

Safety analysis of passing maneuvers using extreme value theory

Farah, Haneen; Azevedo, Carlos Lima

DOI

[10.1016/j.iatssr.2016.07.001](https://doi.org/10.1016/j.iatssr.2016.07.001)

Publication date

2017

Document Version

Final published version

Published in

IATSS Research

Citation (APA)

Farah, H., & Azevedo, C. L. (2017). Safety analysis of passing maneuvers using extreme value theory. *IATSS Research*, 41(1), 12-21. <https://doi.org/10.1016/j.iatssr.2016.07.001>

Important note

To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy

Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.



Safety analysis of passing maneuvers using extreme value theory



Haneen Farah^{a,*}, Carlos Lima Azevedo^b

^a Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands

^b Massachusetts Institute of Technology, 77 Massachusetts Ave, 2139 Cambridge, MA, United States

ARTICLE INFO

Available online 25 July 2016

Keywords:

Road safety
Probabilistic model
Extreme value
Driving behavior
Minimum time-to-collision

ABSTRACT

The increased availability of detailed trajectory data sets from naturalistic, observational, and simulation-based studies, is a key source for potential improvements in the development of detailed safety models that explicitly account for vehicle conflict interactions and various driving maneuvers. Despite the well-recognized research findings on both crash frequency estimation and traffic conflict analysis carried out over the last decades, only recently researchers have started to study and model the link between the two. This link is typically made by statistical association between aggregated conflicts and crashes, which still relies on crash data and ignores heterogeneity in the estimation procedure. More recently, an extreme value (EV) approach has been used to link the probability of a crash occurrence to the frequency of conflicts estimated from observed variability of crash proximity, using a probabilistic framework and without using crash records.

In this study the Generalized Extreme Value distribution used in the block maxima (BM) approach and the Generalized Pareto Distribution used in the peak over threshold approach (POT), are tested and compared for the estimation of head-on collisions in passing maneuvers. The minimum time-to-collision with the opposite vehicle is used in both EV methods. Detailed trajectory data of the passing, passed and opposite vehicles from a fixed-based driving simulator experiment was used in this study. One hundred experienced drivers from different demographic strata participated in this experiment on a voluntary basis. Several two-lane rural highway layouts and traffic conditions were considered in the design of the driving simulator scenarios. Raw data was collected at a resolution of 0.1 s and included the longitudinal and lateral positions, speeds and accelerations of all vehicles in the scenario. From this raw data, both methods were tested for stationary and non-stationary models. The latter allowed not-only for a better modeling performance in estimating the number of expected crashes, but also for a quantified analysis of the detailed driving choices affecting the head-on crash probability during passing maneuvers. The estimation results showed that the BM approach yielded more stable results compared to the POT approach, but the latter was able to produce crash rate estimates more consistently sensitive to the covariates of interest. Finally, the estimated distributions were validated using a second set of data extracted from an additional driving simulator experiment.

The results indicate that this is a promising approach for safety evaluation. On-going work of the authors will attempt to generalize this method to other safety measures related to passing maneuvers, test it for the detailed analysis of the effect of demographic factors on passing maneuvers' crash probability and for its usefulness in a traffic simulation environment.

© 2016 International Association of Traffic and Safety Sciences. Production and hosting by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

1. Introduction

The literature has frequently addressed the advantages of using surrogate safety measures over crash data [1], especially nowadays when advanced sensing technologies which facilitate the collection of

detailed data on vehicles' trajectories are becoming readily available [2]. Crash data suffer from underreporting and frequently poor quality. Crashes are also infrequent, the ratio between conflicts and actual crash frequencies, according to Gettman et al. [3], is generally in the range of thousands to 1 (depending on the definition of conflict). Furthermore, the use of crash data is a reactive approach while using surrogate safety measures is a proactive and time-efficient approach [4]. Finally, the use of crash data to develop safety models is often carried out in an aggregated manner, limiting the insights on heterogeneous crash causations and on the details of driver crash avoidance behaviors. As a result, the use of surrogate safety measures for modeling and estimating safety is considered as a promising approach to achieve

* Corresponding author at: Department Transport & Planning, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands. Fax: +31 15 27 87956.

E-mail addresses: h.farah@tudelft.nl (H. Farah), cam@mit.edu (C.L. Azevedo).

Peer review under responsibility of International Association of Traffic and Safety Sciences.

those targets and have a clear advantage over the use of crash data. Zheng, Ismail, and Meng [5] indicate that the validity of a surrogate safety measure is usually determined by its correlation with crash frequency which is usually assessed using regression analysis. For example, Sayed and Zein [6] found a statistically significant relationship between crashes and conflicts with an R^2 in the range of 0.70–0.77 at signalized junctions. However, the regression analysis still incorporates the use of crash counts which are known to suffer from availability and quality issues. Besides, it is difficult to insure the stability of the crash-to-surrogate ratio and this relationship also hardly reflects the physical nature of crash occurrence [5]. Jonasson and Rootzén [7] concluded that comprehensive and generalized answer to the question “are near-crashes representative for crashes?” may be less useful. Instead careful separate analyses for different types of situations are needed. Recently Songchitruksa and Tarko [8] developed a new and more sophisticated approach based on the extreme value (EV) theory to estimate crash probability based on specific crash proximity measures. The field of EV theory was pioneered by Fisher and Tippet [9]. It is a commonly applied theory in many fields, such as in meteorology, hydrology, and finance [5]. However, Songchitruksa and Tarko [8] indicate that its application in the field of transportation engineering is still limited. According to Tarko et al. [1] the EV approach has three considerable advantages over the aggregated traffic conflict technique: (1) The EV theory abandons the assumption of a fixed coefficient converting the surrogate event frequency into the crash frequency; (2) the risk of a crash given the surrogate event is estimated for any condition based on the observed variability of crash proximity without using crash data; (3) the crash proximity measure precisely defines the surrogate event. This method has the potential to estimate the probability of extreme events from relatively short period of observations and it proposes a single dimension to measure the severity of surrogate events and to identify crashes. The implicit assumption of the EV theory is that the stochastic behavior of the process being modeled is sufficiently smooth to enable extrapolation to unobserved levels [8]. In the context of road safety, the more observable traffic events are used to predict the less frequent crashes, which are often unobservable in a short time period [5]. More recently, Songchitruksa and Tarko [8] used an EV approach to build up relationships between occurrence of right-angle crashes at urban intersections and frequency of traffic conflicts measured by using post-encroachment time. A major improvement of this study is that it links the probability of crash occurrence to the frequency of conflicts estimated from observed variability of crash proximity, using a probabilistic framework and without using crash records. The generic formulation of the application of EV to road safety analysis was then proposed by Tarko [2] and it was only recently applied to other crash types and data sets [5,7].

In this study the time-to-collision or TTC [10] will be used as a surrogate safety measure of the risk to be involved in a head-on collision with the opposite vehicle while passing on two-lane rural highways. According to NHTSA [11] head-on collisions constitute 2.3% of the total crashes on two-lane highways, but they are responsible for 10.4% of the total fatal crashes. Not many studies have focused on the detailed analysis of the link between passing maneuvers and head-on-collisions. The TTC was previously used by Farah et al. [12] to evaluate the risk of passing behavior on two-lane rural highways. The authors defined the minimum TTC, as the remaining gap between the passing vehicle and the opposite vehicle at the end of the passing process. This measure expresses the risk involved in the passing maneuver. The authors developed a Tobit regression model that explains the minimum TTC. Traffic related explanatory variables were found to have the most important effect on the minimum TTC, but also the road geometric design and the driver characteristics were also found to have a significant contribution. Other researchers also used the TTC as a measure for head-on conflicts in studies with a similar purpose [13,14].

There are two families of EV distributions which follow two different approaches to sample extreme events: (1) the Generalized Extreme

Value (GEV) distribution which is used in the block maxima or minima (BM) approach, where maxima over blocks of time (or space) are considered; and (2) the Generalized Pareto Distribution (GPD) which is used in the peak over threshold approach (POT) [15], where all values above some certain level are used. Previous studies suggested that the POT approach is more effective in conditions of short-time observations and from the aspect of estimation accuracy and reliability [5,8]. In this study both distributions will be examined and compared.

2. Research method

This section presents: (1) the modeling approach and (2) the laboratory experiment designed to collect the data, including description of the characteristics of the participants in the study, and a preliminary statistics of the collected data.

2.1. Modeling details

In this study two families of extreme value distributions are used to sample extreme events: (1) BM approach using the GEV distribution; and (2) POT approach using the GPD. The following paragraphs describe those two approaches in more detail.

2.1.1. Block maxima (BM) using the generalized extreme value (GEV)

In the GEV distribution the extreme events are sampled based on the block maxima (BM) approach. Following this approach the observations are aggregated into fixed intervals over time and space, and then the extremes are extracted from each block by identifying the maxima in each single block. Mathematically, the standard GEV function is as follows [5]:

$$G(x) = \exp\left(-\left[1 + \xi\left(\frac{x - \mu}{\sigma}\right)\right]^{\frac{1}{\xi}}\right) \quad (1)$$

where, $\{X_1, X_2, \dots, X_n\}$ is a set of independently and identically distributed random observations with unknown distribution function $F(x) = \Pr(X_i \leq x)$, the maximum $M_n = \max\{X_1, X_2, \dots, X_n\}$ will converge to a GEV distribution when $n \rightarrow \infty$. Three parameters identify this distribution: the location parameter, $-\infty < \mu < \infty$; the scale parameter, $\sigma > 0$; and the shape parameter, $-\infty < \xi < \infty$. If the shape parameter, ξ , is positive, then his would yield the Frechet Cumulative Distribution Function (CDF) with a finite lower endpoint, $(\mu - \sigma/\xi)$, if ξ is negative, this will yield the (reversed) Weibull CDF with finite upper endpoint $(\mu + \sigma/|\xi|)$, and if $\xi = 0$ this yields the Gumbel CDF.

The BM method can also be used to study minima by considering the maxima of the negated values instead of minima of the original values. This is how the minimum TTC is handled in this study.

For the BM approach, and in the case that most blocks have enough observations, the r -largest order statistics is often recommended. It enables the incorporation of more than one extreme from each interval in order to increase the confidence of parameter estimates. Yet, this consideration depends not only on the nature of the phenomenon being modeled, but also on the sample available for estimation. It is usually recommended to have at least a sample of 30 maxima (or minima). The size of the chosen interval should be large enough so that there are enough observations from which a maxima is chosen in which it is truly an extreme value, and small enough to provide a sample larger than 30.

2.1.2. Peak over threshold (POT) using the Generalized Pareto Distribution (GPD)

According to the GPD an observation is identified as an extreme if it exceeds a predetermined threshold. The distribution function of exceedances X over a threshold μ for a set of independently and identically distributed random observations $\{X_1, X_2, \dots, X_n\}$ is: $F_u(x) = \Pr(X - u \leq x | X > u)$. With a high enough

threshold u , the conditional distribution $F_u(x)$ can be approximated by a GPD. The function of GPD is given as follows:

$$G(x) = 1 - \left[1 + \left(\frac{\xi \cdot x}{\sigma} \right) \right]^{-\frac{1}{\xi}} \quad (2)$$

where $\sigma > 0$ is the scale and $-\infty < \xi < \infty$ is the shape parameter, respectively.

Similarly to the BM approach, the determination of the threshold in the POT approach determines the sample size. Therefore, an optimal threshold should be chosen so that the observations that exceed the threshold are real extremes, but still constitute a reasonable sample size with relatively small variance. Choosing a small threshold will bias the results by considering normal observations as extremes, while choosing a high threshold would result in few observations as extremes and thus large variability which would also bias the estimation results of the distribution.

In this study, both models' parameters were estimated using the maximum likelihood method (ML) in R (v3.0.3) using the *extRemes* and *evd* packages [16]. Details on the statistical properties of the GEV and GPD can be found in Coles [17] and on the theoretical background of its applicability for surrogate safety analysis in Tarko [1,2].

2.1.3. Examination of the EV criteria

When using the EV approach there are three main criteria that should be examined and addressed. These are: the sample size, serial dependency, and non-stationarity [5]. With respect to the sample size, in the BM approach the interval size determines the sample size while in the POT approach, the chosen threshold is the main factor. In both approaches the target is to achieve a balance between bias and variance as discussed above. In the case of passing maneuvers, it is possible to assume that the TTCs resulting from different passing maneuvers are independent if only cases where a single vehicle is overtaking another single vehicle are considered. However, since these maneuvers are non-stationary as various factors (road design, traffic conditions, driver characteristics) might affect the measured TTCs and increase the heterogeneity, several covariates should also be tested in the estimation procedure.

2.1.4. Estimation of the risk of passing maneuvers using EV

A passing maneuver is considered to be a risky maneuver as it requires from a fast driver, who wants to pass a slow driver, to search and decide on an appropriate gap in the traffic on the opposite direction and execute this maneuver while maintaining safe distances from all the surrounding vehicles. Therefore, a driver failure to correctly estimate these safe distances might lead for several potential types of collisions, such as a collision with the opposite vehicle, the passed vehicle, or run of the way crashes. This paper will focus on the risk of head-on collisions (i.e., a collision with the opposite vehicle).

A quite often used measure for estimating the risk of a head-on collision is the TTC. The TTC is defined by Hayward [18] as the time left to collision between two vehicles if they remain on their paths and continue with constant speeds. Minderhoud and Bovy [19] defined two TTC indicators for risk. The first is the Time Exposed Time to Collision which is the total sum of the times that a driver spent with sub-critical TTC. The second is the Time integrated TTC which is the time integration of the difference between the critical and actual TTC during the time spent with sub-critical TTC. In this study, the minimum TTC to the opposite vehicle at the end of the passing maneuver will be used as a head-on collision proximity measure [10]. This is actually the most critical time-to-collision during a passing maneuver. This measure has been used by several previous studies [12,20,21], and proved to be a valuable measure for risk of head-on collisions.

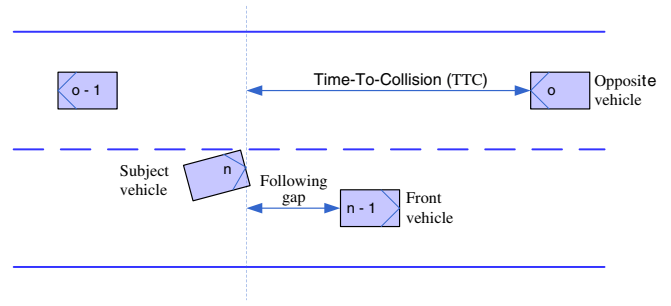


Fig. 1. TTC with the opposite vehicle.

2.2. Laboratory experiment

A laboratory experiment using a driving simulator previously developed by Farah et al. [12] for modeling drivers' passing behavior on two-lane highways was used in order to collect data on the time-to-collision with the opposite vehicle. The simulator used in this experiment, STISIM [22], is a fixed-base interactive driving simulator, which has a 60 horizontal and 40 vertical display. The driving scene was projected onto a screen in front of the driver. The simulator updates the images at a rate of 30 frames per second. The situations that participants encountered were defined by the vehicles shown in Fig. 1. The subject vehicle is passing an impeding vehicle (front vehicle) while another vehicle is approaching from the opposite direction. This paper focuses on the minimum TTC surrogate safety measure while passing on two-lane rural highways. Mathematically, the TTC is calculated by the division of the distance between the fronts of the subject vehicle and the opposite vehicle by the sum of their speeds. The minimum TTC is the TTC value at the end of a successful passing maneuver.

To understand how various infrastructure and traffic factors affect the TTC when passing, a number of simulator scenarios were designed. Each scenario included 7.5 km of two-lane rural highway section, designed on a level terrain, and with no intersections. Daytime and good weather conditions were assumed, which allowed good visibility. However, each scenario design varied according to four main factors of two levels each. The choice of these factors was based on previous studies that showed their significant impact on passing decisions. Two levels were used for each factor. These factors are: speed of the front vehicle (60 or 80 km/h); speed of the opposite vehicle (65 or 85 km/h); opposite lane traffic volume (200 or 400 veh/h); and road curvature, lane and shoulder width (300–400 m, 3.75 m, and 2.25 m or 1500–2500 m, 3.30 m, 1.50 m, respectively). The determination of sight distance in the driving simulator was attempted, but because of the limited resolution of the screens (compared to human eye resolution in reality), this factor was not found to have an impact on the driver behavior. This produces (2^4) 16 different scenarios. The partial confounding method [23] was used to allocate for each driver 4 scenarios out of the 16 scenarios. Detailed information on this experiment can be found in Farah et al. [12].

Table 1
Data summary statistics.

Variable	Mean	Median	15th percentile	85th percentile
Accepted passing gap (s)	21.47	20.75	17.39	28.79
Passing duration (s)	4.98	4.83	3.50	6.48
Passing vehicle speed (m/s)	22.21	21.29	17.27	27.39
Front vehicle speed (km/h)	66.20	60.00	60.00	80.00
Opposite vehicle speed (km/h)	76.28	85.00	65.00	85.00
Following distance from front vehicle when starting to pass (m)	15.47	12.80	8.39	22.92
Minimum TTC (s)	2.37	1.98	0.76	4.10
Gap from passed front vehicle at end of the passing maneuver (s)	2.44	2.24	1.49	3.42

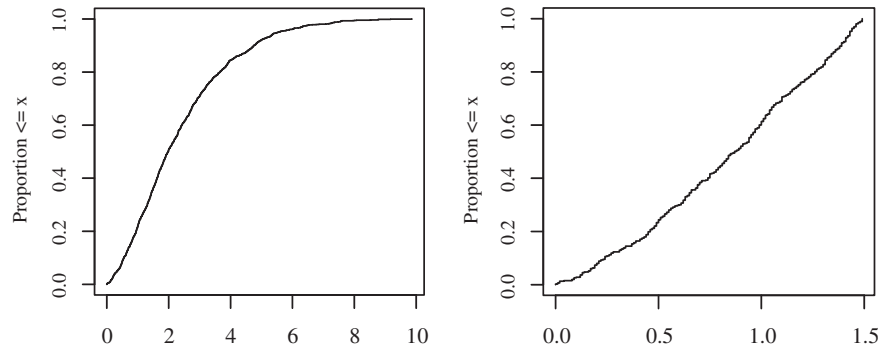


Fig. 2. CDF of minTTC (s) for the full dataset (left) and filtered data (right).

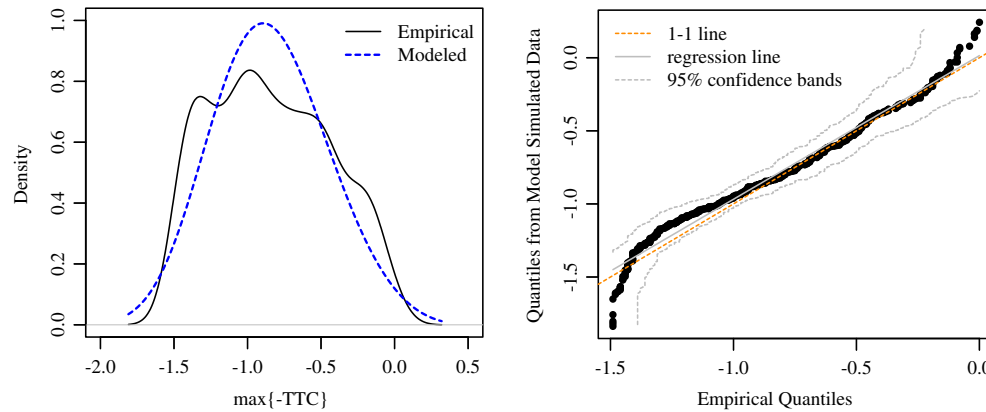


Fig. 3. (Kernel) Probability density plot (left) and simulated QQ plot (right) for the stationary BM model.

2.2.1. Participants

One hundred drivers (64 males and 36 females) with at least 5 years of driving experience participated in the driving simulator experiment on a voluntary base. The drivers' age ranged between 22 and 70 years old. Drivers were instructed to drive as they would normally do in real world. An advertisement on the experiment was published at the Technion campus in Israel and drivers who were interested to participate contacted the researchers.

2.2.2. The data

The data set from the driving simulator experiment resulted in 1287 completed passing maneuvers, in which 9 ended with a collision (these observations were removed from the estimation data sets). Table 1 below presents summary statistics of passing maneuvers related variables.

Passing gaps were defined as the gap between two successive vehicles on the opposite lane at the time the lead opposite vehicle is at the same line with the subject vehicle. The passing duration is measured

from the moment the subject vehicle left front wheel crosses the center line (as shown in Fig. 1) until the passing maneuver ends when the rear left wheel crosses the centerline. Vehicles' speeds as summarized in Table 1 are measured at the beginning of the passing maneuvers. The following distance from front vehicle when starting to pass is measured as the distance between the front of the subject vehicle and the end of the front vehicle as illustrated in Fig. 1. Finally, the minimum TTC is measured at the end of the passing maneuver (since up till this moment there is still a risk of collision) and reflect the risk to collide with the opposite vehicle.

3. Results and analysis

This section presents the results of the analysis following the research method described above. First, the estimation results of the BM using the GEV model is presented, followed by the estimation results according to the POT using the GPD, and finally a validation of the results using a second database.

3.1. Block maxima approach (BM) results

A GEV distribution is fitted using the non-crash passing maneuvers and the respective minimum TTC measurements. For the block intervals we use the annotated time that contain the entire passing maneuver. Both the chosen block interval and the resulting number of observations in each block are variable [7]. In this case, the calculated probability represents the probability of a head-on collision for a single passing maneuver. Furthermore, past studies concluded that with minimum TTC smaller than a low limit (typically, 1 to 1.5 s) are useful as crash surrogates [7,24]. The filtered data according to this approach, and choosing a limit of 1.5 s, resulted in 463 maxima. Fig. 2 (left) presents the CDF of the minimum TTC ($\min\{TTC\}$) for the full data set, while Fig. 2

Table 2

List of covariates tested in the BM Approach.

Acronym	Description
<i>passinggap</i>	The time gap between two opposite vehicles at the time the subject meet the lead opposite vehicle (s)
<i>speedopposing</i>	The speed of the opposite vehicle at the moment of start passing (m/s)
<i>speedfront</i>	The speed of the front vehicle at the moment of start passing (m/s)
<i>followinggap</i>	The time gap between the subject vehicle and the front vehicle at the moment of start passing (s)
<i>passduration</i>	The passing duration (s)
<i>curvature</i>	The road curvature (1/m)

Table 3
Estimation results of the best model for non-stationary BM approach.

Parameter		Estimated value	Standard error
$\hat{\mu}$	$\hat{\mu}_0$	−1.06	(0.139)
	$\hat{\mu}_1$ (speedfront)	0.0245	(0.00644)
	$\hat{\mu}_2$ (followinggap)	0.00274	(0.00179)
	$\hat{\mu}_3$ (passinggap)	−0.0212	(0.00445)
	$\hat{\mu}_4$ (curvature)	−38.1	(13.5)
$\hat{\sigma}$		0.369	(0.0145)
	$\hat{\xi}$	−0.225	(0.0412)
Neg. loglikelihood		215.54	

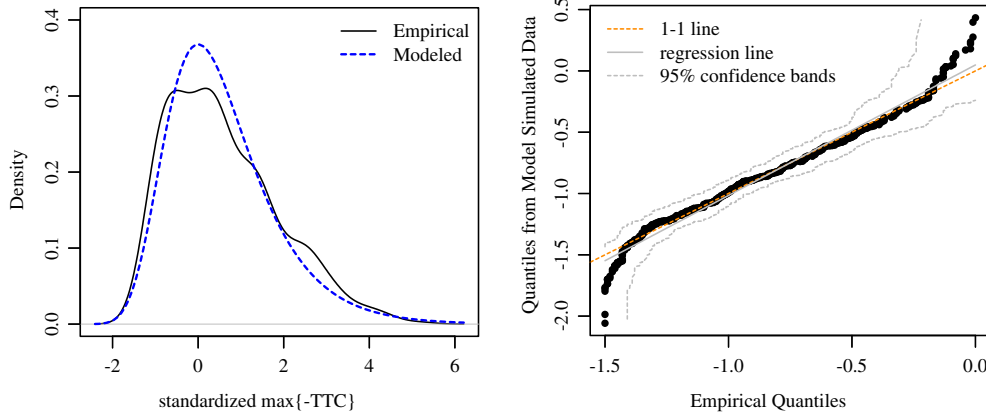


Fig. 4. (Kernel) Probability density function (left) and simulated QQ-plot (right) for the non-stationary BM model.

(right) presents the CDF of the $\min\{\text{TTC}\}$ for the filtered data. For the full data set, 50% of the observations were less than a TTC of about 2 s, while in the filtered data, 50% of the observations were less than a TTC of about 0.9 s. Different values for the filtering threshold were tested and the 1.5 s resulted in the best fitting. Furthermore, this value is consistent with the literature.

We first estimated a stationary block maxima model for the maxima of the negated values instead of minima of the original values, i.e. $\max\{-\text{TTC}\}$. The fitted distribution resulted in the following parameters of the GEV cumulative distribution function: $\hat{\mu} = -0.993$ (0.0212), $\hat{\sigma} = 0.383$ (0.0163) and $\hat{\xi} = -0.236$ (0.0500), where the values in parenthesis are the standard errors. Fig. 3 (left) presents the kernel probability density function of the empirical and modeled negated TTC, and Fig. 3 (right) presents the simulated QQ plot. From these figures it can be concluded that the modeled GEV distribution has satisfactory fitting results to the empirical data since the points fall close to the 45° line in the simulated QQ plot.

With this stationary model the estimated probability of $\max\{-\text{TTC}\} \geq 0$ is 0.0179 with 95% confidence interval (0.0177, 0.0182). The confidence intervals of estimations were computed using simulation, assuming the (independent) normal distribution under regularity conditions of the parameters and a simulation experiment size of 1×10^6 . During the driving simulator experiment, 463 near head-on collisions (using the threshold of 1.5 s) and 9 actual collisions were recorded. In other words, the probability for a head-on collision assuming a near head-on collision during a passing maneuver is $9/472 = 0.0191$, with a 95% confidence interval (0.0088, 0.0359). This value is comparable to the estimate resulting from the fitted GEV distribution.

However, the process of a passing maneuver may be affected by the detailed conditions of each specific passing, such as the relative gaps and speeds between the vehicles surrounding the subject vehicle. To account for the fact that the TTCs are non-stationary observations and

are affected by several factors, we tested the inclusion of different covariates that were collected during the driving simulation experiment in the location parameter of the BM model (see Table 2).

Several linear combinations of these variables were tested during the model estimation task. To test reduced model structures and the inclusion of variables, the likelihood ratio test was used [17]. The final model was also tested against the stationary one, resulting in a p-value (3.741×10^{-8}) significantly smaller than $\alpha = 0.05$.

The results in Table 3 indicate that as the speed of the front vehicle increases the negated TTC increases, and the TTC decreases which is logical since it is more difficult for the subject vehicle to pass the front vehicle. This is also in line with the conclusions from previous studies

[12,25]. Similarly, as the passing gap that is accepted is larger, the negated TTC decreases, and the TTC increases. On the other hand, as drivers start their passing maneuvers from a larger gap from the front vehicle, the negated TTC increases and the TTC decreases. Drivers take longer time to pass the front vehicle, getting closer to the opposite vehicle, and resulting in shorter TTC. The road design impacts the TTC as well. As expected, as the road curvature is larger, the negated TTC is lower, and the TTC is higher. This indicates an adaptation behavior by drivers who compensate for the difficulty of the passing maneuver on complex roads by increasing their safety margins. Previous results by Farah and Toledo [26] found that on roads with larger curvature, drivers accept larger critical gaps, which supports the results of this study. The speed of the opposite vehicle was not found to be significant at the 95% confidence level, however, this variable is indirectly included through the passing gap which is measured in time.

Fig. 4 (left) presents the probability density function of the empirical and modeled standardized¹ maximum negated TTC, and Fig. 4 (right) presents the simulated QQ plot for the non-stationary model. The results indicate a good fit between the modeled GEV distribution and the empirical data, and a better fit compared to the results of the stationary model presented in Fig. 3. Also, the negative log-likelihood has improved from 229.5 to 215.5, maintaining a $\xi > -0.5$ that assures the regular asymptotic properties of the maximum likelihood estimators [17].

To estimate the probability of a head-on-collision during a passing maneuver ($\max\{-\text{TTC}\} \geq 0$), for the non-stationary model, simulated covariates or directly location parameters have to be generated. From the estimated location parameters for the estimation dataset, a normal distribution was fitted with satisfactory results with a mean of -0.996 , standard deviation of 0.115 and a Kolmogorov–Smirnov

¹ For non-stationary models, it is common practice to transform the data to a density function that does not depend on the covariates, using the following function $Z_i = -\log(1 + (\xi/\sigma^*) (X_i - \mu_i))^{(-1/\xi)}$ (Gilleland and Katz, [16]).

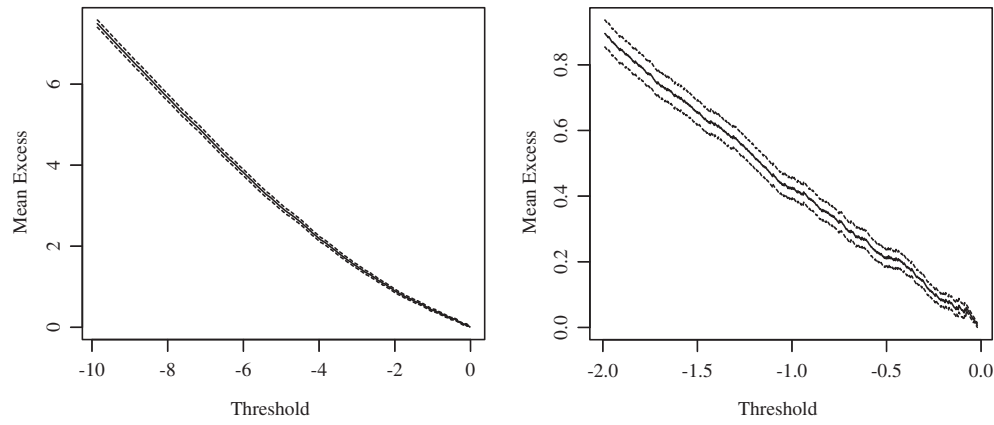


Fig. 5. Mean residual life for the full data set (left) and for negated TTC > -2.0 s (right).

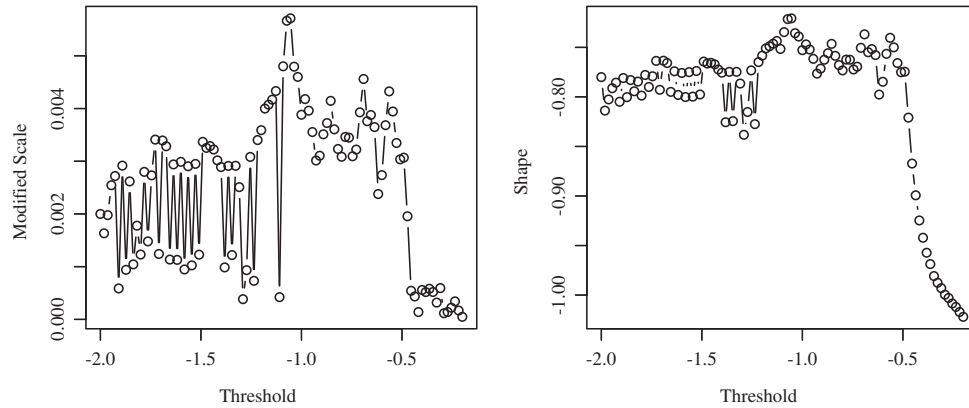


Fig. 6. Stability plot for GPD model modified (modified by subtracting the shape and multiplied by the threshold) scale parameter (left) and shape parameter (right) for different TTC thresholds.

test statistic of 0.0452. The simulated probability of $\max\{-TTC\} \geq 0$ is 0.0190 with 95% confidence interval (0.0188, 0.0193), resulting in a better estimate than the stationary model.

3.2. Peak over threshold (POT) results

In this section the estimation results of the GPD following the POT approach are presented. This analysis was conducted in order to compare with the BM approach results, as previous studies concluded that the POT approach often performs better than the BM approach, especially in situations of short-time observations [5]. As a first step for estimating the GPD, a threshold needs to be determined and selected from the observed maximum negated TTC. To determine the optimal threshold an assessment of mean residual life and stability plots were carried out following Coles [17]. A threshold can be determined when the mean residual life plot is almost linear and the modified scale and shape estimates become constant. In Fig. 5 (left) the mean residual life plot of the maximum negated TTC thresholds is linear starting from a threshold of -2.0 s, where the line becomes more stable, until about -0.2 s. This is better

shown in Fig. 5 (right) where the mean residual life plot of the negated TTC thresholds larger than -2.0 s is presented.

The stability of GPD modified scale and shape parameters were also analyzed. Fig. 6 shows stability plots considering a range between -2.5 and -0.2 s. Both parameters seem to be relatively stable in the range between -1.1 and -0.5 s. Considering the low magnitudes of the variability of the modified scale parameter over the full range of tested threshold values, different stationary models were fitted using the full dataset for the thresholds of $u = -1.5, -1.0, -0.5$ and -0.25 s, using the ML method.

Since the estimated shape parameter is stable and its value is $\hat{\xi} < -0.5$ the estimators from the ML are generally not reliable [27]. (See Table 4) Fig. 7 presents the probability density function of the empirical and modeled negated TTC and the simulated QQ plot for the estimated models. The figures of the probability density functions indicate a good fit between the modeled GPD distribution and the empirical data. It is worth noting that the pdf at $-\min\{TTC\} = 0$ is not zero, but a significantly low value due to the short upper tail for the estimated distribution of excesses and its low estimated upper bound ($u - \hat{\sigma} / \hat{\xi}$).

With these stationary models using the fitted GPD, the estimated probability of head-on collision is 0.00628 with 95% confidence interval (0.00612, 0.00643) for a -0.25 s threshold near-crash; the 0.00240 (0.00234, 0.00254) for a -0.5 s threshold, 0.00107 (0.000972, 0.00109) for a -1.0 s threshold, and 0.000480 (0.000392, 0.000475) for a -1.5 s. threshold. The empirical value stands at 0.00699 (with a 95% binomial confidence of 0.00320, 0.0132), indicating -0.25 s as the suitable threshold for the stationary POT model. However, the instability of the estimated parameters for thresholds greater than -0.5 s and the lower fit for -0.25 s makes this decision less straightforward.

Table 4
Estimation results for two stationary POT models.

Parameter	$u = -0.25$ s	$u = -0.5$ s	$u = -1.0$ s	$u = -1.5$ s
$\hat{\sigma}$	0.181 (0.0393)	0.375 (0.0427)	0.738 (0.0485)	1.150 (0.0136)
$\hat{\xi}$	-0.703 (0.1930)	-0.742 (0.0973)	-0.733 (0.0518)	-0.764 (0.00752)
Neg. log likelihood	-70.68	-81.58	-10.52	175.64

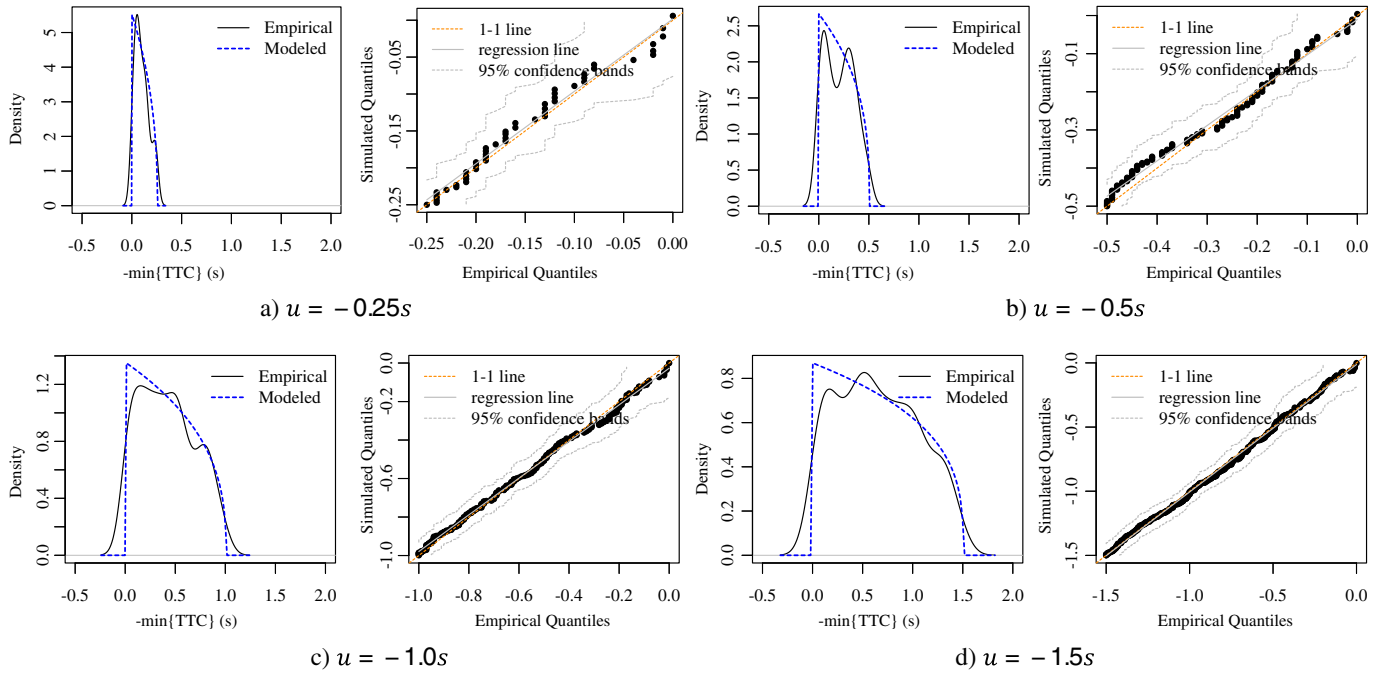


Fig. 7. (Kernel) Probability density plot (left) and simulated QQ plot (right) for the stationary POT model for different thresholds (-0.25 , -0.5 , -1.0 and -1.5 s).

Table 5
Estimation results for the best model for non-stationary POT approach ($u = -0.5$ s).

Parameter	Estimated value	Standard error
$\hat{\sigma}$		
$\hat{\sigma}_0$	0.394	(0.00774)
$\hat{\sigma}_1$ (speedfront)	-0.00559	(2×10^{-8})
$\hat{\sigma}_2$ (passingRate = passingduration/passinggap)	0.451	(0.0934)
$\hat{\xi}$	-0.830	(0.0601)
N	113	
Neg. loglikelihood	-84.27	

Similarly to the BM modeling effort, we tested the inclusion of the same different covariates (see Table 2) in the scale parameter formulation to account for the impact of different factors on TTCs. To test reduced model structures and the inclusion of variables, the likelihood ratio test was used [17]. Non-stationary models for both thresholds of -0.25 s and -0.5 s were considered.

Table 5 presents the results of the best fitted model.² The covariate *passingRate* represents the percentage of the passing gap that was used during the maneuver. From the results, the increase in the speed of the front vehicle reduces the scale parameter, and therefore the variance of the minimum TTC distribution; on the other hand, the increase of the *passingRate* increases the variance of the minimum TTC distribution. The estimated probability of head-on collision is now 0.00711 (0.00660, 0.00765), much closer to the observed 0.00699 than the stationary model. Despite this improved result, the shape parameter is always less than -0.5 s corresponding to a distribution with a very short bounded upper tail, limiting the theoretical robustness of the maximum likelihood approach.

3.3. Validation

This section aims at validating the previous results by applying the previously fitted model to estimate the probability of a head-on collision

² The evd and extremes package support the plotting of non-stationary GDP density functions.

in a different dataset, i.e. a second experiment. In this new experiment different 100 drivers (69 males and 31 females) participated. Their age ranged between 21 and 61 years old. The instructions and experimental conditions were identical to the first experiment. The simulator scenarios included as well rural two-lane road sections each with a total length of 7.5 km. The same two-level four factors as in the first experiment were used to generate the scenarios. However, the values in each level were not fixed but randomly drawn from a specified distribution. Speeds were drawn from truncated uniform distributions, while passing gaps were drawn from truncated negative exponential distributions. More details on the design of the scenarios can be found in Farah and Toledo [26]. A total of 562 passing maneuvers were observed, 9 of which resulted in actual collisions. To check the consistency among covariate data sets, the CDF for each of the covariates considered previously were computed (see Fig. 8). The data plotted in Fig. 8 is filtered for $\min\{\text{TTC}\} < 1.5$ s. It is worth noting that driving speeds in the first experiment were fixed to certain values while in the second experiment were randomly drawn from truncated uniform distributions. This will result in a potential bias in the estimated values, as the estimated model used limited speed-related data.

Recall the estimated BM stationary model; the estimated probability of a head-on collision given a 1.5 s near-collision threshold was 0.0179 (0.0177, 0.0182). In the validation dataset there were 166 near-collision observations (i.e. with $\min\{\text{TTC}\} < 1.5$ s). Therefore, the simulated number of head-on collisions is 2.97. On the other hand, the empirical probability for a head-on collision given a 1.5 s near-collision threshold is $9/166 = 0.0508$, with a 95% binomial confidence interval (0.0235, 0.0943). Fig. 9 presents the probability density function and QQ plot of the validation and the simulated negated TTCs using the BM stationary model.

For the stationary POT approach, the probability of a head-on collision is 0.00240 (0.00234, 0.00254) for a -0.5 s threshold, resulting in a simulated number of head-on collisions of 1.4, even lower than the BM stationary model. The lower estimates of the two models may be due to the different simulator experimental settings, namely to the different speed distributions used. The lower resulting $\min\{\text{TTC}\}$ for validation data set (Kolmogorov–Smirnov statistic $D = 0.99 > 0.12$ for a 0.05 level, rejecting the null hypothesis of being drawn from the

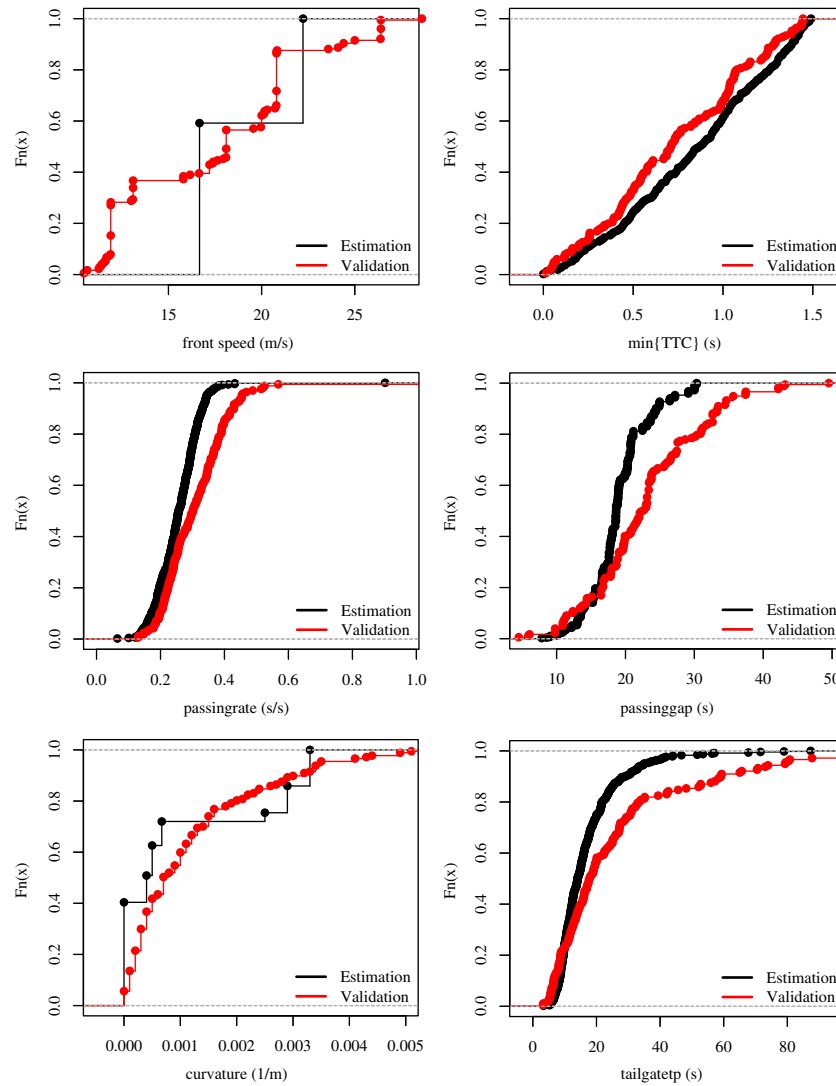


Fig. 8. CDF of the minimum TTC and the covariates considered for both the estimation and the validation data sets.

same distribution) already indicated a possible misfit of a simple stationary model.

The same test was carried out for the non-stationary models. For the non-stationary BM model the simulated number of head-on collisions is 3.4, still far from the observed values (see Fig. 10). However, the non-stationary POT model resulted in a simulated number of collisions of 16.3, mostly due to the difference in *passingRate* and front vehicle speed in the new dataset. Despite overestimating the number of

head-on collisions, the unstable POT model was able to capture the increased risk in the validation dataset.

4. Summary and conclusions

In this study an extreme value (EV) approach was applied for the estimation of the probability of head-on collisions that result from unsuccessful passing maneuvers on two-lane rural highways. Both,

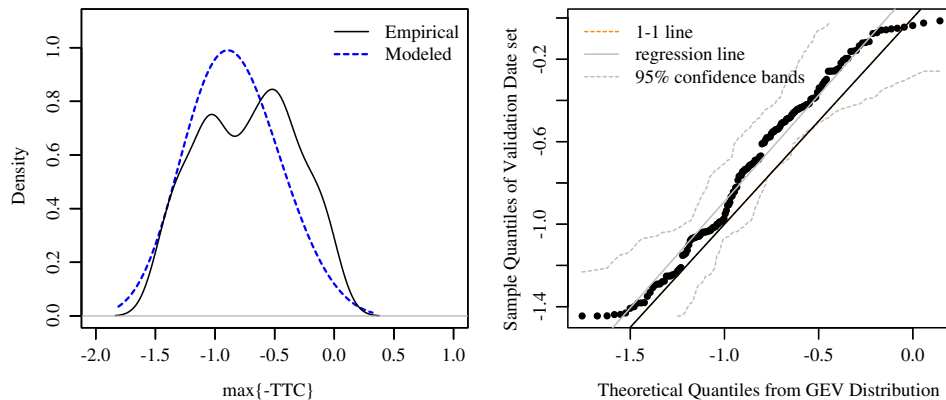


Fig. 9. Probability density plot (left) and QQ plot (right) for the validation set and the stationary BM model.

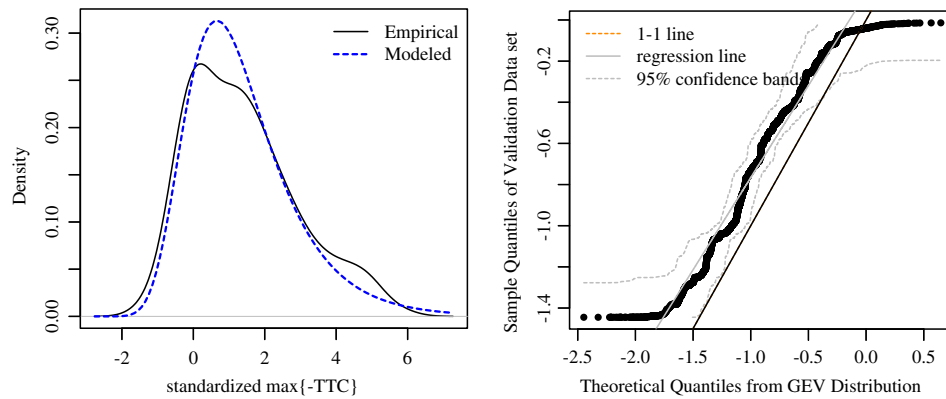


Fig. 10. Probability density plot (left) and QQ plot (right) for the validation set and the non-stationary BM model.

the block maxima (BM) approach using the Generalized Extreme Value (GEV) distribution and the peak over threshold (POT) using the Generalized Pareto Distribution (GPD), were tested and compared using the minimum time-to-collision with the opposite vehicle during passing maneuvers.

This paper brings practical insights to the relatively scarce literature on the use of EV method in detailed road safety analysis. The method, which can be leveraged with the availability of detailed data, shows promising results in quantifying accident probability and in identifying influencing factors. Such knowledge, will bring the necessary capability of not only quantitatively assessing the benefits of interventions targeting such detailed variables (e.g.: safety gap markings, local speed limits and Advanced Driver Assistance Systems) for which safety data is not yet available, but also modeling attributes suitable for integration in accident-free detailed simulators (known to be capable of simulating conflicts, but not accidents).

Our estimations showed that the BM approach yielded more stable results compared to the POT approach, but the latter was able to produce crash rate estimates more consistently sensitive to the covariates of interest. Zheng et al. [5] who conducted a comparative study for the case of using post encroachment time measure for predicting lane-changing maneuver related crashes found that the POT approach performed better than the BM approach. In fact, the data set used in the study by Zheng et al. [5] was relatively limited, and for limited data sets the POT is known to be a more efficient approach than the BM approach. Zheng et al. [5] cite two studies [28,29] which concluded that “the BM approach would work well if the number of observations is large, while the POT approach would have a poor performance”. However, definitive conclusion regarding which method is supreme can not yet be made and further comparative studies are needed in order to reach a firm conclusion. The on-going discussions in the statistics field on the merits of both POT and BM approaches [32] support as well the need for more comparative studies. In general, POT tends to be more efficient than BM in several circumstances, though typically needing a number of exceedances larger than the number of blocks; the BM method may be also preferable when the observations are not exactly independent and identically distributed.

Nevertheless, it was found that the non-stationary BM model performed better than the stationary BM model. This is expected since the introduced covariates significantly affect the TTC and were found to be important explanatory variables in previous studies [12,25]. Furthermore, the predicted probability of head-on collisions based on the BM approach was sufficiently close to the probability of head-on collisions based on the empirical data from the driving simulator. This also indicates that for passing maneuvers the TTC is a good surrogate safety measure for near-crashes of head-on collisions. This is different from the conclusion reached by Jonasson and Rootzén [7] who found severe discrepancy between the rear-striking near-crashes (using the TTC) and rear-striking crashes. However, this can be explained by the mechanism

of crash occurrence and the state of the driver. In passing maneuvers drivers are aware and conscious of their actions and therefore head-on collisions usually result from an error in drivers' judgment of the suitability of the passing gap. On the other hand, in rear-striking collisions, the state of the driver in these collisions might vary a lot. It can result, similarly to passing collisions, from drivers' errors in judging their gap and speed from the front vehicle, but it can also result from the driver being distracted. In the first case, it is most likely to observe an evasive action of the driver to prevent the collision but in the second case no evasive action might be observed. These causes, as Jonasson and Rootzén [7] indicate, a selection bias, and therefore, careful selection of near-crashes is a crucial issue in preventing this from occurring.

The POT models resulted in more accurate predicted probabilities of head-on collisions and a non-stationary model more sensitive to the covariates of interest. This also indicates that the TTC is a good surrogate safety measure for head-on collisions. However, it is worth noting that in all POT models, the shape parameter is less than -0.5 which corresponds to distributions with a very short bounded upper tail. Although this situation is rarely encountered in applications of extreme value modeling, the theoretical limitations of the maximum likelihood approach and the asymptotic properties of its estimators are still at stake.

Despite these promising results, future research by the authors will attempt to expand this work in several possible directions as follows: (1) testing alternative surrogate measures of head-on collisions such as the Time Exposed Time to Collision or Time integrated Time to Collision [19]; (2) developing a more sophisticated measure of risk which accounts for the complexity of the passing maneuver and considers the probability to collide not only with the opposite vehicle but also with the passed vehicle (i.e. when the driver returns too soon to its lane). One possibility is, similarly to Jonasson and Rootzén [7], to use a bivariate model which considers the TTC and the headway between the passing and passed vehicle at the end of the passing maneuver; (3) extending the non-stationary models by including other covariates related to road design (this study accounted only for the road curvature) and drivers' characteristics, such as socio-demographic and driving styles; (4) testing different estimation techniques (e.g.: probability weighted moments) that may result in more robust estimates; (5) examining the transferability of such models and validation of the results with other datasets especially from field studies; (6) applying the developed models in traffic microscopic simulation environments for safety assessment [30,31].

References

- [1] A. Tarko, G. Davis, N. Saunier, T. Sayed, S. Washington, *Surrogate Measures of Safety—White Paper*, Transportation Research Board, Washington, DC, 2009.
- [2] A. Tarko, Use of crash surrogates and exceedance statistics to estimate road safety, *Accid. Anal. Prev.* 45 (2012) 230–240.

- [3] D. Gettman, L. Pu, T. Sayed, S.G. Shelby, Surrogate safety assessment model and validation: final report, Final Report Federal Highway Administration. FHWA-HRT-08-051, 2008.
- [4] J. Archer, Methods for the Assessment and Prediction of Traffic Safety at Urban Intersections and their Application in Micro-Simulation Modelling, Royal Institute of Technology, 2004.
- [5] L. Zheng, K. Ismail, X. Meng, Freeway safety estimation using extreme value theory approaches: a comparative study, *Accid. Anal. Prev.* 62 (2014) 32–41.
- [6] T. Sayed, S. Zein, Traffic conflict standards for intersections, *Transp. Plan. Technol.* 22 (4) (1999) 309–323.
- [7] J.K. Jonasson, H. Rootzén, Internal validation of near-crashes in naturalistic driving studies: a continuous and multivariate approach, *Accid. Anal. Prev.* 62 (2014) 102–109.
- [8] P. Songchitruksa, A.P. Tarko, The extreme value theory approach to safety estimation, *Accid. Anal. Prev.* 38 (4) (2006) 811–822.
- [9] R.A. Fisher, L.H.C. Tippett, Limiting forms of the frequency distribution of the largest or smallest member of a sample, Paper Presented at the Mathematical Proceedings of the Cambridge Philosophical Society, 1928.
- [10] Å. Svensson, C. Hydén, Estimating the severity of safety related behaviour, *Accid. Anal. Prev.* 38 (2) (2006) 379–385.
- [11] NHTSA, Traffic Safety Facts 2003 Data: Pedalcyclists, US Government Printing Office, Washington, DC, 2003.
- [12] H. Farah, S. Bekhor, A. Polus, Risk evaluation by modeling of passing behavior on two-lane rural highways, *Accid. Anal. Prev.* 41 (2009) 887–894, <http://dx.doi.org/10.1016/j.aap.2009.05.006>.
- [13] A. Shariat-Mohaymany, A. Tavakoli-Kashani, H. Nosrati, A. Ranjbari, Identifying significant predictors of head-on conflicts on two-lane rural roads using inductive loop detectors data, *Traffic Inj. Prev.* 12 (6) (2011) 636–641.
- [14] G. Hegeman, Assisted Overtaking: an Assessment of Overtaking on Two-Lane Rural Roads, Netherlands Research School for Transport, Infrastructure, Logistics, Delft, The Netherlands, 2008.
- [15] R. Fuller, B. Potter, Driver control theory, *Handbook of Traffic Psychology*, 1, 2011, pp. 13–26.
- [16] E. Gilleland, R.W. Katz, New software to analyze how extremes change over time, *EOS Trans. Am. Geophys. Union* 92 (2) (2011) 13–14.
- [17] S. Coles, An Introduction to Statistical Modeling of Extreme Values, Springer-Verlag, London, UK, 2001.
- [18] J.C. Hayward, Near-miss determination through use of a scale of danger, *Highw. Res. Rec.* 384 (1972).
- [19] M.M. Minderhoud, P.H. Bovy, Extended time-to-collision measures for road traffic safety assessment, *Accid. Anal. Prev.* 33 (1) (2001) 89–97.
- [20] G. Hegeman, A. Tapani, S. Hoogendoorn, Overtaking assistant assessment using traffic simulation, *Transp. Res. C* 17 (6) (2009) 617–630.
- [21] R.J. Kiefer, C.A. Flannagan, C.J. Jerome, Time-to-collision judgments under realistic driving conditions, *Hum. Factors* 48 (2) (2006) 334–345.
- [22] T. Rosenthal, STISIM Drive user's Manual, Systems Technology Inc., Hawthorne, CA, 1999.
- [23] C.R. Hicks, K.V. Turner, Fundamental Concepts in the Design of Experiments, fifth ed. Oxford University Press, New York, NY, 1999.
- [24] C. Hydén, The development of a method for traffic safety evaluation: the Swedish traffic conflicts technique, 70, Bulletin Lund Institute Of Technology, Department, 1987.
- [25] C. Llorca, A. Garcia, Evaluation of passing process on two-lane rural highways in Spain with new methodology based on video data, *Transp. Res. Rec.* 2262 (1) (2011) 42–51.
- [26] H. Farah, T. Toledo, Passing behavior on two-lane highways, *Transport. Res. F: Traffic Psychol. Behav.* 13 (2010) 355–364, <http://dx.doi.org/10.1016/j.trf.2010.07.003>.
- [27] R.L. Smith, Maximum likelihood estimation in a class of nonregular cases, *Biometrika* 72 (1) (1985) 67–90.
- [28] S. Caires, A Comparative Simulation Study of the Annual Maxima and the Peaks-over-Threshold Methods, Deltares, 2009.
- [29] D. Jarušková, M. Hanek, Peaks over threshold method in comparison with block-maxima method for estimating high return levels of several Northern Moravia precipitation and discharges series, *J. Hydrol. Hydromech.* 54 (4) (2006) 309–319.
- [30] C. Lima Azevedo, J.C. Cardoso, M.E. Ben-Akiva, Probabilistic safety analysis using traffic microscopic simulation, 94th Annual Meeting of the Transportation Research Board, Washington D.C., USA, January 2015, 2014.
- [31] D. Gettman, L. Head, Surrogate safety measures from traffic simulation models, *Transp. Res. Rec. J. Transp. Res. Board* 1840 (1) (2003) 104–115.
- [32] A. Ferreira, L. de Haan, On the block maxima method in extreme value theory: PWM estimators, *Ann. Stat.* 43 (1) (2015) 276–298.