How coordination of production and warehouse decisions affects the product flow between factories and warehouses for manufacturing firms: Mathematical models and a case study at Kraft Heinz

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MSc Thesis in Complex Systems Engineering & Management

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Bas Busser

Executive summary

Introduction

With the prevalence of online ordering, worldwide customer needs have changed over the past decades. The need for fast delivery has increased and in order to cope with this manufacturing firms are required to improve the product flow through their existing supply chain networks. To enhance supply chain agility warehouse activities are outsourced to third-party logistic service providers. In this way, warehouse capacity can be adjusted to the latest needs of the manufacturing companies. Efficiency in terms of optimal and flexible warehouse management is furthermore increased by the close coordination of the expected product flows between the factories and the warehouse. Ideally, the specific product flows can be predicted, resulting in the warehouse manager only having to adjust its warehouse capacity accordingly to minimize warehousing costs. Nonetheless, in practice this is not the case as manufacturing companies have insufficient coordination and synchronisation between production planning and warehousing teams (O'Reilly et al., 2015).

Problem description

The lack of coordination regarding production and warehousing decisions in the ordering strategy may result in unforeseen capacity issues at the warehouse due to misalignment between production plans and warehouse availability. These unforeseen issues can result in additional costs due to production line closure or last-minute stock reallocations. For this reason, it is relevant to study the effects of increasing interdepartmental coordination between production and warehousing departments on product flows between factories and warehouses in the supply chain of a manufacturing company.

Research questions

To structure the research, a main research question is formulated which entails: *"How does the coordination between production and warehouse decisions affect the product flow between factories and warehouses for manufacturing firms?"* In order to find an answer to the main research question two individual mathematical models have been proposed which represent the production and warehouse operations of a manufacturing company. These mathematical models are thereafter combined in two sequential and one simultaneous ordering strategy. In the sequential ordering strategy first one model is optimized after which the output of the first model is used as input for the second model. In the simultaneous ordering strategy which represents central coordination situation, an integrated model simultaneously optimizes the two models. By comparing the different ordering strategies in a case study for Kraft Heinz a popular brand in the food industry, the effects of coordinating production and warehouse decisions can be analysed.

Research methods

In order to find an answer to the main research question and to perform tests with the proposed models, different methods have been used. First, a literature research was conducted to find relevant research papers in the fields of multi-period warehousing, lot sizing and parallel machine scheduling problems. By combing findings from literature, the three mathematical could be proposed. These mathematical models were thereafter programmed as Mixed-integer linear programming problems in IBM ILOG CPLEX Optimization Studio. The models were subsequently tested in a case study at Kraft Heinz to provide a practical setting for the models. Performing a case study helped to reflect on the models' validities as well as generating managerial insights. Finally, an extensive sensitivity analysis was conducted to provide managerial guidelines for practitioners and to weigh which department can have greater impact on the total cost.

Results: Verification

The proposed models need to be verified and possibly improved before the models can be tested in a case study. When the models behave as was intended and thus comply with the formal problem description and the proposed conceptual model the models' verification step is completed. In order to verify the model behaviour, both the individual and integrated models are subjected to a set of three hypothetical scenarios. In which the results of a base scenario are compared with two scenarios in which input parameters are changed. The model behaviour in the three scenarios can be compared to each other to confirm if the model behaves as was intended. Once the individual and integrated models' behaviour were verified. The models were tested in three different demand scenarios to ensure that the proposed models were suitable to be used in different demand scenarios. The results from this test confirmed the models' abilities to adjust their optimization strategy for different demand scenarios. A final step in the model verification was to perform a complexity experiment for the integrated model. The complexity experiment should provide insights into how the integrated model reacts to the introduction of larger problem sets. To this end, the integrated model is tested in hypothetical situations in which the number of products, periods and machines were increased. The effect of this increase on the model running time and optimality gap was captured in an overview. From the complexity experiment, it emerged that increasing the number of products in the problem set increases the model's complexity. Increasing the number of machines has the opposite effect on the complexity. The number of periods considered by the model does not affect the model's complexity but does increase the model running time.

Results: Case study

In the case study the *Sequential Production Strategy, Sequential Warehouse Strategy* and *Simultaneous Strategy* were tested in a real-life scenario of Kraft Heinz, the three ordering strategies were compared on costs, order behaviour and warehousing strategy. From this comparison, it emerged that the *Simultaneous Strategy* was the most cost-effective and reliable strategy to optimize the product between the considered factory and warehouse. The total costs of the S*imultaneous Strategy* were significantly lower compared to the two sequential ordering strategies. Besides the better performance cost-wise, the simultaneous ordering strategy as well emerged to be unsurpassed in ensuring feasible product flows between the factory and warehouse. *Sequential Production Strategy* resulted to be infeasibility in the case study setting due to misalignment between the generated production plans and available warehouse storage capacity. Altogether, it became clear that the *Simultaneous Strategy* was superior in all comparisons due to the simultaneous consideration of warehousing and production constraints. Due to the simultaneous optimization, careful trade-offs could be optimized regarding the timing and quantity of products ordered and the effects on production and warehousing cost. Due to optimizing these trade-offs, superior decisions could be made compared to the sequential ordering strategies in which first one mathematical is optimized after which the outcome is used as input for the second mathematical model. By doing so, the possibilities to make trade-offs in favour of the total product flow are reduced. A final test that was performed in the case study was an extensive sensitivity analysis of the case study cost parameters. From this case study, it emerged that the warehouse inbound capacity is a constraint that limits the optimal outcome of the model and has a great effect on possible model infeasibility. Moreover, the overtime inbound cost and the handling cost were identified as cost parameters with a significant effect on the total cost.

Discussion

The proposed warehousing and production scheduling models represent supply chain activities. The models have been proposed by combining information from literature with information obtained from field experts. By combining information sources, it is ensured that theoretical knowledge is combined with practical knowledge from the field. Nevertheless, in the formulation of the mathematical models, it is required to make assumptions to ensure that the proposed models can be developed and ensure feasible results within the given time and resource constraints. Making modelling assumptions affects the preliminary solution space of the models and may affect the real-life representation. The main model assumptions which affected the real-life representation of the models were the assumptions to consider demand to be deterministic, the assumption to look at the products on recipe level and the assumption to consider a single factory and a warehouse in the case study. Even though these assumptions affect the real-life representation of the used models, they can still be used as a tool to generate useful insight with regards to the high-level product flow between factories and warehouses in the supply chain of manufacturing companies as the models' validities have been accounted for throughout the development process.

Managerial insights

Managerial insights were generated by interpreting the case study and sensitivity analysis results. The managerial insights could be useful for practitioners to improve the product flow between factories and warehouses in a supply chain and the according coordination. A first managerial insight is the fact that the S*imultaneous Strategy* is superior in all tested scenarios. This is mostly related to the simultaneous consideration of all the warehouse and production cost parameters as well as constraints in place. For this reason, it is recommended to consider as many cost parameters and constraints of the supply chain activities as possible when optimizing the size and timing of orders placed. By doing so, a reliable and cost-effective product flow between factories and warehouses can be ensured. Other managerial insights pertain to the outcome of the sensitivity analysis. It is recommended to investigate the opportunities to structurally increase the warehouse inbound capacity with the logistic service provider as this would be beneficial for the total costs.

Conclusion

From the conducted research and the performed studies, it can be concluded that enhancing the coordination of operational warehouse and production decisions reduces the total cost and risk of unforeseen issues in the supply chain. This was demonstrated by the superiority of the *Simultaneous Strategy* over the two sequential strategies in the various comparisons. This result is in line with findings in literature from Bradley and Arntzen (1999) and Atamtürk and Hochbaum (2001). For this reason, it is recommended to enhance central coordination of the ordering strategies of large manufacturing companies to improve the product flows between factories and warehouses.

Further research

This thesis proposes three alternative ordering strategies for large manufacturing companies which make use of three developed MILP models. The proof-of-concept of these strategies is provided by testing them in a case study of a large manufacturing company in the food sector. This proof-of-concept opens doors for further research to expand on the insights obtained in this investigative research and to further improve the models towards possible real-life implementation. A selection of the recommendations for further research entails that it is recommended to further extend the developed models to be suitable for stochastic demand. Besides stochasticity, it would be interesting to increase the level of detail in the production scheduling model such that production schedules can be generated on product level. Another recommendation for further research would be to test the proposed models in a scenario with multiple factories and a single warehouse and compare the results. A final recommendation for further research would be to propose a solving algorithm for the formulated mathematical models and compare its performance against CPLEX. This can be helpful to improve the model running time for larger problem sets.

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1. Introduction

With the rise of the internet and the prevalence of online ordering, the world-wide competition for fast delivery of quality products has increased significantly over the past decades (Ho & Zeng, 2004). In order to comply with the changing customer needs, large (inter)national manufacturing and distribution firms have extended their portfolios with multiple products. In order to ensure that these different products will be delivered on time to the customers, companies have created extensive supply chain networks consisting of factories, warehouses, transportation lanes and customers of which Figure 1 is an illustrative example. Having such extensive supply chain networks helps companies to stay agile and comply with customers expectation to receive quality goods within a limited timeframe (Darwish & Ertogral, 2008). The fast-changing customer needs and the required supply chain agility has increased the importance of coordinating product flows throughout the global supply chain. More specifically, it has enhanced the scrutiny of efficient flows between factories and warehouses because of the critical function the warehouse plays in linking the material flows between suppliers and customers (Ramaa et al., 2012).

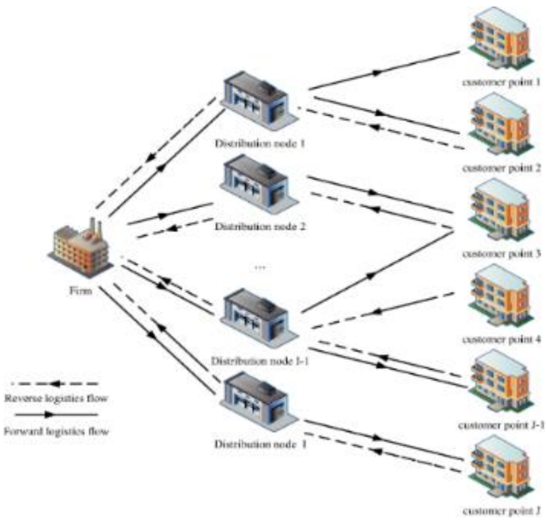

FIGURE 1 SUPPLY CHAIN NETWORK EXAMPLE (L. HUANG, MURONG, & WANG, 2020)

An increasingly popular solution to achieve a higher supply chain agility is to outsource warehouse management to third-party logistic service providers (LSP). By making use of this type of warehousing, companies can reduce costs related to large capital investments and rent or lease warehouse capacity according to their changing needs (Razzaque & Sheng, 1998). Efficiency in terms of optimal and flexible warehouse management is furthermore increased by the close coordination of the expected product flows between the factories and the warehouse. Ideally, the specific product flows can be predicted, resulting in the warehouse manager to only having to adjust its warehouse capacity accordingly to minimize warehousing costs. In practice this is not the case as manufacturing companies have insufficient coordination and synchronisation between production planning and warehousing teams (O'Reilly et al., 2015). The manufacturers production planners focus on optimizing the factory's output and do not account for all warehouse capacity constraints in place. This may result in a mismatch between factory output and available storage capacity at the warehouse. Such issues resulting from the lack of coordination between the two teams can lead to unforeseen costs as a consequence of last-minute stock reallocations or forced line shutdowns. For this reason, it is important that the product flows between factories and warehouses are closely coordinated and that ordering strategies account for all relevant constraints in place (O'Reilly et al., 2015).

Having outlined the coordination and synchronisation problem between production plants and warehousing facilities, this thesis' aim is to analyse the effects of coordinating the production scheduling department with the warehouse management department, against the most common practical case where the two entities hardly communicate. To this end, two optimization models are developed representing the warehouse and factory operations within a supply chain. The first model consists of a multi-period multiproduct lot size and warehouse optimization model for the decision on warehouse capacities while minimizing warehousing costs. The second model is a parallel non-identical machine scheduling model for the optimization of production schedules and the simultaneous minimization of the production costs. Both models are solved to find the optimal order quantities of a portfolio of products in a multiperiod setting, such that either the warehousing cost or production cost are minimized over the time horizon. In other words, in the first model we optimize the cost of the production and based on that calculate the resulting warehousing cost. In the second, we optimize warehousing costs by choosing the capacity to lease and calculate the resulting production scheduling cost. We compare the outcomes of the two models against a model where both decisions are integrated. The modelling decisions are taken in a setting with deterministic demand over time. An extensive sensitivity analysis aims to provide managerial guidelines for practitioners, to weigh which department can have greater impact on the total cost. Also, the integrated model can quantify the benefits of coordinating the production and warehouse departments. A case study at Kraft Heinz, a popular brand in the food industry, is used to provide a practical setting. Testing the models in a practical setting will help to assess the validity of the developed models. Moreover, the case study is a useful means to generate managerial insights for Kraft Heinz regarding the coordination of product flows between factories and warehouses.

In literature this class of problems is known as the dynamic lot sizing problem which was first studied by Wagner & Within (1958). This problem class has been a popular area of study ever after the paper of Wagner & Within. The multi-period warehouse sizing problem under study was first touched upon in the paper of White & Francis (1971). They formulated the multi-period warehouse sizing problem as a linear programming problem and thereafter translated this to a network flow problem in order to find an optimal solution considering stochastic demand (Fan & Wang, 2018). Even though, the individual research areas of dynamic lot sizing and multi-period warehouse sizing have been popular on their own for the last 50 years. Studies towards the joint optimization of the problems are rather scarce. Fan & Wang (2018), Fan et al. (2021) and Atamtürk & Hochbaum (2001) did study the joint optimization of lot sizes and warehouse sizes. Nonetheless, they only considered a single product in their models. Hence, the research gap entails that no model exists, yet which jointly optimizes dynamic lot sizes and warehouse capacity problems for a multi-product range. Similarly, scheduling problems have been studied in many different set-ups ever since the problem was introduced in the 1950s (Nogueira et al., 2019). The type of scheduling problem that is of interested in this research concerns the non-identical parallel machine scheduling problem considering sequence dependent set up times. Most research in the scheduling paradigm focuses on identical parallel machine scheduling, but in practice machines are mostly non-identical (Balin, 2011). Hence the motivation to consider the non-identical parallel machines problem in this thesis. This thesis aims to fill the identified research gaps by modelling and solving the proposed problems.

The remains of this thesis will be structured as follows. The literature review in chapter 2 will further elaborate on the key concepts of this thesis being the lot size problem, the multi-period warehouse sizing problem and the scheduling problem. With this understanding the research gap of this thesis will be expanded. Chapter 3 will elaborate on the research objective, the formulated research questions and the according research approach. Chapter 4 will be dedicated to the problem description which further discusses the problem relevance and gives the formal problem descriptions of the problems. Following, chapter 5 will introduce both the conceptual and the mathematical models. In chapter 6 the proposed models will be verified and their behaviour analysed. Chapter 7 will introduce the case study and analyse the case study results. Chapter 8 will be a discussion chapter in which the research assumptions and limitations as well as the proposed managerial recommendations will be discussed. This thesis will be concluded with chapter 9 in which the research questions will be answered and recommendations for future research will be provided.

2. Literature review

In this section the research relevance will be elaborated on. This will be done through conducting a literature research to the key concepts of the research as well as elaborating on the state of the art in the respective research fields. Understanding the key concepts such as the lot sizing, warehouse capacity and production scheduling problem is of importance as this will help both the author and the reader to better comprehend the requirements for the mathematical model in the following chapters. Moreover, understanding the individual key concepts helps to later approach the model with a holistic view and examine what happens when the 'new' system is studied as a whole, instead of just a summation of its separate parts**.** In this chapter first the key concepts and the state of the art in the respective research field will be elaborated on. Followed by and overview of the key concepts found in the literature. The chapter will be concluded by elaborating on the identified research gap and the societal relevance.

2.1. Multi-period warehouse sizing problem

Three types of warehousing problems exist in literature, being throughput capacity models, warehouse design models and storage capacity models (Cormier & Gunn, 1992). The latter is of interest for this thesis, as a multi-period warehouse size problem will be analysed. The multi-period warehouse size problem concerns the problem of finding the optimal warehouse size or capacity for a certain period *T* in which a warehouse facility is leased from a LSP (White & Francis, 1971). The length of the leasing period is agreed upon by both parties at the start of the leasing period. The contract period T is divided into several sessions, the leased warehouse capacity can be adjusted at the beginning of each session (Fan & Wang, 2018). At these decision moments, the logistic manager can decide to expand or reduce the leased warehouse capacity against a certain penalty cost. The logistic manager bases this decision on the shared demand forecast for the coming period and the paired costs that belong to ordering and storing products in the warehouse. The main aim of this problem is to find the optimal warehouse capacity such that the warehousing costs are minimized, whilst the warehouse capacity does not negatively affect the warehouse performance (Fan & Wang, 2018).

Within the warehouse storage capacity research field, a distinction can be made between static or dynamic problems under consideration. In the static problem interpretation the aim is to find a warehouse size which minimizes warehousing costs whilst satisfying service requirements for a single period (Cormier & Gunn, 1992). Once this warehouse size decision is made it is fixed for the coming period without the possibility to change it. The paper of Jucker et al. (1982) optimizes a warehouse and plant size problem with stochastic demand distribution for a single period. The objective is to find the optimal warehouse and production plant capacity to maximize the expected profit. No stock-outs are allowed and the demand exceeding the capacity is considered to be lost. Construction costs are considered to be non-linear in relation to the capacity. The algorithm that is proposed to solve the problem is based on the Kuhn-Tucker conditions. Another example of a study with a static, single period problem description is the paper of Roll et al. (1989) who developed a simulation model to analyse the relation between container sizes and the required warehouse capacity in a stochastic environment. One more paper that studied the static warehouse size problem is the paper of Roll & Rosenblatt (1988). In their study Roll & Rosenblatt (1988) studied different elements that can affect warehouse capacity in a stochastic environment for a single period. The elements under study are the number of products stored, the respective demand distribution and the applied reordering policy.

On the other side of the warehouse capacity problem is the dynamic warehouse capacity problem, in which the goal is to find the optimal storage capacity at different points in time and so optimize the warehouse size problem over multiple periods. The demand can be either stochastic or deterministic and could vary over time. Having a different demand in each period requires the warehouse capacity to be adjustable as well, resulting in expansion or contraction decisions that need to be considered in this research field (Cormier & Gunn, 1992). The dynamic warehouse capacity problem was first touched upon in the paper of White & Francis (1971). They formulated the multi-period warehouse sizing problem as a linear programming problem considering stochastic demand. Thereafter the problem was translated into a network flow problem in order to find an optimal solution. The problem was thereafter picked up by Lowe et al. (1979) who considered a firm that aimed to minimize the overall warehouse leasing costs. Lowe et al. (1979) consider two types of contracts in their problem formulation. A primary contract or a base scenario which is agreed upon at the beginning of the of the planning horizon and a secondary contract which can additionally be agreed upon if extra storage capacity is required within the coming period. Demand is assumed to be randomly generated and a greedy algorithm is proposed to solve the according network flow problem. This problem formulation differs from the problem under study as this thesis will consider the full warehouse capacity to be leased which is captured in a single contractual agreement. Rao & Rao (1998) formulated a dynamic warehouse size problem which considers the cost function to be concave. They proved that their problem can be solved efficiently by means of a dynamic programming approach. Other well-known variations in the dynamic warehouse size problem are the two 1996 papers of Cormier and Gunn. Cormier & Gunn (1996a) look at the warehouse sizing problem in a deterministic demand setting with the option to lease additional capacity and including inventory policies in their solution strategy. Their aim is to minimize warehousing and operating costs and to provide a planning tool to improve the warehouse size decision making process. In their other paper, Cormier & Gunn (1996b) solve a single and a multi-product alternative of warehouse sizing problem aiming to minimize the overall discounting and operating costs, also considering deterministic demand. In order to find an optimal solution for the formulated problem they use the Newton-Raphson method. A downside of considering deterministic demand in the above problems is that uncertainty and demand variation are not taken into account (Shi et al., 2018). Another method to optimize dynamic warehouse size problems is the queuing method. This method is used for example by Huang et al. (2014) who develop a two-stage network to optimize both the warehouse size and location and solve this problem by means of the queuing method considering stochastic demand. Another example of using the queuing method to solve a warehouse size problem is the paper of Yuan et al. (2016) who study a public warehouse sizing problem in which they maximize the profits for public warehouse by means of a dynamic programming algorithm. The dynamic warehousing problem is of interest for this thesis as this problem is suitable for LSP warehouses with adjustable warehouse capacity. A similar problem will be analysed in the case study in a dynamic deterministic demand scenario. Demand is assumed to be known in advance with certainty but can vary over time. All warehouse capacity will be leased from a LSP and the warehouse capacity can be adjusted at the beginning of each period. The aim is to find the optimal warehouse capacity for the time horizon such that the leasing cost are minimized. This case study scenario can best be represented by the multi-period warehouse sizing problem.

The decision to consider a problem in a static or a dynamic environment is dependent of the type of company or range of products under study. Another factor which can play a role in deciding to study the problem in a static or dynamic environment can be to newness of the study. For example when new parameters are added to the model or when different problems are combined for the first time, it can be wise to study the model in a simpler static environment (Jucker et al., 1982). This would make the model interpretation and analysis easier, moreover the chances of finding a feasible solution are higher in a static problem setting. From the conducted literature research, it did however emerge that the dynamic (multi-period) warehouse sizing problem is the closest representation of the problem under study in this thesis. The main motivation for this is the optimization of the warehouse size over multiple periods which can be compared with a warehouse LSP leasing contract. This problem is better known as the multi-period warehouse sizing problem, since the warehouse size will be optimized for all considered periods. The relation between the warehouse size problem and the lot size problem will be further elaborated on in the following section, also the effects of considering deterministic or stochastic demand will described in more details in the coming section.

2.2. Lot sizing problem

Lot sizes are known by the Economic Times as "*the quantity of an item ordered for delivery on a specific date or manufactured in a single production run*"("Economic Times," n.d.). In this thesis a lot size will be considered as the quantity of a product ordered from a manufacturing site. The rationale behind this, is the fact that this thesis considers both production scheduling and warehousing. The relation between lot sizes and warehouse size is grounded in the fact that the lot size has a direct effect on warehousing costs and warehouse size requirements. Having a large lot size means that large batches of products will be ordered less frequently per period compared to smaller lot sizes (Delaware, n.d.). This will result in low average ordering and transportation costs but will result in high inventory levels which means that the average holding cost will be high. Similarly, ordering large product batches less frequent will require more warehouse space to store the inventory compared to ordering smaller batches which are ordered more frequent (Kemmner, n.d.). On the other hand, having small lot sizes means that you have high ordering and transportation costs but low holding costs, since you will order small batches more frequently (Van Den Heuvel, 2006). This trade-off is known as the basic lot sizing problem. The main objective of this problem is to find the lot size which minimizes the total costs, whist complying with present demand (Erenguc & Aksoy, 1990; W. Lee, 2005). Fan & Wang (2018) even state that; "*Lot sizing problems* lie *at the core of many production and inventory planning applications*" (Fan & Wang, 2018). Due to this link the lot sizing problem has been studied in many different directions for the last decades. Below different classifications of the lot sizing problem are described and key findings in each classification are touched upon.

An important distinction in studying the lot size problem is assuming static or dynamic demand over time. Static demand assumes a constant demand over a finite horizon, which is e.g. applicable in the well-known EOQ model which considers static deterministic demand (Van Den Heuvel, 2006). Whereas, in a dynamic demand situation the demand can have a different value in each period which is know in advance for the specific horizon, this is called dynamic deterministic demand. The dynamic lot sizing problem has been studied thoroughly for many years after the paper of Wagner & Within (1958)**.** Wagner & Within (1958) were the first researchers to drop the assumption of a steady-state demand in the economic lot sizing model and considered varying demand over time which was known in advance. Besides demand, also other parameters can take on different values in different time periods. Even though both the dynamic and static lot sizing problem interpretations are simplifications of reality, dynamic demand better reflects the real-life situation of demand varying over a time horizon. "*It is therefore, not surprising that the dynamic lot-sizing problem has been a popular topic in the academic ever since it was first touched upon by Wagner & Within (1958)*" (Lee et al., 1999). For the same reason, this thesis will consider the lot sizing problem to be dynamic deterministic.

Within the dynamic lot size research area more problem classifications exist. Lot size problems can be divided into a problem class which does not consider resource or capacity constraints from a production or storage point of view. Such a lot size problem is classified as an uncapacitated lot sizing problem. The paper of Wagner & Within (1958) belongs to this class of the lot sizing problems. In the capacitated version of the lot size problem production constraints are taken into account when formulating the mathematical model. This problem class is also referred to as lot sizing problem with bounded inventory (Atamtürk & Küçükyavuz, 2005). These additional constraints increase the model's complexity (Karimi et al., 2003). Studying lot sizes in a capacitated situation is assumed to be a more realistic representation of reality, as it is based on warehouse operations in practice (Fan et al., 2021). Love (1973) was the first to incorporate production capacity constraints by means of time varying inventory bounds in a dynamic lot sizing problem. Atamtürk & Küçükyavuz (2005) conducted a polyhedral analysis of this bounded lot size problem. For this they investigated a lot sizing model with varying inventory bounds in a multi-period situation, to solve their problem a linear programming model was proposed. In their 2008 paper Atamtürk & Kücükyavuz (2008) extend their 2005 research by proposing a dynamic programming algorithm to solve the bounded lot sizing problem more efficiently. Erenguc & Aksoy (1990) came up with an algorithm based on the branch and bound technique to solve the capacitated dynamic lot sizing problem in a deterministic single product nonconvex situation including capacity constraints. Chen et al. (1994) also studied the capacitated dynamic lot size problem and they presented a dynamic programming algorithm to optimize the problem whilst considering piecewise linear inventory and production costs**.** Other examples of papers in the capacitated dynamic lot size research area are Liu & Tu (2008) who considered lot sizing boundaries taking into account lost sales and used a network flow algorithm to find the optimal solution and the paper of Hwang et al. (2013) who consider the a dynamic lot sizing problem with bounded inventory and lost sales and use a polynomial algorithm to solve their problem. A different interpretation of the problem is described by Chu et al. (2013) who formulated a dynamic lot sizing problem for a single-product in which backlogging and outsourcing were allowed. For this research both warehouse and production bounds will be considered in the model as they contribute to the real-life representation of the case study later in the research as was mentioned by Fan et al. (2021). The increased complexity related to including capacity constraints in the model is assumed to not make the problem insolvable but could affect the running time of the model.

It is important to note that joint lot size and warehouse size problems are not the same as lot size models which consider inventory levels. In the classical lot size models, the optimal lot sizes are to be identified which minimize the total inventory costs and thus most often result in the lowest average inventory levels. The classical lot size models consider the inventory level in the objective function as a decision driver to order a new batch of products. The difference with the joint warehouse size and lot size optimization models is that the warehouse capacity is treated as a decision variable over time. In this case both the inventory level as well as the overall warehouse capacity are considered in the objective cost function. The goal is to identify the optimal lot and warehouse size such that the overall warehousing costs are minimized. The difference then is that the peak inventory levels are kept as low as possible, so that the required warehouse size can be minimized.

Another problem classification within the lot sizing research field is the product range under consideration in the problem. Many research papers consider a single product instead of a multi-product range in the researched problems as can be seen in the literature concept overview in Table 1. The latter does increase the complexity of the problem in the dynamic lot sizing problem. The paper of Chen & Thizy (1990) states that the capacitated multi-product dynamic lot sizing problem is strong NP-hard to solve. Nevertheless, with the research objective in mind, it can be assumed that the lot sizing problem can best addressed in a dynamic, capacitated and in a multi-product environment. This will be the closest representation of the real-life scenario for a large manufacturing company in the food sector. Most papers discussed in this section of the literature research consider a single product range. Examples of research papers that do considered a multi-product range in their problem formulation are the papers of Cormier & Gunn (1996a) who propose multi-product lot size and warehouse size problem in continuous time, Cormier & Gunn (1999) who consider a multi-product expansion model in continuous focussing on lot sizes and Minner (2009) who discusses three heuristics to solve a capacitated multi-product dynamic lot size model. The models in these papers will be used as examples to formulate a multi-product problem.

The relationship between lot size and warehouse size was already shortly discussed in the first paragraph of this section. This relationship between lot size and warehouse size is also embedded in literature as there are several papers that both study warehouse size and lot size decisions. However, this joint optimization is not as widely studied as the individual problem settings of the lot sizing and the warehouse sizing problem. The first research that jointly looked into optimizing lot sizes and warehouse size in continuous-time economic order quantity models was the paper of Cormier & Gunn (1996a) who studied a warehouse sizing problem with static demand, which considered inventory polices or lot sizes as a tool to minimize the overall warehousing costs. The problem focussed on a new warehouse which needs to be constructed. Cormier & Gunn (1996b) considered a similar problem environment, but focussed on balancing lot size, warehouse size and leased warehouse size. In their 1999 paper Cormier & Gunn (1999) discussed a similar model which focussed on a warehouse expansion planning, considering constant demand growth. In their problem Cormier and Gunn did not consider warehouse renting costs or penalty cost for capacity expansion or contraction. The paper of Shi et al. (2018) looked into a dynamic warehouse size planning model considering contract flexibility. Their aim was to study the effect of contract flexibility on the subsequent warehouse size planning. For their problem formulation they considered two types of contracts, one long-term nominal capacity contract and one short-term flexible capacity contract which can be dynamically updated. Fan and Wang (2018) studied the warehouse sizing problem over a multi-period planning horizon and were the first to combine this with studying the dynamic lot size problem in a discrete time environment. Fan et al. (2021) extended their previous model by focussing on optimizing both short-term and midterm warehouse management decisions. This extension considered a basic leased storage size which could eventually be increased with additional storage size for a higher cost. A different interpretation of a similar problem was studied by Atamtürk & Hochbaum (2001) who studied decisions on lot-sizing, subcontracting and capacity acquisition. The problem setting is different from the research of Fan & Wang (2018), since Atamtürk & Hochbaum (2001) considered warehouses with a fixed capacity and the possibility to use subcontractors if the regular capacity is not satisfactory. Whereas Fan & Wang (2018) considered the entire warehouse capacity and operations to be carried out by a LSP.

From the literature overview, it emerges that most lot size research papers consider demand to be deterministic. This assumption considers demand to be known in advance without any uncertainty. Whilst assuming that demand is known in advance without any uncertainty is not realistic itself, inventory models considering deterministic demand have proven to be robust with respect to data errors. An example of such a robust model is the well-known EOQ model from Ford Whitman Harris. All input parameters such as demand and holding costs are treated as known whilst in fact they are not, estimates of parameters are used to operate the model and demand forecast are considered to be 100% correct. Whilst, in reality the forecast will be an (close) estimate of the real value. As a result, the outcome of the model should be interpreted as an approximated value. The correctness of the outcome strongly depends on the correctness of the used demand forecast and the parameter estimations. This does not have to form a problem as the model looks at the high level required storage capacity. For example, where one product may be overestimated by the company, other products can be under forecasted which would balance the overall values. Nevertheless, even when data errors are included in inventory models, the outcomes are still representative since inventory models are resilient to data errors within a range of 20 to 30% (Taylor, 2006, p.738). Dagnzo (2005) confirms this statement by mentioning that decision variables and input parameters when estimated within 20-40% of the correct value, do not affect the outcome of the model. Hence the model outcome of deterministic inventory models with correctly estimated input parameters are robust to data errors. Even though the model outcomes remain valid when parameters are estimated with an error percentage of 30%, the goal should be to estimate the parameters as close as possible to the true value. This research will consider demand to be deterministic and varying over time. The reason for this is that it has been widely used in the lot sizing research field. Moreover, the deterministic setting proves to be robust against data errors, so outcomes of the model would still be relevant for managerial insights.

2.3. Scheduling problem

The scheduling problem is a well-known problem in the field of production management. Scheduling problem optimization dates to the 1950s when the first problems were described with industrial application. Ever after the scheduling problem was introduced, it has been applied to many practical situations (Nogueira et al., 2019). The classical scheduling problem looks into what needs to be made at what times and by which resources (Cardon et al., 2000). Production scheduling is an important part of the supply chain as the production schedule greatly determines the production output from a factory. Scheduling which products, in which quantities are produced at what times will also affect how products will be stored further down in the supply chain. In an ideal scenario the production scheduling accounts for warehousing constraints in order to optimize the product flow throughout the supply chain. The scheduling problem assumes that a certain amount of jobs need to be performed on a number of machines which can be identical or non-identical, in most scheduling problems the main objective is to minimize the makespan of production (Balin, 2011). The makespan represents the completion time of the last job in the process. Having a short makespan relates to having a high overall utilization, hence the smaller the makespan the higher the throughput rate (Kashan et al., 2008).

Scheduling problems can be divided into different categories, depending on the machine layout and job flow that is considered (Moon et al., 2002). Three main categories are the job shop problem, the flow shop problem and the open shop problem. A flow shop is a factory layout in which machines must be operated in a sequence. Jobs are required to start on an initial machine, after which they follow a path through subsequent machines in a strict order, until they have been processed by the final machine (Balin, 2011). On the contrary, in a job shop layout the order in which the jobs are processed by the machines is not relevant. Every job has certain operations that need to be performed in an order and not all jobs have to be processed by all machines (Balin,2011)). A production schedule is constructed based on the operations that need to be done, the processing time that it takes to perform a job and precedence requirements of every job. In the case that jobs do not have precedence constraints of other jobs, the shop layout is called an open shop (Balin,2011). The difference between a flow shop layout and a job shop layout is can be seen in Figures 2 and 3.

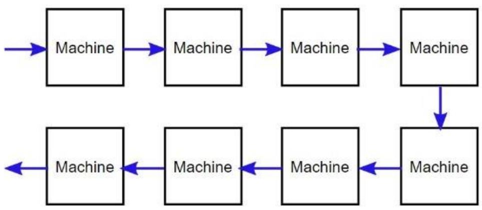

FIGURE 2 FLOW SHOP LAYOUT EXAMPLE (OWEN-HILL, 2017)

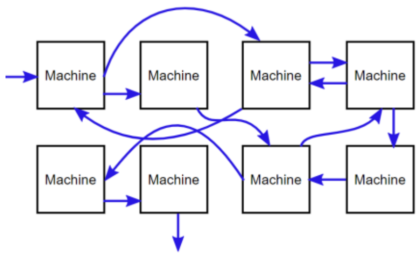

FIGURE 3 JOB SHOP LAYOUT EXAMPLE (OWEN-HILL, 2017)

Min & Cheng (2003) state that practical job shop and open shop problem under uncertain conditions can be simplified into a parallel machine problem. For this reason parallel machine scheduling problems have received much attention in the academic world over the last decades (Min & Cheng, 2003). Another reason for the attention to parallel machine scheduling problems is that the parallel machine scheduling problem is considered to better represent the real-life production scenario compared to the single-machine scheduling problem as most production companies make us of multiple machines in their production process. It was therefore decided to focus on the parallel machine scheduling problem in this thesis.

The parallel machine scheduling research area can be divided into two main problem classes. The first being problems considering machines to be identical and the second class which considers machines to be non-identical. In a classical identical parallel machine scheduling problem the main objective is to minimize the maximum makespan of the machines. Karp (1972) proved that these scheduling problems are NP-hard to solve. Karp did this by changing the problem under study into a bi-partitioning problem. A popular extension of the classical machine scheduling problem is to consider set up times between jobs which are sequence dependent. Guinet (1993) proposed a MIP model to find the minimum overall completion time for this problem extension. Other popular research directions in the field of parallel machine scheduling problems are doing research to the problem and solving the formulated problem by means of metaheuristics such as the genetic algorithm, particle swarm optimisation or tabu search algorithms (J. Kim & Kim, 2021). An example of using metaheuristics to solve scheduling problems is the paper of Yin et al. (2017) who considered the identical parallel machine scheduling problem in a disruptive environment where machines can become unavailable for a certain amount of time due to break-downs or planned maintenance with the objective to minimize the total completion time of jobs under consideration. Polynomial time algorithms were proposed to solve the problem. Another example is the paper of Najat et al.(2019) who proposed a model which minimizes the number of late jobs whilst the system accounts for periodic maintenance on the machines, the authors propose a S-B-C 1 heuristic which combines the earliest due date and the minimum load rule to solve the according problem. The paper of Kim & Kim (2021) proposed a hybrid genetic algorithm which is combined with a travelling salesman problem based heuristic algorithm in order to solve a parallel scheduling problem which considers set up times to be sequence dependent. From the above-mentioned examples, it becomes clear that the parallel identical machine scheduling problem has been researched in many different directions and different algorithms have been proposed to solve the problems under study.

Whilst, most research in the parallel machine scheduling research field considers the machines to be identical, in reality it is more common that machines are not identical (Balin, 2011). The amount of research conducted to non-identical parallel machines is limited compared to its identical counterpart. An example of a research paper in the non-identical paradigm is the paper of Balin (2011) which aimed to minimize the longest processing time for a non-identical machine scheduling problem and proposed a genetic algorithm to solve the problem. Another example is the paper of Li & Yang (2009) which minimized the total weighted completion time of different non-identical scheduling problems and discussed an approximation algorithm to solve the problems. Also the paper of Raja et al. (2008) looks into this problem in which the non-identical scheduling problem considering sequence dependent set up times was optimized with the objective to minimize the overall earliness and tardiness of the jobs. In order to solve the formulated problem a genetic algorithm-fuzzy logic approach was proposed and tested against the performance of other genetic algorithms. One more example is the paper of Hulett et al. (2017) who studied a non-identical scheduling problem which minimized the total tardiness and used particle swarm optimization to solve the formulated problem.

The earlier discussed characteristics of the machine scheduling research field has led to the conclusion to focus on parallel machine scheduling problems with non-identical machines in this thesis. Because the topic has not been researched as extensive as its parallel counterpart and a non-identical parallel machine set-up is more common in practice which is a relevant feature taking into consideration the case study at a later stage in this thesis.

2.4. Literature concept overview

The main concepts of each paper considered in this literature review have been mapped in Table 1. Having a clear overview of main concepts discussed in every paper is helpful for the author in the process of identifying a research gap and finding useful references during the writing process. From Table 1, it emerges that a significant amount of research has been conducted to either the lot sizing problem or the warehouse sizing problem but a joint optimization of the two is scarce. Table 1 also shows that a multi-product range is only considered in merely four research papers and none of these papers optimize both the warehouse as well as the lot size problem. A reason for this low number of papers could be that considering a multi-product range does introduce new forms of complexity to the model which need to be considered when the mathematical model is constructed and might make the problem NP-hard. Another reason is that classically most inventory models were developed under the assumption that a company produces and stores merely a single product. Whilst in reality this assumption does not hold. Most companies have a large product portfolio to comply with different customer needs and in this way sell different products to the same customer (Mousavi et al., 2014).

In the studied production scheduling literature, the papers were selected on the parallel or single machine concept and the consideration of identical or non-identical machines. The solution approach of every paper is also depicted in Table 1. From Table 1 it emerges that most used papers consider parallel identical machines. A reason for the limited amount of research in the non-identical parallel machine scheduling field could be the increased complexity of considering non-identical machines compared to identical machines. As was mentioned in the literature review, the distribution of the amount of research focussed on non-identical and identical machine scheduling is not representative for the number of identical machines used in practice (Balin, 2011). For this reason, it was decided that it was most interesting to consider non-identical parallel machines in this thesis, taking the case study into consideration. Different solution approaches are proposed in the research papers used for the literature review. Nevertheless, the genetic algorithm or heuristics based on the genetic algorithm are a popular solution method for the scheduling problems under consideration.

TABLE 1 LITERATURE CONCEPTS OVERVIEW

Authors	Warehouse size problem	Lot size problem	Multi-period	Multi-product	Outsourcing option	Single or Parallel	Identical	Demand	Solution approach	Capacity constraint
White & Francis (1971)	✓		✓		✓			Stochastic	Network flow algorithm	✓
Lowe et al. (1979)	✓		✓		✓			Stochastic	Network flow algorithm	✓
Cormier & Gunn (1996a)	✓	✓	✓		✓			Deterministic	Planning tool	✓
Jucker et al. (1982)	✓		✓					Stochastic	Kuhn-tucker algorithm	
Hung & Fisk (1984)	✓		✓		✓			Stochastic	Linear programming model	✓
Rao Rao (1998)	✓		✓		✓			Deterministic & Static	Dynamic programming	✓
Cormier & Gunn (1996b)	✓			✓				Deterministic	Newthon-Raphson method	✓
Rolt & Rosenblat (1988)	✓							Stochastic	Minimum cost flow algorithm	
Huang et al. (2014)	✓		✓					Stochastic	Non-linear MIP problem	✓
Yuang et al. (2016)	✓		✓					Stochastic	Queing method/ dynamic programming	✓
Shi et al.(2018)	✓	✓	✓		✓			Stochastic	Dynamic warehouse planning model	✓
Cormier & Gunn (1999)	✓	✓	✓	✓	✓			Deterministic	Dynamic programming	✓
Minner (2009)		✓		✓				Deterministic	Heuristic algorithms	✓
Vaziri et al. (2018)		✓	✓	✓				Deterministic	-	✓
Atamtürk & Hochembaum (2001)	✓	✓	✓		✓			Deterministic	Polynominal time algorithms	✓
Chen & Thizy (1990)		✓	✓	✓				Stochastic	Shortest path algorithm	✓
Wagner & Within (1958)		✓	✓					Determinisic	Forward algorithm	
Love (1973)		✓	✓					Determinisic	Network flow problem	✓
Chu et al. (2013)		✓	✓		✓			Deterministic	Polynominal time algorithms	✓
Hwang et al. (2013)		✓	✓					Deterministic	Polynominal time algorithms	✓
Liu & Tu (2008)		✓	✓					Deterministcic	Network flow algorithm	✓
Loparic et al. (1999)		✓						Deterministic	Polynominal time algorithms	
Atamtürk & Kücukyavuz (2005)		✓	✓					Deterministic	Linear programming model	✓
Chen et al. (1994)		✓	✓					Deterministic	Dynamic programming algorithm	✓
Erenguc & Aksoy		✓	✓					Deterministic	Branch and bound	✓
Fan & Wang (2021)	✓	✓	✓		✓			Deterministic	Dynamic programming algorithm	
Fan & Wang (2018)	✓	✓	✓		✓			Deterministic	Dynamic programming algorithm	✓
Lalla-Ruiz & Voss						Parallel	✓	N/A	Set partitioning problem heuristic	
Kim & Lee (2011)						Parallel	✓	N/A	MIP model to solve single server problem	
Balin (2011)						Parallel		N/A	Genetic algorithm	
Kim & Kim (2021)						Parallel	✓	N/A	Hybrid Genetic algorithm	
Najat et al. (2019)						Parallel	✓	N/A	Multiple heuristics proposed	
Diallo et al. (2014)						Parallel	✓	N/A	Enumeration algorithm	
Nogueira et al. (2019)						Single	N/A	N/A	Multiple MIPs	
Raja et al. (2008)						Parallel		N/A	Genetic algorithm & fuzzy logic approach	
Li & Yang (2009)						Parallel		N/A	Different heuristics	
Liu & Wu (2013)						Parallel	✓	N/A	Evolutionary programming	
Thesis	✓	✓	✓	✓	✓	Parallel	✓	Deterministic	MILP problem	✓

2.5. Research gap & Societal relevance

Limited to the conducted literature research it became clear that to the best of the author's knowledge there does not exist a mathematical model yet, which jointly optimizes lot sizing, production scheduling and the warehouse capacity setting for a multi-product range in a multi-period setting considering dynamic deterministic demand. This research gap was identified by means of mapping the used concepts in literature which are displayed in Table 1.

This thesis aims to fill this research gap by proposing a Mixed-integer linear programming (MILP) model which jointly optimizes dynamic lot sizes and warehouse capacity for a multi-product range as well as optimizing the production schedules for the products under study. The papers of Fan & Wang (2018), Fan et al. (2021) and Cormier & Gunn (1996a) will be used as inspiration for MILP models concerning the joint optimization of lot sizes and warehouse capacities. In order to find a suitable method to include a multi-product range in the proposed MILP model, the papers of Cormier & Gunn (1999), Alle & Pinto (2002), Minner (2009) and Vaziri et al. (2018) will be used as inspiration. Finally, the papers of Kim and Lee (2012), Balin (2011) and Kim & Kim (2021) will be used as inspiration to propose and include a non-identical parallel machine scheduling MILP in the combined MILP.

Besides filling a gap in literature, proposing a combined MILP model which optimizes the three different problems for a multi-product range also has other benefits. The first benefit of the proposed model is that the multi-product aspect of the MILP makes the model suitable for case studies based on real-life situations in which multiple products are produced and stored. This is a new benefit of the proposed model, since from the literature it emerged that combined optimization models in these problem areas were mostly focussed on a single product. Having a mathematical model which is merely focused on a single product can verify the interactions of the model but cannot validate the model's application in a real-life situation and help companies gain useful insights, as most manufacturing companies produce more than one product type (Mousavi et al., 2014). The ability of the new model to be used in case studies should help generate useful managerial insights for logistic managers.

3. Research approach

In this chapter the thesis research approach is discussed. This will be done by first highlighting the research objective and the research questions in section 3.1. Thereafter, the research approach and research methods used to answer the developed research questions are discussed in section 3.2.

3.1. Research objective & questions

The objective of this thesis is to answer the identified research gap by proposing two MILP programming models which optimize multi-period warehouse capacity problems and production scheduling problems for a multi-product range. Besides proposing these new models this thesis also has another objective, namely testing the integrated and sequential optimization models in a real-life scenario by means of a case study. In this thesis the case study will be carried out for a manufacturing company in the food industry, which will be carefully constructed. The second research objective will help to assess if the proposed models can eventually be used by manufacturing companies to improve the coordination between production scheduling and warehousing teams in the future. The research objectives should be completed when the proposed main research question can be answered. The main research question entails:

"How does the coordination between production and warehouse decisions affect the product flow between factories and warehouses for manufacturing firms?"

In order to find an answer to the main research question, sub research questions are formulated to substantiate the main research question. The sub research questions are:

1. *What are available warehouse sizing and production scheduling optimization models in literature?*
2. *How can mathematical warehouse sizing models be extended from a single to a multi-product range?*
3. *How can a warehouse optimization model and a production scheduling optimization model be merged?*
4. *How do production flows from a factory affect warehousing strategies?*
5. *How do the models' input parameters affect the outcome of the proposed models?*
6. *To what extend does the model reflect the real-life scenario for Kraft Heinz?*

3.2. Research approach & methods

Each sub-research question will need to be answered in order to find an answer to the main research question. Finding an answer to each sub-research question will be done in the sequence in which they are listed in the previous section. This is necessary since the answer to the first research question is required to find an answer for the subsequent research questions.

3.2.1. Literature review

The first three sub research questions will be answered through a literature review. A literature review will be performed, since this will form the basis of knowledge on which this thesis builds upon in a later stage. The literature review is a desk research, which will be carried out mainly through articles found on Scopus and Google Scholar. Backward and forward snowballing will used in order to extend the number of relevant papers. The literature review will help the author to describe the state of the art in the respective research areas and elaborate on the research gap. The literature review also serves the purpose of elaborating on the key concepts used in this thesis. These key concepts are dynamic lot sizing, the warehouse capacity problem and the production scheduling problem. In the end of the literature review, the concepts will be linked to each other to find identify the research gap. Once the first research questions are answered, the mathematical models can be constructed. The basis for this also lies in the literature research, as the information on how to construct a mathematical model is obtained by combing knowledge from several other research papers.

3.2.2. Mathematical modelling

The next step in this research approach is setting up the mathematical models, by making use of the answers to the second and third sub research question. The first step in this process is making a detailed problem description which sketches the problem situation and introduces all parameters and variables. The input parameters and decision variables are retrieved via the performed literature research and via interviews from the manufacturing company under study. Once the problem is clear to the reader, the mathematical formulations can be introduced. This includes the proposing the objective function and constraints. The model description will be concluded with an elaboration on the modelling assumptions and limitations. When the problem is clarified on paper, the models can be prepared in an optimization program. Finally, the interaction between the decision variables and the model outcomes can be verified by means of a simple numerical test setting.

Modelling technique

In this thesis it is chosen to formulate the problems as a MILP model. A MILP model is a linear model in which some of the variables can only take on integer values. The MILP approach was chosen as it seemed most fitting to the nature of the two problems. The fact that both problems concern the order, production and storage quantities of a range of products makes it a requirement these variables can only take on integer values. Additionally, the multi-period warehouse capacity problem introduces capacity expansion and contraction decisions which can be modelled by making use of boolean decision variables. The choice for using a MILP model is also supported by literature as the paper of Fang & Wang (2018) also uses a MILP model for a single product range of the problem. The same holds for the papers of Lee et al. (1999), Lee (2005) and Belvaux & Wolsey (2001) who propose MILP models to optimize dynamic lot size problem and the papers of Bradley & Arntzen (1999), Lee & Elsayed (2005) and Sazvar et al. (2016) who use MILP models to optimize the warehouse sizing problem.

Optimization programme

In this thesis it is chosen to use IBM CPLEX optimizer using Optimization Programming Language. CPLEX is an optimization program that can solve large optimization problems via multiple programming languages (C++, Java, C#) and programmes (Python, MATLAB, Excel). CPLEX is suitable for mixed integer programming amongst other programming problems (“IBM CPLEX Optimizer,” n.d.). CPLEX uses a branch and cut algorithm to find the optimal values for the proposed model(“IBM CPLEX Optimizer,” n.d.). The branch and cut algorithm is a combination of the well-known branch and bound algorithm which is also used in the paper of Erenguc & Aksoy (1990) and combines this algorithm with cutting planes or ‘polyhedral algorithms’ which were also used by Atamtürk & Küçükyavuz (2005) to identify the optimal values. Combing the two algorithms into the branch and cut algorithm has proven to be a successful and efficient method to optimally solve many different integer programming problems (Naud et al., 2020). Other research papers have used other solving algorithms to optimize formulated warehouse or lot size problems. An example of such a research is the paper of Lowe et al. (1979) who use a greedy network flow algorithm to solve the warehousing problem. Ahujaakun & Hochbaum (2008) propose a minimum cost flow algorithm to solve a dynamic lot sizing problem. What these examples display is the fact that optimization problems can be solved by means of different algorithms depending on the problem’s characteristics. In this thesis the MILP problem nature and the availability of CPLEX as an optimization program resulted in the usage of the branch and cut algorithm to find the optimal solution. The effectiveness of this algorithm is confirmed by Naud et al. (2020).

3.2.3. Numerical tests

Once the models have been completely verified, numerical tests will be performed with the model in order to test the model's solving capabilities and effectiveness. To this end, the number of products, periods and machines under consideration will be extended as well as all the related input parameters up to the point that the problem becomes insolvable, or the running time exceeds a predetermined amount. The running time of the model will be mapped by increasing the data set under consideration. In this way the effects of increasing the data set on the models' running times can be tested. This can help to identify what the maximum effective solving capability of the model is and to find at till which extend the model is useful for solving warehousing and scheduling problems.

Besides increasing the input data set for the model, an extensive sensitivity analysis will be performed as well. The aim of this analysis is to find out how the values of input parameters affect the values of the objective function and according decision variables. This will be achieved by changing the values of the input parameters one by one and mapping what the effects are for the objective function and decision variables. Studying the effects of changing input parameter values on the objective function and decision variables should help generating useful insights for the concluding managerial recommendations as well as finding an answer to the fifth sub research question.

3.2.4. Case study

After the model is completely verified and the models' behaviour have been thoroughly analysed, the case study will be studied to gain insight into the applicability of the model to a real-world case and to generate managerial insights. Case studies have been widely used as a method to test a model in a real-life scenario (Patton & Appelbaum, 2003). In this thesis a single Kraft Heinz case will be studied in order to obtain useful insights for Kraft Heinz to improve their ordering strategy and to further test the proposed models. It should be noted that carrying out a single case study does have certain limitations, mostly being a reliability issue, since the model is only tested in one single case (Kanama & Kido, 2016). Nonetheless, it was decided to carry out a single case study due to the limited time.

The iterative process of setting up the case study will be supervised by Paul Strijers as a Kraft Heinz representative. Data will be provided through interviews and via the company's data systems to accurately sketch the context in which the model will be tested. The case study shall be carried out on a selected product range in order to test the models. The obtained results will then be analysed and used to generate managerial insights. The interpretation of the case study results should lead to answers for research question four and six. When all the sub research questions have been answered, an answer to the main research question can be provided as well.

4. Problem description

In this chapter the problem background will be elaborated on, this will be done by first discussing the problem relevance in section 4.1. In the subsequent section 4.2 a more detailed and formal problem description of the joint warehouse capacity and lot sizing problem and the production scheduling problem will be provided. Providing a detailed description helps the reader to fully understand the problem and to understand what steps need to be taken and which assumptions will need to be made when constructing the mathematical model. This problem description can be considered as the starting point of the mathematical model. Besides the problem descriptions, both models' assumptions will be listed in section 4.3.

4.1. Business relevance

In an ideal scenario, manufacturing companies plan productions which match the demand in a period and can be produced, transported and stored within the available logistic capacity at hand. In reality however, this is not always the case. For example, at the company under study, orders are placed by demand planners who request a quantity of products in a period, only taking into consideration the needs of the countries who request the products. These order requests are thereafter checked by the supply planning team which translate the orders of different countries into production plans for a factory. These production plans are optimized taking into consideration the factory's availability, the production constraints and the order request of the different countries. Once a production plan is made it is handed over to the factory which will produce according to the shared production plan. When the production is completed, the finished goods need to be moved to a warehouse where they will be stored. The warehouse itself has a capacity which is fixed for a period but can be increased or decreased over time. The last two activities are carried out by the execution team as can be seen in Figure 4.

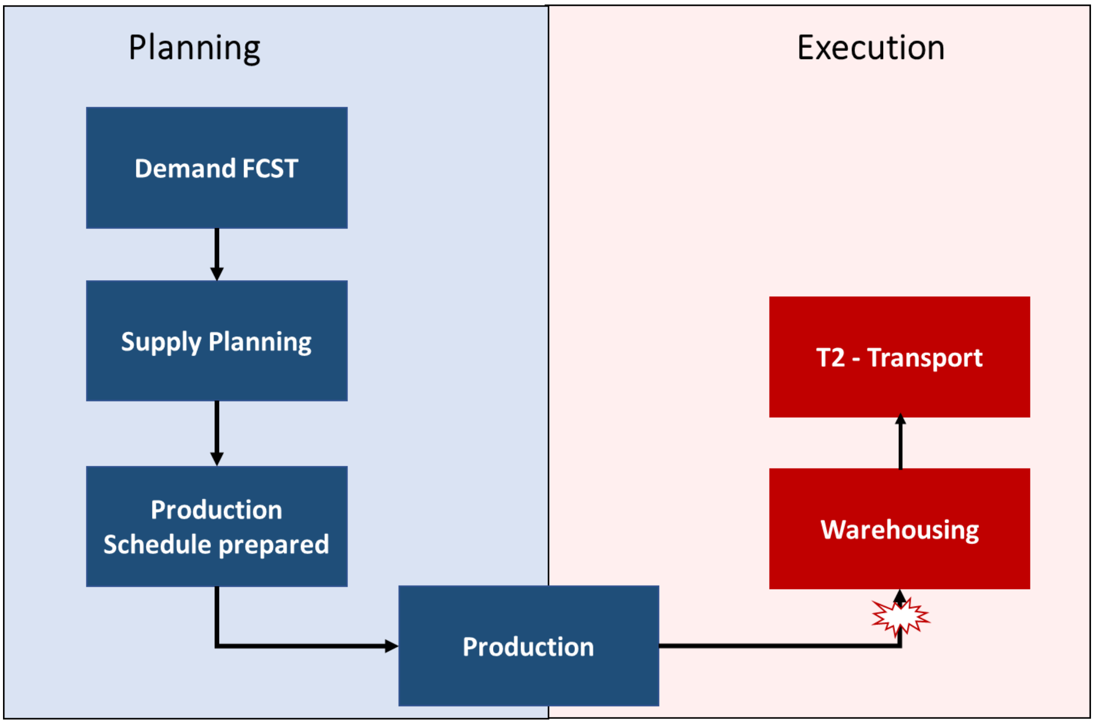

FIGURE 4 CURRENT ORDER GENERATION SEQUENCE

From the above description it becomes clear that the planning and execution teams are not always aligned. The planning team can plan productions which fall within the production constraints, but do not consider the warehouse capacity constraints in place. Hence, the execution team needs to adapt by finding short-term alternative solutions if available capacity is insufficient. In such a set-up, the execution team is dependent of the plans generated by planning team. This could result in unforeseen issues in the factory outbound process as well as the warehouse inbound process. The issues resulting from the unsynchronized production planning process are indicated with a spike mark in Figure 4. To prevent similar issues in the future, three alternative manufacturing order strategies will be proposed in this thesis which generate orders based on shared forecast and considers both production and execution constraints. The proposed ordering strategies should help close the gap between planning and execution teams and decrease the number of unforeseen issues in the product flow between factories and warehouses in the future.

The proposed ordering strategies make use of warehousing and production scheduling optimization models which will be discussed in the following chapter. Two out of the three ordering strategies use a sequential set-up. In other words, first one model is optimized, after which the output of the first model is used as input for the second model in the sequence. The sequential ordering strategies represent the common setting that there is hardly any interdepartmental communication. For the *Sequential Warehouse Strategy* this means that first an order will be placed by the warehouse team based on the demand and their respective warehousing constraints, after which the order will be pushed to the planning team who will translate the order into a production schedule if it can be managed within the production constraints. This ordering strategy is displayed in Figure 6. The second sequential ordering strategy has the opposite sequence in which first the planning team optimizes the production schedule and pushes the order quantities to the warehouse team in the second step. This strategy is displayed in Figure 5 and is called *Sequential Production Strategy*.

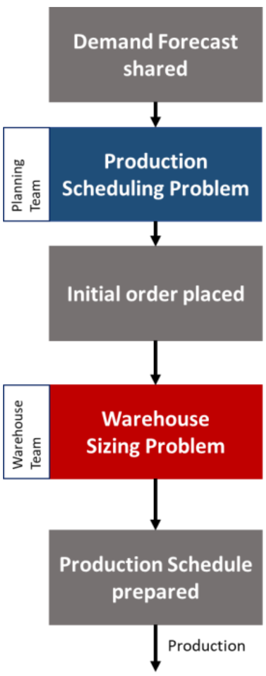

FIGURE 5 SEQUENTIAL PRODUCTION STRATEGY

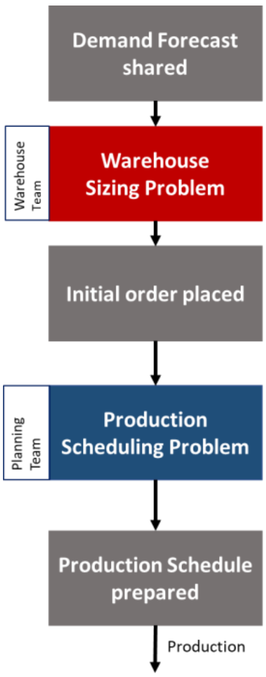

FIGURE 6 SEQUENTIAL WAREHOUSE STRATEGY

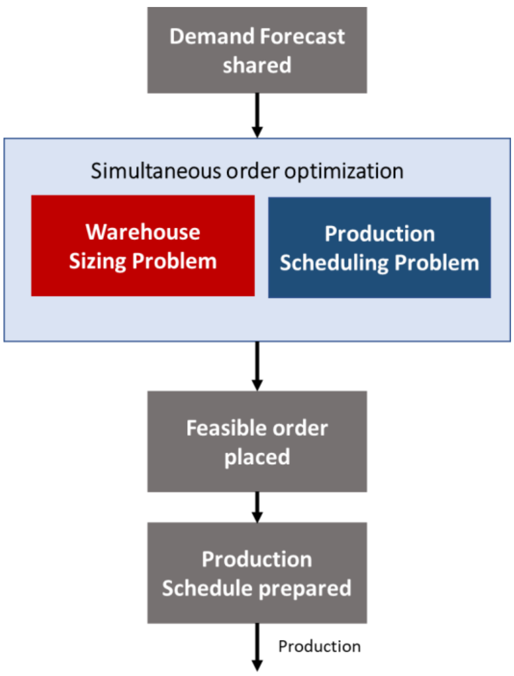

FIGURE 7 SIMULTANEOUS ORDER STRATEGY

The *Simultaneous Ordering Strategy* simultaneously optimizes orders for both the planning and execution departments. In a simultaneous ordering strategy, an interdepartmental decision is made regarding the optimal order quantities which satisfy both the production as well as the warehouse constrains in place. The simultaneous ordering strategy represents the setting in which there is central coordination over the production and warehousing departments. The simultaneous ordering strategy is conceptualized in Figure 7. The three ordering strategies will be tested and compared in a case study in order to find the superior strategy for manufacturing companies.

The case study provides a practical setting to test the three strategies and analyse the effect of the respective strategies on the product flows between the factories and warehouses. In the case study the added value of solving the two models simultaneously over sequential optimization can be demonstrated. This demonstration is relevant for logistic managers in the manufacturing supply chain as the effects of enhancing central coordination through simultaneous optimization can be displayed and tested in a real-life example. The goal is to find the optimal ordering strategy with simultaneous optimization as in literature the papers of Atamtürk & Hochbaum (2001) and Bradley & Arntzen (1999) and Fan & Wang (2018) stated that this leads to superior results. The benefit of simultaneous optimization is the concurrent consideration of the two objective functions and all constraints in place. By considering all the relevant factors at the same time, the model is able to make optimal trade-offs regarding product quantities, available capacity, timing and cost, whilst ensuring a feasible product flow. In this way the optimal strategy for the studied company should will be identified. In the case study the hypotheses from literature will be either confirmed or rejected for the developed production flow models. The simultaneous model optimization will be carried out by optimization software as the studied problem is too large and complicated to be solved by hand. The optimization problem has over 50.000 variables that need to be accounted for and optimized in order to find the optimal warehousing and production scheduling strategies. Each added product or period introduces a set of new variables that needs to be optimized. Solving similar problems by hand would require full-time commitment from an employee. Even then finding the optimal strategy will be difficult or even impossible. For this reason, it is better to model the problems in optimization software which can solve the integrated problem in minutes instead of days or weeks. By preparing the models in software programs, different scenarios can be tested in order to find optimal strategies which are robust. Moreover, the models can be run on a daily bases to ensure that the most recent data is used when finding the optimal strategies. In this way support is provided for logistic managers to improve the product flows between factories and warehouses.

4.2. Formal problem description

The formal problem description provides a more detailed description of the problems under consideration. The detailed description of the problem will be used as a starting point for the mathematical problem that is to be proposed. In this section the lot size and multi-period warehouse problem and the production scheduling problem are described individually.

4.2.1 Lot size & multi-period warehousing problem

For this problem we consider a manufacturing company that produces a variety of products in a single factory. Finished products need to be stored in a warehouse which is not located at the factory. These warehouses are not owned by the manufacturing company itself, but the manufacturing company leases warehouse capacity from a LSP warehouse operator for a certain price *r* per pallet storage space. The leased capacity is agreed upon via a contract for a certain time interval *T.* This time interval T can be subdivided into smaller sessions t which all have a similar length. At the beginning of each session the manufacturing company can choose to either increase, decrease or keep the leased warehouse capacity as it is*.* The leased warehouse capacity *w* is bounded by the actual maximum capacity of the LSP warehouse provider and the minimum agreed warehouse capacity that is to be leased by the company. During the session *t* the warehouse capacity *w* is a fixed value, it can only be changed at the start of a session *t.* The length of the sessions t and the decision moments are agreed upon by the production company and the LSP as the times when the production company can change its warehouse capacity *w* against a certain penalty price *p*.

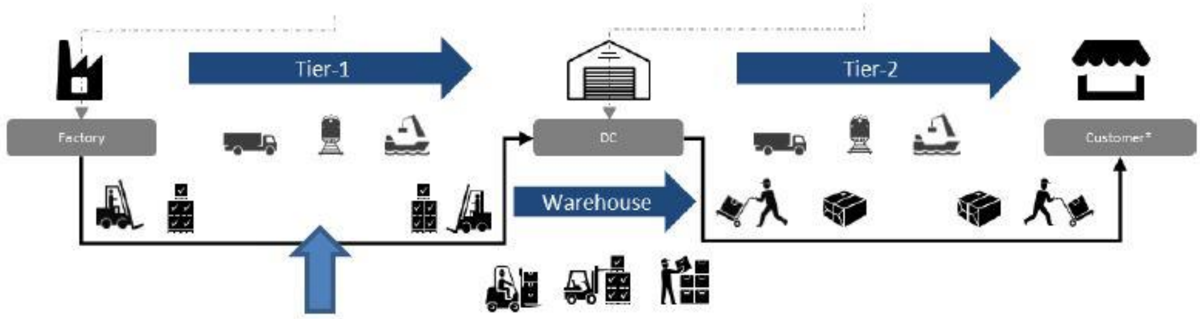

FIGURE 8 SUPPLY CHAIN OVERVIEW (KRAFT HEINZ, 2020)

On a product level we consider a set of products that should be produced before a due date. Products are ordered in order quantities *x* against fixed ordering costs *k.* When these products enter the warehouse, the products are being put into place for handling costs *g* and they stay in place against holding cost *h* per session *t.* The demand *d* for each product is known and no backlog is allowed. The company can produce multiple products at a time, but every factory has a fixed production capacity per period. Once a product is ready, it will be transferred to the warehouse where it is stored. The warehouse has a maximum inbound capacity *b* per period *t.* At the warehouse, at time 0, there is already stock of the different products. Task of the decision maker is to find the optimal warehouse capacity for every interval *t* such that there is sufficient capacity to store all products for that period, but there is no excessive capacity which can result in unnecessary costs. Additionally, the decision maker also wants to find the optimal ordering quantity *x* for the set of products, so to balance production and warehousing cost in the supply chain. The optimal values for warehouse capacity and lot sizes will be solved simultaneously, since the papers of Atamtürk & Hochbaum (2001) and Bradley & Arntzen (1999) proved that making warehouse and lot size decisions simultaneous returns better solutions, than solving the sub-problems sequentially.

4.2.2. Production scheduling problem

The production scheduling problem of parallel machines represents the problem to determine which products should be produced on which lines at what time in a session *t* at the factory. The product demand will be translated into orders via the lot size and warehouse problem, these product orders will thereafter be translated into production flows, which will be moved to the warehouse where the products will be stored before they are sent out to the customers.

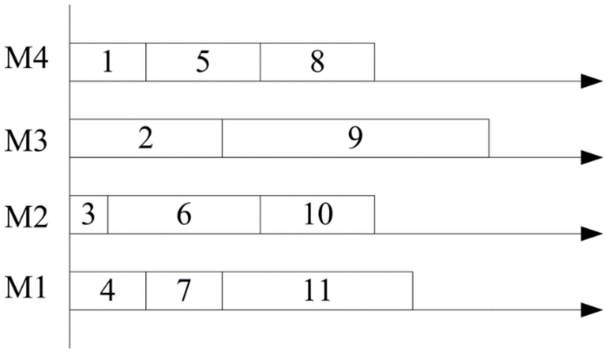

FIGURE 9 ILLUSTRATIVE EXAMPLE PARALLEL MACHINE SCHEDULING (Xu & Nagi, 2013)

The aim for the production scheduling problem is to produce all the orders *x* in a session *t.* This means that all production runs should be finished before the due date D. In the scheduling problem there are *j* independent products which can be produced on *m* machines which are assumed to be always available. Every production line *m* has a production rate PR_j for a product *j* which in known in advance and the same for every session. Every production line *m* is available between the release date *R* and the due date *D* in a session *t*. A production run for product *j* consists of a set up time *S* which is dependent of the product that was produced prior to product *j* on production line m and the actual production time PT_j which depends on the order of product x_j and the production rate PR_j of the machine *m* for product *j*. For every first product *j* which is produced on production line *m* no set-up time *S* is required. Once a production run in finished the production line *m* can be immediately set-up for the next product *j*. A product *j* can only be produced once in every session and only on a single machine *m.* Production lines can run in parallel, but only a single product can be produced on a production at the same time, as no overlap between production runs is allowed. The goal is to produce order x_j within session *t* before the set due date *D* against the lowest possible production and set up costs. The costs are computed by multiplying the production and set-up times by the respective variable production and set-up costs.

4.3. Problem assumptions

The proposed problems do make assumptions with regards to the used data. These assumptions make it easier to translate the real-life situation into a problem, but on the other side also negatively affect the real-life representation of the problem as the problems is a simplification of reality. This holds with the statement of George Box who said: *"All models are wrong, but some are useful"* (Box, 1976). Who emphasized that it is impossible for scientific models to capture the complexity of reality in a model. The aim of this thesis is to come up with a model which is solvable, but at the same time can be used to generate useful insights in current production and warehousing operations. In order to accomplish this, a number of assumptions were made for the formulated lot size and multi-period warehouse problem and the production scheduling problem, these assumptions can be found below. It should be noted that for the combined problem, all assumptions for both the individual problems hold.

The lot size and multi-period warehouse problem assumes that demand is discrete deterministic and varies over time. This means that the demand is known in advance with certainty for every period in the future, during a period the demand is considered to be constant, but every period the demand can be different. At most one order can be placed in a period, products can either be included or excluded from this order. The problem assumes that an order can be fulfilled in the same period as it is placed. The placed order in a period should be produced, nothing more or less than the order can be produced. Full pallets will be used as a unit of measure for all parameters and decision variables. In other words, order quantities and inventory levels are measured as full pallets. The arrival and departure distributions within a period t are not accounted for as the problem merely considers into the overall quantities handled per session *t.*

Besides the above-mentioned assumptions, three additional assumptions were made for the production scheduling problem. The production scheduling problem assumes that all production lines are always available between the release date *R* and due date *D,* since no maintenance or non-availability of lines is considered. Furthermore, the problem looks at complete productions runs, different jobs for each production are not considered. Moreover, the production model assumes that there are sequence dependent set-up times between production runs and that the first production run produced on a line in every period does not require set-up time. This assumption is also frequently used in production scheduling literature of which the paper of Kim & Kim (2021) is an example. A final assumption made is that that raw materials are always available.

5. Model design

This chapter will introduce the models used in this thesis, starting with the conceptual model in section 5.1. The conceptual model gives a clear overview of the models' interactions and the environment it operates in. Section 5.2 will elaborate on the three mathematical models and their respective interpretation.

5.1. Conceptual model

A conceptual model is a helpful tool to understand how the input parameters are being used to optimize the objective function of the model under study within the given constraints. Additionally, the conceptual model forms the basis of the models' verification steps later in this chapter. The conceptual modelling phase succeeds the system description of chapter four which is in line with the model design steps from Robinson (2011) that are as depicted in Figure 10.

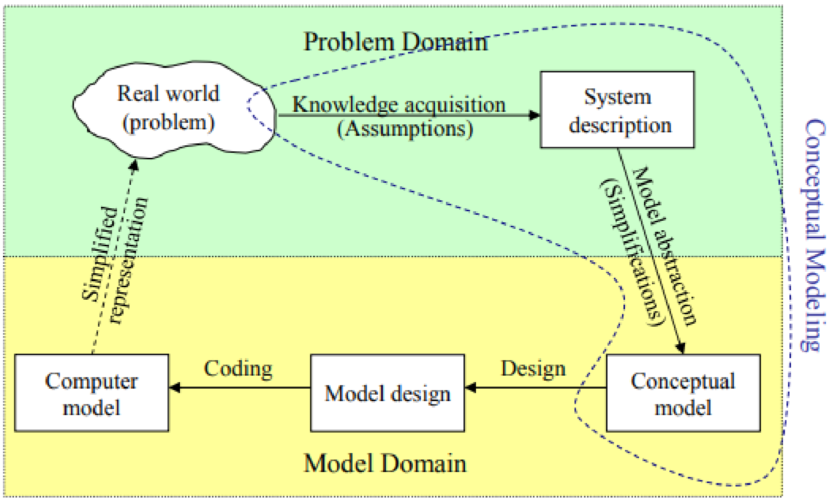

FIGURE 10 MODEL DESIGN STEPS (ROBINSON, 2011)

Based on the formal problem descriptions the conceptual model was realized as depicted by Figure 11. The conceptual model displays the simulation environment of the proposed warehousing and production scheduling models. The starting point of the conceptual model is the inflow of information as input parameters for the model. The input parameters can be subdivided into three main types of parameters, being the product demand, the warehouse cost parameters which concern different warehouse costs such as penalty, holding and handling costs and finally the production scheduling parameters such as production rates, set-up times and production cost. Based on this set of input parameters the mathematical model will be optimized in order to find the optimal values for the decision variables. The mathematical model consists of an objective function in which all cost drivers are captured. The aim is to minimize this objective function such that the overall warehousing and or production costs are minimized whilst satisfying all the set constraints. The set constraints are in place to ensure that inventory and warehouse levels are balanced between periods and that inventory levels are not allowed to exceed the warehouse capacity. The production scheduling constraints ensure that sufficient productions are planned to meet the respective demand in each period within the given capacity constraints. Besides balancing inventory and warehouse sizes between periods and ensuring sufficient productions, the constraints also ensure that decision variables stay within the set bounds and capacity constraints. Moreover, the constraints ensure that the boolean variables are 0 in case that nothing happens and take on value 1 when an expansion, contraction or ordering period takes place in the model.

The mathematical model is optimized by means of an optimization algorithm in IBM ILOG CPLEX Optimization Studio. This optimization algorithm uses the branch-and-cut algorithm to find optimal values for decision variables. This algorithm combines the branch and bound algorithm with a polyhedral algorithm to reach the optimal values (Naud et al., 2020). The branch and cut algorithm uses the cutting planes from the polyhedral algorithm to restrict the solution space and then makes use of the branch and bound technique to arrive at the optimal solution. This algorithm type is suitable to solve linear problems involving integers, hence the model under study. Once the mathematical model is optimized by means of a solver, the resulting decision variables and the overall costs can be interpreted. Making interpretations of the outcomes of the model will result in managerial recommendations for the factory and warehouse under study. The individual cost parameters and the expansion and contraction decisions will be studied in an extensive sensitivity analysis and can be useful to show the effects of changing certain parameters and agreements with the LSP on the total costs as well as on the factory and warehouse performance.

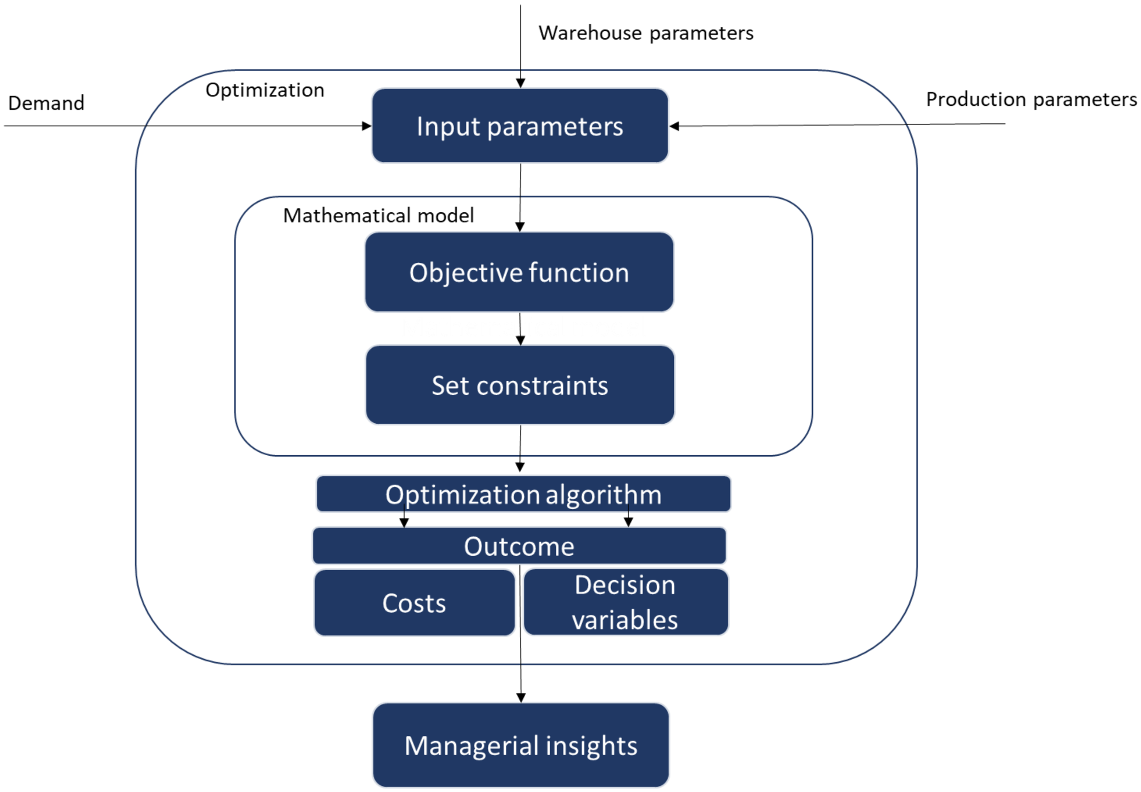

FIGURE 11 CONCEPTUAL MODEL OF COMBINED OPTIMIZATION MODEL

5.2. Mathematical models

Insights from the conceptual model and the formal problem description form the basis for the mathematical models proposed in this section. The lot size and multi-period warehouse model and the production scheduling model are first discussed individually after which the two models are combined into an integrated mathematical model.

5.2.1. Lot size and multi-period warehouse model

The objective function captures the different cost drivers for the warehouse situation. Part of the objective function is summed for every period T as well as for every product J and part of the objective function is only summed over every period T. The reason for these two types of summations is that some parameters have a different value for each product for every period, whilst other parameters are merely measured per period. The 12 constraints are listed below the objective function, the constraints represent conditions that the optimal solution must satisfy. The constraints represent both practical warehouse constraints as well as variable and parameter constraints. The individual constraints will be elaborated on more thoroughly below the mathematical model.

For each period $t \in \{1, \dots \dots \dots, T\}$, and each product $j \in \{1, \dots \dots \dots, J\}$, the following parameters and decision variables are defined.

Parameters

- *D_{tj} Demand in period t for product for product j*
- *K Fixed ordering cost*
- *H Holding cost of a product per period t*
- *G Handling costs per product j*
- *P Fixed cost of changing the warehouse size*
- *R rental cost per unit warehouse size at per period t*
- *N Maximum warehouse inbound capacity per period t*
- *I_j^0 Initial inventory at t=0 for product j*
- *B Maximum warehouse inbound capacity in overtime per period t*
- *W_{max} Maximum warehouse capacity available*
- *W_{min} Minimum warehouse capacity that should be leased*
- *F Variable inbound cost per pallet in OT*
- *SS_j The required safety stock level for every product j*

Decision variables

- *x_{tj} Order quantity of product j in period t*
- *i_{tj} Inventory level of product j at the end of period t*
- *w_t Warehouse size at the end of period t*
- *u_t Warehouse size expansion at the beginning of period t*
- *v_t Warehouse size reduction at the beginning of period t*
- *l_t Inbound pallets handled in overtime*
- *y_t Binary variable for placing an order in period t*
- *z_{t+}Binary variable for warehouse size expansion in period t*
- *z_{t-}Binary variable for warehouse size expansion in period t*

Objective function and constraints

(1) L&WS $Min\sum_{t=1}^{T}(\sum_{j=1}^{J}(Hi_{tj} + Gx_{tj}) + Ky_t + P(z_t^+ + z_t^-) + R(u_t + v_t + w_t) + l_tF)$

Such that

(2) $i_{t-1,j} + x_{tj} - D_{tj} = i_{tj}$ $\forall$ t, j

(3) $w_{t-1} + u_t - v_t = w_t$ $\forall$ t, j

(4) $\sum_{j=1}^{J} i_{tj} \leq w_t$ $\forall$ t, j

(5) $x_{tj} \leq D_{tTj} y_{tj}$ $\forall$ t, j

(6) $u_t \leq \sum_{j=1}^{J} D_{1Tj}\, z_t^+$ $\forall$ t

(7) $v_t \leq \sum_{j=1}^{J} D_{1Tj}\, z_t^-$ $\forall$ t

(8) $\sum_{j=1}^{J} x_{tj} \leq N_t + l_t$ $\forall$ t

(9) $l_t \leq B_t$ $\forall$ t

(10) $W_{min} \leq w_t \leq W_{max}$ $\forall$ t

(11) $i_{tj} \geq SS_j$ $\forall$ t, j

(12) $W_0 = W_{min}$ $t = 0$

(13) $I_{0j} = I_j^0$ $t = 0,\ \forall$j

$x_{tj} \geq 0, I_{tj} \geq 0,$ $\forall$ t, j

$w_t \geq 0, u_t \geq 0, v_t \geq 0,$ $\forall$t

$y_t, z_t^+, z_t^- \in \{0, 1\},$ $\forall$t

Model interpretation

The objective function (1) captures the cost drivers of warehousing such as the holding and handling cost per product per period, the fixed ordering cost, the warehouse renting cost and eventual overtime cost. The aim is to minimize this objective function whilst satisfying all the listed constraints. Constraints (2) and (3) ensure that the inventory and warehouse level in the coming period equals what was added or subtracted from the level of the previous period. These constraints ensure the warehouse capacity and inventory levels are balanced between subsequent periods. Constraint (4) ensures that the sum of inventory in the warehouse cannot exceed the warehouse capacity in the same period. This would be the constraint that bounds the inventory level in the model. Constraint (5), (6) and (7) ensure that the binary variables take on a value of 1 in case of an ordering, expansion or contraction period and 0 if not. Constraint (5) represents that the ordered quantity of product j in period t should be smaller than the forward-looking demand of that product (D_{tTj}). This constraint only holds when Y_t is an ordering period. Constraints (6) and (7) ensure that both u_t and v_t should be smaller than the total demand (D_{1T}) when it is an expansion or a contraction period. Constraint (8) makes sure that the ordered or inbound quantity x_j to the warehouse does not exceed the available unloading capacity of that warehouse in a period, if necessary, the company can use overtime inbound capacity l_t to increase the warehouse inbound capacity up to a limit. Constraint (9) represents the upper bound of possible inbound overtime that can be used in a period t. Constraint (10) represents the minimum and maximum capacity the warehouse can take on. Constraint (11) ensures that the inventory level of every product should never be below the agreed safety stock level. Constraints (12) and (13) set the initial warehouse capacity and inventory levels at time t=0.

5.2.2. Production scheduling model

For each period t $\in \{1, \dots \dots \dots, T\}$, *each product j* $\in \{1, \dots \dots \dots, J\}$, *each product i* $\in \{1, \dots \dots \dots, J\}$, *and each production line m* $\in \{1, \dots \dots \dots, M\}$ *the following parameters and decision variables are defined.*

Parameters

- P_{jm} *Production rate of product j on production line m*
- S_{ij} *Setup time for product j, when product j is produced directly after product i*
- *D Due date in period t*
- *R Release date in period t*
- *SC Set-up cost per time unit for a machine*
- *PC Production cost per time unit for a machine*
- *M is big number M*

Decision variables

- a_{ijmt} *Binary variable which takes the value 1 if product j is produced after product i on production line m in period t, otherwise it takes on 0.* A_{0jmt} *will be the first product produced on a production line.*
- st_{jmt} *Starting time of production for product j on a machine m in period t*
- pt_{jmt} *Production time of product j on production line m*
- x_{jt} *Ordered quantity of product j in period t*

So, a_{3214} =1 if machine 1 produces product 2 after product 3 in the 4th period, A_{3214} is 0 if this is not the case.

Objective function and constraints

(1) Minimize $\sum_{t=1}^{T}(\sum_{j=1}^{J}\sum_{i=1}^{J}\sum_{m=1}^{M} a_{ijmt}(S_{ij}SC + PCpt_{jmt}))$

Such that

(2) $\sum_{j=1}^{J}\sum_{m=1}^{M} a_{ijmt} \leq 1$	∀ t, i ≠j
(3) $\sum_{j}^{J} a_{0jmt} \leq 1$	∀ t, m
(4) $\sum_{i}^{J} a_{ijmt} = \sum_{i}^{J} a_{jimt}$	∀ t, j, m, i ≠j
(5) $a_{iimt} = 0$	∀ t, i, m
(6) $st_{jmt} \geq st_{imt} + pt_{imt} + S_{ij} - M(1 - a_{ijmt})$	∀ t, i, j, m
(7) $X_{jt} = \sum_{m=1}^{M}(pt_{jmt} * P_{jm})$	∀ t, j
(8) $pt_{imt} \leq M * \sum_{j}^{J} a_{jimt}$	∀ t, i, m
(9) $\sum_{1}^{T} x_{tj} \geq \sum_{1}^{T} d_{tj}$	∀ t, j
(10) $R \leq st_{jmt} + pt_{jmt} \leq D$	∀ t, j, m
$a_{ijmt} \in \{0, 1\}$	∀ t, i, j, m
$st_{jmt} \geq 0, pt_{jmt} \geq 0,$	∀ t, i, j, m
$x_{jt} \geq 0$	∀ t, j

Model interpretation

The objective function (1) is aimed at minimizing the overall production and set-up costs for all production lines m in all periods t. Costs for set-up or production are only counted when binary variable a_{ijmt} takes on the value one for the specific combination of the product, production line and period. Constraint (2) ensures that a product j is produced at most once in a period. Constraint (3) ensures that at most one product j can be produced first on every production line. Constraint (4) and (5) ensure that no similar products are scheduled twice in the same period and that every sequence combination is unique. Constraint (6) ensures that the next production can only start once the previous production on a production line has been completed and the line is set-up for the next run. Constraint (7) is in place to ensure that the scheduled production plan exactly fulfils the order for product j in a period. The constraint indicates that if a product j is produced on machine m, the production time multiplied with the production rate should be equal to the order quantity of the product j. Constraint (8) ensures that only a production time is assigned to production runs if they are included in the production schedule. Constraint (9) ensures that sufficient products are ordered and produced in order to cope with the demand. Constraint (10) ensures that all completion times should end between the release date and the due date.

5.2.3. Simultaneous optimization model

For each period t $\in \{1, \dots\dots\dots, T\}$*, each product j* $\in \{1, \dots\dots\dots, J\}$*, each product* $i \in \{1, \dots\dots\dots, J\}$*, and each production line m* $\in \{1, \dots\dots\dots, M\}$ *the following parameters and decision variables are defined.*

Parameters

- P_{jm} *Production rate of product j on production line m*
- S_{ij} *Setup time for product j, when product j is produced directly after product i*
- *D Due date in period t*
- *R Release date in period t*
- *SC Set up cost per time unit for a machine*
- *PC Production cost per time unit for a machine*
- *M is big number M*
- D_{tj} *Demand in period t for product for product j*
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- *P Fixed cost of changing the warehouse size*
- *R rental cost per unit warehouse size per period*
- *N Maximum warehouse inbound capacity per period t*
- I_j^0 *Initial inventory at t=0 for product j*
- *B Maximum warehouse inbound capacity in overtime per period t*
- W_{max} *Maximum warehouse capacity available*
- W_{min} *Minimum warehouse capacity that should be leased*
- *F Variable inbound cost per pallet in OT*
- SS_j *The required safety stock level for every product j*

Decision variables

- *a_{ijmt} Binary variable which takes the value 1 if product j is produced after product i on production line m in period t, otherwise it takes on 0. A_{0jmt} will be the first product produced on a production line.*
- *st_{jmt} Starting time of production for product j on a machine m in period t*
- *pt_{jmt} Production time of product j on production line m*
- *x_{tj} Order quantity of product j in period t*
- *i_{tj} Inventory level of product j at the end of period t*
- *w_t Warehouse size at the end of period t*
- *u_t Warehouse size expansion at the beginning of period t*
- *v_t Warehouse size reduction at the beginning of period t*
- *l_t Inbound pallets handled in overtime*
- *y_t Binary variable for placing an order in period t*
- *z_{t+} Binary variable for warehouse size expansion in period t*
- *z_{t-} Binary variable for warehouse size expansion in period t*

Objective function and constraints

(1) *Minimize* $\sum_{t=1}^{T}(\sum_{j=1}^{J}(Hi_{tj} + Gx_{tj}) + Ky_t + P(z_t^+ + z_t^-) + R(u_t + v_t + w_t) + l_tF + \sum_{i=1}^{J}\sum_{m=1}^{M} a_{ijmt}(S_{ij}SC + pt_{jmt}PC))$

Such that

(2) $i_{t-1,j} + x_{tj} - D_{tj} = i_{tj}$	∀ t
(3) $w_{t-1} + u_t - v_t = w_t$	∀ t
(4) $\sum_{j=1}^{J} i_{tj} \leq w_t$	∀ t
(5) $i_{tj} \geq SS_j$	∀ t
(6) $x_{tj} \leq D_{tTj} y_{tj}$	∀ t, j
(7) $u_t \leq \sum_{j=1}^{J} D_{1Tj}\, z_t^+$	∀ t
(8) $v_t \leq \sum_{j=1}^{J} D_{1Tj}\, z_t^-$	∀ t
(9) $\sum_{j=1}^{J} x_{tj} \leq N_t + l_t$	∀ t
(10) $l_t \leq B$	∀ t
(11) $W_{min} \leq w_t \leq W_{max}$	∀ t
(12) $W_0 = W_{min}$	t=0
(13) $I_{0j} = I_j^0$	t=0, ∀j
(14) $\sum_{j=1}^{J}\sum_{m=1}^{M} a_{ijmt} \leq 1$	∀ t, j, i ≠ t
(15) $\sum_{j}^{J} a_{0jmt} \leq 1$	∀ t, m
(16) $\sum_{i}^{J} a_{ijmt} = \sum_{i}^{J} a_{jimt}$	∀ t, m, j, i ≠j
(17) $a_{iimt} = 0$	∀ t, i, m
(18) $st_{jmt} \geq st_{imt} + pt_{imt} + S_{ij} - M(1 - a_{ijmt})$	∀ t, i, j, m
(19) $X_{jt} = \sum_{m=1}^{M}(pt_{jmt} * P_{jm})$	∀t, j
(20) $pt_{imt} \leq M * \sum_{j}^{J} a_{jimt}$	∀ t, i, m
(21) $\sum_{1}^{T} x_{tj} \geq \sum_{1}^{T} d_{t1j}$	∀ t, j
(22) $R \leq st_{jmt} + pt_{jmt} \leq D$	∀ t, j, m

$st_{jmt} \geq 0, pt_{jmt} \geq 0$ ∀ t, i, j, m
$x_{tj} \geq 0, I_{tj} \geq 0,$ ∀ t, j
$w_t \geq 0, u_t \geq 0, v_t \geq 0,$ ∀ t
$y_t, z_t^+, z_t^- \in \{0, 1\},$ ∀ t
$a_{ijmt} \in \{0, 1\}$ ∀ t i, j, m

Model interpretation

The simultaneous optimization model consists of a summation of both the objective functions and the constraints of the lot size and multi-period warehouse optimization model and the production scheduling model. The interpretation of each constraint can therefore be found in the previous sections in which the mathematical models of the individual optimization models were explained.

6. Model verification

When the mathematical models have been introduced, the models themselves need to be set up, verified, and possibly improved before the models can be tested in a case study. The steps required to perform these actions are depicted in Figure 12. Chapter 4 and chapter 5 pertain to the first step, being the model specification step. The problem description, mathematical model and the model assumptions have been provided in these respective chapters. The subsequent sections of this thesis will be dedicated to the process of verifying the three models. When the models behave as was intended and thus comply with the formal problem description and the proposed conceptual model the models' verification step is completed. Once the models have been completely verified, they will be put into practice in a case study, after which the models and the according results will be analysed on their real-life representation as the validation step for the model.

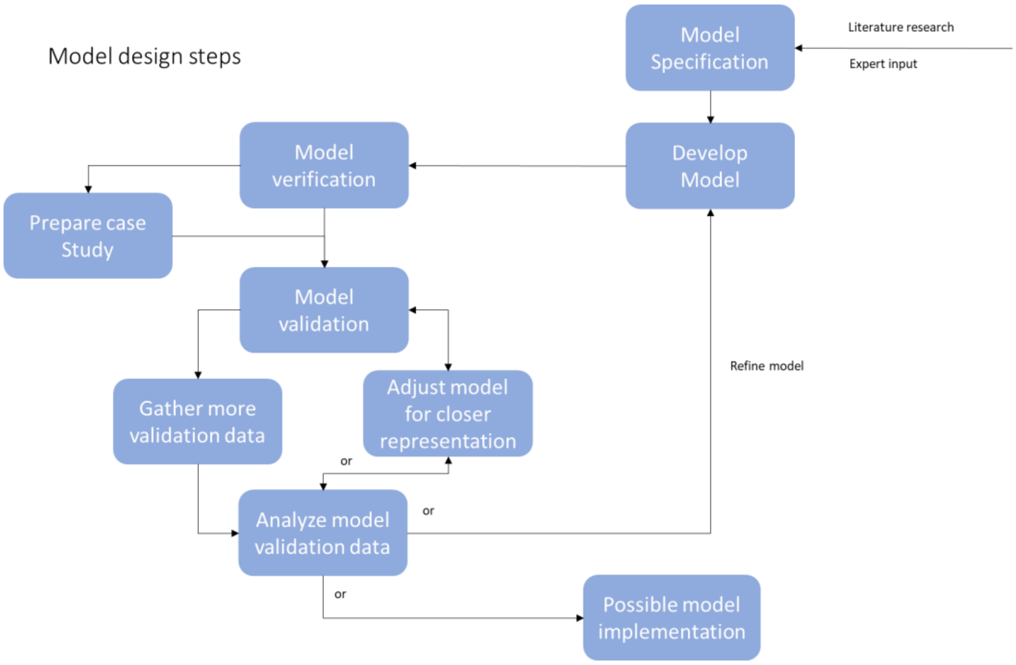

FIGURE 12 MODEL VERIFICATION AND VALIDATION PROCESS

In the verification step, the proposed models are checked if they correctly represent the conceptual models and if the constructed model is built the right way. Questions asked in the verification process are: Does the model correctly represent the conceptual model and the mathematical formulations? Does the outcome of the model make sense based on the input parameters? The three models will be verified by means of simple numerical tests, which can be solved manually as well. The rationale behind this is that the outcomes of the verification tests can be cross checked, to ensure the model's outcomes are correct. Different scenarios will be simulated with the aim of testing the model's behaviour under different circumstances. Below the model verification process will be discussed step by step through the explanation of the different numerical tests per model.

6.1. Lot size & Warehouse model

Input parameters

For the model verification of the lot size and warehouse size model, three simple and hypothetical scenarios were made up in which three products (P1, P2 and P3) are studied for six demand periods. In Table 2 the considered input parameters can be found.

TABLE 2 INPUT PARAMETERS BASE SCENARIO

Product	D0	D1	D2	D3	D4	D5	D6	K	H	P	R	G	N	B	IINV
1	0	100	200	100	300	700	100								400
2	0	400	200	400	200	800	400	15000	5	200	2	4	1500	1000	300
3	0	350	350	350	350	700	350								200

From the table it emerges that the demand varies over time per period and is different for every product. Considering Table 2 to represent the base scenario, two other scenarios have been generated by changing the parameters inbound overtime cost F and the fixed cost of changing warehouse size P. The other parameters do have the same value for every product and for every period in the performed numerical tests. The model outcomes will be verified by analysing the effects of changing input parameters on the model's outcome. A more thorough analysis of the effect of changing input parameters will be conducted in the sensitivity analysis chapter once all the models have been verified.

Model outcome

Figures 13,14 and 15 depict the model's output decision variables after optimizing the mathematical model in CPLEX. The used CPLEX script can be found in Appendix B.

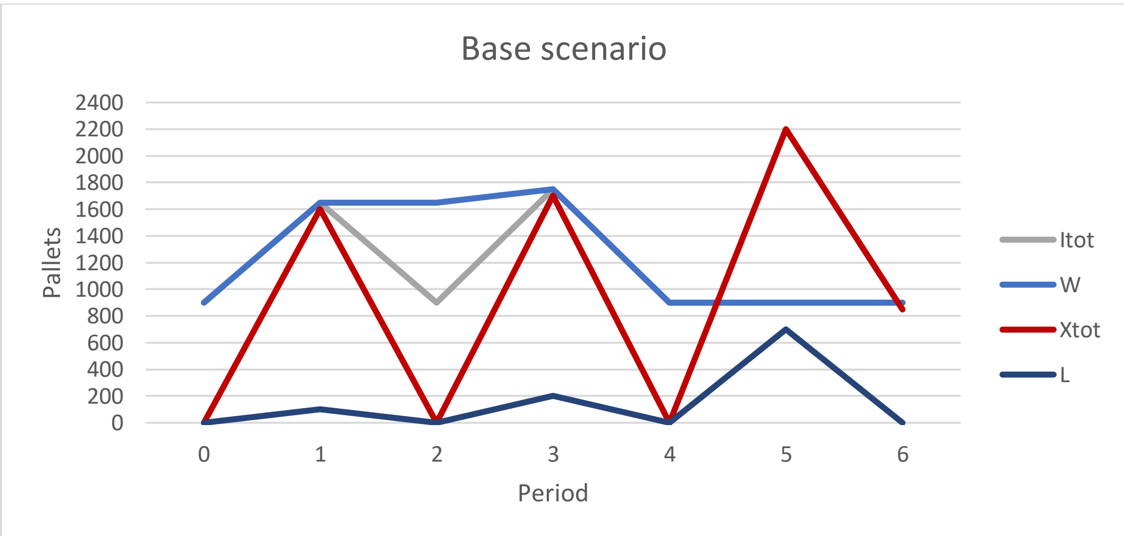

FIGURE 13 MODEL VERIFICATION BASE SCENARIO: *ITOT REPRESENTS INVENTORY, W IS WAREHOUSE CAPACITY, XTOT IS THE QUANTITY ORDERED AND L THE INBOUND OVERTIME CAPACITY USED*

The base scenario outcome in Figure 13 considers the input parameters described in 5.3.1. Based on the described input parameters the model optimizes the ordering and warehousing strategy for the indicated periods. The model outcome can be interpreted as follows, orders are placed in periods 1, 3, 5 and 6 in these periods the binary variable Y takes on the value 1 and fixed ordering cost must be paid. The model aims to combine orders for consecutive periods whenever possible accounting for the objective function to minimize the overall cost, hence the ordering costs as well. An example of this is the order in period 3 which accounts for the demand in both periods 3 and 4. The model can combine the orders due to the relative low demand in both periods without compromising any capacity constraints. As a result, no orders have to be placed in period 4, hence no ordering costs have been incurred. Over the different periods inventory levels *Itot* are increased by placed orders and are being reduced by demand in each period. The resulting warehouse strategy is to change the warehouse size *W* three times, in periods 1,3 and 4. The warehouse size *W* is increased to cope with increasing demand and inventory levels, once the demand is expected to drop, the warehouse size *W* will be reduced in order to minimize the warehousing leasing cost. The decision variable *L* indicates the number of pallets handled in overtime for the inbounding of pallets into the warehouse. This overtime capacity *L* was used in periods 1,3 and 5 as the orders in these periods exceeded the available regular inbound capacity of the warehouse. The pallets inbounded in overtime are incurred for a higher cost than pallets inbounded in the regular capacity, this additional cost is indicated by the parameter F. The base scenario shows that the binary variables are correctly linked to the according decision variables. Moreover, the orders and warehouse size seem to be representative for the changes in demand considering the main objective to minimize the cost. To further verify the model's interactions two other scenarios will be analysed below.

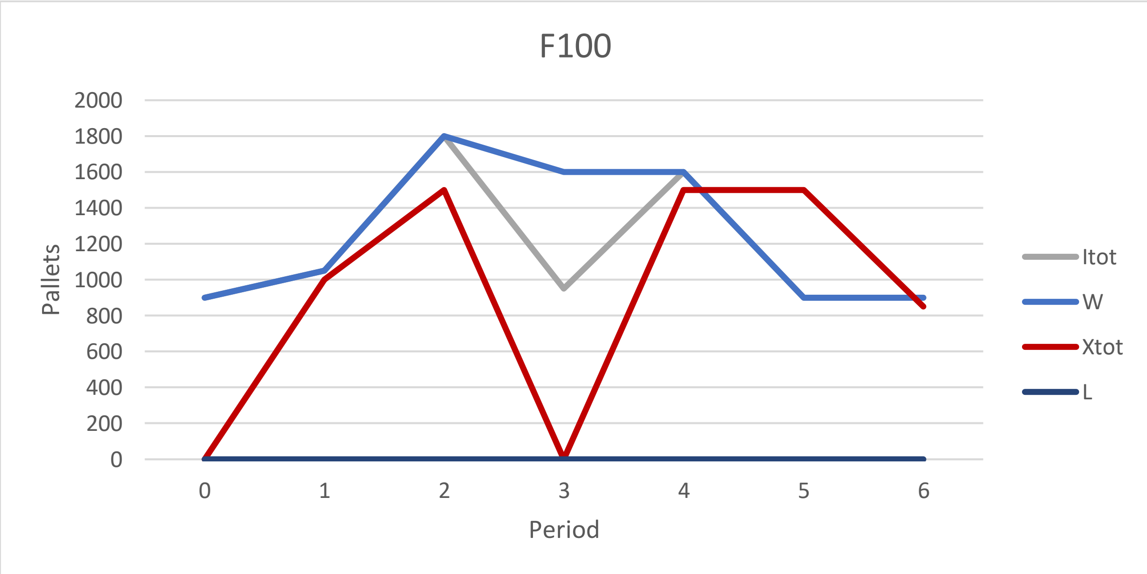

FIGURE 14 MODEL VERIFICATION F =100: *ITOT REPRESENTS INVENTORY, W IS WAREHOUSE CAPACITY, XTOT IS THE QUANTITY ORDERED AND L THE INBOUND OVERTIME CAPACITY USED*

In this second scenario the overtime inbound costs have been increased from 10 euro per pallet to 100 euro per pallet, the model outcome is displayed in Figure 14. Increasing the cost of overtime inbound handling makes this option unattractive for the model, hence the decision variable *L* is not used in any period and thus takes the value 0 in every period. Due to this new cost consideration, the model now has a stricter maximum inbound capacity of 1500 pallets every period, which it will try not to exceed in any period. As a result, less orders can be combined in this scenario due to the inbound capacity constraint. In this scenario now all periods except period 3 are ordering periods. Orders have to be spread out over different periods in order to build inventory for periods in which demand is expected to increase e.g., period 5. This capacity constraint is also the reason why the order sizes are different from the previous scenario. Resulting, the warehousing strategy is also adjusted to cope with the increasing inventory levels as a result of the new ordering strategy in this scenario.

In the third model verification scenario, the base scenario is considered in which the fixed cost of changing warehouse size is increased from 200 to 20.000. Due to the increased cost of changing warehouse size, the model aims to reduce the number of warehouse size adjustments compared to the base scenario if this is will reduce the overall cost. The results of the new scenario can be found in Figure 15. Due to the higher penalty cost of changing warehouse sizes, the model now aims to fulfil demand in every month without adjusting the warehouse size *W*. As a result, the ordering strategy is adjusted to ensure that no additional warehouse space is required in a period, hence stock building will be not be done in this new scenario as this would require additional warehouse capacity. For this reason, the aggregated inventory level is equal to the warehouse size in every period at 900 pallets and orders are placed in every period.

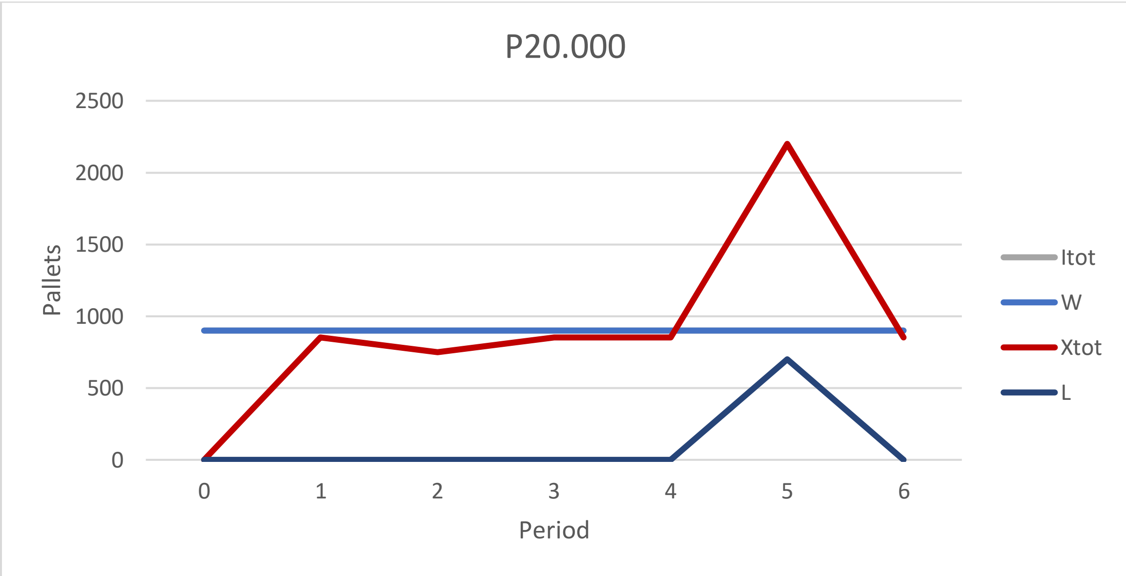

FIGURE 15 MODEL VERIFICATION P =20.000: *ITOT REPRESENTS INVENTORY, W IS WAREHOUSE CAPACITY, XTOT IS THE QUANTITY ORDERED AND L THE INBOUND OVERTIME CAPACITY USED*

Based on the above discussed model outcomes from different hypothetical scenarios it can be concluded that the model is verified and that the model is built correctly. The model minimizes the ordering and warehousing costs for the given set of input parameters within the given constraints. This is in line with the formal problem description and the conceptual model from section 5.1. If a parameter is increased, the model will adjust the ordering and warehousing strategy accordingly to minimize the total cost. The next step is to verify the scheduling model and the combined model before the model's real-life representations will be tested in a case study of a food manufacturing company.

6.2. Scheduling model

Input parameters

In order to verify the scheduling model similar numerical tests will be conducted to analyse the model's behaviour in different scenarios. First the model outcome of a base scenario will be analysed, similar to the lot size and warehouse model. Thereafter, the set-up times and production rates of machines will be adjusted and the new model outcomes will be analysed. Combining knowledge from the three test scenarios will provide the necessary proof of the model's behaviour in order to start verifying the combined model.

The input parameters for the base scenario are displayed in Tables 3, 4 and 5. Other relevant input parameters of the base scenario are that the set-up costs per day are considered to be €18.000, - and the production cost per time unit are €12.000, -. All productions should start and be completed between day 1 and day 30 of every period T.

TABLE 3 ORDER QUANTITIES BASE SCENARIO

Product	X1	X2	X3	X4	X5	X6
1	100	0	100	300	100	100
2	400	0	400	200	400	400
3	350	0	350	350	350	350

TABLE 4 PRODUCTION RATES BASE SCENARIO

Production rate	Machine 1	Machine 2
1	25	20
2	40	25
3	50	70

TABLE 5 SET UP TIMES BASE SCENARIO

Set up time	1	2	3
1	0	3	6
2	2	0	3
3	3	4	0

Model outcome

The main aim of the scheduling model is to produce the order before the due date against the minimal costs within the given constraints. For the verification test we consider six periods, in which three products can be produced on two available production lines. The model outcomes are displayed in Figures 16 and 17. The model treats every period as an individual period, hence there is no interrelation between periods. For this reason, if the order quantity, the production rates and the set-up times are the same for two or more periods, the model will also assign the same optimal production schedule to these periods. This is the case for periods 1,3,5 and 6 in the base scenario. In these periods, product 2 is scheduled first on machine 1 and product 3 is scheduled first on machine 2. Once the production run of product 2 is completed on day 11, the production run of product 1 on machine 1 will be set up and started at day 13. At time 17 all three production runs will be completed. Product 1 is scheduled on machine 1 and not on machine 2 due to the shorter production time it has on machine 1 compared to machine 2. The fact that machine 2 is earlier available than machine 1 does not play a role as sufficient time is still available to complete a production run on machine 1 after the production of product 2. Considering the input parameters, this production schedule is most cost efficient. Set-up costs are only incurred once, between the production of product 2 and product 1, for the minimal possible duration of 2.

In the situation that demand is different in a period as in this model is the case in periods 2 and 4, the production schedule will be different as well. In Table 3 it is visible that no order is placed for period 2, hence the production schedule does not assign a product to a machine in this period. For the same reason period 2 not displayed. In period 4, the order quantities are different compared to the other periods. As a result, the production schedule also differs from the other periods. Product 1 is assigned to machine 1 and product 2 is assigned to machine 2. Once the production of product 2 on machine 2 has finished, the machine will be set up for the production run of product 3 which takes 3 days. After this product 3 can be produced as well. The different demand scenario in period 4 shows the trade-off between minimizing production time and set up time. The increased quantity of product 1 and the decreased order quantity of product 2, resulted in the assignment of different machines to the products. Due to the increased order quantity for product 1 and the assignment on machine 1, the production run now has a duration of 12 instead of 4. Nevertheless, combining the productions of product 2 and product 3 on machine 2 is completed at time 17. Considering the given order quantities in this period it was optimal to assign products to different machines in order to minimize the overall costs, even if this new schedule accounts for an extra day of set up time between productions of product 2 and 3.

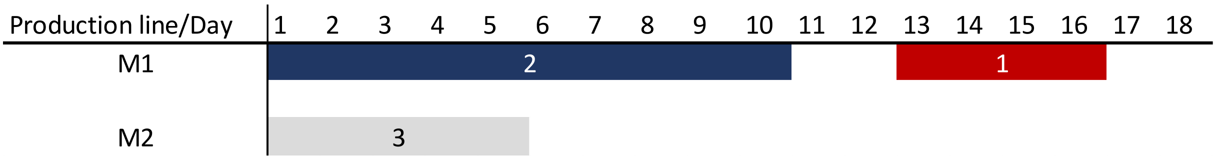

FIGURE 16 PRODUCTION SCHEDULE PERIODS 1, 3, 5 AND 6

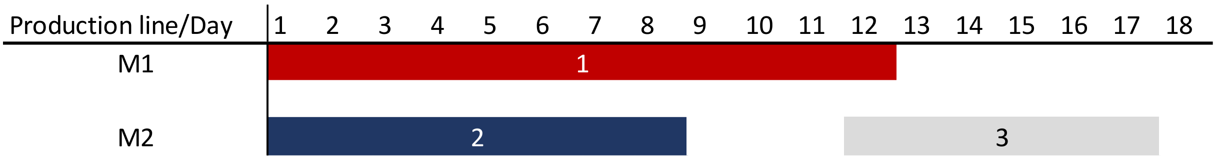

FIGURE 17 PRODUCTION SCHEDULE PERIOD 4

As every period is treated individually by the scheduling model, for the new scenario generation we will only consider period 1 and change the set-up times between the products in order to generate a new scenario. Looking at more than a single period to analyse the model's behaviour is redundant as was seen in the base scenario. The new set-up times considered can be found in Table 6, the order quantities of period 1 from the base scenario will be used.

TABLE 6 ADJUSTED SET-UP TIMES

Set up time adjusted	1	2	3
1	0	6	3
2	4	0	3
3	5	4	0

With the adjusted set-up time the model finds a different optimal production schedule which can be found below in Figure 18. The model now schedules product 1 first on machine 2 and product 2 first on machine 1. The reason for this is that the products which are scheduled first, require no set-up times prior to the production. Since the production of product 3 requires the least amount of set-up time for all options, the model will schedule this production after either the production of product 1 or 2 depending on which production run is competed first. In this way the set-up times between products can determine the sequence in the production schedules, this is however not the only factor determining the production sequence as was visible in period 4 in the base scenario, where the production schedule was different for different order quantities when the set-up times were constant.

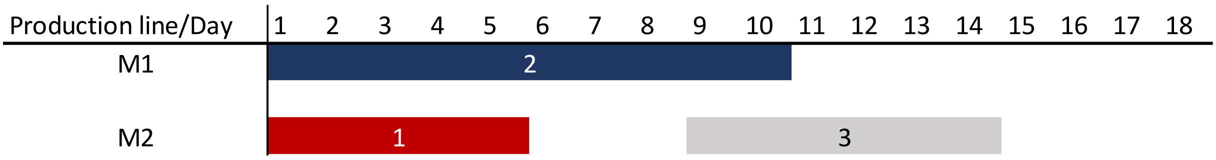

FIGURE 18 PRODUCTION SCHEDULE ADJUSTED SET-UP TIME

A final verification scenario will be generated in which the production rates of the machines will be adjusted. Similar to the set-up time analyses, only period 1 of the base scenario will be considered with the adjusted production rates of the machines which can be found in Table 7.

TABLE 7 ADJUSTED PRODUCTION RATES

Production rate	Machine 1	Machine 2
1	25	50
2	40	50
3	50	70

In the generated scenario, machine 2 has a higher production rate for every product. Hence, it would be fastest to produce every product on machine 2 if the set-up times would not be accounted for. In the new production schedule one can see that the production times have decreased due to the increased production rates. Similar to the base scenario, product 1 is scheduled right after product 2 as this sequence has the lowest set-up time. Nevertheless, what changed compared to the base scenario is that the production runs of product 1 and 2 have now been scheduled on machine 2 instead of on machine 1. The scheduling model has adjusted the production schedule, to ensure that the machine with the highest production rate produces two products and the machine with a lower production rate will produce only a single product. A reason for splitting the productions over two machines instead of scheduling all production runs on machine 2 with the higher production rate, is the model's consideration to the total production cost. If the model would schedule product 3 on machine 2 as well, additional set-up time would be required which would drive up the cost. These considerations resulted in the production schedule which displays the effects of changing the production rates of a machine which can be found below in Figure 19.

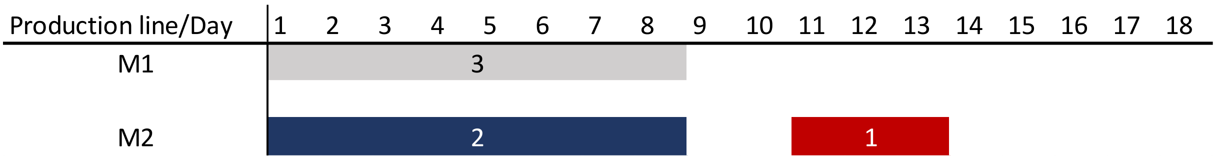

FIGURE 19 PRODUCTION SCHEDULE ADJUSTED PRODUCTION RATES

Based on the model outcomes of the three studied scenarios it can be assumed that the model's optimization behaviour is correct following the formulated problem description, mathematical and conceptual models. The model minimizes the production and set-up cost the given set of input parameters within the given constraints and will adjust the production schedules accordingly when input parameters change. For this reason, it is concluded that the model is built right, hence the model is verified.

6.3. Combined model

Where the previous sections verified the individual model's behaviour, this section will focus on the simultaneous optimization of the two models and how the constraints of both models affect the combined model's outcome. More trivial aspects of the individual models such as the effect of the production rate and warehouse costs on the model outcome will not be discussed in this section, since this was already discussed in the previous sections.

Input parameters

In order to verify the combined model's interactions and behaviour, the base scenarios of the individual models will be combined. By using similar input parameters as in the base scenarios, the model outcome of the combined model can be compared with the model outcomes of the individual models and is this way the combined model can be cross verified. There is one difference with respect to the used input parameters which is that the orders from the scheduling model will not be used as an input parameter in the combined model. The reason for this is that the order size is a decision variable in the combined model which will be optimized so that both the warehousing as well as the production costs will be minimized.

Model outcome

The combined model outcome is divided into the warehouse results and the production scheduling results, this helps to compare the results with the individual models. Nevertheless, the model is solved at once and simultaneously optimizes the warehouse and the production scheduling model. If one compares Figure 20 with Figure 13 it emerges that the ordering and warehousing strategies have changed now that the production constraints are accounted for as well. Instead of four ordering periods, the combined model finds that in the new situation one must order in five of the six periods. This can be accounted to the new production capacity constraints, which may limit the number of orders that can be combined in a single period. As a result of the more equally distributed ordering strategy, the inventory levels and the warehousing strategy also must be adjusted. The simultaneous model decides to make a total of five warehouse size adjustments in order to facilitate the changing inventory levels due to the new order behaviour. Another effect of the more equally distributed ordering strategy is that less inbound overtime capacity is required to inbound the ordered goods into the warehouse which can be seen in Figure 20.

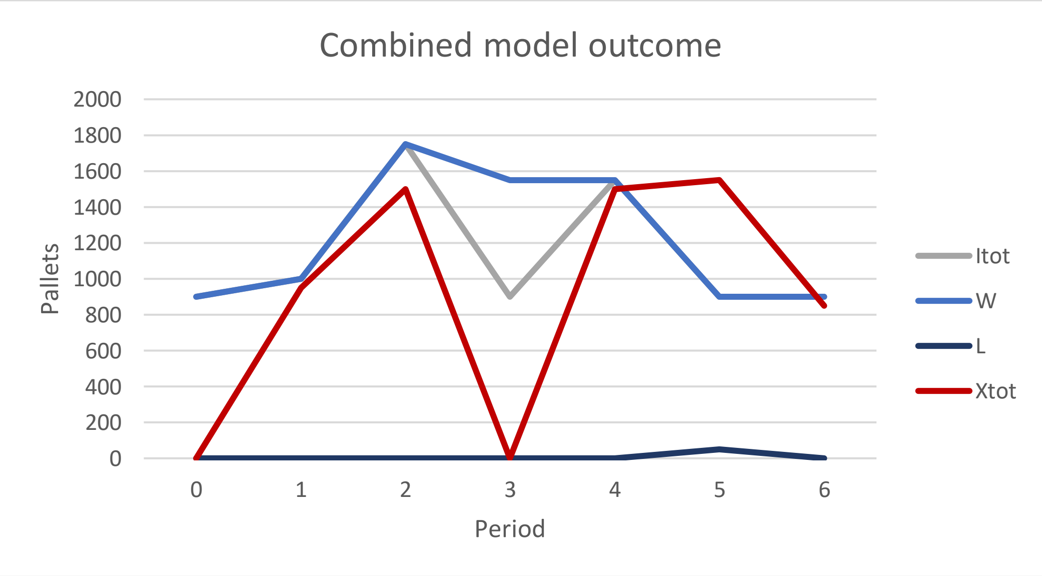

FIGURE 20 VERIFICATION OF WAREHOUSE OUTCOME COMBINED MODEL: *ITOT REPRESENTS INVENTORY, W IS WAREHOUSE CAPACITY, XTOT IS THE QUANTITY ORDERED AND L THE INBOUND OVERTIME CAPACITY USED*

The resulting ordering strategy of the combined model also effects the outcome of the production scheduling strategy which can be found in Figure 21. The first difference compared to the individual model is that no period has the same order size. Consequently, the production schedules are also different in each period. Depending on the quantity of a product ordered in a period the sequence and machine assignment can be different as was seen in the verification of the individual production scheduling model. Despite the fact that the production schedules are different in every period, all production schedules ensure that the complete order quantities are produced within the set time constraints for the minimal possible cost. Period three is not included in Figure 21 since no order is placed in this period.

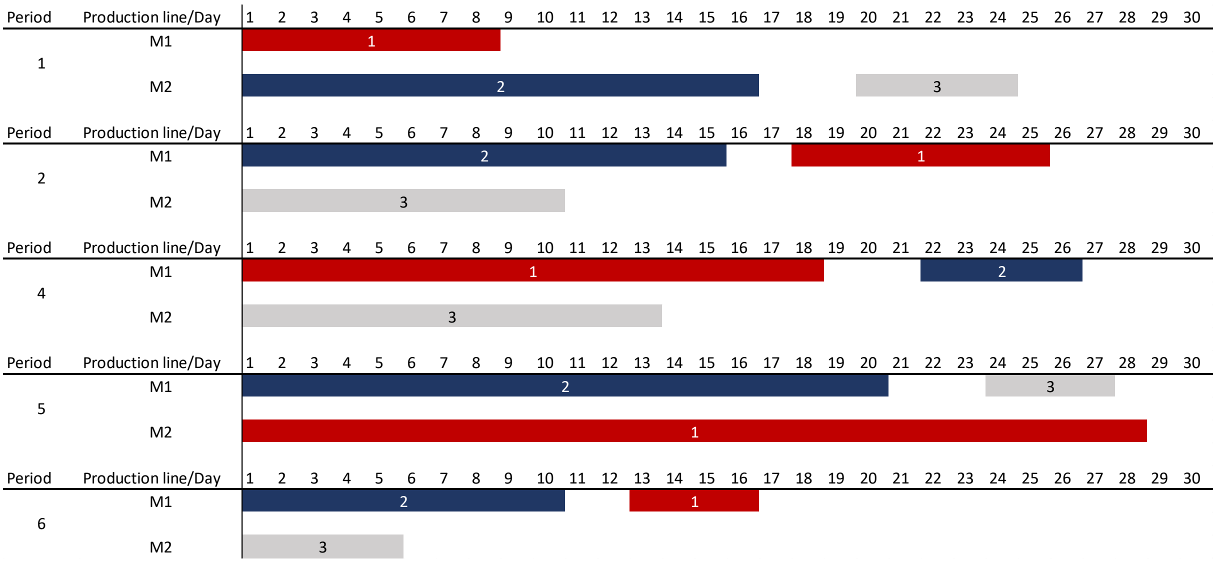

FIGURE 21 VERIFICATION OF PRODUCTION SCHEDULE OUTCOME COMBINED MODEL

Comparing the results of the combined model with the results of the individual models shows the effects of simultaneous optimization versus single or sequential optimization. The effects of simultaneously considering all constraints on the decision variables and model outcome make sense given the set of input parameters used. Combining the insights obtained through the verification of the three models under study, it can now be assumed that all models have been built correctly and their behaviour is verified.

6.4. Strategy Behaviour Analysis

The individual optimization models and the combined model have been verified for a single demand scenario. Testing the models in different demand scenarios is the next step in the verification process as this can prove if the models also behave correctly when the optimization environment changes. When the models and ordering strategies have been verified in different scenarios, the simultaneous ordering strategy will be tested on its complexity. Once both the individual models as well as the according ordering strategies have been verified, they can be tested and validated in the case study.

6.4.1. Demand scenario verification

The three ordering strategies introduced in chapter 4.1 will be tested and compared in three simulated demand scenarios. A base demand scenario which was used in the verification process of the individual models. A demand scenario in which the total demand is spread equally over the three products under study and a third demand scenario in which there is one dominant product which accounts for a significant share of the total demand. The three used demand scenarios can be found in appendix A. The ordering strategies will be compared on total cost and the cost split between warehousing and production related cost. In this way the ordering strategy behaviour in different demand scenarios can be compared and verified. A more detailed analysis of the model outcomes is not relevant as the models have been verified in the previous sections of this chapter and will be more closely studied in the case study chapter.

In Figure 22 the cost comparison of the three ordering strategies in the three demand scenarios is displayed. The input parameters from the previous sections are used as input for the three strategies. If the three ordering strategies are to be compared for the three different demand scenarios it can be seen that the simultaneous ordering strategy is the most cost-effective ordering strategy in all scenarios. The simultaneous ordering strategy is followed by sequential production strategy production which has the second lowest cost in two out of the three analysed scenarios and is infeasible in the third scenario. The sequential warehouse strategy has the highest total cost in all three demand scenarios, if the infeasible solution of the sequential production strategy in the dominant demand scenario is not accounted for. The reason for the different total cost per ordering strategy is accounted for by the different sequence of model optimization between sequential strategies. The model which is optimized first in the sequence will have lower cost than the model that is optimized subsequently. The total cost of the simultaneous strategy is lower than the cost of the two sequential strategies since the simultaneous strategy accounts for the production and warehouse cost parameters at the same time. The model will optimize the trade-off between for example increasing the warehouse capacity to be able to better schedule productions and in this way minimize the total cost. The individual models in the sequential strategies do not have this trade-off and as a result have higher total cost, even though the individual warehouse or production cost for a sequential strategy may be similar to that of the simultaneous strategy in the same scenario.

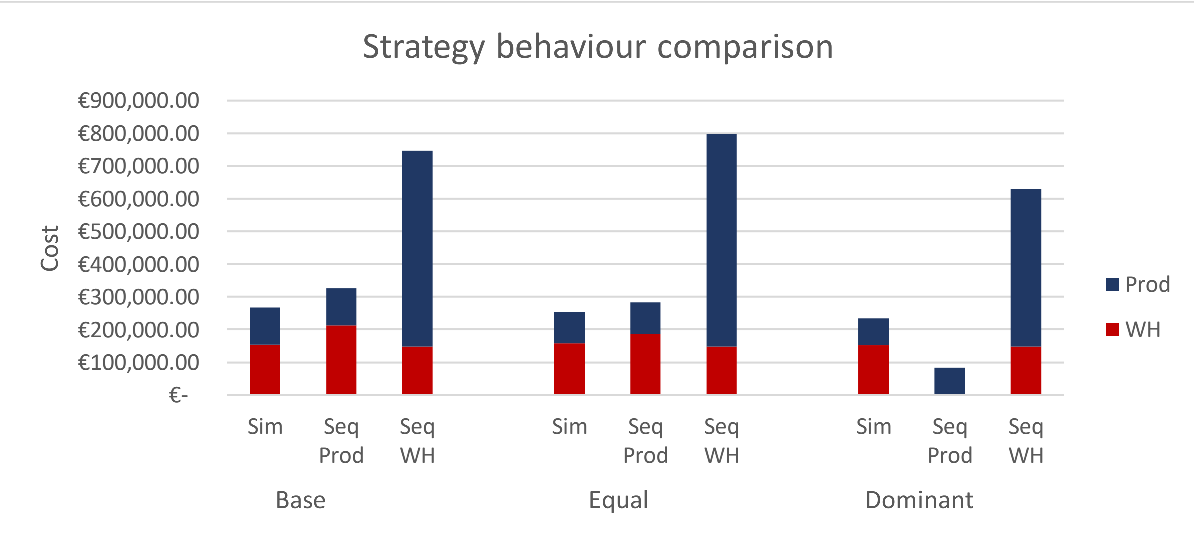

FIGURE 22 STRATEGY BEHAVIOUR COMPARISON VERIFICATION

The demand scenario in place also has an effect on the total cost per ordering strategy. From Figure 22 it emerges that an equal demand distribution is more beneficial for the production cost in the simultaneous and sequential production strategy, as the production cost with equal demand distribution is €96.857, - compared to €114.000, - in the base scenario. This does not hold per definition for the warehousing cost, which are €3.235, - higher for the simultaneous strategy, €25.808, - lower for the sequential production strategy and the same for sequential warehouse strategy if one compares the equal demand scenario results with the base scenario.

The dominant demand scenario has the lowest total cost for all three ordering strategies. The reason for this lower cost is the reduction in production cost compared to the base and equal demand scenario. The production scheduling model is able to better prioritize long production runs of the dominant product on production lines with the highest production rate in a dominant demand scenario. As a result, the total production time in the dominant scenario will be lower than the total production time of the base and equal demand scenarios, hence the production cost will be lower. In the dominant demand scenario, the sequential production strategy proved to be infeasible in the second optimization stage. This was the result of the order placement of the production scheduling model which did not fit within the warehouse constraints in place. This resulted in no result for the warehouse cost in the sequential production strategy. The effects of the order placement on warehousing results will be discussed in more detail below.

That the timing and quantity of the orders are placed has a clear effect on the subsequent model outcome as became clear from Figure 22. This effect can be seen as the production cost are similar for the simultaneous and the sequential production strategy in all three demand scenarios. Nevertheless, the warehousing costs are different in all three scenarios. The reason for this difference is the order placement of the different strategies. The primary focus in the sequential production strategy is to minimize the production and set-up cost and comply with demand. In order to accomplish this the model will aim to aggregate productions as much as possible in the first periods, after which the model can minimize the scheduling cost by producing at most the number of products as the number of production lines available. In this way no set-up costs are incurred and the production costs can be minimized. This ordering behaviour is represented by the red line in Figure 23 and is characterized by the fact that most orders are placed in the first half of the periods under study and no orders placed in the last two periods. The simultaneous model behaves differently under similar input parameters as the model not only accounts for production cost and constraints, but also considers the warehousing costs and constraints. Resulting, the model aims to better balance the orders over the different periods in order to manage the in and outflow of products in the warehouse and the according inventory level. The result of this way of placing orders is that the inventory level and according warehouse capacity are minimized which minimizes that total warehousing cost, hence the difference in warehouse cost between the simultaneous and sequential production strategies. Another possible effect of the order placement of the sequential production strategy is the risk of placing too high orders in the first periods. This may result in warehouse capacity issues and thus into an infeasible result, as can be seen in the dominant demand scenario in Figure 22.

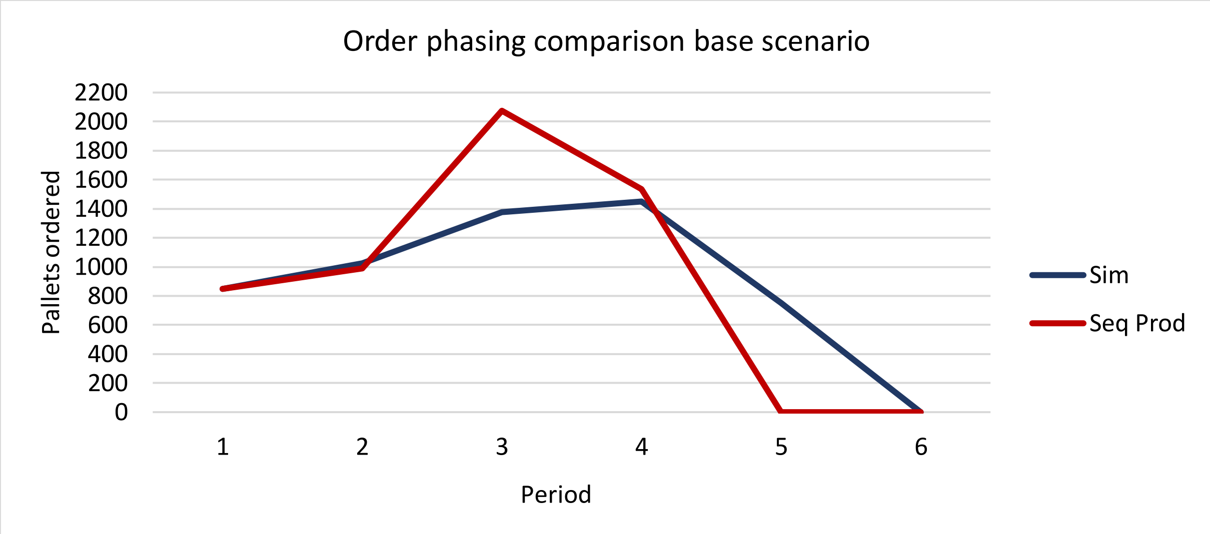

FIGURE 23 STRATEGY ORDER DISTRIBUTION COMPARISON

Based on the outcome of the strategy comparison for different demand scenarios it can be concluded that the individual optimization models and ordering strategies are verified for different scenarios. The model outcomes are as expected based on the used input parameters and the effects of different demand scenarios on the model outcome are clear. The infeasibility of the warehousing model in the dominant demand scenario for the sequential production strategy can be accounted to the effect of distributing the productions in favour of the production scheduling model which did not comply with the warehouse capacity constraints as was explained in the previous section. This does make the sequential production order strategy less suitable for the optimization of ordering strategies with one dominant product.

6.4.2. Complexity experiment

The increased complexity of considering a multi-product range in integrated models was already touched upon shortly in the literature research. The reason for this increased complexity of these models is the increasing number of variables that needs to be considered by the model when multiple products are considered or when two problems are solved simultaneously. Each additional product, period or machine introduces a new set of variables that will need be considered and optimized. In other words, increasing the number of considered products will significantly increase the number of parameters and variables considered by the model, hence the running time as well as the problem-solving chances of the model will be affected due to this. In order to test the complexity of the integrated model, an experiment will be conducted to analyse how the integrated model reacts to increasing the number of variables considered. By mapping the effects on the model running time and optimality gap the increased complexity can be analysed.

For this experiment, the integrated model is tested in a hypothetical environment in which the number of products, periods and machines considered varies. The quantity of the products, periods and machines considered in each tested scenario can be found in the first three columns of Table 8. The running time in seconds and the optimality gap percentage are depicted in the last two columns of the table. The optimality gap percentage indicates the gap between the best feasible solution that was found by the solver and a value which bounds the best possible solution of the MILP problem. A high optimality gap thus indicates that there is a possibility that better solutions can be found by the solver. Further elaboration on the effects of the optimality gap on the reliability of the model outcome will be discussed in the following paragraph. The integrated model complexity experiment was performed in IBM ILOG CPLEX Optimization Studio on a personal computer with an Intel® Core™

i5 processor with 2.4 gigahertz and 8 gigabytes RAM. A solving limit of 250.000 ticks which is a deterministic time unit was set to limit the maximum running time of the integrated model. The results of the experiment can be found in Table 8.

TABLE 8 COMPLEXITY EXPERIMENT INTEGRATED MODEL

Products	Periods	Machines	Time (s)	Gap (%)
3	6	2	1,20	1,06
	12	2	0,78	1,44
	18	2	3,64	0,10
	18	3	3,36	0,19
	18	4	0,61	2,48
	18	5	0,89	0,23
	18	6	0,89	0,04
6	6	2	1,20	1,06
	12	2	238,61	0,16
	18	2	353,93	4,92
	18	3	385,09	0,42
	18	4	145,52	0,03
	18	5	24,00	0,02
	18	6	4,20	0,08
	18	12	15,23	0,00
12	6	2	343,19	39,13
	12	2	504,30	54,81
	18	2	585,02.	70,05
	18	3	342,61	46,83
	18	4	360,33	35,12
	18	5	475,33	18,59
	18	6	404,17	10,46
	18	12	25,97	0,73

From Table 8 it emerges that the running time of the model is affected by the number of products considered. This conclusion was also drawn by Hu et al., 2008 and Gao et al., 2020 in literature. The reason for the increased running time are the new variables that need to be optimized as well. The solver requires more time to find an optimal solution for this larger set of variables. In this way, increasing the number of considered products increases the model running time. Besides the model running time, the introduction of new products also increases the model's complexity. This is indicated by the increased optimality gap when the model considers six and twelve products instead of the original three products. Another observation that can be made from Table 8 is that increasing the number of periods considered affects the running time of the model. This can be accounted to the additional computations the model has to perform to incorporate the additional periods and the related variables in the solution. Different from the number of products and periods, introducing additional machines to the problem has a positive effect on both the model run time as well as the optimality gap in most cases. This is because the same number of products now can be produced on more machines, this increases the problem-solving chances of the model and reduces the model running time. For the situation of 12 products and two machines for a period of 12 and 18 months, the model emerged to be insolvable because of too little machine availability to ensure that all products are produced before the due date.

In order to further test the hypotheses from literature that increasing the number of products in a MILP increases the model's complexity a second experiment is conducted with the integrated model. In a scenario of considering six periods and twelve machines the number of products is increased with three in every step. The remaining optimality gap of the solver after a maximum of 250.000 ticks is mapped in Figure 24, the limit was set to bound running time of the model. The higher the optimality gap, the further the model is from the most optimal solution within the 250.000 ticks. In other words, the more difficult it is to find the optimal solution, due to the increased complexity of the model. From the figure it emerges that when the number of products considered is below the number of available machines, the problem is not complex and can be solved as the optimality gap is close to zero. However, when the number of products considered exceeds the number of available machines the optimality gap will increase. This is a result of the new production scheduling decisions which are introduced in the production scheduling model related to scheduling more products than machines available. The more products are considered by the model, the more of these decisions need to be made, hence the more time will be required to find the optimal solution. This trend is confirmed in Figure 24 and is in line with findings in literature from Hu et al., 2008 and Gao et al., 2020 who found that increasing the number of variables increases the model complexity and solving time of supply chain optimization models.

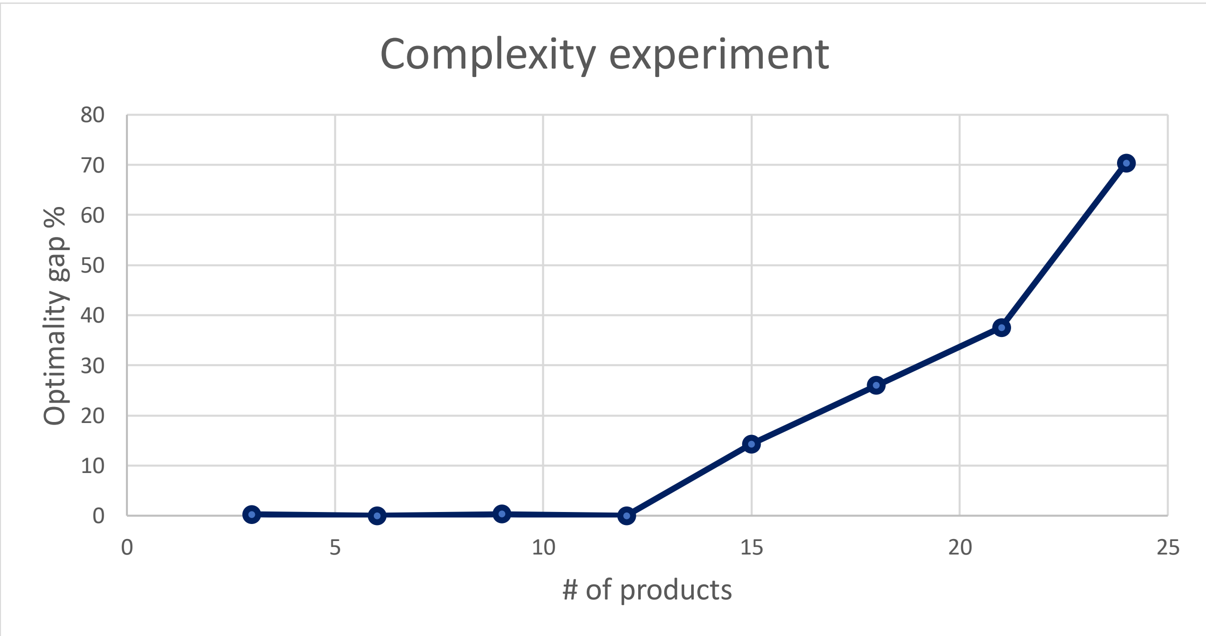

FIGURE 24 OPTIMALITY GAP EXPERIMENT *IN A SCENARIO WITH 12 MACHINES AND 6 PERIODS*

In conclusion, the number of periods considered by the model does not have a significant effect on the model's complexity. Nevertheless, it does affect the running time of the model. The number of machines considered decreases the model's complexity and running time. The number of products considered by the model increases both the model's complexity and running time which is in line with findings from literature. The fact that the performed tests with resulted in optimality gaps up to 70% indicates that the algorithm has not yet covered all the available solution space, hence giving the algorithm more time could result in better solutions with lower optimality gaps. However, for this complexity experiment it was decided to set the limit on 250.000 ticks due to the limited time available.

7. Food manufacturing case study

The discussed models will be tested by means of a case study for Kraft Heinz in a real-life situation to analyse the models' behaviour in a practical situation. In this chapter the case study will be discussed. This will be done by introducing the food manufacturing company under study and through describing the case study background and the according selection process in section 7.1. In section 7.2 the used input parameters for the case study and the case study assumptions will then be discussed to be followed by a detailed analysis of the case study results in section 7.3. This chapter will be concluded with a cost parameter sensitivity analysis in section 7.4.

7.1. Food Manufacturing Case Study

Kraft Heinz is the fifth largest food and beverage company in the world, they manufacture and distribute condiments, sauces, meals, dairy products, soups, meats and beverages across the globe. Well-known brands in the Kraft Heinz portfolio are amongst others Heinz, Kraft, Honig, DeRuijter, Caprisun, Jell-O and Lea & Perrins. In order to supply this wide variety of products to customers across the globe, Kraft Heinz has set out an extensive supply chain world-wide. The focus of Kraft Heinz is to transport their goods as time and cost efficient as possible from the factories to the customers. The Kraft Heinz supply chain consists of internal and external factories, internal and external warehouses and multi-model transportation lanes. The terminology 'internal' or 'external' indicates if the warehouse or factory is owned by Kraft Heinz (internal) or if the factory is owned by a LSP (external). In the latter case Kraft Heinz has an agreement to use or lease the factory or warehouse. By combing internal and external warehouses and factories in their supply chain, Kraft Heinz managed to find the balance between reducing investment costs and staying agile throughout their supply chain.

In the underlying case study, the effects of using different ordering strategies on the costs and product flows between a factory and an external warehouse in the Kraft Heinz supply chain will be analysed. The three different ordering strategies will make use of the developed optimization models in different simulated scenarios. The results of the different tests will be compared and interpreted in order to identify which new ordering strategy can help Kraft Heinz coordinate production and warehousing decisions for product flows between factories and warehouses. To fit within the scope of the research, the real-life situation was simplified. It was decided to test the ordering strategies in a case study that focusses on a product flow between a single factory and warehouse for a 12-month period as this better fitted the designed models and would not overcomplicate the analysis. Insights obtained from the case study between the factory and warehouse are representative for other transportation lanes in the Kraft Heinz supply chain, hence these insights can be used throughout the entire supply chain to improve the ordering process, even though the case study was carried out on a single transportation lane.

The process of selecting a factory and warehouse most suitable for a case study was carried out in cooperation with Paul Strijers (Logistic Manager HUB), Thanos Papakonstantinou (Logistics excellence manager Central Europe) and Luuk Oudendorp (Logistic manager D&E), who are responsible for different warehouses in the European supply chain. Subsequently the rationale behind choosing the desired factory and warehouse will be described. Starting with the selected warehouse, the AGPark15 warehouse was chosen for three main reasons. The first reason why AGPark15 was selected for the case study was the location of the warehouse. AGPark15 has a central location between Kraft Heinz operations in the Netherlands and has a good accessibility due to the multi-model transportation options in the area as can be seen in Figure 25. The AG Park15 warehouse is located close to Elst where Kraft Heinz' largest European sauce factory is located. Moreover, the access to the A15 make that the warehouse is well connected for both east-west as well as north-south travel across the Netherlands and to Germany and Belgium via the A50 with the A12. Figure 26 depicts a close-up of the area surrounding Park15. Across the A15 a rail terminal is being constructed which will be connected to the Betuwe rail line which connects the Port of Rotterdam to Germany. On the south side of the warehouse park a barge terminal is located on the banks of the river Waal. Being located this close to a highway entry, a rail terminal and a barge terminal makes the AG Park15 a suitable multimodal DC in the Kraft Heinz supply chain and therefore an interesting warehouse to perform a case study upon. The second reason which makes this warehouse suitable for the case study is the fact that AGPark15 is an external warehouse in the Kraft Heinz supply chain. Kraft Heinz leases storage capacity at AGPark15 from a LSP and has the option to adjust this capacity to their needs. This makes this warehouse suitable for the multi-period warehouse size optimization model, as this model requires the warehouse to be able to adjust the warehouse capacity over time. The third reason which makes this warehouse most suitable for a case study is the fact that the warehouse was taken into operation on December 1st, 2021. Due to the newness of the warehouse, the chances of making useful recommendations are higher compared to doing the analysis for a warehouse which is already in operation for numerous years and thoroughly studied by different teams.

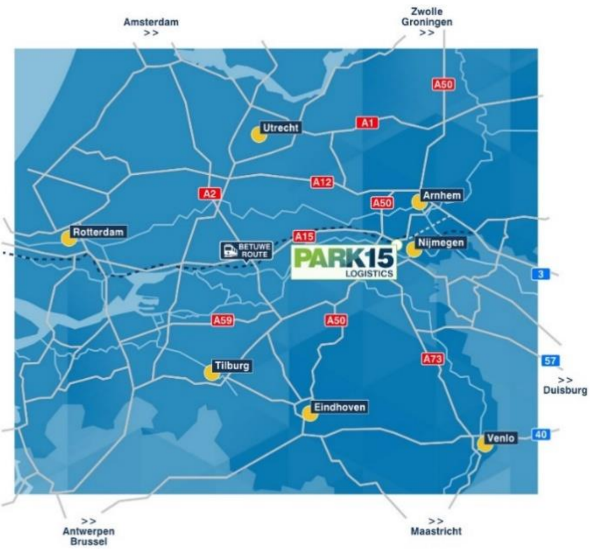

FIGURE 26 AG PARK15 LOCATION IN THE NETHERLANDS (PARK15, N.D.)

FIGURE 26. AG PARK15 AREA (PARK15, N.D.)

Continuing with the desired production facility, the Elst factory was chosen primarily because Elst is the largest sauce factory within Europe and the middle east in the Kraft Heinz portfolio. Due to its size and importance within the Kraft Heinz portfolio it will be most interesting to study and test the proposed models on product flows in which the Elst factory is the source. A second aspect which makes Elst a suitable factory to study is its location which is about 15 minutes driving distance from the AGPark15 warehouse. Due to the short distance between the factory and the external warehouse this link in the supply chain of Kraft Heinz plays a crucial role for the product flow from Elst sourced products to the rest of the world.

Finished products are stored at AG Park15 before they are moved further down the supply chain towards customers. The warehouse functions as a distribution centre and a critical changeover point in the product flow between factories and customers as was seen in chapter 4 in Figure 8. This key position within the supply chain makes it important that the optimal warehouse capacity is determined to ensure an efficient and cost-effective product flow through the warehouse and thus the entire supply chain. The AG Park15 warehouse is used as a distribution centre for three markets of Kraft Heinz being D&E, Nordics and affiliates. Nordics is another naming used within Kraft Heinz to describe the Scandinavian markets, D&E stands for distributers and exports which are mainly focused at countries who do not have their own business units in Europe and lastly affiliates is used for countries not in Europe. From the 700 different products sold in all three markets, roughly 200 products are produced at the Elst factory. In the Elst factory productions are scheduled by the supply planning team who ensure that the demand is translated into feasible and optimized production schedules based on production and material constraints. The Elst factory produces all the products according to the production plan and once a production run is completed the finished products need to be transported to the warehouse where the products will be unloaded and stored before being transported towards the customer. This chain of events describes the different stages of the product flows between the factory and the warehouse.

The described process is vulnerable to different last-minute problems which can negatively affect the product flow between the factory and the warehouse. Two real-life examples from Kraft Heinz are: Insufficient warehouse capacity available to store already produced products and insufficient unloading capacity at the warehouse to unload arriving trucks at the warehouse. These unforeseen issues in the supply chain are the result of not considering warehousing constraints when making production plans. These unforeseen issues can drive up logistic costs as these shipments will need to be redirected last minute to other warehouses against higher cost. These issues are the result of insufficient communication and misalignment between the planning and warehousing teams within Kraft Heinz. The case study is aimed at generating insights in the effects of enhancing synchronization between the planning and warehousing teams. These insights can thereafter by used by logistic managers to improve the product flow between factories and warehouse and reduce the number of logistic issues in the future.

7.2. Case study data & assumptions

The data used to perform the case study has been collected through interviews with warehouse and factory representatives. Information which could not be retrieved via interviews such as current inventory levels or expected future demand were collected through company data sources in order to ensure that the used parameters were up to date and realistic. The case study data is representative for the 2022 cost and demand forecast of Kraft Heinz. With the available data and the described case study setting the actual case study tests can be prepared. Besides using company data, a number of assumptions had to be made as well in order to prepare a business relevant case study which focused specifically on the product flow between the Elst factory and the AGPark15 warehouse. These case study assumptions have been verified by Paul Strijers to be representative for the company setting. It is important that the made assumptions do not significantly affect the case study's real-life representation, as having a case study which is not representative for the real-life scenario also minimizes the chances of making useful managerial insights in this thesis. This can be ensured through using a portion of the actual capacities and costs which is representative for the reduced case study. In this way, the insights obtained in the reduced case study are still relevant for the real-life scaled operations. The rationale behind the data assumptions will be discussed in the following paragraph.

The case study input parameter tables display the used input values for the case study. The warehouse handling, holding, overtime and rent cost parameters considered in the case study are similar to the real-life costs paid by Kraft Heinz for these services. The ordering cost parameter K is however corrected for by the share of products originating from Elst versus the total products stored in the warehouse. The same holds for the warehouse capacities considered in this case study. The rationale behind this is that the case study only accounts for the product flow between Elst and AG Park15 which is about 30% of the total products stored at the warehouse. For this reason, the first case study assumption is that the warehouse capacity and the fixed ordering cost should be reduced to 30% of its real-life value, since normally these capacities and costs are used for the entire product portfolio. Using the 30% share of these input parameters enhances the real-life representation of the case study. The second assumption for the warehousing model is that it is assumed that there is sufficient carrier capacity to transport the finished goods from the factory to the warehouse at all times. The rationale behind this assumption is that considering carrier capacity as well in the model might make the model too complex. In addition to the added complexity, the fact that the warehouse is only 15 minutes away from the factory makes the transportation factor less significant compared to the production and storage capacity issues. Therefore, it was decided to leave this out of the scope of this research.

Warehouse model assumptions:

- The ordering cost and warehouse capacities are considered to be 30% of their real-life value;
- It is assumed there is sufficient carrier capacity to transport the finished goods from the factory to the warehouse at all times.

Similar to the warehouse assumptions, the production scheduling model also requires a number of assumptions in order to ensure a better real-life representation of the model. After conducting interviews and comparing demand data from the Elst factory for the coming two years it became clear the overall demand from the business unites Nordics, D&E and Affiliates together accounts for 25% of the total demand from the Elst factory every month. For this reason, it was decided to only consider a total of three instead of the original 12 production lines in Elst factory for the scheduling problem. In the case study it will be assumed that these three lines will be dedicated to produce all the different products for these markets in a period, whilst in reality the products are produced on all the 12 production lines. Considering all original lines would mean that there are no production constraints at all due to the availability of all the lines and only 25% of the regular products that need to be produced. As a result, only the warehouse model would affect the model outcome, hence it would not be representative for the real-life situation. Another assumption made for the production scheduling model is that production runs are scheduled on a recipe level. An example of this would be that mayonnaise is scheduled prior to ketchup runs and after barbeque sauce. Even though, this is similar to the real-life scenario the proposed scheduling model does not account for bottle changes within the recipe schedules itself. The reason for this is that the focus is on the product flows between the factory and the warehouse and the details of the factory production constraints are relatively less interesting. Due to this assumption, the products are grouped by recipe and are not considered on stock keeping unit level. Resulting, six sauces recipes are accounted for in the case study and this grouping is also considered in the warehousing model for consistency. A third assumption for the production process is to assume that the production lines are available 24/7 without interruptions. The final assumption is that every month consists of 30 days. These last two assumptions were made to increase the production scheduling model's feasibility and to limit the model's complexity.

Production scheduling model assumptions:

- A total of three production lines will be considered for the case study;
- Products are grouped by recipe into a total 6 different recipes;
- It is assumed the factory is available to run 24/7 without interruptions every week ;
- It is assumed that every month consists of 30 days.

7.3. Case study results

In this case study the *Sequential Production Strategy, Sequential Warehouse Strategy and Simultaneous Ordering Strategy* will be tested on their effectiveness and effects on the overall outcome in a practical setting in order generate managerial insights. The names of the sequential strategies indicate which model is optimized first in the sequence. The three strategies will be compared on total cost, order behaviour and warehousing strategy for the 12 months studied. The analysis of the according results will lead to managerial insights regarding a superior ordering strategy which can help Kraft Heinz improve the product flows between factories and warehouses. The first subsection will however be dedicated to describing and analysing the infeasibility of the sequential production strategy, after which the remaining strategy comparisons will be brought forward.

7.3.1. Infeasibility of the Sequential Production Strategy

The sequential production strategy which was introduced in chapter 4 is the first strategy that is tested in the case study. The outcome of this optimization is an optimal production and ordering schedule for the coming 12 months, resulting in total production cost of €79.959, - for all considered periods. The second step was to insert the order quantities in the warehouse model to compute the warehousing cost of the sequential production strategy. The warehousing model could however not be solved for these order quantities within the set constraints. For this reason, no warehousing cost can be given for the sequential production strategy, as the model is infeasible for the given data. In the following paragraph a root cause analysis to the restricting constraints will be carried out to understand why and how the constraints were violated.

Root cause analysis

The first step to analyse why this strategy resulted to be infeasible was to check the aggregated scheduled order quantities X in every period and comparing these with the available warehouse inbound capacity constraint. Aggregating the products ordered in every period resulted in the blue line in Figure 27. The red line in the graph indicates the maximum warehouse inbound capacity every month which is 8.000 pallets. This inbound capacity already includes available overtime which can be scheduled whenever required.

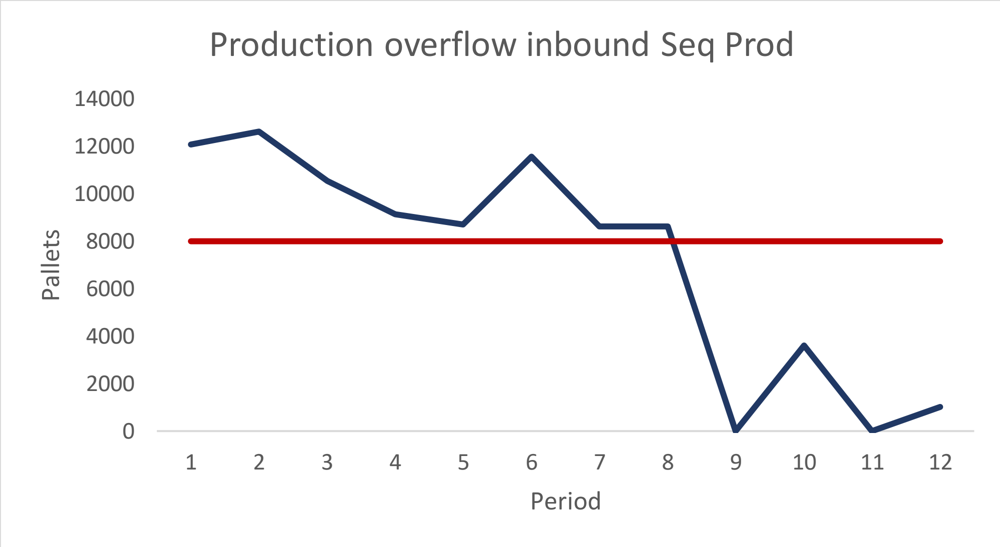

FIGURE 27 INBOUND OVERFLOW SEQUENTIAL PRODUCTION STRATEGY

From Figure 27 and the insights obtained in chapter 6 it emerges that the production scheduling model aims to optimize the production schedules for the coming 12 months by scheduling productions for all six recipes in the first month so that demand for the coming months is covered for most recipes. After the first month, the model schedules at most three recipes on three different machines in every month. By doing this the total set-up and cleaning costs are minimized, as these costs are not incurred for the first recipe on a machine. The side effect of this way of scheduling productions is that the 75% of the total output from the factory is produced in the first six months and the other 25% is produced in the last six months. As a consequence, the warehouse will not be able to process the output of the factory in the first eight months which makes the problem infeasible from a warehouse point of view.

The modelled high factory output not only conflicts with the warehouse inbound capacity, it is also violating the maximum warehouse storage capacity constraint which is set at 6.000 pallets. In order to map the resulting inventory levels from the optimized scheduling orders, both the maximum warehouse as well as the inbound capacity constraints were relaxed. This resulted in Figure 28 which displays the inventory overflow resulting from the sequential production strategy. From Figure 28 it emerges that the warehouse capacity constraint is violated in 11 of the 12 months. This is directly related to the fact that 75% of production runs is planned in the first six months. By scheduling production is this manner, inventory is built in the first six months which will be slowly degraded by demand. As a result, the inventory levels are higher than the available storage capacity in the warehouse.

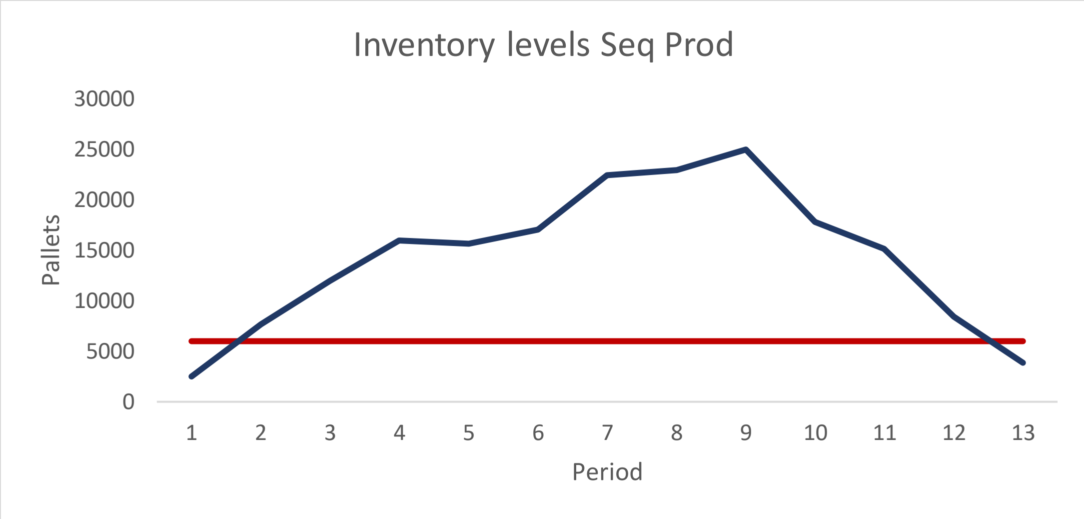

FIGURE 28 INVENTORY OVERFLOW SEQUENTIAL PRODUCTION STRATEGY

The reason for the violation of the inbound and warehouse capacity constraints is the unawareness of the production scheduling model of these constraints. As the production scheduling model solely focusses on minimizing the production scheduling cost within the production scheduling constraints whilst it does not account for warehouse constraints. The unawareness of the constraints of the warehouse model increases the chances of infeasible results. The fact that for the case study the warehouse model is infeasible demonstrates that coordination between the two models or the two different teams is key in finding feasible and cost-effective solutions for optimal product flows between factories and warehouses. Apart from the warehouse storage capacity and the available inbound capacity no other warehouse constraints were violated due to the ordering strategy from the production scheduling model. Running the relaxed warehousing model with the maximum warehouse capacity of 25.000 and a maximum inbound capacity of 13.000 pallets resulted in a total cost for the relaxed sequential production strategy which will be discussed in the following section.

7.3.2. Resulting comparison between the strategies

Cost comparison

The overall cost for producing and storing the six recipes for 12 months is displayed per strategy in Figure 29. From this bar chart it emerges that the simultaneous ordering strategy is more cost efficient than both the sequential strategies, as the total cost of the simultaneous strategy is € 850.490, - against a total cost of €1.615.819, - from the sequential warehouse strategy and a total cost of €1.709.294, - from the relaxed production strategy. For Figure 29 it should be noted that the relaxed model outcome of the sequential production ordering strategy was used for the cost comparison, since the regular model did not return any warehousing cost. Due to this relaxation more holding and warehouse renting cost need to be paid which drive up the warehouse cost. As a result, the warehouse cost of the relaxed sequential production ordering strategy is €1.629.335, - which is higher than the total cost of the two other strategies. With this relaxation in mind, it is surprising that the total cost of the relaxed sequential production ordering strategy is just €93.475, - higher than the total cost of the sequential warehouse ordering strategy.

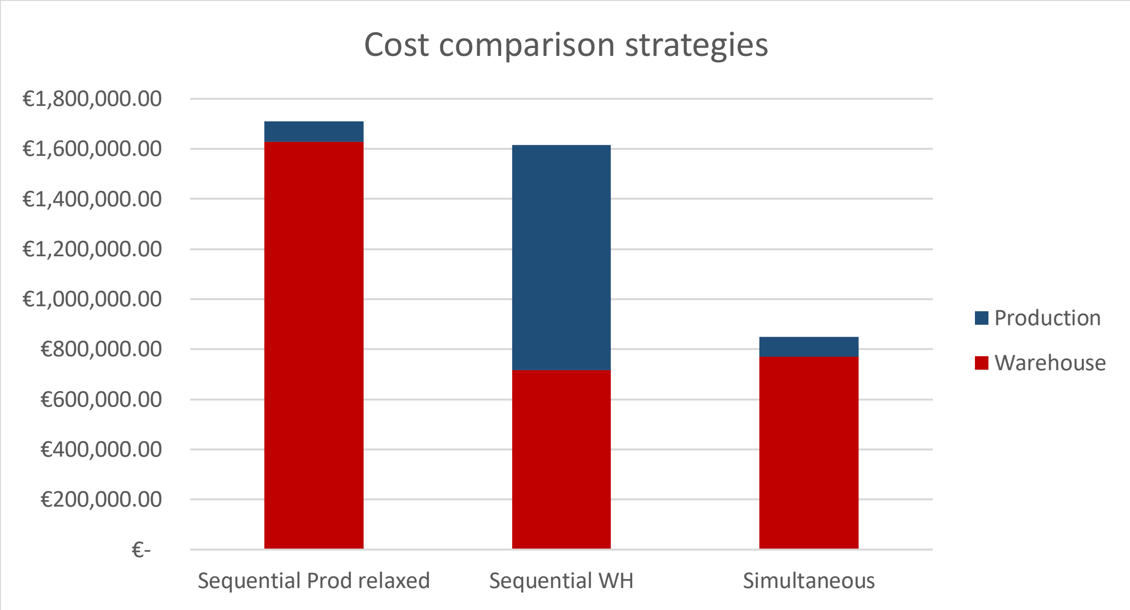

FIGURE 29 COST COMPARISON CASE STUDY

The cost split within each strategy also differs. For example, the total cost of the sequential production strategy consists of only 5% production cost and 95% of warehousing cost, whereas the cost split in the sequential warehouse ordering strategy is 44% of warehousing cost and 56% of production cost. The different split can be accounted to the primary focus of each ordering strategy on the model which is optimized first in the sequence, so the costs related to the first model are minimized and the cost related to the second model in the sequence will be optimized for not optimal input parameters which result in higher cost for the second model in place. The costs are split more equally in the sequential warehouse ordering strategy than in the sequential production ordering strategy for two main reasons. First, the warehouse model has more cost parameters with higher values than the production scheduling model. As a result, the warehousing cost account for a bigger cost share than the production scheduling cost. A second reason for the different split is the fact that the cost of the sequential production ordering strategy are the results from the relaxation with a higher warehouse storage capacity compared to the situation of B, hence the warehousing cost are higher in the relaxed sequential production ordering strategy. The cost split in the simultaneous model is balanced between strategies A and B. The production costs account for 9% of the total cost and the warehousing costs account for 91% of the total costs. This is the result of simultaneously optimizing the two models when accounting for all cost parameters and constraints at the same time. The model finds an optimal trade-off between increasing the cost of the production model and producing orders by evenly spreading them over the 12 months to reduce the warehousing and storing cost. The result of this is a feasible solution and a more balanced split of total cost compared to the sequential ordering strategies.

Comparing the total cost of the three models clarified that the sequence in the sequential ordering strategy has a significant effect on the cost split between warehousing and production cost. Moreover, simultaneously optimizing the models resulted in better balanced cost split which is also more cost efficient for the total cost of the ordering strategy. It should be noted that the costs of the different strategies could not be compared with actual company results, since the product flow between the factory and warehouse has only recently been taken into operation. As a result, no company results are available for comparison at this point in time.

Order placement comparison

In this section the three ordering strategies will be compared on the placed orders. The size and number of orders placed in every month has a significant effect on the overall outcome, as for the sequential models the orders generated by the first model will be used as input parameter in the second model. The second model needs to adjust to the orders placed by the first model which largely determines the total cost as was proven in the previous paragraph.

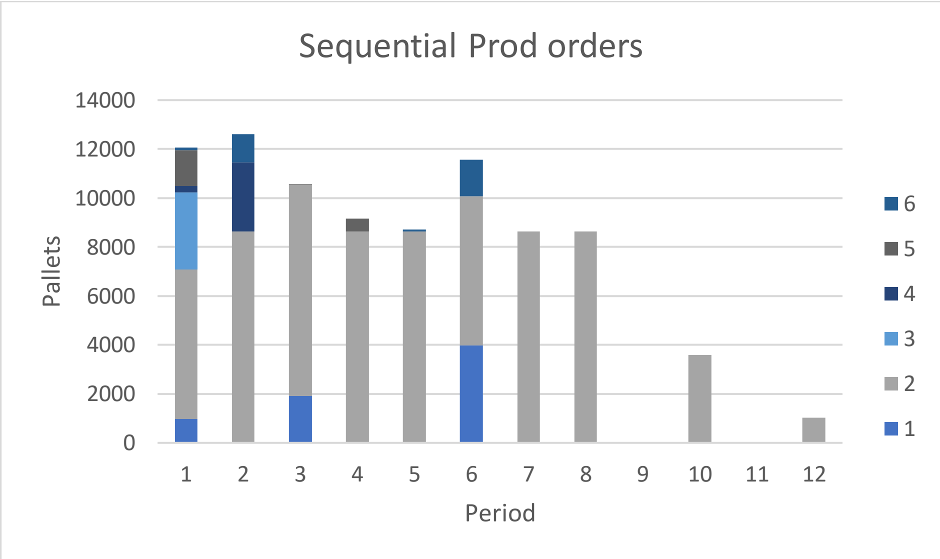

FIGURE 30 ORDERS PLACED BY SEQUENTIAL PRODUCTION STRATEGY CASE STUDY

In Figure 30 the optimized orders from the production scheduling model of the sequential production strategy are depicted. The model aims to minimize the total production and set-up time cost for every individual period based on the input parameters and the given constraints. As a result, the model aims to schedule at most three production runs in every period. The reason for this is that with three available production lines every recipe can be produced on one machine and no cleaning or set-up time cost are incurred. By doing so, the total production costs can be minimized. This does however not hold for the first period as in this period the model schedules all six recipes. The reason for this is that the model aims to produce sufficient quantities of every product to meet the demand in the first period. If possible, the model schedules the productions in such manner that inventory will be built for the coming months. An example of this is product 3 in Figure 30 which is only produced once in the first month, in this production run sufficient products are produced to cope with the forecasted demand for the coming 12 months. The scheduling models aims to increase the overall cost in the first month and produce all the considered products, after which in the consecutive months the model can minimize the production cost by scheduling at most three products in a month whilst meeting the requested demand. This optimization strategy affects the distribution of the orders placed by the model and the resulting production schedules as can be seen in Figure 30.

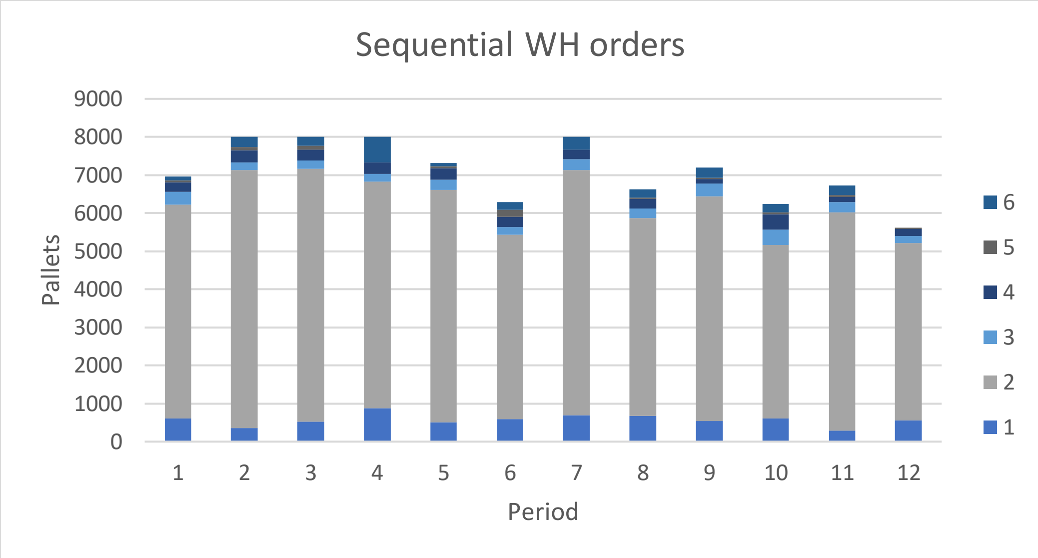

FIGURE 31 ORDERS PLACED BY SEQUENTIAL WAREHOUSE STRATEGY CASE STUDY

In Figure 31 the optimized orders from the warehousing model of the sequential warehouse strategy are displayed. From the bar chart it emerges that due to the different prioritization of the optimization model the order behaviour is different as well. The warehouse optimization model aims to minimize the warehousing costs by keeping the inventory levels as low as possible such that the holding costs as well as the number of warehouse capacity adjustments can be minimized. This different focus results in the model aiming to spread out the orders over all the available periods. Due to this order distribution the inventory levels and the required warehouse capacities can be minimized, whilst demand is met every month. On the other side, this order behaviour affects the production cost as more than three different recipes must be produced every month so the production scheduling model will incur set-up costs in the optimal production schedules. The effect of this order behaviour on the total cost is captured in Figure 29 in the previous section.

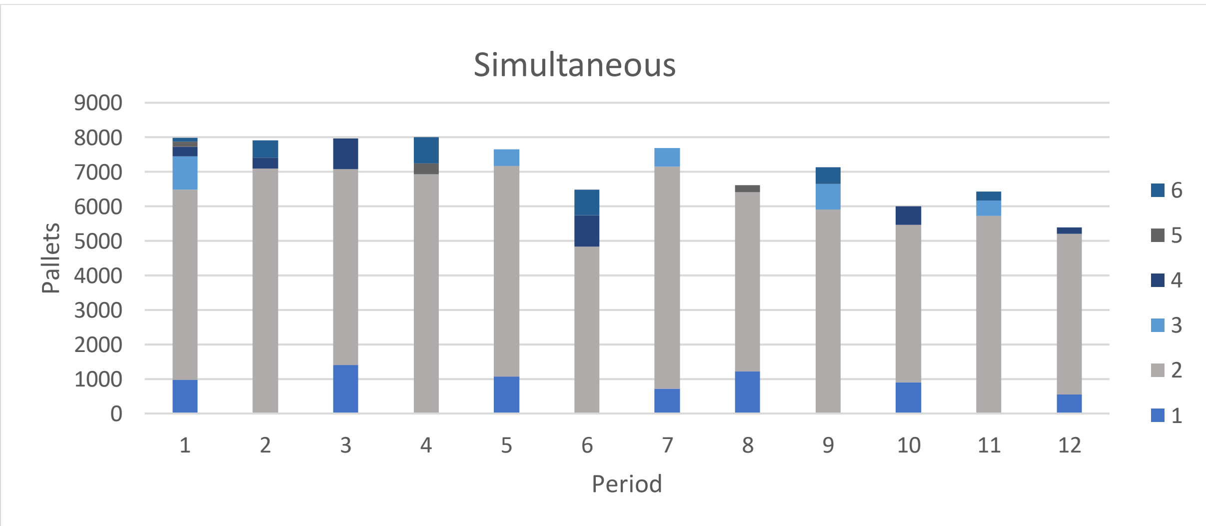

FIGURE 32 SIMULTANEOUS ORDERS PLACED CASE STUDY

In Figure 32 the order behaviour of the simultaneous ordering strategy is displayed. Similar to the cost split comparison, the simultaneous model balances the strategies of the sequential models. Because the model accounts for both the warehousing as well as the production costs the model orders all the six products in the first period incurring higher production cost in this month and requiring storage of the products for the future months. Nevertheless, the sum of the orders placed by the simultaneous model is just below the inbound capacity constraint of 8.000 pallets which was violated by the sequential production strategy in this first period. By ordering all six products in the first period the model enabled itself to optimize the production cost in the following periods as it is possible to schedule a total of three products in every period whilst meeting the requested demand. Placing the orders in this manner is beneficial for both the warehouse as well as the production cost, since no set-up cost are incurred for 11 out of the 12 months, whilst the orders are still spread out enough to keep warehouse cost low and stay within the warehouse capacity limits.

The difference in order behaviour between the three proposed strategies is visualized in Figure 33 in which the total number of production runs for the 12 months per strategy are displayed. Figure 33 confirms that for the production scheduling model it is most cost efficient to minimize the number of productions scheduled, whereas from a warehouse point of view it is most beneficial to spread the orders as much as possible over all the available periods which results in a total of 69 production runs compared to 23 production runs scheduled by the sequential production strategy. Optimizing the models simultaneously results in a more balanced production strategy which behaves like the production scheduling model, whilst accounting for all warehouse capacity constraints in place and thus spreads out the production runs more to minimize the warehousing cost and find a feasible ordering strategy.

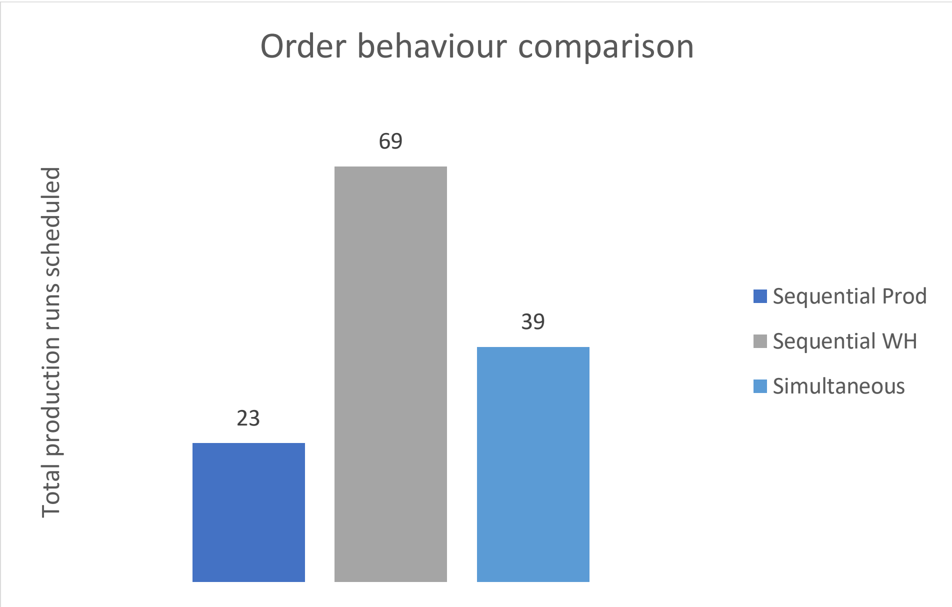

FIGURE 33 ORDER PLACEMENT COMPARISON CASE STUDY

Warehouse behaviour comparison

The final aspect on which the strategies will be compared is the warehouse behaviour resulting from the ordering strategy in place. The Sequential warehouse and simultaneous strategies will be compared on this aspect. The sequential production strategy will be left out of scope due to the model's infeasibility, including the relaxed outcome in the analysis is not helpful as the warehouse and inbound strategy are the violated constraints.

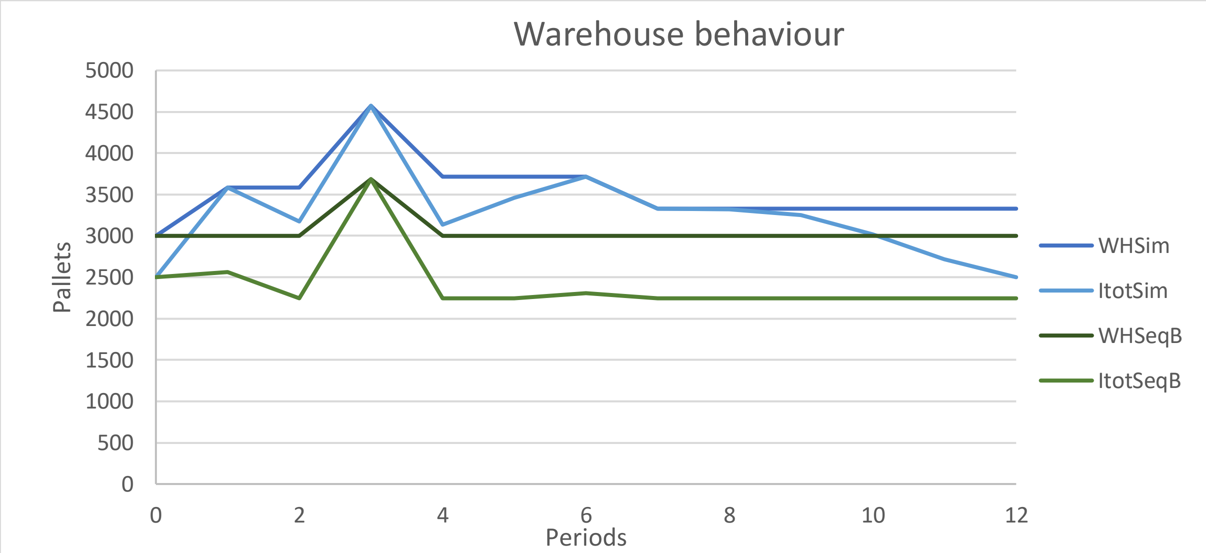

FIGURE 34 WAREHOUSING STRATEGIES CASE STUDY COMPARISON *ITOT REPRESENTS INVENTORY AND WH IS WAREHOUSE CAPACITY*

Figure 34 displays the warehouse capacities *WH* and the inventory levels *Itot* for the sequential warehouse strategy and the simultaneous strategy. The green lines represent the warehouse capacity and inventory levels of the sequential warehouse strategy and the blue lines depict the warehouse behaviour of the simultaneous strategy. The warehouse capacity is at the minimum level for 11 out of the 12 months in the sequential warehouse strategy. The warehouse capacity is only expanded in period three to 3.686 pallets to cope with a peak in demand. From period four onwards the warehouse capacity is back to the minimum level. Different than the sequential warehouse strategy, the simultaneous model does increase and decrease the warehouse capacity two times to cope with increasing in inventory levels. The model decides to have a structurally higher warehouse capacity than the warehouse capacity of the sequential warehouse strategy with a peak in capacity in period three of 4.573 pallets. The simultaneous model chooses for this higher warehouse capacity such that it can minimize the number of production runs and can store more inventory from period to period. Even though this means that the warehousing cost will be higher, the overall cost will be lower as this can facilitate optimal production scheduling as was highlighted in the cost comparison section. Another behavioural treat that emerges from Figure 34 is that even though inventory levels drop below the available warehouse capacity during the last three months the simultaneous model decides to not reduce the warehouse size for these periods. Maintaining a constant higher warehouse capacity is more cost efficient than reducing the capacity for these last three months as penalty cost will be incurred for this.

7.3.3. Concluding remarks

From the case study it becomes clear that using the simultaneous ordering strategy results in the lowest total cost of €850.489, -. The sequential ordering strategies have significant higher total cost due to the primary focus on the first optimization model in the sequence which strongly affects the outcome of the subsequent optimization model. The simultaneous ordering strategy overcomes this by making concessions between the warehousing and production scheduling costs to minimize the total cost. The sequential production ordering strategy resulted to be infeasible in the case study setting due to warehousing capacity constraints which did not allow the production scheduling order distribution. In a demand scenario in which the sequential production strategy would have been feasible the difference between the sequential and simultaneous strategy could have been smaller as was identified in the model verification process. However, this could not be confirmed with the case study results due to the infeasibility of the sequential production strategy.

Both the warehouse behaviour and order placement comparisons confirmed that the simultaneous ordering strategy makes concessions between the warehouse and production scheduling model in order to minimize the total cost. The simultaneous model increases its warehouse capacity and thereby the overall warehouse costs in order to store more inventory in the beginning of the year. In this way the model will be able to better distribute the orders and the production runs so that the total warehouse and production cost can be minimized. This differs from the sole focus of warehouse cost minimization in which the warehouse capacity and inventory levels are kept at the minimal level for 11 out of the 12 periods.

7.4. Cost parameter sensitivity analysis

Now that the models and strategies have been verified and validated in a real-life case study for Kraft Heinz, they will be further analysed by means of a sensitivity analysis on the cost parameters which should help generate useful insights with regards to the effects of the individual parameters on the total cost and the parameters behaviour. These effects will be elaborated on in the following paragraphs as well as in the managerial insights.

The results of the cost parameter sensitivity analysis can be found in Table 9. In the detailed table the cost % change, the cost difference and the solving gap are also displayed. Table 9 does however focus on the main sensitivity analysis outcomes which are needed to compare the three ordering strategies. The first column shows which input parameter is changed compared to the case study base scenario. The second column shows the new value of the input parameter used in the sensitivity analysis. Every parameter is changed multiple times to verify the parameters sensitivity in different scenarios. The parameter % change can be found in the third column, this will later on be used to compute the parameter's sensitivity. The other columns that can be found in the sensitivity analysis table are order strategy specific. The first column for each order strategy displays the total cost of the ordering strategy when the input parameter is changed. The second column for each ordering strategy displays the sensitivity % of the input parameter based on the model's input. The sensitivity % is computed by dividing the % of change of the input parameter by the % change of the model outcome as can be seen in function 7.5. In this way the effect of the individual input parameters on the overall model outcome is mapped in Table 9.

$$\frac{\%\ change\ of\ input\ parameter}{\%\ change\ of\ model\ outcome} = parameter\ sensitivity \quad (7.5)$$

Due to the infeasibility of the sequential production ordering strategy, no sensitivity analysis can be performed for the different input parameters as the model does not have a feasible outcome. For this reason, the sequential production strategy is not included in Table 9. This leaves the sequential warehouse strategy and the simultaneous strategy available for the sensitivity analysis which is described below.

TABLE 9 SENSITIVITY ANALYSIS COST PARAMETERS

Parameter	Value	% Param change	Cost Sequential WH	Sensitivity	Cost Simultaneous	Sensitivity
	Base scenario	0	**€ 1,615,819.00**	**0**	**€ 850,489.00**	**0**
Holding cost (H)	4	51%	€ 1,619,588	0%	€ 907,570	13%
	8	202%	€ 1,847,483	7%	€ 1,076,698	13%
	16	504%	€ 1,983,656	5%	€ 1,414,954	13%
Ordering cost (K)	2000	-50%	€ 1,591,819	3%	€ 826,489	6%
	8000	100%	€ 1,663,819	3%	€ 898,489	6%
	16000	300%	€ 1,759,819	3%	€ 994,489	6%
	50000	1150%	€ 2,167,819	3%	€ 1,402,489	6%
Handling cost (G)	2	-53%	€ 1,409,105	24%	€ 654,522	43%
	8	86%	€ 1,910,793	21%	€ 1,165,740	43%
	16	272%	€ 2,598,377	22%	€ 1,847,364	43%
Rent cost (R)	4	100%	€ 1,697,935	5%	€ 947,339	11%
	8	300%	€ 1,862,167	5%	€ 1,139,707	11%
	16	700%	€ 2,190,631	5%	€ 1,525,227	11%
Penalty cost (P)	0.00001	-100%	€ 1,611,819	0%	€ 841,021	1%
	1000	-50%	€ 1,613,819	0%	€ 846,245	1%
	4000	100%	€ 1,619,819	0%	€ 857,174	1%
	8000	300%	€ 1,627,819	0%	€ 865,254	1%
Warehouse inbound capacity (N)	3000	-50%	Infeasible	Infeasible	Infeasible	Infeasible
	9000	50%	€ 1,486,655	-16%	€ 702,271	-35%
	12000	100%	€ 1,485,486	-8%	€ 701,388	-18%
Overtime inbound cost (F)	2	-80%	€ 1,443,851	13%	€ 740,025	16%
	6	-40%	€ 1,570,471	7%	€ 795,257	16%
	20	100%	€ 1,737,856	8%	€ 988,569	16%
Production cost (PC)	500	-65%	€ 1,261,430	34%	€ 818,983	6%
	2000	41%	€ 1,832,278	33%	€ 870,240	6%
	6000	322%	€ 3,313,585	33%	€ 1,006,924	6%
Set-up cost (SC)	0.0001	-100%	€ 1,235,057	24%	€ 816,985	4%
	1000	-50%	€ 1,430,229	23%	€ 834,489	4%
	4000	100%	€ 1,970,806	22%	€ 882,489	4%
Production rates (PR)	Alt A	-50%	Infeasible	Infeasible	Infeasible	Infeasible
	AltB	100%	€ 1,343,215	-17%	€ 826,193	-3%
	AltC	200%	€ 1,205,420	-13%	€ 814,046	-2%

7.4.1. Sequential ordering strategies

From the results displayed in Table 9 it emerges that input variables warehouse inbound capacity N and production rate PR have a negative sensitivity in the sequential warehouse strategy. This indicates that if these variables are inversely related to the total cost. In other words, if the parameter is to be increased the overall cost will decrease and the other way around. This is not surprising as producing the same quantities with higher production result in shorter production times which will then result in less production cost as these are incurred per hour of production. The negative sensitivity of inbound parameter N can be accounted to the fact that having more inbound capacity at the warehouse during working hours means that larger production runs can be produced which is cost effective from a production point of view as was seen in the case study results. Besides combining production runs, having more inbound capacity during regular working hours also reduces the need of inbound capacity in overtime which has a variable penalty cost, hence the overall cost will be lower. It should however be noted that decreasing the production rates by 50% resulted in an infeasible model outcome as the orders could not be produced within the set time constraints. The same holds for the inbound capacity, as reducing this by 50% also resulted in an infeasible model due to inbound capacity constraints.

The remaining input parameters have positive sensitivities, which indicates that increasing the parameter will result in an increase in overall cost. Parameters fixed ordering cost K, rent cost R and fixed warehouse adjustment penalty cost P have relative low sensitivities. This indicates that changing them will not have a significant effect on the model outcome. This is related to the fact that these variables are fixed cost and are not affected by the number of products handled. As a result, changing the cost parameter only has limited effect on the total cost.

Input parameters with relatively high sensitivities are handling cost G, production cost PC and set-up cost SC. This indicates that these cost parameters are key cost drivers in the models for the sequential warehouse ordering strategy. The high sensitivities of these parameters can be accounted to the fact that handling cost G is incurred for all the products coming into the warehouse, hence it has a significant effect on the overall cost. The production cost and set-up cost also have a large sensitivity as these are the only two cost parameters accounted for in the production model, hence they are both significant cost drivers, reducing or increasing these will strongly affect the production cost.

Holding cost parameter H and overtime inbound cost F do not have constant sensitivities when the input parameters are changed. For example, the holding cost is insensitive for a cost increase of 50%, however increasing the holding cost by 200% or 500% gives a 7% and 5% sensitivity. This means that there is a barrier up to a point that increasing holding cost affects the model's decision and thus the model outcome. The same holds for inbound overtime cost F, where decreasing the cost by 80% results in a sensitivity of 13% and decreasing the cost by 40% results in a sensitivity of 7%, hence the parameter is more sensitive for bigger decreases than smaller decreases.

7.4.2. Simultaneous ordering strategy

Performing the same sensitivity analysis for the simultaneous ordering strategy resulted in a different outcome. Variables inbound warehouse capacity N and production rate PR still have a negative sensitivity which represents their inverse relation to the total cost. Nevertheless, the variable N's sensitivity has more than doubled in the simultaneous strategy compared to the sequential warehouse strategy. A reason for this increased sensitivity is the increased effect of the warehouse inbound capacity on the simultaneous optimization of warehousing and production cost. If both the warehouse capacity and the inbound capacity allow it, the model can optimize the distribution of orders such that production and warehouse cost are minimized. This optimization behaviour was displayed in the case study results section 7.3. As a result of this, increasing the inbound capacity will lead to lower total cost thanks to the enabled optimization space. For the production rate PR the opposite holds, as the parameter still is negatively sensitive, but only for 3 and 2 % instead of the 17 and 13 % it was in the sequential warehouse strategy. This means that changing the parameter does not have a big effect on the total cost. This can be accounted to the fact that if the simultaneous and the sequential warehouse strategy ordering cost are compared, the production cost are merely 9% of the total cost instead of the 56% of the total cost they represented in the sequential warehouse ordering strategy. This cost split is discussed in the case study results and can be found in Figure 29. Due to the smaller share of the total cost, the effect of changing the production rate PR also has less effect on the total cost, hence its sensitivity is smaller. The same holds for the production cost PC and the set-up cost SC which sensitivities dropped from 34 to 6 % and from 23 to 4 % due to the lower cost share of production cost.

The opposite is true for the warehouse cost parameters, which sensitivities have increased due to the different cost split in the simultaneous ordering strategy. The warehouse cost account for 91% of the total cost in the simultaneous ordering strategy, moreover the total costs of the simultaneous strategy are 53% of the total cost of the sequential warehouse strategy. As a result, the fixed ordering cost K, the handling cost G and the rent cost R sensitivities doubled from 3 to 6, 22 to 43 and from 5 to 11 %, due to their increased share in the total cost in the simultaneous ordering strategy. The holding cost H and the inbound overtime cost F have also increased in sensitivities and have a similar sensitivity in all three scenarios which was not the case in the simultaneous strategy. The increased sensitivity of the two variables can be accounted to the changed cost split. The fact that the sensitivity percentage is the same in every tested scenario compared to the different sensitivities in the sequential strategy, can be accounted to the fact that in the sequential warehouse strategy first the warehouse model is optimized. Changing input parameters can have strong effects on the decision variables and the model's optimization behaviour. An example of this is when the inbound overtime cost is increased, the model will aim to not use inbound overtime in order to reduce cost, hence the variable F will be used less and thus has less effect on the overall cost. Therefore, its sensitivity gradually decreases when the parameter is increased in the sequential warehouse ordering strategy. The same theory holds for holding cost H. In the simultaneous optimization strategy this effect is however less strong, as the model also accounts for the production scheduling optimization at the same time. This means that even if the holding cost or the inbound overtime cost are increased, the model is less likely to change its behaviour as keeping inventory or using overtime for inbounding might be more beneficial for the overall optimization than changing its behaviour to merely decrease holding or overtime cost. In other words, the simultaneous optimization model has a higher threshold before the model will adjust its optimization strategy. As a result, the sensitivities are constant in the simultaneous optimization strategy for all tested scenarios for parameters H and F.

7.4.3. Conclusion

From the cost parameter sensitivity analysis, it emerges that the type of ordering strategy used does have an effect on the sensitivity of the input parameters. Most of this difference can be accounted to the difference in total cost and the cost split of the total cost which depends on the ordering strategy that is used. Another effect of using either the sequential or the simultaneous ordering strategy is that the simultaneous model is less likely to change its behaviour due to single parameter changes compared to the sequential optimization strategies. The reason for this is the dual objective of the simultaneous model. If a parameter is to be increased the model will carefully analyse the trade-off between incurring the increased parameter cost or changing its optimal solution. This trade-off makes the model less likely to change its optimal solution due to single cost parameter changes. If it is required to increase the warehousing cost partly in order to minimize the production cost the model will do this. This does not hold for the sequential ordering strategies as these merely focus on either the warehouse or the production model in the first optimization step. As a result of this, parameter cost increases have a stronger effect on the models' optimization behaviour compared to the simultaneous model. For this reason, increasing the cost parameters leads to declining sensitivities in the sequential models, whereas in the simultaneous model the sensitivity is the same for most scenarios.

Another insight which emerged from the cost parameter sensitivity analysis is the fact that the handling cost G has a high sensitivity in both scenarios. This means that it is an important cost parameter which drives the total cost and should be closely monitored or reviewed if the aim is to reduce the overall cost further. Holding cost H and inbound overtime cost F are also cost parameters which have relatively high sensitivities, hence they could be interesting to monitor or minimize as well.

In both the sequential and the simultaneous model, inbound capacity *N* and production rate *PR* have negative sensitivities. Meaning that increasing these parameters will reduce the overall cost. For the production rates this is straight forward as increasing the production rates means that less production time is required to produce the same amount of products, hence less production cost. The biggest negative sensitivity is however from the inbound capacity *N* in both models. This means that this capacity bound value plays a key factor in the model outcome as the parameter value constraints the optimization options. With this in mind it should also be noted that the parameter *N* was one of the two constraints which made the sequential production strategy infeasible. Therefore, it can be concluded that the warehouse inbound capacity plays a crucial role in the total cost generation for the studied case. Possibilities to structurally increase the inbound warehouse capacity should therefore be explored as this will have strong effects on the overall production and warehousing cost.

8. Discussion

For the model outcome interpretation, it should be noted that the case study results are not in principle demonstrative for other scenarios. Consequently, carefulness should be exercised in the generalization of the case study results. This chapter will discuss the limitations as well as the assumptions of the proposed models and their effect on the model results, the validity of the models and the overall generalizability of the results. The chapter will be concluded by discussing the generated managerial insights.

8.1. Critical assumptions and limitations of the research

In the process of describing, conceptualizing, verifying and validating the models and according strategies multiple assumptions have been made. In this section, these assumptions and their effect on the model outcome will be more thoroughly discussed. Followed by an elaboration on the research limitations themselves.

8.1.1. Model assumptions and limitations

The proposed warehousing and production scheduling models represent supply chain activities. The models have been proposed by combining information from literature with information obtained from field experts. By combining information sources, it is ensured that theoretical knowledge is combined with practical knowledge from the field. Nevertheless, in the formulation of the mathematical models it is required to make assumptions to ensure that the proposed models can be developed and will provide feasible results within the given time and resource constraints. Making modelling assumptions affects the preliminary solution space of the models and may affect the real-life representation. For this reason, it is important to reflect on the made assumptions and their effect on the model outcome.

Assumptions

A first model assumption made is that both optimization models consider demand to be deterministic. Assuming that future demand is known with certainty whilst in reality it is not, affects the model outcome as the uncertainty effect of the future is not accounted for. In a deterministic model the correctness of the model outcome strongly depends on the correctness of the used demand forecast and the parameter estimations. Because the used demand forecast is an (close) estimate of the actual demand, the model outcome should be interpreted as an approximated value. In case of perfect demand forecasting, the model assumption does not affect the validity of the model outcome. The likelihood of a perfect 12-month forecast is however non-existent. More plausible is the expectation that the forecast is mostly accurate for the first months under study and that the forecast for consecutive months will be less and less accurate. This does not have to form a problem as the model looks at the high level required storage capacity and product flows. For example, where one product may be over forecasted by the company, other products can be under forecasted which would balance the overall results. A consequence of a wrong forecast may be that excessive warehouse capacity is leased if solely the deterministic model outcomes will be used to determine the optimal company strategy. The opposite is also plausible as too little warehouse capacity might be leased due to wrongly predicted scenarios. This may result in supply chain issues and customer service level risks. For this reason, the model outcomes should be interpreted as approximated values and be used for what-if scenario analyses to see what optimal strategies in case of simulated demand scenarios would be. Updating the models regularly with the latest available forecast should pertain to ensure that model outcomes are as accurate as possible. The negative effect of this assumption on the model outcome could have been resolved by proposing two-stage stochastic optimization models instead of deterministic models. However, due to the limit time available to propose and test the models, it was decided to make this model assumption as this enhanced the project's feasibility. The stochastic modelling approach remains an interesting direction for further research which will be more thoroughly discussed in the subsequent chapter.

A second model assumption made is that no transportation time and capacity is accounted for in the models. This assumption was made as solely the factory and warehouse operations are considered in this study. How and when the finished goods are transported between the factory and the warehouse is not within the scope of this research and thus not considered in the models. This assumption does affect the real-life representation of the model results, as in reality the transportation of goods between factories and warehouses is of interest and might cause issues in the product flow between a factory and a warehouse. It was decided to make this assumption to enhance the model's feasibility and not overcomplicate the model. To enhance the reliability of the case study results it was chosen to study the product flow between a factory and warehouse which are just 8 kilometres apart. Due to the relative short distance between the factory and the warehouse the transportation time and capacity should have less effect on the reliability of the model outcome.

A third model assumption made is that products are grouped by recipe. Both optimization models consider six different product recipes for production and warehouse storage. The products have been grouped by recipe as considering individual products would overcomplicate the production scheduling model due to the introduction of too many new constraints such as bottle change overs, minimal production quantities, minimal remaining shelf-life and short production runs. This was overcome by aggregating the products considered on a recipe level. Not considering these aspects affects the real-life representation of the model outcome as the inclusion of these new constraints would affect the production schedules and according warehousing strategies. When productions would be scheduled on a product level the added the production schedules would be more detailed and the optimal bottle and label changeover sequences would also need to be optimized. This would introduce more variables and so further increase the complexity of the model. By making the assumption to aggregate the products on recipes a less detailed production plan is generated. It should therefore be accounted for in the interpretation of results that the model is based on aggregate information and can be used for a high-level analysis of product flows between factories and warehouses in the supply chain but is not representative for detailed real-life production schedules. A more detailed production scheduling model would generate different optimal production schedules with different production scheduling costs.

A fourth model assumption is that no raw material availability is considered in this model. This assumption was made as the scope of this thesis is to study the product flows between the factory and warehouse and not consider the product flow before the factory and after the warehouse as this would be too much. Not including these product flows in the model does negatively affect the real-life representation of the model and should therefore be taken into account when interpreting the results. Including the raw material availability of the model would introduce a new feature in the production scheduling model as the availability of raw materials would also affect the possibility and timing of the production runs. This feature would make both the production scheduling and the integrated model more complex. Besides the increased complexity the problem-solving chances of the sequential warehouse strategy would also decrease, since the warehouse model does not consider the availability of raw materials and might request products which are not available at certain times. This could result in infeasible results in the second stage of the sequential warehouse strategy. For the above-mentioned reasons, it should be accounted for that in reality the optimal strategy can be affected by a shortage of raw materials which can have an effect on the possible production schedules which is not accounted for in the proposed model.

A fifth and final modelling assumption made is that the proposed models solely consider the product flow between a single factory and warehouse, whilst in practice warehouses receive goods from multiple factories. Due to this assumption, capacity reduction assumptions had to be made in order to make the models capacities representative for the product flow between a single factory and warehouse. Modelling with only part of the actual capacity has the risk that the model outcome is less representative for the real-life scenario. Since in reality, the warehouse inbound capacity and the overall storage capacity has to be shared with products from other factories as well. The product mix inside the warehouse changes continuously depending on the different factory outputs. Whereas, in the case study merely the product flow from a single factory is considered.

Model limitation

A limitation of the used models is that the model outcomes are as reliable as the data used. This model treat was already shortly discussed in the first model assumption elaboration. This model limitation does not only hold for the demand forecast used but holds for all input parameters considered in the proposed models. Therefore, it is of significant importance to ensure that the input data is as accurate as possible. Since making use of inaccurate input data will make the model outcome more unreliable. However, the effects of this model limitation are limited as Daganzo (2005) and Taylor (2006) argued that deterministic inventory models are robust to data errors of forecast and decision variables. Even though the model outcomes remain valid when parameters are estimated differ from the true value, the goal is to estimate the parameters as close as possible to the true value for enhancement of the models' validities.

8.1.2. Research method limitation

The limitation of the used research method to propose and test two optimization models in multiple ordering strategies lies within the fact that the strategies and models have only been tested in a single case study. Carrying out a single case study negatively affects the reliability of the model outcomes, since the models are only tested in one single case (Kanama & Kido, 2016). Nonetheless, it was decided to carry out a single case study due to limited time and resource availability. This research limitation was partly compensated by analysing and testing the different strategy behaviours in three simulated demand scenarios. By doing so, the strategies have been tested and verified in a total of four different scenarios which increases the reliability of the model outcomes. Ideally, the models should have been tested in multiple real-life case studies to further increase the reliability of the model outcomes. This could have been realized by performing similar case studies to other factory, warehouse combinations. However, due to the limited time available this was not feasible.

Another limitation of the research is the that the case study results cannot be compared with previous company results. Comparing the case study results with company results would further validate the case study results and could confirm the dominance of the simultaneous ordering strategy compared to both the sequential and the current Kraft Heinz strategy. The reason for this limitation is the fact that a new warehouse was studied in the case study. Since this warehouse was recently taken into operation and this year demand forecasts were used, no historical results were available which can be used as a comparison for the case study results. In hindsight, this could have been resolved by choosing a different warehouse for the case study or by performing more than one case study in this thesis. By doing so, comparisons between case study results and historical company results could have been realized.

8.2. Reflection on model validities

The validity of the model expresses if the model outcomes are reliable, representative for the real-life scenario and how well the outcome of the model can be generalized for other situations. The validity of the proposed models is ensured throughout different stages of the models' development which will be discussed in the first subsection. This section will be concluded by touching upon the models' generalizability.

8.2.1. Validity

In the first part of this research the validity of the models was ensured by combining findings from different scientific research papers to come up with the mathematical models which form the bases of the proposed models and tested ordering strategies. The findings from the different scientific papers were complemented with model features and constraints inspired by operational problems from Kraft Heinz. Through using findings from literature it is ensured that the proposed models consist of well-known input parameters which make the models suitable for different cases. Combining these findings with model suggestions from field experts, further ensures that the models are representative for real-life operations. During the conducted literature research, it was decided to consider demand to be deterministic in the tested scenarios as this would reduce the models' complexities. Even though, assuming demand to be deterministic whilst in fact it is not, affects the real-life representation of the models. It is argued in literature by Daganzo (2005) and Taylor (2006) that deterministic inventory models are robust to data errors and are useful to generate managerial insights as long as the models' assumptions are accounted for when interpreting the results. An example of such a robust deterministic inventory model is the well-known EOQ model from Ford Whitman Harris.

Once the mathematical models were formulated, they were checked and validated by different logistic managers from Kraft Heinz to ensure the real-life representation of the formulated models and made assumptions. This cross-validation further pertained to the increased validity of the models and the according outcomes.

A third mitigation to ensure the models' validities was to use actual company forecasts and cost calculations as input parameters for the models in order to reduce the possibility of data errors. Kraft Heinz has decades of experience with regards to forecasting the demand for their products in different markets, therefore it is assumed that the forecasted values are a close representation of real demand. Using real-life data further pertains to the real-life representation of the models and their respective outcomes.

A final step in ensuring the model validities was testing the models in a total of four scenarios. First the models were tested in three hypothetical scenarios of which the results verified the models' behaviour. Thereafter, the proposed models were tested in a case study based on real-life input parameters in which the models displayed similar behaviour. Thoroughly testing the formulated models was considered the final step in the model validation as the models behaved similar in all the tested scenarios and can therefore considered to be valid.

One of the objectives of this thesis was to come up with an optimization model to study the real-life product flows between a factory and a warehouse in the supply chain of a manufacturing company. It should be noted however that it is nearly impossible for mathematical and scientific models to capture the complexity of reality. On this note George Box stated in 1976; "*All models are wrong, but some are useful*" (Box, 1976). He sought to emphasize that all scientific models are simplifications of reality, but that simplified models can generate useful insights as well if they are formulated correctly. The same holds for the proposed models in this research. Even though the aim was to study the real-life product flows between a factory and a warehouse in the supply chain of a manufacturing company, several assumptions had to be made to make it possible to study the product flows. These discussed assumptions make the models to be a simplification of reality and thus have an effect on the real-life representation of the models. Nevertheless, the model validities are ensured through the described actions in this section. Altogether, it can be assumed that the simplified models can be a useful tool to generate managerial insights for real-life operations regarding the product flows between factories and warehouses for manufacturing companies as long as the effects of the made assumptions are considered during the interpretation of the model results.

8.2.2. Generalizability

The case study results themselves cannot be generalized for other manufacturing companies that make use of LSP warehouses in their supply chain. The case study results hold specifically for the used set of input parameters which were based on real-life data from Kraft Heinz. Changing the input parameters will affect the model outcome as was demonstrated in chapters six and seven of this research. The proposed models themselves can however be used by other manufacturing companies to analyse the product flows between factories and warehouses as the models' objective functions and constraints have been generally formulated. This general set-up of the models makes the models suitable to be used and tested for other case studies as well. The complexity of setting up and using the proposed models for other manufacturing companies depends on the case study details that should be accounted for such as the number of periods, products, machines and possible missing input data.

8.3. Managerial insights

Preparing and conducting the research and performing the according case study resulted in a number of relevant insights which can be used by logistic managers and researchers to improve the ordering strategy and resulting product flows between factories and warehouses. These managerial insights will be individually discussed on their business relevance, potential effect and possible implication.

A first managerial insight is that simultaneous optimization of the models leads to superior results. The main reason for this is the fact that the simultaneous ordering strategy ensures that the total cost is minimized whilst ensuring that the model outcomes are feasible from both a warehousing as well as production point of view. This does not hold for the sequential ordering strategies, as the optimal outcome of one model might not be optimal or even feasible as input for the other model. As a result, the sequential ordering strategies may lead to infeasible outcomes in some scenarios. The according managerial insight is that it is valuable for business to consider as much cost parameters and constraints of the supply chain activities as possible when optimizing the size and timing of orders placed. Ensuring that the placed production orders satisfy not only the production constraints but also the warehousing constraints will reduce the risk of infeasible results, hence the risk of capacity issues in the supply chain. By simultaneously optimizing multiple segments in the supply chain in-depth trade-offs can be made which are beneficial for ensuring reliable product flows as well as minimizing the total cost throughout the supply chain. This is in line with findings in literature from Bradley and Arntzen (1999) and Atamtürk and Hochbaum (2001) who argued that simultaneous consideration of capacity, inventory and production decisions leads to superior results. As was explained in chapter 4, the simultaneous ordering strategy resembles a company situation in which there is central coordination between the production and warehouse departments and both teams are aware of all the needs and constraints in place of the complete product flow. In this way trade-offs can be made to find the optimal production and warehousing strategies. Because the simultaneous optimization leads to superior results, it is also recommended for logistic managers to enhance central coordination and interdepartmental coordination in order to improve the product flow between factories and warehouses. A possible strategy to facilitate this is introducing key performance indicators which monitor unforeseen supply chain issues due to misalignment between production scheduling and logistic teams. Closely monitoring this key performance indicator and holding multiple teams accountable for this key performance indicator's performance should ensure more interdepartmental collaboration and reduce the future logistic cost and issues. Other strategies to enhance interdepartmental communication would be to organize weekly meetings including both the warehousing and production scheduling team discuss the constraints, capacities and foreseen issues for the near future. Concluding, through interpreting the case study results it emerged that simultaneous optimization the best results are achieved, for business this indicates that enhancing coordination and interdepartmental communication will have a positive effect on the product flow.

In addition to the previous managerial insight that simultaneous optimization is achieving better results than solving the subproblems in a sequence. Another business recommendation would be to invest in an optimization software program to help plan and optimize production scheduling and warehousing planning. The case study results proved that integrated models are most suitable for solving large problem sets and that these problems can easily contain 50.000 variables that need to be optimized. Since solving such problems (partially) by hand is very time consuming and can lead to non-optimal outcomes it is recommended to investigate the possibility to invest in planning optimization applications which consider production and warehousing constraints simultaneously. Investment costs made for the improved planning tools could be saved by decreasing the number of required production planners as well as a reduction in unforeseen logistic costs due to capacity issues or planning errors.

A second managerial insight obtained through the case study results is that the warehouse inbound capacities have a strong effect on the model outcomes and possible (in)feasibility. For this reason, it is of importance that the inbound capacity and overtime inbound capacity cost parameters are correct and representative for the real-life circumstances as wrongly estimating them can have a significant effect on the model outcome. Because of the large and negative sensitivity of the warehouse inbound capacity and the high positive sensitivity of the warehouse inbound overtime cost it is recommended to investigate opportunities to structurally increase the warehouse inbound capacity at the LSP warehouse as this will have a positive effect on both the warehouse as well as the production costs. Besides the positive effect on the costs, increasing the warehouse inbound capacity will also reduce the chance of unforeseen issues at the warehouse due to limited inbounding capacity, since the inbound capacity is limiting the solution space of the studied models and is the first constraint that is violated in case of an infeasible model outcome. To this end, it is recommended to increase the warehouse inbound capacity.

A third managerial insight would be to closely monitor the handling cost and explore options to reduce this cost in the future. The performed sensitivity analysis resulted in the observation that the handling cost have a significant effect on the model outcome for both analysed strategies. Exploring options to reduce this cost parameter in cooperation with the warehouse LSP would be beneficial for the total cost related to the product flows.

The described managerial insights from this research entail recommendations for logistic managers to further improve the product flows between factories and warehouses in supply chains. The first managerial insight is a general managerial insight applicable for all people who are interested in developing or using the proposed model in the future. The second and third proposed managerial insights are related to insights obtained in the sensitivity analysis of the case study. The outcome of this is specific for the case study input parameters, hence different case studies may have different results. Nevertheless, he proposed models and ordering strategies could be used to test different instances which may result in different outcomes.

9. Conclusion

The aim of the final chapter of this research is to summarize the findings and conclude this research. This will be done by first answering the formulated sub research questions. Through combining the answers to the sub research questions, an answer for the main research question can be formulated. This chapter will be concluded by discussing the recommendations for future research.

9.1. Answers to the sub research questions

In this section answers to the sub research questions will be provided. This will be done by simultaneously discussing the answers to the first three sub research questions as these have been answered through the conducted literature research. Thereafter, the final three sub research questions will be answered individually as these questions pertain to the case study outcomes.

The conducted literature review helped to map the state of the art in the research fields of the multi-period warehouse problem, the lot size problem and the scheduling problem. The different problem classifications and solving methods were mapped as such in Table 1. By doing so sub research question one was answered. Furthermore, the conducted literature research helped to identify the necessary adjustments to propose a multi-product version of the combined lot size and multi-period warehouse capacity model. This was done by searching for research papers that focussed on the optimization of multi-product multi-period warehousing models and for papers that proposed integrated lot size and multi-period warehouse problems. From this literature study it emerged that input parameters such as demand and inventory parameters and constraints needed to be made appropriate for multiple products. Besides these adjustments new constraints had to be introduced which captured the effect of considering multiple products on the lot sizes and warehouse strategies. By combining these findings, the second sub research question was answered. The final sub research question which could be answered through consulting research papers was the question how to combine a production scheduling model with a multi-period lot size and warehouse capacity optimization model. By combining findings from papers on the individual models it was decided that the two models could be connected through the generated ordered products that needed to be produced and thereafter stored. As this parameter was present in both models it could be used to connect the two models in a sequential and a simultaneous optimization set-up. In the second phase of this research the fourth, fifth and sixth sub research question were answered. The answers to these sub research questions will discussed in the subsequent paragraphs.

SQ4: "How do product flows from a factory affect warehousing strategies?"

The effect of the product flow on the warehousing strategy depends on the incentive behind the scheduled productions as was displayed in the chapters six and seven. If the production schedules are planned solely to minimize the production scheduling cost such as in the *Sequential Production Strategy*, production runs will be consolidated and the production cost will be minimized. Due to this focus on the production perspective, the factory output will most likely have production peaks followed by periods with lower output. This will result in high inventory levels which requires additional warehouse capacity. In this situation the warehousing strategy is affected by the product flows from the factory as it is reactive to the factory output and will have to adjust its warehouse capacity accordingly. The result of this is an increase in warehousing cost due to high inventory levels. If the warehouse team is integrated in the production flow strategy, the effects for the warehouse would be different. This could be seen in the model outcomes of the *Sequential Warehouse Strategy* and the *Simultaneous Strategy*. If the warehouse cost and constraints are accounted for in the production planning step, the productions will be spread out more over the available periods. In this way a more consistent product flow is ensured and inventory as well as warehouse capacities can be better controlled. In this production planning strategy, the warehouse team is integrated in the decision making which enhances the coordination of operational production and warehousing decisions. The results from the model verification and the case study proved that enhancing this coordination and integrated decision-making leads to superior results and reduces the chances of infeasible outcomes.

SQ5: "How do the models' input parameters affect the outcome of the proposed models?"

The question how the input parameters affect the outcome of the proposed models has been answered by performing a demand scenario behaviour analysis as well as a cost parameter sensitivity analysis in the case study. By performing a sensitivity analysis to the cost parameters, the effects of changing a single cost parameter on the model outcomes became clear. By doing so, managerial insights for the company under study were generated. Main findings form this sensitivity analysis were that the warehouse inbound capacity has a significant effect on the model outcome. Therefore, it is was recommended to investigate the structural increase this parameter as this will improve the model outcome. Additionally, the handling and holding cost were identified as cost drivers of the model outcome which should be accounted for if one aims to improve the model outcomes in the future. The demand scenario behaviour analysis verified the models' behaviour in different demand scenarios. In this way the effects of demand scenarios on the model outcomes could be mapped. A conclusion drawn from this analysis was that the *Simultaneous Ordering Strategy* achieved the best results in every demand scenario. Another insight from this analysis was that in the demand scenario in which one product is dominant the total cost would be lowest for all strategies, the chances of an infeasible result in the *Sequential Production Strategy* are however also higher for dominant demand scenarios.

SQ6: "To what extend does the model reflect the real-life scenario for Kraft Heinz?"

The final sub research question was answered in the discussion chapter. The models' validity and correct representation of the real-life scenario was reflected upon. It was concluded that the validity was ensured through all stages of the model development by means of model validation by field experts as well as using widely known parameters and constraints inspired by real-life operations from Kraft Heinz and using real-life company data in the case study. Nonetheless, a number of assumptions had to be made in order to be able to model the problem. These assumptions did affect the real-life representation of the proposed models, as the due to the made assumptions the models represent a simplified version of reality. Because of these made assumption the model as well as its outcomes should be viewed as an informative simulation of the real-life scenario. The models and the made analyses provide a realistic high-level overview of the operations and product flows between a factory and a warehouse in the supply chain of a large manufacturing company. Nevertheless, it is important to be aware of the high-level representation of the Kraft Heinz operations in the models and the lack of detail in certain model aspects which may affect the model outcome. With this in mind the models can be used to analyse the effects of changing cost or characteristics at the factory or the warehouse and study the effects on the optimal ordering strategies.

9.2. Answer to the main research question

By combining the answers of the sub research questions, an answer to the main research question could be formulated. The main research question entails:

> *"How does the coordination between production and warehouse decisions affect the product flow between factories and warehouses for manufacturing firms?"*

From the conducted research it emerged that increasing interdepartmental coordination between production and warehousing teams has a positive effect on the product flow and according cost. The simultaneous optimization model which accounted for both the warehouse as well as the production constraints simultaneously emerged to be the model which leads to superior results when compared to two sequential alternative strategies. This is in line with findings in literature from Bradley and Arntzen (1999) and Atamtürk and Hochbaum (2001) who found that simultaneous consideration of capacity, inventory and production decisions leads to superior results compared to sequential decision making. Since simultaneous optimization represents a company setting in which there is central coordination, it is important to enhance coordination between warehouse and production decisions through raising awareness for all the constraints and cost drivers in place in this process. Besides the simultaneous model being the most cost-effective model, the simultaneous model also is more reliable to generate feasible results due to the simultaneous consideration of warehousing as well as production constraints. This will translate in practice to a lower risk of capacity issues in the supply chain due to a lack of coordination between production planning and logistic departments. Altogether, this research confirmed the statement in literature that simultaneous optimization achieves better results than sequential optimization. Moreover, this research demonstrated the positive effect of centrally coordinating production and warehousing decisions due to the integrated model's superiority. By doing so, the main research question has been answered and the research objectives have been fulfilled.

9.3. Further research

This thesis proposes three alternative ordering strategies for large manufacturing companies which make use of two developed MILP models. The proof-of-concept of these strategies is provided by testing them in a case study of a large manufacturing company in the food sector. This proof-of-concept opens doors for further research to expand on the insights obtained in this investigative research and to further improve the models towards possible real-life implementation. The recommendations for further research will be discussed separately below.

A first recommendation for further research would be to test the proposed models in a stochastic environment. Now that the models' optimization behaviours have been verified and their real-life representation have been validated, the models could be extended to include demand uncertainty. Adding demand uncertainty does increase the models' complexities. On the other side, it also enhances the models' real-life representation. The developed models could be extended into a two-stage stochastic MIP model and demand can be treated as the uncertain variable. For other possible extension options in the stochastic environment the research of White & Francis (1971), Roll & Rosenblatt (1988) And Huang et al. (2014) may be used as examples of how to include stochasticity in warehousing models. It could be interesting to compare the case study and sensitivity analysis results of the stochastic and deterministic models and describe benefits of the respective models. Moreover, the effects of using different demand distributions on the model outcome could be studied with the stochastic model. Insights obtained from this comparison may help to further improve the models for future real-life implementation.

A second recommendation for further research would be to extend the proposed MILP models to consider multiple factories and a single warehouse in the product flow study. The additional consideration of multiple factories requires an extension of the production scheduling model as certain products may only be produced in specific factories and on specific lines. Moreover, product flows from multiple factories will have to be consolidated and stored in a single warehouse. As a result of this extension, interesting new trade-offs regarding warehouse inbound and storage prioritization can be studied.

A third recommendation for further research would be to expand the mathematical models to include the transportation of the goods between the factory and the warehouse. Including the transportation of the goods, the availability of trucks, the transportation time and cost into the model further increases the real-life representation of the models as well as the model's complexity. This could be included by introducing a carrier capacity and a transportation cost parameter for produced products. A study towards this extension might be interesting to see how this affects the optimal ordering strategies and how this effect is related to the distance between a factory and a warehouse.

A fourth recommendation for further research is to extend the models to analyse the product flows on a product level instead of the current recipe level. The increased level of detail in the models would require a number of new constraints such as the inclusion of bottle and label changeovers which will increase the models' complexity. The product level detail does however introduce new possibilities for the analysis of product flows, since the more detailed production model will affect the ordering strategies due to the added constraints. Other possibilities for extending the production scheduling model would be to include operator assignment and production line maintenance to further improve the models' real-life representation.

A fifth recommendation for further research would be to include arrival and departure intervals of the queuing theories in both the MILP models for the arrival of raw material at the factory and arrival of goods at the warehouse. Including these intervals in the product flow study further increases the level of detail and introduces new subproblems within the periods under study. The complexity of the models will increase but the inclusion of this distribution would also be interesting to see how the ordering strategies would change and what this means for the overall model outcome if the raw materials are not available at all times and if the finished goods arrive with changing intervals at the warehouse.

One more recommendation for further research would be to test the proposed ordering strategies and according optimization models in case studies of other manufacturing companies or for different factory warehouse combinations and compare the results of this case study. By testing the models in different real-life case studies, the model outcomes can be further validated. Moreover, the effects of using different production and warehouse situations on the model outcomes can be compared. For example, other manufacturing companies may require part of the inventory to be stored at specified areas in the warehouse for cooling or other purposes. Studying the effects of these different warehouse layouts on the model outcome might generate new managerial insights. Moreover, performing case studies to warehouses of which historical data is available can help to compare the case study results with company actuals from the past, as this was not available for the Kraft Heinz case study. This contributes to further validation of the strategies as well as highlighting the added value of using a proposed strategy to the business. Another interesting direction for further research would be test the models for manufacturing companies who are not active in the fast moving consumer goods market and investigate if the case study results have similar trends or not.

A final recommendation for further research would be to propose a new optimization algorithm(s) for the proposed mathematical models and test these algorithm's solving performance against the solving times of CPLEX for similar problems. Research in this direction can be of interest to further prepare the proposed models for larger problem sets as this might help increase the solving speed.

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Appendix

A: Tables & Figures

TABLE 10 DEMAND BASE SCENARIO MODEL VERIFICATION

Period	Product 1	Product 2	Product 3
1	100	400	200
2	200	200	350
3	100	400	350
4	300	200	350
5	700	800	650
6	100	400	350

TABLE 11 DEMAND EQUAL SCENARIO VERIFICATION

Period	Product 1	Product 2	Product 3
1	233	233	233
2	250	250	250
3	283	283	284
4	283	284	283
5	717	717	717
6	284	283	283

TABLE 12 DEMAND DOMINANT SCENARIO VERIFICATION

Period	Product 1	Product 2	Product 3
1	100	100	500
2	100	100	550
3	150	100	600
4	100	200	550
5	350	300	1500
6	50	200	600

B: CPLEX scripts Case study

Asequentialprod.Mod

```
// Number of Machines
int nbPeriods = ...;
range Time = 1..nbPeriods;
range T1 = 0..nbPeriods;


// Number of Machines
int nbMachines = ...;
range Machines = 1..nbMachines;

// Number of Jobs
int nbProds = ...;
range Products = 1..nbProds;
range nodes = 0..nbProds;

// Parameters
float PR[Products][Machines] = ...;
int release = ...; //could potentially be changed to time
int due = ...;     //could potentially be changed to time
int SUcost= ...;
int d [T1][Products] = ...;
int S [Products][Products] =...;
int M =...;
int PCost= ...;

// Decision variables
dvar int+ i [T1][Products];
dvar int+ x [T1][Products];
dvar float+ pt [Time][Products][Machines];
dvar float+ st [Time][Products][Machines];
dvar float+ ct [Time][Products][Machines];
dvar boolean a [Time][nodes][nodes][Machines];


 dexpr float SchedCost = sum(t in Time)sum(i in Products)sum(j in
Products)sum(m in Machines)(a[t][i][j][m]*(S[i][j]*SUcost +
pt[t][j][m]*PCost));
 minimize SchedCost;

 subject to{
 forall(i in Products, t in Time){
 c21:sum(m in Machines)sum(j in nodes:j!=i)a[t][i][j][m]<=1;
  }

forall(m in Machines, t in Time){
c22:sum(j in Products)a[t][0][j][m]<=1;
}

 forall(j in Products,m in Machines, t in Time){
 c23:sum(i in nodes)a[t][i][j][m]==sum(i in nodes)a[t][j][i][m];
}

 forall( i in Products,m in Machines, t in Time){
 c24: a[t][i][i][m]==0;                        // ensures that product 1 is
not scheduled after product 1
}
```

```
forall(j in Products, t in Time){
c25:x[t][j]==sum(m in Machines)(pt[t][j][m]*PR[j][m]);
}

forall(j in Products,m in Machines, t in Time){
c26:ct[t][j][m]==st[t][j][m]+pt[t][j][m];
}

forall(j in Products,m in Machines, t in Time){
c27:release<=ct[t][j][m];
}

forall(j in Products,m in Machines, t in Time){
c28:ct[t][j][m]<=due;
}

forall(j in Products,i in Products,m in Machines, t in Time){
 c29:st[t][j][m] >= st[t][i][m]+pt[t][i][m]+S[i][j] - M*(1-a[t][i][j][m]);
 }

forall(j in Products,i in Products,m in Machines, t in Time){
  c30:st[t][j][m] >= release;
}

forall(i in Products, m in Machines, t in Time){
c31: pt[t][i][m]<= M*(sum(j in nodes)a[t][i][j][m]);
}

forall(j in Products, t in Time){
c32:sum(t in 1..t)x[t][j]>= sum(t1 in 1..t)d[t1][j];
}
}

execute
{
var f=new IloOplOutputFile("output2.csv");
f.writeln("x",x);
f.close();
}
```

ASequentialWH.Mod

```
int nbPeriods = ...;
int nbProds = ...;

range T=0..nbPeriods;
range Products=1..nbProds;
range Time=1..nbPeriods;

 // Parameters
 int d [T][Products] = ...; // demand in period t
 int x [T][Products] = ...; // demand in period t
 int K  = ...; // fixed order cost in period t
 float h  = ...; // unit holding cost in period t
 float g = ...; // handling cost
 int p  = ...; // fixed cost of increasing WH size in period t
 float r  = ...;  // WH rental cost per unit in period t
 int b  = ...;
 int MaxWH = ...; //maximum WH capacity
 int MinWH = ...; // Minimum WH capacity
```

```
 int IINV [Products] = ...; // Initial inventory P1
 int n = ...; //max inbound capacity
 int F = ...; //OT cost
 int SS[Products] = ...;
 int y [T] = ...;


//dvar int+ x[T][Products]; //order quantity in period t
dvar int+ i[T][Products]; //inventory lvl at the end of period t of every
product
dvar int+ itot [T]; //sum of inventory at the end of period t
dvar int+ w[T]; // warehouse size at the end of period t
dvar int+ u[T]; // warehouse size expansion at beginning of period t
dvar int+ v[T]; // warehouse size contraction at beginning of period t
dvar int+ l[T]; // Number of pallets inbounded in OT
dvar boolean z1[T]; // binary variable for WH expansion in period t
dvar boolean z2[T]; // binary variable for WH contraction in period t


 //objecTive funcTion
dexpr float ProdCost = sum(t in T)sum(j in Products)(h*i[t][j]+g*x[t][j]);
dexpr float PeriodCost = sum(t in
T)(K*y[t]+r*u[t]+p*z1[t]+r*v[t]+p*z2[t]+r*w[t]+l[t]*F);
dexpr float WHCost = ProdCost+PeriodCost;

minimize WHCost;

//constraints
subject to {

forall(t in Time, j in Products) {
c1: i[t-1][j] + x[t][j] - d[t][j] == i[t][j];      //Inventory balance
constraint
}

forall(t in Time, j in Products) {
c3: i[t][j]>=SS[j]; // Ensures that in all periods t the inventory level
should be as least as high as the required SS
}

forall(t in Time){
c4: w[t-1] + u[t] - v[t] == w[t];   //warehouse balance constraint
}

forall(t in Time){
c5:(sum(j in Products)i[t][j]) <= w[t];    // inventory should be max. the
warehouse level
}

forall(t in Time){
c6:u[t] <= (sum(t in T)sum(j in Products)d[t][j])*z1[t]; // Total demand from
1 to T * z1
}

forall(t in Time){
c7:v[t] <= (sum(t in T)sum(j in Products)d[t][j])*z2[t]; // Total demand from
1 to T  *z2
}

forall(t in Time){
c8:w[t] <= MaxWH; // WH max available capacity
```

```
}

forall(t in Time){
c9:w[t] >= MinWH; // Min WH capacity that is agreed upon
}

forall(t in Time){
c10:(sum(j in Products)x[t][j])<= n + l[t]; //sum of ordered products should
be below inbound capacity
}

forall(t in Time){
c11:l[t] <= b; // upperbound of OT capacity for inbounding pallets in a period
}

forall(t in Time,j in Products){
c12:(sum(j in Products)i[t][j]) == itot[t]; //For visualization of itot
}

forall(j in Products){
c13:i[0][j]== IINV[j];   // initial inv prod
}

c14:w[0]==MinWH;  //initial warehouse size
}
```

BSequentialWH.Mod

```
int nbProds = ...;
int nbPeriods = ...;

range Products = 1..nbProds;
range Time = 1..nbPeriods;
range T=0..nbPeriods;

// Parameters
 int d [T][Products] = ...; // demand in period t
 int K = ...; // fixed order cost in period t
 float h  = ...; // unit holding cost in period t
 float g = ...; // handling cost
 int p = ...; // fixed cost of increasing WH size in period t
 float r = ...;  // WH rental cost per unit in period t
 int b  = ...; // Pallet inbound capacity per period t
 int MaxWH = ...; //maximum WH capacity
 int MinWH = ...; // Minimum WH capacity
 int IINV [Products] = ...; // Initial inventory at t=0
 int n = ...; //max inbound capacity
 int F = ...; // inbound pallet cost in OT
 int SS [Products] =...; //required safetystock per product

 ///////////////////////////////Decision                            variables
WH&LS/////////////////////////////////
dvar int+ x[T][Products]; //order quantity in period t
dvar int+ i[T][Products]; //inventory lvl at the end of period t of every
product
dvar int+ itot [T]; //sum of inventory at the end of period t
dvar int+ w[T]; // warehouse size at the end of period t
dvar int+ u[T]; // warehouse size expansion at beginning of period t
dvar int+ v[T]; // warehouse size contraction at beginning of period t
dvar int+ l[T]; // Number of pallets inbounded in OT
dvar boolean y[T]; // binary variable for ordering in period t
dvar boolean z1[T]; // binary variable for WH expansion in period t
```

```
dvar boolean z2[T]; // binary variable for WH contraction in period t


 ///////////////////////////////objecTive
funcTion/////////////////////////////////
dexpr float ProdCost = sum(t in T)sum(j in Products)(h*i[t][j]+g*x[t][j]);
dexpr          float          PeriodCost          =          sum(t          in
T)(K*y[t]+r*u[t]+p*z1[t]+r*v[t]+p*z2[t]+r*w[t]+l[t]*F);
dexpr float WHCost = ProdCost+PeriodCost;

minimize WHCost;

///////////////////////////////Constraints
LS&WHS//////////////////////////////////
subject to {

forall(t in Time, j in Products) {
c1:  i[t-1][j]  +  x[t][j]  -  d[t][j]  ==  i[t][j];      //Inventory  balance
constraint if order takes 1t to arrive would x[t-1] be the right adjustment?
}

forall(t in Time, j in Products) {
c2:x[t][j]  <=  (sum(t1  in  t..nbPeriods)d[t1][j])*y[t];   //  dtT  Cumulative
demand from t to end T, ensures that Y[t] holds as a binary variable
}

forall(t in Time, j in Products) {
c3:  i[t][j]>=SS[j];  //  Ensures  that  in  all  periods  t  the  inventory  level
should be as least as high as the required SS
}

forall(t in Time){
c4: w[t-1] + u[t] - v[t] == w[t];   //warehouse balance constraint
}

forall(t in Time){
c5:(sum(j  in  Products)i[t][j])  <=  w[t];    //  inventory  should  be  max.  the
warehouse level
}

forall(t in Time){
c6:u[t] <= (sum(t in T)sum(j in Products)d[t][j])*z1[t]; // Total demand from
1 to T * z1
}

forall(t in Time){
c7:v[t] <= (sum(t in T)sum(j in Products)d[t][j])*z2[t]; // Total demand from
1 to T  *z2
}

forall(t in Time){
c8:w[t] <= MaxWH; // WH max available capacity
}

forall(t in Time){
c9:w[t] >= MinWH; // Min WH capacity that is agreed upon
}

forall(t in Time){
c10:(sum(j in Products)x[t][j])<= n + l[t]; //sum of ordered products should
be below inbound capacity
```

```
}

forall(t in Time){
c11:l[t] <= b; // upperbound of OT capacity for inbounding pallets in a period
}

forall(t in Time,j in Products){
c12:(sum(j in Products)i[t][j]) == itot[t]; //For visualization of itot
}

forall(j in Products){
c13:i[0][j]== IINV[j];    // initial inv prod
}

c14:w[0]==MinWH;  //initial warehouse size
}

execute
{
var f=new IloOplOutputFile("output3.csv");
f.writeln("x",x);
}
```

BSequentialProd.Mod

```
int nbPeriods = ...;
range Time = 1..nbPeriods;
range T1 = 0..nbPeriods;


// Number of Machines
int nbMachines = ...;
range Machines = 1..nbMachines;

// Number of Jobs
int nbProds = ...;
range Products = 1..nbProds;
range nodes = 0..nbProds;

// Parameters
float PR[Products][Machines] = ...;
int release = ...;
int due = ...;
int SUcost= ...;
int x [T1][Products] = ...;
int S [Products][Products] =...;
int M =...;
int PCost= ...;

// Decision variables
dvar float+ pt [Time][Products][Machines];
dvar float+ st [Time][Products][Machines];
dvar float+ ct [Time][Products][Machines];
dvar boolean a [Time][nodes][nodes][Machines];


 dexpr float SchedCost = sum(t in Time)sum(i in Products)sum(j in
Products)sum(m in Machines)(a[t][i][j][m]*(S[i][j]*SUcost +
pt[t][j][m]*PCost));
 minimize SchedCost;
```

```
 subject to{
 forall(i in Products, t in Time){
 c21:sum(m in Machines)sum(j in nodes:j!=i)a[t][i][j][m]<=1;
  }

forall(m in Machines, t in Time){
c22:sum(j in Products)a[t][0][j][m]<=1;
}

 forall(j in Products,m in Machines, t in Time){
 c23:sum(i in nodes)a[t][i][j][m]==sum(i in nodes)a[t][j][i][m];
}

 forall( i in Products,m in Machines, t in Time){
 c24: a[t][i][i][m]==0;                        // ensures that product 1 is
not scheduled after product 1
}

forall(j in Products, t in Time){
c25:x[t][j]<=sum(m in Machines)(pt[t][j][m]*PR[j][m]);
}

forall(j in Products,m in Machines, t in Time){
c26:ct[t][j][m]==st[t][j][m]+pt[t][j][m];
}

forall(j in Products,m in Machines, t in Time){
c27:release<=ct[t][j][m];
}

forall(j in Products,m in Machines, t in Time){
c28:ct[t][j][m]<=due;
}

forall(j in Products,i in Products,m in Machines, t in Time){
 c29:st[t][j][m] >= st[t][i][m]+pt[t][i][m]+S[i][j] - M*(1-a[t][i][j][m]);
 }

forall(j in Products,i in Products,m in Machines, t in Time){
  c30:st[t][j][m] >= release;
}

forall(i in Products, m in Machines, t in Time){
c31: pt[t][i][m]<= M*(sum(j in nodes)a[t][i][j][m]);
}
}
```

Simulataneous.Mod

```
int nbMachines = ...;
int nbProds = ...;
int nbPeriods = ...;

range Products = 1..nbProds;
range Machines = 1..nbMachines;
range Time = 1..nbPeriods;
range T=0..nbPeriods;
range nodes = 0..nbProds;

// Parameters
```

```
///////////////////////////////Parameters
LS&WH/////////////////////////////////
 float d [T][Products] = ...; // demand in period t
 int K = ...; // fixed order cost in period t
 float h  = ...; // unit holding cost in period t
 float g = ...; // handling cost
 int p = ...; // fixed cost of increasing WH size in period t
 float r = ...;  // WH rental cost per unit in period t
 int b  = ...; // Pallet inbound capacity per period t
 int MaxWH = ...; //maximum WH capacity
 int MinWH = ...; // Minimum WH capacity
 int IINV [Products] = ...; // Initial inventory at t=0
 int n = ...; //max inbound capacity
 int F = ...; // inbound pallet cost in OT
 int SS [Products] =...; //required safetystock per product
///////////////////////////////Parameters
Schedulling///////////////////////////////
float PR[Products][Machines] = ...;
int release = ...;
int due = ...;
int S [Products][Products] =...;
int M =...;
int PCost= ...;
int SUcost= ...;

//Decision variables
//////////////////////////////Decision                            variables
schedulling///////////////////////////////
dvar float+ pt [Time][Products][Machines]; //Production time
dvar float+ st [Time][Products][Machines]; //Starting time
dvar float+ ct [Time][Products][Machines]; //Completion time
dvar boolean a [Time][nodes][nodes][Machines];

//////////////////////////////Decision                            variables
WH&LS/////////////////////////////////
dvar int+ x[T][Products]; //order quantity in period t
dvar int+ i[T][Products]; //inventory lvl at the end of period t of every
product
dvar int+ itot [T]; //sum of inventory at the end of period t
dvar int+ w[T]; // warehouse size at the end of period t
dvar int+ u[T]; // warehouse size expansion at beginning of period t
dvar int+ v[T]; // warehouse size contraction at beginning of period t
dvar int+ l[T]; // Number of pallets inbounded in OT
dvar boolean y[T]; // binary variable for ordering in period t
dvar boolean z1[T]; // binary variable for WH expansion in period t
dvar boolean z2[T]; // binary variable for WH contraction in period t


 ///////////////////////////////objecTive
funcTion/////////////////////////////////
dexpr  float  SchedCost  =  sum(t  in  Time)sum(i  in  Products)sum(j  in
Products)sum(m      in      Machines)(a[t][i][j][m]*(S[i][j]*SUcost      +
pt[t][j][m]*PCost));
dexpr float ProdCost = sum(t in T)sum(j in Products)(h*i[t][j]+g*x[t][j]);
dexpr         float         PeriodCost          =          sum(t          in
T)(K*y[t]+r*u[t]+p*z1[t]+r*v[t]+p*z2[t]+r*w[t]+l[t]*F);
dexpr float WHCost = ProdCost+PeriodCost;
dexpr float Combi = WHCost+SchedCost;

minimize Combi;
```

```
 //Constraints
 ///////////////////////////////Constraints
schedulling/////////////////////////////////
subject to{
forall(i in Products, t in Time){
c21:sum(m in Machines)sum(j in nodes:j!=i)a[t][i][j][m]<=1;  //ensures that
every product can only be produced on a single mahcine in a period and ensures
that every product is only scheduled once
  }

forall(m in Machines, t in Time){
c22:sum(j in Products)a[t][0][j][m]<=1; // Ensures that at most 1 product
starts on a machine in a period
}

forall(j in Products,m in Machines, t in Time){
c23:sum(i in nodes)a[t][i][j][m]==sum(i in nodes)a[t][j][i][m];  //Flow
constraint, to ensure that if j is scheduled after i, then i cannot be
scheduled after j
}

forall(i in Products,m in Machines, t in Time){
c24: a[t][i][i][m]==0;                          // Ensures that if j is
scheduled after i, then i cannot be scheduled after j
}

forall(j in Products, t in Time){
c25:x[t][j]<=sum(m in Machines)(pt[t][j][m]*PR[j][m]);  // Ensures that
exactly the order will be produced in a period
}

forall(j in Products,m in Machines, t in Time){
c26:ct[t][j][m]==st[t][j][m]+pt[t][j][m];     //introduces that completion
time of a production run
}

forall(j in Products,m in Machines, t in Time){
c27:release<=ct[t][j][m];                     //Ensures that the completion
time of a product should be after the release date
}

forall(j in Products,m in Machines, t in Time){
c28:ct[t][j][m]<=due;                     //Ensures that the completion time
of a product should be before the release date
}

forall(j in Products,i in Products,m in Machines, t in Time){
c29:st[t][j][m] >= st[t][i][m]+pt[t][i][m]+S[i][j] - M*(1-a[t][i][j][m]);
//Ensures that a starting time of a product j is bigger than the startingtime
of the prior product +  the production time and the setup time
 }

forall(j in Products,i in Products,m in Machines, t in Time){
c30: st[t][j][m] >=release;              //Ensures that the starting time of
every production should be bigger than or equal to the release date
}

forall(i in Products, m in Machines, t in Time){
c31: pt[t][i][m]<= M*(sum(j in nodes)a[t][i][j][m]);   // Links production
times to the binary assignment variable
}
```

```
forall(j in Products, t in Time){
c32:sum(t in 1..t)x[t][j]>= sum(t1 in 1..t)d[t1][j];
}
}

 ///////////////////////////////Constraints
LS&WHS////////////////////////////////
subject to {

forall(t in Time, j in Products) {
c1: i[t-1][j] + x[t][j] - d[t][j] == i[t][j];      //Inventory balance
constraint if order takes 1t to arrive would x[t-1] be the right adjustment?
}

forall(t in Time, j in Products) {
c2:x[t][j] <= (sum(t1 in t..nbPeriods)d[t1][j])*y[t];   // dtT Cumulative
demand from t to end T, ensures that Y[t] holds as a binary variable
}

forall(t in Time, j in Products) {
c3: i[t][j]>=SS[j]; // Ensures that in all periods t the inventory level
should be as least as high as the required SS
}

forall(t in Time){
c4: w[t-1] + u[t] - v[t] == w[t];   //warehouse balance constraint
}

forall(t in Time){
c5:(sum(j in Products)i[t][j]) <= w[t];    // inventory should be max. the
warehouse level
}

forall(t in Time){
c6:u[t] <= (sum(t in T)sum(j in Products)d[t][j])*z1[t]; // Total demand from
1 to T * z1
}

forall(t in Time){
c7:v[t] <= (sum(t in T)sum(j in Products)d[t][j])*z2[t]; // Total demand from
1 to T  *z2
}

forall(t in Time){
c8:w[t] <= MaxWH; // WH max available capacity
}

forall(t in Time){
c9:w[t] >= MinWH; // Min WH capacity that is agreed upon
}

forall(t in Time){
c10:(sum(j in Products)x[t][j])<= n + l[t]; //sum of ordered products should
be below inbound capacity
}

forall(t in Time){
c11:l[t] <= b; // upperbound of OT capacity for inbounding pallets in a period
}
```

```
forall(t in Time,j in Products){
c12:(sum(j in Products)i[t][j]) == itot[t]; //For visualization of itot
}

forall(j in Products){
c13:i[0][j]== IINV[j];   // initial inv prod
}

c14:w[0]==MinWH;  //initial warehouse size
}


execute
{
var f=new IloOplOutputFile("output.csv");
f.writeln("d",d);
f.writeln("x",x);
f.writeln("w",w);
f.writeln("itot",itot);
f.writeln("y",y);
f.writeln("l",l);
f.writeln("a",a);
f.close();
```