objective.
Inputs	Description and structure of the module's inputs.
Methods	Thorough description of the methodology used within the module.
Outputs	Description and structure of the module's outputs.
Challenges	Description of the challenges faced during module development, verification and validation.

2.1 Data acquisition and processing

The initial phase in any research project involving statistics and data analysis is the acquisition and preparation of data. The following section describes the methods used and presented in 'progress initiation' and 'flight identification' (Fig. 2.0.0.1).

The following section is structured as follows. Module objectives are introduced in Sec. 2.1.1. This section is followed by a brief description of the module's inputs, after which the methodology is thoroughly described (Sections 2.1.2 and 2.1.3 respectively). Finally Sections 2.1.4 and 2.1.5 describe the module's outputs and elaborate on the implications (challenges) that emerged during the development phase.

2.1.1 Objective

The objective of the following module was to: acquire (import) the datasets introduced in Sec. Background Information: *Data sets*, extract the operational data related to specific component types (part number), and process/prepare the data for analysis. The objectives in terms of outputs per component $\boldsymbol{PN}$ are:

- Set of all operational factors related to $\boldsymbol{PN}$.
- Set of all maintenance activities related to $\boldsymbol{PN}$.
- Assign a $\boldsymbol{type}$ variable to each maintenance events, distinguishing between $\boldsymbol{type} \sim \{\mathbf{0} - \boldsymbol{censored},\ \mathbf{1} - \boldsymbol{complete}\}$.
- Identify which flights potentially related to failure events.
- Compute basic statistics for all, censored, and complete flights ($\boldsymbol{mean}$, $\boldsymbol{std}$).

2.1.2 Inputs

The inputs for this module were the raw datasets identified in Sec. Background Information: *Data sets*. Below is a summary of the datasets in question:

File	Dimensions	Total	Size
FDR Data	$\mathbf{205\,779 \times 1531}$ (*Flights* × *Variables*)	$\mathbf{3.15 \times 10^8}$	$\mathbf{4.56}$ GB
TRAX	$\mathbf{270\,809 \times 50}$ (*Events* × *Variables*)	$\mathbf{1.35 \times 10^7}$	$\mathbf{70.45}$ MB
EHM	$\mathbf{107 \times 52 \times 1}$ (*Engines* × *Weeks* × *Year*)	$\mathbf{5.56 \times 10^3}$	$\mathbf{0.95}$ MB

2.1.3 Methods

2.1.3.1 Data acquisition

In this section the methods used to read and process the data are outlined. Due to the complexity and magnitude of the data it was beneficial to assign variable classes (types) prior to loading the data into memory. This step was essential to avoid dates, strings (factors), and numeric values being observed incorrectly.

Numerals and strings were acquired using a text parser on the first thousand columns. The motivation for using a large number was to avoid columns containing both strings and numbers, such as the columns specifying both serial and part numbers, from being read incorrectly.

2.1.3.2 Data preparation

Once all datasets were loaded into memory the EHM data was merged with the flight data by adding two variables (one per engine) specifying the engine's health status during the flight (Sec.). Unknown health statuses were allocated NaN values.

The next step was to remove all incomplete observations and errors. For this step it was important to use prior knowledge of the datasets (App. A) and determine tolerated values and strings. In TRAX for instance the variable 'schedule category', indicating whether an event was scheduled or unscheduled, included a range of values such as: US, USS, S, UnSched, Sched, na, SS, U, etc. To a certain degree, values such as UnSched and US could easily be assumed to indicate the event was unscheduled. Other values however, such as SS or U, could not.

With the FDR data the problem was different as some columns and rows consisted of more than 50% NaNs. To be safe, these columns and rows were only removed after flights were allocated to each component (discussed in 'Computing general statistics').

2.1.3.3 Data processing

Once all data was readily available for processing, the first step was to identify (per component) which maintenance events were scheduled (censored) and which events were unexpected. It is important to acknowledge that not all unexpected events are failure driven, this occurs for instance when a component (failed or non-failed) requires the removal of another one in order to perform maintenance. TRAX defines eleven reason categories: 'COMP', 'FAILED', 'INOP', 'LEAK', 'MISSING', 'SHIMMY', 'TRANS', 'TROUBLE', 'WTL', 'US', and 'TIMEX'.

In this paper it was assumed that all reasons were related to failures except for TIMEX which, according to research, defines a component removed because it exceeded its tolerated operational time (QantasLink 2014).

2.1.3.4 Flight identification

In the previous module (Sec. 2.1.3.3) a list of maintenance events indicating time/date, number of cycles operated, aircraft registration number, part number, serial number, and schedule type was identified. In this sub-module all flights potentially affiliated with a maintenance event were identified such that they could be used to identify operational factors. The key interests were to identify all successful flights during each component's life cycle and to identify "potential" flights associated with a failure.

In this paper a potential flight is defined as a flight which could have lead to a component's failure. The motivation for identifying multiple potential flights is three-fold:

1. TRAX data does not specify the time of the maintenance event.
2. FDR data does not contain the time of the flight (only date)
3. A failure is not always related to the most recent flight cycle.

The identification of potential flights is discussed further in Sec. 2.1.5.

As the module iterated through each maintenance event corresponding to a part number PN, the grouped number of flights, mean, and standard deviation were updated using the following formulae:

Number of flights,

$$n_{\text{group}} = n_1 + n_2 + \cdots + n_{n_{PN}} = \sum_{i=1}^{n_{PN}} n_i \tag{2.1.1a}$$

Mean,

$$\bar{x}_{\text{group}} = \bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_{n_{PN}} = \frac{\sum_{i=1}^{n_{PN}} \bar{x}_i \, n_i}{\sum_{i=1}^{n_{PN}} n_i} \tag{2.1.1b}$$

Standard deviation,

$$\sigma_{\text{group}} = \sqrt{\frac{\sum_{i=1}^{n_{PN}} n_i \, \sigma_i^2 + n_i(\bar{x}_i - \bar{x}_{\text{group}})^2}{\sum_{i=1}^{n_{PN}} n_i}} \tag{2.1.1c}$$

where n_{PN} denotes the number of events related to part number PN.

2.1.4 Outputs

Once this module was developed, computing it returned the following datasets per part number (component type).

```
eventData {
 colnames = {
  "Id"            numeric
  "SN"            factor
  "Type"          factor
  "Event"         factor
  "Start"         numeric
  "Stop"          numeric
  "Cycles"        numeric
  "flightDate"    date
  "flightID"      numeric
  "failureNo"     numeric
 }
}
```

SAMPLE DATA

id	SN	Type	Event	Start	Stop	Cycles	flightDate	flightID	failureNo
1	n8163	"F"	0	211	212	213	10/10/2009	105106	2
1	n8163	"F"	1	212	213	213	11/10/2009	105107	2
1	n8163	"F"	1	212	213	213	11/10/2009	105108	2
2	n8105	"C"	0	0	1	116	3/11/2009	162101	1
2	n8105	"C"	0	1	2	116	4/11/2009	162102	1
					⋮				
2	n8105	"C"	0	114	115	116	4/12/2009	168113	1
2	n8105	"C"	0	115	116	116	5/12/2009	168114	1

```
flightStats.all | flightStats.cens
{
 rownames = {
  "MEAN"          numeric
  "STD"           numeric
  "n"             numeric
  "NA"            logical
 }
 colnames = {
  "Accn_lat_mean..g.s"
  "Accn_long_mean..g.s"
  "Vz_max..ft..min"
  "flightVars1"
  ...
  "flightVarsN"
 }
}
```

SAMPLE DATA

	TO.Accn_lat	TO.Accn_long	TO.Accn_norm		TD.Vtrue	TD.Aoa
mean	$-1.80\text{E-}03$	$2.63\text{E-}01$	$1.00\text{E-}00$		$7.69\text{E-}01$	$8.08\text{E-}02$
std	$7.06\text{E-}03$	$2.30\text{E-}02$	$2.65\text{E-}03$	...	$5.79\text{E-}00$	$3.33\text{E-}01$
n	$7.45\text{E+}05$	$7.45\text{E+}05$	$7.45\text{E+}05$		$7.45\text{E+}05$	$7.45\text{E+}05$
NA	F	F	F		F	F

For more information on the flight variables see App. A. It is important to note that flight variables consisting of strings were factorised and that the 'flightID' column in 'eventData' is a unique identifier.

2.1.5 Challenges

The challenges in this module were strongly related to the quality of the input data opposed to the computing techniques used. As mentioned briefly in Sections 2.1.3.3 and 2.1.3.4, the flight data was filled with NaN values and not all maintenance event times were specified. In addition flight data did not specify the departure time of flights (only dates).

As a solution to unallocated flight times it was assumed that, in the raw dataset, flights associated with one tail number were chronologically distributed on a daily basis. The validity of this assumption was verified using random sampling. In addition, risk analysis indicated that the possible consequences of this assumption being invalid were minimal as neither time-based nor time-independent COX models were directly affected. Its effect on time-dependent COX models was minimal.

Missing values in the flight data were only corrected once the flights related to component maintenance events were identified and flights statistics were computed. This increased the total computation time, yet minimized the chance that a flight variable was overseen or excluded from analysis. To solve missing values in the flight variables the number of NaN values per column were computed. If a flight variable consisted of more than 80% NaNs, the variable was removed. To save time, columns (flight variables) with no values other than NaNs were identified during the computation of flight statistics ('flightStats.all' & 'flightStats.cens').

The identification and correction of missing values in individual flights was more complicated. Flights could not be completely removed as they were essential in the computation of time-dependent COX models. Ergo all missing values were replaced with expected values computed from trend-lines derived from historical data. The computation of trends is discussed further in Sec. 2.4.

The last challenge was to overcome the problem arising from TRAX data not specifying the time of maintenance events. For scheduled (censored) maintenance events the consequence of selecting an incorrect flight was minimal and hence it was assumed that the component was removed the night prior to the date that the maintenance was executed.

For flights related to failure events the risk of incorrectly specifying the causal flight was higher. As a solution an analysis/optimisation technique was introduced in Sec. 2.2. In preparation of this technique flights that could have 'potentially' lead to the failure were identified. Potential flights for maintenance event j were selected using the following logic:
Let the following variables denote

$$\begin{aligned}
\text{mDate} &\sim \text{Date of maintenance event } j \\
\text{mTime} &\sim \text{Time of maintenance event } j \\
\text{mAC} &\sim \text{Tail number of AC corresponding maintenance event } j \\
\text{F.id} &\sim \text{Vector of unique flight identifiers} \\
\text{F.ac} &\sim \text{Vector of flight tail numbers corresponding to each identifier} \\
\text{F.date} &\sim \text{Vector of flight dates corresponding to each identifier}
\end{aligned} \tag{2.1.2a}$$

Then the flight IDs of potential flights could be computed using

$$F_{ID} = \begin{cases} \text{F.id}\,[\ \overbrace{\text{F.date} = \text{mDate} - 1}^{\text{Flights occuring day before mDate}} \ \&\ \overbrace{\text{F.ac} = \text{mAC}}^{\text{Flights corresponding to mAC}}\ ], & \textbf{IF } \text{mTime} \leq 05:30 \\ \text{F.id}\,[\ \overbrace{\text{F.date} = \text{mDate}}^{\text{Flights occuring on mDate}} \ \&\ \overbrace{\text{F.ac} = \text{mAC}}^{\text{Flights corresponding to mAC}}\ ], & \textbf{ELSE} \end{cases} \tag{2.1.2b}$$

Research showed that the first flight of the day departs at $05:30$, hence any unscheduled maintenance occurring between $24:00$ and $05:30$ must be related to a flight that occurred the day before (Qantas 2015). The above assumptions fixed the issue of unknown maintenance and flight times, however not without imposing other limitations. The power of the analysis performed in Sec. 2.2 is reduced significantly if the number of flights marked as potential is large.

2.2 Data analysis

The previous section showed how flight data was prepared such that analysis could be performed. The primary objective in this module was to determine which flight variables were most likely to affect component reliability. In the literature study, cluster analysis and logistic regression were selected as potential techniques for the identification of flight variables (de Boer 2015). Both techniques were however disregarded due to their poor capabilities when using censored data (Sec. 2.2.5).

In the following section the data analysis module's objectives are introduced and the input data (retrieved from the previous module) are restated. The methods used with regard to each objective identified in Sec. 2.2.1 are described in Sec. 2.2.3 and examples of their outcomes are presented in Sec. 2.2.4. The challenges related to each method, including the motivation behind discarding cluster and logistic regression analysis, are presented in Sec. 2.2.5.

2.2.1 Objective

The primary objective for this module was to identify which operational factors were 'most likely' to affect component reliability. The secondary objective was to obtain a reduced set of operational factors to minimise the computation time and complexity of the reliability models. The module identified the operational factors using two different techniques and then reduced these in preparation for modelling.

The first method, labelled *Extreme Value Analysis (EVA)* (Sec. 2.2.3.1), identified operational variables by running an analysis on potential 'failure' flights and selecting an appropriate flight (per failure) that maximised the occurrence of unexpected (least 'likely') values.

In the second, labelled *Maximum Difference Analysis (MDA)* (Sec. 2.2.3.2), failure cycles were analysed as a whole and compared to censored (incomplete) cycles using Z-tests (Sec. Background Information:*Data Analysis*).

To solve the secondary objective, "obtain a reduced set of operational factors to minimise the computational time and complexity of reliability models", the identified flight variables were tested for linear correlation (Sec. 2.2.3.3).

2.2.2 Inputs

The inputs to this section followed the results from the previous module. To summarise:

eventData A 'data.frame' listing all flights associated maintenance events (and prior). The list distinguished failure from censored events and linked each event to a unique flight ID f_{ID} which could be used to retrieve the operational factors associated with f_{ID}.

flightStats.all (flightStats.cens) A 'data.frame' containing the mean, standard deviation, number, and a NA row for all (censored) flights. The logical NA row denoted whether more than 80% of the observations of an operational factor was missing.

2.2.3 Methods

2.2.3.1 Extreme value analysis

The focus of this module was to narrow down the number of potential fail flights and assign one flight per failure event based on the occurrence of extreme values. In general terms, this module assessed (to a certain significance level) which operational factors were abnormally high.

In an ideal scenario (perfect input data) flights could be identified using a linear optimisation problem of the form:

$$\textbf{Maximise} \quad Z = \sum_{i \in N} \sum_{j \in M_i} f_{ij} \times \sum_{v \in V} p_{ij,v} \tag{2.2.3a}$$

subject to,

$$\begin{aligned} \sum_{j \in M_i} f_{ij} &= 1, \quad \forall\, i \in N \\ f_{ij} &\in \{0, 1\}, \quad \forall\, i \in N, \quad j \in M_i \end{aligned} \tag{2.2.3b}$$

where,

$$\begin{aligned} N &\sim \text{Set of unscheduled 'failed' maintenance events.} \\ V &\sim \text{Set of operational factors (variables).} \\ M_i &\sim \text{Flights potentially related to 'failed' maintenance event } i. \\ f_{ij} &\sim \text{Binary decision variable:} \\ &\quad 1 - \text{Potential flight } j \text{ corresponding to event } i \text{ is cause of failure.} \\ &\quad 0 - \text{Potential flight } j \text{ corresponding to event } i \text{ is not cause of failure.} \\ p_{ij,v} &\sim \text{Probability variable } v \text{ in flight } f_{ij} \text{ belongs to group } C. \end{aligned} \tag{2.2.3c}$$

The nature of FDR data, however, consisting of means, maximums, and minimums throughout various flight phases, implicated the optimisation process (Sec. 2.2.5). In an attempt to find a solution two different techniques were applied, one of which yielded better results. The other technique is discussed further in Sec. 2.2.5.

The technique used in Extreme Value Analysis (EVA) was based on the successive implementation of linear optimisation problems to select the optimal combination of flights. This maximised the probability that a variable did not cohere to the censored data population (Group C).

Lets define the following parameters:

$$\begin{aligned} \mu_v &\sim \text{Mean of variable } v \text{ in non-fail population.} \\ \sigma_v &\sim \text{Std of variable } v \text{ in non-fail population.} \\ x_{i,v} &\sim \text{Value of variable } v \text{ during flight } i. \\ p_{i,v} &\sim \text{Probability } x_{i,v} \text{ belonging to } N(\mu_v, \sigma_v). \end{aligned} \tag{2.2.4}$$

In contrast to (2.2.5a), EVA optimised one flight variable at a time, searching for optimals in both the positive and negative direction. When optimising in the positive (negative) direction, flights with observation values x below (above) the mean μ were penalised by assigning a negative p value. This increases the probability that the flights selected experienced similar extremities in the operational variables. The optimisation problem could be derived from the expression above (2.2.5a).

$$\begin{gathered} \text{Selected Flights,} \quad \underset{N\times M}{F} = \begin{bmatrix} f^D_{11,v} & \cdots & f^D_{1M,v} \\ \vdots & \ddots & \vdots \\ f^D_{N1,v} & \cdots & f^D_{NM,v} \end{bmatrix} = F^D_v \\ \text{for} \quad v \text{ and } D \quad \text{where} \quad z^D_v = \textbf{Max} \underset{V_n\times 2}{Z} \\ Z = \begin{bmatrix} z^-_1 & z^-_2 & \cdots & z^-_{V_n} \\ z^+_1 & z^+_2 & \cdots & z^+_{V_n} \end{bmatrix}^T \end{gathered} \tag{2.2.5a}$$

In the above, z^D_v and $f^D_{ij,v}$ denote the objective function value and selected solution (decision variables) to the optimisation problem,

$$\textbf{Maximise} \quad z^D_v = \sum_{i\in N}\sum_{j\in M_i} f^D_{ij,v} \times p^D_{ij,v}, \quad \forall\ v \in V, \quad D \in \{\text{-}, \text{+}\} \tag{2.2.5b}$$

subject to,

$$\begin{gathered} \sum_{j\in M_i} f^D_{ij,v} = 1, \quad \forall\ i \in N, \quad v \in V, \quad D \in \{\text{-}, \text{+}\} \\ f^D_{ij,v} \in \{0, 1\}, \quad \forall\ i \in N, \quad j \in M_i, \quad v \in V, \quad D \in \{\text{-}, \text{+}\} \end{gathered} \tag{2.2.5c}$$

In addition to the sets and variables defined in (2.2.3c), let:

$$\begin{aligned} p^D_{ij,v} &\sim \text{Probability variable } v \text{ in flight } f_{ij} \text{ belongs to group } C \\ f^D_{ij,v} &\sim \text{DV representing optimal flight selection for variable } v \text{ in optimisation direction } D. \\ & \qquad 1 - \text{Flight } j \text{ corresponding to event } i \text{ is cause of failure.} \\ & \qquad 0 - \text{Flight } j \text{ corresponding to event } i \text{ is not cause of failure} \\ D &\sim \text{Optimisation direction for variable } v \\ & \qquad \text{-} - \text{Penalise } p \text{ values of variables } v \text{ during flight } f_{ij} \text{ if observed value } x_{ij} \text{ above } \mu_v. \\ & \qquad \text{+} - \text{Penalise } p \text{ values of variables } v \text{ during flight } f_{ij} \text{ if observed value } x_{ij} \text{ below } \mu_v. \end{aligned} \tag{2.2.5d}$$

To improve convergence appropriate objective coefficients $p^D_{ij,v}$ needed to be assigned. As mentioned earlier the objective coefficients for a particular operational factor (variable) v was derived from the probability that an observation x_v belonged to μ_v, the mean of variable v derived from all flights unrelated to failures. The value was computed by assigning Z-score and computing the associated probability as discussed in Sec. Background Information:*Data Analysis*. The assumption that variable v is normally distributed with μ_v and σ_v is valid as the number of flights was significantly large.

The initial p-value was computed using (2.2.6).

$$\text{p-value} \quad p_{ij,v} = 1 - 2P\Big(z > \Big|\frac{x_{ij,v} - \mu_v}{\sqrt{\frac{\sigma^2_v}{n}}}\Big|\Big) \tag{2.2.6}$$

Where i represents an unscheduled maintenance ($i \in N$) and j is a flight that is 'potentially' related to maintenance event i ($j \in M_i$) (2.2.3c)(2.2.5d). Hence subscript ij represents 'potential' flight j related to unscheduled maintenance event i.

Note that in the above p is a positive value in the interval $[0 \quad 1]$. To specify an optimisation direction D all p values were computed such that, depending on the direction, observations $x_{ij,v}$ below (or above) μ_v were penalised (negated). Hence,

$$p_{ij,v}^{D} = \begin{cases} D \times \left(1 - 2P\left(z > \left|\frac{x_{ij,v} - \mu_v}{\sqrt{\frac{\sigma_v^2}{n}}}\right|\right)\right) & \text{IF} \quad x_{ij,v} \geq \mu_v \\ -D \times \left(1 - 2P\left(z > \left|\frac{x_{ij,v} - \mu_v}{\sqrt{\frac{\sigma_v^2}{n}}}\right|\right)\right) & \text{ELSE} \quad (x_{ij,v} < \mu_v) \end{cases} \tag{2.2.7a}$$

where,

$$D \in \{-1, \quad 1\} \quad \text{or} \quad D \in \{\text{negative}, \quad \text{positive}\} \tag{2.2.7b}$$

To calibrate the model, optimise the probability that extreme values are selected, objective function coefficients $p_{ij,v}^{D}$ in (2.2.7a) can be reformulated to include weights (2.2.8).

$$p_{ij,v}^{D} = \begin{cases} D \times \left(1 - 2P\left(z > \left|\frac{x_{ij,v} - \mu_v}{\sqrt{\frac{\sigma_v^2}{n}}}\right|\right)\right)^{C} & \text{IF} \quad x_{ij,v} \geq \mu_v \\ -D \times \left(1 - 2P\left(z > \left|\frac{x_{ij,v} - \mu_v}{\sqrt{\frac{\sigma_v^2}{n}}}\right|\right)\right)^{C} & \text{ELSE} \quad (x_{ij,v} < \mu_v) \end{cases} \tag{2.2.8}$$

By increasing C the optimal solution is driven to select flights with operational factors deviating largely from non-failure related flights. Tests showed that the flights selected by settings $C = 2$ were most optimal. It remains a challenge to select the appropriate critical value for selecting operational factors, however tests showed that operational factors with a probability of deviating from the non-failure flights of 60% (or more) had minimal affect on the reliability modelling procedure. Section 2.2.4 describes the output obtained by this module.

2.2.3.2 Maximum difference analysis

The maximum difference module was important for time-independent COX models, which focus on mean values during a component's fail cycle. Its application was straight forward:

1. Compute mean (per operational factor) of all flights related to failure events (Group F).
2. Extract mean and standard deviation (per operational factor) of all flights related to censored events from flightStats.cens (Group C).
3. Compute probability (per operational factor) of F belonging to C using Z-test (large population size and known standard error).
4. Extract operational factors that are least likely to belong to Group C.

2.2.3.3 Variable reduction

The process of variable reduction was used to speed up the overall modelling process and avoiding errors during fitting procedures, which arose with highly correlated variables (see Sec. 2.2.5). Selection of the appropriate critical value for variables reduction remained a challenge. Nonetheless tests showed that critical values of 90% had minimal effects on modelling procedures.

2.2.4 Outputs

Successful execution of the above modules would have produced a selection of flights associated with failure events along with a list of reduced operational factors that are likely to be the root cause of failures. As a result the following datasets were obtained:

```
flightParam   {
 type =          vector
 class =         logical
 length =
   {"Number␣potential␣flights"}
}
```

SAMPLE DATA

US Maintenance Event		1			2		i	N	
Potential Flight	1	2	3	1	...	M_2	...	$M_N - 1$	M_N
flightParam	1	0	0	0	...	1	...	1	0

The above dataset (flightParam), extracted from EVA, was equivalent to the flight selection decision variables specified earlier as f_{ij} (2.2.5a). In the final output the matrix f_{ij} was transformed to a vector of length $\sum M_i$ (total number of potential flights). This vector was used during model preparation to remove the flights unrelated to US maintenance events.

```
flightVars, flightVars.reduced
{
 type =          vector
 class =         string
 length =
   {"Number␣operational␣factors"}
}
```

SAMPLE DATA

$[1]$	TO_Torque_rhs_mean	TD_NormalForce_lhs_max
$[3]$	TD_Accn_long_mean	TD_Brake_press_rhs_mean
	$\vdots$	
$[n-1]$	D_Vz_mean	D_Pitch_cmd_FO_force_mean

The operational factors identified as potential failure drivers were listed in flightVars and flightVars.reduced. Due to non-linear correlation among variables (e.g. pressure with altitude) the resulting reduced list remained considerably large $n > 20$. In this case variables were removed during the reliability modelling process. Both cluster analysis and logistic regression were tested post-analysis, yet remained inaccurate for this censoring degree.

2.2.5 Challenges

In the previous section a multitude of challenges were introduced such as: the nature of the flight data imposing a challenge on the optimisation procedure, the magnitude of flight variables affecting the computational time, and challenges related to the selection of valid critical values. This section will focus on explaining the resolution process such that the reader is informed to what extent solutions were solved and tested.

In the literature review both cluster analysis and logistic regression were identified as potential techniques to identify operational factors related to failures (de Boer 2015). Due to the preventative nature of aviation related maintenance activities a high level of censorship was present in the data (Sec. Background Information:*Data Sets*). Testing logistic regression and cluster analysis on a multitude of components showed that high censoring levels had a direct effect on the analysis techniques.

To benefit from the large datasets, using the central limit theorem, it was assumed that operational factors were normally distributed. As a result, Z-tests were introduced to compute the probability that operational factors deviated from the general (non-fail) population.

Now that an analysis technique was identified (Z-tests) we had to overcome the challenges imposed by analysing pre-analysed data. To clarify, data servers used to acquire data from FDRs pre-analyse the data by computing means, standard deviations, maxima & minima, and other statistics throughout phases of the flight. As a result this means, when analysing maxima, observations below the average should be disregarded. To be exact, if observations of an operational factor are measured to be abnormally high (or low) ($P(x \sim N(\mu, \sigma)) < 5\%$) during many flights prior to the occurrence of the failure, then the operational factor should be disregarded as the factor clearly did not affect the component's reliability during flights preceding the failure.

A solution to this problem would not be complex if the data existed purely of maxima and minima observations. However, due to observation of means, there is an interest to assess whether an operational factor could have been abnormally high (or low) during a certain flight phase for a majority of the failure-related flights. In contrast to maxima and minima the operational factor is considered significant if the mean for a majority of the failure-related flights is above or below the general population's (not both). An optimisation problem of this form would be formulated as follows:

$$\textbf{Maximise} \quad Z = \sum_{i \in N} \sum_{j \in M_i} f_{ij} \times \left(\sum_{v \in V_{max}} p_{ij,v} - \sum_{w \in V_{min}} p_{ij,w} + \sum_{u \in V_{other}} D_u p_{ij,u} \right) \tag{2.2.9a}$$

subject to,

$$\begin{aligned} \sum_{j \in M_i} f_{ij} = 1, &\quad \forall\, i \in N \\ f_{ij} \in \{0, 1\}, &\quad \forall\, i \in N, \;\; j \in M_i \\ D_u \in \{-1, 1\}, &\quad \forall\, u \in V_{other} \end{aligned} \tag{2.2.9b}$$

where, in addition to the sets, variables, and parameters identified in (2.2.3c) and (2.2.5d),

$$\begin{aligned} \boldsymbol{V_{min}} &\sim \text{Set of operational factors observing minima.} \\ \boldsymbol{V_{max}} &\sim \text{Set of operational factors observing maxima.} \\ \boldsymbol{V_{other}} &\sim \text{Set of operational factors observing means and others (e.g. duration).} \\ \boldsymbol{D_u} &\sim \text{DV defining optimisation direction of variable } \boldsymbol{u}. \\ &\quad -1 \;\; \text{Observations of variable } \boldsymbol{u} \text{ above } \boldsymbol{\mu_u} \text{ are penalised.} \\ &\quad +1 \;\; \text{Observations of variable } \boldsymbol{u} \text{ below } \boldsymbol{\mu_u} \text{ are penalised.} \end{aligned} \tag{2.2.9c}$$

The optimisation problem above could easily have been solved with an optimiser, such as IBM's *CPLEX* (IBM 2015). However R has its limitations, one of which is quadratic optimisation problems. As a solution an iterative linear optimisation problem was designed (Sec. 2.2.3.1).

2.3 Reliability modelling

In the previous module operational factors and flights 'potentially' related to US (failure) maintenance events were computed using EVA and MDA (Sec. 2.2). These outputs were used to prepare datasets for reliability modelling. It is assumed that the reader is familiar with the techniques used to prepare and structure data, hence this step is omitted here. The final structure of the datasets are however presented in Sec. 2.3.2.

The main objective of this module is to atomically obtain one (or more) statistically significant reliability models that comfortably accommodate the failure events. The focus of this section is to explore new reliability models such as time-based general repair, complex time-independent, and complex time-dependent (fully-parametric) proportional hazard models (PHMs) not yet applied in the industry.

In this section independent COX (indCOX) and dependent COX (dCOX) will refer to proportional hazard models with time-independent and time-dependent covariates respectively. The structure of this section is similar to that of Sections 2.1 and 2.2:

Section	**Description**
2.3.1	Introduction of modules objectives.
2.3.2	Description of modules inputs.
2.3.3	Explanation of the techniques used in each respective sub-module.
2.3.3.1	Explanation of the derivation and fitting of time-based reliability models. Brief introduction of challenges (further described in Sec. 2.3.5).
2.3.3.2	Explanation of the derivation and fitting of proportional hazard models with time-independent covariates (indCOX). Brief introduction of challenges (further described in Sec. 2.3.5).
2.3.3.3	Explanation of the derivation and fitting of proportional hazard models with time-dependent covariates (dCOX). Brief introduction of challenges (further described in Sec. 2.3.5).
2.3.3.4	Explanation of the computation of standard errors and confidence intervals related to the parameters estimated using Maximum Likelihood Estimation (MLE).
2.3.4	Brief overview of structure of obtained output.
2.3.5	Elaboration on the challenges introduced in Sec. 2.3.3 and description of the applied solutions.

2.3.1 Objective

The primary objective of this module is to obtain one (or more) statistically significant reliability models to represent QantasLinks reliability data in an autonomic manner. In context of this thesis, and in view of the risk avoidance in this industry, a statistically significant model is defined as a reliability model with a significance level of 95% (or more). These significance levels can be derived from standard errors computed from the Hessian during the fitting process (Sec. 2.3.3).

Although there are other reliability models available in practice (e.g. accelerated hazard models), the application of general-repair models and complex time-independent and -dependent PHM on aviation related reliability data is a new concept. In this thesis the scope will be limited to addressing normal, log-normal, logistic, gamma, exponential, and Weibull distributed general-repair models, as well as PHM models with similar underlying distributions and both time-independent and -dependent covariates.

2.3.2 Inputs

As time-based, indCOX, and dCOX differ in nature the datasets for each are slightly different. In general the structure for each will be as followed:

$$\text{Model data,} \quad \text{Mdl} = \left[\begin{array}{c|c} \underset{\text{Failure data}}{[\boldsymbol{N_F} \times \textbf{colnames}(\text{eventData})]} & \\ \hline & \underset{\text{Covariates}}{[(\boldsymbol{N_F}+\boldsymbol{N_C}) \times \text{flightVars.reduced}]} \\ \underset{\text{Censored data}}{[\boldsymbol{N_C} \times \textbf{colnames}(\text{eventData})]} & \end{array} \right] \tag{2.3.10}$$

where $\boldsymbol{N_F}$ and $\boldsymbol{N_C}$ denote the number of failure and censored maintenance events for time-based and indCOX modelling data and denote the number of flights related to failure and maintenance events for dCOX modelling data respectively.

For both time-based and indCOX the failure and censored data is the same database containing only one item per maintenance event describing the observation id, SN, number of cycles, and type. These values can be extracted directly from eventData, a dataset specifying all flights related to a maintenance event $\boldsymbol{id}$, described in Sec. 2.1.4. In this dataset the parameters required: id, SN, number of cycles, and type are identical for each unique $\boldsymbol{id}$.

The failure and censoring data for dCOX modelling is identical to the set derived in eventData, specifying a start and stop column. This column is used for the step-wise computation of the cumulative hazard function required in dCOX modelling (Sec. 2.3.3.3).

The operational factors, labelled as Covariates in (2.3.10), are used in PHMs to model the effect of additional factors on the component's reliability. Hence for time-based reliability models this dataset is empty. For time-independent covariates the operational factors need to be reduced such that each operational factor has only ONE value per maintenance event. A logical solution would be to use the average of all flights related to the maintenance event as the covariate. In this paper the averages are taken over all flights prior to the failure flight. In other words, potential flights related to the failure are excluded from the average. The potential flights are included in dCOX models, which use the operational factors during most recent flights to compute the instantaneous hazard.

Time-dependent covariates must be specified per flight (interval of time), hence do not require reduction. It is essential however that missing operation factors are predicted such that certain flights are not excluded during the computation of the cumulative hazard function. The prediction of operational factors during these phases is described in Sec. 2.4.

Tests proved that normalisation of the covariates $\begin{bmatrix} \mathbf{0} & \mathbf{1} \end{bmatrix}$ did not affect the outcome of the modelling procedures, yet was a practical procedure to compare the estimated parameters (same scale) (Bolstad 2015). Operational factors with all measured values above (below) zero were normalised using (2.3.11a) ((2.3.11b)). Operational factors with measured values above and below zero were normalised using (2.3.11c).

$$x_{norm} = \frac{x - x_{min}}{x_{max} - x_{min}} \tag{2.3.11a}$$

$$x_{norm} = \frac{x - x_{max}}{x_{max} - x_{min}} \tag{2.3.11b}$$

$$x_{norm} = \frac{x}{\textbf{max}(\, x_{max}, |x_{min}|\,)} \tag{2.3.11c}$$

2.3.3 Methods

2.3.3.1 Time-based models

Previously discussed in Sec. Background Information: *Reliability Modelling* is the advantage of Kijiama's G-Renewal process II (GRP) over other point processes such as HPPs, NHPPs and RPs (Kijima & Sumita 1986, Kaminskiy & Krivtsov 2006, Kaminskiy & Krivtsov 2010). Its advantages primarily arise from the introduction of a restoration coefficient q which equates RP and NHPPs when q is 0 and 1 respectively. In addition the process has proven to fit more accurately in comparison with the G-Renewal Process I (Mettas & Zhao 2005). The renewal equation (2.3.12a) represents the n^{th} failures distribution function given that its restored (virtual) age after the $(n-1)^{th}$ repair is V_{n-1}.

$$F(t|V_{n-1}=v) = \frac{F(t+v)-F(v)}{1-F(v)} \tag{2.3.12a}$$

In (2.3.12a) $F(t)$ denotes the cumulative distribution function of the first failure, also referred to as the underlying distribution function. Its virtual age V_n is given by,

$$\begin{aligned} V_0 &= 0 \\ V_1 &= q(V_0+t_1) = qt_1 \\ V_2 &= q(V_1+t_2) = q^2t_1 + qt_2 \\ &\vdots \\ V_n &= q(V_{n-1}+t_n) = q^nt_1 + q^{n-1}t_2 + \cdots + q^0t_n \end{aligned} \tag{2.3.12b}$$

With the assumption that a component's reliability at $t=0$ is 100% it can logically be shown that (2.3.12a) becomes $F(t)$ and $F(T)$ if q is 0 and 1 respectively (2.3.12b). Here t and T denote inter-arrival and system-total times respectively. Those uncommon with the two aforementioned terms can use the following annotation:

$$\begin{aligned} \text{Inter-arrival times,} \quad & t \in (t_1, t_2, \ldots, t_n) \\ \text{System-total times,} \quad & T \in (T_1, T_2, \ldots, T_n) \end{aligned} \tag{2.3.12c}$$

where

$$\begin{aligned} T_1 &= t_1 \\ T_2 &= t_1 + t_2 \\ &\vdots \\ T_n &= t_1 + t_2 + \cdots + t_n = \sum_{i=1}^{n} t_n \end{aligned} \tag{2.3.12d}$$

It is stressed that the inter-arrival times are ordered according to occurrence X.

The derivation of the Probability Distribution Function (PDF) of (2.3.12a) is straight forward:

$$\begin{gathered} f(t) = \frac{\partial F(t)}{\partial t} \quad \text{or} \quad f(t|V_{n-1}=v) = \frac{\partial F(t|V_{n-1}=v)}{\partial t} \\ f(t|V_{n-1}=v) = \frac{\partial}{\partial t}\frac{(F(t+v)-F(v))}{1-F(v)} = \frac{f(t+v)}{1-F(v)} \end{gathered} \tag{2.3.13}$$

Let θ and q denote the underlying distributions characteristic parameters and restoration coefficient respectively. Studies show that for large sample sizes Maximum Least Square Estimation is advantageous over other parameter estimators (Myung 2003). To compute the estimates of θ and q for right censored data we reformulate (BG10) in (Sec. Background

Information *Censoring*) to:

$$\begin{aligned}\text{Likelihood,}\quad L &= \prod_{d\in D}^{n_d} f(t_d) \prod_{r\in R}^{n_r} S(t_r)\\ &= \prod_{i=1}^{n} \big(f(t_i;\theta,q)\big)^{\delta_i}\big(R(t_i;\theta,q)\big)^{1-\delta_i}\\ &= \prod_{i=1}^{n} \big(\lambda(t_i;\theta,q)\big)^{\delta_i} R(t_i;\theta,q)\end{aligned} \tag{2.3.14a}$$

To address the issue of multiple components (same part number) (2.3.14a) can be reformulated to include the observations t_i related to each component of type PN. To do so we introduce the sets SN and N_s. SN denotes the set of components, by serial number, of type PN (Part Number). N_s denotes the failure events related to component s ($s \in SN$).

Again the importance of ordering each failure occurrence in N_s in chronological order is stressed:

$$N_s = \{\overset{X_{1,s}}{(T_{1,s}, t_{1,s}, \delta_{1,s})}, \overset{X_{2,s}}{(T_{2,s}, t_{2,s}, \delta_{2,s})}, \ldots, \overset{X_{n_s,s}}{(T_{n_s,s}, t_{n_s,s}, \delta_{n_s,s})}\} \tag{2.3.14b}$$

where

$$0 \le T_{1,s} \le T_{2,s} \le \cdots \le T_{n_s,s} \tag{2.3.14c}$$

The likelihood for component PN can then be computed using,

$$\begin{aligned} L &= \prod_{s\in SN}\prod_{i\in N_s} \big(f(t_i;\theta,q)\big)^{\delta_i}\big(R(t_i;\theta,q)\big)^{1-\delta_i}\\ &= \prod_{s\in SN}\prod_{i\in N_s} \big(\lambda(t_i;\theta,q)\big)^{\delta_i} R(t_i;\theta,q)\end{aligned} \tag{2.3.14d}$$

and its logarithm,

$$\log(L) = \mathcal{L} = \sum_{s\in SN}\left\{\sum_{i\in N_s}\Big(\delta_i \log\big(\lambda(t_i;\theta,q)\big)\Big) + \sum_{i\in N_s}\Big(\log\big(R(t_i;\theta,q)\big)\Big)\right\} \tag{2.3.14e}$$

In this module underlying normal, log-normal, logis, exponential, gamma, and weibull distributions were considered. The derivation of the likelihood function will be shown for logis distributions. Derivations of the log likelihood function for the other functions can be derived in a similar fashion.

Lets start by defining the underlying logistic distribution functions:

$$\text{PDF,}\quad f(t) = \frac{e^{-\frac{t-\mu}{s}}}{s\big(1+e^{-\frac{t-\mu}{s}}\big)^2},\qquad \text{CDF,}\quad F(t) = \frac{1}{1+e^{-\frac{t-\mu}{s}}} \tag{2.3.15a}$$

where μ and s are the location and scale characteristic parameters of the logistic distribution. Using (2.3.13) and (2.3.12).

$$\begin{aligned} f(t|v) &= \frac{e^{-\frac{t+v-\mu}{s}}}{s\big(1+e^{-\frac{t+v-\mu}{s}}\big)^2}\frac{1}{1-\frac{1}{1+e^{-\frac{v-\mu}{s}}}} = \frac{\big(e^{\frac{\mu}{s}}+e^{\frac{v}{s}}\big)e^{\frac{t+v}{s}}}{s\big(e^{\frac{\mu}{s}}+e^{\frac{t+v}{s}}\big)^2}\\ F(t|v) &= \frac{\frac{1}{1+e^{-\frac{t+v-\mu}{s}}}-\frac{1}{1+e^{-\frac{v-\mu}{s}}}}{1-\frac{1}{1+e^{-\frac{v-\mu}{s}}}} = \frac{e^{\frac{t+v}{s}}-e^{\frac{v}{s}}}{e^{\frac{t+v}{s}}+e^{\frac{\mu}{s}}}\\ R(t|v) &= \frac{e^{\frac{\mu}{s}}+e^{\frac{v}{s}}}{e^{\frac{\mu}{s}}+e^{\frac{t+v}{s}}}\\ \lambda(t|v) &= \frac{f(t|v)}{R(t|v)} = \frac{\big(e^{\frac{\mu}{s}}+e^{\frac{v}{s}}\big)e^{\frac{t+v}{s}}}{s\big(e^{\frac{\mu}{s}}+e^{\frac{t+v}{s}}\big)^2}\frac{e^{\frac{\mu}{s}}+e^{\frac{t+v}{s}}}{e^{\frac{\mu}{s}}+e^{\frac{v}{s}}} = \frac{e^{\frac{t+v}{s}}}{s\big(e^{\frac{\mu}{s}}+e^{\frac{t+v}{s}}\big)}\end{aligned} \tag{2.3.15b}$$

Substituting (2.3.15b) into (2.3.14d) yields

$$L = \prod_{s \in SN} \prod_{i \in N_s} \left(\overbrace{\frac{e^{\frac{t_i + V_{i-1,s}}{s}}}{s(e^{\frac{\mu}{s}} + e^{\frac{t_i + V_{i-1,s}}{s}})}}^{\lambda(t_i|V_{i-1,s})} \right)^{\delta_i} \overbrace{\frac{e^{\frac{\mu}{s}} + e^{\frac{V_{i-1,s}}{s}}}{e^{\frac{\mu}{s}} + e^{\frac{t_i + V_{i-1,s}}{s}}}}^{R(t_i|V_{i-1,s})} \tag{2.3.15c}$$

and the natural logarithm,

$$\begin{aligned} \log(L) = \mathcal{L} = \sum_{s \in SN} \Bigg\{ \sum_{i \in N_s} \Bigg[\delta_i \Big(\frac{t_i + V_{i-1,s}}{s} - \log(s) - \log(e^{\frac{\mu}{s}} + e^{\frac{t_i + V_{i-1,s}}{s}}) \Big) \Bigg] \\ + \sum_{i \in N_s} \Bigg[\log(e^{\frac{\mu}{s}} + e^{\frac{V_{i-1,s}}{s}}) - \log(e^{\frac{\mu}{s}} + e^{\frac{t_i + V_{i-1,s}}{s}}) \Bigg] \Bigg\} \end{aligned} \tag{2.3.15d}$$

Where s denotes both the characteristic scale parameter of the logis function and an element of SN when used in subscript N_s. $V_i = V_i(q)$, a function of restoration/repair coefficient q, denotes the components virtual age after failure i (2.3.12b). The maximum of 2.3.15d does not contain a closed form solution and hence numerical algorithms are used for optimisation. Computation of the likelihood function in R is rather straight forward.

Based on the research performed by Nash on best practices for optimisation in R this problem was tested on three components using three different algorithms: 'Nelder-Mead', 'BFGS', 'CG', and 'SANN' (Nash 2014). The results for components were identical to those of component PN *3-1573-1* 'MLG wheel & tire assy' shown in Tab. 2.3.3.1, showing that *Nelder-Mead*'s optimisation algorithms converged quickest and most accurately for normal, lognormal, logis, exponential, and gamma functions. It was concluded, based on the results from Tab. 2.3.3.1, that Weibull distributions are best optimised with *BFGS* methods.

Table 2.3.3.1: Comparison of optimisation methods for component PN *3-1573-1*.

	NORM		LNORM		LOGIS	
Method	**Value**	**Time (sec)**	**Value**	**Time (sec)**	**Value**	**Time (sec)**
Nelder-Mead	$-1,761.319$	31.586	$-1,763.946$	31.020	$-1,761.342$	34.100
BFGS	$-1,761.319$	115.254	$-1,781.779$	31.906	$-1,762.630$	138.300
CG	< -2500	> 300	$-1,781.778$	50.820	< -1926	> 300
SANN	< -2000	> 300	< -1764	> 300	< -2000	> 300

	EXP		GAMMA		WEIBULL	
Method	**Value**	**Time (sec)**	**Value**	**Time (sec)**	**Value**	**Time (sec)**
Nelder-Mead	$-1,778.219$	9.324	$-1,762.101$	72.006	$-5,723.770$	8.040
BFGS	$-2,090.787$	12.505	$-1,770.435$	60.540	$-1,761.723$	350.850
CG	$-1,778.219$	18.194	< -1880	> 300	$-3,952.705$	49.412
SANN	< -1800	> 300	< -1900	> 300	< -3600	> 300

The goodness-of-fit of the various distributions was tested using the methods described in Sec. Background Information *Goodness-of-Fit*.

2.3.3.2 Proportional hazard models (time-independent)

GRP time-based reliability models described in Sec. 2.3.3.1 are extended by introducing time-independent covariates (indCOX). The underlying distribution function of the proportional hazard functions (Sec.) now become the GRP function described in Sec. 2.3.3.1 for repairables, or a standard statistical distribution function $q = 0$ for non-repairables.

Derivations of the likelihood function are similar to that described in the previous section, however with a different hazard function (BG9). From (BG9) and using general reliability theory the hazard rate, cumulative hazard rate, and reliability function can be formulated as (2.3.16a), (2.3.16b), and (2.3.16c) respectively.

indCOX Hazard rate,

$$\lambda(t) = \lambda_0(t)\mathrm{e}^{\beta^T Z} \tag{2.3.16a}$$

indCOX Cumulative hazard rate,

$$\begin{aligned}\Lambda(t) &= \int_0^t \lambda(u)\,\mathrm{d}u = \int_0^t \lambda_0(u)\mathrm{e}^{\beta^T Z}\,\mathrm{d}u = \mathrm{e}^{\beta^T Z}\int_0^t \lambda_0(u)\,\mathrm{d}u \\ &= \Lambda_0(t)\mathrm{e}^{\beta^T Z}\end{aligned} \tag{2.3.16b}$$

indCOX Reliability function,

$$R(t) = \mathrm{e}^{-\Lambda(t)} = \mathrm{e}^{-\Lambda_0(t)\mathrm{e}^{\beta^T Z}} \tag{2.3.16c}$$

where in (2.3.16a), (2.3.16b), and (2.3.16c) $\lambda_0(t)$ and $\Lambda_0(t)$ denote the underlying hazard rate equal to $\lambda(t|v)$ and $\int \lambda(u|v)\mathrm{d}u$ (Sec. 2.3.3.1). In Section 2.3.3.1 the cumulative hazard rate was not yet defined. Although the values can be computed using numerical integration, it was found that formulating the cumulative hazard function in terms of the parameters and X & V vectors reduced computational time significantly.

In general, the likelihood function of GRP models (2.3.14e) can be reformulated to (2.3.19e) for time-independent PH models.

$$\begin{aligned}L &= \prod_{s\in SN}\prod_{i\in N_s}\left(\lambda(t_i;\theta,q)\right)^{\delta_i} R(t_i;\theta,q) \\ &= \prod_{s\in SN}\prod_{i\in N_s}\left(\lambda_0(t_i;\theta,q)\mathrm{e}^{\beta^T Z_i}\right)^{\delta_i}\mathrm{e}^{-\Lambda_0(t_i;\theta,q)\mathrm{e}^{\beta^T Z_i}}\end{aligned} \tag{2.3.17a}$$

and its natural logarithm,

$$\begin{aligned}\log(L) = \mathcal{L} = \sum_{s\in SN}\Bigg\{&\sum_{i\in N_s}\delta_i \log\left(\lambda_0(t_i;\theta,q)\mathrm{e}^{\beta^T Z_i}\right) \\ &- \sum_{i\in N_s}\left(\Lambda_0(t_i;\theta,q)\mathrm{e}^{\beta^T Z_i}\right)\Bigg\}\end{aligned} \tag{2.3.17b}$$

As an example the derivation of the likelihood function of a logistic general repair time-independent PH model will be shown. We start by expressing the underlying cumulative hazard function in terms of t, V, θ, and q, such that the indCOX reliability function can be formulated (2.3.18b).

Substituting the hazard rate (2.3.15b) in (BG7),

$$\begin{aligned}\Lambda_0(t|v) &= \int_0^t \frac{\mathrm{e}^{\frac{u+v}{s}}}{s(\mathrm{e}^{\frac{\mu}{s}} + \mathrm{e}^{\frac{u+v}{s}})}\,\partial u = \left[\log\left(\mathrm{e}^{\frac{m}{s}} + \mathrm{e}^{\frac{u+v}{s}}\right) + c\right]_0^t \\ &= \log\left(\mathrm{e}^{\frac{m}{s}} + \mathrm{e}^{\frac{t+v}{s}}\right) - \log\left(\mathrm{e}^{\frac{m}{s}} + \mathrm{e}^{\frac{v}{s}}\right)\end{aligned} \tag{2.3.18a}$$

and substituting (2.3.18a) in (2.3.16c),

$$R(t|v) = \mathrm{e}^{-\Lambda_0(t|v)\mathrm{e}^{\beta^T Z}} = \mathrm{e}^{\left[\log\left(\mathrm{e}^{\frac{m}{s}}+\mathrm{e}^{\frac{t+v}{s}}\right)-\log\left(\mathrm{e}^{\frac{m}{s}}+\mathrm{e}^{\frac{v}{s}}\right)\right]\mathrm{e}^{\beta^T Z}} \tag{2.3.18b}$$

Finally the likelihood function can be formulating by substituting (2.3.15b), (2.3.16a), and (2.3.18b) into (2.3.14d) but now formulated for time-independent proportional hazard instead of GRP models.

$$L = \prod_{s\in SN}\prod_{i\in N_s}\left[\frac{\mathrm{e}^{t_i+V_{i-1,s}}}{s(\mathrm{e}^{\frac{m}{s}} + \mathrm{e}^{\frac{t_i+V_{i-1,s}}{s}})}\mathrm{e}^{B^T Z_i}\right]^{\delta_i}\mathrm{e}^{\left[\log\left(\mathrm{e}^{\frac{m}{s}}+\mathrm{e}^{\frac{t_i+V_{i-1,s}}{s}}\right)-\log\left(\mathrm{e}^{\frac{m}{s}}+\mathrm{e}^{\frac{V_{i-1,s}}{s}}\right)\right]\mathrm{e}^{\beta^T Z_i}} \tag{2.3.18c}$$

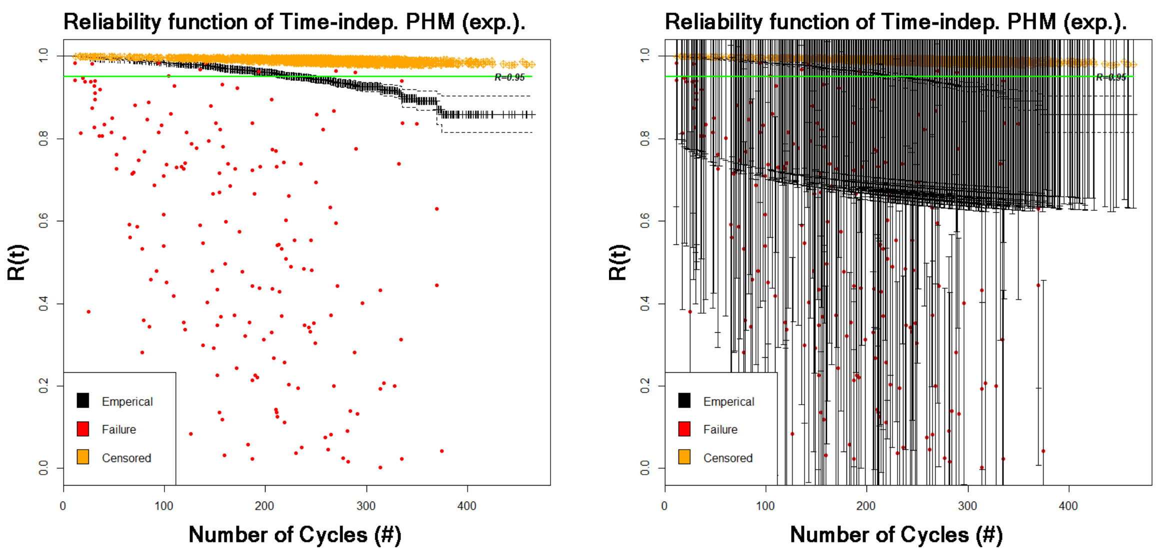
and its logarithm,

$$\begin{aligned}\mathcal{L} = \sum_{s \in SN} \Bigg\{ & \sum_{i \in N_s} (\delta_i \beta^T Z_i) + \sum_{i \in N_s} \delta_i (t_i + V_{i-1,s}) \\ & - \sum_{i \in N_s} \delta_i \log(s) - \sum_{i \in N_s} \delta_i \log\left(e^{\frac{m}{s}} + e^{\frac{t_i + V_{i-1,s}}{s}}\right) \\ & + \sum_{i \in N_s} \left[\log\left(e^{\frac{m}{s}} + e^{\frac{t_i + V_{i-1,s}}{s}}\right) - \log\left(e^{\frac{m}{s}} + e^{\frac{V_{i-1,s}}{s}}\right)\right] e^{\beta^T Z_i} \Bigg\}\end{aligned} \tag{2.3.18d}$$

Similarly to Sec. 2.3.3.1 the solution to maximising 2.3.18d is not readily available in closed form. Hence numerical optimisation algorithms were used to compute the parameter estimates.

In a similar fashion to Sec. 2.3.3.1 the appropriate optimisation method was selected based on best optimisation practices and testing (Nash 2014). Optimisation results showed that *Nedler-Mead* converged quickest and with most optimised MLE values in all cases.

The next issue that arises is that associated with confidence intervals and significance levels.



(a) Without confidence intervals

(b) With confidence intervals

Figure 2.3.3.2: Cost of using a high number of covariates.

The hazard (and reliability) functions of indCOX models multiply the product of an underlying hazard (cumulative hazard) distribution by the exponential of $\beta^T Z$. In general this means the standard error increases significantly with each added parameter.

As an example consider Fig. 2.3.3.2. If standard errors and confidence intervals are not considered, as shown in Fig. 2.3.3.2a, the time independent PHM model indicates that for 94.24% of the failures events the component's reliability is less than 95% and for all (100%) of the scheduled (censored) events the component has a reliability above 95%. These results would suggest that if maintenance were scheduled once a component's reliability is expected to drop below 95% then 94.24% of the failures would have been avoided and all scheduled maintenance events could have be postponed (cost effective). However at a confidence level of 95%, see Fig. 2.3.3.2b, the model suggests the reliability of ALL components would be below 95% even during the first flight cycle. Information on the computation of confidence intervals and standard errors is explained in Sec. 2.3.3.4.

The results (Fig. 2.3.3.2) suggest that the number of operational factors (covariates) should be reduced. Literature shows that numerous techniques are available ranging from step-wise procedures to criterion tests (Miller 1984, Weisberg 2014). To limit the scope, the foundation of two step-wise techniques was identified and tested on highly censored components. The findings indicated that forward selection was the most optimal procedure (in terms of time and accuracy) to reduce the number of covariates 2.3.5.

2.3.3.3 Proportional hazard models (time-dependent)

Time-dependent proportional hazard models are another method to incorporate operational factors into reliability models. Unlike time-independent PHMs however they allow operational factors to vary over time ($\boldsymbol{Z} \rightarrow \boldsymbol{Z}(t)$). Evidently the hazard rate identified for indCOX can be reformulated to (Lin 1994, Arasan & Ehsani 2011):

$$\lambda(t; \boldsymbol{\theta}, \boldsymbol{q}) = \lambda_0(t; \boldsymbol{\theta}, \boldsymbol{q}) \mathrm{e}^{\boldsymbol{\beta}^T \boldsymbol{Z}(t)} \tag{2.3.19a}$$

The second time-varying components make integration very complex, in fact for may distributions no closed-form of the solution is available. Instead, integration can be transformed into the piecewise sum of the hazard rate from $[\boldsymbol{0} \quad \boldsymbol{t}]$ (2.3.19b).

$$\Lambda(t; \boldsymbol{\theta}, \boldsymbol{q}) = \int_0^t \lambda(u; \boldsymbol{\theta}, \boldsymbol{q}) \, \partial u = \sum_{i \in M(t)} \lambda(t_i; \boldsymbol{\theta}, \boldsymbol{q}) = \sum_{i \in M(t)} \lambda_0(t_i; \boldsymbol{\theta}, \boldsymbol{q}) \mathrm{e}^{\boldsymbol{\beta}^T \boldsymbol{Z}(t_i)} \tag{2.3.19b}$$

where $\boldsymbol{M}(\boldsymbol{t})$ denotes a set of flights related to a maintenance event up to time $\boldsymbol{t}$. Using the above formulation we readily find the reliability function

$$R(t; \boldsymbol{\theta}, \boldsymbol{q}) = \mathrm{e}^{-\sum_{i \in M(t)} \lambda(t; \boldsymbol{\theta}, \boldsymbol{q})} = \mathrm{e}^{-\sum_{i \in M(t)} \lambda_0(t_i; \boldsymbol{\theta}, \boldsymbol{q}) \mathrm{e}^{\boldsymbol{\beta}^T \boldsymbol{Z}(t)}} \tag{2.3.19c}$$

It follows that the maximum likelihood function for time-independent PHMs with censored data (2.3.19d) can be formulated for time-dependent covariates.

$$\begin{aligned} L &= \prod_{s \in SN} \prod_{i \in N_s} \left(\lambda(t_i; \boldsymbol{\theta}, \boldsymbol{q})\right)^{\delta_i} R(t_i; \boldsymbol{\theta}, \boldsymbol{q}) \\ &= \prod_{s \in SN} \prod_{i \in N_s} \left(\lambda_0(t_i; \boldsymbol{\theta}, \boldsymbol{q}) \mathrm{e}^{\boldsymbol{\beta}^T \boldsymbol{Z}(t_i)}\right)^{\delta_i} \mathrm{e}^{-\sum_{i \in M(t)} \lambda_0(t_i; \boldsymbol{\theta}, \boldsymbol{q}) \mathrm{e}^{\boldsymbol{\beta}^T \boldsymbol{Z}(t_i)}} \end{aligned} \tag{2.3.19d}$$

and its logarithm:

$$\begin{aligned} \mathcal{L} = \sum_{s \in SN} \Bigg\{ & \sum_{i \in N_s} \delta_i \log \left(\lambda_0(t_i; \boldsymbol{\theta}, \boldsymbol{q}) \mathrm{e}^{\boldsymbol{\beta}^T \boldsymbol{Z}(t_i)}\right) \\ & - \sum_{i \in N_s} \sum_{i \in M(t)} \lambda_0(t_i; \boldsymbol{\theta}, \boldsymbol{q}) \mathrm{e}^{\boldsymbol{\beta}^T \boldsymbol{Z}(t_i)} \Bigg\} \end{aligned} \tag{2.3.19e}$$

The only requirements for computing (2.3.19e) are the underlying hazard rate function $\boldsymbol{hGRP}$ and an appropriately formulated dataset.

Similar to the other models, the appropriate optimisation method had to be selected. Similar tests to those performed in Sections 2.3.3.1 and 2.3.3.2 were conducted and, based on model convergence and time elapsed, the findings suggested, that *Nedler-Mead* was the best choice.

In time-dependent models the hazard rate for all flights related to a maintenance event are computed. Each observation (flight) is subject to some error, ergo, it follows that the error of the computed reliability increases cumulatively. To limit the total error and computational time, based on the findings in Sec. 2.3.3.2, a forward selection approach was implemented with a maximum of two iterations (two covariates).

2.3.3.4 Standard errors and confidence intervals

To complement the modelling modules described in Sections 2.3.3.1, 2.3.3.2, and 2.3.3.3, a module was created to assist in the computation of standard errors and confidence intervals. The module is based on the findings of Geyer and Greene, which highlight the use of Fisher's information matrices in computing confidence intervals (Geyer 2003, Greene 2012).

Fisher's information matrices are a way of quantifying the amount of information an observed variable $\boldsymbol{R}$ carries on the unknown distribution parameters that model $\boldsymbol{R}$. Studies have shown that, in the case of maximum likelihood estimation, the matrix can be approximated using the Hessian matrix. This is especially true when n, the number of observations, is large $n >> 0$ (Geyer 2003). Often, when computing the maximum likelihood estimates, the function is reversed (multiplied by -1) such that the problem is transformed into a minimisation problem.

Hence the Hessian is found using:

$$\text{Hessian,} \quad H = -\nabla^2\lambda(X;\theta,q,\beta) \tag{2.3.20a}$$

It follows that the Fisher information matrix I is approximated to:

$$I \approx H = -\nabla^2\lambda(X;\theta,q,\beta) \tag{2.3.20b}$$

In equations (2.3.20a) and (2.3.20b) $\lambda(X;\theta,q,\beta)$ represents the maximum likelihood function. In addition $\nabla f(x)$ and $\nabla^2 f(x)$ represent the first and second-order partial derivatives of function $f(x)$.

The standard errors of θ, q, and (if present) β can now be computed by square-rooting the diagonal elements of the inverse of the information matrix I , which is an estimator of the asymptotic covariance matrix (Powell 2007, Yuen 2012). Letting θ denote all unknown parameters (θ, q, and β) we find that,

$$\hat{\sigma}_\theta = \sqrt{I^{-1}_{jj}} \tag{2.3.20c}$$

Finally the standard error can be transformed into confidence intervals (CI) by multiplying $\hat{\sigma}_\theta$ by the appropriate z-critical value z_α. Let α denote the required significance level (Geyer 2003).

$$\text{CI} = \hat{\theta} \pm z_\alpha \hat{\sigma}_\theta \tag{2.3.20d}$$

To evaluate the residual effect of the errors associated to each parameter on the computed reliability model two techniques were assessed. Literature revealed that primarily two techniques are used to identify the standard error. The first, the Delta method, is primarily effective for smooth continuous functions, the second, Bootstrapping, can be used more widely and is related to Monte Carlo simulation (Department of Mathematics 2015). In this module the Delta method was used as the primary technique to compute confidence intervals and Bootstrapping as a fail-safe implemented only when the Delta method ran into errors.

Delta method The Delta method refers to the first-order approximation to the variance of a transformed parameter. Its derivation is well-documented, therefore we will only focus on it's application.

Let $R_{SN}(\theta,q,\beta)$ denote the reliability of a component SN at time t, where $\theta \sim N(\hat{\theta},\sigma_\theta)$, $q \sim N(\hat{q},\sigma_q)$, and $\beta \sim N(\hat{\beta},\sigma_\beta)$. The variance of each parameter σ^2_Θ ($\Theta = \theta,q,\beta$) can be extracted from Σ, the co-variance matrix (Yuen 2012). Furthermore, suppose $R_{SN}(\theta,q,\beta)$ is normally distributed with mean $\hat{R}_{SN}$ and standard deviation (error) $\sigma_{R_{SN}}$ (2.3.21a) (Department of Mathematics 2015, Muller, Gong & Muñoz 1980, Seber 1982).

$$R_{SN} \sim \left(R_{SN}(\hat{\theta},\hat{q},\hat{\beta}), \sqrt{R'_{SN}(\hat{\theta},\hat{q},\hat{\beta})\, \Sigma\, R'_{SN}(\hat{\theta},\hat{q},\hat{\beta})^T}\right) \tag{2.3.21a}$$

where Σ denotes the covariance matrix of parameters Θ and R'_{SN} denotes the first partial derivatives of function $R'_{SN}(\hat{\theta},\hat{q},\hat{\beta})$,

$$R'_{SN}(\hat{\theta},\hat{q},\hat{\beta}) = \begin{bmatrix} \frac{\partial R_{SN}}{\partial \theta} & \frac{\partial R_{SN}}{\partial q} & \frac{\partial R_{SN}}{\partial \beta} \end{bmatrix} \tag{2.3.21b}$$

The confidence interval for $R_{SN}(\theta,q,\beta)$ can then be computed using,

$$R_{SN} = \hat{R}_{SN} \pm z_\alpha \sigma_{R_{SN}} \tag{2.3.21c}$$

Where z_α is the z-critical value at α level of significance.

Bootstrapping In terms of parameter confidence interval estimation, Bootstrapping relies on the random sampling of estimated parameters $\Theta = \theta, q, \beta$. The underlying assumptuion is that each estimated parameter is normally distributed with mean $\hat{\Theta}$ and standard error σ_{Θ}. Using simulation, n random samples can be extracted from each parameter and consequently be used to evaluate the desired (transposed) expression for n sampled parameters. The mean and standard deviation of the evaluated expression can then be used to assign confidence intervals using (2.3.20d).

The primary disadvantage of using Bootstrapping over the Delta method is considerable increases in computational time (Department of Mathematics 2015).

2.3.4 Outputs

Similar to results in Sections 2.1 and 2.2 the focus of this section is to outline the format and content of the results obtained from this module. Prior to addressing the overall outcome the general reliability model structure is described:

```
Model {
 value            = numeric      # MLE value
 Class            = class        # Defines model class: GRP, indCOX, or dCOX
 Dist             = character    # Describes underlying distribution: e.g. norm, lnorm, etc
 Par              = vector       # Vector of parameter estimates
 hessian          = matrix       # Hessian matrix as computed by numderviv
 MLE.input        = list         # MLE inputs such as X, V, Z, di, etc
 Confidence.Intervals {
   CI             = table        # Confidence intervals per parameter
   P              = table        # Significance probability per parameter
 }
 GOF              = table        # Table containing statistical test results
}
```

In general the 'Model' contains sufficient information to generate plots, define tests, and to rebuild input data. The sizes of each data structure is limited and ranges from 0.3 to 320 MB depending on the type of model and size of the input data. This may be considered substantial however nowadays TB hard-drives are readily available at low costs.

To assist in model selection the models are summarised in a table containing data similar to that shown in Tab. 2.3.5.2. For each model class the following table is generated:

```
summaryModels {
 colnames     {
       "DIST.model1"                 # Identifier for model in table column
       "DIST.mode2"                  # e.g. norm.step1
       "DIST.modeN"                  # any other model
 }
 rownames     {
       "Dist"                        # Name of distribution
       "Step"                        # Step number if forward selection procedure
       "MLE"                         # MLE value
       "Time"                        # Computational time in minutes
       "Kolmogorov-Smirnov"          # Kolmogorov-Smirnov test statistic
       "Cramer-von.Mises-Smirnov"    # Cramer-von Mises-Smirnov test statistic
       "Anderson-Darling"            # Anderson-Darling test statistic
       "NRR"                         # NRR test statistic
 }
}
```

The following section will focus on the computation and prediction of flight variables and component reliability.

2.3.5 Challenges

In the previous section, Sec. 2.3.3, a number of challenges were identified and solved. To remain concise this section will only elaborate on the major challenges identified in Sec. 2.3.3.2 and 2.3.3.3. The challenges, tests, and solutions associated with optimisation method selection (e.g. *Nelder-Mead* and *BGFS*) were covered in Sec. 2.3.3.1 based on best practices for optimisation in R (Nash 2014).

The major challenge in this section was to obtain a 'satisfying' degree of certainty with respect to the reliability model whilst minimising computational time. The challenge was first introduced in Sec. 2.3.3.2 using a figure (Fig. 2.3.3.2) to graphically outline the cost (in accuracy) related to an increasing number of covariates. A variety of methods exists to assist in the selection of variables in regression models, however some require prior knowledge of the relationships between the independent and dependent variables. To automatise the selection process two step-wise procedures were tested: forward selection and backward elimination (Miller 1984, Weisberg 2014).

The two techniques, forward selection and backward elimination, are both iterative. In the first, forward selection, the reliability model is estimated using each individual covariate. On completion the covariate with the largest probability of being significant is selected and the process is repeated, computing the reliability with the selected and each individual covariate(s). This process continues until the probability of all covariates being significant reaches a certain threshold.

The second technique, backward elimination, computes the reliability model for all covariates and then removes the covariate with the largest probability of insignificance. This process is repeated until the probability of all covariates being significant is above a certain threshold, or until all covariates are removed.

The techniques were tested for a variety of underlying distributions (normal, exponential, and weibull), initial covariates, rules, and significance levels. The results are shown in Tab. 2.3.5.2 where the model results are given for each case. The significance level was set at 99% after it was established that the number of covariates remaining at a significance level of 95% was too high.

In Table 2.3.5.2 general modelling statistics such as MLE value, number of iterations, and time taken are given. In addition the test scores of the Kolmogorov (KS), Cramer-von Mises-Smirnov (CS), Anderson-Darling (AD), and Nikulin-Rao-Robson (NRR) statistical tests are provided (Sec.). The rules, column 3) refers to the elimination process: rule A, the standard form of backward-elimination, refers to a process where only one variable is eliminated per iteration. In rule B all variables with significance probabilities below 50% are eliminated.

The terms Vars refers to the number of covariates included during the process. If Vars was categorised 'All', then all the variables identified during data analysis (Sec. 2.2) were included. The category 'reduced' refers to a subset of 'all' variables, which were obtained with semi-parametric COX Proportional Hazard Models. Semi-parametric PHMs focus on estimating hazard ratios, which evidently means the underlying baseline function is eliminated from the equation. In fact the focus of semi-parametric PHMs is to determine which (and to what extent) covariates affect the hazard rate (Lin 1994, Fan & Jiang 2009).

The tests scores concur with literature, in which research has proven the tests to yield poor results for highly censored data (Nikulin et al. 2010). Nonetheless the scores are an effective measure to compare the performance of multiple models.

To assist the reader, each table (Table 2.3.5.2 a and b) was colour coded to indicate which value (per column) was the best fit (absolute minimum) amongst all tested models with a common underlying distribution (coloured: Norm, Exp, and Weibull) and in general (**bold font**). The following observations can be made:

- Backward tests fail to reduce the number of variables below 4 for both exponential and Weibull underlying distributions.
- Forward tests yield that no more than two covariates can be selected for a significance level of 99%.
- The tests scores from forward selection were better for all tests except for AD (all) and CS (exp).

Table 2.3.5.2: Comparing forward selection and backward elimination variable selection processes.

(a) Backward elimination.

Vars	SL	Rule	Dist	MLE	Covs #	KS	CS	AD	NRR	Its #	Time (min)
All	99%	A	***Norm***	−1184.49	2	50.71	72.15	−316.20	228.08	22	111
			Exp	−1261.44	7	**50.05**	**69.73**	−256.12	**91.12**	16	59
			Weibull	−1194.71	5	51.60	77.04	−280.47	189.33	20	92
	99%	B	***Norm***	**-1181.95**	3	51.05	72.06	−313.5	225.96	16	76
			Exp	−1269.21	4	50.87	72.94	−253.5	91.33	12	58
			Weibull	−1192.83	6	51.98	77.08	−280.46	202.83	14	76
Reduced	99%	A	***Norm***	−1183.99	2	50.68	71.95	−318.15	228.56	12	41
			Exp	−1269.21	6	52.88	80.27	−247.94	95.94	7	23
			Weibull	−1192.83	6	51.14	77.83	−277.63	200.05	8	40
	99%	B	***Norm***	−1464.72	**1**	52.61	108.68	**-222.38**	174.43	**3**	**5**
			Exp	−1269.03	6	52.87	80.3	−247.92	95.35	6	14
			Weibull	−1192.74	6	51.76	77.78	−275.86	177.68	7	34

(b) Forward selection.

Vars	SL	Steps	Dist	MLE	Covs #	KS	CS	AD	NRR	Its #	Time (min)
All	99%	1	***Norm***	−1186.81	1	49.41	71.5	−308.45	191.66	22	9
			Exp	−1242.54	1	**43.84**	77.91	**-263.8**	**61.76**	22	16
			Weibull	−1193.2	1	48.63	77.42	−292.68	154.91	22	12
	99%	2	***Norm***	**-1181.12**	2	51.5	**71.2**	−303.8	172.19	43	20
			Exp	−	−	−	−	−	−	−	−
			Weibull	−1189.46	2	49.29	75.21	−293.42	131.27	43	22
Reduced	99%	1	***Norm***	−1186.81	1	49.41	71.5	−308.45	191.66	12	**5**
			Exp	−1242.54	1	**43.84**	77.91	**-263.8**	**61.76**	12	9
			Weibull	−1193.2	1	48.63	77.42	−292.68	154.91	12	7
	99%	2	***Norm***	**-1181.12**	2	51.5	**71.2**	−303.8	172.19	13	11
			Exp	−	−	−	−	−	−	−	−
			Weibull	−1189.46	2	49.29	75.21	−293.42	131.27	13	13

- The time taken to complete the entire process was significantly less, on average 76.3%, during the forward selection procedure.

The tests revealed that a majority of the tests scored better during the forward selection procedure. Further investigation of the reliability models obtained from both procedures revealed that models obtained from forward selection were more accurate. Evidently this is due to the increasing number of covariates present in the backwards elimination model.

2.4 Future predictions

The focus of this chapter is to inform the reader on techniques used to forecast component reliability. In this thesis, forecasting of flight variables was limited to the use of trends in past data. However, in reality a multitude of techniques exists which will be discussed in Sec. 4.

To remain consistent this sections structure is identical to that of the previous section. First the primary objectives and inputs will be stated (Sections 2.4.1 and 2.4.2). Secondly the underlying logic and methodology for obtaining the elementary trend-line function is discussed in Sec. 2.4.3. Finally the section is concluded with an outline of the results and challenges (Sections 2.4.4 and 2.4.5).

2.4.1 Objective

The primary objective of this module is to ascertain a technique to forecast an operational factor during a certain flight phase for a certain route. The underlying idea is to use historical data to estimate the effect of seasonal and annual changes on each operational factor. Techniques behind deriving seasonal and annual effects will be presented in Sec. 2.4.3.1.

This thesis was limited to generating forecasts based on trends in historic data. In reality there are a multitude of techniques available. These techniques will be discussed in Sec. 4.

2.4.2 Inputs

The input to this module was directly obtained from the FDRs and therefore is identical to the dataset labelled 'FDR Data' in Sec. 2.2.2. For more information regarding the structure and content of the data the reader is referred to Sec. Background Information *Data sets* and App. A. In short, in the dataset a set of observations (columns), one per operational factor during each flight phase, is given per flight (rows).

2.4.3 Methods

Similar to reliability modelling, trend-fitting refers to the process of fitting a pre-determined model to a set of data. In trend-fitting however the underlying model is not related to a statistical distribution and the input data consists of a complete set of observations. This section will consist of two parts. In the first, the model used to forecast operational factors will be defined. In the second, the prediction requirements, set by QantasLink, will be introduced and the errors associated with these, using time-independent and -dependent PHMs, will be evaluated.

2.4.3.1 The model

Operational factors are highly dependent on external factors and hence forecasting them can be a complex task. The operational factors measured by the on-board FDRs can be categorised into the following groups: *environmental*, *technical*, and *other*.

Environmental factors consists of variables that describe environmental conditions such temperature, pressure, and air density. These factors often directly (or indirectly) affect other operational factors.

Technical factors are referred to as factors that describes the aircraft' state. These factors include control surface inputs, angular rotations, velocities, & accelerations, and aircraft loading. The data captured by FDRs consist predominantly of technical factors. Technical factors are highly dependent on external factors, hence are difficult to predict.

Other factors refer to factors that cannot be described quantitatively and therefore are not measured by the FDR. These factors include, but are not limited to:

- personal traits, aspirations, and state of mind of flight staff
- personal traits, aspirations, and state of mind of ground staff
- working conditions
- passengers
- etc

Complex relationships between technical and external factors make it difficult to accurately forecast operational factors. Investigation of these relationships is beyond the scope of this paper.

To quantify the potential benefits of this solution, component reliability must be predicted. As such, operational factor forecasts must be ascertained. To do so this paper assumes operational factors are only affected by annual and seasonal changes. Logically these include variations in atmospheric conditions caused by the earth's orbit about the sun and green house gasses. These models also capture changes related to seasonal/annual changes in demand, which directly affects aircraft utilisation, routes, and aircraft loading.

To account for seasonal and annual changes constant, linear, and cyclic distributions were used. Routes with a large number of missing values resulting in less than 50 observations were estimated using constants (overall mean). If sufficient data was found ($n > 50$) then annual and seasonal changes were estimated.

In this section the method will be outlined progressively, ergo: first, the constant model used for $n \leq 50$ is introduced, secondly, a model that accounts for annual changes is outlined, and finally, the complete model accounting for annual and seasonal changes is described.

Constant Model $(n \leq 50)$

$$f(x) = \theta_0 \tag{2.4.22}$$

In the constant model one constant value is returned for every flight. Logically this implies that the constant θ_0 is equal to the mean, which is the central value (Jr. et al. 2009).

The reliability models defined in Sec. 2.3 are primarily affected by extreme value occurrences, hence accepting θ_0 as an operational factor's value will generally improve the computed reliability. Ergo during reliability forecasting the constant θ_0 can be 'readjusted' to account for worst case scenarios by adding an error term that is derived from the operational factor's standard deviation and z-critical value (2.3.20d). In this context the standard deviation is extracted from the data corresponding to flights with the same departures and destinations (city pairs).

Annual changes $(n > 50)$

$$f(x) = \theta_1 x + \theta_0 \tag{2.4.23}$$

Linear models can be fit to data which are expected to experience constant (gradual) changes throughout time (θ_1). The values for β can be estimated using the Best Linear Unbiased Estimator (BLUE), which is discussed subsequently.

Similarly to the constant model, the consequence of using $f(x)$ in reliability modelling is a higher computed component reliability. Hence to account for worst-case scenarios $f(x)$ can be 'readjusted' using its standard error and a z-critical value (2.3.20d).

Annual and Seasonal changes $(n > 50)$

$$f(x) = \theta_3 \sin\left(\frac{2\pi}{365.25}x + \theta_2\right) + \theta_1 x + \theta_0 \tag{2.4.24}$$

To account for seasonal changes a cyclic function, e.g. the sinusoidal function, was introduced to the model. In the model θ_3 denotes the strength (amplitude) of the seasonal changes and θ_2 allows for shifts in the x-axis. The constant term $2\pi/365.25$ was set such that the period of the sine curve was equal to the number of days in one year. The term was fixed (made constant) once tests revealed that setting it as an unknown yielded poor results. It can easily be shown that for non-seasonal and (or) -annually affected operational factors the coefficients β_3 and (or) β_1 become zero.

Similar to the aforementioned models the computed values of $f(x)$ were readjusted to account for worst-case scenarios.

Estimating θ and σ_θ To estimate coefficients of linear models, e.g. annual change (2.4.23), Best Linear Unbiased Estimators (BLUE) can be used. The reader is expected to be familiar with BLUE hence its application and derivation are not covered in this paper. However if required, more information on BLUE can be found in Srivastava book on regression analysis (Sen & Srivastava 1990).

A variety of algorithms exists to estimate the coefficients of non-linear function, e.g. (2.4.24). However to remain consistent the same optimisation module used in reliability modelling was used to minimise the residual sum of error (2.4.25).

$$\textbf{Maximise} \quad R^2 = \sum_{i=1}^{n} \left(\left(\theta_3 \sin\left(\frac{2\pi}{365.25} x_i + \theta_2 \right) + \theta_1 x_i + \theta_0 \right) - Y_i \right)^2 \tag{2.4.25}$$

where x_i and Y_i denote the time (day) and observed value of operational factor i and n denotes the total number of observations of operational factor Y for flights related to a city-pair. The standard error of the estimate can easily be computed using,

$$\sigma_{\hat{Y}} = \sqrt{\frac{R^2}{n}} \tag{2.4.26}$$

In addition it is essential that the optimisation procedure is initiated with properly formulated initial parameters. Tests revealed that the following initial parameters Θ_0 generated optimal results.

$$\text{Initial Par,} \quad \Theta_0 \begin{cases} \theta_0 = \bar{Y} & \text{Mean of operational factor Y} \\ \theta_1 = 0 & \text{No annual change in variable} \\ \theta_2 = 0 & \text{No shift in x-axis} \\ \theta_3 = \frac{\max(\max(Y), |\min(Y)|) - \bar{Y}}{2} & \text{Half of maximal diff. with mean} \end{cases} \tag{2.4.27}$$

2.4.3.2 Forecasting requirements and errors

The scheduling of maintenance activities requires time such that administrative tasks can be completed and the inventory can be restocked. Hence it is common that an operator would like to introduce a minimum forecasting period. To allow for sufficient time to organise and restock the inventory, the requirement imposed by QantasLink was two weeks. At an average operation frequency of 6.44 cycles per day, two weeks corresponds to 90.18 cycles. To be safe the number of cycles operated on a daily basis was rounded-up to seven. At this frequency, QantasLink operates approximately 100 cycles per two weeks ($14 \times 7 = 98$ cycles).

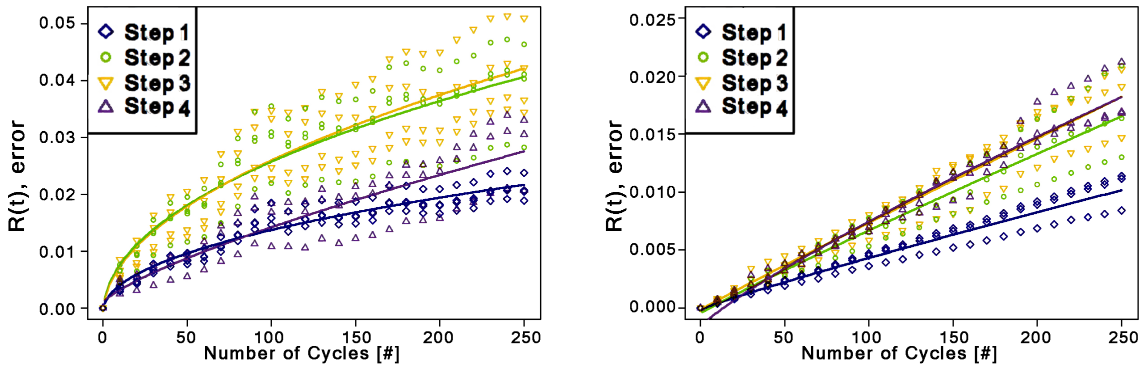
The next step is to verify that the errors generate by the aforementioned forecasting module is minimum. To do so, numerous reliability models were tested and values obtained using actual and predicted data (Sec. 2.4.3.1) were compared.

Figure 2.4.3.3 shows that the average error, imposed on each reliability model, increases along with the number of cycles predicted. In addition, the figures also demonstrates that the error is highly dependent on the covariates included within the model. When comparing time-independent with -dependent PHMs we readily find that the error generated by time-independent PHMs is larger.

When 100 cycles are predicted, the maximum error generated by the tested time-independent and -dependent PHMs was approximately 3.5% and 0.75%. Given that both errors are below 5% we can conclude that the model used to forecast operational factors 100 cycles in advance is relatively accurate. In addition, by using worst-case scenarios, when computing component reliability, the impact of the errors can be reduced.

2.4.4 Outputs

The objective of this module was to predict the value of a certain operational factor on a certain flight and day. To do so it was assumed that the operational factors were related to seasonal and annual changes only and a non-linear model was fit to the FDR's data. To be able to compute the value at any instance of time the model parameters and standard deviation were stored in a list with p number of elements, where p denotes the number of unique city pairs (see 'paramData'). A support function was written such that flight variables



(a) Time-independent PHM. (b) Time-dependent PHM.

Figure 2.4.3.3: Errors generated when the number of cycles predicted and covariates increases.

could be predicted using a simple call (e.g. *predict*$(\boldsymbol{day}, \boldsymbol{city.pair}, \boldsymbol{var})$) which returned the value and corresponding standard error.

```
paramData = list  {
  length          = numeric        # Number city pairs
  names           = character      # List of city pairs formatted using ICAO codes DEP-ARR
  city.pair1 = list {              # e.g. YSSY-YBBN Sydney-Brisbane
      var1 = list {                # e.g. Vertical speed during take-off
          par0  = numeric          # Parameter 0
          par1  = numeric          # Parameter 1
          par2  = numeric          # Parameter 2
          par3  = numeric          # Parameter 3
          std   = numeric          # Standard error
      }
      var2        = list           # List similar to var1
  }
}
```

2.4.5 Challenges

There were no challenges specifically related to the techniques described in this module. Instead there are a variety of challenges associated to accurately predicting operational factors in general. In this module only annual and seasonal changes were investigated for correlation with operational factors. In reality however operational factors can be highly dependent on other variables and conditions, e.g. correlation amongst control surface deflections and crosswinds.

Accuracy can be drastically improved by using weather forecasts and investigating the relationships among the variables. Despite its importance, the scope of this thesis was limited to identifying what operational factors affected component reliability and whether they could be used to reduce the number of unscheduled occurrences. Further investigating the process of variable prediction can be considered a thesis of its own.

Other limitations and techniques related to variable prediction will be discussed in Sec. 4.

3

Results

Although the software was designed to perform data analysis and reliability modelling on any component, this section will focus on the results derived from the following top ten components, w.r.t. URRs identified by QantasLink (Bombardier 2014*a*, Bombardier 2014*b*):

Section	**PN**	**Name**
3.1	**697071003**	Blade assembly and bearing
3.2	**174260-08**	Crew oxygen mask
3.3	**1152106-3**	DC starter generator
3.4	**903-1342**	Hand microphone
3.5	**3-1573-1**	MLG wheel & tire assembly
3.6	**3-1574**	NLG wheel & tire assembly
3.7	**92003-051-052-001**	Sensor high-level, fuel
3.8	**728809-1**	Thermal actuator
3.9	**10-105-31**A-N-**2**	VHF antenna
3.10	EVR**716-11-0350**A	VHF transceiver

Ideally, when evaluating the effectiveness of reliability models, a ‘new’ (untouched) set of data is used. However, due to the secure nature of aviation data, a second set of operational data was not available. As a solution, in most projects, the input data is split into two sets: one for modelling, the other for evaluating. In this project however several components had less than 50 observations. Splitting the data of these components into two separate sets would strongly effect the quality of the final models and evaluation results. As a result, the data used for modelling also had to be used in the evaluation process.

In Appendix B the reader will find a complete overview of the results obtained during modelling. To keep the report concise, the results of the first component, PN:**697071003**, are discussed elaborately. For the remaining components this section will only summarise and discus the results, which are presented in Appendix B.

The reader is reminded that this section discusses component-specific results. A generalised discussion on the research proposal is provided in Sec. 4.

3.1 697071003 Blade assembly and bearing

This section will focus on the results obtained from analysing 'blade assembly and bearing' components corresponding to part number **697071003**. As mentioned earlier, an elaborate analysis of the results related to component PN **697071003** will be presented such that the reader is informed on the various outputs generated by the models and has an idea on how the evaluation process is executed.

Unlike other components, all results (tables and figures) related to component PN **697071003** will be presented in this section and not in the Appendix.

Generally the results consist out of six parts:

Part I Overview of component's input data.

Part II Overview of operational factors identified during preliminary data analysis and reduction.

Part III Overview of time-based reliability models to validate assumptions and assess current practices.

Part IV *a.* Overview of time-independent PHMs modelling results (tests and MLE scores).
b. Overview of time-dependent PHMs modelling results (tests and MLE scores).

Part V *a.* Results obtained from evaluating time-independent PHMs with historical data*.
b. Results obtained from evaluating time-dependent PHMs with historical data*.

Part VI Overview of operational factors identified during modelling procedures.

* *Historical data is limited to the data used during modelling procedures.*

The reader is notified that the data used in this section is identical to the data used during modelling.

3.1.1 Results

Part I *Overview of input data:*
To understand the limitations and nature of the component the user is given an overview of the total number of observations used in the solution. Table 3.1.1.1 shows that a total number of 1597 maintenance events were recorded since 2004 of which 282 related to flights after 2010 (FDR data). Out of these 282 flights, 218 were scheduled (approximately 77%) at an average cycle time of 1970 cycles per event. Generally when the censoring levels are high it implies that component failure induces high costs. This is something the user should keep in mind when selecting an appropriate reliability model and threshold.

Table 3.1.1.1: General overview of component inputs.

Name	**Value**
Part Number	697071003
Total # (A, F, C)	**1597, 396, 1201**
Registered # (A, F, C)	**282, 64, 218**
Related Flights # (A, F, C)	**548353, 118865, 429488**
Avg. Cycles (A, F, C)	**1944.51, 1857.27, 1970.13**
% Censored	**77.3**

Table 3.1.1.2: Overview of analysis input and output.

	# Variables
ALL	**1531**
EVA	**38**
MDA	**78**
Combined	**116**
reduced Corr.	**83**
reduced semi-COX	**0**
Take-Off related	**24**
Cruise related	**25**
Touch-Down related	**34**

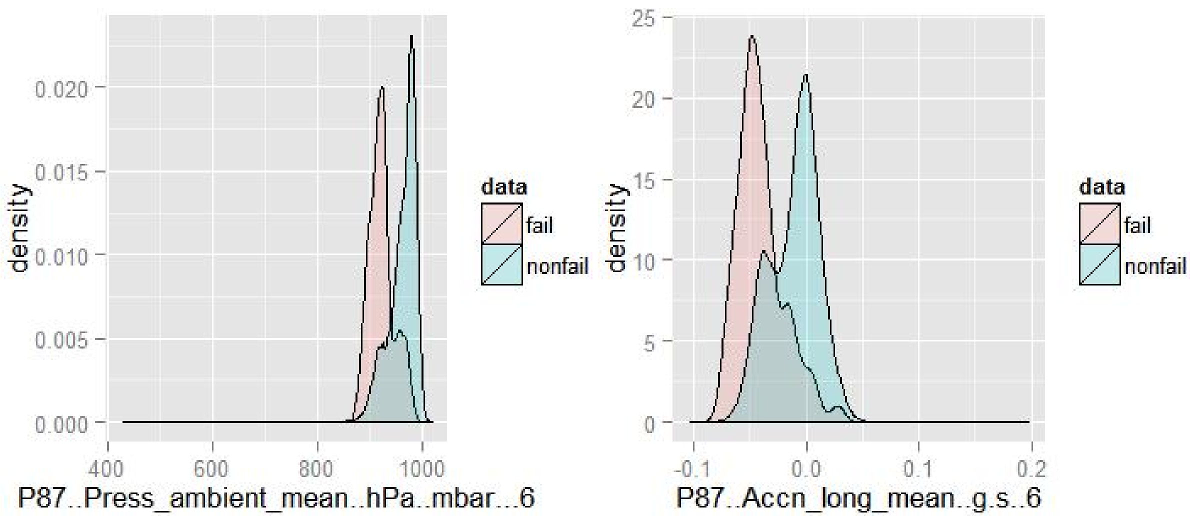
Part II *Overview of preliminary data-analysis:*
To grasp an idea of how many operational factors were included during modelling the program presents an overview of the analysis inputs and outputs in tables similar to Tables 3.1.1.2 and 3.1.1.3. In these tables the user is notified of the number of significant variables identified using EVA and MDA variable analysis procedures (Sec. 2.2) and how many of these

were strongly correlated. The label 'reduced semi-COX' specifies how many variables were reduced using semi-parametric PHMs. The tables further inform the user on the nature and distribution of the preliminarily identified operational factors.

Table 3.1.1.3: Overview of identified operational factors during all flight phases.

Variable	**Count**	**Variable**	**Count**	**Variable**	**Count**
Yaw_rate	8	Pressure_dynamic	3	Pitch_cmd_FO_force	1
Pitch_rate	6	Accn_norm	2	Rudder_cmd_force	1
Torque_lhs	5	Accn_lat	2	Elevator_Rin	1
Vtrue	5	Brake_press_lhs	2	Aileron_Rin	1
Roll_rate	5	Accn_long	2	Density_total	1
Vz	5	Vcal	2	Prop_spd_rhs	1
Aoa	5	Pressure_total	2	Headwind	1
Rudder_low	4	Press_ambient	1	NormalForce_nose	1
Elevator_Lin	3	Brake_press_rhs	1	Torque_rhs	1
Roll	3	Ttot	1		
NormalForce_rhs	3	Pitch_cmd_force	1		

For blade assembly and bearing parts analysis, procedures reduced the number of operational factors from 1531 to 83. Notice that semi-parametric PHMs were unable to reduce the number of operational factors any further. This is often the case when the number of input factors (i.e. 83) out-weights the number of failures (i.e. 64) (Tab. 3.1.1.2). The operational factors identified prior to semi-parametric PHM reduction were somewhat evenly distributed amongst the flight phases. Furthermore, density plots shown in Figure 3.1.1.1 give a sample graphical representation of the differences among operational factors between non-failure and failure related flights.



(a) EVA 1. (b) MDA 1.

Figure 3.1.1.1: Graphical overview of top operational factors identified by EVA and MDA.

Part III *Overview time-based reliability models:*
The next step is two-fold: first the user must test whether time-based reliability models with an exponential distribution (industry standard) model the component's reliability function effectively, and secondly, the assumption of one failure mode must be validated graphically. The first part is assessed numerically using Table 3.1.1.4. This table shows overall MLE and GOF test results obtained from evaluating time-based reliability models with underlying normal, log-normal, logistic, exponential, Weibull, and gamma distributions.

In Table 3.1.1.4 the absence of a 'gamma' column indicates that a time-based model with an underlying gamma distribution could not converge using the empirical data. Furthermore,

Table 3.1.1.4: Overview of results obtained by MLE optimisation and GOF tests.

	Distributions				
	norm	**lnorm**	**logis**	**exp**	**weibull**
MLE	**−641.98**	**−645.01**	**−642.6**	**−643.57**	**−643.59**
Kolmogorov-Smirnov	**4.25**	**5.39**	**3.93**	**4.85**	**3.13**
Cramer-von-Mises Smirnov	**40.61**	**39.44**	**41.02**	**40.07**	**41.66**
Anderson-Darling	**−121.43**	**−122.9**	**−121.14**	**−122.07**	**−120.84**
NRR	**48.86**	**43.76**	**50.06**	**44.45**	**63.01**

Maximum Likelihood Estimators (MLE), Kolmogorov-Smirnov, Cramer-von-Mises Smirnov, Anderson-Darling, and Nikulin-Rao-Robson scores imply that a diverse selection of reliability models fit the empirical data rather well. However all tests showed that an underlying exponential distribution did not model the data effectively. Figure 3.1.1.2 shows how the diverse time-based reliability models fit the empirical data. In this figure, for repairables, the virtual life (Sec. Background Information:*Reliability modelling*), computed using restoration coefficients, was averaged.

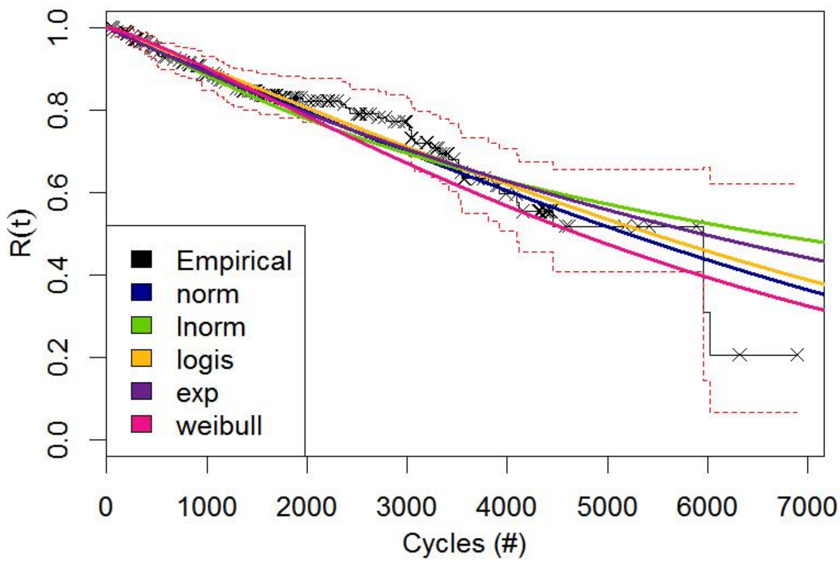
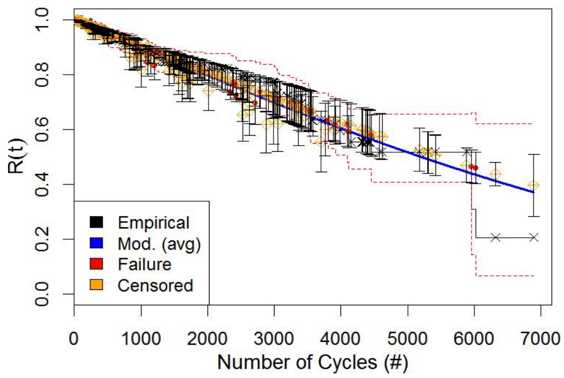


Figure 3.1.1.2: Overview of overall fit of multiple GRP models.

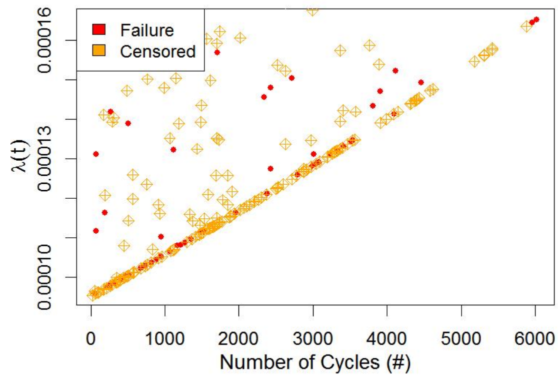
The second step, testing of the one failure mode assumption, is essential. If multiple failure modes are present complex reliability models will converge poorly. If the user finds that the assumption is violated he is advised to separate the input data accordingly such that failures related to one failure mode are isolated. The assumption is tested graphically using Figures 3.1.1.3, 3.1.1.4, 3.1.1.5, 3.1.1.6, and 3.1.1.7. Multiple failure modes are often present when failure data is unevenly distributed such that two (or more) groups can be differentiated. To benefit the user, confidence intervals are hidden in hazard plots. In the figures, failures are evenly distributed, implying that failures are most likely related to one failure mode.

Part IV *Overview time-(in)dependent PH models:*
If the assumption of one failure mode is not violated then the user can continue to establish whether component reliability can be improved using more complex reliability models. First the user is advised to use MLE and GOF test statistics to determine which type of PHMs, time-independent or -dependent, fit the empirical data better (Tables 3.1.1.5 and 3.1.1.6). Research and tests have shown that the GOF tests behave poorly when the input data is highly censored (Nikulin et al. 2010). Hence in the decision logic diagram, Sec. 4, the user is advised to use historical data (Part V) and the reliability figures to determine which model is most effective.

Comparing the time-based, time-independent and -dependent models using Tables 3.1.1.4,

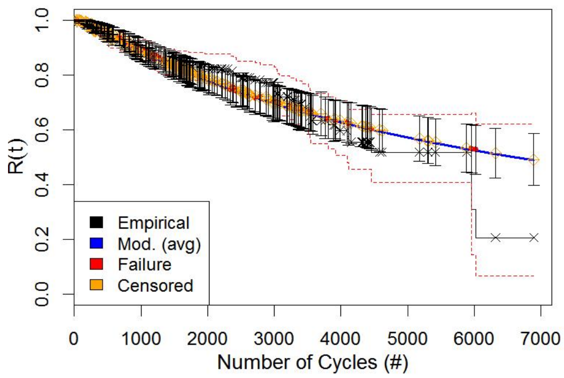


(a) Reliability function.

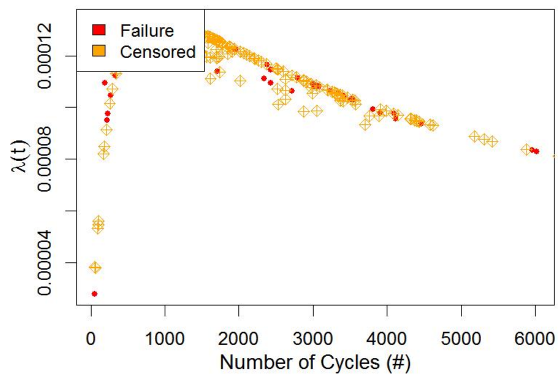


(b) Hazard function.

Figure 3.1.1.3: Computed reliability for time-based models with underlying norm distribution.

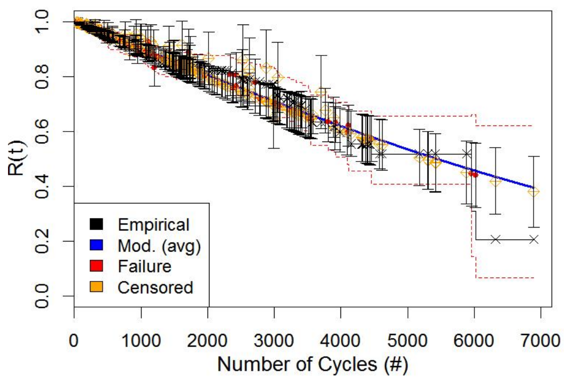


(a) Reliability function.

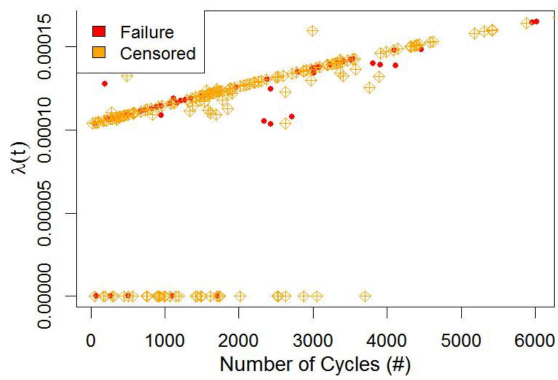


(b) Hazard function.

Figure 3.1.1.4: Computed reliability for time-based models with underlying lnorm distribution.



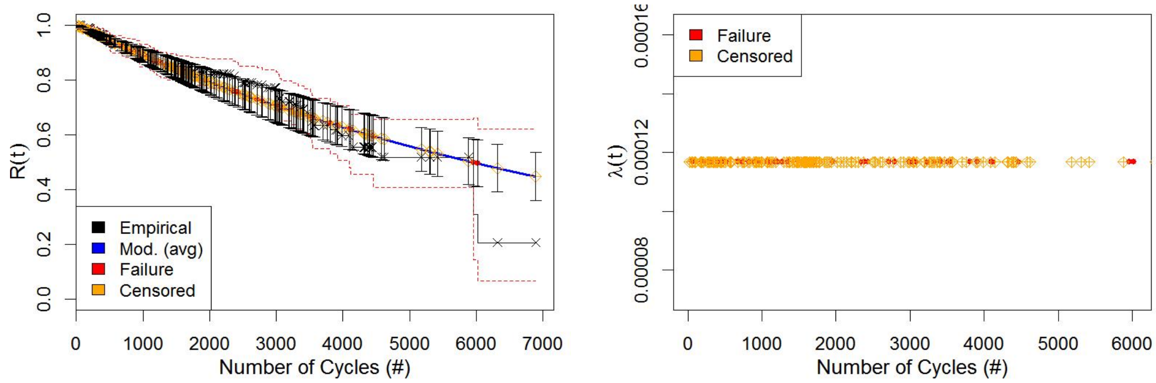
(a) Reliability function.



(b) Hazard function.

Figure 3.1.1.5: Computed reliability for time-based models with underlying logis distribution.

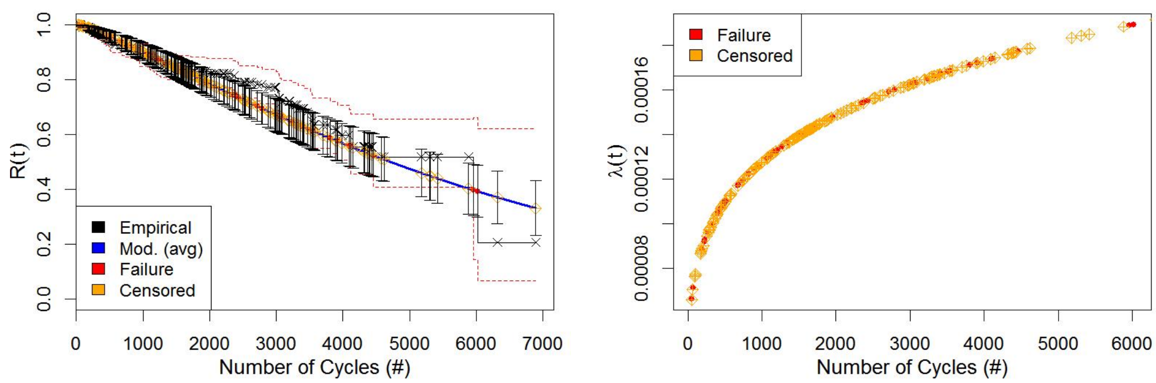
3.1.1.5, and 3.1.1.6 we find that time-dependent models fit the empirical data most accurately, scoring better in terms of its Maximum Likelihood Estimator and Kolmogorov-Smirnov (KS), Cramer-von-Mises Smirnov (CS), Anderson-Darling (AD), and Nikulin-Rao-Robson (NRR) test scores. However due to high censoring levels, the tests favour a diverse selection of reliability models. It is therefore up to the user to select which model best fits operational needs. To do so the user is given two tools: The first is the use of graphical representations of each model; Figures 3.1.1.8, 3.1.1.9, & 3.1.1.10 and Figures 3.1.1.11, & 3.1.1.12 for time-independent and -dependend models respectively. The second is using the results generated in part V.



(a) Reliability function.

(b) Hazard function.

Figure 3.1.1.6: Computed reliability for time-based models with underlying exp distribution.



(a) Reliability function.

(b) Hazard function.

Figure 3.1.1.7: Computed reliability for time-based models with underlying weibull distribution.

Table 3.1.1.5: Overview of time-independent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	norm	norm	norm	lnorm	lnorm	lnorm	lnorm
Step #	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
MLE	**−628.74**	**−624.72**	**−617.28**	**−611.66**	**−628.4**	**−619.73**	**−611.67**	**−602.25**
Time (min)	**11.42**	**21.29**	**28.05**	**34.35**	**9.62**	**16.53**	**22.99**	**29.79**
Kolmogorov-Smirnov	**7.94**	**10.71**	**11.77**	**8.5**	**8.27**	**5.84**	**11.73**	**13.06**
Cramer-von Mises-Smirnov	**40.28**	**36.69**	**32.47**	**36.31**	**39.96**	**41.44**	**41.98**	**41.67**
Anderson-Darling	**−124.3**	**−129.31**	**−130.83**	**−125.56**	**−125.34**	**−124**	**−127.23**	**−131.04**
NRR	**43.9**	**36.85**	**35.08**	**34.49**	**40.55**	**41.07**	**38.67**	**29.81**
Distribution	logis	logis	logis	logis	exp	exp	exp	exp
Step #	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
MLE	**−630.94**	**−622.75**	**−614.65**	**−609.91**	**−632.38**	**−624.5**	**−618.57**	**−614.67**
Time (min)	**10.35**	**19.94**	**27.02**	**33.08**	**6.35**	**8.7**	**11.34**	**13.51**
Kolmogorov-Smirnov	**7.74**	**4.59**	**8.37**	**10.54**	**8.13**	**6.47**	**8.66**	**11.22**
Cramer-von Mises-Smirnov	**39.62**	**41.98**	**39.15**	**37.76**	**39.39**	**41.13**	**42.06**	**41.94**
Anderson-Darling	**−125.07**	**−122.19**	**−126.76**	**−126.85**	**−125.5**	**−123.71**	**−125.33**	**−127.5**
NRR	**37.65**	**58.85**	**27.41**	**23.75**	**37.32**	**32.43**	**35.41**	**27.21**
Distribution	weibull	weibull						
Step #	**1**	**2**						
MLE	**−630.03**	**−624.19**						
Time (min)	**12.47**	**20.2**						
Kolmogorov-Smirnov	**5.67**	**8.73**						
Cramer-von Mises-Smirnov	**39.5**	**40.25**						
Anderson-Darling	**−124.73**	**−125.14**						
NRR	**42.83**	**43.88**						

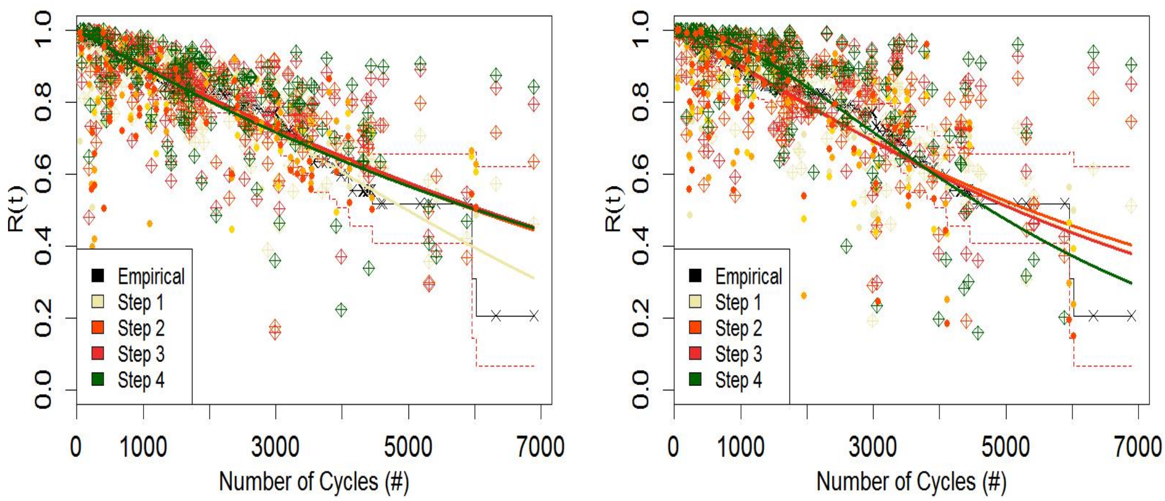
Table 3.1.1.6: Overview of time-dependent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	norm	norm	lnorm	lnorm	lnorm	lnorm	exp
Step #	1	2	3	1	2	3	4	1
MLE	−581.11	−511.96	−453.16	−582.7	−513.53	−455.41	−383.89	−582.27
Time (min)	250.9	733.95	1484.24	317.35	787.72	1482.96	2294.44	46.1
Kolmogorov-Smirnov	4.48	4.2	4.09	4.79	5.48	7.44	13.81	4.87
Cramer-von Mises-Smirnov	40.23	39.77	40.83	39.85	38.31	33.37	30.92	40.17
Anderson-Darling	−122.02	−122.43	−122.54	−122.44	−124.27	−130.32	−133.63	−122.48
NRR	178.17	227.2	−605.9	−722.14	−487.67	48.31	52.86	104.25

Distribution	exp	exp	exp	exp	exp	exp	exp	exp
Step #	2	3	4	5	6	7	8	9
MLE	−501.99	−425.06	−387.06	−361.65	−338.72	−319.69	−308.49	−292.94
Time (min)	89.73	152.15	217.91	289.8	384.09	505.03	618.48	700.59
Kolmogorov-Smirnov	5.82	7.21	9.38	13.85	14.98	15.05	14.75	14.99
Cramer-von Mises-Smirnov	38.42	37.43	38.11	37.54	36.17	32.97	31.54	30
Anderson-Darling	−125.01	−128.53	−127.85	−133.3	−137.4	−141.35	−146.16	−145.67
NRR	76.37	46.38	60.14	4295.93	2284.57	6151.13	117254.1	8139.05

Distribution	exp	weibull	weibull	weibull	weibull	weibull	weibull	weibull
Step #	10	1	2	3	4	5	6	7
MLE	−286.95	−580.03	−499.66	−443.6	−378.59	−344.6	−302.84	−285.49
Time (min)	741.62	120.69	244.14	389.3	549.9	720.35	929.45	1171.43
Kolmogorov-Smirnov	14.57	4.19	5.14	4.65	13.8	13.76	15.16	15.13
Cramer-von Mises-Smirnov	28.65	40.74	39.09	39.78	36.79	34.67	28.87	25.69
Anderson-Darling	−149.44	−121.74	−124.14	−124.9	−132.35	−138.33	−151.07	−152.04
NRR	12614.88	121.43	76.86	54.22	133.44	532.19	16654.75	25247.2

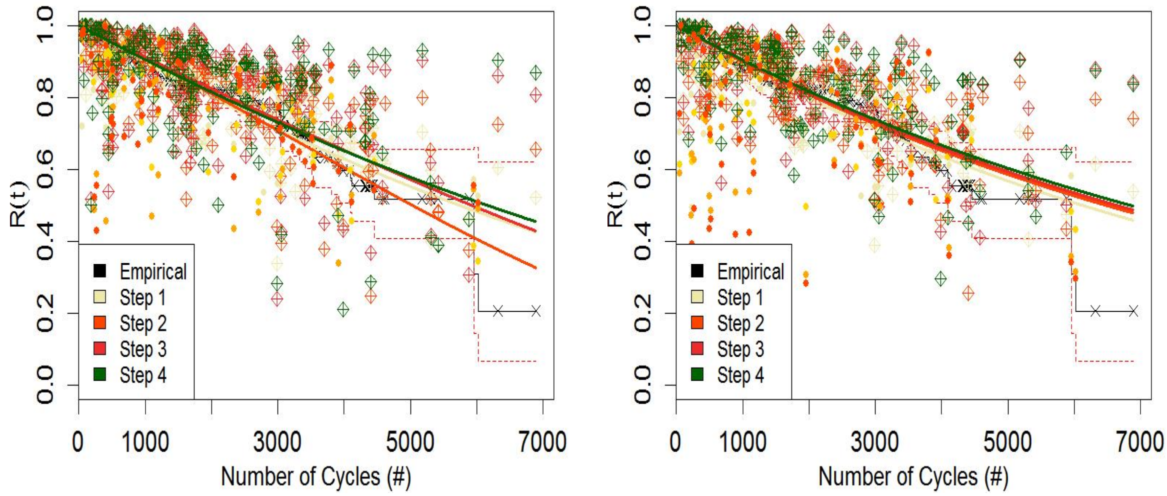
Distribution	weibull
Step #	8
MLE	−273.96
Time (min)	1382.89
Kolmogorov-Smirnov	15.04
Cramer-von Mises-Smirnov	23.53
Anderson-Darling	−152.55
NRR	7709.85



(a) With an underlying norm distribution. (b) With an underlying lnorm distribution.

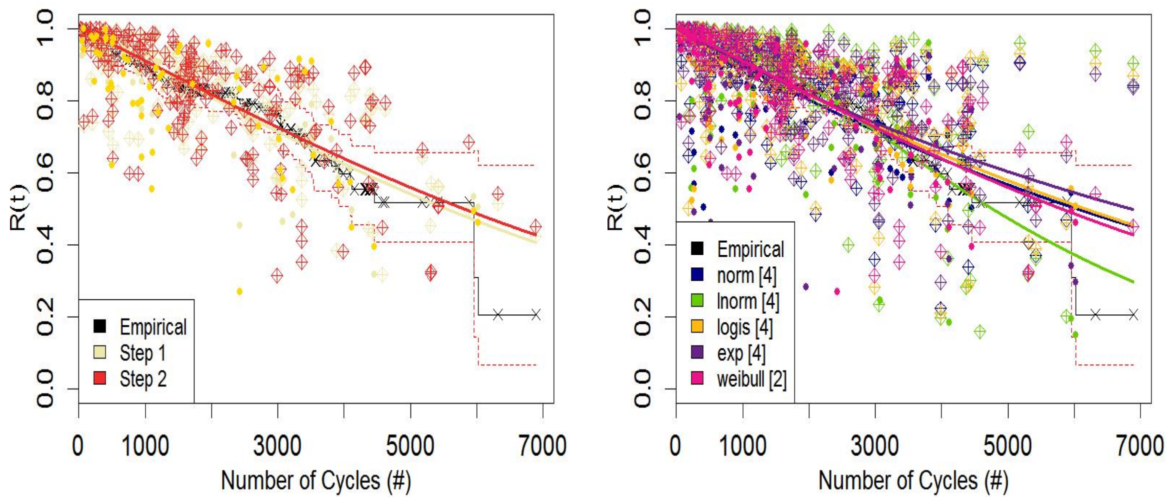
Figure 3.1.1.8: Time-independent PHMs with an underlying norm and lnorm distribution.

To truly understand the effectiveness of each model the estimated time till failure, for each event, was computed using data available 100 cycles before an estimated failure (Tables 3.1.1.7, 3.1.1.8, 3.1.1.9, and 3.1.1.10). The values in the tables were computed for general and worst-case scenarios and denote the percentage of failure [censored] events below [above] the indicated reliability levels. It is expected that the reader is familiar with the overall objective, however, for clarification, the aim is to maximise the number of failure and scheduled (censored) events below and above the reliability threshold respectively. In addition, for each model and scenario the estimated Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) are given in Tables 3.1.1.11, 3.1.1.12, 3.1.1.13, & 3.1.1.14 and in Tables 3.1.1.15, 3.1.1.17, 3.1.1.16, & 3.1.1.18 for time-independent and -dependent PHMs respectively. When addressing



(a) With an underlying logis distribution. (b) With an underlying exp distribution.

Figure 3.1.1.9: Time-independent PHMs with an underlying logis and exp distribution.



(a) With an underlying weibull distribution. (b) All distributions (last step).

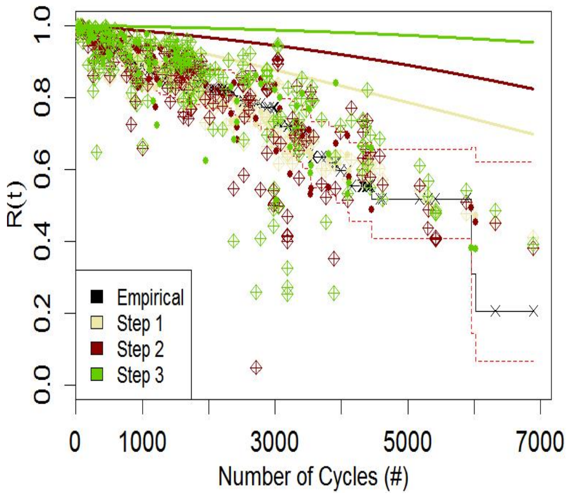
Figure 3.1.1.10: Figures containing a weibull distribution and all time-independent PHMs.

censored events it is important to be aware that it is uncertain whether rescheduled events would still prevent failures.

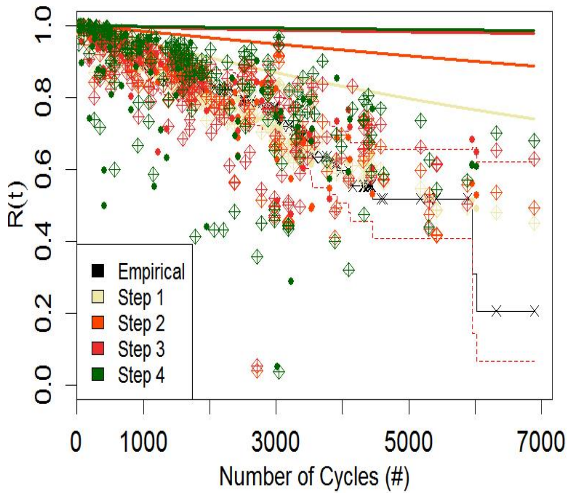
As mentioned in Part I, high censoring levels generally imply that costs associated to blade assembly and bearing failures are critical. Keeping this in mind the user is expected to select a relatively high reliability threshold. Furthermore, since failures are critical, the user is advised to use worst-case scenarios, which effectively reduce costs associated to incorrect parameter estimations.

Comparing time-independent and -dependent worst-case scenario tables we readily find, at equal reliability thresholds, that time-independent models generally predict more failures and postpone an equal number of scheduled events. It becomes evident, at high reliability levels $R_L \geq$40%, that logistic and exponential distribution with four covariates maximise the number of failure & scheduled events prevented & postponed respectively. At lower reliability levels a log-normal distribution with four covariates performs best.

Given that our intention is to select a high reliability threshold and time-independent logistic and exponential PHMs perform best at high reliability levels, we can disregard the other models and continue our analysis. The next step is to determine at what reliability threshold we would like to plan maintenance. Generally speaking if failures jeopardise flight safety,

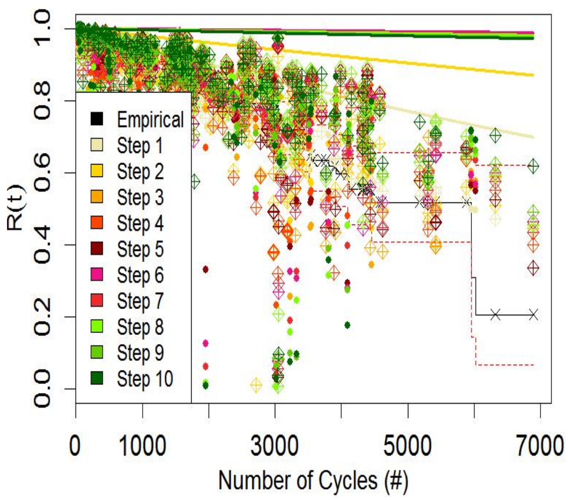


(a) With an underlying norm distribution.

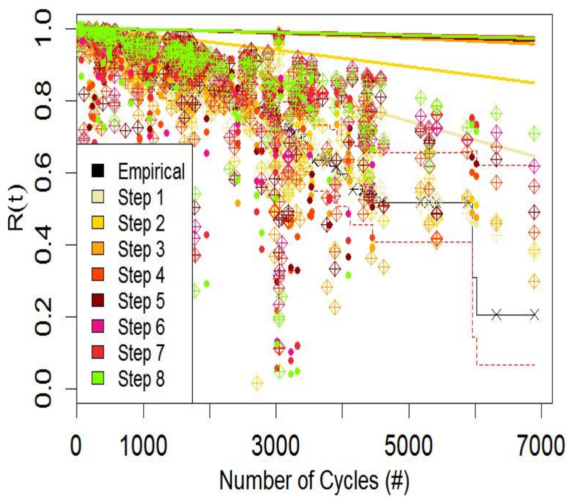


(b) With an underlying lnorm distribution.

Figure 3.1.1.11: Time-dependent PHMs with an underlying norm and lnorm distribution.



(a) With an underlying exp distribution.

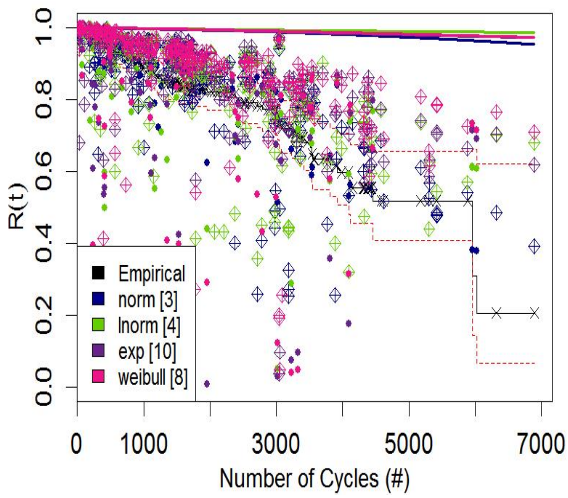


(b) With an underlying weibull distribution.

Figure 3.1.1.12: Time-dependent PHMs with an underlying exp and weibull distribution.

Table 3.1.1.7: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
norm	1	0 [84]	0 [84]	0 [84]	0 [83]	5 [82]	8 [78]	17 [73]	28 [62]	41 [45]	69 [26]	100 [0]
	2	0 [84]	0 [84]	0 [84]	2 [83]	5 [81]	8 [80]	19 [71]	42 [61]	53 [52]	75 [30]	100 [0]
	3	0 [84]	0 [84]	0 [84]	0 [82]	2 [80]	6 [77]	20 [72]	34 [68]	56 [58]	83 [31]	100 [0]
	4	0 [85]	0 [85]	0 [85]	0 [85]	3 [85]	12 [83]	23 [76]	36 [63]	64 [48]	86 [32]	100 [0]
lnorm	1	0 [84]	0 [84]	0 [83]	0 [82]	11 [81]	11 [79]	22 [71]	31 [62]	45 [45]	64 [29]	100 [0]
	2	0 [85]	0 [85]	0 [84]	2 [84]	9 [80]	14 [77]	22 [73]	31 [65]	45 [47]	64 [33]	100 [0]
	3	0 [85]	0 [85]	6 [85]	8 [85]	11 [83]	17 [82]	22 [71]	31 [62]	44 [50]	69 [38]	100 [0]
	4	2 [85]	8 [85]	9 [85]	9 [83]	11 [81]	17 [81]	23 [77]	33 [70]	47 [60]	72 [46]	100 [0]
logis	1	0 [85]	0 [85]	0 [84]	0 [84]	0 [82]	8 [81]	17 [74]	31 [60]	48 [45]	77 [24]	100 [0]
	2	0 [85]	0 [85]	0 [85]	5 [84]	6 [79]	8 [78]	22 [72]	28 [62]	44 [46]	72 [30]	100 [0]
	3	0 [85]	0 [85]	2 [84]	2 [83]	2 [81]	17 [79]	19 [75]	36 [62]	45 [51]	83 [36]	100 [0]
	4	0 [85]	0 [85]	0 [85]	2 [85]	2 [84]	12 [81]	25 [76]	39 [64]	62 [51]	84 [35]	100 [0]
exp	1	0 [85]	0 [85]	0 [85]	0 [85]	0 [82]	11 [81]	20 [75]	31 [62]	53 [44]	77 [25]	100 [0]
	2	0 [85]	0 [85]	0 [85]	0 [84]	6 [82]	12 [81]	20 [74]	33 [63]	52 [48]	81 [30]	100 [0]
	3	0 [85]	0 [85]	0 [85]	8 [85]	8 [85]	14 [83]	23 [75]	38 [63]	61 [49]	77 [33]	100 [0]
	4	0 [85]	0 [85]	6 [85]	8 [85]	11 [85]	19 [83]	28 [80]	45 [71]	59 [54]	80 [32]	100 [0]
weibull	1	0 [85]	0 [83]	0 [83]	2 [82]	3 [82]	11 [77]	20 [74]	27 [67]	41 [53]	66 [32]	100 [0]
	2	0 [85]	0 [85]	0 [85]	0 [83]	6 [82]	12 [78]	22 [74]	30 [67]	44 [58]	69 [36]	100 [0]



(a) All distributions (last step).

Figure 3.1.1.13: Figure of all time-dependent PHMs.

Table 3.1.1.8: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
norm	1	0 [83]	0 [82]	6 [81]	9 [79]	9 [77]	16 [70]	23 [63]	33 [51]	56 [32]	80 [20]	100 [0]
	2	2 [81]	2 [81]	5 [81]	6 [79]	16 [78]	22 [73]	33 [66]	58 [55]	67 [40]	92 [20]	100 [0]
	3	2 [82]	2 [81]	5 [80]	6 [79]	19 [77]	31 [72]	44 [67]	58 [56]	75 [42]	92 [18]	100 [0]
	4	0 [85]	0 [84]	2 [84]	6 [82]	9 [79]	27 [76]	36 [71]	47 [57]	78 [46]	92 [24]	100 [0]
lnorm	1	3 [80]	3 [80]	6 [78]	8 [78]	14 [76]	19 [66]	25 [60]	36 [51]	58 [36]	73 [21]	100 [0]
	2	3 [82]	6 [81]	9 [79]	11 [77]	14 [77]	20 [70]	28 [62]	38 [50]	53 [41]	80 [27]	100 [0]
	3	5 [82]	8 [80]	12 [79]	14 [78]	17 [79]	22 [76]	34 [66]	39 [50]	59 [41]	73 [30]	100 [0]
	4	8 [84]	12 [84]	14 [82]	14 [81]	16 [80]	23 [76]	33 [73]	44 [60]	61 [55]	78 [41]	100 [0]
logis	1	0 [85]	0 [82]	3 [81]	9 [79]	8 [79]	17 [77]	27 [68]	36 [51]	61 [34]	91 [20]	100 [0]
	2	0 [85]	0 [85]	5 [85]	5 [82]	12 [79]	12 [76]	23 [70]	36 [56]	50 [45]	78 [28]	100 [0]
	3	0 [82]	2 [82]	6 [82]	11 [81]	12 [78]	23 [74]	30 [68]	52 [55]	69 [44]	89 [28]	100 [0]
	4	0 [84]	0 [84]	3 [82]	8 [81]	16 [79]	30 [76]	45 [69]	55 [56]	80 [44]	91 [27]	100 [0]
exp	1	0 [85]	0 [83]	3 [81]	5 [79]	12 [79]	17 [75]	27 [69]	36 [54]	62 [35]	94 [20]	100 [0]
	2	0 [85]	0 [84]	8 [82]	8 [82]	14 [80]	22 [79]	28 [69]	42 [58]	69 [43]	86 [25]	100 [0]
	3	0 [85]	5 [85]	6 [84]	12 [83]	19 [83]	25 [81]	33 [73]	48 [54]	67 [43]	86 [29]	100 [0]
	4	0 [85]	6 [85]	8 [84]	19 [82]	22 [82]	28 [80]	44 [78]	55 [64]	62 [41]	86 [29]	100 [0]
weibull	1	0 [84]	0 [82]	5 [81]	9 [80]	12 [79]	17 [76]	25 [70]	28 [56]	50 [48]	73 [22]	100 [0]
	2	0 [84]	0 [83]	8 [83]	9 [82]	12 [80]	22 [76]	27 [68]	42 [61]	53 [50]	78 [23]	100 [0]

operators are advised to select very high reliability levels. To do so, the user can use current reliability thresholds, estimated by finding the reliability level at MTTRep using the empirical distribution (R_L ≈80%, Fig. 3.1.1.2). Furthermore, as suggested in Sec. 4, to prevent maintenance activities from deviating too much, the user can also select reliability models and thresholds based on the supplied MTTRep tables.

Table 3.1.1.7 shows that at current reliability levels, $\mathbf{76\% \leq R_L \%84}$, an underlying exponential distribution with four covariates can prevent between 45 to 60% of the failures (general-case). In Tab. 3.1.1.12 we can find that the corresponding MTTRep of this distribution and $\mathbf{76\% \leq R_L \%84}$ is between 1938 and 2700 cycles. This suggests that a reliability level of 76% and a time-independent PHM with an underlying exponential distribution can effectively prevent 45% of the failures and delay 71% of the maintenance events. However, since costs related to blade assembly and bearing failures are significant we want to minimise the chance that our estimates are wrong, hence use worst-case scenarios. Evidently, the corresponding tables, Tables 3.1.1.10 and 3.1.1.14, show us that 55 & 64% of the failures and scheduled events and would have been prevented and postponed (respectively) using worst-case scenarios and a reliability level of 76%.

Lastly, prior to addressing operational factor identification, the user is reminded that custom reliability levels and models can be evaluated using the 'EvaluateRM' function.

Table 3.1.1.9: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
	1	0 [85]	0 [85]	0 [85]	0 [85]	5 [85]	6 [79]	19 [75]	30 [59]	42 [48]	66 [25]	100 [0]
norm	2	0 [85]	0 [85]	0 [84]	0 [82]	6 [79]	11 [76]	14 [69]	27 [61]	30 [49]	52 [30]	100 [0]
	3	0 [85]	0 [84]	0 [82]	5 [79]	5 [79]	6 [77]	16 [71]	17 [64]	31 [52]	44 [34]	100 [0]
	1	0 [85]	0 [85]	0 [85]	0 [85]	5 [85]	6 [81]	22 [74]	33 [59]	45 [45]	64 [25]	100 [0]
lnorm	2	0 [85]	0 [85]	0 [84]	0 [82]	5 [79]	11 [78]	16 [76]	28 [63]	36 [52]	55 [29]	100 [0]
	3	0 [85]	0 [85]	0 [85]	0 [81]	0 [80]	8 [78]	16 [71]	28 [63]	47 [52]	61 [33]	100 [0]
	4	0 [85]	0 [85]	0 [82]	0 [81]	0 [79]	0 [77]	9 [76]	20 [70]	38 [56]	47 [40]	100 [0]
	1	2 [81]	2 [81]	2 [81]	2 [81]	6 [81]	8 [78]	22 [70]	33 [57]	44 [41]	75 [25]	100 [0]
	2	2 [81]	2 [81]	2 [81]	3 [78]	6 [77]	12 [72]	17 [70]	30 [63]	41 [44]	62 [28]	100 [0]
	3	2 [81]	2 [79]	3 [77]	6 [76]	9 [75]	11 [73]	16 [71]	23 [65]	36 [49]	53 [36]	100 [0]
	4	2 [81]	2 [81]	2 [77]	2 [76]	2 [76]	11 [74]	17 [69]	27 [66]	38 [49]	53 [35]	100 [0]
exp	5	2 [81]	2 [81]	2 [81]	2 [78]	5 [77]	9 [77]	11 [74]	20 [69]	34 [58]	50 [36]	100 [0]
	6	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	6 [82]	9 [76]	19 [73]	33 [66]	52 [36]	100 [0]
	7	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [81]	8 [77]	14 [72]	33 [56]	52 [36]	100 [0]
	8	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	3 [81]	11 [76]	30 [66]	52 [38]	100 [0]
	9	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	14 [74]	31 [64]	52 [38]	100 [0]
	10	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	11 [76]	31 [64]	52 [37]	100 [0]
	1	2 [81]	2 [81]	2 [81]	2 [81]	8 [80]	8 [76]	19 [70]	31 [57]	39 [46]	67 [24]	100 [0]
	2	2 [81]	2 [81]	2 [80]	3 [78]	5 [76]	12 [75]	17 [72]	25 [60]	39 [46]	53 [28]	100 [0]
	3	2 [81]	2 [80]	2 [78]	3 [76]	8 [76]	14 [75]	17 [67]	22 [62]	31 [54]	42 [38]	100 [0]
weibull	4	2 [81]	2 [81]	2 [78]	5 [76]	6 [76]	11 [75]	17 [74]	22 [66]	34 [57]	50 [36]	100 [0]
	5	2 [80]	2 [80]	2 [79]	2 [75]	3 [75]	6 [74]	12 [73]	20 [66]	27 [58]	42 [40]	100 [0]
	6	2 [81]	2 [81]	2 [81]	2 [80]	2 [81]	2 [78]	5 [75]	14 [72]	23 [60]	42 [42]	100 [0]
	7	2 [81]	2 [81]	2 [81]	2 [81]	2 [80]	2 [76]	3 [75]	9 [74]	23 [65]	42 [41]	100 [0]
	8	2 [81]	2 [81]	2 [81]	2 [81]	2 [81]	2 [81]	5 [81]	12 [77]	22 [69]	41 [45]	100 [0]

Table 3.1.1.10: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
	1	0 [85]	0 [85]	0 [85]	0 [85]	5 [85]	6 [79]	19 [75]	30 [58]	42 [46]	66 [25]	100 [0]
norm	2	0 [85]	0 [85]	0 [84]	0 [82]	6 [79]	11 [76]	14 [69]	27 [61]	33 [49]	52 [29]	100 [0]
	3	0 [85]	0 [84]	0 [81]	5 [79]	5 [79]	6 [77]	16 [71]	17 [64]	31 [52]	44 [34]	100 [0]
	1	0 [85]	0 [85]	0 [85]	0 [85]	5 [85]	6 [80]	23 [73]	34 [59]	45 [45]	64 [25]	100 [0]
lnorm	2	0 [85]	0 [85]	0 [84]	0 [82]	5 [79]	12 [78]	16 [76]	28 [63]	36 [52]	56 [28]	100 [0]
	3	0 [85]	0 [85]	0 [85]	0 [81]	0 [79]	8 [78]	17 [71]	28 [63]	47 [52]	61 [32]	100 [0]
	4	0 [85]	0 [85]	0 [82]	0 [81]	0 [79]	0 [77]	11 [76]	20 [69]	38 [55]	47 [38]	100 [0]
	1	2 [81]	2 [81]	2 [81]	2 [81]	6 [81]	8 [79]	19 [74]	33 [59]	42 [42]	69 [26]	100 [0]
	2	2 [81]	2 [81]	2 [81]	3 [78]	3 [77]	12 [74]	16 [71]	30 [65]	38 [44]	53 [30]	100 [0]
	3	2 [81]	2 [79]	3 [77]	6 [76]	9 [75]	11 [73]	16 [71]	23 [65]	36 [49]	53 [36]	100 [0]
	4	2 [81]	2 [81]	2 [77]	2 [77]	2 [76]	11 [74]	17 [71]	27 [67]	38 [51]	53 [35]	100 [0]
exp	5	2 [81]	2 [81]	2 [81]	2 [78]	3 [77]	9 [77]	11 [74]	20 [70]	33 [59]	48 [37]	100 [0]
	6	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	6 [82]	9 [76]	19 [73]	33 [66]	50 [36]	100 [0]
	7	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [81]	8 [77]	14 [72]	33 [56]	52 [36]	100 [0]
	8	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	3 [81]	11 [76]	30 [66]	52 [38]	100 [0]
	9	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	14 [74]	31 [64]	52 [38]	100 [0]
	10	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	2 [82]	11 [76]	31 [64]	50 [37]	100 [0]
	1	2 [81]	2 [81]	2 [81]	2 [81]	6 [81]	8 [78]	17 [73]	30 [58]	39 [50]	58 [29]	100 [0]
	2	2 [81]	2 [81]	2 [80]	3 [78]	3 [77]	12 [75]	17 [72]	25 [60]	34 [46]	48 [30]	100 [0]
	3	2 [81]	2 [80]	2 [78]	3 [76]	8 [76]	14 [74]	17 [67]	22 [63]	31 [56]	42 [38]	100 [0]
weibull	4	2 [81]	2 [81]	2 [78]	5 [76]	6 [76]	8 [75]	17 [74]	22 [63]	34 [57]	50 [36]	100 [0]
	5	2 [80]	2 [80]	2 [79]	2 [75]	3 [76]	6 [74]	12 [74]	19 [66]	27 [58]	41 [41]	100 [0]
	6	2 [81]	2 [81]	2 [81]	2 [80]	2 [81]	2 [78]	5 [75]	14 [72]	23 [61]	42 [42]	100 [0]
	7	2 [81]	2 [81]	2 [81]	2 [81]	2 [80]	2 [76]	3 [75]	9 [74]	23 [65]	42 [41]	100 [0]
	8	2 [81]	2 [81]	2 [81]	2 [81]	2 [81]	2 [81]	5 [81]	12 [77]	20 [69]	39 [45]	100 [0]

Table 3.1.1.11: Average MTTF values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
norm	1	1857.3	1857.3	1857.3	1857.3	1654.4	1607.5	1395.2	1141.5	812.6	584.4	NaN
	2	1857.3	1857.3	1857.3	1883	1842.2	1801.6	1529.7	918.2	766.8	568.9	NaN
	3	1857.3	1857.3	1857.3	1857.3	1883	1822.6	1400.7	1199	894.8	627.8	NaN
	4	1857.3	1857.3	1857.3	1857.3	1827.5	1496.4	1251.9	1028.8	912.7	430.1	NaN
lnorm	1	1857.3	1857.3	1857.3	1857.3	1562.3	1562.3	1351.2	1087.2	747.8	571.7	NaN
	2	1857.3	1857.3	1857.3	1791.1	1567.7	1538.5	1335.8	1148.7	871.5	642.8	NaN
	3	1857.3	1857.3	1649.3	1607.5	1590.7	1452.1	1352.5	1198.8	978.3	712.6	NaN
	4	1869.3	1622.1	1597.1	1597.1	1556.4	1415.2	1338.7	1065.6	869.8	786	NaN
logis	1	1857.3	1857.3	1857.3	1857.3	1857.3	1607.5	1395.2	1087.2	680.9	531.8	NaN
	2	1857.3	1857.3	1857.3	1654.4	1616.8	1574.4	1268.6	1107.1	770.4	541.1	NaN
	3	1857.3	1857.3	1824.6	1824.6	1824.6	1517.4	1468.8	1203.4	1010.6	492.3	NaN
	4	1857.3	1857.3	1857.3	1869.3	1869.3	1496.4	1340.6	1140.1	976.4	508.9	NaN
exp	1	1857.3	1857.3	1857.3	1857.3	1857.3	1562.3	1399.3	1087.2	696.9	531.8	NaN
	2	1857.3	1857.3	1857.3	1857.3	1616.8	1515.2	1331.2	1221.9	819.8	410.9	NaN
	3	1857.3	1857.3	1857.3	1607.5	1607.5	1509.8	1445.6	1185.8	897.4	692.4	NaN
	4	1857.3	1857.3	1649.3	1607.5	1629.1	1453.9	1409.5	1371.4	1356.3	529.7	NaN
weibull	1	1857.3	1857.3	1857.3	1815.9	1795.6	1509.5	1293.7	1125	911.5	520.8	NaN
	2	1857.3	1857.3	1857.3	1857.3	1607.7	1478.1	1248.8	1156.5	1170.3	540.9	NaN

Table 3.1.1.12: Average MTTRep values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
norm	1	4861.7	4772.3	4643.1	4462.8	4206.4	3738.1	3258.3	2551.1	1702.7	865.8	20
	2	4860.8	4782.7	4638.7	4444.5	4181.8	3831.2	3326.6	2693.1	1861.8	907.5	20
	3	4786.8	4652.9	4547.5	4393	4100	3779.4	3428.6	2909.2	2001.9	1034.3	20
	4	4887.8	4819	4707.7	4534.7	4253.4	3939.6	3452.3	2763.8	1867.7	1024.3	20
lnorm	1	4904.7	4831	4718.2	4517.2	4135.6	3714.6	3170.7	2542.6	1700.3	992.5	20
	2	4920.6	4884.6	4807.6	4681	4350.2	3900.1	3338.1	2626.3	1872.2	1129.1	20
	3	4916.2	4911.8	4855.8	4754.5	4461.1	4108.6	3431	2719.2	2036.7	1313.1	20
	4	4810.3	4724.3	4586.7	4464.4	4298.1	4003.4	3523.9	3014	2355.7	1551.4	20
logis	1	4918.5	4896.4	4819.7	4681.5	4425.1	3880.2	3316.4	2474.9	1565	768.3	20
	2	4897.2	4828.1	4688.8	4516.6	4195.6	3847.7	3278.5	2654.6	1899.7	1000.3	20
	3	4869.2	4798.9	4680.9	4546.4	4337.9	3942.7	3580.9	2903.7	2003.2	1136.3	20
	4	4896.7	4836.7	4727.8	4522.6	4284.9	4013.5	3534.1	2798.2	1968.7	1077.7	20
exp	1	4920.9	4914.9	4855.7	4717.6	4469.7	3907.2	3296.6	2476.7	1495.6	749.4	20
	2	4920.9	4920.9	4895.4	4794.3	4576.8	4109.6	3453.4	2578.4	1762.5	940	20
	3	4919.8	4918.3	4915.6	4860.1	4731.2	4256.6	3513.3	2644.4	1838.3	1068.8	20
	4	4917.8	4916.2	4906.1	4833.4	4626.3	4275.2	3772.2	2772.4	1938.6	1030.7	20
weibull	1	4899.1	4800.3	4693	4573.3	4282.3	3908.9	3395.1	2736	1964.9	1056.8	20
	2	4919.8	4905.8	4825.2	4705.8	4455.8	3858.5	3401.4	2736.1	2046.4	1113.1	20

Table 3.1.1.13: Average MTTF values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
norm	1	1857.3	1857.3	1681	1631.1	1631.1	1492.7	1294.4	1057.1	795.6	661.5	NaN
	2	1883	1883	1845.1	1836.8	1603.3	1611.8	1384.9	831.3	787.8	388.8	NaN
	3	1883	1883	1931.5	1941	1711.1	1538.2	1366.2	1113.1	864.9	494.6	NaN
	4	1857.3	1857.3	1869.3	1663.7	1573.2	1286	1213.5	1034.1	770.4	508.4	NaN
lnorm	1	1819.1	1819.1	1775.5	1607.5	1540.4	1406.7	1272.6	1045.4	698.1	601.5	NaN
	2	1819.1	1681	1564.6	1523.4	1468.3	1344.8	1247.1	1174.3	926.9	580.2	NaN
	3	1654.4	1607.5	1512	1519.5	1473.8	1410.3	1377.3	1211.7	928.7	730.4	NaN
	4	1622.1	1527.4	1499.2	1499.2	1454.5	1419.4	1329.3	1090.4	943.3	772.7	NaN
logis	1	1857.3	1857.3	1819.1	1631.1	1607.5	1447.1	1204.7	993.5	732	378.2	NaN
	2	1857.3	1857.3	1654.4	1654.4	1477.1	1477.1	1245.8	934.8	821	501.6	NaN
	3	1857.3	1883	1810.6	1591.1	1577.6	1462	1496.2	1146.8	834.8	633.3	NaN
	4	1857.3	1857.3	1895.6	1687.9	1643.6	1464.9	1268.7	1101.7	1025.8	535.8	NaN
exp	1	1857.3	1857.3	1819.1	1845.1	1586	1447.1	1204.7	993.5	725.5	507.8	NaN
	2	1857.3	1857.3	1607.5	1607.5	1468.3	1367	1259.7	1150.9	830.5	415	NaN
	3	1857.3	1654.4	1649.3	1512	1533.7	1514.8	1489.1	1293.7	925.4	699.8	NaN
	4	1857.3	1663.7	1607.5	1581.2	1586.4	1553.6	1480.1	1512.6	1300.6	459.4	NaN
weibull	1	1857.3	1857.3	1775.8	1558.3	1467.3	1357.6	1193.3	1119.9	852	519.5	NaN
	2	1857.3	1857.3	1607.5	1558.3	1467.3	1292.4	1252.5	1120.3	1098.6	586.6	NaN

Table 3.1.1.14: Average MTTRep values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
norm	1	**4623.5**	**4509.6**	**4326.4**	**4040**	**3705**	**3223.9**	**2769.4**	**2107.7**	**1205.6**	**542**	**20**
	2	**4514.4**	**4416**	**4226.3**	**4050.8**	**3796.4**	**3351.9**	**2757.1**	**2105.4**	**1177**	**481.1**	**20**
	3	**4412.5**	**4291.5**	**4083.3**	**3913.3**	**3533.7**	**3168.6**	**2770.1**	**1973.1**	**1170.9**	**466.4**	**20**
	4	**4688.5**	**4563.7**	**4414.5**	**4263.8**	**3945.6**	**3489.7**	**3011.3**	**2389.6**	**1498.6**	**591.1**	**20**
lnorm	1	**4507.2**	**4399.7**	**4159.5**	**3971.6**	**3521.4**	**3080.7**	**2607.6**	**2033.5**	**1235.7**	**644.9**	**20**
	2	**4716.7**	**4629.8**	**4255.6**	**4023.5**	**3730.7**	**3281.9**	**2711.8**	**2038.1**	**1432.8**	**790.3**	**20**
	3	**4824.3**	**4618.7**	**4389.7**	**4148.1**	**3846.7**	**3457.6**	**2821.4**	**1984.8**	**1499.7**	**967.8**	**20**
	4	**4605.3**	**4382.9**	**4213.2**	**4070.3**	**3841.6**	**3496.1**	**3042.7**	**2462.3**	**1958.5**	**1243.9**	**20**
logis	1	**4763.1**	**4624.4**	**4459.6**	**4237.7**	**3951.3**	**3427.8**	**2846.9**	**2090.8**	**1082**	**495.6**	**20**
	2	**4852.8**	**4736**	**4554.5**	**4331.7**	**4000.8**	**3584.1**	**3036**	**2415.4**	**1667.3**	**831.9**	**20**
	3	**4673.2**	**4498.7**	**4329.3**	**4179.7**	**3940.7**	**3494.6**	**2917**	**2226.5**	**1462.6**	**719.9**	**20**
	4	**4639.1**	**4552.2**	**4344.8**	**4192.1**	**3818.5**	**3346.2**	**2873.7**	**2226.7**	**1391.4**	**627.8**	**20**
exp	1	**4805.8**	**4652.6**	**4470**	**4233.5**	**3944.6**	**3423.8**	**2886.5**	**2055**	**1092.3**	**477**	**20**
	2	**4900.4**	**4869.8**	**4709.9**	**4502.3**	**4038.5**	**3595.8**	**2971.6**	**2165.7**	**1339.2**	**718.2**	**20**
	3	**4919.3**	**4910.3**	**4806.4**	**4644.8**	**4259.3**	**3723.3**	**3002.4**	**2070.7**	**1449.2**	**815.3**	**20**
	4	**4917.5**	**4868.4**	**4774.2**	**4383.7**	**4063.4**	**3636.4**	**3031**	**2351.2**	**1451.5**	**775**	**20**
weibull	1	**4800.6**	**4612.3**	**4386.8**	**4244.3**	**3924.6**	**3535.8**	**2988**	**2304.2**	**1553.8**	**774.4**	**20**
	2	**4817.6**	**4686.4**	**4543**	**4337.9**	**3985.6**	**3412.4**	**2890.4**	**2229.4**	**1638.5**	**738.1**	**20**

Table 3.1.1.15: Average MTTF values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
norm	1	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1654.4**	**1607.7**	**1281**	**1016.3**	**715.3**	**401.2**	NaN
	2	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1607.7**	**1501.3**	**1422.4**	**1108.2**	**1028.8**	**606**	NaN
	3	**1857.3**	**1857.3**	**1857.3**	**1654.4**	**1654.4**	**1623.1**	**1396.4**	**1377.8**	**1036**	**792**	NaN
lnorm	1	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1654.4**	**1607.7**	**1199.3**	**942**	**657.5**	**422.4**	NaN
	2	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1759.4**	**1501.3**	**1392.9**	**1065.4**	**893.1**	**558.4**	NaN
	3	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1727.5**	**1551.7**	**1103.5**	**801.4**	**653**	NaN
	4	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1588.6**	**1324**	**914.1**	**773.1**	NaN
exp	1	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1676.6**	**1629.4**	**1273.9**	**991**	**805.2**	**317.2**	NaN
	2	**1881.6**	**1881.6**	**1881.6**	**1840**	**1788.5**	**1532.6**	**1431.4**	**1081.8**	**873.5**	**669.2**	NaN
	3	**1881.6**	**1881.6**	**1840**	**1784.1**	**1723.6**	**1648.1**	**1473.9**	**1245.6**	**1043.1**	**741.1**	NaN
	4	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1562.4**	**1424.1**	**1223.1**	**1041.5**	**741.1**	NaN
	5	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1827.2**	**1612.3**	**1573.8**	**1347.7**	**1097.5**	**779.6**	NaN
	6	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1676.6**	**1597.8**	**1396.3**	**1039.7**	**751.6**	NaN
	7	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1640.5**	**1477.9**	**1043.1**	**778.7**	NaN
	8	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1814.8**	**1572.9**	**1165.5**	**756.2**	NaN
	9	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1504.8**	**1053.2**	**820.7**	NaN
	10	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1583.5**	**1116.1**	**820.7**	NaN
weibull	1	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1644.9**	**1629.4**	**1340.5**	**1023.8**	**858.9**	**404.8**	NaN
	2	**1881.6**	**1881.6**	**1881.6**	**1840**	**1771.4**	**1532.6**	**1405.5**	**1210.5**	**890.3**	**613.1**	NaN
	3	**1881.6**	**1881.6**	**1881.6**	**1840**	**1629.4**	**1491.3**	**1424.1**	**1298.9**	**1074.6**	**842.2**	NaN
	4	**1881.6**	**1881.6**	**1881.6**	**1812**	**1783.3**	**1653.3**	**1418.5**	**1293.1**	**983.3**	**771.5**	NaN
	5	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1840**	**1783.3**	**1522.3**	**1345.6**	**1178.3**	**895.1**	NaN
	6	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1807.7**	**1514.7**	**1262.4**	**845.4**	NaN
	7	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1850.5**	**1591.8**	**1259.4**	**845.4**	NaN
	8	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1807.7**	**1537.6**	**1292.6**	**852.1**	NaN

Table 3.1.1.16: Average MTTF values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
norm	1	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1654.4**	**1607.7**	**1281**	**1016.3**	**715.3**	**401.2**	NaN
	2	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1607.7**	**1501.3**	**1422.4**	**1108.2**	**952.6**	**606**	NaN
	3	**1857.3**	**1857.3**	**1857.3**	**1654.4**	**1654.4**	**1623.1**	**1396.4**	**1377.8**	**1036**	**792**	NaN
lnorm	1	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1654.4**	**1607.7**	**1162.2**	**908.7**	**657.5**	**422.4**	NaN
	2	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1759.4**	**1470.5**	**1392.9**	**1065.4**	**893.1**	**536.5**	NaN
	3	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1727.5**	**1514.3**	**1103.5**	**801.4**	**653**	NaN
	4	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1857.3**	**1554.4**	**1324**	**914.1**	**773.1**	NaN
exp	1	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1676.6**	**1629.4**	**1351.7**	**991**	**829.7**	**387.9**	NaN
	2	**1881.6**	**1881.6**	**1881.6**	**1840**	**1840**	**1532.6**	**1449.9**	**1081.8**	**896**	**747**	NaN
	3	**1881.6**	**1881.6**	**1840**	**1784.1**	**1723.6**	**1648.1**	**1473.9**	**1245.6**	**1043.1**	**741.1**	NaN
	4	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1562.4**	**1424.1**	**1223.1**	**1041.5**	**741.1**	NaN
	5	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1854.9**	**1612.3**	**1573.8**	**1347.7**	**1153.7**	**788.3**	NaN
	6	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1676.6**	**1597.8**	**1396.3**	**1039.7**	**764.8**	NaN
	7	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1640.5**	**1477.9**	**1043.1**	**778.7**	NaN
	8	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1814.8**	**1572.9**	**1165.5**	**756.2**	NaN
	9	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1504.8**	**1053.2**	**820.7**	NaN
	10	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1583.5**	**1116.1**	**820.8**	NaN
weibull	1	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1676.6**	**1629.4**	**1387.1**	**1061.3**	**858.9**	**533.2**	NaN
	2	**1881.6**	**1881.6**	**1881.6**	**1840**	**1840**	**1532.6**	**1405.5**	**1210.5**	**976.6**	**780.1**	NaN
	3	**1881.6**	**1881.6**	**1881.6**	**1840**	**1629.4**	**1491.3**	**1424.1**	**1298.9**	**1074.6**	**842.2**	NaN
	4	**1881.6**	**1881.6**	**1881.6**	**1812**	**1783.3**	**1758.8**	**1418.5**	**1293.1**	**983.3**	**771.5**	NaN
	5	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1840**	**1783.3**	**1522.3**	**1387.3**	**1178.3**	**884.8**	NaN
	6	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1807.7**	**1514.7**	**1262.4**	**845.4**	NaN
	7	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1850.5**	**1591.8**	**1259.4**	**845.4**	NaN
	8	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1881.6**	**1807.7**	**1537.6**	**1336.7**	**858.9**	NaN

Table 3.1.1.17: Average MTTRep values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
norm	1	**4920.9**	**4902.8**	**4835.5**	**4677.3**	**4307.3**	**3772.2**	**3143**	**2420.3**	**1676.9**	**848.1**	**20**
	2	**4572.8**	**4497.4**	**4344.6**	**4080.7**	**3885.3**	**3521**	**3061.7**	**2545.1**	**1880.5**	**1089.2**	**20**
	3	**4535.6**	**4430.1**	**4248.7**	**4176.1**	**3981**	**3702.1**	**3253.3**	**2730.5**	**2099.6**	**1271.2**	**20**
lnorm	1	**4920.9**	**4920.9**	**4917**	**4815.9**	**4442.3**	**3800.1**	**3071.6**	**2315.4**	**1625.3**	**892.7**	**20**
	2	**4605.4**	**4544.5**	**4421**	**4209.6**	**3967.4**	**3726.8**	**3221.7**	**2580.7**	**1875.5**	**1103.1**	**20**
	3	**4732.6**	**4605.4**	**4472.6**	**4324.7**	**4132.6**	**3822.2**	**3269.1**	**2692.2**	**1845.1**	**1030.8**	**20**
	4	**4920.9**	**4873.3**	**4559.7**	**4331.5**	**4075.9**	**3820.8**	**3595.2**	**3011.3**	**2277.1**	**1304.4**	**20**
exp	1	**4548.2**	**4548.1**	**4514.8**	**4428.7**	**4159.6**	**3632.6**	**3009.3**	**2232.1**	**1479.1**	**741.6**	**20**
	2	**4301.7**	**4305.4**	**4203**	**4008.3**	**3735.3**	**3505.2**	**3007.4**	**2436.4**	**1697.8**	**898.2**	**20**
	3	**4470**	**4315.4**	**4120.2**	**3923.7**	**3727.3**	**3573.1**	**3261.1**	**2668.4**	**2001.3**	**1149.6**	**20**
	4	**4470**	**4340.7**	**4187**	**4003.6**	**3703.7**	**3510.1**	**3146.1**	**2638.4**	**1928.8**	**1092.4**	**20**
	5	**4631.2**	**4630.2**	**4474.8**	**4206.9**	**3995.9**	**3890.6**	**3596.8**	**2881.3**	**2154.1**	**1217.9**	**20**
	6	**4705.1**	**4705.1**	**4705.1**	**4705**	**4623.7**	**4485.8**	**4086**	**3511**	**2558.4**	**1348.7**	**20**
	7	**4705.1**	**4705.1**	**4705.1**	**4669.1**	**4614.3**	**4441.7**	**4096.3**	**3449.8**	**2427.1**	**1283.6**	**20**
	8	**4705.1**	**4705.1**	**4705.1**	**4705**	**4694.8**	**4579.1**	**4316**	**3734.3**	**2734.7**	**1496.8**	**20**
	9	**4705.1**	**4705.1**	**4705.1**	**4705**	**4697.4**	**4572.2**	**4318.5**	**3643.3**	**2613.9**	**1429**	**20**
	10	**4705.1**	**4705.1**	**4705.1**	**4705**	**4702**	**4626**	**4429.1**	**3801**	**2785.8**	**1447.9**	**20**
weibull	1	**4548.2**	**4524.1**	**4438**	**4242.4**	**3917.1**	**3446.9**	**2943.9**	**2325.5**	**1635.9**	**927.9**	**20**
	2	**4294.8**	**4263.3**	**4144.1**	**3934.6**	**3675.8**	**3426.7**	**3011.3**	**2455**	**1820.1**	**966.4**	**20**
	3	**4431.1**	**4312.6**	**4145.3**	**3975.6**	**3869.9**	**3564.8**	**3145.7**	**2719.6**	**2104.4**	**1375.7**	**20**
	4	**4532.6**	**4448.6**	**4270.9**	**4112.6**	**3846.5**	**3603.2**	**3310.4**	**2851.1**	**2182.7**	**1297.6**	**20**
	5	**4501.8**	**4436.1**	**4331.4**	**4175.8**	**3996**	**3820.5**	**3586.3**	**3055.3**	**2449.2**	**1490.1**	**20**
	6	**4532.6**	**4532.6**	**4510.2**	**4480.4**	**4421.5**	**4238.5**	**3881.3**	**3390.8**	**2624.8**	**1656.4**	**20**
	7	**4548.2**	**4548.2**	**4535.6**	**4478.6**	**4350.3**	**4182.4**	**3925.6**	**3582.8**	**2780.9**	**1737.5**	**20**
	8	**4548.2**	**4548.2**	**4548.2**	**4548.1**	**4548**	**4512.6**	**4255.9**	**3827.7**	**2946.4**	**1793**	**20**

Table 3.1.1.18: Average MTTRep values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	21%	29%	37%	44%	52%	60%	68%	76%	84%	92%	100%
	1	4920.9	4902.6	4833.8	4671.2	4299.4	3760.8	3126.4	2401.6	1659.4	829	20
norm	2	4572.4	4494.9	4337.7	4074.3	3875.9	3505	3046.9	2524.9	1871.8	1066.3	20
	3	4535.1	4428.5	4246.7	4175.2	3978.7	3694.4	3240.2	2712.1	2083	1253.6	20
	1	4920.9	4920.9	4916.9	4812.5	4434.7	3788.4	3058	2297.1	1607.2	874.1	20
lnorm	2	4605.4	4543.7	4417.9	4207.7	3962.3	3714.1	3206.3	2561.4	1850.9	1077.9	20
	3	4731.3	4605.4	4470.4	4322.6	4130.9	3817.4	3256.7	2669.8	1820.4	1009.5	20
	4	4920.9	4872.6	4558.6	4324.2	4074.4	3817.3	3563.6	2979.6	2241	1258.9	20
	1	4570.5	4570.5	4536.7	4455.4	4205.9	3696.9	3107.4	2345.1	1579	847.3	20
	2	4320.9	4318.4	4230.5	4032	3783.9	3569.5	3078.8	2521.3	1778.3	969.8	20
	3	4470	4317.1	4120.7	3922.4	3728.2	3610.7	3264.5	2679.7	2009.6	1164	20
	4	4496.3	4368.6	4227.8	4036.8	3708.4	3566.4	3208.1	2718.7	1974.4	1128.6	20
	5	4633.3	4633.3	4487.1	4247.6	4027.1	3912.7	3620	2965.9	2206.6	1275	20
exp	6	4705.1	4705.1	4705.1	4705	4624.8	4487.4	4090.1	3517.5	2565.8	1358.6	20
	7	4705.1	4705.1	4705.1	4669.9	4614.8	4446.4	4103.4	3463.8	2446	1300.6	20
	8	4705.1	4705.1	4705.1	4705	4695	4580.2	4320.5	3743.7	2747.4	1513	20
	9	4705.1	4705.1	4705.1	4705	4697.5	4574.9	4323.4	3653.4	2630.7	1440.8	20
	10	4705.1	4705.1	4705.1	4705	4702.4	4627.5	4432.9	3811.7	2800.9	1468.6	20
	1	4570.5	4546.2	4478.6	4312.5	4005.5	3539.9	3035.5	2416.2	1735.7	1037.4	20
	2	4320	4284.4	4167.1	3959.4	3726.8	3486.3	3068.5	2515.3	1873.8	1133.9	20
	3	4432.7	4315.4	4146.9	3979.1	3878.8	3561	3056.1	2723.6	2133.6	1407.4	20
weibull	4	4532.6	4448.6	4270.9	4112.9	3849.1	3587.6	3304.5	2829.6	2192	1300.4	20
	5	4501.8	4436.9	4332.2	4177.5	4000.6	3822.1	3603.3	3069.9	2448.9	1522.8	20
	6	4532.6	4532.6	4510.2	4480.7	4422	4238.4	3886	3401.7	2627	1672.6	20
	7	4548.2	4548.2	4535.6	4478.9	4350.7	4183.9	3927.1	3587.9	2794.3	1752.2	20
	8	4548.2	4548.2	4548.2	4548.1	4548	4512.9	4258.3	3827.2	2946.4	1830.8	20

Part VI *Identified operational factors:*
Tables 3.1.1.19, 3.1.1.20, 3.1.1.21, and 3.1.1.22 list the operational factors identified by time-independent and -dependent PH models. The term 'scaled value' represents the effect the operational factor had on the maximum likelihood function.

Using Table 3.1.1.20 the user can identify the covariates corresponding to a time-independent PHM with an underlying d distribution and n number of covariates. Continuing from the decisions made in Part V, we find that time-independent PHMs with an underlying exponential or logistic distribution and four covariates included:

Exp. & Logis.	**Group A**	Vertical velocity during landing
Logis.	**Group B**	Dynamic pressure during landing
Logis.	**Group C**	RHS inner aileron input during landing
Exp. & Logis.	**Group D**	Rudder input during take-off
Exp.	**Group E**	Angle of attack during take-off
Exp.	**Group F**	Rudder command force during various phases

The identified factors will be discussed in validation (Sec. 3.1.2). Recall that variables in the same group are highly correlated (linear).

Table 3.1.1.19: Variables identified by time-(in)dependent PHM models.

	Time-independent PHM			**Time-dependent PHM**	
	Variable	Scaled Value		Variable	Scaled Value
①	*Group A*	$\mathbf{-12.01}$	①	*Group I*	$\mathbf{-30.14}$
②	*Roll rate max deg sec 2*	$\mathbf{8.19}$	②	*Group K*	$\mathbf{-22.35}$
③	*Group B*	$\mathbf{8.15}$	③	*Vz min ft min*	$\mathbf{-17.68}$
④	*Group D*	$\mathbf{6.39}$	④	*Roll mean deg*	$\mathbf{-16.71}$
⑤	*Group E*	$\mathbf{-4.82}$	⑤	*Pitch rate mean deg sec 8*	$\mathbf{16.21}$
⑥	*Group C*	$\mathbf{-2.71}$	⑥	*Group H*	$\mathbf{-12.43}$
⑦	*Group F*	$\mathbf{0.92}$	⑦	*Brake press lhs mean psi 8*	$\mathbf{11.87}$
			⑧	*Group J*	$\mathbf{-11.59}$
			⑨	*Group L*	$\mathbf{-11.11}$
			⑩	*Group M*	$\mathbf{11.09}$
			⑪	*Group O*	$\mathbf{-10.11}$
			⑫	*Group N*	$\mathbf{-7.68}$
			⑬	*Accn lat mean g s*	$\mathbf{7.02}$
			⑭	*Group G*	$\mathbf{-6.62}$

Table 3.1.1.20: Variables identified by each step by time-(in)dependent PHMs (in order).

	PHM Variables	
	Time-independent	*Time-dependent*
norm	① ③ ⑥ ④	⑭ ④ ⑥
lnorm	① ④ ⑤ ③	⑭ ④ ① ⑥
logis	① ④ ③ ⑥	
exp	① ④ ⑤ ⑦	⑧ ④ ② ⑨ ⑩ ⑫ ⑪ ① ③ ⑥
weibull	② ⑦	⑧ ④ ② ⑦ ⑤ ① ⑭ ⑬

Table 3.1.1.21: Number of times variables identified by each step by time-(in)dependent PHMs.

Key				**Key**			
indep	*dep*	*Variable*	*Count*	*indep*	*dep*	*Variable*	*Count*
①		Group A	4		⑨	Group L	1
③		Group B	3		⑩	Group M	1
⑥		Group C	2		⑫	Group N	1
④		Group D	4		⑪	Group O	1
⑤		Group E	2		⑬	Accn lat mean g s	1
⑦		Group F	2		⑦	Brake press lhs mean psi 8	1
	⑭	Group G	3		⑤	Pitch rate mean deg sec 8	1
	⑥	Group H	3		④	Roll mean deg	4
	①	Group I	3	②		Roll rate max deg sec 2	1
	⑧	Group J	2		③	Vz min ft min	1
	②	Group K	2				

3.1.2 Validation

The following section will focus on validating the results using logical arguments derived from aviation and aircraft theory. Blade assembly and bearing parts are found in the aircraft' engines. Hence, logically, the factors identified by the models should be directly or indirectly related to engine performance. In addition, as for any component, failures could be related to abnormal forces and stresses experienced by the aircraft. In this paper validation will consist

Table 3.1.1.22: Variables belonging to each group identified in 3.1.1.19.

Group	*Variables*	*Group*	*Variables*
Group A	Vz mean ft min 5, Vz mean ft min 6	Group I	Torque lhs mean 5, Torque rhs mean 5, Torque lhs mean 6, Torque rhs mean 6
Group B	Pressure dynamic max hPa mbar 8, Pressure dynamic min hPa mbar 5	Group J	Pressure total max hPa mbar 5, Alt pres min ft 5, Alt pres min ft 6, Press ambient max hPa mbar 6, Pressure total mean hPa mbar 6, Pressure total max hPa mbar 6, Alt pres mean ft 8, Alt pres max ft 8, Alt pres min ft 8, Press ambient mean hPa mbar 8, Press ambient max hPa mbar 8, Press ambient min hPa mbar 8, Pressure total mean hPa mbar 8, Pressure total max hPa mbar 8, Pressure total min hPa mbar 8, Pressure total max hPa mbar 1
Group C	Aileron Rin max deg TEU 5, Aileron Rin max deg TEU 6, Aileron Rin max deg TEU 2, Aileron Rin mean deg TEU 6, Aileron Rin mean deg TEU 8	Group K	Vcal mean knots , Vtrue mean knots
Group D	Rudder low mean deg TER 1, Rudder low mean deg TER 2	Group L	Accn long mean g s 8, Brake press rhs mean psi 8
Group E	Aoa min deg , Aoa mean deg	Group M	Accn norm mean g s 1, Pitch rate mean deg sec 1
Group F	Rudder cmd force mean lbs Nose Right 3, Rudder cmd force mean lbs Nose Right 2, Rudder cmd force mean lbs Nose Right 4, Rudder cmd force mean lbs Nose Right 6	Group N	Vtrue mean knots 4, Vcal mean knots 4, Pressure dynamic mean hPa mbar 4
Group G	Roll mean deg 1, Yaw rate mean deg sec 1	Group O	Vz min ft min 5, Vz min ft min 6
Group H	Accn long mean g s 6, Accn long mean g s 5, Pitch mean deg 6, Aoa min deg 5		

of identifying whether a potential relationship exists between operational factors and the affected component.

The reader should keep in mind that the factors identified during modelling are limited to the parameters recorded by on-board FDRs and the reduction procedures described in Sec. 2.

The identified variables (Tab. 3.1.1.19) were categorised in the following manner:

Engine related:	Torque
Roll/Pitch/Yaw related:	Roll (angle/rate), pitch (angle), angle of attack
	Control: aileron & rudder input, rudder command force
Interaction:	Vertical & true velocity, pressure, lateral & longitudinal acceleration
Other:	Brake pressure

Since a variety of operational factors were identified it is likely that failures were caused by the interaction of two (or more) of the identified variables. The operational factors were selected in a stepwise manner, hence the first operational factor generally drives the selection of the subsequent variables. The first variables identified by time-independent and dependent PHMs were vertical velocity (descent/landing) and total/ambient/altitude pressure (descent/landing) & roll and yaw angle (take-off) respectively. This could indicate that two (or more) failure modes are present.

For a majority of the time-independent PHMs dynamic pressure was identified during climb. Dynamic pressure is the kinetic energy per unit volume of a fluid. In in-compressible fluid dynamics theory the parameter is a function of density and velocity. The negative scale value (Tab. 3.1.1.19) implies that, on average, the dynamic pressure observed by failure related flights was lower than that of non-failed flights. Hence the air density and (or) velocity was less than usual. To conclude, a study would have to be performed to determine whether component failures were a consequence of low air densities and velocities or an operational procedure performed to correct for these (e.g. increase in thrust).

Interestingly the remainder of variables are all indirectly related, such as rolling & pitching motions, lateral & longitudinal accelerations, and aileron & rudder inputs. The variables predominately were identified during take-off/climb and descent/landing, both of which are flight phases in which the blade assembly and bearing are operated irregularly.

The variables identified by the most effective reliability models, i.e. time-independent PHMs with an underlying exponential (or logistic) distribution and four covariates, indicate that in combination to aileron and rudder inputs being abnormal during various flight phases the dynamic pressure and vertical velocity during landing were far from the norm. Perhaps the aircraft was subject to adverse weather conditions or was improperly loaded. The true nature of the failure can be investigated by the airline in the future.

To conclude, although the operational factors identified by the solution have been validated, the true cause for component failures is still uncertain. The variables suggest that the aircraft, during failure related flights, endured abnormal roll, yaw, and pitching angles during take-off, climb, and touchdown and as a consequences used significantly different thrust (torque) during these phases. It is very likely that, as a consequence, the stresses and strains experienced by the structure and component were higher than usual. The aforementioned observations could be related to numerous scenarios such as: abnormal aircraft loading, weather conditions, delays, etc.

Nonetheless, model evaluation using historical data suggests that reliability can be improved significantly based on a variety of models. As an example, at a reliability level of 76 or 84%, a time-independent PHM with an underlying exponential or logistic distribution and four covariates could have prevented 50 to 80% of the failures. In addition, up to 64% of the scheduled events could have been delayed, saving costs related to performing unnecessary maintenance.

3.2 174260-08 Crew oxygen mask

This section will summarise the results obtained from analysing 'crew oxygen mask' components corresponding to part number **174260-08**. This section will act as a summary only and unlike component PN:**697071003** all results are listed in Appendix B.2. The reader is advised to read Sec. 3.1 prior to this one to understand how the components are evaluated.

3.2.1 Results

Table B.2.1 shows that in total 373 maintenance events were recorded since 2004 of which 62 were related to flights after 2010 (FDR data). About 40% of the occurrences were scheduled at an average cycle time of 2223 cycles per event. The failures, on average, occurred every 1758 cycles indicating that current maintenance activities are scheduled too late.

Analysis procedures reduced the number of operational factors from 1531 to 32, after identifying significant variables extracted from semi-parametric PHMs (Tab. B.2.2). 18 and 13 out of the 41 operational factors identified prior to semi-parametric PHM reduction were related to take-off and landing procedures respectively. The remaining 10 were identified during clean-wing flight.

Goodness-of-Fit tests regarding time-based reliability models indicated that an exponential model best-fit the empirical data (Fig. B.2.2 and Tab. B.2.4). In addition, the reliability and hazard functions imply that a majority of the failures occurred within the first 1000 flight cycles, after which failure events were distributed more evenly. This would imply that potentially two failure modes are present. The majority of failures are however related to the first flight cycle, hence the assumption, one failure-mode, can temporarily be accepted. It is likely that the operational factors identified by the reliability models relate to just one of the failure modes. The computed mean time till failure is approximately 2250 cycles, verifying that maintenance is currently scheduled in accordance with an underlying exponential distribution.

Time-independent and -dependent MLE and GOF test results (Tables B.2.5 and B.2.12) indicate that time dependent models generate significantly better results than time-independent PHMs, based on MLE, Kolmogorov Smirnov, Cramer-von Mises-Smirnov, and Anderson-Darling test results. In terms of Kolmogorov Smirnov and Anderson-Darling tests, time-based models scored better than time-independent and -dependent PHMs, verifying that potentially two (or more) failure modes are present in the data. Table B.2.12 illustrates a perfect example of the complications arising when identifying the best time-dependent PHM model as the GOF tests and MLE scores are inconsistent (i.e. numerous models are identified).

To truly understand the effectiveness of each model the estimated time till failure, for each individual component, is computed using data available 100 cycles prior to an estimated failure (Table B.2.6, B.2.7, B.2.13, and B.2.14). The tables can effectively be used to select an appropriate model based on the component's nature and importance. In addition, once an appropriate model has been selected, the covariates can be used to assess the importance of external factors (variables).

Although aircraft will carry spare oxygen masks in case of failure, the costs associated are critical in terms of crew safety. Hence there is enough argument to select a model and reliability level that minimises the number of failures. Logically these conditions are met with high reliability levels, such that components are removed once the computed reliability has fallen below a certain 'high' threshold. Comparing the reliability functions of time-independent and -dependent PHMs in Figures B.2.8, B.2.9, & B.2.10 and Figures B.2.11, and B.2.12 respectively, we quite readily find that predominately late occurring events can be anticipated. This could potentially imply two (or more) failure modes exists.

Taking this into consideration we find that time-independent and -dependent PHMs with three and two covariates and an underlying log-normal and exponential distribution maximises the number of failures prevented. At the same reliability level currently used, R_L =53%, these models could prevent up to 43% of the failures and postpone 80% of the scheduled events (general case). However, since failure induced costs are significant, modelling errors must be minimised by using worst-case scenarios. In the output report you will find that the MTTRep computed using worst-case operational factors are significantly lower, indicating that, in the worst case, components must be repaired more frequently.

By restraining MTTRep from dropping below 2000 cycles the reliability level could be dropped to R_L =35%. At this threshold, 57% of the failure would have been prevented and 76% of the maintenance events would be postponed.

If an employee decides that costs induced by failures are irrelevant then a log-normal distribution can be selected with a reliability level of 25%. These criteria would have prevented 24% of the failures and allowed for the rescheduling of 80% of the maintenance events. Please acknowledge that new failures would occur due to delayed maintenance events.

3.2.2 Validation

Despite the potential presence of two-failure modes an attempt is made in this section to validate the operational factors identified by the solution. The primary variables identified by the time-independent and -dependent PHMs mentioned in the previous section were labelled: 1, 2, & 4 for time-independent and 6 & 8 for time-dependent PHMs (Tables B.2.19, B.2.20, B.2.21, and B.2.22).

Dis functionality of crew oxygen masks could be related to a variety of factors. However, logically one would expect the root cause to be changes in atmospheric conditions. Hence one would expect failures to occur when atmospheric conditions are changing dynamically (e.g. climb and descent). The aforementioned variables correspond to yaw rates during descent and approach, dynamic pressure (departure airport), rudder inputs, and roll angle during approach. The remaining variables did not contribute significantly to component reliability, nonetheless, one could analyse whether they are related to alternative failure-modes.

The most suitable models, identified in Sec. 3.2.1, have identified operational factors that occurred during descent and touchdown. It is safe to assume that the reliability of oxygen masks, which are vulnerable during these flight phases, are directly or indirectly related to these factors. .

In addition to improving component reliability, QantasLink could use these results to improve overall safety by notifying flight staff on complications arising during descents.

3.3 1152106-3 DC starter generator

This section will summarise the results obtained from analysing 'DC starter generator' components corresponding to part number **1152106-3**. This section will act as a summary only and unlike component PN:**697071003** all results are listed in Appendix B.3. The reader is advised to read Sec. 3.1 prior to this one to understand how the components are evaluated.

3.3.1 Results

Between the years 2004 and 2015 a total number of 1618 maintenance events were recorded of which 273 were related to flights operated post 2010 (Table B.3.23). About 78% of these occurrences were scheduled with an average cycle time of 1311 cycles per event. The remaining unscheduled events (i.e. failures), on average, occurred every 605 cycles indicating that current maintenance activities are scheduled too late.

Analysis procedures reduced the number of operational factors from **1531** to **11**, after identifying significant variables extracted from semi-parametric PHMs (Tab. B.3.24). **12** out of the **24** operational factors identified prior to semi-parametric PHM reduction were related to take-off procedures. The remaining 12 referred to cruise and landing.

Goodness-of-Fit tests regarding time-based reliability models indicated that an underlying log-normal distribution fits the empirical data exceptionally well (Fig. B.3.15 and Tab. B.3.26). Although the failures are rather evenly distributed, the reliability and hazard functions show that failures were separated into two groups, between 0 & 650 and 800 & 1550 cycles, indicating that 'potentially' two failure modes are present in the dataset. Scheduled events were primarily planned between 1200 and 1600 cycles. Graphical examination confirms that time-based reliability models with an underlying log-normal distribution best fit the empirical data.

Time-independent and -dependent MLE and GOF test results (Tables B.3.27 and B.3.34) indicate that time dependent and independent models generate similar results, based on MLE, Kolmogorov Smirnov, and Anderson-Darling tests. This could possibly be the result of similar operational factors being identified during both modelling procedures B.3.41.

To truly understand the effectiveness of each model the estimated time till failure, for past events, was computed using data available 100 cycles prior to estimated failures (Tables B.3.28, B.3.29, B.3.35, and B.3.36). The tables can effectively be used to select an appropriate model based on the component's nature and importance. In addition once, an appropriate model has been selected the covariates can be used to assess the importance of external factors (variables).

Table B.3.23 shows that approximately 78% of the maintenance occurrences were scheduled. This implies that costs associated to DC starter generator failures are considered relevant, hence failure prevention is prioritised. As such it can be argued that a model and reliability level is selected such that the total number of failures is minimised. Logically these conditions are met with high reliability levels such that components are removed once the computed reliability has fallen below a certain 'high' threshold. Taking this into consideration Tables B.3.28, B.3.29, B.3.35, and B.3.36 indicate that both time-independent and -dependent models can be used to reduce the number of failures significantly.

Comparing the scenarios we find that time-independent models are slightly more effective at reducing the number of failures. The input data suggested that currently maintenance activities are scheduled from 1200 cycles onward, which coincides with a reliability level of approximately 78% (empirical distribution: Fig. B.3.15). Maintaining approximately the same reliability level, R_L =77%, the user will find that, using general-case operational factors, time-independent PHMs with an underlying log-normal distribution and two covariates could have prevented 23% of the failures. However, since costs associated to component-failures are significant, the errors associated to model parameters should be minimised. To do so, we can use worst-case scenarios and slightly higher reliability thresholds. Historical data suggests that, increasing the reliability level to 82% and using worst-case scenarios, up to 34% of the failures (21 out of 61 events) can be prevented using the same model. In addition 53% of the scheduled events would have been postponed.

Since operators would like to avoid 'over'-maintenance, selecting reliability models and thresholds for high-value components remains a difficult task. Generally by selecting a high reliability value, an engineer can guarantee that the overall number of failures prevented is maximised.

3.3.2 Validation

The performance of DC starter generators is essential as faulty performance can cause failures to on-board electrical systems. This is confirmed by the high censoring level present in the failure data. Tables B.3.41, B.3.42, B.3.43, and B.3.44 indicate that the primary factors identified during modelling were labelled 1 & 2 (time-independent PHMs) and 1, 3, & 4 (time-dependent PHMs).

Interestingly, time-independent PHMs identified operational factors during take-off, i.e. pitch angle & rate and normal acceleration, and time-dependent PHMs identified factors during landing, i.e. elevator input, braking pressure, and lateral acceleration. Furthermore, Figures B.3.22, B.3.23, & B.3.24 (time-independent PHMs) and Figures B.3.26, B.3.27, & B.3.28 (time-dependent PHMs) show that time-independent PHMs were more effective at identifying early failures. This coincides with an earlier observation which suggested that potentially two failure-modes were present.

A majority of operational factors relate to longitudinal and lateral motions (control surface inputs and aircraft response) during take-off. These could have caused abnormal stresses and strains to interact with the DC starter generators or were a result of alternative take-off procedures, such as those occurring when aircraft loads are significant. The same can be said of variables identified during descent and landing.

To conclude, the operational factors identified by PHMs strongly indicate that DC starter generator failures are related to structural stress experienced during take-off and landing. This hypothesis can be tested by the manufacturer by running structural tests.

Results show that if a time-independent PHM model is used with an underlying log-normal distribution and a reliability threshold of 82%, then 21 of the previously recorded failures would have been prevented. In addition approximately 58% of the scheduled maintenance occurrences would have been postponed.

3.4 903-1342 Hand microphone

This section will summarise the results obtained from analysing 'hand microphone' components corresponding to part number **903-1342**. This section will act as a summary only and unlike component PN:**697071003** all results are listed in Appendix B.4. The reader is advised to read Sec. 3.1 prior to this one to understand how the components are evaluated.

3.4.1 Results

Between the years 2004 and 2015 a total number of 199 maintenance events were recorded of which 33 were related to flights operated post 2010 (Table B.4.45). About 6% of these occurrences were scheduled (i.e. censored) with an average cycle time of 1177 cycles per event. The remaining unscheduled events (i.e. failures), on average, occurred every 1629 cycles indicating that scheduled maintenance activities preceded most failures.

Analysis procedures reduced the number of operational factors from 1531 to 74. Semi-parametric PHMs identified no significant operational factors since the number of input parameters outweighed the number of observations (Tab. B.4.46). The identified operational factors were distributed evenly throughout all of the flight phases, however a majority were related to aircraft roll, yaw, and pitch motions.

Goodness-of-Fit tests regarding time-based reliability models indicated that all models fit the empirical data effectively (Fig. B.4.30 and Tab. B.4.48). This observation was derived from GOF scores which indicated exponential, Weibull, and log-normal distributions scored best regarding KS, CS & AD, and NRR test scores respectively. In addition the reliability and hazard functions imply a few failures were grouped around 1200 flight cycles, before and after which failure events were distributed evenly. Since the dataset is small there are no indicators that imply more than one failure mode is present. The computed mean time till failure is approximately every 1200 cycles.

Time-independent and -dependent MLE and GOF test results (Tables B.4.49 and B.4.56) indicate that time dependent models produced better MLE, CS, and AD scores, yet time-independent models generated significantly better results in terms of KS and NRR tests. Since the data is not highly censored the results from GOF tests can be considered relevant (Nikulin et al. 2010). Table B.4.49 illustrates a perfect example of the complications arising when identifying the best time-independent and -dependent PHM model.

To truly understand the effectiveness of each model the estimated time till failure, for each individual component, is computed using data available 100 cycles prior to an estimated failure (Tables B.4.50, B.4.51, B.4.57, and B.4.58). The tables can effectively be used to select an appropriate model based on the components nature and importance. In addition once an appropriate model has been selected the covariates can be used to assess the importance of external factors (variables).

Costs associated with the failures of hand microphones are minimal, hence it can be argued that a model and reliability level is selected such that a component is replaced once it is almost certain it will fail. Logically these conditions are met when components are only removed when the computed reliability is poor. Furthermore, since the focus is not on failure prevention, the user is advised to compare general-case tables instead of worst-case.

Taking this into consideration we readily find, using Tables B.4.50, that at a low reliability levels, $R_L \leq$20%, time-independent and -dependent PHMs with two (or three) covariates and an underlying Weibull and exponential distribution respectively would prevent up to 23% of the failures. However the user will find that, at this reliability level, the MTTRep has increased by a factor of two. By limiting the overall change in MTTRep, the user can use the model and general- or worst-case scenarios to prevent 48% or 58% of the failures at a reliability level of 48%, or 29% respectively. In addition, at this reliability level, all scheduled maintenance events would have been postponed.

Further comparison shows that the Mean Time Till Failure (MTTF) and Mean Time Till Repair (MTTRep) is significantly larger for the time-independent PHM, implying this model was more successful at distinguishing failure from censored events. Tables B.4.63, B.4.64, B.4.65, and B.4.66 show the variables identified by each PHM, which will be discussed in 3.4.2.

3.4.2 Validation

Tables B.4.63, B.4.64, B.4.65, and B.4.66 show the operational factors identified by the solution. The tables show that, for the time-independent PHM identified in the previous section, as well as other time-independent PHMs, the first covariates identified were the duration of and roll rate during touchdown till end-rollout. In addition the third covariate 'Group C' corresponds to the dynamic pressure and true velocity during departure (taxiing and take-off).

Since the hand microphone is primarily used to notify passengers when the aircraft is grounded and descending the results are valid. It remains however uncertain whether the roll rate, dynamic pressure, and true velocity during taxiing and take-off are critical as the variables could be related to delays, parking, or slowing down of the aircraft which as a consequence would require the pilot to use the microphone more frequently.

The solutions propose that initially maintenance activities should be scheduled using a critical reliability level of approximately 20% and a time-independent PHM with an underlying Weibull distribution and three covariates: *duration touchdown till end rollout* and *minimum roll rate 8*. In this 'general' case 23% of the failures could have been avoided using data computed 100 cycles prior to the estimated failure. By assuming worst-case the number of failures prevented can be increased to 45%.

3.5 3-1573-1 MLG wheel & tire assembly

This section will summarise the results obtained from analysing 'Main Landing Gear (MLG) wheel and tire assembly' components corresponding to part number **3-1573-1**. This section will act as a summary only and unlike component PN:**697071003** all results are listed in Appendix B.5. The reader is advised to read Sec. 3.1 prior to this one to understand how the components are evaluated.

3.5.1 Results

Between the years 2004 and 2015 a total number of 18809 maintenance events were recorded of which 3082 were related to flights operated post 2010 (Table B.5.67). About 94% of these occurrences were scheduled (i.e. censored) with an average cycle time of 257.2 cycles per event. The remaining unscheduled events (i.e. failures), on average, occurred every 171.51 cycles indicating that current maintenance activities are scheduled too late.

Analysis procedures reduced the number of operational factors from 1531 to 12, after identifying significant variables extracted from semi-parametric PHMs (Tab. B.5.68). Furthermore, the table shows, prior to semi-COX reduction, that a majority of the operational factors were related to take-off and cruise, 9 and 8 out of the 22 respectively.

Goodness-of-Fit tests regarding time-based reliability models indicated that an underlying logistic distribution fits the empirical data exceptionally well (Tab. B.5.70); scoring best w.r.t. KS, AD, and NRR GOF tests. This statement can be confirmed graphically with Fig. B.5.43. The reliability and hazard functions, Figures. B.5.44 and B.5.45, show that the failures are evenly distributed throughout the observed operational time. Hence the underlying assumption, one failure-mode, has not been violated.

The time-independent and -dependent GOF test results, Tables B.5.71 and B.5.78, verify the conclusions made by Nikulin, stating that GOF tests behave poorly in high-censored data samples (Nikulin et al. 2010). Since these tests failed to indicate which models and distributions fit the empirical data best the user will need to evaluate each model using estimated failure times derived from analysing historical data (Tables B.5.72, B.5.73, B.5.79, & B.5.80 and Figures B.5.50, B.5.51, & B.5.52 (time-independent PHMs), and B.5.54, B.5.55, and B.5.56(time-independent PHMs)).

Table B.5.67 shows that approximately 94% of the maintenance occurrences were scheduled. This implies that costs saved by preventing unscheduled maintenance of MLG wheel & tire assembly parts are considerable. As such it can be argued that a model and reliability level are selected such that the total number of failures is minimised. Graphical analysis shows that Logically these conditions are met with high reliability thresholds, such that components are removed on-time. The user is advised to use worst-case scenario tables when evaluating high-risk components.

When comparing worst-case scenario Tables B.5.73, and B.5.80 the user will find that, on average, the number of failures prevented by time-independent PHMs is similar to that of time-dependent PHMs. The minor differences primarily comes from the fact that the covariates identified by both models coincide to a large extent. Generally, with large difference, the user is advised to disregard the less effective models. In this case however, both models must be evaluated. To do so, we first determine which models maximised the number of failures and scheduled events prevented and postponed respectively.

At the same reliability levels the table indicate that, on average, the number of failures prevented using time-independent PHMs is higher, but the number of scheduled events postponed is not. The empirical data shows that QantasLink currently removes/replaces components, on average, every 257 cycles (MTTRep), which coincides with a reliability threshold of approximately 93% (Fig. B.5.43). Tables B.5.73, and B.5.80 show that, at this reliability level, the maximum number of failures is maximised by time-dependent PHMs which maximise the number of failures prevented between R_L =91% and R_L =97%.

Focussing on time-dependent PHMs, at a reliability level of 96% up to 30% of the previous recorded failures could have been prevented while keeping MTTRep approximately equal to 250 cycles. Since component failures contribute significantly to costs, it is essential that the selection procedures focus on limiting the increases in MTTRep. As such, we can use worst-case MTTRep figures, Table B.5.82, to select a model that has an MTTRep of approximately 250. This table, in combination with Table B.5.80, show that a log-normal time-dependent PHM with five covariates can prevent up to 28% of the failures and postpone 54% of the scheduled events at a MTTRep of 270 cycles. This value can be increased further to 40% if the MTTRep is reduced to 225 cycles. Overall, the results suggest that the number of failures can be reduced significantly.

Since operators wish to avoid 'over'-maintenance, selecting reliability models and thresholds for high-value components remains a difficult task. Generally, by selecting a high reliability value, an engineer can guarantee that the overall number of failures prevented is maximised.

3.5.2 Validation

This section will focus on validating the variables identified by the solution. Since component PN **3-1573-1** corresponds to the Main Landing Gear (MLG) wheel and tire assembly we would expect the operational factors to be related to flight phases where the landing gear is extended. In terms of time-independent PHMs, historical data and Tables B.5.85, B.5.86, B.5.87, and B.5.88 show that covariates labelled 1, 2, 3, & 4 were most effective at modelling component reliability. Since these variables relate to both take-off and landing, the models would suggest that either two (or more) failure modes are present, or an external factor, such as aircraft loading, is causing the abnormalities.

Time-dependent PHM results obtained from historical data, summarised in the previous section, indicated that a log-normal distribution with five covariates was extremely effective at modelling component reliability. Similar to time-independent PHMs, these five covariates, i.e. breaking pressure LHS (landing), roll and yaw rate (take-off), roll rate (cruise), and normal-force (landing), indicate that either two (or more) failure-modes are present or the failures are related to an unidentified operational factor affecting both take-off and landing.

Since forward selection procedures maximise the probability of a variable significantly affecting component reliability, the order of retrieval is important. It is logical that MLG wheel and tyre assembly failures are related to roll rates during take-off and normal forces during landing.

3.6 3-1574 NLG wheel & tire assembly

This section will summarise the results obtained from analysing 'Nose Landing Gear (NLG) wheel and tire assembly' components corresponding to part number **3-1574**. This section will act as a summary only and unlike component PN:**697071003** all results are listed in Appendix B.6. The reader is advised to read Sec. 3.1 prior to this one to understand how the components are evaluated.

3.6.1 Results

Between the years 2004 and 2015 a total number of 19504 maintenance events were recorded of which 3092 were related to flights operated post 2010 (Table B.6.89). About 95% of these occurrences were scheduled (i.e. censored) with an average cycle time of 129.4 cycles per event. The remaining unscheduled events (i.e. failures), on average, occurred every 94.3 cycles indicating that current maintenance activities are scheduled too late.

Analysis procedures reduced the number of operational factors from 1531 to 31, after identifying significant variables extracted from semi-parametric PHMs (Tab. B.6.90). Furthermore, the table shows, prior to semi-COX reduction, that a large majority of the operational factors were related to take-off and touch-down, 18 (32%) and 27 (48%) out of the 56 respectively.

Goodness-of-Fit tests regarding time-based reliability models indicated that an underlying normal and logistic distribution fits the empirical data exceptionally well (Tab. B.6.92), scoring best w.r.t. KS, AD, and NRR GOF tests. This statement can be confirmed graphically with Fig. B.6.58, which also shows that log-normal, exponential, and gamma distributions behave well for early-on failures, but fail to match later occurring events. The reliability and hazard functions, Figures. B.6.59 and B.6.60, show that the failures are evenly distributed throughout the observed operational time. Hence the underlying assumption, one failure-mode, has not been violated.

The time-independent and -dependent GOF test results, Tables B.6.93 and B.6.100, verify the conclusions made by Nikulin; stating that GOF tests behave poorly in high-censored data samples (Nikulin et al. 2010). Since these tests fail at indicating which models and distributions fit the empirical data best the user will need to evaluate each model using estimated failure times derived from analysing historical data (Tables B.6.94, B.6.95, B.6.101, & B.6.102 and Figures B.6.65, B.6.66, & B.6.67 (time-independent PHMs), and B.6.68, and B.6.69 (time-dependent PHMs)).

Table B.6.89 shows that approximately 95% of the maintenance occurrences were scheduled. This implies that costs saved from preventing NLG wheel & tire assembly parts are considerably large. As such it can be argued that a model and reliability level is selected such that the total number of failures is minimised. Graphical analysis shows that logically these conditions are met with high reliability thresholds, such that components are removed on-time. The user is advised to use worst-case scenario tables when evaluating high-risk components.

When comparing worst-case scenario Tables B.6.95, and B.6.102 the user will find that, on average, the number of failures prevented by time-independent PHMs is larger than time-dependent PHMs. The minor difference primarily comes from the fact that the covariates identified by time-independent and -dependent PHMs are relatively similar. Generally with large differences the user is advised to disregard the less-effective models. However if an interested user would evaluate time-dependent models they would find that PHMs including torque (landing), yaw rate (final taxiing), and roll rate (take-off) modelled component reliability most effectively (Tables B.6.107, B.6.108, B.6.109, and B.6.110).

The results obtained from time-independent PHMs show that the normal distribution was most effective at distinguishing failure from non-failure events and the Weibull distributions at identifying early-on failures. Upon analysis of the identified variables, Tables B.6.107, B.6.108, B.6.109, and B.6.110, it is found that the braking pressure (LHS) and roll & pitch rate during final taxiing contributed most to component failures.

At current reliability thresholds, approximately R_L =95%, the tables (Tab. B.6.94 and B.6.95) show that an underlying normal distributions with three covariates can prevent and postpone up to 27% and 70% of the failures and scheduled events respectively (General Case). This is at the cost of MTTRep, which, using this model and threshold, is significantly lower than that currently used, i.e. 129 cycles (Tab. B.6.98). We find, at this reliability level, that a normal distribution with six covariates is more effective. Using general case scenarios, this model is capable of preventing 23% of the failures and postponing 78% at a MTTRep of 145.3. Minimising errors by assuming worst-case scenarios indicate that up to 26% of the failures can be prevented at a MTTRep of 128. This would suggest that new maintenance schedules could 'potentially' reduce costs significantly.

Since operators wish to avoid 'over'-maintenance, selecting reliability models and thresholds for high-value components remains a difficult task. Generally by selecting a high reliability value, an engineer can guarantee that the overall number of failures prevented is maximised.

3.6.2 Validation

This section will focus on validating the variables identified by the solution. Since component PN **3-1574** corresponds to the Nose Landing Gear (NLG) wheel and tire assembly we would expect the operational factors to be related to flight phases 1, 2, 5, 6, 7, and 8 (take-off to clean-wing & clean-wing to landing). Tables B.6.107, B.6.108, B.6.109, and B.6.110 show that all of the identified operational factors occurred during take-off and landing, confirming our previous statement.

Time-dependent PHM results obtained from historical data, summarised in the previous section, indicated that factors directly and indirectly related to the force acting on the NLG during landing significantly affected component reliability. This observation coincided with the operational factors identified by time-independent PHMs, which indicated that the three major contributors to component failures were LHS breaking pressure (final taxiing), pitch & roll rate (final taxing), and approach speed (landing). In addition the the previously identified factors, the models also suggest that a few failures were related to pitch rate, normal acceleration, and velocity during take-off. This would suggest that the data could contain a second failure mode.

Finally, by evaluating reliability figures obtained from time-independent PHMs (Figures B.6.65, B.6.66, & B.6.67), we find that steps 6 and 7 from the normal underlying distribution significantly affected the reliability of a few components that failed relatively early. Overlapping covariates in Table B.6.108 suggest that the underlying cause for these failures was vertical velocity during climb and torque during descent. It can be proven that the operational factors identified by the solution have a direct (or indirect) effect on the NLG wheel and tyre assemblies. Hence the derived solution is valid.

3.7 92003-051-052-001 Sensor high-level, fuel

This section will focus on the results obtained from analysing 'high-level fuel sensor' components corresponding to part number **92003-051-052-001**.

3.7.1 Results

From the nineteen maintenance events recorded since 2004, none were solely affiliated to flights operated post 2010. This was because, on average, 15040 cycles were operated before maintenance events (scheduled and unscheduled). Each aircraft operated approximately 6.4 cycles per day (FDR data), implying that high-level fuel sensor related maintenance occurred, on average, every 2350 working days ($\frac{15040}{6.4}$). Logically, for all maintenance events, the flights operated prior to removal dated back before 2010. Since QantasLink started recording operational factors, using FDRs, after 2010, insufficient data was available to initiate the analysis and modelling process.

3.8 728809-1 Thermal actuator

This section will summarise the results obtained from analysing 'thermal actuator' components corresponding to part number **728809-1**. This section will act as a summary only and unlike component PN:**697071003** all results are listed in Appendix B.8. The reader is advised to read Sec. 3.1 prior to this one to understand how the components are evaluated.

3.8.1 Results

Between the years 2004 and 2015 a total number of 732 maintenance events were recorded of which 135 were related to flights operated post 2010 (Table B.8.111). About 36% of these occurrences were scheduled (i.e. censored) with an average cycle time of 1168 cycles per event. The remaining unscheduled events (i.e. failures), on average, occurred every 2220 cycles indicating that scheduled maintenance activities preceded most failures.

Analysis procedures reduced the number of operational factors from 1531 to 13, after identifying significant variables extracted from semi-parametric PHMs (Tab. B.8.112). Pitch rate, true velocity, and yaw rate make up 50% of the identified variables, which were, for a majority, related to cruise, descent, and landing flight phases.

Goodness-of-Fit tests and MLE scores retrieved from time-based reliability models indicated that logistic, exponential, and Weibull distributions performed best (Tab. B.8.114). Due to censoring these results are inconclusive, hence the user is advised to perform graphical analysis using Fig. B.8.72. The figure shows that exponential distributions, currently used in the industry, perform worse than the logistic and Weibull distributions. Graphical analysis suggests that the empirical data is slightly better represented using logistic distributions. Furthermore, the reliability and hazard functions (Figures B.8.73 and B.8.74), indicate that a majority of the unscheduled and scheduled events occurred around 2200 and 1100 flight cycles respectively. Since the failures are evenly distributed along the observed operational time, it can be presumed that the assumption of one-failure mode is valid.

Time-independent and -dependent MLE and GOF test results (Tables B.8.115 and B.8.122) indicate that time dependent models performed better w.r.t. MLE, KS, CS, and AD. However research has shown that modified GOF tests behave poorly for highly censored datasets (Nikulin et al. 2010). Since 36% of the input data was censored, the time-dependent models should be evaluated using historical data (Tables B.8.116, B.8.117, B.8.123, & B.8.124 and Figures B.8.78, B.8.79, & B.8.80 (time-independent PHMs) and B.8.81, B.8.82, and B.8.83 (time-dependent PHMs)).

To comprehensively understand the effectiveness of each model the estimated time till failure, for each individual component, was computed using data available 100 cycles before an estimated failure (Tables B.8.116, B.8.117, B.8.123, and B.8.124). The tables can effectively be used to select an appropriate model based on the component's nature and importance. In addition once an appropriate model has been selected the covariates can be used to assess the importance of external factors (variables).

Since a large number of scheduled maintenance events occurred, on average, 1000 cycles prior to failures, it can be deduced that costs associated to component failures are significantly high. Keeping this into consideration a reliability engineer must select a model and reliability level such that the number of expected time till failures is maximised. This logic guarantees that new reliability procedures will reduce failure-related costs. Since a majority of the scheduled events occurred before the observed failure times, one would expect the models to post-pone a large sum of these.

The results obtained from worst-case scenario Table B.8.124 indicate that a time-dependent PHM with an underlying log-normal and Weibull distribution maximises the number of failures prevented at high reliability levels. Maintenance is currently scheduled every 1200 cycles coinciding with a reliability threshold of approximately 90%. At approximately the same reliability level (89%), worst-case scenario tables indicate that a log-normal distribution with one covariate can prevent up to 92% of the failures and postpone 61% of the scheduled events. At this level, MTTRep is approximately 1162 cycles (Tab. B.8.128). At a reliability threshold of 79%, MTTRep increases to 1556 cycles, hence ideally a reliability threshold between 79% and 89% is selected such that the worst-case scenario MTTRep is approximately the same as that currently used. To test alternative reliability levels the user can use the 'EvaluateRM' function. It was found, using 'EvaluateRM', that a time-dependent PHM with a log-normal distribution and one covariate can prevent 81% of the failures at a reliability level of 85% with a MTTRep of 1237.

The significant impact of exponential and Weibull time-dependent PHMs with less than two covariates respectively imply that roll rate (take-off), normal force (landing), normal force (take-off), and rudder input (landing) strongly affected the component's reliability (Tables B.8.129, B.8.130, B.8.131, and B.8.132),

3.8.2 Validation

Thermal actuators act as motors controlling mechanisms and (or) systems. As one would suspect, failures would generally be related to exposure to abnormal atmospheric conditions, vibrations or loads. The input data (App. A) shows that a large number of variables is not measured, hence when evaluating the identified factors we must consider the correlation of the variables with other factors.

During forward selection the first factor is selected based on the probability that it affects the maximum likelihood function. The consecutive variables are then selected based on their probability of belonging to a reliability model with the first identified factor. Tables B.8.117 and B.8.124 showed that, using historical data, a log-normal (one covariate) time-independent and Weibull (one covariates) time-dependent PHM would have maximised the number of failures prevented. In addition, time-dependent PHMs with underlying exponential distributions were the only models initially selecting a different set of factors. Implying that failures could be related to high pitch rates during take-off.

The results suggest that thermal actuators have been failing due to high approach speeds and torque during landing. The underlying cause can take multiple forms, hence, the user is advised to further investigate these results and address the component's manufacturer with this news. Failure could perhaps be prevented by small structural changes or by 'further' restricting approach speeds.

Retaining current reliability thresholds (90%), it can be shown that adapted scheduling procedures, adopted from new reliability models, (e.g. one cov. Weibull time-dependent PHM) can prevent (post-pone) up to 95% of the failures (maintenance events).

3.9 10-105-31A-N-2 VHF antenna

This section will summarise the results obtained from analysing 'VHF antenna' components corresponding to part number **10-105-31**A-N-**2**. This section will act as a summary only and unlike component PN:**697071003** all results are listed in Appendix B.9. The reader is advised to read Sec. 3.1 prior to this one to understand how the components are evaluated.

3.9.1 Results

Between the years 2004 and 2015 a total number of 253 maintenance events were recorded of which 39 were related to flights operated post 2010 (Table B.9.133). About 5% of these occurrences were scheduled (i.e. censored) with an average cycle time of 2616.5 cycles per event. The remaining unscheduled events (i.e. failures), on average, occurred every 1718.7 cycles indicating that current maintenance activities are scheduled too late.

Analysis procedures reduced the number of operational factors from 1531 to 53. Semi-parametric PHMs identified no significant operational factors since the number of input operational factors outweighed the number of observations (Tab. B.9.134). The identified operational factors were distributed evenly throughout all of the flight phases, however a majority were related to aircraft yawing & pitching motions, as well as accelerations in the longitudinal axis.

Goodness-of-Fit tests regarding time-based reliability models indicated that models with an underlying normal and logistic distribution fit the empirical data exceptionally well (Tab. B.9.136). This can be verified graphically using Fig. B.9.85. The reliability and hazard functions imply a few failures were grouped around 1500 flight cycles, however a majority of failures were evenly distributed throughout the observed time. Hence the underlying assumption of one-failure mode has not been violated.

Comparing time-independent and -dependent PHM MLE and GOF test results (Tables B.9.137 and B.9.144) we can find that time dependent models generated significantly better results w.r.t. MLE, Kolmogorov Smirnov, Cramer-von Mises-Smirnov, Anderson-Darling, and NRR. Time-dependent GOF test results, Tab. B.9.144, shows that models with underlying normal, log-normal, and Weibull distributions model the component's reliability most effectively.

To truly understand the effectiveness of each model the estimated time till failure, for each individual component, was computed using data available 100 cycles prior to an estimated failure (Tables B.9.138, B.9.139, B.9.145, and B.9.146). The tables can effectively be used to select an appropriate model based on the component's nature and importance. In addition once an appropriate model has been selected the covariates can be used to assess the importance of external factors (variables).

Generally, due to redundancy and relatively low prices, costs associated to VHF antenna failures are low. To support this argument the reader can examine the data where you will find a that only 5% of the events were scheduled. Furthermore, the estimated time till next scheduled repair is considerably larger than the average failure time implying that unscheduled maintenance events are preferred over scheduling unnecessary events. Keeping this in consideration a reliability engineer must select a model and reliability level such that a component is only repaired once it is very certain it will fail. This logic guarantees that new reliability procedures will not accumulate costs by scheduling unnecessary maintenance events. Since failures are not critical the user is advised to use results obtained from evaluating general-case scenarios.

With a focus on reducing costs over failure prevention, when evaluating historical data using time-dependent models, Table B.9.145, we readily find, at low reliability levels (R_L ≤12%), PHMs with an underlying logistic distribution and one covariate maximises the number of failures prevented and minimise the costs associated to over-maintenance. To a large certainty these models would prevent 5 of the 37 failures (14%) and delay 1 out of the 2 scheduled events (50%). If the operators prefers to use a reliability threshold that generates a similar

MTTRep as that currently used, then a reliability level of 31% (MTTRep = 2495) can be utilised.

3.9.2 Validation

Similar to other components the root cause for antenna failures can be related to a variety of factors ranging from weather conditions (e.g. high temperatures) to operating conditions (e.g. high vibrations caused by propellers). The covariates identified during forward selection of time-dependent PHMs were: angle of attack (take-off), braking pressure (landing), vertical velocity (climb), propeller speed (take-off), roll angle (take-off), and yaw rate (take-off). Notice is that models with one covariate perform best at low reliability levels (Table B.9.145). This would imply that the first covariate identified, covariate Group B, affected component reliability the most. The variables identified (per model) are summarised in Tables B.9.151, B.9.152, B.9.153, and B.9.154.

The results imply that the flights subject to antenna failures, in general, operated in significantly different take-off/climbing and landing conditions. Noticeable in Table B.9.151 are the negative values associated with braking pressure, vertical velocity, angle of attack, propeller speed, and longitudinal accelerations. These values imply that the measured operational factors were significantly lower during failure related flights. Thus it can be concluded that either: the factors identified by time-dependent PHMs are directly affecting VHF antennas, or the factors are an indicator of an underlying issue, which was either eliminated during variable reduction or not measured by FDR.

3.10 EVR716-11-0350A VHF transceiver

This section will summarise the results obtained from analysing 'VHF transceiver' components corresponding to part number EVR716-11-0350A. This section will act as a summary only and unlike component PN:697071003 all results are listed in Appendix B.10. The reader is advised to read Sec. 3.1 prior to this one to understand how the components are evaluated.

3.10.1 Results

Between the years 2004 and 2015 a total number of 368 maintenance events were recorded of which 60 were related to flights operated post 2010 (Table B.10.155). About 5% of these occurrences were scheduled (i.e. censored) with an average cycle time of 2039 cycles per event. The remaining unscheduled events (i.e. failures), on average, occurred every 1465 cycles indicating that current maintenance activities are scheduled too late.

Analysis procedures reduced the number of operational factors from 1531 to 90. Semi-parametric PHMs identified no significant operational factors since the number of input operational factors outweighed the number of observations (Tab. B.10.156). The identified operational factors were distributed evenly throughout all of the flight phases, however a majority were related to aircraft rolling, yawing & pitching motions, as well as dynamic pressure and accelerations in the longitudinal axis.

Goodness-of-Fit tests, performed on time-based reliability models, were inconsistent, indicating that numerous distributions, normal, logistic, exponential, and log-normal, fit the empirical data well (Tab. B.10.158). This was supported using graphical analysis, which showed that normal & logistic distributions were effective in modelling early to mid-life failures and exponential & Weibull distributions were slightly less effective early-on, but improved as operational life increased (Fig. B.10.98). Reliability and hazard functions, Figures B.10.99 and B.10.100, show, with the exception of three failures, that all failures were evenly distributed till approximately 3100 cycles. Hence we can not disregard the underlying assumption: one failure-mode.

GOF and MLE scores derived from testing time-independent and -dependent PHMs, Tables B.10.159 and B.10.166, indicate that time dependent models generated significantly better results, w.r.t. MLE, KS, CS, AD, and NRR. The tables illustrates a perfect example of the complications arising with modified GOF tests and censored data. Since 25% of the data is censored the user is advised to evaluate each model, time-independent and -dependent, using results obtained from evaluating historical data.

For each model the estimated time till failure, for each individual component, was computed using data available 100 cycles prior to an estimated failure (Tables B.10.160, B.10.161, B.10.167, and B.10.168). This would guarantee that the operator had sufficient time to (re) schedule maintenance events such that failures could be avoided. Models should be selected based on the risk associated with the component. Once an appropriate model has been selected the covariates can be used to assess the importance of external factors (variables).

Generally, due to redundancy and relatively low prices, costs associated to VHF antenna failures are low. To support this argument the reader can examine the data where relatively low number of scheduled events, 15 out of 60, and considerably high scheduled average time till next repair imply that unscheduled maintenance events are preferred over scheduling unnecessary events. Keeping this in mind, reliability engineers are advised to select low reliability levels, such that components are removed once it is almost certain that a failure will occur within the next few flight cycles. This guarantees that costs associated to un-necessary (premature) maintenance events are minimised. Since failure induced costs are non critical the user is advised to evaluate historical data using general-case scenarios (Tables B.10.160 and B.10.167).

Initial comparison of Tables B.10.160 and B.10.167 shows that the number of failures (scheduled events) prevented (postponed) are similar using both time-independent and -dependent PHMs. Closer analysis shows that time-independent and -dependent PHMs maximises the values at low and high reliability levels respectively. Furthermore, using Tables B.10.173 and

B.10.174, we can find that there are small deviations between the variables identified by each model, implying that failures are either related to a combination of both variables, or there are two (or more) failure modes. Assuming the first, the user needs to identify, for each PHM type, which covariates affected component reliability the most.

From Table B.10.160 it is evident that time-independent PHMs with normal (two covariates), log-normal (two covariates) and Weibull (two covariates) maximise the number of failures (scheduled events) prevented (postponed). With respect to the aforementioned models, the most frequently reoccurring covariates are labelled 1, and 6 in Tables B.10.173, B.10.174, B.10.175, and B.10.176. This implies that VHF transceiver failures are related to abnormally high roll rates and pitch angles.

From Table B.10.167 it is evident that time-dependent PHMs with normal, log-normal and exponential distributions with less than three covariates maximise the number of failure (scheduled events) prevented (postponed). The corresponding covariates labelled 2, 3, 6, 10, 18, & 22 in Tables B.10.173, B.10.174, B.10.175, and B.10.176 indicate that VHF transceiver failures are primarily related to rolling & yawing (take-off), vertical velocity (take-off), torque (descent).

Combining the results obtain from time-independent and -dependent PHMs we can establish that failures are directly or indirectly related to abnormal rolling and yawing motions, which, as a result, affect pitch angle and vertical velocity. These motions are controlled by rudder and elevator inputs (also identified) and, as a consequence, cause longitudinal and lateral accelerations (also identified).

Current maintenance activities are scheduled approximately every 2040 cycles. Using the supplemented MTTR module (or Tab. B.10.172), we readily find that a log-normal (one covariate), log-normal (eight covariate), and exponential (nine covariate) time-dependent PHM can prevent 33, 38, & 31% of the failures and postpone 53, 60, & 80% of the scheduled events at a reliability level of 39, 79, and 69% with an average MTTRep of 2078, 2031, and 2235 cycles respectively. Any of the above reliability models would suffice, however, following the decision logic described in Sec. 4, it is advised that the number of postponed maintenance events are maximised since costs associated to component-failures are minimum. As such, a time-dependent PHM with an underlying exponential distribution and nine covariates, could have prevented 31% of the failures (14/45) and reduced costs associated to early scheduling of 80% of the maintenance activities (12/15).

3.10.2 Validation

Similar to VHF antennas the root cause for VHF transceiver failures can be related to a variety of factors ranging from environmental (e.g. high temperatures) to operating conditions (e.g. high vibrations caused by propellers). As discussed earlier, time-dependent PHMs with 8 (or more) covariates were effective at preventing and postponing over 30% and 80% of the failures and scheduled events respectively at high reliability levels ($R_L \geq$70%). These covariates coincide with the variables identified by the other models, hence this section will focus on validating them. The covariates identified by the time-dependent exponential PHM were: roll angle and rate (take-off), roll angle (descent), vertical and longitudinal velocity (climb), torque (descent), and elevator input (landing) (Tables B.10.173, B.10.174, B.10.175, and B.10.176).

Normally with such a diverse quantity of operational factors occurring at a variety of flight phases it would be difficult to establish which operational factors affected failures the most. In this case, time-dependent PHMs with an underlying logistic distribution and two covariate could distinguish $\approx$ 16% of the failures from all scheduled events at a reliability level of 20%. This implies that the underlying covariates, velocity and torque, affected component reliability considerably. The challenge arises when the user has to clarify why the true velocity (descent) was significantly larger during failure-related flights (scaled value is 21.11), but the torque (descent), i.e. Group G, was significantly smaller (scaled value is -21.32). This could be related to a steep descent angle.

The solution indicates that, using current reliability thresholds, considerable amounts of costs can be saved in the prevention of failures and postponing maintenance events scheduled far too early. Historical data suggests that QantasLink could even increase the component's reliability threshold to 69%, which would have prevented 14 (out of 45) failures and postponed 12 (out of 15) maintenance events. The company has nothing to loose: assuming worst-case, all postponed events result in failures, the company still effectively prevented two failures.

4

Solution Decision Logic

For the convenience of the reader and future users, this section presents the decision logic to be used to improve overall component reliability. The decision logic is based on the logic used to evaluate the results in Sec. 3.

In general the procedure consists of: preliminary analysis, assumption validation, model and covariate identification, and schedule modification. Figure 4.0.0.1 covers preliminary analysis and assumption validation. The flow charts continues into Fig. 4.0.0.2, which presents model and covariate identification. Finally, upon validation, Figure 4.0.0.3 outlines the final steps to adapt current maintenance schedules such that overall reliability can be improved.

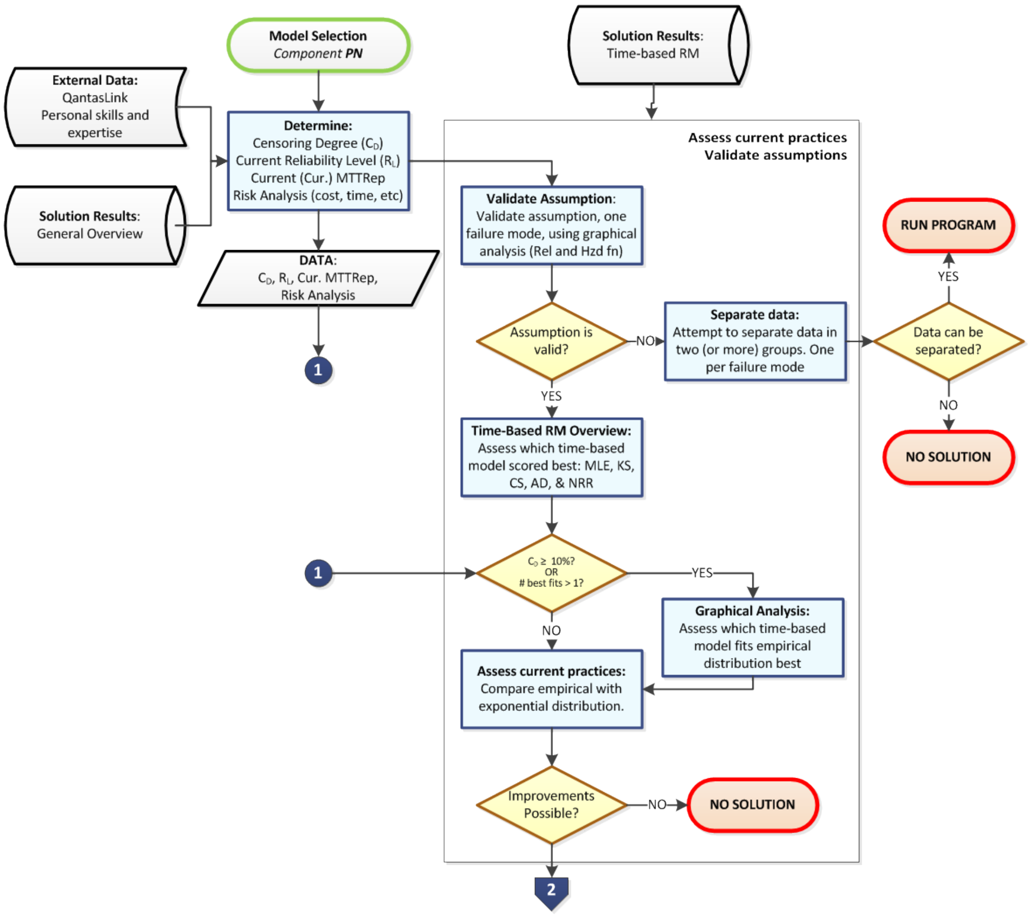


Figure 4.0.0.1: Flow chart describing decision logic used during reliability model selection. (Part 1/3)

Preliminary analysis The purpose of preliminary analysis is to derive the nature of the component which is being evaluated. In this phase the user will perform risk analysis on the component, such that a balance between failure prevention and over-maintenance can be established. In general overview tables produced by the solution, the user will find that, generally:

- High censoring degrees indicate that risks associated to component failures are high.
- When, on average, the scheduling time (MTTRep) is far less than the failure time (MTTF), the risks associated to component failures are high.
- Current reliability levels indicate the company's perspective with respect to the component's liability.

As a reminder, current reliability levels can be estimated by cross-referencing the average scheduling time with the empirical data's reliability function (Kaplan Meier).

The results obtained in this section will be recalled further on in the decision logic tree.

Assumption validation The purpose of 'assumption validation' is to verify whether the assumption of one failure mode is valid. To do so the user can inspect the reliability and hazard functions of the best-fit time-based (GRP) models. As a rule of thumb, if failures are clustered in two (or more) groups then the assumption is violated. In the case that the assumption is violated, the user can attempt to separate the failure data into an appropriate number of groups, such that each group corresponds to one failure mode. Once separated, the solution will have to be re-runned using the separated datasets.

If the assumption is not violated the user can assess whether current reliability models, an exponential distribution, models the empirical data appropriately. If not, continue to model and covariate identification.

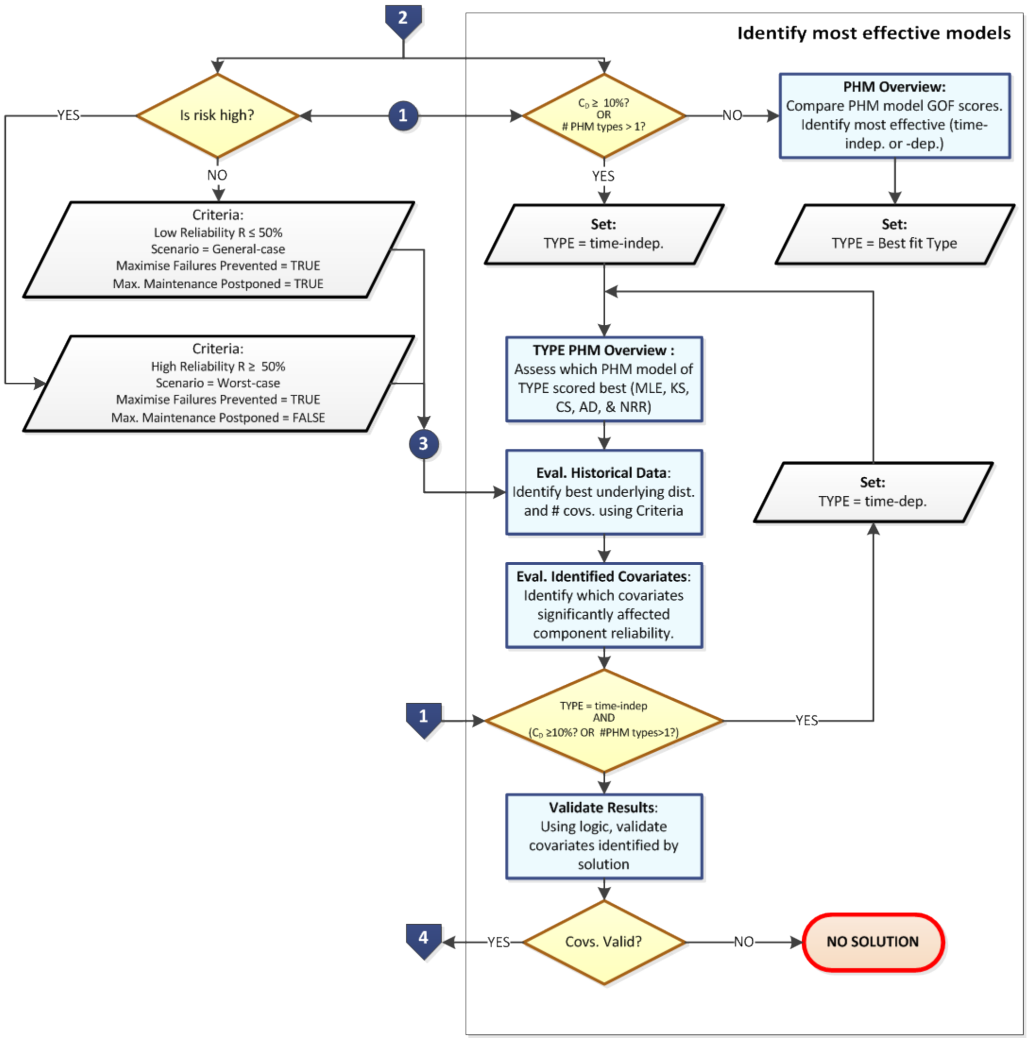


Figure 4.0.0.2: Flow chart describing decision logic used during reliability model selection. (Part 2/3)

Model and covariate identification For the next phase the user is required to use his knowledge of the component's nature. The first step is to establish whether the input data was highly censored, $C_L \leq 10\%$. This is important since research has shown modified GOF tests, such as Kolmogorov Smirnov (KS), Cramer-von Mises-Smirnov (CS), Anderson Darling (AD), and Nikulin-Rao-Robson (NRR) (Sec. Background Information: *Goodness-of-Fit Tests*), perform poorly when the data is highly censored. In this paper, to be safe, it is proposed to set the boundary between normal and highly censored data at 10%.

Most often, when the data is not highly censored, the best GOF test scores are related to one PH model type (e.a. time-independent or time-dependent). In this case the user is advised to continue analysis using only the one PHM type (discussed shortly).

If GOF tests point towards both time-independent and -dependent PHMs, the user will have to address both model types. Here the first step is to identify which covariates affected component reliability most, for each model type. In order to do so, we look at the results obtained from evaluating historical data. In these tables, failure times are computed using flight data available 100 cycles prior to the event, ten working days (two weeks) assuming ten cycles per day. This would guarantee that the model selected contributes to improvements in reliability practices.

Based on the nature of the component the user can determine whether to use worst- or general-case scenario tables. Logically, if the costs associated to component failures are significant, worst-case scenario tables should be used, otherwise general-case. Furthermore, if the risk and current reliability levels are high, then the user should focus on maximising the number of failures prevented, not necessarily the number of maintenance events postponed. On the contrary, if risks and reliability levels are low, the user should aim to maximise the difference between the number of failures prevented and number of censoring events scheduled prematurely.

The next step is for the user to identify the distributions and steps, in the appropriate table, that meet the criteria described in the logic above for all reliability levels. Upon identification, the user will find that only a few distributions and steps meet the specified criteria. By establishing which operational factors are included in each model the user can identify, to a 99% significance level, which operational factors affected component reliability the most.

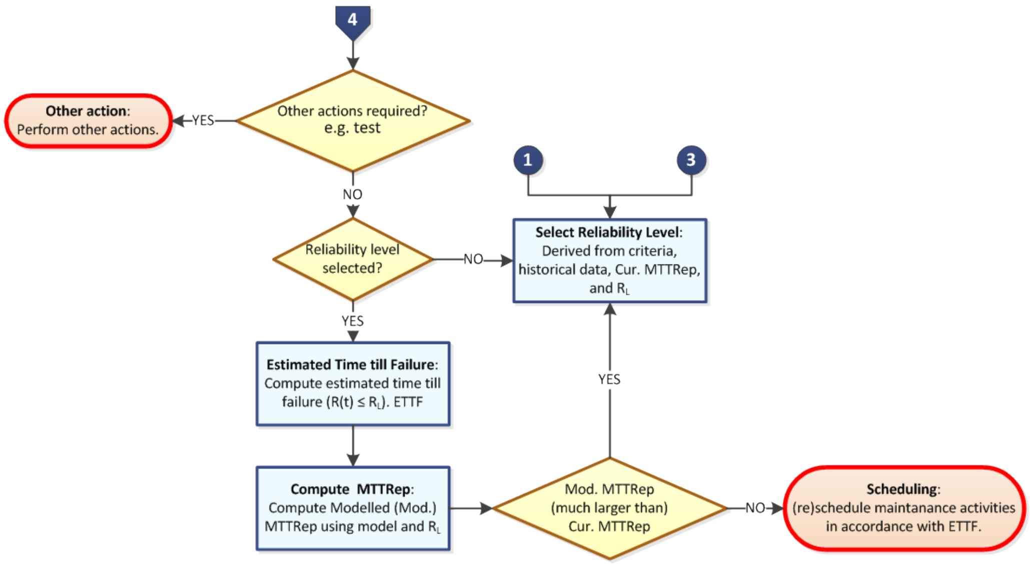


Figure 4.0.0.3: Flow chart describing decision logic used during reliability model selection. (Part 3/3)

Schedule modification After identifying the significant operational factors for both time-independent and -dependent PHMs (if required), the user needs to validate them logically. If considered logical, we can continue with model selection. At this point the objective is to identify which model (type, distribution, and covariates) to use for scheduling future maintenance activities. To do so, the user can either use the currently used reliability level, or select a new threshold based on the results, MTTRep and desired outcome.

During implementation, it is advised that the modelled MTTRep is restrained from deviating too far from the current MTTRep. This minimises the probability of observing a significant number of unforeseen failures, related to postponing scheduled events, in the future. As shown in Fig. 4.0.0.3, if the computed MTTRep is too large, then the reliability level should be re-establish.

Once a reliability level is selected, a model can be assigned by identifying which model (type, distribution, and step) meets the aforementioned criteria at this level. Finally the estimated time till failure for future events can be computed using the program and maintenance can be scheduled accordingly.

Discussion and Conclusion

In the previous sections the methodology and results were introduced of a complex solution by performing large-scale reliability modelling on aviation related data. Section 4 will discuss the solution and its findings with respect to the research objective. Final remarks are presented in the conclusion (Sec. 4).

Discussion

As discussed in Sec. *Introduction* the primary objective was to identify factors affecting component reliability and assess whether they can be used to reduce the number of Unscheduled Removals (URs). It was evident, based on results obtained from analysing the top ten components [1] , that QantasLink's current maintenance practices are based on Mean Time Till Failures (MTTF) derived from models with constant hazard rates (i.e. exponential). These models are based on the underlying assumption that failures are unrelated to external factors and the component's operational age. The data showed that the Suggested Time Till Repair (STTR) was adjusted depending on the costs associated with component failures.

The models used in this solution explored a multitude of alternatives such as repair effects, time-independent and -dependent covariates derived from operational factors recorded during flights. In essence, a broad spectrum of solutions is evaluated and a selection of models and reliability levels is returned based on their GOF test performance and predicted results.

The list of distributions is then returned to the reliability engineer such that an informed decision can be made on the scheduling of future maintenance events. To assist the engineers two functions are supplied. The first evaluates historic events (general and worst-case) using a selected model, reliability level, and a certain number of cycles in advance. The second predicts the occurrence of the next failure derived from flight data available up to the most recent date.

Analysis of the top 10 components [1] (Sec. 3) indicated that a multitude of improvements can be made to current reliability practices to reduce the number of URRs. During time-based modelling it became evident that a majority of components was better represented using a normal, log-normal, logistic, Weibull, and (or) gamma distribution opposed to the standard exponential distribution used in current practices. Further investigation showed that more complex reliability models, including covariates, could effectively reduce the number of failures by 10 to 90% without accumulating any additional costs (Tab. DC1).

Although the results showed that the proposed solution has great potential in improving overall airline reliability, the challenges encountered during this research project indicated that current reliability levels are far from being optimal. This conclusion was derived from challenges associated with the input data, failure data, and operational factor predictions.

Challenge I: Input data As for any project in the domain of data analysis, the quality of input data is vital. The primary concern with respect to input data in this thesis is related to the maintenance data provided by TRAX. In addition to containing a large number of missing and incorrect values the maintenance data often incorrectly specified the time and location of repairs. As a result it was impossible to deduce on what flights specific failure events occurred. Identifying these flights is essential for an analysis that focusses on identifying the factors related to component failures.

A solution was proposed in Sec. 2 which used linear optimisation to identify which variables in most recent flights were significantly different from the norm. To maximise the probability of identifying relevant variables during modelling the aforementioned solution (linear optimisation) produced a broad range of operational factors.

Challenge II: Failure data In the aviation industry maintenance has a preventative nature, hence challenges associated with highly censored data are inevitable. Challenge II refers to the difficulty of identifying multiple failure modes numerically, a step which is essential during reliability modelling. Ideally reliability modelling is performed on each unique mode such that the operational factors directly related to the mode can be identified. As a result the findings provide valuable insight into the nature of each failure.

Research showed that currently the methods used to distinguish failure modes are graphical, hence can not be automated. The derivation of a numerical method can be considered a stand-alone thesis project, and therefore not within the scope of this thesis. As a consequence, it was assumed that component failures were related to one failure mode only. When multiple failure modes are present during reliability modelling operational factors will be identified such that the likelihood is maximised for all failure-modes. This implies that if the failure data contains one dominant failure mode (e.g. 80% of the failures) it is very likely that significant operational factors are identified. If not, the operational factors identified are generalised and hence do not provide significant insight into the origin of failures.

If this challenge were to be overcome by the implementation of a numerical technique that can identify multiple failure modes and separate the failure data accordingly, the validity of the results is expected to increase drastically. For now it is up to the reliability engineer to use the hazard functions and reliability plots to determine whether multiple failure modes are present.

Challenge III: Forecasting To limit the scope, the techniques used to predict future operational data were limited to trend lines computed using historical data (Sec. 2.4). For time-based and time-independent PH models the seasonal and annual oscillating moving average works effectively. However time-dependent PHMs compute hazard rates derived from instantaneous operational factors. As a result, computing failures using time-dependent PHMs and moving averages will compute times assuming the flights till failure were not subject to abnormalities. In the proposed solution the error can be minimised by using worst-case scenarios when evaluating time-dependent Proportional hazard models.

To improve the overall solution's reliability significantly, operational factors should be analysed such that correlations are established amongst operational variables and environmental variables that can be predicted using a higher certainty. As an example, weather forecasts and (or) similar services can be used to obtain better predictions of environmental factors. Operational factors can then be analysed such that correlations amongst environmental and operational factors are identified and used for forecasting.

Research objective and academic contribution

Research indicates that the analysis techniques and reliability models applied in this solution were predominately used in medical research. Recent papers suggest that the application of complex reliability models in the aviation industry is a growing trend, yet remains limited to relatively non-complex time-based exponential models. Research performed by T. Schotte indicated that time-independent PHM had potential to improve component reliability, yet failed to further develop and quantify this due to limited input data (Schotte 2015).

The results obtained by analysing the top 10 components with respect to URRs (Sec. 3), show that time-independent and -dependent PHMs with underlying GRP hazard functions produce significantly better models than time-based modelling, hence confirming and verifying that complex reliability models will in fact improve overall reliability in the aviation industry. The results further suggest that time-dependent PHMs generally fit the empirical data better, however due to high censoring-levels the exact difference computed using Kolmogorov-Smirnov, Cramer-von Mises-Smirnov, Anderson-Darling, and Nikulin-Rao-Robson GOF tests (Sec. Background Information: *Goodness-of-fit tests*) cannot be quantified (D'Agostino 1986, Chimitova et al. 2010, Nikulin et al. 2010, Balakrishnan et al. 2014).

Results obtained from evaluating each component, using the models and reliability levels identified in Sec. 3, suggest that 16 to 81% of the failures can be reduced (Table DC1). In addition, the results show that the MTTF of all components decreases on average by 33%. This suggests that, predominantly, later occurring failures are prevented, causing a decrease in the average time between failures (MTTF). Logically this is related to the fact that component reliability decreases over time due to age, and also, exposure to operational factors. Evidently, the minor increase in MTTRep (5%) implies that a significant number of failures

Table DC1: Comparing actual (Empirical - Emp.) to modelled (Mod.) results using historical data (summary).

	697071003						174260-08					
		Mod. (GC)[1][2]			**Mod. (Adj.)**[3]			**Mod. (GC)**[1][2]			**Mod. (Adj.)**[3]	
	Emp. [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]	**Emp.** [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]
Failures	64	31	35	38	29	−54.7	37	18	29	35	18	−51.4
Scheduled	218	244	247	251	253	+16.1	25	27	33	44	44	+76.0
MTTRep	1970	2332	2773	4921	2352	+19.4	2223	2354	3196	4947	2106	−5.3
MTTF	1857	1256	1372	1858	1513	−18.5	1757	1147	1395	1513	1344	−23.5

	1152106-3						903-1342					
		Mod. (GC)[1][2]			**Mod. (Adj.)**[3]			**Mod. (GC)**[1][2]			**Mod. (Adj.)**[3]	
	Emp. [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]	**Emp.** [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]
Failures	61	41	48	55	40	−34.4	31	17	22	26	14	−54.8
Scheduled	212	218	225	232	233	+9.9	2	7	11	16	19	+850.0
MTTRep	1311	1370	1857	2657	1420	+8.3	1176	1225	2335	3556	1249	+6.2
MTTF	605	422	470	542	408	−32.6	1629	833	871	1147	830	−49.0

	3-1573-1						3-1574					
		Mod. (GC)[1][2]			**Mod. (Adj.)**[3]			**Mod. (GC)**[1][2]			**Mod. (Adj.)**[3]	
	Emp. [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]	**Emp.** [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]
Failures	191	150	165	181	149	−22.0	153	64	75	89	113	−26.1
Scheduled	2891	2901	2917	2932	2933	+1.5	2939	3003	3017	3028	2979	+1.4
MTTRep	257	247	300	390	244	−5.1	129	131	146	193	128	−0.8
MTTF	172	149	159	166	148	−14.0	94	74	78	84	76	−19.1

	728809-1						10-105-31A-N-2					
		Mod. (GC)[1][2]			**Mod. (Adj.)**[3]			**Mod. (GC)**[1][2]			**Mod. (Adj.)**[3]	
	Emp. [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]	**Emp.** [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]
Failures	86	13	20	28	16	−81.4	37	30	32	34	31	−16.2
Scheduled	49	107	115	122	119	+142.9	2	5	7	9	8	+300.0
MTTRep	1168	1249	1362	1566	1237	+5.9	2616	2634	2641	3502	2523	−3.6
MTTF	2220	856	999	1115	856	−61.4	1719	1411	1539	1549	1539	−10.5

	EVR-716-11-0350A						ALL					
		Mod. (GC)[1][2]			**Mod. (Adj.)**[3]			**Mod. (GC)**[1][2]			**Mod. (Adj.)**[3]	
	Emp. [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]	**Emp.** [#]	LB [#]	Val. [#]	UB [#]	Val. [#]	Diff. [%]
Failures	45	31	33	36	32	−28.9	705	395	459	522	442	−37.3
Scheduled	15	24	27	29	28	+86.7	6353	6536	6599	6663	6616	+4.1
MTTRep	2039	2034	2668	3457	2130	+4.5	1432	1508	1920	2799	1504	+5.0
MTTF	1465	904	983	1120	927	−36.7	1280	784	874	1010	857	−33.0

[1] Component reliability was computed using general-case scenarios (GC).

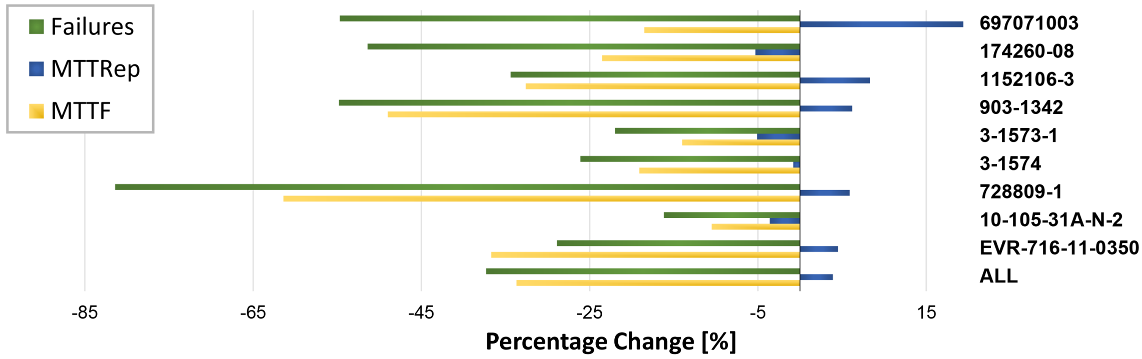
[2] The lower and upper bounds, LB and UB respectively, indicate the error produced by the model parameters at a significance level of 99% (see Sec. 2.3.3.4).

[3] In order to minimise costs associated to errors, the model's inputs (worst0case) and reliability threshold (increased) was adjusted (Adj.).

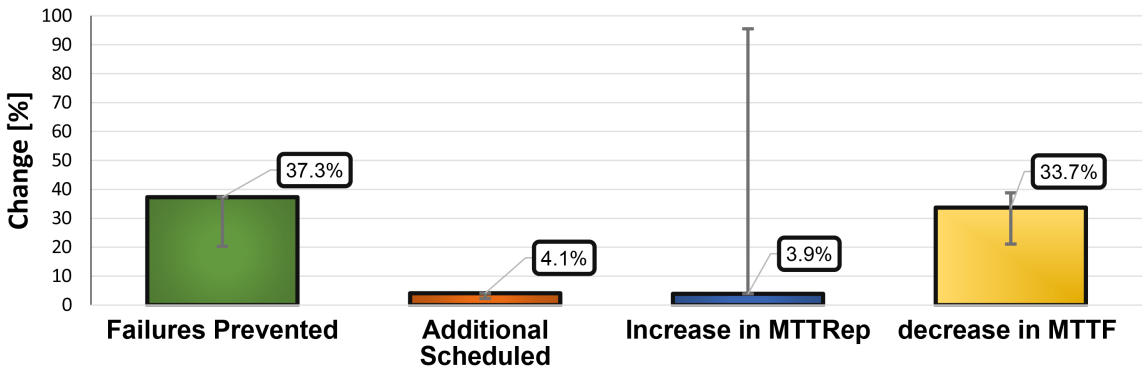
can be prevented without altering current maintenance times significantly.

In the table Mod. Adj. presents the results when the 'general case' reliability model, labelled Mod. GC, is adjusted to minimise the error. Logically, based on previous statements, this implies that in these scenarios the previously identified reliability model is used in combination with 'worst-case' operational factors and, if necessary, an increased reliability threshold. Generally the aim is to minimise the error associated to planning maintenance events too late. Hence, in all cases, we desire the modelled MTTRep to be close to the Lower-Bound (LB) of the error. Results show that worst-case scenarios and increases in reliability thresholds are effective in minimising the costs associated with errors.

Noticeably, in Table DC1 and Figure DC1, the error computed using the delta method and a significance level of 99% (Sec. 2.3.3.4) can be large. This is due to the positive relationship between the error and the number of covariates present in the model. To compensate for the error, generally, worst-case scenario models are used, which compute the estimated failure times using worst-case operational factors (Sec. 2.4). Essentially, errors can be further reduced by restraining MTTRep from deviating too far from current MTTReps. This can be done by selecting higher reliability thresholds, which, as a consequence, will minimise the number of unforeseen failures.



(a) Top 10 components.



(b) Summary of top 10 components.

Figure DC1: Summary of potential outcome using historical data.

In this example, the focus during reliability model and threshold selection was not solely on maximising the number of failures prevented, but also on limiting the deviation between current and modelled MTTRep values. This is because, when this solution is implemented for the first time, minimising the increase in MTTRep should be prioritised as this limits the number of unforeseen maintenance events, caused by postponing activities. However, once the proposed solution has been integrated into current practices, reliability thresholds can be readjusted significantly based on the significance of the component with respect to failure prevention.

For instance, consider component PN:**903-1342**. The component's actual and modelled MTTRep is approximately 1200 cycles. However, due to its irrelevance, only 2 out of the 33 maintenance events were scheduled, one could argue that the reliability threshold should be decreased, such that maintenance events are postponed. In doing so, the MTTRep could be increased significantly to 3054 cycles and 19% of the failures could have been prevented.

The same logic can be applied to critical components such as PN:$\mathbf{3-1574}$ of which 95% of the maintenance occurrences were scheduled. This implies that costs related to component failures are significant. At the current MTTRep (129 cycles) and reliability level (Tab. DC1), 26% of the failures were prevented. However, the results in App. B.6 suggest that if MTTRep is reduced to 111 up to 40% of the failures can be prevented.

Overall, the results suggest that up to 37% of the failures could have been prevented, which implies that the identified operational factors are an underlying cause of one (or more) of the failure modes related to each component. An attempt can be made to improve the reliability of components which yielded poor improvements, such as PN: **10-105-31**A-N-**2**. In the attempt, the number of failure modes would be investigated to determine whether the underlying assumption of one failure-mode was violated. In the case it was violated, the input data can be split into two (or more) sets, one per failure mode, such that new factors can be identified.

Validation of the top ten components [1] showed that identifying the true cause of component failures was a difficulty process. Nevertheless, it was possible to show that the factors obtained during forward selection procedures (Sec. 3) were directly or indirectly affecting the component's reliability. Furthermore, Table DC1 shows that, in combination with reliability models, the factors can be used to reduce the number of failures and to increase MTTRef, which logically improves the Unscheduled Removal Rate.

Aside from identifying operational factors and reducing the number of failures encountered, the solution also proposes new opportunities for improving component reliability. For a few components the solution identified that factors driving component failures were related to accelerations on and about the longitudinal, lateral, and vertical axis. These variables would suggest that component failures are the result of abnormal forces and strains endured by the component. Further investigation and (or) testing could identify new solutions to improve structural integrity and hence improve the component's reliability.

The primary research objective was to identify whether operational factors affecting component reliability could be identified and, if so, whether these could be used to reduce the number of unscheduled occurrences using a decision support model. Despite facing several challenges, solved using the assumptions defined in Sec. 1, a strategic solution was implemented to 'preliminarily' identify operational factors, estimate time-based and time-(in)dependent proportional hazard models, and finally evaluate which factors had a significant effect on component reliability.

The operational factors and models identified using the described solution have proven to model component reliability far better than models used to date, yet understanding the true nature and origin of the identified operational factors remains difficult. The root cause of these complications is related to variable reduction using linear correlation. For instance, LHS and RHS braking pressure could have a combined (or individual) effect on a component's reliability. However due to the variables being highly correlated only one is included during reliability modelling.

Despite the solution's limitations in identifying the exact root cause related to component failures, a reliability engineer can use the outcome to understand the underlying cause. In addition evaluating the reliability models using the historical data of the top 10 components [1] identified by QantasLink (Sec. 3) has proven that the solution will have a significant impact on future reliability practices. To reduce the model's limitations, progress must be made in numerical identification of multiple failure modes. Currently, due to security restriction, the solution was limited to a desktop environment. However by transforming the solution to work on a server-side environment the computational time can be reduced significantly, allowing for less 'comprehensive' variable reduction techniques.

Conclusion

With meagre returns the aviation industry is grasping for opportunities to reduce costs (The Economist 2014). At the hope of reducing the cost associated with unscheduled maintenance (e.g. delays) this paper proposes a solution to identify factors related to component failures and improve overall reliability practices at airlines. The solution differentiates itself from current practices by introducing complex reliability models which utilise operational data extracted from on-board Flight Data Recorders (FDRs) provided by *QantasLink*.

The primary research objective was to determine whether operational factors affecting component reliability could be identified (to a certain significance level) and if so, whether these could be used to reduce the number of unscheduled maintenance occurrences at QantasLink. To test the overall quality of the solution the identified models, and corresponding operational factors, were evaluated using historical data. Finally, the quality of the model is based on the reduction of unnecessary scheduled maintenance events and the number of failures avoided.

The solution consists of four modules: data acquisition & preparation, data analysis, reliability modelling, and future predictions.

In data acquisition & preparation maintenance data and operational factors are extracted and prepared for analysis. To circumvent challenges associated with missing values and errors a few assumptions were proposed to produce a clean set of data for analysis.

The second module, data analysis, focussed on reducing the number of operational factors using two analysis techniques to identify variables significantly different during failure and non-failure related flights. The first, Extreme Value Analysis (EVA), identified operational factors that were significantly different on flights operated prior to the failure. The second, Maximum Difference Analysis (MDA), identified operational factors that were overall significantly different throughout the entire failure-cycle (begin repair to failure). To solve the complications arising from the statistical nature (e.g. mean, std, max, and min) of FDR data and lack of non-linear optimisation functionality in R, EVA's optimisation algorithm was formulated using successive loops.

In an attempt to reduce the overall computational time the variables were further reduced using linear correlation. Furthermore, if the number of failures outweighed the number of operational factors, semi-parametric COX modelling was used to further reduce the number of variables (Lin 1994).

The third module, reliability modelling, uses failure data and the preliminary identified operational factors to compute MLEs for time-based, time-independent PH, and -dependent PH models subject to underlying hazard models derived from normal, log-normal, logistic, exponential, Weibull, and gamma distributions based on Kijiama's type two General Repair Processes (GRPs) (Kijima & Sumita 1986, Kaminskiy & Krivtsov 2006). Tests revealed that computational time was minimised and accuracy was maximised using 'Nelder-Maed' optimisation algorithms and forward-selection procedures (Nash 2014, Miller 1984).

To assess the Goodness of Fit (GOF) of aviation related failure data, common tests had to be modified for high censoring levels (>50%) (D'Agostino 1986). In practice the modified Kolmogorov-Smirnov, Cramer-von Mises-Smirnov, Anderson-Darling, and Nikulin-Rao-Robson GOF tests have proven to behave poorly for data subject to censoring levels beyond 20% (Nikulin et al. 2010, Chimitova et al. 2010). Despite the complications, the tests gave a good indication of which models behaved poorly with respect to others. Since the operational factors identified during the modelling process generally coincided with varying distributions, the researcher suggests that an experienced engineer uses the final outcome to select an appropriate model based on preferences regarding safety and costs associated to component failures (Sec. 3 & App. B).

Finally the fourth module, future predictions, used historical data to forecast operational factors in order to predict a component's reliability in the future. Although a variety of forecasting techniques exist, this module was limited to basic trend-line forecasting using sinusoidal and linear models to address seasonal and annual changes. In future this module can be improved to include external data sources and variable correlations such that accuracy is improved. For practical purposes the number of predicted cycles was limited to 100 to minimise errors associated to inaccurate forecasts.

Results derived from analysing and modelling the top 10 components [1] , in terms of URRs, confirm that current reliability practices are based on an underlying assumption that hazard rates remain constant over time. Although this assumption does hold for a few components, in general the results obtained from time-based modelling indicate the assumption is violated and reliability is better modelled using an alternative distribution.

Once a selection of models has been identified the solution returns a list of identified variables with their corresponding values. Assessment of the variables was simplified using scaling, transforming all variables on to a null to one scale. Generally, covariates with large values have a strong effect on the component's reliability. The variables affecting component reliability for the top 10 components [1] have been validated using theory. Nonetheless it is important for the user to be aware that forward variable selection disregards variable correlations and that limitations are introduced by variable reduction and the number of observed factors. As a result, the identified components can have only an indirect effect on the component's reliability. As an example, assume LHS braking pressure was identified during modelling procedures. Disregarding the prior mentioned limitations one could assume LHS braking pressure is directly related to a component's reliability, or indirectly due to a torque induced about the vertical axis. However, due to variable reduction, LHS and RHS braking pressure was identified as being linearly dependent and highly correlated and therefore RHS braking pressure was removed prior to modelling. Hence the failure could be related to abnormal longitudinal accelerations caused by the application of LHS and RHS braking.

Once the components have been validated a reliability engineer generally has three options: adjust maintenance schedule based on new reliability models, (re)stock inventory in preparation of future failures, and (or) run tests to diagnose new solutions to improve reliability.

Finally, the last step to answering the research question is to determine whether the operational factors can be used, in conjunction with reliability models, to improve overall reliability. In an ideal study, new failure data would be used to assess the model's predictive power. In this case however, new failure data was not readily available. As a consequence, the study had to be performed with the original data, which could not be split into two due to the limited number of failure events.

Table DC1 (Sec. 4) shows that the number of failures can be reduced significantly at the same reliability levels currently used by the operator. The current level was obtained by identifying which reliability level corresponds to the component's current average scheduling time, using the reliability function derived from empirical data. In Table DC1, it was shown that the error induced by estimated model parameters can be minimised by computing failure times using worst-case scenario operational factors and increasing the reliability threshold. The results suggest that the total number of failures can be reduced from 16 to 81% depending on the components criticality, with respect to failure costs, and the reliability level used.

Since the exact benefits in terms of cost reductions cannot be specified, it is valid to assume that an overall failure reduction of 26% (worst-case) to 44% (best-case) in combination with a limited increase in MTTRep, 5%, will reduce the company's total expenditure. Nonetheless, due to a total censoring level of approximately 90%, it is uncertain whether unforeseen failures would impact costs enough to reduce the overall benefits of the solution. The modelled solution suggests that over 263 of the postponed events must fail in order to break-even with the number of failures prevented. This scenario is very unlikely considering that MTTRep only increased by 5%. Be that as it may, the uncertainty can be further reduced (minimised) by increasing modelled reliability levels.

It is advised that the solution is tested with 'new' historic data, prior to actual implementation. Once it is verified that the models reduce the number of failures, the maintenance schedule can be adapted with caution.

Notes:

[1] Analysis was performed on QantasLink's top ten components, w.r.t. unscheduled removal rates. Due to insufficient data, high-level fuel sensors (PN: **92003-051-052-001**) could not be analysed/modelled (see Sec. 3.7).

Recommendations and Suggestions

In an ideal world, components would never fail. In reality, however, they do and it is our objective to minimise the impact of failures on primary and secondary stakeholders (i.e. employees, passengers, and communities). Since component failures are inevitable, for the future we hope to optimise maintenance such that components are removed just before they are likely to fail. Although the solution in this paper has proposed techniques to identify factors related to component-failures and introduced models to improve future predictions, it is far from perfect. It should be regarded as an important stepping stone on its way to fully optimising reliability practices. In this section we will outline the steps required towards a fully optimised maintenance schedule.

It speaks for itself that an increase in quantity and quality of input data would improve reliability forecasts. However, to make real progress, future developments should focus on the identification of multiple failure modes and the effective forecasting of operational factors.

Multiple failure modes The results showed that models obtained from processing failure data comprised of two (or more) failure modes yield poor forecasts and failed at identifying the factors affecting component reliability. To identify the primary factors affecting component reliability, a method must be developed that can 'effectively' distinguish between failure modes. If done correctly, component reliability can be derived from the factors associated to each failure mode.

Forecasting operational factors In this paper, operational factor forecasting was heavily simplified by the assumption that all operational factors follow seasonal and annual trends. In reality, however, operational factors are highly correlated and are affected by a variety of external factors (e.g. weather, flight crew, etc). Forecasting can be improved significantly by studying the correlation among flight variables and assessing which factors can be predicted more accurately (e.g. atmospheric pressure, temperature, etc).

Once the relationships among the operational factors have been established, future reliability programs could enhance their forecasting accuracy by using data collected on some of these factors, from internal or external sources (e.g. weather stations), to predict other factors. Improved forecasting methods would drastically improve the accuracy of time-dependent reliability models, which use instantaneous flight data to compute component reliability.

Other suggestions Besides developing techniques to cope with multiple failure modes and forecasting, airlines should focus on recording data as accurately as possible. This is especially the case for systems that depend on human input, such as TRAX. In addition, to identify the factors associated to each failure-mode, the input data must be sufficiently large. The study showed that, currently, datasets are limited in size. Since it is very unlikely that individual airlines will have sufficiently large datasets in the near future, it is recommended that airlines, operating similar aircraft, collaborate.

Implementation

DISCLAIMER
It is strongly advised that the strategy proposed in this paper is first tested using trial runs prior to implementation.

This section proposes a strategy to integrate the proposed solution into current reliability practices. The implementation strategy is driven by the solution's limitations and QantasLink's requirements with respect to maintenance scheduling. These limitations and requirements include:

- I1 Built upon the notion that QantasLink requires a minimum of five working days to schedule maintenance events (Sec. 2.4), the solution was designed such that it can predicted up to two weeks in advance (100 cycles).
- I2 Built upon requirement I1, maintenance events with scheduled times beyond two weeks should remain static (unchanged) until a failure is expected to occur within the next two weeks.
- I3 Components with a reliability level within 5% of the threshold should be listed, such that they are put on a watch list.

Requirements I2 and I3 are company specific requirements and can easily be adapted to suit the company's own needs. Requirement I1 on the other hand is a limitation imposed by the solution's forecasting module. Future developments in variable forecasting can extend this barrier.

In Sec. Executive Summary, both a non-invasive and invasive implementation strategy are described. This section will focus on the implementation of the invasive strategy since it covers the aspects of the non-invasive method.

Solution Integration

In the following section the integration strategy is described in four steps. The first, preparation, explains the program's installation process and requirements. The second, program initiation, explains the process of populating the program with FDR and TRAX data, such that reliability models can be ascertained. It is important that the user is aware that program initiation can be executed multiple times a year, such that reliability models are improved using new data. The third step, model selection, explains the process of identifying the reliability model and threshold for computing maintenance scheduling times. Finally step four, failure forecasting, explains how maintenance events should be (re)scheduled.

1. Preparation

The first step is to prepare your computer to use the program. To do so, the programming language environment needs to be installed on either a Microsoft Windows OS personal computer, or preferably, a server (The R Project 2016). To install the program, locate the setup file, 'setup.exe', in the installation folder. Installing the program via the setup file is essential because the process verifies that prerequisites, such as R and .NET framework 4.0, are installed (Fig. LT1).

Once installed, the program can be initiated using 'CoBaPreM.QLK.exe' in the program's install directory. To protect QantasLink's data, the first time the program is executed the user is prompted for a password. The password will be distributed to authorised personnel and is required to decrypt the pre-existing operational and maintenance data that was installed with the program.
As described in Sec. Background Information: *R Programming language*, the program depends on a few existing R libraries. An attempt to download and install these libraries will

Figure LT1: Program's setup wizard.

be made by the program on execution, however, if the process fails, the user must manually install the libraries listed in this section.

2. Program Initiation

Per default, the input and output directories have been set by the program. However if you would like to change the installation (or sub directories), or run the *.R scripts manually please set the main directory ('mainDir') in 'Run.R', 'EvaluateRM.R', and 'EvaluatePN.R' and configure settings in 'LTsettings.R'. The latter, 'LTsettings.R' can also be used to change the name/location of the FDR and TRAX data files.

To process the data and create reliability models for the components you would like to start scheduling please use the first function provided in the program executable 'LTpredictQLK.exe' (Fig. LT2). Alternatively (not recommended), specify the components and inputs in the 'Run.R' file and source it using a compiler.

Since the program takes a substantial amount of time to analyse and model component reliability, it is advised to run multiple instances of the program for diverse Part Numbers (PNs).

3. Model Selection

In order to select the appropriate reliability model and threshold the user is redirected to Sec. 4. In this section a decision logic diagram is presented which, in combination with the results in the output directory, can be used to identify the factors that affect component reliability and select an appropriate model and threshold.

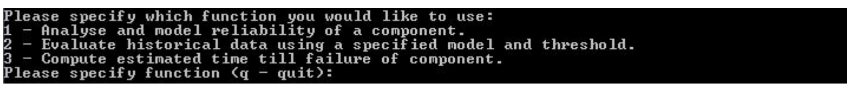


Figure LT2: Program's menu.

The user can use the second function, see Fig. LT2, for assessing reliability models at alternative reliability thresholds.

4. Failure Forecasting

Now that a reliability model and threshold are selected, failure times can be extrapolated and maintenance schedules can be adjusted. To obtain initial estimates for the time till next repair, the user can use function two (Fig. LT2), which uses historical data to compute the past-observed MTTF and MTTRep.

Once all initial maintenance times are submitted, the operator can start evaluating component reliability on an interval-basis (e.g. weekly). Based on the reliability computed the user is notified whether a component's reliability level has either fallen below the threshold, or is far above its anticipated value. If it has fallen below the threshold the user is 'advised' to schedule a maintenance event within the next two weeks. If it is far above its anticipated value the user is 'advised' to delay the original scheduling date. By default, this event is triggered when the component's reliability is 5% above its anticipated value.

The user can compute the estimated time till failure of any individual component using function three (Fig. LT2), or alternatively, not recommended, manually using the 'EvaluatePN' script. Function three requires the user to import a flight schedule (future flights). When importing the schedule, the user should keep in mind that the schedule should follow from the last observed date in the operational factor datasets (FDR and TRAX), and that flights can only be predicted up-to the last date specified in the schedule. To conform to the aforementioned requirements, the schedule should contain at least two weeks of data. It is important that the user is aware that the error accumulates as the number of predicted flights increases (see Sec. 2.4).

As per I1, the maximum amount of time the program can compute a component's reliability accurately is two weeks. As such it is required that at minimum, the components are evaluated every two weeks. Logically the model's effectiveness increases with frequency, hence it is advised to evaluate the components every week.

References

Adler, D., Gläser, C., Nenadic, O., Oehlschlägel, J. & Zucchini, W. (2015), 'Package 'ff'', CRAN documentation.

Arasan, J. & Ehsani, S. (2011), 'Modeling repairable system failures with repair effect and time dependent covariates', *International Journal of Applied Mathematics* **41**(3).

Balakrishnan, N., Chimitova, E. & Vedernikova, M. (2014), 'An empirical analysis of some non-parametric goodness-of-fit tests for censored data', *Communications in Statistics - Simulation and Computation* .

Bolstad, B. (2015), 'Data normalization and standardization', Statistics presentation by University of California.

Bombardier (2014*a*), Q series: Semi-annual fracas report (covering q100, q200, q300), FRACAS Report 43, Bombardier.

Bombardier (2014*b*), Q400 quarterly fracas report, FRACAS Report 117, Bombardier.

Bombardier (2015*a*), *Flight Data Recorder Q400 Parameter Data Map*, dh8-400-sl-31-008 edn, Bombardier, 123 Garratt Blvd., Toronto, Ontario. Service Letter.

Bombardier (2015*b*), Oceania regonal review 2015, *in* 'Oceania regional review: Spring 2015', Bombardier.

Chimitova, E., Tsivinskaya, A. & Vedernikova, M. (2010), 'Investigation of goodness-of-fit test statistic distributions by random censored samples', Novosibirsk State Technical University Presentation.

Citrix Systems (2015), 'Xenapp', Online documentation, Accessed on: 03/02/2015. URL:**https://www.citrix.com/content/dam/citrix/en_us/documents/oth/xenapp-reviewers-guide.pdf**

D'Agostino, R. B. (1986), *Goodness-of-fit-techniques (statistics: a series of textbooks and monographs)*, CRC Press.

de Boer, L. W. (2015), 'Component-based predictive maintenance to reduce unscheduled maintenance occurrences.', Literature Review.

Department of Mathematics (2015), 'Interval estimation', Online cource documents. University of Arizona.

Fan, J. & Jiang, J. (2009), 'Non- and semi- parametric modeling in survival analysis', *Research Gate* **10**(1142).

Geyer, C. J. (2003), 'Fisher information and confidence intervals using maximum likelihood', Statistic notes.

Ghobbar, A. A. (2011), 'Reliability, availability, maintainability, & supportability', TU Delft Course AE4440 - reliability, availability, maintainability, & supportability Lectures.

Greene, W. H. (2012), *Econometric analysis*, 7 edn, Pearson Education Inc.

IATA (2015), 'Strong traffic growth continues in may', Press release. URL:**https://www.iata.org/pressroom/pr/Pages/2015-07-02-01.aspx**

IBM (2015), 'Cplex optimizer', Online documentation. URL:**http://www-01.ibm.com/software/commerce/optimization/cplex-optimizer/**

Ionescu, D. & Limnios, N. (1999), *Statistical and probabilistic models in reliability*, Springer.

Johnson, R. A. & Wichern, D. W. (2007), *Applied multivariate statistical analysis*, 6 edn, Pearson Education Inc.

Jr., J. F. H., Black, W. C., Babin, B. J. & Anderson, R. E. (2009), *Multivariate data analysis*, Prentice Hall.

Kaminskiy, M. & Krivtsov, V. (2006), 'A monte carlo approach to estimation of g-renewal process in warranty data analysis', Online document.

Kaminskiy, M. & Krivtsov, V. (2010), 'G1-renewal process as repairable system model', *arXiv preprint arXiv:1006.3718* .

Kijima, M. & Sumita, U. (1986), 'A useful generalization of renewal theory: counting processes governed by non-negative markovian increments', *Journal of Applied Probability* **23**(1), 71–88.

Kopecky, K. A. (2007), 'Numerical differentiation', Lecture notes.

Lin, D. (1994), 'Cox regression analysis of multivariate', *Statistics in Medicine* **13**, 2233–2247.

Mettas, A. & Zhao, W. (2005), Modeling and analysis of repairable systems with general repair, *in* '2005 Annual Reliability and Maintainability Symposium', ReliaSoft Corporation.

MiKTeX (2016), 'Miktex project page', Online resource.
URL:**http://miktex.org/**

Miller, A. J. (1984), 'Selection of subsets of regression variables', *Journal of the Royal Statistical Society* **147**(3), 389–425.

Miranda, M. J. & Fackler, P. L. (2004), *Applied computational economics and finance*, MIT Press.

Muenchen, R. A. (2015), 'The popularity of data analysis software', Research paper.
URL:**http://r4stats.com/articles/popularity/**

Muller, R., Gong, G. & Muñoz, A. (1980), Survival analysis, Technical Report 58, Stanford University.

Myung, J. (2003), 'Tutorial on maximum likelihood estimation', *Journal of Mathematical Psychology* **47**, 90–100.

Nash, J. C. (2014), 'On best practice optimization methods in r', *Journal of Statistical Software* **60**(2).

Nikulin, M., Lemeshko, B., Chimitova, E. & Tsivinskaya, A. (2010), 'Nonparametric goodness-of-fit tests for censored data', Research paper.

Nocedal, J. & Wright, S. J. (2006), *Numerical optimization*, Springer.

OAG (2012), 'Low-cost carriers grow at incredible rate, reports oag', Online article.
URL:**https://www.oag.com/**

Orme, J. G. & Combs-Orme, T. (2009), *Multiple regression with discrete dependent variables*, Oxford University Press.

Powell, L. A. (2007), 'Approximating variance of demographic parameters using the delta method: a reference for avian biologists', *The Condor* **109**(4).

Qantas (2015), 'Qantaslink timettime.', Online pdf timetable.

QantasLink (2014), *Sunstate Airlines Reliability Manual 400 Series*, 2.1 edn, Qantas Airways Limited, Qantas HQ Sydney.

ReliaSoft (2015), 'Resource portal for professionals in reliability engineering and related fields.', Resource Portal.

Schotte, T. (2015), 'Development of a pro-active maintenance framework for failure prediction.', Graduation project.

Seber, G. (1982), *The estimation of animal abundance and related parameters*, New York Chapman.

Sen, A. & Srivastava, M. (1990), *Regression analysis: Theory, methods and applications*, Springer.

Stevens, J. R. (2013), 'Survival analysis: left-truncated data', Statistics 5500/6500 Presentation Survival Analysis, Utah State University.

The Economist (2014), 'Why airlines make such meagre profits', Report.

The R Project (2016), 'The r project for statistical computing', Online Documentation.
URL:**https://www.r-project.org/**

TRAX Systems (2015*a*), 'Trax maintenance software', Virtual application (XenApp environment).

TRAX Systems (2015*b*), 'Trax systems standards manual', Online documentation, Accessed on: 04/02/2015.
URL:**http://qlinkmanuals.qantas.com.au/publish/Trax/TRAXOnlineHelp/index.htm**

Verhagen, W. (2014), 'Ae4465 maintenance, modelling and analysis - lecture series'.

Weisberg, S. (2014), *Applied linear regression*, 4 edn, WILEY. Chapter 10.

Yuen, K.-V. (2012), *Relationship between the Hessian and Covariance Matrix for Gaussian Random Variables*, John Wiley & Sons.

Zhang, D. (2005), 'Analysis of survival data', Lecture Notes: ST745 Analysis of survival data, North Carolina State University.

Appendix A

Data Sets QantasLink

This appendix describes the various sources made available by QantasLink. The following sources are relevant to this study: aircraft maintenance management system (TRAX) (Sec. A.1), Flight Data Recorders (FDR) (Sec. A.2), Engine Health Monitoring (EHM) (Sec. A.3), Original Equipment Manufacturer (OEM), technical information services (Sec. A.4) and AC technical delay reports (Sec. A.5).

Each source has been analysed in a systematic manner. The following list outlines the structure in which each source is presented.

1. Description of source
2. Overview of data provided
3. Methodology behind data acquisition
4. Quality and usefulness of data

A.1 TRAX

The following section introduces the reader to TRAX, its outputs, methods and validity.

A.1.1 Description

TRAX is an airline maintenance management suite(TRAX Systems 2015*b*) aimed at assisting airlines in tracking, monitoring, and planning maintenance and engineering activities. The data is accessible using Citrix's XenApp and XenDesktop (Citrix Systems 2015), a virtual application and desktop environment providing access to TRAX's software. In order to access the virtual desktop environment, one must be connected to the Qantas network (or VPN), have a user account authorising access to the virtual environment, and have a user account to access TRAX.

In essence, TRAX is just a secured environment allowing you to create SQL queries and manipulate data on TRAX owned SQL databases. TRAX will allow its users to input schedules, defects, incidents, component utilisation and shop data, Minimum Equipment Lists (MELs), technical defect and supplementary maintenance data, and Significant Defect Reports (SDR). Using a query, various items can be extracted from the TRAX database. Figure A.1.2 presents the SQL query used to extract various parameters related to Part Number (PN) transaction (install and removal) history from TRAX. In case more parameters are required, the user can specify them by adding or editing the query.

A.1.2 Data Extracted

Table A.1.2 contains a list of the parameters returned from the query shown in Figure A.1.2. As an aid to the reader, for each parameter an example and description are given. Although maintenance and defect data exists prior to 2004, it is only since then that Qantas has been utilising and inputting data into TRAX in a consistent manner. Hence, TRAX data from 2004 onwards will be utilised. In addition to date restrictions, TRAX data is also subject to human error as the database is populated by ground crew and other airline employees (See Sec. A.1 "*Data Acquisition*"). Each transaction displays the part subjected to the transaction (pn & sn), the aircraft and position where it was previously installed (ac & $position$), and the date the transaction occurred ($transaction\_date$). The reason why it occurred and whether it was unscheduled or scheduled is given by parameters $removal_reason$ and $reason_category$ respectively, and the time operated since last transaction is given in both number of cycles and minutes. For each part the corresponding ATA chapter, section, and paragraph are given. In addition, engine parts indicate part and serial number of higher and lower level assemblies.

```
SELECT ”AC_PN_TRANSACTION_HISTORY”.”TRANSACTION” AS ”TRANSACTION”,
       ”AC_PN_TRANSACTION_HISTORY”.”TRANSACTION_ITEM” AS ”TRANSACTION_ITEM”,
       ”AC_PN_TRANSACTION_HISTORY”.”TRANSACTION_TYPE” AS ”TRANSACTION_TYPE”,
       ”AC_PN_TRANSACTION_HISTORY”.”AC” AS ”AC”,
       ”AC_PN_TRANSACTION_HISTORY”.”TRANSACTION_DATE” AS ”TRANSACTION_DATE”,
       ”AC_PN_TRANSACTION_HISTORY”.”TRANSACTION_HOUR” AS ”TRANSACTION_HOUR”,
       ”AC_PN_TRANSACTION_HISTORY”.”TRANSACTION_MINUTE” AS ”TRANSACTION_MINUTE”,
       ”AC_PN_TRANSACTION_HISTORY”.”DEFECT_TYPE” AS ”DEFECT_TYPE”,
       ”AC_PN_TRANSACTION_HISTORY”.”DEFECT” AS ”DEFECT”,
       ”AC_PN_TRANSACTION_HISTORY”.”DEFECT_ITEM” AS ”DEFECT_ITEM”,
       ”AC_PN_TRANSACTION_HISTORY”.”WO” AS ”WO”,
       ”AC_PN_TRANSACTION_HISTORY”.”TASK_CARD” AS ”TASK_CARD”,
       ”AC_PN_TRANSACTION_HISTORY”.”GOODS_RCVD_BATCH” AS ”GOODS_RCVD_BATCH”,
       ”AC_PN_TRANSACTION_HISTORY”.”PN” AS ”PN”,
       ”AC_PN_TRANSACTION_HISTORY”.”SN” AS ”SN”,
       ”AC_PN_TRANSACTION_HISTORY”.”POSITION” AS ”POSITION”,
       ”AC_PN_TRANSACTION_HISTORY”.”REASON_CATEGORY” AS ”REASON_CATEGORY”,
       ”AC_PN_TRANSACTION_HISTORY”.”SCHEDULE_CATEGORY” AS ”SCHEDULE_CATEGORY”,
       ”AC_PN_TRANSACTION_HISTORY”.”HOURS_INSTALLED” AS ”HOURS_INSTALLED”,
       ”AC_PN_TRANSACTION_HISTORY”.”MINUTES_INSTALLED” AS ”MINUTES_INSTALLED”,
       ”AC_PN_TRANSACTION_HISTORY”.”CYCLES_INSTALLED” AS ”CYCLES_INSTALLED”,
       ”AC_PN_TRANSACTION_HISTORY”.”DAYS_INSTALLED” AS ”DAYS_INSTALLED”,
       ”AC_PN_TRANSACTION_HISTORY”.”NHA_PN” AS ”NHA_PN”,
       ”AC_PN_TRANSACTION_HISTORY”.”NHA_SN” AS ”NHA_SN”,
       ”AC_PN_TRANSACTION_HISTORY”.”NLA” AS ”NLA”,
       ”AC_PN_TRANSACTION_HISTORY”.”NOTES” AS ”NOTES”,
       ”AC_PN_TRANSACTION_HISTORY”.”CREATED_BY” AS ”CREATED_BY”,
       ”AC_PN_TRANSACTION_HISTORY”.”CREATED_DATE” AS ”CREATED_DATE”,
       ”AC_PN_TRANSACTION_HISTORY”.”MODIFIED_BY” AS ”MODIFIED_BY”,
       ”AC_PN_TRANSACTION_HISTORY”.”MODIFIED_DATE” AS ”MODIFIED_DATE”,
       ”AC_PN_TRANSACTION_HISTORY”.”BATCH” AS ”BATCH”,
       ”AC_PN_TRANSACTION_HISTORY”.”NON_SN” AS ”NON_SN”,
       ”AC_PN_TRANSACTION_HISTORY”.”STATION” AS ”STATION”,
       ”AC_PN_TRANSACTION_HISTORY”.”RECURRENT” AS ”RECURRENT”,
       ”AC_PN_TRANSACTION_HISTORY”.”STATUS” AS ”STATUS”,
       ”AC_PN_TRANSACTION_HISTORY”.”TRANSACTION_TYPE_CONTROL” AS ”TRANSACTION_TYPE_CONTROL”,
       ”AC_PN_TRANSACTION_HISTORY”.”CHAPTER” AS ”CHAPTER”,
       ”AC_PN_TRANSACTION_HISTORY”.”SECTION” AS ”SECTION”,
       ”AC_PN_TRANSACTION_HISTORY”.”PARAGRAPH” AS ”PARAGRAPH”,
       ”AC_PN_TRANSACTION_HISTORY”.”TAG_NO” AS ”TAG_NO”,
       ”AC_PN_TRANSACTION_HISTORY”.”REMOVAL_REASON” AS ”REMOVAL_REASON”,
       ”AC_PN_TRANSACTION_HISTORY”.”NLA_POSITION” AS ”NLA_POSITION”,
       ”AC_PN_TRANSACTION_HISTORY”.”INTERFACE_ECTM_TRANSFER_BY” AS ”INTERFACE_ECTM_TRANSFER_BY”,
       ”AC_PN_TRANSACTION_HISTORY”.”INTERFACE_ECTM_TRANSFER_DATE” AS ”INTERFACE_ECTM_TRANSFER_DATE”,
       ”AC_PN_TRANSACTION_HISTORY”.”REMOVE_AS_SERVICEABLE” AS ”REMOVE_AS_SERVICEABLE”,
       ”AC_PN_TRANSACTION_HISTORY”.”BLOB_NO” AS ”BLOB_NO”,
       ”AC_PN_TRANSACTION_HISTORY”.”DOCUMENT_NO” AS ”DOCUMENT_NO”,
       ”AC_PN_TRANSACTION_HISTORY”.”TASK_CARD_PN” AS ”TASK_CARD_PN”,
       ”AC_PN_TRANSACTION_HISTORY”.”TASK_CARD_SN” AS ”TASK_CARD_SN”,
       ”AC_PN_TRANSACTION_HISTORY”.”NHA_PN_PRORATED” AS ”NHA_PN_PRORATED”
  FROM AC_PN_TRANSACTION_HISTORY
 WHERE ( ”AC_PN_TRANSACTION_HISTORY”.”TRANSACTION_DATE” >= to_Date ( '2015/01/01','yyyy/mm/dd' )
         AND ”AC_PN_TRANSACTION_HISTORY”.”TRANSACTION_DATE” < to_date ( '2020/01/01','yyyy/mm/dd' ) )
```

Figure A.1.2: TRAX PN transaction history SQL query(TRAX Systems 2015*a*).

A.1.3 Data Acquisition

As mentioned in Sec. A.1 ”*Data Extracted*”, the database is populated by airline employees and hence subject to human error. In order to minimise human error, limitations in terms of character types, lengths, date/time formats, and default values have been implemented, yet parameters such as serial and part numbers are dependent on each individual manufacturer's preference and can vary in size, style, and format. The aforementioned information implies that any data obtained by TRAX must first be filtered and prepared before commencing with data analysis and comparison.

Table A.1.2: List containing abbreviations, examples, and descriptions from parameters exported out of TRAX.

Parameter	Example	Description
ac	VH-TQX	Aircraft registration number of part which is installed/removed.
batch	552041	Store batch number.
chapter	61	ATA chapter number.
created_by	SIN07	TRAX user ID of event creator.
created_date	2/01/2005 13:16:09	Date and time when event was created in TRAX.
cycles_installed	815	Flight cycles operated since installation.
days_installed	119	Days operated since installation.
defect	0041423	Defect number.
defect_type	PILOT	Party responsible for detecting the defect (e.g. Pilot or Maintenance).
goods_rcvd_batch	521831	Store batch number.
hours_installed	855	Hours operated since installation.
minutes_installed	55	Minutes operated since installation.
modified_by	SIN07	TRAX user ID of event modifier.
modified_date	2/01/2005 13:16:43	Date and time when event was modified in TRAX.
nha_pn	93A100-80	Next higher assembly engine part number.
nha_sn	25680	Next higher assembly engine serial number.
nla	Y	Next lower assembly.
non_sn	0	Serial number
paragraph	3	ATA paragraph number.
pn	H321ALM1	Part identifier (part number).
position	CL	Position of part on AC. Ceter Line, Auxilary, Inboard, Outboard, Seat number, etc
reason_category	US	Reason part removed.
removal_reason	INOP	Inoperative failed leaking
schedule_category	UN/SCHEDULE	Scheduled or unscheduled transaction.
section	21	ATA section number.
sn	6775	Part's unique identifier (serial number).
station	SYD	Station where transaction occurred.
status	CLOSED	Status open or clossed.
task_card	AD/INST/9	Task card number.
transaction	2/01/200513:16:09:597.0	Date and time issue created in TRAX. (Format: dd/mm/yyyyHH:mm:ss:ms)
transaction_date	1/01/2005 20:30:00	Date and time transaction occurred. (Format: dd/mm/yyyyHH:mm:ss:ms)
transaction_hour	20	Hour transaction occurred.
transaction_item	1	Type of transaction item. Either 2 (Installed) or 1(Removed)
transaction_minute	30	Minute transaction occurred.
transaction_type	REMOVE	Type of transaction.
transaction_type_code	REMOVE/INSTALL	Transaction type options.
wo	7483	Work order number.

A.1.4 Quality and Usefulness of Data

Presence of human error suggests that TRAX data will contain noise, which will need to be filtered prior to analysis. More data regarding part/component failures will be available from the OEMs and other operators, yet TRAX data will be vital when it comes to comparing how Qantas owned parts are performing in comparison to manufacturer standards and other competitors. TRAX data can be combined with AC Delay reports (Sec. A.5) to determine the maintainability and supportability. These three can be combined to give an indication of an items availability (Verhagen 2014).

A.2 Flight Data Recorder

The following section introduces the reader to Flight Data Recorders (FDRs) used at QantasLink, the recorders' outputs, methods, and validity.

A.2.1 Description

Flight Data Recorders (FDR) are used to monitor in-flight conditions such as flight crew inputs and outputs measured by sensors on-board the aircraft. The data recorded by the FDR is also saved on the black-box to assist the aircraft incident investigators in understanding why an aircraft failed.

QantasLink's fleet consists of 3, 16, and 31 Bombardier Dash8 Q-200s, -300s and -400s respectively, most of which are equipped with a Universal SSFDR data recorder and a few with Quick Access recorders. Only two of the aircraft are equipped with a Flight Acquisition Storage and Transmission (FAST) box, allowing flight data to be extracted after each sector, via a wireless connection (QantasLink 2014). Data on the other aircraft are extracted every evening. The data files outputted by both recorders use the same data map outlined in Sec. A.2 *Data Extracted*(Bombardier 2015*a*).

A.2.2 Data Extracted

Service letter (DH8-400-SL-31-008)(Bombardier 2015*a*) outlines the data map used on-board the Dash 8 Q400 aircraft. The Q200 and 300 series use the same data map but contain less variables. The data extracted from the FDRs on the Q400 compromise of 273 different parameters. Figure A.2.3 shows an example of how the data map defines each parameter recorded by the recorder. The first few words up to *category* define the source, name, and unit of the parameter recorded (see Table A.2.3 for a list of abbreviations). The *category* is almost always normal, meaning the data is recorded against time DATE and GMT. DATE and GMT are the only two parameters categorised as ***base***. *Type*, *word*, *sample*, and *res* refers to the data type recorded (signed, unsigned, discrete, raw, or BCD), the number of word(s) used (1=normal, 2=combined), number of samples that occur in one second/subframe, and the decimal locator respectively. In order to compute the real value, Slope or Coeff. and Offset are used to indicate what coefficient to apply to the obtained value to retrieve the engineering value and what offset to add to obtain the real value. The min and max define the upper and lower limit of the range.

ACCN NORM ACCN NORMAL (g) Category : Normal Type : Signed Analog
Words: 1 Sample/SF : 8 Res : 4 Slope : 3.90625e-003 Offset : 0.
Min : -3. Max : 6.

Sub-frame	Word No.	Start Bit	End Bit	Frame ID
1234	5	1	12	0
1234	21	1	12	0
1234	37	1	12	0
1234	53	1	12	0
1234	69	1	12	0
1234	85	1	12	0
1234	101	1	12	0
1234	117	1	12	0

AFCS ALT SEL AFCS ALT SEL MODE () Category: Normal Type: Discrete
Words: 1 Sample/SF : 1 Res : 0

Sub-frame	Word No.	Start Bit	End Bit	Frame ID	Discrete Text
1234	25	11	11	0	0 : NOT ACTIVE
					1 : ACTIVE

Figure A.2.3: Example of two FDR data map entries (Bombardier 2015*a*).

Access to FDR data is subject to various regulatory, security, and technical constraints. At Qantas, in order to gain access to the data one must get security clearance, approval from the chief pilot and fleet technical dep., and retrieve the data under supervision of a level

Table A.2.3: List of definitions from abbreviations used in the FDR data map (Bombardier 2015*a*).

Abbreviation	Definition	Abbreviation	Definition
AC	Alternating Current	LONG	Longitude (Longitudinal)
ACCN	Acceleration	LPLT	Left Hand Pilot
ADU	Air Data Unit	MFD	Multifunction Display
AF	Auto Feather	MKR	Marker
AFC	Automatic Frequency Control	MLG	Main Landing Gear
AFCS	Automatic Flight Control System	MLS	Microwave Landing System
AFIS	Airborne Flight Information System	MSTR	Master
AFRME	Airframe	NAV	Navigation
AHRS	Attitude Heading Reference System	ND	Navigation Display (Nose Down)
AIL	Aileron	NH	Engine high speed
ALRT	Alert	NL	Engine low speed
ALT	Altitude	NLG	Nose Landing Gear
ANG	Angle	NO	Normal
AOA	Angle of Attack	NORM	Normal
AP	Auto Pilot	NP	Prop speed
APU	Auxiliary Power Unit	NU	Nose Up
ARC	Arc	OAT	Outside Air Temperature
ARM	Armed	OPRTN	Operation
AZ	Aximuth	OR	Operation
BARO	Barometer	OTBD	Outbound
BATT	Battery	PB	Park Break
BC	Back Course	PDL	Pedal
BCN	Beacon	PEDL	Pedal
BRK	Brake	PFD	Primary Flight Display
CAL	Calibrated	PIT	Pitch
CHK	Check	PL	Pedal
CLA	Climb Angle	PLA	Power Lever Angle
CMD	Command	PLT	Pilot
COL	Column	PLT	Pitch Limit Trim
CPLT	Complete	POS	Position
CPT	Copilot	PR	Pressure
CRS	Course	PRESS	Pressure
CTL	Control	PREV	Prevention
CTN	Caution	PROP	Propeller
DC	Direct Current	PT	Pitch Trim
DET	Detected	PTR	Pointer
DEV	Deviation	PTT	Push to Talk
DH	Decision Height	R	Right
DME	Distance Measuring Equipment	RA	Resolution Advistories
DNLOCKED	Down-Locked	RAD	Radar
ED	EICAS Display	RCPT	Right Hand Copilot
EFIS	Electronic Flight Instrument System	REC	Recorded
EGPWS	Enhanced Group Proximity Warning System	REF	Reference
EL	Elevation	RET	Retract
ELEV	Elevator	RG	Range
EN	Enabled	RH	Right Hand
EXT	Extend	RPLT	Right Hand Pilot
FADEC	Full Authority Digital Electronic Control	RUD	Rudder
FLT	Flight	RWY	Runway
FMS	Flight Management System	SB	Standby
FOR	Force	SEL	Select
FREQ	Frequency	SELC	Selective Calling
GA	Go Around	SET	Setting
GD	Ground	SHKER	Shaker
GEN	Generator	SPD	Speed
GPWS	Ground Proximity Warning System	SPLR	Spoiler
GS	Glide Slope	SRCE	Source
GS	Ground Speed	STBY	Standby
HDG	Heading	STCK	Stick
HF	High Frequency	TA	Traffic Advisory
HF	Human Factors	TCAS	Traffic allert Collision Avoidance System
HSI	Horizontal Situation Indicator	TCH	Touched
HUD	Head-Up Display	TCS	Tower Communication System
HWHEEL	Hand Wheel	TERR	Terrain
HWL	Hand Wheel	TGT	Target
HYD	Hydraulic	TQ	Torque
IAS	Indicated Airspeed	TRU	Transform Rectify Unit
ILS	Instrument Landing System	UPLOCKED	Up-Locked
INBD	Inbound	VERT	Vertical
INCR	Increase	VHF	Very High Frequency
ITT	Interstage Turbine Temperatures	VNAV	Vertical Navigation
L	Left	VOR	VHF Omnidirectional Range
LAT	Lateral	VRT	Vertical
LAT	Latitude	VS	Vertical Speed
LCPT	Left Hand Copilot	WCP	Warning Caution Panel
LG	Landing Gear	WOW	Weight on Wheels
LH	Left Hand	WRN	Warning
LNAV	Lateral Navigation	WX/TERR	Weather/Terrain
LOC	Localiser	YD	Yaw Damper

2 manager. Once the above requirements have formally been acquired the measurements need to be defined in terms of:

- (What?) parameter and measurement (e.g. average, maximum, minimum)
- (When?) flight phase and interval (e.g. every minute/nautical mile/etc)
- (Which?) flight(s)

From the 273 parameters recorded by the Q400 universal SSFDRs, 45 were selected as potential indicators for failures. If the study shows significant results, a larger set of indicators can be retrieved. The parameters were selected in collaboration with Qantas Reliability engineers and Bombardier employees and are listed below:

Table A.2.4: Parameters extracted from FDR during nine different flight phases.

1	Flight Record	**22**	Roll_rate (deg/sec)	**43**	Pitch_cmd_FO_force (lbs+=Nose up)
2	Download Record Creation Date (Local)	**23**	Drift (deg)	**44**	Roll_cmd_force (lbs+=RWD)
3	Flight Date (Exact) (UTC)	**24**	Yaw_rate (deg/sec)	**45**	Roll_cmd_FO_force (lbs+=RWD)
4	Flight Date (Month/Year) (UTC)	**25**	Elevator_Lin (deg+=TEU)	**46**	Rudder_cmd_force (lbs+=Nose Right)
5	Flight Number	**26**	Elevator_Rin (deg+=TEU)	**47**	Brake_press_lhs (psi)
6	Tail Number	**27**	Aileron_Rin (deg+=TEU)	**48**	Brake_press_rhs (psi)
7	City Pair	**28**	Rudder_low (deg+=TER)	**49**	NormalForce_lhs (lbs)
8	Takeoff Airport Code	**29**	Density_ambient (kg/m^3)	**50**	NormalForce_rhs (lbs)
9	Takeoff Runway ID	**30**	Density_total (kg/m^3)	**51**	NormalForce_nose (lbs)
10	Landing Airport Code	**31**	Press_ambient (hPa (mbar))	**52**	Start of TakeOff (Seconds (From Start of File))
11	Landing Runway ID	**32**	Pressure_dynamic (hPa (mbar))	**53**	Duration TakeOff (Seconds)
12	Accn_lat (g's)	**33**	Pressure_total (hPa (mbar))	**54**	Duration LiftOff→CleanWing (Seconds)
13	Accn_long (g's)	**34**	Tout (deg C)	**55**	Duration CleanWing → TransitionCruise (Seconds)
14	Accn_norm (g's)	**35**	Ttot (deg C)	**56**	Duration Cruise (Seconds)
15	Vtrue (knots)	**36**	Crosswind (knots)	**57**	Duration Cruise → CleanWing (Seconds)
16	Vcal (knots)	**37**	Headwind (knots)	**58**	Duration GearExt→ Touchdown (Seconds)
17	Alt_pres (ft)	**38**	Torque_lhs (%)	**59**	Duration CleanWing → Touchdown (Seconds)
18	Aoa (deg)	**39**	Torque_rhs (%)	**60**	Duration StartGoAround → Touchdown (Seconds)
19	Pitch (deg)	**40**	Prop_spd_lhs (%)	**61**	Duration Touchdown → end rollout +10 (Seconds)
20	Pitch_rate (deg/sec)	**41**	Prop_spd_rhs (%)		
21	Roll (deg)	**42**	Pitch_cmd_force (lbs+=Nose up)		

Each flight phase is subject to component failures, hence data will be retrieved during nine intervals: take-off, lift-off to clean wing, clean wing to cruise, cruising, cruise to clean wing, gear extension to touchdown, clean wing to touchdown, start go-around to touchdown and touchdown to roll out (+10 seconds). FD is recorded and stored for a maximum of three years before removal.

A.2.3 Data Acquisition

Unlike data from TRAX, FDR data comes from on-board sensors and systems on the aircraft. The data is not subject to human failure, but instead to noise, technical failures, and bugs. Aircraft containing a FAST box will have the data retrieved after every sector. Aircraft without the box will have their data retrieved at the end of each day.

FDR data is stored in a secure environment and is inaccessible as a whole. A system allows users to create profiles used to extract certain measurements and indicators. Once the profile is executed, the system outputs a comma-separated value (*.csv) file with the results.

A.2.4 Quality and Usefulness of Data

FDR data comes from a variety of systems and sensors on-board the aircraft which are regularly tested, calibrated, and validated. Hence the quality of FDR data can be considered quite high. Its usefulness will depend on whether relationships can to an acceptable significance level be established between flight conditions and defects.

A.3 Engine Health Monitoring (EHM)

The following section introduces the reader to EHM, its output, components, and validity.

A.3.1 Description

EHM is a method of assessing the health of an engine. The method monitors five different engine related components (see Sec. A.3 *Date Extracted*). Once each component has been assessed the engine is given a grade defining how often it will be inspected and in case it has to be removed, the maximum amount of Flight Hours (FH) before removal has to take place.

A.3.2 Data Extracted

Engine Health Monitoring (EHM) consists of five different components:

1. Engine Condition Trend Monitoring (ECTM)

   ECTM process consists of monitoring/recording torque, propeller Revolutions Per Minute (RPM), NH, NL, fuel flow, temperature, altitude, and airspeed on the first flight of the day after 15 minutes of stable flight. On the Q200 and 300, the data is recorded manually on a QL-2 form by the first officer. Q400 record the data automatically using Engine Gateway Monitoring (EGM). On a weekly basis the data is analysed and trends are monitored. ECTM data is collected and saved on servers located in Montreal.

2. Oil consumption

   On a daily basis, the amount of oil remaining in the engine is recorded and in-putted into TRAX. This data can then be used to determine whether the engine is using unusual amounts of oil. This process is done manually by maintenance.

3. Start temperature

   The start temperature of an engine is used to determine its condition. Hence on a daily basis the start temperatures are recorded and this data is then analysed on a weekly basis.

4. Baroscope imaging (BSI)

   Baroscoping is done during every structural inspection and consists of internal imaging using a baroscope that is sent to QantasLink for inspection. During image inspections: tears, corrosion, and air pockets are identified and assessed on severity. If the severity is unknown, the images are forwarded to Bombardier.

5. Patch samples

   After every structural inspection, the amount and size of debris is determined by inspecting patch samples from the chip detectors (two magnetic poles). Depending on the size of the debris an alert is raised whether the engines should be inspected/removed or not.

The above five components are used to rate each engine and finally compute an engine health rating as shown in Table A.3.5. The colour together with Flight Hours (FH) indicate the number of FH the engine is allowed to operate until the next inspection. If rated a 60 or below, the engine must be removed within the indicated FH.

Table A.3.5: EHM results in tabular form.

					Color Coding
61	**XX0001**	**100 FH**	**HS ”dd” ”mmm” ”yyyy”**	**Removed regoNo. - ”dd” ”mmm” ”yyyy”**	**600 FH**
62	**XX0002**				**400 FH**
63	**XX0003**				**300 FH**
64	**XX0004**			**Removed regoNo. - ”dd” ”mmm” ”yyyy”**	**100 FH**
1	**YY0001**	**300 FH**	**CL ”dd” ”mmm” ”yyyy”**	**Fitted regoNo. - ”dd” ”mmm” ”yyyy”**	**60 FH**
2	**YY0002**	**600 FH**	**CL ”dd” ”mmm” ”yyyy”**	**Fitted regoNo. - ”dd” ”mmm” ”yyyy”**	**10 FH**
3	**YY0003**				**0 FH**
4	**YY0004**			**Removed regoNo. - ”dd” ”mmm” ”yyyy”**	
5	**YY0005**	**600 FH**	**HS ”dd” ”mmm” ”yyyy”, CL ”dd” ”mmm” ”yyyy”**	**Fitted regoNo. - ”dd” ”mmm” ”yyyy”**	

A.3.3 Data Acquisition

Although ECTM, oil consumption, and start temperatures could all be analysed using statistical methods, currently all components are analysed visually and subjectively by Qantas engineers. Normally this would lead to quite some uncertainty when establishing the EHR of an engine. However, since EHR identifies 7 easily distinguishable levels of quality, the final ratings are accurate.

A.3.4 Quality and Usefulness of Data

Similar to TRAX data, EHM results are subject to human error. The final result is a simple scaling system determining the status of the engine from perfect to terrible condition. Despite the fact that the final EHM result is subjective, there could be a potential relationship between Engine Health Ratings (EHRs) and engine failures.

A.4 OEM Technical Information Services

The following section introduces the reader to Original Equipment Manufacturer (OEM) Technical Information Services.

A.4.1 Description

QantasLink's fleet consists of 50 turo-prop Dash-8 Series aircraft. Although the manufacturer is Bombardier, the ac consists of many parts and components from various vendors and providers. Bombardier being the manufacturer, however, means that they will provide the design, parts, assemblies, and technical services to their customers. Bombardier provides many services, one of which are quarterly and semi-annual Failure Reporting, Analysis, and Corrective Action System (FRACAS) reports (Bombardier 2014*b*, Bombardier 2014*a*). The FRACAS reports provide information regarding aircraft statistics, utilization, system & component statistics, and subsystem/component Dispatch Reliability Rates (DIR) & Cancellation Rates.

During a side meeting with Bombardier and Horizon officials at the 2015 Oceania Regional Review (Bombardier 2015*b*), it was established that Bombardier would provide to me all Horizon's technical data with direct links to the data sources used to compile the FRACAS reports in support of the present thesis. Hence, the following section will primarily focus on the data extracted from FRACAS reports published by Bombardier. The reports contain data from analysis and tests done by Bombardier and other manufacturers, technical data from other Dash 8 operators. For a more thorough overview of FRACAS Data see Sec. A.4 *Data Extracted*.

A.4.2 Data Extracted

Figure A.4.4 gives a visual overview of the data presented in FRACAS reports published by Bombardier. The data used to provide the analysis is collected from various Bombardier Dash 8 operators, vendors, and Bombardier.

The system and component data is an amendment to the FRACAS report providing an overview of all significant components and their corresponding number of (un)scheduled removals, Unscheduled Removal Rates (URR), Total Removal Rates (TRR), Mean Time Between Unscheduled Removals (MTBUR), and Mean Time Between Removals (MTBR).

A.4.3 Data Acquisition

OEM FRACAS data is a summation of the operators and vendors statistics on a 12-month basis. Since some operators may consider some items disposable/dispensable or not significant enough to monitor, not all components are covered. However, since this study primarily focusses on identifying/predicting high-value component failures, the components under question are rarely not reported on. Statistical data can also be acquired from vendors and other manufacturers if the operator's data is missing.

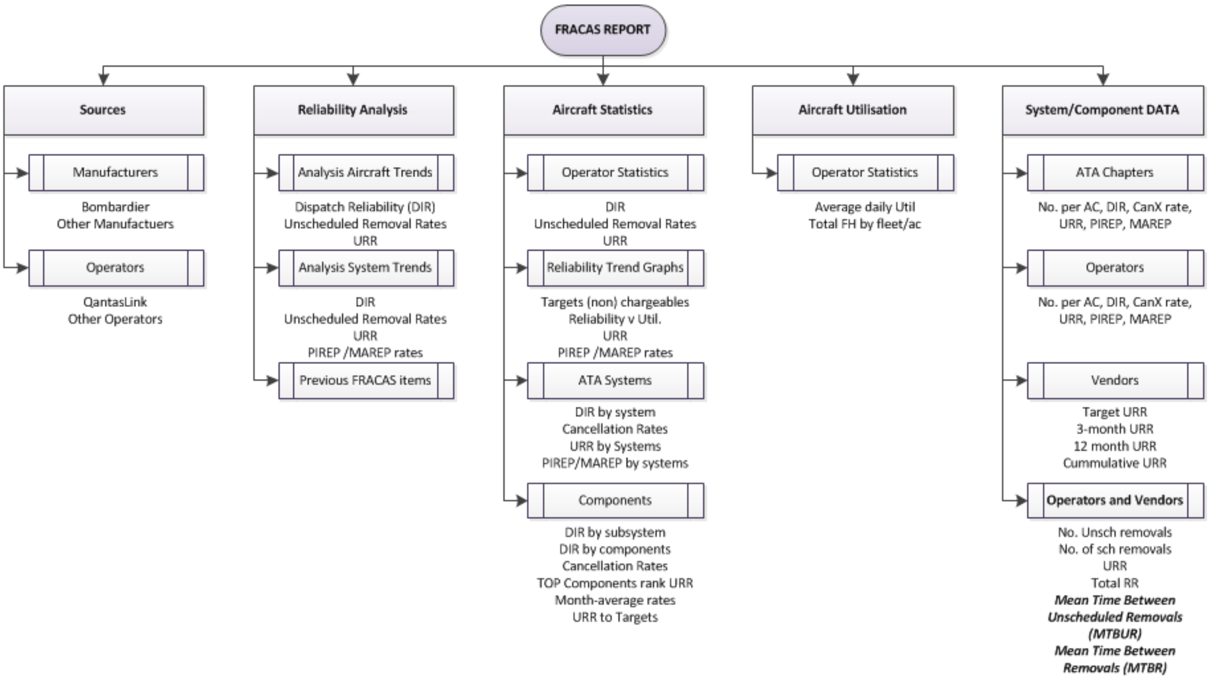


Figure A.4.4: Summary of data presented in Fracas report (Bombardier 2014*a*, Bombardier 2014*b*).

A.4.4 Quality and Usefulness of Data

Data collected from FRACAS reports is rated high in usefulness. It can be used to assess QantasLink's performance in comparison to that of other operators. In addition, it can help identify flaws/issues with the current maintenance programs and with AC operations.

A.5 AC Delay Reports

The following section introduces the reader to AC delay reports made by QantasLink's maintenance watch.

A.5.1 Description

AC delay reports track the delays that occur during the day. For each delay/defect, it is logged when the issue was raised (in terms of time, flight, aircraft, and station), why the delay occurred (e.g. component or system failure, warning indicators, failed check-ups, etc.), and when & how the event was resolved (e.g. part was replaced with component B). The log/report is updated as soon as an incident is reported by the maintenance watch. "Maintenance watch" refers to a department in charge of being available for maintenance related issues 24/7.

A.5.2 Data Extracted

Table A.5.6 shows five example entries listed in the maintenance watch event log. The layout of the data has been adjusted manually in order to fit the contents to one A4 sized sheet. The logs' most valuable entries are the event descriptions, which help identify whether the event was unscheduled (US), whether the aircraft remained unserviceable (U/S), and how long it took for the event to be resolved (Duration).

Table A.5.6: Example of data recorded in maintenance watch event log.

ITEM	DATE	Month/Year	Flight No: QFAxxxx	Aircraft (VH-XXX)	Series (200/300/400)	Station	Time Informed (HH:MM)	Event Defect Description	Defered Defect
1	42018	1/01/2015	2010	VH-QOK	400	SYD	14:28	AWAIT COMPANY STOCK (SICK BAGS, CLEANING PRODUCTS ETC) ENGINEERING TRANSFER ITEM TO TMW CLEANERS.	NO
2	42018	1/01/2015	2490	VH-QOS	400	CNS	15:00	SBW HP BLEED VLV US. AIRCRAFT SWAP TO QOS	NO
3	42018	1/01/2015	1420	VH-LQB	400	CBR	15:23	CREW REPORT HIGHT FUEL TEMP WITHIN SL LIMITS	YES
4	42018	1/01/2015	1491	VH-QON	400	SYD	15:43	CREW REPORT AFT SERVICE DOOR JAMMED WHEN CONDUCTING EP's. ENTERED IN QL3	
5	42018	1/01/2015	2030	VH-TQK	300	SYD	15:30	AIRCRAFT CHANGE DUE QOE PITOT HEAT DEFECT	NO

ITEM	Primary QL3 Number	Part Number	Part Description	Cancelation Qty (1,2,3)	Duration (HH:MM)	Delay Code for Event	Cause of Delay	Comments	MEL/NAD/DWL Reference
1	NIL QL3	NIL PART	NIL PART		0:06	450	MAINT		NIL ACTION REQD.
2	2029567	G728888	VALVE		2:05	440	MAINT	PART ROBBED FROM AOG TQE TO SERVICE SBW. DELAY RECODE 460 TO 440	DEFECT CLEARED
3	2034401	G728888	VALVE		0:00				DWL
4					0:00				
5	2023820	4100S018-07	TMU		0:14	460	MAINT		DEFECT CLEARED

ITEM	Engineer sent to station	Part available at station	Part shipped to station	Engineer at station	CH	SEC	Date Resolved / Defered	Event Resolution or Rectification	Time Resolved / Defered
1	N	N	Y	Y	2	00	14/01/2015	MOCO RANG MW CHASING STORES. STOREMAN JUST LEAVING HANGAR. FLIGHT LOCKED UP. STOREMEN ARRIVED LATE AND LOADED ITEMS	14:34
2	N	N	Y	Y	2	00	14/01/2015	SBW HP BLEED VLV US. AIRCRAFT SWAP TO QOS	17:05
3		Y		Y	73	20	14/01/2015	DWL IAW PWC -SL-PW150-034	15:30
4		Y		Y	52	40	14/01/2015	ENGINEERS ADVISE U/S - TAKEN TO HANGAR...REMAINS U/S OVERNIGHT	23:59
5		Y		Y	2	00	14/01/2015	AIRCRAFT CHANGE DUE QOE PITOT HEAT DEFECT	15:44

A.5.3 Data Acquisition

Data retrieved in the AC delay log is acquired by the Maintenance Watch staff, whom will log an event and continuously update its status as long as the event remains unresolved. Although subject to human error, data listed in the delay log are cross-checked by at least two staff members.

A.5.4 Quality and Usefulness of Data

As mentioned in Sec. A.5 *Data Acquisition*, the AC Delay Log is a LIVE document, meaning it is updated on a continuous basis. The event times reports (informed, resolved, duration) are more reliable then the times listed in TRAX, which are updated on an irregular basis. In addition, events in the log are cross-checked by at least two staff members and are reviewed twice (once raised, once resolved). The duration of an event can be used to establish the reliability of a particular part, by assessing how quick an item can be restored to a state of functioning.

Appendix B

Component results

In the following section a full overview of the results obtained by the software are provided for each component in the top 10 components, based on URRs, identified by QantasLink (Sec. 3). The results are published using an automated script, hence are shown in a systematic manner. Results, on a component level, are further discussed in Sec. 3.

B.1 697071003 Blade assembly and bearing

The analysis and modelling results related to component PN **697071003** are thoroughly described in Sec. 3.1.1.

B.2 174260-08 Crew oxygen mask

Table B.2.1 provides a summary of the input data related to the component. The number of registered maintenance events is less than the total number of events due to the fact that TRAX data stretches back to 2004/2005 and FDR data only to 2011. Maintenance events with insufficient data, regarding operational factors, cannot be evaluated, hence are not registered during the modelling process.

Table B.2.1: General overview of component inputs.

Name	**Value**
Part Number	174260-08
Total # (A, F, C)	**373, 226, 147**
Registered # (A, F, C)	**62, 37, 25**
Related Flights # (A, F, C)	**120599, 65027, 55572**
Avg. Cycles (A, F, C)	**1945.15, 1757.49, 2222.88**
% Censored	**40.32**

In Tab. B.2.1 (A, F, C) denotes statistics regarding All (A), Failed (F), and Censored (C) events respectively. Ergo A will always be the sum or mean derived from F and C.

Analysis
Tables B.2.2 and B.2.3 summarise the results from EVA and MDA. In addition the variables obtained by semi-parametric PHM modelling (labelled 'reduced semi-COX') are also presented if applicable. Table B.2.3 provides an overview of the specific operational factors identified during all flight phases. In this case high counts indicate operational factors that were significantly different during multiple flight phases.

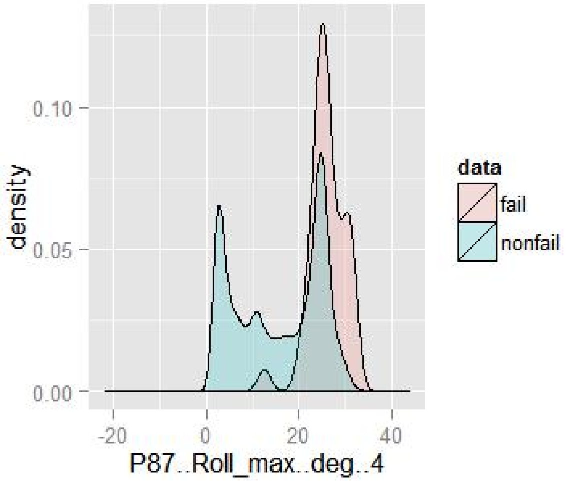
Table B.2.2: Overview of analysis input and output.

	# Variables
ALL	**1531**
EVA	**24**
MDA	**22**
Combined	**46**
reduced Corr.	**41**
reduced semi-COX	**32**
Take-Off related	**18**
Cruise related	**10**
Touch-Down related	**13**

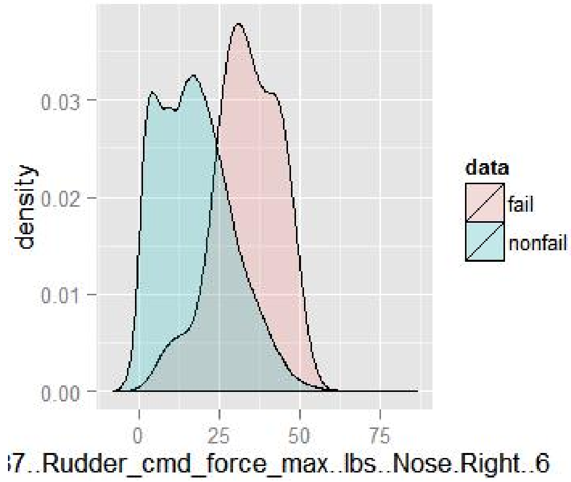
A multitude of factors were identified during EVA and MDA. Figure B.2.1 give a general overview of the top operational factors identified by EVA and MDA.

Table B.2.3: Overview of identified operational factors during all flight phases.

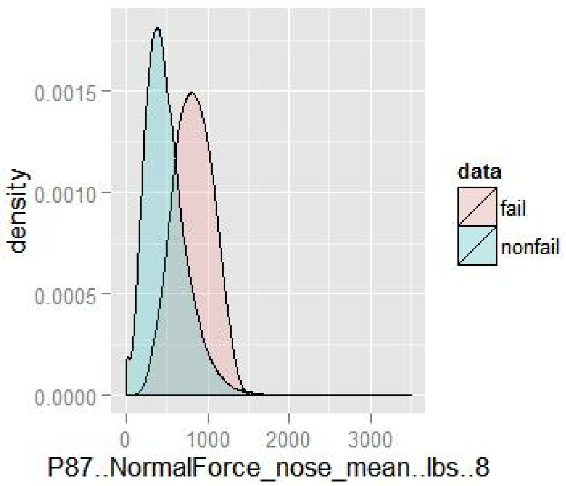
Variable	Count	Variable	Count	Variable	Count
Aoa	4	Roll	2	Vcal	1
Rudder_cmd_force	3	Pressure_dynamic	2	Brake_press_rhs	1
Accn_long	3	Torque_rhs	2	Aileron_Rin	1
Roll_rate	3	Rudder_low	2	Crosswind	1
Torque_lhs	2	NormalForce_nose	1	Ttot	1
Pitch	2	NormalForce_rhs	1	Roll_cmd_force	1
Yaw_rate	2	NormalForce_lhs	1	Vtrue	1
Pitch_rate	2	Prop_spd_rhs	1	Drift	1



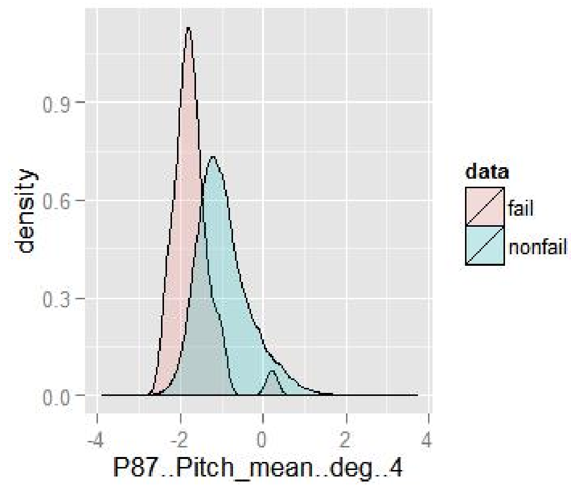
(a) EVA 1.



(b) EVA 2.



(c) MDA 1.



(d) MDA 2.

Figure B.2.1: Graphical overview of top operational factors identified by EVA and MDA.

Time-based reliability modelling
Table B.2.4 reports the maximum likelihood and goodness-of-fit tests results obtained from time-based reliability modelling. To show the overall fit Fig. B.2.2 shows the computed reliability function using an averaged virtual age V for all fitted models.

In addition Figures B.2.3, B.2.4, B.2.5, B.2.6, and B.2.7 present the reliability and hazard functions computed for each underlying distribution evaluated in the program.

Time independent proportional hazard reliability modelling

Table B.2.4: Overview of results obtained by MLE optimisation and GOF tests.

	Distributions				
	norm	**lnorm**	**logis**	**exp**	**weibull**
MLE	**−336.44**	**−336.76**	**−336.47**	**−336.3**	**−340.92**
Kolmogorov-Smirnov	**1.22**	**1.23**	**1.12**	**0.87**	**1.79**
Cramer-von-Mises Smirnov	**24.32**	**23.69**	**24.26**	**24.12**	**22.6**
Anderson-Darling	**−74.08**	**−73.88**	**−73.96**	**−73.7**	**−76.59**
NRR	**34.94**	**26.99**	**34.88**	**25.25**	**70.99**

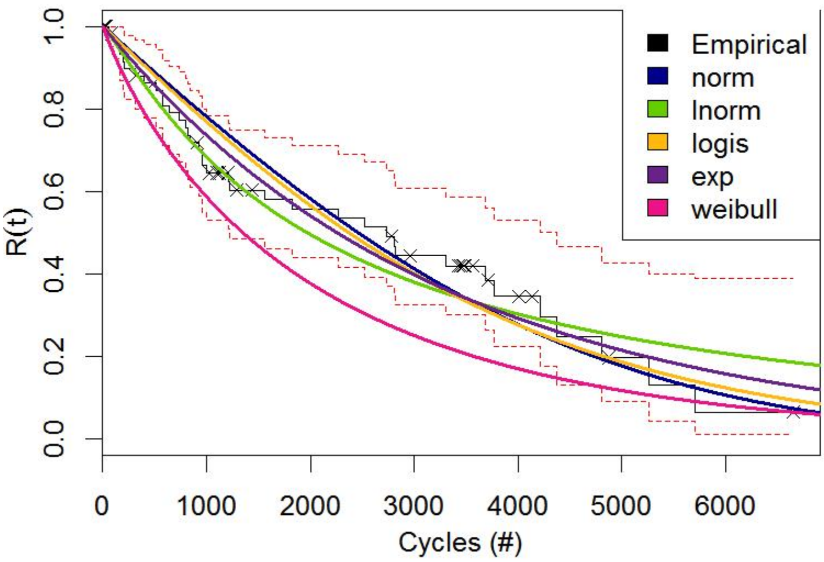
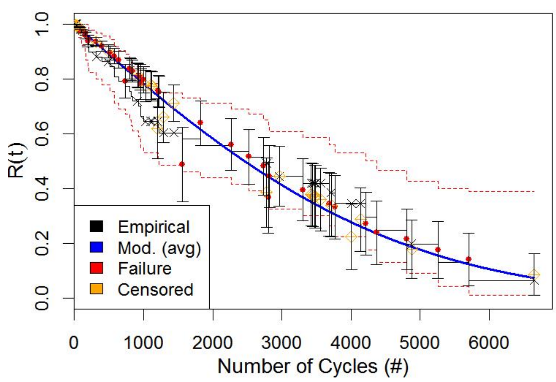
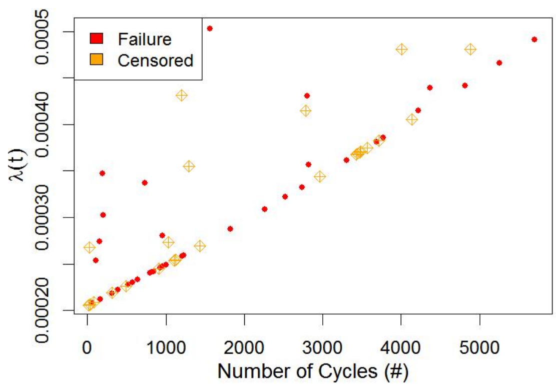


Figure B.2.2: Overview of overall fit of multiple GRP modells.



(a) Reliability function.

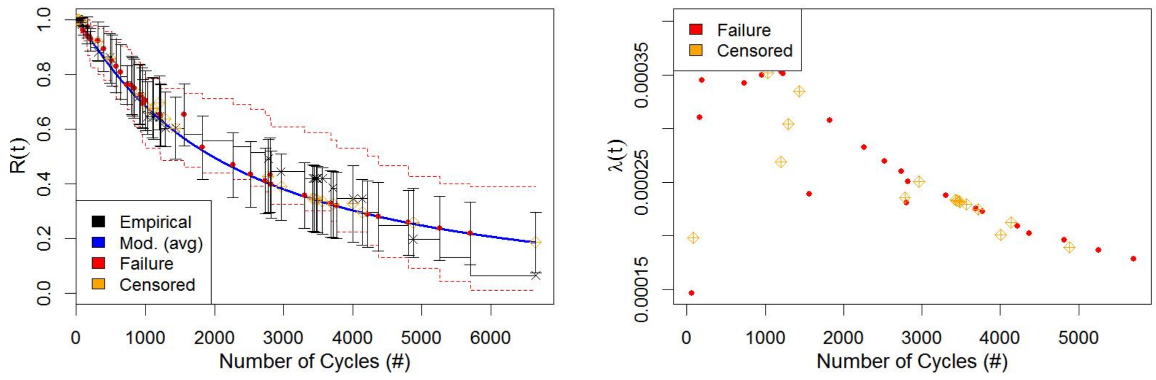


(b) Hazard function.

Figure B.2.3: Computed reliability for time-based models with underlying norm distribution.

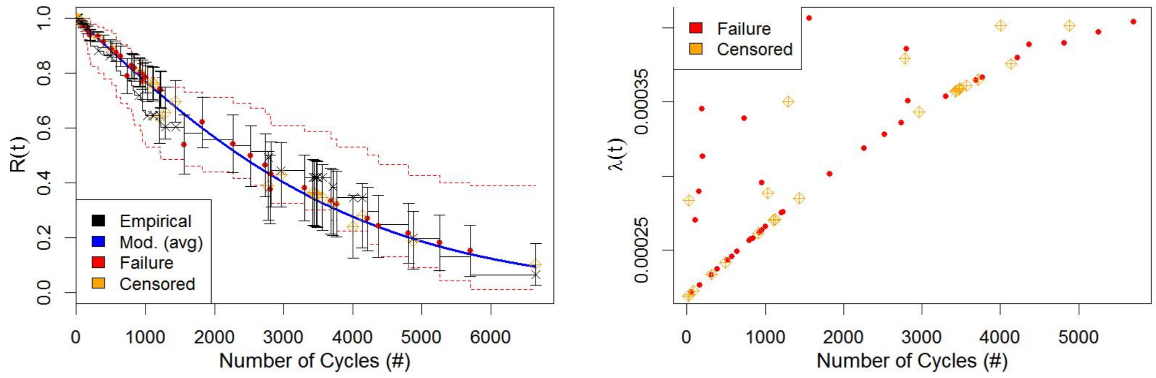
Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.2.5 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.2.8, B.2.9, and B.2.10 as well as a general overview in Figure B.2.10b.
Tables B.2.6 and B.2.7 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.



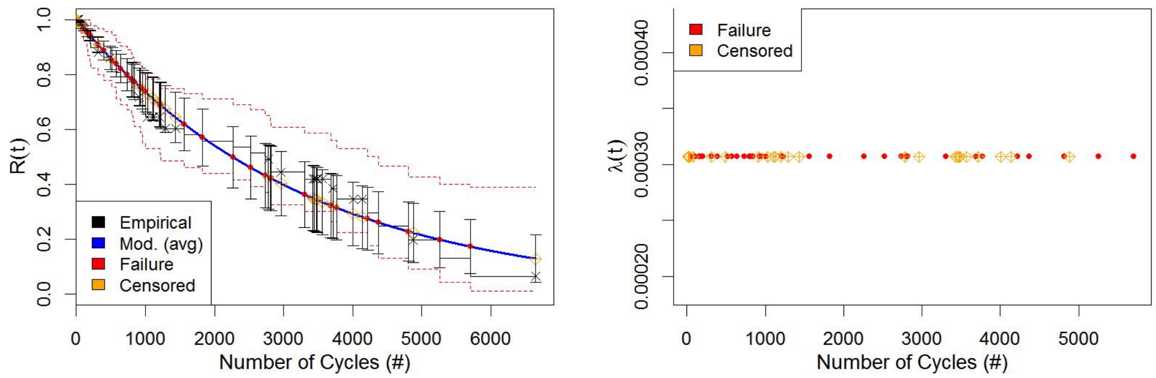
(a) Reliability function. (b) Hazard function.

Figure B.2.4: Computed reliability for time-based models with underlying lnorm distribution.



(a) Reliability function. (b) Hazard function.

Figure B.2.5: Computed reliability for time-based models with underlying logis distribution.



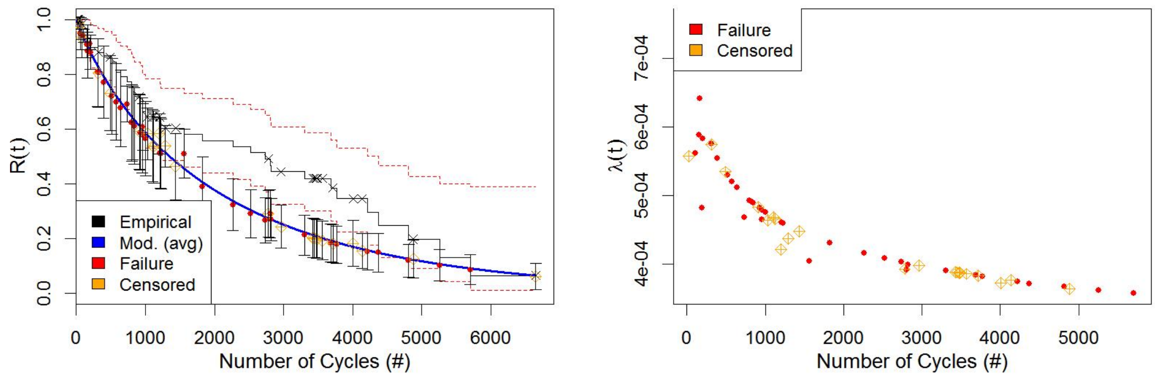
(a) Reliability function. (b) Hazard function.

Figure B.2.6: Computed reliability for time-based models with underlying exp distribution.

To assist in the selection of models, Tables B.2.8, B.2.9, B.2.10, and B.2.11 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.

Time dependent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.2.12 gives a general overview of all the models obtained during each step in the process.



(a) Reliability function.

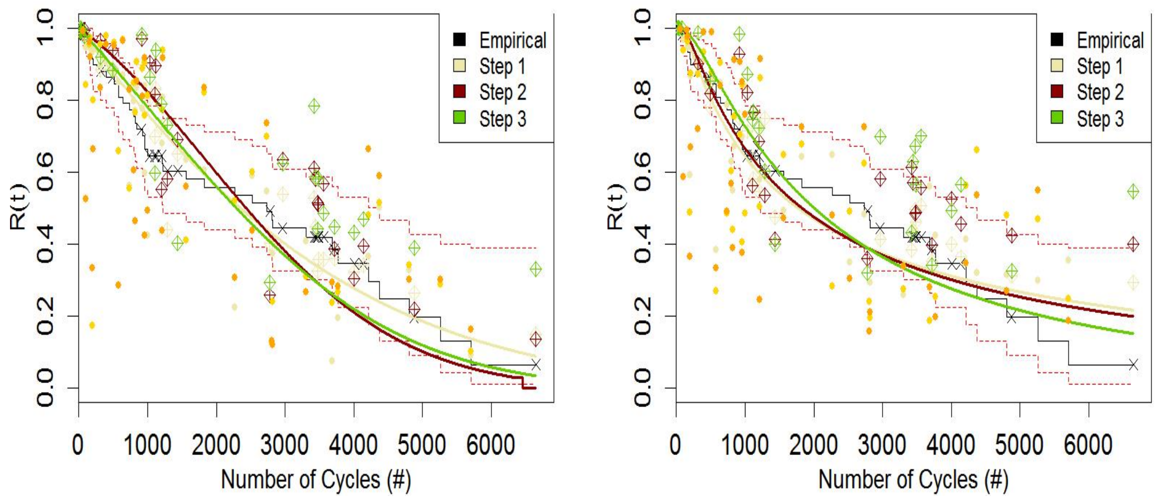
(b) Hazard function.

Figure B.2.7: Computed reliability for time-based models with underlying weibull distribution.

Table B.2.5: Overview of time-independent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	norm	norm	lnorm	lnorm	lnorm	logis	logis
Step #	**1**	**2**	**3**	**1**	**2**	**3**	**1**	**2**
MLE	**−332.96**	**−327.32**	**−321.31**	**−332.71**	**−327.93**	**−323.84**	**−329.32**	**−324.13**
Time (min)	**0.66**	**1.48**	**2.5**	**0.59**	**1.26**	**1.97**	**0.68**	**1.44**
Kolmogorov-Smirnov	**2.48**	**5.84**	**4.59**	**3.37**	**4.27**	**4.33**	**4.7**	**5.62**
Cramer-von Mises-Smirnov	**22.96**	**19.53**	**14.31**	**23.37**	**21.76**	**14.37**	**20.88**	**14.72**
Anderson-Darling	**−76.54**	**−80.32**	**−83.14**	**−77.21**	**−82**	**−83.44**	**−77.81**	**−84.68**
NRR	**28.64**	**27.43**	**24.81**	**18.66**	**26.45**	**19.62**	**23.03**	**17.39**

Distribution	exp	exp	weibull	weibull	weibull
Step #	**1**	**2**	**1**	**2**	**3**
MLE	**−329.72**	**−324.94**	**−329.36**	**−326.9**	**−323.4**
Time (min)	**0.55**	**1.04**	**0.78**	**1.44**	**2.53**
Kolmogorov-Smirnov	**4.87**	**5.52**	**5.23**	**3.68**	**3.63**
Cramer-von Mises-Smirnov	**20.75**	**16.34**	**20.85**	**21.42**	**14.8**
Anderson-Darling	**−78.22**	**−81.89**	**−78.34**	**−76.35**	**−79.47**
NRR	**17.1**	**22.03**	**20.75**	**29.48**	**28.71**



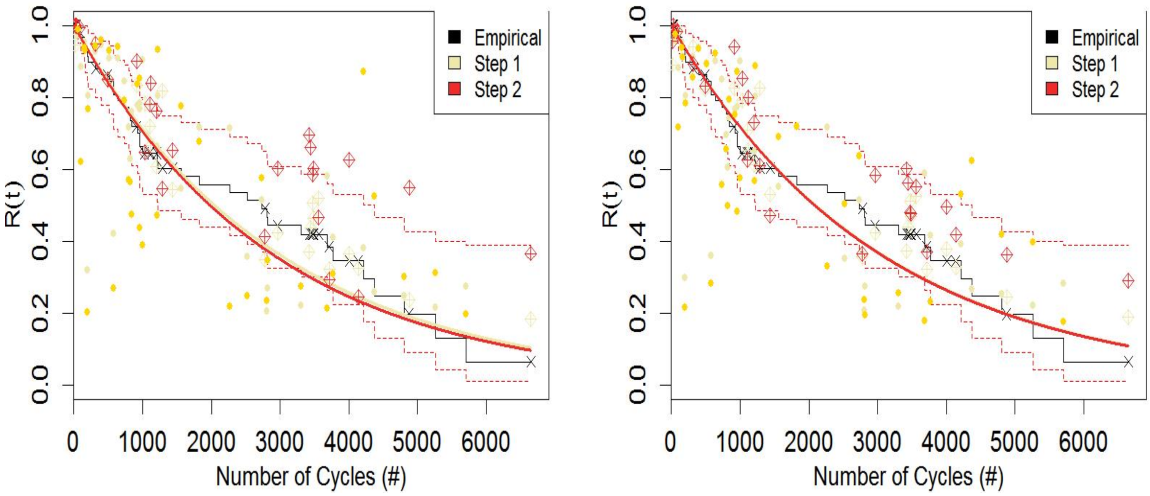
(a) With an underlying norm distribution.

(b) With an underlying lnorm distribution.

Figure B.2.8: Time-independent PHMs with an underlying norm and lnorm distribution.

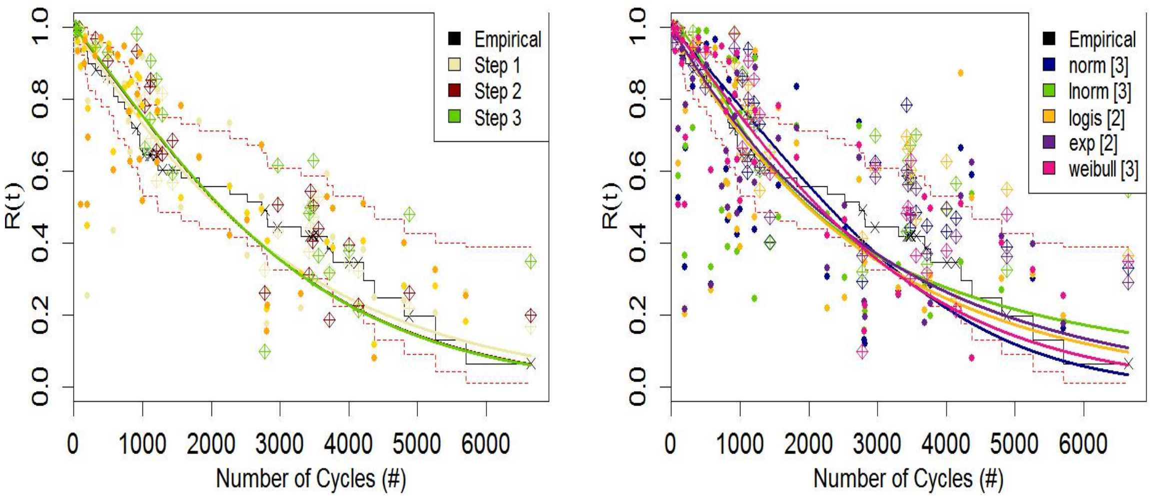
Graphical representation of the computed reliability per model are shown in Figures B.2.11, and B.2.12 as well as a general overview in Figure B.2.13a.

Tables B.2.13 and B.2.14 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the



(a) With an underlying logis distribution. (b) With an underlying exp distribution.

Figure B.2.9: Time-independent PHMs with an underlying logis and exp distribution.



(a) With an underlying weibull distribution. (b) All distributions (last step).

Figure B.2.10: Figures containing a weibull distribution and all time-independent PHMs.

Table B.2.6: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	3 [100]	5 [96]	11 [96]	16 [80]	30 [60]	32 [56]	43 [48]	51 [40]	62 [32]	84 [16]	100 [8]
norm	2	0 [100]	3 [96]	8 [92]	19 [84]	22 [76]	32 [68]	41 [44]	49 [44]	59 [40]	76 [28]	100 [8]
	3	0 [100]	0 [100]	11 [100]	27 [92]	32 [80]	41 [60]	49 [48]	59 [44]	70 [36]	89 [28]	100 [8]
	1	0 [100]	0 [100]	0 [100]	19 [88]	27 [68]	38 [52]	46 [40]	62 [32]	76 [20]	86 [20]	100 [8]
lnorm	2	0 [100]	3 [100]	5 [100]	22 [100]	27 [76]	41 [68]	51 [40]	62 [36]	68 [24]	86 [24]	100 [8]
	3	0 [100]	5 [100]	11 [100]	24 [92]	30 [80]	43 [80]	54 [56]	65 [44]	70 [32]	81 [24]	100 [8]
logis	1	0 [100]	0 [100]	11 [92]	19 [80]	27 [68]	35 [52]	41 [48]	51 [40]	70 [28]	84 [20]	100 [8]
	2	0 [100]	0 [100]	8 [96]	22 [92]	35 [84]	46 [76]	51 [56]	59 [44]	70 [32]	78 [20]	100 [8]
exp	1	0 [100]	0 [100]	5 [92]	19 [80]	32 [68]	35 [52]	43 [52]	51 [36]	68 [28]	86 [20]	100 [8]
	2	0 [100]	3 [100]	11 [100]	19 [92]	30 [76]	38 [68]	49 [40]	65 [40]	78 [32]	89 [24]	100 [8]
	1	0 [100]	0 [100]	11 [92]	19 [76]	27 [68]	32 [56]	41 [48]	49 [40]	65 [28]	84 [20]	100 [8]
weibull	2	0 [100]	3 [100]	5 [88]	14 [76]	27 [68]	35 [56]	38 [52]	43 [44]	68 [36]	78 [24]	100 [8]
	3	0 [100]	3 [96]	3 [92]	11 [84]	19 [72]	35 [60]	41 [56]	54 [44]	68 [36]	84 [28]	100 [8]

model's effectiveness can be assessed.

To assist in the selection of models, Tables B.2.15, B.2.16, B.2.17, and B.2.18 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various

Table B.2.7: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	3 [100]	8 [96]	14 [96]	19 [80]	30 [56]	32 [48]	49 [44]	51 [36]	62 [32]	84 [16]	100 [8]
norm	2	5 [100]	14 [96]	19 [88]	27 [76]	35 [72]	41 [56]	54 [36]	62 [36]	73 [36]	92 [24]	100 [8]
	3	22 [92]	35 [92]	41 [92]	51 [80]	57 [60]	62 [52]	73 [40]	84 [36]	95 [28]	97 [24]	100 [8]
	1	0 [100]	0 [100]	3 [100]	22 [80]	38 [60]	46 [40]	57 [36]	73 [20]	84 [20]	95 [16]	100 [8]
lnorm	2	5 [100]	8 [100]	14 [100]	24 [96]	41 [64]	46 [52]	59 [40]	73 [32]	76 [24]	95 [16]	100 [8]
	3	22 [92]	35 [92]	46 [88]	57 [76]	65 [76]	70 [60]	76 [52]	81 [28]	84 [28]	97 [24]	100 [8]
logis	1	0 [100]	3 [100]	16 [92]	24 [72]	35 [64]	43 [48]	54 [40]	62 [36]	78 [20]	95 [20]	100 [8]
	2	11 [100]	16 [100]	27 [96]	38 [84]	46 [72]	54 [68]	70 [40]	73 [40]	81 [24]	95 [16]	100 [8]
exp	1	0 [100]	3 [100]	16 [92]	30 [76]	35 [64]	43 [48]	57 [40]	68 [36]	81 [20]	97 [20]	100 [8]
	2	5 [96]	22 [96]	27 [92]	41 [68]	54 [64]	62 [52]	68 [40]	81 [36]	92 [24]	100 [20]	100 [8]
	1	0 [100]	3 [96]	16 [92]	24 [72]	35 [64]	43 [44]	54 [40]	62 [40]	78 [24]	95 [20]	100 [8]
weibull	2	0 [100]	5 [100]	14 [80]	22 [76]	30 [64]	38 [52]	43 [40]	65 [40]	73 [28]	92 [20]	100 [8]
	3	3 [100]	11 [96]	24 [92]	30 [72]	41 [56]	57 [52]	70 [40]	78 [36]	89 [28]	97 [24]	100 [8]

Table B.2.8: Average MTTF values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	1703.9	1589.7	1503.2	1275.4	976.3	840.5	628	525.3	406.4	146	NaN
norm	2	1757.5	1647.9	1553.9	1205.8	1133.4	818	772.8	690.2	658.7	396.8	NaN
	3	1757.5	1757.5	1515.5	1096.4	1067.1	1114.7	935.4	603.9	656	312	NaN
	1	1757.5	1757.5	1757.5	1198.3	942.2	869	713.8	456.6	323.2	174.8	NaN
lnorm	2	1757.5	1703.9	1687.9	1368	1426.7	1086.4	756.7	477.8	464.4	174.8	NaN
	3	1757.5	1600	1412.5	1158.5	979.7	722.2	709.2	624.8	514	330.9	NaN
logis	1	1757.5	1757.5	1447.8	1198.3	942.2	898	741	680.8	458	197.7	NaN
	2	1757.5	1757.5	1569.8	1241.7	1147.4	832.5	833	811.5	792.1	426.9	NaN
exp	1	1757.5	1757.5	1570.2	1198.3	971.6	898	737.5	680.8	436.7	204.8	NaN
	2	1757.5	1703.9	1486.3	1355.8	1041.5	943.9	767.3	627.9	466.4	178	NaN
	1	1757.5	1757.5	1447.8	1198.3	942.2	845.5	741	741.2	552.9	257	NaN
weibull	2	1757.5	1684.9	1570	1314.8	899.2	727.2	660.3	627	489.2	293.1	NaN
	3	1757.5	1684.9	1684.9	1420.4	1198.3	770.3	748.2	620.8	483.5	362.7	NaN

Table B.2.9: Average MTTF values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	1703.9	1569.8	1385.9	1284.4	976.3	840.5	561.4	525.3	406.4	146	NaN
norm	2	1589.7	1480.7	1378.3	1126.4	1131.5	843.9	807.4	702.1	545.6	559	NaN
	3	1758.1	1606.2	1535.4	1218.2	1253.1	1061.6	900.1	894.3	616	392	NaN
	1	1757.5	1757.5	1660.2	1152.6	776.8	713.8	564.6	355	197.7	82	NaN
lnorm	2	1687.9	1708.1	1518.8	1382.7	1101.7	1110.5	550.3	505.9	468.8	82	NaN
	3	1456.5	1558.3	1411.7	1276.7	1134.1	870	789	594.4	676	392	NaN
logis	1	1757.5	1660.2	1250.7	1059.2	858.5	767.1	638.8	562.3	398.2	225.5	NaN
	2	1754	1652.2	1291	1301.9	1130.3	1017.6	792.1	860.8	1030.9	225.5	NaN
exp	1	1757.5	1660.2	1250.7	1096.5	858.5	767.1	668.6	575.8	432.9	392	NaN
	2	1687.9	1630.7	1452.2	1513.5	1241.9	1108.6	700.8	528.9	445.7	NaN	NaN
	1	1757.5	1660.2	1250.7	1059.2	858.5	767.1	638.8	619.1	398.2	225.5	NaN
weibull	2	1757.5	1570	1350.1	1079.9	846.7	660.3	627	528.4	475.4	323	NaN
	3	1684.9	1547.7	1384.6	1323	908.6	790.4	783.2	561.9	651.5	392	NaN

Table B.2.10: Average MTTRep values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	5307.3	5010.3	4128.1	3477.2	2731	2191.5	1621.1	1187.7	772.1	333.6	30
norm	2	5253.4	4877.8	4187.2	3596.9	3226.1	2659.2	1987.2	1579.3	1149.5	615.1	30
	3	5379.2	5358.5	4666.2	3621.9	2802.1	1974.1	1511.5	1170	788.3	411.1	30
	1	5379.2	5379.2	5271.1	3770.1	2598.4	1745.1	1205.8	845.5	551.5	302.1	30
lnorm	2	5379.2	5177.8	5018.7	4283	3051.7	2005.8	1322.8	955.9	667.5	354.8	30
	3	5338.8	5040.9	4848.1	3903.5	3126.6	2467.8	1527.4	1052.6	845	484.5	30
logis	1	5379.2	5273.4	4289.7	3441.1	2699.2	2026.9	1568.8	1125.8	763.6	392.8	30
	2	5333.4	5296.1	4732.9	3969.9	3029.1	2316.3	1593	1132.6	733.1	380.9	30
exp	1	5379.2	5283.5	4424.6	3467.7	2589.8	2054.1	1543.3	1079.3	758.8	338.5	30
	2	5379.2	5184	4804.4	3862.7	3051.3	2176.7	1538.3	1009	659.1	332	30
	1	5379.2	5147.8	4230.5	3385	2753.3	2161.3	1645	1194.5	811.9	413	30
weibull	2	5348.1	5036.5	4277.3	3526.6	2991.7	2375.8	1825.2	1352.2	900.5	480	30
	3	5362.7	4895.9	4661.5	4065.7	3292.9	2247.4	1689.4	1195.9	802	381.3	30

time-independent PHMs and scenarios.

The operational factors identified during time-independent and time-dependent PHM mod-

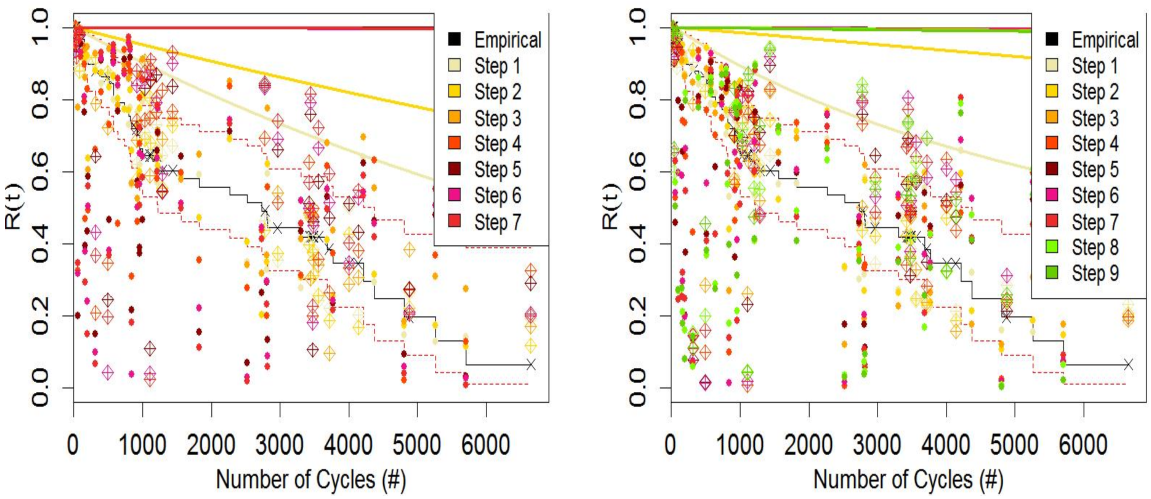
Table B.2.11: Average MTTRep values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
norm	1	5245.3	4805.3	4089.4	3281.5	2640.8	2102.5	1537.1	1112.9	700.1	273.2	30
	2	5106	4343	3688.2	3091.9	2510.7	2029	1526.3	1174.3	806.2	332.2	30
	3	3656.1	3396.2	3036.8	2302.3	1733.4	1233.4	936.1	606.4	402.6	178.5	30
lnorm	1	5379.2	5379.2	5125	3180.5	2203.6	1431.4	956.4	679.8	435.4	217.6	30
	2	5008.5	4826.1	4662.5	3994.4	2131.4	1435.2	1041.7	695.1	417.6	213	30
	3	3925.5	3304.5	2972.5	2360	1957.1	1386.2	984.1	686.3	482.7	250.7	30
logis	1	5379.2	5108	4001.3	3056.6	2369.1	1712.7	1268	854.8	578.1	228.5	30
	2	4623.2	4427.3	3870.6	2961.8	2281.7	1673.1	1037	704.9	397.4	212.8	30
exp	1	5379.2	5106.2	4082.7	2907.9	2373.2	1726.3	1211.8	810.5	508.3	185.7	30
	2	4781.1	4148.5	3559.6	2473.8	1808.8	1454.5	987.3	690.8	327.3	133.6	30
weibull	1	5333.6	4934.2	3931.5	3051.9	2357.6	1787.8	1298.7	906.6	600.7	232.7	30
	2	5323.6	4779.3	3968.2	3152.4	2615.8	2082.7	1510.7	1007.9	655	290.5	30
	3	5268	4469.7	3686.9	2915.7	2177.3	1454.8	982.9	628.6	369	209.9	30

Table B.2.12: Overview of time-dependent PHM results obtained by MLE optimisation and GOF tests.

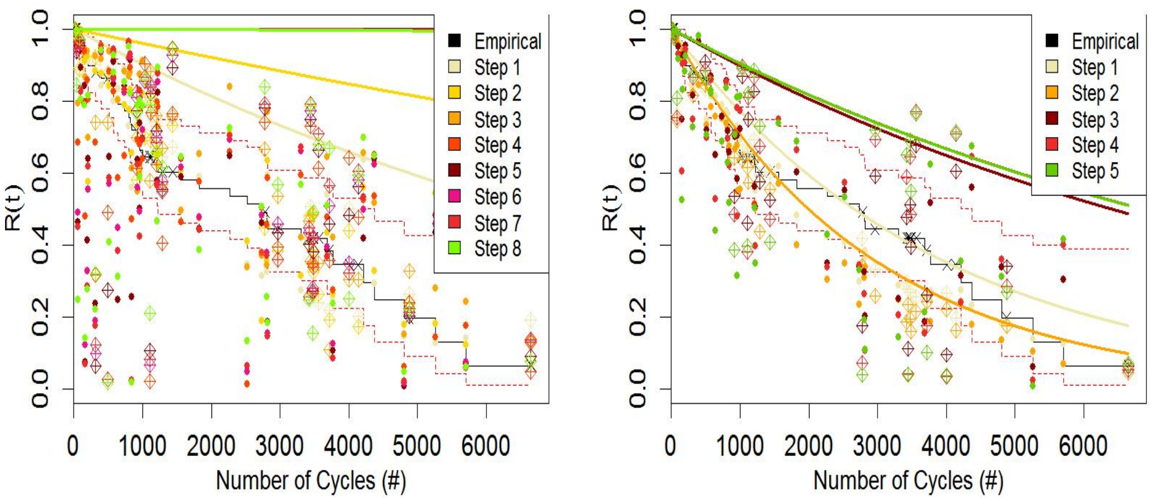
Distribution	norm	norm	norm	norm	norm	norm	norm	lnorm
Step #	1	2	3	4	5	6	7	1
MLE	−300.85	−264.29	−231.96	−188.99	−153.91	−141.12	−138.17	−300.87
Time (min)	20.56	43.1	66.59	97.55	129.11	161.61	176.05	15.48
Kolmogorov-Smirnov	2.71	2.26	3.16	5.19	6.17	6.16	6.41	2.86
Cramer-von Mises-Smirnov	24.29	24.76	24.18	23.81	22.38	21.19	20.63	24.36
Anderson-Darling	−75.26	−74.98	−79.13	−80.79	−84.45	−87.66	−88.88	−75.44
NRR	69.01	4.37	66.14	23.11	8.52	1757.74	463.54	113.15
Distribution	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm
Step #	2	3	4	5	6	7	8	9
MLE	−254.94	−203.63	−171.76	−143.71	−129.37	−121.03	−115.71	−111.42
Time (min)	34.97	64.65	94.8	131.26	169.42	210.82	249.02	279.75
Kolmogorov-Smirnov	2.36	3.55	4.9	5.87	6.34	6.39	6.47	6.5
Cramer-von Mises-Smirnov	23.79	22.9	21.61	17.51	13.03	13.84	13.33	13.62
Anderson-Darling	−76.08	−77.25	−83.61	−87.97	−92.34	−92.63	−93.9	−93.56
NRR	60.87	7.16	1026.2	1864.08	5254.31	217.94	100.99	732.68
Distribution	logis	logis	logis	logis	logis	logis	logis	logis
Step #	1	2	3	4	5	6	7	8
MLE	−300.85	−263.75	−226.7	−179.5	−148.27	−139.64	−138.77	−129.46
Time (min)	5.49	12.27	22.06	37.14	46.81	59.21	68.4	83.99
Kolmogorov-Smirnov	2.7	2.54	2.84	3.95	5.31	5.47	5.54	5.81
Cramer-von Mises-Smirnov	24.29	24.75	24.41	23.09	20.05	18.08	17.64	17.04
Anderson-Darling	−75.26	−75.26	−78.19	−79.62	−85.31	−88.1	−89.26	−89.2
NRR	36.77	15.66	22.5	2413.5	140.66	5449.47	9.56	1596.53
Distribution	exp	exp	exp	exp	exp			
Step #	1	2	3	4	5			
MLE	−310.62	−301.96	−281.49	−265.06	−253.9			
Time (min)	1.71	2.11	2.66	3.26	3.9			
Kolmogorov-Smirnov	1.54	2.05	2.49	3.14	3.39			
Cramer-von Mises-Smirnov	23.91	23.34	22.42	20.7	19.98			
Anderson-Darling	−74.12	−75.63	−76.91	−80.43	−82.33			
NRR	57.76	66.94	56.3	85.54	69.48			

elling are shown in Tables B.2.19, B.2.20, B.2.21, and B.2.22.



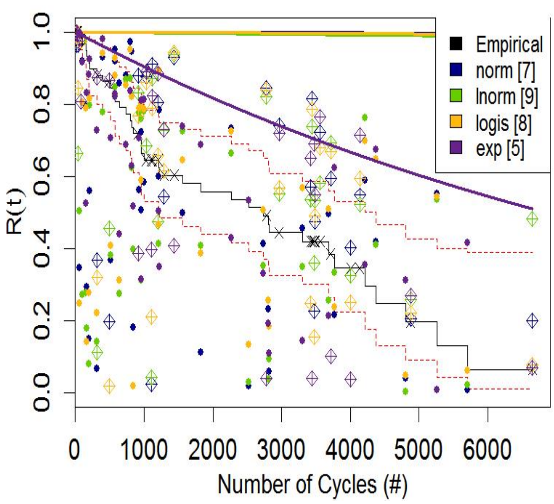
(a) With an underlying norm distribution. (b) With an underlying lnorm distribution.

Figure B.2.11: Time-dependent PHMs with an underlying norm and lnorm distribution.



(a) With an underlying logis distribution. (b) With an underlying exp distribution.

Figure B.2.12: Time-dependent PHMs with an underlying logis and exp distribution.



(a) All distributions (last step).

Figure B.2.13: Figure of all time-dependent PHMs.

Table B.2.13: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	0 [100]	8 [100]	8 [84]	22 [80]	24 [64]	24 [52]	41 [52]	49 [36]	62 [24]	81 [20]	100 [8]
	2	0 [100]	3 [96]	11 [84]	19 [76]	27 [56]	32 [52]	38 [40]	49 [32]	65 [28]	84 [24]	100 [8]
	3	0 [100]	0 [96]	0 [92]	11 [84]	14 [68]	19 [64]	32 [52]	41 [44]	46 [36]	65 [32]	100 [8]
norm	4	0 [100]	0 [100]	5 [100]	11 [88]	19 [80]	24 [72]	32 [56]	38 [52]	41 [40]	62 [28]	100 [8]
	5	0 [100]	0 [100]	3 [100]	5 [96]	14 [92]	19 [76]	24 [64]	35 [56]	41 [44]	49 [28]	100 [8]
	6	0 [100]	0 [100]	0 [96]	3 [96]	5 [96]	11 [84]	19 [68]	35 [56]	41 [44]	46 [32]	100 [8]
	7	0 [100]	0 [100]	0 [96]	0 [96]	5 [96]	8 [96]	19 [72]	24 [64]	38 [48]	46 [40]	100 [8]
	1	0 [100]	3 [100]	8 [84]	24 [80]	24 [64]	24 [52]	43 [44]	49 [32]	65 [24]	81 [20]	100 [8]
	2	0 [100]	5 [100]	14 [84]	19 [80]	24 [72]	27 [52]	38 [48]	41 [36]	57 [28]	76 [20]	100 [8]
	3	0 [100]	0 [100]	8 [92]	19 [80]	24 [80]	30 [64]	35 [44]	41 [36]	54 [32]	78 [24]	100 [8]
	4	0 [100]	0 [100]	0 [96]	3 [88]	8 [84]	22 [80]	27 [60]	38 [48]	49 [40]	65 [20]	100 [8]
lnorm	5	0 [100]	0 [100]	0 [100]	0 [100]	5 [96]	14 [84]	30 [72]	35 [48]	41 [40]	65 [24]	100 [8]
	6	0 [100]	0 [100]	0 [100]	0 [100]	3 [96]	5 [88]	16 [68]	30 [60]	35 [40]	51 [32]	100 [8]
	7	0 [100]	0 [100]	0 [96]	0 [96]	0 [96]	5 [84]	11 [72]	27 [64]	41 [40]	54 [32]	100 [8]
	8	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	5 [92]	8 [76]	24 [60]	35 [48]	51 [32]	100 [8]
	9	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	5 [92]	8 [76]	22 [60]	30 [52]	46 [32]	100 [8]
	1	0 [100]	8 [100]	8 [84]	22 [80]	24 [64]	24 [52]	41 [52]	49 [36]	62 [24]	81 [20]	100 [8]
	2	0 [100]	3 [96]	11 [84]	19 [80]	24 [56]	32 [52]	41 [40]	49 [36]	65 [28]	84 [24]	100 [8]
	3	0 [100]	0 [92]	0 [92]	8 [80]	19 [72]	24 [64]	27 [52]	41 [44]	43 [36]	68 [32]	100 [8]
logis	4	0 [96]	0 [96]	3 [96]	8 [88]	14 [72]	19 [64]	32 [56]	41 [44]	49 [36]	62 [32]	100 [8]
	5	0 [100]	0 [96]	3 [96]	5 [88]	8 [84]	14 [76]	27 [60]	32 [52]	43 [40]	51 [28]	100 [8]
	6	0 [100]	0 [96]	0 [96]	3 [96]	5 [88]	11 [76]	22 [64]	27 [52]	38 [44]	46 [32]	100 [8]
	7	0 [100]	0 [96]	0 [96]	3 [96]	5 [88]	8 [80]	22 [64]	24 [52]	38 [44]	46 [32]	100 [8]
	8	0 [100]	0 [96]	0 [96]	0 [92]	5 [92]	5 [88]	14 [68]	24 [56]	35 [40]	43 [32]	100 [8]
	1	0 [100]	5 [96]	14 [80]	24 [56]	35 [52]	38 [52]	41 [40]	54 [24]	70 [24]	81 [16]	100 [8]
	2	0 [96]	11 [96]	22 [60]	32 [52]	38 [48]	41 [48]	51 [32]	59 [24]	78 [24]	84 [16]	100 [8]
exp	3	3 [96]	3 [84]	8 [84]	19 [76]	27 [72]	30 [60]	41 [40]	49 [36]	70 [28]	81 [20]	100 [8]
	4	3 [84]	5 [84]	14 [80]	16 [76]	27 [72]	30 [60]	38 [56]	49 [40]	70 [36]	84 [20]	100 [8]
	5	3 [88]	8 [80]	14 [80]	19 [76]	27 [64]	32 [64]	41 [60]	49 [48]	65 [36]	86 [20]	100 [8]

Table B.2.14: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	0 [100]	8 [100]	8 [84]	22 [80]	24 [64]	24 [52]	41 [52]	49 [36]	62 [24]	81 [20]	100 [8]
	2	0 [100]	3 [96]	11 [84]	19 [76]	27 [56]	32 [52]	38 [40]	49 [32]	65 [28]	84 [24]	100 [8]
	3	0 [100]	0 [96]	0 [92]	11 [84]	14 [68]	19 [64]	32 [52]	41 [44]	46 [36]	65 [32]	100 [8]
norm	4	0 [100]	0 [100]	5 [100]	11 [88]	19 [80]	24 [68]	32 [56]	38 [52]	41 [40]	62 [28]	100 [8]
	5	0 [100]	0 [100]	3 [100]	5 [96]	14 [92]	19 [76]	24 [64]	35 [56]	41 [44]	49 [28]	100 [8]
	6	0 [100]	0 [100]	0 [96]	3 [96]	5 [96]	11 [84]	19 [68]	38 [56]	41 [44]	49 [32]	100 [8]
	7	0 [100]	0 [100]	0 [96]	0 [96]	5 [96]	8 [96]	19 [72]	27 [64]	38 [48]	43 [40]	100 [8]
	1	0 [100]	3 [100]	8 [84]	24 [80]	24 [64]	27 [52]	43 [44]	51 [32]	65 [24]	81 [20]	100 [8]
	2	0 [100]	5 [100]	14 [84]	19 [80]	24 [64]	27 [52]	38 [48]	43 [36]	59 [28]	78 [20]	100 [8]
	3	0 [100]	0 [100]	8 [92]	19 [80]	24 [80]	30 [64]	35 [44]	43 [36]	54 [32]	78 [24]	100 [8]
	4	0 [100]	0 [100]	0 [96]	3 [88]	8 [84]	22 [80]	27 [60]	38 [48]	51 [40]	65 [20]	100 [8]
lnorm	5	0 [100]	0 [100]	0 [100]	0 [100]	5 [96]	14 [84]	30 [68]	35 [48]	41 [40]	65 [24]	100 [8]
	6	0 [100]	0 [100]	0 [100]	0 [100]	3 [96]	5 [84]	16 [68]	30 [60]	35 [40]	54 [32]	100 [8]
	7	0 [100]	0 [100]	0 [96]	0 [96]	0 [96]	5 [84]	11 [72]	24 [64]	38 [40]	51 [32]	100 [8]
	8	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	5 [92]	8 [76]	24 [60]	35 [48]	51 [32]	100 [8]
	9	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	5 [92]	8 [76]	22 [60]	30 [52]	46 [32]	100 [8]
	1	0 [100]	8 [100]	8 [84]	22 [80]	24 [64]	24 [52]	41 [52]	49 [36]	62 [24]	81 [20]	100 [8]
	2	0 [100]	3 [96]	11 [84]	19 [80]	24 [56]	32 [52]	41 [40]	49 [36]	65 [28]	84 [24]	100 [8]
	3	0 [100]	0 [92]	0 [92]	8 [80]	19 [72]	24 [64]	27 [52]	41 [44]	43 [36]	68 [32]	100 [8]
logis	4	0 [96]	0 [96]	3 [96]	8 [88]	14 [72]	19 [64]	32 [56]	41 [44]	49 [36]	62 [32]	100 [8]
	5	0 [100]	0 [96]	3 [96]	5 [88]	8 [84]	14 [76]	27 [60]	32 [52]	43 [40]	51 [28]	100 [8]
	6	0 [100]	0 [96]	0 [96]	3 [96]	5 [88]	11 [76]	22 [64]	27 [52]	38 [44]	49 [32]	100 [8]
	7	0 [100]	0 [96]	0 [96]	3 [96]	5 [88]	8 [80]	22 [64]	24 [52]	38 [44]	46 [32]	100 [8]
	8	0 [100]	0 [96]	0 [96]	0 [92]	5 [92]	5 [88]	14 [68]	24 [56]	35 [40]	43 [32]	100 [8]
	1	0 [100]	5 [96]	14 [80]	24 [56]	35 [52]	38 [52]	41 [40]	54 [24]	70 [24]	81 [16]	100 [8]
	2	0 [96]	11 [96]	22 [60]	32 [52]	38 [48]	41 [48]	51 [32]	59 [24]	78 [24]	84 [16]	100 [8]
exp	3	3 [96]	3 [84]	8 [84]	19 [76]	27 [72]	30 [60]	41 [40]	51 [36]	70 [28]	81 [16]	100 [8]
	4	3 [84]	5 [84]	14 [80]	16 [76]	27 [72]	30 [60]	41 [56]	49 [40]	70 [36]	84 [16]	100 [8]
	5	3 [88]	8 [80]	14 [80]	19 [76]	30 [64]	32 [64]	41 [60]	49 [48]	65 [36]	86 [20]	100 [8]

Table B.2.15: Average MTTF values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	1757.5	1448.6	1448.6	1112.6	1062.2	1062.2	655.1	544.6	401.8	169.7	NaN
	2	1757.5	1647.9	1409.5	1169	965	800.4	679.3	578.2	423.5	164.3	NaN
	3	1757.5	1757.5	1757.5	1409.5	1350.3	1221.2	1051.7	752.8	676.9	430.5	NaN
norm	4	1757.5	1757.5	1557.5	1475.2	1228.4	1153	912.7	760.2	666.5	446.2	NaN
	5	1757.5	1757.5	1647.9	1616.7	1530.8	1332	1205.4	910.7	691.9	558.1	NaN
	6	1757.5	1757.5	1757.5	1647.9	1557.5	1521.5	1232.1	866.2	691.9	572.1	NaN
	7	1757.5	1757.5	1757.5	1757.5	1557.5	1557.2	1234.9	1032.4	760.2	572.1	NaN
	1	1757.5	1660.2	1448.6	1062.2	1062.2	1062.2	640.6	544.6	355.5	169.7	NaN
	2	1757.5	1557.5	1284.8	1166.8	1062.2	1043.6	717	694.5	481.6	293.2	NaN
	3	1757.5	1757.5	1474.7	1166.8	1062.2	955	818.8	693.9	493.9	237	NaN
	4	1757.5	1757.5	1757.5	1672.6	1474.7	1050.8	948.3	799.5	539.5	391.6	NaN
lnorm	5	1757.5	1757.5	1757.5	1757.5	1557.5	1363.5	876.8	768.5	607.3	389.7	NaN
	6	1757.5	1757.5	1757.5	1757.5	1647.9	1557.5	1260.1	876.8	768.5	533.3	NaN
	7	1757.5	1757.5	1757.5	1757.5	1757.5	1557.5	1454.7	1094.8	689.9	493.4	NaN
	8	1757.5	1757.5	1757.5	1757.5	1757.5	1557.5	1492.4	1218.6	768.5	512.7	NaN
	9	1757.5	1757.5	1757.5	1757.5	1757.5	1557.5	1492.4	1303.7	921.5	635.2	NaN
	1	1757.5	1448.6	1448.6	1112.6	1062.2	1062.2	655.1	544.6	401.8	169.7	NaN
	2	1757.5	1647.9	1409.5	1169	1062.2	800.4	701	578.2	423.5	164.3	NaN
	3	1757.5	1757.5	1757.5	1448.6	1221.2	1096.4	1043.6	752.8	654.3	405.3	NaN
logis	4	1757.5	1757.5	1647.9	1448.6	1335.8	1249.9	850	691.9	563.7	393.2	NaN
	5	1757.5	1757.5	1647.9	1557.5	1492.4	1335.8	1071.4	850	590.5	512.7	NaN
	6	1757.5	1757.5	1757.5	1647.9	1557.5	1378.3	1147.6	977.1	703.5	572.1	NaN
	7	1757.5	1757.5	1757.5	1647.9	1557.5	1492.4	1147.6	1032.4	703.5	572.1	NaN
	8	1757.5	1757.5	1757.5	1757.5	1557.5	1557.5	1451.2	1162.4	768.5	590.5	NaN
	1	1757.5	1544.7	1270.7	970.3	698	648.9	607.3	472.8	306.5	169.7	NaN
	2	1757.5	1380.8	1031.2	771.1	648.9	607.3	497.9	415.9	197.5	146	NaN
exp	3	1660.2	1660.2	1566.6	1241	1015.4	909.6	634.8	608.4	321	203.4	NaN
	4	1660.2	1627.1	1429.4	1385	1015.4	1017.6	848.9	614.2	321	185.3	NaN
	5	1660.2	1577.8	1473.5	1270.8	1063.7	1032.6	849.3	614.2	504.7	190	NaN

Table B.2.16: Average MTTF values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	1757.5	1448.6	1448.6	1112.6	1062.2	1062.2	655.1	544.6	401.8	169.7	NaN
	2	1757.5	1647.9	1409.5	1169	965	800.4	679.3	578.2	423.5	164.3	NaN
	3	1757.5	1757.5	1757.5	1409.5	1350.3	1221.2	1051.7	752.8	676.9	430.5	NaN
norm	4	1757.5	1757.5	1557.5	1475.2	1228.4	1153	912.7	760.2	666.5	446.2	NaN
	5	1757.5	1757.5	1647.9	1616.7	1530.8	1332	1205.4	910.7	691.9	558.1	NaN
	6	1757.5	1757.5	1757.5	1647.9	1557.5	1521.5	1232.1	760.2	691.9	538.5	NaN
	7	1757.5	1757.5	1757.5	1757.5	1557.5	1557.2	1234.9	966.7	760.2	590.5	NaN
	1	1757.5	1660.2	1448.6	1062.2	1062.2	1043.6	640.6	530.4	355.5	169.7	NaN
	2	1757.5	1557.5	1284.8	1166.8	1062.2	1043.6	717	640.6	449.9	238.2	NaN
	3	1757.5	1757.5	1474.7	1166.8	1062.2	955	818.8	639.9	493.9	237	NaN
	4	1757.5	1757.5	1757.5	1672.6	1474.7	1050.8	948.3	799.5	528.7	391.6	NaN
lnorm	5	1757.5	1757.5	1757.5	1757.5	1557.5	1363.5	876.8	768.5	607.3	389.7	NaN
	6	1757.5	1757.5	1757.5	1757.5	1647.9	1557.5	1260.1	876.8	768.5	493.4	NaN
	7	1757.5	1757.5	1757.5	1757.5	1757.5	1557.5	1454.7	1121	758.3	533.3	NaN
	8	1757.5	1757.5	1757.5	1757.5	1757.5	1557.5	1492.4	1218.6	768.5	512.7	NaN
	9	1757.5	1757.5	1757.5	1757.5	1757.5	1557.5	1492.4	1303.7	921.5	635.2	NaN
	1	1757.5	1448.6	1448.6	1112.6	1062.2	1062.2	655.1	544.6	401.8	169.7	NaN
	2	1757.5	1647.9	1409.5	1169	1062.2	800.4	701	578.2	423.5	164.3	NaN
	3	1757.5	1757.5	1757.5	1448.6	1221.2	1096.4	1043.6	752.8	654.3	405.3	NaN
logis	4	1757.5	1757.5	1647.9	1448.6	1335.8	1249.9	850	691.9	563.7	393.2	NaN
	5	1757.5	1757.5	1647.9	1557.5	1492.4	1335.8	1071.4	850	590.5	512.7	NaN
	6	1757.5	1757.5	1757.5	1647.9	1557.5	1378.3	1147.6	977.1	703.5	563.7	NaN
	7	1757.5	1757.5	1757.5	1647.9	1557.5	1492.4	1147.6	1032.4	703.5	572.1	NaN
	8	1757.5	1757.5	1757.5	1757.5	1557.5	1557.5	1451.2	1162.4	768.5	590.5	NaN
	1	1757.5	1544.7	1270.7	970.3	698	648.9	607.3	472.8	306.5	169.7	NaN
	2	1757.5	1380.8	1031.2	771.1	648.9	607.3	497.9	415.9	197.5	146	NaN
exp	3	1660.2	1660.2	1566.6	1241	1015.4	909.6	634.8	589.1	321	203.4	NaN
	4	1660.2	1627.1	1429.4	1385	1015.4	1017.6	688.7	614.2	321	185.3	NaN
	5	1660.2	1577.8	1473.5	1270.8	1017.6	1032.6	849.3	614.2	504.7	190	NaN

Table B.2.17: Average MTTRep values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

						Reliability Level						
Dist.	Step	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	5268.1	4871.5	4195	3373.3	2756.9	2171.5	1673.1	1213.9	800	392.9	30
	2	5303.6	4766.3	3950.9	3226	2664.3	2189.3	1683.5	1238.2	836.7	456.3	30
	3	5379.2	5242.8	4745.9	3963.3	3329.4	2780	2230.3	1845.7	1324.3	798.2	30
norm	4	5379.2	5244.8	5033.2	4288.4	3438	2703.6	2174	1786.3	1324.7	799.7	30
	5	5293.3	5264.6	5181.8	4808.6	4096.1	3555.5	2901.8	2313.8	1685.4	988.7	30
	6	5141.7	5131.3	4990.8	4794.7	4389.8	3596.1	3127.1	2586.6	1986.5	1125.2	30
	7	5141.7	5126.2	4999.5	4808.5	4567.3	3916.9	3321.1	2904.6	2260.8	1367.1	30
	1	5321.5	4991.3	4318.2	3369.6	2689.1	2043.3	1530.4	1116.5	742	380.5	30
	2	5317.2	4825.3	4150.5	3456.8	2827.2	2254.6	1784.8	1370.6	1002.7	570.4	30
	3	5295.8	4939.4	4447	3735.9	3087.5	2531.3	2029.1	1623.1	1158.3	635.3	30
	4	5379.2	5379.2	5220.7	4620.4	4039.4	3393.5	2579.3	1928.7	1351.4	742.7	30
lnorm	5	5379.2	5379.2	5379.2	5268.4	4830.1	4004.9	3048.8	2211.2	1566.7	776.5	30
	6	5379.2	5379.2	5379.2	5372	5132.1	4507.3	3701.8	2955.4	2076.7	1069.7	30
	7	5379.2	5379.2	5351.6	5063.2	4854	4321.9	3578.3	2785.1	1961	1030	30
	8	5359.8	5355.4	5355.4	5353.6	5034.3	4491.5	3811.2	2855.6	2106.7	1171	30
	9	5364.8	5359.4	5356.2	5344.1	5148.6	4594.7	3837.4	2845.8	2103.1	1157.3	30
	1	5266.6	4866.1	4189	3365.7	2751.1	2166.8	1668.5	1211.3	798	391.9	30
	2	5318.6	4849.1	4031.9	3306.8	2698.4	2252	1692	1279.3	862.1	470.3	30
	3	5352	5147.1	4647	3882.6	3349	2773.2	2289.7	1844.8	1370.2	801.6	30
logis	4	5238.8	4988.6	4749.7	4271.9	3627.8	3101.6	2611.6	2029.2	1429.5	795	30
	5	5379.2	5259.4	5015.7	4592.9	4140.3	3586.4	2794.9	2355.5	1745	908.9	30
	6	5227.5	5174.3	4998.8	4641	4169.7	3647.8	3062.8	2577.6	1953.2	1073.9	30
	7	5227.5	5171.6	5007	4632.4	4203.1	3680.9	3208.2	2766.6	2112.9	1244.3	30
	8	5198	5015	4955.2	4766.1	4444.2	3953.8	3260.2	2657.2	2145.2	1255.9	30
	1	5290.3	4487.3	3573.1	2797.4	2226.3	1717.8	1264.2	903.8	583.1	278.5	30
	2	5069.5	3971.3	3139.4	2399.8	1874.4	1458.7	1057.5	758.1	493.1	234	30
exp	3	4887.1	4427.8	3976.2	3445.2	2933.7	2426.1	1794.7	1250.2	817	390.2	30
	4	4644.2	4107.7	3728.2	3310.3	2963.2	2500.3	2026.4	1463.4	931.5	426.9	30
	5	4597.2	3993.8	3625	3321.8	2946	2440.8	2072.8	1515.8	918.8	424.1	30

Table B.2.18: Average MTTRep values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

						Reliability Level						
Dist.	Step	7%	16%	25%	35%	44%	53%	63%	72%	81%	91%	100%
	1	5266.9	4865.4	4184.9	3357.3	2741.3	2154.3	1652.8	1197.6	780.7	373.7	30
	2	5303.6	4766	3950.5	3225.5	2663.3	2189.1	1682.6	1237.5	836.3	455.9	30
	3	5379.2	5242	4744.8	3963	3328.6	2779.6	2229.6	1845.2	1324.1	798	30
norm	4	5379.2	5244.8	5033	4288.4	3437.4	2701.9	2166.9	1785.2	1324.2	799.6	30
	5	5289.3	5264.6	5035.9	4804.9	4090.4	3546.1	2881.4	2253.7	1670.3	977.7	30
	6	5141.6	5126	4989.7	4793.1	4323.1	3580.5	3075.8	2593.2	1970	1108.3	30
	7	5141.7	5126.4	5000.6	4809.2	4570.8	3922.9	3329.6	2898.4	2234.9	1370	30
	1	5320.5	4986.7	4310.3	3355.5	2672.3	2011	1507.9	1088.5	724.9	363.7	30
	2	5315.9	4815.6	4138.9	3427.5	2805.5	2222	1760.9	1361.8	977.8	551.7	30
	3	5295.8	4935.4	4441.9	3728.5	3080.6	2526	2024.6	1622.9	1155.3	633	30
	4	5379.2	5379.2	5219.7	4614.8	4036.6	3386.9	2576.7	1922.7	1335	741.3	30
lnorm	5	5379.2	5379.2	5379.2	5266.3	4827.4	4002.7	3000.7	2205.8	1558	769.4	30
	6	5379.2	5379.2	5379.2	5365.6	5129	4498.4	3690.2	2935.1	2063.6	1065.6	30
	7	5379.2	5379.2	5351.6	5075.4	4863.4	4347	3607.8	2824	1981.4	1058.9	30
	8	5359.8	5355.4	5355.4	5354.6	5036.2	4493.2	3815.6	2861.4	2114	1184.9	30
	9	5364.8	5359.4	5356.2	5344.2	5151.3	4600.1	3841.1	2854.4	2106.7	1169.4	30
	1	5266.3	4857.4	4175	3350.1	2733.7	2149.6	1649	1194.5	778.4	372.9	30
	2	5318.6	4848.6	4030.9	3305.8	2697.1	2251.1	1690.9	1278.8	861.6	469.8	30
	3	5352	5146.6	4646.1	3881.7	3347.8	2772.3	2289.3	1843.1	1369.8	801.2	30
logis	4	5238.8	4988.4	4748.3	4266.6	3625.2	3097.4	2609.2	2025.2	1426.9	793.4	30
	5	5379.2	5259.1	5013.6	4592.2	4138.2	3584.1	2790.5	2351.6	1742.8	907.8	30
	6	5227.5	5174.3	4994.3	4639.4	4160.8	3640.1	3055.3	2570.1	1943.8	1061.1	30
	7	5227.5	5170.6	5005	4627.8	4192.2	3677.6	3203.6	2758.5	2103.1	1240.1	30
	8	5197.3	5015	4952.5	4762.2	4438	3946.7	3255.4	2646.5	2127.7	1250.1	30
	1	5288.9	4476.9	3562.4	2785.1	2212.7	1705.5	1253.1	891.8	570.2	266.5	30
	2	5064.4	3956.8	3123.2	2383.7	1857.8	1442.3	1041.2	742.4	476	217.4	30
exp	3	4883.2	4405.5	3973.1	3430.8	2922.2	2410.8	1782.8	1236.2	804.8	377.4	30
	4	4643.1	4094	3724.5	3306	2949.4	2492.1	2079.2	1450	917	417.5	30
	5	4594.8	3992.7	3621	3318.8	2917.8	2433.8	2057.5	1502.1	905.2	408.1	30

Table B.2.19: Variables identified by time-(in)dependent PHM models.

	Time-independent PHM Variable	Scaled Value
①	*Group A*	**−8.55**
②	*Yaw rate max deg sec 4*	**4.8**
③	*Accn long mean g s 1*	**−4.35**
④	*Pressure dynamic mean hPa mbar*	**3.48**
⑤	*Pitch mean deg 1*	**−3.14**
⑥	*Group C*	**2.4**
⑦	*Group B*	**−1.91**

	Time-dependent PHM Variable	Scaled Value
①	*Group C*	**−39.93**
②	*Group B*	**30.6**
③	*Group D*	**−18.7**
④	*Roll max deg 4*	**15.77**
⑤	*Yaw rate max deg sec 4*	**14.62**
⑥	*Group E*	**−12.25**
⑦	*Rudder low mean deg TER 2*	**11.06**
⑧	*Group F*	**10.37**
⑨	*Rudder cmd force max lbs Nose Right 8*	**9.93**
⑩	*NormalForce lhs mean lbs 1*	**9.88**
⑪	*Pressure dynamic mean hPa mbar*	**−8.02**
⑫	*Prop spd rhs min*	**−6.62**
⑬	*Roll rate mean deg sec 1*	**5.95**

Table B.2.20: Variables identified by each step by time-(in)dependent PHMs (in order).

	PHM Variables *Time-independent*	*Time-dependent*
norm	④ ① ⑦	③ ② ① ⑥ ⑧ ⑤ ⑫
lnorm	① ④ ②	③ ④ ② ① ⑥ ⑨ ⑫ ⑩ ⑬
logis	① ③	③ ② ① ⑨ ⑥ ⑤ ⑪ ⑩
exp	① ④	⑥ ⑧ ⑦ ④ ③
weibull	① ⑤ ⑥	

Table B.2.21: Number of times variables identified by each step by time-(in)dependent PHMs.

Key *indep*	*dep*	*Variable*	*Count*	**Key** *indep*	*dep*	*Variable*	*Count*
①		Group A	5	⑤		Pitch mean deg 1	1
⑦	②	Group B	4	④	⑪	Pressure dynamic mean hPa mbar	4
⑥	①	Group C	4		⑫	Prop spd rhs min	2
	③	Group D	4		④	Roll max deg 4	2
	⑥	Group E	4		⑬	Roll rate mean deg sec 1	1
	⑧	Group F	2		⑨	Rudder cmd force max lbs Nose Right 8	2
③		Accn long mean g s 1	1		⑦	Rudder low mean deg TER 2	1
	⑩	NormalForce lhs mean lbs 1	2	②	⑤	Yaw rate max deg sec 4	3

Table B.2.22: Variables belonging to each group identified in B.2.19.

Group	*Variables*	*Group*	*Variables*
Group A	Yaw rate max deg sec 5, Yaw rate max deg sec 6	Group D	Pitch mean deg 4, Accn long mean g s 4
Group B	Accn long mean g s 5, Pitch mean deg 5, Accn long mean g s 6, Pitch mean deg 6	Group E	Roll min deg 6, Yaw rate min deg sec 6, Yaw rate mean deg sec 5
Group C	Torque rhs mean 5, Torque lhs mean 5, Torque lhs mean 6, Torque rhs mean 6	Group F	Rudder cmd force max lbs Nose Right 6, Rudder cmd force max lbs Nose Right 5

B.3 1152106-3 DC starter generator

Table B.3.23 provides a summary of the input data related to the component. The number of registered maintenance events is less than the total number of events due to the fact that TRAX data stretches back to 2004/2005 and FDR data only to 2011. Maintenance events with insufficient data, regarding operational factors, cannot be evaluated, hence are not registered during the modelling process.

Table B.3.23: General overview of component inputs.

Name	Value
Part Number	1152106-3
Total # (A, F, C)	1618, 342, 1276
Registered # (A, F, C)	273, 61, 212
Related Flights # (A, F, C)	314916, 36933, 277983
Avg. Cycles (A, F, C)	1153.54, 605.46, 1311.24
% Censored	77.66

In Tab. B.3.23 (A, F, C) denotes statistics regarding All (A), Failed (F), and Censored (C) events respectively. Ergo A will always be the sum or mean derived from F and C.

Analysis

Tables B.3.24 and B.3.25 summarise the results from EVA and MDA. In addition the variables obtained by semi-parametric PHM modelling (labelled 'reduced semi-COX') are also presented if applicable. Table B.3.25 provides an overview of the specific operational factors identified during all flight phases. In this case high counts indicate operational factors that were significantly different during multiple flight phases.

Table B.3.24: Overview of analysis input and output.

	# Variables
ALL	1531
EVA	12
MDA	21
Combined	33
reduced Corr.	24
reduced semi-COX	11
Take-Off related	12
Cruise related	6
Touch-Down related	6

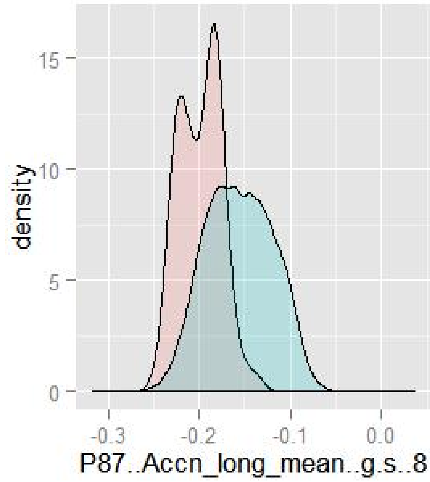
Table B.3.25: Overview of identified operational factors during all flight phases.

Variable	Count	Variable	Count	Variable	Count
Pressure_dynamic	3	Accn_norm	1	Pitch_rate	1
Rudder_cmd_force	3	Torque_rhs	1	Pitch_cmd_FO_force	1
Brake_press_rhs	2	Roll_rate	1	Roll_cmd_force	1
Accn_long	2	Roll	1	Ttot	1
Elevator_Lin	2	Vz	1	Roll_cmd_FO_force	1
Accn_lat	1	Pitch	1		

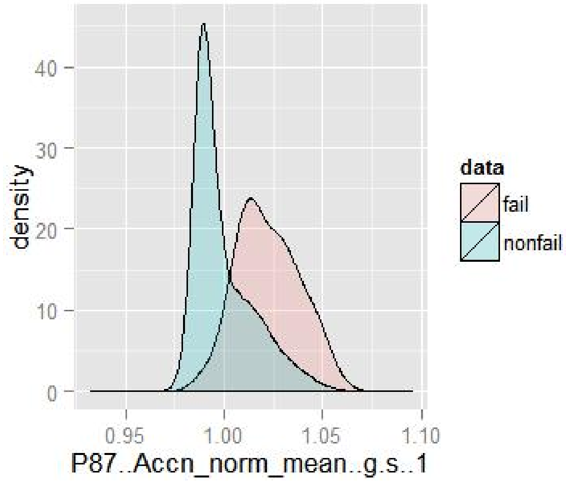
A multitude of factors were identified during EVA and MDA. Figure B.3.14 give a general overview of the top operational factors identified by EVA and MDA.

Time-based reliability modelling

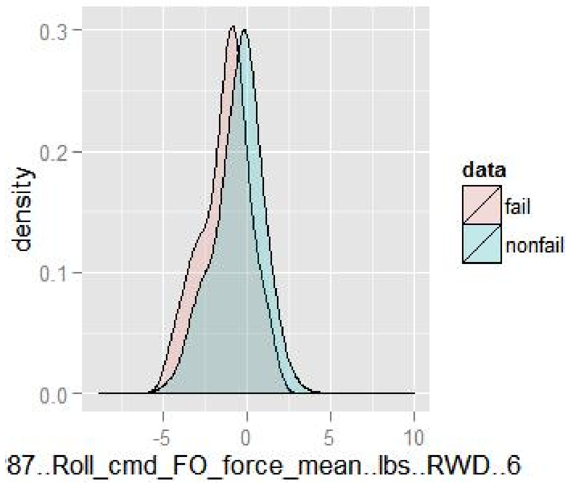
Table B.3.26 reports the maximum likelihood and goodness-of-fit tests results obtained from



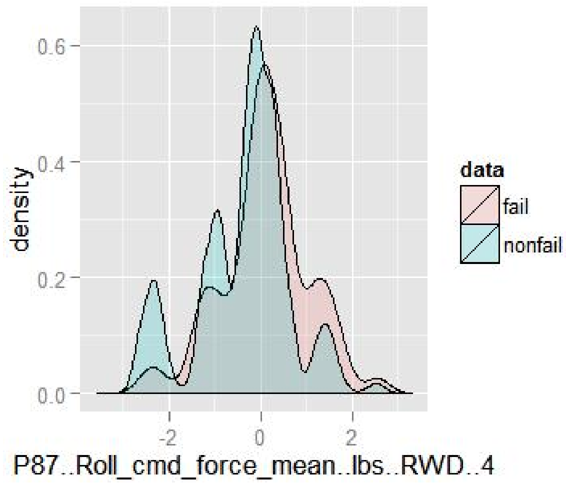
(a) EVA 1.



(b) EVA 2.



(c) MDA 1.



(d) MDA 2.

Figure B.3.14: Graphical overview of top operational factors identified by EVA and MDA.

Table B.3.26: Overview of results obtained by MLE optimisation and GOF tests.

	Distributions					
	norm	**lnorm**	**logis**	**exp**	**weibull**	**gamma**
MLE	**−585.35**	**−579.92**	**−585.36**	**−582.5**	**−583.23**	**−582.45**
Kolmogorov-Smirnov	**2.26**	**0.45**	**2.34**	**0.71**	**0.6**	**0.74**
Cramer-von-Mises Smirnov	**29.69**	**31.74**	**29.71**	**32.11**	**33.15**	**32.02**
Anderson-Darling	**−72.53**	**−72.21**	**−72.54**	**−72.29**	**−72.27**	**−72.28**
NRR	**41**	**35.43**	**41.06**	**38.15**	**39.17**	**38.22**

time-based reliability modelling. To show the overall fit Fig. B.3.15 shows the computed reliability function using an averaged virtual age V for all fitted models.

In addition Figures B.3.16, B.3.17, B.3.18, B.3.19, B.3.20, and B.3.21 present the reliability and hazard functions computed for each underlying distribution evaluated in the program.

Time independent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.3.27 gives a general overview of all the models obtained during each step in the process.

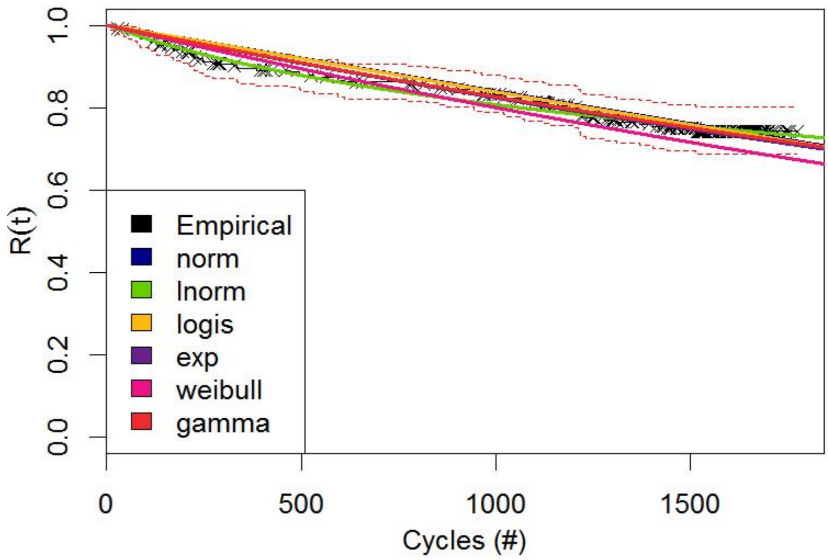
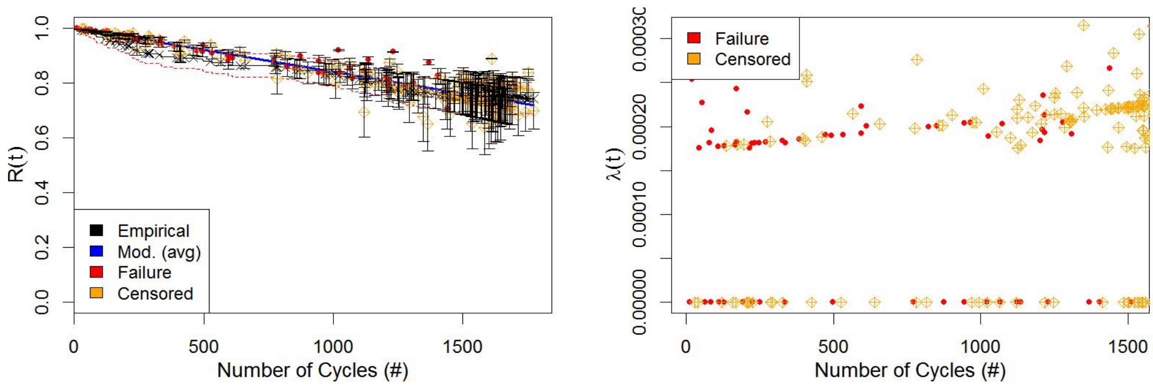


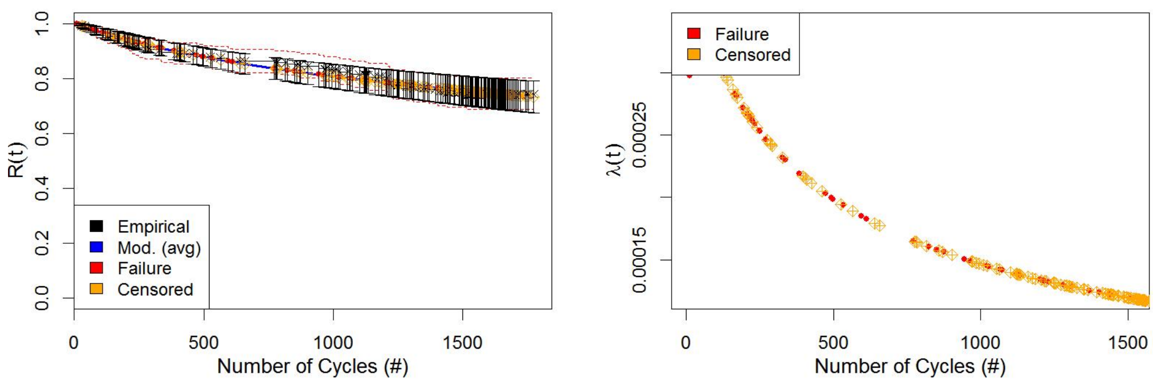
Figure B.3.15: Overview of overall fit of multiple GRP modells.



(a) Reliability function.

(b) Hazard function.

Figure B.3.16: Computed reliability for time-based models with underlying norm distribution.

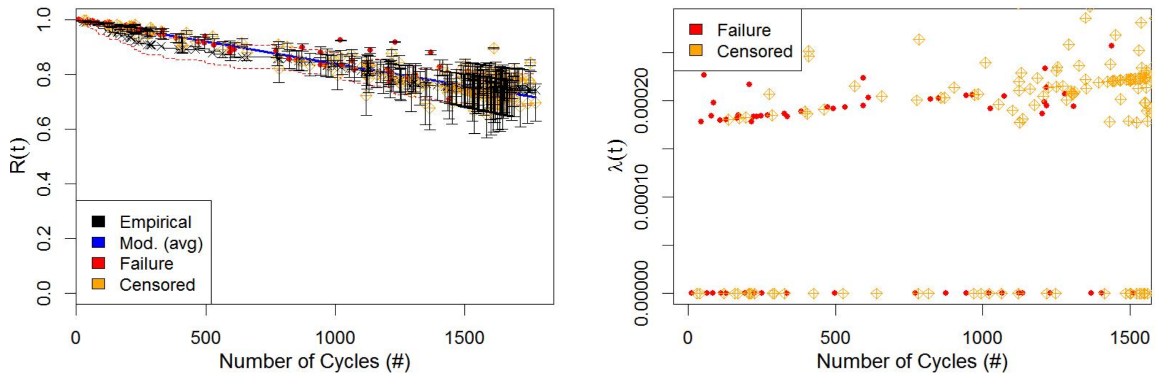


(a) Reliability function.

(b) Hazard function.

Figure B.3.17: Computed reliability for time-based models with underlying lnorm distribution.

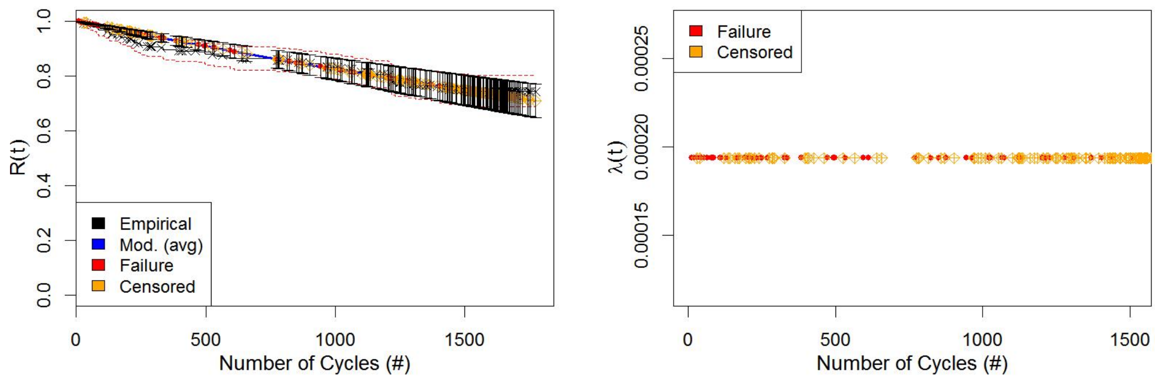
Graphical representation of the computed reliability per model are shown in Figures B.3.22, B.3.23, and B.3.24 as well as a general overview in Figure B.3.25a.
Tables B.3.28 and B.3.29 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables



(a) Reliability function.

(b) Hazard function.

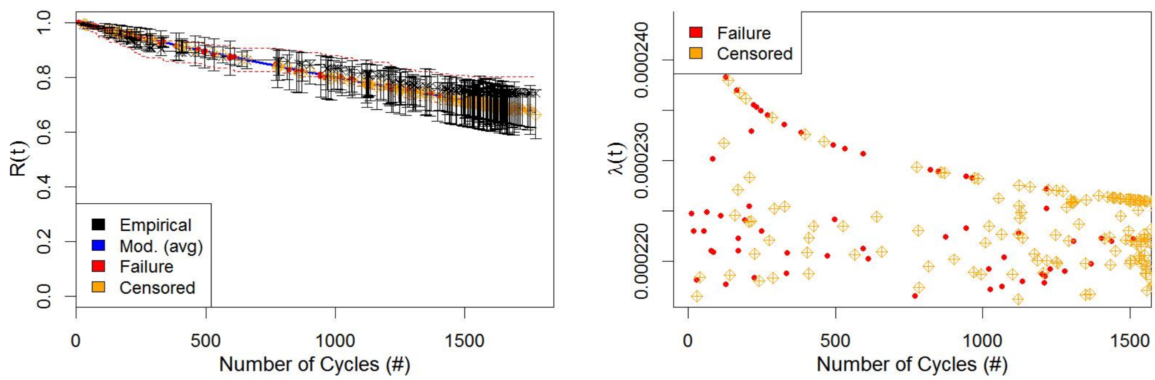
Figure B.3.18: Computed reliability for time-based models with underlying logis distribution.



(a) Reliability function.

(b) Hazard function.

Figure B.3.19: Computed reliability for time-based models with underlying exp distribution.



(a) Reliability function.

(b) Hazard function.

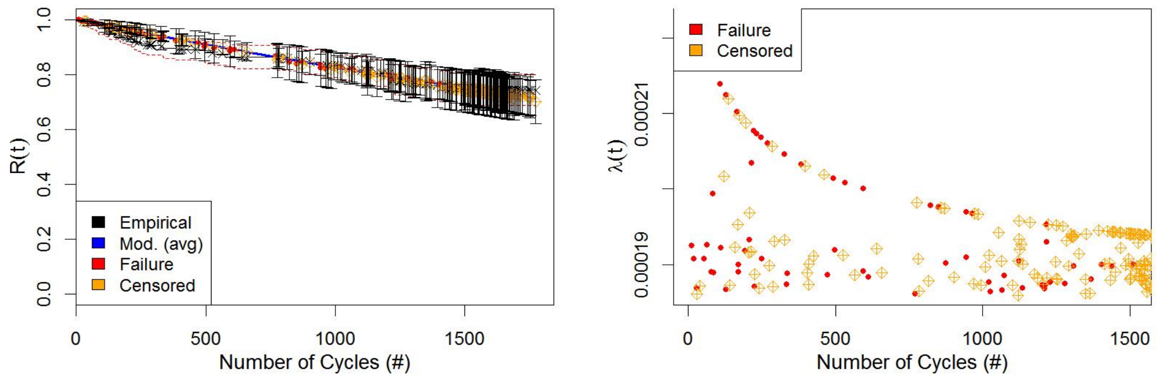
Figure B.3.20: Computed reliability for time-based models with underlying weibull distribution.

were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

To assist in the selection of models, Tables B.3.30, B.3.31, B.3.32, and B.3.33 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.

Time dependent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification

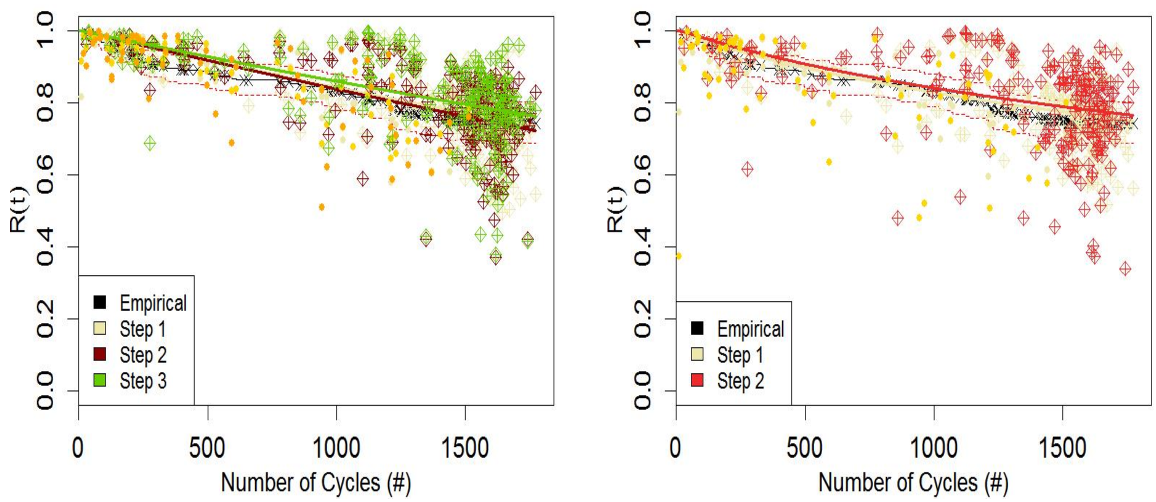


(a) Reliability function. (b) Hazard function.

Figure B.3.21: Computed reliability for time-based models with underlying gamma distribution.

Table B.3.27: Overview of time-independent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	norm	norm	lnorm	lnorm	logis	exp	weibull	weibull	gamma	gamma
Step #	**1**	**2**	**3**	**1**	**2**	**1**	**1**	**1**	**2**	**1**	**2**
MLE	**−573.75**	**−566.04**	**−561.29**	**−571.53**	**−559.65**	**−578.86**	**−586.71**	**−574.14**	**−561.28**	**−572.34**	**−561.34**
Time (min)	**0.76**	**1.52**	**2.2**	**0.69**	**1.34**	**0.72**	**0.51**	**0.86**	**1.6**	**27.8**	**49.93**
Kolmogorov-Smirnov	**3.59**	**2.58**	**13.4**	**3.69**	**10.3**	**2.91**	**3.42**	**5.12**	**11.9**	**4.59**	**12.05**
Cramer-von Mises-Smirnov	**29.71**	**30.01**	**27.84**	**28.37**	**28**	**28.1**	**31.54**	**29.05**	**27.51**	**27.38**	**27.57**
Anderson-Darling	**−72.82**	**−72.66**	**−73.31**	**−72.78**	**−73.49**	**−72.51**	**−72.74**	**−72.79**	**−73.28**	**−72.72**	**−73.27**
NRR	**41.89**	**40.32**	**35.83**	**38.28**	**32.14**	**41.13**	**40.74**	**42.91**	**32.05**	**35.81**	**32.06**

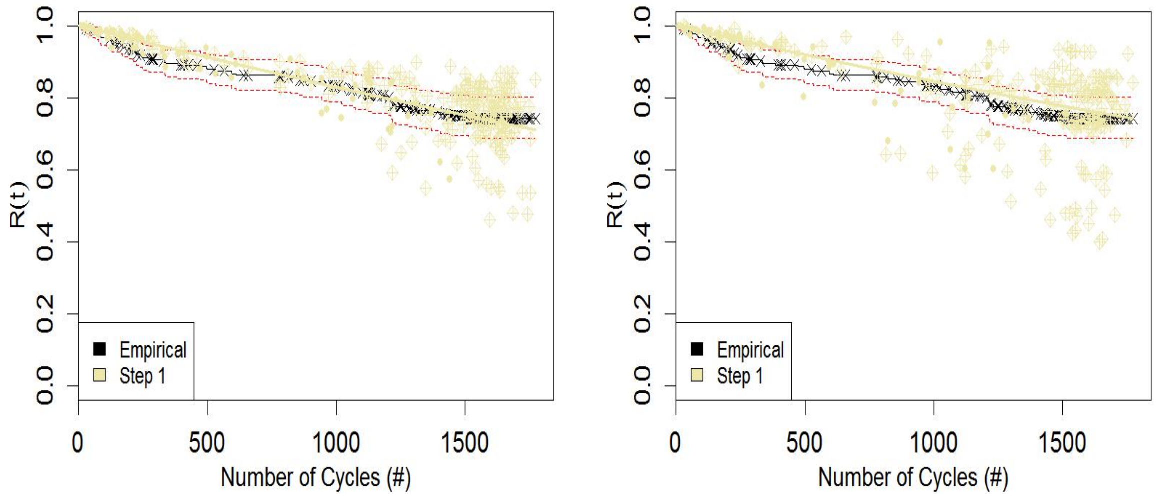


(a) With an underlying norm distribution. (b) With an underlying lnorm distribution.

Figure B.3.22: Time-independent PHMs with an underlying norm and lnorm distribution.

techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.3.34 gives a general overview of all the models obtained during each step in the process.

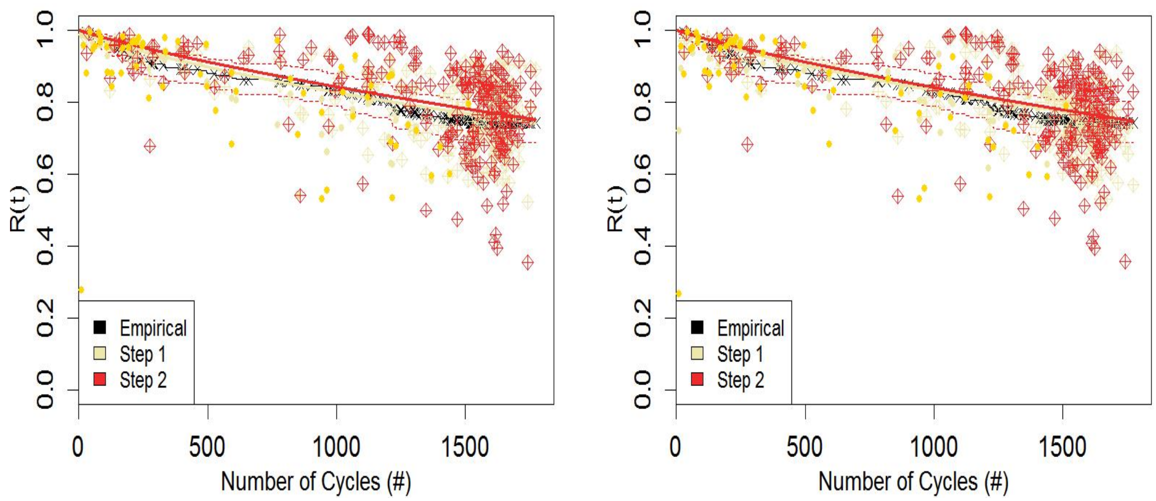
Graphical representation of the computed reliability per model are shown in Figures B.3.26, B.3.27, and B.3.28 as well as a general overview in Figure B.3.28b.



(a) With an underlying logis distribution.

(b) With an underlying exp distribution.

Figure B.3.23: Time-independent PHMs with an underlying logis and exp distribution.



(a) With an underlying weibull distribution.

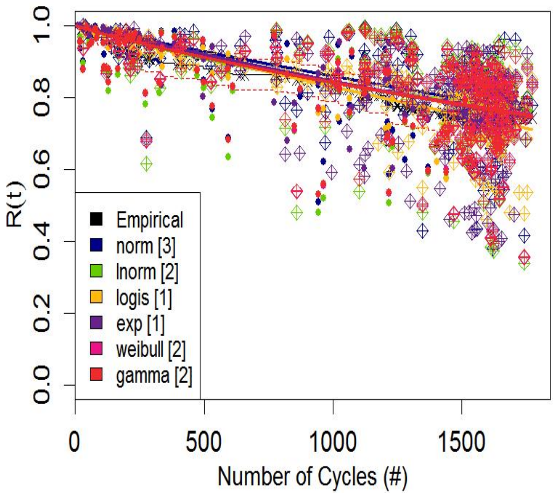
(b) With an underlying gamma distribution.

Figure B.3.24: Time-independent PHMs with an underlying weibull and gamma distribution.

Table B.3.28: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
	1	5 [67]	13 [58]	13 [52]	26 [41]	26 [35]	33 [31]	39 [28]	49 [20]	62 [10]	72 [8]	97 [0]
norm	2	3 [69]	10 [59]	11 [56]	18 [45]	25 [34]	30 [27]	39 [24]	49 [17]	52 [9]	70 [5]	97 [0]
	3	13 [71]	13 [68]	21 [63]	26 [55]	30 [47]	33 [38]	36 [32]	49 [24]	61 [15]	69 [9]	97 [0]
lnorm	1	5 [71]	15 [60]	16 [51]	28 [38]	33 [34]	44 [27]	46 [24]	56 [17]	69 [10]	79 [7]	97 [0]
	2	15 [71]	21 [68]	23 [64]	26 [55]	36 [47]	48 [36]	52 [32]	56 [21]	64 [12]	79 [10]	97 [0]
logis	1	5 [69]	5 [58]	11 [45]	15 [32]	18 [23]	28 [17]	39 [16]	48 [11]	61 [8]	72 [5]	97 [0]
exp	1	8 [67]	13 [56]	18 [50]	23 [42]	26 [36]	34 [23]	43 [20]	48 [16]	61 [9]	75 [5]	97 [0]
weibull	1	15 [67]	21 [56]	23 [50]	33 [38]	38 [30]	43 [23]	46 [18]	52 [10]	72 [8]	84 [2]	97 [0]
	2	13 [69]	18 [67]	20 [62]	30 [52]	34 [45]	44 [35]	54 [28]	56 [20]	64 [12]	79 [8]	97 [0]
gamma	1	5 [71]	13 [58]	16 [52]	26 [38]	33 [33]	43 [28]	46 [23]	54 [17]	69 [10]	82 [6]	97 [0]
	2	11 [69]	18 [66]	20 [62]	30 [50]	36 [43]	46 [35]	54 [28]	56 [20]	64 [12]	79 [9]	97 [0]

Tables B.3.35 and B.3.36 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.



(a) All distributions (last step).

Figure B.3.25: Figure of all time-independent PHMs.

Table B.3.29: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
norm	1	7 [60]	15 [53]	20 [46]	30 [38]	33 [33]	44 [29]	46 [25]	56 [18]	70 [10]	80 [6]	97 [0]
	2	5 [67]	11 [58]	13 [54]	18 [43]	26 [34]	34 [26]	43 [24]	49 [17]	54 [9]	77 [5]	97 [0]
	3	15 [70]	21 [65]	23 [61]	30 [51]	31 [46]	36 [36]	39 [31]	56 [22]	64 [14]	74 [9]	97 [0]
lnorm	1	13 [64]	20 [54]	26 [47]	34 [35]	43 [33]	46 [26]	54 [22]	70 [13]	77 [9]	90 [3]	97 [0]
	2	20 [69]	26 [65]	26 [63]	34 [53]	38 [45]	48 [35]	54 [31]	61 [20]	72 [12]	80 [9]	97 [0]
logis	1	5 [68]	5 [58]	11 [45]	15 [31]	18 [23]	28 [17]	39 [16]	48 [11]	61 [7]	72 [5]	97 [0]
exp	1	8 [66]	15 [54]	20 [49]	23 [41]	28 [34]	34 [23]	43 [20]	48 [15]	62 [8]	77 [5]	97 [0]
weibull	1	15 [68]	20 [59]	23 [55]	31 [40]	38 [32]	39 [26]	44 [18]	51 [10]	69 [8]	82 [2]	97 [0]
	2	16 [68]	23 [64]	26 [61]	31 [49]	36 [42]	49 [34]	54 [28]	59 [18]	72 [12]	84 [7]	97 [0]
gamma	1	10 [62]	16 [54]	26 [47]	33 [35]	39 [33]	44 [26]	54 [22]	66 [13]	77 [9]	92 [2]	97 [0]
	2	18 [68]	23 [64]	25 [59]	33 [48]	36 [42]	52 [32]	54 [27]	59 [18]	72 [12]	84 [7]	97 [0]

Table B.3.30: Average MTTF values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
norm	1	572.1	514.9	514.9	393.7	393.7	356	331.4	282.5	175.1	179.9	15
	2	582.1	540.4	527.6	468.4	416.5	379.2	276.9	199	192.1	133.6	15
	3	525.5	513.5	498.3	469.1	448.6	411.2	397.7	309.1	206.5	191.4	15
lnorm	1	572.1	504.2	489.1	389.2	356	325.9	320.7	237.2	177.1	175	15
	2	520.2	469.7	468.4	423.6	391.4	345.8	258.9	200.4	204.1	170.8	15
logis	1	566.8	566.8	526.7	504.2	471.3	376.2	280.6	203.4	136.2	97	15
exp	1	562.1	515.4	469.8	441.1	410	328	272.1	244.3	138	87.2	15
weibull	1	508.8	458.2	445.3	349.2	316.5	284.8	247.2	207	99.2	57.9	15
	2	526.4	500.8	484.9	439.6	417.6	321.5	231.7	200.4	204.1	170.8	15
gamma	1	572.1	514.9	479.4	404.2	365.8	340.1	299.1	242.5	177.1	153.5	15
	2	527.6	500.8	484.9	439.6	391.4	294.4	231.7	200.4	204.1	170.8	15

Table B.3.31: Average MTTF values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
norm	1	579.9	546.4	498.3	408.3	386.4	342.1	327.6	249.1	180.7	168.7	15
	2	575.9	527.6	517.5	468.4	412.6	325.2	251.7	199	194.4	126.7	15
	3	543.9	498.3	497.6	464.3	452.8	412.6	381.2	296.4	191.5	189.1	15
lnorm	1	558.1	512.1	463.3	418.2	364.8	327.6	320.4	180.7	174.7	191.5	15
	2	527	485.7	485.7	407.3	393.1	345.8	231.7	208.2	192.8	170.8	15
logis	1	566.8	566.8	526.7	504.2	471.3	376.2	280.6	203.4	136.2	97	15
exp	1	562.1	500.1	463.6	441.1	397.9	328	272.1	244.3	132.3	79.6	15
weibull	1	508.8	464.6	445.3	368	316.5	310.6	257.4	211.2	113.1	62.8	15
	2	538.9	503.6	465.5	423.3	391.4	318.1	231.7	205	192.8	162.8	15
gamma	1	583.2	532	470.5	407.2	374	342.1	283.2	178.1	174.7	221	15
	2	537.8	503.6	480.1	408.6	391.4	258.9	231.7	205	192.8	162.8	15

Table B.3.32: Average MTTRep values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
norm	1	**1868.8**	**1668.1**	**1544.2**	**1312.1**	**1190**	**978.3**	**843.8**	**629.6**	**415.9**	**246.4**	**20**
	2	**1866.1**	**1645**	**1509.2**	**1311.5**	**1165.4**	**964.2**	**828.1**	**606.2**	**379.4**	**223**	**20**
	3	**2097**	**1912.3**	**1721.1**	**1495**	**1357**	**1129.9**	**966.6**	**721.5**	**495.8**	**314.5**	**20**
lnorm	1	**2013.7**	**1731.3**	**1547**	**1283**	**1108.2**	**862**	**727.9**	**546.5**	**342.9**	**181.1**	**20**
	2	**2210**	**1924.5**	**1763.9**	**1506.8**	**1297.2**	**1002.7**	**876.9**	**640.9**	**435.7**	**281.9**	**20**
logis	1	**1671**	**1476.7**	**1335.6**	**1138.7**	**1012.5**	**828.6**	**703.2**	**514.2**	**319.4**	**190.8**	**20**
exp	1	**1846.1**	**1627.7**	**1480.6**	**1259.4**	**1121.1**	**914.3**	**766**	**550.7**	**352.7**	**208**	**20**
weibull	1	**1820.6**	**1567.4**	**1412.3**	**1175.2**	**1014.8**	**775.8**	**628.6**	**409.9**	**224.5**	**116.1**	**20**
	2	**1998.1**	**1801.5**	**1660.3**	**1393.1**	**1230.9**	**997.9**	**853.6**	**629.2**	**406.5**	**233.7**	**20**
gamma	1	**1943.2**	**1692.5**	**1530.3**	**1279.6**	**1104.7**	**863.3**	**748.1**	**536.3**	**320.9**	**167**	**20**
	2	**1969.3**	**1763.1**	**1629.9**	**1363.8**	**1203.2**	**990**	**846.2**	**625.1**	**402**	**233.6**	**20**

Table B.3.33: Average MTTRep values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
norm	1	**1772.4**	**1562.9**	**1435.2**	**1209.9**	**1081.9**	**875.6**	**760.3**	**558**	**335.1**	**182.3**	**20**
	2	**1828.4**	**1615.7**	**1472.8**	**1282.3**	**1131.8**	**941.1**	**801.4**	**582.4**	**355.8**	**204.9**	**20**
	3	**2000.7**	**1784.5**	**1649.2**	**1415.7**	**1266.5**	**1043.3**	**896.5**	**662**	**441**	**260.9**	**20**
lnorm	1	**1858.6**	**1569.7**	**1412.6**	**1134.2**	**985.1**	**773**	**641**	**464.3**	**261.9**	**130**	**20**
	2	**2077.4**	**1825**	**1671.7**	**1419.4**	**1228.3**	**955.8**	**839.2**	**585.2**	**391.1**	**247.9**	**20**
logis	1	**1656.7**	**1462.3**	**1321.1**	**1124.9**	**999.3**	**816.1**	**691.7**	**503.9**	**309.9**	**181.5**	**20**
exp	1	**1816.5**	**1594.7**	**1444.5**	**1231.1**	**1091.2**	**886.1**	**732**	**529.5**	**333.5**	**190.5**	**20**
weibull	1	**1871**	**1623.5**	**1461.3**	**1219.2**	**1057.8**	**817.7**	**668.8**	**446.6**	**253.4**	**144.1**	**20**
	2	**1906.1**	**1694.4**	**1557.1**	**1332.4**	**1180.4**	**941.4**	**808.7**	**583**	**358.8**	**200.7**	**20**
gamma	1	**1800**	**1566.3**	**1389.3**	**1146.3**	**993.6**	**795.8**	**648.8**	**457.1**	**241.4**	**111.6**	**20**
	2	**1862.1**	**1667.5**	**1548.7**	**1316.6**	**1157.8**	**927.3**	**789.6**	**577.3**	**356.2**	**199**	**20**

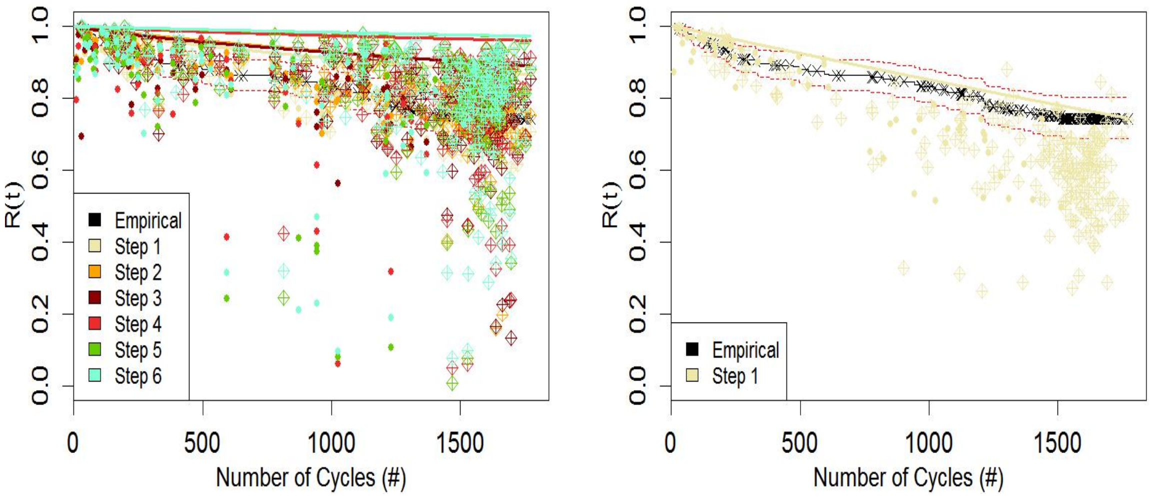
Table B.3.34: Overview of time-dependent PHM results obtained by MLE optimisation and GOF tests.

Distribution	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm	logis	exp
Step #	**1**	**2**	**3**	**4**	**5**	**6**	**1**	**1**
MLE	**−518.48**	**−466.06**	**−416.86**	**−368.74**	**−336.04**	**−330.05**	**−545.34**	**−574.96**
Time (min)	**29.75**	**71.17**	**107.47**	**155.21**	**197.25**	**256.27**	**5.01**	**3.89**
Kolmogorov-Smirnov	**1.94**	**2.17**	**4.91**	**15.37**	**16.17**	**14.88**	**5.11**	**2.57**
Cramer-von Mises-Smirnov	**31.36**	**33.43**	**32.35**	**31.44**	**27.41**	**28.66**	**37.72**	**28.32**
Anderson-Darling	**−72.47**	**−72.46**	**−72.77**	**−75.76**	**−77.52**	**−76.66**	**−73.3**	**−72.61**
NRR	**116.78**	**96.49**	**76.09**	**32.55**	**23.79**	**0.5**	**110.29**	**242.09**

Distribution	weibull	weibull	weibull	gamma	gamma	gamma	gamma	gamma
Step #	**1**	**2**	**3**	**1**	**2**	**3**	**4**	**5**
MLE	**−532.59**	**−491.83**	**−469.76**	**−531.25**	**−499.06**	**−489.86**	**−485.57**	**−479.75**
Time (min)	**12.53**	**22.97**	**31.07**	**86.27**	**172.44**	**234.4**	**275.19**	**311.46**
Kolmogorov-Smirnov	**2.1**	**3.01**	**3.3**	**1.97**	**2.58**	**3.07**	**3.19**	**3.73**
Cramer-von Mises-Smirnov	**35.29**	**34.95**	**35.42**	**34.11**	**36.52**	**36.97**	**37.09**	**37.31**
Anderson-Darling	**−72.52**	**−72.57**	**−72.72**	**−72.51**	**−72.74**	**−73.33**	**−73.09**	**−73.52**
NRR	**179.01**	**276.37**	**106.74**	**389.45**	**241.09**	**309.54**	**301.98**	**218.21**

To assist in the selection of models, Tables B.3.37, B.3.38, B.3.39, and B.3.40 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.

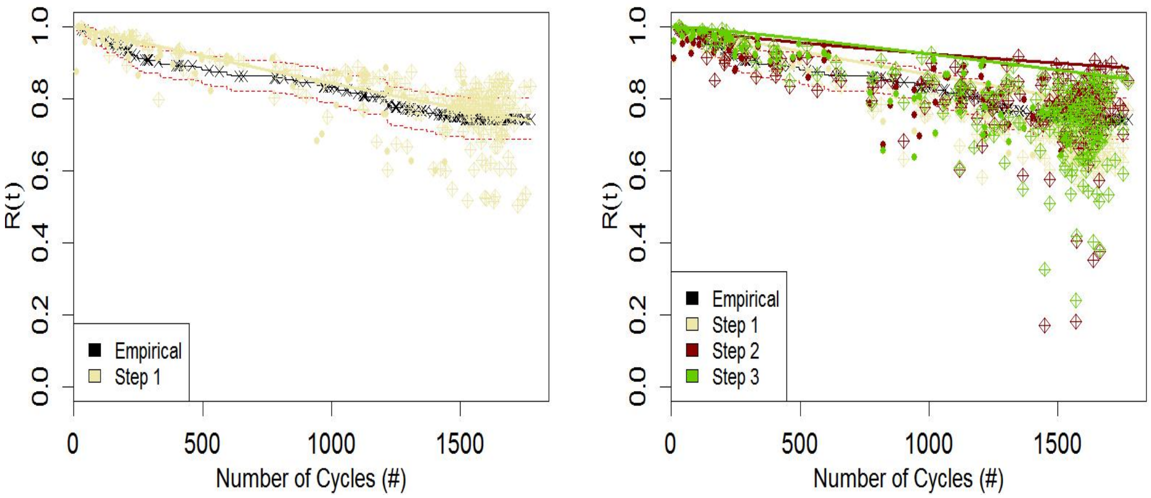
The operational factors identified during time-independent and time-dependent PHM modelling are shown in Tables B.3.41, B.3.42, B.3.43, and B.3.44.



(a) With an underlying lnorm distribution.

(b) With an underlying logis distribution.

Figure B.3.26: Time-dependent PHMs with an underlying lnorm and logis distribution.



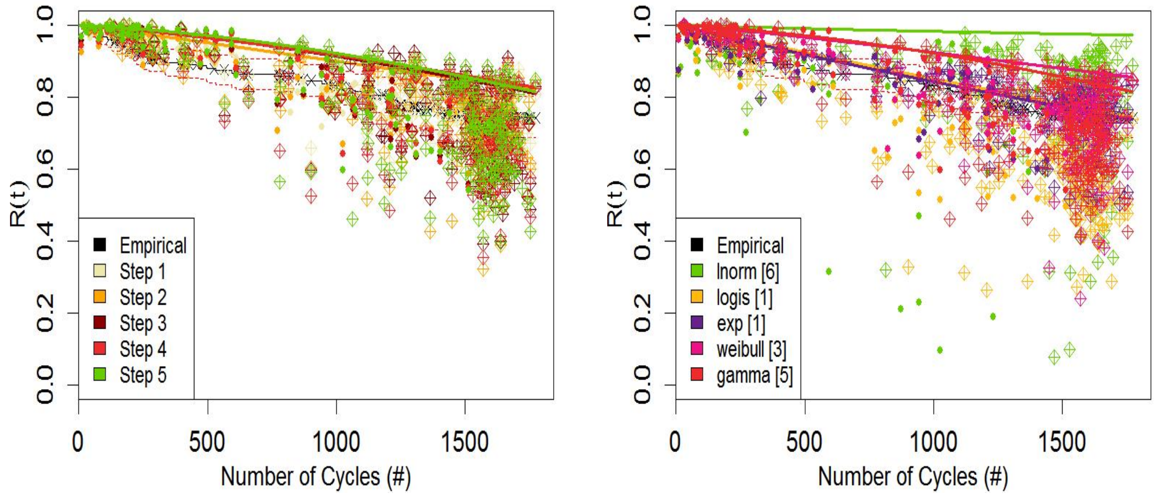
(a) With an underlying exp distribution.

(b) With an underlying weibull distribution.

Figure B.3.27: Time-dependent PHMs with an underlying exp and weibull distribution.

Table B.3.35: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
lnorm	1	8 [69]	15 [54]	20 [43]	31 [32]	36 [23]	43 [16]	44 [13]	56 [7]	67 [5]	80 [3]	97 [0]
	2	7 [82]	13 [68]	15 [54]	26 [31]	30 [26]	38 [17]	43 [14]	52 [10]	59 [7]	75 [5]	97 [0]
	3	5 [79]	8 [72]	16 [65]	16 [54]	21 [39]	38 [25]	51 [18]	56 [11]	62 [7]	77 [4]	97 [0]
	4	7 [84]	11 [76]	16 [76]	20 [65]	21 [57]	28 [44]	33 [35]	48 [22]	59 [10]	64 [6]	97 [0]
	5	3 [84]	7 [79]	10 [73]	13 [65]	16 [61]	21 [44]	25 [39]	41 [24]	54 [15]	66 [8]	97 [0]
	6	10 [81]	11 [79]	11 [76]	16 [69]	20 [60]	25 [45]	30 [36]	44 [22]	59 [11]	67 [8]	97 [0]
logis	1	18 [28]	28 [19]	31 [18]	36 [17]	41 [15]	43 [10]	44 [10]	52 [8]	64 [6]	72 [4]	97 [0]
exp	1	8 [80]	11 [59]	15 [43]	18 [30]	26 [23]	39 [18]	43 [13]	57 [12]	62 [8]	77 [4]	97 [0]
weibull	1	7 [45]	16 [32]	20 [27]	28 [19]	36 [18]	41 [15]	43 [12]	48 [10]	62 [8]	70 [5]	97 [0]
	2	8 [82]	11 [68]	15 [56]	25 [38]	28 [30]	38 [18]	39 [16]	49 [12]	64 [8]	77 [4]	97 [0]
	3	11 [67]	16 [55]	18 [42]	25 [31]	30 [26]	33 [15]	38 [15]	43 [13]	57 [11]	61 [9]	97 [0]
gamma	1	3 [70]	7 [56]	15 [44]	20 [26]	26 [23]	36 [18]	41 [16]	43 [12]	56 [9]	64 [6]	97 [0]
	2	2 [59]	5 [49]	5 [39]	15 [26]	23 [22]	31 [18]	36 [14]	41 [13]	44 [9]	54 [6]	97 [0]
	3	0 [53]	2 [43]	3 [41]	5 [35]	8 [29]	18 [22]	21 [19]	33 [15]	41 [12]	43 [8]	97 [0]
	4	0 [52]	2 [42]	3 [38]	5 [33]	11 [30]	20 [20]	26 [17]	38 [14]	41 [11]	44 [8]	97 [0]
	5	0 [47]	2 [42]	3 [41]	5 [35]	10 [29]	18 [24]	23 [18]	28 [15]	41 [12]	43 [9]	97 [0]



(a) With an underlying gamma distribution.

(b) All distributions (last step).

Figure B.3.28: Figures containing a gamma distribution and all time-dependent PHMs.

Table B.3.36: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
	1	8 [67]	16 [53]	20 [42]	31 [32]	36 [23]	43 [16]	44 [13]	56 [6]	72 [5]	84 [2]	97 [0]
	2	7 [80]	13 [66]	16 [53]	26 [31]	30 [24]	39 [17]	44 [13]	54 [10]	66 [7]	77 [3]	97 [0]
lnorm	3	5 [79]	8 [70]	16 [64]	18 [51]	25 [38]	43 [24]	52 [18]	59 [9]	69 [6]	82 [2]	97 [0]
	4	7 [84]	11 [76]	16 [76]	20 [64]	21 [57]	28 [44]	33 [34]	49 [22]	59 [10]	66 [6]	97 [0]
	5	3 [84]	7 [79]	10 [74]	13 [65]	16 [62]	21 [46]	25 [39]	41 [24]	54 [16]	64 [8]	97 [0]
	6	10 [81]	11 [79]	11 [76]	16 [69]	20 [61]	25 [47]	30 [36]	41 [24]	59 [11]	67 [8]	97 [0]
logis	1	16 [28]	26 [20]	31 [18]	36 [17]	41 [15]	43 [12]	43 [10]	52 [8]	64 [6]	72 [5]	97 [0]
exp	1	8 [78]	11 [58]	15 [42]	18 [30]	26 [23]	39 [17]	43 [13]	57 [12]	62 [8]	79 [4]	97 [0]
	1	7 [45]	16 [33]	20 [27]	28 [19]	34 [18]	41 [15]	43 [12]	48 [10]	61 [8]	69 [5]	97 [0]
weibull	2	8 [82]	11 [68]	15 [56]	25 [38]	28 [30]	38 [18]	39 [16]	49 [12]	64 [8]	77 [4]	97 [0]
	3	11 [66]	16 [55]	18 [42]	25 [31]	30 [24]	33 [15]	38 [15]	43 [13]	57 [11]	61 [9]	97 [0]
	1	3 [70]	7 [57]	15 [44]	18 [26]	26 [24]	36 [18]	39 [16]	43 [12]	56 [9]	64 [6]	97 [0]
	2	2 [59]	5 [49]	5 [39]	15 [26]	21 [22]	31 [18]	36 [14]	41 [13]	44 [9]	52 [6]	97 [0]
gamma	3	0 [58]	0 [47]	3 [42]	3 [38]	5 [34]	16 [26]	20 [22]	30 [15]	41 [12]	43 [11]	97 [0]
	4	0 [54]	2 [46]	2 [39]	5 [35]	10 [32]	18 [21]	25 [20]	34 [15]	41 [12]	44 [10]	97 [0]
	5	0 [53]	0 [44]	2 [42]	3 [37]	7 [33]	15 [26]	21 [21]	28 [15]	39 [12]	41 [12]	97 [0]

Table B.3.37: Average MTTF values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
	1	541.1	481.4	445.1	362.4	298.9	246.2	229.3	167.4	120.5	69.7	15
	2	553.6	500.2	486.5	392.5	364.9	277.7	237.7	183.8	154.1	85.7	15
lnorm	3	567.4	538.9	486.8	486.8	447.3	301.4	214.3	172.4	147.4	79.6	15
	4	564.7	539.8	496.6	476.1	466.3	406.4	397.8	274.5	154.1	138.9	15
	5	589.4	569.5	536.5	516.1	492.3	453.3	418.1	337.1	209.7	124.1	15
	6	551	533.2	533.2	494.9	461.6	426.5	409	324.7	174.3	118.7	15
logis	1	472.3	391.5	362.5	311.2	241.7	233.4	222.8	172.1	125	100.1	15
exp	1	552.9	528.6	504.2	471.3	404.9	280.6	238.5	148.3	132.8	79.6	15
	1	558.9	489.9	452.5	385	309.3	241.7	233.4	199	131.3	104	15
weibull	2	557.1	534.2	508.6	429.9	398.2	303.6	284.2	204.8	130.1	83	15
	3	526.1	489.8	475.3	418.6	387.1	350	287.7	231.6	148.3	139	15
	1	584.3	558.9	503.9	452.5	400.1	309.3	241.7	233.4	154.9	125	15
	2	590.3	567.1	567.1	506.1	432.7	357.9	312.6	241.7	222.8	169.3	15
gamma	3	605.5	593.7	578.1	567.1	550	462.2	436.8	332.9	241.7	231.6	15
	4	605.5	590.3	578.1	567.1	521.8	443.7	396.2	283.7	241.7	222.8	15
	5	605.5	590.3	578.1	567.1	533.8	459.1	426	382.9	241.7	231.6	15

Table B.3.38: Average MTTF values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
	1	**541.1**	**465.7**	**445.1**	**362.4**	**298.9**	**246.2**	**229.3**	**167.4**	**103.8**	**57.9**	**15**
	2	**553.6**	**500.2**	**476**	**392.5**	**364.9**	**264.4**	**220.5**	**169.1**	**121.6**	**79.9**	**15**
lnorm	3	**567.4**	**538.9**	**486.8**	**474.1**	**427.1**	**280.9**	**210.1**	**154.1**	**110.9**	**62.8**	**15**
	4	**564.7**	**539.8**	**496.6**	**476.1**	**466.3**	**406.4**	**397.8**	**276.1**	**154.1**	**134.5**	**15**
	5	**589.4**	**569.5**	**536.5**	**516.1**	**492.3**	**453.3**	**418.1**	**337.1**	**209.7**	**133.7**	**15**
	6	**551**	**533.2**	**533.2**	**494.9**	**461.6**	**426.5**	**409**	**328.9**	**174.3**	**118.7**	**15**
logis	1	**489.9**	**403.8**	**362.5**	**311.2**	**241.7**	**233.4**	**233.4**	**172.1**	**125**	**100.1**	**15**
exp	1	**552.9**	**528.6**	**504.2**	**471.3**	**404.9**	**280.6**	**238.5**	**148.3**	**132.8**	**75.8**	**15**
	1	**558.9**	**489.9**	**452.5**	**385**	**331.9**	**241.7**	**233.4**	**199**	**139.8**	**109.4**	**15**
weibull	2	**557.1**	**534.2**	**508.6**	**429.9**	**398.2**	**303.6**	**284.2**	**204.8**	**130.1**	**83**	**15**
	3	**526.1**	**489.8**	**475.3**	**418.6**	**387.1**	**350**	**287.7**	**231.6**	**148.3**	**139**	**15**
	1	**584.3**	**558.9**	**503.9**	**472.3**	**400.1**	**309.3**	**262.8**	**233.4**	**154.9**	**125**	**15**
	2	**590.3**	**567.1**	**567.1**	**506.1**	**452.2**	**357.9**	**312.6**	**241.7**	**222.8**	**176.7**	**15**
gamma	3	**605.5**	**605.5**	**578.1**	**578.1**	**567.1**	**478.2**	**443.7**	**364.7**	**241.7**	**231.6**	**15**
	4	**605.5**	**590.3**	**590.3**	**567.1**	**531.7**	**462.2**	**414.3**	**314.4**	**241.7**	**222.8**	**15**
	5	**605.5**	**605.5**	**590.3**	**578.1**	**559**	**493.1**	**433.2**	**382.9**	**267.7**	**241.7**	**15**

Table B.3.39: Average MTTRep values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
	1	**1738.9**	**1463.9**	**1287.1**	**1038.2**	**895.1**	**690.4**	**571**	**397.6**	**244.2**	**155.1**	**20**
	2	**1823.1**	**1567.6**	**1407.2**	**1162.9**	**1014.6**	**799.7**	**663.9**	**470.8**	**299.4**	**193.1**	**20**
lnorm	3	**2212.1**	**1908**	**1668.6**	**1374.4**	**1176.5**	**903.3**	**727.2**	**495.6**	**293.1**	**180.1**	**20**
	4	**2651.2**	**2368.3**	**2158.5**	**1839.3**	**1622.9**	**1278.1**	**1031.3**	**725.3**	**435.3**	**274.4**	**20**
	5	**2712**	**2424.2**	**2226.9**	**1899.5**	**1697**	**1363.9**	**1158.4**	**815.6**	**513.8**	**322.4**	**20**
	6	**2500.9**	**2263.7**	**2099.5**	**1809.9**	**1613.8**	**1315.1**	**1092.3**	**767.8**	**477.6**	**316.3**	**20**
logis	1	**1012.1**	**899**	**822.5**	**718.2**	**655.4**	**543.7**	**467.6**	**358.4**	**245.4**	**165.3**	**20**
exp	1	**1684.4**	**1477.2**	**1340.6**	**1139.8**	**1007.7**	**810.1**	**681.5**	**485.9**	**297.2**	**176.4**	**20**
	1	**1359.9**	**1182.7**	**1076.8**	**920**	**826**	**681.3**	**580.9**	**437.5**	**289.6**	**189.4**	**20**
weibull	2	**1933**	**1655.7**	**1479**	**1204.1**	**1036.4**	**804.9**	**665**	**462.9**	**278.8**	**172.4**	**20**
	3	**1576.1**	**1410**	**1303.8**	**1138.3**	**1021.8**	**869.5**	**773.4**	**609.7**	**422.1**	**290**	**20**
	1	**1645**	**1447.3**	**1304.4**	**1112.4**	**983.6**	**812.2**	**698.8**	**520.8**	**348.1**	**227.6**	**20**
	2	**1473.3**	**1316.5**	**1213.4**	**1056.2**	**962.4**	**811.9**	**715.6**	**570**	**398.4**	**282**	**20**
gamma	3	**1429.8**	**1305.7**	**1227**	**1107.2**	**1022.6**	**906.7**	**813.1**	**681.2**	**525.6**	**392.3**	**20**
	4	**1422.6**	**1296.9**	**1216.1**	**1085.9**	**1001.5**	**876.1**	**783.8**	**647.5**	**487.2**	**358.7**	**20**
	5	**1395.6**	**1290.8**	**1219.6**	**1111.1**	**1036.5**	**925.9**	**834.2**	**697.5**	**555.2**	**422.7**	**20**

Table B.3.40: Average MTTRep values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	74%	77%	79%	82%	84%	87%	89%	92%	95%	97%	100%
	1	**1725.1**	**1448.1**	**1272.8**	**1024.4**	**880.8**	**677.9**	**559.5**	**382.9**	**228.9**	**141.9**	**20**
	2	**1806**	**1546.1**	**1380.8**	**1140**	**990.9**	**777.6**	**642**	**450.5**	**274.9**	**168.6**	**20**
lnorm	3	**2184.4**	**1874**	**1633.4**	**1338.2**	**1143.2**	**857.6**	**690.9**	**451.7**	**251.3**	**144.2**	**20**
	4	**2644.9**	**2358.8**	**2149.9**	**1827.6**	**1610.7**	**1264.3**	**1018.3**	**708.1**	**424**	**259.2**	**20**
	5	**2729**	**2439.8**	**2243.8**	**1922.7**	**1713.3**	**1390.8**	**1178.5**	**838.3**	**537**	**347.6**	**20**
	6	**2513.7**	**2274.9**	**2115**	**1828.9**	**1632.2**	**1334.3**	**1115.5**	**797.6**	**499.4**	**339.9**	**20**
logis	1	**1017.2**	**907.5**	**830.6**	**726.3**	**663.6**	**552.3**	**476.7**	**366.6**	**254.5**	**174.1**	**20**
exp	1	**1675.8**	**1468.1**	**1330.9**	**1130.2**	**997.7**	**799.4**	**670.7**	**475.4**	**286.8**	**165.8**	**20**
	1	**1369.2**	**1192**	**1085.8**	**928.4**	**831.4**	**690.2**	**590**	**447**	**298.5**	**198.8**	**20**
weibull	2	**1934.2**	**1657.8**	**1481.3**	**1206.4**	**1038.6**	**807.2**	**667.4**	**465.4**	**281.4**	**175.1**	**20**
	3	**1573.7**	**1407.7**	**1301.5**	**1135.9**	**1019.1**	**867.1**	**770.8**	**607.1**	**419.1**	**287.2**	**20**
	1	**1653.7**	**1456.1**	**1313.8**	**1118.6**	**993.1**	**820.3**	**705.4**	**530.8**	**357.1**	**237.2**	**20**
	2	**1477.2**	**1320.2**	**1216.6**	**1059.4**	**962.4**	**815.1**	**718.9**	**572.9**	**401.8**	**284.6**	**20**
gamma	3	**1486.7**	**1364.6**	**1283.3**	**1161.3**	**1079.8**	**959.8**	**872**	**732.5**	**583.4**	**452.4**	**20**
	4	**1473.6**	**1350.7**	**1263.7**	**1138.3**	**1052.6**	**925.6**	**832**	**698.1**	**539.8**	**410.9**	**20**
	5	**1450.2**	**1339.8**	**1272.8**	**1161.8**	**1086.9**	**974.3**	**890.8**	**751.5**	**606.3**	**477.2**	**20**

Table B.3.41: Variables identified by time-(in)dependent PHM models.

	Time-independent PHM			Time-dependent PHM	
	Variable	Scaled Value		Variable	Scaled Value
①	*Group A*	**9.1**	①	*Group G*	**28.3**
②	*Pitch mean deg*	**3.7**	②	*Accn long mean g s*	**20.54**
③	*Group D*	$\mathbf{-2.59}$	③	*Group F*	**17.93**
④	*Group B*	**1.54**	④	*Group E*	**12.36**
⑤	*Group C*	$\mathbf{-0.9}$	⑤	*Group A*	**9.39**
			⑥	*Elevator Lin mean deg TEU*	$\mathbf{-3.18}$
			⑦	*Group C*	$\mathbf{-3.12}$
			⑧	*Group B*	**1.83**

Table B.3.42: Variables identified by each step by time-(in)dependent PHMs (in order).

	PHM Variables	
	Time-independent	*Time-dependent*
norm	① ② ④	
lnorm	① ②	② ④ ③ ① ⑤ ⑧
logis	⑤	①
exp	③	⑦
weibull	② ①	① ④ ③
gamma	① ②	① ④ ⑤ ⑥ ③

Table B.3.43: Number of times variables identified by each step by time-(in)dependent PHMs.

Key				Key			
indep	*dep*	*Variable*	*Count*	*indep*	*dep*	*Variable*	*Count*
①	⑤	Group A	6		③	Group F	3
④	⑧	Group B	2		①	Group G	4
⑤	⑦	Group C	2		②	Accn long mean g s	1
③		Group D	1		⑥	Elevator Lin mean deg TEU	1
	④	Group E	3	②		Pitch mean deg	4

Table B.3.44: Variables belonging to each group identified in B.3.41.

Group	*Variables*
Group A	Accn norm mean g s 1, Pitch rate mean deg sec 1
Group B	Pitch cmd FO force mean lbs Nose up 3, Pitch cmd FO force mean lbs Nose up 4, Pitch cmd FO force mean lbs Nose up 8
Group C	Roll cmd FO force mean lbs RWD 6, Roll cmd FO force mean lbs RWD 5, Roll cmd FO force mean lbs RWD 3
Group D	Brake press rhs mean psi 5, Brake press rhs mean psi , Brake press rhs mean psi 6
Group E	Brake press rhs mean psi 8, Brake press lhs mean psi 8
Group F	Elevator Lin max deg TEU 4, Elevator Lin mean deg TEU 4
Group G	Accn lat mean g s 5, Accn lat mean g s 6

B.4 903-1342 Hand microphone

Table B.4.45 provides a summary of the input data related to the component. The number of registered maintenance events is less than the total number of events due to the fact that TRAX data stretches back to 2004/2005 and FDR data only to 2011. Maintenance events with insufficient data, regarding operational factors, cannot be evaluated, hence are not registered during the modelling process.

Table B.4.45: General overview of component inputs.

Name	**Value**
Part Number	903-1342
Total # (A, F, C)	199, 188, 11
Registered # (A, F, C)	33, 31, 2
Related Flights # (A, F, C)	52862, 50509, 2353
Avg. Cycles (A, F, C)	1601.88, 1629.32, 1176.5
% Censored	6.06

In Tab. B.4.45 (A, F, C) denotes statistics regarding All (A), Failed (F), and Censored (C) events respectively. Ergo A will always be the sum or mean derived from F and C.

Analysis
Tables B.4.46 and B.4.47 summarise the results from EVA and MDA. In addition the variables obtained by semi-parametric PHM modelling (labelled 'reduced semi-COX') are also presented if applicable. Table B.4.47 provides an overview of the specific operational factors identified during all flight phases. In this case high counts indicate operational factors that were significantly different during multiple flight phases.

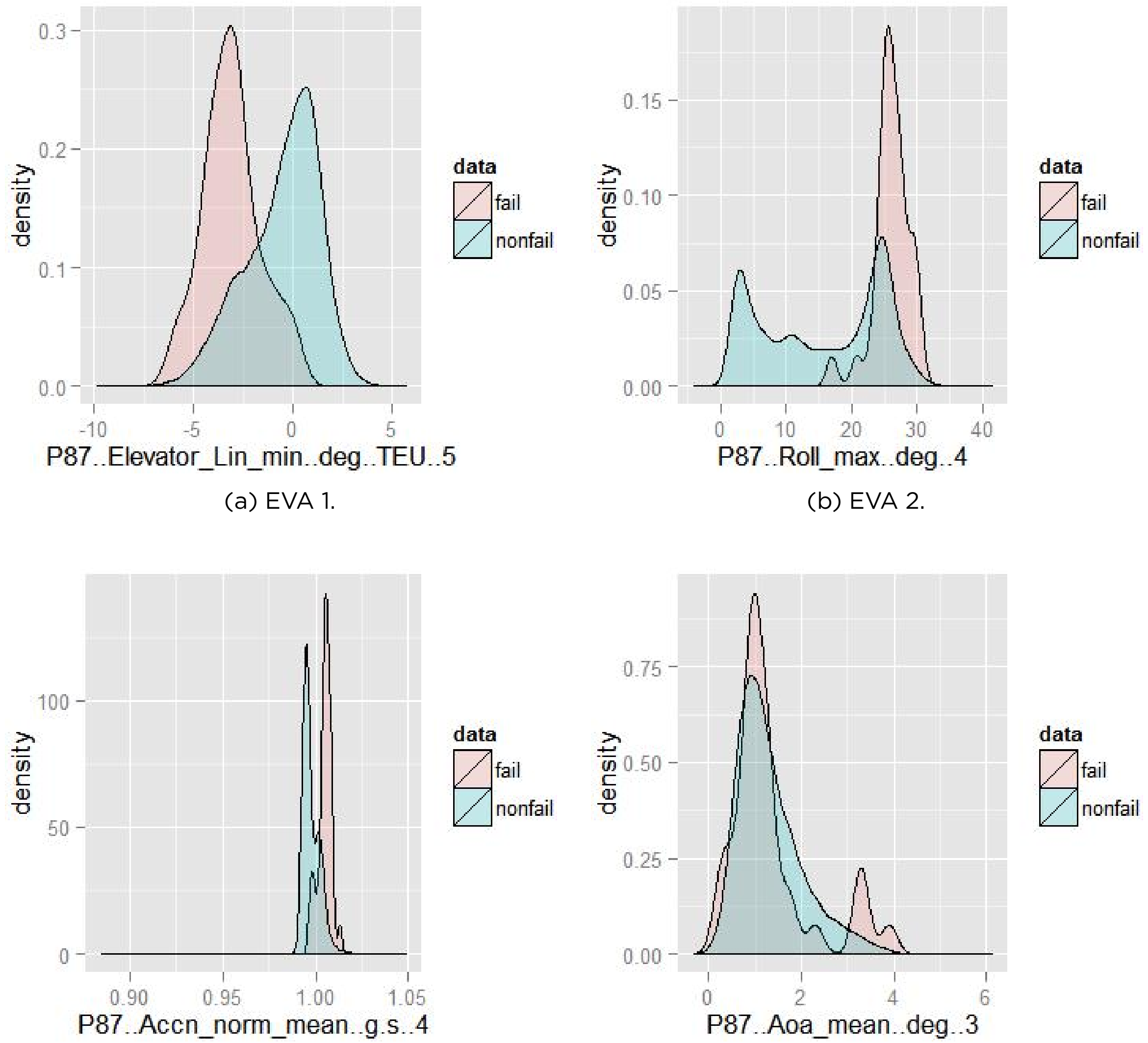
Table B.4.46: Overview of analysis input and output.

	# Variables
ALL	1531
EVA	65
MDA	20
Combined	85
reduced Corr.	74
reduced semi-COX	0
Take-Off related	23
Cruise related	24
Touch-Down related	27

Table B.4.47: Overview of identified operational factors during all flight phases.

Variable	**Count**	**Variable**	**Count**	**Variable**	**Count**
Roll_rate	7	NormalForce_rhs	2	Prop_spd_rhs	1
Roll	6	Pitch_rate	2	Crosswind	1
Rudder_low	5	Prop_spd_lhs	2	Accn_lat	1
Torque_rhs	5	Ttot	2	Pressure_total	1
Headwind	4	Accn_norm	1	Brake_press_lhs	1
Yaw_rate	4	Vcal	1	NormalForce_nose	1
Aoa	4	Vtrue	1	Pitch	1
Rudder_cmd_force	4	Drift	1	Roll_cmd_FO_force	1
Elevator_Rin	3	Accn_long	1	Aileron_Rin	1
Elevator_Lin	3	Pitch_cmd_FO_force	1	Brake_press_rhs	1
Pressure_dynamic	2	NormalForce_lhs	1		

A multitude of factors were identified during EVA and MDA. Figure B.4.29 give a general overview of the top operational factors identified by EVA and MDA.



(a) EVA 1. (b) EVA 2.

(c) MDA 1. (d) MDA 2.

Figure B.4.29: Graphical overview of top operational factors identified by EVA and MDA.

Time-based reliability modelling

Table B.4.48 reports the maximum likelihood and goodness-of-fit tests results obtained from time-based reliability modelling. To show the overall fit Fig. B.4.30 shows the computed reliability function using an averaged virtual age V for all fitted models.

Table B.4.48: Overview of results obtained by MLE optimisation and GOF tests.

	Distributions				
	norm	**lnorm**	**logis**	**exp**	**weibull**
MLE	**−262.11**	**−265.53**	**−261.83**	**−261.69**	**−261.76**
Kolmogorov-Smirnov	**1.39**	**0.99**	**1.04**	**0.62**	**0.71**
Cramer-von-Mises Smirnov	**20.98**	**21.26**	**21.11**	**21.05**	**20.91**
Anderson-Darling	**−63.82**	**−62.93**	**−62.93**	**−62.32**	**−62.43**
NRR	**27.36**	**14.52**	**22.93**	**20.12**	**21.52**

In addition Figures B.4.31, B.4.32, B.4.33, B.4.34, and B.4.35 present the reliability and hazard functions computed for each underlying distribution evaluated in the program.

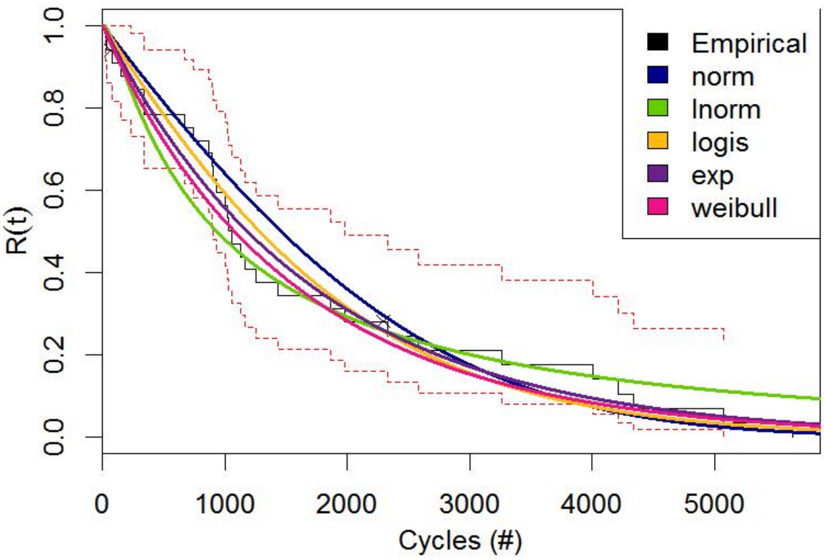
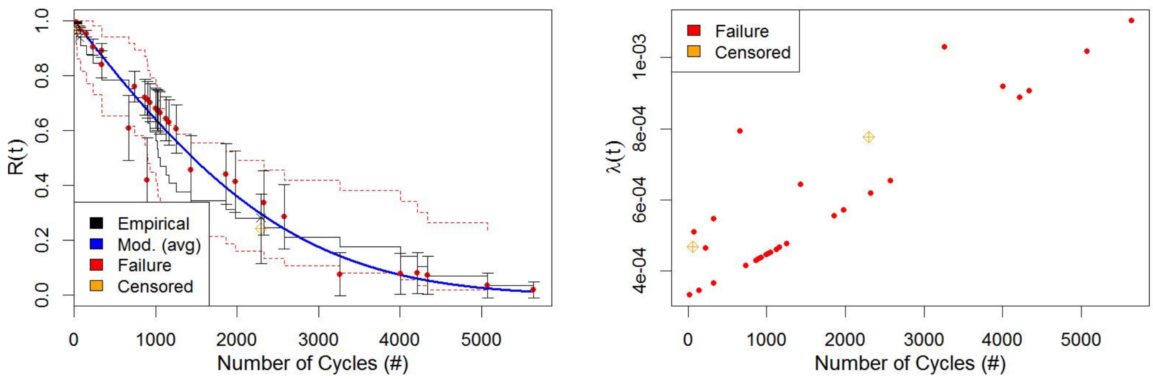
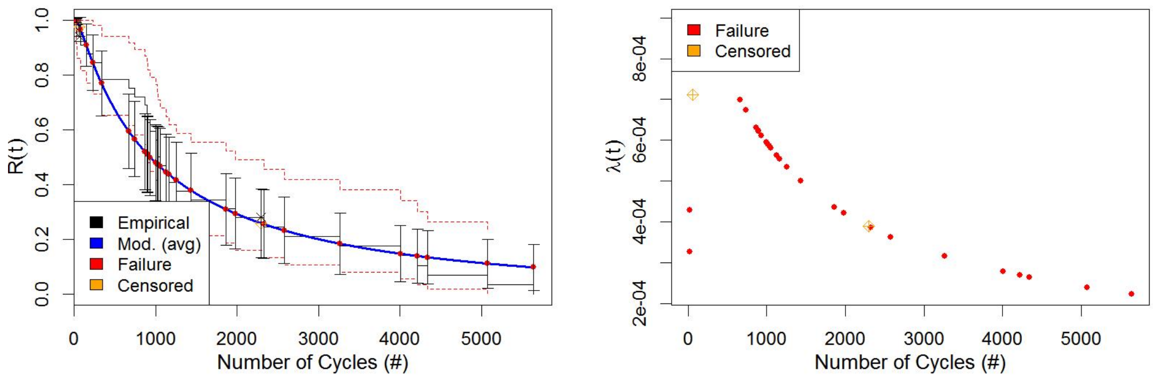


Figure B.4.30: Overview of overall fit of multiple GRP modells.



(a) Reliability function. (b) Hazard function.

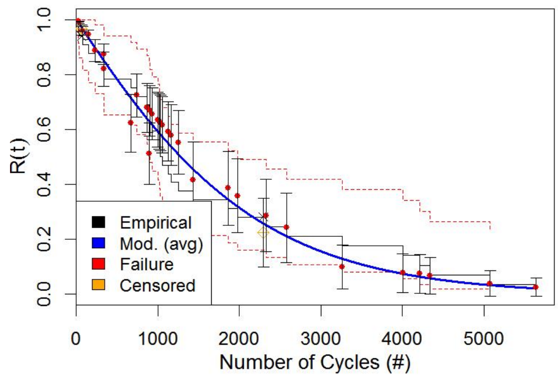
Figure B.4.31: Computed reliability for time-based models with underlying norm distribution.



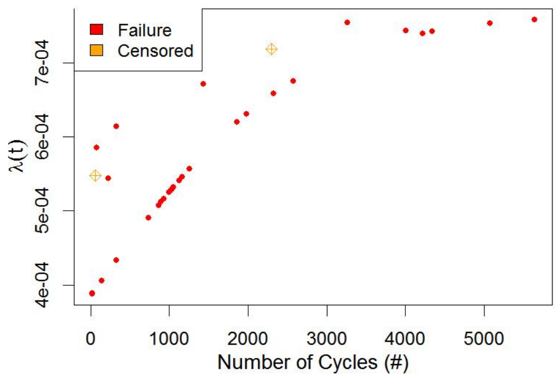
(a) Reliability function. (b) Hazard function.

Figure B.4.32: Computed reliability for time-based models with underlying lnorm distribution.

Time independent proportional hazard reliability modelling
Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.4.49 gives a general overview of all the models obtained during each

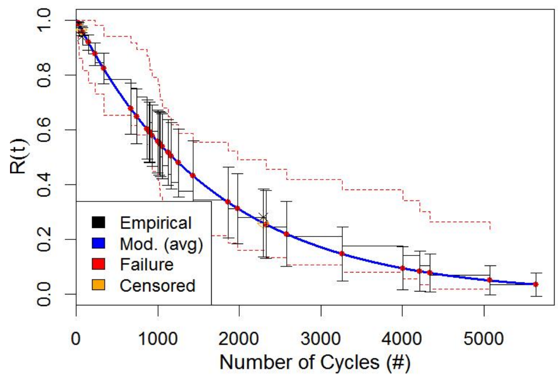


(a) Reliability function.

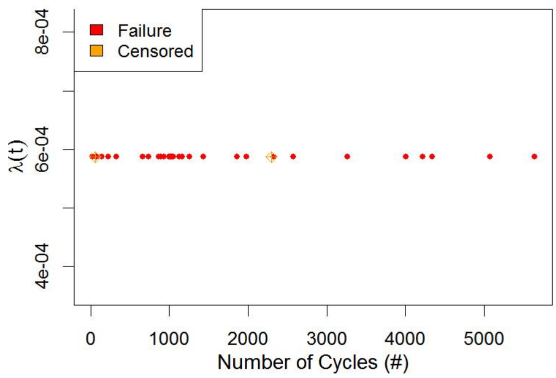


(b) Hazard function.

Figure B.4.33: Computed reliability for time-based models with underlying logis distribution.

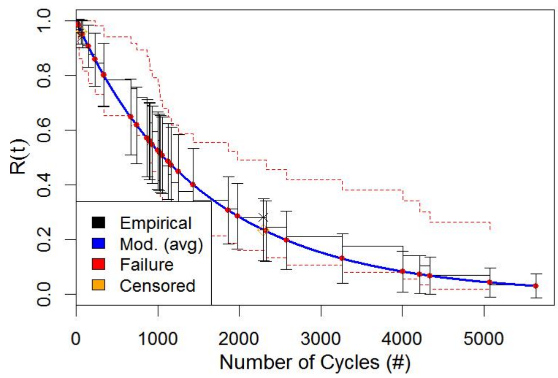


(a) Reliability function.

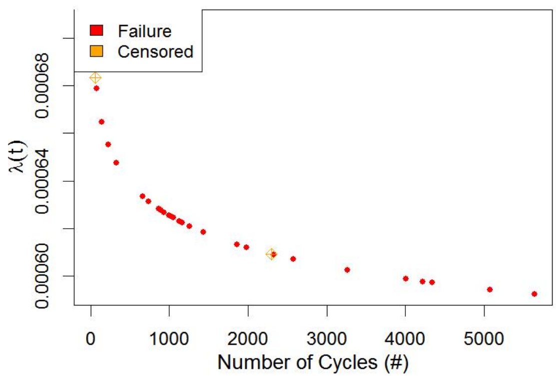


(b) Hazard function.

Figure B.4.34: Computed reliability for time-based models with underlying exp distribution.



(a) Reliability function.



(b) Hazard function.

Figure B.4.35: Computed reliability for time-based models with underlying weibull distribution.

step in the process.

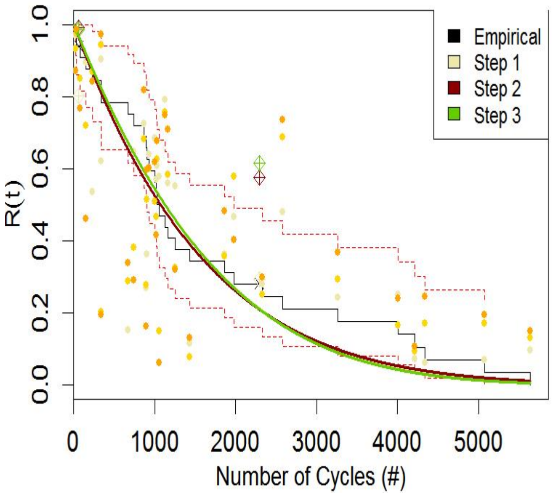
Graphical representation of the computed reliability per model are shown in Figures B.4.36, B.4.37, and B.4.38 as well as a general overview in Figure B.4.38b.

Tables B.4.50 and B.4.51 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

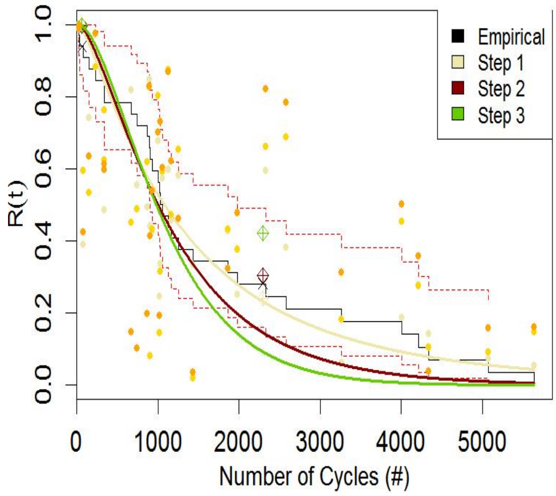
Table B.4.49: Overview of time-independent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	norm	norm	lnorm	lnorm	lnorm	logis	exp
Step #	1	2	3	1	2	3	1	1
MLE	−257.11	−253.27	−252.03	−259.72	−253.43	−248.3	−256.91	−257.02
Time (min)	0.63	1.36	1.93	0.58	1.03	1.42	0.6	0.47
Kolmogorov-Smirnov	3.47	3.36	3.4	3	3.17	3.58	2.61	2.1
Cramer-von Mises-Smirnov	20.08	19.22	19.07	20.37	19.48	18.55	20.4	19.95
Anderson-Darling	−67.43	−69.62	−72.52	−66.84	−72.84	−77.42	−65.73	−66.19
NRR	14.09	9.67	10.99	12.69	16.8	17.53	13.5	14.32

Distribution	weibull	weibull
Step #	1	2
MLE	−256.62	−253.62
Time (min)	0.71	1.29
Kolmogorov-Smirnov	2.33	3.41
Cramer-von Mises-Smirnov	20.23	19.65
Anderson-Darling	−66.78	−69.33
NRR	16.5	11.61

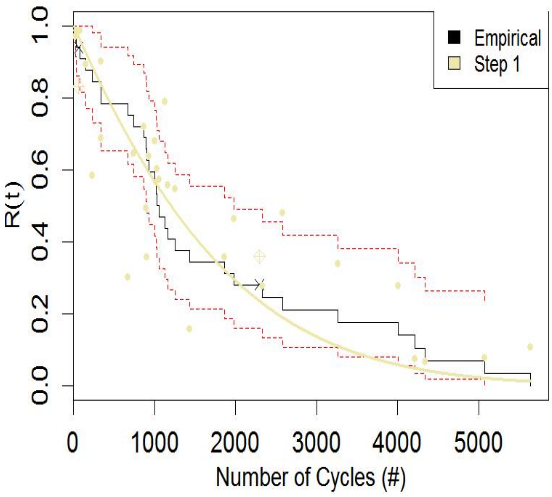


(a) With an underlying norm distribution.

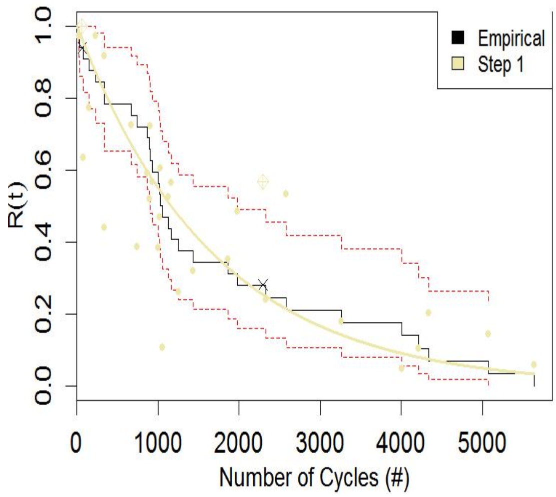


(b) With an underlying lnorm distribution.

Figure B.4.36: Time-independent PHMs with an underlying norm and lnorm distribution.



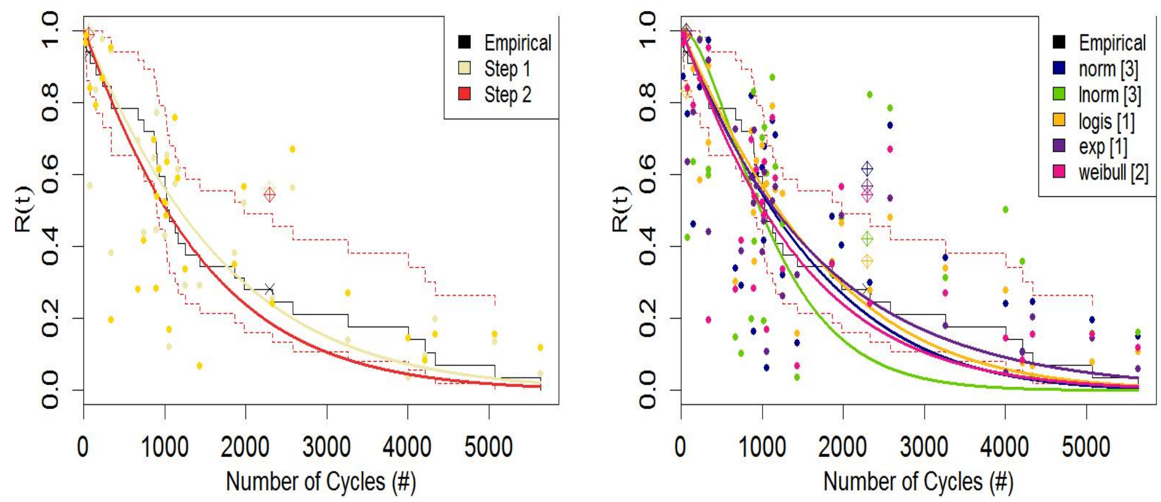
(a) With an underlying logis distribution.



(b) With an underlying exp distribution.

Figure B.4.37: Time-independent PHMs with an underlying logis and exp distribution.

To assist in the selection of models, Tables B.4.52, B.4.53, B.4.54, and B.4.55 indicate the



(a) With an underlying weibull distribution. (b) All distributions (last step).

Figure B.4.38: Figures containing a weibull distribution and all time-independent PHMs.

averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.

Table B.4.50: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
	1	0 [100]	13 [100]	16 [100]	29 [50]	32 [50]	39 [50]	55 [50]	65 [50]	84 [50]	84 [50]	94 [0]
norm	2	0 [100]	3 [100]	19 [100]	29 [100]	39 [100]	48 [100]	61 [50]	77 [50]	81 [50]	84 [50]	94 [50]
	3	0 [100]	0 [100]	13 [100]	29 [100]	35 [100]	45 [100]	55 [50]	61 [50]	81 [50]	87 [50]	94 [50]
	1	0 [100]	3 [100]	13 [100]	19 [50]	29 [50]	39 [50]	52 [50]	68 [50]	71 [50]	84 [50]	90 [50]
lnorm	2	0 [100]	3 [100]	16 [100]	19 [100]	23 [50]	32 [50]	42 [50]	58 [50]	71 [50]	84 [50]	94 [50]
	3	0 [100]	6 [100]	10 [100]	16 [100]	29 [100]	35 [50]	42 [50]	48 [50]	65 [50]	81 [50]	90 [50]
logis	1	0 [100]	13 [100]	16 [100]	23 [100]	29 [50]	35 [50]	58 [50]	65 [50]	84 [50]	84 [50]	90 [0]
exp	1	0 [100]	10 [100]	19 [100]	29 [100]	39 [100]	45 [100]	65 [50]	74 [50]	84 [50]	84 [50]	94 [50]
weibull	1	0 [100]	10 [100]	19 [100]	26 [100]	35 [100]	42 [100]	52 [50]	74 [50]	81 [50]	84 [50]	94 [50]
	2	0 [100]	6 [100]	19 [100]	29 [100]	39 [100]	45 [100]	55 [50]	74 [50]	81 [50]	84 [50]	94 [50]

Table B.4.51: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
	1	0 [100]	13 [100]	16 [100]	29 [50]	39 [50]	48 [50]	61 [50]	77 [50]	84 [50]	90 [50]	94 [0]
norm	2	0 [100]	26 [100]	39 [100]	52 [100]	65 [100]	71 [50]	81 [50]	87 [50]	90 [50]	90 [50]	94 [0]
	3	0 [100]	35 [100]	48 [100]	65 [100]	74 [100]	81 [50]	84 [50]	87 [50]	90 [50]	90 [0]	94 [0]
	1	0 [100]	13 [100]	23 [100]	26 [50]	39 [50]	55 [50]	65 [50]	77 [50]	81 [50]	90 [50]	94 [50]
lnorm	2	0 [100]	23 [100]	32 [100]	42 [50]	42 [50]	55 [50]	61 [50]	81 [50]	90 [50]	94 [50]	94 [0]
	3	0 [100]	23 [100]	29 [100]	45 [100]	58 [50]	61 [50]	65 [50]	74 [50]	87 [50]	90 [50]	90 [0]
logis	1	0 [100]	13 [100]	16 [100]	23 [100]	39 [50]	48 [50]	61 [50]	71 [50]	84 [50]	90 [50]	94 [0]
exp	1	0 [100]	16 [100]	39 [100]	48 [100]	58 [100]	74 [100]	84 [50]	87 [50]	87 [50]	90 [50]	94 [50]
weibull	1	0 [100]	19 [100]	35 [100]	48 [100]	61 [100]	71 [100]	77 [50]	87 [50]	87 [50]	90 [50]	94 [50]
	2	0 [100]	16 [100]	35 [100]	55 [100]	61 [100]	71 [50]	77 [50]	87 [50]	90 [50]	90 [50]	94 [0]

Time dependent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.4.56 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.4.39, B.4.40, and B.4.41 as well as a general overview in Figure B.4.41b.

Tables B.4.57 and B.4.58 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the

Table B.4.52: Average MTTF values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
	1	1629.3	1156.9	1146.3	888.3	841.9	779.4	564.1	443.9	119.6	119.6	23.5
norm	2	1629.3	1543.1	1032	870.8	797.3	795.5	775	278.6	137.5	150.2	23.5
	3	1629.3	1629.3	1278.7	978.6	850.5	787.9	744.1	707.3	269.5	151.5	23.5
	1	1629.3	1514.4	1192.1	1086.8	836.2	793.1	665.5	371.3	301.9	136.2	91.3
lnorm	2	1629.3	1635.9	1269.3	1189.5	1063.4	881.5	810.3	794.8	447.4	136	23.5
	3	1629.3	1542.7	1561.5	1269.3	1087.2	1069.5	896.3	811.9	798.1	137.5	91.3
logis	1	1629.3	1156.9	1146.3	978	833.9	785.1	526.6	443.9	119.6	119.6	40.3
exp	1	1629.3	1308.9	1090.3	878.9	791.8	781.8	520.9	298.6	150.2	150.2	23.5
weibull	1	1629.3	1308.9	1090.3	895.2	802	794.8	654.7	298.6	274.8	150.2	23.5
	2	1629.3	1401.8	1032	870.8	797.3	792.1	814.2	352	137.5	150.2	23.5

Table B.4.53: Average MTTF values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
	1	1629.3	1156.9	1146.3	888.3	779.2	644.8	492.8	351.6	119.6	40.3	23.5
norm	2	1629.3	1392.5	1012.3	834.6	847.6	705.6	688.7	131	126.3	126.3	23.5
	3	1629.3	1513.1	1407.9	1113.3	749.2	808.8	797.4	739.2	126.3	126.3	23.5
	1	1629.3	1315.2	1049.6	1057.6	798.2	648.9	656.5	330.1	361	91.3	23.5
lnorm	2	1629.3	1510	1234.4	1146	1146	897.7	930.3	416	91.3	23.5	23.5
	3	1629.3	1519.7	1334	1551.3	1213.5	981.1	964.3	1048.2	650.5	91.3	91.3
logis	1	1629.3	1156.9	1146.3	978	779.2	644.8	492.8	335.7	119.6	40.3	23.5
exp	1	1629.3	1328.8	915.8	830.6	788.6	376.5	150.2	131	131	126.3	23.5
weibull	1	1629.3	1379	986.5	878	843.2	680.3	382.7	131	131	126.3	23.5
	2	1629.3	1466.5	1008.3	830.4	838.5	705.6	751	131	126.3	126.3	23.5

Table B.4.54: Average MTTRep values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
	1	4117.5	3780.5	2669.3	2193.3	1698.2	1333.6	1097.2	842.4	567	324.6	70.4
norm	2	4117.5	3877.7	3471.8	2372.5	1634.7	1197.1	885.6	769.4	503.7	301.1	79.7
	3	4117.5	3823	3351.8	2389.9	2046.6	1418.4	1017.9	757.6	550.4	367.3	103.7
	1	4117.5	3747.7	3174.2	2332	1802.4	1199.4	994.4	846.8	640.9	402	167.7
lnorm	2	4117.5	2594.7	2830.1	2039.5	1911.2	1754.4	1352.1	961.1	820.8	584.9	243.5
	3	4117.5	3375.8	1598.2	2151.9	2013.6	1517.5	1376.5	1212.7	839	697.9	315.8
logis	1	4117.5	4045.8	2912.4	2430.9	1979.4	1497.2	1144	872.3	592.7	333.3	73.3
exp	1	4117.5	4121	3031.9	2251.3	1422.7	1063.9	849.7	619.5	410.4	205.3	80.8
weibull	1	4117.5	3890.8	2832.5	2363.9	1530.2	1125	907.9	703.1	473.2	303.2	90.2
	2	4117.5	4310.2	3324.4	2339.1	1627.9	1151.4	874.5	739.4	554.5	325.4	89.6

Table B.4.55: Average MTTRep values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

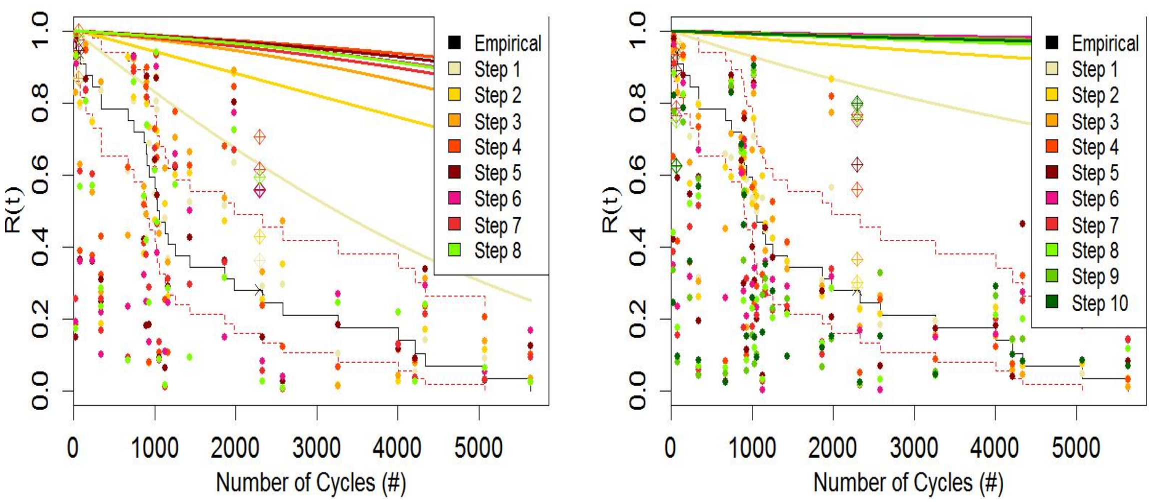
		Reliability Level										
Dist.	Step	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
	1	4117.5	3626	2544	2056.8	1523.3	1235.8	981.8	711.6	467.7	212.8	35.6
norm	2	4117.5	1851.4	1664.5	1225.6	679.3	565.4	375.1	277.9	170.6	88.7	32.9
	3	4117.5	1181.2	1025.9	874.2	770.5	485	334.4	259.2	181.7	122.8	37.4
	1	4117.5	3079.2	2482.6	1407.9	1267.9	1027.1	775.2	648.3	436.8	282.6	107.8
lnorm	2	4117.5	1875.1	1854.5	1223.8	960.5	1051.8	798.9	746.8	489	297.8	130.1
	3	4117.5	1652.9	1291.9	901.7	1007	972.9	848.6	620.9	511.7	382.6	150
logis	1	4117.5	3891.7	2792.4	2266.7	1699.6	1333	1023.5	774.8	484.1	226.3	36.4
exp	1	4117.5	2458.9	1818.6	1228.3	792.5	704.9	476.9	331.4	187.1	94.4	35.8
weibull	1	4117.5	1740.5	1760.7	1217.9	780.3	620.6	535	339	218.3	120.4	43.5
	2	4117.5	2507.7	1750.9	1249	849.7	637.1	431.7	357.3	193.3	104.4	37.1

model's effectiveness can be assessed.

To assist in the selection of models, Tables B.4.59, B.4.60, B.4.61, and B.4.62 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.
The operational factors identified during time-independent and time-dependent PHM modelling are shown in Tables B.4.63, B.4.64, B.4.65, and B.4.66.

Table B.4.56: Overview of time-dependent PHM results obtained by MLE optimisation and GOF tests.

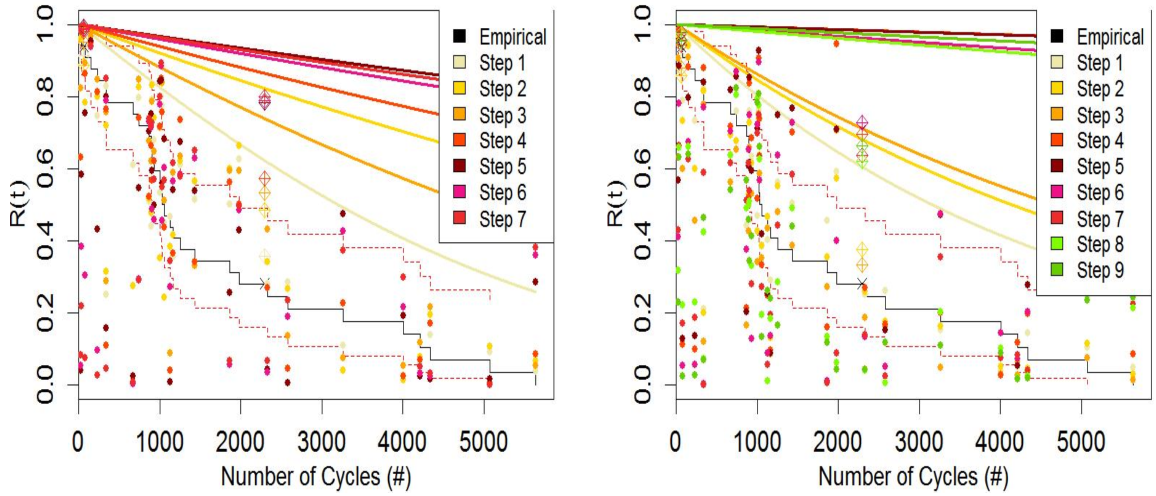
Distribution	norm	norm	norm	norm	norm	norm	norm	norm
Step #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
MLE	**−229.26**	**−196.98**	**−165.01**	**−141.87**	**−125.29**	**−114.75**	**−104.15**	**−98.91**
Time (min)	**17.73**	**39.89**	**72.43**	**111.18**	**137.03**	**165.88**	**195.22**	**222.8**
Kolmogorov-Smirnov	**3.18**	**3.48**	**3.54**	**3.29**	**3.71**	**3.21**	**3.52**	**3.55**
Cramer-von Mises-Smirnov	**20.4**	**19.99**	**19.57**	**19.25**	**18.79**	**18.39**	**18.74**	**18.96**
Anderson-Darling	**−66.45**	**−71.15**	**−72.74**	**−74.3**	**−74.58**	**−73.01**	**−74.98**	**−74.56**
NRR	**20.13**	**43.51**	**58.61**	**−187.52**	**108.31**	**270.67**	**1291.52**	**871.03**
Distribution	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm
Step #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
MLE	**−228.51**	**−198.98**	**−160.8**	**−133.9**	**−115.97**	**−101.87**	**−92.13**	**−89.12**
Time (min)	**12.07**	**36.83**	**57.37**	**89.54**	**139.22**	**193.38**	**254.61**	**328.22**
Kolmogorov-Smirnov	**2.89**	**3.39**	**2.92**	**3.16**	**2.81**	**3.33**	**3.47**	**3.45**
Cramer-von Mises-Smirnov	**20.58**	**20.26**	**20.32**	**19.46**	**19.12**	**18.61**	**17.07**	**16.71**
Anderson-Darling	**−63.91**	**−65.99**	**−66.06**	**−70.52**	**−72.12**	**−74.5**	**−75.62**	**−74.69**
NRR	**71.38**	**31.16**	**64.55**	**20.39**	**3288.48**	**6262**	**1346.17**	**2131.73**
Distribution	lnorm	lnorm	logis	logis	logis	logis	logis	logis
Step #	**9**	**10**	**1**	**2**	**3**	**4**	**5**	**6**
MLE	**−81.93**	**−76.71**	**−229.26**	**−197.19**	**−162.36**	**−133.62**	**−105.17**	**−97.26**
Time (min)	**415.96**	**510.5**	**5.87**	**15.39**	**23.89**	**38.06**	**59.52**	**76.18**
Kolmogorov-Smirnov	**3.64**	**3.99**	**3.25**	**2.87**	**4.32**	**4.28**	**5.35**	**5.26**
Cramer-von Mises-Smirnov	**14.13**	**14.61**	**20.4**	**19.57**	**19.82**	**19.32**	**18.57**	**17.45**
Anderson-Darling	**−75.52**	**−76.1**	**−66.38**	**−69.07**	**−74.5**	**−75.27**	**−75.89**	**−78.13**
NRR	**41.84**	**51.29**	**21.7**	**43.92**	**12854.07**	**184.86**	**75694.13**	**56900.68**
Distribution	logis	exp	exp	exp	exp	exp	exp	exp
Step #	**7**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
MLE	**−93.18**	**−229.23**	**−201.99**	**−177.18**	**−152.5**	**−130.48**	**−110.45**	**−99.16**
Time (min)	**89.97**	**1.86**	**3.86**	**5.25**	**7.07**	**9.69**	**13.28**	**17.78**
Kolmogorov-Smirnov	**5.07**	**3.48**	**3.51**	**3.88**	**4.31**	**4.09**	**4.55**	**3.8**
Cramer-von Mises-Smirnov	**16.88**	**20.34**	**20.27**	**19.67**	**18.7**	**16.98**	**15.41**	**15.02**
Anderson-Darling	**−81.24**	**−66.09**	**−67.05**	**−69.77**	**−76.45**	**−78.89**	**−78.27**	**−72.66**
NRR	**127952557160384320**	**18.72**	**21.67**	**27.44**	**26.01**	**2987.06**	**1355.02**	**8398.6**
Distribution	exp	exp	weibull	weibull	weibull	weibull	weibull	weibull
Step #	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**
MLE	**−93.09**	**−86.98**	**−229.13**	**−198.49**	**−165.59**	**−132.36**	**−117.51**	**−107.03**
Time (min)	**23.07**	**29.86**	**3.99**	**14.28**	**21.08**	**33.17**	**50.62**	**62.87**
Kolmogorov-Smirnov	**3.54**	**4.72**	**3.53**	**2.92**	**2.54**	**4.79**	**4.51**	**4.3**
Cramer-von Mises-Smirnov	**15.74**	**14.45**	**20.21**	**20.19**	**20.11**	**18.06**	**15.53**	**17.55**
Anderson-Darling	**−72.24**	**−74.45**	**−66.19**	**−67.5**	**−71.17**	**−76.53**	**−76.25**	**−76.44**
NRR	**6246.75**	**2370.93**	**24.05**	**16.14**	**1574457.3**	**201.64**	**5755.65**	**2925.94**
Distribution	weibull	weibull						
Step #	**7**	**8**						
MLE	**−103.33**	**−97.92**						
Time (min)	**69.99**	**75.31**						
Kolmogorov-Smirnov	**4.37**	**4.53**						
Cramer-von Mises-Smirnov	**13.63**	**14.27**						
Anderson-Darling	**−80.17**	**−82.5**						
NRR	**−4848.43**	**1893.4**						



(a) With an underlying norm distribution.

(b) With an underlying lnorm distribution.

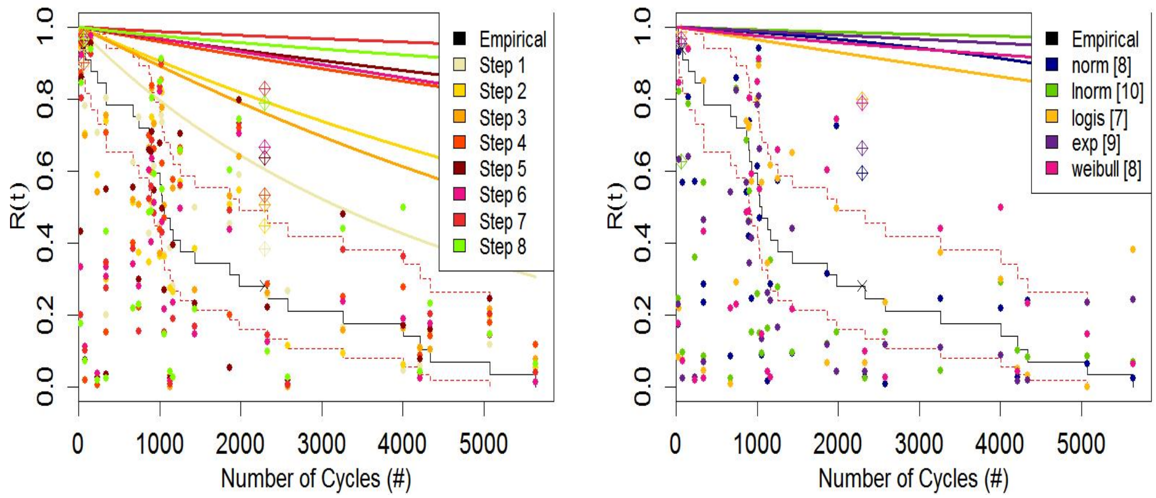
Figure B.4.39: Time-dependent PHMs with an underlying norm and lnorm distribution.



(a) With an underlying logis distribution.

(b) With an underlying exp distribution.

Figure B.4.40: Time-dependent PHMs with an underlying logis and exp distribution.



(a) With an underlying weibull distribution.

(b) All distributions (last step).

Figure B.4.41: Figures containing a weibull distribution and all time-dependent PHMs.

Table B.4.57: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
	1	0 [100]	16 [100]	19 [100]	19 [50]	29 [50]	29 [50]	52 [50]	61 [50]	65 [50]	81 [50]	90 [50]
	2	0 [100]	13 [100]	19 [100]	23 [100]	29 [100]	32 [50]	45 [50]	52 [50]	55 [50]	68 [50]	77 [50]
	3	0 [100]	13 [100]	19 [100]	19 [100]	23 [100]	26 [100]	32 [50]	39 [50]	58 [50]	61 [50]	87 [50]
norm	4	0 [100]	6 [100]	16 [100]	19 [100]	26 [100]	29 [100]	35 [100]	39 [100]	42 [50]	55 [50]	77 [50]
	5	0 [100]	3 [100]	6 [100]	16 [100]	26 [100]	32 [50]	35 [50]	42 [50]	52 [50]	71 [50]	84 [50]
	6	0 [100]	3 [100]	10 [100]	13 [100]	26 [100]	26 [50]	35 [50]	45 [50]	52 [50]	71 [50]	84 [50]
	7	0 [100]	0 [100]	0 [100]	13 [100]	19 [100]	23 [100]	32 [50]	39 [50]	45 [50]	61 [50]	84 [50]
	8	0 [100]	0 [100]	0 [100]	6 [100]	19 [100]	26 [100]	29 [50]	32 [50]	45 [50]	55 [50]	84 [50]
	1	0 [100]	16 [100]	19 [100]	26 [50]	29 [50]	35 [50]	52 [50]	68 [50]	77 [50]	81 [50]	90 [50]
	2	0 [100]	16 [100]	19 [100]	26 [50]	32 [50]	35 [50]	45 [50]	61 [50]	74 [50]	81 [50]	87 [50]
	3	0 [100]	13 [100]	19 [100]	23 [100]	32 [50]	39 [50]	42 [50]	45 [50]	71 [50]	81 [50]	87 [50]
	4	0 [100]	10 [100]	13 [100]	16 [100]	26 [100]	32 [100]	35 [50]	52 [50]	55 [50]	74 [50]	87 [50]
lnorm	5	0 [100]	3 [100]	10 [100]	13 [100]	19 [100]	35 [100]	39 [100]	45 [50]	58 [50]	68 [50]	84 [50]
	6	0 [100]	0 [100]	3 [100]	10 [100]	10 [100]	16 [100]	42 [100]	48 [100]	55 [50]	71 [50]	84 [50]
	7	0 [100]	0 [100]	0 [100]	3 [100]	10 [100]	13 [100]	26 [100]	48 [100]	55 [50]	68 [50]	84 [50]
	8	0 [100]	0 [100]	0 [100]	0 [100]	13 [100]	19 [100]	23 [100]	42 [100]	55 [50]	65 [50]	84 [50]
	9	0 [100]	0 [100]	0 [100]	0 [100]	3 [100]	13 [100]	26 [100]	42 [100]	55 [100]	61 [50]	87 [50]
	10	0 [100]	0 [100]	0 [100]	0 [100]	6 [100]	10 [100]	19 [100]	35 [100]	48 [100]	58 [50]	87 [50]
	1	0 [100]	16 [100]	19 [100]	19 [100]	29 [50]	32 [50]	52 [50]	61 [50]	68 [50]	81 [50]	90 [50]
	2	0 [100]	13 [100]	16 [100]	23 [100]	32 [100]	42 [50]	48 [50]	61 [50]	65 [50]	77 [50]	87 [50]
	3	0 [100]	6 [100]	16 [100]	26 [100]	29 [100]	29 [100]	35 [50]	48 [50]	65 [50]	81 [50]	87 [50]
logis	4	0 [100]	3 [100]	10 [100]	19 [100]	26 [100]	35 [100]	42 [50]	52 [50]	61 [50]	74 [50]	87 [50]
	5	0 [100]	0 [100]	0 [100]	3 [100]	6 [100]	10 [100]	26 [100]	32 [100]	45 [50]	65 [50]	87 [50]
	6	0 [100]	0 [100]	0 [100]	0 [100]	3 [100]	10 [100]	19 [100]	29 [100]	45 [100]	58 [50]	87 [50]
	7	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	3 [100]	13 [100]	26 [100]	45 [100]	52 [50]	87 [50]
	1	0 [100]	10 [100]	19 [100]	23 [100]	29 [50]	45 [50]	55 [50]	61 [50]	77 [50]	81 [50]	90 [50]
	2	0 [100]	13 [100]	23 [100]	32 [100]	35 [50]	45 [50]	55 [50]	58 [50]	77 [50]	84 [50]	90 [50]
	3	0 [100]	19 [100]	23 [100]	32 [100]	39 [50]	45 [50]	55 [50]	61 [50]	81 [50]	84 [50]	90 [50]
	4	0 [100]	6 [100]	16 [100]	23 [100]	23 [100]	29 [100]	35 [100]	45 [100]	52 [50]	65 [50]	87 [50]
exp	5	0 [100]	3 [100]	6 [100]	13 [100]	19 [100]	23 [100]	32 [100]	39 [100]	42 [50]	58 [50]	84 [50]
	6	0 [100]	0 [100]	0 [100]	10 [100]	19 [100]	26 [100]	32 [100]	35 [100]	52 [50]	65 [50]	84 [50]
	7	0 [100]	0 [100]	3 [100]	13 [100]	19 [100]	29 [100]	39 [100]	52 [50]	55 [50]	71 [50]	84 [50]
	8	0 [100]	0 [100]	0 [100]	13 [100]	19 [100]	26 [100]	35 [100]	48 [50]	52 [50]	68 [50]	87 [50]
	9	0 [100]	0 [100]	0 [100]	6 [100]	13 [100]	19 [100]	26 [100]	45 [50]	52 [50]	55 [50]	84 [50]
	1	0 [100]	10 [100]	19 [100]	23 [100]	32 [50]	45 [50]	55 [50]	61 [50]	77 [50]	84 [50]	90 [50]
	2	0 [100]	10 [100]	16 [100]	26 [100]	39 [100]	45 [50]	55 [50]	65 [50]	68 [50]	87 [50]	87 [50]
	3	0 [100]	6 [100]	16 [100]	19 [100]	26 [100]	29 [50]	39 [50]	61 [50]	65 [50]	81 [50]	87 [50]
weibull	4	0 [100]	0 [100]	10 [100]	19 [100]	23 [100]	26 [100]	39 [50]	48 [50]	65 [50]	71 [50]	87 [50]
	5	0 [100]	3 [100]	13 [100]	13 [100]	19 [100]	23 [100]	29 [100]	42 [50]	52 [50]	71 [50]	87 [50]
	6	0 [100]	6 [100]	6 [100]	13 [100]	19 [100]	23 [100]	29 [100]	42 [50]	52 [50]	61 [50]	87 [50]
	7	0 [100]	3 [100]	10 [100]	10 [100]	10 [100]	19 [100]	26 [100]	39 [100]	48 [50]	58 [50]	87 [50]
	8	0 [100]	3 [100]	6 [100]	10 [100]	10 [100]	13 [100]	26 [100]	35 [100]	45 [50]	61 [50]	87 [50]

Table B.4.58: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
norm	1	0 [100]	16 [100]	19 [100]	19 [50]	29 [50]	32 [50]	52 [50]	61 [50]	68 [50]	81 [50]	90 [50]
	2	0 [100]	13 [100]	19 [100]	23 [100]	29 [100]	32 [50]	45 [50]	52 [50]	55 [50]	68 [50]	77 [50]
	3	0 [100]	13 [100]	19 [100]	19 [100]	23 [100]	26 [100]	32 [50]	39 [50]	58 [50]	61 [50]	87 [50]
	4	0 [100]	6 [100]	16 [100]	19 [100]	26 [100]	29 [100]	35 [100]	39 [100]	42 [50]	55 [50]	77 [50]
	5	0 [100]	3 [100]	10 [100]	16 [100]	26 [100]	32 [50]	35 [50]	42 [50]	52 [50]	71 [50]	84 [50]
	6	0 [100]	3 [100]	10 [100]	13 [100]	26 [100]	26 [50]	35 [50]	45 [50]	52 [50]	71 [50]	84 [50]
	7	0 [100]	0 [100]	0 [100]	13 [100]	19 [100]	23 [100]	32 [50]	39 [50]	45 [50]	61 [50]	84 [50]
	8	0 [100]	0 [100]	0 [100]	6 [100]	19 [100]	26 [100]	29 [50]	32 [50]	45 [50]	58 [50]	84 [50]
lnorm	1	0 [100]	16 [100]	23 [100]	26 [50]	29 [50]	35 [50]	52 [50]	68 [50]	77 [50]	84 [50]	90 [50]
	2	0 [100]	16 [100]	19 [100]	26 [50]	32 [50]	35 [50]	45 [50]	61 [50]	74 [50]	81 [50]	87 [50]
	3	0 [100]	13 [100]	19 [100]	23 [100]	32 [50]	39 [50]	42 [50]	45 [50]	71 [50]	81 [50]	87 [50]
	4	0 [100]	10 [100]	13 [100]	16 [100]	26 [100]	32 [100]	35 [50]	52 [50]	58 [50]	74 [50]	87 [50]
	5	0 [100]	3 [100]	10 [100]	13 [100]	19 [100]	35 [100]	42 [100]	45 [50]	58 [50]	68 [50]	84 [50]
	6	0 [100]	0 [100]	3 [100]	10 [100]	10 [100]	16 [100]	42 [100]	52 [100]	55 [50]	74 [50]	84 [50]
	7	0 [100]	0 [100]	0 [100]	3 [100]	10 [100]	16 [100]	26 [100]	48 [100]	55 [50]	68 [50]	84 [50]
	8	0 [100]	0 [100]	0 [100]	3 [100]	13 [100]	19 [100]	23 [100]	45 [100]	55 [50]	68 [50]	84 [50]
	9	0 [100]	0 [100]	0 [100]	0 [100]	3 [100]	13 [100]	26 [100]	42 [100]	58 [100]	65 [50]	87 [50]
	10	0 [100]	0 [100]	0 [100]	3 [100]	10 [100]	13 [100]	23 [100]	35 [100]	48 [100]	58 [50]	87 [50]
logis	1	0 [100]	16 [100]	19 [100]	23 [100]	29 [50]	35 [50]	52 [50]	61 [50]	68 [50]	81 [50]	90 [50]
	2	0 [100]	13 [100]	16 [100]	23 [100]	32 [100]	42 [50]	48 [50]	61 [50]	65 [50]	77 [50]	87 [50]
	3	0 [100]	6 [100]	16 [100]	26 [100]	29 [100]	32 [50]	35 [50]	55 [50]	68 [50]	84 [50]	90 [50]
	4	0 [100]	3 [100]	10 [100]	19 [100]	26 [100]	35 [100]	45 [50]	52 [50]	65 [50]	81 [50]	87 [50]
	5	0 [100]	0 [100]	0 [100]	3 [100]	6 [100]	10 [100]	26 [100]	32 [100]	45 [50]	65 [50]	87 [50]
	6	0 [100]	0 [100]	0 [100]	0 [100]	3 [100]	10 [100]	19 [100]	29 [100]	45 [50]	58 [50]	87 [50]
	7	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	6 [100]	13 [100]	26 [100]	45 [100]	52 [50]	87 [50]
exp	1	0 [100]	10 [100]	19 [100]	23 [100]	29 [50]	45 [50]	55 [50]	61 [50]	77 [50]	81 [50]	90 [50]
	2	0 [100]	13 [100]	23 [100]	32 [100]	35 [50]	45 [50]	55 [50]	58 [50]	77 [50]	87 [50]	90 [50]
	3	0 [100]	19 [100]	26 [100]	32 [100]	39 [50]	45 [50]	55 [50]	65 [50]	81 [50]	84 [50]	90 [50]
	4	0 [100]	6 [100]	16 [100]	23 [100]	23 [100]	29 [100]	35 [100]	45 [100]	52 [50]	65 [50]	87 [50]
	5	0 [100]	3 [100]	6 [100]	13 [100]	19 [100]	23 [100]	32 [100]	39 [100]	42 [50]	58 [50]	84 [50]
	6	0 [100]	0 [100]	0 [100]	10 [100]	19 [100]	26 [100]	32 [100]	35 [100]	52 [50]	65 [50]	84 [50]
	7	0 [100]	0 [100]	3 [100]	13 [100]	19 [100]	29 [100]	39 [100]	52 [50]	55 [50]	71 [50]	84 [50]
	8	0 [100]	0 [100]	0 [100]	13 [100]	19 [100]	26 [100]	35 [100]	48 [50]	52 [50]	68 [50]	87 [50]
	9	0 [100]	0 [100]	0 [100]	6 [100]	13 [100]	19 [100]	29 [100]	45 [50]	52 [50]	55 [50]	84 [50]
weibull	1	0 [100]	10 [100]	19 [100]	23 [100]	32 [50]	45 [50]	55 [50]	61 [50]	77 [50]	87 [50]	90 [50]
	2	0 [100]	10 [100]	16 [100]	26 [100]	39 [100]	45 [50]	58 [50]	65 [50]	68 [50]	87 [50]	90 [50]
	3	0 [100]	6 [100]	16 [100]	19 [100]	29 [100]	32 [50]	39 [50]	65 [50]	65 [50]	84 [50]	90 [50]
	4	0 [100]	0 [100]	10 [100]	23 [100]	23 [100]	29 [100]	39 [50]	48 [50]	68 [50]	77 [50]	87 [50]
	5	0 [100]	3 [100]	13 [100]	13 [100]	19 [100]	26 [100]	29 [50]	42 [50]	58 [50]	74 [50]	90 [50]
	6	0 [100]	6 [100]	6 [100]	13 [100]	23 [100]	23 [100]	29 [100]	45 [50]	52 [50]	65 [50]	87 [50]
	7	0 [100]	3 [100]	10 [100]	10 [100]	10 [100]	19 [100]	26 [100]	39 [100]	48 [50]	55 [50]	87 [50]
	8	0 [100]	3 [100]	6 [100]	10 [100]	10 [100]	13 [100]	26 [100]	32 [100]	45 [50]	55 [50]	84 [50]

Table B.4.59: Average MTTF values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
norm	1	**1629.3**	**1088.8**	**958.8**	**958.8**	**813.5**	**813.5**	**644.7**	**481.1**	**440**	**137.5**	**40.3**
	2	**1629.3**	**1236.4**	**958.8**	**950.1**	**813.5**	**811**	**705.7**	**620.7**	**601.2**	**481.8**	**213.1**
	3	**1629.3**	**1236.4**	**1095**	**1095**	**959.8**	**950.9**	**880.5**	**791.9**	**620.9**	**568.2**	**66.5**
	4	**1629.3**	**1491.1**	**1248.1**	**1124.5**	**950.9**	**876.9**	**804.9**	**791.9**	**756.3**	**615.9**	**213.1**
	5	**1629.3**	**1550.2**	**1409.1**	**1088.8**	**889.5**	**820.8**	**803.5**	**776.2**	**737.3**	**386.3**	**98.6**
	6	**1629.3**	**1550.2**	**1329.1**	**1169.4**	**889.5**	**889.5**	**809.3**	**748.3**	**679.5**	**386.3**	**98.6**
	7	**1629.3**	**1629.3**	**1629.3**	**1169.4**	**958.8**	**891.3**	**813.3**	**776.5**	**689.7**	**568.2**	**98.6**
	8	**1629.3**	**1629.3**	**1629.3**	**1401.8**	**958.8**	**889.5**	**882.1**	**813.3**	**689.7**	**614.9**	**98.6**
lnorm	1	**1629.3**	**1047.5**	**958.8**	**828.8**	**781.9**	**730.1**	**609.7**	**377.3**	**165.4**	**137.5**	**40.3**
	2	**1629.3**	**1047.5**	**958.8**	**828.8**	**750.9**	**725.7**	**664.8**	**554.1**	**392**	**411.8**	**66.5**
	3	**1629.3**	**1156.9**	**958.8**	**959.8**	**774.7**	**719.4**	**701**	**668.5**	**461.3**	**137.5**	**66.5**
	4	**1629.3**	**1308.9**	**1236.4**	**1088.8**	**950.9**	**880.5**	**852.9**	**657.5**	**624.1**	**271.8**	**66.5**
	5	**1629.3**	**1543.1**	**1308.9**	**1284.1**	**1146.5**	**973**	**977.2**	**768.4**	**545.4**	**382.4**	**98.6**
	6	**1629.3**	**1629.3**	**1543.1**	**1308.9**	**1308.9**	**1138.3**	**853.2**	**700**	**554**	**342.9**	**98.6**
	7	**1629.3**	**1629.3**	**1629.3**	**1543.1**	**1308.9**	**1271.1**	**946.2**	**700**	**554**	**381**	**98.6**
	8	**1629.3**	**1629.3**	**1629.3**	**1629.3**	**1271.1**	**996.2**	**955.3**	**734.3**	**554**	**440**	**98.6**
	9	**1629.3**	**1629.3**	**1629.3**	**1629.3**	**1543.1**	**1244.3**	**934.6**	**853.2**	**554**	**456**	**66.5**
	10	**1629.3**	**1629.3**	**1629.3**	**1629.3**	**1560.1**	**1414.3**	**1141.8**	**800.6**	**597.2**	**500.2**	**66.5**
logis	1	**1629.3**	**1088.8**	**958.8**	**958.8**	**813.5**	**802.2**	**644.7**	**481.1**	**381**	**137.5**	**40.3**
	2	**1629.3**	**1236.4**	**1088.8**	**954.9**	**810.3**	**781.1**	**768.8**	**481.1**	**494.5**	**246.1**	**66.5**
	3	**1629.3**	**1491.1**	**1088.8**	**964.3**	**890.9**	**890.9**	**811**	**611.4**	**440**	**231.8**	**66.5**
	4	**1629.3**	**1550.2**	**1308.9**	**958.8**	**881.1**	**786**	**715.4**	**594.4**	**507.8**	**344.1**	**66.5**
	5	**1629.3**	**1629.3**	**1629.3**	**1543.1**	**1458.3**	**1308.9**	**844**	**758**	**661.6**	**408.2**	**66.5**
	6	**1629.3**	**1629.3**	**1629.3**	**1629.3**	**1543.1**	**1308.9**	**958.8**	**829.4**	**661.6**	**495.3**	**66.5**
	7	**1629.3**	**1629.3**	**1629.3**	**1629.3**	**1629.3**	**1543.1**	**1196.7**	**884.3**	**661.6**	**570.7**	**66.5**
exp	1	**1629.3**	**1308.9**	**958.8**	**901.7**	**813.5**	**705.7**	**601.2**	**481.1**	**165.4**	**137.5**	**40.3**
	2	**1629.3**	**1196.7**	**950.1**	**811**	**758.5**	**687.6**	**601.2**	**495.3**	**165.4**	**98.6**	**40.3**
	3	**1629.3**	**958.8**	**950.1**	**811**	**745.9**	**687.6**	**601.2**	**560.1**	**137.7**	**98.6**	**40.3**
	4	**1629.3**	**1409.1**	**1239.2**	**1125.5**	**1125.5**	**891.3**	**799**	**719.4**	**636.1**	**552.7**	**66.5**
	5	**1629.3**	**1550.2**	**1563.5**	**1491.1**	**1211.1**	**1125.5**	**811**	**780.9**	**774.7**	**621.9**	**98.6**
	6	**1629.3**	**1629.3**	**1629.3**	**1586**	**1208.4**	**1055.2**	**877.4**	**804.9**	**592.1**	**470.4**	**98.6**
	7	**1629.3**	**1629.3**	**1550.2**	**1196.7**	**986.2**	**935.4**	**743.2**	**592.1**	**544.8**	**379.9**	**98.6**
	8	**1629.3**	**1629.3**	**1629.3**	**1196.7**	**986.2**	**935.3**	**821**	**671.4**	**592.1**	**441.5**	**66.5**
	9	**1629.3**	**1629.3**	**1629.3**	**1446.7**	**1196.7**	**986.2**	**947.2**	**682.9**	**592.1**	**561.8**	**119.6**
weibull	1	**1629.3**	**1308.9**	**958.8**	**901.7**	**802.2**	**705.7**	**601.2**	**481.1**	**165.4**	**98.6**	**40.3**
	2	**1629.3**	**1342.9**	**1117.1**	**879.3**	**788.6**	**753.7**	**638.1**	**457.7**	**413.7**	**66.5**	**66.5**
	3	**1629.3**	**1409.1**	**1117.1**	**958.8**	**878.6**	**873.3**	**781.3**	**491.8**	**457.7**	**137.5**	**66.5**
	4	**1629.3**	**1629.3**	**1296.9**	**1104.5**	**939**	**878.6**	**781.3**	**682.8**	**457.7**	**419.8**	**66.5**
	5	**1629.3**	**1495.6**	**1196.7**	**1196.7**	**996.2**	**978**	**870.8**	**703.2**	**670.5**	**419.8**	**66.5**
	6	**1629.3**	**1401.8**	**1401.8**	**1196.7**	**958.8**	**901.7**	**878**	**758.6**	**684.8**	**532**	**66.5**
	7	**1629.3**	**1539**	**1296.9**	**1296.9**	**1296.9**	**996.2**	**998.3**	**854.3**	**683.6**	**547.2**	**254.8**
	8	**1629.3**	**1539**	**1397.5**	**1296.9**	**1296.9**	**1196.7**	**902.1**	**784.3**	**759.8**	**673.5**	**254.8**

Table B.4.60: Average MTTF values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

							Reliability Level					
Dist.	Step	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
	1	1629.3	1088.8	958.8	958.8	813.5	802.2	644.7	481.1	381	137.5	40.3
	2	1629.3	1236.4	958.8	950.1	813.5	811	705.7	620.7	601.2	481.8	213.1
	3	1629.3	1236.4	1095	1095	959.8	950.9	880.5	791.9	620.9	568.2	66.5
norm	4	1629.3	1491.1	1248.1	1124.5	950.9	876.9	804.9	791.9	756.3	615.9	213.1
	5	1629.3	1550.2	1278.2	1088.8	889.5	820.8	803.5	776.2	737.3	386.3	98.6
	6	1629.3	1550.2	1329.1	1169.4	889.5	889.5	809.3	748.3	679.5	386.3	98.6
	7	1629.3	1629.3	1629.3	1169.4	958.8	891.3	813.3	776.5	689.7	568.2	98.6
	8	1629.3	1629.3	1629.3	1401.8	958.8	889.5	882.1	813.3	689.7	611	98.6
	1	1629.3	1047.5	891.3	828.8	781.9	730.1	609.7	377.3	165.4	98.6	40.3
	2	1629.3	1047.5	958.8	828.8	750.9	725.7	664.8	554.1	392	411.8	66.5
	3	1629.3	1156.9	958.8	959.8	774.7	719.4	701	668.5	461.3	137.5	66.5
	4	1629.3	1308.9	1236.4	1088.8	950.9	880.5	852.9	657.5	575.7	271.8	66.5
lnorm	5	1629.3	1543.1	1308.9	1284.1	1146.5	973	790.4	768.4	545.4	382.4	98.6
	6	1629.3	1629.3	1543.1	1308.9	1308.9	1138.3	853.2	574.8	554	257	98.6
	7	1629.3	1629.3	1629.3	1543.1	1308.9	1153.2	946.2	700	554	381	98.6
	8	1629.3	1629.3	1629.3	1543.1	1271.1	996.2	955.3	711.4	554	381	98.6
	9	1629.3	1629.3	1629.3	1629.3	1543.1	1244.3	934.6	853.2	539.8	430.4	66.5
	10	1629.3	1629.3	1629.3	1543.1	1414.3	1393.4	1008.5	800.6	597.2	500.2	66.5
	1	1629.3	1088.8	958.8	901.7	813.5	786	644.7	481.1	381	137.5	40.3
	2	1629.3	1236.4	1088.8	954.9	810.3	781.1	768.8	481.1	494.5	246.1	66.5
	3	1629.3	1491.1	1088.8	964.3	890.9	822.5	811	573.1	450.7	98.6	40.3
logis	4	1629.3	1550.2	1308.9	958.8	881.1	786	718.3	594.4	440	231.8	66.5
	5	1629.3	1629.3	1629.3	1543.1	1458.3	1308.9	844	758	661.6	408.2	66.5
	6	1629.3	1629.3	1629.3	1629.3	1543.1	1308.9	958.8	829.4	661.6	495.3	66.5
	7	1629.3	1629.3	1629.3	1629.3	1629.3	1401.8	1196.7	884.3	661.6	570.7	66.5
	1	1629.3	1308.9	958.8	901.7	813.5	705.7	601.2	481.1	165.4	137.5	40.3
	2	1629.3	1196.7	950.1	811	758.5	687.6	601.2	495.3	165.4	66.5	40.3
	3	1629.3	958.8	879.3	811	745.9	687.6	601.2	529.4	137.7	98.6	40.3
	4	1629.3	1409.1	1239.2	1125.5	1125.5	891.3	799	719.4	636.1	552.7	66.5
exp	5	1629.3	1550.2	1563.5	1491.1	1211.1	1125.5	811	780.9	774.7	621.9	98.6
	6	1629.3	1629.3	1629.3	1586	1208.4	1055.2	877.4	804.9	592.1	470.4	98.6
	7	1629.3	1629.3	1550.2	1196.7	986.2	935.4	743.2	592.1	544.8	379.9	98.6
	8	1629.3	1629.3	1629.3	1196.7	986.2	935.3	821	671.4	592.1	441.5	66.5
	9	1629.3	1629.3	1629.3	1446.7	1196.7	986.2	841.8	682.9	592.1	561.8	119.6
	1	1629.3	1308.9	958.8	901.7	802.2	705.7	601.2	481.1	165.4	66.5	40.3
	2	1629.3	1342.9	1117.1	879.3	788.6	753.7	590.7	457.7	413.7	66.5	40.3
	3	1629.3	1409.1	1117.1	958.8	873.3	792.1	781.3	457.7	457.7	98.6	40.3
weibull	4	1629.3	1629.3	1296.9	939	939	873.3	781.3	682.8	400.5	444.7	66.5
	5	1629.3	1495.6	1196.7	1196.7	996.2	878.6	870.8	703.2	542.3	430.8	40.3
	6	1629.3	1401.8	1401.8	1196.7	901.7	901.7	878	686.8	684.8	466.4	66.5
	7	1629.3	1539	1296.9	1296.9	1296.9	996.2	998.3	854.3	683.6	574.7	254.8
	8	1629.3	1539	1397.5	1296.9	1296.9	1196.7	902.1	869.7	759.8	660.1	232.8

Table B.4.61: Average MTTRep values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
norm	1	**4117.5**	**3557.6**	**2968.6**	**2425.4**	**1962.2**	**1570.3**	**1142.3**	**882.1**	**603.5**	**396.5**	**123.6**
	2	**4117.5**	**3319.2**	**3023.4**	**2299.3**	**1969.8**	**1574.4**	**1187.1**	**867.3**	**611.4**	**434.9**	**200.7**
	3	**4117.5**	**3575.2**	**2952.1**	**2567.2**	**2403**	**1962.3**	**1614.9**	**1341.6**	**943.2**	**586.9**	**248.3**
	4	**4117.5**	**3692**	**3249.9**	**3022.2**	**2581.4**	**2269.2**	**1896.9**	**1449.4**	**1114.1**	**758.3**	**356.6**
	5	**4117.5**	**3504.3**	**3688.5**	**3385**	**2773.4**	**2309.2**	**1890.1**	**1437.7**	**987.3**	**748.9**	**273**
	6	**4117.5**	**3396.3**	**3645**	**3414.7**	**2818.8**	**2452.2**	**1917**	**1425**	**1066.8**	**774.6**	**288.4**
	7	**4117.5**	**3676**	**3147.5**	**3834.3**	**3519.1**	**3012.8**	**2177.1**	**1685.2**	**1175.4**	**775.9**	**284.4**
	8	**4117.5**	**4117.5**	**3407**	**3770**	**3580.9**	**2875.3**	**2207**	**1876.5**	**1327.5**	**842.5**	**329.4**
lnorm	1	**4117.5**	**3665.1**	**2833.1**	**2132.9**	**1608.1**	**1270.2**	**977.8**	**749.2**	**535.4**	**346.3**	**142.1**
	2	**4117.5**	**3653**	**2670.4**	**2063.4**	**1519.2**	**1196.8**	**925.9**	**677.2**	**478**	**321**	**163.6**
	3	**4117.5**	**3712.3**	**2775.8**	**1934.9**	**1646.2**	**1300.5**	**1023.1**	**863.2**	**690.4**	**455.7**	**199.1**
	4	**4117.5**	**3946.2**	**3247.5**	**2826.7**	**2126.6**	**1732.9**	**1385.5**	**1097.8**	**785.3**	**586**	**222.1**
	5	**4117.5**	**3844**	**3925.2**	**3075.3**	**2445.8**	**1907.7**	**1554.6**	**1352.2**	**989.7**	**668.3**	**241.4**
	6	**4117.5**	**4117.5**	**3961.7**	**3899.2**	**3231.2**	**2918.4**	**1860.3**	**1437.3**	**1053**	**676.6**	**234.2**
	7	**4117.5**	**4117.5**	**4117.5**	**3954**	**3917**	**2851.8**	**2287.6**	**1497.2**	**1130.2**	**737.8**	**257.2**
	8	**4117.5**	**4117.5**	**4117.5**	**4057.5**	**3661**	**3200.1**	**2447**	**1672.9**	**1156.8**	**750.5**	**273.5**
	9	**4117.5**	**4117.5**	**4117.5**	**4117.5**	**3925**	**3783.7**	**2835.4**	**1659.5**	**1267.6**	**850.7**	**306.2**
	10	**4117.5**	**4117.5**	**4117.5**	**4117.5**	**3206**	**3419.6**	**2957.2**	**2075.8**	**1394.6**	**978.6**	**316.2**
logis	1	**4117.5**	**3574.9**	**2937.2**	**2393.8**	**1919.4**	**1481.7**	**1108.3**	**857.6**	**600.7**	**385.3**	**121.7**
	2	**4117.5**	**3536.5**	**3166.6**	**2401.6**	**2005.8**	**1504.8**	**1120.9**	**941**	**632.8**	**456.2**	**165.6**
	3	**4117.5**	**3372**	**3441**	**2545.3**	**2217.5**	**1776.9**	**1377.2**	**1095.2**	**782.7**	**518.3**	**160.9**
	4	**4117.5**	**3881.7**	**3651.6**	**3458.5**	**2622.1**	**1959.5**	**1492.3**	**1111.3**	**763.9**	**530.5**	**209.2**
	5	**4117.5**	**4117.5**	**4117.5**	**3897**	**3871.5**	**3922.6**	**3163.5**	**2220**	**1363.6**	**848.5**	**286.2**
	6	**4117.5**	**4117.5**	**4117.5**	**4117.5**	**4044.7**	**4067.6**	**3578.2**	**2407**	**1437.2**	**863**	**278.3**
	7	**4117.5**	**4117.5**	**4117.5**	**4117.5**	**4117.5**	**4076**	**3724.3**	**2666**	**1535.5**	**920.2**	**302.8**
exp	1	**4117.5**	**3543**	**2979.8**	**2324.8**	**1803.9**	**1316.9**	**999.7**	**754.1**	**561.4**	**330.5**	**111.3**
	2	**4117.5**	**3469.2**	**2626**	**1934.4**	**1513.5**	**1147**	**866.4**	**689**	**513**	**325.5**	**121.3**
	3	**4117.5**	**3063.2**	**2166.7**	**1653.5**	**1260.6**	**986.2**	**758.6**	**566**	**459.8**	**276.6**	**113.8**
	4	**4117.5**	**4034.5**	**3286.6**	**2687.2**	**2266**	**2058.7**	**1528.8**	**1096.4**	**753.6**	**525.4**	**256.6**
	5	**4117.5**	**3945**	**2929.8**	**2492.7**	**2861.2**	**2504.9**	**2107.2**	**1440.3**	**964.8**	**639.6**	**269.7**
	6	**4117.5**	**4117.5**	**4117.5**	**2404**	**2978**	**2647.5**	**2156**	**1675.3**	**1099.1**	**661.4**	**265**
	7	**4117.5**	**4117.5**	**4011**	**4098.2**	**3345.2**	**2331.8**	**1865.5**	**1355.7**	**935.9**	**589.9**	**231.4**
	8	**4117.5**	**4117.5**	**4117.5**	**4018.8**	**3373.5**	**2490.5**	**1832.5**	**1295.9**	**914.8**	**601.8**	**241.7**
	9	**4117.5**	**4117.5**	**4117.5**	**3890.8**	**3752.3**	**3025.5**	**2131.4**	**1499.2**	**1053.2**	**634.4**	**273.8**
weibull	1	**4117.5**	**3606.4**	**3030.2**	**2335.2**	**1718.8**	**1285.4**	**963.7**	**716.2**	**525.4**	**304.5**	**106.1**
	2	**4117.5**	**3703.6**	**3174.6**	**2417.5**	**1718.7**	**1310.1**	**1015.9**	**794**	**572.8**	**390.2**	**145.9**
	3	**4117.5**	**4456.8**	**3583.4**	**3097.1**	**2334.6**	**1834.9**	**1367.8**	**1043.1**	**706.3**	**471.9**	**154.9**
	4	**4117.5**	**4117.5**	**4145.2**	**3117.6**	**2557.4**	**2005.3**	**1559.7**	**1161.8**	**814.6**	**500.2**	**208**
	5	**4117.5**	**4531.3**	**3843**	**3321.5**	**2813.1**	**2229.7**	**1651.6**	**1201.6**	**756.8**	**528.4**	**226.3**
	6	**4117.5**	**4150.2**	**3700.8**	**3155.5**	**2788**	**2406**	**1746.5**	**1224.6**	**837.9**	**599.2**	**279.1**
	7	**4117.5**	**3963.7**	**3967**	**3541.6**	**3094**	**2985.6**	**1969.2**	**1330.7**	**969.4**	**654.8**	**234.6**
	8	**4117.5**	**3963.7**	**4354.2**	**3765.8**	**3367.6**	**3003.7**	**2224.6**	**1660.8**	**924.5**	**533.1**	**226.3**

Table B.4.62: Average MTTRep values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	19%	29%	39%	48%	58%	68%	78%	87%	97%
norm	1	**4117.5**	**3547**	**2956.1**	**2414.2**	**1944.6**	**1506.8**	**1126.8**	**865.4**	**600.6**	**377.4**	**114.5**
	2	**4117.5**	**3318.7**	**3022.6**	**2295.2**	**1965.5**	**1566.5**	**1178.9**	**860.5**	**607.8**	**430.2**	**194.7**
	3	**4117.5**	**3573.5**	**2946.5**	**2564**	**2396**	**1957**	**1610.2**	**1333.9**	**928.9**	**579**	**241.8**
	4	**4117.5**	**3688.2**	**3245.9**	**3013.6**	**2579.1**	**2267.4**	**1890**	**1437.7**	**1105.7**	**749.5**	**342.8**
	5	**4117.5**	**3503.3**	**3959.8**	**3380.4**	**2766.7**	**2304.2**	**1879.6**	**1435**	**982.6**	**744.1**	**262.6**
	6	**4117.5**	**3392**	**3644.6**	**3409.5**	**2805.5**	**2448**	**1911.6**	**1420**	**1049.7**	**766.9**	**274.1**
	7	**4117.5**	**3673.5**	**3095**	**3826.7**	**3497.6**	**2998.4**	**2144.1**	**1679.6**	**1153.3**	**758.1**	**271.2**
	8	**4117.5**	**4110.5**	**3397.5**	**3759**	**3557.5**	**2863**	**2187.3**	**1867.5**	**1317.5**	**820.6**	**309.3**
lnorm	1	**4117.5**	**3652**	**2792.8**	**2120.2**	**1594.5**	**1258**	**965.5**	**737.2**	**518.2**	**333.6**	**125.6**
	2	**4117.5**	**3651.6**	**2668.9**	**2061.1**	**1511.8**	**1191**	**924**	**672.9**	**476.2**	**317.5**	**159.5**
	3	**4117.5**	**3709.5**	**2770.4**	**1930.2**	**1645.8**	**1295.1**	**1021**	**854.4**	**683.8**	**449.7**	**191.7**
	4	**4117.5**	**3936.2**	**3235.2**	**2823.9**	**2118.6**	**1728**	**1376.5**	**1086.1**	**800**	**576.9**	**211.8**
	5	**4117.5**	**3844**	**3925.2**	**3064.8**	**2439.4**	**1903**	**1734.8**	**1348.1**	**977.5**	**653.4**	**226.1**
	6	**4117.5**	**4117.5**	**3957.3**	**3872.6**	**3228.6**	**2910.1**	**1842.4**	**1491.1**	**1041.3**	**674.5**	**215.8**
	7	**4117.5**	**4117.5**	**4117.5**	**3940.3**	**3909.6**	**3050.3**	**2278.2**	**1488.6**	**1107.1**	**709.8**	**232.9**
	8	**4117.5**	**4117.5**	**4117.5**	**4104.3**	**3658.5**	**3156.6**	**2439.1**	**1603.5**	**1117.2**	**728.2**	**241.8**
	9	**4117.5**	**4117.5**	**4117.5**	**4117.5**	**3921.7**	**3691.7**	**2772.5**	**1614.9**	**1205.4**	**797.4**	**284.5**
	10	**4117.5**	**4117.5**	**4117.5**	**3688.3**	**3628.4**	**3075.5**	**3081.9**	**2064.7**	**1374.5**	**888.4**	**282.2**
logis	1	**4117.5**	**3556.4**	**2927.8**	**2374**	**1900.7**	**1438.4**	**1092.4**	**837.5**	**586.3**	**368.2**	**113.1**
	2	**4117.5**	**3527.8**	**3147.6**	**2379.2**	**1987.3**	**1484.7**	**1102.4**	**919.4**	**610.5**	**428.7**	**134.3**
	3	**4117.5**	**3352**	**3395**	**2520.5**	**2175.9**	**1777.8**	**1339.5**	**1037.4**	**718.4**	**481.6**	**108.2**
	4	**4117.5**	**3881.7**	**3596.2**	**3410.6**	**2600.1**	**1927.6**	**1412.7**	**1085.6**	**749.2**	**503.1**	**145.7**
	5	**4117.5**	**4117.5**	**4117.5**	**3897**	**3861.2**	**3901.8**	**3139.8**	**2191.1**	**1342.4**	**824.6**	**256**
	6	**4117.5**	**4117.5**	**4117.5**	**4117.5**	**4035**	**4044.8**	**3550.2**	**2375.2**	**1410.8**	**833.5**	**245.3**
	7	**4117.5**	**4117.5**	**4117.5**	**4117.5**	**4117.5**	**4452**	**3709**	**2634.5**	**1508.3**	**895.3**	**262.7**
exp	1	**4117.5**	**3533.8**	**2962.5**	**2313**	**1786.3**	**1299.4**	**983.4**	**737.6**	**540.6**	**313.2**	**104.7**
	2	**4117.5**	**3438.7**	**2614.8**	**1916.4**	**1494.1**	**1126.6**	**846**	**670.1**	**498.3**	**296.3**	**106.1**
	3	**4117.5**	**3054.8**	**2200.7**	**1650.1**	**1253.4**	**974.3**	**749.4**	**571.4**	**451**	**266.5**	**109.8**
	4	**4117.5**	**4034.5**	**3285.7**	**2687.1**	**2264.4**	**2054.3**	**1528.1**	**1096.2**	**750.9**	**522**	**254.9**
	5	**4117.5**	**3945**	**2929.8**	**2492.5**	**2860.5**	**2504.8**	**2103.9**	**1438.8**	**962.3**	**637.5**	**267.3**
	6	**4117.5**	**4117.5**	**4117.5**	**2404**	**2975**	**2646.6**	**2153.9**	**1672.6**	**1092.1**	**656.5**	**259.4**
	7	**4117.5**	**4117.5**	**4008.3**	**4091.3**	**3340.2**	**2326.9**	**1859.2**	**1349.6**	**930.7**	**584.6**	**224.6**
	8	**4117.5**	**4117.5**	**4016.7**	**4016.7**	**3362.9**	**2478.3**	**1827.4**	**1287.8**	**904.8**	**593.1**	**229.7**
	9	**4117.5**	**4117.5**	**4117.5**	**3885.8**	**3738.8**	**3021**	**2224.6**	**1488.2**	**1046.7**	**629.5**	**259.3**
weibull	1	**4117.5**	**3597.6**	**3013.6**	**2322.7**	**1706.3**	**1266.5**	**942.1**	**698.9**	**507.5**	**285.8**	**102.4**
	2	**4117.5**	**3701.2**	**3149**	**2389.1**	**1699.9**	**1291.8**	**1006**	**782.3**	**556.8**	**370.1**	**122.3**
	3	**4117.5**	**4450**	**3551.7**	**3052.8**	**2179.5**	**1851.8**	**1324.9**	**996.1**	**654.5**	**424.5**	**112.9**
	4	**4117.5**	**4117.5**	**4144.4**	**3315.8**	**2517.1**	**1893.9**	**1519**	**1125.8**	**771.9**	**460.5**	**155.9**
	5	**4117.5**	**4531.3**	**3828.5**	**3306.2**	**2773**	**2273.9**	**1621.2**	**1140.1**	**785.7**	**473.6**	**169.5**
	6	**4117.5**	**4146.8**	**3680.2**	**3082.8**	**2703.3**	**2341.4**	**1694.9**	**1209.6**	**793**	**581.9**	**202.4**
	7	**4117.5**	**3998**	**3993.8**	**3563.8**	**3113.2**	**3006.1**	**1975.6**	**1363.6**	**977.7**	**667.3**	**258.6**
	8	**4117.5**	**3998**	**4385.2**	**3788.8**	**3419**	**3014.7**	**2328.3**	**1598.7**	**938.8**	**573.4**	**259.4**

Table B.4.63: Variables identified by time-(in)dependent PHM models.

	Time-independent PHM			Time-dependent PHM	
	Variable	Scaled Value		Variable	Scaled Value
①	*Duration Cruise CleanWing Seconds*	**10.09**	①	*Group C*	**−19.19**
②	*Duration Touchdown end rollout 10 Seconds*	**5.98**	②	*Group F*	**−16.34**
③	*Roll rate min deg sec 8*	**−5.62**	③	*Rudder low mean deg TER 8*	**13.64**
④	*Group B*	**−2.3**	④	*Aoa max deg 2*	**13.6**
⑤	*Group A*	**−1.22**	⑤	*Accn long mean g s 8*	**13.03**
			⑥	*Pressure dynamic mean hPa mbar 8*	**12.62**
			⑦	*Yaw rate mean deg sec 8*	**12.16**
			⑧	*Yaw rate min deg sec 8*	**−12.09**
			⑨	*Roll rate mean deg sec 8*	**−11.53**
			⑩	*Rudder cmd force min lbs Nose Right 8*	**−10.17**
			⑪	*Pitch cmd FO force max lbs Nose up 8*	**10.17**
			⑫	*Aoa min deg*	**−9.65**
			⑬	*Group G*	**−9.33**
			⑭	*Group H*	**9.25**
			⑮	*Roll mean deg*	**−9.06**
			⑯	*Headwind max knots 3*	**7.93**
			⑰	*Group E*	**−7.86**
			⑱	*Group A*	**7.22**
			⑲	*Group D*	**6.62**
			⑳	*Group I*	**−5.36**
			㉑	*Headwind max knots 2*	**4.85**
			㉒	*Rudder cmd force mean lbs Nose Right*	**−4.6**
			㉓	*Prop spd lhs mean*	**−3.07**

Table B.4.64: Variables identified by each step by time-(in)dependent PHMs (in order).

	PHM Variables	
	Time-independent	*Time-dependent*
norm	③ ② ⑤	⑮ ⑱ ① ⑩ ⑲ ⑧ ⑰ ⑫
lnorm	① ③ ④	② ⑱ ① ⑮ ⑪ ⑬ ④ ⑥ ⑨ ㉑
logis	③	⑮ ⑯ ⑰ ⑧ ⑦ ⑨ ⑬
exp	②	⑮ ⑬ ⑱ ⑤ ⑧ ① ② ㉒ ⑨
weibull	② ③	⑮ ⑭ ③ ⑨ ⑳ ㉒ ㉓ ⑩

Table B.4.65: Number of times variables identified by each step by time-(in)dependent PHMs.

Key indep	Key dep	Variable	Count
5	18	Group A	4
4		Group B	1
	1	Group C	3
	19	Group D	1
	17	Group E	2
	2	Group F	2
	13	Group G	3
	14	Group H	1
	20	Group I	1
	5	Accn long mean g s 8	1
	4	Aoa max deg 2	1
	12	Aoa min deg	1
1		Duration Cruise CleanWing Seconds	1
2		Duration Touchdown end rollout 10 Seconds	3
	21	Headwind max knots 2	1
	16	Headwind max knots 3	1
	11	Pitch cmd FO force max lbs Nose up 8	1
	6	Pressure dynamic mean hPa mbar 8	1
	23	Prop spd lhs mean	1
	15	Roll mean deg	5
	9	Roll rate mean deg sec 8	4
3		Roll rate min deg sec 8	4
	22	Rudder cmd force mean lbs Nose Right	2
	10	Rudder cmd force min lbs Nose Right 8	2
	3	Rudder low mean deg TER 8	1
	7	Yaw rate mean deg sec 8	1
	8	Yaw rate min deg sec 8	3

Table B.4.66: Variables belonging to each group identified in B.4.63.

Group	*Variables*
Group A	Roll mean deg 1, Yaw rate mean deg sec 1
Group B	Ttot min deg C 2, Tout mean deg C 3, Ttot mean deg C 3, Ttot max deg C 3, Ttot min deg C 3
Group C	Yaw rate min deg sec 2, Roll min deg 2
Group D	Rudder low max deg TER , Rudder low mean deg TER
Group E	Headwind mean knots 3, Headwind min knots 3
Group F	Roll min deg 4, Yaw rate min deg sec 4
Group G	Accn lat mean g s 5, Accn lat mean g s 6
Group H	Drift mean deg 6, Drift mean deg 5, Crosswind mean knots 6
Group I	Elevator Rin min deg TEU 6, Elevator Rin min deg TEU 5

B.5 3-1573-1 MLG wheel & tire assembly

Table B.5.67 provides a summary of the input data related to the component. The number of registered maintenance events is less than the total number of events due to the fact that TRAX data stretches back to 2004/2005 and FDR data only to 2011. Maintenance events with insufficient data, regarding operational factors, cannot be evaluated, hence are not registered during the modelling process.

Table B.5.67: General overview of component inputs.

Name	**Value**
Part Number	3-1573-1
Total # (A, F, C)	18809, 1132, 17677
Registered # (A, F, C)	3082, 191, 2891
Related Flights # (A, F, C)	776321, 32758, 743563
Avg. Cycles (A, F, C)	251.89, 171.51, 257.2
% Censored	93.8

In Tab. B.5.67 (A, F, C) denotes statistics regarding All (A), Failed (F), and Censored (C) events respectively. Ergo A will always be the sum or mean derived from F and C.

Analysis
Tables B.5.68 and B.5.69 summarise the results from EVA and MDA. In addition the variables obtained by semi-parametric PHM modelling (labelled 'reduced semi-COX') are also presented if applicable. Table B.5.69 provides an overview of the specific operational factors identified during all flight phases. In this case high counts indicate operational factors that were significantly different during multiple flight phases.

Table B.5.68: Overview of analysis input and output.

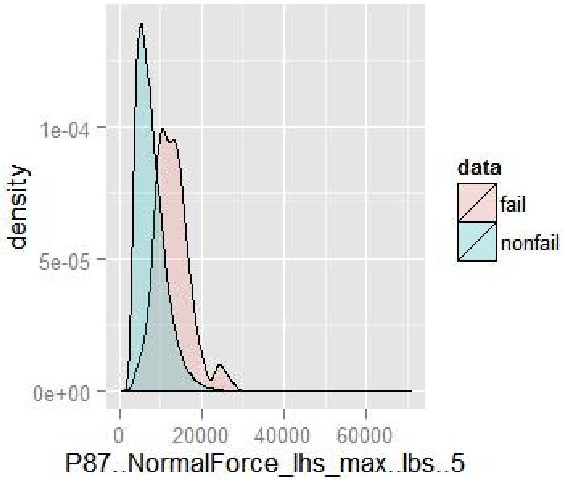
	# Variables
ALL	1531
EVA	20
MDA	8
Combined	28
reduced Corr.	22
reduced semi-COX	12
Take-Off related	9
Cruise related	8
Touch-Down related	5

Table B.5.69: Overview of identified operational factors during all flight phases.

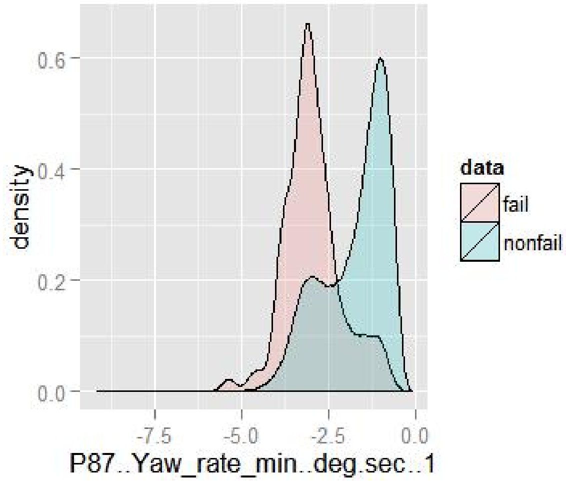
Variable	**Count**	**Variable**	**Count**	**Variable**	**Count**
Accn_long	3	Vcal	2	Brake_press_lhs	1
Yaw_rate	3	NormalForce_lhs	1	Aileron_Rin	1
Torque_rhs	2	Pitch_rate	1	Headwind	1
Roll_rate	2	Brake_press_rhs	1	Pitch_cmd_FO_force	1
Vz	2	Torque_lhs	1		

A multitude of factors were identified during EVA and MDA. Figure B.5.42 give a general overview of the top operational factors identified by EVA and MDA.

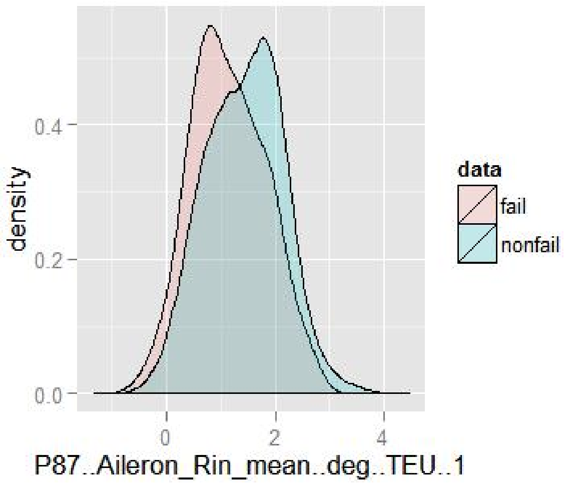
Time-based reliability modelling
Table B.5.70 reports the maximum likelihood and goodness-of-fit tests results obtained from time-based reliability modelling. To show the overall fit Fig. B.5.43 shows the computed re-



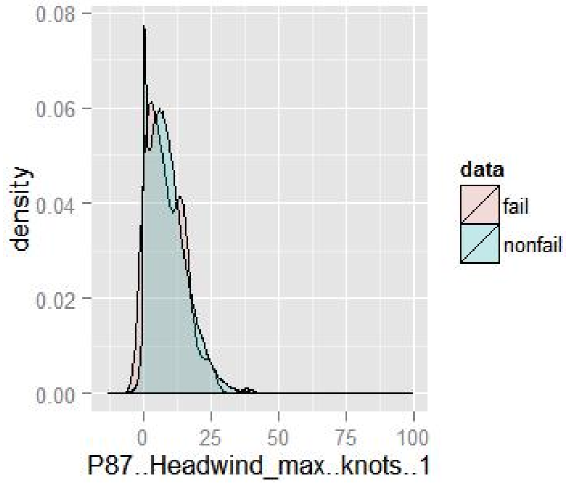
(a) EVA 1.



(b) EVA 2.



(c) MDA 1.



(d) MDA 2.

Figure B.5.42: Graphical overview of top operational factors identified by EVA and MDA.

Table B.5.70: Overview of results obtained by MLE optimisation and GOF tests.

	Distributions					
	norm	**lnorm**	**logis**	**exp**	**weibull**	**gamma**
MLE	**−1761.32**	**−1765.75**	**−1761.34**	**−1778.22**	**−1761.75**	**−1762.15**
Kolmogorov-Smirnov	**1.51**	**2.57**	**1.41**	**2.98**	**2.07**	**2.16**
Cramer-von-Mises Smirnov	**82.93**	**80.21**	**83.18**	**78.58**	**81.54**	**81.3**
Anderson-Darling	**−204.08**	**−204.14**	**−204.07**	**−204.21**	**−204.1**	**−204.11**
NRR	**159.07**	**174.83**	**158.71**	**163.14**	**182.39**	**181.97**

liability function using an averaged virtual age V for all fitted models.

In addition Figures B.5.44, B.5.45, B.5.46, B.5.47, B.5.48, and B.5.49 present the reliability and hazard functions computed for each underlying distribution evaluated in the program.

Time independent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.5.71 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.5.50,

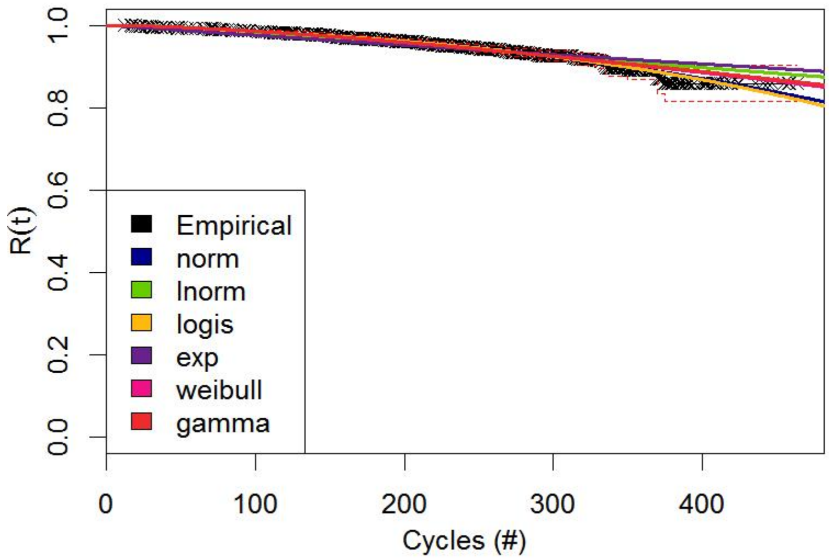
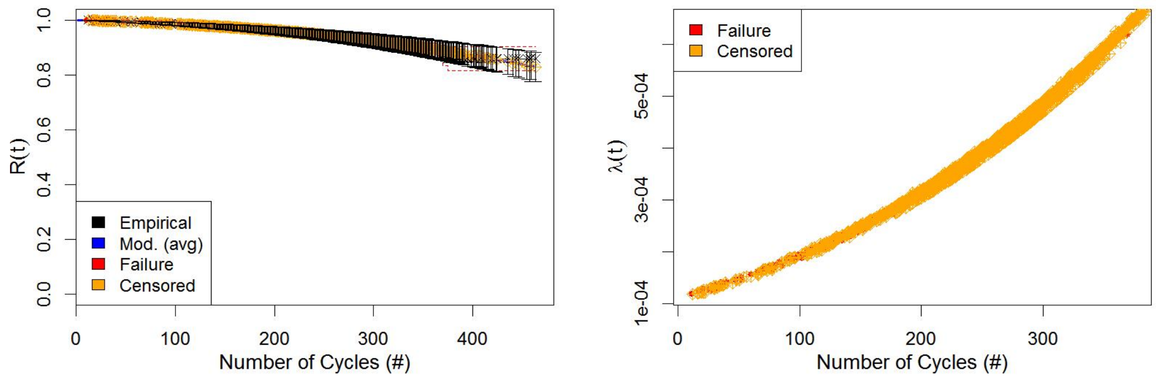


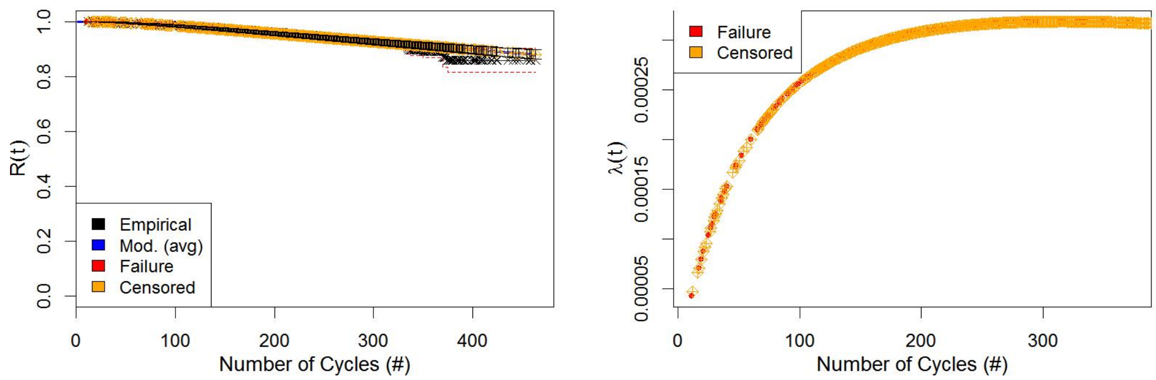
Figure B.5.43: Overview of overall fit of multiple GRP modells.



(a) Reliability function.

(b) Hazard function.

Figure B.5.44: Computed reliability for time-based models with underlying norm distribution.

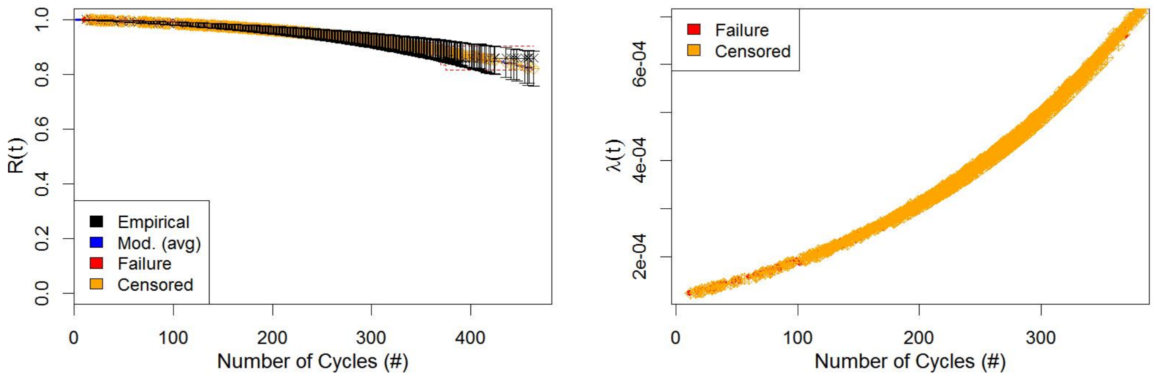


(a) Reliability function.

(b) Hazard function.

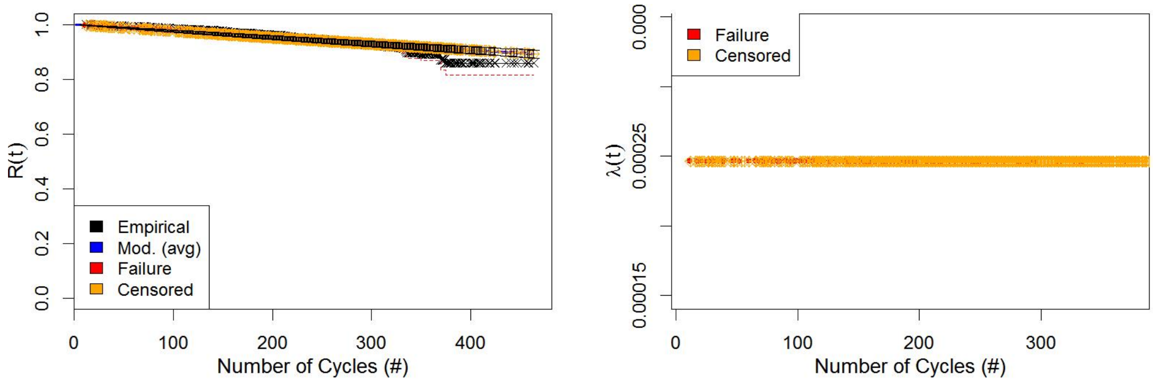
Figure B.5.45: Computed reliability for time-based models with underlying lnorm distribution.

B.5.51, and B.5.52 as well as a general overview in Figure B.5.53a.
Tables B.5.72 and B.5.73 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables



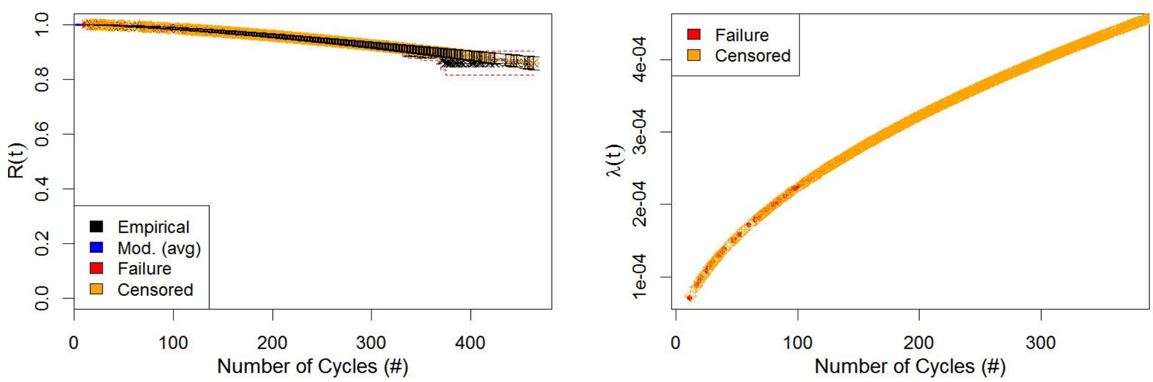
(a) Reliability function. (b) Hazard function.

Figure B.5.46: Computed reliability for time-based models with underlying logis distribution.



(a) Reliability function. (b) Hazard function.

Figure B.5.47: Computed reliability for time-based models with underlying exp distribution.



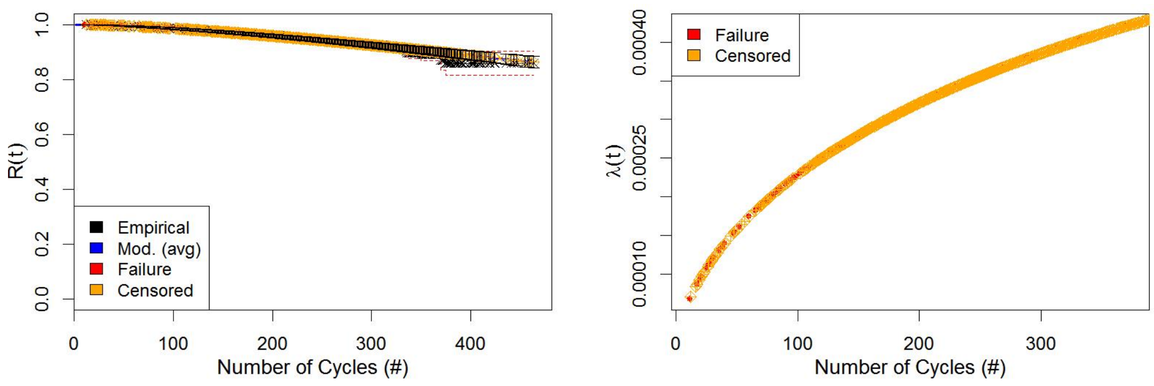
(a) Reliability function. (b) Hazard function.

Figure B.5.48: Computed reliability for time-based models with underlying weibull distribution.

were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

To assist in the selection of models, Tables B.5.74, B.5.75, B.5.76, and B.5.77 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.

Time dependent proportional hazard reliability modelling
Discussed in Sec. 2 is the step-wise application of forward selection variable identification



(a) Reliability function.

(b) Hazard function.

Figure B.5.49: Computed reliability for time-based models with underlying gamma distribution.

Table B.5.71: Overview of time-independent PHM results obtained by MLE optimisation and GOF tests.

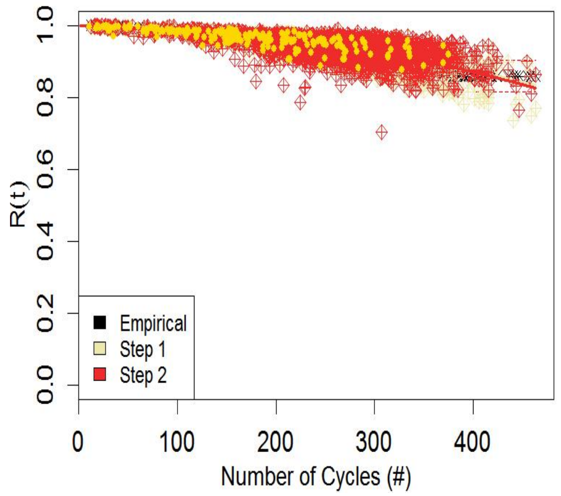
Distribution	norm	norm	lnorm	lnorm	lnorm	lnorm	lnorm	logis
Step #	1	2	1	2	3	4	5	1
MLE	−1750.42	−1741.37	−1753.05	−1743.13	−1738.68	−1734.96	−1727.93	−1750.45
Time (min)	4.93	9.72	6.49	11.61	17.61	22.53	25.89	4.26
Kolmogorov-Smirnov	3.49	3.99	4.09	5.04	10.41	13.95	11.64	3.55
Cramer-von Mises-Smirnov	76.78	76.86	75.08	73.86	71.4	68.91	71.69	76.61
Anderson-Darling	−204.35	−204.31	−204.39	−204.51	−204.73	−204.86	−204.66	−204.35
NRR	177.98	172.11	176.2	164.78	164.25	168.52	156.55	174.2

Distribution	logis	logis	exp	exp	weibull	weibull	weibull	gamma
Step #	2	3	1	2	1	2	3	1
MLE	−1744.05	−1738.24	−1771.93	−1767.49	−1751.02	−1741.99	−1736.08	−1751.69
Time (min)	8.52	12.51	3.24	6.67	5.05	8.9	13.41	110.85
Kolmogorov-Smirnov	8.6	10.25	14.52	21.47	3.8	4.39	4.89	3.95
Cramer-von Mises-Smirnov	75.14	74.17	74.63	70.05	76.04	75.44	72.84	75.62
Anderson-Darling	−204.41	−204.41	−204.37	−204.47	−204.35	−204.39	−204.55	−204.35
NRR	173.93	157.03	170.51	171.93	162.5	171.45	165.32	172.25

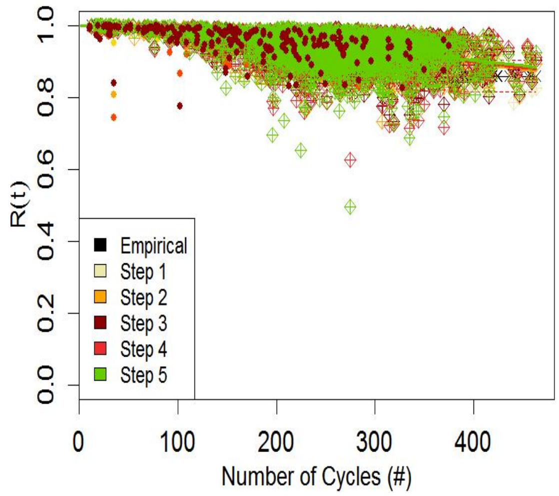
Distribution	gamma	gamma	gamma
Step #	2	3	4
MLE	−1744.16	−1740.23	−1733.6
Time (min)	194.78	231.16	258.65
Kolmogorov-Smirnov	7.7	13.52	14.56
Cramer-von Mises-Smirnov	73.87	71.69	67.54
Anderson-Darling	−204.45	−204.53	−204.83
NRR	168.96	154.92	163.27

Table B.5.72: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
norm	1	1 [98]	1 [97]	2 [92]	3 [89]	3 [85]	9 [68]	15 [51]	38 [25]	52 [10]	84 [4]	98 [0]
	2	0 [99]	0 [98]	1 [93]	2 [92]	3 [86]	10 [65]	17 [50]	36 [27]	51 [12]	84 [5]	98 [0]
lnorm	1	0 [99]	0 [99]	1 [92]	1 [91]	1 [86]	6 [66]	9 [51]	28 [20]	41 [8]	73 [4]	98 [0]
	2	0 [99]	0 [98]	0 [94]	0 [93]	1 [86]	6 [66]	10 [52]	27 [26]	38 [13]	70 [4]	98 [0]
	3	0 [98]	0 [98]	1 [95]	1 [92]	2 [85]	9 [70]	15 [58]	28 [28]	42 [15]	71 [5]	98 [0]
	4	0 [98]	0 [96]	0 [93]	1 [89]	2 [85]	8 [65]	13 [56]	24 [31]	37 [15]	66 [5]	98 [0]
	5	1 [97]	1 [96]	2 [94]	5 [89]	6 [83]	11 [61]	15 [53]	30 [34]	41 [22]	69 [4]	98 [0]
logis	1	1 [99]	1 [97]	2 [93]	3 [90]	3 [85]	9 [70]	14 [54]	37 [25]	52 [12]	84 [4]	98 [0]
	2	1 [95]	1 [95]	2 [91]	2 [84]	3 [77]	7 [64]	13 [53]	38 [28]	50 [15]	84 [5]	98 [0]
	3	0 [95]	0 [95]	2 [92]	2 [89]	3 [82]	9 [61]	16 [48]	35 [30]	49 [14]	84 [5]	98 [0]
exp	1	0 [99]	0 [99]	0 [99]	0 [97]	0 [94]	6 [67]	17 [41]	51 [11]	68 [7]	92 [2]	98 [0]
	2	0 [100]	0 [100]	0 [97]	0 [96]	1 [93]	8 [72]	20 [49]	51 [14]	66 [6]	91 [3]	98 [0]
weibull	1	1 [99]	1 [97]	1 [91]	3 [89]	3 [86]	10 [68]	16 [50]	39 [21]	53 [10]	83 [5]	98 [0]
	2	0 [99]	0 [98]	1 [94]	3 [90]	4 [84]	12 [67]	18 [49]	40 [25]	53 [12]	82 [5]	98 [0]
	3	0 [98]	1 [98]	2 [95]	3 [91]	6 [85]	14 [71]	18 [56]	40 [26]	52 [14]	81 [5]	98 [0]
gamma	1	0 [100]	1 [98]	1 [96]	3 [93]	3 [87]	9 [70]	17 [54]	40 [20]	53 [8]	83 [2]	98 [0]
	2	0 [98]	0 [98]	1 [94]	1 [88]	1 [86]	4 [70]	7 [55]	24 [24]	39 [12]	72 [3]	98 [0]
	3	0 [98]	0 [96]	1 [92]	2 [90]	2 [85]	5 [72]	9 [60]	31 [24]	45 [10]	73 [3]	98 [0]
	4	0 [98]	0 [96]	1 [90]	2 [87]	3 [84]	10 [75]	14 [64]	28 [31]	43 [16]	73 [4]	98 [0]

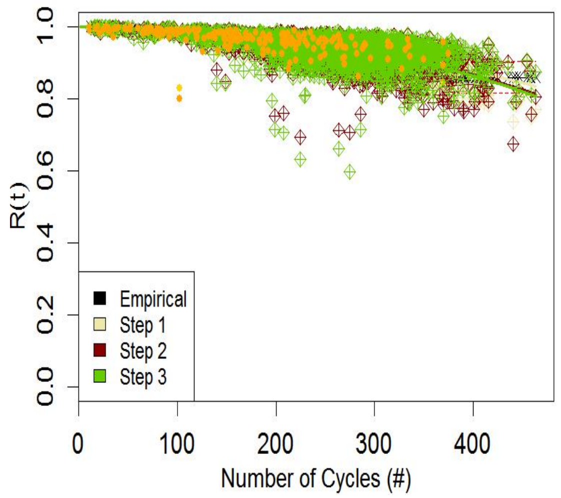


(a) With an underlying norm distribution.

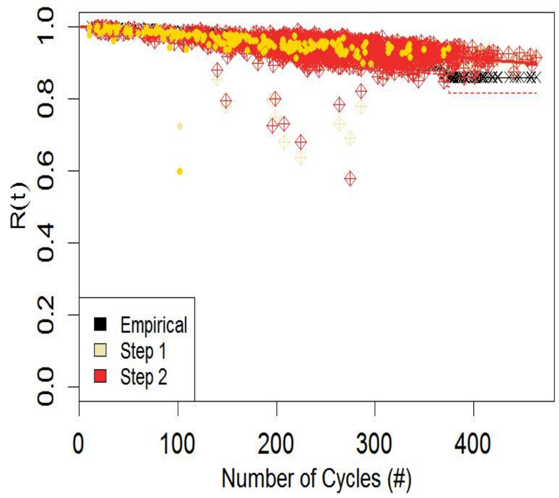


(b) With an underlying lnorm distribution.

Figure B.5.50: Time-independent PHMs with an underlying norm and lnorm distribution.



(a) With an underlying logis distribution.

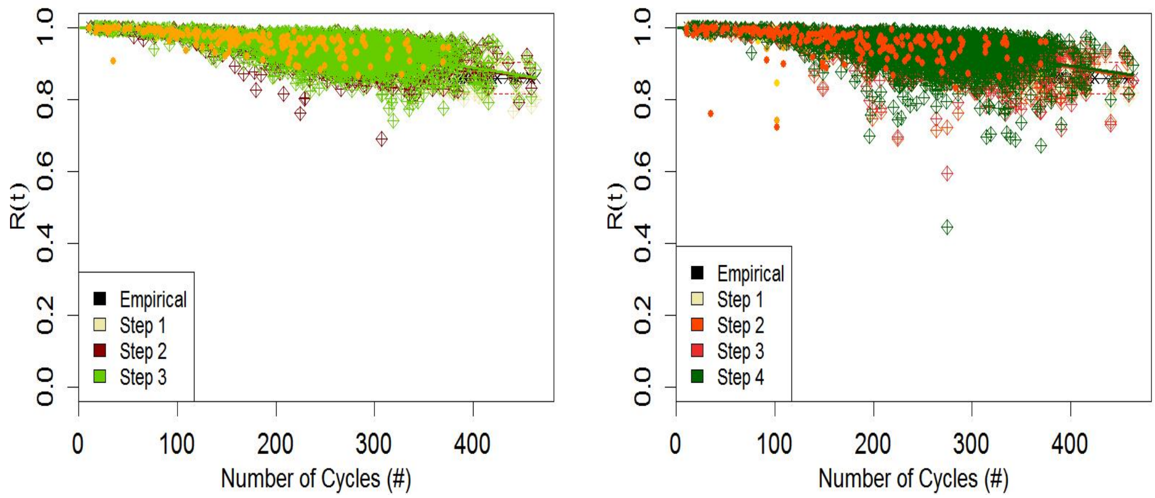


(b) With an underlying exp distribution.

Figure B.5.51: Time-independent PHMs with an underlying logis and exp distribution.

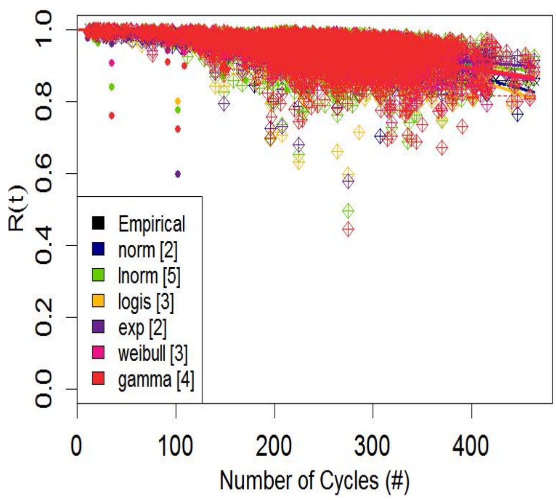
Table B.5.73: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
norm	1	1 [96]	1 [95]	3 [88]	3 [85]	5 [75]	16 [56]	23 [40]	50 [17]	61 [7]	90 [4]	98 [0]
	2	1 [93]	3 [93]	5 [79]	9 [75]	15 [63]	26 [37]	37 [32]	60 [9]	74 [8]	91 [2]	98 [0]
	1	0 [99]	1 [96]	1 [90]	1 [83]	4 [76]	9 [51]	15 [35]	38 [9]	53 [7]	80 [4]	98 [0]
	2	0 [93]	0 [90]	2 [74]	6 [68]	9 [57]	17 [33]	26 [27]	48 [11]	58 [8]	82 [4]	98 [0]
lnorm	3	1 [93]	2 [91]	6 [78]	8 [73]	13 [68]	20 [44]	27 [32]	50 [13]	59 [9]	83 [4]	98 [0]
	4	1 [92]	3 [87]	4 [76]	6 [69]	10 [61]	17 [43]	23 [34]	43 [16]	55 [9]	80 [4]	98 [0]
	5	2 [93]	5 [89]	7 [81]	8 [74]	11 [65]	18 [51]	27 [41]	42 [20]	54 [10]	80 [4]	98 [0]
	1	1 [96]	1 [95]	3 [88]	3 [85]	5 [78]	14 [56]	23 [40]	50 [17]	61 [7]	90 [4]	98 [0]
logis	2	1 [91]	2 [90]	3 [78]	3 [73]	7 [68]	19 [47]	29 [38]	50 [15]	63 [11]	93 [4]	98 [0]
	3	1 [91]	2 [87]	7 [72]	12 [66]	14 [58]	26 [36]	36 [29]	57 [11]	73 [8]	93 [2]	98 [0]
exp	1	0 [99]	0 [99]	0 [97]	0 [97]	0 [88]	9 [56]	26 [31]	60 [9]	75 [6]	95 [2]	98 [0]
	2	0 [100]	0 [100]	0 [97]	0 [97]	1 [94]	8 [76]	15 [53]	50 [14]	64 [7]	91 [4]	98 [0]
	1	1 [97]	1 [95]	3 [89]	4 [85]	5 [76]	17 [54]	25 [39]	51 [14]	62 [7]	87 [4]	98 [0]
weibull	2	1 [92]	3 [91]	8 [81]	12 [71]	15 [61]	29 [34]	41 [30]	62 [9]	75 [8]	89 [4]	98 [0]
	3	1 [97]	2 [93]	8 [83]	10 [77]	14 [73]	25 [52]	32 [38]	54 [15]	64 [11]	87 [4]	98 [0]
	1	1 [98]	1 [96]	3 [92]	4 [86]	5 [82]	17 [54]	25 [40]	52 [11]	65 [6]	87 [2]	98 [0]
gamma	2	0 [94]	0 [94]	1 [86]	2 [82]	2 [76]	9 [52]	17 [37]	39 [12]	52 [6]	81 [3]	98 [0]
	3	0 [96]	1 [95]	2 [89]	2 [86]	3 [82]	9 [64]	15 [53]	39 [16]	51 [8]	79 [3]	98 [0]
	4	1 [92]	2 [90]	4 [84]	5 [82]	8 [76]	15 [60]	20 [48]	42 [18]	53 [9]	81 [2]	98 [0]



(a) With an underlying weibull distribution. (b) With an underlying gamma distribution.

Figure B.5.52: Time-independent PHMs with an underlying weibull and gamma distribution.



(a) All distributions (last step).

Figure B.5.53: Figure of all time-independent PHMs.

Table B.5.74: Average MTTF values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
norm	1	**170.5**	**170.5**	**168.7**	**167.5**	**167**	**158.2**	**151.8**	**119.7**	**100.5**	**41.5**	**14.5**
	2	**171.5**	**171.5**	**170.7**	**168.1**	**166.8**	**158.1**	**150.1**	**123.9**	**105**	**41.2**	**14.5**
lnorm	1	**171.5**	**171.5**	**170.5**	**170.5**	**169.6**	**163.8**	**159.7**	**134.2**	**114.3**	**62.6**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**171.5**	**170.7**	**164.7**	**160.1**	**140.3**	**122.9**	**70.1**	**14.5**
	3	**171.5**	**171.5**	**170.7**	**170.1**	**169.5**	**162.2**	**156.2**	**137**	**119.9**	**69**	**14.5**
	4	**171.5**	**171.5**	**171.5**	**170.1**	**170.1**	**164.3**	**160**	**145.7**	**127.5**	**78.1**	**14.5**
	5	**170.7**	**170.7**	**169.1**	**167.1**	**166.1**	**161.4**	**157.7**	**134.8**	**120.5**	**73.4**	**14.5**
logis	1	**170.5**	**170.5**	**168.7**	**167.5**	**167**	**158.2**	**153.8**	**120.9**	**100.5**	**41.5**	**14.5**
	2	**170.5**	**170.5**	**169**	**169**	**168.1**	**162.5**	**154.4**	**121.5**	**104.4**	**43.2**	**14.5**
	3	**171.5**	**171.5**	**169**	**169**	**167.1**	**161.2**	**154.2**	**126.4**	**108.7**	**46.1**	**14.5**
exp	1	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**162.6**	**148.2**	**101.4**	**72.9**	**26.8**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**171.5**	**170.9**	**160.4**	**147.2**	**101.1**	**76.6**	**27.8**	**14.5**
weibull	1	**170.5**	**170.5**	**169.6**	**167**	**167**	**157.6**	**151**	**117.6**	**99.4**	**44.5**	**14.5**
	2	**171.5**	**171.5**	**170.7**	**167.6**	**166.1**	**158.4**	**151.8**	**120.3**	**102.2**	**47.1**	**14.5**
	3	**171.5**	**170.7**	**168.5**	**167**	**164**	**157.2**	**151**	**123.3**	**104.3**	**49.5**	**14.5**
gamma	1	**171.5**	**170.5**	**169.6**	**167.5**	**167**	**158.9**	**150.7**	**115.2**	**98.8**	**42.3**	**14.5**
	2	**171.5**	**171.5**	**170.5**	**170.5**	**169.9**	**166.5**	**162.7**	**141.1**	**120.3**	**66.5**	**14.5**
	3	**171.5**	**171.5**	**170.5**	**169**	**169**	**165.6**	**159.7**	**134.8**	**110.7**	**64.8**	**14.5**
	4	**171.5**	**171.5**	**170.1**	**169**	**167.7**	**161.9**	**158.2**	**140.2**	**117.5**	**66.8**	**14.5**

Table B.5.75: Average MTTF values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
norm	1	**170.5**	**169.6**	**167**	**167**	**164**	**151.4**	**142.1**	**104.1**	**86.1**	**28.4**	**14.5**
	2	**170.7**	**167.6**	**165.3**	**160.6**	**155.7**	**142**	**128.2**	**95.2**	**73.9**	**30.9**	**14.5**
lnorm	1	**171.5**	**170.5**	**170.5**	**169.6**	**166.6**	**160**	**153.2**	**119.6**	**98.8**	**50.1**	**14.5**
	2	**171.5**	**171.5**	**169.8**	**165.7**	**162.8**	**154.3**	**144.3**	**114.4**	**96.9**	**47**	**14.5**
	3	**170.2**	**169.7**	**165.9**	**164.1**	**160**	**152.6**	**144.8**	**113.1**	**95.6**	**45.1**	**14.5**
	4	**170.9**	**169.6**	**169.5**	**166.8**	**162.9**	**155.8**	**150.2**	**121.9**	**103**	**52.2**	**14.5**
	5	**170.3**	**168.1**	**165.8**	**164.4**	**162**	**152.8**	**143.6**	**121.9**	**102.4**	**53.3**	**14.5**
logis	1	**170.5**	**169.6**	**167**	**167**	**164**	**153.8**	**142.1**	**104.1**	**86.1**	**28.4**	**14.5**
	2	**169.9**	**169**	**168.1**	**167.6**	**162.6**	**149.9**	**137.9**	**108.4**	**89.1**	**26.1**	**14.5**
	3	**170.1**	**169**	**165.5**	**161.1**	**159.4**	**143.8**	**131.5**	**104.3**	**84.1**	**29.8**	**14.5**
exp	1	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**158.8**	**137.8**	**87.7**	**61.2**	**23**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**171.5**	**170.9**	**160.4**	**151**	**102.8**	**80.4**	**27.9**	**14.5**
weibull	1	**170.5**	**169.6**	**167**	**166.4**	**164**	**150.9**	**140.8**	**103.4**	**84.2**	**33.9**	**14.5**
	2	**170.1**	**168.2**	**163.2**	**161.2**	**156.8**	**141.2**	**124.3**	**92.5**	**67.9**	**32.8**	**14.5**
	3	**170.2**	**168.5**	**165.2**	**162.9**	**157.2**	**144.8**	**136.2**	**102.9**	**86.4**	**37.2**	**14.5**
gamma	1	**170.5**	**170.5**	**167**	**166.4**	**165.1**	**150.9**	**140.8**	**101.5**	**77.5**	**32.8**	**14.5**
	2	**171.5**	**171.5**	**169.9**	**169**	**168.4**	**161.5**	**152.6**	**123.9**	**103.9**	**47.7**	**14.5**
	3	**171.5**	**170.5**	**169**	**169**	**167.6**	**160.1**	**154.1**	**120.5**	**102.9**	**51.8**	**14.5**
	4	**170.9**	**169.6**	**167.9**	**167.4**	**164.7**	**157.3**	**152.6**	**122.2**	**101**	**48.2**	**14.5**

Table B.5.76: Average MTTRep values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
norm	1	**427.2**	**411.8**	**375.1**	**355.4**	**335.2**	**289**	**263.6**	**208.3**	**173.2**	**77.6**	**20**
	2	**421.9**	**406.1**	**372.4**	**352.9**	**332.5**	**287.5**	**261.6**	**207.6**	**173.1**	**78.9**	**20**
lnorm	1	**534.1**	**503.1**	**433.8**	**400.1**	**365.3**	**295.7**	**264.3**	**206**	**175.3**	**97**	**20**
	2	**528.6**	**496.7**	**436.9**	**406.2**	**374**	**304.5**	**270.5**	**210.9**	**183**	**101.9**	**20**
	3	**574.5**	**539.8**	**466.9**	**428.7**	**390.5**	**308.6**	**272.9**	**212.4**	**180**	**100.2**	**20**
	4	**561**	**529**	**462.2**	**423.1**	**388**	**312.7**	**275.5**	**213.5**	**184**	**104.9**	**20**
	5	**588.9**	**550.9**	**479**	**432.3**	**388.2**	**316.8**	**279.2**	**213.3**	**181.2**	**103.4**	**20**
logis	1	**428.5**	**413.3**	**377.1**	**357.4**	**337.9**	**291.2**	**265.9**	**209.5**	**174.2**	**76.1**	**20**
	2	**407.2**	**393.7**	**363.2**	**346.9**	**328**	**288.4**	**265.4**	**212.6**	**178.4**	**81.4**	**20**
	3	**405.5**	**393**	**363.7**	**348.3**	**329.8**	**286.1**	**259.7**	**211.6**	**177.3**	**82.3**	**20**
exp	1	**615**	**569.1**	**479.9**	**435.1**	**389.8**	**300.5**	**256.5**	**169.5**	**126.4**	**42**	**20**
	2	**631.9**	**587.2**	**491.3**	**449.1**	**400.9**	**303.1**	**254.3**	**170.6**	**128.1**	**42.7**	**20**
weibull	1	**464.3**	**442.9**	**396**	**368.2**	**344.5**	**289.7**	**260.9**	**204.2**	**169.9**	**84.6**	**20**
	2	**470.1**	**450.6**	**401.8**	**373**	**345.3**	**286.6**	**257.4**	**202.4**	**170.8**	**86**	**20**
	3	**480.5**	**458**	**406.5**	**380**	**347.6**	**288.8**	**261.3**	**202.5**	**173.1**	**88.2**	**20**
gamma	1	**497**	**470.2**	**412.7**	**381.2**	**349.7**	**287.2**	**256.6**	**196.7**	**162.7**	**79.6**	**20**
	2	**473.2**	**449.6**	**401.2**	**375**	**347.9**	**291.3**	**262.6**	**211.1**	**179.5**	**94.7**	**20**
	3	**487.2**	**462.1**	**411.6**	**380.9**	**350.3**	**293.4**	**266**	**206.5**	**177.8**	**91.7**	**20**
	4	**503.9**	**478.4**	**420.8**	**390.5**	**360.8**	**294.4**	**265.3**	**208.9**	**180.1**	**97.8**	**20**

Table B.5.77: Average MTTRep values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
norm	1	**406.7**	**388.1**	**351**	**332.3**	**310.2**	**263.8**	**238.5**	**181**	**144.5**	**56.7**	**20**
	2	**376.9**	**358.4**	**322.9**	**301.7**	**275.2**	**229.1**	**203.2**	**146.2**	**110.4**	**41.8**	**20**
lnorm	1	**498.8**	**464.3**	**398**	**363.6**	**327.9**	**264.4**	**235.2**	**181.5**	**149.8**	**79.5**	**20**
	2	**442.8**	**413**	**349.6**	**317.1**	**283.3**	**224.7**	**199.8**	**151.3**	**123.4**	**64.8**	**20**
	3	**487.7**	**446.3**	**372.8**	**336.1**	**300.1**	**237.7**	**208.6**	**156.7**	**129.8**	**67.6**	**20**
	4	**481.9**	**442.9**	**375.7**	**340.7**	**301.9**	**242.6**	**216.8**	**164.2**	**135.7**	**72.9**	**20**
	5	**515.5**	**474.3**	**398.5**	**365**	**327.9**	**262.1**	**225.6**	**172**	**145.4**	**76.5**	**20**
logis	1	**408.5**	**390.5**	**353.2**	**334.5**	**312.7**	**265.6**	**240.6**	**181.8**	**144.1**	**55**	**20**
	2	**378**	**363.4**	**331.5**	**314.4**	**294.4**	**251.6**	**227.8**	**170.2**	**133.5**	**51.5**	**20**
	3	**362.5**	**349.2**	**307.9**	**287.5**	**267**	**224.8**	**199.3**	**142.6**	**107.5**	**41.5**	**20**
exp	1	**591.7**	**547.4**	**457.8**	**412**	**367.9**	**277.9**	**234.8**	**146.9**	**105.2**	**34.5**	**20**
	2	**639.8**	**594.8**	**498.8**	**456.2**	**408.8**	**310.2**	**264.6**	**178.6**	**135.6**	**46.3**	**20**
weibull	1	**437.3**	**415.2**	**364.6**	**340.9**	**316**	**259.9**	**234.2**	**176.9**	**143.2**	**66.7**	**20**
	2	**405.7**	**379.3**	**325.6**	**294.7**	**271.8**	**217.5**	**194**	**141.1**	**110**	**50.5**	**20**
	3	**434.2**	**407.7**	**345.8**	**320.4**	**295.4**	**243.9**	**216.3**	**165.3**	**132.9**	**61**	**20**
gamma	1	**467.1**	**437.2**	**377.7**	**346.7**	**317.9**	**256.6**	**228.7**	**169.6**	**136.4**	**62.9**	**20**
	2	**431.4**	**406.7**	**355**	**328.5**	**304**	**250.8**	**226.7**	**172.5**	**140.9**	**68.5**	**20**
	3	**463.9**	**439**	**383.9**	**354**	**326.1**	**272.9**	**243.2**	**189.2**	**157.8**	**77.7**	**20**
	4	**462.6**	**429.5**	**363**	**338.1**	**308.4**	**256.7**	**228.5**	**176.5**	**148.6**	**73.5**	**20**

techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.5.78 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.5.54,

B.5.55, and B.5.56 as well as a general overview in Figure B.5.56b.

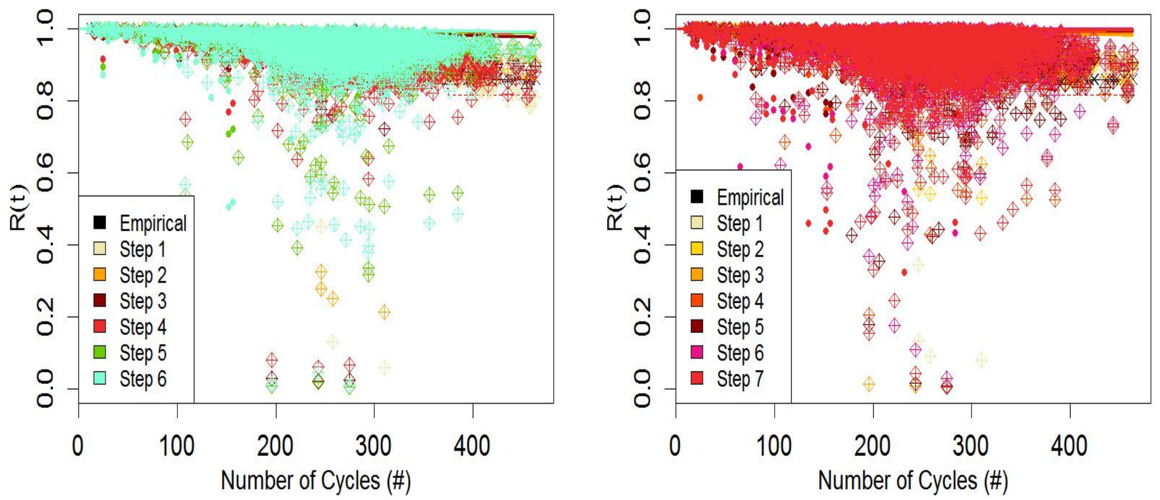
Table B.5.78: Overview of time-dependent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	norm	norm	norm	norm	norm	lnorm	lnorm
Step #	1	2	3	4	5	6	1	2
MLE	−1623.12	−1484.78	−1376.53	−1309	−1232.55	−1179.59	−1628.56	−1473.75
Time (min)	133.02	276.35	397.82	534.27	610.38	644	76.94	166.26
Kolmogorov-Smirnov	2.63	3.85	3.81	9.92	13.95	25.2	3.1	3.64
Cramer-von Mises-Smirnov	86.41	77.06	79.03	81.97	75.95	79.91	79.73	79.31
Anderson-Darling	−204.1	−204.54	−204.37	−204.44	−205.1	−205.2	−204.23	−204.38
NRR	235.31	479.56	−117.59	−1387.93	454.98	3179.01	434.49	578.3

Distribution	lnorm	lnorm	lnorm	lnorm	lnorm	logis	logis	exp
Step #	3	4	5	6	7	1	2	1
MLE	−1382.61	−1283.51	−1223.08	−1144.11	−1123.98	−1620.73	−1560.44	−1641.1
Time (min)	300.22	424.12	548.65	680.16	779.62	42.5	76.33	14.78
Kolmogorov-Smirnov	4.02	10.66	27.78	45.35	41.33	2.77	3.01	3.32
Cramer-von Mises-Smirnov	78.29	74.64	77.83	78.67	82.23	84.8	83.28	77.45
Anderson-Darling	−204.42	−204.75	−205.07	−206.13	−206.19	−204.11	−204.25	−204.33
NRR	453.11	421.38	494.2	407.39	287.36	103.13	1367.31	276.67

Distribution	exp	exp	exp	exp	weibull	weibull	weibull	weibull
Step #	2	3	4	5	1	2	3	4
MLE	−1571.65	−1551.14	−1517.21	−1511.62	−1612.81	−1481.74	−1345.96	−1267.35
Time (min)	24.04	32.76	39.11	44.75	53.68	85.68	119.01	158.69
Kolmogorov-Smirnov	3.49	4.1	5.05	4.62	3.47	4.4	4.96	10.14
Cramer-von Mises-Smirnov	82.17	83.4	79.66	80.42	79.03	82.66	86.9	84.15
Anderson-Darling	−204.26	−204.35	−204.31	−204.32	−204.41	−204.61	−204.63	−204.68
NRR	263.59	366.83	280.09	307.33	216.16	155.48	240.65	226.45

Distribution	weibull	weibull	weibull
Step #	5	6	7
MLE	−1207.66	−1150.13	−1134.52
Time (min)	194.87	233.89	265.36
Kolmogorov-Smirnov	19.05	19.2	23.88
Cramer-von Mises-Smirnov	81.82	89.84	93.98
Anderson-Darling	−204.97	−205.39	−205.38
NRR	179.18	292.51	173.81



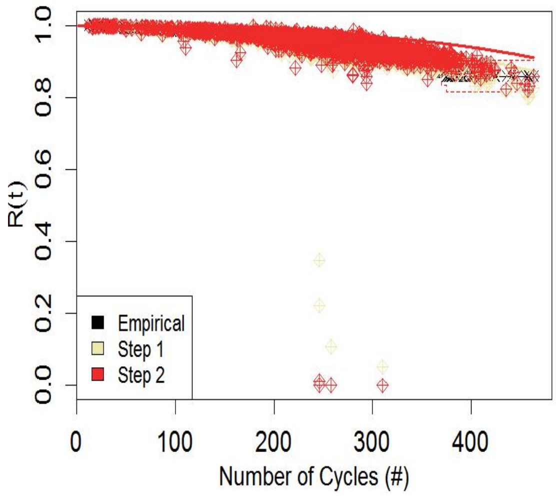
(a) With an underlying norm distribution.

(b) With an underlying lnorm distribution.

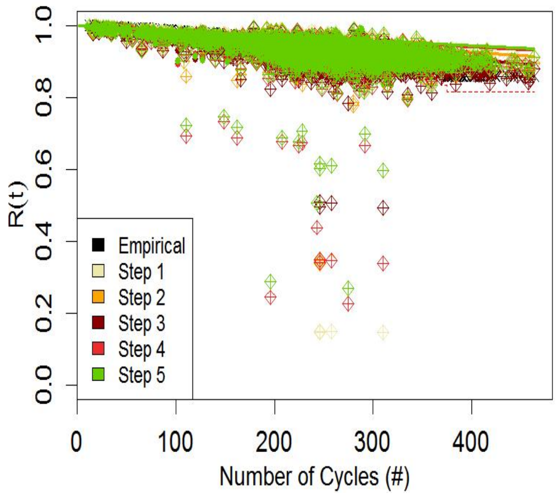
Figure B.5.54: Time-dependent PHMs with an underlying norm and lnorm distribution.

Tables B.5.79 and B.5.80 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

To assist in the selection of models, Tables B.5.81, B.5.82, B.5.83, and B.5.84 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.

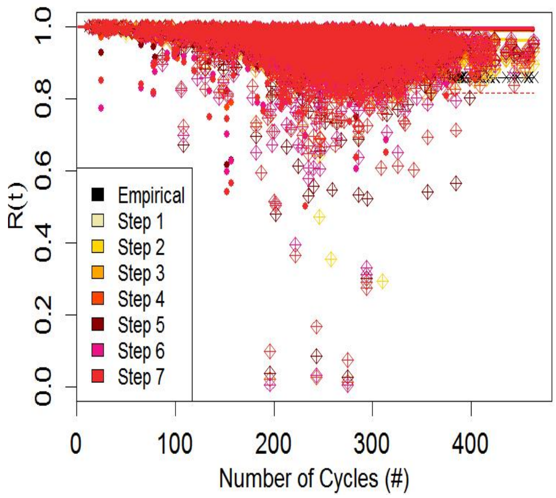


(a) With an underlying logis distribution.

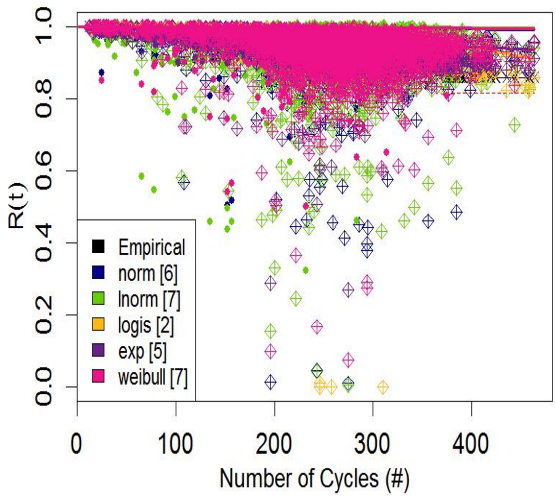


(b) With an underlying exp distribution.

Figure B.5.55: Time-dependent PHMs with an underlying logis and exp distribution.



(a) With an underlying weibull distribution.



(b) All distributions (last step).

Figure B.5.56: Figures containing a weibull distribution and all time-dependent PHMs.

The operational factors identified during time-independent and time-dependent PHM modelling are shown in Tables B.5.85, B.5.86, B.5.87, and B.5.88.

Table B.5.79: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
	1	0 [100]	0 [100]	1 [100]	3 [99]	3 [96]	7 [74]	13 [62]	28 [23]	42 [14]	74 [5]	98 [0]
	2	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	1 [99]	1 [91]	6 [50]	13 [32]	61 [7]	98 [0]
	3	0 [100]	0 [100]	0 [99]	1 [99]	1 [98]	3 [96]	6 [81]	17 [48]	30 [25]	67 [6]	98 [0]
norm	4	0 [98]	0 [98]	1 [98]	1 [98]	2 [96]	5 [86]	6 [76]	17 [50]	30 [29]	62 [7]	98 [0]
	5	1 [98]	1 [97]	1 [97]	1 [97]	1 [97]	1 [90]	2 [86]	5 [72]	10 [54]	49 [11]	98 [0]
	6	1 [96]	1 [96]	1 [96]	1 [95]	1 [92]	3 [88]	4 [80]	7 [68]	17 [52]	49 [12]	98 [0]
	1	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	3 [87]	9 [68]	30 [18]	45 [10]	75 [5]	98 [0]
	2	0 [100]	0 [100]	1 [100]	1 [100]	1 [99]	3 [94]	4 [81]	16 [44]	28 [25]	64 [7]	98 [0]
	3	0 [100]	0 [100]	0 [99]	0 [99]	1 [98]	2 [96]	5 [83]	14 [54]	21 [27]	59 [8]	98 [0]
lnorm	4	0 [99]	0 [99]	1 [98]	2 [96]	2 [95]	7 [87]	8 [82]	17 [64]	26 [46]	55 [6]	98 [0]
	5	0 [97]	0 [97]	1 [96]	3 [96]	4 [93]	7 [85]	12 [80]	19 [65]	29 [48]	60 [14]	98 [0]
	6	2 [96]	2 [96]	2 [92]	2 [91]	3 [91]	5 [87]	7 [87]	12 [68]	21 [58]	42 [25]	98 [0]
	7	2 [96]	2 [95]	2 [92]	2 [90]	3 [89]	4 [87]	6 [81]	12 [63]	19 [54]	40 [19]	98 [0]
logis	1	0 [100]	0 [100]	0 [100]	2 [100]	3 [99]	7 [76]	10 [66]	26 [26]	42 [14]	75 [5]	98 [0]
	2	0 [100]	0 [100]	0 [100]	0 [100]	1 [100]	3 [97]	3 [88]	14 [55]	26 [28]	68 [7]	98 [0]
	1	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	3 [94]	9 [68]	35 [14]	57 [8]	75 [5]	98 [0]
	2	0 [100]	0 [100]	0 [98]	1 [96]	2 [86]	14 [52]	24 [25]	53 [9]	68 [7]	75 [5]	98 [0]
exp	3	0 [100]	0 [100]	2 [91]	4 [72]	10 [59]	27 [24]	38 [12]	65 [7]	72 [5]	75 [5]	98 [0]
	4	0 [100]	0 [99]	0 [98]	0 [96]	1 [90]	6 [61]	18 [41]	42 [10]	61 [9]	75 [5]	98 [0]
	5	0 [100]	0 [99]	0 [96]	0 [93]	1 [89]	8 [58]	18 [40]	42 [10]	61 [9]	75 [5]	98 [0]
	1	1 [99]	1 [98]	2 [95]	2 [92]	2 [91]	3 [78]	8 [63]	25 [33]	38 [19]	69 [6]	98 [0]
	2	1 [99]	1 [99]	2 [95]	2 [95]	2 [94]	3 [83]	6 [79]	19 [48]	27 [30]	66 [9]	98 [0]
	3	1 [99]	1 [99]	2 [97]	2 [96]	2 [97]	3 [88]	5 [80]	13 [59]	19 [45]	51 [11]	98 [0]
weibull	4	1 [98]	1 [97]	1 [96]	1 [96]	1 [96]	4 [87]	4 [81]	8 [67]	16 [47]	47 [12]	98 [0]
	5	1 [97]	1 [97]	1 [96]	1 [96]	1 [95]	3 [88]	3 [80]	6 [69]	14 [55]	43 [14]	98 [0]
	6	1 [96]	1 [94]	1 [94]	1 [93]	1 [92]	2 [89]	2 [84]	6 [67]	11 [59]	41 [17]	98 [0]
	7	1 [96]	1 [95]	1 [94]	1 [92]	1 [91]	2 [89]	4 [81]	7 [66]	11 [56]	42 [13]	98 [0]

Table B.5.80: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
	1	0 [100]	1 [100]	2 [99]	3 [97]	4 [86]	9 [66]	15 [56]	32 [18]	48 [11]	75 [5]	98 [0]
	2	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	1 [95]	3 [80]	8 [44]	17 [22]	67 [5]	98 [0]
	3	0 [100]	0 [100]	1 [99]	1 [99]	1 [98]	5 [86]	10 [70]	23 [31]	40 [14]	75 [5]	98 [0]
norm	4	0 [98]	1 [98]	1 [97]	2 [94]	4 [90]	7 [75]	12 [62]	27 [33]	40 [18]	73 [5]	98 [0]
	5	0 [98]	0 [98]	0 [98]	0 [98]	0 [98]	0 [92]	0 [89]	2 [79]	5 [63]	37 [16]	98 [0]
	6	0 [97]	0 [97]	0 [97]	0 [97]	0 [95]	0 [92]	1 [87]	4 [76]	8 [61]	39 [17]	98 [0]
	1	0 [100]	0 [100]	0 [100]	0 [100]	0 [99]	5 [75]	13 [60]	35 [14]	52 [8]	75 [5]	98 [0]
	2	0 [100]	1 [100]	1 [100]	1 [100]	1 [99]	3 [86]	9 [74]	23 [33]	36 [16]	72 [6]	98 [0]
	3	0 [100]	0 [99]	1 [99]	1 [99]	1 [98]	4 [85]	8 [72]	19 [38]	32 [19]	71 [6]	98 [0]
lnorm	4	0 [99]	1 [99]	2 [96]	3 [95]	5 [92]	9 [82]	11 [71]	24 [55]	36 [38]	66 [5]	98 [0]
	5	0 [97]	1 [97]	3 [95]	4 [93]	5 [91]	12 [80]	13 [75]	28 [54]	40 [45]	72 [7]	98 [0]
	6	1 [96]	1 [96]	2 [93]	2 [91]	3 [91]	4 [88]	4 [87]	10 [72]	16 [63]	35 [28]	98 [0]
	7	0 [96]	1 [95]	1 [92]	2 [90]	3 [90]	4 [88]	4 [83]	8 [67]	13 [57]	31 [21]	98 [0]
logis	1	0 [100]	0 [100]	1 [100]	2 [99]	3 [94]	8 [71]	13 [61]	32 [18]	47 [11]	75 [5]	98 [0]
	2	0 [100]	0 [100]	0 [100]	1 [100]	2 [100]	3 [87]	7 [74]	19 [45]	31 [17]	71 [5]	98 [0]
	1	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	3 [83]	13 [60]	41 [11]	61 [7]	75 [5]	98 [0]
	2	0 [100]	0 [100]	0 [97]	2 [91]	3 [73]	18 [38]	31 [18]	62 [8]	72 [5]	75 [5]	98 [0]
exp	3	0 [100]	0 [99]	2 [80]	7 [63]	14 [49]	35 [13]	47 [9]	70 [6]	75 [5]	75 [5]	98 [0]
	4	0 [99]	0 [99]	0 [96]	0 [93]	1 [85]	14 [51]	22 [29]	54 [9]	70 [6]	75 [5]	98 [0]
	5	0 [99]	0 [99]	0 [95]	0 [90]	2 [85]	16 [48]	22 [26]	54 [10]	70 [6]	75 [5]	98 [0]
	1	0 [100]	0 [100]	0 [100]	0 [99]	1 [99]	1 [93]	1 [90]	8 [64]	15 [50]	46 [13]	98 [0]
	2	0 [100]	0 [100]	0 [100]	0 [99]	0 [97]	1 [95]	1 [89]	7 [73]	13 [54]	39 [13]	98 [0]
	3	1 [99]	1 [100]	1 [97]	1 [97]	2 [97]	2 [91]	3 [86]	8 [69]	15 [55]	38 [16]	98 [0]
weibull	4	0 [99]	0 [99]	0 [98]	0 [98]	0 [98]	2 [95]	3 [87]	6 [72]	8 [60]	36 [19]	98 [0]
	5	0 [98]	0 [98]	0 [97]	0 [97]	0 [97]	1 [92]	1 [88]	4 [76]	8 [65]	34 [23]	98 [0]
	6	0 [98]	0 [95]	0 [95]	0 [95]	0 [93]	1 [91]	1 [88]	3 [74]	8 [65]	30 [24]	98 [0]
	7	0 [97]	0 [96]	0 [95]	0 [94]	0 [92]	1 [90]	1 [86]	4 [73]	7 [60]	32 [22]	98 [0]

Table B.5.81: Average MTTF values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
norm	1	**171.5**	**171.5**	**169.4**	**166.4**	**166.4**	**158.4**	**151.2**	**130.2**	**112.9**	**59.8**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**170.7**	**170.7**	**161.6**	**152.1**	**84.9**	**14.5**
	3	**171.5**	**171.5**	**171.5**	**170.7**	**170.7**	**167.4**	**161.8**	**146.9**	**131.2**	**73.8**	**14.5**
	4	**171.5**	**171.5**	**170.7**	**169.7**	**168.9**	**164.4**	**161.9**	**147.7**	**132.7**	**83.8**	**14.5**
	5	**171.2**	**170.6**	**170.9**	**170.9**	**170.9**	**171.2**	**170.3**	**166.4**	**160.4**	**108.9**	**14.5**
	6	**170.6**	**170.6**	**170.9**	**170.9**	**170.3**	**169.8**	**169.4**	**164.2**	**152.5**	**108.3**	**14.5**
lnorm	1	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**166.4**	**156.1**	**129.4**	**108.8**	**57.1**	**14.5**
	2	**171.5**	**171.5**	**170.7**	**170.7**	**169.7**	**166.3**	**164.8**	**148.3**	**133**	**79.1**	**14.5**
	3	**171.5**	**171.5**	**171.5**	**171.5**	**170.7**	**169.1**	**163**	**151.3**	**142.1**	**86.7**	**14.5**
	4	**171.5**	**171.5**	**170.2**	**169.3**	**168.7**	**163.7**	**161**	**150.5**	**140.3**	**96.6**	**14.5**
	5	**171.5**	**171.5**	**170.8**	**168.3**	**167**	**162.7**	**157.2**	**150.1**	**135.9**	**86.6**	**14.5**
	6	**170.1**	**170**	**170.2**	**170.2**	**169.3**	**168.3**	**165.6**	**158.8**	**150.5**	**122.7**	**14.5**
	7	**170.1**	**170.2**	**170.5**	**170.3**	**169.9**	**169.1**	**167**	**159.4**	**155**	**125.2**	**14.5**
logis	1	**171.5**	**171.5**	**171.5**	**167.4**	**166.4**	**158.4**	**154.1**	**132.4**	**112.9**	**58**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**171.5**	**170.5**	**166.4**	**166.4**	**149.7**	**133.9**	**70.1**	**14.5**
exp	1	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**166.4**	**156.2**	**123.1**	**91.4**	**57.1**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**169.4**	**167.5**	**149.1**	**138.1**	**97.4**	**70.1**	**57.1**	**14.5**
	3	**171.5**	**171.5**	**168.5**	**165**	**155.1**	**132.9**	**118.9**	**76.3**	**62.8**	**57.1**	**14.5**
	4	**171.5**	**171.5**	**171.5**	**171.5**	**170.5**	**161.6**	**145.4**	**114.2**	**84.4**	**57.1**	**14.5**
	5	**171.5**	**171.5**	**171.5**	**171.5**	**170.5**	**158.3**	**144.9**	**115.2**	**83.8**	**57.1**	**14.5**
weibull	1	**170.3**	**170.3**	**169.7**	**169.7**	**169.7**	**168.3**	**161**	**138.5**	**121.1**	**71.6**	**14.5**
	2	**170.3**	**170.3**	**169.7**	**169.7**	**169.7**	**168.6**	**165.8**	**147.9**	**135.7**	**77.5**	**14.5**
	3	**170.3**	**170.3**	**169.2**	**169.2**	**169.2**	**167.9**	**167.1**	**155.6**	**147.2**	**102.9**	**14.5**
	4	**170.6**	**170.6**	**170.9**	**170.3**	**170.9**	**168.2**	**167.5**	**163.5**	**152.3**	**109.7**	**14.5**
	5	**170.6**	**170.6**	**170.9**	**170.9**	**170.3**	**170.1**	**169.3**	**165.6**	**156.9**	**114.1**	**14.5**
	6	**170.6**	**170.6**	**170.9**	**170.9**	**170.9**	**170.6**	**170.3**	**166.3**	**161.3**	**119.7**	**14.5**
	7	**170.6**	**170.6**	**170.9**	**170.9**	**170.3**	**170**	**168.7**	**164.4**	**160.6**	**116.7**	**14.5**

Table B.5.82: Average MTTF values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
norm	1	**171.5**	**170.4**	**167.4**	**166.4**	**163.7**	**155.4**	**148**	**125.4**	**103.5**	**57.1**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**170.7**	**167.5**	**158.6**	**146.8**	**73.7**	**14.5**
	3	**171.5**	**171.5**	**170.7**	**170.7**	**169.7**	**163**	**155.7**	**140.1**	**117.5**	**58**	**14.5**
	4	**171.5**	**170.7**	**169.7**	**169.1**	**164.8**	**160.5**	**154.9**	**136.6**	**118.3**	**64.1**	**14.5**
	5	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**170.1**	**166.5**	**124.5**	**14.5**
	6	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**170.6**	**166.9**	**162.7**	**119.2**	**14.5**
lnorm	1	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**163.1**	**150.4**	**123.6**	**97.4**	**57.1**	**14.5**
	2	**171.5**	**170.7**	**170.7**	**169.7**	**169.7**	**166.3**	**157.4**	**139.7**	**121.7**	**64.9**	**14.5**
	3	**171.5**	**171.5**	**170.7**	**170.7**	**170.7**	**165.7**	**159.7**	**144.4**	**127.5**	**66.9**	**14.5**
	4	**171.5**	**170.9**	**169.3**	**167.4**	**165.5**	**160.3**	**157.7**	**142**	**129**	**76.8**	**14.5**
	5	**171.5**	**171.4**	**167.7**	**166.6**	**165.7**	**156.9**	**156**	**137.1**	**124.2**	**64.6**	**14.5**
	6	**170.6**	**170.6**	**170.8**	**170.8**	**169.3**	**168.1**	**168.1**	**160.7**	**153.6**	**131.6**	**14.5**
	7	**171.5**	**171.1**	**171.1**	**170.3**	**169.9**	**168.9**	**168.1**	**164.5**	**159.3**	**135.2**	**14.5**
logis	1	**171.5**	**171.5**	**169.4**	**167.4**	**166.4**	**156.9**	**150.4**	**127.2**	**105.4**	**57.1**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**170.4**	**168.5**	**165.6**	**158.4**	**141.8**	**128**	**64.8**	**14.5**
exp	1	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**166.4**	**151.2**	**114.5**	**83.6**	**57.1**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**168.5**	**165.6**	**145.4**	**128.7**	**82.2**	**62.8**	**57.1**	**14.5**
	3	**171.5**	**171.5**	**167.4**	**159.2**	**149.5**	**123.6**	**105.8**	**66.7**	**57.1**	**57.1**	**14.5**
	4	**171.5**	**171.5**	**171.5**	**171.5**	**169.4**	**150.7**	**140.1**	**96.7**	**68.5**	**57.1**	**14.5**
	5	**171.5**	**171.5**	**171.5**	**171.5**	**167.7**	**147.9**	**140.1**	**98.1**	**68.5**	**57.1**	**14.5**
weibull	1	**171.5**	**171.5**	**171.5**	**171.5**	**170.9**	**170.3**	**170.3**	**159.7**	**150.1**	**109**	**14.5**
	2	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**170.9**	**170.5**	**161.7**	**152.9**	**119.8**	**14.5**
	3	**170.9**	**170.9**	**170.3**	**170.4**	**169.2**	**168.6**	**168.1**	**161.9**	**154.8**	**121.7**	**14.5**
	4	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**168.6**	**168.2**	**165.2**	**161.9**	**125.9**	**14.5**
	5	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**171**	**169.9**	**166.8**	**161.8**	**127.4**	**14.5**
	6	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**171**	**171**	**168.4**	**162.3**	**132.5**	**14.5**
	7	**171.5**	**171.5**	**171.5**	**171.5**	**171.5**	**171**	**171**	**167.7**	**163**	**129.1**	**14.5**

Table B.5.83: Average MTTRep values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
	1	410.6	397.8	370	354.3	338.6	302.9	280.8	230.9	197.6	107.6	20
	2	626	599.4	542.9	513.3	479.4	406.1	368.7	278.3	236.8	136.6	20
norm	3	566.2	544.2	489.9	460	432	370.1	336.6	257.8	218.3	127.5	20
	4	539.3	518.3	470.4	443.4	414.3	354.5	324.2	256.7	218.4	137.5	20
	5	641.4	614.3	564.7	540.5	512	438.7	392.9	311.1	267.5	159.3	20
	6	602	580.6	535.4	514.4	488.4	414	377.4	301.2	258.3	160.8	20
	1	568.6	533.9	464.1	429.4	395	325.1	291.2	220	183.8	101.4	20
	2	693.3	645.3	551.1	512.1	465.9	381.4	338.4	256.9	221.6	138.8	20
	3	669.2	629.8	550.1	509.2	466.9	389	349.9	269.3	230.8	146.5	20
lnorm	4	803.4	761.7	667.4	614.4	566.5	443.7	392.5	293.9	246	147.2	20
	5	834.9	792.7	689.1	634.8	582.6	467	408.3	306.7	259.5	152.3	20
	6	899.6	871.5	768.3	725.2	671.1	559.1	492.8	366.2	298.3	187.4	20
	7	814.5	766.8	694.3	642.9	596.2	490.6	428.3	327.8	274.5	174.7	20
logis	1	425.2	412.3	383.5	366.9	349.5	311.6	288.3	233.6	197.6	104.6	20
	2	452	440.3	415.1	400.5	384.6	346	324	269.8	235.1	133.6	20
	1	655.8	610.7	522.9	476.9	430.9	338.1	291.9	203	159.8	101	20
	2	513.1	477.7	405.7	368.4	333.5	264.6	230.2	164.7	131.5	100.2	20
exp	3	436.8	406.1	344	313.6	284.8	225.3	196.3	139.1	110.6	100	20
	4	546.1	508.5	436.2	397.7	361.3	288.4	252.6	182	147.9	100.8	20
	5	537.9	500.9	429.6	392.2	357.6	286.5	250.7	181.6	149	100.9	20
	1	470.6	451.9	407.9	385.8	365	315.4	289.8	230.1	198.8	118.1	20
	2	501.3	480.4	434.6	413.4	390.4	335	309.8	249.4	219.1	136.9	20
	3	541.5	518.1	474.7	448.7	428.7	367.6	332.4	278.2	242.3	161.3	20
weibull	4	553.6	533.4	492.7	469.1	447.9	380.6	347.4	290.3	253.2	167.4	20
	5	581	558.8	524.1	499.5	472.4	403.9	368.2	306.3	266.3	175	20
	6	595.4	568.5	530	507.4	488.8	426.2	391	315.8	275.8	187.7	20
	7	552.9	533.8	501.6	484.2	460.8	404.5	365.6	306	269.3	185.8	20

Table B.5.84: Average MTTRep values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	86%	87%	89%	90%	91%	93%	94%	96%	97%	99%	100%
	1	398.4	385.9	357	342.6	327	291.3	269.7	219.3	186.1	102	20
	2	603.7	576.5	521.2	488.6	455.6	382.5	341.3	257	214.9	117.3	20
norm	3	540.9	516.5	459.7	431.4	404.3	340.5	304.2	231.1	191.6	104.7	20
	4	512.6	485.6	437.5	410	379.3	321.4	292.5	226.3	190.6	109.2	20
	5	678.5	652.9	596.9	568.1	539.8	474.9	439.7	347.7	296.7	187.2	20
	6	637.9	611.6	561.9	542.8	520.1	459.5	422.7	335.8	286.7	188.6	20
	1	555.6	520.7	450.2	415.2	380.2	311.2	277.3	207.1	171	101	20
	2	671.6	621.8	531.8	486	439.8	357	311.5	235.3	197.6	111.5	20
	3	649.2	605.1	521.8	480.8	439.3	359.8	318	242.1	205.2	115.4	20
lnorm	4	785.1	734.7	637.1	578.9	522.2	410.6	360.4	261.2	211.9	122.7	20
	5	806.6	758.9	649.7	593.8	544.6	422.8	375.9	272.9	222.5	119.8	20
	6	934.3	895.1	793.8	747.5	686	577.2	520.2	385.4	319	201.7	20
	7	863.6	803.8	716	657.6	610.7	508.8	451.3	346.8	295.6	192.6	20
logis	1	413.4	399.9	370.5	354.2	337.7	299.7	276	220	185.4	101.3	20
	2	433.8	422.2	396.2	380.8	363.6	327.2	306.9	251.4	215.8	114.7	20
	1	642.7	598.6	509.1	462.8	416.9	323.8	278.5	188.9	146.2	100.5	20
	2	494.8	459	383.8	347.9	313	244.3	211.7	145.3	112.2	100	20
exp	3	415.3	384	322.8	294.2	264.7	204.6	175.7	118.2	101.2	100	20
	4	524.9	487.1	412.9	375.6	339.6	267.6	232.2	160.6	125.4	100.2	20
	5	515.8	477.9	404.7	368.6	333.4	264.1	228.7	158.6	125.1	100.2	20
	1	540.2	519.6	478.8	455.6	429.6	375.6	351.9	294.5	261.7	180.7	20
	2	558.6	539.4	495.3	471.4	443.3	393.1	362.5	300.2	269.9	186.7	20
	3	567.1	551.4	497.1	482.7	455.6	394	366.2	302.6	267.5	187.9	20
weibull	4	582.7	564.1	521.3	500.9	475.9	414.6	381.2	316.7	281.4	192.5	20
	5	614.9	589	545.1	526	501.5	437.8	409.3	332.9	296.9	201.9	20
	6	622.7	598.8	551.4	528	503.6	450.8	417.7	343.6	301.3	205.4	20
	7	584.4	563.1	525.1	502.4	483.6	433.7	400.8	334.6	294.8	205.3	20

Table B.5.85: Variables identified by time-(in)dependent PHM models.

	Time-independent PHM	
	Variable	Scaled Value
①	*Yaw rate min deg sec 1*	−4.04
②	*Roll rate min deg sec 3*	3.94
③	*Group B*	3.75
④	*Group A*	−3.37
⑤	*Vz mean ft min 4*	2.52

	Time-dependent PHM	
	Variable	Scaled Value
①	*Brake press lhs mean psi 8*	−30.42
②	*Roll rate mean deg sec 1*	−12.74
③	*Yaw rate min deg sec 1*	−10.12
④	*Roll rate min deg sec 3*	−9.86
⑤	*Group C*	−9.83
⑥	*Group B*	9.43
⑦	*Pitch rate max deg sec 2*	7.88
⑧	*Vz mean ft min 4*	3.46

Table B.5.86: Variables identified by each step by time-(in)dependent PHMs (in order).

	PHM Variables	
	Time-independent	*Time-dependent*
norm	④ ①	② ① ⑥ ④ ⑤ ⑦
lnorm	④ ① ② ③ ⑤	② ① ⑥ ④ ③ ⑤ ⑦
logis	④ ③ ①	② ④
exp	③ ②	② ④ ⑥ ⑧ ③
weibull	④ ① ②	⑤ ② ① ⑥ ④ ③ ⑦
gamma	④ ③ ② ①	

Table B.5.87: Number of times variables identified by each step by time-(in)dependent PHMs.

Key				**Key**			
indep	*dep*	*Variable*	*Count*	*indep*	*dep*	*Variable*	*Count*
④		Group A	5		②	Roll rate mean deg sec 1	5
③	⑥	Group B	8	②	④	Roll rate min deg sec 3	9
	⑤	Group C	3	⑤	⑧	Vz mean ft min 4	2
	①	Brake press lhs mean psi 8	3	①	③	Yaw rate min deg sec 1	8
	⑦	Pitch rate max deg sec 2	3				

Table B.5.88: Variables belonging to each group identified in B.5.85.

Group	*Variables*	*Group*	*Variables*
Group A	Headwind max knots 1, Headwind mean knots 1	Group C	Vcal mean knots 2, Pressure dynamic mean hPa mbar 2
Group B	NormalForce lhs max lbs 6, NormalForce lhs max lbs 5		

B.6 3-1574 NLG wheel & tire assembly

Table B.6.89 provides a summary of the input data related to the component. The number of registered maintenance events is less than the total number of events due to the fact that TRAX data stretches back to 2004/2005 and FDR data only to 2011. Maintenance events with insufficient data, regarding operational factors, cannot be evaluated, hence are not registered during the modelling process.

Table B.6.89: General overview of component inputs.

Name	**Value**
Part Number	3-1574
Total # (A, F, C)	**19504, 989, 18515**
Registered # (A, F, C)	**3092, 153, 2939**
Related Flights # (A, F, C)	**394788, 14430, 380358**
Avg. Cycles (A, F, C)	**127.68, 94.31, 129.42**
% Censored	**95.05**

In Tab. B.6.89 (A, F, C) denotes statistics regarding All (A), Failed (F), and Censored (C) events respectively. Ergo A will always be the sum or mean derived from F and C.

Analysis
Tables B.6.90 and B.6.91 summarise the results from EVA and MDA. In addition the variables obtained by semi-parametric PHM modelling (labelled 'reduced semi-COX') are also presented if applicable. Table B.6.91 provides an overview of the specific operational factors identified during all flight phases. In this case high counts indicate operational factors that were significantly different during multiple flight phases.

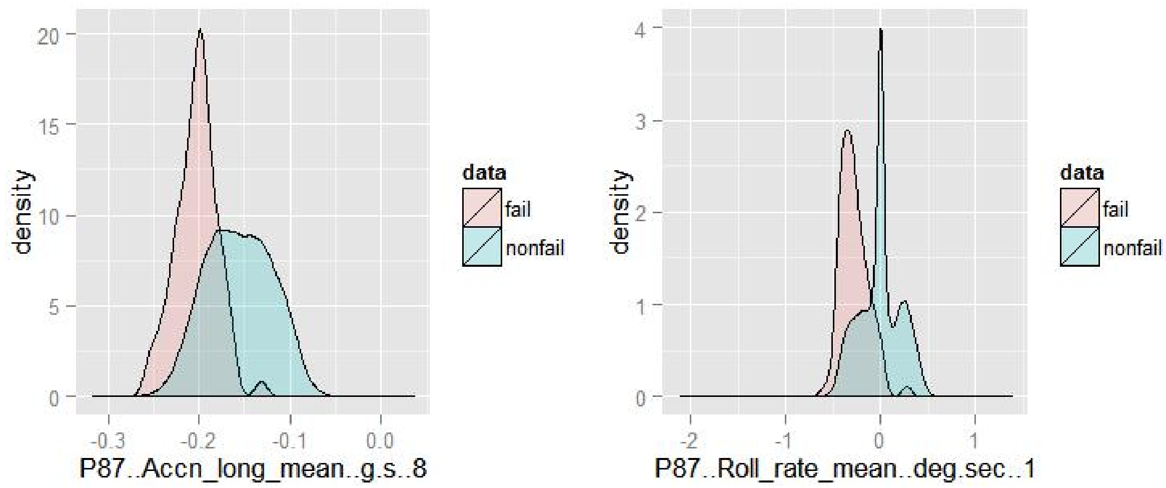
Table B.6.90: Overview of analysis input and output.

	# Variables
ALL	**1531**
EVA	**70**
MDA	**0**
Combined	**70**
reduced Corr.	**56**
reduced semi-COX	**31**
Take-Off related	**18**
Cruise related	**11**
Touch-Down related	**27**

Table B.6.91: Overview of identified operational factors during all flight phases.

Variable	**Count**	**Variable**	**Count**	**Variable**	**Count**
Yaw_rate	**7**	Pitch	**3**	Rudder_cmd_force	**1**
Vtrue	**6**	NormalForce_lhs	**2**	Brake_press_lhs	**1**
Vz	**5**	Torque_rhs	**2**	Brake_press_rhs	**1**
Pressure_dynamic	**4**	Accn_lat	**2**	Roll	**1**
Pitch_rate	**3**	NormalForce_rhs	**2**	Crosswind	**1**
Roll_rate	**3**	Vcal	**2**	Headwind	**1**
Aoa	**3**	Elevator_Rin	**1**	Accn_norm	**1**
Accn_long	**3**	NormalForce_nose	**1**		

A multitude of factors were identified during EVA. Figure B.6.57 give a general overview of the top operational factors identified by EVA.



(a) EVA 1. (b) EVA 2.

Figure B.6.57: Graphical overview of top operational factors identified by EVA.

Time-based reliability modelling

Table B.6.92 reports the maximum likelihood and goodness-of-fit tests results obtained from time-based reliability modelling. To show the overall fit Fig. B.6.58 shows the computed reliability function using an averaged virtual age V for all fitted models.

Table B.6.92: Overview of results obtained by MLE optimisation and GOF tests.

	Distributions					
	norm	**lnorm**	**logis**	**exp**	**weibull**	**gamma**
MLE	-1337.08	-1338.14	-1337.09	-3522.96	$-1.8e+100$	-1336.46
Kolmogorov-Smirnov	1.63	3.12	1.55	8.73	$9e+99$	2.53
Cramer-von-Mises Smirnov	67.77	64.85	68.03	51	$9e+99$	66.1
Anderson-Darling	-165.44	-165.54	-165.44	-219.99	$9e+99$	-165.48
NRR	77.1	82.92	76.44	153	$9e+99$	78.06

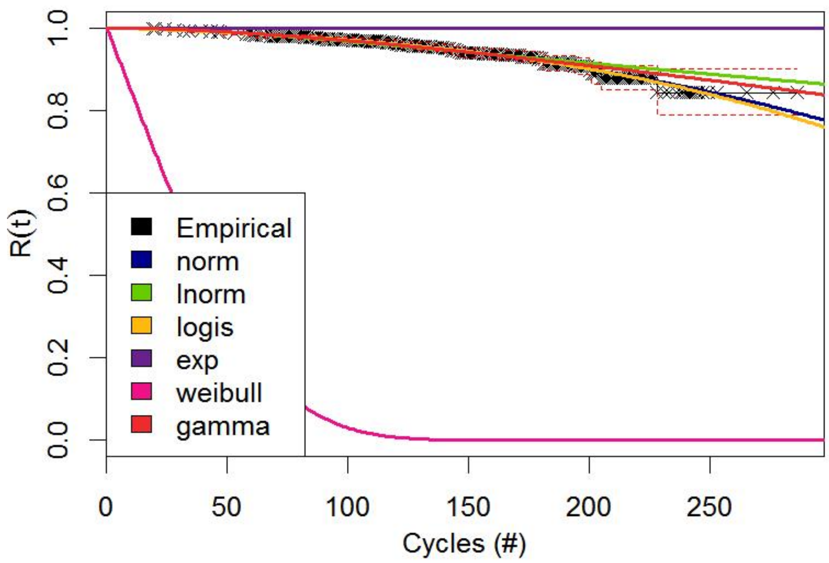
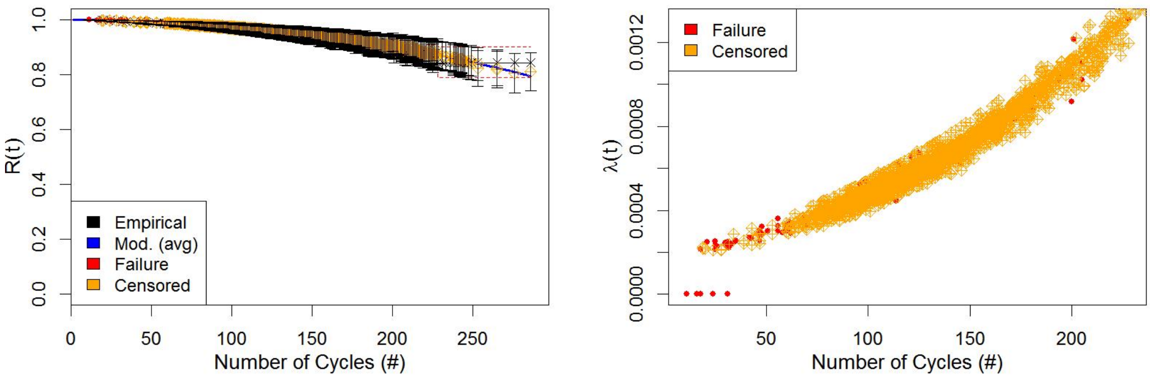


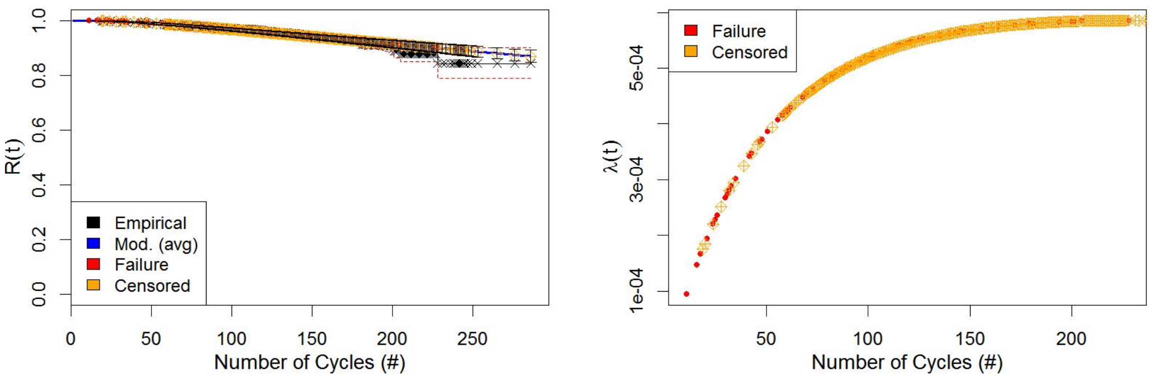
Figure B.6.58: Overview of overall fit of multiple GRP modells.

In addition Figures B.6.59, B.6.60, B.6.61, B.6.62, B.6.63, and B.6.64 present the reliability and hazard functions computed for each underlying distribution evaluated in the program.



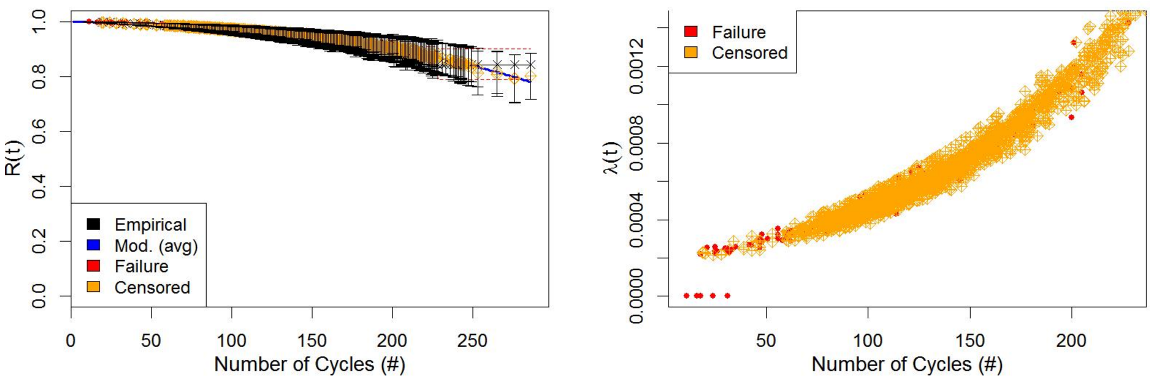
(a) Reliability function. (b) Hazard function.

Figure B.6.59: Computed reliability for time-based models with underlying norm distribution.



(a) Reliability function. (b) Hazard function.

Figure B.6.60: Computed reliability for time-based models with underlying lnorm distribution.

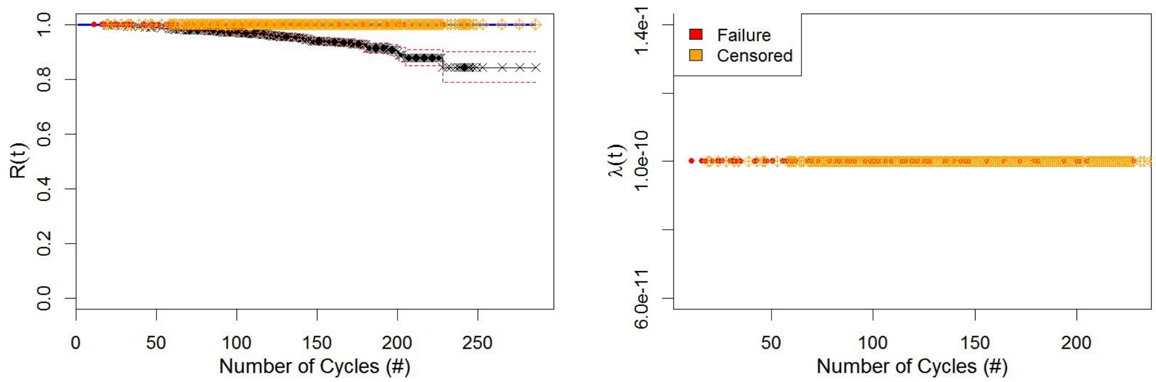


(a) Reliability function. (b) Hazard function.

Figure B.6.61: Computed reliability for time-based models with underlying logis distribution.

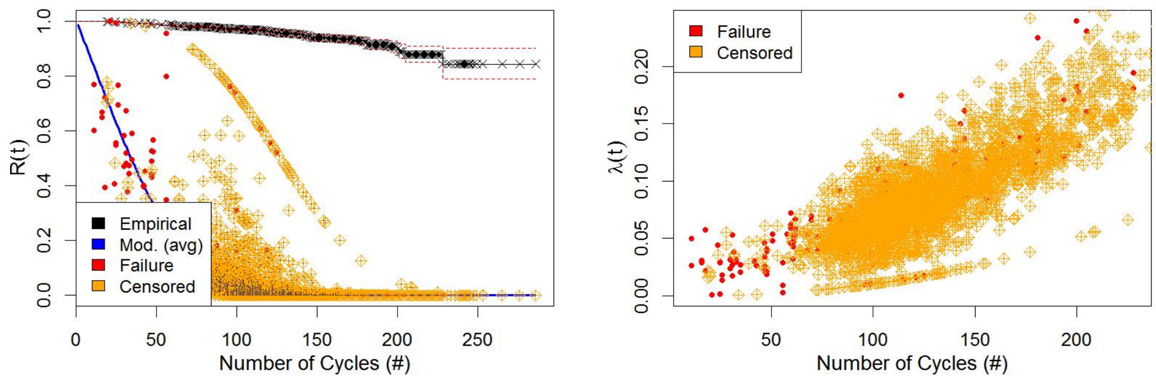
Time independent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.6.93 gives a general overview of all the models obtained during each step in the process.



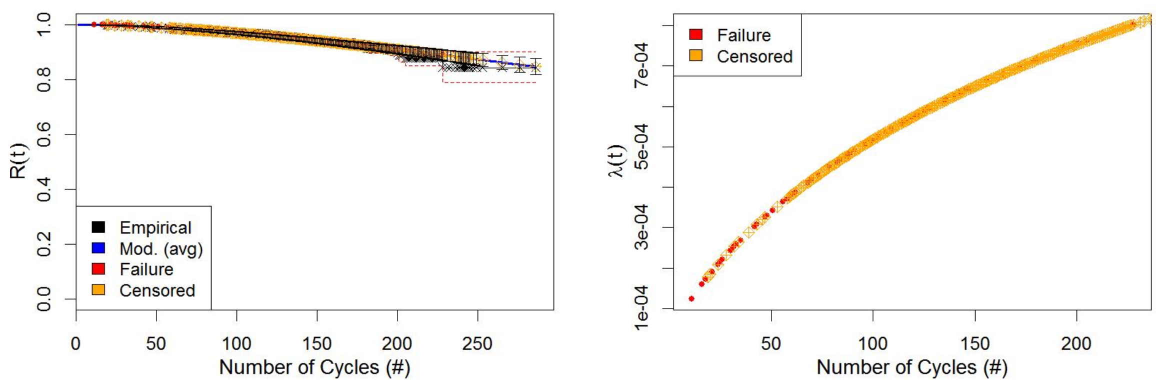
(a) Reliability function. (b) Hazard function.

Figure B.6.62: Computed reliability for time-based models with underlying exp distribution.



(a) Reliability function. (b) Hazard function.

Figure B.6.63: Computed reliability for time-based models with underlying weibull distribution.



(a) Reliability function. (b) Hazard function.

Figure B.6.64: Computed reliability for time-based models with underlying gamma distribution.

Graphical representation of the computed reliability per model are shown in Figures B.6.65, B.6.66, and B.6.67 as well as a general overview in Figure B.6.67b.

Tables B.6.94 and B.6.95 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

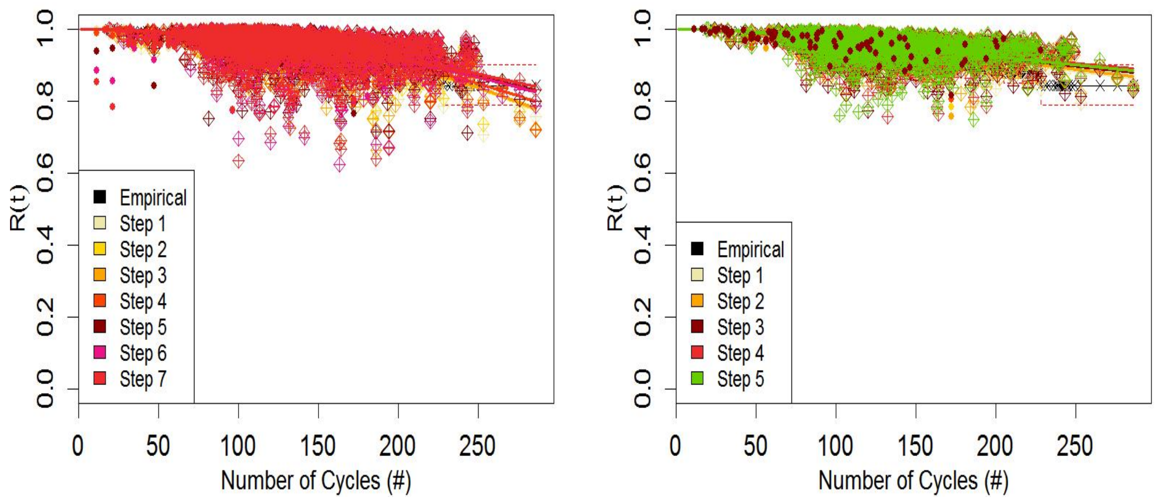
Table B.6.93: Overview of time-independent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	norm	norm	norm	norm	norm	norm	lnorm
Step #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**1**
MLE	**−1325.6**	**−1318.06**	**−1312.94**	**−1304.41**	**−1296.02**	**−1287.9**	**−1284.41**	**−1327.3**
Time (min)	**10.02**	**21.12**	**32.48**	**41.38**	**48.68**	**54.21**	**58.64**	**9.75**
Kolmogorov-Smirnov	**5.54**	**9.16**	**7.04**	**7.69**	**9.09**	**11.05**	**11.92**	**4.85**
Cramer-von Mises-Smirnov	**65.25**	**67.41**	**65.37**	**60.38**	**53.03**	**44.31**	**40.65**	**62.58**
Anderson-Darling	**−165.68**	**−165.78**	**−165.8**	**−166.14**	**−166.24**	**−166.45**	**−166.4**	**−165.83**
NRR	**78.19**	**78.6**	**78.32**	**77.22**	**75.85**	**72.78**	**70.62**	**77.93**

Distribution	lnorm	lnorm	lnorm	lnorm	logis	logis	logis	logis
Step #	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**
MLE	**−1320.68**	**−1314.49**	**−1310.97**	**−1307.35**	**−1325.57**	**−1318.17**	**−1312.84**	**−1304.06**
Time (min)	**18.9**	**28.24**	**36.95**	**43.91**	**8.63**	**17.93**	**27.45**	**35.68**
Kolmogorov-Smirnov	**8.11**	**9.59**	**6.9**	**6.41**	**5.58**	**9.88**	**7.35**	**9.14**
Cramer-von Mises-Smirnov	**64.14**	**60.77**	**59.32**	**58.91**	**65.55**	**67.24**	**65.26**	**60.88**
Anderson-Darling	**−165.92**	**−166.3**	**−166.44**	**−166.54**	**−165.67**	**−165.81**	**−165.81**	**−166.12**
NRR	**81.94**	**76.54**	**75.08**	**74.19**	**77.84**	**78.3**	**76.86**	**78.95**

Distribution	logis	weibull	weibull	weibull	weibull	weibull	weibull	gamma
Step #	**5**	**1**	**2**	**3**	**4**	**5**	**6**	**1**
MLE	**−1300.3**	**−1325.16**	**−1318.02**	**−1310.69**	**−1301.57**	**−1295.7**	**−1291.41**	**−1325.49**
Time (min)	**42.24**	**13.89**	**24.42**	**34.39**	**43.37**	**51.49**	**58.35**	**216.83**
Kolmogorov-Smirnov	**8.19**	**3.47**	**3.88**	**5**	**5.95**	**8.4**	**7.34**	**5.2**
Cramer-von Mises-Smirnov	**58.47**	**68.8**	**64.49**	**62.5**	**56.68**	**54.23**	**49.73**	**63.39**
Anderson-Darling	**−166.31**	**−165.63**	**−165.81**	**−165.93**	**−165.95**	**−166.3**	**−166.34**	**−165.77**
NRR	**77.85**	**86.72**	**87.34**	**81.92**	**78.98**	**77.65**	**78.16**	**80.01**

Distribution	gamma	gamma	gamma
Step #	**2**	**3**	**4**
MLE	**−1318.48**	**−1310.85**	**−1308.99**
Time (min)	**404.4**	**562.13**	**725.52**
Kolmogorov-Smirnov	**8.84**	**9.61**	**9.96**
Cramer-von Mises-Smirnov	**65.35**	**61.56**	**62.38**
Anderson-Darling	**−165.87**	**−166.19**	**−166.16**
NRR	**81.53**	**80.56**	**80.12**



(a) With an underlying norm distribution.

(b) With an underlying lnorm distribution.

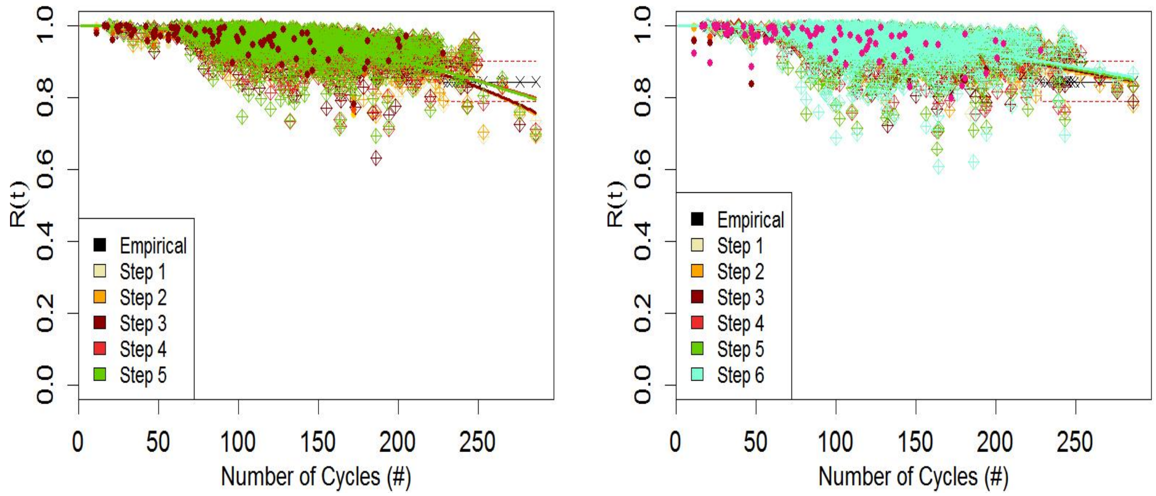
Figure B.6.65: Time-independent PHMs with an underlying norm and lnorm distribution.

To assist in the selection of models, Tables B.6.96, B.6.97, B.6.98, and B.6.99 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.

Time dependent proportional hazard reliability modelling

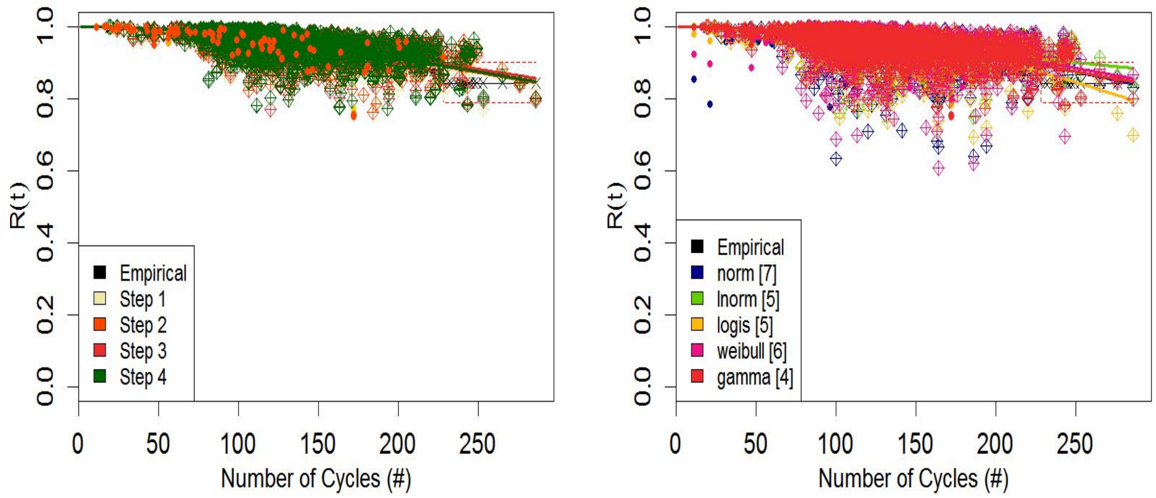
Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.6.100 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.6.68,



(a) With an underlying logis distribution. (b) With an underlying weibull distribution.

Figure B.6.66: Time-independent PHMs with an underlying logis and weibull distribution.



(a) With an underlying gamma distribution. (b) All distributions (last step).

Figure B.6.67: Figures containing a gamma distribution and all time-independent PHMs.

and B.6.69 as well as a general overview in Figure B.6.70a.
Tables B.6.101 and B.6.102 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

To assist in the selection of models, Tables B.6.103, B.6.104, B.6.105, and B.6.106 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.
The operational factors identified during time-independent and time-dependent PHM modelling are shown in Tables B.6.107, B.6.108, B.6.109, and B.6.110.

Table B.6.94: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	0 [99]	0 [98]	1 [96]	4 [93]	7 [87]	9 [86]	18 [79]	20 [70]	41 [32]	58 [10]	96 [0]
	2	1 [99]	1 [97]	3 [96]	4 [92]	5 [89]	7 [87]	12 [80]	22 [73]	39 [34]	58 [11]	96 [0]
	3	3 [96]	4 [96]	4 [95]	8 [91]	9 [88]	9 [84]	19 [79]	27 [70]	46 [36]	56 [16]	96 [0]
norm	4	3 [97]	4 [97]	5 [92]	8 [91]	10 [88]	12 [87]	20 [81]	25 [73]	44 [44]	55 [19]	96 [0]
	5	4 [95]	4 [95]	4 [94]	7 [92]	11 [91]	12 [88]	18 [81]	20 [74]	39 [42]	50 [24]	96 [0]
	6	3 [96]	4 [96]	4 [95]	7 [93]	9 [93]	12 [90]	18 [84]	23 [78]	36 [50]	51 [23]	96 [0]
	7	3 [96]	4 [96]	5 [95]	5 [92]	9 [92]	12 [90]	18 [82]	24 [77]	37 [47]	51 [22]	96 [0]
	1	0 [99]	0 [99]	0 [99]	1 [96]	3 [90]	8 [87]	18 [78]	22 [64]	45 [20]	61 [4]	96 [0]
	2	0 [100]	1 [99]	1 [99]	1 [95]	5 [91]	5 [89]	13 [81]	22 [69]	44 [27]	61 [9]	96 [0]
lnorm	3	1 [100]	1 [99]	1 [98]	1 [95]	4 [92]	7 [90]	14 [82]	24 [77]	42 [33]	59 [9]	96 [0]
	4	1 [100]	3 [99]	3 [97]	5 [96]	8 [93]	8 [91]	20 [84]	27 [75]	44 [30]	63 [10]	96 [0]
	5	3 [99]	1 [97]	3 [96]	4 [94]	7 [91]	8 [90]	20 [81]	28 [71]	46 [30]	59 [12]	96 [0]
	1	0 [99]	0 [98]	1 [96]	4 [93]	7 [87]	9 [86]	18 [79]	20 [70]	40 [33]	59 [10]	96 [0]
	2	1 [99]	1 [97]	3 [96]	4 [92]	5 [88]	7 [87]	13 [80]	21 [74]	38 [35]	60 [12]	96 [0]
logis	3	3 [96]	4 [95]	4 [93]	8 [91]	9 [89]	10 [84]	20 [80]	25 [70]	45 [36]	56 [16]	96 [0]
	4	4 [97]	4 [95]	4 [93]	7 [92]	10 [89]	14 [87]	20 [83]	25 [78]	41 [43]	56 [19]	96 [0]
	5	3 [95]	4 [93]	4 [93]	5 [91]	11 [91]	15 [85]	22 [82]	24 [71]	42 [41]	54 [19]	96 [0]
	1	0 [99]	0 [99]	1 [99]	3 [97]	5 [92]	5 [89]	13 [79]	25 [69]	43 [28]	59 [8]	96 [0]
	2	0 [99]	0 [99]	0 [98]	1 [96]	7 [93]	8 [91]	20 [83]	24 [73]	42 [41]	55 [13]	96 [0]
weibull	3	1 [99]	3 [98]	3 [98]	5 [96]	10 [91]	14 [88]	22 [78]	24 [69]	47 [42]	60 [9]	96 [0]
	4	1 [96]	4 [96]	4 [97]	5 [95]	10 [91]	12 [88]	20 [80]	24 [75]	39 [43]	58 [18]	96 [0]
	5	1 [97]	4 [94]	7 [94]	7 [92]	9 [91]	14 [87]	18 [79]	22 [75]	43 [40]	52 [23]	96 [0]
	6	1 [97]	3 [94]	5 [94]	7 [92]	10 [89]	13 [87]	16 [80]	19 [73]	43 [45]	51 [26]	96 [0]
	1	0 [99]	0 [99]	0 [99]	1 [96]	5 [90]	8 [87]	18 [78]	22 [66]	42 [23]	58 [8]	96 [0]
gamma	2	1 [99]	1 [99]	1 [98]	3 [93]	5 [90]	5 [88]	14 [81]	24 [70]	42 [30]	61 [10]	96 [0]
	3	1 [99]	1 [98]	1 [97]	1 [94]	4 [92]	9 [88]	16 [81]	25 [76]	39 [38]	56 [10]	96 [0]
	4	1 [99]	1 [98]	1 [97]	1 [94]	5 [92]	7 [88]	14 [82]	23 [75]	38 [37]	54 [11]	96 [0]

Table B.6.95: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	0 [97]	1 [96]	4 [92]	5 [89]	9 [85]	14 [82]	22 [63]	33 [53]	54 [12]	67 [2]	96 [0]
	2	1 [99]	1 [98]	1 [97]	4 [92]	5 [89]	5 [87]	10 [81]	20 [78]	37 [41]	50 [14]	96 [0]
	3	3 [96]	4 [96]	5 [94]	8 [91]	9 [88]	10 [84]	20 [79]	29 [67]	48 [33]	60 [15]	96 [0]
norm	4	3 [97]	4 [97]	4 [96]	7 [91]	10 [89]	10 [87]	18 [83]	24 [78]	40 [51]	50 [24]	96 [0]
	5	3 [98]	4 [95]	4 [95]	5 [92]	9 [92]	10 [90]	16 [84]	19 [79]	33 [53]	47 [29]	96 [0]
	6	4 [96]	7 [93]	7 [93]	10 [93]	15 [90]	19 [88]	23 [77]	26 [63]	48 [32]	60 [13]	96 [0]
	7	3 [96]	4 [96]	7 [95]	5 [92]	11 [92]	14 [90]	20 [82]	23 [75]	40 [45]	52 [18]	96 [0]
	1	0 [99]	1 [98]	1 [96]	4 [91]	8 [85]	10 [82]	24 [61]	35 [46]	58 [11]	64 [2]	96 [0]
	2	0 [100]	1 [99]	1 [99]	1 [95]	4 [91]	5 [89]	12 [82]	21 [74]	42 [29]	60 [10]	96 [0]
lnorm	3	0 [100]	1 [99]	1 [99]	1 [98]	1 [94]	3 [93]	8 [88]	16 [81]	33 [52]	47 [23]	96 [0]
	4	1 [100]	1 [100]	1 [100]	4 [98]	5 [93]	8 [93]	15 [88]	23 [82]	40 [46]	51 [18]	96 [0]
	5	3 [99]	1 [97]	3 [96]	3 [93]	7 [92]	8 [90]	19 [81]	25 [69]	45 [32]	60 [11]	96 [0]
	1	0 [96]	1 [96]	4 [92]	5 [88]	9 [85]	14 [82]	22 [67]	31 [53]	53 [12]	67 [2]	96 [0]
	2	1 [99]	1 [96]	1 [96]	4 [92]	5 [88]	7 [87]	12 [80]	21 [75]	38 [38]	56 [12]	96 [0]
logis	3	3 [96]	4 [95]	8 [92]	8 [91]	9 [87]	12 [83]	22 [76]	31 [65]	50 [31]	61 [14]	96 [0]
	4	3 [98]	4 [98]	4 [98]	4 [93]	5 [90]	9 [90]	15 [85]	20 [82]	35 [56]	48 [30]	96 [0]
	5	3 [96]	4 [94]	4 [93]	4 [92]	9 [91]	11 [85]	18 [83]	24 [76]	39 [48]	50 [25]	96 [0]
	1	0 [100]	0 [99]	0 [99]	0 [99]	3 [97]	4 [95]	5 [85]	10 [81]	32 [61]	43 [24]	96 [0]
	2	0 [100]	0 [100]	0 [99]	0 [99]	0 [98]	1 [96]	7 [93]	9 [89]	22 [74]	34 [55]	96 [0]
weibull	3	1 [100]	3 [100]	3 [99]	3 [98]	3 [97]	7 [95]	14 [86]	19 [84]	29 [60]	44 [40]	96 [0]
	4	1 [99]	1 [99]	3 [98]	4 [98]	4 [94]	5 [93]	12 [87]	18 [83]	28 [60]	39 [42]	96 [0]
	5	1 [96]	4 [95]	5 [95]	7 [92]	9 [92]	11 [89]	16 [82]	20 [77]	37 [50]	50 [28]	96 [0]
	6	1 [98]	1 [96]	1 [94]	7 [94]	7 [92]	10 [91]	16 [85]	16 [81]	25 [68]	46 [40]	96 [0]
	1	0 [98]	1 [97]	1 [95]	7 [90]	8 [86]	10 [81]	24 [61]	34 [49]	56 [11]	64 [2]	96 [0]
gamma	2	1 [99]	1 [99]	1 [99]	1 [93]	5 [91]	5 [89]	13 [82]	22 [74]	39 [35]	56 [11]	96 [0]
	3	0 [99]	1 [99]	1 [99]	1 [98]	1 [94]	3 [94]	9 [89]	13 [84]	31 [62]	42 [32]	96 [0]
	4	0 [99]	1 [99]	1 [99]	1 [98]	2 [94]	3 [93]	8 [87]	14 [85]	31 [60]	40 [30]	96 [0]

Table B.6.96: Average MTTF values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
norm	1	94.3	94.3	93.3	90.3	87.6	85.1	75.5	74.4	57.6	45.1	15
	2	93.3	93.3	91.5	90.3	88.9	87.6	83.5	73.7	60.7	47.9	15
	3	94.5	93.4	93.4	90.1	88.5	88.5	79.2	73.2	59.4	52.6	15
	4	94.5	93.4	93.2	90.4	89.6	86.7	79.9	75.7	61.7	52.8	15
	5	93.4	93.4	93.4	91.8	87.5	85.4	82	79.5	67.2	55.1	15
	6	94.5	93	93	91.7	88.9	86.7	81.1	77.1	66.4	54.9	15
	7	94.5	93	92.8	92.6	88.9	86.6	81.1	76.1	66.9	54.6	15
lnorm	1	94.3	94.3	94.3	93.3	92.1	86.7	75.5	72.7	55.1	41.7	15
	2	94.3	93.3	93.3	93.3	88.9	88.9	83.3	76.9	56.2	43.4	15
	3	93.3	93.3	93.3	93.3	90.7	89.1	83.1	76.4	56.9	45.1	15
	4	95.3	94.3	94.3	93.2	91.3	91.2	81.2	76.1	61.4	45.8	15
	5	94.5	95.3	94.3	93.4	92.5	91.2	81.1	75.7	58.7	47.6	15
logis	1	94.3	94.3	93.3	90.3	87.6	85.1	75.5	74.4	58.8	44.7	15
	2	93.3	93.3	91.5	90.3	88.9	87.6	81.7	74.5	61.9	47.1	15
	3	94.5	93.4	93.4	90.1	88.5	88.4	79.1	74.3	60.7	52.6	15
	4	93.4	93.4	93.4	92	87.7	85.1	79.9	76.5	63.1	51.9	15
	5	94.5	93.4	93.4	93.2	88.1	83.6	78.8	77.7	61.5	52.6	15
weibull	1	94.3	94.3	92.5	91.5	88.9	88.9	83	72.3	56.1	44	15
	2	94.3	94.3	94.3	93.3	87.9	87.2	79.1	74.9	58	46.6	15
	3	95.3	94.5	94.5	93.1	87.8	86	80.2	78.5	57	45	15
	4	95.3	93	93	92.8	86.9	86.2	79.9	76.1	62.9	48	15
	5	95.3	93	91.7	91.7	89	84.6	82	78.5	62.3	52.1	15
	6	95.3	94.3	91.9	91.6	87.6	84.6	82.6	81.1	60.9	53.8	15
gamma	1	94.3	94.3	94.3	93.3	88.8	86.7	75.5	72.7	57	45.1	15
	2	93.3	93.3	93.3	91.5	88.9	88.9	81.8	74.8	57.2	43.4	15
	3	93.3	93.3	93.3	93.3	90.7	86.6	81.6	75.4	61.7	46.5	15
	4	93.3	93.3	93.3	93.3	90.4	88.1	81.1	76.3	61.3	48.7	15

Table B.6.97: Average MTTF values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
norm	1	94.3	93.3	90.3	88.8	85.1	79.1	72.7	65.7	49.1	40.3	15
	2	93.3	93.3	93.3	90.3	88.9	88.9	84.3	75.3	63.1	52.6	15
	3	94.5	93.4	92.2	90.1	88.5	86.9	78.5	71.4	57.3	50.1	15
	4	94.5	93.4	93.4	91.7	89.6	89.6	81.4	76.6	64.9	55.4	15
	5	94.5	93.4	93.4	93.2	89	87.6	82.6	80.4	68.5	57.3	15
	6	93	91.7	91.7	88.1	85.1	80.8	77.1	75.2	59	50.6	15
	7	94.5	93	91.7	92.6	88	85.8	78.4	77.1	64.2	54.4	15
lnorm	1	94.3	93.3	93.3	91.4	86.7	84.2	72	63.4	45.1	40.9	15
	2	94.3	93.3	93.3	93.3	90.3	88.9	84.3	79	57.5	43.8	15
	3	94.3	93.3	93.3	93.3	93.3	91.9	88.6	81.6	67.2	54	15
	4	95.3	95.3	95.3	93.4	93.2	91.3	86.1	78.4	64.9	53.5	15
	5	94.5	95.3	94.3	94.3	92.5	91	83.5	76.9	59.4	47.4	15
logis	1	94.3	93.3	90.3	88.8	85.1	79.1	72.7	66.7	50.1	40.3	15
	2	93.3	93.3	93.3	90.3	88.9	87.6	83.5	74.5	61.9	50.5	15
	3	94.5	93.4	90.1	90.1	88.5	86	78.4	71.1	57.5	48.8	15
	4	94.5	93.4	93.4	93.4	92.2	89.7	84.5	79.9	67.1	55.7	15
	5	94.5	93.4	93.4	93.4	89.5	88.1	81.4	77.7	63.4	55.4	15
weibull	1	94.3	94.3	94.3	94.3	91.5	90.3	88.9	84.1	65.3	56.1	15
	2	94.3	94.3	94.3	94.3	94.3	93.3	87.9	85.9	75.3	64.8	15
	3	95.3	94.5	94.5	94.5	94.5	90.7	86	81.6	70.4	58.4	15
	4	95.3	95.3	94.5	93	93	92.8	86.1	81.2	71.3	62.2	15
	5	95.3	93	92.8	91.7	89	87.5	82.6	79.5	65.8	55.1	15
	6	95.3	95.3	95.3	91.7	91.7	87.6	82.8	82.6	73.5	58.8	15
gamma	1	94.3	93.3	93.3	88	86.7	84.2	72	64.3	46.4	40.9	15
	2	93.3	93.3	93.3	93.3	88.9	88.9	82.8	75.3	60.7	48	15
	3	94.3	93.3	93.3	93.3	93.3	91.9	86.6	82.9	70	56.9	15
	4	94.3	93.3	93.3	93.3	92.6	91.9	87.6	81.1	68.8	58.9	15

Table B.6.98: Average MTTRep values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	**247.5**	**233.2**	**224.5**	**207.7**	**188.1**	**177.2**	**153.8**	**139.6**	**102.9**	**78.6**	**20**
	2	**247.2**	**233.6**	**226.3**	**209.9**	**190.6**	**179.9**	**156.4**	**143.2**	**106**	**81.5**	**20**
	3	**241.1**	**226.5**	**219.3**	**198.9**	**181.3**	**171.1**	**149.4**	**135.5**	**100.6**	**78.6**	**20**
norm	4	**253**	**236.6**	**226.3**	**205.8**	**185.7**	**175**	**151.7**	**138.1**	**104.7**	**83.2**	**20**
	5	**273.5**	**255.4**	**245.9**	**221.9**	**197.6**	**185.6**	**157**	**142.6**	**107.8**	**86.8**	**20**
	6	**272.9**	**252.1**	**242.6**	**219.8**	**198.4**	**186.7**	**160.2**	**145.3**	**112.3**	**89.3**	**20**
	7	**255.6**	**244.9**	**235.2**	**221.8**	**201.2**	**187.7**	**162.2**	**146.3**	**110.6**	**87.9**	**20**
	1	**327.6**	**293.6**	**276.3**	**240.6**	**205.8**	**187.5**	**153.4**	**135.2**	**95.7**	**74.8**	**20**
	2	**345.2**	**303.9**	**288.5**	**253.8**	**215.7**	**198**	**159**	**138.7**	**99.2**	**76.5**	**20**
lnorm	3	**414.2**	**361.4**	**336.2**	**288.1**	**241.7**	**217.4**	**168.4**	**145.1**	**103.1**	**79.1**	**20**
	4	**475.2**	**408.1**	**375.6**	**304.9**	**247.8**	**221.3**	**166.8**	**141.8**	**98.8**	**76.5**	**20**
	5	**433.5**	**379.4**	**346.6**	**291.3**	**235.3**	**211.5**	**161**	**137.1**	**98**	**77**	**20**
	1	**242.6**	**229.8**	**221.9**	**206.3**	**187.8**	**177.4**	**154.9**	**140.7**	**103.5**	**78.5**	**20**
	2	**241.9**	**230.1**	**223.1**	**208.1**	**189.8**	**179.7**	**157.3**	**144.3**	**106.2**	**80.6**	**20**
logis	3	**235**	**222.6**	**215.7**	**196.4**	**180.4**	**169.6**	**148.7**	**136**	**100.3**	**78.1**	**20**
	4	**256.1**	**239.9**	**230.4**	**210**	**189.3**	**177.4**	**153.2**	**139.3**	**106**	**83.9**	**20**
	5	**254.9**	**237**	**229.1**	**209.2**	**188**	**176.9**	**152.3**	**138.2**	**107.5**	**84.7**	**20**
	1	**287.4**	**264.5**	**252**	**225.4**	**200.2**	**185.2**	**155.1**	**139.2**	**101.9**	**78.9**	**20**
	2	**302.7**	**279.9**	**267.7**	**239.1**	**209.9**	**194.3**	**160.1**	**145.5**	**106.7**	**84.3**	**20**
	3	**299.2**	**270.4**	**259**	**229.2**	**197.8**	**180.2**	**149.2**	**135.1**	**99.8**	**79.3**	**20**
weibull	4	**302**	**273.2**	**263.2**	**233.1**	**203.2**	**187.8**	**156.5**	**143**	**106**	**84.8**	**20**
	5	**306.1**	**275.1**	**258.6**	**231.3**	**202.4**	**187.6**	**158.8**	**143.6**	**106.7**	**86.6**	**20**
	6	**317.4**	**288**	**271**	**237.2**	**207.8**	**193.9**	**164.3**	**147.4**	**111.3**	**88.1**	**20**
	1	**288.8**	**264.8**	**252.7**	**225.3**	**197.5**	**182.1**	**153.1**	**136.5**	**98.9**	**77**	**20**
gamma	2	**293**	**269.6**	**257.3**	**231.8**	**203.4**	**188.5**	**156.7**	**138.9**	**101.7**	**78.8**	**20**
	3	**327.2**	**297.7**	**282.7**	**253.6**	**220.5**	**199.9**	**163**	**143.8**	**105.5**	**82.4**	**20**
	4	**314.6**	**287.2**	**273.3**	**246.5**	**213.6**	**197.4**	**164.4**	**144.2**	**108.7**	**85.5**	**20**

Table B.6.99: Average MTTRep values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	**235.3**	**219.4**	**211**	**193.3**	**172.9**	**163**	**137.1**	**120.7**	**85.1**	**63.8**	**20**
	2	**250.2**	**236.7**	**229.8**	**213**	**194.4**	**183.9**	**160.7**	**147.9**	**110.7**	**86.1**	**20**
	3	**239.1**	**224.9**	**216.6**	**197.3**	**179.4**	**169.3**	**147**	**132.6**	**98.6**	**76.7**	**20**
norm	4	**258.9**	**241.9**	**233.9**	**213.8**	**191**	**179.9**	**158.3**	**143.9**	**110.2**	**89.1**	**20**
	5	**285.7**	**264.5**	**255**	**231.4**	**208.8**	**196.4**	**167.2**	**152.2**	**118.4**	**94.7**	**20**
	6	**253.8**	**231.5**	**222.9**	**202.6**	**180.5**	**168.8**	**143.2**	**127.9**	**96**	**74.6**	**20**
	7	**253.8**	**243.5**	**231.9**	**219.5**	**196.5**	**185.1**	**159.6**	**143.3**	**108.7**	**85.2**	**20**
	1	**305.2**	**270**	**253.4**	**217.2**	**183.4**	**166.3**	**132.6**	**114.8**	**81.2**	**63.5**	**20**
	2	**351.4**	**308.4**	**292.4**	**258**	**221.7**	**202.9**	**163.2**	**141.8**	**103**	**79.7**	**20**
lnorm	3	**446.9**	**389.2**	**362.5**	**312.1**	**265.6**	**243.2**	**191**	**166.5**	**119.5**	**93.9**	**20**
	4	**503.7**	**432.6**	**402.9**	**329.3**	**272.5**	**243.2**	**183.4**	**158.3**	**111**	**86.9**	**20**
	5	**435.7**	**379.3**	**347.4**	**296.4**	**238.4**	**213.6**	**161**	**139.8**	**99.6**	**77.6**	**20**
	1	**231.3**	**216.8**	**209.1**	**192.5**	**173.2**	**163.5**	**137.9**	**121.6**	**85**	**63.4**	**20**
	2	**242.6**	**230.7**	**223.7**	**208.7**	**190.7**	**180.4**	**157.5**	**145**	**107.2**	**81.2**	**20**
logis	3	**232.4**	**219**	**209.2**	**192.8**	**176.6**	**165.5**	**144.2**	**129.4**	**95.1**	**74.1**	**20**
	4	**270.7**	**251.9**	**241.6**	**224**	**202.8**	**189.9**	**165.2**	**151.9**	**119.9**	**96.8**	**20**
	5	**259.9**	**241.5**	**233.6**	**215.6**	**193.1**	**181.3**	**157.1**	**142.5**	**113.3**	**88.9**	**20**
	1	**309.5**	**286.9**	**275.5**	**250**	**221.5**	**207.7**	**177.6**	**161.6**	**124.2**	**99.1**	**20**
	2	**342.4**	**319.8**	**308.5**	**281.4**	**252.9**	**236.2**	**203.5**	**184.9**	**144.6**	**120.1**	**20**
	3	**324.3**	**298.4**	**285.6**	**260.8**	**231.5**	**215.5**	**176.3**	**160.2**	**126.1**	**101.5**	**20**
weibull	4	**327.9**	**303.5**	**287.1**	**259.9**	**231.7**	**214.9**	**180.6**	**163.8**	**127.4**	**104.8**	**20**
	5	**313.7**	**283.9**	**268.9**	**240.5**	**210.3**	**195.8**	**165.8**	**151.1**	**115.4**	**92.6**	**20**
	6	**337.3**	**312.1**	**298.4**	**260.6**	**232.6**	**213.3**	**182.8**	**165.7**	**130.7**	**105**	**20**
	1	**271**	**245.7**	**233**	**205.1**	**177.7**	**163.2**	**133.6**	**117.1**	**83.3**	**64.9**	**20**
gamma	2	**297.1**	**273.9**	**262.3**	**236.5**	**207.9**	**193.3**	**161.2**	**143.8**	**105.7**	**82.5**	**20**
	3	**356.8**	**322.3**	**308.4**	**277.8**	**245.5**	**228.8**	**188.6**	**169.5**	**126.1**	**102.3**	**20**
	4	**339.8**	**309.6**	**295.1**	**267.3**	**237.6**	**220.9**	**184.6**	**166.8**	**126.1**	**103.4**	**20**

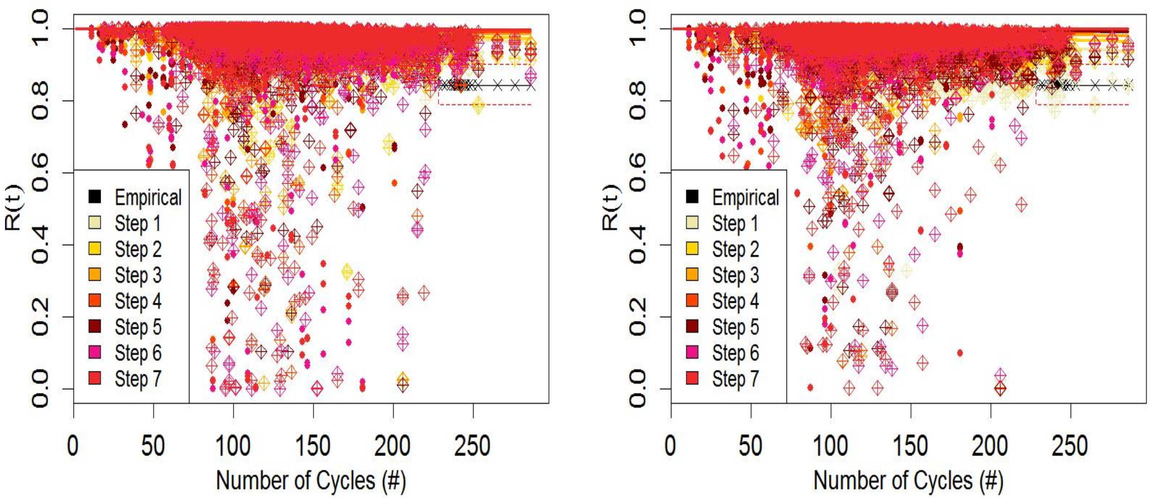
Table B.6.100: Overview of time-dependent PHM results obtained by MLE optimisation and GOF tests.

Distribution	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm	logis
Step #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**1**
MLE	**−1223.37**	**−1100.96**	**−945.18**	**−798.89**	**−671.78**	**−575.14**	**−494.78**	**−1283.23**
Time (min)	**112.6**	**286.43**	**500.39**	**843.9**	**1154.66**	**1550.82**	**2432.1**	**42.18**
Kolmogorov-Smirnov	**4.63**	**5.45**	**50.74**	**46.62**	**54.12**	**54.23**	**53.45**	**6.79**
Cramer-von Mises-Smirnov	**61.34**	**59.75**	**52.49**	**55.62**	**52.49**	**61.75**	**57.14**	**60.06**
Anderson-Darling	**−165.78**	**−166.22**	**−167.83**	**−169.24**	**−172.92**	**−187.68**	**−189.61**	**−166.08**
NRR	**−80.8**	**191.24**	**216.88**	**242.94**	**37937.42**	**96740.22**	**26275.15**	**353.24**

Distribution	logis	logis	logis	logis	logis	logis	exp	exp
Step #	**2**	**3**	**4**	**5**	**6**	**7**	**1**	**2**
MLE	**−1115.03**	**−922.27**	**−817.44**	**−706.47**	**−605.07**	**−551.02**	**−1244.2**	**−1146.88**
Time (min)	**184.48**	**352.13**	**540.38**	**724.58**	**986.23**	**1176.91**	**37.3**	**87.07**
Kolmogorov-Smirnov	**10.14**	**39.79**	**32.19**	**48.03**	**54**	**54.02**	**5.67**	**5.16**
Cramer-von Mises-Smirnov	**56.93**	**54.99**	**48.31**	**52.01**	**48.76**	**42.16**	**58.13**	**58.79**
Anderson-Darling	**−166.1**	**−167.27**	**−169.19**	**−170.98**	**−175.24**	**−180.85**	**−166.11**	**−165.95**
NRR	**1386.97**	**89.02**	**50.46**	**2194.9**	**18534.29**	**15738.46**	**191.78**	**254.43**

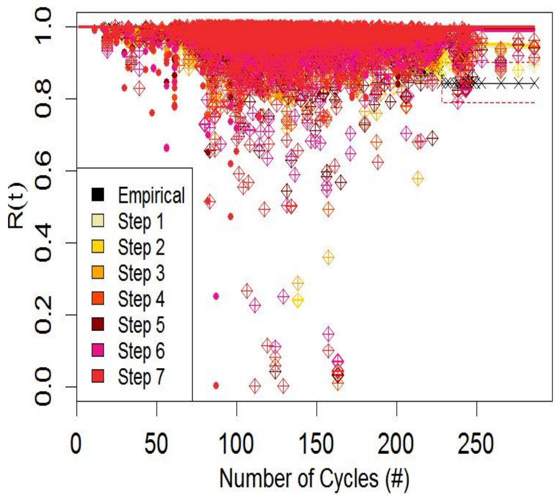
Distribution	exp	exp	exp	exp	exp	weibull	weibull	weibull
Step #	**3**	**4**	**5**	**6**	**7**	**1**	**2**	**3**
MLE	**−1035.83**	**−924.17**	**−842.77**	**−785**	**−685.37**	**−1226.44**	**−1101.17**	**−969.11**
Time (min)	**159.6**	**256.99**	**379.99**	**521.87**	**712.29**	**58.62**	**222.06**	**393.69**
Kolmogorov-Smirnov	**35.08**	**27.47**	**47.1**	**46.93**	**54.06**	**4.73**	**27.7**	**43.21**
Cramer-von Mises-Smirnov	**52.98**	**52.72**	**53.92**	**53.81**	**50.19**	**60.93**	**52.62**	**52.01**
Anderson-Darling	**−167.69**	**−167.66**	**−167.66**	**−168.13**	**−171.42**	**−165.85**	**−166.57**	**−167.02**
NRR	**234.22**	**415.41**	**638.18**	**207.98**	**6639.73**	**−388.08**	**1011.41**	**319.84**

Distribution	weibull
Step #	**4**
MLE	**−853.01**
Time (min)	**654.51**
Kolmogorov-Smirnov	**21.54**
Cramer-von Mises-Smirnov	**42.89**
Anderson-Darling	**−168.2**
NRR	**192.79**

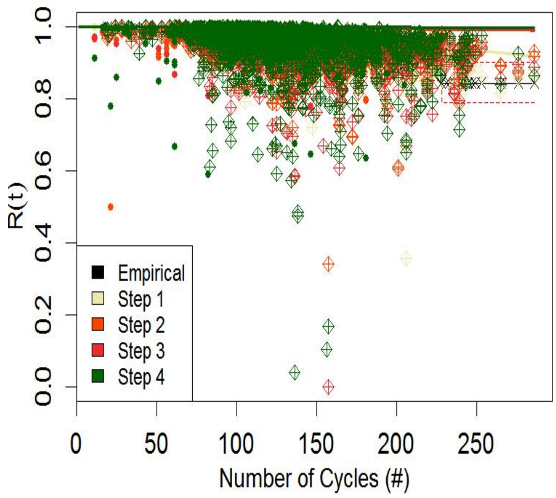


(a) With an underlying lnorm distribution. (b) With an underlying logis distribution.

Figure B.6.68: Time-dependent PHMs with an underlying lnorm and logis distribution.

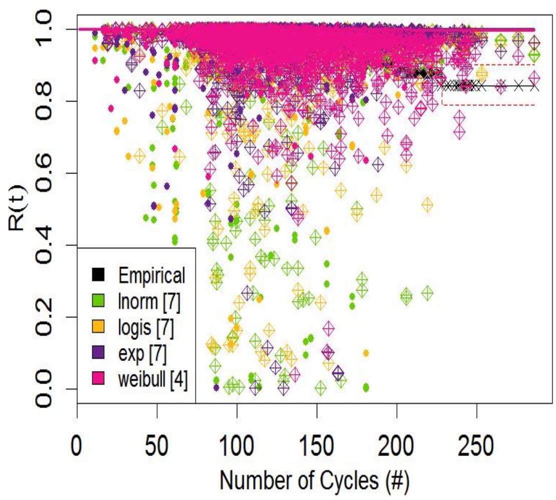


(a) With an underlying exp distribution.



(b) With an underlying weibull distribution.

Figure B.6.69: Time-dependent PHMs with an underlying exp and weibull distribution.



(a) All distributions (last step).

Figure B.6.70: Figure of all time-dependent PHMs.

Table B.6.101: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	1 [100]	0 [100]	0 [100]	0 [99]	0 [98]	0 [96]	4 [94]	10 [89]	21 [71]	31 [44]	96 [0]
	2	0 [100]	0 [100]	0 [100]	0 [99]	0 [98]	1 [97]	3 [95]	4 [92]	14 [82]	20 [68]	96 [0]
	3	1 [99]	1 [99]	1 [99]	0 [99]	0 [99]	0 [98]	0 [95]	3 [94]	6 [87]	13 [76]	96 [0]
lnorm	4	1 [100]	1 [100]	1 [99]	1 [99]	1 [98]	1 [97]	3 [97]	3 [95]	4 [93]	10 [90]	96 [0]
	5	1 [100]	1 [100]	1 [100]	1 [100]	1 [99]	1 [98]	3 [97]	3 [97]	3 [95]	4 [89]	96 [0]
	6	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	0 [99]	0 [99]	1 [96]	1 [94]	96 [0]
	7	1 [100]	1 [100]	3 [100]	3 [100]	3 [100]	3 [99]	3 [97]	3 [97]	5 [95]	7 [90]	96 [0]
	1	0 [100]	0 [99]	0 [99]	0 [97]	0 [96]	0 [94]	1 [85]	1 [79]	2 [51]	3 [31]	96 [0]
	2	0 [99]	0 [99]	0 [99]	0 [98]	1 [98]	1 [96]	1 [95]	5 [92]	20 [72]	31 [47]	96 [0]
	3	0 [99]	1 [99]	1 [99]	3 [99]	1 [98]	3 [98]	3 [96]	5 [95]	11 [83]	21 [76]	96 [0]
logis	4	1 [100]	1 [100]	1 [100]	1 [100]	1 [100]	1 [100]	1 [95]	3 [94]	6 [85]	14 [76]	96 [0]
	5	0 [99]	0 [99]	0 [99]	1 [99]	1 [99]	1 [98]	3 [96]	3 [93]	5 [88]	16 [84]	96 [0]
	6	3 [100]	3 [100]	3 [100]	3 [100]	3 [98]	3 [97]	3 [97]	3 [96]	8 [94]	11 [92]	96 [0]
	7	1 [100]	1 [100]	3 [100]	3 [99]	3 [98]	3 [98]	5 [97]	4 [96]	5 [95]	8 [93]	96 [0]
	1	0 [100]	0 [100]	0 [100]	0 [100]	0 [99]	0 [97]	1 [90]	9 [84]	34 [47]	42 [26]	96 [0]
	2	0 [100]	0 [99]	0 [98]	0 [98]	0 [96]	0 [93]	4 [85]	12 [78]	33 [48]	42 [26]	96 [0]
	3	0 [100]	0 [99]	0 [98]	0 [98]	0 [97]	0 [96]	1 [95]	3 [95]	7 [86]	12 [79]	96 [0]
exp	4	0 [100]	0 [100]	0 [99]	0 [98]	0 [96]	0 [95]	1 [94]	1 [94]	8 [86]	16 [75]	96 [0]
	5	0 [100]	0 [99]	0 [99]	0 [97]	1 [97]	1 [97]	1 [94]	1 [93]	5 [88]	12 [78]	96 [0]
	6	0 [100]	0 [100]	0 [100]	0 [100]	1 [98]	1 [97]	3 [94]	5 [92]	8 [86]	17 [76]	96 [0]
	7	0 [99]	0 [99]	0 [99]	0 [99]	0 [99]	0 [98]	0 [96]	0 [96]	4 [92]	4 [89]	96 [0]
	1	0 [100]	0 [100]	0 [100]	0 [99]	0 [97]	0 [96]	4 [93]	8 [90]	24 [66]	41 [42]	96 [0]
weibull	2	1 [100]	1 [100]	1 [100]	3 [100]	4 [100]	3 [97]	3 [93]	6 [92]	10 [79]	19 [66]	96 [0]
	3	1 [100]	3 [100]	3 [100]	3 [100]	3 [99]	3 [98]	3 [95]	4 [92]	9 [87]	16 [85]	96 [0]
	4	3 [100]	4 [99]	4 [99]	4 [99]	4 [99]	5 [98]	5 [94]	5 [94]	7 [88]	9 [88]	96 [0]

Table B.6.102: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	1 [100]	0 [100]	0 [99]	0 [98]	0 [96]	0 [96]	10 [90]	13 [85]	27 [52]	40 [32]	96 [0]
	2	0 [100]	0 [100]	0 [100]	0 [99]	1 [97]	4 [97]	6 [92]	9 [88]	20 [66]	28 [43]	96 [0]
	3	1 [99]	1 [99]	1 [99]	0 [99]	0 [99]	0 [97]	1 [95]	5 [93]	8 [81]	16 [73]	96 [0]
lnorm	4	1 [100]	1 [100]	1 [99]	1 [99]	1 [98]	1 [97]	3 [96]	3 [95]	7 [91]	14 [86]	96 [0]
	5	1 [100]	1 [100]	1 [100]	1 [99]	1 [99]	3 [98]	3 [97]	3 [97]	3 [90]	7 [86]	96 [0]
	6	0 [99]	0 [99]	0 [99]	0 [100]	0 [98]	0 [97]	0 [96]	0 [95]	1 [89]	3 [82]	96 [0]
	7	1 [100]	1 [100]	3 [100]	3 [100]	3 [100]	3 [100]	3 [100]	3 [100]	4 [98]	4 [93]	96 [0]
	1	0 [100]	0 [99]	0 [99]	0 [98]	0 [96]	0 [95]	1 [88]	1 [82]	2 [63]	3 [43]	96 [0]
	2	0 [99]	0 [99]	0 [99]	0 [99]	1 [98]	0 [96]	1 [94]	7 [91]	20 [66]	33 [44]	96 [0]
	3	0 [99]	1 [99]	1 [99]	3 [99]	1 [97]	3 [98]	4 [96]	5 [94]	12 [83]	22 [71]	96 [0]
logis	4	1 [100]	1 [100]	1 [100]	1 [100]	1 [100]	1 [99]	3 [95]	3 [91]	7 [83]	17 [72]	96 [0]
	5	0 [99]	0 [99]	1 [99]	1 [99]	1 [98]	1 [98]	3 [93]	4 [91]	10 [82]	22 [74]	96 [0]
	6	3 [100]	3 [100]	3 [100]	3 [100]	3 [99]	3 [98]	4 [97]	3 [97]	7 [95]	10 [92]	96 [0]
	7	1 [100]	1 [100]	3 [100]	3 [99]	3 [99]	3 [99]	3 [99]	3 [98]	5 [95]	8 [93]	96 [0]
	1	0 [100]	0 [100]	0 [100]	0 [100]	0 [97]	0 [97]	4 [88]	14 [79]	37 [40]	42 [26]	96 [0]
	2	0 [100]	0 [98]	0 [98]	0 [96]	0 [93]	0 [90]	10 [80]	21 [66]	37 [34]	42 [26]	96 [0]
	3	0 [100]	0 [99]	0 [98]	0 [98]	0 [97]	0 [96]	1 [95]	3 [95]	9 [86]	14 [76]	96 [0]
exp	4	0 [100]	0 [100]	0 [99]	0 [98]	0 [96]	0 [95]	1 [94]	1 [94]	9 [84]	16 [75]	96 [0]
	5	0 [100]	0 [99]	0 [99]	0 [97]	1 [97]	1 [97]	1 [94]	1 [92]	5 [84]	13 [77]	96 [0]
	6	0 [100]	0 [100]	0 [100]	0 [100]	1 [97]	1 [96]	4 [93]	5 [92]	8 [82]	20 [75]	96 [0]
	7	0 [99]	0 [99]	0 [99]	0 [99]	0 [99]	0 [98]	0 [96]	1 [95]	4 [91]	4 [87]	96 [0]
	1	0 [100]	0 [100]	0 [100]	0 [100]	0 [98]	0 [97]	1 [95]	4 [91]	20 [74]	29 [54]	96 [0]
weibull	2	1 [100]	1 [100]	1 [100]	3 [100]	1 [100]	4 [98]	3 [95]	5 [94]	8 [90]	12 [79]	96 [0]
	3	1 [100]	3 [100]	3 [100]	3 [100]	3 [99]	3 [98]	3 [97]	4 [95]	4 [91]	12 [89]	96 [0]
	4	3 [100]	4 [99]	4 [99]	4 [99]	4 [99]	4 [98]	5 [96]	5 [94]	7 [88]	8 [89]	96 [0]

Table B.6.103: Average MTTF values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	93.5	94.3	94.3	94.3	94.3	94.3	90.3	83.9	74.4	66.1	15
	2	94.3	94.3	94.3	94.3	94.3	93.5	92.3	90.9	79.5	74.9	15
	3	93.5	93.5	93.5	94.3	94.3	94.3	94.3	91.6	89.5	82.7	15
lnorm	4	92.9	92.9	92.9	92.9	92.9	92.9	91.7	91.7	90.3	85.4	15
	5	92.9	92.9	92.9	92.9	92.9	92.9	91.7	91.7	91.7	91	15
	6	94.3	94.3	94.3	94.3	94.3	94.3	94.3	94.3	94	93.3	15
	7	92.9	92.9	91.7	91.7	91.4	91.4	91.4	91.4	90.1	89.8	15
	1	94.3	94.3	94.3	94.3	94.3	94.3	92.9	92.9	92.2	91.5	15
	2	94.3	94.3	94.3	94.3	93.5	93.5	93	88.7	75.3	65.6	15
	3	94.3	93.2	93.2	92.3	93.2	92.3	92.3	90.4	84.2	74.2	15
logis	4	93.2	93.2	93.2	93.2	93.2	93.2	93.2	92.3	89.1	81.9	15
	5	94.3	94.3	94.3	93.2	93.2	93.2	91.7	91.7	89.6	79.2	15
	6	91.7	91.7	91.7	91.7	91.7	91.7	91.7	91.7	89	87	15
	7	92.9	92.9	91.7	91.7	91.7	91.7	90.5	91.1	90.2	89.5	15
	1	94.3	94.3	94.3	94.3	94.3	94.3	92.9	84	62.7	55.7	15
	2	94.3	94.3	94.3	94.3	94.3	94.3	89.9	81.1	63.9	55.7	15
	3	94.3	94.3	94.3	94.3	94.3	94.3	92.9	91.7	88.2	84.3	15
exp	4	94.3	94.3	94.3	94.3	94.3	94.3	92.9	92.9	87.7	80	15
	5	94.3	94.3	94.3	94.3	92.9	92.9	92.9	92.9	88.7	83.1	15
	6	94.3	94.3	94.3	94.3	93.2	93.2	91.7	90.2	87	79.2	15
	7	94.3	94.3	94.3	94.3	94.3	94.3	94.3	94.3	90.6	90.6	15
	1	94.3	94.3	94.3	94.3	94.3	94.3	90	85.8	71.8	56.8	15
weibull	2	92.9	92.9	92.9	92.1	90.8	91.7	91.4	89.6	87.3	78.4	15
	3	92.9	92.1	92.1	92.1	92.1	92.1	92.7	91.8	87.3	82.5	15
	4	91.7	90.8	90.8	90.8	90.8	90.6	90.8	90.1	88.9	87.5	15

Table B.6.104: Average MTTF values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	93.5	94.3	94.3	94.3	94.3	94.3	83.3	81.6	69.4	58.5	15
	2	94.3	94.3	94.3	94.3	93.2	90.9	89.2	85.8	75.3	68.7	15
	3	93.5	93.5	93.5	94.3	94.3	94.3	93.7	90.4	87.1	79.6	15
lnorm	4	92.9	92.9	92.9	92.9	92.9	92.9	91.7	91.7	88.9	81.4	15
	5	92.9	92.9	92.9	92.9	92.9	91.7	91.7	91.7	91.7	88.8	15
	6	94.3	94.3	94.3	94.3	94.3	94.3	94.3	94.3	93.3	91.9	15
	7	92.9	92.9	91.7	91.7	91.4	91.4	91.4	91.4	91	91	15
	1	94.3	94.3	94.3	94.3	94.3	94.3	93.6	92.9	92.2	91.5	15
	2	94.3	94.3	94.3	94.3	93.5	94.3	93	87.8	75.3	64.2	15
	3	94.3	93.2	93.2	92.3	93.2	92.3	90.9	90.4	84	73.6	15
logis	4	93.2	93.2	93.2	93.2	93.2	93.2	92.3	91.8	88.9	78.7	15
	5	94.3	94.3	93.2	93.2	93.2	93.2	91.7	90.3	85.6	73.7	15
	6	91.7	91.7	91.7	91.7	91.7	91.7	90.8	91.7	89.7	88.4	15
	7	92.9	92.9	91.7	91.7	91.7	91.7	91.4	91.4	90.2	89.5	15
	1	94.3	94.3	94.3	94.3	94.3	94.3	90.4	78.7	60.4	55.7	15
	2	94.3	94.3	94.3	94.3	94.3	94.3	83.3	74	60	55.7	15
	3	94.3	94.3	94.3	94.3	94.3	94.3	92.9	91.7	87.2	81.7	15
exp	4	94.3	94.3	94.3	94.3	94.3	94.3	92.9	92.9	87.3	80	15
	5	94.3	94.3	94.3	94.3	92.9	92.9	92.9	92.9	88.7	82.6	15
	6	94.3	94.3	94.3	94.3	93.2	93.2	90.6	90.2	87	76.5	15
	7	94.3	94.3	94.3	94.3	94.3	94.3	94.3	93.2	90.6	90.6	15
	1	94.3	94.3	94.3	94.3	94.3	94.3	92.9	90	74.5	67.2	15
weibull	2	92.9	92.9	92.9	92.1	92.9	90.8	91.7	90.5	88.4	85.9	15
	3	92.9	92.1	92.1	92.1	92.1	92.1	92.7	91.8	90.8	86.2	15
	4	91.7	90.8	90.8	90.8	90.8	90.8	90.8	90.1	88.9	88.6	15

Table B.6.105: Average MTTRep values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	653.9	557.8	511.2	400.3	324.4	285.6	221.3	187.6	137.9	113.4	20
	2	555.9	477.7	444.3	382.5	322.5	293.4	241.1	214.6	163.7	137.5	20
	3	342.2	340.8	338.7	338.5	329.2	321.3	301.4	274.3	200.5	157.8	20
lnorm	4	748.4	650.6	606.7	515.6	436.3	395.7	320.7	291.4	223	184.2	20
	5	680.8	610	571.9	492.6	427.5	392.3	334.1	305.7	241.4	204.5	20
	6	416.6	399.6	389	367.9	347.7	336.1	308.6	292.9	251.2	223	20
	7	561.6	530.5	505.5	460.3	411.5	387.4	343.4	317	258.2	217.8	20
	1	392.7	347.5	326.9	286.7	245.5	225.7	184.1	163.1	122.3	106.8	20
	2	550	482.5	448	390.1	320.5	289.8	237.1	205.7	143.3	114	20
	3	641.2	564.6	526.3	451.8	389.7	350.1	288.1	254.1	181.9	150	20
logis	4	751.9	662	620.9	540	454.1	407	323.9	282.5	199.1	160.5	20
	5	500.5	457.5	435	387.5	348.2	328.5	278.3	251.1	194.4	168.1	20
	6	957.5	884.6	827.5	719.4	603.7	551.2	440.4	379.6	261.4	211.2	20
	7	1076.9	965	897.7	779.2	663.4	598.5	460.7	401.5	284.7	224.5	20
	1	479.4	421.2	392.9	335.2	282.7	255.3	199.4	173.6	120.2	101.1	20
	2	441.2	391.3	366.7	317.2	266.1	241.5	190.1	167.3	120.5	101.5	20
	3	764	691	647.9	565	484.5	446.9	360.4	316.8	224.5	174.1	20
exp	4	738.3	659.6	619.5	543.5	465.8	428.6	344.1	303.9	210.8	166.5	20
	5	780.2	702.9	658.5	572.9	480	440.6	363.6	325.8	225.9	180.4	20
	6	731.9	648.8	611.4	526.4	440.8	402.2	328.1	284.9	203.7	161.1	20
	7	1258.8	1134.9	1077.8	927.9	792.2	728.9	588.8	520.7	352.4	273.4	20
	1	342.6	317.9	304.1	276.9	248.2	233.4	198.9	180.4	136.8	111.3	20
weibull	2	565.4	505.8	477.2	407.7	345	317.7	253.1	218.4	160.4	136.5	20
	3	387.4	361.3	353.4	320	290.1	272.3	241.6	221.3	183.2	159.9	20
	4	395.2	368.6	355.2	331.2	305.4	286.8	260	247.8	201.9	181.9	20

Table B.6.106: Average MTTRep values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	84%	86%	87%	89%	91%	92%	94%	95%	97%	98%	100%
	1	641.9	541.4	492.3	390.3	306.3	269.6	198.4	170.5	121.1	104.6	20
	2	536.3	457.9	421.5	361.3	299.1	267.3	214.9	186.8	139.8	114.6	20
	3	341.9	340.7	338.5	336.8	328.6	319.6	297.1	269	189.5	149.8	20
lnorm	4	739.8	642.2	598.2	508.1	427.3	386.8	311.8	282	208.3	173.3	20
	5	671.1	598.3	559.5	480.3	414.4	377.5	321.7	293	227.2	185.3	20
	6	374.6	362.7	349.9	331.1	305.9	295.8	271.8	258	213.8	182.9	20
	7	593.4	555.8	530.7	494	445.8	425.7	377.9	351.4	290.6	250.1	20
	1	403.1	358.9	337.9	296.7	256.5	236.3	195.1	174.6	133.8	113.3	20
	2	548.3	479.9	446.5	387.7	321	292.2	235.3	201.9	141	112.6	20
	3	633.5	554.1	517	442.6	379.1	342.2	279.7	247.9	172.8	141.8	20
logis	4	740.4	648.5	608.4	529.4	441.5	394.4	308.3	272.6	186.2	151.5	20
	5	484	440	415.1	371.6	332.7	311.5	259.2	233.2	175.4	148.2	20
	6	991.3	899.5	839.4	731.1	623.2	561.9	444.6	388.5	273.4	222	20
	7	1104.4	975.3	911.6	800.4	681.8	615.7	484.8	422.2	296.4	234.7	20
	1	469.3	412.2	382.5	326.2	272.4	244.8	188.8	162.7	110.7	101	20
	2	424.7	373.6	346.5	297.9	246.9	221.3	171.2	146.5	105	100.9	20
	3	759.5	686.1	642.8	560	479.5	442.3	355.2	310.7	213.2	168.5	20
exp	4	732.5	655.2	614.3	538.5	461.3	423.5	339.6	298.4	202.4	161	20
	5	772.6	694.5	652.1	565.7	473.4	434.3	356.9	317.2	218.2	170.8	20
	6	726.3	640	603.9	518.7	433.6	394.7	316.5	274.2	194.9	152.6	20
	7	1249.7	1125.2	1064.6	916	781.2	715.3	574.3	498	337.9	257.4	20
	1	354	329.9	316.7	288.2	260.2	245.1	211.2	193	149.8	123.7	20
weibull	2	596.5	531.2	497.3	429.5	374.5	336	277.5	244.6	186.1	157.2	20
	3	409.2	383	368.9	342.4	312.4	294.1	264.3	248.1	209.7	183	20
	4	404.6	380.3	361.3	340.7	316.3	300.6	270.5	258.6	216.7	194.1	20

Table B.6.107: Variables identified by time-(in)dependent PHM models.

	Time-independent PHM			Time-dependent PHM	
	Variable	Scaled Value		Variable	Scaled Value
①	*Pitch rate mean deg sec 8*	6.1	①	*Group A*	−34.89
②	*Yaw rate min deg sec 2*	−5.08	②	*Group B*	26.26
③	*Group A*	−4.73	③	*Torque rhs mean 5*	−22.85
④	*Torque rhs mean 5*	−4.09	④	*Vz mean ft min 4*	−20.68
⑤	*Group B*	2.99	⑤	*Pitch rate mean deg sec 8*	19.57
⑥	*Brake press lhs mean psi 8*	2.97	⑥	*Roll rate mean deg sec 1*	−14.1
⑦	*Roll rate mean deg sec 8*	2.8	⑦	*Brake press lhs mean psi 8*	13.28
⑧	*Vz min ft min*	2.19	⑧	*Group C*	9.88
⑨	*Vcal mean knots 6*	1.59	⑨	*Roll rate mean deg sec 8*	8.36
			⑩	*Yaw rate max deg sec 8*	7.81
			⑪	*Group D*	−6.01

Table B.6.108: Variables identified by each step by time-(in)dependent PHMs (in order).

	PHM Variables	
	Time-independent	*Time-dependent*
norm	⑥ ⑦ ③ ① ④ ⑤ ⑧	
lnorm	⑥ ⑦ ① ③ ②	③ ⑦ ⑤ ② ⑥ ④ ①
logis	⑥ ⑦ ③ ① ②	⑤ ③ ⑦ ⑥ ② ① ⑨
exp		⑩ ⑥ ⑤ ⑧ ⑪ ⑦ ③
weibull	⑦ ① ③ ④ ⑥ ⑧	⑤ ① ⑩ ④
gamma	⑥ ⑦ ① ⑨	

Table B.6.109: Number of times variables identified by each step by time-(in)dependent PHMs.

Key				Key			
indep	*dep*	*Variable*	*Count*	*indep*	*dep*	*Variable*	*Count*
③	①	Group A	7	⑦	⑨	Roll rate mean deg sec 8	6
⑤	②	Group B	3	④	③	Torque rhs mean 5	5
	⑧	Group C	1	⑨		Vcal mean knots 6	1
	⑪	Group D	1		④	Vz mean ft min 4	2
⑥	⑦	Brake press lhs mean psi 8	8	⑧		Vz min ft min	2
①	⑤	Pitch rate mean deg sec 8	9		⑩	Yaw rate max deg sec 8	2
	⑥	Roll rate mean deg sec 1	3	②		Yaw rate min deg sec 2	2

Table B.6.110: Variables belonging to each group identified in B.6.107.

Group	*Variables*	*Group*	*Variables*
Group A	Accn norm mean g s 1, Pitch rate mean deg sec 1	Group C	NormalForce lhs max lbs 6, NormalForce lhs max lbs 5
Group B	Torque rhs mean 4, Torque lhs mean 4	Group D	Crosswind mean knots 5, Drift mean deg 5

B.7 92003-051-052-001 Sensor high-level, fuel

No maintenance events could be registered for the high-level fuel sensor component. This was directly related to the significantly high number of cycles operator prior to maintenance events. As a result the required cycles dated back further than the FDR data. Reliability models using operational factors (covariates) can not be evaluated without the complete history of the component, hence the program automatically terminated itself.

B.8 728809-1 Thermal actuator

Table B.8.111 provides a summary of the input data related to the component. The number of registered maintenance events is less than the total number of events due to the fact that TRAX data stretches back to 2004/2005 and FDR data only to 2011. Maintenance events with insufficient data, regarding operational factors, cannot be evaluated, hence are not registered during the modelling process.

Table B.8.111: General overview of component inputs.

Name	**Value**
Part Number	728809-1
Total # (A, F, C)	**732, 468, 264**
Registered # (A, F, C)	**135, 86, 49**
Related Flights # (A, F, C)	**248126, 190918, 57208**
Avg. Cycles (A, F, C)	**1837.97, 2219.98, 1167.51**
% Censored	**36.3**

In Tab. B.8.111 (A, F, C) denotes statistics regarding All (A), Failed (F), and Censored (C) events respectively. Ergo A will always be the sum or mean derived from F and C.

Analysis
Tables B.8.112 and B.8.113 summarise the results from EVA and MDA. In addition the variables obtained by semi-parametric PHM modelling (labelled 'reduced semi-COX') are also presented if applicable. Table B.8.113 provides an overview of the specific operational factors identified during all flight phases. In this case high counts indicate operational factors that were significantly different during multiple flight phases.

Table B.8.112: Overview of analysis input and output.

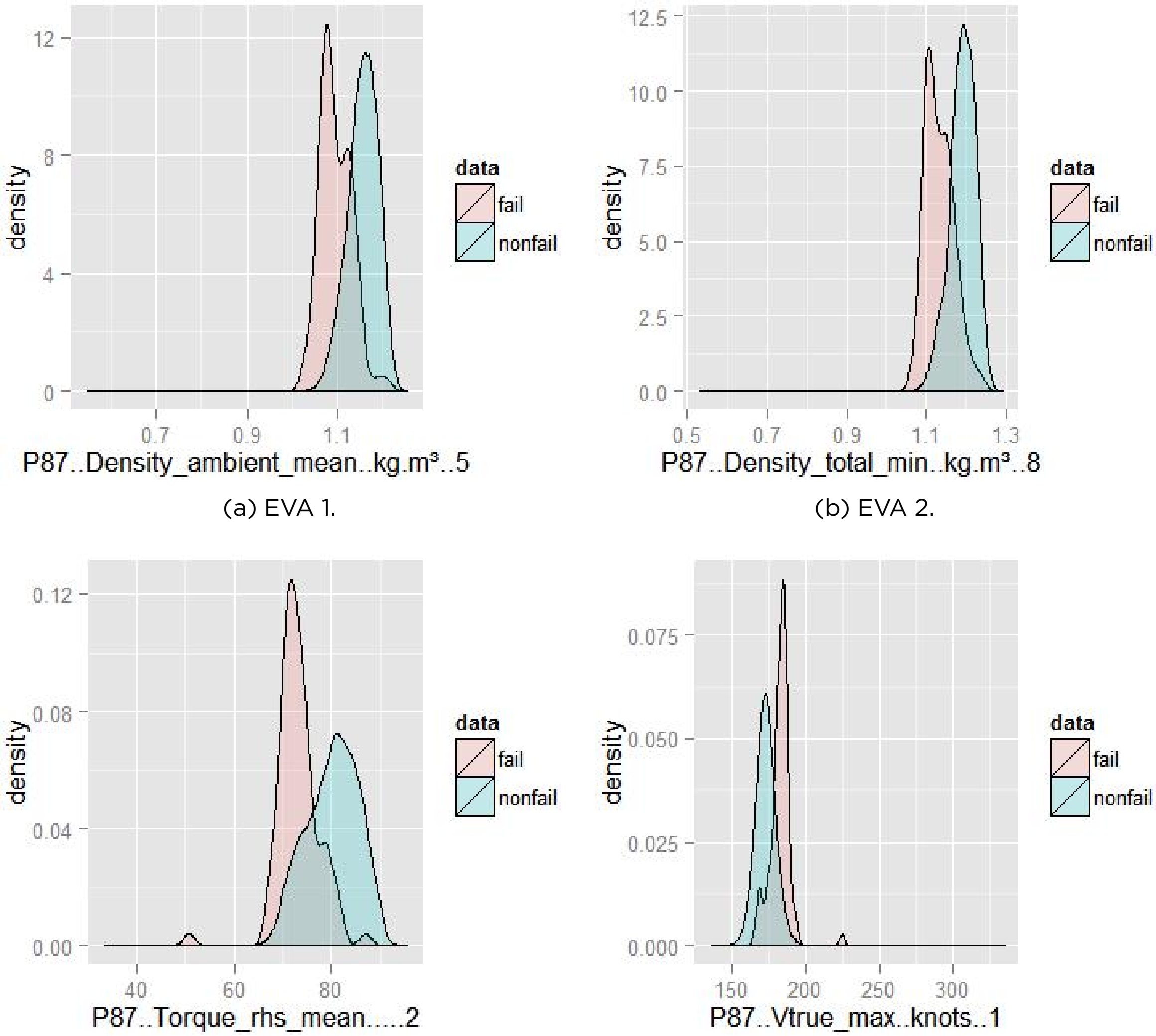
	# Variables
ALL	**1531**
EVA	**47**
MDA	**80**
Combined	**127**
reduced Corr.	**55**
reduced semi-COX	**13**
Take-Off related	**16**
Cruise related	**18**
Touch-Down related	**21**

A multitude of factors were identified during EVA and MDA. Figure B.8.71 give a general overview of the top operational factors identified by EVA and MDA.

Time-based reliability modelling
Table B.8.114 reports the maximum likelihood and goodness-of-fit tests results obtained from time-based reliability modelling. To show the overall fit Fig. B.8.72 shows the computed reliability function using an averaged virtual age V for all fitted models.

Table B.8.113: Overview of identified operational factors during all flight phases.

Variable	Count	Variable	Count	Variable	Count
Vtrue	8	NormalForce_lhs	2	Torque_lhs	1
Yaw_rate	8	NormalForce_rhs	2	Alt_pres	1
Pitch_rate	8	Aileron_Rin	2	Ttot	1
Roll_rate	4	Accn_norm	2	Density_total	1
Torque_rhs	3	Pressure_total	2	Brake_press_lhs	1
Vz	3	Rudder_cmd_force	1		
Headwind	3	Aoa	1		



(a) EVA 1. (b) EVA 2.

(c) MDA 1. (d) MDA 2.

Figure B.8.71: Graphical overview of top operational factors identified by EVA and MDA.

Table B.8.114: Overview of results obtained by MLE optimisation and GOF tests.

	Distributions				
	norm	**lnorm**	**logis**	**exp**	**weibull**
MLE	−724.57	−767.76	−723.33	−771.19	−731.99
Kolmogorov-Smirnov	5.11	3.34	5.61	3.06	4.5
Cramer-von-Mises Smirnov	57.07	57	57.03	55.12	57.42
Anderson-Darling	−178.3	−179.3	−178.42	−182.52	−176.71
NRR	79.3	20.58	66.85	17.38	48.96

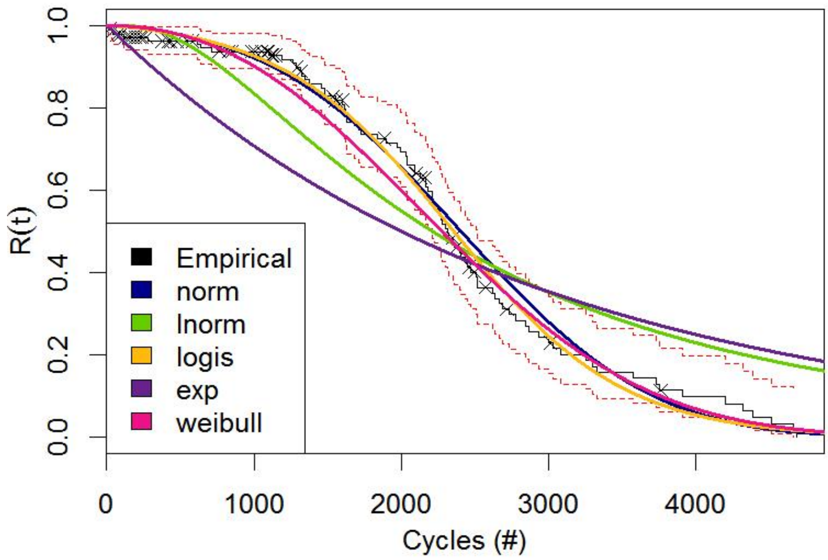
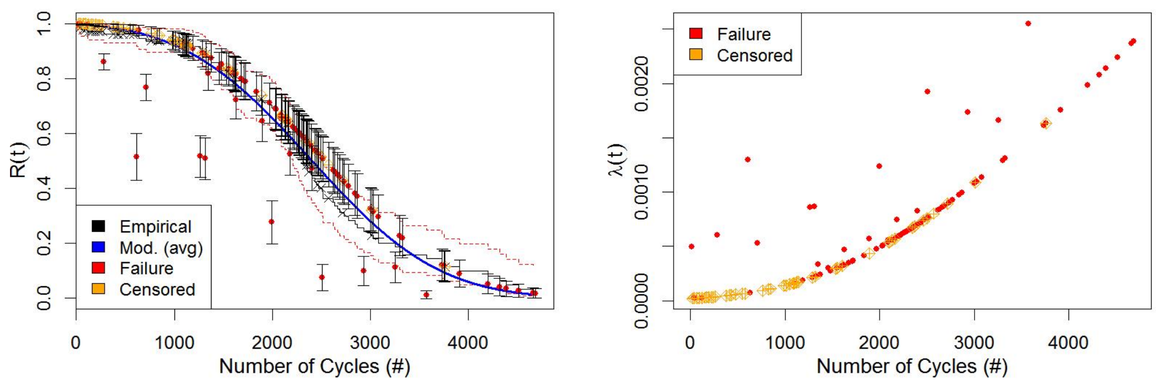


Figure B.8.72: Overview of overall fit of multiple GRP modells.

In addition Figures B.8.73, B.8.74, B.8.75, B.8.76, and B.8.77 present the reliability and hazard functions computed for each underlying distribution evaluated in the program.



(a) Reliability function.

(b) Hazard function.

Figure B.8.73: Computed reliability for time-based models with underlying norm distribution.

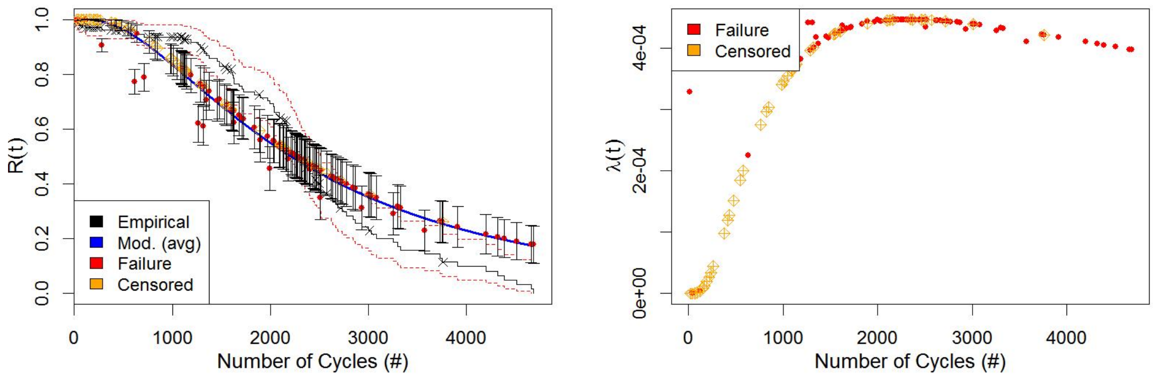
Time independent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.8.115 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.8.78, B.8.79, and B.8.80 as well as a general overview in Figure B.8.80b.

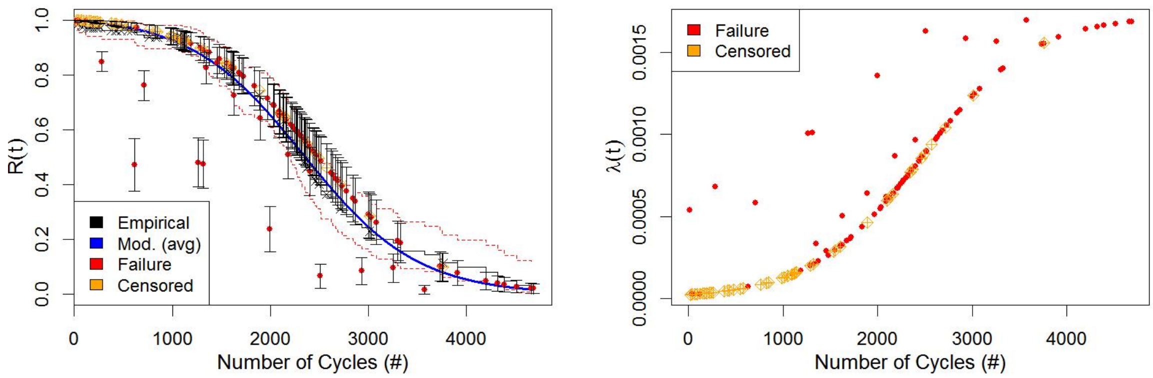
Tables B.8.116 and B.8.117 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

To assist in the selection of models, Tables B.8.118, B.8.119, B.8.120, and B.8.121 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.



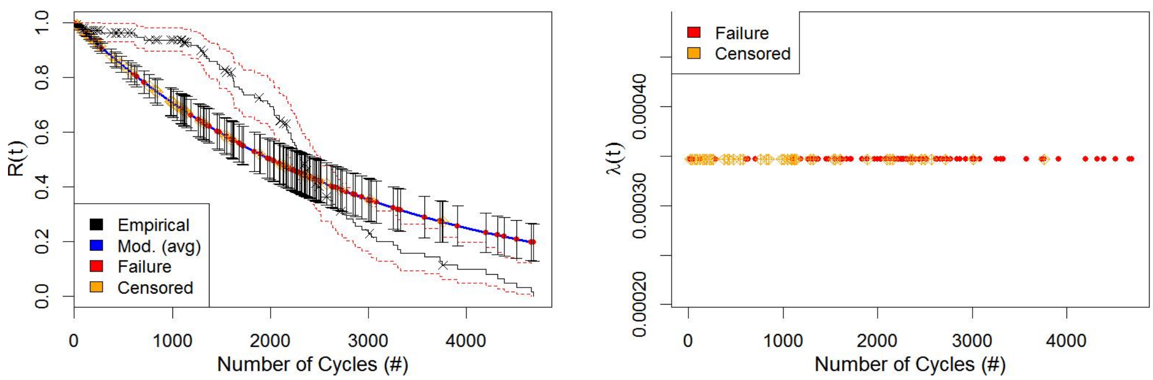
(a) Reliability function. (b) Hazard function.

Figure B.8.74: Computed reliability for time-based models with underlying lnorm distribution.



(a) Reliability function. (b) Hazard function.

Figure B.8.75: Computed reliability for time-based models with underlying logis distribution.



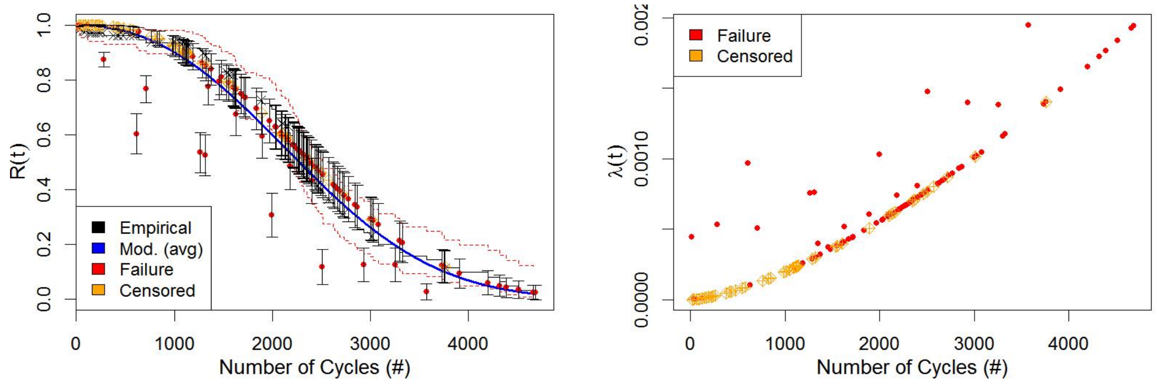
(a) Reliability function. (b) Hazard function.

Figure B.8.76: Computed reliability for time-based models with underlying exp distribution.

Time dependent proportional hazard reliability modelling
Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.8.122 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.8.81, B.8.82, and B.8.83 as well as a general overview in Figure B.8.83b.

Tables B.8.123 and B.8.124 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In



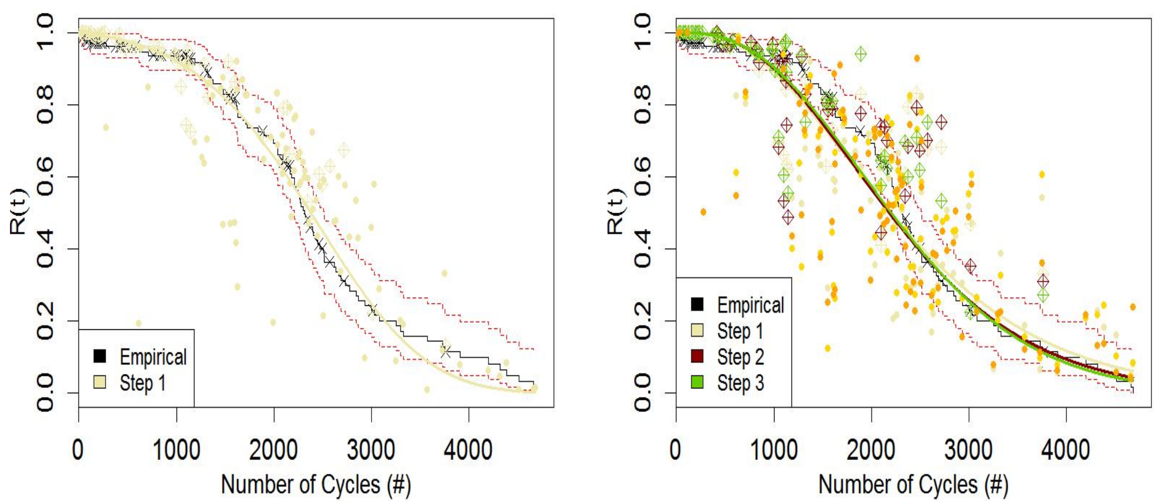
(a) Reliability function. (b) Hazard function.

Figure B.8.77: Computed reliability for time-based models with underlying weibull distribution.

Table B.8.115: Overview of time-independent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	lnorm	lnorm	lnorm	logis	exp	exp	exp
Step #	1	1	2	3	1	1	2	3
MLE	−717.05	−746.94	−743.25	−739.16	−715.79	−1312.24	−826.91	−814.2
Time (min)	0.78	0.98	1.79	2.85	0.84	0.75	1.3	1.8
Kolmogorov-Smirnov	8.84	7.97	8.14	6.59	7.16	11.63	11.19	10.17
Cramer-von Mises-Smirnov	55.14	55.11	54.06	54.73	55.09	30.78	42.51	26.94
Anderson-Darling	−185.88	−187.75	−192.17	−189.22	−185.92	−677.87	−236.72	−228.5
NRR	81.29	27.2	29.05	34.42	79.22	169.06	36.05	31.58

Distribution	weibull
Step #	1
MLE	−724.61
Time (min)	0.9
Kolmogorov-Smirnov	8.26
Cramer-von Mises-Smirnov	55.55
Anderson-Darling	−185.03
NRR	40.67

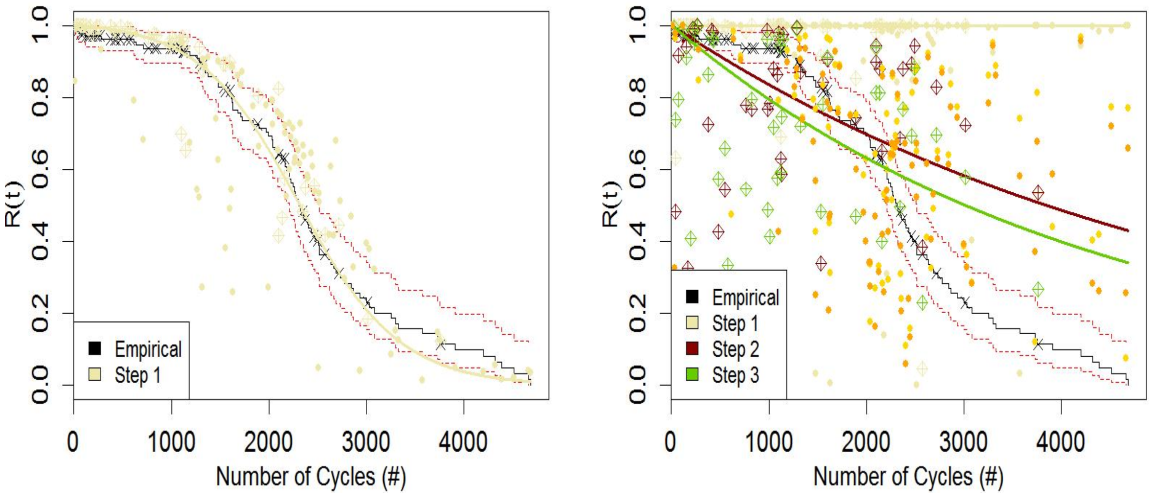


(a) With an underlying norm distribution. (b) With an underlying lnorm distribution.

Figure B.8.78: Time-independent PHMs with an underlying norm and lnorm distribution.

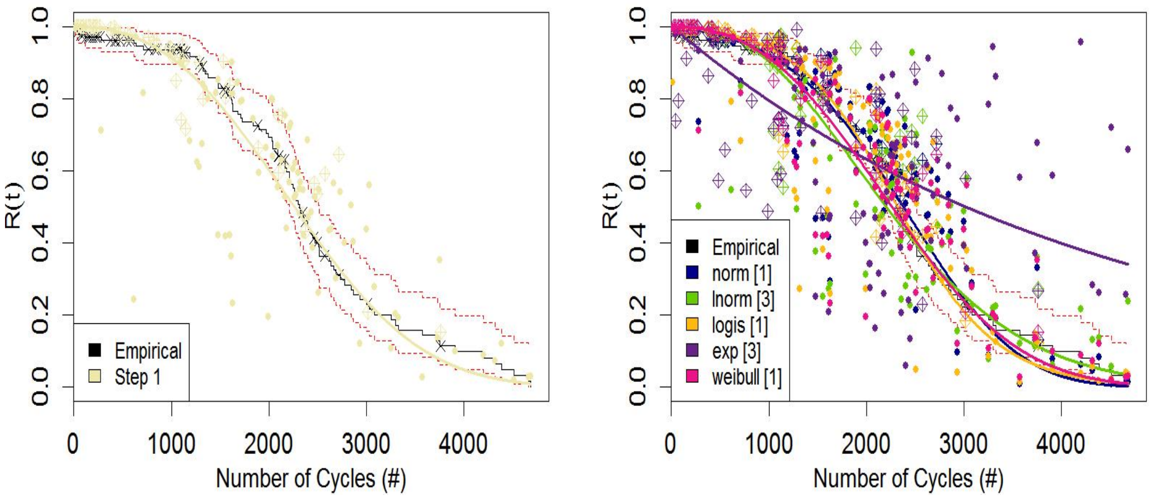
general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

To assist in the selection of models, Tables B.8.125, B.8.126, B.8.127, and B.8.128 indicate the av-



(a) With an underlying logis distribution. (b) With an underlying exp distribution.

Figure B.8.79: Time-independent PHMs with an underlying logis and exp distribution.



(a) With an underlying weibull distribution. (b) All distributions (last step).

Figure B.8.80: Figures containing a weibull distribution and all time-independent PHMs.

Table B.8.116: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	0 [100]	10 [100]	13 [98]	21 [96]	23 [96]	36 [94]	52 [90]	65 [80]	76 [69]	92 [61]	95 [22]
	1	0 [100]	6 [100]	12 [100]	21 [100]	37 [98]	43 [94]	62 [92]	76 [76]	84 [63]	92 [55]	95 [29]
lnorm	2	0 [100]	7 [100]	10 [100]	23 [100]	34 [96]	48 [94]	59 [88]	70 [84]	86 [59]	93 [55]	95 [29]
	3	0 [100]	5 [100]	12 [100]	23 [96]	34 [96]	47 [96]	58 [92]	66 [76]	84 [65]	91 [59]	95 [29]
logis	1	0 [100]	13 [100]	17 [96]	21 [96]	27 [96]	31 [84]	47 [78]	64 [76]	80 [73]	86 [63]	97 [29]
	1	0 [100]	3 [98]	5 [96]	6 [96]	6 [96]	7 [96]	8 [94]	8 [94]	8 [94]	9 [90]	22 [82]
exp	2	0 [100]	1 [98]	2 [98]	13 [96]	22 [98]	31 [90]	38 [80]	47 [78]	62 [53]	85 [29]	98 [6]
	3	0 [100]	0 [100]	1 [98]	12 [96]	23 [92]	37 [86]	48 [73]	59 [65]	69 [41]	86 [24]	98 [4]
weibull	1	0 [100]	8 [100]	14 [98]	21 [96]	27 [96]	40 [92]	59 [86]	71 [78]	79 [69]	92 [61]	95 [29]

eraged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.
The operational factors identified during time-independent and time-dependent PHM modelling are shown in Tables B.8.129, B.8.130, B.8.131, and B.8.132.

Table B.8.117: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	0 [100]	12 [100]	17 [98]	21 [96]	27 [96]	37 [94]	52 [88]	69 [80]	78 [69]	92 [61]	95 [16]
	1	0 [100]	7 [100]	15 [100]	28 [100]	40 [98]	55 [94]	66 [82]	80 [69]	87 [61]	92 [55]	95 [29]
lnorm	2	0 [100]	9 [100]	19 [100]	30 [98]	41 [94]	52 [90]	65 [86]	73 [78]	87 [59]	93 [55]	95 [29]
	3	0 [100]	8 [100]	17 [100]	29 [96]	42 [96]	52 [94]	64 [80]	77 [69]	86 [61]	92 [55]	95 [27]
logis	1	0 [100]	12 [100]	17 [96]	20 [96]	26 [96]	30 [86]	42 [78]	56 [78]	77 [73]	84 [67]	95 [29]
	1	0 [100]	5 [98]	5 [96]	6 [96]	6 [96]	8 [96]	8 [94]	8 [94]	8 [94]	9 [90]	24 [82]
exp	2	0 [100]	13 [80]	20 [76]	30 [78]	42 [76]	47 [65]	55 [61]	63 [55]	76 [39]	90 [18]	99 [4]
	3	0 [100]	13 [80]	26 [78]	36 [71]	45 [69]	51 [49]	65 [45]	76 [35]	84 [22]	95 [12]	99 [2]
weibull	1	0 [100]	9 [100]	15 [98]	24 [96]	30 [96]	41 [92]	63 [86]	71 [76]	84 [67]	92 [59]	95 [29]

Table B.8.118: Average MTTF values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	2220	1992.5	1956.7	1916.3	1893	1787.1	1563.1	1392.3	1240.8	508.6	51.8
	1	2220	2107.1	2017.9	1905.6	1775.1	1713.4	1484.3	1253.3	987.6	473.4	51.8
lnorm	2	2220	2077.1	2070.3	1907.7	1894.8	1764.9	1581.5	1434.7	891.8	323.3	51.8
	3	2220	2129.1	2026.5	1891.5	1818.9	1674.2	1605.5	1475.9	980.6	760.9	51.8
logis	1	2220	1973	1885.4	1879.1	1812.4	1771.3	1667.1	1423.3	983	780.2	57.7
	1	2220	2207.3	2216.2	2216.6	2216.6	2216.7	2205.7	2205.7	2205.7	2202.4	2119.9
exp	2	2220	2191.3	2172.9	2150	2147	2057.1	2088.1	2048.9	1822.7	1449.9	65.5
	3	2220	2220	2217.9	2162.1	2103.3	2113	2051.2	1810.1	1698.9	1325.2	65.5
weibull	1	2220	2032.7	1961.2	1916.3	1876.2	1758.9	1458.3	1314.6	1111.9	508.6	51.8

Table B.8.119: Average MTTF values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	2220	1980.1	1942.1	1916.3	1891.6	1791.5	1563.1	1378.9	1160.6	508.6	51.8
	1	2220	2077.1	1991.2	1891.2	1768.4	1661.5	1459.1	1175.1	989.4	473.4	51.8
lnorm	2	2220	2089.6	1987.6	1932.2	1869.3	1721.8	1483.2	1381.7	853.5	323.3	51.8
	3	2220	2113.4	2046.8	1870.9	1801.9	1685.3	1542.4	1256.9	928.2	630	51.8
logis	1	2220	1989.9	1885.4	1892.1	1809.5	1767.7	1676.4	1542	1111.8	907.9	51.8
	1	2220	2216.2	2216.2	2216.6	2216.6	2205.7	2205.7	2205.7	2205.7	2202.4	2080
exp	2	2220	2224.6	2227.1	2222.6	2154.6	2173.5	2095.7	2116.8	2061.7	1721.7	12
	3	2220	2231.2	2275.4	2221	2192.7	2135.7	2062	2001.5	1926.2	1240.5	12
weibull	1	2220	2008.6	1942.9	1913.9	1873.3	1746.4	1411.3	1314.6	935.3	508.6	51.8

Table B.8.120: Average MTTRep values for time-independent PHMs computed using historical data and assuming general-case scenarios.

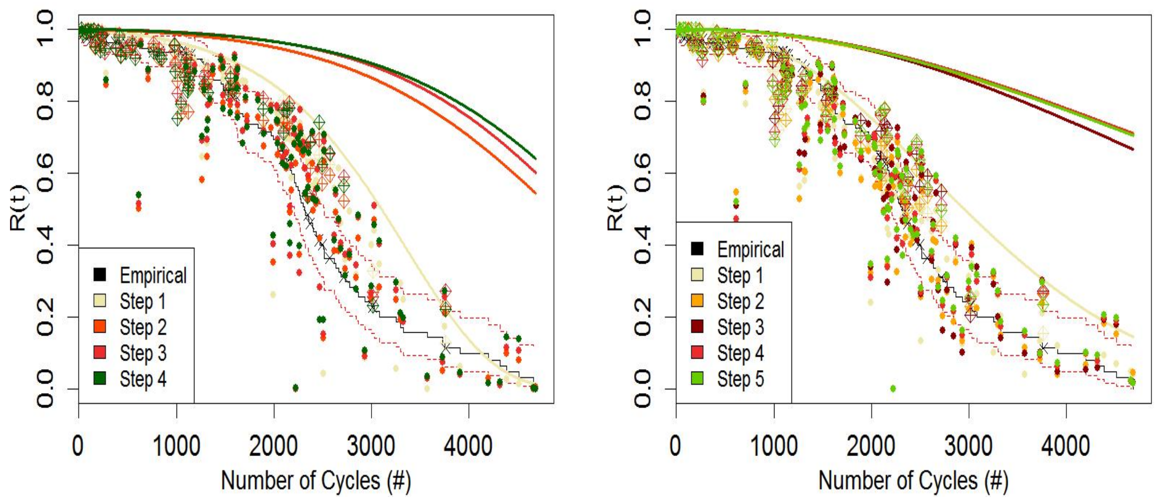
		Reliability Level										
Dist.	Step	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	4836.2	3385.8	3068.5	2761.9	2548.1	2341.9	2171.9	1940.9	1611.3	1176.2	302.2
	1	4836.2	3961.5	3387	2967.6	2608.4	2327.6	2034.5	1776.6	1440.6	1050.1	476.6
lnorm	2	4836.2	3951.5	3321.5	2928.4	2526.5	2261.5	2028.1	1803.8	1462.2	1091.7	500.6
	3	4836.2	3726.3	3208.5	2869.5	2541.8	2291.7	2012.9	1782.3	1483.6	1089.9	487.9
logis	1	4836.2	3234.7	2912.4	2630	2433.4	2250.7	2056.2	1901.1	1669.8	1314.2	403.4
	1	4836.2	4491.5	4366.6	4289.8	4240.8	4154.1	4033	3973.1	3865.5	3659.6	2793.7
exp	2	4836.2	4729.3	4452.1	3894.1	3417.9	2916.9	2394.4	1992.6	1414.6	846	221.7
	3	4836.2	4577.1	4310.1	3765.8	3164.8	2555.3	2058.9	1684.8	1025.9	662.7	146
weibull	1	4836.2	3479.6	3069.8	2735.1	2475.3	2261.7	2067.8	1821.4	1512.6	1130	450

Table B.8.121: Average MTTRep values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	4836.2	3343.7	2984.8	2723.5	2477.8	2289.9	2126.2	1887.1	1563.7	1127.8	254.1
	1	4836.2	3847.9	3250.2	2809.7	2507.7	2191.1	1917.2	1660.2	1331.9	975	442.4
lnorm	2	4836.2	3746.6	3158.4	2722	2392.8	2168.1	1947.6	1689.5	1357.4	1028.6	468.2
	3	4836.2	3515.7	2993.4	2697.3	2386.9	2141.8	1881.8	1668.8	1363.3	1004.3	443.9
logis	1	4836.2	3293	2958.4	2677	2492.5	2301.3	2111.8	1937.2	1718.5	1366.1	463.5
	1	4836.2	4422.5	4351.5	4270.8	4221.6	4102.3	4005.2	3949.6	3843.2	3628.6	2696.8
exp	2	4836.2	3633	3068.3	2720.5	2288	1851.6	1653.8	1375.6	894.5	548.3	70
	3	4836.2	3331.8	2690.9	2294.8	1965.6	1474.4	1180	788.3	539.8	334.1	32.8
weibull	1	4836.2	3444.9	3030.6	2658	2411	2219.7	2024.5	1778.1	1476.4	1090.1	424.8

Table B.8.122: Overview of time-dependent PHM results obtained by MLE optimisation and GOF tests.

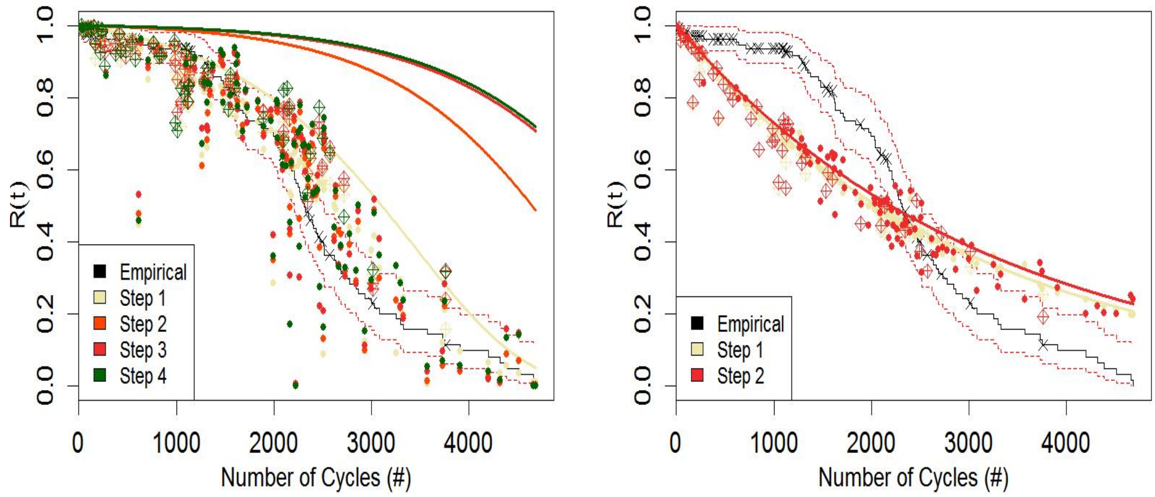
Distribution	norm	norm	norm	norm	lnorm	lnorm	lnorm	lnorm
Step #	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
MLE	**−664.21**	**−566.31**	**−535.49**	**−529.2**	**−667.31**	**−576.53**	**−540.86**	**−528.19**
Time (min)	**23.01**	**59.07**	**96.83**	**114.2**	**13.11**	**27.1**	**45.22**	**61.07**
Kolmogorov-Smirnov	**6.67**	**5.25**	**6.52**	**6.5**	**5.68**	**4.74**	**6.52**	**6.53**
Cramer-von Mises-Smirnov	**56.19**	**56.75**	**56.06**	**56.26**	**57.13**	**57**	**56.56**	**56.61**
Anderson-Darling	**−184.2**	**−179.12**	**−186.74**	**−184.77**	**−178.03**	**−178.57**	**−184.74**	**−184.71**
NRR	**5453.86**	**15411.94**	**49264.29**	**35688.85**	**48.83**	**65.69**	**928.83**	**1670.78**
Distribution	lnorm	logis	logis	logis	logis	exp	exp	weibull
Step #	**5**	**1**	**2**	**3**	**4**	**1**	**2**	**1**
MLE	**−525.42**	**−660.19**	**−568.1**	**−538.56**	**−523.14**	**−763.41**	**−758.18**	**−667.87**
Time (min)	**75.64**	**4.82**	**12.15**	**21.72**	**28.22**	**1.95**	**2.64**	**5.62**
Kolmogorov-Smirnov	**6.53**	**5.86**	**5.54**	**6.48**	**6.52**	**2.97**	**3.8**	**4.55**
Cramer-von Mises-Smirnov	**56.65**	**56.75**	**56.57**	**56.09**	**56**	**55.16**	**55.21**	**57.42**
Anderson-Darling	**−184.62**	**−179.08**	**−180.49**	**−184.9**	**−186.1**	**−182.62**	**−182.38**	**−176.71**
NRR	**1341.32**	**3644.06**	**60458.12**	**51876.54**	**32423.14**	**32.48**	**37.04**	**718.4**
Distribution	weibull	weibull	weibull	weibull	weibull			
Step #	**2**	**3**	**4**	**5**	**6**			
MLE	**−627.15**	**−615.15**	**−607.65**	**−603.13**	**−600.14**			
Time (min)	**12.38**	**20.03**	**28.19**	**35.41**	**40.76**			
Kolmogorov-Smirnov	**6.54**	**6.53**	**6.54**	**6.54**	**6.54**			
Cramer-von Mises-Smirnov	**56.99**	**56.92**	**56.92**	**56.84**	**56.75**			
Anderson-Darling	**−183.7**	**−183.66**	**−185.03**	**−184.84**	**−185.75**			
NRR	**1146.38**	**662.14**	**1105.96**	**772**	**922.6**			



(a) With an underlying norm distribution.

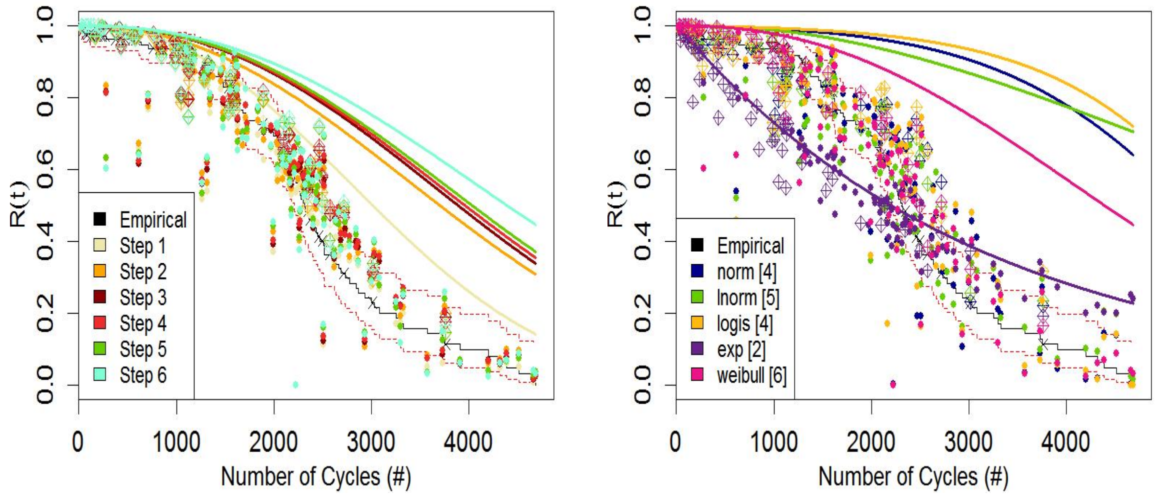
(b) With an underlying lnorm distribution.

Figure B.8.81: Time-dependent PHMs with an underlying norm and lnorm distribution.



(a) With an underlying logis distribution. (b) With an underlying exp distribution.

Figure B.8.82: Time-dependent PHMs with an underlying logis and exp distribution.



(a) With an underlying weibull distribution. (b) All distributions (last step).

Figure B.8.83: Figures containing a weibull distribution and all time-dependent PHMs.

Table B.8.123: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
	1	0 [100]	14 [100]	15 [98]	17 [98]	22 [96]	28 [96]	35 [94]	48 [86]	67 [80]	86 [69]	95 [27]
norm	2	0 [100]	9 [100]	13 [100]	19 [98]	26 [96]	30 [96]	42 [92]	57 [86]	73 [73]	88 [61]	95 [24]
	3	0 [100]	9 [100]	14 [100]	17 [98]	22 [96]	30 [96]	36 [94]	42 [88]	66 [82]	83 [65]	95 [27]
	4	0 [100]	8 [100]	13 [100]	16 [96]	22 [96]	30 [96]	37 [94]	49 [90]	67 [82]	83 [57]	95 [27]
	1	0 [100]	8 [100]	14 [98]	22 [96]	28 [96]	41 [94]	59 [84]	71 [76]	81 [73]	94 [61]	95 [31]
	2	0 [100]	7 [100]	13 [100]	22 [96]	30 [96]	37 [94]	53 [86]	66 [80]	79 [73]	91 [55]	95 [31]
lnorm	3	0 [100]	8 [100]	15 [100]	21 [96]	28 [96]	35 [96]	43 [92]	62 [84]	78 [73]	90 [59]	95 [31]
	4	0 [100]	6 [100]	10 [100]	17 [96]	27 [96]	35 [96]	45 [90]	63 [86]	77 [76]	88 [57]	95 [31]
	5	0 [100]	5 [100]	10 [100]	17 [96]	27 [96]	36 [96]	45 [92]	63 [86]	77 [76]	88 [57]	95 [31]
	1	0 [100]	8 [100]	14 [98]	17 [98]	23 [96]	30 [96]	44 [88]	66 [80]	81 [73]	94 [47]	95 [12]
logis	2	0 [100]	9 [100]	13 [100]	17 [98]	22 [96]	30 [96]	38 [94]	50 [86]	69 [76]	86 [61]	95 [24]
	3	0 [100]	8 [100]	13 [100]	16 [100]	26 [96]	30 [96]	36 [92]	47 [88]	67 [76]	86 [55]	95 [24]
	4	0 [100]	7 [100]	13 [100]	15 [100]	24 [96]	30 [94]	37 [92]	49 [88]	66 [82]	84 [57]	95 [27]
exp	1	0 [100]	0 [100]	2 [100]	12 [98]	24 [94]	65 [76]	79 [65]	91 [49]	92 [37]	94 [24]	97 [8]
	2	0 [100]	0 [100]	0 [98]	13 [98]	34 [92]	55 [80]	73 [67]	88 [51]	91 [37]	94 [24]	97 [8]
	1	0 [100]	10 [100]	16 [98]	20 [98]	29 [96]	38 [92]	58 [82]	70 [76]	79 [69]	91 [61]	95 [31]
	2	0 [100]	8 [100]	14 [100]	19 [98]	26 [96]	35 [96]	53 [88]	65 [78]	78 [71]	92 [61]	95 [31]
weibull	3	0 [100]	12 [100]	14 [100]	19 [98]	24 [96]	35 [96]	47 [90]	65 [80]	76 [76]	88 [63]	95 [31]
	4	0 [100]	9 [100]	14 [98]	19 [98]	26 [96]	35 [94]	45 [90]	65 [82]	74 [76]	87 [61]	95 [31]
	5	0 [100]	10 [100]	15 [98]	20 [98]	23 [96]	37 [94]	47 [88]	63 [84]	76 [76]	86 [65]	95 [31]
	6	0 [100]	9 [100]	15 [98]	19 [98]	27 [96]	37 [94]	48 [88]	65 [84]	76 [76]	87 [65]	95 [31]

Table B.8.124: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	0 [100]	10 [100]	15 [98]	15 [98]	22 [96]	24 [96]	30 [96]	44 [90]	66 [80]	83 [71]	95 [27]
	2	0 [100]	8 [100]	13 [100]	19 [98]	23 [96]	30 [96]	41 [92]	57 [86]	71 [73]	86 [61]	95 [24]
	3	0 [100]	8 [100]	13 [100]	17 [100]	22 [96]	30 [96]	35 [94]	42 [90]	65 [84]	81 [67]	95 [29]
	4	0 [100]	8 [100]	13 [100]	16 [96]	22 [96]	30 [96]	36 [94]	45 [90]	67 [82]	83 [57]	95 [27]
lnorm	1	0 [100]	6 [100]	14 [98]	22 [96]	28 [96]	33 [96]	52 [84]	67 [80]	78 [73]	92 [61]	95 [33]
	2	0 [100]	6 [100]	13 [100]	22 [96]	29 [96]	37 [94]	51 [86]	66 [80]	78 [73]	91 [55]	95 [31]
	3	0 [100]	8 [100]	14 [100]	20 [96]	27 [96]	34 [96]	41 [92]	62 [84]	74 [73]	88 [63]	94 [33]
	4	0 [100]	6 [100]	10 [100]	16 [96]	27 [96]	35 [96]	44 [92]	62 [86]	74 [76]	87 [57]	95 [31]
	5	0 [100]	5 [100]	10 [100]	16 [98]	27 [96]	35 [96]	43 [92]	63 [88]	76 [76]	87 [57]	95 [31]
logis	1	0 [100]	8 [100]	14 [98]	16 [98]	22 [96]	28 [96]	42 [92]	59 [82]	74 [73]	93 [51]	95 [22]
	2	0 [100]	9 [100]	13 [100]	15 [98]	22 [96]	30 [96]	37 [94]	48 [86]	66 [78]	84 [61]	95 [24]
	3	0 [100]	8 [100]	13 [100]	15 [100]	24 [96]	30 [96]	35 [94]	47 [88]	66 [76]	84 [55]	95 [24]
	4	0 [100]	7 [100]	13 [100]	15 [100]	21 [96]	30 [96]	36 [92]	49 [90]	64 [84]	83 [57]	95 [27]
exp	1	0 [100]	0 [100]	2 [100]	12 [98]	24 [94]	65 [76]	79 [65]	91 [49]	92 [37]	94 [24]	97 [8]
	2	0 [100]	0 [100]	0 [98]	13 [98]	34 [92]	55 [80]	74 [67]	88 [51]	91 [37]	94 [24]	97 [8]
weibull	1	0 [100]	10 [100]	16 [98]	20 [98]	29 [96]	38 [92]	58 [80]	70 [76]	81 [69]	91 [59]	95 [31]
	2	0 [100]	9 [100]	14 [100]	19 [98]	26 [96]	35 [94]	55 [88]	69 [78]	80 [69]	93 [59]	95 [29]
	3	0 [100]	12 [100]	14 [98]	19 [98]	26 [96]	35 [92]	51 [88]	66 [78]	77 [76]	90 [61]	95 [31]
	4	0 [100]	12 [100]	14 [98]	19 [98]	26 [96]	35 [94]	48 [90]	65 [78]	76 [76]	88 [59]	95 [31]
	5	0 [100]	10 [100]	15 [98]	21 [98]	24 [96]	37 [94]	49 [88]	67 [84]	76 [73]	87 [61]	95 [31]
	6	0 [100]	9 [100]	15 [98]	21 [98]	29 [94]	38 [94]	50 [88]	67 [82]	78 [73]	88 [59]	95 [29]

Table B.8.125: Average MTTF values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	2220	1949.3	1924.5	1903.8	1839	1790.7	1714	1591	1245.1	849.3	51.8
	2	2220	2017.1	1966.6	1869.5	1789.3	1739.1	1660.9	1449.2	1100.6	781.5	51.8
	3	2220	2046.4	1963.2	1899.9	1843.9	1749.8	1707.6	1639.3	1285.5	875	51.8
	4	2220	2067.7	1970.5	1919.3	1836.8	1749.8	1661.9	1553.6	1273.4	875	51.8
lnorm	1	2220	2032.7	1949.3	1839	1755.8	1643.9	1410.1	1153.2	883.8	168.4	51.8
	2	2220	2067.7	1955.8	1831.2	1739.1	1654.4	1462.7	1268.7	972.5	591.4	51.8
	3	2220	2067.7	1962.6	1873.3	1772.8	1692.1	1630	1346.9	1011.4	687.7	51.8
	4	2220	2112.6	2034.9	1918.3	1786.2	1692.1	1625.4	1319.9	1047	766.8	51.8
	5	2220	2139.5	2034.9	1918.3	1786.2	1680.7	1600.3	1347.7	1047	766.8	51.8
logis	1	2220	2032.7	1949.3	1891.9	1821.4	1764.1	1593.6	1258.3	896.2	168.4	51.8
	2	2220	2017.1	1966.6	1885.4	1824	1739.1	1674	1549.6	1216.4	734	51.8
	3	2220	2067.7	1970.5	1919.3	1802.7	1749.8	1726.8	1583.8	1273.4	734	51.8
	4	2220	2094.3	1970.5	1938.6	1805	1749.8	1694.1	1578.5	1274.7	843	51.8
exp	1	2220	2220	2161.6	1963	1785.5	1184.6	865.6	305.9	248	97.8	29.3
	2	2220	2220	2220	1960.6	1690.9	1422	1045.2	473.4	305.9	97.8	29.3
weibull	1	2220	1986.2	1905.4	1852	1742.1	1657.8	1381.2	1171.3	1011.2	576	51.8
	2	2220	2067.7	1963.2	1880.6	1785.6	1679.7	1483.3	1243.9	992.2	506	51.8
	3	2220	1983.1	1963.2	1880.6	1800.8	1679.7	1553.9	1279.4	1041.9	758.4	51.8
	4	2220	2038	1963.2	1880.6	1784.5	1679.7	1571.8	1279.4	1072.4	836.1	51.8
	5	2220	2005.8	1944.9	1863.2	1818.3	1671.8	1558.1	1348.2	1053.1	901.9	51.8
	6	2220	2038	1944.9	1880.6	1782.9	1652.3	1550.1	1318.3	1053.1	836.1	51.8

Table B.8.126: Average MTTF values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	2220	2010.7	1924.5	1924.5	1839	1840	1764.1	1618.7	1258.3	844.1	51.8
	2	2220	2048.6	1966.6	1869.5	1811.3	1739.1	1653.1	1449.2	1151.1	734	51.8
	3	2220	2067.7	1970.5	1899.9	1843.9	1749.8	1688	1639.3	1316.3	900.8	51.8
	4	2220	2067.7	1970.5	1919.3	1836.8	1749.8	1674.4	1598.3	1273.4	875	51.8
lnorm	1	2220	2088.5	1949.3	1839	1755.8	1710.6	1505.2	1245.1	1006.9	447	51.8
	2	2220	2088.2	1955.8	1831.2	1754.2	1654.4	1496.7	1268.7	1002.9	591.4	51.8
	3	2220	2067.7	1981.1	1882.6	1786.2	1705.1	1650.6	1346.9	1105.9	780.9	168.4
	4	2220	2112.6	2034.9	1928.9	1786.2	1692.1	1617.9	1356.1	1114.5	844.9	51.8
	5	2220	2139.5	2034.9	1928.9	1786.2	1692.1	1639.3	1347.7	1071	844.9	51.8
logis	1	2220	2032.7	1949.3	1905	1839	1755.8	1629.5	1404.3	1083	187.3	51.8
	2	2220	2017.1	1966.6	1923.5	1824	1739.1	1691.8	1583.4	1268.7	821.8	51.8
	3	2220	2067.7	1970.5	1944.5	1817.6	1749.8	1706.9	1583.8	1285.5	839.4	51.8
	4	2220	2094.3	1970.5	1938.6	1867	1749.8	1718.3	1578.5	1326	875	51.8
exp	1	2220	2220	2161.6	1963	1785.5	1184.6	865.6	305.9	248	97.8	29.3
	2	2220	2220	2220	1960.6	1690.9	1422	1019.1	473.4	305.9	97.8	29.3
weibull	1	2220	1986.2	1905.4	1852	1742.1	1657.8	1381.2	1171.3	948.4	576	51.8
	2	2220	2046.4	1963.2	1880.6	1785.6	1679.7	1463.1	1161.4	987.9	323.3	51.8
	3	2220	1983.1	1963.2	1880.6	1786.9	1679.7	1510.3	1265.7	1020.1	690	51.8
	4	2220	1983.1	1963.2	1880.6	1784.5	1679.7	1537.4	1279.4	1041.9	758.4	51.8
	5	2220	2005.8	1944.9	1846.5	1806	1671.8	1523.7	1264.6	1053.1	836.1	51.8
	6	2220	2038	1944.9	1846.5	1761.2	1642.4	1521.3	1264.6	992.2	758.4	51.8

Table B.8.127: Average MTTRep values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	**4836.2**	**3351.1**	**3123.5**	**2921.7**	**2732.7**	**2548.8**	**2365.6**	**2151.6**	**1875**	**1369.8**	**313.1**
	2	**4836.2**	**3508.3**	**3190.4**	**2952.8**	**2694.2**	**2478.6**	**2239.4**	**2013.8**	**1661.2**	**1132.8**	**262**
	3	**4836.2**	**3474.1**	**3203.3**	**2970.5**	**2734**	**2549**	**2340.6**	**2130.9**	**1829.8**	**1299**	**322.9**
	4	**4836.2**	**3511.8**	**3238.8**	**2991.2**	**2741.5**	**2545.8**	**2351.7**	**2093.6**	**1749**	**1198.6**	**287.5**
lnorm	1	**4836.2**	**3541.1**	**3095.5**	**2771.8**	**2524.7**	**2272.8**	**2035.8**	**1804**	**1492**	**1096.6**	**448.7**
	2	**4836.2**	**3694.8**	**3229.3**	**2833.1**	**2556.9**	**2313.7**	**2061.4**	**1807.1**	**1485**	**1095.3**	**467.2**
	3	**4836.2**	**3648.9**	**3184.3**	**2860.5**	**2615.9**	**2398.2**	**2157.9**	**1965.7**	**1650.1**	**1225.4**	**532.2**
	4	**4836.2**	**3727.5**	**3288.4**	**2928.3**	**2647.6**	**2406.8**	**2142.8**	**1943.2**	**1614.6**	**1185.6**	**502.7**
	5	**4836.2**	**3739.6**	**3327.5**	**2966.3**	**2666.7**	**2416.4**	**2166.5**	**1936.4**	**1619.9**	**1179**	**491.8**
logis	1	**4836.2**	**3461.6**	**3158.6**	**2926**	**2675.2**	**2446.1**	**2215.3**	**1951.1**	**1546.4**	**990.9**	**173.1**
	2	**4836.2**	**3489.4**	**3209.4**	**3002.4**	**2756.6**	**2548.7**	**2323.6**	**2093.9**	**1748.6**	**1207.9**	**267.9**
	3	**4836.2**	**3530.6**	**3260.3**	**3004.6**	**2731.5**	**2529.5**	**2284.4**	**2050.7**	**1691**	**1137.8**	**264**
	4	**4836.2**	**3526**	**3301.7**	**3045.6**	**2781.9**	**2541.3**	**2319.9**	**2048.3**	**1708.1**	**1185.8**	**286.3**
exp	1	**4836.2**	**4705.3**	**3839.2**	**3141.4**	**2525.6**	**1960**	**1448.5**	**1058.6**	**679.3**	**338.6**	**100**
	2	**4836.2**	**4653.7**	**3742.7**	**3108.9**	**2495.9**	**1950.1**	**1467.9**	**1086**	**700.4**	**350.3**	**100**
weibull	1	**4836.2**	**3493**	**3097**	**2823.2**	**2560.4**	**2299.4**	**2054.9**	**1808.4**	**1495.9**	**1126.5**	**421.8**
	2	**4836.2**	**3580.1**	**3183.7**	**2897.4**	**2632.7**	**2376.8**	**2105.5**	**1873.1**	**1545**	**1138.2**	**415.1**
	3	**4836.2**	**3576**	**3181.5**	**2914.5**	**2666.3**	**2422.6**	**2173.3**	**1925.2**	**1624**	**1217.2**	**466.8**
	4	**4836.2**	**3584.8**	**3211.2**	**2940**	**2691.7**	**2442.1**	**2185.7**	**1945**	**1639.2**	**1218**	**473**
	5	**4836.2**	**3563.4**	**3195.6**	**2924.8**	**2666.9**	**2417.9**	**2184.1**	**1943.3**	**1645.2**	**1222.9**	**479.9**
	6	**4836.2**	**3533.5**	**3194.5**	**2912.8**	**2641.8**	**2411.4**	**2155.3**	**1921.7**	**1633.5**	**1202.4**	**457.7**

Table B.8.128: Average MTTRep values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	40%	50%	60%	69%	79%	89%	99%
norm	1	**4836.2**	**3394.9**	**3175.2**	**2979.4**	**2780**	**2589.2**	**2410**	**2221.1**	**1944.5**	**1453.8**	**379.1**
	2	**4836.2**	**3507.7**	**3211.2**	**2970.4**	**2716.9**	**2501.1**	**2268.9**	**2036**	**1682.3**	**1165**	**284.6**
	3	**4836.2**	**3487.8**	**3232.3**	**2989.7**	**2749.9**	**2565**	**2373.7**	**2150**	**1845**	**1320.2**	**343.8**
	4	**4836.2**	**3526.9**	**3253.1**	**3007**	**2756.5**	**2562**	**2368.3**	**2109**	**1770.1**	**1218.6**	**308.8**
lnorm	1	**4836.2**	**3559.2**	**3142.6**	**2822.2**	**2577.1**	**2340.4**	**2095.7**	**1865.3**	**1556**	**1162.3**	**515.6**
	2	**4836.2**	**3710.9**	**3246.8**	**2849.6**	**2572.2**	**2334.8**	**2085.8**	**1830.9**	**1505.6**	**1117.4**	**491.4**
	3	**4836.2**	**3667.4**	**3201**	**2882.6**	**2636.2**	**2420.8**	**2180.8**	**1985.2**	**1673.7**	**1245.9**	**556.5**
	4	**4836.2**	**3743.2**	**3305.1**	**2950.7**	**2667.6**	**2429.1**	**2174**	**1958.7**	**1634.2**	**1203.4**	**528.5**
	5	**4836.2**	**3758.6**	**3346.2**	**2990.6**	**2686.9**	**2439.8**	**2182.3**	**1961.1**	**1643.5**	**1199.5**	**520.9**
logis	1	**4836.2**	**3509.9**	**3208.5**	**2978.2**	**2719.3**	**2530.5**	**2273.9**	**2007.7**	**1612**	**1061.1**	**237.6**
	2	**4836.2**	**3508.7**	**3228.9**	**3013.3**	**2776.3**	**2579**	**2340.7**	**2111.6**	**1769.7**	**1228.6**	**292.3**
	3	**4836.2**	**3543**	**3272.7**	**3006.5**	**2745**	**2547.7**	**2322.9**	**2066.6**	**1709.7**	**1149.4**	**281.1**
	4	**4836.2**	**3542**	**3316.1**	**3061.8**	**2778.8**	**2554.1**	**2322.1**	**2064.5**	**1718.9**	**1201.9**	**304**
exp	1	**4836.2**	**4704.4**	**3837.1**	**3138.5**	**2522.3**	**1956.4**	**1444.9**	**1055**	**675.7**	**335**	**100**
	2	**4836.2**	**4651.4**	**3737.7**	**3102.3**	**2487.5**	**1941.7**	**1460.1**	**1077.3**	**691.1**	**341.2**	**100**
weibull	1	**4836.2**	**3481**	**3085.3**	**2810.6**	**2545.4**	**2284.1**	**2038.8**	**1793.5**	**1479.1**	**1107.6**	**407.6**
	2	**4836.2**	**3566.5**	**3163.2**	**2877.6**	**2613.7**	**2353.4**	**2086.5**	**1853.8**	**1512.2**	**1115.4**	**393.8**
	3	**4836.2**	**3554.4**	**3154**	**2888.2**	**2634.3**	**2392.8**	**2137**	**1891.9**	**1588.4**	**1181.2**	**439.8**
	4	**4836.2**	**3578.5**	**3182.7**	**2911.5**	**2660.9**	**2413.5**	**2156.8**	**1911.7**	**1603.2**	**1184.5**	**443.1**
	5	**4836.2**	**3538.5**	**3165.4**	**2893.1**	**2635.2**	**2386.7**	**2153.5**	**1909.3**	**1606.5**	**1184.3**	**445.5**
	6	**4836.2**	**3483.7**	**3140.9**	**2870.9**	**2583.5**	**2364.7**	**2103.8**	**1868**	**1576.1**	**1154.9**	**409.9**

Table B.8.129: Variables identified by time-(in)dependent PHM models.

	Time-independent PHM			Time-dependent PHM	
	Variable	Scaled Value		Variable	Scaled Value
①	*Group C*	**−18.61**	①	*Group E*	**−29.73**
②	*Group D*	**14.25**	②	*Group A*	**20.98**
③	*Vtrue mean knots 6*	**6.08**	③	*Group F*	**7.82**
④	*Group A*	**3.22**	④	*Pitch rate min deg sec 2*	**−6.68**
⑤	*Yaw rate mean deg sec 8*	**3.1**	⑤	*Vtrue mean knots 6*	**−6.27**
⑥	*Group E*	**2.87**	⑥	*Headwind mean knots 6*	**4.42**
⑦	*Group B*	**2.66**	⑦	*Group C*	**−4.11**

Table B.8.130: Variables identified by each step by time-(in)dependent PHMs (in order).

	PHM Variables	
	Time-independent	*Time-dependent*
norm	④	② ① ③ ④
lnorm	③ ④ ⑦	② ① ③ ⑥ ④
logis	⑤	② ① ③ ⑥
exp	① ② ⑥	④ ⑦
weibull	④	① ③ ⑥ ④ ⑦ ⑤

Table B.8.131: Number of times variables identified by each step by time-(in)dependent PHMs.

Key				Key			
indep	*dep*	*Variable*	*Count*	*indep*	*dep*	*Variable*	*Count*
④	②	Group A	6		③	Group F	4
⑦		Group B	1		⑥	Headwind mean knots 6	3
①	⑦	Group C	3		④	Pitch rate min deg sec 2	4
②		Group D	1	③	⑤	Vtrue mean knots 6	2
⑥	①	Group E	5	⑤		Yaw rate mean deg sec 8	1

Table B.8.132: Variables belonging to each group identified in B.8.129.

Group	*Variables*	*Group*	*Variables*
Group A	Vtrue max knots , Pressure dynamic max hPa mbar , Vtrue min knots 1	Group D	Aileron Rin max deg TEU 1, Aileron Rin max deg TEU 4, Aileron Rin max deg TEU 5, Aileron Rin max deg TEU 6
Group B	Accn norm mean g s 6, Accn norm mean g s 5	Group E	Torque lhs mean 5, Torque lhs mean 6
Group C	Aileron Rin min deg TEU 1, Aileron Rin min deg TEU 4	Group F	Rudder cmd force max lbs Nose Right 6, Rudder cmd force max lbs Nose Right 5

B.9 10-105-31A-N-2 VHF antenna

Table B.9.133 provides a summary of the input data related to the component. The number of registered maintenance events is less than the total number of events due to the fact that TRAX data stretches back to 2004/2005 and FDR data only to 2011. Maintenance events with insufficient data, regarding operational factors, cannot be evaluated, hence are not registered during the modelling process.

Table B.9.133: General overview of component inputs.

Name	**Value**
Part Number	10-105-31A-N-2
Total # (A, F, C)	**253, 239, 14**
Registered # (A, F, C)	**39, 37, 2**
Related Flights # (A, F, C)	**68824, 63591, 5233**
Avg. Cycles (A, F, C)	**1764.72, 1718.68, 2616.5**
% Censored	**5.13**

In Tab. B.9.133 (A, F, C) denotes statistics regarding All (A), Failed (F), and Censored (C) events respectively. Ergo A will always be the sum or mean derived from F and C.

Analysis
Tables B.9.134 and B.9.135 summarise the results from EVA and MDA. In addition the variables obtained by semi-parametric PHM modelling (labelled 'reduced semi-COX') are also presented if applicable. Table B.9.135 provides an overview of the specific operational factors identified during all flight phases. In this case high counts indicate operational factors that were significantly different during multiple flight phases.

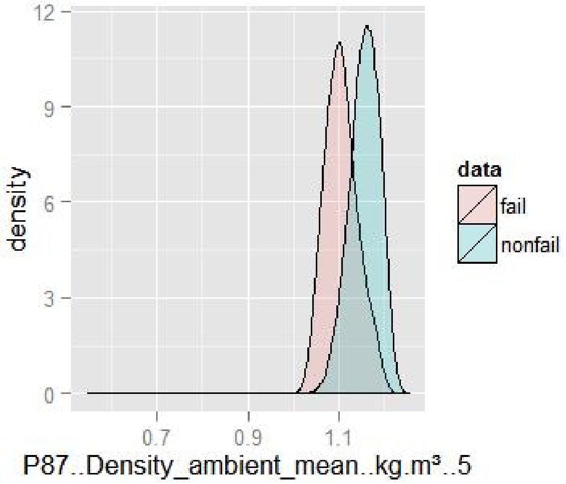
Table B.9.134: Overview of analysis input and output.

	# Variables
ALL	**1531**
EVA	**33**
MDA	**64**
Combined	**97**
reduced Corr.	**53**
reduced semi-COX	**0**
Take-Off related	**21**
Cruise related	**16**
Touch-Down related	**16**

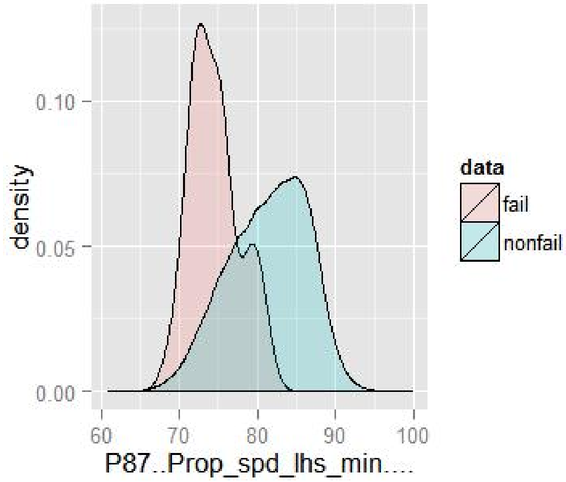
Table B.9.135: Overview of identified operational factors during all flight phases.

Variable	**Count**	**Variable**	**Count**	**Variable**	**Count**
Pitch_rate	8	Brake_press_lhs	2	Pressure_dynamic	1
Yaw_rate	4	Accn_norm	2	NormalForce_rhs	1
Accn_long	4	Pitch	1	Aileron_Rin	1
Torque_lhs	4	Elevator_Lin	1	Pitch_cmd_force	1
Roll	4	Rudder_low	1	Ttot	1
Vtrue	3	Prop_spd_rhs	1	Torque_rhs	1
Pressure_total	3	Aoa	1	Density_total	1
Prop_spd_lhs	2	Vcal	1	Headwind	1
Roll_rate	2	Vz	1		

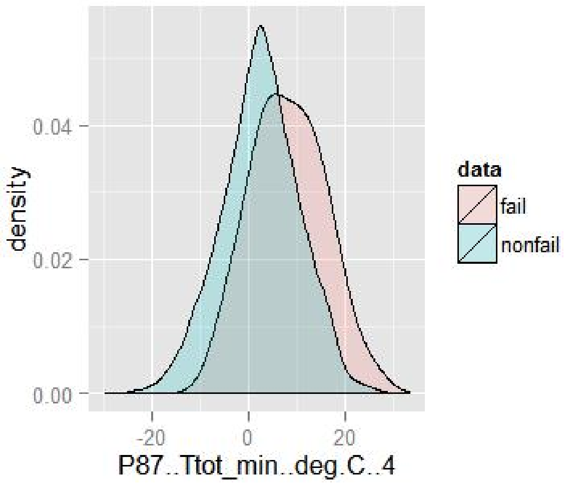
A multitude of factors were identified during EVA and MDA. Figure B.9.84 give a general overview of the top operational factors identified by EVA and MDA.



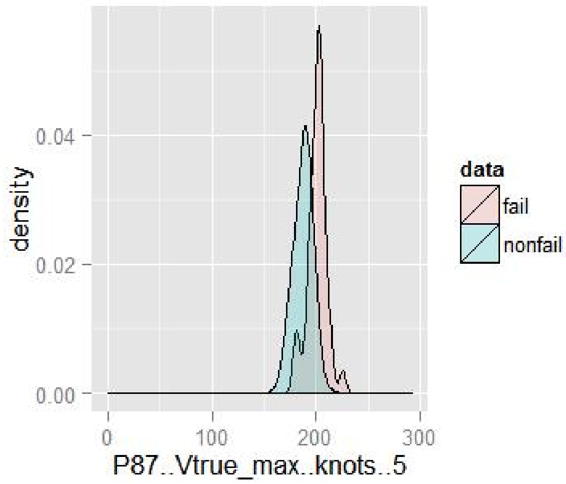
(a) EVA 1.



(b) EVA 2.



(c) MDA 1.



(d) MDA 2.

Figure B.9.84: Graphical overview of top operational factors identified by EVA and MDA.

Time-based reliability modelling

Table B.9.136 reports the maximum likelihood and goodness-of-fit tests results obtained from time-based reliability modelling. To show the overall fit Fig. B.9.85 shows the computed reliability function using an averaged virtual age V for all fitted models.

Table B.9.136: Overview of results obtained by MLE optimisation and GOF tests.

	Distributions					
	norm	**lnorm**	**logis**	**exp**	**weibull**	**gamma**
MLE	-308.77	-316.04	-308.65	-315.55	-309.66	-310.94
Kolmogorov-Smirnov	0.64	1.15	0.56	1.49	0.76	0.87
Cramer-von-Mises Smirnov	24.17	24.75	24.35	21.47	24.65	24.7
Anderson-Darling	-74.1	-75.29	-74.07	-76.19	-74.34	-74.53
NRR	16.64	10.32	18.49	8.95	15.89	13.6

In addition Figures B.9.86, B.9.87, B.9.88, B.9.89, B.9.90, and B.9.91 present the reliability and hazard functions computed for each underlying distribution evaluated in the program.

Time independent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification

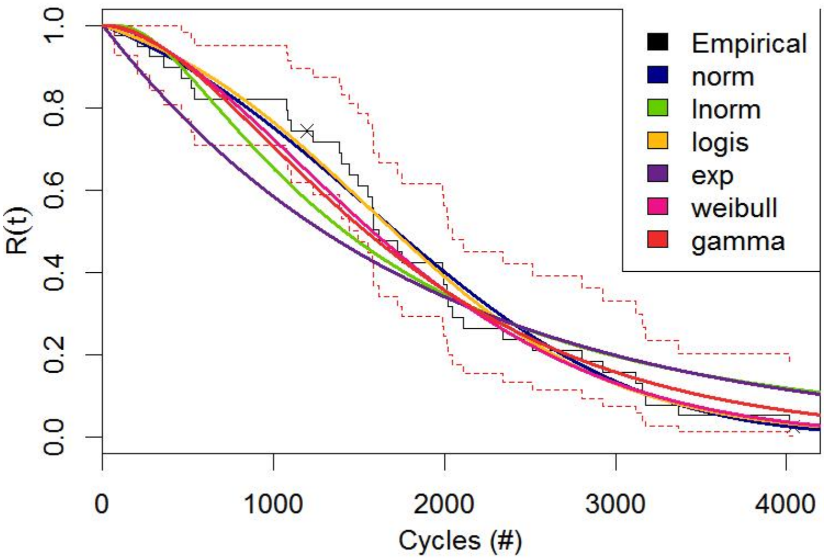
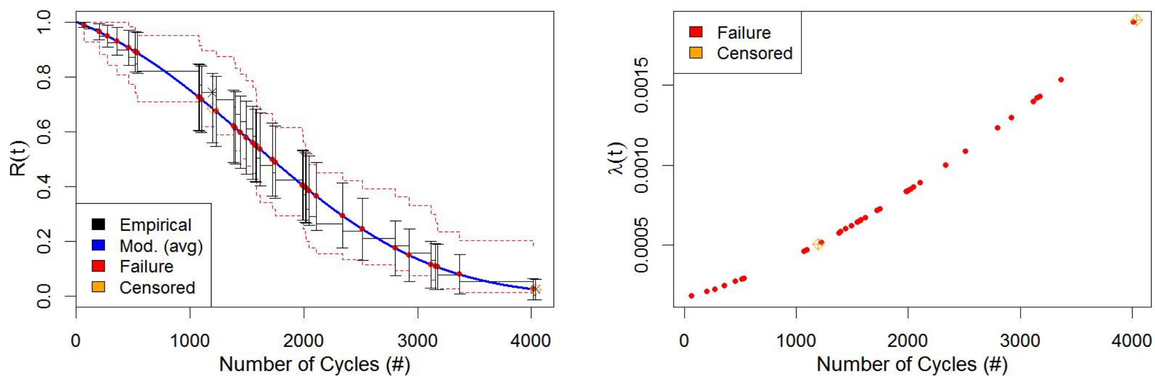
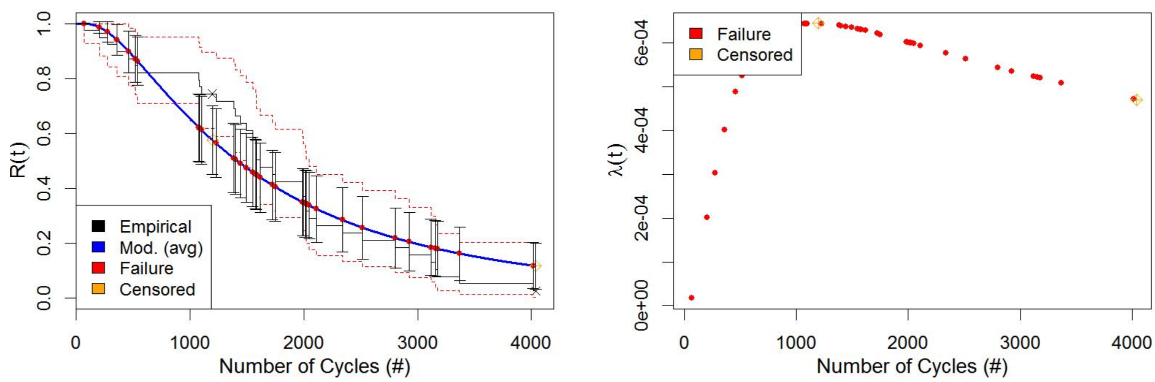


Figure B.9.85: Overview of overall fit of multiple GRP modells.



(a) Reliability function. (b) Hazard function.

Figure B.9.86: Computed reliability for time-based models with underlying norm distribution.

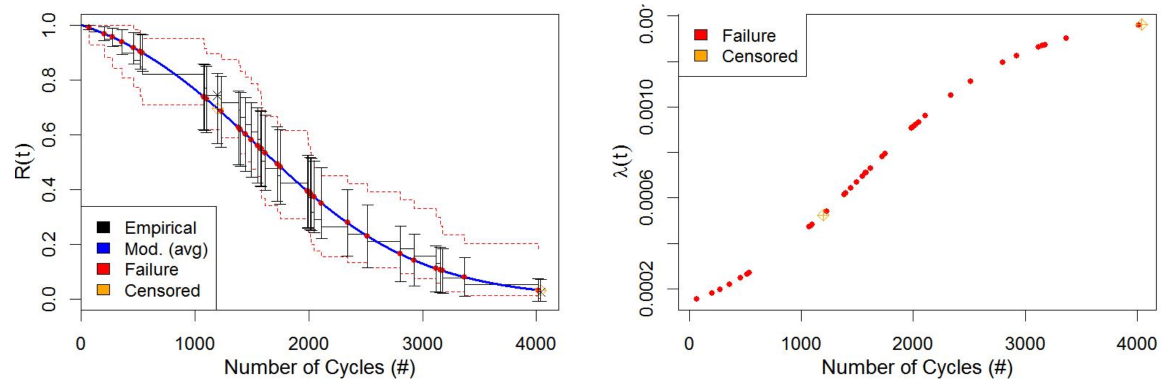


(a) Reliability function. (b) Hazard function.

Figure B.9.87: Computed reliability for time-based models with underlying lnorm distribution.

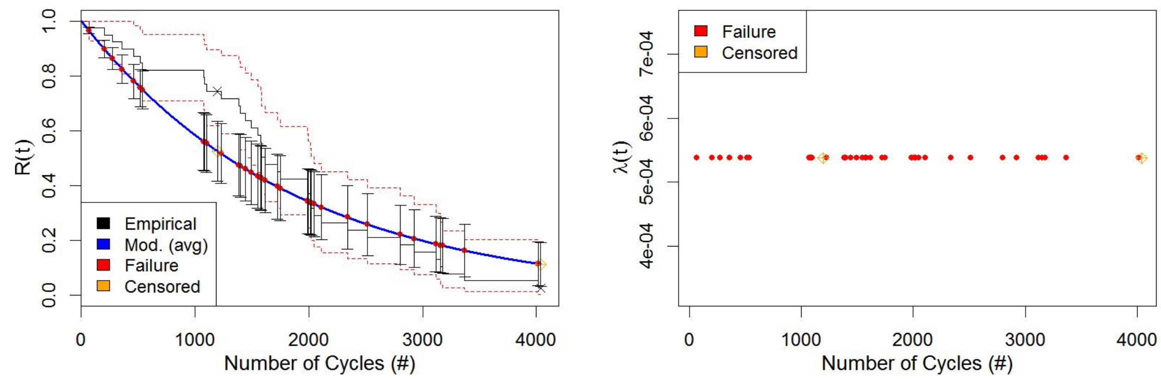
techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.9.137 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.9.92



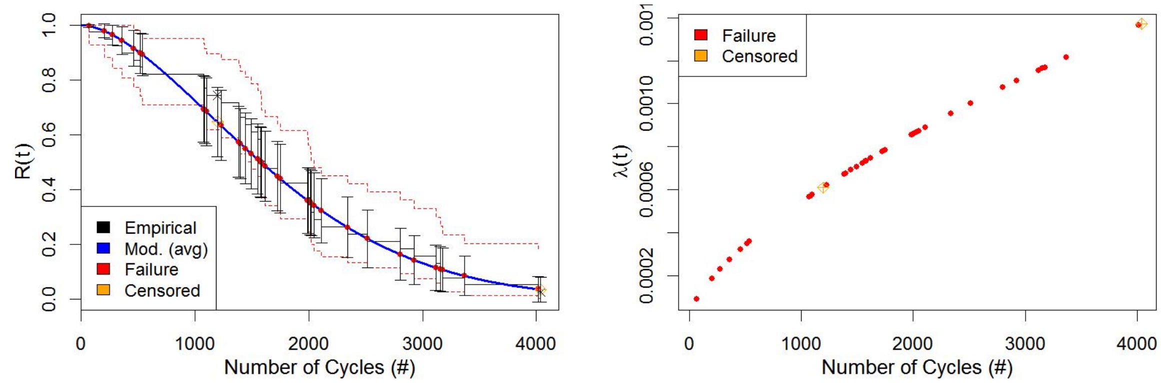
(a) Reliability function. (b) Hazard function.

Figure B.9.88: Computed reliability for time-based models with underlying logis distribution.



(a) Reliability function. (b) Hazard function.

Figure B.9.89: Computed reliability for time-based models with underlying exp distribution.



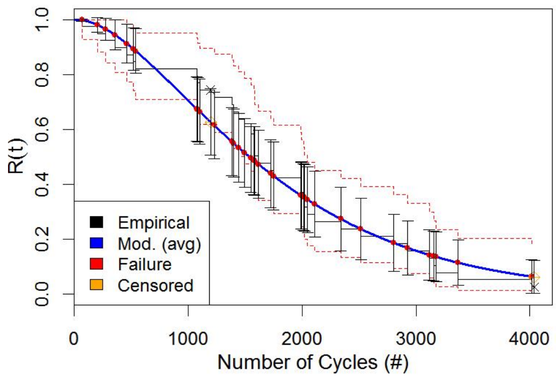
(a) Reliability function. (b) Hazard function.

Figure B.9.90: Computed reliability for time-based models with underlying weibull distribution.

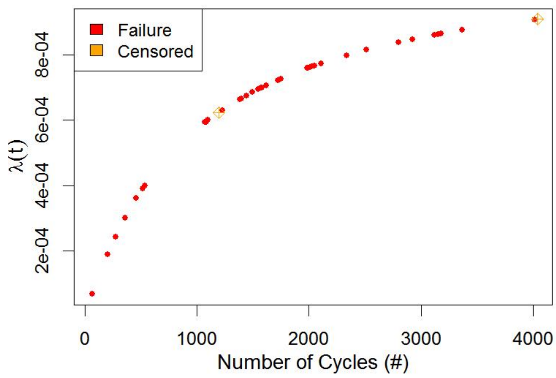
as well as a general overview in Figure B.9.92b.

Tables B.9.138 and B.9.139 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

To assist in the selection of models, Tables B.9.140, B.9.141, B.9.142, and B.9.143 indicate the



(a) Reliability function.

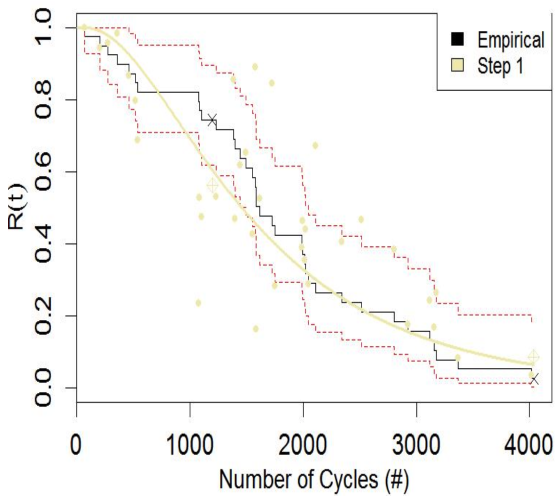


(b) Hazard function.

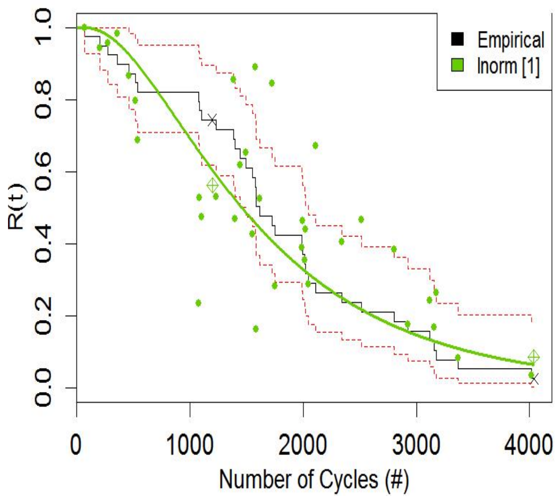
Figure B.9.91: Computed reliability for time-based models with underlying gamma distribution.

Table B.9.137: Overview of time-independent PHM results obtained by MLE optimisation and GOF tests.

Distribution	lnorm
Step #	**1**
MLE	**−310.89**
Time (min)	**1.09**
Kolmogorov-Smirnov	**3.53**
Cramer-von Mises-Smirnov	**22.33**
Anderson-Darling	**−81.01**
NRR	**14.93**



(a) With an underlying lnorm distribution.



(b) All distributions (last step).

Figure B.9.92: Figures containing a lnorm distribution and all time-independent PHMs.

averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.

Table B.9.138: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
lnorm	1	0 [100]	3 [100]	11 [50]	27 [50]	38 [50]	51 [50]	59 [0]	76 [0]	78 [0]	89 [0]	95 [0]

Time dependent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected

Table B.9.139: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	**3%**	**12%**	**22%**	**31%**	**41%**	**50%**	**60%**	**69%**	**78%**	**88%**	**97%**
lnorm	**1**	**3 [100]**	**11 [50]**	**16 [50]**	**27 [50]**	**41 [50]**	**54 [0]**	**68 [0]**	**76 [0]**	**84 [0]**	**92 [0]**	**95 [0]**

Table B.9.140: Average MTTF values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
lnorm	1	**1718.7**	**1654.9**	**1518.9**	**1365.6**	**1205.7**	**1065.1**	**976.7**	**729.1**	**755.2**	**225.2**	**212.5**

Table B.9.141: Average MTTF values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
lnorm	1	**1654.9**	**1607.4**	**1514.9**	**1365.6**	**1196.9**	**1081.6**	**967.8**	**729.1**	**643.2**	**233**	**212.5**

Table B.9.142: Average MTTRep values for time-independent PHMs computed using historical data and assuming general-case scenarios.

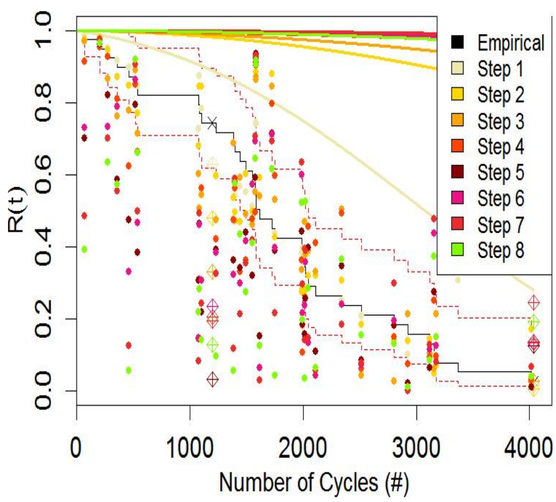
		Reliability Level										
Dist.	Step	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
lnorm	1	**6173**	**4205.3**	**2205.2**	**1743.6**	**1604.7**	**1323.6**	**1071.3**	**923**	**740.5**	**572.9**	**319.9**

Table B.9.143: Average MTTRep values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

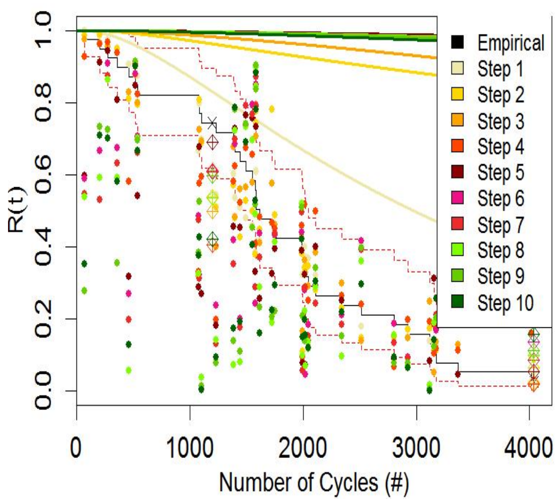
		Reliability Level										
Dist.	Step	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
lnorm	1	**5148.3**	**1739.7**	**1571.4**	**1430**	**1462.6**	**1195**	**945.2**	**843.1**	**653.8**	**504.5**	**271.1**

for modelling. Table B.9.144 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.9.93, B.9.94, and B.9.95 as well as a general overview in Figure B.9.96a.



(a) With an underlying norm distribution.



(b) With an underlying lnorm distribution.

Figure B.9.93: Time-dependent PHMs with an underlying norm and lnorm distribution.

Tables B.9.145 and B.9.146 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the

Table B.9.144: Overview of time-dependent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	norm	norm	norm	norm	norm	norm	norm
Step #	1	2	3	4	5	6	7	8
MLE	−267.35	−220.28	−192.57	−167.81	−146.7	−128.81	−121.67	−114.19
Time (min)	28.66	73.23	121.49	180.53	254.59	294.25	317.54	327.19
Kolmogorov-Smirnov	1.7	2.74	3.88	4.79	4.86	4.47	4.36	3.93
Cramer-von Mises-Smirnov	23.72	23.58	22.51	21.11	21.06	21.8	21.02	20.91
Anderson-Darling	−75.35	−78.59	−80.6	−83.96	−86.13	−84.84	−89.1	−88.96
NRR	−1992.83	113.44	25.99	31.68	18	884.64	591.29	2031.76

Distribution	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm
Step #	1	2	3	4	5	6	7	8
MLE	−272.38	−233.32	−203.89	−177.66	−149.54	−131.22	−123.4	−117.43
Time (min)	15.93	36.89	64.19	100.51	134.99	174.06	213.79	257.68
Kolmogorov-Smirnov	1.32	2.93	2.63	2.18	2.98	4.58	4.64	4.43
Cramer-von Mises-Smirnov	24.65	23.86	23.21	23.11	20.28	13.98	12.3	12.58
Anderson-Darling	−75.14	−78.1	−76.76	−77.58	−82.77	−87.9	−90.65	−89.75
NRR	15.6	−41.58	39.93	546.64	25.57	194.07	10835.52	558.78

Distribution	lnorm	lnorm	logis	logis	logis	logis	logis	logis
Step #	9	10	1	2	3	4	5	6
MLE	−111.52	−108.17	−267.37	−225.35	−192.8	−167.66	−149.67	−132.73
Time (min)	303.66	336.12	6.84	16.15	28.78	43.81	57.01	66.69
Kolmogorov-Smirnov	4.58	4.65	1.7	3.44	3.74	4.64	3.75	4.52
Cramer-von Mises-Smirnov	14.36	14.3	23.64	22.25	22.82	21.5	21.91	21.67
Anderson-Darling	−90.57	−90.96	−75.33	−80.05	−80.02	−83.36	−80.13	−85.36
NRR	3906.4	7149.46	−254.78	14.68	31.65	161.3	280.79	8.68

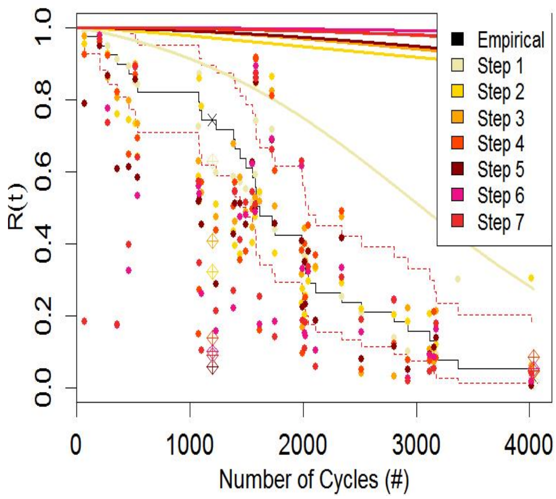
Distribution	logis	exp	exp	exp	exp	exp	exp	exp
Step #	7	1	2	3	4	5	6	7
MLE	−126.79	−275.06	−231.93	−199.75	−173.99	−156.4	−142.73	−133.66
Time (min)	73.31	2.57	4.11	6.13	8.95	12.18	15.34	19.04
Kolmogorov-Smirnov	4.49	2.4	2.33	2.86	3.82	4.64	4.66	4.66
Cramer-von Mises-Smirnov	21.41	21.09	21.95	22.12	20.52	19.92	17.82	14.98
Anderson-Darling	−85.53	−77.22	−77.99	−80.52	−84.84	−88.36	−89.77	−89.52
NRR	436.22	16.15	22.03	14.61	28.68	14734.25	22086.97	42814.72

Distribution	weibull	weibull	weibull	weibull	weibull	weibull	weibull	weibull
Step #	1	2	3	4	5	6	7	8
MLE	−268.47	−221.14	−189.18	−160.82	−136.77	−118.87	−110.38	−105.69
Time (min)	8.77	18.25	27.86	39.38	51.34	63.73	72.82	80.74
Kolmogorov-Smirnov	1.78	2.73	3.37	2.94	4.3	4.76	3.96	4.08
Cramer-von Mises-Smirnov	24.46	23.82	22.55	23.49	21.9	19.17	17.88	15.44
Anderson-Darling	−75.52	−78.77	−82.84	−80.51	−85.33	−91.22	−89.33	−91.18
NRR	−58.78	21.39	652.97	1713.94	10028.98	2314.28	48.81	2405.08

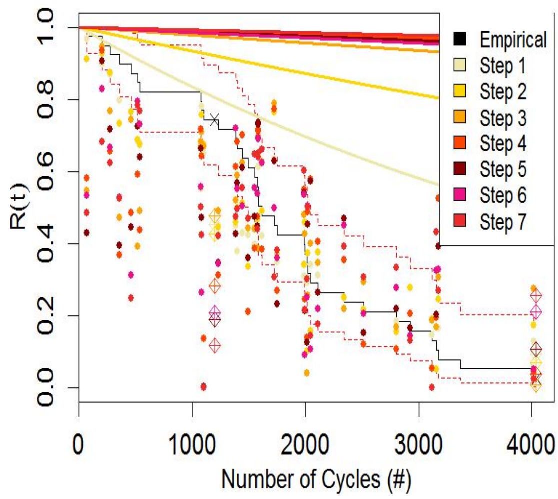
Distribution	gamma	gamma	gamma	gamma	gamma	gamma	gamma	gamma
Step #	1	2	3	4	5	6	7	8
MLE	−269.37	−223.74	−194.07	−170	−142.93	−130.19	−119.17	−112.41
Time (min)	37.16	80.43	133.04	195.12	249.43	313.42	365.91	391.75
Kolmogorov-Smirnov	1.94	3.16	4.1	4.53	3.55	4.43	4.53	4.33
Cramer-von Mises-Smirnov	24.45	23.57	23.05	22.18	21.94	20.55	17.43	17.83
Anderson-Darling	−75.72	−79.44	−81.14	−83.57	−82.56	−87.07	−89.91	−89.3
NRR	11.22	−20.45	23.33	14.11	45.08	513.21	779.64	2104.49

model's effectiveness can be assessed.

To assist in the selection of models, Tables B.9.147, B.9.148, B.9.149, and B.9.150 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.
The operational factors identified during time-independent and time-dependent PHM modelling are shown in Tables B.9.151, B.9.152, B.9.153, and B.9.154.

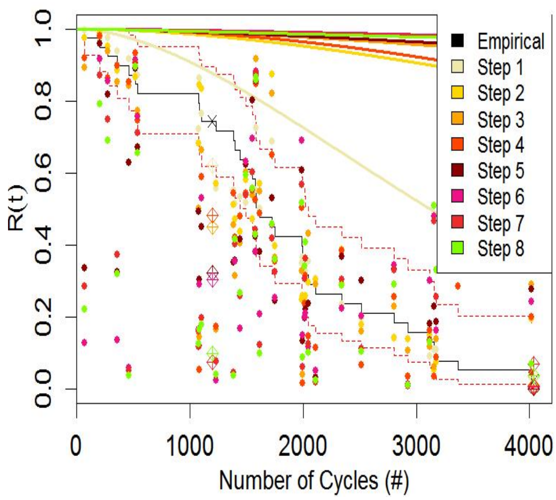


(a) With an underlying logis distribution.

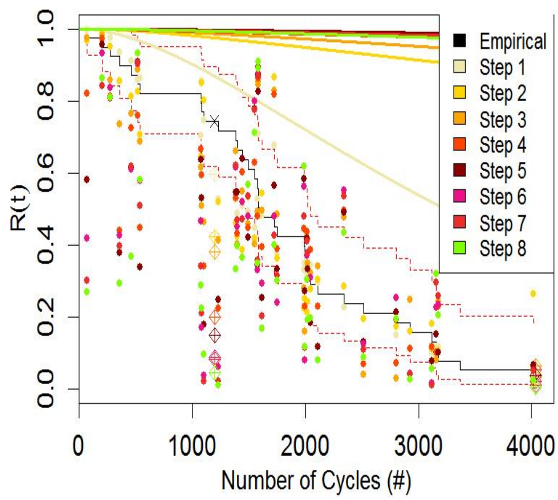


(b) With an underlying exp distribution.

Figure B.9.94: Time-dependent PHMs with an underlying logis and exp distribution.

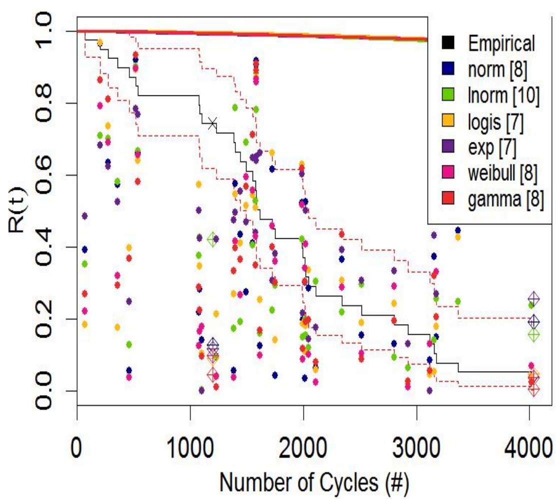


(a) With an underlying weibull distribution.



(b) With an underlying gamma distribution.

Figure B.9.95: Time-dependent PHMs with an underlying weibull and gamma distribution.



(a) All distributions (last step).

Figure B.9.96: Figure of all time-dependent PHMs.

Table B.9.145: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
	1	0 [50]	14 [50]	16 [50]	24 [50]	38 [50]	43 [50]	57 [50]	65 [0]	76 [0]	84 [0]	97 [0]
	2	0 [50]	5 [50]	16 [50]	24 [50]	38 [50]	43 [50]	54 [0]	65 [0]	68 [0]	70 [0]	92 [0]
	3	0 [100]	11 [50]	14 [50]	16 [50]	24 [50]	32 [50]	49 [0]	62 [0]	68 [0]	78 [0]	92 [0]
norm	4	0 [100]	3 [100]	14 [50]	14 [50]	27 [50]	38 [50]	54 [50]	65 [0]	70 [0]	81 [0]	92 [0]
	5	0 [100]	0 [100]	3 [50]	16 [50]	27 [50]	32 [50]	41 [50]	51 [50]	68 [0]	86 [0]	92 [0]
	6	3 [100]	3 [100]	5 [50]	14 [50]	22 [50]	27 [50]	32 [50]	41 [50]	54 [50]	81 [0]	92 [0]
	7	0 [100]	0 [100]	3 [100]	8 [50]	8 [50]	22 [50]	35 [50]	38 [50]	54 [50]	78 [0]	92 [0]
	8	0 [100]	0 [100]	0 [100]	3 [100]	8 [50]	14 [50]	30 [50]	38 [50]	54 [50]	76 [0]	89 [0]
	1	0 [50]	3 [50]	19 [50]	24 [50]	41 [50]	51 [50]	62 [50]	76 [0]	78 [0]	81 [0]	92 [0]
	2	0 [100]	0 [50]	19 [50]	24 [50]	43 [50]	51 [50]	62 [0]	68 [0]	70 [0]	78 [0]	89 [0]
	3	0 [50]	3 [50]	14 [50]	22 [50]	38 [50]	51 [50]	57 [0]	65 [0]	70 [0]	81 [0]	89 [0]
	4	0 [50]	3 [50]	14 [50]	24 [50]	32 [50]	41 [0]	54 [0]	62 [0]	70 [0]	86 [0]	89 [0]
lnorm	5	0 [100]	5 [50]	8 [50]	16 [50]	27 [50]	41 [50]	49 [50]	54 [50]	62 [0]	78 [0]	86 [0]
	6	0 [100]	3 [100]	3 [50]	16 [50]	22 [50]	30 [50]	41 [50]	49 [50]	62 [50]	76 [0]	86 [0]
	7	0 [100]	0 [100]	3 [50]	8 [50]	22 [50]	27 [50]	38 [50]	49 [50]	57 [50]	68 [0]	86 [0]
	8	0 [100]	3 [100]	3 [50]	5 [50]	14 [50]	24 [50]	38 [50]	46 [50]	49 [50]	68 [50]	86 [0]
	9	0 [100]	0 [100]	3 [50]	3 [50]	11 [50]	19 [50]	35 [50]	46 [50]	54 [50]	70 [50]	86 [0]
	10	0 [100]	0 [100]	3 [100]	3 [50]	5 [50]	16 [50]	35 [50]	41 [50]	54 [50]	70 [50]	86 [0]
	1	3 [50]	14 [50]	16 [50]	24 [50]	38 [50]	43 [50]	57 [50]	65 [0]	76 [0]	86 [0]	97 [0]
	2	0 [100]	0 [50]	11 [50]	30 [50]	43 [0]	49 [0]	62 [0]	68 [0]	70 [0]	89 [0]	95 [0]
	3	0 [100]	11 [50]	14 [50]	16 [50]	27 [50]	38 [50]	49 [0]	62 [0]	68 [0]	84 [0]	92 [0]
logis	4	0 [100]	8 [100]	14 [50]	16 [50]	27 [50]	35 [50]	49 [50]	62 [0]	68 [0]	81 [0]	92 [0]
	5	0 [100]	3 [50]	16 [50]	22 [50]	22 [50]	38 [50]	49 [50]	65 [0]	76 [0]	86 [0]	95 [0]
	6	0 [100]	0 [100]	5 [50]	16 [50]	22 [50]	27 [50]	35 [50]	59 [50]	65 [0]	84 [0]	92 [0]
	7	0 [100]	3 [100]	3 [50]	11 [50]	22 [50]	27 [50]	38 [50]	59 [50]	68 [0]	76 [0]	92 [0]
	1	0 [100]	0 [50]	16 [50]	27 [50]	46 [50]	65 [0]	68 [0]	81 [0]	84 [0]	92 [0]	97 [0]
	2	0 [100]	5 [50]	16 [50]	27 [50]	41 [50]	54 [0]	65 [0]	70 [0]	84 [0]	92 [0]	97 [0]
	3	3 [50]	5 [50]	16 [50]	24 [50]	41 [50]	54 [50]	65 [0]	70 [0]	84 [0]	92 [0]	97 [0]
exp	4	3 [100]	8 [50]	14 [50]	22 [50]	35 [50]	41 [50]	59 [50]	68 [0]	86 [0]	86 [0]	97 [0]
	5	3 [100]	3 [100]	8 [50]	19 [50]	19 [50]	38 [50]	51 [50]	65 [0]	86 [0]	89 [0]	95 [0]
	6	3 [100]	3 [100]	8 [100]	11 [50]	22 [50]	27 [50]	46 [50]	65 [0]	78 [0]	89 [0]	92 [0]
	7	3 [100]	3 [100]	8 [100]	11 [50]	19 [50]	24 [50]	43 [50]	62 [50]	68 [0]	81 [0]	92 [0]
	1	0 [100]	14 [50]	19 [50]	22 [50]	43 [50]	46 [50]	65 [50]	65 [0]	76 [0]	86 [0]	92 [0]
	2	0 [50]	5 [50]	16 [50]	27 [50]	38 [50]	49 [50]	54 [0]	65 [0]	68 [0]	70 [0]	92 [0]
	3	0 [100]	3 [50]	11 [50]	22 [50]	30 [50]	41 [50]	54 [0]	62 [0]	68 [0]	70 [0]	92 [0]
weibull	4	3 [50]	3 [50]	14 [50]	24 [50]	35 [50]	38 [50]	51 [50]	65 [0]	70 [0]	76 [0]	92 [0]
	5	3 [100]	5 [50]	5 [50]	16 [50]	16 [50]	30 [50]	41 [0]	54 [0]	65 [0]	78 [0]	92 [0]
	6	3 [100]	8 [100]	11 [100]	16 [50]	19 [50]	22 [50]	30 [50]	46 [50]	68 [0]	78 [0]	92 [0]
	7	3 [100]	3 [100]	3 [100]	3 [100]	14 [50]	22 [50]	32 [50]	46 [50]	62 [50]	84 [0]	92 [0]
	8	3 [100]	3 [100]	3 [100]	3 [100]	3 [50]	11 [50]	22 [50]	41 [50]	54 [50]	81 [0]	92 [0]
	1	0 [100]	8 [50]	19 [50]	24 [50]	43 [50]	46 [50]	65 [50]	68 [0]	78 [0]	86 [0]	92 [0]
	2	0 [100]	3 [50]	14 [50]	32 [50]	41 [50]	49 [0]	65 [0]	65 [0]	68 [0]	81 [0]	92 [0]
	3	0 [100]	11 [50]	14 [50]	19 [50]	30 [50]	43 [0]	57 [0]	59 [0]	70 [0]	84 [0]	92 [0]
gamma	4	3 [100]	8 [50]	14 [50]	24 [50]	24 [50]	38 [0]	54 [0]	65 [0]	78 [0]	84 [0]	95 [0]
	5	0 [100]	0 [50]	8 [50]	16 [50]	22 [50]	32 [0]	49 [0]	57 [0]	65 [0]	78 [0]	95 [0]
	6	0 [100]	0 [100]	0 [50]	8 [50]	24 [50]	27 [50]	38 [0]	49 [0]	62 [0]	78 [0]	92 [0]
	7	0 [100]	0 [100]	0 [50]	0 [50]	11 [50]	22 [50]	30 [50]	43 [0]	62 [0]	76 [0]	89 [0]
	8	0 [100]	0 [100]	0 [50]	0 [50]	11 [50]	22 [50]	27 [50]	43 [50]	59 [0]	73 [0]	89 [0]

Table B.9.146: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
norm	1	3 [50]	14 [50]	19 [50]	24 [50]	38 [50]	43 [50]	59 [50]	65 [0]	76 [0]	86 [0]	97 [0]
	2	0 [50]	5 [50]	16 [50]	24 [50]	38 [50]	43 [50]	54 [0]	65 [0]	68 [0]	76 [0]	92 [0]
	3	0 [100]	11 [50]	14 [50]	16 [50]	24 [50]	32 [0]	49 [0]	62 [0]	68 [0]	78 [0]	92 [0]
	4	0 [100]	3 [100]	14 [50]	14 [50]	27 [50]	38 [50]	54 [50]	65 [0]	70 [0]	81 [0]	92 [0]
	5	0 [100]	0 [100]	3 [50]	16 [50]	27 [50]	32 [50]	41 [50]	51 [50]	68 [0]	81 [0]	89 [0]
	6	3 [100]	3 [100]	5 [50]	14 [50]	22 [50]	24 [50]	30 [50]	41 [50]	54 [50]	76 [0]	89 [0]
	7	0 [100]	0 [100]	3 [100]	8 [100]	8 [50]	19 [50]	32 [50]	38 [50]	54 [50]	70 [0]	89 [0]
	8	0 [100]	0 [100]	0 [100]	3 [100]	5 [50]	11 [50]	27 [50]	38 [50]	54 [50]	70 [0]	89 [0]
lnorm	1	0 [50]	3 [50]	19 [50]	24 [50]	43 [50]	54 [50]	68 [50]	76 [0]	78 [0]	84 [0]	92 [0]
	2	0 [100]	3 [50]	19 [50]	24 [50]	43 [50]	54 [50]	65 [0]	68 [0]	76 [0]	81 [0]	92 [0]
	3	0 [50]	3 [50]	14 [50]	22 [50]	38 [50]	51 [50]	57 [0]	65 [0]	70 [0]	81 [0]	89 [0]
	4	0 [50]	3 [50]	14 [50]	24 [50]	32 [50]	41 [0]	54 [0]	62 [0]	70 [0]	86 [0]	89 [0]
	5	0 [100]	5 [50]	8 [50]	16 [50]	27 [50]	41 [50]	49 [50]	54 [50]	62 [0]	78 [0]	86 [0]
	6	0 [100]	3 [100]	3 [50]	16 [50]	22 [50]	30 [50]	41 [50]	49 [50]	62 [50]	76 [0]	86 [0]
	7	0 [100]	0 [100]	3 [50]	8 [50]	22 [50]	27 [50]	38 [50]	49 [50]	57 [50]	68 [0]	86 [0]
	8	0 [100]	3 [100]	3 [50]	5 [50]	14 [50]	24 [50]	38 [50]	46 [50]	49 [50]	65 [50]	86 [0]
	9	0 [100]	0 [100]	3 [50]	3 [50]	11 [50]	19 [50]	35 [50]	46 [50]	54 [50]	68 [50]	86 [0]
	10	0 [100]	0 [100]	3 [100]	3 [50]	5 [50]	16 [50]	35 [50]	41 [50]	54 [50]	70 [50]	86 [0]
logis	1	3 [50]	14 [50]	19 [50]	24 [50]	38 [50]	43 [50]	59 [50]	65 [0]	76 [0]	86 [0]	97 [0]
	2	0 [100]	0 [50]	16 [50]	30 [50]	43 [0]	51 [0]	62 [0]	68 [0]	73 [0]	89 [0]	95 [0]
	3	0 [100]	11 [50]	14 [50]	16 [50]	27 [50]	41 [50]	51 [0]	62 [0]	68 [0]	84 [0]	92 [0]
	4	0 [100]	8 [100]	14 [50]	16 [50]	27 [50]	35 [50]	49 [50]	62 [0]	68 [0]	81 [0]	92 [0]
	5	0 [100]	3 [50]	16 [50]	22 [50]	22 [50]	35 [50]	49 [50]	65 [50]	73 [0]	86 [0]	92 [0]
	6	0 [100]	0 [100]	5 [50]	16 [50]	22 [50]	27 [50]	35 [50]	54 [50]	65 [0]	78 [0]	89 [0]
	7	0 [100]	3 [100]	3 [50]	11 [50]	22 [50]	27 [50]	38 [50]	57 [50]	65 [0]	70 [0]	89 [0]
exp	1	0 [100]	0 [50]	19 [50]	30 [50]	46 [50]	65 [0]	68 [0]	81 [0]	84 [0]	95 [0]	97 [0]
	2	0 [100]	5 [50]	16 [50]	27 [50]	41 [50]	54 [0]	65 [0]	70 [0]	84 [0]	92 [0]	97 [0]
	3	3 [50]	5 [50]	16 [50]	24 [50]	41 [50]	51 [50]	65 [0]	70 [0]	84 [0]	89 [0]	97 [0]
	4	3 [100]	8 [50]	14 [50]	22 [50]	35 [50]	41 [50]	57 [50]	68 [0]	86 [0]	86 [0]	97 [0]
	5	3 [100]	3 [100]	8 [50]	19 [50]	19 [50]	38 [50]	51 [50]	62 [0]	86 [0]	89 [0]	95 [0]
	6	3 [100]	3 [100]	8 [100]	11 [50]	22 [50]	27 [50]	41 [50]	59 [0]	78 [0]	86 [0]	92 [0]
	7	3 [100]	3 [100]	8 [100]	11 [50]	19 [50]	24 [50]	43 [50]	62 [50]	68 [0]	84 [0]	92 [0]
weibull	1	0 [100]	14 [50]	19 [50]	24 [50]	43 [50]	46 [50]	65 [50]	65 [0]	76 [0]	86 [0]	92 [0]
	2	0 [50]	5 [50]	16 [50]	27 [50]	38 [50]	49 [50]	54 [0]	65 [0]	68 [0]	78 [0]	92 [0]
	3	0 [100]	3 [50]	11 [50]	22 [50]	32 [50]	41 [50]	57 [0]	62 [0]	68 [0]	70 [0]	92 [0]
	4	3 [50]	3 [50]	14 [50]	24 [50]	35 [50]	38 [50]	51 [50]	65 [0]	70 [0]	76 [0]	92 [0]
	5	3 [100]	5 [50]	5 [50]	16 [50]	16 [50]	30 [50]	41 [0]	54 [0]	65 [0]	78 [0]	92 [0]
	6	3 [100]	8 [100]	11 [100]	16 [50]	19 [50]	22 [50]	30 [50]	46 [50]	68 [0]	78 [0]	92 [0]
	7	3 [100]	3 [100]	3 [100]	3 [50]	14 [50]	22 [50]	32 [50]	46 [50]	62 [50]	84 [0]	92 [0]
	8	3 [100]	3 [100]	3 [100]	3 [100]	3 [50]	11 [50]	19 [50]	41 [50]	54 [50]	81 [0]	92 [0]
gamma	1	0 [100]	11 [50]	19 [50]	24 [50]	43 [50]	46 [50]	65 [50]	68 [0]	78 [0]	86 [0]	92 [0]
	2	0 [100]	5 [50]	14 [50]	35 [50]	41 [50]	49 [0]	65 [0]	65 [0]	68 [0]	84 [0]	92 [0]
	3	0 [100]	11 [50]	14 [50]	19 [50]	30 [50]	46 [0]	59 [0]	59 [0]	70 [0]	84 [0]	92 [0]
	4	3 [100]	8 [50]	14 [50]	22 [50]	24 [50]	35 [0]	51 [0]	62 [0]	78 [0]	78 [0]	92 [0]
	5	0 [100]	0 [50]	8 [50]	16 [50]	22 [50]	32 [0]	49 [0]	54 [0]	65 [0]	78 [0]	92 [0]
	6	0 [100]	0 [100]	0 [50]	8 [50]	24 [50]	27 [50]	38 [0]	49 [0]	62 [0]	78 [0]	89 [0]
	7	0 [100]	0 [100]	0 [50]	0 [50]	11 [50]	22 [50]	30 [50]	43 [0]	62 [0]	76 [0]	89 [0]
	8	0 [100]	0 [100]	0 [50]	0 [50]	11 [50]	22 [50]	27 [50]	43 [50]	59 [0]	73 [0]	89 [0]

Table B.9.147: Average MTTF values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
norm	1	1718.7	1475	1432.1	1291.9	1130.1	1059.5	921.6	812.4	511.4	407.5	66
	2	1718.7	1635.9	1432.1	1352.5	1190	1059.5	957.9	812.4	790.5	815.1	180.7
	3	1718.7	1548.5	1499.5	1445.4	1353.5	1291.3	1128.8	853.1	788.4	708.1	180.7
	4	1718.7	1654.9	1498.8	1498.8	1397.4	1297.9	1203.1	988.9	1019.8	735	180.7
	5	1718.7	1718.7	1696.5	1482.3	1344.4	1292.8	1301.6	1202	1053.8	396.6	233
	6	1718.6	1718.6	1684.1	1529.6	1423.9	1330.1	1230.6	1270.3	981.1	506.4	233
	7	1718.7	1718.7	1678.1	1572.7	1572.7	1490.9	1234.8	1281	1038	508.1	233
	8	1718.7	1718.7	1718.7	1654.9	1613.2	1487	1314.9	1242.6	1048.2	651.9	225.2
lnorm	1	1718.7	1654.9	1396	1291.9	1110.6	972.7	817.8	511.4	437.9	345.9	180.7
	2	1718.7	1718.7	1463.4	1341	1091.6	1015.9	863.9	790.5	762.4	437.9	225.2
	3	1718.7	1678.8	1513.9	1366.7	1175	998.9	939.7	798	705.9	345.9	225.2
	4	1718.7	1672.8	1496.2	1310.5	1204.4	1168.6	956.5	849	705.9	287.8	225.2
	5	1718.7	1662.9	1625.7	1527.7	1435.5	1206.5	1052.1	970.4	871.9	632.9	287.8
	6	1718.7	1710.3	1710.3	1527.9	1457	1350.1	1219.5	1052.1	887.6	557.4	287.8
	7	1718.7	1718.7	1710.3	1625.7	1457	1386.7	1252.9	1052.1	950.4	810.6	287.8
	8	1718.7	1710.3	1710.3	1675.5	1530.6	1455.1	1230	1087	1068.2	771.8	287.8
	9	1718.7	1718.7	1710.3	1710.3	1578.6	1470.9	1250.5	1105.5	1008.5	705.9	287.8
	10	1718.7	1718.7	1710.3	1710.3	1662.9	1486.1	1184.4	1171.6	1008.5	731.5	287.8
logis	1	1654.9	1475	1432.1	1291.9	1130.1	1059.5	921.6	812.4	511.4	272.6	66
	2	1718.7	1718.7	1551.9	1289.3	1151.2	1117.4	995.1	790.5	815.1	406	134
	3	1718.7	1548.5	1499.5	1445.4	1328.8	1224.3	1128.8	853.1	788.4	417.2	180.7
	4	1718.7	1567.1	1498.8	1445.4	1328.8	1306.2	1232.2	1043.2	979.7	735	180.7
	5	1718.7	1654.9	1445.4	1374.4	1374.4	1197.8	1137.2	865.9	718.1	396.6	170
	6	1718.7	1718.7	1612	1496.3	1416	1338	1283.9	962.9	815.9	417.2	233
	7	1718.7	1678.8	1678.8	1621.7	1443.3	1338	1159.9	946.9	767.4	549.7	233
exp	1	1718.7	1718.7	1432.1	1315.2	1133.7	812.4	790.5	423.6	407.5	180.7	66
	2	1718.7	1669.3	1498.1	1314.2	1187.1	964.9	812.4	672.6	407.5	180.7	66
	3	1718.6	1710.3	1543.8	1410.1	1265.1	993.2	941.7	854.5	410.5	243.3	66
	4	1718.6	1619.2	1543.2	1472.6	1400.7	1334.4	1039.1	964.3	272.6	272.6	66
	5	1718.6	1718.6	1627.6	1475.1	1475.1	1321.5	1157.2	979.2	272.6	225.2	170
	6	1718.6	1718.6	1641.4	1614.9	1415.1	1400	1255.7	1002.8	475.4	225.2	233
	7	1718.6	1718.6	1641.4	1594.8	1471.8	1393.8	1183.8	968.6	839.8	469	233
weibull	1	1718.7	1475	1396	1363.5	1059.5	1035.1	812.4	812.4	511.4	272.6	180.7
	2	1718.7	1635.9	1432.1	1328.1	1190	1013.6	957.9	812.4	790.5	815.1	180.7
	3	1718.7	1678.8	1551.9	1471.5	1302.6	1210.9	957.9	842	790.5	705.9	233
	4	1718.6	1718.6	1580.1	1355.3	1196.5	1146.9	976	752.5	666	511.4	233
	5	1718.6	1684.1	1684.1	1563.8	1531.4	1344.5	1103	951.4	831.5	440.1	233
	6	1718.6	1726.7	1690.3	1631.7	1615.7	1636.5	1551.5	1127.2	802.9	517.6	233
	7	1718.6	1718.6	1718.6	1718.6	1643.8	1568.7	1401	1162.6	911.4	326.5	233
	8	1718.6	1718.6	1718.6	1718.6	1718.6	1628.6	1518.1	1301.5	1013.3	345.9	233
gamma	1	1718.7	1567.8	1396	1336.8	1059.5	1035.1	812.4	736.7	437.9	272.6	180.7
	2	1718.7	1678.8	1512.8	1261.2	1164.5	1117.4	812.4	812.4	790.5	426.1	180.7
	3	1718.7	1548.5	1499.5	1427.3	1301.2	1215.9	1058.2	1020.9	762.4	417.2	180.7
	4	1718.6	1663.3	1543.2	1428.5	1390.4	1209.2	992.9	939.2	700.9	407.5	170
	5	1718.7	1718.7	1573.4	1451.5	1374.4	1242.1	1049.4	917.6	825.5	630.6	170
	6	1718.7	1718.7	1718.7	1585.4	1361.1	1353.1	1198.4	1052.6	865.3	640.1	233
	7	1718.7	1718.7	1718.7	1718.7	1544.8	1374.4	1328.5	1105.4	865.3	620.3	225.2
	8	1718.7	1718.7	1718.7	1718.7	1559.1	1374.4	1277.6	1105.4	944.1	775.1	225.2

Table B.9.148: Average MTTF values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
norm	1	**1654.9**	**1475**	**1396**	**1291.9**	**1130.1**	**1059.5**	**878**	**812.4**	**511.4**	**272.6**	**66**
	2	**1718.7**	**1635.9**	**1432.1**	**1352.5**	**1190**	**1059.5**	**957.9**	**812.4**	**790.5**	**814.2**	**180.7**
	3	**1718.7**	**1548.5**	**1499.5**	**1445.4**	**1353.5**	**1291.3**	**1128.8**	**853.1**	**788.4**	**708.1**	**180.7**
	4	**1718.7**	**1654.9**	**1498.8**	**1498.8**	**1397.4**	**1297.9**	**1203.1**	**988.9**	**1019.8**	**735**	**180.7**
	5	**1718.7**	**1718.7**	**1696.5**	**1482.3**	**1344.4**	**1292.8**	**1301.6**	**1202**	**1053.8**	**735**	**225.2**
	6	**1718.6**	**1718.6**	**1684.1**	**1529.6**	**1423.9**	**1402.9**	**1312.8**	**1270.3**	**981.1**	**745.2**	**225.2**
	7	**1718.7**	**1718.7**	**1678.1**	**1572.7**	**1572.7**	**1553.5**	**1320.2**	**1281**	**1038**	**705.9**	**225.2**
	8	**1718.7**	**1718.7**	**1718.7**	**1654.9**	**1611.4**	**1544.1**	**1391**	**1242.6**	**1048.2**	**820.8**	**225.2**
lnorm	1	**1718.7**	**1654.9**	**1396**	**1291.9**	**1067.8**	**941.9**	**736.7**	**511.4**	**437.9**	**313.8**	**180.7**
	2	**1718.7**	**1678.1**	**1463.4**	**1341**	**1091.6**	**972.8**	**824.1**	**790.5**	**580.4**	**423.6**	**180.7**
	3	**1718.7**	**1678.8**	**1513.9**	**1366.7**	**1175**	**998.9**	**939.7**	**798**	**705.9**	**345.9**	**225.2**
	4	**1718.7**	**1672.8**	**1496.2**	**1310.5**	**1204.4**	**1168.6**	**956.5**	**849**	**705.9**	**287.8**	**225.2**
	5	**1718.7**	**1662.9**	**1625.7**	**1527.7**	**1435.5**	**1206.5**	**1052.1**	**970.4**	**871.9**	**632.9**	**287.8**
	6	**1718.7**	**1710.3**	**1710.3**	**1527.9**	**1457**	**1350.1**	**1219.5**	**1052.1**	**887.6**	**557.4**	**287.8**
	7	**1718.7**	**1718.7**	**1710.3**	**1625.7**	**1457**	**1386.7**	**1252.9**	**1052.1**	**950.4**	**810.6**	**287.8**
	8	**1718.7**	**1710.3**	**1710.3**	**1675.5**	**1530.6**	**1455.1**	**1230**	**1087**	**1068.2**	**831.5**	**287.8**
	9	**1718.7**	**1718.7**	**1710.3**	**1710.3**	**1578.6**	**1470.9**	**1250.5**	**1105.5**	**1008.5**	**771.8**	**287.8**
	10	**1718.7**	**1718.7**	**1710.3**	**1710.3**	**1662.9**	**1486.1**	**1184.4**	**1171.6**	**1008.5**	**731.5**	**287.8**
logis	1	**1654.9**	**1475**	**1396**	**1291.9**	**1130.1**	**1059.5**	**878**	**812.4**	**511.4**	**272.6**	**66**
	2	**1718.7**	**1718.7**	**1496.8**	**1289.3**	**1151.2**	**1091.9**	**995.1**	**790.5**	**842.8**	**406**	**134**
	3	**1718.7**	**1548.5**	**1499.5**	**1445.4**	**1328.8**	**1173.7**	**1103.9**	**853.1**	**788.4**	**417.2**	**180.7**
	4	**1718.7**	**1567.1**	**1498.8**	**1445.4**	**1328.8**	**1306.2**	**1232.2**	**1043.2**	**979.7**	**735**	**180.7**
	5	**1718.7**	**1654.9**	**1445.4**	**1374.4**	**1374.4**	**1288.3**	**1137.2**	**865.9**	**666.5**	**396.6**	**180.7**
	6	**1718.7**	**1718.7**	**1612**	**1496.3**	**1416**	**1338**	**1283.9**	**1022.9**	**815.9**	**708.1**	**225.2**
	7	**1718.7**	**1678.8**	**1678.8**	**1621.7**	**1443.3**	**1338**	**1159.9**	**1019.7**	**827.5**	**737.2**	**225.2**
exp	1	**1718.7**	**1718.7**	**1396**	**1284.5**	**1133.7**	**812.4**	**790.5**	**423.6**	**407.5**	**134**	**66**
	2	**1718.7**	**1669.3**	**1498.1**	**1314.2**	**1187.1**	**964.9**	**812.4**	**672.6**	**407.5**	**180.7**	**66**
	3	**1718.6**	**1710.3**	**1543.8**	**1410.1**	**1265.1**	**1048.5**	**941.7**	**854.5**	**410.5**	**272.2**	**66**
	4	**1718.6**	**1619.2**	**1543.2**	**1472.6**	**1400.7**	**1334.4**	**1098.4**	**964.3**	**272.6**	**272.6**	**66**
	5	**1718.6**	**1718.6**	**1627.6**	**1475.1**	**1475.1**	**1321.5**	**1157.2**	**923.7**	**272.6**	**225.2**	**170**
	6	**1718.6**	**1718.6**	**1641.4**	**1614.9**	**1415.1**	**1400**	**1263.1**	**982.3**	**475.4**	**272.6**	**233**
	7	**1718.6**	**1718.6**	**1641.4**	**1594.8**	**1471.8**	**1393.8**	**1183.8**	**968.6**	**839.8**	**513.5**	**233**
weibull	1	**1718.7**	**1475**	**1396**	**1336.8**	**1059.5**	**1035.1**	**812.4**	**812.4**	**511.4**	**272.6**	**180.7**
	2	**1718.7**	**1635.9**	**1432.1**	**1328.1**	**1190**	**1013.6**	**957.9**	**812.4**	**790.5**	**588**	**180.7**
	3	**1718.7**	**1678.8**	**1551.9**	**1471.5**	**1261.2**	**1210.9**	**916.6**	**842**	**790.5**	**705.9**	**233**
	4	**1718.6**	**1718.6**	**1580.1**	**1355.3**	**1196.5**	**1146.9**	**976**	**752.5**	**666**	**511.4**	**233**
	5	**1718.6**	**1684.1**	**1684.1**	**1563.8**	**1531.4**	**1344.5**	**1103**	**951.4**	**831.5**	**440.1**	**233**
	6	**1718.6**	**1726.7**	**1690.3**	**1631.7**	**1615.7**	**1636.5**	**1551.5**	**1127.2**	**802.9**	**517.6**	**233**
	7	**1718.6**	**1718.6**	**1718.6**	**1718.6**	**1643.8**	**1568.7**	**1401**	**1162.6**	**911.4**	**326.5**	**233**
	8	**1718.6**	**1718.6**	**1718.6**	**1718.6**	**1718.6**	**1628.6**	**1537.9**	**1301.5**	**1013.3**	**345.9**	**233**
gamma	1	**1718.7**	**1526.6**	**1396**	**1336.8**	**1059.5**	**1035.1**	**812.4**	**736.7**	**437.9**	**272.6**	**180.7**
	2	**1718.7**	**1635.9**	**1512.8**	**1228.4**	**1164.5**	**1117.4**	**812.4**	**812.4**	**790.5**	**407.5**	**180.7**
	3	**1718.7**	**1548.5**	**1499.5**	**1427.3**	**1301.2**	**1159.8**	**1020.9**	**1020.9**	**762.4**	**417.2**	**180.7**
	4	**1718.6**	**1663.3**	**1543.2**	**1452.1**	**1390.4**	**1299.2**	**1125**	**987.7**	**700.9**	**700.9**	**180.7**
	5	**1718.7**	**1718.7**	**1573.4**	**1451.5**	**1374.4**	**1242.1**	**1049.4**	**954.7**	**825.5**	**630.6**	**180.7**
	6	**1718.7**	**1718.7**	**1718.7**	**1585.4**	**1361.1**	**1353.1**	**1198.4**	**1052.6**	**865.3**	**640.1**	**225.2**
	7	**1718.7**	**1718.7**	**1718.7**	**1718.7**	**1544.8**	**1374.4**	**1328.5**	**1105.4**	**865.3**	**620.3**	**225.2**
	8	**1718.7**	**1718.7**	**1718.7**	**1718.7**	**1559.1**	**1374.4**	**1277.6**	**1105.4**	**944.1**	**775.1**	**225.2**

Table B.9.149: Average MTTRep values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
	1	3780	2994.4	2495.4	2304.9	1986.9	1748.6	1442.4	1190	969	605.6	217.2
	2	3420	2891.2	2669.8	2243.5	1931.4	1755.6	1454.3	1230.4	981.9	644	358.8
	3	3830	2845	2475.3	2333.4	2022.7	1715.8	1480.1	1333.3	1042.3	696.5	333.1
norm	4	4458.5	3977.3	3016.6	2651.4	2042.6	1684.8	1383.8	1139.2	882.1	690.3	345
	5	5079	4114.5	3142	2801.9	2304.3	1996.6	1577.8	1224.9	1003.6	719.6	385.7
	6	3206	3205.3	2926.2	2898	2579.9	2232.8	1975.7	1631	1430.9	930.3	484.1
	7	4087	4087	3692.7	3320.4	2957	2122.7	1964.5	1652.1	1434.7	973.6	520.6
	8	4366	4365	4222	4027.7	2959.6	2613.3	1969.8	1712.2	1471.3	933.7	524.5
	1	4478.5	3463.3	2574.2	2238.4	1856.8	1554.1	1296.8	1088.3	875.5	616.4	321.7
	2	5044.5	3342.5	2507.2	2224.5	1828.8	1558.5	1283.1	1062.3	872.1	651.4	370.9
	3	3871.5	3062.7	2639.7	2378.6	1937.5	1645.5	1405.3	1207.9	974.4	736.3	363.3
	4	4051	3247.7	2842.9	2534.5	2184.2	1763.9	1541.5	1314.5	1058.8	774.6	365.3
lnorm	5	5116.5	3173.8	2829.2	2570.4	2082.3	1899.2	1733	1473.8	1193.5	840.4	410.7
	6	5263	3428	2929	2493.4	2157.5	2016.8	1830.2	1634.9	1320.4	956.4	446.5
	7	5388.5	4932	3261.7	2958.4	2461.3	2159.9	1950.1	1794.5	1426.9	970	469.6
	8	6173	3907.7	3469	2862.2	2797.3	2227.9	2009.9	1807	1515.8	1107.7	553.3
	9	6173	5229.5	3592.3	3009.7	2813.3	2506.6	2174.5	1893.7	1493.9	1135.5	548.8
	10	6173	5709	3810.7	3091	3191.5	2847.9	2415.1	1961.6	1537.4	1065.1	528.3
	1	3835.3	2987.9	2495.9	2305.9	1992.2	1753.8	1442	1185.7	958.2	603.1	206.4
	2	4784	3065.5	2459.5	2109.2	1650.1	1339.5	1098.4	927	669	518.8	241.3
	3	4119	2844.7	2457.7	2280.1	1949.1	1704.8	1441.7	1310.3	1053.9	774.8	355.5
logis	4	4166	3507.8	2943.1	2706.2	2244.2	1809.6	1428.8	1205.6	943.4	723	360.3
	5	4567	3599	2813	2422.7	2124.3	1815	1555.2	1340	1020.5	728	338.3
	6	5066	3669	3044.8	2543.1	2377.7	2104.4	1735.6	1548.5	1290	828.5	460.4
	7	4882	3212.3	2619.3	2192.2	2287.2	2124.7	1890.6	1570	1205.5	881.5	432.4
	1	6173	3554	2560.9	1967.1	1528.9	1269.5	948.7	757.4	511	296.4	112.7
	2	4821.5	2143.2	2089.9	1873	1535.9	1359.7	1055.5	836.7	595.9	371.2	141.9
	3	3288	2070	1923.8	1924.3	1602.2	1403.9	1037.7	845.6	718.5	458.6	187.9
exp	4	3921	2985	2471.3	1979.2	1541.2	1372.2	1307.8	1021.9	842.6	525	224.3
	5	4264	3221.3	2604	2325.7	1790.1	1550.1	1276	1101	881.7	542.6	232.9
	6	4627.3	4236.3	2430	2305.2	2138	1547.1	1322.5	1094.1	870.7	573.2	219
	7	4627.3	4627.3	3161	2690.7	2235.9	1787.6	1494.4	1194.9	910.9	567.8	228.7
	1	4167	3035.4	2443.4	2119.5	1898.7	1647.6	1362.6	1136.6	955.1	664.9	313.4
	2	3437.5	2922.8	2656.9	2168.9	1866.6	1665.8	1401.3	1190.9	978.1	674.1	393.3
	3	4057.5	3136.7	2709.8	2166.1	2032.6	1697.5	1526.4	1303.2	1034.7	743.5	391.3
weibull	4	2846.7	2617.3	2501	2483.7	2204	1954.8	1654	1429.1	1178.2	924.1	499.2
	5	3953.3	2887.8	2521	2283.4	2199.5	2088.8	1790.9	1445.5	1149.8	832.5	384.7
	6	4627.3	2922.4	2485.8	1963.9	1942.9	1588.5	1518.8	1595.5	1283.8	890.8	418.1
	7	4627.3	4627.3	4152	2810	2293.6	2021.9	1916.2	1640.3	1300.2	850.7	394
	8	4627.3	4627.3	4595.7	3480.7	3048.3	2587.3	2058.8	1712.2	1259.5	867.6	375.6
	1	4432.5	3090.8	2417.9	2067.9	1850.9	1572.5	1301.5	1107.1	915.7	638.1	313.5
	2	4115.5	3042	2504.7	2124	1723.5	1421.8	1282	1066.4	884.3	672.1	376
	3	4556.5	2842.3	2467.9	2219.9	1865.2	1545.1	1347.1	1150	1001.7	741.7	392.2
gamma	4	3165.3	2417.2	2388.7	2079.9	1809.5	1638.7	1369.6	1179	946.5	655.8	362.5
	5	4362	3381	2959.6	2496.4	2182.8	1918.3	1597	1375.4	1118.5	797	397.2
	6	5670	4114.5	3421.5	2974.6	2354	1899.8	1726.3	1479.5	1169.3	805.6	396.4
	7	6173	4232.5	3722.5	3123	2740.2	2349.7	1791.9	1625.8	1314.4	895.4	419.1
	8	6012	4321.5	3972	3552.5	2745.3	2459.5	2169.4	1714.7	1360.2	891	457.5

Table B.9.150: Average MTTRep values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	3%	12%	22%	31%	41%	50%	60%	69%	78%	88%	97%
norm	1	3820	2966.3	2482.7	2284	1969.1	1731.1	1428.3	1168.7	946.7	597.6	196.3
	2	3419	2879	2626.9	2229.6	1913	1733.3	1429.3	1215	968.9	632	334.6
	3	3811.5	2838.5	2464.6	2324.8	2002.3	1689.9	1454.6	1306.8	1017.1	671.6	309.6
	4	4458.5	3977.3	3017.3	2651.6	2043	1679.6	1385.6	1141.1	881.3	692.7	348.2
	5	5079	4114.5	3147.3	2804.5	2341	1998	1597.5	1236.6	1027.9	740.9	405.7
	6	3239.3	3238.7	2955.2	2917.6	2603.5	2253	2013.6	1662.2	1445.6	980.4	510.2
	7	4087	4087	3694.7	3440.8	2962.8	2129.2	1989.8	1664.6	1468.2	1048.4	550.5
	8	4366	4365	4224	4036.7	3337	2725.2	1987.6	1749.9	1493.1	994.4	533.2
lnorm	1	4459.5	3435	2548.9	2211.3	1839.8	1527.1	1271.3	1063.2	847	590	297.8
	2	5019	3273.3	2491.3	2205.1	1809.5	1549.2	1266.4	1044.5	892.1	628.8	349.8
	3	3885.5	3069	2647.6	2386	1951.1	1654.4	1415	1221.5	987.3	745	381.5
	4	4051	3274	2845.3	2539.5	2191.7	1772.2	1548	1324.2	1068.4	784.6	375.3
	5	5116.5	3177	2830.8	2572.4	2084.5	1900.9	1733.6	1478.5	1197.2	846.4	417.2
	6	5277	3428.3	2930	2494	2161	2020	1833.7	1635.8	1326.4	960.7	448.2
	7	5389	4932	3261.7	2961.6	2463.6	2161.2	1951.9	1797	1429.8	973.9	476.8
	8	6173	3907.7	3469	3036.5	2801.4	2229.2	2055.4	1811.2	1524.3	1092.7	559.2
	9	6173	5229.5	3599.7	3240	2814.2	2508.6	2182.1	1897.1	1503	1129.9	554.1
	10	6173	5718.5	3810.7	3321.7	3192	2852.9	2422.1	1970	1541.9	1079.2	533.4
logis	1	3797.3	2948.9	2483.7	2284.2	1972.9	1735	1429.2	1165	935.8	582.7	186.2
	2	4768.5	3050	2425.8	2082.2	1627.2	1330.4	1072.6	909.3	642.9	495.5	204.5
	3	4100	2838	2441.4	2273.1	1929.8	1716.7	1414.1	1282.3	1029.1	745.4	331.6
	4	4166	3493.8	2947.7	2707.1	2244.2	1812.9	1430.2	1207.8	944.6	725.4	363.3
	5	4567.5	3601.7	2837.9	2441.3	2127.9	1804.4	1587.8	1361	1078.8	743.6	354.5
	6	5066	3669	3044.8	2555.6	2388	2114.8	1752.4	1600.3	1349.5	895.8	485.6
	7	4896.5	3274	2623.7	2211	2301.2	2137.5	1940.9	1575.5	1262.2	911.7	457.2
exp	1	6173	3505.5	2537.7	1963.7	1507.2	1252.1	925.9	734.9	485.2	280.4	106.6
	2	4821	2141.8	2081.4	1868	1531.2	1357.8	1054	834.7	592.9	369.1	139.8
	3	3288	2071.8	1935.5	1929.7	1605.8	1385.3	1060.3	867.3	728.2	474.6	209.8
	4	3924	2992.4	2479.4	1987.8	1545.7	1375.6	1278.8	1032.1	852.1	534.2	243
	5	4297.3	3254.7	2630.6	2339	1803.4	1562.2	1280.8	1163.7	888.1	550	246.5
	6	4660.7	4276.3	2720.6	2323.7	2156.2	1555	1351.8	1148.5	938.5	586.2	233.5
	7	4660.7	4660.3	3186.2	2691.5	2241.3	1803.5	1500.8	1201.6	925.4	563.5	243.3
weibull	1	4150.5	3023	2424.1	2098.5	1878.9	1630.6	1343.7	1116.8	932.9	644.7	297.3
	2	3430	2905.5	2623.2	2144.2	1841	1643.1	1383.6	1166.4	962.6	716.9	371.7
	3	4057	3112.7	2695.8	2153.4	2011.3	1667	1510.3	1270.1	983.3	722.3	369.6
	4	2846.7	2612	2500.6	2472.3	2188.9	1946.3	1649.6	1423.5	1162.8	902.9	485
	5	3962.7	2892.8	2521	2293.9	2206.9	2081.5	1811.5	1454.2	1164.9	842.9	398.2
	6	4627.3	2925.4	2486	1963.9	1942.9	1591	1520.7	1587.3	1288	898	427.6
	7	4660.7	4660.7	4219.7	3100	2299.3	2024.4	1919.1	1644.4	1302.2	857.8	410.3
	8	4660.7	4660.7	4632.7	3405.3	3119.3	2614.7	2094.4	1717.4	1288.5	877.3	389.9
gamma	1	4404	3046.8	2390.3	2026.7	1831.8	1549.1	1284.7	1086.1	897.7	616.8	298
	2	4114.5	3066.2	2491.3	2097.7	1709.3	1408.8	1268.1	1042.5	849.4	645.5	354.6
	3	4528.5	2830.3	2426.7	2204.7	1816.3	1558.7	1337.3	1124.1	973	714.2	366.2
	4	3198.7	2460.8	2407	2118.9	1820.6	1610	1403.7	1180.8	983.7	687.3	382.7
	5	4362	3387.5	2987.6	2499.6	2185	1922.9	1607	1378.5	1122.3	820.5	428.3
	6	5670	4114.5	3439	2995.4	2356.7	1904.8	1732.7	1490.8	1202.4	822.5	420.2
	7	6173	4232.5	3722.5	3137	2746	2356.5	1793.3	1629.8	1320.8	902.1	425.7
	8	6012.5	4335	3972.5	3555.5	2760.5	2467.7	2175.6	1746.3	1375.2	896.1	464

Table B.9.151: Variables identified by time-(in)dependent PHM models.

Time-independent PHM

	Variable	Scaled Value
①	*Group A*	3.65

Time-dependent PHM

	Variable	Scaled Value
①	*Aoa mean deg 1*	−32.44
②	*Group C*	−30.89
③	*Elevator Lin max deg TEU*	17.31
④	*Vz mean ft min 2*	−17.25
⑤	*Group E*	16.18
⑥	*Accn long mean g s*	15.46
⑦	*Accn long mean g s 8*	15.46
⑧	*Pitch rate mean deg sec 8*	15.13
⑨	*Group D*	13
⑩	*Yaw rate min deg sec 1*	−12.99
⑪	*Yaw rate max deg sec 2*	12.82
⑫	*Prop spd rhs min*	−12.48
⑬	*Accn long mean g s 4*	−11.98
⑭	*Prop spd lhs min*	−10.28
⑮	*Rudder low max deg TER*	9.65
⑯	*Group B*	−6.66
⑰	*Roll rate mean deg sec 1*	−4

Table B.9.152: Variables identified by each step by time-(in)dependent PHMs (in order).

	PHM Variables	
	Time-independent	*Time-dependent*
norm		⑯ ② ⑮ ③ ⑫ ④ ⑧ ⑥
lnorm	①	⑩ ② ⑨ ⑤ ⑫ ⑬ ⑪ ③ ⑧ ⑮
logis		⑯ ② ⑮ ③ ⑭ ⑧ ⑬
exp		⑯ ⑦ ⑫ ⑮ ⑬ ④ ⑰
weibull		⑯ ② ① ④ ⑭ ③ ⑮ ⑪
gamma		⑯ ② ⑮ ⑭ ⑨ ① ⑪ ③

Table B.9.153: Number of times variables identified by each step by time-(in)dependent PHMs.

Key				Key			
indep	*dep*	*Variable*	*Count*	*indep*	*dep*	*Variable*	*Count*
①		Group A	1		③	Elevator Lin max deg TEU	5
	⑯	Group B	5		⑧	Pitch rate mean deg sec 8	3
	②	Group C	5		⑭	Prop spd lhs min	3
	⑨	Group D	2		⑫	Prop spd rhs min	3
	⑤	Group E	1		⑰	Roll rate mean deg sec 1	1
	⑥	Accn long mean g s	1		⑮	Rudder low max deg TER	6
	⑬	Accn long mean g s 4	3		④	Vz mean ft min 2	3
	⑦	Accn long mean g s 8	1		⑪	Yaw rate max deg sec 2	3
	①	Aoa mean deg 1	2		⑩	Yaw rate min deg sec 1	1

Table B.9.154: Variables belonging to each group identified in B.9.151.

Group	*Variables*	*Group*	*Variables*
Group A	Vtrue min knots 4, Vtrue max knots 6	Group D	Vtrue mean knots 2, Accn long mean g s 2, Pitch mean deg 2
Group B	Roll mean deg 1, Yaw rate mean deg sec 1	Group E	Torque lhs mean , Torque rhs mean
Group C	Brake press lhs mean psi 8, Brake press rhs mean psi 8		

B.10 EVR716-11-0350A VHF transceiver

Table B.10.155 provides a summary of the input data related to the component. The number of registered maintenance events is less than the total number of events due to the fact that TRAX data stretches back to 2004/2005 and FDR data only to 2011. Maintenance events with insufficient data, regarding operational factors, cannot be evaluated, hence are not registered during the modelling process.

Table B.10.155: General overview of component inputs.

Name	**Value**
Part Number	EVR716-11-0350A
Total # (A, F, C)	368, 283, 85
Registered # (A, F, C)	60, 45, 15
Related Flights # (A, F, C)	96496, 65913, 30583
Avg. Cycles (A, F, C)	1608.27, 1464.73, 2038.87
% Censored	25

In Tab. B.10.155 (A, F, C) denotes statistics regarding All (A), Failed (F), and Censored (C) events respectively. Ergo A will always be the sum or mean derived from F and C.

Analysis
Tables B.10.156 and B.10.157 summarise the results from EVA and MDA. In addition the variables obtained by semi-parametric PHM modelling (labelled 'reduced semi-COX') are also presented if applicable. Table B.10.157 provides an overview of the specific operational factors identified during all flight phases. In this case high counts indicate operational factors that were significantly different during multiple flight phases.

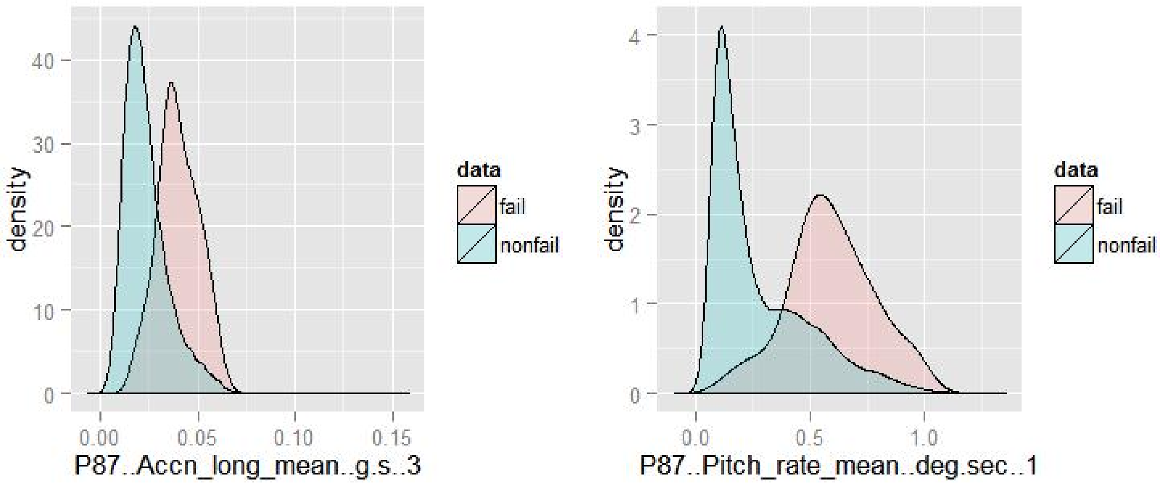
Table B.10.156: Overview of analysis input and output.

	# Variables
ALL	1531
EVA	116
MDA	42
Combined	158
reduced Corr.	90
reduced semi-COX	2
Take-Off related	27
Cruise related	27
Touch-Down related	36

Table B.10.157: Overview of identified operational factors during all flight phases.

Variable	**Count**	**Variable**	**Count**	**Variable**	**Count**
Roll_rate	9	Accn_lat	3	Torque_lhs	2
Roll	8	Aoa	3	Pressure_dynamic	2
Pitch_rate	6	Elevator_Rin	3	Drift	2
Elevator_Lin	6	Rudder_cmd_force	3	Press_ambient	1
Accn_long	5	Vcal	3	NormalForce_nose	1
Vtrue	5	Pitch	2	Aileron_Rin	1
Yaw_rate	4	NormalForce_rhs	2	Pitch_cmd_force	1
Vz	4	Rudder_low	2	Density_total	1
Torque_rhs	3	Brake_press_rhs	2		
NormalForce_lhs	3	Brake_press_lhs	2		

A multitude of factors were identified during EVA and MDA. Figure B.10.97 give a general overview of the top operational factors identified by EVA and MDA.



(a) EVA 1. (b) EVA 2.

(c) MDA 1. (d) MDA 2.

Figure B.10.97: Graphical overview of top operational factors identified by EVA and MDA.

Time-based reliability modelling

Table B.10.158 reports the maximum likelihood and goodness-of-fit tests results obtained from time-based reliability modelling. To show the overall fit Fig. B.10.98 shows the computed reliability function using an averaged virtual age V for all fitted models.

Table B.10.158: Overview of results obtained by MLE optimisation and GOF tests.

	Distributions				
	norm	**lnorm**	**logis**	**exp**	**weibull**
MLE	-385.95	-394.07	-386.65	-390.18	-389.86
Kolmogorov-Smirnov	2.19	1.3	1.45	0.54	0.68
Cramer-von-Mises Smirnov	29.24	30.04	29.51	30.17	29.64
Anderson-Darling	-92.5	-91.38	-92.37	-90.51	-90.55
NRR	22.67	18.28	25.32	27.52	23.03

In addition Figures B.10.99, B.10.100, B.10.101, B.10.102, and B.10.103 present the reliability and hazard functions computed for each underlying distribution evaluated in the program.

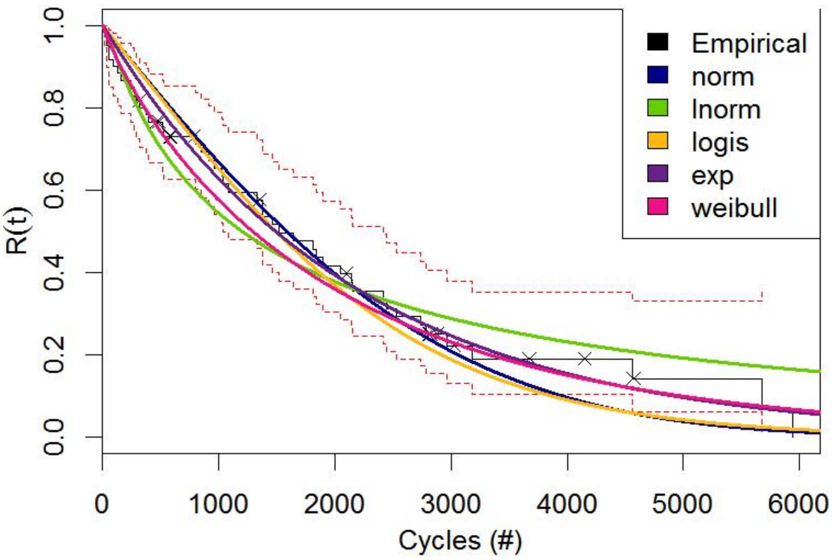
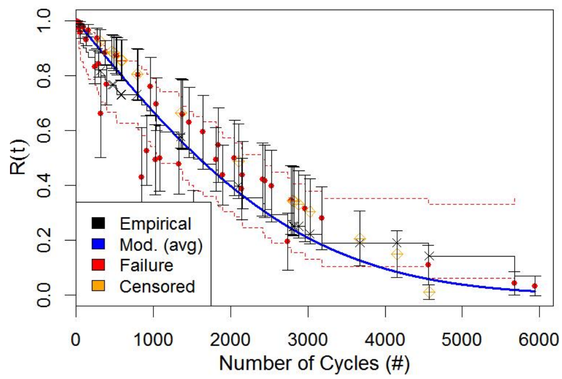
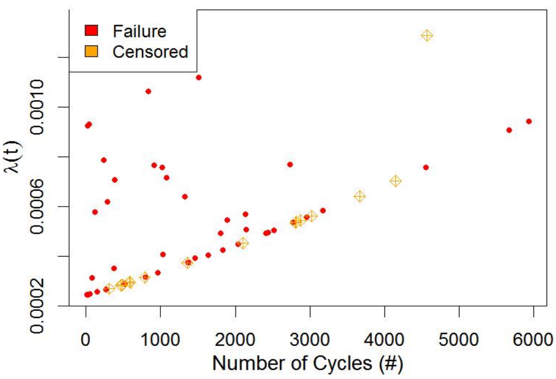


Figure B.10.98: Overview of overall fit of multiple GRP modells.

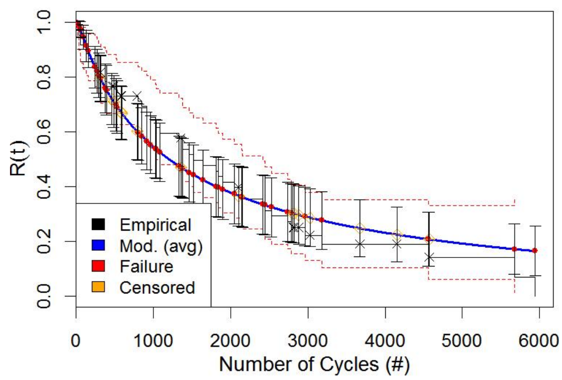


(a) Reliability function.

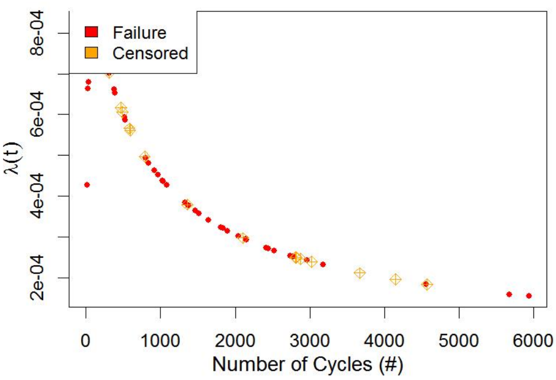


(b) Hazard function.

Figure B.10.99: Computed reliability for time-based models with underlying norm distribution.



(a) Reliability function.

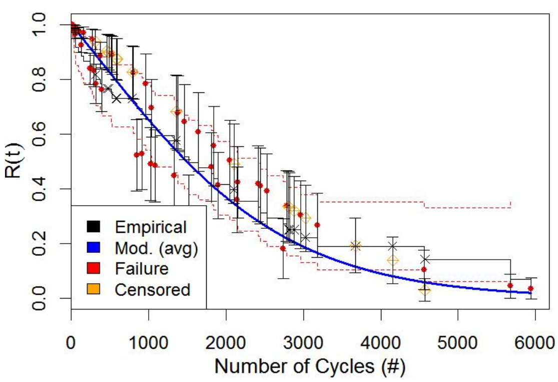


(b) Hazard function.

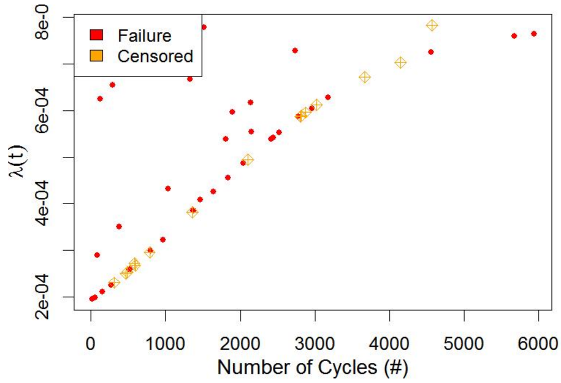
Figure B.10.100: Computed reliability for time-based models with underlying lnorm distribution.

Time independent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected

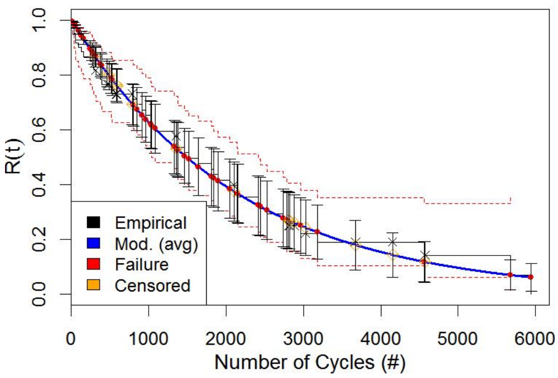


(a) Reliability function.

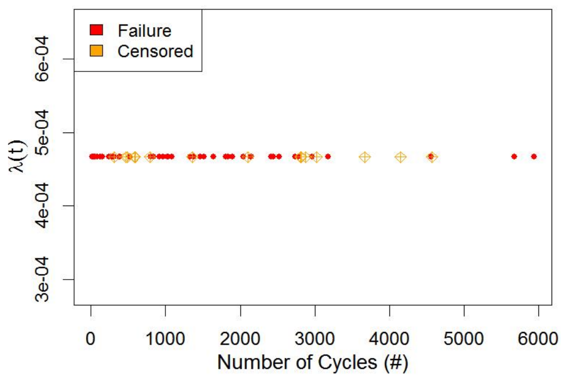


(b) Hazard function.

Figure B.10.101: Computed reliability for time-based models with underlying logis distribution.

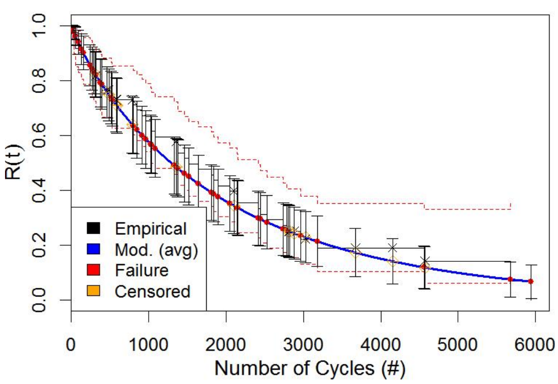


(a) Reliability function.

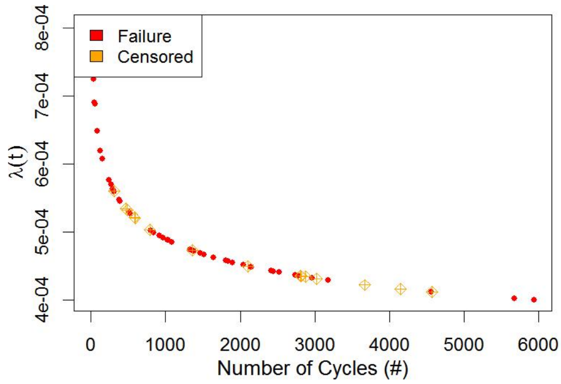


(b) Hazard function.

Figure B.10.102: Computed reliability for time-based models with underlying exp distribution.



(a) Reliability function.



(b) Hazard function.

Figure B.10.103: Computed reliability for time-based models with underlying weibull distribution.

for modelling. Table B.10.159 gives a general overview of all the models obtained during each step in the process.

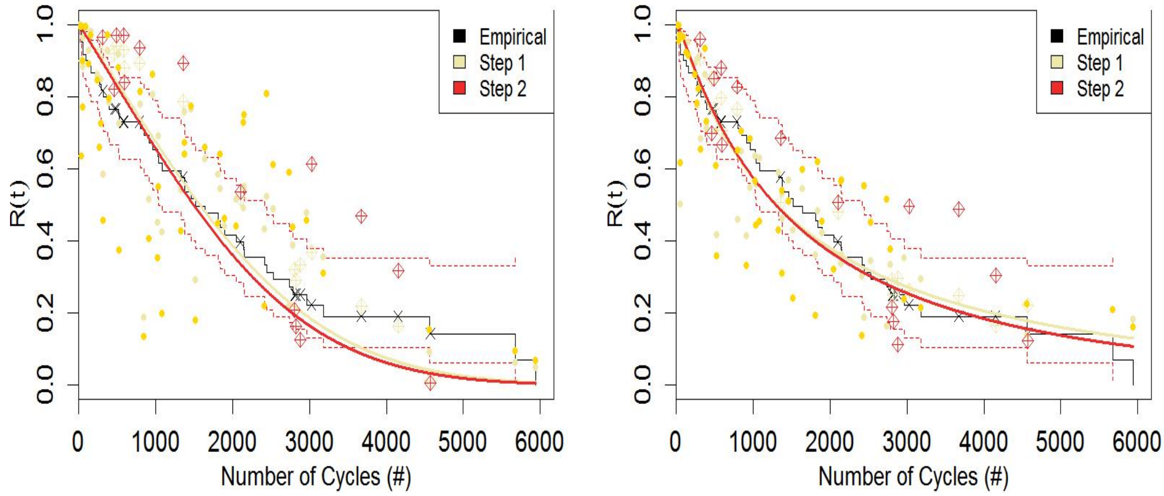
Graphical representation of the computed reliability per model are shown in Figures B.10.104, B.10.105, and B.10.106 as well as a general overview in Figure B.10.106b.

Tables B.10.160 and B.10.161 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables were computed using data available 100 cycles prior to expected failure events such that the

Table B.10.159: Overview of time-independent PHM results obtained by MLE optimisation and GOF tests.

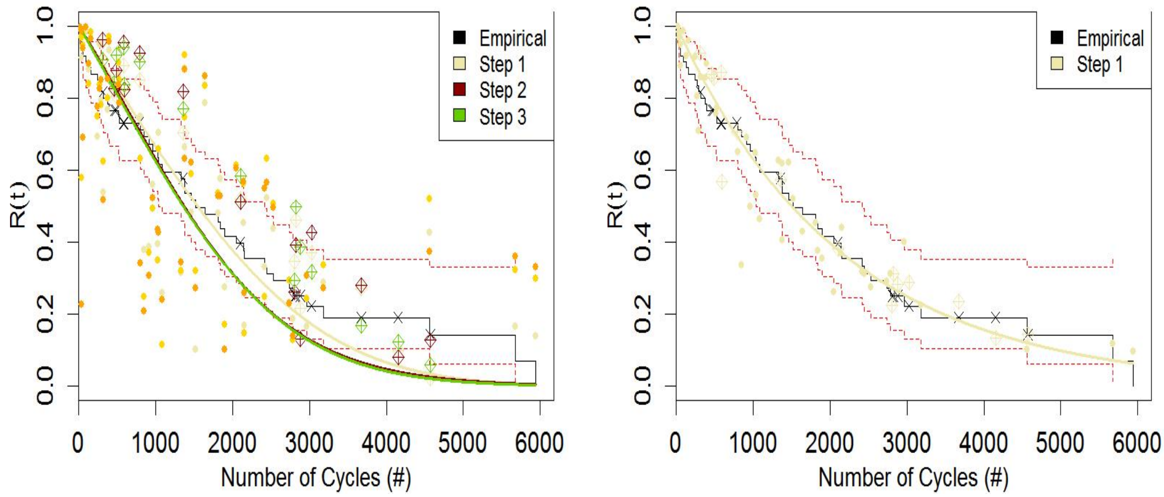
Distribution	norm	norm	lnorm	lnorm	logis	logis	logis	exp
Step #	1	2	1	2	1	2	3	1
MLE	−381.51	−377.01	−390.55	−385.56	−381.75	−377.42	−373.79	−388.58
Time (min)	1.56	3.38	1.54	2.88	1.42	2.92	3.89	1.31
Kolmogorov-Smirnov	3.93	4.33	3.37	2.91	3.07	3.73	5.74	2.76
Cramer-von Mises-Smirnov	25.76	17.96	28.84	27.81	25.89	17.12	19.17	29.8
Anderson-Darling	−94.77	−100.23	−93.5	−95.18	−97.22	−101.94	−103.07	−91.88
NRR	23.34	23.36	20.74	15.98	25.41	19.1	26.4	23.12

Distribution	weibull	weibull
Step #	1	2
MLE	−385.69	−383.08
Time (min)	1.63	3.06
Kolmogorov-Smirnov	2.82	3.43
Cramer-von Mises-Smirnov	28.55	26.39
Anderson-Darling	−93.4	−95.24
NRR	21.05	19.55



(a) With an underlying norm distribution. (b) With an underlying lnorm distribution.

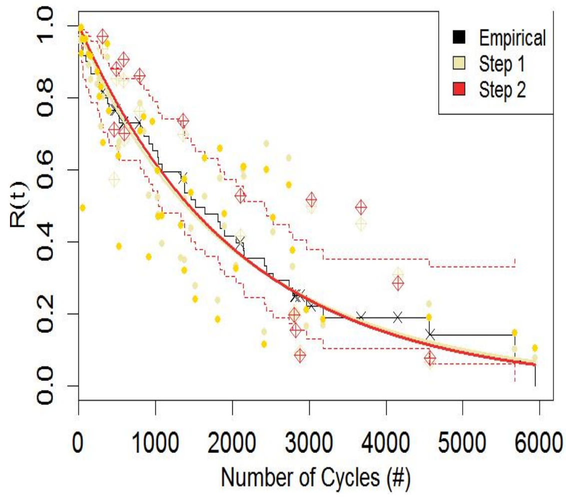
Figure B.10.104: Time-independent PHMs with an underlying norm and lnorm distribution.



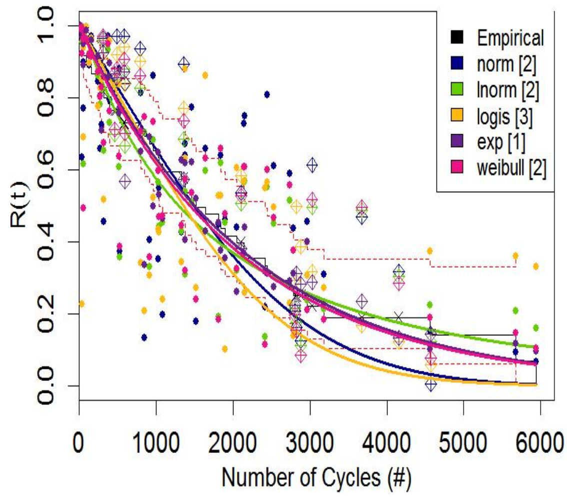
(a) With an underlying logis distribution. (b) With an underlying exp distribution.

Figure B.10.105: Time-independent PHMs with an underlying logis and exp distribution.

model's effectiveness can be assessed.



(a) With an underlying weibull distribution.



(b) All distributions (last step).

Figure B.10.106: Figures containing a weibull distribution and all time-independent PHMs.

To assist in the selection of models, Tables B.10.162, B.10.163, B.10.164, and B.10.165 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.

Table B.10.160: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
norm	1	0 [100]	7 [87]	9 [80]	11 [67]	16 [53]	24 [53]	40 [47]	56 [47]	67 [27]	73 [13]	93 [0]
	2	0 [100]	2 [87]	9 [80]	16 [73]	18 [67]	27 [53]	40 [47]	51 [47]	64 [47]	78 [20]	93 [0]
lnorm	1	0 [100]	0 [100]	7 [93]	16 [53]	29 [53]	44 [47]	58 [40]	69 [40]	76 [13]	80 [7]	96 [0]
	2	0 [100]	0 [100]	7 [80]	18 [73]	24 [67]	38 [47]	56 [47]	69 [40]	76 [27]	80 [7]	93 [0]
logis	1	0 [100]	0 [93]	9 [80]	16 [67]	20 [53]	40 [53]	49 [47]	56 [33]	67 [27]	76 [0]	91 [0]
	2	0 [100]	0 [87]	2 [80]	13 [60]	29 [53]	29 [53]	40 [47]	56 [40]	64 [33]	71 [13]	89 [0]
	3	0 [100]	0 [93]	2 [73]	11 [60]	29 [60]	33 [53]	47 [47]	60 [40]	64 [20]	80 [13]	91 [0]
exp	1	0 [100]	2 [100]	9 [87]	16 [53]	31 [47]	44 [47]	51 [33]	62 [33]	71 [27]	78 [7]	96 [0]
weibull	1	0 [100]	2 [93]	11 [87]	18 [73]	27 [60]	38 [47]	53 [47]	60 [33]	71 [20]	80 [7]	96 [0]
	2	0 [100]	0 [93]	11 [73]	18 [67]	24 [67]	38 [53]	51 [47]	64 [47]	73 [27]	78 [13]	98 [0]

Table B.10.161: Percentage of failure [censored] events below [above] numerous reliability levels for various time-independent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
norm	1	0 [100]	7 [87]	9 [73]	11 [67]	18 [53]	29 [53]	42 [47]	56 [47]	69 [27]	76 [13]	93 [0]
	2	0 [100]	4 [87]	13 [73]	16 [73]	18 [53]	31 [53]	42 [47]	56 [47]	71 [40]	84 [7]	96 [0]
lnorm	1	0 [100]	0 [100]	11 [73]	16 [53]	31 [53]	49 [47]	64 [40]	73 [40]	80 [13]	84 [0]	96 [0]
	2	0 [100]	2 [87]	9 [73]	18 [67]	31 [53]	47 [47]	67 [47]	71 [27]	80 [20]	87 [7]	98 [0]
logis	1	0 [100]	0 [87]	9 [80]	16 [60]	24 [53]	40 [53]	49 [40]	56 [33]	67 [27]	76 [0]	93 [0]
	2	0 [100]	0 [87]	4 [67]	22 [53]	29 [53]	29 [47]	44 [40]	58 [40]	67 [27]	78 [7]	93 [0]
	3	0 [100]	0 [80]	7 [67]	18 [60]	31 [53]	40 [53]	56 [40]	62 [40]	71 [20]	84 [7]	93 [0]
exp	1	0 [100]	4 [93]	9 [73]	18 [53]	33 [47]	49 [47]	51 [33]	64 [33]	71 [27]	84 [0]	96 [0]
weibull	1	0 [100]	2 [93]	11 [80]	18 [67]	27 [60]	38 [47]	53 [47]	62 [33]	71 [20]	84 [7]	98 [0]
	2	0 [100]	2 [80]	11 [73]	18 [67]	31 [53]	40 [47]	56 [47]	69 [33]	80 [27]	89 [7]	98 [0]

Time dependent proportional hazard reliability modelling

Discussed in Sec. 2 is the step-wise application of forward selection variable identification techniques to ensure that all variables, which satisfy a certain significant level, are selected for modelling. Table B.10.166 gives a general overview of all the models obtained during each step in the process.

Graphical representation of the computed reliability per model are shown in Figures B.10.107, B.10.108, and B.10.109 as well as a general overview in Figure B.10.109b.

Tables B.10.167 and B.10.168 give an overview of the results obtained from computing the expected time till failure for each component and model at a defined reliability critical level. In general this provides great insights on the overall predictability of the models. These tables

Table B.10.162: Average MTTF values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
norm	1	**1464.7**	**1184.2**	**1192.6**	**1154**	**1057.9**	**918.1**	**689.1**	**438.4**	**238.8**	**174**	**43.7**
	2	**1464.7**	**1362.9**	**1192.6**	**1111.1**	**1116.5**	**975.5**	**898.4**	**680.7**	**408.2**	**177.3**	**43.7**
lnorm	1	**1464.7**	**1464.7**	**1254.1**	**1025.5**	**761.2**	**602.4**	**379.4**	**210.7**	**131.8**	**92.3**	**22.5**
	2	**1464.7**	**1464.7**	**1294.7**	**1023.3**	**931.3**	**836.9**	**450.8**	**210.7**	**131.8**	**102.1**	**25**
logis	1	**1464.7**	**1464.7**	**1184.2**	**1065.4**	**1057.6**	**733.4**	**582.7**	**451.1**	**285.4**	**156.9**	**45.5**
	2	**1464.7**	**1464.7**	**1434.8**	**1285.9**	**1000.2**	**1000.2**	**719.6**	**443.5**	**367.1**	**266.5**	**135**
	3	**1464.7**	**1464.7**	**1434.8**	**1361.9**	**933.2**	**934.2**	**718.9**	**399.3**	**382.1**	**140.2**	**130.8**
exp	1	**1464.7**	**1362.9**	**1135.6**	**1032.8**	**803**	**554.8**	**470.8**	**257.1**	**163.8**	**117.4**	**22.5**
weibull	1	**1464.7**	**1362.9**	**1143.6**	**1023.3**	**919.2**	**797.4**	**535.2**	**308.4**	**163.8**	**102.1**	**21.5**
	2	**1464.7**	**1464.7**	**1103.7**	**1023.3**	**931.3**	**836.9**	**618.5**	**253.8**	**143**	**116**	**13**

Table B.10.163: Average MTTF values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
norm	1	**1464.7**	**1184.2**	**1192.6**	**1154**	**1057.2**	**880.1**	**703.7**	**438.4**	**187.1**	**185.4**	**43.7**
	2	**1464.7**	**1262.7**	**1164.1**	**1111.1**	**1116.5**	**1011.4**	**859.9**	**677.8**	**404.1**	**157.7**	**49.5**
lnorm	1	**1464.7**	**1464.7**	**1095.5**	**1025.5**	**727.4**	**553.3**	**235.8**	**143**	**150**	**104.4**	**22.5**
	2	**1464.7**	**1443.2**	**1215.1**	**1023.3**	**940.1**	**787.5**	**249.9**	**196.8**	**150**	**115.2**	**13**
logis	1	**1464.7**	**1464.7**	**1184.2**	**1065.4**	**1024.1**	**733.4**	**582.7**	**451.1**	**285.4**	**156.9**	**32**
	2	**1464.7**	**1464.7**	**1443**	**1032.8**	**1000.2**	**1000.2**	**680**	**450.4**	**375.5**	**277.1**	**43.7**
	3	**1464.7**	**1464.7**	**1407.4**	**1338.4**	**937.5**	**843.5**	**601**	**419.9**	**408.9**	**157.7**	**43.7**
exp	1	**1464.7**	**1288.6**	**1135.6**	**995.5**	**793.7**	**495.4**	**470.8**	**255**	**163.8**	**97.4**	**22.5**
weibull	1	**1464.7**	**1362.9**	**1143.6**	**1023.3**	**919.2**	**797.4**	**535.2**	**269.9**	**163.8**	**91.3**	**13**
	2	**1464.7**	**1443.2**	**1103.7**	**1023.3**	**940.1**	**827.8**	**606.7**	**210.7**	**150**	**107.8**	**13**

Table B.10.164: Average MTTRep values for time-independent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
norm	1	**5833.3**	**3860.7**	**3056.2**	**2570.7**	**2283.9**	**1922.6**	**1562.8**	**1195.3**	**839.4**	**483**	**95.8**
	2	**5833.3**	**4127.1**	**3407.4**	**2768.1**	**2309.2**	**1956.2**	**1424.4**	**1196.3**	**919.3**	**543.9**	**107.7**
lnorm	1	**5833.3**	**5523.5**	**3985.8**	**2666.6**	**2019.8**	**1386.3**	**961.6**	**645.4**	**402.9**	**239.1**	**57.3**
	2	**5833.3**	**5690.7**	**4292.4**	**3268.3**	**2340.7**	**1535.2**	**1202.4**	**784.1**	**473.2**	**281.8**	**63.4**
logis	1	**5833.3**	**3643.8**	**3214.5**	**2679.4**	**2190.2**	**1801.6**	**1451.1**	**1141.6**	**799.7**	**480.8**	**88.5**
	2	**5833.3**	**3630.5**	**2960.6**	**2437**	**2138.8**	**1800.2**	**1625.8**	**1269.5**	**894.2**	**539.1**	**135.4**
	3	**5833.3**	**3854.9**	**2931**	**2369.1**	**2297.5**	**1862.5**	**1557.5**	**1243.6**	**883.4**	**514.5**	**107.7**
exp	1	**5833.3**	**4525**	**3486.8**	**2576.5**	**2011.3**	**1594.1**	**1155.4**	**817.6**	**532.8**	**287.7**	**58.6**
weibull	1	**5833.3**	**5222.2**	**3917.5**	**2938.8**	**2239.5**	**1556.9**	**1185.1**	**867.5**	**538.3**	**273**	**34.3**
	2	**5833.3**	**5110.6**	**4091.9**	**3027.9**	**2400.9**	**1659.3**	**1315.5**	**956.5**	**595.9**	**328.4**	**56.6**

Table B.10.165: Average MTTRep values for time-independent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
norm	1	**5833.3**	**3756.6**	**2982.5**	**2502.2**	**2168.8**	**1833.1**	**1475.1**	**1150.5**	**780.8**	**436.4**	**69.9**
	2	**5833.3**	**4024.5**	**3024.5**	**2576.7**	**2125.5**	**1612.2**	**1310.6**	**1068.5**	**748.6**	**432.2**	**50.1**
lnorm	1	**5833.3**	**4903.9**	**3481.8**	**2329.2**	**1781.9**	**1135.4**	**790.4**	**523.8**	**286.7**	**141.2**	**34.8**
	2	**5833.3**	**4592.9**	**3902.1**	**2682**	**1824.6**	**1289.7**	**957.6**	**597.3**	**324.9**	**167.2**	**30.9**
logis	1	**5833.3**	**3570.7**	**3153**	**2602.9**	**2072.7**	**1745.1**	**1409.1**	**1100**	**760.3**	**444.6**	**61.6**
	2	**5833.3**	**3378.5**	**2604.2**	**2442.8**	**1988.4**	**1680.6**	**1490.1**	**1130.5**	**748.4**	**434.5**	**73.2**
	3	**5833.3**	**3321.9**	**2576.3**	**2060.5**	**2041.9**	**1680.6**	**1368.5**	**1023.6**	**637.4**	**368.9**	**50.5**
exp	1	**5833.3**	**4187.8**	**3094.6**	**2370.5**	**1800.5**	**1426.7**	**1031.1**	**710.3**	**457.7**	**232**	**39.9**
weibull	1	**5833.3**	**5054.8**	**3734.6**	**2788.6**	**2107.3**	**1450**	**1111.7**	**803.3**	**482.3**	**227**	**23.8**
	2	**5833.3**	**4266.7**	**3548.7**	**2628.6**	**1895**	**1396.8**	**1036.4**	**727.1**	**419.4**	**187.6**	**25.9**

were computed using data available 100 cycles prior to expected failure events such that the model's effectiveness can be assessed.

To assist in the selection of models, Tables B.10.169, B.10.170, B.10.171, and B.10.172 indicate the averaged Mean Time Till Failure (MTTF) and Mean Time Till (next) Repair (MTTRep) for various time-independent PHMs and scenarios.
The operational factors identified during time-independent and time-dependent PHM modelling are shown in Tables B.10.173, B.10.174, B.10.175, and B.10.176.

Table B.10.166: Overview of time-dependent PHM results obtained by MLE optimisation and GOF tests.

Distribution	norm	norm	norm	norm	norm	norm	norm	norm
Step #	1	2	3	4	5	6	7	8
MLE	−344.51	−263.92	−219.19	−187.25	−159.93	−140.82	−125.35	−116.47
Time (min)	54.06	143.04	283.5	443.81	620.73	774.9	931.86	1099.58
Kolmogorov-Smirnov	3.39	5.47	4.72	3.09	4.09	4.98	5.74	4.93
Cramer-von Mises-Smirnov	27.9	26.07	24.66	22.56	21.79	17	17.46	16.07
Anderson-Darling	−94.56	−97.83	−97.22	−96.92	−99.51	−102.92	−111.93	−111.51
NRR	62.23	250.85	230.21	−303.6	1722.13	991.45	71315.05	8882.5

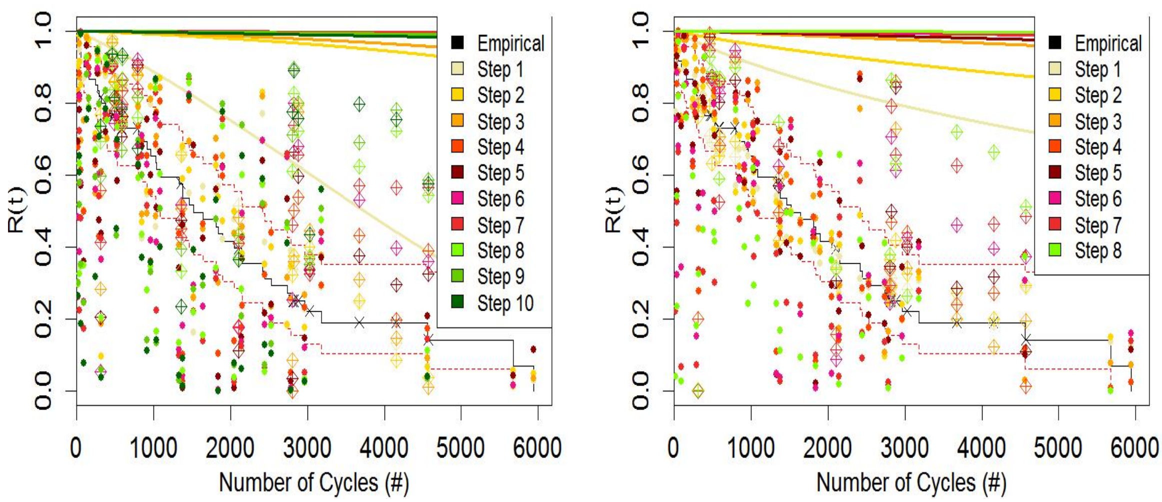
Distribution	norm	norm	lnorm	lnorm	lnorm	lnorm	lnorm	lnorm
Step #	9	10	1	2	3	4	5	6
MLE	−111.27	−105.26	−348.38	−301.68	−253.58	−206.28	−168.87	−135.28
Time (min)	1215.22	1327.29	39.85	106.65	203.4	360.56	564.41	866.09
Kolmogorov-Smirnov	5.57	6.33	1.63	1.77	3.5	4.25	4.13	4.43
Cramer-von Mises-Smirnov	14.78	15.38	29.19	29.46	27.37	25.3	19.76	17.37
Anderson-Darling	−114.04	−115.56	−91.16	−92.01	−95	−97.9	−101.01	−102.95
NRR	14684.31	6357.66	174.64	118.8	78.5	934.19	507774.35	6245.98

Distribution	lnorm	lnorm	logis	logis	logis	logis	logis	logis
Step #	7	8	1	2	3	4	5	6
MLE	−116.29	−103.06	−344.1	−258.81	−200.24	−176.1	−161.08	−150.12
Time (min)	1183.76	1619.05	24.83	68.05	111.18	152	192.39	228.34
Kolmogorov-Smirnov	6.44	7	2.92	7.24	7.51	7.47	6.86	6.57
Cramer-von Mises-Smirnov	16.69	17.64	28.28	26.06	26.84	26.16	22.6	21.11
Anderson-Darling	−107.71	−110.01	−93.79	−101.35	−113.02	−112.42	−111.95	−112.56
NRR	401.72	35166.99	70.6	24.81	9426.83	7386.22	10198.97	4250.46

Distribution	logis	logis	exp	exp	exp	exp	exp	exp
Step #	7	8	1	2	3	4	5	6
MLE	−142.8	−132.91	−349.66	−292.89	−245.04	−202.46	−169.32	−142.69
Time (min)	265.79	291.91	9.38	18.81	33.92	57.31	91.47	129.16
Kolmogorov-Smirnov	6.21	6.19	1.28	1.94	2.52	6.02	6.13	5.73
Cramer-von Mises-Smirnov	19.83	17.2	28.13	25.14	22.46	21.25	20.53	17.88
Anderson-Darling	−111.1	−109.84	−91.05	−92.06	−93.75	−98.87	−104.5	−107.37
NRR	8238.05	16491.43	33.88	74.81	114.58	3392.75	3661.99	19803.28

Distribution	exp	exp	exp	weibull	weibull	weibull	weibull	weibull
Step #	7	8	9	1	2	3	4	5
MLE	−129.63	−123.48	−118.89	−347.04	−285.39	−240.9	−201.91	−165.5
Time (min)	174.82	224.49	281.69	29.11	59.29	107.97	179.8	279.4
Kolmogorov-Smirnov	6.18	6.54	6.92	1.81	2.71	2.86	4.35	6.66
Cramer-von Mises-Smirnov	16.72	18.56	19.66	26.9	19.29	19.36	17.57	22.42
Anderson-Darling	−106	−105.16	−105.33	−92.07	−94.41	−97.26	−100.93	−105.69
NRR	62683.73	69867.95	70698.62	46.18	11091.77	43847.96	31398.21	6743.92

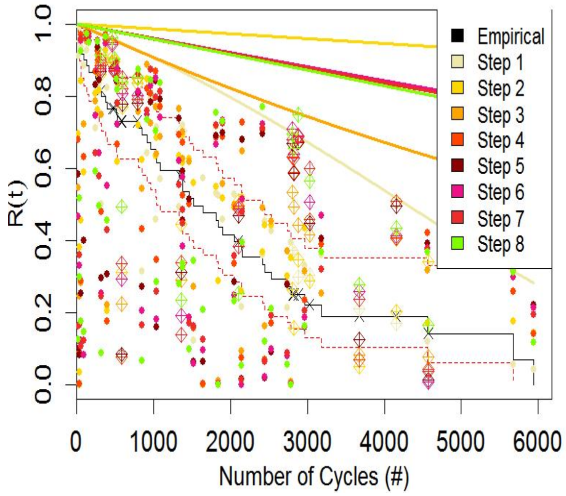
Distribution	weibull	weibull	weibull	weibull
Step #	6	7	8	9
MLE	−144.45	−125.81	−113.6	−106.16
Time (min)	377.85	486.89	608.64	728.15
Kolmogorov-Smirnov	6.43	6.08	6.66	6.94
Cramer-von Mises-Smirnov	21.09	19.43	20.51	20.56
Anderson-Darling	−109.18	−110.3	−114.95	−115.78
NRR	48396.86	2836.55	5842.21	2779.78



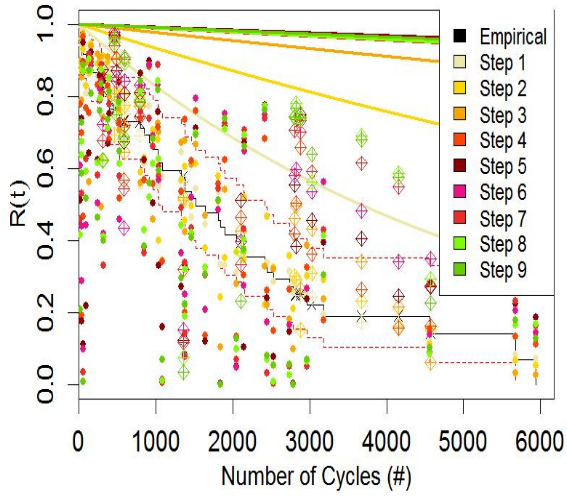
(a) With an underlying norm distribution.

(b) With an underlying lnorm distribution.

Figure B.10.107: Time-dependent PHMs with an underlying norm and lnorm distribution.

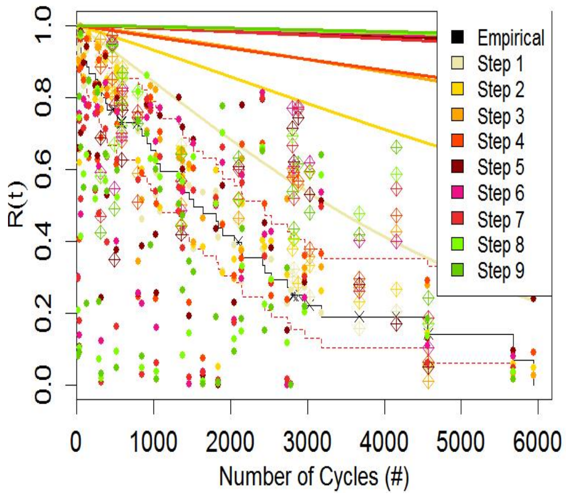


(a) With an underlying logis distribution.

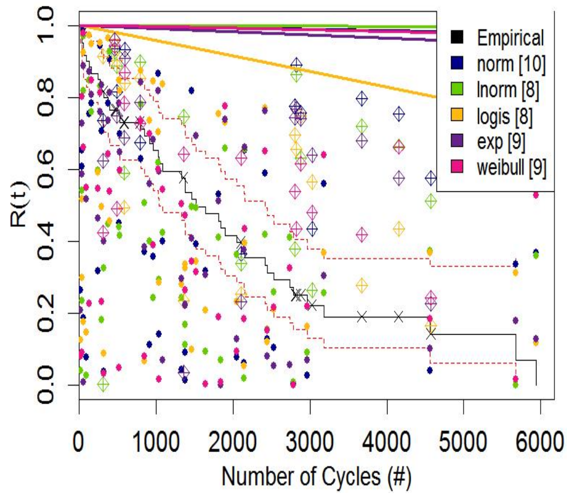


(b) With an underlying exp distribution.

Figure B.10.108: Time-dependent PHMs with an underlying logis and exp distribution.



(a) With an underlying weibull distribution.



(b) All distributions (last step).

Figure B.10.109: Figures containing a weibull distribution and all time-dependent PHMs.

Table B.10.167: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (General case).

		Reliability Level										
Distribution	**Step**	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
	1	0 [100]	4 [93]	9 [80]	9 [73]	16 [60]	29 [60]	40 [53]	53 [40]	60 [27]	69 [7]	84 [0]
	2	0 [100]	7 [87]	11 [80]	16 [73]	22 [67]	33 [53]	38 [53]	49 [53]	58 [47]	64 [13]	82 [7]
	3	0 [100]	4 [87]	7 [73]	9 [73]	18 [67]	24 [60]	38 [53]	44 [47]	58 [33]	67 [13]	84 [0]
	4	0 [100]	0 [93]	0 [93]	4 [73]	13 [73]	13 [60]	33 [47]	44 [47]	58 [40]	69 [27]	78 [0]
norm	5	0 [100]	0 [93]	0 [87]	4 [80]	7 [73]	16 [60]	22 [60]	40 [40]	51 [40]	69 [20]	82 [0]
	6	0 [100]	0 [100]	0 [93]	2 [80]	9 [80]	11 [73]	18 [67]	29 [53]	47 [33]	60 [33]	80 [0]
	7	0 [100]	0 [100]	0 [100]	2 [100]	4 [100]	7 [87]	9 [80]	20 [67]	38 [47]	56 [27]	78 [0]
	8	0 [100]	0 [100]	0 [100]	0 [100]	4 [100]	9 [93]	9 [93]	20 [80]	38 [60]	58 [33]	76 [7]
	9	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	2 [100]	7 [93]	13 [93]	27 [73]	44 [47]	78 [7]
	10	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	2 [100]	2 [93]	13 [93]	24 [67]	47 [53]	76 [7]
	1	0 [100]	0 [100]	7 [87]	18 [73]	33 [53]	40 [47]	53 [40]	64 [33]	69 [7]	78 [0]	87 [0]
	2	0 [100]	2 [100]	13 [93]	16 [80]	22 [53]	38 [40]	51 [40]	58 [40]	71 [13]	76 [0]	87 [0]
	3	0 [100]	7 [93]	13 [87]	13 [67]	27 [53]	33 [53]	42 [53]	53 [47]	58 [40]	67 [13]	80 [0]
lnorm	4	0 [100]	4 [93]	7 [93]	9 [73]	16 [60]	27 [53]	42 [47]	44 [47]	53 [47]	60 [13]	73 [7]
	5	0 [100]	0 [93]	7 [93]	7 [93]	11 [67]	16 [53]	29 [53]	40 [47]	49 [47]	60 [27]	80 [7]
	6	0 [100]	0 [100]	2 [100]	2 [80]	2 [80]	11 [67]	16 [60]	33 [47]	42 [40]	58 [40]	78 [0]
	7	0 [100]	0 [100]	0 [100]	0 [100]	2 [93]	7 [73]	18 [73]	29 [60]	36 [47]	58 [40]	76 [0]
	8	0 [100]	0 [100]	0 [100]	0 [93]	0 [93]	4 [87]	9 [80]	18 [67]	38 [60]	58 [40]	80 [0]
	1	0 [100]	4 [87]	7 [80]	11 [67]	18 [53]	29 [53]	47 [47]	51 [47]	62 [33]	67 [7]	87 [0]
	2	0 [100]	7 [100]	16 [93]	16 [73]	22 [53]	27 [53]	42 [47]	58 [40]	67 [33]	76 [27]	82 [0]
	3	0 [100]	0 [100]	0 [100]	2 [100]	7 [87]	13 [67]	27 [47]	38 [47]	60 [40]	69 [27]	87 [0]
logis	4	0 [100]	0 [100]	2 [100]	4 [93]	7 [93]	9 [80]	18 [60]	22 [40]	49 [40]	64 [20]	84 [0]
	5	0 [100]	0 [100]	0 [100]	2 [93]	4 [93]	9 [80]	16 [60]	22 [47]	44 [40]	67 [20]	84 [0]
	6	0 [100]	0 [100]	0 [100]	2 [93]	4 [93]	7 [80]	16 [73]	22 [60]	44 [40]	62 [27]	82 [0]
	7	0 [100]	0 [100]	0 [100]	0 [93]	2 [93]	7 [87]	16 [73]	20 [53]	47 [47]	62 [27]	82 [0]
	8	0 [100]	0 [100]	0 [100]	0 [93]	4 [93]	4 [87]	13 [87]	18 [67]	44 [47]	58 [27]	82 [0]
	1	0 [100]	4 [93]	9 [87]	18 [60]	29 [47]	36 [47]	53 [40]	64 [40]	64 [27]	73 [0]	87 [0]
	2	0 [100]	7 [93]	9 [80]	9 [73]	29 [47]	38 [47]	51 [40]	62 [40]	71 [40]	73 [0]	87 [0]
	3	0 [100]	7 [87]	13 [87]	16 [80]	27 [73]	31 [53]	42 [40]	53 [33]	69 [27]	76 [13]	87 [0]
	4	0 [100]	2 [93]	11 [80]	11 [80]	18 [73]	29 [60]	36 [40]	51 [40]	62 [33]	71 [20]	82 [0]
exp	5	0 [100]	2 [93]	2 [93]	7 [80]	13 [73]	31 [73]	29 [53]	44 [40]	53 [40]	69 [27]	84 [0]
	6	0 [100]	9 [93]	9 [93]	18 [93]	16 [80]	20 [80]	29 [73]	33 [60]	53 [40]	62 [40]	82 [7]
	7	0 [100]	9 [93]	11 [93]	13 [87]	18 [87]	22 [87]	27 [80]	33 [80]	49 [53]	60 [33]	80 [7]
	8	0 [100]	9 [93]	11 [93]	13 [93]	18 [87]	20 [87]	27 [87]	29 [80]	44 [60]	60 [40]	80 [13]
	9	0 [100]	9 [93]	11 [93]	13 [87]	11 [87]	16 [87]	27 [80]	31 [80]	44 [67]	53 [33]	80 [13]
	1	0 [100]	7 [93]	7 [80]	13 [60]	20 [53]	36 [47]	47 [47]	58 [33]	62 [20]	69 [7]	84 [0]
	2	0 [100]	7 [93]	9 [93]	11 [80]	18 [53]	24 [53]	38 [47]	58 [40]	62 [33]	69 [13]	84 [0]
	3	0 [100]	7 [93]	7 [93]	7 [93]	16 [80]	18 [60]	29 [47]	38 [33]	53 [33]	64 [20]	80 [0]
	4	0 [100]	2 [93]	7 [93]	11 [93]	18 [87]	24 [73]	27 [47]	40 [33]	58 [33]	69 [13]	84 [0]
weibull	5	0 [100]	0 [93]	2 [87]	9 [87]	11 [87]	13 [87]	27 [67]	31 [47]	49 [33]	60 [20]	78 [0]
	6	0 [100]	0 [100]	0 [93]	2 [93]	11 [93]	18 [87]	24 [80]	29 [73]	49 [33]	58 [27]	78 [0]
	7	0 [100]	0 [100]	0 [100]	0 [100]	7 [93]	13 [87]	22 [80]	33 [53]	40 [40]	58 [27]	78 [13]
	8	0 [100]	0 [100]	0 [100]	0 [100]	0 [93]	4 [93]	11 [80]	24 [60]	42 [47]	53 [33]	76 [7]
	9	0 [100]	0 [100]	0 [100]	0 [100]	2 [100]	4 [93]	9 [80]	22 [67]	38 [53]	53 [47]	73 [7]

Table B.10.168: Percentage of failure [censored] events below [above] numerous reliability levels for various time-dependent PHMs computed 100 cycles in advance (Worst case).

		Reliability Level										
Distribution	**Step**	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
norm	1	0 [100]	4 [93]	9 [80]	9 [73]	16 [60]	29 [60]	38 [53]	53 [40]	60 [27]	69 [7]	82 [0]
	2	0 [100]	7 [87]	11 [80]	16 [73]	22 [67]	33 [53]	38 [53]	49 [53]	58 [47]	64 [13]	82 [7]
	3	0 [100]	4 [87]	7 [73]	9 [73]	18 [67]	24 [60]	38 [53]	44 [47]	58 [33]	67 [13]	84 [0]
	4	0 [100]	0 [93]	0 [93]	4 [73]	13 [73]	13 [60]	33 [47]	44 [47]	58 [40]	69 [27]	78 [0]
	5	0 [100]	0 [93]	0 [87]	4 [80]	7 [73]	16 [60]	22 [60]	40 [40]	51 [40]	69 [20]	82 [0]
	6	0 [100]	0 [100]	0 [93]	2 [80]	9 [80]	11 [73]	18 [67]	29 [53]	47 [33]	60 [33]	80 [0]
	7	0 [100]	0 [100]	0 [100]	2 [100]	4 [100]	7 [87]	9 [80]	20 [67]	38 [47]	56 [27]	78 [0]
	8	0 [100]	0 [100]	0 [100]	0 [100]	4 [100]	9 [93]	9 [93]	20 [80]	38 [60]	58 [33]	76 [7]
	9	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	2 [100]	7 [93]	13 [93]	27 [73]	44 [47]	78 [7]
	10	0 [100]	0 [100]	0 [100]	0 [100]	0 [100]	2 [100]	2 [93]	13 [93]	24 [67]	47 [53]	76 [7]
lnorm	1	0 [100]	0 [100]	7 [87]	18 [73]	33 [53]	40 [47]	58 [40]	64 [33]	71 [7]	78 [0]	87 [0]
	2	0 [100]	4 [100]	13 [93]	16 [80]	22 [53]	38 [40]	51 [40]	60 [40]	71 [13]	76 [0]	87 [0]
	3	0 [100]	7 [93]	13 [87]	13 [67]	27 [53]	33 [53]	42 [53]	53 [47]	58 [40]	67 [13]	80 [0]
	4	0 [100]	4 [93]	7 [93]	9 [73]	16 [60]	27 [53]	42 [47]	44 [47]	53 [47]	60 [13]	73 [7]
	5	0 [100]	0 [93]	7 [93]	7 [93]	11 [67]	18 [53]	29 [53]	40 [47]	49 [47]	60 [27]	80 [7]
	6	0 [100]	0 [100]	2 [100]	2 [80]	2 [80]	11 [67]	16 [60]	33 [47]	42 [40]	58 [40]	78 [0]
	7	0 [100]	0 [100]	0 [100]	0 [100]	2 [93]	7 [73]	18 [73]	29 [60]	36 [47]	58 [40]	76 [0]
	8	0 [100]	0 [100]	0 [100]	0 [93]	0 [93]	4 [87]	9 [80]	18 [67]	38 [60]	58 [40]	80 [0]
logis	1	0 [100]	4 [87]	7 [80]	11 [67]	18 [60]	27 [53]	38 [47]	51 [47]	62 [33]	67 [13]	82 [0]
	2	0 [100]	7 [100]	16 [93]	16 [73]	20 [53]	27 [53]	42 [47]	58 [40]	64 [33]	76 [27]	80 [0]
	3	0 [100]	0 [100]	0 [100]	2 [100]	7 [87]	13 [67]	24 [47]	33 [47]	58 [40]	69 [27]	82 [0]
	4	0 [100]	0 [100]	2 [100]	4 [93]	7 [93]	9 [80]	18 [60]	22 [40]	49 [40]	64 [20]	82 [0]
	5	0 [100]	0 [100]	0 [100]	2 [93]	4 [93]	7 [80]	16 [67]	22 [47]	44 [40]	62 [20]	82 [0]
	6	0 [100]	0 [100]	0 [100]	2 [93]	4 [93]	7 [80]	16 [73]	22 [60]	42 [40]	62 [27]	82 [0]
	7	0 [100]	0 [100]	0 [100]	0 [93]	2 [93]	7 [87]	16 [80]	20 [53]	42 [47]	58 [27]	82 [0]
	8	0 [100]	0 [100]	0 [100]	0 [100]	4 [93]	4 [87]	13 [87]	18 [67]	44 [47]	56 [27]	80 [0]
exp	1	0 [100]	4 [93]	9 [87]	18 [53]	29 [47]	36 [47]	53 [40]	64 [40]	67 [27]	76 [0]	87 [0]
	2	0 [100]	7 [93]	9 [80]	9 [73]	27 [47]	40 [47]	51 [40]	62 [40]	71 [40]	73 [0]	84 [0]
	3	0 [100]	7 [87]	13 [87]	16 [80]	27 [73]	31 [53]	42 [47]	53 [33]	69 [27]	73 [13]	84 [0]
	4	0 [100]	2 [93]	11 [80]	11 [80]	18 [73]	27 [60]	36 [40]	51 [40]	62 [33]	71 [20]	82 [0]
	5	0 [100]	2 [93]	2 [93]	7 [80]	13 [73]	31 [73]	29 [53]	44 [40]	53 [40]	69 [27]	84 [0]
	6	0 [100]	9 [93]	9 [93]	18 [93]	16 [80]	20 [80]	29 [73]	33 [60]	53 [40]	62 [40]	82 [7]
	7	0 [100]	9 [93]	11 [93]	13 [87]	18 [87]	22 [87]	27 [80]	31 [80]	47 [53]	60 [33]	80 [13]
	8	0 [100]	9 [93]	11 [93]	11 [93]	16 [87]	20 [87]	27 [87]	29 [80]	44 [60]	56 [40]	80 [13]
	9	0 [100]	9 [93]	11 [93]	11 [87]	11 [87]	16 [87]	22 [80]	29 [80]	42 [67]	51 [33]	80 [13]
weibull	1	0 [100]	7 [93]	7 [80]	13 [60]	20 [53]	36 [47]	49 [47]	58 [33]	64 [20]	69 [7]	84 [0]
	2	0 [100]	7 [93]	9 [93]	11 [80]	18 [53]	27 [53]	42 [47]	58 [40]	64 [33]	69 [13]	84 [0]
	3	0 [100]	7 [93]	7 [93]	9 [87]	16 [80]	18 [60]	29 [47]	42 [33]	58 [33]	67 [13]	82 [0]
	4	0 [100]	2 [93]	7 [93]	11 [93]	18 [87]	24 [73]	27 [47]	40 [33]	58 [33]	69 [13]	84 [0]
	5	0 [100]	0 [93]	2 [87]	9 [87]	11 [87]	16 [87]	27 [67]	31 [47]	49 [33]	60 [20]	80 [0]
	6	0 [100]	0 [100]	0 [93]	2 [93]	11 [93]	18 [87]	24 [80]	29 [73]	49 [33]	58 [27]	78 [0]
	7	0 [100]	0 [100]	0 [100]	0 [100]	7 [93]	13 [87]	20 [80]	33 [53]	40 [40]	58 [27]	78 [13]
	8	0 [100]	0 [100]	0 [100]	0 [100]	0 [93]	4 [93]	11 [80]	24 [67]	42 [47]	53 [33]	76 [7]
	9	0 [100]	0 [100]	0 [100]	0 [100]	2 [100]	4 [93]	7 [87]	18 [67]	36 [53]	56 [47]	73 [7]

Table B.10.169: Average MTTF values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
	1	1464.7	1262.7	1146.3	1146.3	1002.1	845.7	690.1	498.2	332.7	207.5	55.6
	2	1464.7	1184.2	1167.2	1148.3	1012.9	847.4	767	578.1	442.2	292.4	67.6
	3	1464.7	1262.7	1184.2	1151.5	1044.3	946.6	776.2	680.4	442.2	276.8	55.6
	4	1464.7	1464.7	1464.7	1288.6	1067.3	1067.3	762.9	560.8	404.7	237.3	104.8
norm	5	1464.7	1464.7	1464.7	1262.7	1184.2	1031.7	945.7	681.7	545.3	227.8	78.8
	6	1464.7	1464.7	1464.7	1394.4	1169	1160.4	1038.1	924.6	614.3	339.2	103.7
	7	1464.7	1464.7	1464.7	1394.4	1384.8	1381.7	1365.5	1007.1	752.5	424.9	119.9
	8	1464.7	1464.7	1464.7	1464.7	1384.8	1365.5	1365.5	1080.7	766.6	369.8	130.9
	9	1464.7	1464.7	1464.7	1464.7	1464.7	1394.4	1381.7	1302.8	888.9	607.2	131.2
	10	1464.7	1464.7	1464.7	1464.7	1464.7	1394.4	1394.4	1282.7	903.1	592.3	130.9
	1	1464.7	1464.7	1184.2	980.8	688.3	579.9	380.9	216.1	172.9	104.8	43.5
	2	1464.7	1435.8	1071.3	1032.9	894.4	663.6	423.5	333.9	156	121.6	43.5
	3	1464.7	1184.2	1071.3	1071.3	845.6	754.3	684.1	498.2	442.2	247.7	89.7
lnorm	4	1464.7	1262.7	1184.2	1151.5	1138.1	904.3	695.6	647.4	514.1	465.9	155.6
	5	1464.7	1464.7	1184.2	1184.2	1110.7	1013.5	796	661.6	546.7	422.4	92.3
	6	1464.7	1464.7	1434.8	1434.8	1434.8	1191	1050.6	769.3	574.1	404.7	117.4
	7	1464.7	1464.7	1464.7	1464.7	1434.8	1384.4	1106.5	840.9	655.4	344.1	133.1
	8	1464.7	1464.7	1464.7	1464.7	1464.7	1426.1	1378.1	1106.5	664.8	350.8	116.2
	1	1464.7	1262.7	1184.2	1154	1021.2	824.5	548.8	508.1	288.7	209.7	43.5
	2	1464.7	1505.1	1194.7	1194.7	1096.2	1003.1	639.4	413.7	291.2	153.2	67.6
	3	1464.7	1464.7	1464.7	1362.9	1184.2	1046.6	810.7	628.6	299.7	188.6	43.5
logis	4	1464.7	1464.7	1362.9	1262.7	1232.6	1185.1	1009.9	931.3	564.9	247.2	55.6
	5	1464.7	1464.7	1464.7	1362.9	1262.7	1212.7	1031.7	931.3	599.4	195.4	55.6
	6	1464.7	1464.7	1464.7	1362.9	1262.7	1232.6	1031.7	931.3	556.6	289.3	67.6
	7	1464.7	1464.7	1464.7	1464.7	1362.9	1184.2	1031.7	930.7	516.8	276.1	67.6
	8	1464.7	1464.7	1464.7	1464.7	1262.7	1262.7	1076.5	979.5	535.4	331.6	67.6
	1	1464.7	1262.7	1135.6	960.9	778.6	654.4	406.1	216.1	216.1	137.5	43.5
	2	1464.7	1235.3	1154.2	1154.2	842.2	717.1	481.5	296.6	163.8	144.8	43.5
	3	1464.7	1235.3	1080.7	1025.5	869.5	857.9	712.9	519.3	261.4	129.6	43.5
	4	1464.7	1443.2	1103.7	1103.7	995.5	908.8	775.7	558.5	429.8	163.8	81.8
exp	5	1464.7	1443.2	1443.2	1235.3	1122.1	848.5	850.5	602.4	453.8	220.9	71.7
	6	1464.7	1483	1483	1181.5	1174.4	1095.8	934	886	509.7	322.6	78.8
	7	1464.7	1483	1406.1	1418.8	1219.5	1068.7	982.3	902.7	576	415.5	86.9
	8	1464.7	1483	1406.1	1418.8	1157.2	1095.8	982.3	926.1	636.6	386	86.9
	9	1464.7	1483	1406.1	1418.8	1406.1	1174.4	982.3	911.2	636.6	525.2	86.9
	1	1464.7	1184.2	1184.2	1052.3	920.5	682	554.2	328.6	260.1	204	59
	2	1464.7	1226.5	1145.3	1094.5	973.8	858.6	643.8	328.6	260.1	204	59
	3	1464.7	1226.5	1226.5	1226.5	1031.7	979.5	847.4	723.6	438.8	247.2	89.7
	4	1464.7	1362.9	1226.5	1183.7	1015.2	908.6	880.4	719.8	356.6	172.9	55.6
weibull	5	1464.7	1464.7	1456.9	1230.2	1147	1111.6	837.6	865.8	655.7	350.1	118.5
	6	1464.7	1464.7	1464.7	1451.5	1147	1068.6	941.9	915.2	580.4	362.8	118.5
	7	1464.7	1464.7	1464.7	1464.7	1244	1177.8	1027.3	859.4	833.1	386.4	131.1
	8	1464.7	1464.7	1464.7	1464.7	1464.7	1420.6	1122.7	963.8	753.7	586	143.4
	9	1464.7	1464.7	1464.7	1464.7	1451.5	1420.6	1319.9	1035	832.3	562.6	155.3

Table B.10.170: Average MTTF values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
norm	1	1464.7	1262.7	1146.3	1146.3	1002.1	845.7	724.1	498.2	332.7	207.5	67.6
	2	1464.7	1184.2	1167.2	1148.3	1012.9	847.4	767	578.1	442.2	292.4	67.6
	3	1464.7	1262.7	1184.2	1151.5	1044.3	946.6	776.2	680.4	442.2	276.8	55.6
	4	1464.7	1464.7	1464.7	1288.6	1067.3	1067.3	762.9	560.8	404.7	237.3	104.8
	5	1464.7	1464.7	1464.7	1262.7	1184.2	1031.7	945.7	681.7	545.3	227.8	78.8
	6	1464.7	1464.7	1464.7	1394.4	1169	1160.4	1038.1	924.6	614.3	339.2	103.7
	7	1464.7	1464.7	1464.7	1394.4	1384.8	1381.7	1365.5	1007.1	752.5	424.9	119.9
	8	1464.7	1464.7	1464.7	1464.7	1384.8	1365.5	1365.5	1080.7	766.6	369.8	130.9
	9	1464.7	1464.7	1464.7	1464.7	1464.7	1394.4	1381.7	1302.8	888.9	607.2	131.2
	10	1464.7	1464.7	1464.7	1464.7	1464.7	1394.4	1394.4	1282.7	903.1	592.3	130.9
lnorm	1	1464.7	1464.7	1184.2	980.8	688.3	579.9	319.1	216.1	156	104.8	43.5
	2	1464.7	1412.5	1071.3	1032.9	894.4	663.6	423.5	305.7	156	121.6	43.5
	3	1464.7	1184.2	1071.3	1071.3	845.6	754.3	684.1	498.2	442.2	247.7	89.7
	4	1464.7	1262.7	1184.2	1151.5	1138.1	904.3	695.6	647.4	514.1	465.9	155.6
	5	1464.7	1464.7	1184.2	1184.2	1110.7	992	796	661.6	546.7	422.4	92.3
	6	1464.7	1464.7	1434.8	1434.8	1434.8	1191	1050.6	769.3	574.1	404.7	117.4
	7	1464.7	1464.7	1464.7	1464.7	1434.8	1384.4	1106.5	840.9	655.4	344.1	133.1
	8	1464.7	1464.7	1464.7	1464.7	1464.7	1426.1	1378.1	1106.5	664.8	350.8	116.2
logis	1	1464.7	1262.7	1184.2	1154	1021.2	839.8	670.5	508.1	288.7	209.7	67.6
	2	1464.7	1505.1	1194.7	1194.7	1132.8	1003.1	639.4	413.7	289.6	153.2	86.9
	3	1464.7	1464.7	1464.7	1362.9	1184.2	1046.6	858.6	704	334.6	188.6	67.6
	4	1464.7	1464.7	1362.9	1262.7	1232.6	1185.1	1009.9	931.3	564.9	247.2	67.6
	5	1464.7	1464.7	1464.7	1362.9	1262.7	1232.6	1031.7	931.3	599.4	310.9	67.6
	6	1464.7	1464.7	1464.7	1362.9	1262.7	1232.6	1031.7	931.3	598.3	289.3	67.6
	7	1464.7	1464.7	1464.7	1464.7	1362.9	1184.2	1031.7	930.7	567.6	348.9	67.6
	8	1464.7	1464.7	1464.7	1464.7	1262.7	1262.7	1076.5	979.5	535.4	381.6	102.1
exp	1	1464.7	1262.7	1135.6	960.9	778.6	654.4	406.1	216.1	196.5	123.6	43.5
	2	1464.7	1235.3	1154.2	1154.2	872.4	732.9	432.3	296.6	163.8	144.8	59
	3	1464.7	1235.3	1080.7	1025.5	869.5	857.9	712.9	519.3	261.4	151.5	59
	4	1464.7	1443.2	1103.7	1103.7	995.5	936.1	775.7	558.5	429.8	163.8	81.8
	5	1464.7	1443.2	1443.2	1235.3	1122.1	848.5	850.5	602.4	453.8	220.9	71.7
	6	1464.7	1483	1483	1181.5	1174.4	1095.8	934	886	509.7	322.6	78.8
	7	1464.7	1483	1406.1	1418.8	1219.5	1068.7	982.3	903	615	415.5	86.9
	8	1464.7	1483	1406.1	1406.1	1174.4	1095.8	982.3	926.1	636.6	465.1	86.9
	9	1464.7	1483	1406.1	1406.1	1406.1	1174.4	1075.5	961.7	651.5	547.9	86.9
weibull	1	1464.7	1184.2	1184.2	1052.3	920.5	682	499.6	328.6	216.1	204	59
	2	1464.7	1226.5	1145.3	1094.5	973.8	810.7	608	328.6	216.1	204	59
	3	1464.7	1226.5	1226.5	1145.3	1031.7	979.5	847.4	640.6	356.6	242.9	84.9
	4	1464.7	1362.9	1226.5	1183.7	1015.2	908.6	880.4	719.8	356.6	172.9	55.6
	5	1464.7	1464.7	1456.9	1230.2	1147	1067.7	837.6	865.8	655.7	350.1	89.7
	6	1464.7	1464.7	1464.7	1451.5	1147	1068.6	941.9	915.2	580.4	362.8	118.5
	7	1464.7	1464.7	1464.7	1464.7	1244	1177.8	1068.9	859.4	833.1	386.4	131.1
	8	1464.7	1464.7	1464.7	1464.7	1464.7	1420.6	1122.7	963.8	753.7	586	143.4
	9	1464.7	1464.7	1464.7	1464.7	1451.5	1420.6	1312.9	1031.4	838.9	539.5	155.3

Table B.10.171: Average MTTRep values for time-dependent PHMs computed using historical data and assuming general-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
norm	1	5833.3	3897.6	3262.5	2752.3	2418.6	1960.3	1561.1	1205.5	887.1	530.6	136.1
	2	5833.3	4056.2	3193.4	2538.9	2242	1857.3	1565.6	1272.5	920.6	622.8	199
	3	5833.3	4025.8	3322.1	2702.9	2316.5	2017.1	1696.4	1351.5	995.2	682.8	205
	4	5833.3	4647.8	4127.4	3510.8	2943.4	2442	1905.2	1494.1	992.6	661.2	242.9
	5	5833.3	5038.3	4317.9	3807.7	3187.8	2573.2	1937.8	1540.3	1088.6	715.2	291.2
	6	5833.3	5287.2	4445.7	3750.1	3496.2	3020.6	2475.3	1902.9	1360.4	918.1	342.6
	7	5833.3	5800.5	5307.7	4745.4	4083	3377.9	2784.6	2363.8	1710	1132.8	356.1
	8	5833.3	5833.3	5648.4	5342.8	4592.1	3711.8	3144.3	2635.7	1893	1283	376.7
	9	5833.3	5833.3	5833.3	5740.3	5425.3	5018.1	3953.6	3087.7	2520.1	1614.3	434.8
	10	5833.3	5833.3	5833.3	5661.9	5431.5	5184	4553.7	3193.5	2491.6	1569.8	431.7
lnorm	1	5833.3	5431.1	3983.5	2779.2	2078	1490.7	1060.1	729.3	471	267.2	101.9
	2	5833.3	5366.2	3828.4	2816.2	2203.4	1645	1185.1	816.3	521.8	301	117.1
	3	5833.3	4387.8	3198.5	2659.1	2237.4	1890.9	1499	1190.3	886.7	639.8	222.4
	4	5833.3	4164.6	3350.9	2598.7	2013.7	1892.2	1636.2	1385	1107.2	780.2	333.5
	5	5833.3	5304.3	4356.2	3491.9	2848.6	2350.1	1948.1	1466.2	1118.1	762	289
	6	5833.3	5636.9	4857.8	3950.1	3400	2936.1	2307.9	1771.8	1383.8	892.6	320.5
	7	5833.3	5650.9	5442.5	5306.3	4781.4	3838	3007.3	2316.9	1702.4	1032.2	351.9
	8	5833.3	5752.9	5210.5	4756.7	4519.6	4046.8	3241.7	2743.4	2031.5	1224.8	368.4
logis	1	5833.3	3925	3300.6	2609.5	2294.1	1927	1553.1	1166.2	825.4	487	124.9
	2	5833.3	4221	3299.2	2575	2070	1754.9	1495.3	1113.7	794.2	442.9	152.2
	3	5833.3	5369.4	4635.9	3947.2	3444.7	2686.3	2076.4	1560.2	995.7	570.8	135.1
	4	5833.3	5492.3	4965.6	4367.2	3715.3	3009.3	2291.6	1736.1	1194.4	729.9	185.3
	5	5833.3	5829.2	5303.1	4620.2	3900.6	2995.4	2381.5	1792.7	1270.3	779.6	191
	6	5833.3	5527.7	5198.3	4777.9	4083.8	3341.2	2577.6	1908.5	1397.2	836.3	205.3
	7	5833.3	5373.2	5212.7	4835.9	4390.9	3789.7	2883.2	2170	1506.2	909.9	211
	8	5833.3	5349.9	5219.3	4974.6	4534.2	3838.8	3072.6	2244	1602.3	964.1	227.1
exp	1	5833.3	4556.5	3475.7	2667.7	2092.6	1619.3	1192.5	846.3	551.7	318.9	101.5
	2	5833.3	3638.7	3085.9	2570.8	2096.9	1665.8	1234.3	886.5	628.6	383.8	139
	3	5833.3	4036.2	3150.2	2626.5	2059.3	1579.1	1342	1041.7	724.3	456.2	179
	4	5833.3	4435.4	3335.7	2816.6	2316.3	1699.1	1428.1	1123.9	784.6	487.4	195.5
	5	5833.3	4561.6	3940.3	3119.2	2729.9	2154.7	1781.9	1415.9	1000.3	665.8	206.2
	6	5833.3	4312.9	4152.6	3300.3	3199.9	2556.8	2105.5	1613.9	1286.7	882.9	235.7
	7	5833.3	4312.9	4178.6	3802.2	3413.9	2922.8	2428.4	1931.2	1504.1	1029.7	275.2
	8	5833.3	4313	4261	4031.4	3698.1	3174	2616.4	2114	1584.5	1100.3	302.7
	9	5833.3	4313	4293.1	3995.9	3958.7	3512	2789.1	2236.9	1699.4	1137	317.7
weibull	1	5833.3	4035.9	3216.2	2611	2199.6	1790.5	1374.1	1061	727.7	449.2	143.6
	2	5833.3	4269.1	3504.8	2845.8	2375	1948.3	1585.1	1200.9	831.6	518.8	172
	3	5833.3	4524.7	3621.7	3032.8	2549.5	2159.8	1777.1	1401.3	1099.4	707.3	232.5
	4	5833.3	5360.8	4253	3302.7	2711.4	2148.1	1692.1	1311.7	1019	645.7	190.3
	5	5833.3	5675.2	4722	3956.8	3316.4	2712.9	2129.6	1540.8	1070	802.3	272.9
	6	5833.3	5324.2	5041.1	4431.8	3876.3	3072.2	2420.9	1828.3	1379.2	979	272
	7	5833.3	5108.9	5056.1	4536.6	4183.1	3205.6	2567.2	1834.1	1309.4	1009.8	291.4
	8	5833.3	5033.1	4903.6	4833.1	4232.3	3315.6	3069.7	2392.2	1671.6	1069.4	358.4
	9	5833.3	5115	4798.7	4327.2	3965	3337.5	2833.2	2459.8	1790.7	1192.3	418.7

Table B.10.172: Average MTTRep values for time-dependent PHMs computed using historical data and assuming worst-case scenarios.

		Reliability Level										
Dist.	Step	0%	10%	20%	30%	39%	49%	59%	69%	79%	88%	98%
	1	5833.3	3905.6	3273.2	2762.9	2427.1	1969.7	1570.1	1216.5	897.5	540.2	144.8
	2	5833.3	4054.8	3192	2537.3	2241	1856.4	1564.4	1271.5	919.8	621.9	197.7
	3	5833.3	4025.8	3322.6	2702.9	2316.8	2017.7	1697.4	1351.6	995.5	683.4	206
	4	5833.3	4647.8	4126.7	3508.8	2942.7	2436.9	1905	1493.6	991.6	661	242.4
norm	5	5833.3	5038.3	4317	3807.1	3186.7	2572.5	1937.7	1539.3	1087.1	714.5	289.7
	6	5833.3	5287.1	4445.1	3749.5	3495.9	3020.5	2473.2	1902.5	1360	917.4	333.7
	7	5833.3	5799.5	5307.5	4743.2	4082.9	3373.6	2780.8	2360.5	1707.4	1128.6	342.8
	8	5833.3	5833.3	5648.3	5337.4	4584.6	3709.2	3141.6	2628.6	1879.3	1276.6	343.7
	9	5833.3	5833.3	5833.3	5739.9	5424.1	5017.4	3951	3081.9	2514.9	1609	413.4
	10	5833.3	5833.3	5833.3	5660.4	5427	5183.2	4535.8	3182.6	2480.4	1559.1	407.6
	1	5833.3	5425.1	3968.8	2760.1	2059.6	1475.4	1040.6	710.3	447.4	248.8	100.6
	2	5833.3	5179.5	3817.5	2801.1	2188	1619.3	1163.7	798.8	498.8	284.2	105
	3	5833.3	4384.8	3196.6	2656.7	2235	1888.8	1497.9	1187.3	879.8	637.2	218.8
lnorm	4	5833.3	4160.8	3346.7	2597.8	2012.6	1891	1635.7	1384.2	1105.4	777	328.7
	5	5833.3	5303.2	4354	3489.5	2844.3	2323.9	1946.8	1463.9	1115.6	758.4	285.3
	6	5833.3	5636.9	4857.4	3949.8	3398.1	2932.7	2306.8	1770.8	1380.8	889.3	306.9
	7	5833.3	5650.9	5442.5	5306.3	4778.5	3835.8	3004.5	2313.6	1701.1	1028.4	335.5
	8	5833.3	5752.9	5205.9	4756.7	4519.4	4046.7	3238.2	2740.1	2011.3	1221.7	349.2
	1	5833.3	3963.6	3354.1	2668	2353.3	2006.4	1637.2	1226.4	885.7	551.1	176.5
	2	5833.3	4233.6	3320.7	2598.8	2080	1774.6	1520.4	1146.7	841.3	473.8	188.3
	3	5833.3	5381.7	4670.3	3986.8	3494.3	2743.4	2122.1	1606.9	1058.1	636	198
logis	4	5833.3	5492.3	4980.8	4378.6	3735.4	3041.2	2332.2	1777.1	1239.1	768.6	233.9
	5	5833.3	5831.5	5319.1	4637.1	3932.8	3071.8	2411.4	1824.3	1307.5	804	234.7
	6	5833.3	5534.8	5203.7	4807.1	4109.4	3370.9	2607.6	1938	1426.4	872.6	246.2
	7	5833.3	5378.5	5217.9	4852.9	4410.4	3821.7	2914.4	2217.1	1563.6	939.4	257.8
	8	5833.3	5350.3	5224.4	4992.1	4552.2	3861.4	3098	2280.6	1636.5	986.3	264.3
	1	5833.3	4547.2	3463.5	2650.3	2075.9	1602	1176	834	528.4	302.9	100.6
	2	5833.3	3643.4	3090.8	2580.1	2127.8	1630.8	1278.4	897.2	645.8	392	156.8
	3	5833.3	4038.8	3197.8	2628.5	2062.6	1572.7	1344.3	1045.9	727.1	451.4	183.8
	4	5833.3	4435.4	3337.5	2817.9	2319.7	1706	1430.5	1128.7	786.3	492.3	202
exp	5	5833.3	4561.7	3941.8	3112.4	2732.5	2158.4	1784.7	1417.7	1023.3	668.1	210
	6	5833.3	4312.9	4152.6	3304.3	3210.1	2568.5	2111.6	1627.6	1298	905.9	249.3
	7	5833.3	4343.5	4181.5	3808.8	3422.8	2928.2	2437.9	1973.6	1516.4	1043.7	290.1
	8	5833.3	4403.9	4347.9	4251.4	3841.5	3206.8	2645.2	2129.4	1599.3	1105.2	312
	9	5833.3	4403.9	4380.1	4233.1	4043.2	3602.2	2884.2	2258	1750.9	1160.3	330.2
	1	5833.3	4025.5	3203.9	2594.4	2185.3	1774.8	1366	1044	713.3	435	131.4
	2	5833.3	4251.8	3480.8	2829.3	2354.7	1926	1492.7	1176.2	784.6	497.2	151.4
	3	5833.3	4510.2	3604.3	3050.5	2517.8	2131.5	1742.4	1392.9	1048.7	669.2	199.4
	4	5833.3	5343.3	4247.1	3242.3	2697.9	2131.8	1676.9	1300.7	996.4	622.9	167.6
weibull	5	5833.3	5671.4	4711.3	3911.5	3291.9	2693.6	2127	1529.4	1043.7	777.6	252.9
	6	5833.3	5322.3	5035.7	4421	3858.6	3055.4	2362.8	1792.2	1346.3	931.1	244.6
	7	5833.3	5127.9	5056.5	4521.1	4190.9	3222.3	2574.1	1842.1	1331.3	1015.7	299.1
	8	5833.3	5054.1	4922.5	4834.3	4167.3	3319.9	3075.4	2428.4	1679.2	1076.8	366.5
	9	5833.3	5136.1	4819.6	4348.1	3984.2	3342.5	3085.6	2599.7	1825.4	1202.2	422.1

Table B.10.173: Variables identified by time-(in)dependent PHM models.

Time-independent PHM

	Variable	Scaled Value
①	*Roll rate min deg sec 3*	−5.13
②	*Vtrue mean knots 5*	4.49
③	*Group B*	4.08
④	*Group A*	−3.81
⑤	*Pitch mean deg 1*	3.19
⑥	*NormalForce rhs mean lbs 1*	3.02

Time-dependent PHM

	Variable	Scaled Value
①	*NormalForce nose mean lbs 8*	−28.41
②	*Vz mean ft min 2*	−23.02
③	*Group G*	−21.32
④	*Vtrue mean knots 4*	21.11
⑤	*Group L*	−21.1
⑥	*Roll min deg 2*	−18.92
⑦	*Duration Cruise CleanWing Seconds*	18.78
⑧	*Group A*	−15.97
⑨	*Roll rate mean deg sec 8*	15.87
⑩	*Group F*	13.39
⑪	*Accn long mean g s*	−12.43
⑫	*Accn lat mean g s 4*	12.4
⑬	*Roll rate min deg sec 1*	−11.83
⑭	*Roll rate min deg sec*	−11.83
⑮	*Roll rate min deg sec 4*	−10.68
⑯	*Group C*	9.51
⑰	*Group J*	−9.24
⑱	*Roll mean deg*	−8.96
⑲	*Group D*	−8.74
⑳	*Accn lat mean g s 8*	7.71
㉑	*Group E*	7.56
㉒	*Roll rate mean deg sec 1*	7.55
㉓	*Roll mean deg 8*	−7.51
㉔	*Group K*	5.42
㉕	*Group H*	−5.12
㉖	*Accn long mean g s 8*	4.87
㉗	*Group I*	2.35

Table B.10.174: Variables identified by each step by time-(in)dependent PHMs (in order).

	PHM Variables	
	Time-independent	*Time-dependent*
norm	4 6	22 6 20 16 23 12 19 21 26 13
lnorm	1 6	10 6 22 23 9 16 1 12
logis	3 4 5	4 8 3 23 25 13 14 27
exp	2	3 2 18 22 13 17 24 15 14
weibull	6 1	3 7 23 9 5 13 11 16 12

Table B.10.175: Number of times variables identified by each step by time-(in)dependent PHMs.

Key				Key			
indep	*dep*	*Variable*	*Count*	*indep*	*dep*	*Variable*	*Count*
4	8	Group A	3		7	Duration Cruise CleanWing Seconds	1
3		Group B	1		1	NormalForce nose mean lbs 8	1
	16	Group C	3	6		NormalForce rhs mean lbs 1	3
	19	Group D	1	5		Pitch mean deg 1	1
	21	Group E	1		18	Roll mean deg	1
	10	Group F	1		23	Roll mean deg 8	4
	3	Group G	3		6	Roll min deg 2	2
	25	Group H	1		22	Roll rate mean deg sec 1	3
	27	Group I	1		9	Roll rate mean deg sec 8	2
	17	Group J	1		14	Roll rate min deg sec	2
	24	Group K	1		13	Roll rate min deg sec 1	4
	5	Group L	1	1		Roll rate min deg sec 3	2
	12	Accn lat mean g s 4	3		15	Roll rate min deg sec 4	1
	20	Accn lat mean g s 8	1		4	Vtrue mean knots 4	1
	11	Accn long mean g s	1	2		Vtrue mean knots 5	1
	26	Accn long mean g s 8	1		2	Vz mean ft min 2	1

Table B.10.176: Variables belonging to each group identified in B.10.173.

Group	*Variables*	*Group*	*Variables*
Group A	Torque lhs mean 3, Vcal mean knots 3, Torque rhs mean 3	Group G	Torque rhs mean 4, Torque lhs mean 4
Group B	Rudder low max deg TER , Rudder low mean deg TER	Group H	Roll min deg 4, Yaw rate min deg sec 4
Group C	Rudder cmd force max lbs Nose Right , Rudder cmd force mean lbs Nose Right	Group I	Roll mean deg 1, Yaw rate mean deg sec 1
Group D	Pressure dynamic min hPa mbar 3, Aoa max deg 3	Group J	Vcal mean knots 2, Pressure dynamic mean hPa mbar 2
Group E	Elevator Lin mean deg TEU 8, Elevator Rin mean deg TEU 8	Group K	Elevator Lin max deg TEU 8, Elevator Rin max deg TEU 8
Group F	Roll max deg 2, Yaw rate max deg sec 2	Group L	Torque lhs mean 5, Torque lhs mean 6