THE SPATIAL ORIGIN OF -5/3 SPECTRA IN GRID-GENERATED TURBULENCE

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Abstract A combined wind tunnel and computational study of grid-generated turbulence along the centreline shows that the close to $-5/3$ power law signature of energy spectra in the frequency domain originates in the production region close to the grid where the velocity derivative statistics become quite suddenly isotropic but also where the turbulent fluctuating velocities are very intermittent and non-Gaussian. As the inlet flow velocity increases, these power laws are increasingly well defined and increasingly close to $-5/3$ over an increasing range of frequencies. However, this range continuously decreases with streamwise distance from the grid even though the local Reynolds number first increases and then decreases along the same streamwise extent. The intermittency at the point of origin of the close to $-5/3$ power spectra consists of alternations between intense vortex tube clusters with shallow broad-band spectra and quiescent regions where the velocity fluctuations are smooth with steep energy spectra.

INTRODUCTION

Grid-generated turbulence is arguably the oldest canonical test bed of turbulence theories, and more specifically Kolmogorov's 1941 theory of small-scale turbulence (see Batchelor 1953 "The theory of homogeneous turbulence", CUP). In grid-generated turbulent flows, the turbulence intensity along a centreline streamwise axis increases first (production region) till it reaches a peak at a distance x_{peak} from the grid beyond which it continuously decays further downstream (decay region). The Kolmogorov 1941 prediction that the energy spectrum scales as the $-5/3$ power of wavenumber is verified in the near-field decay region of the flow where the turbulence is not in Richardson-Kolmogorov equilibrium (see Vassilicos 2015 Ann. Rev. Fluid Mech. 47, 95-114 and references therein) but not in the far downstream region where the dissipation coefficient C_ϵ acquires a constant value (e.g. see most recent works by Isaza, Salazar & Warhaft 2014, J. Fluid Mech. 753, 402-426 and Hearst & Lavoie 2014, J. Fluid Mech. 741, 567-584, but also references therein to previous works by Corrsin and coworkers and others). In the decay region, the Reynolds number continuously drops with increasing downstream distance from the grid. It is therefore often thought that the absence of a -5/3 spectrum in the far field is simply caused by the low local Reynolds number there, typically about 100 and below for the Taylor length-scale based Reynolds number Re_λ.

There are two possible avenues for progress beyond this state of affairs. One is to attempt to generate a far downstream field with very high local Re_λ values, $C_\epsilon = Const$ and a well-defined $-5/3$ energy spectrum. It is yet unknown whether such a far field exists at high enough Reynolds numbers, so such an avenue of research is clearly welcome.

The other avenue is to go in the opposite direction and study the near field. The aim of this avenue is to determine the streamwise position closest to the grid where $-5/3$ spectra first appear. The mechanism responsible for $-5/3$ spectra in grid-generated turbulence remains unknown, and one might expect the cause of the $-5/3$ to be present at the streamwise point closest to the grid where this $-5/3$ power law first appears. We do already know that well-defined $-5/3$ power-law spectra are present in the near-field decay region where C_ϵ is not constant (see Vassilicos 2015 and references therein). Could it be that the -5/3 scaling appears, in fact, even closer to the grid, i.e. in the production region?

It must be stressed that these two research avenues are complementary. It is conceivable, for example, that $-5/3$ power-law spectra occur as a result of a mechanism different from Kolmogorov's 1941 equilibrium cascade yet the presence of such spectra may themselves cause an evolution towards a Richardson-Kolmogorov equilibrium cascade very far downstream if the Reynolds numbers are high enough. This scenario would nevertheless require a non-equilibrium turbulence

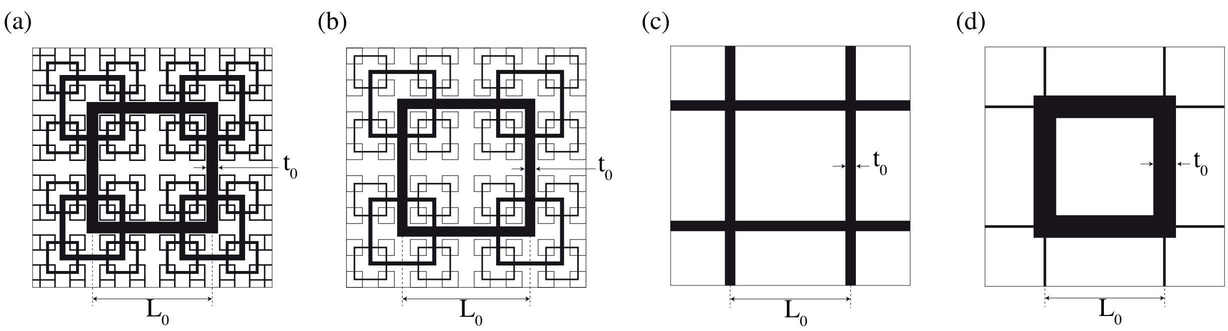

Figure 1. Turbulence generating grids. From left to right, FSG8, FSG17, RG230 and SSG.

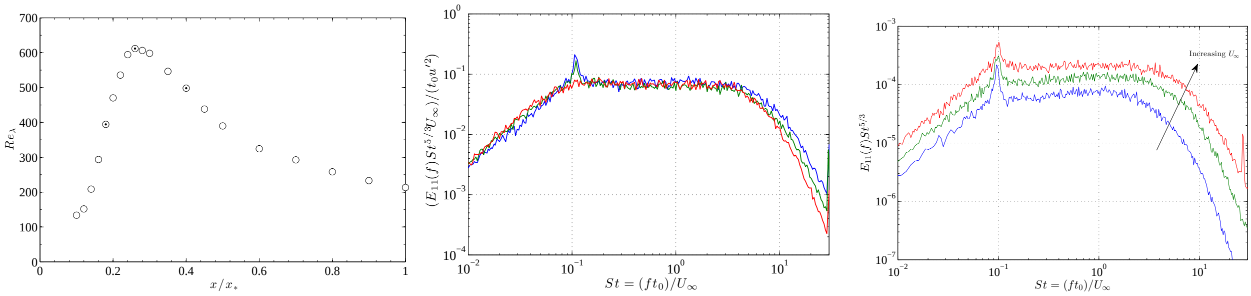

Figure 2. Wind tunnel measurements with the FSG8 grid. Left: Streamwise evolution of Re_λ along the centreline normalised by the wake-interaction length-scale $x_* = L_0^2/t_0$ (L_0 and t_0 are shown in figure 1) at $U_\infty = 10m/s$. Middle: Log-log plot of compensated energy spectra $E_{11}(f)St^{5/3}U_\infty/(t_0u'^2)$ (where u' is the rms streamwise fluctuating velocity, f is a frequency) versus the non-dimensional frequency $St = ft_0/U_\infty$. $U_\infty = 10m/s$. The three curves correspond to the three centreline positions x marked with black circles on the left plot. The curve (in blue) with the most intense peak at St close to 10^{-1} and with the most energy at the highest frequencies is for $x = 0.18x_* = x_{53}$, the location closest to the grid where $E(f) \sim f^{-5/3}$ over a significant range. The curve without a peak at St close to 10^{-1} and with the least energy at the highest frequencies is for $x = 0.4x_*$ (in red). The other curve is for $x = 0.26x_*$ (in green). Right: Log-log plot of $E_{11}(f)St^{5/3}$ versus St at $x = 0.2x_*$ for $U_\infty = 2.5m/s$ (blue), $5m/s$ (green) and $10m/s$ (red).

to evolve towards an equilibrium turbulence as the Reynolds number *decays*. It is therefore also conceivable that the turbulence never evolves towards a Richardson-Kolmogorov equilibrium cascade far-downstream, however high the Reynolds numbers. These are important open questions which delineate the long term scope of our investigations.

In this particular work we report progress on the second research avenue. We carry out direct numerical simulations (DNS) and hot wire anemometry (HWA) measurements of grid-generated turbulence. We do not use classical regular grids because the production region of such grids is extremely short, in fact confined to the immediate neighbourhood of the grids, and because the turbulence intensity in the production region of such grids is extremely high (up to and above 20%). To facilitate the use of HWA measurements and limit the demands on our DNS resolution in the production region we use four different passive grids all designed such that the production region upstream of the decay region is quite long and with moderate turbulence intensities. We therefore conduct wind tunnel experiments of turbulence generated by two different fractal grids (FSG8 and FSG17 in figure 1) and two different single mesh grids (RG230 and SSG in figure 1). We also carry out DNS of turbulence generated by the FSG8 and SSG grids. These different grids generate different Reynolds numbers and different (in)homogeneity profiles thus permitting general conclusions.

RESULTS

Given the very large size of our simulations, we solve the incompressible Navier-Stokes equations on a Cartesian mesh with the parallel version (see Laizet & Li 2011, Int. J. Numer. Methods Fluids 67, 1735-1757) of our numerical code **Incompact3d** which is based on sixth-order compact schemes in space and a third order Adams-Bashforth scheme in time. Full details on the code, its validations and its application to grid-generated turbulence can be found in Laizet & Lamballais (2009, J. Comp. Phys. 228, 5989-6015) and Laizet & Vassilicos (2011, Flow, Turbulence and Combustion 87, 673-705). HWA measurements were taken in our 0.4572m $\times$ 0.4572m blow down wind tunnel. The test section is 3.5m long and the background turbulence level at the wind tunnel speeds we applied is about 0.1%. For a description of the experimental set-up, HWA and wind tunnel, see Valente & Vassilicos (2011, J. Fluid Mech. 687, 300-340).

Our HWA and DNS results along the centreline show that $-5/3$ frequency spectra (see figure 2) originate inside the production region $x < x_{peak}$ around a position x_{53} rather close to the grid where the velocity derivative statistics become quite suddenly isotropic but also where the turbulent fluctuating velocities are very intermittent and non-Gaussian. The streamwise location x_{53} is independent of the inlet flow velocity U_∞ but does vary from grid to grid. Our DNS results show that the intermittency at x_{53} consists of alternations between vortex tube clusters with shallow broad-band spectra and quiescent regions where the velocity fluctuations are smooth with steep energy spectra. As x increases beyond x_{53} the range over which the $-5/3$ spectrum is present diminishes even if Re_λ does not diminish (see figure 2). It is the inlet Reynolds number based on U_∞ and not the local Reynolds number Re_λ which determines the extent of the $-5/3$ frequency spectrum, see figure 2 (right).

We therefore now know the spatial origin of $-5/3$ spectra in grid-generated turbulence and properties characterising these spectra and the flow at that spatial origin. The next step is to educe the mechanism responsible for the $-5/3$ power-law spectra and work in this direction is under way.