A Survey on Multilateral Negotiation Models with Alternating Offers Protocol

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Abstract—This survey contains some recent extensions and applications of multilateral negotiation models using alternating offers protocol. We show that this protocol still plays a major role in modern-day multilateral negotiation models, though (seriously) adjusted to meet specific needs. Such needs include to accommodate bargaining in (sequential or simulations) networks, resource reallocation, bargaining with resource uncertainty and coalitional bargaining. In addition, it is observed that recent bargaining models are by no means solely studied in the field of bargaining theory. Many applications can be found in computer sciences (i.e. multi-agent systems, automated negotiation and grid computing), supply-chain networks, exchange economies and water- or environmental management. Though, there are still some major challenges left. In general, there is a need to narrow the gap between theoretical literature and empiric and experimental research. Moreover, some research areas could be combined to create more realistic multilateral models – i.e. literature on bargaining in (sequential) networks and coalition formation. Finally, imperfect information in general is seriously underexposed in recent multilateral bargaining models and should be further examined.

Index Terms—Multilateral negotiation, bargaining, alternating offers, survey, networks, coalition, imperfect information.

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1 INTRODUCTION

FOR decades, bilateral negotiations have undergone major attention in economics, game theory, social sciences and various academic niches such as bargaining theory and automated negotiation. The reasons for studying and formalizing primarily *bilateral* negotiations have been twofold [1, p. 335-336]. First, such situations are more basic and prevalent in everyday life. Secondly, until recently, academic literature on multilateral negotiation models have been extremely small and under-developed. However, because of its empirical relevance [2, p. 269-271] and technological advances – especially in terms of computing power – literature on multilateral negotiation models have had a substantial impulse, as illustrated by Fig. 1.

Arguably, many of such models are derived from (or extensions of [1, p. 337]) Rubinstein's bilateral bargaining model, based on the *alternating offers* protocol [3]. Put simply, in this model, two players negotiate over a piece of a cake of size π by making offers and counteroffers in turns until agreement is reached. Unfortunately, despite the growing body of knowledge on multilateral negotiation models using this protocol, there is no recent study providing an overview of this field. Therefore, to further stimulate development, structure the debate and indicate potential research areas, we provide an overview of recently proposed multilateral negotiation models using the alternating offers procedure.

Though, this cannot be adequately done by first describing multilateral models that have been proposed a decade after the introduction of Rubinstein's model in 1982.

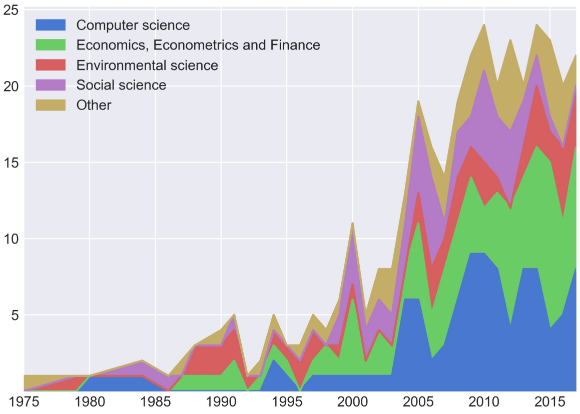

Fig. 1. Number of peer-reviewed articles in Scopus® on multilateral bargaining models categorized by subject area (see Appendix A).

Therefore, in Section 2, we describe some historic developments on multilateral bargaining models, as well as some proposed extensions. In Section 3, we introduce various recent research areas including multilateral bargaining in networks, coalitional bargaining and resource reallocation. Finally, in Section 4 and Section 5 we provide a short summary, conclusion and consider some future research directions.

2 BEFORE THE 00S

It was Rubinstein himself who, together with Osborn, introduced the first multilateral extension of the alternating offers procedure [4, p. 63-65]. Though, the use of "multilateral" is a bit premature as this extension was only applicable to three players. The protocol assumes three players $A = [a_1, a_2, a_3]$ taking the roles of proposer and accepters alternately. In short, if a_1 is the proposer, a_2 and

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a_3 either accepts or rejects a_1's offer $x = \{x_1,\ x_2,\ x_3\}$. Only when both accept (i.e. unanimous), the negotiation ends in agreement. If not, the role of proposer is altered in the (fixed) order of A until agreement is reached. Disappointingly, this procedure leads to multiple equilibrium solutions[1].

2.1 Regaining uniqueness of equilibrium solution

The property of uniqueness can be regained by changing the way players interact during the negotiations, as shown by among others Chae and Yang [5], [6] and Jun [7]. Both protocols describe multilateral bargaining as series of bilateral negotiations, in which any player accepting an offer temporarily leaves the negotiation table until agreement is reached in *all* sequential negotiations. If any player i rejects his personal offer x_i in the meantime, all players enter the negotiation arena again for the second try.

Another procedure was introduced by Krishna and Serrano [8] in which a single player makes one offer $x = \{x_1, \cdots, x_n\}$ to all other players involved in the negotiations. In return, the accepters in parallel decide whether they agree with their personal share of the pie π. If so, they leave with *partial agreements*. Note that using this procedure, negotiations have become multilateral. Note too that these negotiations do not necessarily end in unanimous agreement any more, as only a subset of $A = \{a_1, \cdots, a_n\}$ is needed to win the game. As a result, also this protocol regained uniqueness of equilibrium solution [8, p. 68-76].

In addition, two other solutions have been proposed regaining uniqueness. These do not rely on player interaction, but on bargaining strategies and deadlines respectively. As among others demonstrated in the Baron-Ferejohn model [9, p. 1189-1191] of legislative bargaining, using *stationary* bidding strategies uniqueness of the equilibrium outcome is regained. This implies that the offer x is both independent of time and of historic offers in previous time periods. Finally, uniqueness can be regained by imposing *finite* time horizons. This is, among others, demonstrated by Dutta and Gevers [10]. Though, one can imagine especially the first solution to be rather restrictive for practical applications.

2.2 Multiple issues, agenda and coalitions

Not all work has been only focusing on regaining uniqueness of the equilibrium solution for multilateral bargaining models. Rausser and Simon [11] introduce a multilateral multi-issue model to deal with collective decision-making problems. This model consists of a number of single-issue negotiation rounds, each with their own finite time horizons and "admissible coalitions". The latter describes those players needed to reach agreement for a single issue[2]. Moreover, players still alternate their offers, only the order in which day do so is derived from their "access probabilities" [11, p. 1-2] – i.e relationship between more important players and the likelihood that their offer is considered by others.

The introduction of multiple issues has added another layer of complexity [1, p. 339]. In such negotiation settings, not only the order of interaction between players is important, but also the order of issues or negotiation *agenda*. Put differently, the agenda can be seen as a protocol on its own. Bac and Raff [13] were one of the first to formalize two procedures dealing with multiple issue, namely "issue-by-issue" and "complete package". Moreover, the agenda also introduces new strategies as the order of issue might be exploited and manipulated in favor of a player's own gains. One of the first authors formalizing this problem was Fershtman [14], though for a rather simple bilateral two-pies setup. In a series of papers of Fatima, Wooldridge and Jennings studying optimal agendas has undergone some major research in the field of automated negotiation (e.g [15], [16]). Unfortunately, all their research has been focusing on bilateral settings and, to our best knowledge, no other work dealing with multilateral negotiations has been published[3]. Therefore, the negotiation agenda is out of scope in Section 3.

Finally, the introduction of multiple players has paved the way for another major challenge, namely that of coalition formation. In a series of consecutive papers, Binmore started to formalize this as the three-player/three-cake problem [19], in which each combination of players – that is $\{a_1,\ a_2\}$, $\{a_1,\ a_3\}$ and $\{a_2,\ a_3\}$ – control the division of different cakes, but only one cake is divided. In Binmore's words [19, p. 269]: *"This problem is, of course, a paradigm for a much wider class of problems concerning the conditions under which coalitions will or will not form"*. For a n-person coalitional bargaining setting, Chatterjee et al. (1993) were one of the first extending Rubinstein's model to examine under which circumstances the *grand coalition*[4] will form (or not) and how they should divide pie π [20]. Though, under strict (stationary) strategy assumptions.

2.3 Asymmetric information

Many of the multilateral negotiation models in this and the previous section assumed perfect information (e.g. [9], [11], [20]) to obtain one or more efficient properties[5]. Obviously, in everyday bargaining situations, at least one of the players involved knows something others do not. Although this also applies to bilateral settings, the magnitude of the problem changes substantially in multilateral negotiations. After all, more players implies more private information and, thus, more difficulties approaching multilateral negotiation settings from a meta perspective. As we will see later on, there are only a handful of articles dealing with this phenomenon in a multilateral setting.

2.4 Side notes

In the above sections, we have deliberately only introduced developments related to the introduction of multiple (contrary to two) players in the bargaining game. Of course, for the bilateral setting, many other directions of research have

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[1] This, in contrast to Rubinstein's protocol for bilateral bargaining, which always has a unique equilibrium.

[2] Thus, the admissible coalition prescribes certain characteristics of the *winning coalition* (see also [12, p. 177]). These are, for example, derived from a player's veto rights.

[3] One could argue that the work of Elkind and Fatima concerns agendas in multilateral negotiation [17]. However, that article discusses a (sequential) auction setting, using a distinctly different negotiation protocol (see also [18, p. 139-145]).

[4] The *grand coaltion* refers to the coalition containing all players.

[5] Among others uniqueness of the equilibrium solution.

been explored and concepts have been formalized. Some also affecting multilateral negotiations are:

- *Risk of breakdown*: danger negotiations break down in a random manner (e.g. players may get fed up);
- *Outside options*: strategic decision of a player to opt out as he or she has a better alternative;
- *Inside options*: payoff a player obtains while temporarily disagreeing with one another.

For more on these concepts, we refer to ”Bargaining Theory with Applications” authored by Muthoo [1], who has done a beautiful job of synthesizing the extensive literature on bilateral negotiations.

3 RECENT MULTILATERAL NEGOTIATION MODELS

In this section, we present some recent developments in multilateral negotiation models using the alternating offers procedure. Remark that this section is by no means exhaustive as research has shattered into many academic (sub)fields.

3.1 Exchange economies and networks

In his work, Penta applies the alternating offers protocol to exchange economies with players A and respective resources Π [21, p. 418]. Thereby, his work generalizes Dávila and Eeckhout’s [22] work to exchange economies with more than two players involved. In his model, every player has their own distinct resources $\pi_i \in \Pi$. Each period of time a proposer is chosen to announce *prices* for (slices of) his resource π_i and *maximum trading constraints*, limiting buyer’s demanded quantity. If an agreement is reached – thus, all buyers agree upon both the price and their share – the proposer is the ”residual claimant” of a central market[6]. This implies that it is now up to the remaining players to simultaneously make their purchases, subject to the resources they have and the trading constraints imposed by the proposer. Finally, the market is cleared at the established prices. Or, if no deal is struck, another proposer is chosen.

Under stationary strategies and frictionless bargaining[7], this model has shown there is a unique sub-game perfect equilibrium implementing true competitive (or Walrasian) allocation or resources Π over the players A [21, p 423]. Again, the assumption of stationary strategies is a strong one and, as argued by the author, should only be chosen in market settings in which players are (extremely) bounded rational [21, p. 424]. Moreover, the role of different bargaining procedures – as extensions of the alternating offers protocol – are still relatively unknown.

3.1.1 Bargaining in simultaneous networks

Another obvious limitation is that trade is centralized, involving all players. Therefore, Manea [24] and Abreu and Manea [25][26] analyze network structures between the players involved. In other words, they do not allow all players to trade with any other player in the multilateral negotiation model. Instead, only players linked to each other can make transactions in the exchange economy [25, p. 43-44]. Not only does this decentralize trades, it also becomes more realistic as in many markets players (i.e. buyers and sellers) need to have specific relationships with each other. This includes, among others, social contracts, joint business opportunities, trade agreements and technological capabilities [25, p. 44]. Trade between relationships again occurs according to the alternating offers protocol, similar to [21]. Thus, for each period of time proposers are selected who announce both prices and certain trading constraints. Note that they only propose to one (linked) buyer. Therefore, the models in [25], [26] and [24] are all decentralized implementations of multiple simultaneous bilateral negotiations.

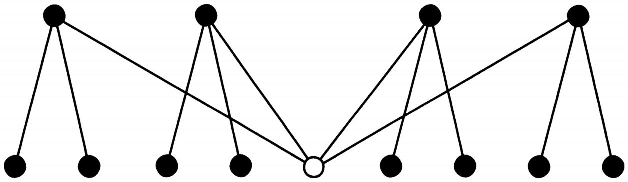

Fig. 2. Is the white player the most powerful in the trade network? Adapted from [24, p. 2043]

The main purpose of the authors above is to analyze i) the relative strength of firms affected by the network structure; ii) possible partnerships in the equilibrium and iii) whether or not efficient allocation of goods is achievable [26, p. 2]. They demonstrated, among others, that a player’s power in the network is highly interdependent. Their bargaining power does not only depend on the number of links, but also the identity and power of the player connected by that link (for example, see Fig. 2). Subsequently, also their power depends on the number of links and identify and power of players connected to their links, and so forth [24, p. 2045-2049].

Despite their novel approaches, there are still some deficiencies. First, they are aiming for efficient global (network) allocations, whereas offers, counteroffers and transactions are inherently local. This creates the obvious tension between a player’s individual optimal allocation versus global (network) efficiency [25, p. 58]. Secondly, it is unclear whether certain useful characterizations of network structures lead to definite generalized efficiency equilibrium allocations [26, p. 13]. Finally, the overall literature on bargaining in decentralized networks is rather limited. So, there are still many interesting gaps to explore.

3.1.2 Bargaining in sequential networks

In contrast to the simultaneous decentralized multilateral bargaining models introduced in the previous section, also some work has been done molding vertical (supply-chain) relationships in networks. Such relations occur, for example, when upstream producers negotiate with different retailers who all face the end user market. Not only is there an interdependency between the demand of consumers and number of orders retailers place at producers, also the producer itself may choose to bargaining with intermediates in sequential order [27, p. 411-412]. Assume, for example, a_1 is the producer and $\{a_2,\, a_3\}$ retailers. Then, the outcome of price/demand negotiations between a_1 and a_2 provide

[6] The residual claimant refers to the player who has the remaining claim on the complete market’s net flows [23, p. 302-303].

[7] There are no transaction costs between the proposer and its buyers.

a (dynamic) outside option in the subsequent negotiations with a_3 [27, p. 411].

Guo and Iyer (2013) have implemented a multilateral bargaining model analyzing the strategic options producers have in such bargaining network [27]. They might either negotiate with retailers one-by-one in random order, all together or choose specific retailers at certain periods in time. This type of research is derived from a stream of literature studying three-party negotiations, first introduced by Aghion and Bolton (1987, [28]) and further refined by Marx and Shaffer (2007, 2010) [29], [30] respectively. The aforementioned authors were, similar to [27], interested in the order of subsequent negotiations. Though, whereas Marx and Shaffer have implemented the "take-it-or-leave-it" protocol[8] [30, p. 452], Guo introduced the alternating offers protocol to analyze the same problem.

The most obvious limitation of the last research paper is that they still only model settings with one producer and two intermediate retailers. Whereas, it would be interesting to model multiple producers negotiating with multiple (linked) retailers, similar to the decentralized networks we have seen in Section 3.1.1. This relates to reality as retailers often carry products of multiple suppliers [27, p. 428].

3.2 Resource reallocation

Similar to the exchange economies introduced in Section 3.1, the exchange of goods has been researched in the field of automated negotiation in terms of resource *re*allocation. Though, where in exchange economies in every time period a single resource is distributed over multiple customers[9], Sandholm (1998) introduced a problem of reallocating multiple resources – belonging to different players – over various others [31]. Another essential difference is that to reallocate resources, players are allowed to make side-payments[10] to compensate potential losses [31, p. 71-72].

The reallocation of resources among fixed number of players has received major attention within the study of multi-agent systems, especially in terms of task allocation. Both Endriss et al. (2003, [33]) and Dunne et al. (2005, [34]) introduce a similar protocol (based upon alternating offers) dealing with resource reallocation. Let us denote the distribution of resources over n players as $\Pi = \{\pi_1, \cdots, \pi_n\}$ and the accompanying side-payments as $\bar{p} = \{\bar{p}_1, \cdots, \bar{p}_n\}$ then the general protocol studied in this work reads:

1) Start with initial allocation of resources Π^0 and with $\sum_{i=1}^{n} \bar{p}_i = 0$ (no side payments);
2) Random player is selected and asked if he wants to make a proposal deal $(\Pi, \Pi', \bar{p})$;
3) If all players agree with Π' and $\bar{p}$ – that is, the proposed allocation of resources and side payments – and a certain termination condition is satisfied, then negotiation terminates and both Π' and $\bar{p}$ are accepted;
4) If every player agrees but termination rule is not satisfied, then Π' and $\bar{p}$ are still accepted but we return to step 2;
5) If any player does not agree, then Π' and $\bar{p}$ are rejected and we return to step 2.

In which Π' denotes the proposed distribution of resources. Moreover, the termination rule is often an optimal global allocation, such as one of various measures of social welfare (e.g. egalitarian, utilitarian and Nash product). Clearly, step 2, 3, 4 and 5 entail a modified version of the alternating offers protocol. Though, in this case, not a single offer but a *deal* is proposed containing one or more players. It is this deal (or "contract" [31]) that has been the major unit of analysis as finding an optimal deal is NP-hard [18, p. 153-155]. Especially considering the fact that if $\Pi \rightarrow \Pi'$, players should not be worse off[11]. Therefore, various restricted, computationally easy types of deals have been proposed by [31], including:

- *O-contracts* only involve exchanging one resource and side payment to another player;
- *C-contracts* involve exchanging multiple resources and one side payment to another player;
- *S-contracts* concern swapping a resource (and making side payment) from one player to another;
- *M-contracts* comprise of transferring a single resource.

In [34], it is shown that even a sequence of these contract types does not result in optimal allocations and determining whether a particular allocation can be reached is also NP-hard. For a recent survey on approximability and inapproximability of optimal resource allocation in multi-agent systems, see Nguyen et al. (2013, [35]). More recent applications of optimal reallocation using similar contracts and protocols have among other been focusing on traffic routing in grid computing (e.g. [36]). Also the reallocation of indivisible resources have been encountered (e.g. [37]).

3.3 Imperfect information in multilateral bargaining

In Section 2.3, we already touched upon imperfect information in bilateral bargaining settings. Most early multilateral closed form models assumed perfect information. In this subsection, we will present some novel techniques to learn from the opponent, as an alternative approach to reduce the impact of incomplete information caused by a player's private values. Moreover, we present papers introducing stochastic uncertainty in the negotiation resources, as another example of imperfect information.

3.3.1 Opponent modeling

From a player perspective, opponent models can be created to deal with the information imbalance between players involved in bargaining games. Such an *opponent model* is an abstract description of a player and/or its behavior [38] often estimating what opponents want (i.e. *preference estimation*), what opponents will do (i.e. *strategy prediction*) or what type of player the opponents are and how to

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[8] In which, contrary to the alternating offers procedure, only a single offer is made.

[9] Or, when decentralized, a single resource is bilaterally exchanged multiple times.

[10] Introduced by Von Neumann and Morgenstern's theory of cooperative games [32]. For an excellent introduction, see [12, p. 127-132]

[11] That is, players are believed to be *individual rational* (see also [12, p. 150]).

act correspondingly (i.e. *opponent classification*, [39, p. 864-865]). Automated negotiation has come up with an extensive set of modeling techniques, ranging from neural networks predicting the most likely next opponent counteroffer x^{t+1} (e.g [40], [41]) to Bayesian learning estimating the likelihood offers are accepted (e.g. [42]). Most of these techniques use information from the offers and counteroffers and, therewith, exploit the alternating offers procedure to make predictions. This is also referred to as *online* modeling [39, p. 865]. In contrary, *offline* opponent models use historical data obtained from previous negotiations or other events. We refer to Baarslag et al. (2015) for a comprehensive survey of opponent modeling techniques in automated negotiation models [39].

Unfortunately, many of the surveyed models are still only applied in bilateral settings. A major challenge scaling these models is that they need to be constructed from a limited number of offers and counteroffers [39, p. 865], which is even harder in multilateral settings[12]. In addition, computational limitations and real-time deadlines impose the challenge of creating these models as fast as possible [39, p. 866].

3.3.2 Exogenous resource uncertainty

Not all incomplete information stems from opponents. In [43], Sgobbi and Carraro introduce a multilateral multi-issue negotiation model – in itself based upon [11] – in which the to be negotiated resources are partly uncertain. This implies that all players have their own perception of the size of pie π and, therefore, value each slice (the offer x) differently. As the model is applied to the Piave River Basin in Italy, the latter is believed to represent the increasing concern over the availability of water resources. As a consequence of introducing (stochastic) uncertainty, the model has no closed form and, thus, could not be solved analytically [43, p. 122]. Though, the numerical solutions still provides useful insights in describing how players strategies and the resulting agreement are affected by stochastic uncertainty.

The work above is further extended by Goodhue et al. (2016) in which a similar multilateral multi-issue negotiation model with uncertain resources has been applied to study the future of California's Sacramento-San Joaquin Delta [44]. They try to systematically assess the influence of different sources of bargaining power on the negotiation outcome under uncertainty. Note that more work covering uncertain resources in multilateral negotiation settings is extremely small, but – in our opinion – an important area for future research[13]. Clearly, because many (international) negotiations nowadays face similar, uncertain characteristics. Most obvious examples include climate change negotiations and refugee crises.

3.4 Coalitional bargaining

In Section 2.2 on page 2, we have already seen that whenever negotiations involve more than two players, the possibility of coalition formation emerges. Two prominent papers have been extending the model introduced by Chatterjee et al. (1993, [20]). Before we present those extensions, let us first describe the initial model briefly.

Again, we have a set of n players $A = \{a_1. \cdots, a_n\}$ and pie π. Instead of only bargaining over a share of pie π, a player's offer x in [20] also consist of a proposed coaltion $S \subseteq A$. For example, if $n = 6$ and player a_1 makes an offer to $S = \{a_1,\ a_2,\ a_3,\ a_5\}$, the three other members $S \backslash \{a_1\}$ need to agree or disagree with their (personal) offers $x = \{x_1,\ , x_2,\ x_3,\ ,x_5\}$. If everyone agrees, coalition S will form and leaves the negotiation table (thus $A' = A \backslash S$). If not, then the first who disagrees[14] is allowed to make a counteroffer. Note that the remaining players A' continue to negotiate until every one agrees and π is divided. Thus, the model provides (stylized) answers to two fundamental questions[15]: which coalitions will form? And, how does it divide the winnings among its members? They classify efficient outcomes whenever the grand coalition forms. The authors show that if the discount factor is sufficiently large enough, there will always be an order of A for which the grand coalition forms. Though, again, the major limitation of this model is that strategies are assumed to be stationary in order to do so. Additional empiric research of Bolton, Chatterjee and McGinn (2003, [48]) has shown that, in contrary to the theoretical model, real-life experiments were much closer to efficient outcomes as participants were able to react upon each other. Or, to paraphrase *"(...) whenever C is aware of A's or B's offers, she 'jumps in' and makes the efficient offer involving all three players"* [49, p. 80].

Another limitation of the above model is that no externalities occur [49, p. 81-82] – i.e. that the valuation of an offer x does not depend on the coalition S or other coalitions that have been formed. This is, of course, not true in many business and political situations. Therefore, Huang and Sjöström (2010, [50]) have extended [20] by allowing externalities. Moreover, their model does allow "inefficient" outcomes i.e. the grand coalitions does not always have to form. As, in some cases, large coalitions can be extremely costly [50, p. 67].

A final extension examines the influence of private information on coalitional bargaining. Okdada (2009, [51]) is one of the first (and only very view) researchers introducing incomplete information in coalitional bargaining models. Therefore, he alters the model of [20] in such a way that besides players agreeing or disagreeing on offers, they also agree or disagree on information (which Okada denotes as *informational objection*). This implies that players should also evaluate to what extend *"(...) proposer's private information is credibly transmitted to responders"* [51, p. 4]. Roughly translated, this incorporates the trustworthiness of the proposer and his shared information in deciding whether or not to form coaltion.

Apart from the model of Chatterjee et al. and its extensions, also slightly different approaches to coalitional bargaining have been taken. Note that the model description in the introduction of this section only mentions the emerge of coalitions, not the potential *collapse* during bargaining.

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[12] From our own experience, although the total number of offers and counteroffers increase in multilateral negotiations, the average number for each player decreases, especially in finite-time horizon negotiations.

[13] Although, the authors of [45] have frequently adjusted their model in different settings (e.g. [46], [47]), they have never introduced exogenous resource uncertainty.

[14] Note that players are selected equal to the order of A.

[15] See [32].

The most well known model introducing the latter is described by Ray and Vohra's [52] *Equilibrium Binding Agreement* (EBA). However, in their turn, they assume coalitions do not re-emerge. So, Diamantoudi and Xue's [53] extended the EBA model[16] allowing the coalitional bargaining model to do both. A related, though slightly distinct model is introduced by Funaki and Yamato [54]. They prescribe a step-by-step approach restricting the coalition structures that can change every bargaining offer: either a *merger* of two separate coalitions into a single coalition occurs or a coalition collapses in two separate coalitions [54, p. 1-2]. By doing so, they are able to identify certain stable coalition structures; those who dominate others by being better off with the final outcome. Such models have also appeared in the field of automated negotiations. For example, Fatima, Michalak and Wooldridge (2016, [55]) and Hussin and Fatima (2017, [56]) have provided an automated negotiation environment and heuristic methods for analyzing and/or optimizing coalitional structures respectively. In this environment, the to be divided resources have been replaced by preferences of the potential structure of coalitions allowing players to *"(...) resolve such conflicts and partition themselves into non-overlapping coalitions.". [55, p. 899].*

4 DISCUSSION AND CONCLUSION

In this survey, we have seen that Rubinstein's initial alternating offers protocol has still a major influence on modern-day multilateral negotiation models. However, it has been altered countless of times to meet specific needs. Such needs, for example, stem from different application areas. We have described literature using multilateral negotiation models in different types of markets, networks and supply-chain relations. Moreover, we have seen altered protocols to allow for resource reallocation in large groups of players, among others applied to grid computing. In addition, some recent techniques have been (very) briefly discussed dealing with imperfect information settings by modeling opponents. Also real-life applications of multilateral bargaining models with exogenous resources uncertainty are considered, among others applied to the Sacramento-San Joaquin Delta in California. Finally, we survey some coalitional bargaining models in which the alternating offers protocol is used to form coalitions and divide the winnings among their members. It is discussed that recent models allow for externalities, have introduced private information or support the collapse or re-emerge of coalitions during the bargaining process.

To conclude, we observe that bargaining theory is by no means the only field studying multilateral negotiation models using (extensions of) the alternating offers protocol. Currently, these models are being studied in computer science related fields, such as multi-agent systems, automated negotiation and grid computing. We have also seen many studies in exchange economies and supply-chain management. Finally, applications can be found in water- and environment management related topics. Note that, as studies are so wide spread in various fields of research, this survey is by no means exhaustive.

5 DIRECTIONS FOR FUTURE RESEARCH

By conducting this survey, we identified a number of directions for future research. First, and far most important, we follow Bolton and Croson (2012, [49, p. 85-86]) that, in general, there is a need to diminish the gap between theoretical literature on multilateral bargaining models and empiric and experimental research. Either data needs to be gathered and simulated with the theory in mind or theories need to be extended even further to meet reality.

Secondly, we observe that mainly in coalitional bargaining the alternating offers protocol still allows every player to interact with all other players. This is, of course, not true in, for example, political settings. Therefore, linking the theory on multilateral bargaining in networks with coalitional bargaining can in potential lead to very interesting models, more close to real negotiation settings.

Moreover, in Section 2.2 we have not found any research on agenda optimization in multilateral negotiation settings. Also, such research might contribute to shortening the gap between theoretical models and experimental research, as agendas optimizing social welfare, for example, could be used to design more fair negotiation settings.

Thirdly, literature on bargaining in sequential networks could be further extended, both theoretically (i.e. > 3 players) as well as by applying such models in other negotiation settings. For example, political negotiations often happen in sequential phases and multiple arenas. Therefore, assuming outcomes of one phase or arena not influencing other outcomes is seriously flawed.

Finally, both in simulation and empirical settings, opponent modeling could be used as an alternative approach to reduce the impact of incomplete information caused by a player's private values. Though, some work still needs to be done in order to improve the scalability of opponent modeling techniques. In general, imperfect information is seriously underexposed in recent multilateral bargaining models. Especially, compared to the overwhelming amount of literature on imperfect information in bilateral negotiations (for an oversight, see [1, p. 251-294]).

APPENDIX A

The data in Fig. 1 were collected using Scopus® abstract and citation database of peer-reviewed literature. In total, 340 scientific articles, conference papers, book chapters and reviews were obtained by searching for *("multilateral" AND ("negotiation" OR "bargaining") AND "model")* in titles, abstracts and keywords. These were categorized in their respective research areas for each year.

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[16] Which they (uncreatively) named *Extended Equilibrium Binding Agreement* (EEBA).

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