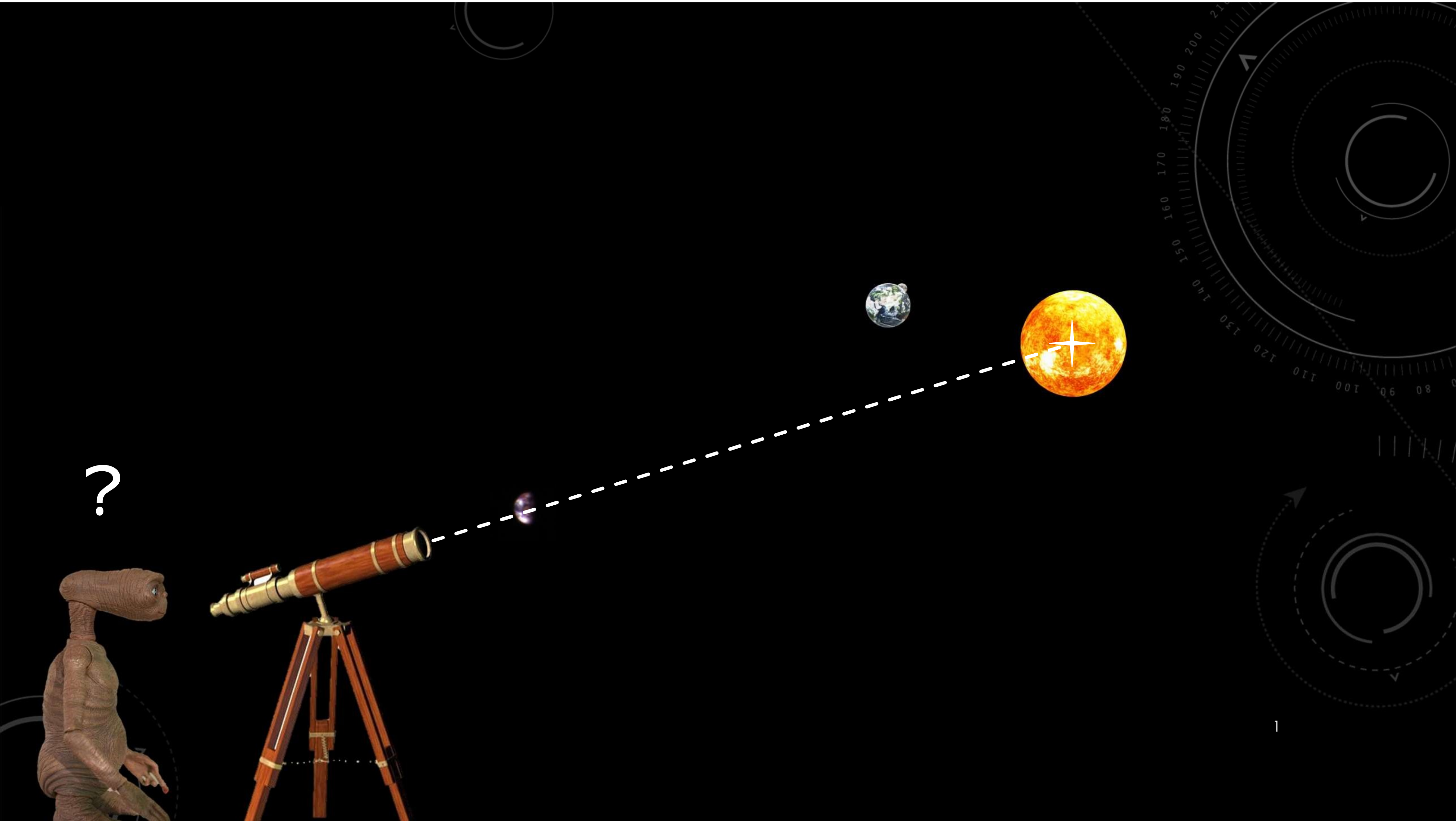


EXO-CARTOGRAPHY

BACHELOR THESIS SIMON VAN OOSTEROM

PRESENTATION CONTENT

01
What is the map and the signal

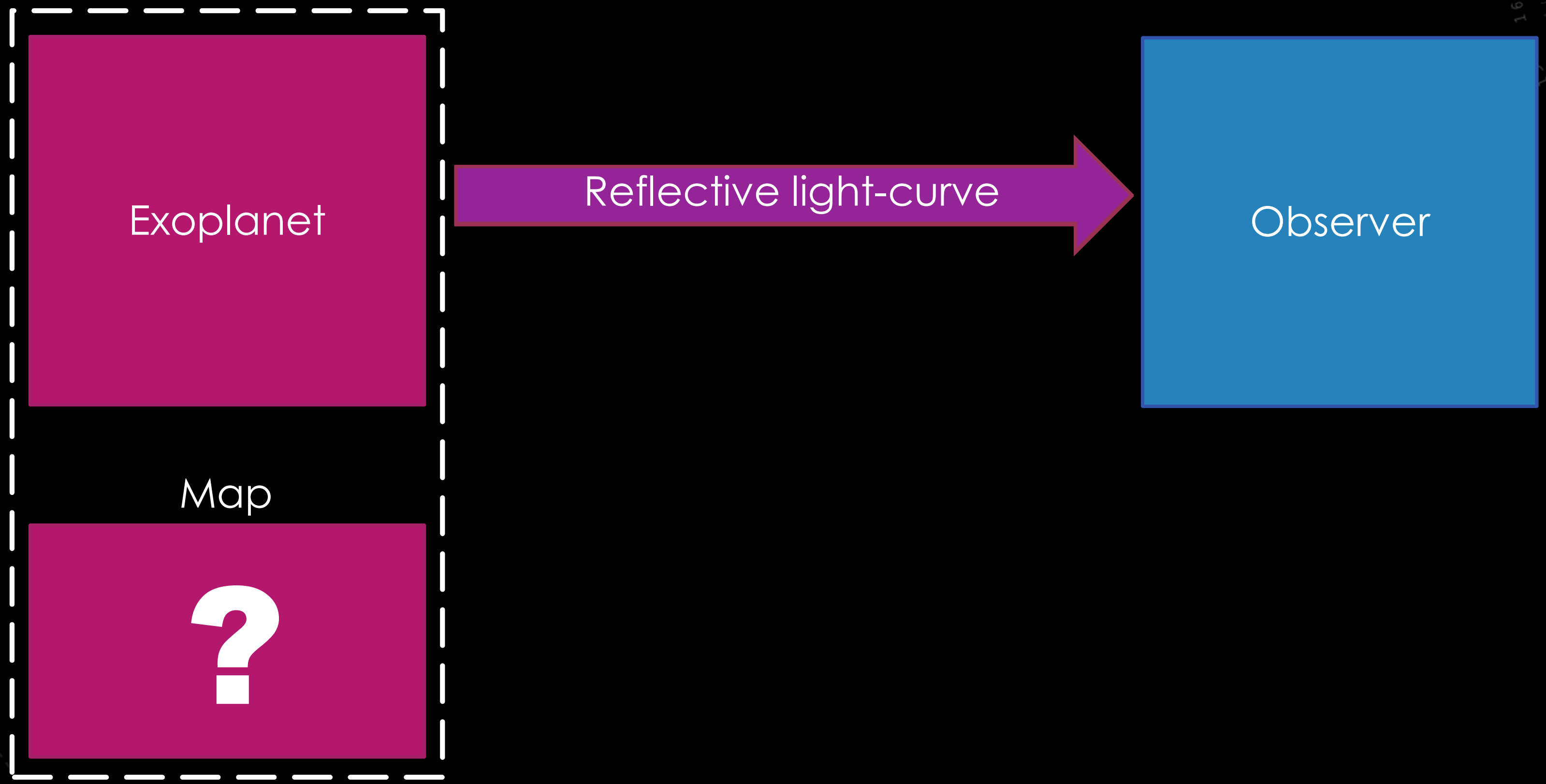
02
How orientation determines the signal

03
How do we calculate this signal

04
Retrieve the map with known orientation

05
Retrieve the map with unknown orientation

TO RECONSTRUCT A SURFACE MAP

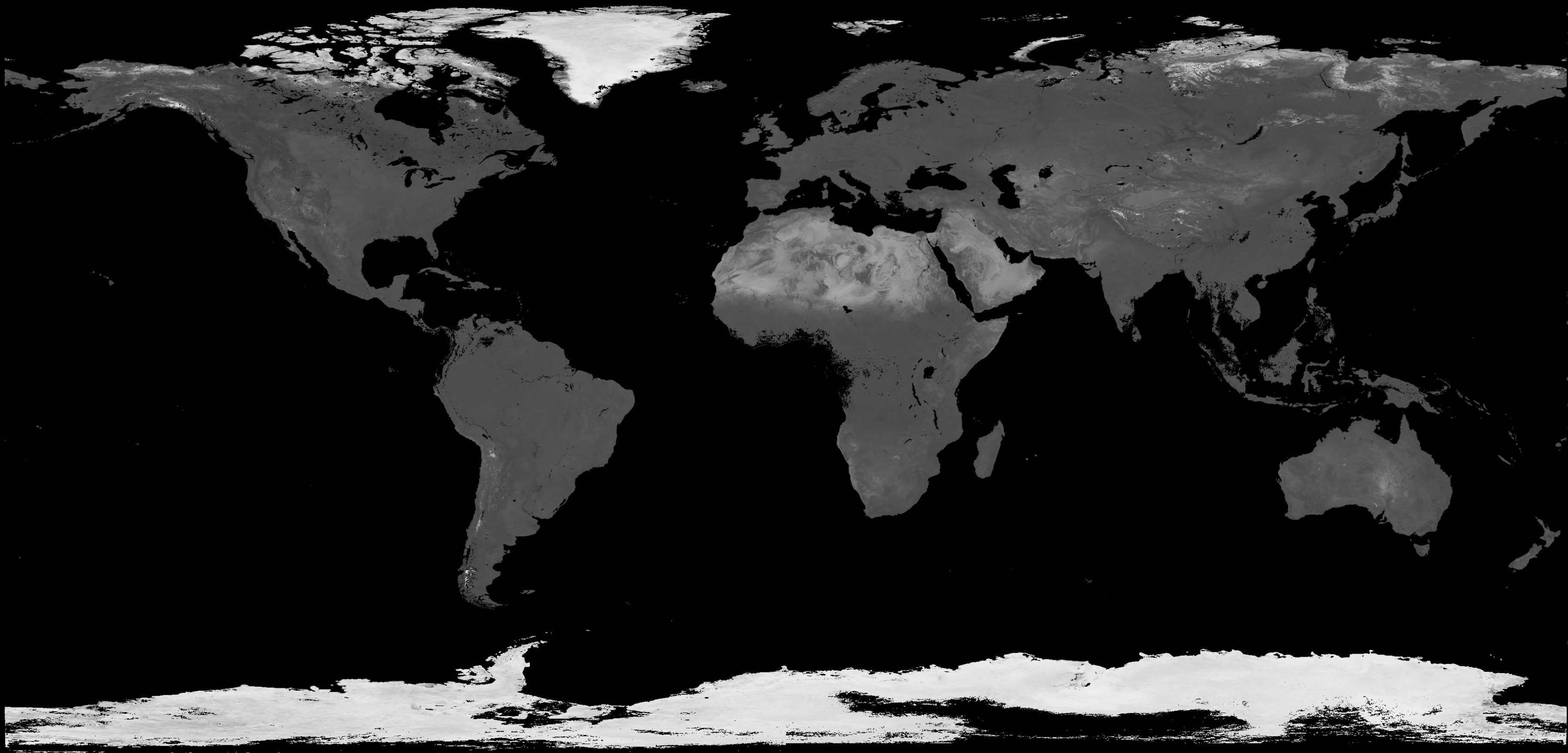


01

ALBEDO

- **Albedo** (/ælˈbiːdoʊ/) (Latin: *albedo*, meaning 'whiteness')
- Absorption of light at the surface
- Expressed as a number between 0 and 1
  - 0: surface absorbs all light
  - 1: surface reflects all light

01

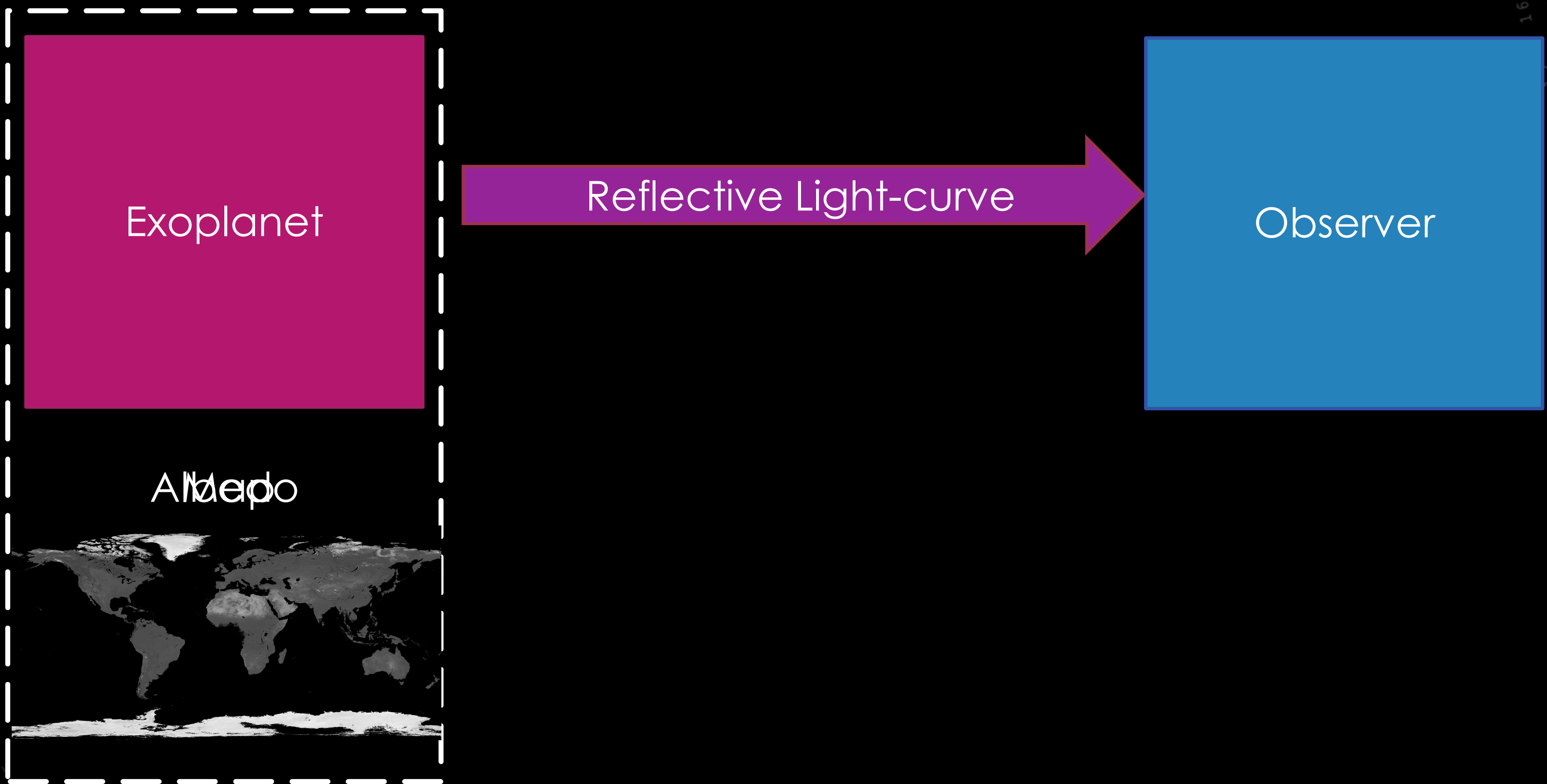


ALBEDO MAP OF EARTH

Source: NASA

01

TO RECONSTRUCT A SURFACE MAP



01

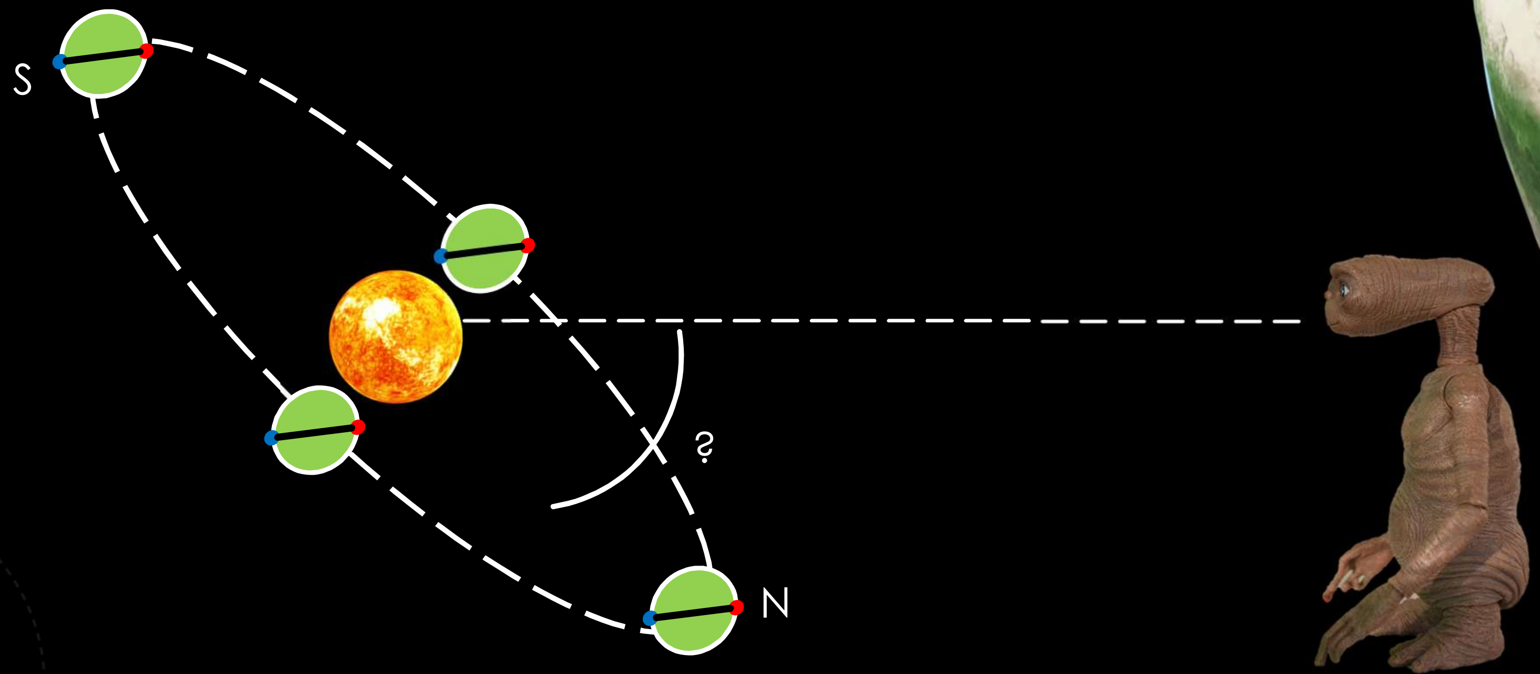
WHAT DETERMINES THE LIGHT-CURVE: THE MAP

- Half of the planet is illuminated by the star
- Half of the planet is visible to ET
- Rotation gives a signal, dependent on the map

02

WHAT DETERMINES THE LIGHT-CURVE: ORIENTATION

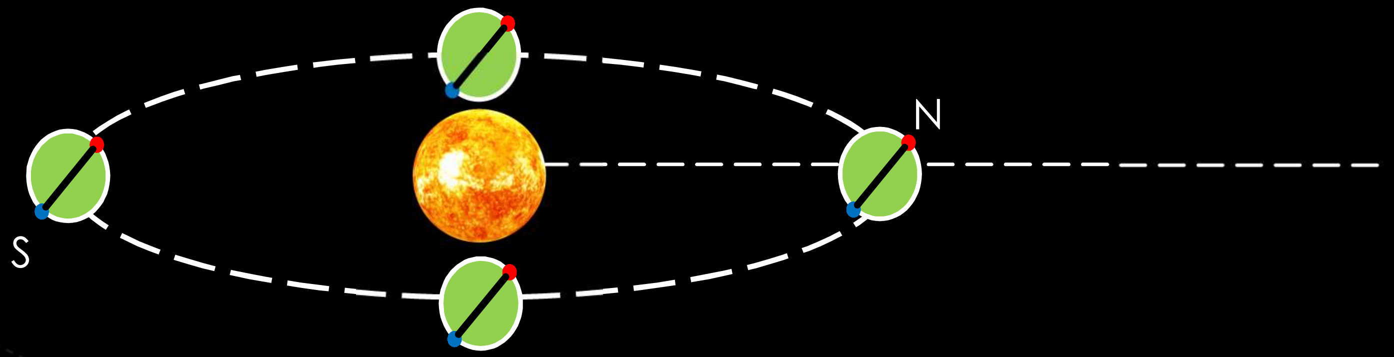
- north-south axis
- Observer perspective



02

WHAT DETERMINES THE LIGHT-CURVE: ORIENTATION

- north-south axis
- Observer perspective: Edge-on observation



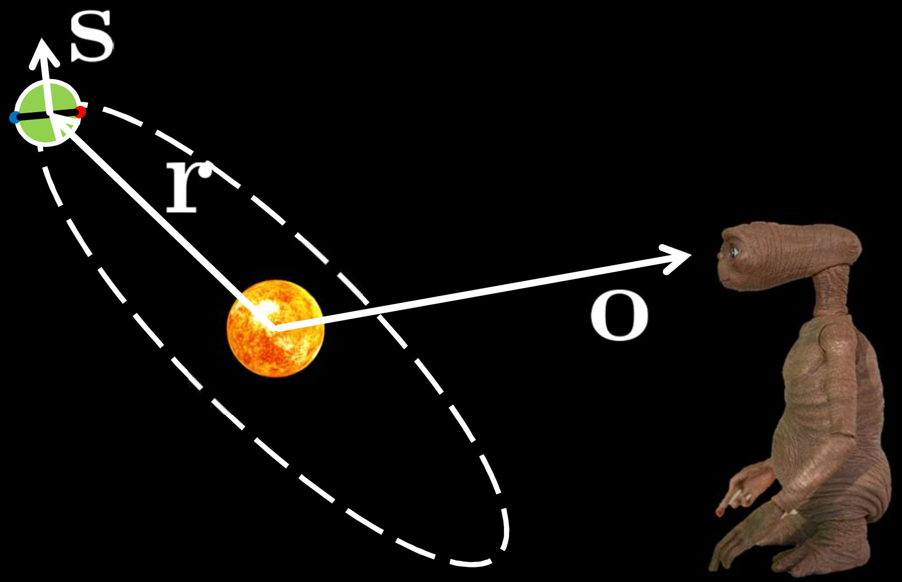
02

NEXT GEAR

03

THE REFLECTIVE LIGHT CURVE EQUATION

- Vectors
  - Planet $\mathbf{r}$
  - Surface $\mathbf{s}$
  - Observer $\mathbf{o}$
- Albedo Map $M(\mathbf{s})$
- Light-curve $f(t)$



$$f(t) = \frac{1}{\pi R^2} \iint_{\text{vis}} (-\hat{\mathbf{r}} \cdot \hat{\mathbf{s}})(\hat{\mathbf{s}} \cdot \hat{\mathbf{o}}) M(\mathbf{s}) d^2 S$$

03

THE REFLECTIVE LIGHT CURVE EQUATION

$$f(t) =$$

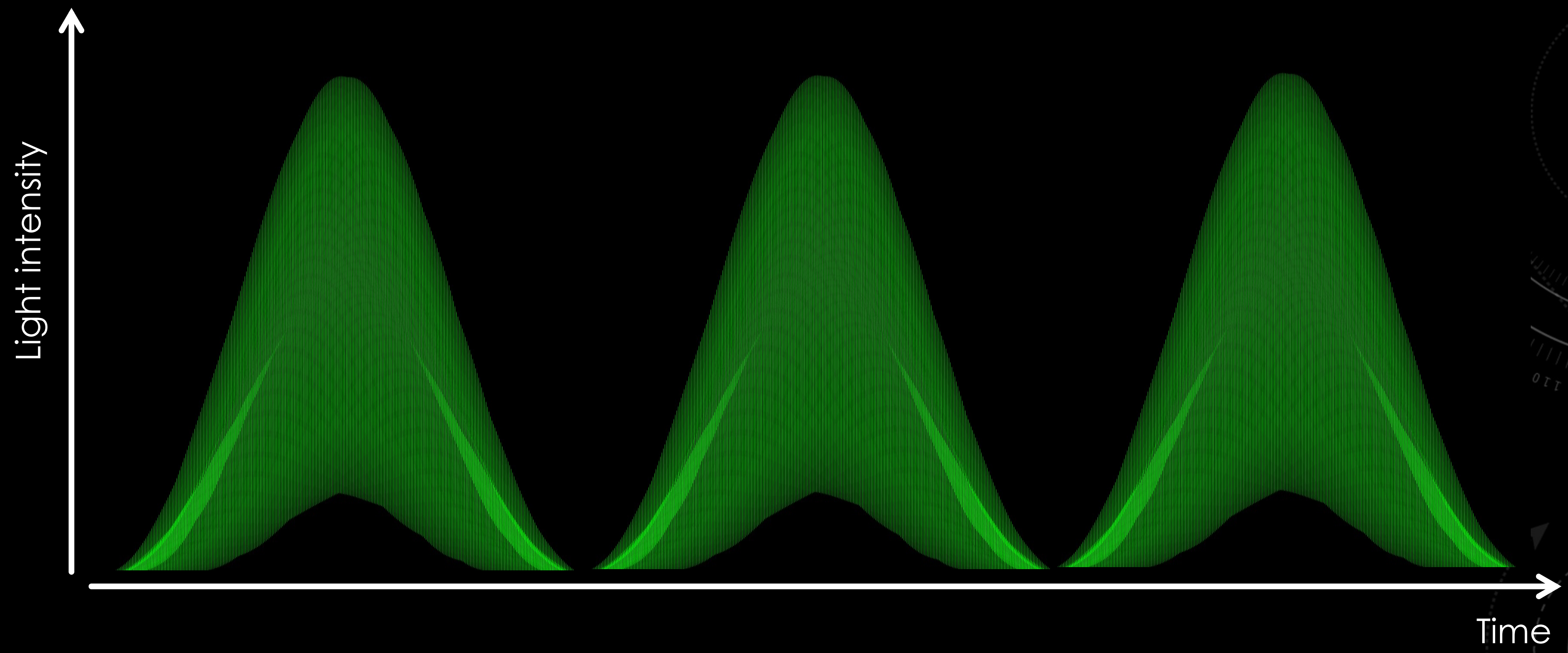
The light-curve

03

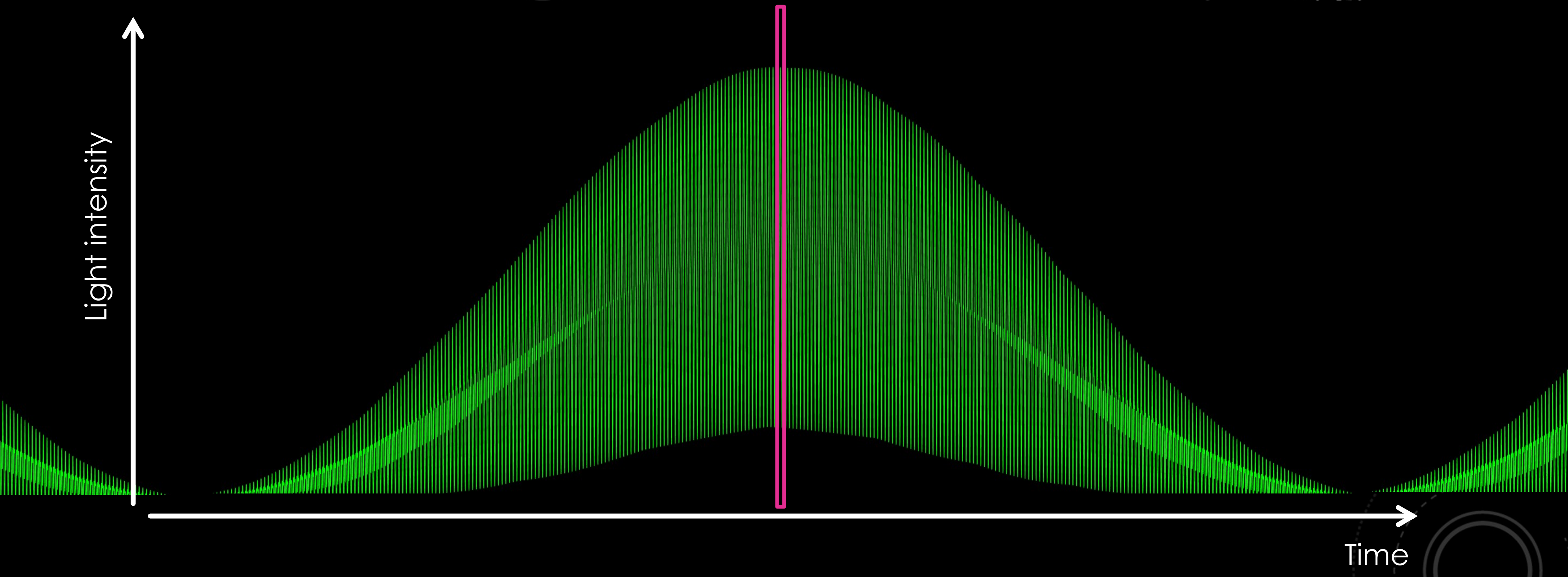
THE REFLECTIVE LIGHT CURVE EQUATION

$$f(t) = \frac{1}{\pi R^2} \iint_{\text{vis}} (-\hat{\mathbf{r}} \cdot \hat{\mathbf{s}})(\hat{\mathbf{s}} \cdot \hat{\mathbf{o}}) M(\mathbf{s}) d^2 S$$

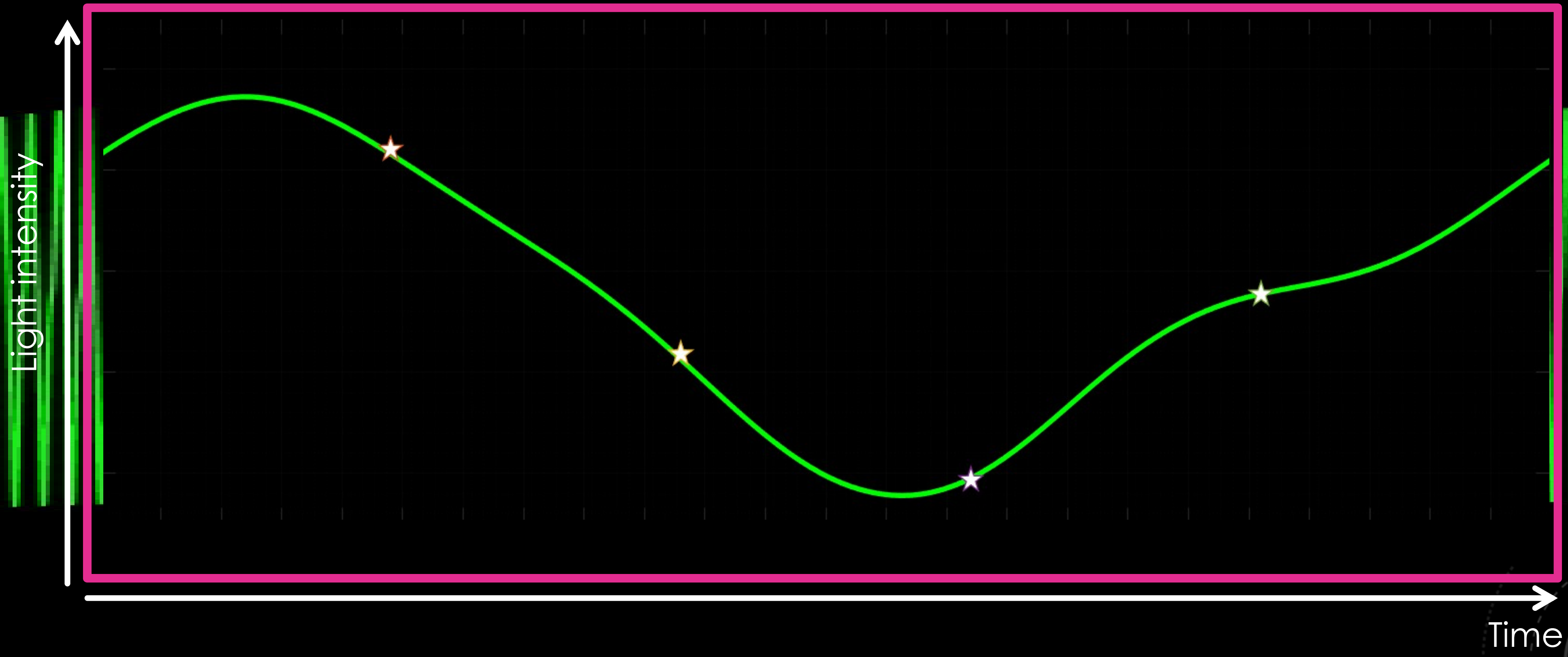
$$f(t) = A(M)$$



03 Light-curve, multiple years



03 Light-curve, one year



17

03 Light-curve, one day

HOW TO INVERT WITH KNOWN TILT

$$f(t) = \frac{1}{\pi R^2} \iint_{\text{vis}} (-\hat{\mathbf{r}} \cdot \hat{\mathbf{s}})(\hat{\mathbf{s}} \cdot \hat{\mathbf{o}}) M(\mathbf{s}) d^2 S$$

$$f(t) = A(M)$$

$$A(M_1 + M_2) = A(M_1) + A(M_2)$$

$$A(cM_1) = cA(M_1)$$

$A(M)$ is a linear transformation

04

$$f(t) = A(M)$$

$$A(M_1 + M_2) = A(M_1) + A(M_2)$$

$$A(cM_1) = cA(M_1)$$

$A(M)$ is a linear transformation

$f(t) = \sum_\nu f_\nu e^{i\nu t}$ Fourier series

$M(\mathbf{s}) = \sum_{l,m} M_l^m Y_l^m(\mathbf{s})$ SpH series

$$\sum f_\nu e^{i\nu t} = \sum_{l,m} M_l^m A(Y_l^m)$$

04

$$M(\mathbf{s}) = \sum_{l,m} M_l^m Y_l^m(\mathbf{s}) \text{ SpH series}$$

$$\sum f_\nu e^{i\nu t} = \sum_{l,m} M_l^m A(Y_l^m)$$

Projecting

$$f_{\nu_0} = \sum_{l,m} A_l^m(\nu_0) M_l^m$$

04

$$f_{\nu_0} = \sum_{l,m} A_l^m(\nu_0) M_l^m$$
$$f_{\nu_1} = \sum_{l,m} A_l^m(\nu_1) M_l^m$$
$$f_{\nu_2} = \sum_{l,m} A_l^m(\nu_2) M_l^m$$
$$\vdots$$
$$f_{\nu_N} = \sum_{l,m} A_l^m(\nu_N) M_l^m$$

$$\mathbf{f} = A\mathbf{M}$$

04

$$f_{\nu_0} = \sum_{l,m} A_l^m(\nu_0) M_l^m$$
$$f_{\nu_1} = \sum_{l,m} A_l^m(\nu_1) M_l^m$$
$$f_{\nu_2} = \sum_{l,m} A_l^m(\nu_2) M_l^m$$
$$\vdots$$
$$f_{\nu_N} = \sum_{l,m} A_l^m(\nu_N) M_l^m$$

$$\mathbf{f} = A\mathbf{M}$$

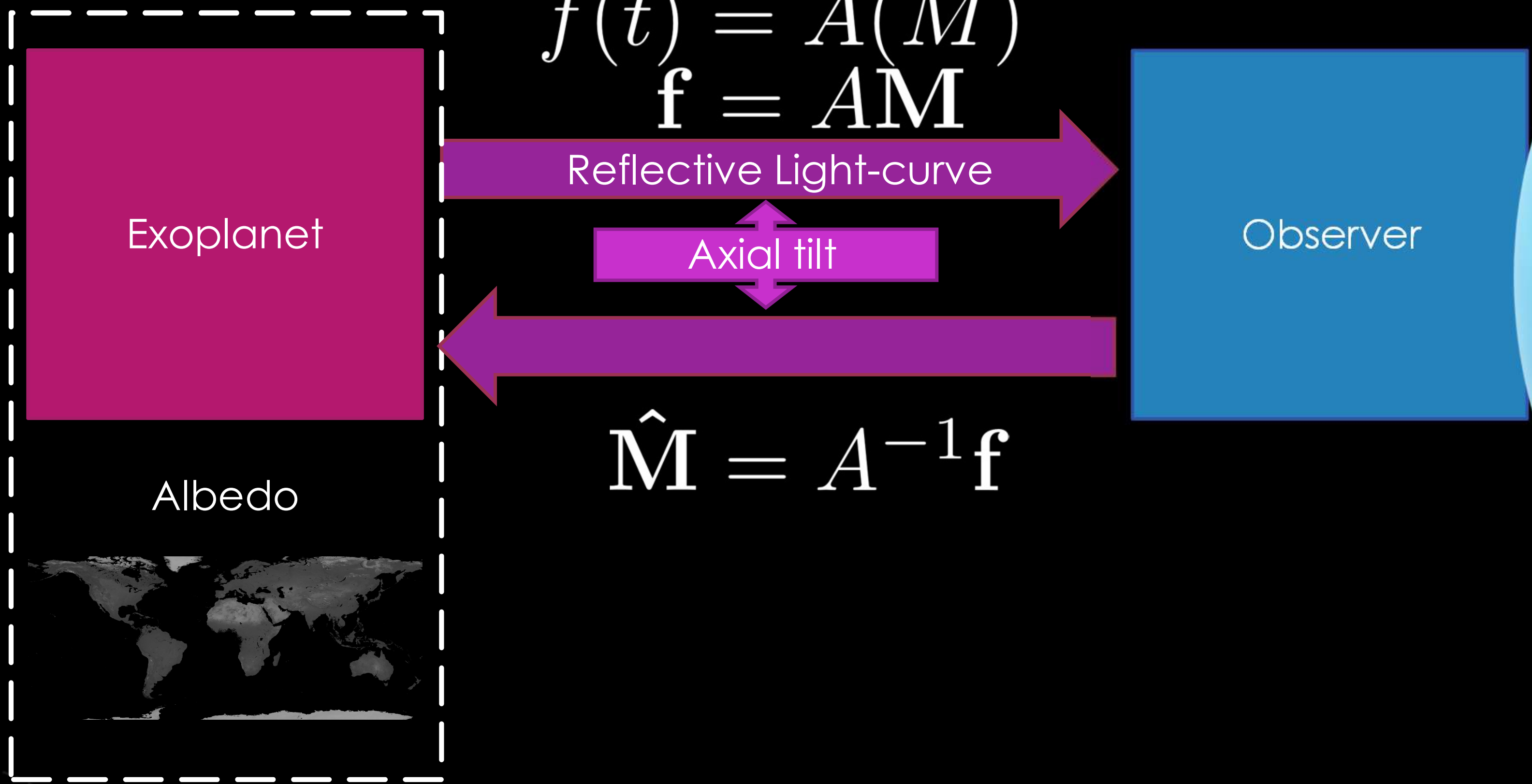
$$\hat{\mathbf{M}} = A^{-1}\mathbf{f}$$

04

TO RECONSTRUCT A SURFACE MAP

$$f(t) = A(M)$$

$$\mathbf{f} = A\mathbf{M}$$

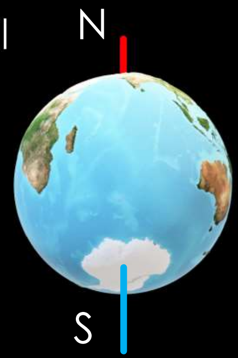


$$\hat{\mathbf{M}} = A^{-1}\mathbf{f}$$

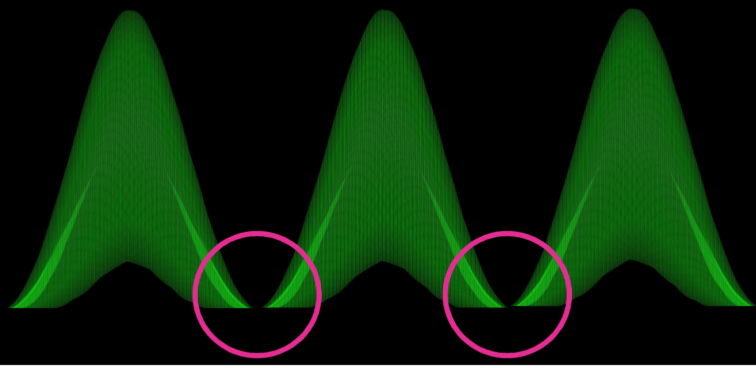
04

IS THE TRANSFORMATION INVERTIBLE?

- A is <u>not</u> injective ⇔ Multiple maps give the same signal
  - Observer cannot see the north pole
  - Change the map on the north pole➔
  - Same signal

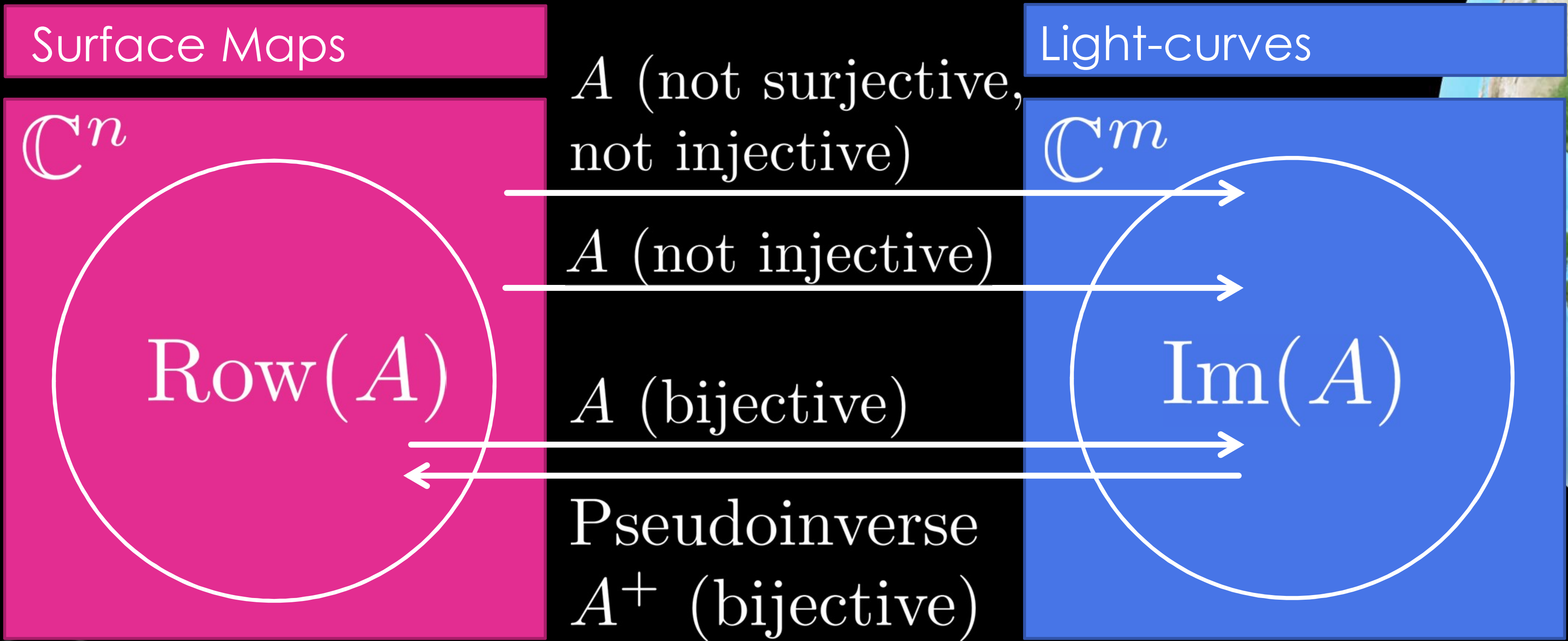


- A is <u>not</u> surjective ⇔ Not all light-curves can occur
  - When the planet is between the star and the observer
  - Nothing is illuminated and visible ➔
  - For this t: $f(t) = 0$
- A solution does exist



04

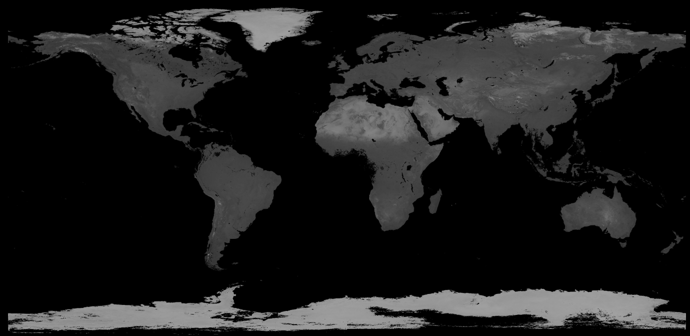
IS THE TRANSFORMATION INVERTIBLE?

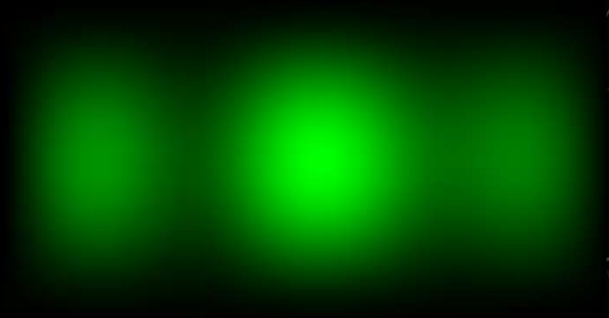


04

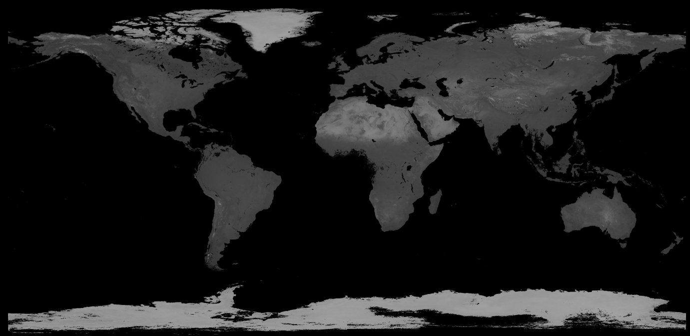
RECONSTRUCTING EARTH'S SURFACE MAP

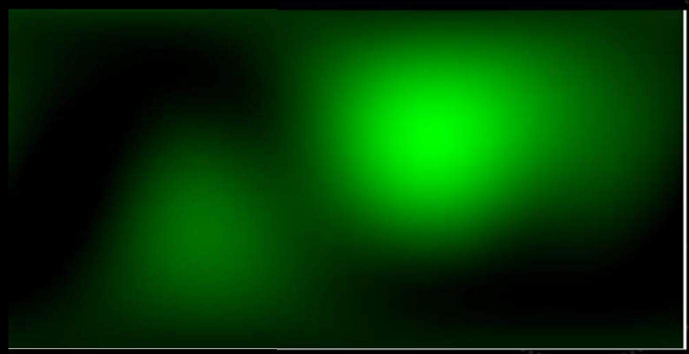
Non-tilted





tilted



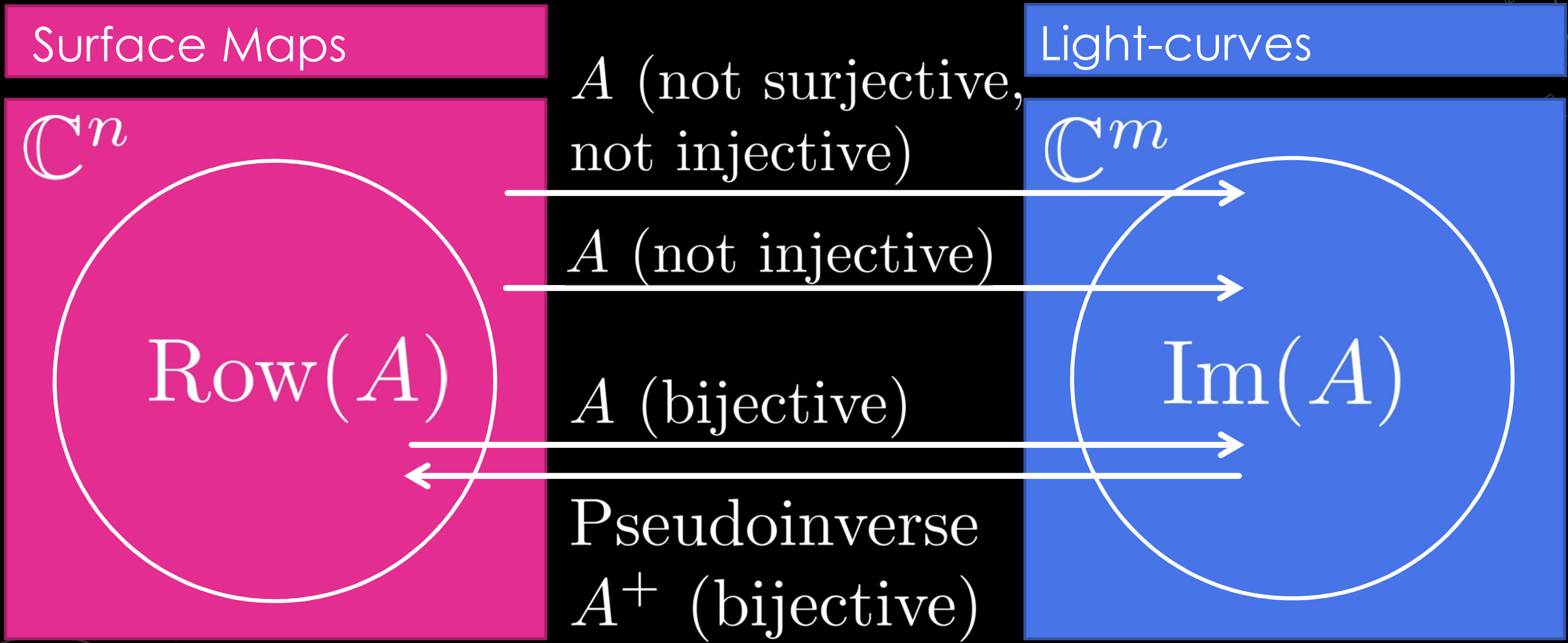


04

HOW TO INVERT WITH UNKNOWN TILT

05

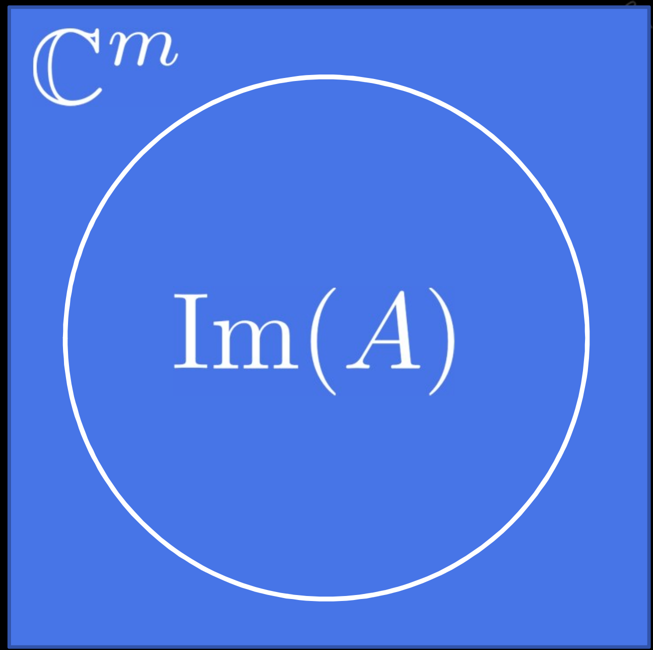
UNKNOWN TILT



04

UNKNOWN TILT

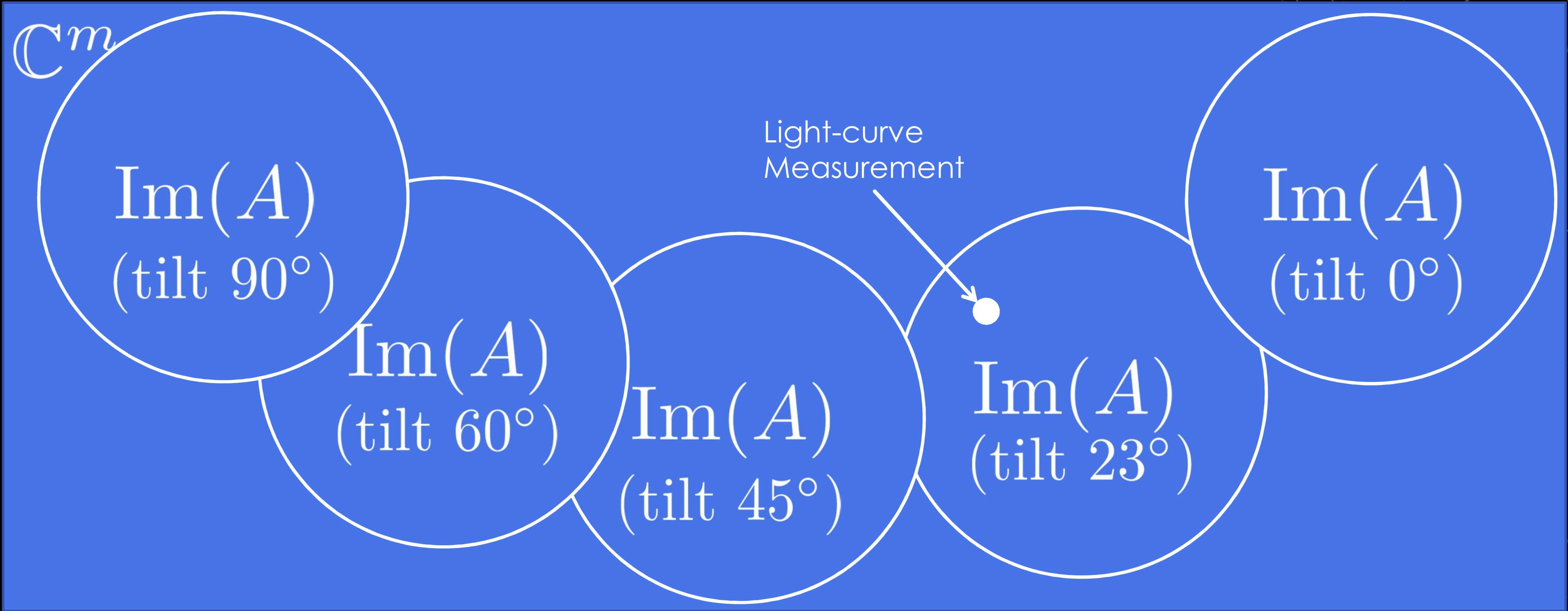
Light-curves



04

UNKNOWN TILT?

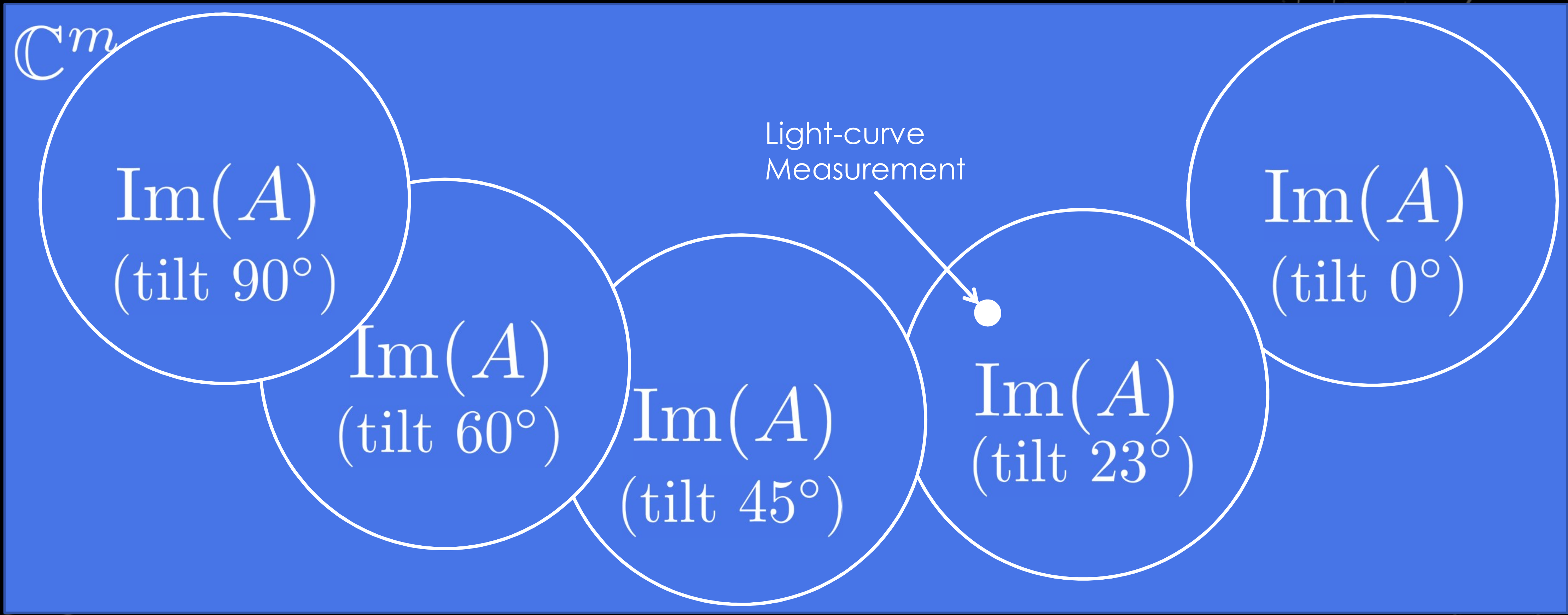
Light-curves



05

DOES TILT AFFECT THE IMAGE?

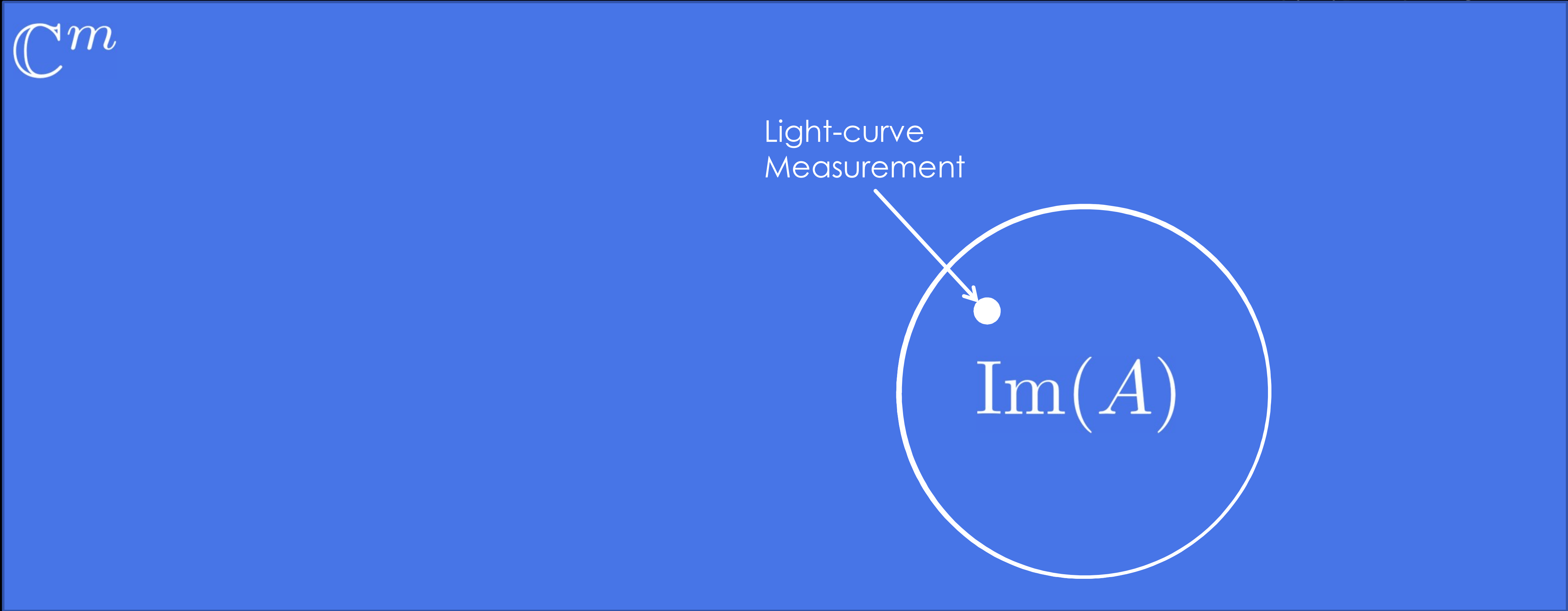
Light-curves



05

DOES TILT AFFECT THE IMAGE? NO!

Light-curves



05

TO RECONSTRUCT A SURFACE MAP

$$f(t) = A(M)$$

$$\mathbf{f} = A\mathbf{M}$$

Exoplanet

Reflective Light-curve

Axial tilt

observer

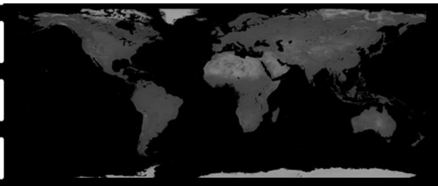
Albedo

$$\hat{\mathbf{M}} = A^{+1}\mathbf{ff}$$

- We do not know the axial tilt.
- Do not know the pseudoinverse

Recovered Albedo

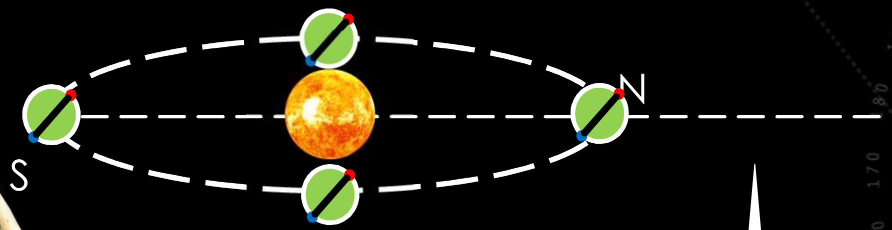
05



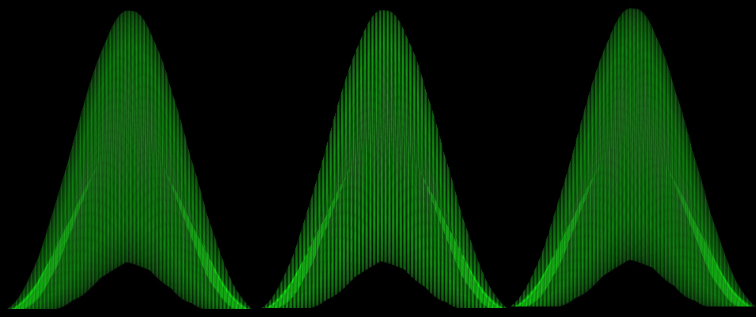
- We do not know the axial tilt.
- Do not know the pseudoinverse

CONCLUSIONS

- We can reconstruct the albedo map of an exoplanet if we know the tilt
- The image of the transformation is not a function of the tilt ➔
  - No reconstructions of maps with unknown tilt



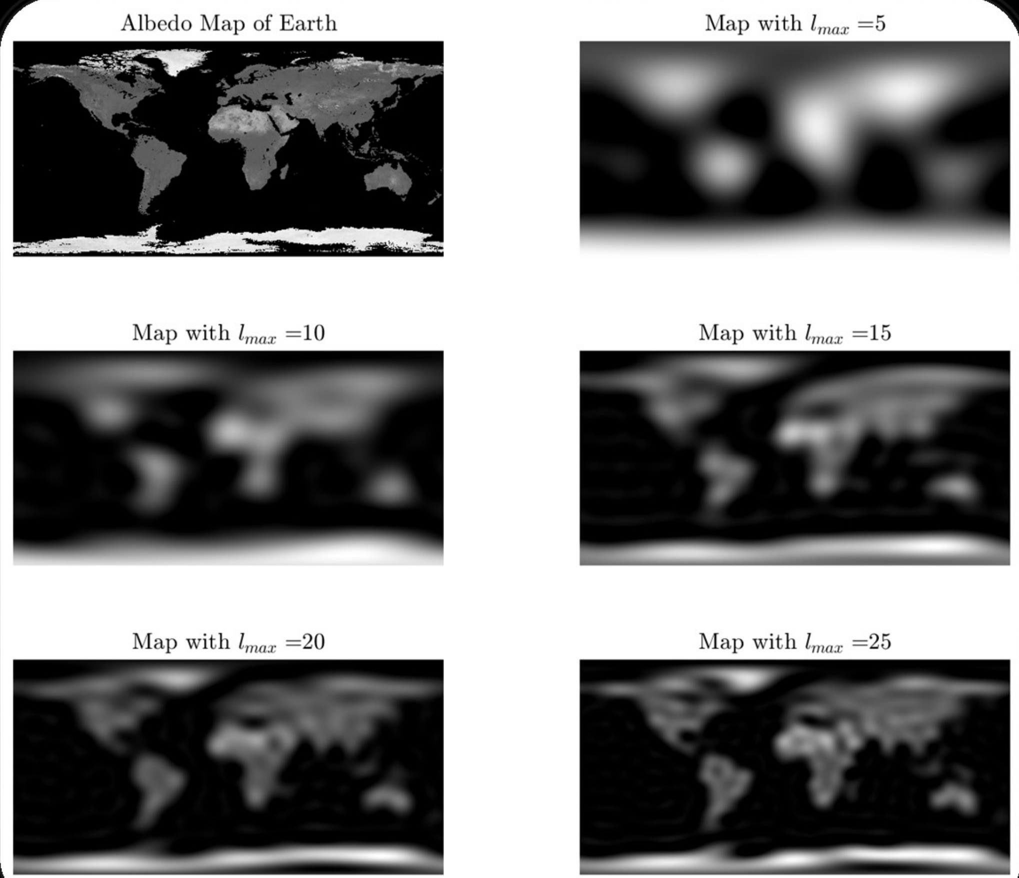
Questions?



IMPORTANT REFERENCES

- Fujii, Y. and Kawahara, H. (2010). Mapping clouds and terrain of earth-like planets from photometric variability: demonstration with planets in face-on orbits. The Astrophysical Journal Letters, 739(2).
- Fujii, Y. and Kawahara, H. (2012). Mapping earth analogs from photometric variability: Spin-Orbit Tomography for planets in inclined orbits. The Astrophysical Journal, 755(2).
- Visser, P.M. and van der Bult, F. (2015). Fourier spectra from exoplanets with polar caps and ocean glint. Astronomy and Astrophysics, 597.

DECOMPOSITION OF THE MAP



$$M(\theta_p, \phi_p) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} M_l^m Y_l^m(\theta_p, \phi_p)$$

OTHER PLANETS OR MOONS?

- What to do if there is more tha one planet in the solar system?
- Other frequencies

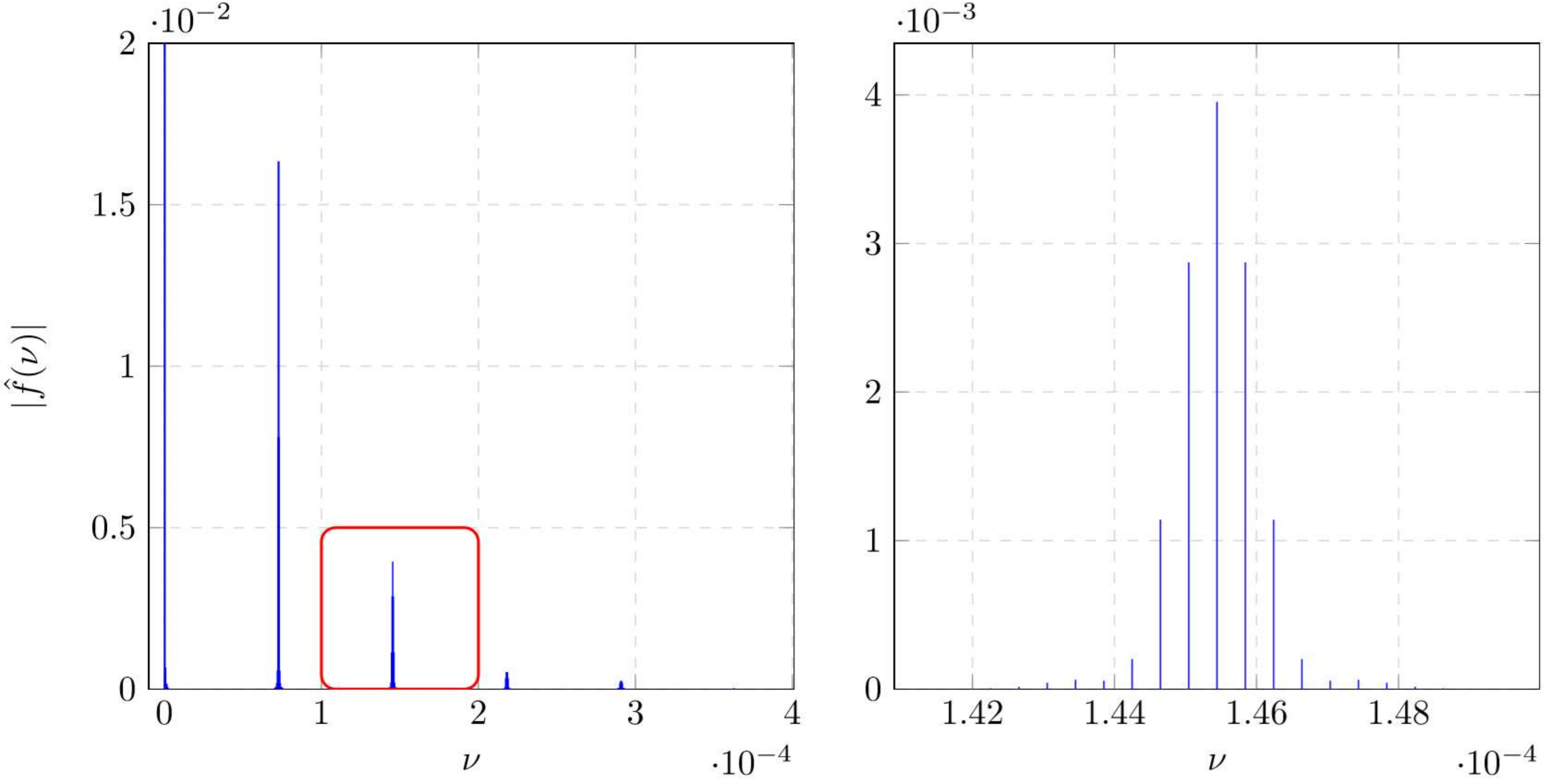
IS DE AARDE TE ZIEN

- Could aliens see earth this way?
- No: to much noise from the sun

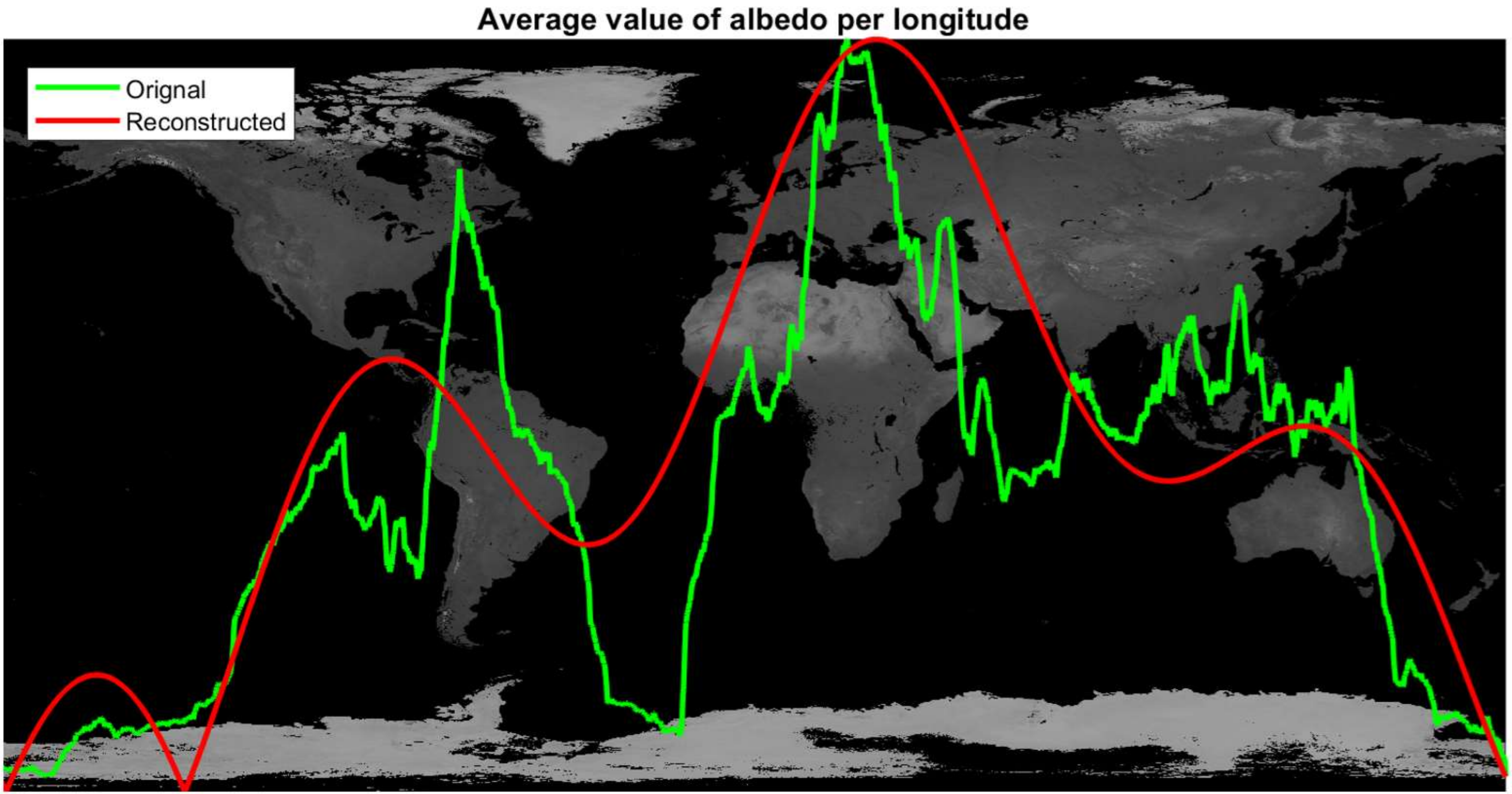
WHY DO WE WANT THIS MAP?

- Information about extraterestial life
- Insight in plate tectonics

01



$\cdot 10^{-2}$
2
1.5
1
0.5
0
$|\hat{f}(\nu)|$
0
1
2
3
4
ν
$\cdot 10^{-4}$
$\cdot 10^{-3}$
4
3
2
1
0
1.42
1.44
1.46
1.48
ν
$\cdot 10^{-4}$


Average value of albedo per longitude
Orignal
Reconstructed