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Geotechnical Ultimate Limit State Design Using Finite Elements

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Abstract. Displacement-based finite element calculations are primarily used for serviceability limit state (SLS) analysis, but the finite element method also offers possibilities for ultimate limit state (ULS) design in geotechnical engineering. The combined use of SLS and ULS calculations with partial safety factors according to the different design approaches in the Eurocode 7 can be time-consuming and prone to error. In this paper a Design Approaches facility is presented for an efficient use of partial safety factors in a finite element environment. In addition to a description of the methods used in this facility, an example is elaborated involving the geotechnical design of a sheet-pile wall supported excavation using different design approaches.

Keywords. Finite element method, SLS, ULS, Eurocode 7, Design Approaches

1. Introduction

Although the finite element method is primary used to calculate displacements, it can very well be used in a geotechnical context for ultimate limit state situations, providing solutions for bearing capacity, soil failure and safety factors. For many years, the method of phi-c reduction or strength reduction (Brinkgreve & Bakker, 1991; Griffith & Lane, 1999) was *the* way to calculate safety factors in FEM. The introduction of Eurocode 7 stimulated the use of partial safety factors. Different Design Approaches have been formulated in which combinations of partial factors for Actions (Loads), Materials and Resistances have to be applied (depending on local design rules). Factoring Resistances is difficult to achieve when simulating soil behaviour based on equilibrium and constitutive relations, as done in the finite element method. Alternatively, partial factors can be applied on Action Effects, i.e. (structural) forces obtained from numerical analysis. Meanwhile, some authors have shown finite element calculations according to Eurocode 7 using different design approaches and constitutive models (e.g. Schweiger 2010), but, in general, this is time-consuming and error-prone. In this paper a Design Approaches method is described to facilitate the use of partial factors in FEM. This facility has been implemented in the finite element program PLAXIS and is available since 2012.

Section 2 gives a description of the concept of the Design Approaches facility and the possible calculation schemes. Section 3 gives an elaboration of an example case, after which the main conclusions are drawn.

2. Design Approaches facility

The idea of the Design Approaches facility is to provide an environment to define and apply a set of partial factors on Loads (*Actions*), Model Parameters (*Materials*) and/or Structural Forces (*Action Effects*) in a finite element model, in correspondence with the applicable design regulation (Eurocode 7 or otherwise). The facility consists of the following parts:

- Defining Design Approaches, i.e. coherent sets of partial factors ('labels')
- Assigning partial factors ('labels')
- Applying Design Approach in calculation phases

If a calculation in which a coherent set of partial factors has been applied finishes successfully, it can be concluded that the design (represented by the finite element model) complies with the design regulation.

2.1. Defining sets of partial factors

In this context, a 'Design Approach' is a coherent set of partial factors according to a particular design regulation. Different partial factors may be defined for different types of *Loads*, *Materials* and/or *Structural Forces*. In the category Loads (Actions), for example, Eurocode 7 distinguishes between combinations of *permanent* vs. *variable* loads and *unfavourable* vs. *favourable* loads. In the framework of Design Approaches, each loading type may be identified by a 'label' with corresponding partial factor. Similarly, in the category Materials, distinction can be made, for example, between effective strength parameters and undrained shear strength, so each parameter type may be identified by a 'label' with corresponding partial factor. Considering Structural Forces (Action Effects), different types of structures (walls, anchors or other structural elements) may be identified by 'labels' with a corresponding partial factor.

The Design Approaches facility involves an environment to create and modify Design Approaches, to create 'labels' for loads, materials and structural forces (Figure 1), and to specify the corresponding partial factors. Preferably, labels must be given names that are common in different design approaches.

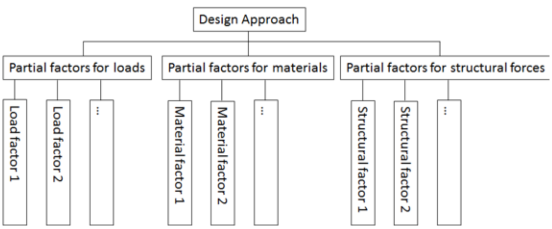

Figure 1. Schematic overview of a Design Approach (coherent set of partial factors).

2.2. Assigning partial factors

As indicated above, a completed Design Approach consists of a coherent set of partial factors ('labels') that can be assigned to loads, model parameters and structural elements in a finite element model. Design Approaches that have been defined (and validated!) in a particular project can be re-used in other projects. This makes the use of the Design Approaches facility more efficient and lesser prone to error than crudely applying partial factors in individual projects.

In a finite element model, representative values (cautious estimates, characteristic values) are normally used for loads and model parameters to calculate a serviceability limit state (SLS) situation. This practice shall be maintained when using Design Approaches, since the partial factors are supposed to be applied to representative values. With the creation of the finite element model, the aforementioned 'labels' must be assigned to the various loads, model parameters and structural elements in the model to indicate which partial factors must be applied during ULS calculations.

2.3. Defining sets of partial factors

After defining and completing the finite element calculation process using representative values of loads and model parameters (SLS calculation), the calculation is continued with similar calculation phases in which the applicable Design Approach is applied (ULS calculation). The facility involves the selection of the 'Reference' situation (SLS, using reference values) or a predefined Design Approach (ULS) in each calculation phase. In the latter case, reduced values of loads and model parameters will be used during the calculation and/or increased values of structural forces will be calculated according to the assigned 'labels' with partial factors. If the ULS calculation finishes successfully (stable result without failure), it can be concluded that the design complies with the design regulation.

2.4. Calculation schemes

There are two possible calculation schemes to perform ULS calculations in relation to SLS calculations (Bauduin et al., 2000). In the example below, the Phases 4, 5 and 6 are similar to the Phases 1, 2 and 3, respectively, except that the first three phases are 'Reference' (SLS) calculations (using reference values of loads and model parameters), whereas the latter three phases are 'Design' (ULS) calculations (using a predefined Design Approach).

Scheme 1:
0. Initial state
1. Phase 1 (SLS) ➢ 4. Phase 4 (ULS)
2. Phase 2 (SLS) ➢ 5. Phase 5 (ULS)
3. Phase 3 (SLS) ➢ 6. Phase 6 (ULS)

In the first scheme the design calculations (ULS) are performed for each serviceability state calculation separately. This means that Phase 4 starts from the resulting stress state of Phase 1, Phase 5 starts from Phase 2, and Phase 6 starts from Phase 3.

Scheme 2:
0. Initial state ➢ 4. Phase 4 (ULS)
1. Phase 1 (SLS) 5. Phase 5 (ULS)
2. Phase 2 (SLS) 6. Phase 6 (ULS)
3. Phase 3 (SLS)

In the second scheme, the design calculations (ULS) start from the initial situation and are performed subsequently. This means that Phase 4 starts from the Initial state, Phase 5 starts from the resulting stress state of Phase 4, and Phase 6 starts from Phase 5.

In general, it is recommended to establish the initial stress field from representative values of K_0; see also Frank et al. (2004).

3. Case: ULS design of a sheet pile wall

3.1. Introduction

This case presents the calculation of the Structural and Ground Limit State of a sheet pile wall supported excavation. The case is based on example 9.2 from the Eurocode 7 document (European Committee for Standardization, 2004). The sheet pile wall has a nominal excavation depth of 5 m. An additional excavation depth of 0.4 m due to accidental over-dig is foreseen. The wall is supported by a row of anchors (out of plane) at an elevation depth of 1.0 m. The anchor has a downward inclination of 10 degrees. The free length is 11 m and the anchor body length is approximately 6.5 m. The situation is depicted in Figure 2.

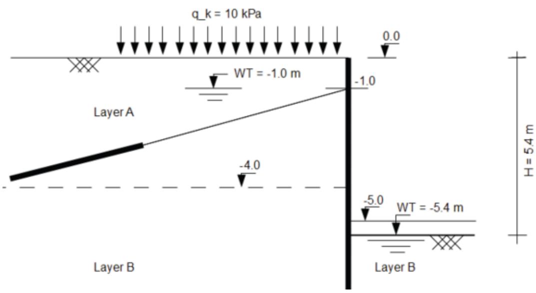

Figure 2. Geometry of sheet-pile wall supported excavation.

3.2. Soil properties and hydraulic conditions

The soil profile consists of two layers. A relatively soft soil layer (Layer B) is overlain by a 4 m thick stiffer layer (Layer A). The soil behaviour in these layers is represented by the Hardening Soil model. The initial phreatic level is 1.0 m below ground surface. In this example the long term situation is analysed, so only drained behaviour is considered and effective stress parameters are used. The applied model parameters are listed in Table 1. Other parameters used in the calculation are:

- Variable unfavourable surcharge load: 10 kN/m^2
- Sheet pile wall: depth 12 m; weight 1.4 kN/m/m; EA $3.675 \cdot 10^6$ kN/m; EI $5 \cdot 10^6$ kNm^2/m
- Anchor: EA $16.5 \cdot 10^3$ kN/m; pre-stress force 100 kN/m (one phase only)

This case has been presented previously in Brinkgreve & Post (2013). The current paper presents the results of a recalculation with the most recent PLAXIS version (2D 2015). The anchor body is represented here by an embedded beam row instead of a geogrid as in Brinkgreve & Post (2013). The used finite element model is presented in Figure 3.

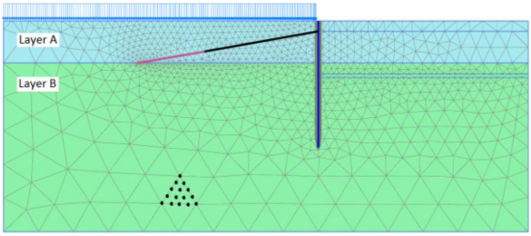

Figure 3. Geometry used to create finite element model.

Table 1. Model parameters.

Parameter	Layer A	Layer B	Unit
Soil model	Hard. Soil	Hard. Soil	-
Drainage type	Drained	Drained	-
Unit weight $\gamma_{unsat}/\gamma_{sat}$	18/20	20/20	kN/m^3
Triaxial stiffnss $E_{50}{}^{ref}$	20000	12000	kN/m^2
Oedometer stiff $E_{oed}{}^{ref}$	20000	8000	kN/m^2
Unloading stiff. $E_{ur}{}^{ref}$	60000	36000	kN/m^2
Reference stress p^{ref}	100	100	kN/m^2
Power m	0.5	0.8	-
Poisson's ratio ν	0.2	0.2	-
Cohesion c	1.0	5.0	kN/m^2
Friction angle φ	35	24	°
Dilatancy angle ψ	0	0	°
NC stress ratio $K_0{}^{nc}$	0.50	0.59	-
Failure ratio R_f	0.9	0.9	°
Tensile strength σ_t	0.0	0.0	kN/m^2
Interf. strength ratio	0.67	0.67	-
Initial stress ratio K_0	0.67	0.67	-
Permeability k	1.0	0.001	m/day

3.3. Design philosophy

The design philosophy is introduced using the following starting points and assumptions:

- For this example it is chosen to use both Eurocode 7 – Design Approach 2 (EC7-DA2) as well as EC7-DA3 for the structural (STR) and ground (GEO) Limit State verification. The partial factors are taken from EC7, appendix A, as presented in Table 2 and 3.
- No partial factors are applied to the properties of structural elements (only elastic behaviour).
- It is assumed that all water levels are strictly controlled, so no additional surcharge is applied on water conditions during ULS.
- Accidental over-dig is taken into account, so an additional excavation depth is applied in the ULS calculations.
- In this example no stiffness variation for soil and structural elements is applied during the ULS calculations.
- In this example only an unfavourable load factor is used for the (variable) surcharge load.

Table 2. Partial factors on actions (STR/GEO LS, default values according to EC7, annex A).

Action	EC7-DA2	EC7-DA3
Permanent unfavourable	1.35	1.00
Permanent favourable	1.00	1.00
Variable unfavourable	1.50	1.30
Variable favourable	0.00	0.00

Table 3. Factors on soil parameters (STR/GEO LS, default values according to EC7, annex A).

Soil parameter	EC7-DA2	EC7-DA3
Angle of shear resistance	1.00	1.25
Effective cohesion	1.00	1.25
Undrained shear strength	1.00	1.40
Weight density	1.00	1.00

The Design Approach facility, as introduced in the previous chapter, has been used to elaborate this case; both according to EC7-DA3 and EC7-DA2. In the latter case, partial factors are used on *Action effects* by multiplying the resulting structural forces (obtained with representative values for the soil properties) with the corresponding partial factor for action effects. This approach is often indicated as EC7-DA2*. The following practical method is applied:

- A partial factor of 1.00 is used for the *Permanent Unfavourable* loads (instead of 1.35)
- A factor of 1.50 / 1.35 = 1.11 is used for the *Variable Unfavourable* loads (instead of 1.50).
- A partial factor of 1.35 is used for structural forces (*Action Effects*) in the wall and the anchor.

3.4. Calculation process

The calculation scheme is presented in Table 4. This scheme is applied for both EC7-DA2* and EC7-DA3 design approaches.

Table 4. Calculation scheme

Phase	State	Start from phase
0. Initial phase	SLS	
1. Activate wall	SLS	0
2. Surcharge 10kPa + excav. to -1m	SLS	1
3. Anchor + pre-stressing 100 kN/m	SLS	2
4. Full excav. + dewatering to -5.0m	SLS	3
5. Full excav. + dewatering to -5.4m	SLS	4
6. ULS of Phase 2 (scheme 1)	ULS	2
7. ULS of Phase 3 (scheme 1)	ULS	3
8. ULS of Phase 4 (scheme 1)	ULS	4
9. ULS of Phase 5 (scheme 1)	ULS	5
10. ULS of Phase 2 (scheme 2)	ULS	1
11. ULS of Phase 3 (scheme 2)	ULS	10
12. ULS of Phase 4 (scheme 2)	ULS	11
13. ULS of Phase 5 (scheme 2)	ULS	12

The first series of six calculation phases is defined as 'Reference' calculations (SLS). The activation of the wall is considered to be part of the initial situation, but requires and additional phase. The excavation process consists of four phases (2-5). In addition to the 'Reference' calculations, four phases have been defined according to Scheme 1 and four according to Scheme 2, all for one design approach (EC7-DA3). The whole scheme was repeated for the other design approach (EC7-DA2*). In practical applications, it is, of course, sufficient to use only one calculation scheme and one design approach (according to the applicable design regulation).

Note that in Schweiger (2010) a slightly different approach is presented for EC7-DA2*, where calculations are performed without (case 1) and with (case 2) the variable unfavourable load. The design values of structural forces are then obtained by the sum of the results for case 1 with the partial factor for permanent unfavourable load, and the difference in results between case 2 and case 1 with the partial factor for variable unfavourable load, i.e.

$$F_{design} = F_{case1}*1.35 + (F_{case2} - F_{case1})*1.5 \quad (1)$$

The latter approach would be more difficult to automate than the approach described herein. It should be noted that both approaches are approximations of the original Eurocode 7 Design Approach 2.

3.5. Results

The results of the calculations in terms of design values of structural forces are presented in Table 5 (for EC7-DA3) and Table 6 (for EC7-DA2*). The results are similar and consistent with what was presented in Brinkgreve & Post (2013).

Table 5. SLS values and design values of structural forces (design values according to EC7-DA3)

Phase	Max anchor force [kN/m]	Max bending moment [kNm/m]
0. Initial phase		
1. Activate wall		
2. Surch. 10kPa + excav. to -1m		-4
3. Anchor + pre-stress. 100 kN/m	100	34
4. Full excav. + dewater. to -5.0m	118	174
5. Full excav. + dewater. to -5.4m	128	216
6. ULS of Phase 2 (scheme 1)		-6
7. ULS of Phase 3 (scheme 1)	100	36
8. ULS of Phase 4 (scheme 1)	136	238
9. ULS of Phase 5 (scheme 1)	161	334
10. ULS of Phase 2 (scheme 2)		-7
11. ULS of Phase 3 (scheme 2)	100	28
12. ULS of Phase 4 (scheme 2)	144	262
13. ULS of Phase 5 (scheme 2)	166	353

Table 6. SLS values and design values of structural forces (design values according to EC7-DA2*)

Phase	Max anchor force [kN/m]	Max bending moment [kNm/m]
0. Initial phase		
1. Activate wall		
2. Surch. 10kPa + excav. to -1m		-4
3. Anchor + pre-stress. 100 kN/m	100	34
4. Full excav. + dewater. to -5.0m	118	174
5. Full excav. + dewater. to -5.4m	128	216
6. ULS of Phase 2 (scheme 1)		-6
7. ULS of Phase 3 (scheme 1)	135	49
8. ULS of Phase 4 (scheme 1)	163	246
9. ULS of Phase 5 (scheme 1)	178	306
10. ULS of Phase 2 (scheme 2)		-6
11. ULS of Phase 3 (scheme 2)	135	45
12. ULS of Phase 4 (scheme 2)	161	236
13. ULS of Phase 5 (scheme 2)	175	294

From the results, some general observations can be made:

- Using scheme 1 or scheme 2 gives fairly similar values in structural forces both for EC7-DA2* and EC7-DA3 (in this example). However, it should be realised that differences may be larger in other cases.

- For a number of phases EC7-DA2* gives relatively large values for the anchor force compared to EC7-DA3, which is the result of the fact that the pre-stress force is entered as a characteristic value in EC7-DA3.

4. Conclusions

In this paper a facility is presented to enhance ultimate limit state (ULS) calculations in a finite element environment, in addition to normal serviceability limit state (SLS) finite element calculations. The focus has been on the components relevant for defining Design Approaches, i.e. coherent sets of partial factors ('labels') for loads, materials and structural forces, as well as assigning partial factors ('labels') to these components in the finite element model and applying a Design Approach in calculation phases. An example has been elaborated in which both EC7-DA2 (or actually EC7-DA2*) and EC7-DA3 have been used in order to show the possibilities for working with Eurocode 7. EC7-DA1 has not been considered, but this can be regarded as a combination of DA2 and DA3.

The purpose of this contribution is to demonstrate how the Design Approaches facility can be used as an efficient facility in a finite element environment to perform geotechnical ultimate limit state design calculations. It is NOT the authors' intention to advocate a particular design approach.

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