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## URBAN TRAVEL TIME RELIABILITY AT DIFFERENT TRAFFIC CONDITIONS

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# Urban Travel Time Reliability at different Traffic Conditions 

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## AbSTRACT

The decision making of travelers for route choice and departure time choice depends on the expected travel time and its reliability. A common understanding of reliability is that it is related to several statistical properties of the travel time distribution, especially to the standard deviation of the travel time and also to the skewness. For an important corridor in Changsha (P.R. China) the travel time reliability has been evaluated and a linear model is proposed for the relationship between travel time, standard deviation, skewness and some other traffic characteristics. Statistical analysis is done for both simulation data from a delay distribution model and for real life data from Automated Number Plate Recognition (ANPR) cameras. ANPR data give unbiased travel time data, which is more representative than probe vehicles.

The relationship between the mean travel time and its standard deviation is verified with an analytical model for travel time distributions as well as with the ANPR travel times. Average travel time and the standard deviation are linearly correlated for single links as well as corridors. Other influence factors are related to skewness and travel time standard deviations, such as vehicle density and degree of saturation. Skewness appears to be less well to explain from traffic characteristics than the standard deviation is.

Keywords: Travel time reliability, Skewness, Travel time standard deviation, Automated number plate recognition, Urban traffic.

## Introduction

The reliability of travel time is an important characteristic of a trip. It reflects the quality of traffic operations both for freeways and urban networks. In the choice of routes, departure time, travel mode and destinations, travel time reliability plays a role at least as important as the expected travel time (Bates et al., 2001; Bogers, 2009). In certain situations, travelers seem to place a higher relative value on reducing travel time variability than on the mean travel time (Bates et al., 2001).

In order to quantify travel time reliability, a range of reliability measures have been proposed in the past decades, e.g., statistical range in the form of expected travel time plus/minus the standard deviation multiplied by a factor (Bates et al., 2001), $95^{\text {th }}$ percentile (FHWA 2015), buffer index, planning time index, tardy trip measures (Lomax et al., 2003). Among all these measures, the standard deviation is most often used to represent variability and is included in the route choice utility function in studies of the value of reliability (Liu et al., 2004). Sen et al. (2001) proposed a mean-variance approach which employs the variance of travel time as the travel time reliability measure to investigate travelers' route choice behavior. Different methods to determine the value of reliability have been reviewed by e.g. Carrion and Levinson(2012) and De Jong and Bliemer (2015).

Mahmassani et al. (2012) analyzed the characteristics of the travel time reliability for an urban road network by establishing a linear regression model describing the relationship between the standard deviation of travel time per unit distance and the corresponding mean value. Van Lint et al.(2008) argued that the travel time distribution is often wide and skewed, particularly during the periods when congestion occurs, sets in or dissolves. Given a skewed distribution, applying classic measures based on the mean or variance of travel times may lead to a biased estimate of reliability. They suggested that skewness should be considered as another measure of unreliability. The importance of skewness is emphasized by Bogers
(2009). She showed that route choice in an experimental situation was especially determined by the skewness of the travel time distribution.

There are large differences in the traffic processes that determine travel time reliability between urban roads and freeways (Tu, 2008). In this article we focus on urban travel time reliability. The travel time reliability (or variability) in urban networks has received a lot of attention during the past years. With the development of various monitoring techniques, travel times can be derived from different data sources (e.g., from Automatic Number Plate Recognition data, GPS/mobile phone equipped floating car data, and Bluetooth). Different statistical models such as Normal, Lognormal, Gamma and Weibull distributions (Al-Deek \& Emam 2006;Arroyo \& Kornhauser, 2005; Pu, 2011) are applied to describe the travel time variability. Due to the complexity of urban traffic conditions, a single distribution model couldn't well represent the travel time distribution of an urban road. Guo et al. ( 2010) proposed a mixed distribution model to capture multi-state traffic conditions for urban roads. Chen et al. (2017) proposed a copula-based model to estimation path travel time for urban arterial roads considering stochastic characteristics of segments. Yang et al. (2017) proposed a Gaussian mixture model to estimate travel time distributions for urban arterial road. Apart from modeling travel time variability with field data, Kim et al. (2013) employed traffic simulation models to derive travel time distributions under different scenarios considering various demand and supply uncertainty factors, such as weather, traffic incidents, work zones, traffic control. In urban context, travel time variability can be further recognized in different temporal scales. Robinson and Polak (2007) applied the K-Nearest Neighbor classification method to characterize travel time variability using loop detector data. They disaggregated travel time variability into three components: vehicle-to-vehicle, period-to-period and day-today. In order to capture both the vehicle-to-vehicle and day-to-day variability in travel time data, Kim and Mahmassani (2014) proposed a Gamma-Gamma mixture distribution model.

Based on their proposed model, the heterogeneity within these two variability types under different weather conditions can be described as well.

Up to now, most research on urban travel time reliability was done with simulation models to produce data for reliability analysis. Though Mahmassani et al. (2012; 2013) used both simulated data and field GPS trajectory data to model travel time reliability, the investigation with field data is rather limited. Moreover, the GPS data they used is only from a sample of the total traffic on the network and no flow data is available in their study area. The question is to what extent the derived relationship between the travel time variability and the network flow or density deduced from GPS sample data can represent the real situation. With the development of traffic monitoring systems, different field data sources including Automated Number Plate Recognition (ANPR) camera data, GPS taxi data, loop detector data and signal timing data are now available.

In ( Zheng \& van Zuylen, 2011), the authors derived an analytical model which describes the travel time distribution for signalized roads. In that model the variation in travel time between different vehicles is due to differences in arrival moment at the intersection and variations in the arrival rate. Zheng et al. (2017) further extended the model to include also the effect of spill back. Their model has been validated with respect to the travel time distribution both with simulated data from VISSIM and real data from probe vehicles.

In this paper, firstly we give some observation for the use of simulation programs, probe vehicle data and data from ANPR cameras. Then we apply the travel time distribution model to derive two reliability measures: the standard deviation and the skewness of travel time. The flow data from cameras and signal timing data extracted from the SCATS traffic control system in Changsha are used as input to the travel time distribution model. We investigate the relationship between the expected travel time and the standard deviation, as well as the effect of changing traffic states on the travel time variability, i.e. to show the changes of the standard
deviation and the skewness during the day. Consecutively, field travel times are calculated from ANPR data. The relationship between the mean travel time and the standard deviation is shown. Also the connection between the travel time variability (e.g., the standard deviation, skewness) and different traffic states is established. Finally, some discussion and conclusions are provided.

## Some observations regarding anpr, simulation and probe vehicles

For the calculation of travel time reliability the travel time distribution on links, routes and/or whole networks are needed. The source of the necessary data is often a micro simulation program, speed measurements, probe vehicles or ANPR data. In the study reported in this article we used data from a route in Changsha, the capital of Hunan province in China (Figure 1), consisting of ANPR data, traffic volumes, and GPS data from 7200 taxis. These data were collected for three days from 20, April 2015 to 22, April 2015. Several input variables and parameters such as traffic flow rates, signal control plans and the saturation flow rates have to be determined in order to derive the delay and travel time distributions. Here we mainly use ANPR data to estimate travel time distributions, traffic flow rates, and saturation flow rates for the study area. Before analyzing travel time data, the different empirical data sources have been analyzed.

In Figure 1 the part of the network that is used for the travel time analysis is shown. It consists of 9 intersections which are all provided with loop detectors located close to the stop line on every lane. These detectors are a part of the traffic control system SCATS. The intersections denoted with italic numbers have ANPR cameras, one camera per lane. Taxis equipped with GPS devices send their position data every 30 seconds.

[^0]
## Analysis of the quality of the ANPR data

The ANPR cameras record the number plates of all vehicles passing the stop line. Lighting conditions appear to have an influence on the quality of the registrations. In the evening and early morning the failure rate of the cameras, in terms of missed number plates, is much larger than during the day time. Some number plates appear to be registered more than once at an intersection, sometimes by the same camera (up to 8\%) and sometimes by adjacent cameras (up to $0.06 \%$ ).

The ANPR cameras count traffic per lane, just as the loop detectors of the SCATS traffic control system do. The values from the cameras were in general close to the loop detector counts on the intersections in the study area. Since drivers do not always follow paths indicated by the lane marking on the intersection, they can 'escape' from being counted by the loop detectors or cameras. Especially left turning traffic is often miscounted.

Also the volumes of taxis counted from ANPR data (taxis have a special number plate and can be separately identified by the cameras) are close to those counted from the GPS taxi data.

An analysis of the volumes from ANPR cameras shows that some cameras did not perform well during these three days and at some intersections incidents have happened during one of the days. In the further analysis only intersections with regular and consistent data have been chosen.

## Travel time measurement

For the travel time measurement, two data sources are available: ANPR and probe vehicles (taxis). A comparison was made for travel times measured from these two data sources. It appears that travel times measured from taxi GPS data can be rather different from the travel
times determined from ANPR data, as shown in Figure 2. The reason is probably that taxi drivers are more experienced and thus faster than other drivers, but sometimes stop half way on a road to let passengers alight or get in, or just to take a break. The records from taxis stopping for more than 2 minutes or making a detour from a link have been removed. Such elimination of outliers, i.e. vehicles stopping between two intersections for reasons that have nothing to do with the traffic conditions, is also necessary for ANPR data. The outliers in ANPR travel times are visible in the graph of travel times as function of the arrival time at the end of a link. Normally travel times of two consecutive arriving cars are only slightly different, with the exception of the transition between a car that just passes the end of the green time and its follower. ANPR journey times that were more than twice the journey time of the vehicle arriving before or after were considered as drivers who had some activities during their journey between two cameras. About $5 \%$ of the journey times were identified as outliers and removed. The elimination of outliers in the ANPR travel times is very important to get a consistent statistical analysis.

The fact that during some time of the day (e.g. between 16:00 and mid-night) taxis drivers appear to be much faster than the other drivers, makes it preferable to use ANPR data for the measurement of travel time.
\# Figure 2 \#

## Conclusions with respect to simulations and probe vehicles

A simulation represents aspects of real world traffic. Most simulations are calibrated and validated on aspects like mean travel time and traffic volumes. In a study of travel time reliability, the simulations should give travel time distributions which is proven to be valid.

Zheng et al. (2010, 2011, and 2015) explicitly show the validity of their travel time model with respect to the distribution of travel time, both with measured real travel time data and simulation data.

Another drawback of simulation is that certain aspects of reality are ignored. Saturation flows, which have an important role in the urban traffic process, are assumed to be constant, independent from time and traffic volume. In most articles about the application of simulation programs the issue of saturation flow rates is not discussed at all, so that probably default values of saturation flow rates are used. Based on these findings we firstly analyzed the travel time reliability with a mesoscopic simulation program developed by Zheng (2011), which gives validated travel time distributions. For this simulation study the traffic volumes that have been measured in reality are used. The cycle times and green splits for the model calculation were taken from the traffic control system SCATS. Afterwards the travel times measured by ANPR cameras are analyzed. The probe vehicle data from the taxis are not used because of the reasons mentioned above.

## Modelling delay and travel time distributions

Microscopic simulation models try to represent the behaviour of traffic on a detailed level: the behaviour of individual vehicles interacting with each other and responding to road conditions and traffic management measures. For the purpose of evaluation of the traffic situation, such simulations can provide the necessary characteristics; however, for the optimization of the traffic system it is more effective to have a macroscopic model that gives the relation between parameters of the traffic system, such as traffic volumes, the timing of signals, and the performance, e.g. travel time.

A meta-model can be derived from microscopic simulation. An example is the model developed by Webster (1958) to describe the macroscopic relation between traffic parameters
and signal timing and the delay. He developed a mathematical model consisting of three terms:

1. A term that gives the delay for a continuous flow of vehicles, stopped during the red phase when the queue builds up and crossing the stop line when the queue is released;
2. Since the intersection is also a capacity bottleneck, the second term in the delay model describes the delay at a bottleneck with arrivals according to a Poisson statistics.
3. The third term is a correction, a rather complicated product of parameters that play a role in the first two terms.

There are some disadvantages of meta-models because the validity of the model is restricted to a certain range of the parameters. The Webster model is only valid for undersaturated conditions and becomes infinite for fully saturated situations. The integration of the oversaturated and undersaturated model has been developed by several researchers (e.g. Akcelik, 1988). All of these models give a smooth transition between undersaturated and oversaturated conditions.

Fu and Hellinga (2000) continued in that line and developed an analytic expression for the standard deviation of the delay, assuming stationary arrivals rates and zero initial queue. The difficulty in applying a formula like the one derived by Fu and Hellinga (2000) and Akcelik (1988) in practice is, that often the initial queue is not zero and that the queue is not always growing but also shrinking. Although the mathematical model for the delay probability distribution developed by Viti (2006) covers all these situations, it does not provide a closed mathematical expression for the situation that flows and queues are changing over time.

From the examples mentioned above one can find that the most important parameters that determine the delay and its standard deviation are the effective green to cycle time ratio, the flow and saturation flow and the degree of saturation $x$. The offset between signal control on different intersections is another important factor determining travel time and its standard deviation. This is not taken into account in the studies mentioned above.

Another approach to determine the statistical properties of travel time is to use a mesoscopic model for the distribution of the delay. For the delay on an isolated intersection the distribution $P(W)$ of the delay $(W)$ when there is no overflow queue (a remain queue at the start of the red phase) is given by (Van Zuylen and Viti, 2007; Zheng et al., 2010):

$$
\begin{equation*}
P(W)=\alpha \delta(W)+\beta, \quad 0<W<t_{r} \tag{1}
\end{equation*}
$$

where $\alpha=1-t_{r} /\left\{t_{c}(1-q / s)\right\}$ and $\beta=\left\{t_{c}(1-q / s)\right\}^{-1}$.
where $t_{c}$ is the cycle time and $t_{r}$ is the duration of the red phase; $q$ is the flow of the arriving traffic and $s$ is the saturation flow. The function $\delta$ is a distribution function defined as:

$$
\begin{aligned}
& \delta(x)=0 \text { if } x \neq 0, \\
& \delta(0)=\infty, \text { and } \\
& \int \delta\left(x-x_{0}\right) f(x) d x=f\left(x_{0}\right)
\end{aligned}
$$

For this uniform delay distribution the expectation value and the standard deviation can be calculated:

$$
\begin{align*}
& E\{W\}=\int_{0}^{t_{r}} W P(W) d W=0.5 \beta \cdot t_{r}^{2}=\frac{t_{r}^{2}}{2 t_{C}(1-q / s)}  \tag{2}\\
& \sigma\{W\}=\sqrt{E\left\{W^{2}\right\}-E^{2}\{W\}}  \tag{3}\\
& E\left\{W^{2}\right\}=\int_{0}^{t_{r}} W^{2} P(W) d W=\frac{t_{r}^{3}}{3 t_{C}(1-q / s)}  \tag{4}\\
& \sigma\{W\}=E\{W\} \sqrt{\frac{4 t_{C}(1-q / s)}{3 t_{r}}-1}=E\{W\} \sqrt{\frac{2 t_{r}}{3 E\{W\}}-1} \tag{5}
\end{align*}
$$

This shows that the standard deviation of the (uniform) delay for undersaturated conditions can be considered as a function of the expectation value and the duration of the red phase.

That means that also the standard deviation will become larger when the ratio of the traffic flow and saturation flow $q / s$ becomes larger, e.g., closer to the value 1 .

For larger values of $q / s$ the uniform delay is only a small part of the real delay and the delay due to variations in the arrivals and oversaturation has to be included. The mathematical expression for the initial or overflow queue is the representation of a Markov process in which the distribution of the overflow queue length in a cycle depends on the distribution in the previous cycle and the probability distribution of the arrivals (Olszewski, 1990; Viti and van Zuylen, 2006; 2009, 2010).

Zheng and van Zuylen $(2011$; 2014) further extended the single intersection model to two intersection models which consider the signal coordination between the upstream intersection and the downstream intersection, and the overflow queue at the beginning of the red phase. The delay probability distribution function is given as:

$$
\begin{equation*}
P_{d}\left(W \mid n_{0}\right)=\alpha\left(n_{0}\right) \delta(W)+\sum_{N} \beta B\left(W, W_{2 N+1}\left(n_{0}\right), W_{2 N+2}\left(n_{0}\right)\right) \tag{6}
\end{equation*}
$$

Where

$$
\begin{equation*}
\alpha=\max \left(\frac{s t_{g}^{\prime}-n_{0}-1}{q t_{C}}-\frac{t_{r}+\frac{\left(n_{0}+1\right)}{s}}{t_{C}\left(1-\frac{q}{s}\right)}, 0\right), \beta=\frac{1}{\tau_{C}\left(1-\frac{q}{s}\right)}, \tag{7}
\end{equation*}
$$

$B\left(w, w_{2 N+1}, w_{2 N+2}\right)$ is a box-shaped function with the property:

$$
B\left(w, w_{2 N+1}, w_{2 N+2}\right)=\left\{\begin{array}{lr}
1 & w_{2 N+1}<w<w_{2 N+2}  \tag{8}\\
0 & \text { otherwise }
\end{array}\right.
$$

$W_{2 N+1}, W_{2 N+2}$ are delays at transition moments as shown in Figure 3;
$n_{0}$ is the overflow queue at the beginning of the red time;
$t_{g}$ ' is the 'effective' green time at the downstream intersection, which is calculated as the green time of the downstream intersection minus the mismatched green time between the upstream intersection and the downstream intersection due to bad coordination.
\# Figure 3 \#

The next section uses the delay (travel time) distribution models as developed by van Zuylen and Viti (2007), Zheng and Van Zuylen (2010; 2011). Equations (1) to (8)) are used to determine the standard deviation and skewness of delays and travel times for a single intersection and two linked intersections. The leading principle in this article is that a relation is searched between the standard deviation and skewness and traffic parameters like flow, average or expected delay, volumes and signal control parameters. This is done by both simulation and empirical data. From the discussion above we can expect that the relevant parameters are $E[W], q / s, x, t_{C}$, and $t_{r}$.

## The Model-Simulated standard deviation and skewness

The calculations with the mesoscopic model are executed using the actual traffic variables for the corridor shown in figure 1. The data from the ANPR cameras were used for this purpose. The cycle times and green splits were obtained from the log files of the SCATS control system, but the offsets had to be estimated from the ANPR data. Traffic flow rates, saturation flow rates and signal timings are aggregated into 15 minutes time intervals starting from 7:00 AM until midnight. For each 15 min time interval, the delay distribution, expectation, standard deviation and skewness are computed by Equations (1-8). Since the cycle time of the traffic control was variable and in the order of 3 minutes, a shorter time interval would give too much influence of the control process on the aggregated traffic characteristics.

Figure 4 illustrates the correlation between the expectation and the standard deviation of delays calculated based on the delay distribution model (equations $1-8$ ) using the measured
traffic flow rates and signal setting. Each 'star' in the figure represents the expectation vs. the standard deviation for a 15 minutes time interval. The correlation coefficients ( $R$-square) of west bound links 50-113 (a-b), 113-51 (b-c), 51-24 (c - d), 24-54 (d -e) are large (>0.85), which suggest that a strong linear relation exists between the expectation and the standard deviation. These results confirm the findings by Mahmassani et al. (2013) who also found a linear relation between the mean travel time and the standard deviation.
\# Figure 4 \#

If we try to find a similar relation for two linked intersections, the correlation is still visible but becomes much weaker, as visible in the lower two graphs in Figure 4. This indicates that the signal coordination between intersections has a significant influence on travel time reliability.

Although the relation between the expected value of the travel time and the standard deviation is consistent for all links, the regression coefficients differ per link.

The relation between the expectation value of travel times and skewness is shown in Figures 5 and 6. It is clear that the skewness becomes less when the travel time becomes larger, which means that for higher travel times the distribution of the travel times becomes more symmetrical. However, when the degree of saturation becomes close to 1 , i.e. larger than 0.8, the skewness increases with the increase of the expectation value as shown in Figure 6, which indicates that travel times become more skewed with a longer tail to the right for heavy traffic conditions.
\# Figure 5 \#

## \# Figure 6 \#

Apparently the degree of saturation plays an important role in the skewness. Therefore, the analysis of skewness is repeated with the degree of saturation as an explanatory variable. Figure 7(a)(b) show the relation between the degree of saturation $x$ and the skewness. Each dot represents a skewness value with respect to a certain degree of saturation for a time interval of 15 min. From Figure 7 (a), we can see that when the degree of saturation is low (e.g., in free flow conditions), the skewness value is larger than zero which indicates that the travel time distribution is right-skewed with a longer tail towards high travel time values. For undersaturated situations (e.g. $x<0.8$ ) the skewness decreases with the degree of saturation; while for near saturated conditions (e.g. $x>0.8$, Figure 7 (b)), the skewness increases with the increase of the degree of saturation. This indicates that traffic becomes more uncertain near saturated conditions, which is consistent with what we have found in the previous study (Zheng and van Zuylen, 2010). No oversaturation ( $x>1$ ) was observed for the intersections within the study corridor. The regression coefficient reflects how much the skewness will decrease when the degree of saturation increases.
\# Figure 7 \#

In the following section we analyze the travel times and travel time distributions as estimated from the ANPR observations and determine the relations between traffic states and the standard deviation and skewness.

## travel time analysis from the anpr registrations

The number plate recognition data were used to determine travel times in the East - West direction (see figure 1) for traffic travelling on the Renmin road as follows:

- Intersection 50 to Intersection 113 (a to b)
- Intersection 113 to Intersection 51 (b to c)
- Intersection 51 to Intersection 24 (c to d)
- Intersection 24 to Intersection 54 (d to e)
- Intersection 54 to Intersection 59 (e to f)

The travel times over the day are shown in Figure 8. The data of Monday 20 April 2015 appear to be different from the other two days, Tuesday and Wednesday 21 and 22 April. This is probably a post-weekend / beginning of the week effect. These differences are very well visible on the link $113-51(\mathrm{~b}-\mathrm{c})$, while on the following link, $51-24(\mathrm{c}-\mathrm{d})$ the difference is especially in the morning, while on link $24-54(\mathrm{~d}-\mathrm{e})$ the difference is more in the afternoon.

## \# Figure 8 \#

The correlation between TTSD and skewness has been analyzed for every link for three days. Figure 9 gives the analysis for link $113-51(\mathrm{~b}-\mathrm{c})$, triangle shaped dots represent data collected on $20^{\text {th }}$ April, square dots are from $21^{\text {st }}$ April, and asterisk shaped dots represent data collected on $22^{\text {nd }}$ April respectively. The relation between the mean travel time $T$ and the Travel Time Standard Deviation TTSD is assumed to have the form as:

$$
\begin{equation*}
T T S D=b+a * T \tag{9}
\end{equation*}
$$

\# Figure 9\#

The coefficients in the regression formula are slightly different for the different days and there is a negative correlation $\left(\mathrm{R}^{2}=0.908\right)$ between the slope and constants. The analysis for each link separately (see Table 1) shows that the regression parameters differ from link to link and from day to day, while the square of the correlation coefficient $R^{2}$ varies between 0.6681 and 0.9187 . It can be concluded that the there is a relation between mean travel time $T$ and $T T S D$, but that the linear relation is not a generic linear function. However, if we analyze the regression parameters $a$ and $b$ we can see that for 4 of the 5 links, these parameters have a strong correlation with the link length. Only the link 113-51 (b-c) does not fit in this relation.

Ignoring the link 113-51, we can write the parameter $a$ as $a=0.403-0.0002 L$ while for $b$ we derived $b=-9.1885+0.0221 L$, where $l$ is the length of the link. The $R^{2}$ is 0.815 for $a$ and 0.9222 for $b$.

Although we analyzed only a small number of links, we can assume that there is a linear relation between mean travel time and its standard deviation and that the parameters in that linear relation depend on the length of the link.

\# Table 1 \#

## Traffic density as explanatory variable

The average traffic density of link $i \hat{k}_{i}$ in time interval $T$ is calculated as:

$$
\begin{equation*}
\hat{k}_{i}=\frac{\sum_{j=1}^{n} \frac{t_{i j}}{l_{i}}}{T} \quad\left(T=T_{2}-T_{1}\right) \tag{10}
\end{equation*}
$$

Where

$$
\begin{aligned}
& t_{i j}: \text { travel time of vehicle } j \text { on link } i \\
& T_{1} \text { and } T_{2}: \text { start and end of the time interval. }
\end{aligned}
$$

The traffic density is calculated for all lanes of a link. Traffic densities on Monday 20 April were in general higher than on the other days. On that day the travel times were also higher and density and mean travel time are related as visible in equation (10).

## Degree of saturation

Previous research (e.g. Fu and Hellinga 2000) indicates that the degree of saturation is an important characteristic of a link that determines the standard deviation of the travel time.

In this study, the degree of saturation is also estimated for every 15 minutes using the flow rates, saturation flow rates determined with the ANPR data and the green time and cycle time from the SCATS traffic control. The statistical analysis of the relation between TTSD and the degree of saturation, saturation flow rate, traffic density, mean travel time and skewness has initially be done using the Pearson correlation coefficient. The variation in the saturation flow rates appears to have no significant relation with the variation in the TTSD. Density and travel times have a significant correlation which is obvious from the way density is calculated by eq. (10) and on most intersections skewness is also significantly related to TTSD. For the skewness, the most important correlated factors are TTSD, degree of saturation (consistent with the analysis of the model data in Figure 6) and Traffic density.

A further analysis has been executed with partial correlation analysis. In that analysis the most important correlated parameter is first used in a linear regression. The remaining errors are then analyzed to find possible correlations with the remaining variables. This analysis method removes the effect of collinearity. Table 2 shows the result of this analysis. The role of the degree of saturation appears not to be significant for most links (with the exception of 113-51) and also the explanatory value of the traffic density is relatively small (traffic density is correlated with the mean travel time as can be seen in its definition in eq. 10).
\# Table 2 \#

This analysis is further extended by a step wise linear regression. In that regression method the function to be estimated is

$$
\begin{equation*}
T T S D=a_{1} \times T+a_{2} \times S+a_{3} \times k+b \tag{11}
\end{equation*}
$$

Where:
$S$ : Travel time skewness,
$k$ : Traffic density,
b: Constant,
$a_{i}$ : Coefficient of independent variable $i$.
The parameters are tested in every step of the regression whether their explanatory values for the residuals that remain from the previous regression step can be sufficiently explained by the remaining variables. Variables that don't contribute significantly to the quality of the regression are eliminated from formula (11). The results are shown in 3 and 4.
\# Table 3 \#
\# Table 4 \#

The adjusted $R^{2}$ represents the proportion of the total variation in the series that is explained by the linear regression model. Compared with $R^{2}$, the adjusted $R^{2}$ eliminates the influence of dependent variables and series size on the coefficients.

The conclusion is that the mean travel time, TTSD and travel time skewness $S$ are mutually significantly related and that the traffic density plays a minor role in explaining the variations
of the standard deviation TTSD. Therefore, we can eliminate the traffic density from equation (11) as:

$$
\begin{equation*}
T T S D=a_{1} \times T+a_{2} \times S+b \tag{12}
\end{equation*}
$$

Table 5 shows the regression coefficients and their estimation errors for every link and on the three dates. Coefficients $a_{1}$ of Link 50-113, 113-51(a-b, b-c), and 24-54(d-e) are rather consistent over different days and just varies between 0.35 and 0.2 for most days. However, coefficient $a_{1}$ of Link $51-24(\mathrm{c}-\mathrm{d})$ and $54-59(\mathrm{e}-\mathrm{f})$ fluctuates observably, with an upper boundary of 0.554 , while the lower boundary is around 0.260 .

The coefficient $a_{2}$ varies strongly over the days and differs per link.

$$
\text { \# Table } 5 \text { \# }
$$

A next question is whether the regression coefficients are the same for different links. For the simple regression (eq. 9) it was shown that the regression coefficients depend on the link length. For the coefficients in eq. (12) we can find similar results. For the coefficients found for the three days together we can derive the following linear relations between the coefficients and the link length:

$$
\begin{array}{ll}
b=0.0161 L-8.0739 & R^{2}=0.6751 \\
a_{1}=-0.0002 L+0.4069 & R^{2}=0.9928 \\
a_{2}=0.0242 L-7.2406 & R^{2}=0.8456 \tag{15}
\end{array}
$$

Just as in the case of the simple regression we had to ignore the data from link 113 - 51 (b c), which are far from the linear relations.

## The travel time reliability analysis for a route

In the previous section the travel time reliability analysis has been done for single links. In this section travel time over a route of several links is analyzed. The full route from intersection 50 (a) to intersection 59 (f) appears to have too few complete trips. Since very few vehicles can be followed over these links before 6:00, the analysis period was confined to the period 6:00 and 0:00. Figure 8 (f) shows the route travel time for three days.

The variation of travel time between different periods of the day and between days is rather large. We will first analyze the travel time reliability within a time period, just as we did for the single links. The partial correlations are shown in Table 6.

\# Table 6\#

For this route, negative correlation exists between skewness and mean travel time. That is consistent with the results for undersaturated intersections in the model based analysis. The degree of saturation of the four intersections is mostly between 0.7 and 0.9 . Furthermore, the traffic density and volumes have a significant correlation with the travel time, standard deviation and skewness, which was not found for single links.

A stepwise linear regression of the Travel Time Standard Deviation (TTSD) as a function of the mean travel time $T$ gives:

$$
\begin{equation*}
T T S D=0.120 \times T+26.566 \tag{16}
\end{equation*}
$$

This regression equation explains $44.5 \%$ of the variation of the $T T S D$. If the skewness $S$ is also included in the regression formula, it becomes:

$$
\begin{equation*}
T T S D=0.126 \times T+10.369 \times S+20.3 \tag{17}
\end{equation*}
$$

## Comparison with a link on another route

The analysis performed on the West bound links of the Renmin Road have been repeated on different links in the network. In general a similar relation between Travel time $T$, Skewness $S$ and Travel time standard deviations TTSD is found with significant values for the coefficients $a_{1}, a_{2}$, and $b$ in eq. (12). The relation between these coefficients and the link length is not well represented by eq. 13, 14 and 15. Probably the coefficients depend on more traffic characteristics than only the link length.

## DISCUSSION AND CONCLUSIONS

Travel time (un)reliability can be characterized by two important quantities: the standard deviation of the travel time and the skewness. Most reliability studies concentrate on the TTSD, but several studies show that traveler's route choice and departure time behavior is determined also by the travel time skewness. For this reason the travel times of a route in Changsha have been analyzed with respect to mean travel time, TTSD and skewness. The analysis was aimed on travel time reliability within time periods of 15 minutes.

Two methods have been used for this study:

- A model based calculation of travel time distributions,
- Travel times obtained from Automated Number Plate Recognition cameras.

The model based reliability analysis gives results that are qualitatively consistent with the results for the real ANPR data. However, the regression coefficients from the model outcomes differ quantitatively from the ANPR outcomes. Also the regression of the ANPR measurements differs between links. Still there are significant features in the relation between travel time, travel time standard deviation and skewness. On single links TTSD is positively correlated with the mean travel time. On routes with several links this relationship is also found. The skewness is negatively correlated with the mean travel time and positively with $T T S D$. For the route the influence of volumes and density on travel time, standard deviation and skewness are significant.

The real travel times, obtained from the ANPR cameras are probably influenced by traffic management made by the traffic police in the field. Even though high degrees of saturation and high traffic densities occur, no spill back phenomena could be observed. In our two analysis methods we took this automatically into account because we used the data that came directly from the real traffic situation. Simulation programs that ignore the influence of traffic management measures probably give results that differ from the empirical data. The standard deviation of the travel time is dominated by the mean travel time.

The linear relationship between mean travel time and TTSD is not the same for every link and every day. There is some evidence that the regression parameters for a link depend linearly on the link length.

In the ANPR the taxis were removed because their travel times were not representative for the rest of the traffic. The remaining travel times contain outliers, travel times that are much longer than travel times of vehicles arriving just before or just after a vehicle. These are not representative for the traffic conditions because the drivers apparently had some activities to
do between the two observation sites. The elimination of these outliers changed the statistics of the travel times considerably.

The consequences of the findings for practice is first of all that the improvement of the travel time reliability has no apparent conflict with the reduction of delays, since both characteristics are linearly related. Also the skewness and mean travel time optimization does not have a conflict. Traffic management aimed at the reduction of travel time standard deviation and skewness are expected to optimize the mean travel time as well. The study reported in this article used only a small part of the available data. Only 7 intersections with ANPR cameras have been analyzed, while the cameras are installed on 120 intersections in Changsha. The main reason for this selection was that not all ANPR cameras worked sufficiently well over longer time periods. That limitation made it possible to analyze in detail the relation between travel time reliability and other traffic characteristics. Further research will be done to identify the network wide travel time reliability, e.g. the reliability on origin destination basis. Furthermore the data will be used to analyze also the day to day and within day travel time reliability.

## ACKNOWLEDGEMENT

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## List of figures and tables



Figure 1 The network of Changsha for which the travel time distributions have been determined. The intersections with a number in italics have ANPR cameras. The area in the red dashed rectangle is the Renmin Road, where the travel times have been analyzed on the corridor 50-59 (a-f)

(a)

(b)

Figure 2 Comparison of travel times as measured by taxi with GPS and by cameras.

(a) Undersaturated condition


(b) Oversaturated condition

Figure 3 Relationship between the arrival moment and the delay, delay probability distribution and cumulative distribution for both undersaturated and oversaturated conditions


Figure 4 The relation between expectation value of travel time and its standard deviation for single links and routes with two coordinated intersections


Figure 5 Relation between expectation value of the travel time and skewness for isolated intersections with a degree of saturation lower than 0.8


Figure 6 Relation between skewness and expectation value of the travel time for intersections with a degree of saturation $>0.8$

(a) degree of saturation $\mathbf{x}<0.8$


Figure 7 (b) Degree of saturation $x>0.8$
Figure 7 Relation between degree of saturation and skewness

(a)

(b)

(c)

(d)

(e)

(f)

Figure 8 Travel times as function of the time of day and day of the week


Figure 9 The graphical relation between TTSD and average travel time

Table 1 The parameters of a linear function used to describe the relation between travel time $T$ and $T T S D: T T S D=a \times T+b$. Number is the number of recognized vehicles.

| Link |  | $50-113(\mathrm{a}-$ <br> b) | $\begin{aligned} & 113-51 \\ & (b-c) \end{aligned}$ | $\begin{aligned} & 51-24 \\ & (c-d) \end{aligned}$ | $\begin{aligned} & 24-54 \\ & (\mathrm{~d}-\mathrm{e}) \end{aligned}$ | $\begin{aligned} & 54-59 \\ & (e-f) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Length (m) |  | 449 | 486 | 326 | 718 | 671 |
| Apr. $20{ }^{\text {th }}$ | $a$ | 0.2800 | 0.2095 | 0.3050 | 0.2904 | 0.2468 |
|  | $b$ | 2.1846 | 16.531 | 0.3125 | 4.0841 | 9.6628 |
|  | $R^{2}$ | 0.9187 | 0.7710 | 0.7944 | 0.8765 | 0.7121 |
|  | Number | 6937 | 11482 | 10005 | 7271 | 4362 |
| Apr. $21{ }^{\text {st }}$ | $a$ | 0.3515 | 0.2616 | 0.5337 | 0.2767 | 0.4219 |
|  | $b$ | -2.6692 | 13.633 | -13.532 | 7.655 | -14.465 |
|  | $R^{2}$ | 0.9078 | 0.6708 | 0.7211 | 0.8582 | 0.7702 |
|  | Number | 7549 | 10216 | 10541 | 7731 | 4670 |
| Apr. $22^{\text {nd }}$ | $a$ | 0.2868 | 0.2530 | 0.4637 | 0.2952 | 0.3541 |
|  | $b$ | 4.2786 | 13.046 | -9.2114 | 4.7014 | -4.3405 |
|  | $R^{2}$ | 0.9078 | 0.7218 | 0.7161 | 0.8928 | 0.6681 |
|  | Number | 7434 | 10457 | 10881 | 7355 | 4707 |
| Integrated <br> Data | $a$ | 0.3103 | 0.2243 | 0.3607 | 0.2862 | 0.271 |
|  | $b$ | 1.1674 | 16.145 | -3.1195 | 5.6170 | 6.8523 |
|  | $R^{2}$ | 0.8661 | 0.7245 | 0.7102 | 0.8748 | 0.6883 |
|  | Number | 21920 | 32155 | 31427 | 22357 | 13739 |

Table 2 Partial correlation analysis between Travel Time Standard Deviation (TTSD) / skewness resp. and other factors ( 3 days data)

| Partial correlations between TTSD and other factors |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Link | Degree of saturation | Density | Travel Time | Skewness |
| 50-113 (a - b) | . 026 | -. 124 | .673** | .383** |
| 113-51 (b-c) | .304** | -.229** | .464** | .308** |
| 51-24 (c-d) | .199* | -. 117 | .467** | -. 161 |
| 24-54 (d-e) | .178* | -.179* | .632** | .573** |
| 54-59 (e - f) | . 065 | -. 005 | .556** | .231* |
| Partial correlation between skewness and other factors |  |  |  |  |
| Link | Degree of saturation | Density | Travel Time | TTSD |
| 50-113 (a - b) | . 011 | -. 093 | -.230** | .383** |
| 113-51 (b-c) | -.240** | . 083 | -.178** | .308** |
| 51-24 (c-d) | . 048 | . 020 | -. 138 | -. 161 |
| 24-54 (d-e) | -. 102 | . 099 | -.321** | .573** |
| 54-59 (e - f) | . 050 | . $397 * *$ | -.625** | .231* |

[^1]*. Correlation is significant at the 0.05 level (2-tailed).

Table 3 Coefficients in the multiple linear regression formula for TTSD

| Link | Coefficient of travel time $T$ |  | Coefficient of Skewness |  | Coefficient of density |  | Coefficientrof <br> Saturation flow <br> rate |  | Constant |  | $\left\lvert\, \begin{array}{r} \text { Adjusted } \\ R^{2} \end{array}\right.$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $a_{1}$ | Std. Error | $a_{2}$ | Std. Error | $a_{3}$ | Std. Error | $a_{4}$ | Std. Error | Constant | Std. Error |  |
| 50-113 (a - b) | . 329 | . 014 | 7.443 | 1.452 |  |  |  |  | -3.585 | 2.136 | 814 |
| 113-51 (b-c) | . 312 | . 041 | 4.396 | . 931 | -. 498 | . 145 | 30.995 | 6.649 | 2.288 | 4.758 | 668 |
| 51-24 (c-d) | . 361 | . 017 |  |  |  |  |  |  | -3.329 | 1.301 | 770 |
| 24-54 (d-e) | . 255 | . 008 | 14.384 | 1.519 |  |  |  |  | 8.585 | 1.791 | 862 |
| 54-59 (e - f) | . 271 | . 019 | 5.789 | 2.058 |  |  |  |  | 3.162 | 4.417 | 699 |

* Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100).

Table 4 Stepwise linear regression of travel time skewness

| Link | Coefficient of TTSD |  | Coefficient of $T$ |  | Coefficient of V/C |  | Coefficient of Density $k$ |  | Constant |  | Adjusted$R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $a_{1}$ | Std. Error | $a_{2}$ | Std. Error | $a_{3}$ | Std. <br> Error | $a_{4}$ | Std. Error | Constant | Std. Error |  |
| 50-113 (a - b) | . 022 | . 004 | -. 010 | . 001 |  |  |  |  | . 687 | . 102 | .252 |
| 113-51 (b-c) |  |  |  |  | -1.382 | . 321 |  |  | 1.103 | . 244 | 0.075 |
| 51-24 (c-d) |  |  | -. 016 | . 002 |  |  |  |  | 2.378 | . 133 | . 390 |
| 24-54 (d-e) | . 022 | . 002 | -. 005 | . 001 |  |  |  |  | -. 153 | . 074 | . 343 |
| 54-59 (e-f) | . 010 | . 004 | -. 013 | . 001 |  |  | . 018 | . 004 | 1.853 | . 140 | . 474 |

Table 5 Regression coefficients for equation (9) calculated for different days and different links

| Link | Name | 50-113 (a - b) |  | 113-51 (b-c) |  | 51-24 (c-d) |  | 24-54 (d-e) |  | 54-59 (e-f) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Length | 448.77 m |  | 485.789 m |  | 325.92 m |  | 718.93 m |  | 671.16m |  |
| Date | coefficient | value | Std | value | Std | value | Std | value | Std | value | Std |
| $\begin{aligned} & \text { Apr. } \\ & 20^{\text {th }} \end{aligned}$ | $b$ | . 142 | 2.035 | 14.995 | 2.436 | . 915 | 1.966 | 3.287 | 1.899 | 5.780 | 5.955 |
|  | $a_{1}$ | . 291 | . 014 | . 216 | . 012 | . 302 | . 018 | . 283 | . 010 | . 260 | . 024 |
|  | $a_{2}$ | 2.692 | 1.964 | 2.785 | 1.780 | -. 346 | . 892 | 8.486 | 1.626 | 3.353 | 3.052 |
|  | $R^{2}$ | . 919 |  | . 772 |  | . 790 |  | . 906 |  | . 733 |  |
| $\begin{aligned} & \text { Apr. } \\ & 21^{\mathrm{st}} \end{aligned}$ | $b$ | -5.268 | 2.085 | 12.506 | 2.811 | -16.122 | 3.031 | 7.127 | 2.173 | -15.609 | 4.183 |
|  | $a_{1}$ | . 361 | . 014 | . 269 | . 021 | . 554 | . 039 | . 257 | . 011 | . 421 | . 028 |
|  | $a_{2}$ | 3.539 | 1.519 | 1.426 | 1.725 | 1.056 | . 848 | 14.494 | 2.666 | 1.298 | 1.427 |
|  | $R^{2}$ | . 912 |  | . 666 |  | . 720 |  | . 892 |  | . 766 |  |
| $\begin{aligned} & \text { Apr. } \\ & 22^{\text {nd }} \end{aligned}$ | $b$ | -2.493 | 2.293 | 11.647 | 2.010 | -2.841 | 3.005 | 2.211 | 1.730 | . 555 | 4.992 |
|  | $a_{1}$ | . 318 | . 015 | . 265 | . 015 | . 404 | . 036 | . 296 | . 009 | . 288 | . 034 |
|  | $a_{2}$ | 8.925 | 1.722 | 5.142 | 1.131 | -1.963 | . 725 | 7.667 | 1.704 | 7.609 | 1.523 |
|  | $R^{2}$ | . 841 |  | . 767 |  | . 731 |  | . 918 |  | . 665 |  |
| 3-day | $b$ | -3.368 | 1.301 | 14.916 | 1.316 | -1.084 | 1.407 | 4.291 | 1.115 | 3.287 | 1.899 |
|  | $a_{1}$ | . 331 | . 009 | . 231 | . 008 | . 347 | . 015 | . 279 | . 006 | . 283 | . 010 |
|  | $a_{2}$ | 5.987 | 1.042 | 2.888 | . 882 | -. 947 | . 493 | 9.428 | 1.117 | 8.486 | 1.626 |
|  | $R^{2}$ | . 883 |  | . 732 |  | . 712 |  | . 902 |  | . 906 |  |

Table 6 Partial Correlations analysis of 3 links $113 \Rightarrow 51 \Rightarrow 24 \Rightarrow 54$. Volume and density are averaged over the 3 links.

|  | TT_mean | TT_std | TT_skew | Volume | Density |
| :--- | :---: | :--- | :---: | :---: | :---: |
| TT_mean | 1 | $.650^{* *}$ | $-.236^{* *}$ | - | .89 |
| TTSD | $.650^{* *}$ | 1 | $.405^{* *}$ | $.522^{* *}$ | $.305^{* *}$ |
| TT_skew | $-.236^{* *}$ | $.405^{* *}$ | 1 | - |  |
| Volume | $-.522^{* *}$ | $.305^{* *}$ | $-.327^{* *}$ | $.327^{* *}$ | 1 |
| Density | $.899^{* *}$ | $-.398^{* *}$ | $.230^{* * *}$ | $.761^{* *}$ | .76 |
|  |  |  |  | 1 |  |

Dear editor
We have modified our article 'urban travel time reliability at different traffic conditions' (GITS-2015-0259) according to your recommendations. We have added two references to publications in your journal (by Yang and by Viti and van Zuylen) and a reference to a recently published article (Zheng, F., Van Zuylen, H.J., Liu, X.B. (2017)). We have followed the guidelines for authors of your journal.

Thank you for publishing our article.
Sincerely yours
Henk J. van Zuylen
(on behalf of all authors)


[^0]:    \# Figure 1\#

[^1]:    **. Correlation is significant at the 0.01 level (2-tailed).

