BTA/TG/12-30 The relation between particle settling velocity and shape: a critical review

17-12-2012 Suzanne Boekhout

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Title	:	The relation between particle settling velocity and shape: a critical review
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TA Report number	:	BTA/TG/12-30

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The relation between particle settling velocity and shape: a critical review

BSC thesis Applied Earth Sciences
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17-12-2012

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Contents

Abstract

This research has tried to find a relation between particle settling velocity and shape. This was done by taking the settling velocity log-contrast of a non-spherical particle and an equivalent spherical particle. This log-contrast should be the logarithmic function of a certain shape factor. Settling data of several shapes were taken from previous authors and with those data several existing and new shape factors were tested to form a single relation.

Recently a simple equation was developed which can explicitly predict the settling velocity of spherical particles for any Reynolds number. This equation was tested to be fairly accurate for spherical data. However, the spherical data showed a systematic error which needs to be resolved for the equation to be perfectly accurate.

The equation also showed to be fairly accurate for non-spherical sieve diameter data. However sieve diameter data cannot be used to define the shape of a particle as there is no direct relation between sieve diameter and actual particle size and shape.

In order to find a relation between particle settling velocity and shape, a correct shape factor must be found. For that matter several shape factors from previous authors were tested on data directly available from previous authors. The existing shape factors were proved to not be able to combine all investigated shapes into a single relation for settling velocity and a new shape factor was found with which a single relation might be formed. This new equation is a generalization of the Corey Shape Factor, the Boekhout Shape Factor: $BSF = \frac{D_s}{{D_l}^n D_i^{1-n}}$. However due to the small amount of data, important data gaps exist so that nothing can be concluded with certainty. Therefore more settling experiments should be done over a large range of Reynolds numbers to be able to conclude whether the Boekhout Shape Factor is the correct link between particle settling velocity and shape.

1. Research Goal

Different equations exist for the calculation of the terminal settling velocity of sediment particles in water. These equations describe the different flow regimes characterised by the grain Reynolds number. For a low Reynolds number (<0.5) and high Reynolds numbers (>10^3) well-defined relations are known (Stokes Law and the so-called 'Impact Law') in which the settling velocity of particles is a function of the submerged density and diameter of the particle, the viscosity of the fluid and for the high flow regime also the drag coefficient. Unfortunately the settling of most sediment particles such as quartz grains occurs in a transitional flow regime.

Recently Ferguson and Church (2004) have presented an equation which can explicitly predict the settling velocity of spherical particles in all Reynolds regimes, including the regime in between Stokes Law and the Impact Law, by combining the flow equations of the two extreme regimes.

Based on the Ferguson and Church (2004) equation Weltje and Bloemsma (2013) have developed a new parameterisation of the settling velocity in terms of log ratios. With this equation relative settling velocities can easily be predicted in terms of grain size and density contrasts.

However settling velocities predicted with this equation do not apply to non-spherical grains. The shape of a grain is a factor which can adjust the settling velocity significantly, for the fluid flow geometry affects the shear and pressure drag by controlling the contact area between the fluid and particle, the pressure distribution around it, and the point of separation of the wake (Stringham et al, 1969).

The Weltje and Bloemsma (2013) equation presents a way to find a relation between shape and velocity by comparing the settling velocity of a grain and that of a grain equivalent sphere, which can be predicted with the Ferguson and Church (2004) Formula. The difference between those two velocities must be due to the shape of the grain.

The goal of this Bachelor thesis is to find a relation between shape and velocity in terms of log ratios using the Ferguson and Church (2004) and the Weltje and Bloemsma (2013) Equations. The first step in this process is to find out what is known and investigated by previous authors about settling velocity and the relation between settling velocity and shape of grains. Existing shape factors and their relation with settling velocity will be investigated on data from previous authors. Furthermore new ideas on shape factors will be formed and investigated as well.

2. Settling Velocity of Sedimentary Particles: General Theorem

The settling of sedimentary particles is controlled by many factors. This chapter will describe what is known about settling of particles and how that has been translated into equations.

2.1 An equation for Settling Velocity

Settling Velocity is controlled by both particle properties, such as density and particle diameter, and properties of the medium, such as density and viscosity. These properties control the flow regime in which the particles settle. The flow regime can be described best by defining a dimensionless number that includes all properties that have influence on the flow regime, the Reynolds Number:

$$Re = \frac{wD}{\nu} \tag{1}$$

In which the symbols are defined as:

Re Reynolds number [-]
w Particle velocity [m/s]
D Particle diameter [m]
v Kinematic viscosity of the medium [m^2/s]

Two types of flow regimes exist dependent on the Reynolds number: Laminar flow and turbulent flow. In laminar flow water molecules flow in streamlines without lateral mixing. This only happens at low Reynolds numbers. In turbulent flow, which exists at high Reynolds numbers, lateral mixing does occur and the flow becomes chaotic.

In laminar flow the velocity can easily be calculated from the force balance between gravitational force and drag resistance. The following formula follows from this balance, called Stokes' Law (Ferguson and Church, 2004):

$$w = \frac{RgD^2}{18\nu} \tag{2}$$

With:

R submerged density of the particle [-], R = $\frac{\rho_s - \rho_f}{\rho_f}$ with ρ_s [kg/m^3] being the density of the particle and ρ_f [kg/m^3] the density of the fluid medium
g gravitational acceleration [m/s^2]

For turbulent flow a different equation exists (Ferguson and Church, 2004), the impact law:

$$w = \sqrt{\frac{4RgD}{3C}} \tag{3}$$

With C [-] being the asymptotic value of the drag coefficient with an empirically determined value of around 0.4 for smooth spheres.

Unfortunately the boundary between these two regimes is not sharp. Pure laminar flow only exists for Reynolds numbers smaller than approximately 1, while pure turbulent flow exists for Re>10^3. The settling of common sand grains in water mostly takes place in the transitional regime, from silt particles up to boulders with settling velocities of 0.02 m/s and higher.

Many relations have been proposed to fill this gap, for example Gibbs et al (1971) or Cheng (1997). However most of them were empirical, large and difficult equations. Ferguson and Church (2004) presented a formula that asymptotically reduces to the two equations above by combining the two in one relation:

$$w = \frac{RgD^2}{18\nu + (0.75CRgD^3)^{0.5}} \tag{4}$$

This relation was experimentally tested and showed good results in comparison with former equations.

2.2 Relative Settling Velocity

As relative settling velocities give enough information to for example correlate between wells in a reservoir and many of the medium characteristics cancel out, Weltje and Bloemsma (2013) developed a new relation based on the ideas of Ferguson and Church (2004) to put together the two equations for laminar and turbulent flow regimes. They looked at the log-ratio of two particle velocities. Taking the logarithm of a function has many advantages as it produces a linearized function in which all variables are separated.

Weltje and Bloemsma (2013) rewrote equation (2) and (3) as the relative fall velocity of two particles *i* and *j*, having particle diameters D_i and D_j:

For equation (2):

$$\log\left(\frac{w_i}{w_j}\right) = \log\left(\frac{R_i\, g\, D_i^2 18\,\nu}{R_j\, g\, D_j^2 18\,\nu}\right) = \log\left(\frac{R_i}{R_j}\right) + \; 2\log\left(\frac{D_i}{D_j}\right) \tag{5}$$

And for equation (3):

$$\log\left(\frac{w_i}{w_j}\right) = \log\left(\left(\frac{4R_i\, g\, D_i C\,\nu}{4R_j\, g\, D_j C\,\nu}\right)^{0.5}\right) = \frac{1}{2}\log\left(\frac{R_i}{R_j}\right) + \; \frac{1}{2}\log\left(\frac{D_i}{D_j}\right) \tag{6}$$

These two relations only differ in coefficients and can be generalized into one relation by inserting two coefficients α and β:

$$\log\left(\frac{w_i}{w_j}\right) = \alpha\log\left(\frac{R_i}{R_j}\right) + \; \beta\log\left(\frac{D_i}{D_j}\right) \tag{7}$$

Weltje and Bloemsma (2013) found that α and β are dependent on the Reynolds numbers of the two particles, α varying between 0.5 and 1 and β between 0.5 and 2. They defined statistical distribution functions of both coefficients from which α and β can be determined. β may be calculated from α

However equation (7) is not yet complete as a difference in velocity could also be caused by a difference in particle shape. In the next chapters it will be investigated whether shape can be added into the equation.

3. Settling velocity of spheres

Most research has focussed on settling velocity of spherical particles and therefore much is known about their settling behaviour. Therefore spheres are the basis with which every shape can be compared. Consequently, to be able to predict the settling velocity of non-spherical particles properly, first the settling behaviour of spheres should be defined.

Many researchers defined equations for the settling velocity of spherical particles, Ferguson and Church (2004) being one of the most recent. As stated in chapter 2 they combined the two theoretical, proved relations for low and high Reynolds numbers, Stokes' Law and the Impact Law, into one single relation which asymptotically approaches those two relations (equation (4)). This equation was tested against data from literature to check whether the equation predicts the settling velocity of spheres well.

3.1 Spherical Data

However many research has been done on the settling velocity of spheres, directly available data with all needed parameters for the application of the Ferguson and Church (2004) formula are scarce. Le Roux (2004) produced tables in his papers that included spherical data. He combined data of different researchers.

Stringham et al (1969) tested 56 spheres of which Le Roux used 16. This research used the same 16 data points, for not all the necessary data could be obtained from the original. Stringham et al (1969) used nylon, steel and Teflon spheres, which were produced for this specific research, thereby being secured of the density and the length of the axes of the particles.

Le Roux (2004) added to these data 16 data points from Gibbs et al (1971). Gibbs et al (1971) used glass spheres of which he checked the sphericity by rolling them down an incline. The spheres that rolled farthest and in the straightest line were considered spherical.

Furthermore Le Roux (2004) used 4 data points of Williams (1965). These were the only spherical particles available in that article. Williams (1965) produced the spheres himself from commercially available plastics.

Le Roux (2004) also used 4 other data points which were not retraceable, the total being 40.

The data correspond to Reynolds numbers as given in Figure 3.1. As can be found from this histogram the only data available at low Reynolds numbers are Gibbs' data. At higher Reynolds numbers the data are obtained from mixed sources.

The data described above have various sources and therefore various measurement methods. This could lead to variations in outcome.

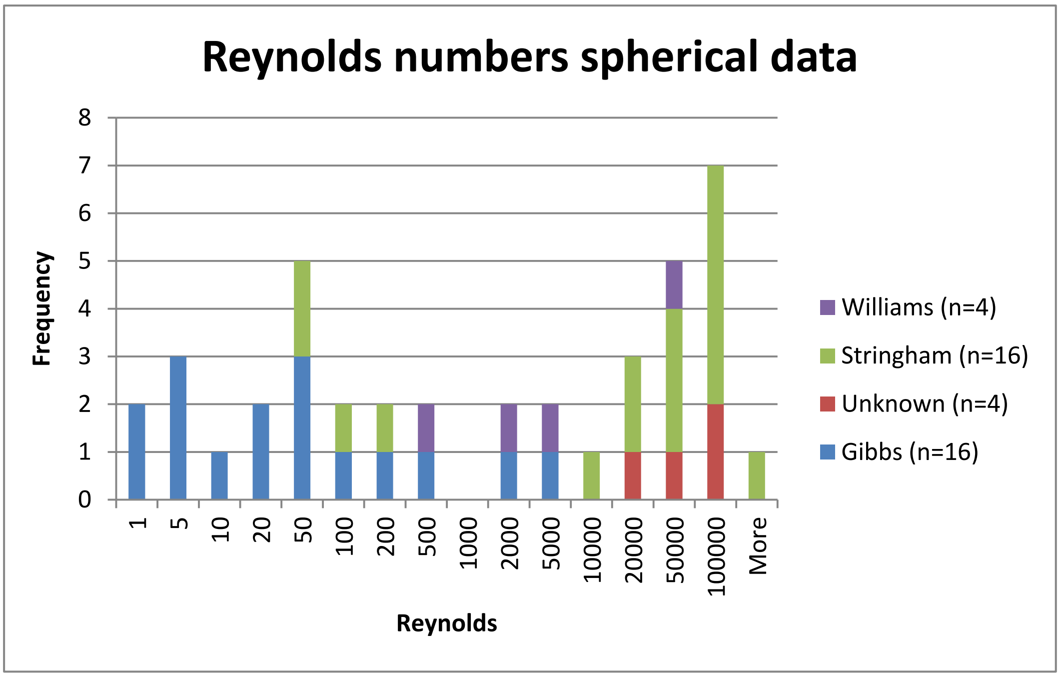

Fig. 3.1: Reynolds numbers for the spherical data. The different sources of the data are represented by different colours.

<u>3.2 The Ferguson and Church equation for spherical data</u>

With the data described above the Ferguson and Church (2004) equation (4) was tested on accuracy with the values for C=0.4 as proposed by Ferguson and Church (2004). For that matter the settling velocity was predicted with the Ferguson and Church (2004) equation (w_p, the predicted velocity) and compared to the true, measured settling velocity (w_o, the observed velocity) by plotting them against each other. When the Ferguson and Church (2004) equation predicts the settling velocity well, the difference should be zero and a linear relation must exist between the two settling velocities with a slope of 1. Therefore the logarithm of the ratio of the 2 velocities, called the settling velocity log-contrast, must be 0. Therefore it is tested whether the Ferguson and Church (2004) equation with C=0.4 shows an average log-contrast of 0. Deviation from zero is probably

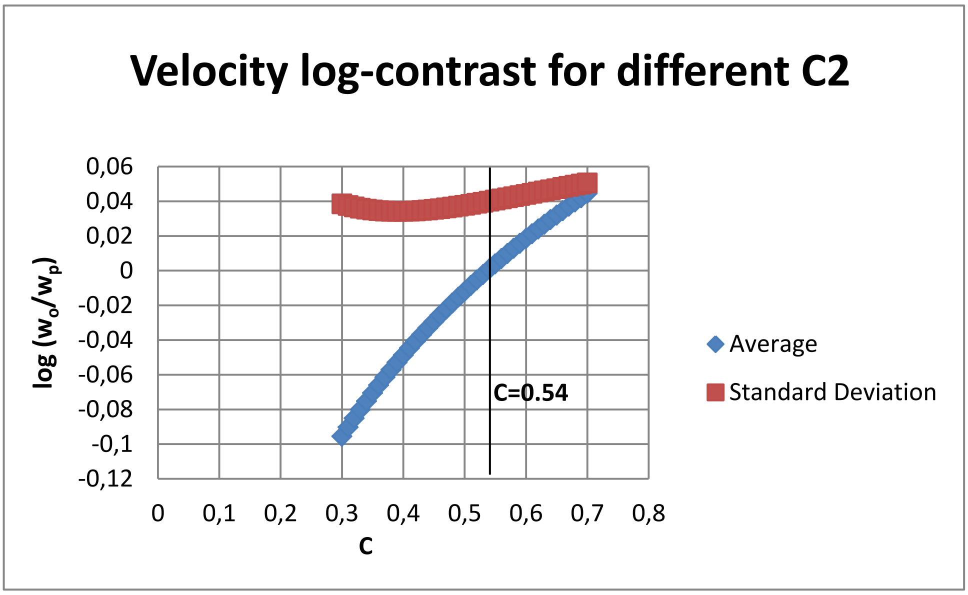

Fig. 3.2: The averaged and standard deviation of the settling velocity log-contrast for spherical data with different values of C. At C=0.54, the average log-contrast is 0.

due to a wrong constant C as it is empirically determined. Therefore the velocity log-contrast is calculated for different values of C, thereby giving the average and standard deviation.

Figure 3.2 shows the average and standard deviation of the velocity log-contrast.

As can be seen from figure 3.2, the average of the log-contrast does become zero, but not at C=0.4. C=0.4 does give approximately the lowest standard deviation. At C=0.54 the average is 0 with a standard deviation of approximately 0.04. Therefore it can be concluded that 0.4 is not the correct value for C. The correct value for these data C=0.54. This value will be used in further calculations with the Ferguson and Church (2004) formula.

The standard deviation of 0.04 corresponds with a ratio of 1.096. This means that the standard deviation of the data corresponds with a deviation of w_p of 9,6%.

In figure 3.3 the data are plot against the Reynolds number on a logarithmic axis. The plot shows that the data deviate around zero in a sinusoidal shape. From this observation can be concluded that the Ferguson and Church (2004) formula predicts the settling velocity most accurately at Reynolds numbers around 1, 300 and 100,000 and that the data deviate from 0 with a maximum 0.08, which corresponds with a deviation of w_p from w_o of almost 20%. Furthermore this plot shows that the different datasets follow the same trend which means that the deviation from zero is not due to the data, but to a systematic error in the Ferguson and Church (2004) formula.

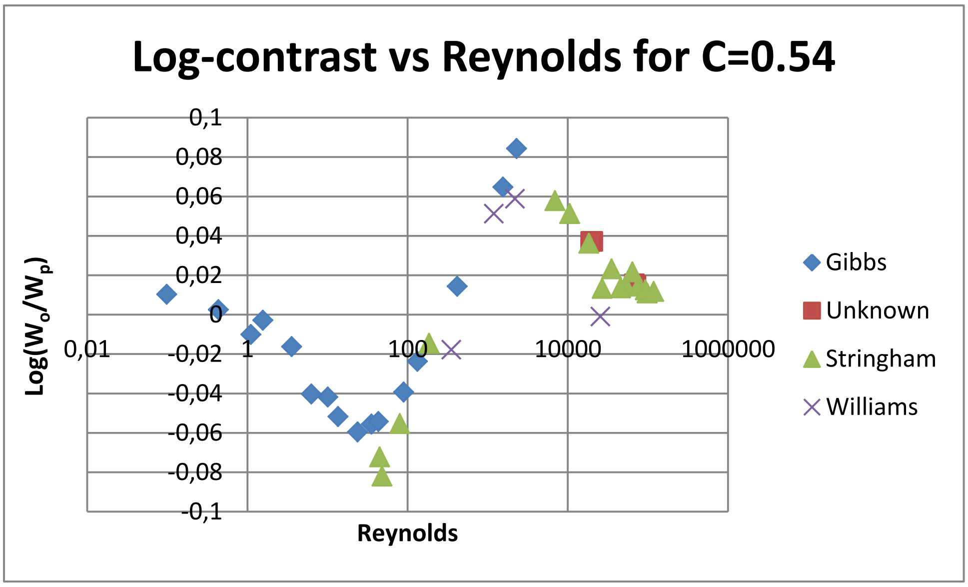

Fig. 3.3: The settling velocity log-contrast for C=0.54 at different Reynolds numbers. This plot shows that the data deviate around zero in a sinusoidal shape

A polynomial trend line was fit to these data to get a better view on the systematic error, shown in figure 3.4. As can be seen from this figure, a sixth order polynomial is needed to fit the data, which is too much for the amount of curves in the data (2 or maybe 3). Therefore this trend line is probably not realistic and more data are needed to fit a proper trend line and to obtain a realistic view on the systematic error of the Ferguson and Church(2004) formula.

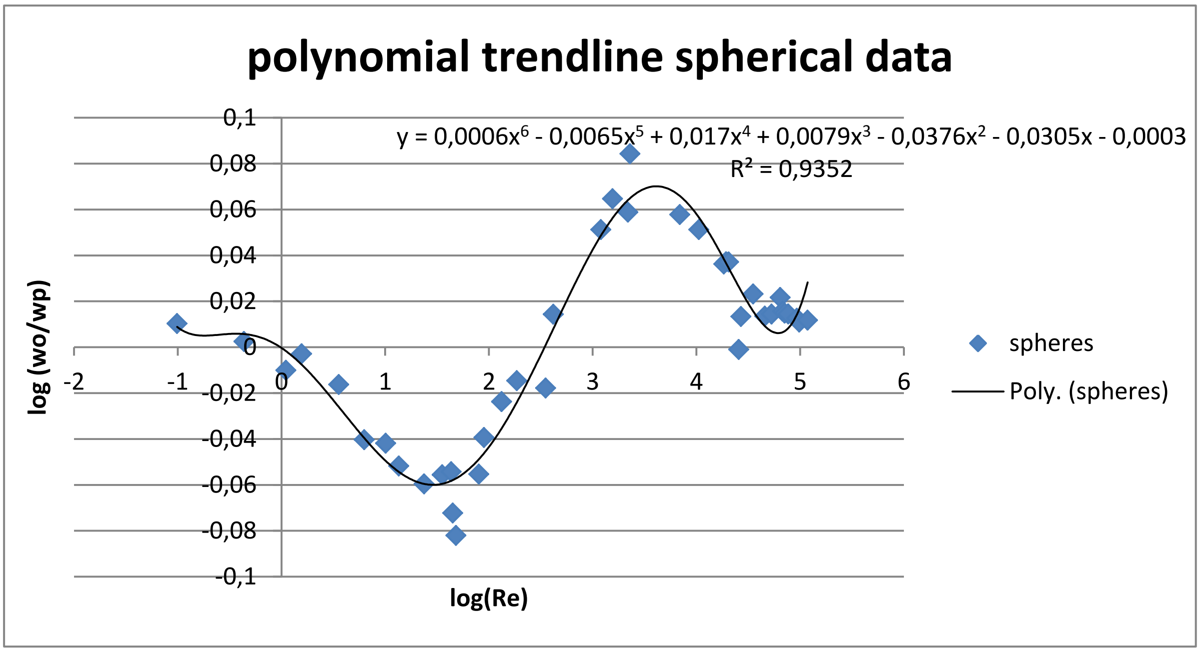

Fig. 3.4: The fitting of a polynomial trend line to the spherical data to get a view on the systematic error. A sixth order polynomial is needed to fit the data with R^2=0.9352

<u>3.3 Conclusions</u>

With spherical data it is proved that the Ferguson and Church (2004) formula can predict the settling velocity of spherical particles fairly accurate when C=0.54. As the data originate from different datasets, this conclusion can be considered valid for smooth spheres with various characteristics. However, only a small amount of data was used, so to be sure about the best value for C2, more research should be done to validate this conclusion.

Furthermore the Ferguson and Church (2004) equation shows to have a systematic error which causes the data to deviate around zero in a sinusoidal shape when the Reynolds numbers are plotted on a logarithmic axis. This means that however quite accurate, the Ferguson and Church (2004) formula does not predict the settling velocity of particles perfectly for each Reynolds number and gives a standard deviation of 9,6%.

4. Settling Velocity of Sieve Diameter Data

An important application of the Ferguson and Church (2004) formula would be in the prediction of settling velocities of particles of which only the sieve diameter is known. Many data are available which use the sieve diameter of natural particles instead of three axes as they are relatively easy to obtain. Therefore those data were investigated as well as to the working of the Ferguson and Church (2004) formula. Also it was investigated whether sieve diameter data can be directly coupled to particle shape and size.

<u>4.1 Data analysis</u>

Ferguson and Church (2004) themselves used several data to test their formula which were not directly available from their article. From these data only the data of Hallermeier (1981) could be found. Hallermeier (1981) collected many data of natural sand particles from previous authors and measured everything that is needed for the Ferguson and Church (2004) formula.

The data of Hallermeier (1981), which consist of 115 measurements of 13 different researches, were put into the Ferguson and Church (2004) formula. Then the log-contrast of the predicted (w_p) and measured (w_o) velocities was taken. Again if the Ferguson and Church (2004) formula is correct, the average value of this log-contrast should be zero. Therefore this was done for a range of values for C. The average and standard deviation of these calculations were plotted against C. The result can be found in figure 4.1.

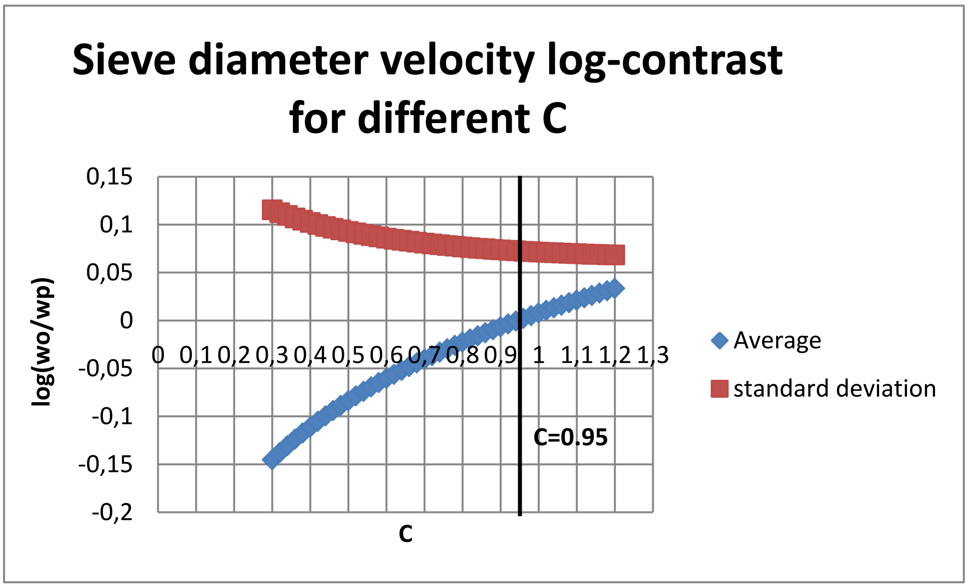

Fig. 4.1: The average and standard deviation of the settling velocity log-contrast of sieve diameter data for different values of C. At C=0.95, the average log-contrast is 0.

As can be seen from figure 4.1, sieve diameter data need a much higher value of C in order to predict the velocity of the particles as good as possible. The optimum of 0 as can be found from the plot is C = 0.95. This gives a standard deviation of 0.073, which gives a difference between w_o and w_p of about 18%. That is quite a large uncertainty. The velocity log-contrast with C=0.95 for different Reynolds numbers is shown in figure 4.2.

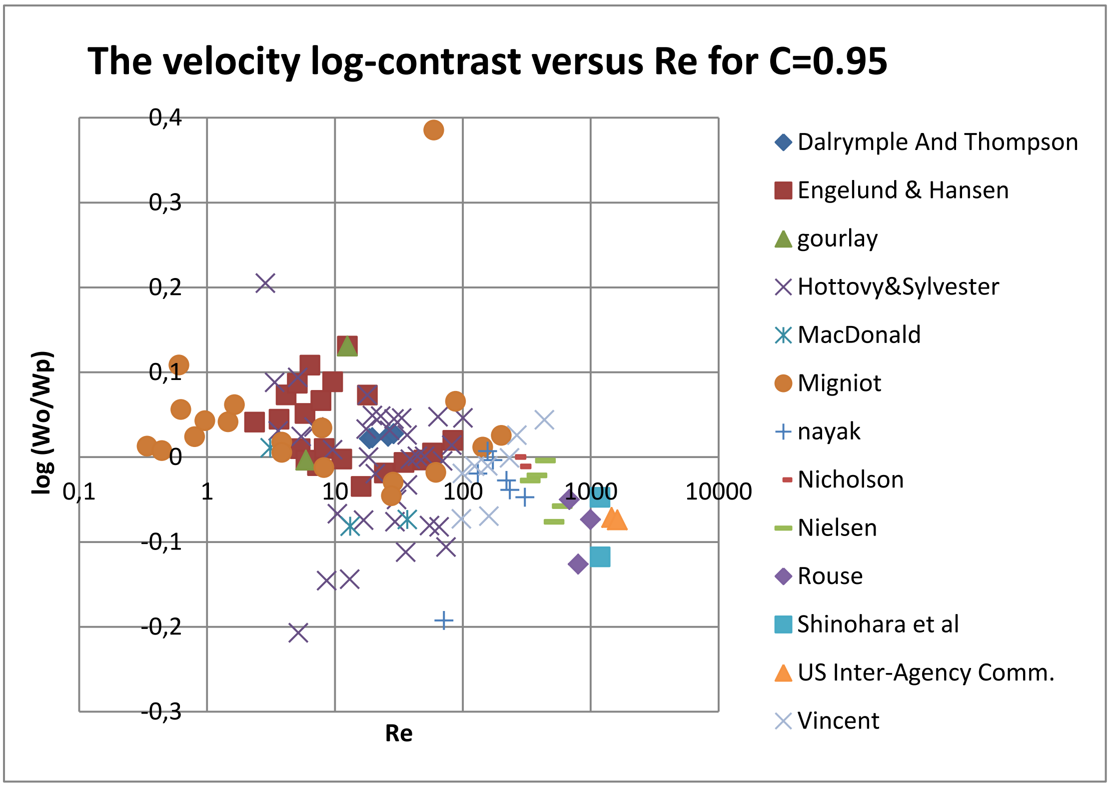

Fig. 4.2: The settling velocity log-contrast of sieve diameter data with C=0.95 for different Reynolds numbers

As can be seen from figure 4.2, the data deviate around zero up to Re=500. After that, the data are a little below zero. However all the data for Re>500 are from very small datasets with only 2 or 3 data points and nothing is known about the research method by which the data were obtained. Therefore nothing can be concluded from this deviated data. Furthermore an uncertainty is present for all data as 13 different researches can produce 13 different measurements of which is not at all sure that the methods were precise.

<u>4.2 The relation between sieve diameter and particle shape and size</u>

As it is much easier to do a sieve analysis instead of measuring all the axes of every particle, the relation between sieve diameters and size and shape of particles has been researched.
Sieving sorts particles both by shape and size. According to Komar and Cui (1984) the sorting is mostly dependent on the intermediate diameter of a particle. However the short axis is also important. Komar and Cui (1984) state that when an ellipsoid is to pass through a square sieve with sides D_{sv}, then the limit of the size is controlled by the grain orientation at which the intermediate axis has a size of D_{sv} to $\sqrt{2}\, D_{sv}$, while the short axis should have a size between D_{sv} and 0 respectively. Any grain in between these ranges can pass through the sieve. Consequently a size and shape range with different ratios of the intermediate and short axis will be left behind on a certain sieve, especially as the long axis can be of any length. This means that the size and shape of a grain cannot be measured directly from sieve analysis.

4.3 Conclusions

From this analysis can be concluded that the Ferguson and Church (2004) equation seems to work for non-spherical particles as well when only the sieve diameter of the particles is known. However with a value of C of 0.54 as used with the spherical data, the predicted velocities are too high. This is probably due to shape and to the fact that sieve diameters do not give a very precise measurement of the particle diameters. With a higher value of C, 0.95, the Ferguson and Church (2004) formula does predict the settling velocity, however the uncertainty is large with a standard deviation of 18%. However as many different data sources have been put together in one analysis without knowing the methods which were used, nothing can be concluded with certainty.

The sieve diameter data cannot be linked directly to shape and size of the particles, as the ratio between the short and intermediate axes play an important role in the sieve sorting process and nothing can be said about the long axis.

5. Non-spherical particles – Shape Factors and Settling Velocity

In the previous sections was investigated how the Ferguson and Church (2004) formula predicts the settling velocity of spheres and sieve diameter data. Non-spherical particles are a lot more complex. This chapter will explain what is known about the shape of non-spherical particles and how that should be translated into an equation for settling velocity.

5.1 Existing Shape Factors and Equations

The effect of shape on the settling velocity of a grain has been studied by many (McNown and Malaika (1950), Williams (1966), Komar and Reimers (1978)). Following these studies shape can be described by two factors: sphericity and angularity, which both have influence on the settling velocity. Baba and Komar (1981) found that the non-sphericity of particles contributes most to a lower settling velocity. However some reduction is also due to the angularity of particles.

To obtain a good relation between the shape of particles and their settling velocity, the manner at which non-spherical particles fall through a fluid first has to be determined. Many researchers (Briggs et al, (1962), Komar and Reimers (1978), and others) observed that ellipsoidal grains tend to settle with their maximum projection area normal to the settling direction. This observation was explained as the result of a pressure gradient which develops on the front of grains oriented oblique to the settling direction. This pressure gradient causes the grains to turn to a position with their maximum projection area normal to the settling direction. However, Stringham et al (1969) observed that when the Reynolds number exceeds certain values dependent on their shape, grains tend to oscillate or rotate about their vertical axis. Moreover he also observed that at very low Reynolds numbers, the grains tend to settle at their initial orientation, not perpendicular to the settling direction. These last two observations complicate the definition of a general relation between particle shape and settling velocity.

In order to quantify the shape of a particle many different shape factors have been introduced which can be divided in static and dynamic shape factors.

Dynamic shape factors deal with the velocity difference between a particle of a certain shape and the volume-equivalent sphere, defined by Le Roux (1997) as:

$$DSF = \frac{w_o}{w_n} \tag{8}$$

In which w_o is the observed velocity for a certain grain and w_n is the settling velocity of the nominal sphere in m/s. The nominal sphere can be calculated with the nominal diameter:

$$D_n = \sqrt[3]{D_l D_i D_s} \tag{9}$$

In which D_l, D_i and D_s are the long, intermediate and short axis of the grain respectively.

Static shape factors do not include movement of the grain but only deal with the sphericity of a grain by estimating grains to be ellipsoids with a long, intermediate and short axis. As most sedimentary particles are rounded by transportation so that their shapes are described best as an ellipsoid, this is probably a good definition of shape. However there are exceptions such as micas. These minerals are very platy and therefore their shape can hardly be said to approximate an ellipsoid, though even for micas three axes can be defined. Apart from this exception, minerals generally have an ellipsoidal shape.

The most important static shape factors from previous authors are described in the next sections.

5.1.1 The Corey Shape Factor (CSF)

Corey and McNown and Mailaika (1950) developed a shape factor in 1950 which since then has been investigated very often (Komar and Reimers (1978), Le Roux (1997)). It was experimentally determined by Komar and Reimers (1978) to be the best shape factor and was used very commonly for many years to describe particle shape. This shape factor was based on the thought that all three axes influence the settling velocity of a particle, the short axis more than the others because that axis causes the largest deviation from a sphere. It is called the Corey Shape Factor (CSF):

$$CSF = \frac{D_s}{\sqrt{D_l D_i}} \tag{10}$$

In which D_s, D_l and D_i are the short, long and intermediate axes of the grain respectively. When the particle is spherical, the axes all have the same length and the CSF is 1.

5.1.2 The Hofmann Shape Entropy (HSE)

Hofmann (1994) introduced another shaper factor, called the Hofmann Shape Entropy (HSE). He reasoned that the shape factor must indeed be a relation between the short, intermediate and long axes of a grain and came up with the following equation:

$$HSE = \frac{p_l \ln(p_l) + p_i \ln(p_i) + p_s \ln(p_s)}{\ln(3)} \tag{11}$$

In which p_l, p_i and p_s are the proportions of the long, intermediate and short axes. For example:

$$p_i = \frac{D_i}{D_l + D_i + D_s} \tag{12}$$

5.1.3 Comparison of Existing Shape Factors by Le Roux (1997)

Le Roux (1997) compared the CSF, HSE and other factors with each other by using the Dynamic Shape Factor. He used existing data and plotted the square of the DSF against the different shape factors. If the shape factors would be exactly right, a straight line could be drawn between all the data points with no scattering at all. Therefore Le Roux (1997) examined which shape factor produced the least scattering. From this it followed that the Corey Shape Factor was not a good factor at all and the Hofmann Shape Entropy was found to give the best results.

5.1.4 The Le Roux Shape Factor (LRSF)

After this comparison Le Roux (2004) came up with another shape factor. He reasoned that if particles fall in a stable position, some function of $(1 - \frac{D_a}{D_b})$ should approach zero if the settling velocity of a particle approaches the settling velocity of a nominal sphere, D_a and D_b being two of the 3 (long, intermediate and short) axes of the grain. That is, changes in $\frac{D_a}{D_b}$ can be considered to be proportional to deviations from the projected equivalent sphere in 2D.

Following the observations made by Briggs et al (1962) and others that most grains tend to fall with their maximum projection area normal to the flow and finding all the other combinations of axes having less correlation, Le Roux (2004) came up with the following shape factor for a ellipsoidal grain, further called the Le Roux Shape Factor (LRSF):

$$LRSF = (1 - \frac{D_s}{D_l})^{2.5} \tag{13}$$

The exponent in the LRSF depends on the shape of a particle as well; if the grain is not ellipsoidal, the exponent will be different from 2.5.

Le Roux (2004) tested the LRSF and found that it was an even better shape factor than the HSE.

5.1.5 Settling Velocity Equations for existing shape factors

In the previous section was shown that there are two empirical shape factors that seem to be quite well in agreement with experimental data: the Hofmann Shape entropy (HSE) and the Le Roux Shape Factor (LRSF). Le Roux (1992, 2002, 2004) found a way to determine an equation between these shape factors and the settling velocity of a particle. He used 5 different empirical formulas for different size ranges and many calculation steps to come to a valid solution. A more detailed explanation about this method can be found in Appendix A.

The method Le Roux (1992, 2002, 2004) used is a very difficult method to predict the settling velocity of a non-spherical particle with many different empirical relations for different dimensionless diameters. The Weltje and Bloemsma (2013) equation provides a more straight-forward way to relate shape to settling velocity.

<u>5.2 A new relation between settling velocity and shape</u>

As stated in chapter 2, the last term in the Weltje and Bloemsma (2013) equation, a term which defines the shape of the particle, is not yet included. When included the equation should become:

$$\log\left(\frac{w_i}{w_j}\right) = \alpha \log\left(\frac{R_i}{R_j}\right) + \beta \log\left(\frac{D_{n,i}}{D_{n,j}}\right) + \gamma \log\left(\frac{SF_i}{SF_j}\right) \tag{14}$$

With $D_{n.i}$ and $D_{n,j}$ the nominal diameters of non-spherical particle i and j, SF_i and SF_j being the shape factors of particles i and j respectively and γ a certain function dependent on the Reynolds number just like α and β.

As the shape factor yet has to be found, it is necessary to simplify this equation. If two particles are compared which have exactly the same nominal diameter and density and settle in the same liquid, the first 2 terms are zero and disappear from the equation. If two particles are compared of which one is the nominal sphere of a certain non-spherical particle, the following equation can be derived:

$$\log\left(\frac{w_o}{w_p}\right) = \gamma \cdot \log(SF) \tag{15}$$

With w_o being the observed settling velocity of the particle {m/s], w_p the predicted settling velocity of the equivalent nominal sphere [m/s] and SF the shape factor of the non-spherical particle. As the shape factor of the nominal sphere is 1, it disappears from the equation. It is important to note that this equation is nothing else but a relation between the dynamic and static shape factors.

As tested in chapter 3, the Ferguson and Church (2004) formula was proved to be able to accurately predict the settling velocity of spheres when C=0.54 is taken. Therefore that formula with C=0.54 can be used to calculate w_p.

Now w_p can be calculated when the non-spherical particle dimensions are known and w_o can be derived from experiments. This means that if it is possible to find a shape factor that produces a single function for γ dependent on the Reynolds number, than a way is found to predict the settling velocity of a particle when its shape is known.

The CSF, HSE and LRSF will be investigated using equation (15) and new ideas on the shape factor will be presented and investigated as well in the next chapters.

6. Non-Spherical Particles – Data Analysis

To be able to test the shape factors in equation (15), data were acquired of prolate and oblate spheroids and ellipsoids all having smooth surfaces. The shape factors as described in chapter 5 were tested on these data.

6.1 Available Data

Various datasets are available for testing equation (15).

6.1.1 Data by Le Roux (2004)

Most of the available data are given and used in the research by Le Roux (2004). The tables which included the data were directly available from his article. Therefore the datasets as given by Le Roux are used. Le Roux investigated ellipsoids and oblate and prolate spheroids. With oblate spheroids, the long and intermediate axes both have the same length. With prolate spheroids the intermediate and small axes have the same length. Below some characteristics of the data are given.

Ellipsoidal Data

Komar and Reimers (1978) measured the settling velocity of smooth, natural ellipsoidal pebbles, of which they measured the long, intermediate and short axes with a micrometer or a caliper, depending on the size. In their data also the densities of the settling medium and the particles and the viscosity of the medium were given, as well as the Reynolds number. The pebbles were dropped in a 33 cm inside diameter PVC cylinder and multiple runs were done with each pebble, the results being very close.
The dataset of Komar and Reimers contained 51 pebbles, of which Le Roux used 49. Two were left out for unknown reasons. Unfortunately the data in the article of Komar and Reimers were not directly available, thus the two ellipsoids that were not used by Le Roux could not be obtained.
Le Roux did not use any other ellipsoidal data.

In figure 6.1 a histogram is shown for the Reynolds numbers which were included in the dataset of Komar and Reimers. The figure shows that the data only include low Reynolds numbers. Hence the data are not diverse which makes analysing the data difficult.

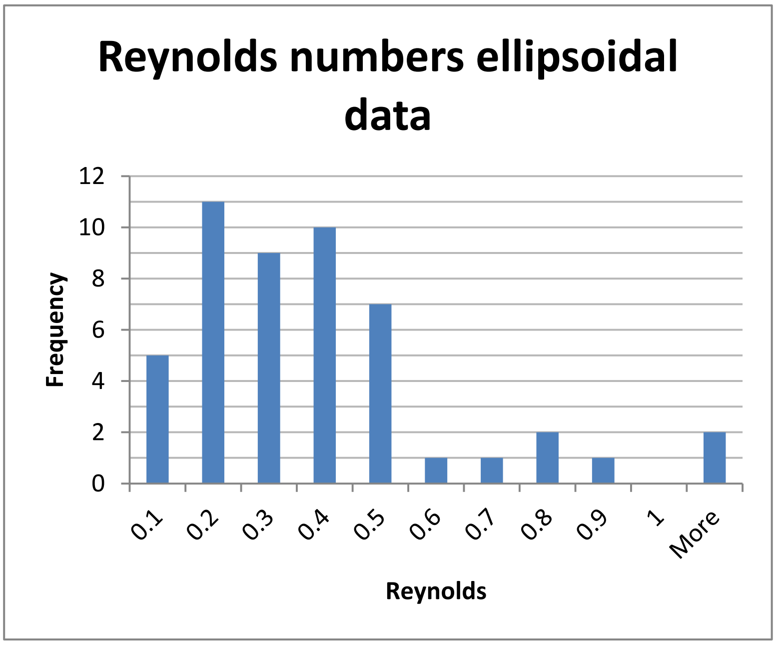

Fig. 6.1: Histogram that shows the spread of Reynolds numbers of the ellipsoidal data. It shows that the data only include very low Reynolds numbers.

Data for Prolate and Oblate Spheroids
Stringham et al (1969) produced data for both oblate and prolate spheroids, which he produced himself, thereby being secured of the density and the length of the axes of the particles. He gave all the characteristics of the particles and the settling medium needed for the calculation. For the experiments Stringham et al (1969) used a column of 40 cm diameter.
Le Roux did not use all of the data from Stringham et al (1969), but only used 36 of the 40 oblate spheroids and 29 of the 40 prolate spheroids for unknown reasons. Unfortunately the unused data were not obtainable from the article of Stringham et al (1969).

Le Roux (2004) added to these data some data from Komar (1980). For unknown reasons he used 4 cylinders from the 29 cylindrical data of Komar (1980) as being prolate spheroids. As cylinders are mostly no good approximation of natural grains as they have sharp edges and are not alike prolate spheroids, it is not clear why Le Roux (2004) used them. Furthermore it is not clear why Le Roux (2004) used exactly those 4 data.

An analysis was made of the data to check what the distribution of the data is considering their Reynolds numbers. In figures 6.2 and 6.3 the results are shown for the prolate spheroids and for oblate and prolate spheroids together. The oblate spheroids are not shown as those data all have the same source.

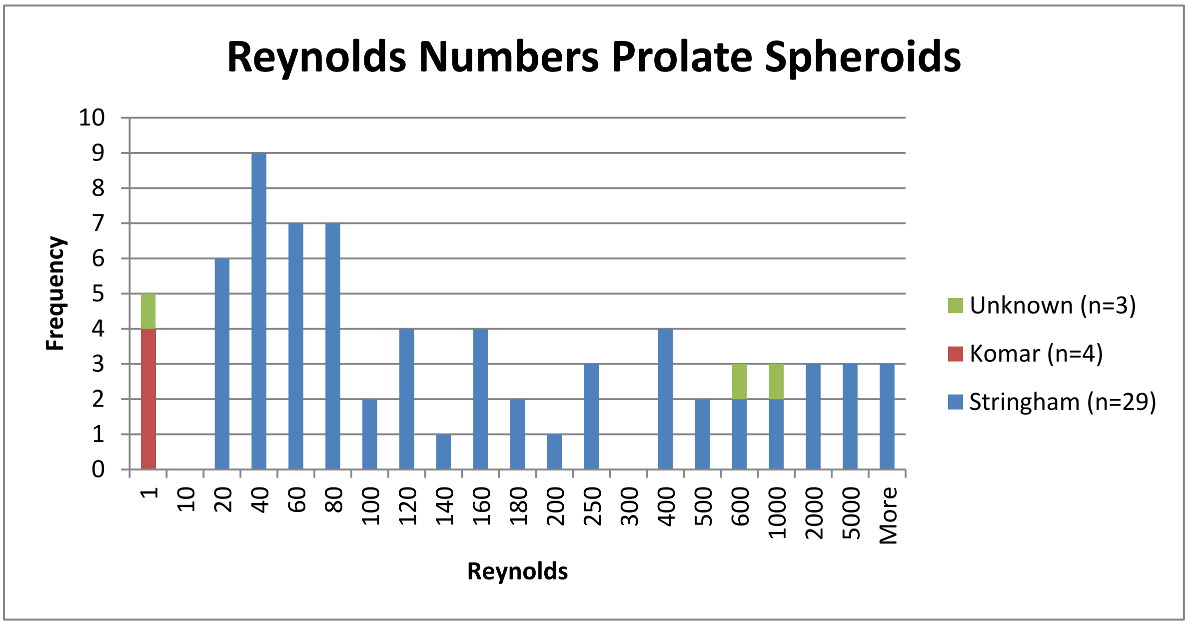

Fig. 6.2: The spread of Reynolds numbers of the prolate spheroids. It shows a data gap between Reynolds numbers of 1 and 10 and a different source for Reynolds numbers smaller than 1.

As can be seen from figure 6.2 the data for the lowest Reynolds numbers are from a completely different dataset than most of the other data. Moreover these data do not have a reliable source as the data from Komar (1980) are supposed to be cylinders and 1 datapoint was not retraceable at all. Also there is a data gap between Reynolds numbers of 1 and 10.

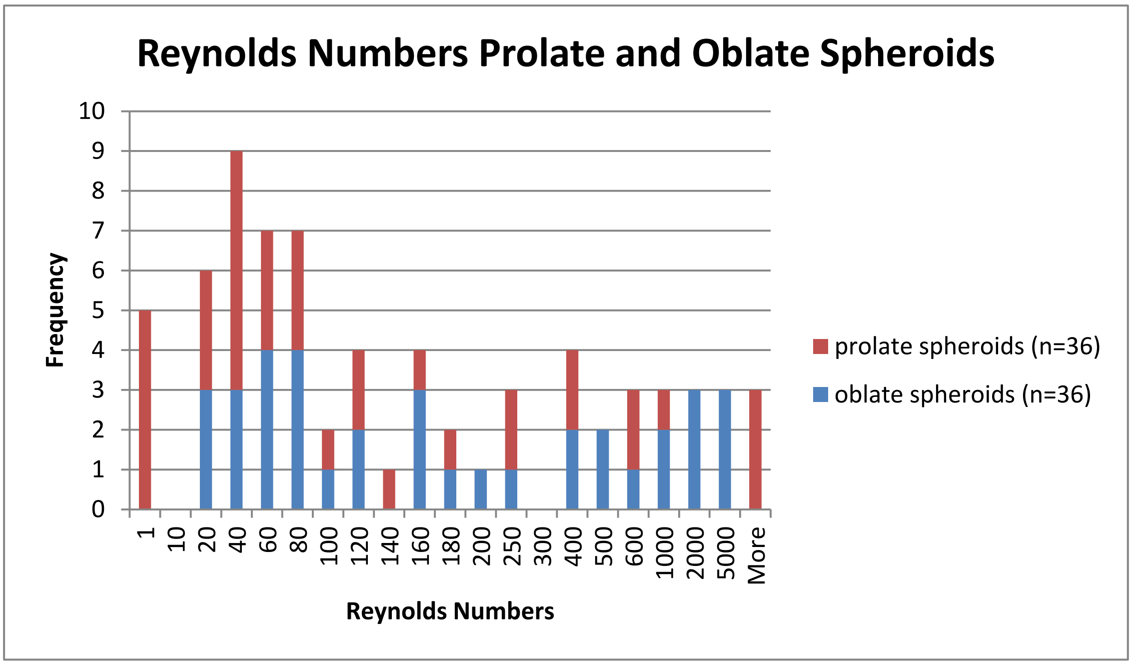

Fig. 6.3: The spread in Reynolds numbers of both prolate and oblate spheroids.

Figure 6.3 shows that again there is a data gap for Reynolds numbers between 1 and 10 and between 250 and 300. Furthermore the very low and very high data only include prolate spheroids. For the data in between the data more or less evenly divided between prolate and oblate spheroids.

6.1.2 Data by Baba and Komar (1981)

Baba and Komar (1981) did settling experiments with glass particles in glycerine. The glass particles were taken from Oregon beaches and were fragments of broken bottles, rounded by the sea in various degrees. They analysed 72 fragments in total.

Unfortunately the article of Baba and Komar (1981) does not clearly define Glycerine as in density and viscosity. Glycerine is usually a mixture of 80% glycerol and 20% glycerine, but it is also often used for the word glycerol itself (Wikipedia, 2012). As nothing is said of a mixture with water, we presume that the authors meant pure glycerol, which has a density of 1.2613 g/cm3.

The authors did not know the precise density of the particles as they only knew the weight and three axes of the particles and could not measure the exact volume, so they measured different types of bottle glass densities and took the average, which is 2.41 (measurements ranged from 2.39-2.44) g/cm3. This can lead to a certain error, especially for the viscosity has to be calculated with this averaged density from the known Reynolds numbers.

As stated above there are some uncertainties is this dataset which can lead to major errors in the analysis of these data. Therefore it is important that these uncertainties are taken into account while interpreting the results of the analysis.

The data are distributed by Reynolds number as given in figure 6.4.

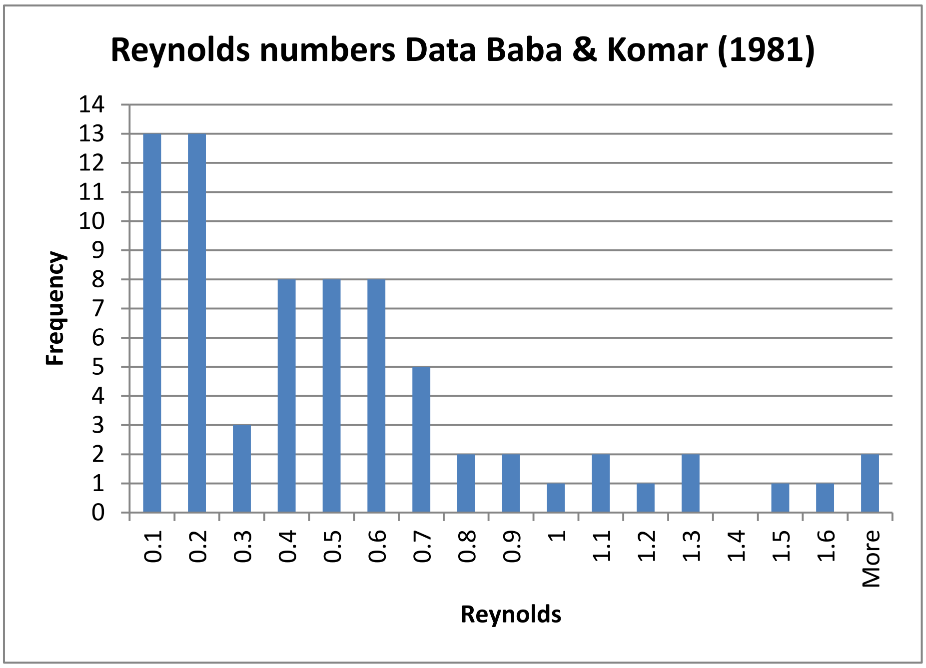

Fig. 6.4: Spread in Reynolds numbers of the ellipsoidal data by Baba and Komar (1981).

From Figure 6.4 it is clear that the data of Baba and Komar (1981) only include very low Reynolds numbers. As these Reynolds numbers are mostly within the Stokes regime, this dataset might not give enough information to be used to validate a certain relation between settling velocity and shape.

6.1.3 Other Datasets

All the data described above include particle materials and different settling media. Other data for fall velocity measurements were available, but all those data were not directly available from the article (Dietrich (1982), Stringham et al (1969)) or did not include measurements of the three axes (Hallermeier (1981), Raudkivi (1990), Van Rijn (1990)). They only included the mean sieve diameter and/or the nominal diameter.

6.2 Testing Shape Factors

The data described in paragraph 6.1 were used to test the various shape factors with equation 15:

$$\log\left(\frac{w_o}{w_p}\right) = \gamma \cdot \log(SF)$$

6.2.1 Method

To decide whether a certain shape factor is correct, the following procedure was followed:
The available data included measured settling velocity, lengths of the three axes, the viscosity and density of the fluid and the density of the particle. First the predicted settling velocity was calculated with equation 4, using the nominal diameter of the particle as explained in section 5.2. The logarithm of the measured settling velocity against the predicted settling velocity was taken. Next the shape factor and its logarithm were calculated. Then γ was calculated by dividing the two logarithms.
Thus γ is:

$$\gamma = \frac{\log\left(\frac{w_o}{w_p}\right)}{\log\ (SF)} \tag{16}$$

To be able to decide whether the shape factor was correct, γ was plotted against the Reynolds number of the particle as γ should be a function of the Reynolds number (section 5.2). If all data follow one single relation, the right shape factor is found.

As the data for prolate and oblate spheroids do not overlap with the data for ellipsoids, first only the spheroidal data were taken. If a shape factor is correct, the data for prolate spheroids should produce the same equation for settling velocity as the data for oblate spheroids.

6.2.2 Testing existing Shape Factors

As the Le Roux Shape Factor and Hofmann Shape Entropy were found to be accurate by Le Roux (1997, 2004), those shape factors were tested first. Furthermore the Corey Shape Factor was tested as according to Le Roux (1997) it was widely used by many authors.

The Le Roux Shape Factor
As the Le Roux Shape Factor was found by Le Roux (2004) to be the best relation, this shape factor was tested first. The results can be found in figure 6.5. Clearly the prolate and oblate spheroids do not follow the same trend, which they should be if the shape factor would have been right. This proves that the LRSF is not an accurate shape factor for the prediction of settling velocity.

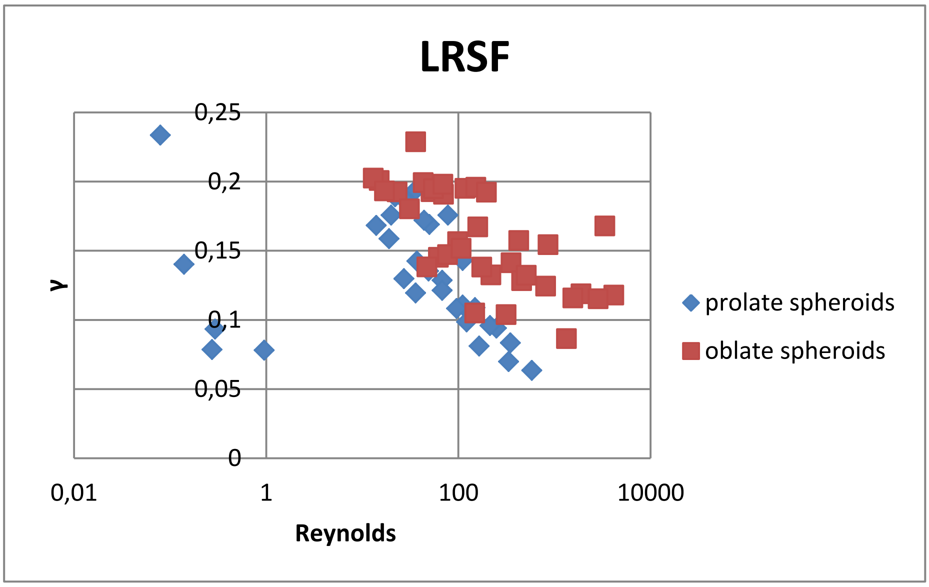

Fig. 6.5: γ versus Reynolds for the Le Roux Shape Factor, applied to the oblate and prolate spheroids. The oblate and prolate spheroids show 2 different trends.

The Hofmann Shape Entropy

The Hofmann Shape Entropy was proved by Le Roux (1997) to be very accurate as well. The results of the analysis of the HSE are in figure 6.6. As is clear from this figure the results are even further apart than the LRSF. The prolate spheroids obviously show a different relation for γ than the oblate spheroids. Therefore it can be concluded that the HSE is not an accurate shape factor either.

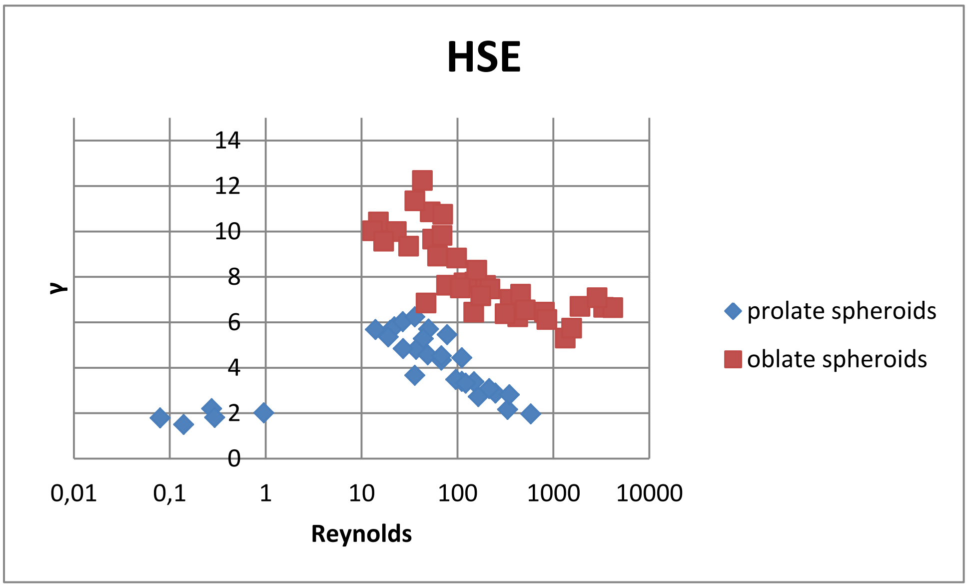

Fig. 6.6: γ versus Reynolds for the Hofmann Shape Entropy, applied to the oblate and prolate spheroids. The oblate and prolate spheroids show 2 different trends.

The Corey Shape Factor

As the Corey Shape Factor has been used very commonly, it is important that this factor is considered as well. The results of the analysis are in figure 6.7. These results show that the CSF, though closer than the LRSF and HSE, still produces 2 different relations for the oblate and prolate spheroids. That means that the CSF is not a very accurate shape factor.

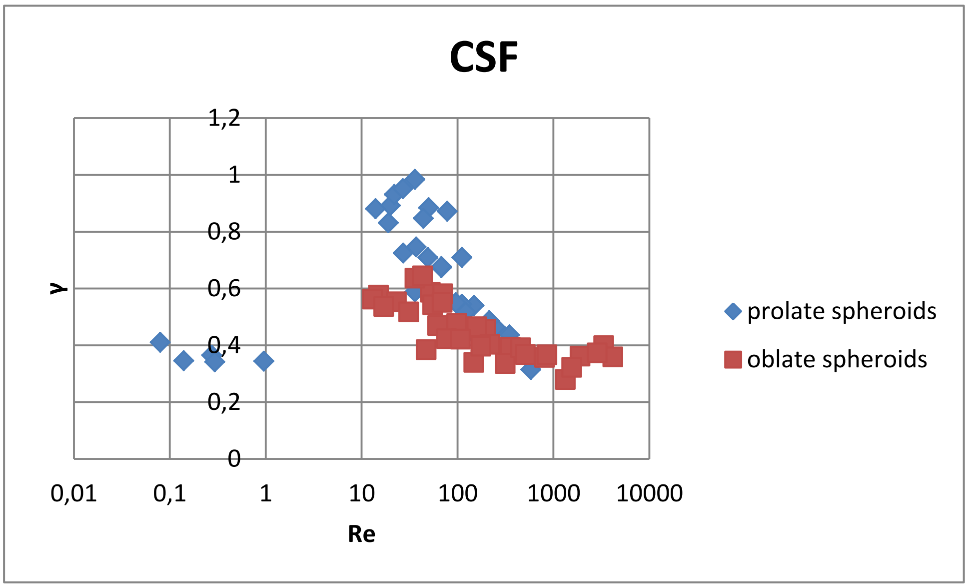

Fig. 6.7: γ versus Reynolds for the Corey Shape Factor, applied to the oblate and prolate spheroids. The oblate and prolate spheroids show 2 different trends.

6.2.3 Testing new Shape Factors

As the existing shape factors clearly do not present the best way to define shape, some new ideas were formed and tested on the data.

The ratio between long and short axis

The deviation from sphericity is most clear from the relation between the long and the short axis of a particle. Therefore the difference in settling velocity between a particle and its nominal sphere might be described best by some function of the ratio between the long and short axis of the particle. When such a shape factor is filled in in equation 15, the following can be obtained:

$$\log\left(\frac{w_o}{w_p}\right) = \gamma \cdot \log\left(\frac{D_l}{D_s}\right) \tag{17}$$

Thus if we can find a fitting relation between γ and the Reynolds number with this shape factor, it is proved that the relation between the settling velocity and shape of a particle can be represented by a function of the ratio of the long and short axis of a particle.

This shape factor was tested for the available data. The results are in figure 6.8.

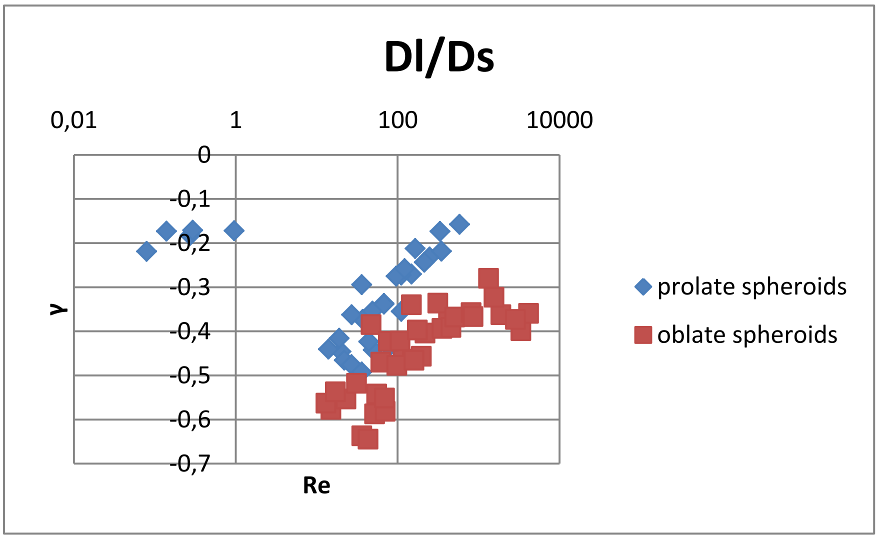

Fig. 6.8: γ versus Reynolds for the ratio of D_l/D_s, applied to the oblate and prolate spheroids. The oblate and prolate spheroids show 2 different trends.

As can be seen clearly there is a different relation between γ and Re for the prolate spheroids than for the oblate spheroids. This means that the two different shapes are not accounted for by just taking the ratio of the long and the short axis. Therefore the suggested definition of shape is not correct and yet another relation must be found.

The Statistical Approach

Another way to look at the shape of a particle is the deviation of a particle from a perfect sphere. In other words that is the deviation of each axis from the nominal diameter. The product or the sum of the deviations of the 3 axes from the nominal diameter might be a good way to define shape:

$$D^* = \prod_{x=l,i,s}(D_x - D_n)^2 \tag{18}$$

Or:

$$D^* = \sum_{x=l,i,s}(D_x - D_n)^2 \tag{19}$$

The relation between shape and settling velocity then might be:

$$\log\left(\frac{w_o}{w_p}\right) = \gamma \cdot \log(D^*) \tag{20}$$

γ again being some function of the Reynolds number.
These two formulas were tested on the data. Figure 6.9 shows the results.
Clearly the prolate and oblate spheroids show different functions. Therefore this definition of the shape is not the correct definition.

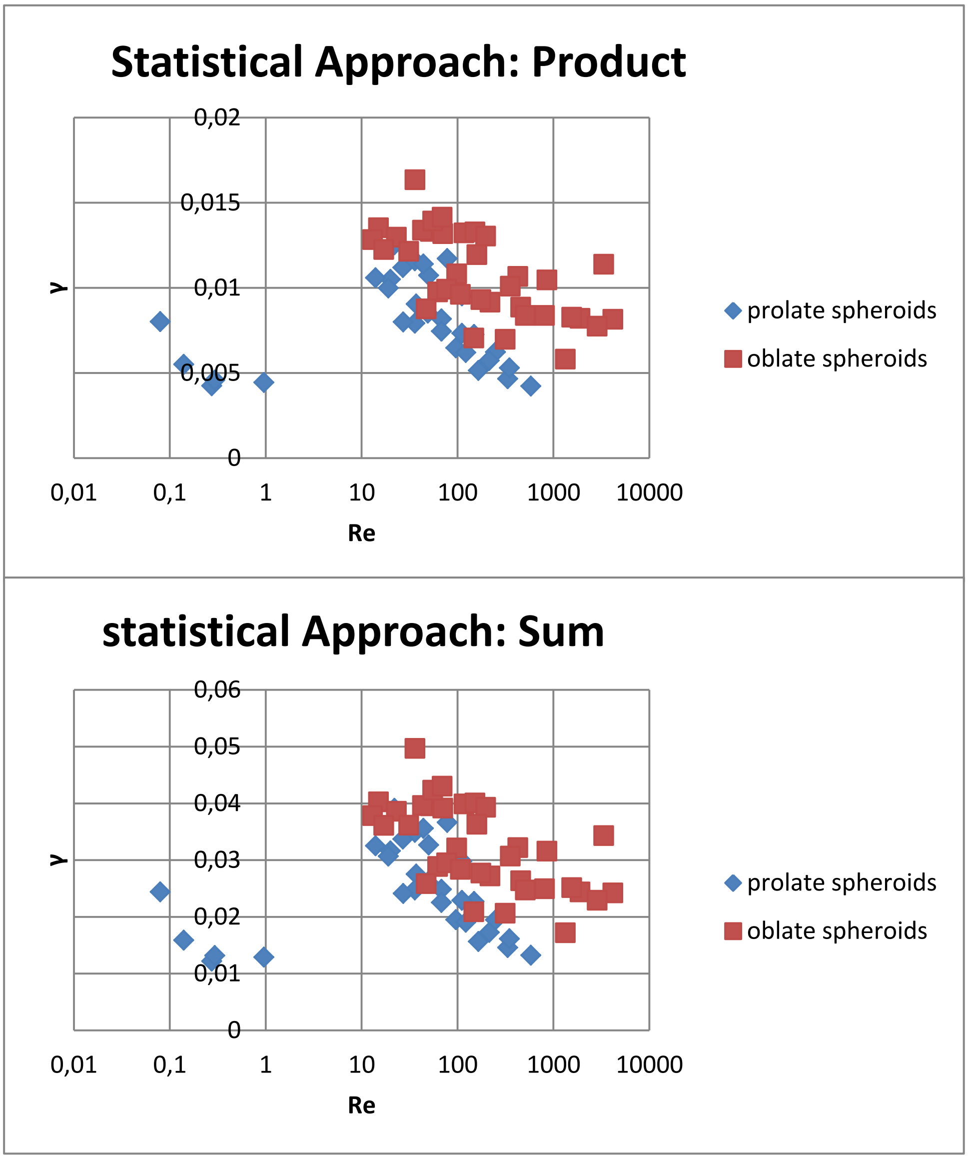

Fig. 6.9: γ versus Reynolds for the statistical approach, the product and the sum, applied to the oblate and prolate spheroids. The oblate and prolate spheroids show 2 different trends.

The Boekhout Shape Factor: a generalization of the Corey Shape Factor

As all the above shape factors did not seem to work, they were examined again.
What is clearly visible is that when the Corey Shape Factor (10) is used, the two datasets of prolate and oblate spheroids seem to be very close to each other (figure 6.7), though they still do not follow the same trend. Thus it might be that the shape factor searched for is a shape factor which uses all three axes.

Therefore first two relations were looked at which only differ from the Corey Shape factor from the position of all the axes:

$$SF = \frac{D_i}{\sqrt{D_l D_s}} \tag{21}$$

$$SF = \frac{D_l}{\sqrt{D_s D_i}} \tag{22}$$

However for equation (21) prolate and oblate spheroids will never be combined into one relation. For with oblate spheroids, $D_l = D_i$, which means that in (21) the shape will always be more than one. With prolate spheroids however $D_i = D_S$, which means that (21) will always turn out to be less than one. As the logarithm of numbers less than 1 is a negative number and the logarithm of number more than one a positive number, these numbers will never combine into one formula for both prolate and oblate spheroids.

Equation (22) could be possible, but as tested it was concluded that also this factor gave significantly different numbers for the different shapes. This can also be explained as a result of the difference in D_i for the two shapes.

Therefore Corey was right in putting the smallest axis in the numerator because then all numbers will be smaller than one.

Hence maybe when the relative contributions of the different axes in the Corey Shape Factor (equation 10) are changed by changing the power of the axes, there might be a match between the two datasets. Of course if the shape is a sphere, there must be no difference so the total power in the denominator as well as in the numerator should be 1.

The Corey Shape Factor may be generalized as follows:

$$BSF = \frac{D_S}{{D_l}^n {D_i}^{1-n}} \tag{23}$$

In which BSF is the new shape factor, called the Boekhout Shape Factor and n is a number between 0 and 1. In the Corey Shape Factor n=0.5. A new n needs to be found which gives a fitting relation between γ and Re as follows:

$$\log\left(\frac{w_o}{w_p}\right) = \gamma \cdot \log(BSF) \tag{24}$$

In an iterative process it is found that for n=0.6 figure 6.10 arises.

In figure 6.10 can be found that the relation between γ and Re could be the same for both prolate and oblate spheroids as the two datasets seem to match each other.

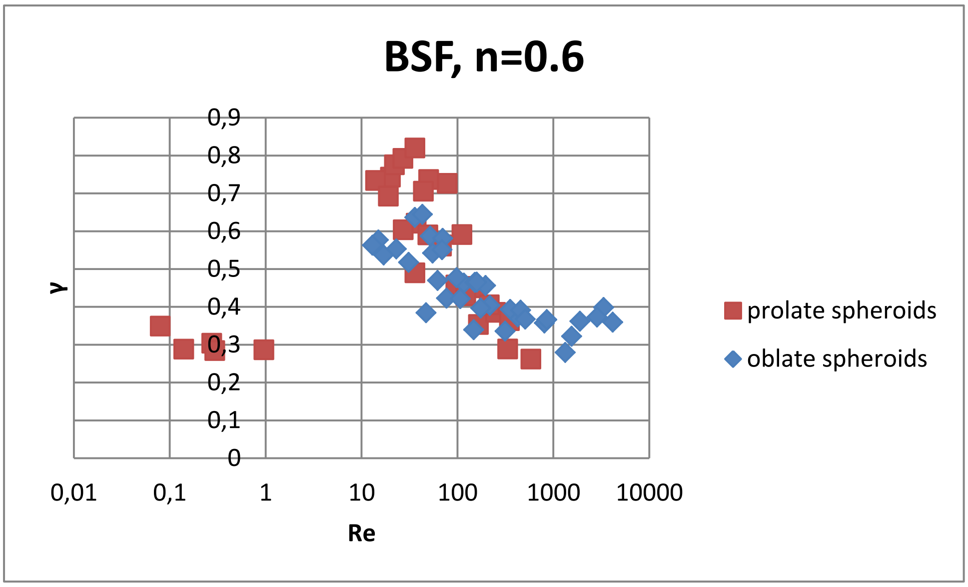

Fig. 6.10: *γ versus Reynolds for the Boekhout Shape Factor, applied to the oblate and prolate spheroids. The two datasets overlap each other.*

Now the relation for shape which gives a fit for spheroids is:

$$BSF = \frac{D_s}{{D_l}^{0.6} D_i^{0.4}} \quad (25)$$

This new shape factor might be an accurate way to define shape at least for both types of spheroids.

6.3 Analysing the Boekhout Shape Factor on all data

To check whether the BSF really is a good general shape factor, the ellipsoidal data from Komar and Reimers (1978) and from Baba and Komar (1981) were included in the analysis. If the BSF is accurate, one single relation most be found between γ and Re for all rounded shapes including ellipsoids. The results including these data can be found in figure 6.11.

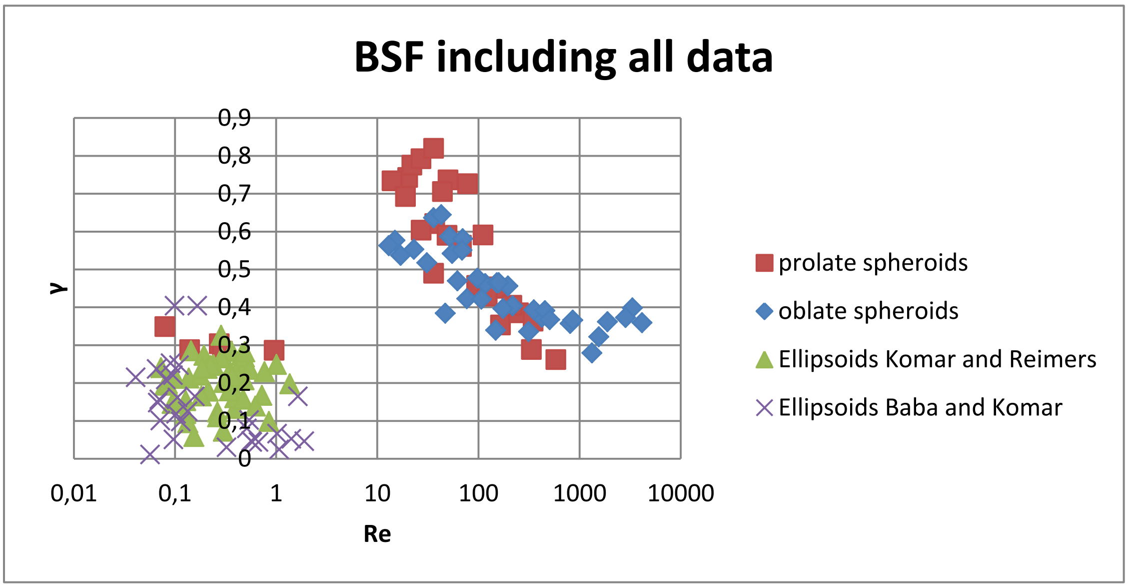

Fig. 6.11: *γ versus Reynolds for the Boekhout Shape Factor, all data included. Two groups of data form. However it cannot be said whether the two are connected for there is a large data gap between Reynolds numbers of 1 and 10*

As can be seen from this figure, the ellipsoidal data only include very low Reynolds numbers, a fact already stated in paragraph 6.1. It is also clear that is does not seem to be related to the spheroidal data in the higher Reynolds regimes, however this is not certain at all as there is a large data gap in the important range of Reynolds numbers between 1 and 10 and only 4 data points of the spheroids overlap the Reynolds range of the ellipsoidal data. Furthermore the ellipsoidal data do not produce a visible trend, but a cloud of data.

What is clearly visible is that the data of Baba and Komar (1981) produce three groups of data on the outside borders of the data of Komar and Reimers(1978). The reasons for this strange arrangement of the data might be found in the data themselves. Therefore the reliability of the data should be investigated. This will be done in chapter 7.

6.4 The Testing Method of Le Roux (1997)

The results from this research are ambiguous as they give opposite results from the research of Le Roux (1997, 2004). Therefore the results were checked with the testing method of Le Roux (1997)

Le Roux (1997) tested various shape factors on their ability to describe shape in order to use in settling velocity equations. He came to a conclusion opposite from this research. Therefore the testing method of Le Roux (1997) was applied to the HSE, LRSF, CSF and BSF. A detailed description of this test can be found in Appendix B. What can be concluded from this test is that Le Roux (1997) came to the wrong conclusion. He left out some essential data which contradict his conclusion that the HSE and LRSF are better shape factors than the CSF. Furthermore this test confirmed that the BSF might be a good shape factor.

Other testing methods to check whether the BSF is a good shape factor were searched for, but unfortunately not found. Formulas for predicting settling velocities for specified shapes such as oblate and prolate spheroids were available (Dressel (1985), lecture notes of Ahmadi, Clarkson University). However they were only valid in the Stokes regime and therefore could not be used in this research to compare the outcomes of those formulas with the BSF.

6.5 Conclusions

From this data analysis some important conclusions can be drawn.

First of all the already existing shape factors as given by Le Roux (1997, 2004), the CSF, HSE and LRSF, seem to be no accurate shape factors to form a single true relation between particle settling velocity and shape.

Secondly the available data are not very good as they produce an important data gap between Reynolds numbers of 1 and 10. Furthermore they come from many different researches which makes comparing them difficult.

Finally the Boekhout Shape Factor (BSF) seems to be an accurate shape factor to predict settling velocity for both types of spheroidal data. The data for ellipsoids do not seem to fit the relation found for spheroids. However the two datasets for spheroids and ellipsoids do not overlap and therefore not much can be said about the accuracy of the shape factor for ellipsoidal data. The test of Le Roux (1997) confirms that the BSF might be a good shape factor.

7. Discussion

The results of this research are largely dependent on the reliability and homogeneity of the data. In chapter 6 was stated that nothing can be concluded with certainty as the data are from many different resources and some of those resources are dubious. This chapter will explain the main problems of the obtained data set and how these problems influence the results.

The results of this research were all obtained with data found directly in only a few articles from a few authors. The data themselves were obtained by many authors. Unfortunately other data were not directly obtainable from tables in the articles and therefore could not be used in this research. This causes three problems.

First of all each author uses different research and measuring methods and as such each research will probably obtain slightly different results. Therefore it might not be very accurate to include so many differently obtained data in one single research. For example it is clear from figure 6.2 that the data for prolate spheroids for Reynolds numbers smaller than 1 might not be valid as they have a completely different source than all the other spheroidal data.

Secondly there were only limited data directly available from the articles: only 36 oblate spheroids, 36 prolate spheroids and 121 ellipsoids. This means that the total amount of data used is not very large, which is the main reason that conclusions cannot be made with certainty as it gives four difficulties.

The first difficulty is that the data for oblate and prolate spheroids do not overlap with the data for ellipsoids. The ellipsoids only include very low Reynolds numbers up to numbers of 1 while the prolate and oblate spheroids only include Reynolds numbers higher than 10. Therefore nothing can ever be concluded with certainty for all shapes as the ellipsoids might follow a completely different trend than the oblate and prolate spheroids.

The second is that there is an important data gap between Reynolds numbers of 1 and 10. No data exist in between these Reynolds numbers, although those Reynolds numbers are very common in normal flow and settling regimes as those numbers present the settling of silt up to medium gravel (sizes between 0.01 mm and 20 mm) with settling velocities between 0.02 and 2 m/s. The data gap could exactly link the data for ellipsoids with the data for spheroids and therefore is very important.

Thirdly all non-spherical grains have nominal diameters between 6.4 and 31 mm. These sizes all can be classified as gravel, which is relatively large. Sizes like sand and silt are not included in the data, but are very important in common sedimentary environments.

The last difficulty is that apparently for both the prolate and the oblate spheroids the ratio between the long and short axis only covers a very small range of numbers. This is further explained in Appendix B. This means that the data cover a very limited amount of shapes and no extreme shapes are included.

Furthermore all the data were obtained from only two different authors.

Most data were obtained from the article of Le Roux (2004). It was already found that Le Roux (2204) made some interesting choices in the data he used as is explained in Appendix B and paragraph 6.1 and some sources of data were not even found. For example he used cylinders as being spheroids without explaining why, thereby only using 4 of them which data were in a Reynolds regime of which he had only 1 other data point from an author which could not be retraced. This example shows that the data might not be trusted at all.

The other ellipsoidal data were obtained from the article of Baba and Komar (1981). The uncertainty associated with their data are quite large for three reasons.

Firstly Baba and Komar (1981) took many different types of rounded glass particles with three different colours and thus characteristics from a beach and averaged the densities. From figure 6.11 it becomes clear that three groups of data form, which could be exactly the three differently coloured particle groups. Those three groups probably have different densities but as the densities were averaged, the results will never be accurate. This might explain the three groups forming.

Secondly the settling experiments were done in glycerine, which properties were not defined well in the article.

Finally the data of Baba and Komar (1981) present a very small Reynolds range which is about the same as the range of the ellipsoidal data from Le Roux (2004), which means these two datasets add no information to the other.

As becomes clear from this analysis of the data is that the results of this research are influenced greatly by the availability and accuracy of the available data. This uncertainty is too important to be overlooked and therefore the conclusions of this research are dubious and cannot be verified.

8. Conclusions

From this research on settling velocities of particles with different shapes, the following can be concluded.

The Ferguson and Church (2004) formula (4) presents a simple and fairly accurate way to predict the settling velocity of spherical particles with a constant C=0.54. However the method does include a systematic error which needs to be solved and which causes the predicted value to deviate from the true settling velocity with a standard deviation of 9.6%.
For sieve diameter data the Ferguson and Church (2004) equation could also be applied with C=0.95. However the equation gives a standard deviation of 18%, which is too much to ignore. As there is no direct link between sieve diameter and grain size and shape, sieving analysis cannot be used to define shape.

As can be obtained from the Weltje and Bloemsma (2013) equation (14), a logical manner to relate particle shape to settling velocity is by using the following logarithmic equation:

$$\log\left(\frac{w_o}{w_p}\right) = \gamma \cdot \log(SF)$$

With w_o being the measured settling velocity of an ellipsoidal grain, w_p the predicted settling velocity of the nominal sphere, SF a certain shape factor and γ a certain function dependent on the Reynolds number. Previously introduced shape factors, such as the Corey Shape Factor and the Le Roux Shape Factor, do not present a single relation for γ for different shapes. However the generalized form of the Corey Shape Factor, the Boekhout Shape Factor, does seem to form a single relation for γ, at least for oblate and prolate spheroids:

$$BSF = \frac{D_s}{{D_l}^n D_i^{1-n}}$$

With BSF being the Boekhout Shape Factor and D_s, D_i and D_l the short, intermediate and long axes of the ellipsoid respectively. This shape factor might be an accurate factor to define shapes of particles with n=0.6. This is confirmed by the test of Le Roux (1997). However nothing can be concluded with certainty as the available data are dubious in many ways, having data gaps, no data overlap for different shapes and many different sources and measuring methods.

The goal of this research was to find a relation between shape and velocity in terms of log ratios using the Ferguson and Church (2004) and the Weltje and Bloemsma (2013) Equations. A shape factor might have been found which could form a relation, however with the amount of data available it is not possible to find a good equation for γ or to be certain about the Boekhout Shape Factor. Therefore, however the first steps have been set in the right direction, the goal has not been reached in the amount of time given for this research and more data should be acquired to come to a solid conclusion.

9. Recommendations

As this research has shed little light on the subject of shape and settling velocity, many more research should be done to uncover the many dark corners of this still largely undiscovered subject.

Firstly research should be done on the reason for the error in the Ferguson and Church (2004) equation. The error is clearly systematic, so there might be a function or parameter that should be added to the equation to correct the error.

However most important is that more settling experiments should be done with both spheres, spheroids and ellipsoids. The available datasets are very small and as such nothing can be concluded with certainty. Especially the non-spherical data contain a very important data gap and no overlap between the spheroids and the ellipsoids. When these data gaps are filled, only then a conclusion can be drawn on the ability of the BSF to describe shape. Therefore a large settling experiment should be set up of ellipsoidal grains in a Reynolds range between zero and 1000 with an uniform data coverage for each logarithmic scale.

The settling experiments should be done with a glass vertical tube with high-definition cameras set up around it which will be able to follow the particles during their whole settling path and form a 3D view of the particles, so that the size and shape can be determined. To obtain a large amount of Reynolds numbers, several different size ranges and fluids should be used. As silt particles might be too small to measure their shape and size with enough precision, sand and gravel should be used in water and in a fluid with a higher viscosity such as oil. It is also important to not just take different sizes but also different ratios between the different axes in order to obtain a large shape range, from flat (D_s/D_l very small) to spherical (D_s/D_l =1) and from platy (D_s/D_i very small) to round (D_s/D_i = 1).

During the experiments the temperature of the fluid should be kept constant so that the fluid viscosity does not change during the experiments. It is also important to design a system with which the particles fall into the water in the same way, so that such variations are limited.

The experimental set-up should be able to measure fluid viscosity and density and particle density and three-axial shape. Furthermore camera should follow the settling path of the particle so that the settling velocity can be determined.

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Websites

http://nl.wikipedia.org/wiki/Glycerol
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Appendix A

The Le Roux method for calculating settling velocity with the HSE and LRSF

Le Roux (1992, 2002, 2004) found a way to determine an equation between the shape factors he found most reliable, the HSE and LRSF, and the settling velocity of a particle.

For the HSE this relation is as follows:

$$w_p = w_n\left(\frac{HSE-0.5833}{0.4167}\right) \tag{A1}$$

And for the LRSF the relation is:

$$w_p = -w_n(0.572(1-\frac{D_s}{D_l})^{2.5}-1) \tag{A2}$$

The difficulty in using these relationships lies in the determination of w_n, the velocity of a nominal sphere, for of course this velocity is as well as the real particle velocity dependent on the flow regime during settling.

Le Roux (2002) found a way to determine w_n with empirical formulas and the use of the dimensionless diameter D_{ds} and dimensionless velocity w_{ds}.

The dimensionless diameter of the nominal sphere is calculated as follows:

$$D_{ds} = D_n\sqrt[3]{\frac{g(\rho_g-\rho_f)}{\mu^2}} \tag{A3}$$

In which ρ_g is the grain density and ρ_f the fluid density. With the dimensionless diameter the dimensionless velocity can be calculated with the following formulas:

$w_{ds} = (0.2354D_{ds})^2$	For	$D_{ds} < 1.2538$	(A4)
$w_{ds} = (0.208D_{ds}-0.0652)^{3/2}$	For	$1.2538 < D_{ds} < 2.9074$	(A5)
$w_{ds} = (0.2636D_{ds}-0.37)$	For	$2.9074 < D_{ds} < 22.9866$	(A6)
$w_{ds} = (0.8255D_{ds}-5.4)^{2/3}$	For	$22.9866 < D_{ds} < 134.9215$	(A7)
$w_{ds} = (2.531D_{ds}+160)^{1/2}$	For	$134.9215 < D_{ds} < 1750$	(A8)

In which w_{ds} is the dimensionless velocity. As can be seen above five different equations are needed for five different ranges of the dimensionless diameter. Now the settling velocity of the nominal sphere can be calculated from the dimensionless velocity with the following formula:

$$w_n = \frac{w_{ds}}{\sqrt[3]{\rho_f^2/\mu g(\rho_g-\rho_f)}} \tag{A9}$$

Appendix B

Comparison of results with research of Le Roux (1997)

Le Roux (1997) compared different shape factors with each other as well, including the Corey Shape Factor and the Hofmann Shape Entropy. What is interesting after what was found in this research, is that he found that the Hofmann Shape Factor was the best shape factor to use, after which he introduced an, according to him, even better shape factor (Le Roux, 2004). Both of these shape factors do not appear to be the best shape factors in this research.

The difference in his conclusion might be caused by the different investigation method and/or by the fact that he only used ellipsoidal data and a few of the data for prolate spheroids. He did not use the oblate spheroids at all. Therefore it is checked whether with the comparison method of Le Roux (1997) the results agree with the results of Le Roux (1997) or with this research. The Le Roux Shape Factor (2004) is also included in this analysis as well as the Corey Shape Factor and the new Boekhout Shape Factor.

For the comparison of shape factors Le Roux (1997) plots the normalized shape factor against the ratio of the observed settling velocity and the settling velocity of an equivalent sphere, which is a modified form of the Dynamic Shape Factor by McCulloch et al (1960).

The settling velocity of an equivalent sphere he calculates via numerous empirical formulas for different ranges of the Reynolds numbers, a method which he developed himself (1992) and which is explained in Appendix A.

This method provides a way to check whether all the different shapes, defined by a certain shape factor, follow the same trend. If so the shape factor is proved to be accurate for defining shape.

While investigating the paper of Le Roux (1997), an error was found in the calculation of the normalized shape factor. In equation (13) of his paper he states that the normalized shape factor for a certain shape factor in a dataset is:

$$SF_n = (SF_{max} - SF_{\mathrm{min})}/(SF - SF_{\mathrm{min}}) \tag{B1}$$

Thereby stating that using this approach all shape factors will lie between 0 and 1 and as such can be compared with each other, while that is harder when you do not normalize the shape factors. Of course this is not a valid statement, for in this approach the normalized shape factors will always be more than 1, as $SF_{max} – SF_{min}$ is always a larger number than $SF – SF_{min}$.

Therefore it is assumed that Le Roux (1997) meant:

$$SF_n = (SF - SF_{\mathrm{min})}/(SF_{max} - SF_{min}) \tag{B2}$$

For then all normalized shape factors in a certain dataset do lie between 0 and 1.

In this investigation the Le Roux (1997) method was used to compare 4 shape factors: the LRSF, HSE, CSF and BSF.
The results of these calculations can be found in figure B1.

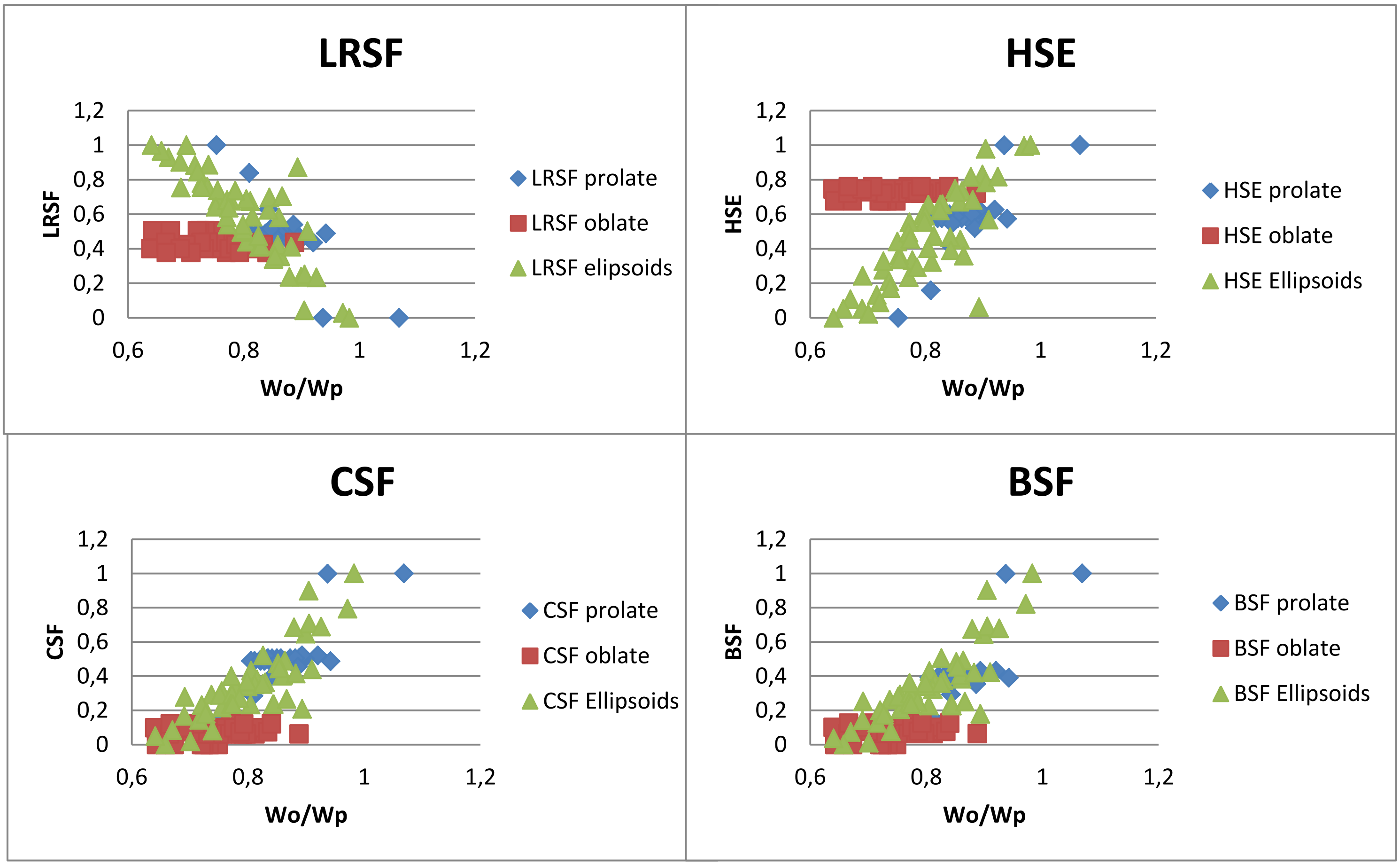

Fig. B1: The Le Roux Shape Factor, Hofmann Shape Entropy, Corey Shape Factor and Boekhout Shape Factor plotted against the ratio of the observed and predicted settling velocity for the oblate and prolate spheroids and ellipsoids. The oblate spheroids fall completely out of the trend with the LRSF and the HSE.

What is conspicuous from these graphs is that apparently for both the prolate and the oblate spheroids the ratio between the long and short axis only covers a very small range of numbers. This was checked in the data and proved to be right. For the prolate spheroids almost all data have a ratio between the long and short axis between 1.95 and 2.1. For the oblate spheroids the ratios between the long and short axis are all between 1.8 and 2.05. From this can be concluded that the dataset might not be diverse enough to form a proper view on the shape.

However what stands out most is that for the LRSF and the HSE the data for oblate spheroids completely fall out of the trend, while for the CSF and the BSF these data do fall approximately within the visible trend.

From these observations can be concluded not only that the CSF and BSF probably are better shape factors than the LRSF and the HSE, but also that Le Roux left an essential part of the available data out of his comparative research, namely the oblate spheroids. Had he included them in his research, the conclusions would have been very different.

The LRSF and HSE might give slightly better results for just ellipsoids and prolate spheroids, but that is not at all the case for oblate spheroids. Therefore these results clearly present the same conclusion as was investigated in the previous chapter: the LRSF and HSE do not provide an accurate way to describe shape, whereas the CSF and BSF might describe the shape of a particle more accurately. Interesting in the work of Le Roux (2004) is that he needs extra variables in the predicted settling velocity calculation with the LRSF and the HSE to define the shape correctly:

$$w_P = w_n\left(\frac{HSE-z}{y}\right) \tag{B3}$$

And:

$$w_p = -w_n(0.572(1 - D_s/D_l)^x - 1) \tag{B4}$$

In which he determines x, y and z experimentally. These numbers defer for each shape Le Roux investigates and therefore he contradicts himself in stating that the HSE or the LRSF are accurate shape factors.

The formulas for settling velocity as given above are not at all simple to apply in predicting the settling velocity as for each particle you need to know much more than just the lengths of the three axes. This is also explained in Appendix A. Furthermore we cannot use these formulas in developing an equation such as (15). Therefore it can be concluded that using the LRSF or the HSE would not be the best way to describe shape.

Appendix C – Data

Ellipsoids (Komar and Reimers, 1978)

										F&C (C=0.54)	
ρs (kg/m^3)	ρ (kg/m^3)	ν (m^2/s)	Dl (m)	Di (m)	Ds (m)	Dn (m)	Wo (m/s)	Re		Wp (m/s)	log (Wo/Wp)
3493	1259.5	0.00169	0.023	0.015	0.005	0.012	0.051	0.362		0.074	-0.165
3095	1263.6	0.00169	0.018	0.013	0.007	0.012	0.050	0.351		0.060	-0.082
3333	1258.2	0.00169	0.022	0.013	0.007	0.013	0.064	0.483		0.077	-0.082
3162	1263.8	0.00169	0.017	0.012	0.008	0.012	0.056	0.391		0.061	-0.035
2769	1258.6	0.00169	0.034	0.017	0.008	0.017	0.085	0.850		0.095	-0.049
3628	1262.5	0.00169	0.023	0.020	0.005	0.013	0.059	0.455		0.089	-0.177
3309	1262.5	0.00169	0.024	0.013	0.007	0.013	0.064	0.498		0.081	-0.103
3622	1261.6	0.00169	0.023	0.015	0.006	0.013	0.064	0.486		0.087	-0.133
2993	1262.7	0.00169	0.022	0.017	0.008	0.014	0.065	0.547		0.080	-0.094
3527	1257.3	0.00169	0.020	0.011	0.006	0.011	0.046	0.288		0.060	-0.119
3248	1255.5	0.00169	0.026	0.015	0.009	0.015	0.083	0.724		0.096	-0.064
2863	1258.9	0.00169	0.018	0.014	0.008	0.013	0.052	0.390		0.059	-0.053
2926	1256.6	0.00170	0.019	0.010	0.006	0.010	0.035	0.214		0.042	-0.074
2925	1257.9	0.00169	0.010	0.007	0.006	0.007	0.021	0.094		0.023	-0.025
3397	1282.1	0.00168	0.012	0.010	0.004	0.008	0.027	0.128		0.031	-0.055
3271	1259.9	0.00169	0.021	0.012	0.006	0.011	0.043	0.286		0.060	-0.148
3355	1264.5	0.00169	0.017	0.012	0.005	0.010	0.044	0.263		0.050	-0.058
3805	1254.5	0.00170	0.017	0.011	0.003	0.008	0.029	0.144		0.043	-0.175
3632	1257.2	0.00169	0.012	0.007	0.003	0.006	0.019	0.073		0.024	-0.104
3404	1258.6	0.00169	0.014	0.010	0.004	0.008	0.028	0.137		0.035	-0.103
3347	1261.5	0.00169	0.017	0.012	0.005	0.010	0.040	0.241		0.051	-0.109
3375	1260.1	0.00169	0.010	0.008	0.004	0.007	0.020	0.079		0.024	-0.078
2870	1262.3	0.00169	0.016	0.010	0.008	0.011	0.041	0.264		0.044	-0.027
3431	1254.1	0.00170	0.011	0.008	0.004	0.007	0.021	0.084		0.025	-0.078
3262	1260.2	0.00169	0.014	0.009	0.004	0.008	0.028	0.134		0.031	-0.043
3721	1258.7	0.00169	0.019	0.010	0.004	0.009	0.036	0.194		0.050	-0.144
3785	1265.4	0.00169	0.014	0.010	0.003	0.007	0.024	0.102		0.032	-0.135
2884	1261.5	0.00169	0.013	0.009	0.006	0.009	0.029	0.154		0.030	-0.014
3545	1258.4	0.00169	0.018	0.011	0.004	0.010	0.037	0.208		0.050	-0.129
3491	1260.2	0.00169	0.013	0.011	0.005	0.009	0.034	0.177		0.042	-0.094
3219	1259.3	0.00169	0.026	0.023	0.009	0.017	0.098	1.007		0.128	-0.114
3377	1258.8	0.00169	0.028	0.021	0.007	0.016	0.083	0.768		0.113	-0.133
3269	1260.3	0.00169	0.020	0.014	0.006	0.012	0.055	0.386		0.066	-0.084
2954	1265.2	0.00169	0.012	0.011	0.007	0.010	0.039	0.223		0.038	0.006

3504	1261.1	0.00169	0.026	0.013	0.006	0.013	0.063	0.490		0.087	-0.138
3464	1258	0.00169	0.019	0.009	0.006	0.010	0.039	0.226		0.049	-0.097
3432	1259.4	0.00169	0.020	0.011	0.006	0.011	0.048	0.302		0.059	-0.092
3297	1256.9	0.00169	0.018	0.015	0.007	0.012	0.064	0.473		0.073	-0.058
3452	1256.9	0.00169	0.018	0.012	0.007	0.012	0.058	0.400		0.069	-0.073
3330	1259.8	0.00169	0.019	0.015	0.006	0.012	0.053	0.369		0.067	-0.101
3311	1263.2	0.00169	0.016	0.014	0.006	0.011	0.052	0.344		0.060	-0.069
3009	1260.6	0.00169	0.017	0.013	0.005	0.010	0.032	0.187		0.042	-0.121
3141	1256.4	0.00169	0.018	0.011	0.006	0.011	0.048	0.302		0.051	-0.026
3261	1260.4	0.00169	0.011	0.009	0.005	0.008	0.027	0.128		0.031	-0.051
3258	1261.9	0.00169	0.020	0.016	0.008	0.014	0.076	0.625		0.086	-0.050
3402	1259.6	0.00169	0.013	0.011	0.005	0.009	0.035	0.183		0.040	-0.065
2779	1260.7	0.00169	0.015	0.011	0.010	0.012	0.050	0.346		0.049	0.007
3233	1257.3	0.00169	0.026	0.022	0.011	0.019	0.124	1.363		0.145	-0.068
3302	1263.6	0.00169	0.011	0.007	0.005	0.007	0.023	0.099		0.026	-0.039

Ellipsoids (Baba and Komar, 1981)

										F&C (C=0.54)	
ρs (kg/m3)	ρ (kg/m3)	nu (m2/s)	Dl (m)	Di (m)	Ds (m)	Dn (m)	Wo (m/s)	Re		Wp (m/s)	log (Wo/Wp)
2410	1261.3	8.30E-04	0.023	0.011	0.010	0.014	0.106	1.786766		0.097	0.038
2410	1261.3	8.30E-04	0.024	0.011	0.007	0.012	0.070	1.023753		0.075	-0.028
2410	1261.3	8.30E-04	0.015	0.011	0.006	0.010	0.063	0.761195		0.054	0.068
2410	1261.3	8.30E-04	0.023	0.011	0.007	0.012	0.080	1.167319		0.075	0.027
2410	1261.3	8.28E-04	0.017	0.011	0.008	0.012	0.069	0.961977		0.069	0.003
2410	1261.3	8.31E-04	0.020	0.008	0.003	0.008	0.034	0.323726		0.035	-0.019
2410	1261.3	8.30E-04	0.017	0.012	0.005	0.010	0.060	0.723986		0.053	0.050
2410	1261.3	8.31E-04	0.020	0.014	0.011	0.014	0.095	1.642477		0.102	-0.033
2410	1261.3	8.30E-04	0.016	0.010	0.006	0.010	0.060	0.728517		0.053	0.051
2410	1261.3	8.30E-04	0.015	0.010	0.005	0.009	0.050	0.537003		0.043	0.066
2410	1261.3	8.30E-04	0.017	0.010	0.005	0.010	0.045	0.514674		0.048	-0.033
2410	1261.3	8.30E-04	0.025	0.019	0.007	0.015	0.105	1.902296		0.110	-0.023
2410	1261.3	8.30E-04	0.023	0.018	0.006	0.014	0.086	1.413642		0.092	-0.029
2410	1261.3	8.30E-04	0.020	0.016	0.004	0.010	0.053	0.67178		0.057	-0.031
2410	1261.3	8.30E-04	0.023	0.016	0.006	0.013	0.081	1.242781		0.081	0.001
2410	1261.3	8.30E-04	0.020	0.016	0.006	0.012	0.073	1.073277		0.075	-0.013
2410	1261.3	8.31E-04	0.016	0.011	0.004	0.009	0.050	0.565454		0.047	0.034
2410	1261.3	8.30E-04	0.019	0.012	0.003	0.009	0.045	0.489096		0.044	0.003
2410	1261.3	8.30E-04	0.016	0.011	0.005	0.010	0.054	0.647328		0.052	0.021
2410	1261.3	8.29E-04	0.012	0.009	0.006	0.009	0.042	0.43843		0.040	0.023
2410	1261.3	8.29E-04	0.020	0.011	0.004	0.010	0.045	0.540451		0.052	-0.058
2410	1261.3	8.30E-04	0.011	0.007	0.002	0.006	0.013	0.090579		0.018	-0.144
2410	1261.3	8.27E-04	0.011	0.004	0.003	0.005	0.013	0.081015		0.015	-0.068
2410	1261.3	8.31E-04	0.017	0.010	0.005	0.010	0.051	0.591179		0.049	0.021
2410	1261.3	8.29E-04	0.011	0.006	0.003	0.006	0.016	0.109872		0.020	-0.101
2410	1261.3	8.30E-04	0.011	0.009	0.005	0.008	0.036	0.32901		0.031	0.065
2410	1261.3	8.32E-04	0.011	0.009	0.005	0.008	0.037	0.344662		0.032	0.060
2410	1261.3	8.29E-04	0.014	0.007	0.006	0.008	0.040	0.404505		0.038	0.018
2410	1261.3	8.30E-04	0.014	0.009	0.006	0.009	0.047	0.501944		0.042	0.047
2410	1261.3	8.30E-04	0.007	0.006	0.003	0.005	0.014	0.086735		0.014	0.003
2410	1261.3	8.30E-04	0.018	0.014	0.006	0.011	0.070	0.951356		0.067	0.019
2410	1261.3	8.29E-04	0.019	0.011	0.005	0.010	0.053	0.632192		0.052	0.008
2410	1261.3	8.30E-04	0.019	0.010	0.005	0.010	0.048	0.567726		0.051	-0.021
2410	1261.3	8.30E-04	0.016	0.006	0.005	0.008	0.039	0.373096		0.035	0.049
2410	1261.3	8.31E-04	0.019	0.009	0.005	0.010	0.049	0.581321		0.051	-0.020
2410	1261.3	8.30E-04	0.014	0.009	0.004	0.008	0.038	0.376307		0.037	0.016
2410	1261.3	8.28E-04	0.011	0.008	0.006	0.008	0.039	0.381595		0.036	0.035
2410	1261.3	8.29E-04	0.016	0.009	0.004	0.008	0.040	0.39198		0.035	0.056
2410	1261.3	8.31E-04	0.018	0.008	0.004	0.008	0.040	0.401706		0.037	0.035

2410	1261.3	8.28E-04	0.014	0.011	0.004	0.008	0.041	0.410789		0.038	0.025
2410	1261.3	8.29E-04	0.014	0.007	0.005	0.008	0.032	0.289889		0.032	0.003
2410	1261.3	8.29E-04	0.013	0.005	0.004	0.006	0.025	0.194984		0.023	0.033
2410	1261.3	8.30E-04	0.011	0.009	0.004	0.007	0.029	0.254796		0.029	0.002
2410	1261.3	8.30E-04	0.014	0.012	0.006	0.010	0.055	0.647943		0.051	0.029
2410	1261.3	8.30E-04	0.015	0.012	0.005	0.010	0.050	0.582136		0.050	0.001
2410	1261.3	8.29E-04	0.013	0.008	0.004	0.007	0.033	0.286217		0.028	0.068
2410	1261.3	8.30E-04	0.011	0.008	0.004	0.007	0.028	0.22852		0.026	0.038
2410	1261.3	8.30E-04	0.015	0.011	0.004	0.009	0.042	0.434301		0.040	0.015
2410	1261.3	8.28E-04	0.011	0.007	0.002	0.006	0.020	0.136289		0.018	0.037
2410	1261.3	8.30E-04	0.015	0.010	0.004	0.008	0.038	0.365599		0.036	0.023
2410	1261.3	8.29E-04	0.011	0.008	0.004	0.007	0.033	0.279523		0.027	0.084
2410	1261.3	8.29E-04	0.015	0.012	0.004	0.009	0.041	0.434528		0.041	0.004
2410	1261.3	8.28E-04	0.013	0.005	0.004	0.006	0.014	0.099018		0.020	-0.152
2410	1261.3	8.29E-04	0.009	0.006	0.005	0.007	0.021	0.166342		0.025	-0.085
2410	1261.3	8.25E-04	0.008	0.006	0.003	0.005	0.012	0.070138		0.014	-0.070
2410	1261.3	8.37E-04	0.008	0.005	0.003	0.005	0.012	0.071775		0.013	-0.032
2410	1261.3	8.28E-04	0.011	0.006	0.004	0.006	0.020	0.157276		0.023	-0.051
2410	1261.3	8.32E-04	0.007	0.007	0.003	0.005	0.017	0.107833		0.016	0.038
2410	1261.3	8.29E-04	0.007	0.005	0.005	0.005	0.017	0.112764		0.017	0.002
2410	1261.3	8.27E-04	0.011	0.005	0.003	0.005	0.019	0.123804		0.017	0.039
2410	1261.3	8.27E-04	0.007	0.005	0.004	0.005	0.016	0.102928		0.017	-0.021
2410	1261.3	8.32E-04	0.009	0.006	0.003	0.006	0.014	0.095738		0.017	-0.081
2410	1261.3	8.31E-04	0.009	0.007	0.004	0.006	0.019	0.133453		0.020	-0.040
2410	1261.3	8.26E-04	0.007	0.005	0.002	0.004	0.010	0.049128		0.009	0.032
2410	1261.3	8.28E-04	0.010	0.007	0.003	0.006	0.017	0.114324		0.019	-0.054
2410	1261.3	8.34E-04	0.009	0.007	0.003	0.006	0.016	0.105654		0.018	-0.071
2410	1261.3	8.31E-04	0.009	0.005	0.003	0.005	0.015	0.096531		0.016	-0.018
2410	1261.3	8.32E-04	0.007	0.006	0.004	0.006	0.019	0.127101		0.018	0.035
2410	1261.3	8.37E-04	0.007	0.005	0.003	0.005	0.011	0.065848		0.013	-0.067
2410	1261.3	8.26E-04	0.008	0.004	0.003	0.004	0.011	0.056685		0.011	-0.004
2410	1261.3	8.25E-04	0.009	0.005	0.002	0.004	0.008	0.04095		0.011	-0.129
2410	1261.3	8.26E-04	0.008	0.004	0.003	0.005	0.012	0.067652		0.013	-0.040

Prolate Spheroids

Stringham et al (1969)
Komar (1980)
Unknown

										F&C (C=0.54)	
ρs (kg/m^3)	ρ (kg/m^3)	ν (m^2/s)	Dl (m)	Di (m)	Ds (m)	Dn (m)	Wo (m/s)	Re		Wp (m/s)	log (Wo/Wp)
10150	931.5	8.27E-07	0.025	0.013	0.013	0.0164	1.78	35222.14		1.978	-0.046
3502	1259.4	1.69E-03	0.014	0.005	0.005	0.0068	0.01981	0.08		0.025	-0.104
10150	931.5	8.27E-07	0.025	0.013	0.013	0.0164	1.78	35222.14		1.978	-0.046
10150	995.9	8.33E-07	0.039	0.019	0.019	0.0243	1.96	57213.71		2.326	-0.074
2810	999.5	8.30E-07	0.025	0.013	0.013	0.016	0.752	14452.01		0.834	-0.045
10150	1252.6	3.52E-04	0.019	0.010	0.01	0.0123	0.572	20.00		0.777	-0.133
10150	1273.7	3.59E-04	0.025	0.013	0.013	0.0164	0.813	37.00		1.047	-0.110
10150	1259	3.56E-04	0.039	0.019	0.019	0.0243	1.142	78.00		1.547	-0.132
2810	1264.5	3.64E-04	0.038	0.019	0.019	0.0239	0.336	22.00		0.465	-0.141
10150	1242	2.83E-04	0.019	0.010	0.01	0.0123	0.621	27.00		0.861	-0.142
10150	1254.1	2.80E-04	0.025	0.013	0.013	0.0164	0.855	50.00		1.153	-0.130
10150	1250.2	2.81E-04	0.039	0.019	0.019	0.0243	1.28	111.00		1.638	-0.107
2810	1254.5	2.77E-04	0.025	0.013	0.013	0.016	0.243	14.00		0.327	-0.130
2810	1256	2.82E-04	0.038	0.019	0.019	0.0239	0.425	36.00		0.522	-0.089
10150	1247.6	2.30E-04	0.039	0.019	0.019	0.0243	1.41	148.99		1.701	-0.082
10150	1262.8	2.32E-04	0.025	0.013	0.013	0.0164	0.965	68.00		1.212	-0.099
10150	1242.8	2.28E-04	0.019	0.010	0.01	0.0123	0.667	36.00		0.936	-0.147
2810	1286.5	2.23E-04	0.025	0.013	0.013	0.016	0.266	19.00		0.352	-0.122
2810	1253.6	2.28E-04	0.038	0.019	0.019	0.0239	0.42	44.00		0.565	-0.128
2810	1231	1.22E-04	0.018	0.010	0.01	0.012	0.276	27.13		0.350	-0.103
2810	1238	1.21E-04	0.025	0.013	0.013	0.016	0.372	49.00		0.473	-0.104
2810	1250.8	1.20E-04	0.038	0.019	0.019	0.0239	0.556	111.00		0.673	-0.083
10150	1220.4	1.21E-04	0.019	0.010	0.01	0.0123	0.944	96.00		1.140	-0.082
10150	1245.1	1.21E-04	0.025	0.013	0.013	0.0164	1.22	165.00		1.408	-0.062
10150	1227.3	1.21E-04	0.039	0.019	0.019	0.0243	1.658	333.99		1.870	-0.052
2810	1215.7	6.25E-05	0.018	0.010	0.01	0.0119	0.357	68.00		0.446	-0.096
2810	1221.6	6.25E-05	0.025	0.013	0.013	0.016	0.478	122.01		0.569	-0.076
2810	1222.7	6.23E-05	0.038	0.019	0.019	0.0239	0.65	249.02		0.764	-0.070
10150	1219.7	6.25E-05	0.019	0.010	0.01	0.0123	1.085	214.01		1.283	-0.073
10150	1231.8	6.24E-05	0.025	0.013	0.013	0.0164	1.328	347.99		1.539	-0.064
10150	1215.1	7.33E-05	0.039	0.019	0.019	0.0243	1.753	581.99		1.956	-0.048
2490	1236	8.33E-04	0.010	0.004	0.004	0.006	0.01926	0.14		0.023	-0.073
2490	1236	8.33E-04	0.011	0.005	0.005	0.0075	0.0303	0.27		0.035	-0.057
2490	1236	8.33E-04	0.012	0.005	0.005	0.0077	0.0317	0.29		0.036	-0.060
2490	1236	8.33E-04	0.017	0.008	0.008	0.0117	0.06823	0.96		0.077	-0.055
1090	997.5	9.12E-07	0.005	0.004	0.004	0.0052	0.1153	658.73		0.101	0.058
1090	997.5	9.12E-07	0.005	0.004	0.004	0.0052	0.1011	577.16		0.101	0.001

Oblate Spheroids (Stringham et al (1969))

										F&C (C=0.54)	
ρs (kg/m^3)	ρ (kg/m^3)	v (m^2/s)	Dl (m)	Di (m)	Ds (m)	Dn (m)	Wo (m/s)	Re		Wp (m/s)	log (Wo/Wp)
10150	1253.7	3.59E-04	0.025	0.025	0.013	0.0204	0.915	52.00		1.322	-0.160
10150	1254.3	3.58E-04	0.039	0.039	0.019	0.0309	1.355	117.00		1.883	-0.143
2810	1261.7	3.65E-04	0.025	0.025	0.013	0.0205	0.267	15.00		0.388	-0.162
2810	1257.3	3.62E-04	0.038	0.038	0.02	0.0313	0.416	36.00		0.632	-0.182
10150	1308	2.74E-04	0.019	0.019	0.01	0.0159	0.74	43.00		1.095	-0.170
10150	1299.1	2.80E-04	0.025	0.025	0.013	0.0204	0.958	70.00		1.379	-0.158
10150	1299.1	2.80E-04	0.039	0.039	0.019	0.0309	1.377	152.00		1.917	-0.144
2810	1251.8	2.69E-04	0.019	0.019	0.01	0.0156	0.224	13.00		0.324	-0.161
2810	1258.1	2.76E-04	0.025	0.025	0.013	0.0205	0.31	23.00		0.443	-0.155
2810	1242.6	2.78E-04	0.038	0.038	0.02	0.0313	0.488	55.00		0.697	-0.155
2810	1223.7	2.32E-04	0.019	0.019	0.01	0.0156	0.253	17.00		0.360	-0.153
2810	1252.6	2.28E-04	0.025	0.025	0.013	0.0205	0.345	31.00		0.482	-0.145
2810	1241.7	2.31E-04	0.038	0.038	0.02	0.0313	0.51	69.00		0.732	-0.157
10150	1242.2	2.31E-04	0.019	0.019	0.01	0.0159	0.9	62.00		1.197	-0.124
10150	1249.2	2.29E-04	0.025	0.025	0.013	0.0204	1.1	98.00		1.483	-0.130
10150	1241.8	2.31E-04	0.039	0.039	0.019	0.0309	1.465	196.00		2.027	-0.141
2810	1233.1	1.20E-04	0.019	0.019	0.01	0.0156	0.362	47.00		0.466	-0.110
2810	1238.5	1.21E-04	0.025	0.025	0.013	0.0205	0.454	77.00		0.597	-0.119
2810	1228.8	1.22E-04	0.038	0.038	0.02	0.0313	0.619	159.00		0.840	-0.133
10150	1230.3	1.21E-04	0.019	0.019	0.01	0.0159	1.13	147.99		1.389	-0.090
10150	1233.6	1.21E-04	0.025	0.025	0.013	0.0204	1.289	217.99		1.660	-0.110
10150	1226.4	1.21E-04	0.039	0.039	0.019	0.0309	1.665	424.99		2.172	-0.115
2810	1214.2	6.23E-05	0.019	0.019	0.01	0.0156	0.427	107.00		0.563	-0.120
2810	1216.8	6.25E-05	0.025	0.025	0.013	0.0205	0.534	175.01		0.690	-0.111
2810	1213.8	6.26E-05	0.038	0.038	0.02	0.0313	0.708	353.99		0.917	-0.112
10150	1222.8	6.27E-05	0.019	0.019	0.01	0.0159	1.238	313.98		1.518	-0.089
10150	1217.1	6.25E-05	0.025	0.025	0.013	0.0204	1.392	455.00		1.779	-0.106
2810	1160.4	1.66E-05	0.019	0.019	0.01	0.0156	0.541	508.11		0.689	-0.105
2810	1145.7	1.64E-05	0.025	0.025	0.013	0.0205	0.647	807.98		0.815	-0.100
10150	1174.1	6.41E-05	0.039	0.039	0.019	0.0309	1.775	855.96		2.303	-0.113
10150	1203	1.66E-05	0.019	0.019	0.01	0.0159	1.39	1329.71		1.647	-0.074
10150	1179.5	1.64E-05	0.025	0.025	0.013	0.0204	1.518	1890.14		1.905	-0.099
10150	1181.4	1.65E-05	0.039	0.039	0.019	0.0309	1.778	3339.13		2.360	-0.123
2810	1175.9	1.64E-05	0.038	0.038	0.02	0.0313	0.812	1550.18		1.003	-0.092
10150	995	8.31E-06	0.025	0.025	0.013	0.0204	1.689	4150.99		2.116	-0.098
10150	992.1	8.29E-06	0.019	0.019	0.01	0.0159	1.485	2850.47		1.863	-0.098

Sieve Diameter Data

Source	Dsieve (m)	R	v (m^2/s)	Wo (m/s)	Re		F&C (C=0.95) Wp (m/s)	log(Wo/Wp)
Dalrymple & Thompson (1976)	4.00E-04	1.65	1.20E-06	0.056	18.63		0.053	0.022
	4.00E-04	1.65	1.16E-06	0.057	19.62		0.054	0.023
	4.00E-04	1.65	9.50E-07	0.062	26.11		0.059	0.025
	4.00E-04	1.65	8.90E-07	0.064	28.76		0.060	0.028
Engelund &Hansen (1967)	1.47E-04	1.65	1.00E-06	0.016	2.35		0.015	0.041
	2.08E-04	1.65	1.31E-06	0.023	3.65		0.021	0.045
	2.08E-04	1.65	1.00E-06	0.028	5.82		0.025	0.052
	2.50E-04	1.65	1.31E-06	0.028	5.34		0.027	0.010
	2.90E-04	1.65	1.31E-06	0.033	7.31		0.034	-0.010
	2.50E-04	1.65	1.00E-06	0.033	8.25		0.032	0.010
	2.90E-04	1.65	1.00E-06	0.039	11.31		0.039	-0.002
	4.20E-04	1.65	1.31E-06	0.050	16.03		0.054	-0.035
	4.20E-04	1.65	1.00E-06	0.058	24.36		0.061	-0.019
	5.90E-04	1.65	1.31E-06	0.077	34.68		0.078	-0.006
	5.90E-04	1.65	1.00E-06	0.084	49.56		0.085	-0.003
	7.60E-04	1.65	1.31E-06	0.100	58.02		0.099	0.005
	7.60E-04	1.65	1.00E-06	0.110	83.60		0.105	0.020
	2.20E-04	1.67	1.00E-06	0.027	5.94		0.027	-0.004
	6.86E-05	0.739	1.60E-07	0.010	4.15		0.008	0.073
	6.86E-05	0.794	1.50E-07	0.011	5.08		0.009	0.087
	6.86E-05	0.857	1.40E-07	0.013	6.37		0.010	0.108
	8.90E-05	0.739	1.60E-07	0.014	7.79		0.012	0.067
	8.90E-05	0.794	1.50E-07	0.016	9.61		0.013	0.089
	8.90E-05	0.857	1.40E-07	0.020	12.52		0.015	0.131
	1.27E-04	0.739	1.60E-07	0.023	17.94		0.019	0.073
	8.90E-05	0.739	1.60E-07	0.014	7.79		0.012	0.067
Gourlay (1980)	2.20E-04	1.67	1.00E-06	0.027	5.94		0.027	-0.004
	8.90E-05	0.857	1.40E-07	0.020	12.52		0.015	0.131
Hottovy & Sylvester (1979)	1.27E-04	0.739	1.60E-07	0.023	17.94		0.019	0.073
	1.27E-04	0.794	1.50E-07	0.023	19.64		0.021	0.049
	1.27E-04	0.857	1.40E-07	0.025	22.86		0.023	0.048
	1.64E-04	0.739	1.60E-07	0.028	29.01		0.026	0.045
	1.64E-04	0.794	1.50E-07	0.031	33.35		0.027	0.046
	1.64E-04	0.857	1.40E-07	0.031	36.78		0.030	0.026
	2.13E-04	0.739	1.60E-07	0.033	44.20		0.033	0.001
	2.13E-04	0.794	1.50E-07	0.035	50.27		0.035	0.001
	2.13E-04	0.857	1.40E-07	0.042	64.05		0.038	0.048

	2.73E-04	0.739	1.60E-07	0.041	69.62		0.041	-0.005
	2.73E-04	0.794	1.50E-07	0.045	82.08		0.044	0.014
	2.73E-04	0.857	1.40E-07	0.052	100.43		0.046	0.046
	4.60E-04	0.25	9.50E-07	0.011	5.18		0.017	-0.207
	5.40E-04	0.25	9.50E-07	0.015	8.64		0.021	-0.146
	6.50E-04	0.25	9.50E-07	0.019	13.07		0.027	-0.144
	2.60E-04	0.4	1.00E-06	0.013	3.38		0.011	0.088
	2.10E-04	1.6	1.00E-06	0.026	5.46		0.025	0.024
	8.60E-04	0.03	1.00E-06	0.008	6.88		0.007	0.035
	6.40E-04	0.12	1.00E-06	0.015	9.60		0.015	0.009
	6.00E-04	0.4	1.00E-06	0.028	16.80		0.033	-0.074
	1.48E-03	0.03	1.00E-06	0.014	20.72		0.015	-0.020
	4.90E-04	1.6	1.00E-06	0.062	30.38		0.070	-0.050
	1.18E-03	0.12	1.00E-06	0.025	29.50		0.030	-0.076
	7.40E-04	0.59	1.00E-06	0.050	37.00		0.054	-0.033
	1.10E-03	0.4	1.00E-06	0.050	55.00		0.060	-0.081
	2.00E-03	0.12	1.00E-06	0.037	74.00		0.047	-0.106
	1.37E-04	1.66	1.00E-06	0.021	2.86		0.013	0.205
	4.83E-04	0.393	1.00E-06	0.022	10.48		0.025	-0.067
	1.07E-03	0.054	1.00E-06	0.017	17.57		0.015	0.033
	5.20E-04	1.65	1.00E-06	0.075	38.91		0.075	-0.002
	9.15E-04	0.393	1.00E-06	0.039	35.79		0.051	-0.112
	1.22E-03	0.392	1.00E-06	0.054	65.39		0.065	-0.082
	4.20E-04	1.61	1.00E-06	0.064	26.88		0.060	0.031
	1.75E-04	1.65	1.00E-06	0.021	3.61		0.019	0.031
	1.90E-04	1.65	1.00E-06	0.027	5.13		0.022	0.094
	3.60E-04	1.65	1.00E-06	0.051	18.36		0.051	0.000
MacDonald (1977)	5.50E-04	1.65	1.00E-06	0.067	36.85		0.079	-0.074
	1.50E-04	1.65	8.40E-07	0.018	3.13		0.017	0.011
	3.00E-04	1.65	8.40E-07	0.037	13.18		0.045	-0.082
Migniot (1977)	3.00E-04	0.29	8.90E-07	0.011	3.84		0.011	0.018
	2.00E-04	1.66	1.14E-06	0.022	3.84		0.022	0.006
	4.25E-04	1.65	8.90E-07	0.060	28.56		0.064	-0.029
	2.40E-04	1.65	1.00E-06	0.033	7.92		0.030	0.035
	3.90E-04	0.46	1.00E-06	0.021	8.19		0.022	-0.012
	4.60E-04	1.65	1.00E-06	0.060	27.60		0.067	-0.045
	6.30E-04	0.46	1.00E-06	0.094	59.22		0.039	0.385
	1.10E-03	0.38	1.00E-06	0.056	61.60		0.058	-0.018
	1.20E-03	0.38	1.00E-06	0.073	87.60		0.063	0.066
	8.90E-05	1.65	1.31E-06	0.005	0.34		0.005	0.013
	8.90E-05	1.65	1.00E-06	0.007	0.62		0.006	0.056
	1.26E-04	1.65	1.31E-06	0.010	0.96		0.009	0.043
	1.47E-04	1.65	1.31E-06	0.013	1.46		0.012	0.042
	1.26E-04	1.65	1.00E-06	0.013	1.64		0.011	0.062

	8.20E-05	1.65	1.00E-06	0.005	0.44	0.005	0.008
	7.50E-05	1.65	8.40E-07	0.007	0.60	0.005	0.109
	1.00E-04	1.65	1.00E-06	0.008	0.80	0.008	0.024
	1.25E-03	1.65	1.31E-06	0.150	143.13	0.146	0.012
	1.25E-03	1.65	1.00E-06	0.160	200.00	0.151	0.026
Nayak (1970)	1.80E-03	1.65	1.31E-06	0.170	233.59	0.186	-0.039
	1.80E-03	1.65	1.00E-06	0.170	306.00	0.190	-0.047
	1.55E-03	0.34	1.00E-06	0.046	71.30	0.072	-0.193
	3.99E-04	0.739	1.60E-07	0.053	131.42	0.055	-0.019
	3.99E-04	0.794	1.50E-07	0.059	156.41	0.058	0.007
	3.99E-04	0.857	1.40E-07	0.060	172.14	0.061	-0.004
	5.49E-04	0.739	1.60E-07	0.064	219.60	0.068	-0.028
Nicholson (1968)	5.49E-04	0.794	1.50E-07	0.071	260.96	0.071	0.000
	5.49E-04	0.857	1.40E-07	0.073	285.48	0.075	-0.011
Nielsen (1979)	7.16E-04	0.739	1.60E-07	0.075	336.97	0.080	-0.028
	7.16E-04	0.794	1.50E-07	0.080	379.96	0.084	-0.022
	7.16E-04	0.857	1.40E-07	0.087	442.90	0.087	-0.004
	1.01E-03	0.739	1.60E-07	0.082	517.63	0.098	-0.076
	1.01E-03	0.794	1.50E-07	0.089	599.27	0.102	-0.058
Rouse (1938)	1.01E-03	0.857	1.40E-07	0.095	681.75	0.106	-0.050
	1.44E-03	0.739	1.60E-07	0.089	798.30	0.119	-0.126
	1.44E-03	0.794	1.50E-07	0.104	998.40	0.123	-0.074
Shinohara et al (1959)	1.44E-03	0.857	1.40E-07	0.115	1182.86	0.128	-0.047
	1.84E-03	0.739	1.60E-07	0.103	1184.50	0.135	-0.118
US Inter-Agency Committee (1957)	1.84E-03	0.794	1.50E-07	0.119	1459.73	0.140	-0.071
	1.84E-03	0.857	1.40E-07	0.123	1616.57	0.146	-0.074
Vincent (1958)	1.24E-03	0.59	1.00E-06	0.080	99.20	0.084	-0.019
	1.50E-03	0.4	1.00E-06	0.065	97.50	0.077	-0.073
	1.00E-03	1.6	1.00E-06	0.124	124.00	0.127	-0.011
	1.60E-03	0.59	1.00E-06	0.098	156.80	0.100	-0.010
	2.00E-03	0.4	1.00E-06	0.080	160.00	0.094	-0.070
	2.00E-03	0.59	1.00E-06	0.116	232.00	0.116	-0.001
	1.50E-03	1.6	1.00E-06	0.177	265.50	0.167	0.026
	1.99E-03	1.6	1.00E-06	0.219	435.81	0.198	0.044

Spheres (Le Roux, 2004)

Stringham et al (1969)	
Gibbs et al (1971)	
Williams (1966)	
Unknown	

F&C (C=0.54)

ρs (kg/m^3)	ρ (kg/m^3)	v (m^2/s)	Dl (m)	Di (m)	Ds (m)	Dn (m)	Wo (m/s)	Re	Wp (m/s)	log (Wo/Wp)
2488	998.2	1.00E-06	5.0E-05	5.0E-05	5.0E-05	5.0E-05	0.002	0.099	0.0019	0.0103
2488	998.2	1.00E-06	8.4E-05	8.4E-05	8.4E-05	8.4E-05	0.005	0.436	0.0052	0.0025
2488	998.2	1.00E-06	1.2E-04	1.2E-04	1.2E-04	1.2E-04	0.009	1.105	0.0096	-0.0101
2488	998.2	1.00E-06	1.3E-04	1.3E-04	1.3E-04	1.3E-04	0.012	1.563	0.0119	-0.0029
2488	998.2	1.00E-06	1.8E-04	1.8E-04	1.8E-04	1.8E-04	0.020	3.573	0.0203	-0.0163
2488	998.2	1.00E-06	2.3E-04	2.3E-04	2.3E-04	2.3E-04	0.027	6.290	0.0297	-0.0403
2488	998.2	1.00E-06	2.8E-04	2.8E-04	2.8E-04	2.8E-04	0.036	10.121	0.0395	-0.0419
2488	998.2	1.00E-06	3.2E-04	3.2E-04	3.2E-04	3.2E-04	0.042	13.547	0.0474	-0.0518
2488	998.2	1.00E-06	4.2E-04	4.2E-04	4.2E-04	4.2E-04	0.057	23.762	0.0656	-0.0596
2488	998.2	1.00E-06	5.0E-04	5.0E-04	5.0E-04	5.0E-04	0.071	35.415	0.0808	-0.0556
2488	998.2	1.00E-06	5.5E-04	5.5E-04	5.5E-04	5.5E-04	0.079	43.285	0.0895	-0.0543
2488	998.2	1.00E-06	7.8E-04	7.8E-04	7.8E-04	7.8E-04	0.115	89.548	0.1257	-0.0393
2488	998.2	1.00E-06	9.5E-04	9.5E-04	9.5E-04	9.5E-04	0.140	132.496	0.1479	-0.0237
2755	998.2	1.00E-06	1.7E-03	1.7E-03	1.7E-03	1.7E-03	0.251	417.081	0.2425	0.0144
2240	998.2	1.00E-06	4.0E-03	4.0E-03	4.0E-03	4.0E-03	0.390	1555.678	0.3364	0.0647
2240	998.2	1.00E-06	5.0E-03	5.0E-03	5.0E-03	5.0E-03	0.461	2294.764	0.3794	0.0843
2650	998	9.80E-07	2.0E-02	2.0E-02	2.0E-02	2.0E-02	0.983	20463.297	0.9023	0.0372
5170	998	9.80E-07	1.4E-02	1.4E-02	1.4E-02	1.4E-02	1.313	19324.095	1.2053	0.0372
2650	998	9.80E-07	4.7E-02	4.7E-02	4.7E-02	4.7E-02	1.420	67865.580	1.3695	0.0156
7680	998.2	1.00E-06	1.9E-02	1.9E-02	1.9E-02	1.9E-02	1.854	35277.105	1.7575	0.0232
5170	998	9.80E-07	3.6E-02	3.6E-02	3.6E-02	3.6E-02	1.961	71035.525	1.8947	0.0149
7680	998.2	1.00E-06	2.5E-02	2.5E-02	2.5E-02	2.5E-02	2.096	53036.498	2.0276	0.0144
7900	998.2	1.00E-06	3.8E-02	3.8E-02	3.8E-02	3.8E-02	2.586	97895.327	2.5216	0.0110
14950	998.2	1.00E-06	2.5E-02	2.5E-02	2.5E-02	2.5E-02	3.030	76670.128	2.9307	0.0145
14950	990.5	8.28E-07	2.5E-02	2.5E-02	2.5E-02	2.5E-02	3.030	92979.105	2.9434	0.0126
7900	1004.8	8.36E-07	3.8E-02	3.8E-02	3.8E-02	3.8E-02	2.586	117992.688	2.5170	0.0117
7680	1026.9	8.28E-07	2.5E-02	2.5E-02	2.5E-02	2.5E-02	2.096	64247.349	1.9940	0.0217
7680	958	7.72E-07	1.9E-02	1.9E-02	1.9E-02	1.9E-02	1.854	45701.797	1.7971	0.0135
2150	990.3	8.28E-07	2.6E-02	2.6E-02	2.6E-02	2.5E-02	0.875	26914.724	0.8485	0.0133
1140	991.2	8.27E-07	1.9E-02	1.9E-02	1.9E-02	1.9E-02	0.299	6909.369	0.2617	0.0578
2150	987.1	8.31E-07	1.9E-02	1.9E-02	1.9E-02	1.9E-02	0.800	18375.468	0.7360	0.0362
1140	988.8	8.29E-07	2.5E-02	2.5E-02	2.5E-02	2.5E-02	0.344	10567.793	0.3057	0.0512
2150	1257.8	3.54E-04	3.8E-02	3.8E-02	3.8E-02	3.8E-02	0.445	48.044	0.5375	-0.0820
7680	1264.9	3.56E-04	1.9E-02	1.9E-02	1.9E-02	1.9E-02	0.839	44.861	0.9909	-0.0723
7680	1270.2	3.62E-04	2.5E-02	2.5E-02	2.5E-02	2.5E-02	1.140	79.912	1.2949	-0.0553

7900	1245	3.70E-04	3.8E-02	3.8E-02	3.8E-02	3.8E-02	1.800	185.698		1.8615	-0.0146
1170	997.5	9.12E-07	3.2E-03	3.2E-03	3.2E-03	3.2E-03	0.100	352.591		0.1045	-0.0178
2480	997.5	9.12E-07	3.1E-03	3.1E-03	3.0E-03	3.0E-03	0.358	1197.835		0.3185	0.0512
1150	997.5	9.12E-07	9.5E-03	9.5E-03	9.5E-03	9.5E-03	0.210	2189.941		0.1835	0.0588
2490	997.5	9.12E-07	2.5E-02	2.5E-02	2.5E-02	2.5E-02	0.943	25599.431		0.9456	-0.0010