Combining Blind Equalization with Constant Modulus Properties

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Abstract

This paper presents an approach to multi-user blind space-time equalization exploiting the constant modulus (e.g. BPSK, m-PSK or QAM) modulation properties of the source signals. This is a problem that asks for both a blind equalization and a blind source separation based on the modulation properties. Previously proposed algorithms have consisted of two steps in sequence: equalization to linear mixtures of the source signals followed by separation of these mixtures, or separation into constant modulus source signals at several delays followed by assigning outputs to corresponding users. In this paper we combine both types of properties into a one-stage algebraic technique.

1. Introduction

Blind equalization has been an active research area during the last few years, fueled by the growth of wireless communications and by the upcoming third generation standards of wideband CDMA. Temporal and spatial oversampling techniques (using fractional sampling and antenna arrays, respectively) provide a multichannel data representation with a rich structure enabling several leverages for blind equalization [1, 2].

We consider an application wherein several co-channel users are received over an FIR convolutive channel with a delay spread of at most 2 symbols. The signals themselves can be modulated by periodic CDMA codes, and we can employ multiple transmission and receiver antennas. Knowledge of the codes is not assumed, in order not to confuse the additional possibilities that this would give. We thus arrive at a model where temporal (chiprate) and spatial oversampling makes sense.

From an algebraic perspective, oversampling an FIR convolutive channel leads to a low-rank model for the received data matrix. The structure present in this model (subspaces generated by Toeplitz and Hankel matrices) enables blind equalization in a variety of ways. Generally speaking, we can classify algorithms into two categories: "column-span methods" that first estimate the channel, as in [3], and "row-span methods" that directly estimate the equalizers to recover the symbols, e.g., [4, 5]. Here, we consider in particular the subspace-intersection formulation in [5], in which

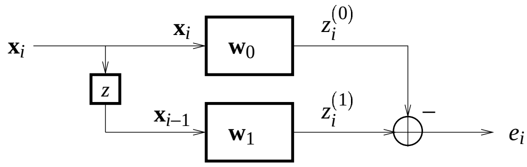

Figure 1. Mutually referenced equalizers

several shifts of the row span of the data matrix are intersected. This procedure is such that only the symbol sequence that is present in all shifts will remain, thus removing the ISI. As shown in [6], the algorithm is essentially identical to the "mutually referenced equalizer" (MRE) technique by Gesbert e.a. [7]. The MRE idea is illustrated in figure 1: if the received data vector $\{\mathbf{x}_i\}$ at time i is obtained by the convolution of a source symbol sequence by a filter of length 2, then there are two possible equalizer outputs, $z_i^{(0)}$ and $z_i^{(1)}$, one a delay of the other. By forcing this property, the equalizers are defined blindly.

A separate class of blind algorithms are those that separate souces based on their modulation properties, e.g., constant modulus (CM) or finite alphabet. In this paper we consider that the sources have a constant-modulus modulation (e.g., BPSK or QAM or m-PSK). The algebraic constant-modulus algorithm (ACMA) in [8] constructs a beamformer (or equalizer) to recover such a source based on forcing the property

$$s_k \in \mathrm{CM} \qquad \Rightarrow \qquad |s_k|^2 = 1\,.$$

In a multi-user FIR-MIMO scenario, blind equalization needs to be combined with blind source separation. In previous publications [4, 5], it was shown that this can be done in two separate stages: first blind equalization, which (ideally) reduces the problem to an instantaneous mixture of sources, and secondly a blind instantaneous MIMO source separation stage based on the constant modulus property.

We propose algorithms that solve this problem in a single stage. To this end, we explore how the MRE-conditions on the equalizer outputs can be combined with the constant modulus condition $|z_k^{(0)}|^2 = 1$ and $|z_k^{(1)}|^2 = 1$.

Notation $\;^T$ denotes a matrix transpose, * the matrix complex conjugate transpose, $\mathbf{0}$ a vector of all 0s, $\mathbf{1}$ a vector of all 1s, $\mathrm{vec}(\mathbf{A})$ a stacking of the colums of a matrix $\mathbf{A}$, $\otimes$ a

Kronecker product, and $\circ$ a Khatri-Rao product: $\mathbf{A} \circ \mathbf{B} := [\mathbf{a}_1 \otimes \mathbf{b}_1 \quad \mathbf{a}_2 \otimes \mathbf{b}_2 \quad \cdots]$. We will use the property, for vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$,

$$\mathbf{a}^* \mathbf{b} \mathbf{c}^* \mathbf{d} = (\mathbf{c} \otimes \overline{\mathbf{b}})^* (\mathbf{d} \otimes \overline{\mathbf{a}}) \tag{1}$$

2. Data model

We first consider a single source, and subsequently generalize to Q sources.

A digital symbol sequence $[s_i]$ is transmitted through a medium and received by an array of $M \geq 1$ sensors. The received signals are sampled $P \geq 1$ times faster than the symbol rate, here normalized to $T = 1$. Hence, during each symbol period, a total of MP measurements are available, which can be stacked into MP-dimensional vectors $\mathbf{x}_i = [x_i^1, \cdots, x_i^{MP}]^T$. Assuming an FIR channel, we can model $\mathbf{x}_i$ as the output of an MP-dimensional vector channel with impulse response $[\mathbf{h}_0, \mathbf{h}_1, \cdots, \mathbf{h}_{L-1}]$, where L denotes the channel length. In the noise free case, $\mathbf{x}_i$ is then given by

$$\mathbf{x}_i = \sum_{k=0}^{L-1} \mathbf{h}_k s_{i-k}\,. \tag{2}$$

Although it is not hard to generalize this, we assume from now on a simplified case where the channel has length $L = 2$ symbols, since this situation applies to CDMA systems after some preprocessing. Thus consider a finite block of data and define the $MP \times N$ data matrix

$$\mathbf{X}^{(i)} = \left[\begin{array}{cccc} \mathbf{x}_i & \mathbf{x}_{i+1} & \ldots & \mathbf{x}_{i+N-1} \end{array}\right].$$

From (2), $\mathbf{X}^{(i)}$ has a factorization as $\mathbf{X}^{(i)} = \mathbf{H}\mathbf{S}^{(i)}$, where $\mathbf{H}$ is an $MP \times 2$ channel matrix and $\mathbf{S}^{(i)}$ is a $2 \times N$ signal matrix,

$$\begin{array}{rcl} \mathbf{H} & = & \left[\begin{array}{cc} \mathbf{h}_0 & \mathbf{h}_1 \end{array}\right] \\ \mathbf{S}^{(i)} & = & \left[\begin{array}{cccc} s_i & s_{i+1} & \cdots & s_{i+N-1} \\ s_{i-1} & s_i & \cdots & s_{i+N-2} \end{array}\right]. \end{array} \tag{3}$$

We will assume that $\mathbf{H}$ is tall and full column rank 2, and $\mathbf{S}^{(i)}$ is wide and full row rank 2, so that this is a low rank factorization. (If $\mathbf{H}$ is not tall, then it can be made tall by shifting and stacking rows of $\mathbf{X}$ [5].) A low-rank factorization is essential because it ensures the existence of (zero-forcing) equalizers $\mathbf{w}$ that can reconstruct rows of $\mathbf{S}$ via $\mathbf{w}^*\mathbf{X}$.

The above model is readily extended to Q sources:

$$\mathbf{x}_i \;=\; \sum_{q=1}^{Q} \sum_{k=0}^{L-1} \mathbf{h}_k^q s_{i-k}^q \tag{4}$$

$$\mathbf{X}^{(i)} = \sum_{q=1}^{Q} \mathbf{H}^q \mathbf{S}^{q(i)} = \left[\mathbf{H}^1, \cdots, \mathbf{H}^Q\right] \left[\begin{array}{c} \mathbf{S}^{1(i)} \\ \vdots \\ \mathbf{S}^{Q(i)} \end{array}\right]$$

where q indicates a source index, and with obvious definitions of $\mathbf{H}^q$ and $\mathbf{S}^{q(i)}$. $\mathbf{X}^{(i)}$ has a low-rank factorization enabling ZF equalization if $MP \geq 2Q$. A low-rank factorization can be obtained by shifting and stacking whenever $MP > Q$ and sufficiently large N [5].

To avoid equalizers in the null space of $\mathbf{X}$, in all algorithms to follow a preprocessing is necessary, consisting of a prewhitening and dimension reduction to the rank of $\mathbf{X}$. The processing consists of computing a singular value decomposition of $\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}$, and replacing $\mathbf{X}$ by the first $2Q$ rows of $\mathbf{V}$. Refer to [8, 5, 7] for further details.

3. Algorithm derivation

3.1. Mutually referenced equalizers

We consider $Q = 1$ for now, and drop the index q for readability. An equalizer can be viewed as a vector $\mathbf{w}$ acting on $\mathbf{X}^{(i)}$ to produce an output sequence $\mathbf{z} = \mathbf{w}^*\mathbf{X}^{(i)}$. Since $\mathbf{S}^{(i)}$ has two rows, there are two different equalizers, $\mathbf{w}_0$ and $\mathbf{w}_1$, to recover the source symbols at different delays, viz.

$$\left\{ \begin{array}{rcl} \mathbf{w}_0^*\mathbf{X}^{(i)} & = & [s_i \;\; s_{i+1} \;\; \cdots \;\; s_{i+N-1}] \\ \mathbf{w}_1^*\mathbf{X}^{(i)} & = & [s_{i-1} \;\; s_i \;\; \cdots \;\; s_{i+N-2}] \end{array} \right.$$

or

$$\mathbf{w}_0^*\mathbf{x}_k = \mathbf{w}_1^*\mathbf{x}_{k+1}\,. \tag{5}$$

Taking two delays of the inputs, we can write

$$\mathbf{w}_1^*\mathbf{X}^{(1)} = [s_0 \;\; s_1 \;\; \cdots \;\; s_{N-1}] = \mathbf{w}_0^*\mathbf{X}^{(0)} \tag{6}$$

Thus, the equalizer outputs can be paired, which is the idea behind the MRE technique. The equalizers can be found in various ways, adaptively or using subspace intersections, cf. [6], essentially by solving

$$\min_{\mathbf{w}_0, \mathbf{w}_1} \| [\mathbf{w}_0^* \quad \mathbf{w}_1^*] \left[\begin{array}{c} \mathbf{X}^{(0)} \\ -\mathbf{X}^{(1)} \end{array}\right] \|^2$$

with a suitable norm constraint on $[\mathbf{w}_0^* \;\; \mathbf{w}_1^*]$. The solution is given by the left singular vector corresponding to the smallest singular value of $\left[\begin{smallmatrix} \mathbf{X}^{(0)} \\ -\mathbf{X}^{(1)} \end{smallmatrix}\right]$. The corresponding right singular vector is the source sequence $\alpha[s_0 \;\; s_1 \;\; \cdots \;\; s_{N-1}]$, where α is an indetermined scaling.

With Q users, we similarly find a basis of Q row vectors in the intersection, with each vector an arbitrary linear combination of the Q symbol sequences. Identification of the Q source symbol sequences from this basis cannot be done using blind equalization, but only using the constant modulus property.

3.2. Forcing the constant-modulus property

The constant-modulus property can be expressed as

$$|s_k|^2 = 1\,, \qquad k = 0, \cdots, N-1\,.$$

In this equation we can substitute the equalizer outputs $z_k^{(0)} = \mathbf{w}_0^*\mathbf{x}_k$ and $z_k^{(1)} = \mathbf{w}_1^*\mathbf{x}_k$ and require $z_k^{(0)} z_k^{(0)*} = 1$ and $z_k^{(1)} z_k^{(1)*} = 1$, i.e.

$$\left\{ \begin{array}{rcl} \mathbf{w}_0^*[\mathbf{x}_k\mathbf{x}_k^*]\mathbf{w}_0 & = & 1\,, \qquad k = 0, \cdots, N-1 \\ \mathbf{w}_1^*[\mathbf{x}_k\mathbf{x}_k^*]\mathbf{w}_1 & = & 1 \end{array} \right. \tag{7}$$

Using property (1), we can rewrite this as

$$\begin{cases} [\mathbf{x}_k \otimes \overline{\mathbf{x}}_k]^* (\mathbf{w}_0 \otimes \overline{\mathbf{w}}_0) &= 1\,, \quad k = 0, \cdots, N-1 \\ [\mathbf{x}_k \otimes \overline{\mathbf{x}}_k]^* (\mathbf{w}_1 \otimes \overline{\mathbf{w}}_1) &= 1 \end{cases}$$

These expressions can be written more compactly using Khatri-Rao products,

$$\mathbf{P}_{0,0} := \left[\mathbf{X}^{(0)} \circ \overline{\mathbf{X}}^{(0)}\right]^* := \begin{bmatrix} [\mathbf{x}_0 \otimes \overline{\mathbf{x}}_0]^* \\ \vdots \\ [\mathbf{x}_{N-1} \otimes \overline{\mathbf{x}}_{N-1}]^* \end{bmatrix}$$

which gives

$$\begin{cases} \mathbf{P}_{0,0} (\mathbf{w}_0 \otimes \overline{\mathbf{w}}_0) &= \mathbf{1} \\ \mathbf{P}_{0,0} (\mathbf{w}_1 \otimes \overline{\mathbf{w}}_1) &= \mathbf{1} \end{cases} \tag{8}$$

This leads to the ACMA technique [8] where the problem is solved in two steps. First solve the unstructured problem

$$\mathbf{P}_{0,0}\mathbf{y} = \mathbf{1} \qquad \Leftrightarrow \qquad [\mathbf{1} \quad \mathbf{P}_{0,0}] \begin{bmatrix} -1 \\ \mathbf{y} \end{bmatrix} = \mathbf{0}\,. \tag{9}$$

There is a two-dimensional subspace of solutions, since $\mathbf{w}_0 \otimes \overline{\mathbf{w}}_0$ and $\mathbf{w}_1 \otimes \overline{\mathbf{w}}_1$ are solutions, but also linear combinations of these two vectors (with proper scaling). In particular, let $\{\mathbf{y}_0, \mathbf{y}_1\}$ be a basis of the solution subspace of (9), then

$$\mathbf{y}_i = \lambda_{i,0} (\mathbf{w}_0 \otimes \overline{\mathbf{w}}_0) + \lambda_{i,1} (\mathbf{w}_1 \otimes \overline{\mathbf{w}}_1)\,, \qquad i = 0, 1\,.$$

The second step is to identify the structured vectors from this basis. This is done by reshaping the vectors $\mathbf{y}_i$ into square matrices $\mathbf{Y}_i$ such that $\mathrm{vec}(\mathbf{Y}_i) = \overline{\mathbf{y}}_i$. Using (1), we can write

$$\begin{aligned} \mathbf{Y}_i &= \lambda_{i,0} \mathbf{w}_0 \mathbf{w}_0^* + \lambda_{i,1} \mathbf{w}_1 \mathbf{w}_1^* \\ &= [\mathbf{w}_0 \;\; \mathbf{w}_1] \begin{bmatrix} \lambda_{i,0} & \\ & \lambda_{i,1} \end{bmatrix} [\mathbf{w}_0 \;\; \mathbf{w}_1]^* \end{aligned}$$

Thus, we have two matrices $\mathbf{Y}_0$ and $\mathbf{Y}_1$ with structure

$$\mathbf{Y}_0 = \mathbf{W}\mathbf{\Lambda}_0\mathbf{W}^*\,, \qquad \mathbf{Y}_1 = \mathbf{W}\mathbf{\Lambda}_1\mathbf{W}^*$$

where $\mathbf{W} = [\mathbf{w}_0 \;\; \mathbf{w}_1]$ is the solution of interest, and $\mathbf{\Lambda}_i$ is diagonal. This is a joint diagonalization problem that can be solved using various techniques, in this case e.g. via an eigendecomposition of $\mathbf{Y}_0\mathbf{Y}_1^{-1}$.

Note that the ACMA technique produces two equalizers, but does not tell which is $\mathbf{w}_0$ and which is $\mathbf{w}_1$: they are not related, and both solve the same system of equations (8). To identify which equalizer is which, we have to compare the corresponding output sequences to see which one is a delay of the other, cf. equation (6).

For $Q > 1$ users, the ACMA technique can still be used. We form the same equations (9), but in this case, we find a subspace of $2Q$ solution vectors. After solving the joint diagonalization problem for $2Q$ matrices, we obtain an unordered set of $2Q$ equalizers. The correct pairing into $\{(\mathbf{w}_0^q, \mathbf{w}_1^q)\}$ follows from solving a combinatorial problem that involves correlating all $2Q$ output sequences with their shifts.

3.3. Combining both parts

We now show how the MRE equations (5) can be combined with the CM conditions (7), so that they are automatically related and the combinatorial problem can be avoided. We derive several versions, beginning with a simple solution and extending it to involve more relations.

Version 1 Let us start with (7), $\mathbf{w}_0^*[\mathbf{x}_k\mathbf{x}_k^*]\mathbf{w}_0 = 1$, and combine with (5), viz. $\mathbf{w}_0^*\mathbf{x}_k = \mathbf{w}_1^*\mathbf{x}_{k+1}$. This produces

$$\mathbf{w}_0^*[\mathbf{x}_k\mathbf{x}_{k+1}^*]\mathbf{w}_1 = 1\,, \qquad k = 0, \cdots, N-2 \tag{10}$$

As before, we can rewrite this equation as

$$[\mathbf{x}_{k+1} \otimes \overline{\mathbf{x}}_k]^* (\mathbf{w}_1 \otimes \overline{\mathbf{w}}_0) = 1\,, \quad k = 0, \cdots, N-2$$

and collect the equations compactly by defining

$$\mathbf{P}_{1,0} := \left[\mathbf{X}^{(1)} \circ \overline{\mathbf{X}}^{(0)}\right]^* := \begin{bmatrix} [\mathbf{x}_1 \otimes \overline{\mathbf{x}}_0]^* \\ \vdots \\ [\mathbf{x}_{N-1} \otimes \overline{\mathbf{x}}_{N-2}]^* \end{bmatrix}$$

which gives

$$\mathbf{P}_{1,0}(\mathbf{w}_1 \otimes \overline{\mathbf{w}}_0) = \mathbf{1} \tag{11}$$

As before, this is solved as

$$\mathbf{P}_{1,0}\mathbf{y} = \mathbf{1}\,, \qquad \mathbf{y} = \mathbf{w}_1 \otimes \overline{\mathbf{w}}_0$$

For $Q = 1$ user, the linear equation has a single unique solution $\mathbf{y}$. To factor it into $\mathbf{y} = \mathbf{w}_1 \otimes \overline{\mathbf{w}}_0$, we construct a square matrix $\mathbf{Y}$ such that $\mathrm{vec}(\mathbf{Y}) = \overline{\mathbf{y}}$, which is such that

$$\mathbf{Y} = \mathbf{w}_0\mathbf{w}_1^*$$

Thus, $\mathbf{Y}$ is a rank-1 matrix and we can easily find the factors $\mathbf{w}_0$ and $\mathbf{w}_1$, e.g. via a singular value decomposition.

For $Q > 1$ users, this immediately generalizes. We form the linear system $\mathbf{P}_{1,0}\mathbf{y} = \mathbf{1}$ and compute a basis $\{\mathbf{y}_1, \cdots, \mathbf{y}_Q\}$ of the subspace of solutions. Each $\mathbf{y}_i$ is an arbitrary linear combination of the desired structured solutions $\mathbf{w}_1^q \otimes \overline{\mathbf{w}}_0^q$, $q = 1, \cdots, Q$. We unstack the $\overline{\mathbf{y}}_i$ into matrices $\mathbf{Y}_i$, with structure

$$\mathbf{Y}_i = \sum_{q=1}^{Q} \lambda_{q,i} \mathbf{w}_0^q \mathbf{w}_1^{q*} = \mathbf{W}_0 \mathbf{\Lambda}_i \mathbf{W}_1^*\,, \qquad i = 1, \cdots, Q$$

where $\mathbf{W}_0 = [\mathbf{w}_0^1 \cdots \mathbf{w}_0^Q]$, $\mathbf{W}_1 = [\mathbf{w}_1^1 \cdots \mathbf{w}_1^Q]$. This is a joint diagonalization problem of the Q matrices $\mathbf{Y}_i$, and can be solved using the same techniques as in [8]. Note that, unlike in ACMA, the matrices $\mathbf{Y}_i$ are non-symmetric. A Gauss-Newton optimization procedure for problems of this type is considered in [9].

After solving, we have all equalizers available in $\mathbf{W}_0$ and $\mathbf{W}_1$, and moreover we know for each user which equalizer is $\mathbf{w}_0^q$ and $\mathbf{w}_1^q$: the equalizers are automatically paired and no combinatorial search is needed. Moreover, the matrix $\mathbf{P}_{1,0}$ has the same size as in ACMA, but we have to find and decouple only Q solutions from it. From a complexity point of view, it is thus more attractive.

Version 2 To obtain improved accuracy, we can extend the system with additional equations. Similar to (5), consider $\mathbf{w}_0^*\mathbf{x}_{k+1} = \mathbf{w}_1^*\mathbf{x}_{k+2}$. From this we can derive

$$\mathbf{w}_0^*[\mathbf{x}_{k+1}\mathbf{x}_{k+1}^*]\mathbf{w}_1 \;=\; \mathbf{w}_1^*[\mathbf{x}_{k+2}\mathbf{x}_k^*]\mathbf{w}_0$$

Along with the conjugate of this equation, and the conjugate of (11), we obtain the set

$$\begin{cases} \mathbf{w}_0^*[\mathbf{x}_k\mathbf{x}_{k+1}^*]\mathbf{w}_1 &= 1, \qquad k=0,\cdots,N-2 \\ \mathbf{w}_1^*[\mathbf{x}_{k+1}\mathbf{x}_k^*]\mathbf{w}_0 &= 1 \end{cases}$$
$$\begin{cases} \mathbf{w}_0^*[\mathbf{x}_{k+1}\mathbf{x}_{k+1}^*]\mathbf{w}_1 &= \mathbf{w}_1^*[\mathbf{x}_{k+2}\mathbf{x}_k^*]\mathbf{w}_0 \\ \mathbf{w}_1^*[\mathbf{x}_{k+1}\mathbf{x}_{k+1}^*]\mathbf{w}_0 &= \mathbf{w}_0^*[\mathbf{x}_k\mathbf{x}_{k+2}^*]\mathbf{w}_1 \end{cases}$$

$$\Leftrightarrow \quad \left[\begin{array}{cc} \mathbf{P}_{1,0} & 0 \\ 0 & \mathbf{P}_{0,1} \\ \hline \mathbf{P}_{1,1} & -\mathbf{P}_{0,2} \\ -\mathbf{P}_{2,0} & \mathbf{P}_{1,1} \end{array}\right] \begin{bmatrix} \mathbf{w}_1 \otimes \overline{\mathbf{w}}_0 \\ \mathbf{w}_0 \otimes \overline{\mathbf{w}}_1 \end{bmatrix} = \left[\begin{array}{c} \mathbf{1} \\ \mathbf{1} \\ \hline \mathbf{0} \\ \mathbf{0} \end{array}\right] \qquad (12)$$

where $\mathbf{P}_{a,b} := \left[\mathbf{X}^{(a)} \circ \overline{\mathbf{X}}^{(b)}\right]^*$.

This is again a linear system of the form $\mathbf{P}\mathbf{y} = \mathbf{e}$ and for Q users we expect a subspace of Q solutions. After solving for the subspace, we can split each basis vector $\mathbf{y}_i$ into two components $\mathbf{y}_{i,0}$ and $\mathbf{y}_{i,1}$, unstack to obtain $\mathbf{Y}_{i,0} = \mathbf{W}_0\mathbf{\Lambda}_i\mathbf{W}_1^*$ and $\mathbf{Y}_{i,1} = \mathbf{W}_1\mathbf{\Lambda}_i\mathbf{W}_0^*$, and combine both parts to $\mathbf{Y}_i = \mathbf{Y}_{i,0} + \mathbf{Y}_{i,1}^*$. For this to work we have to ensure that $\mathbf{\Lambda}_i$ is real, which can be done by playing with the conjugate structure in (12) which allows to map it to an equation with real entries. (We omit the details.)

Version 3 Continuing along these lines, we can introduce equations involving all variants of the parameter vectors, $\mathbf{w}_0 \otimes \overline{\mathbf{w}}_0$, $\mathbf{w}_1 \otimes \overline{\mathbf{w}}_1$, $\mathbf{w}_0 \otimes \overline{\mathbf{w}}_1$ and $\mathbf{w}_1 \otimes \overline{\mathbf{w}}_0$. This leads to the following linear system of equations:

$$\left[\begin{array}{cccc} \mathbf{P}_{0,0} & 0 & 0 & 0 \\ 0 & \mathbf{P}_{1,0} & 0 & 0 \\ 0 & 0 & \mathbf{P}_{0,1} & 0 \\ 0 & 0 & 0 & \mathbf{P}_{0,0} \\ \hline \mathbf{P}_{1,0} & 0 & 0 & -\mathbf{P}_{2,1} \\ \mathbf{P}_{0,1} & 0 & 0 & -\mathbf{P}_{1,2} \\ 0 & -\mathbf{P}_{1,1} & \mathbf{P}_{0,2} & 0 \\ 0 & -\mathbf{P}_{2,0} & \mathbf{P}_{1,1} & 0 \\ \mathbf{P}_{1,0} & -\mathbf{P}_{2,0} & 0 & 0 \\ 0 & 0 & -\mathbf{P}_{0,2} & \mathbf{P}_{1,2} \end{array}\right] \begin{bmatrix} \mathbf{w}_0 \otimes \overline{\mathbf{w}}_0 \\ \mathbf{w}_1 \otimes \overline{\mathbf{w}}_0 \\ \mathbf{w}_0 \otimes \overline{\mathbf{w}}_1 \\ \mathbf{w}_1 \otimes \overline{\mathbf{w}}_1 \end{bmatrix} = \left[\begin{array}{c} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \hline \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{array}\right]$$

This system is again of the form $\mathbf{P}\mathbf{y} = \mathbf{e}$, and for Q users it has a basis of Q solutions $\{\mathbf{y}_i\}$. As before, we reshape these into square matrices $\mathbf{Y}_i$, satisfying

$$\mathbf{Y}_i = \sum_{q=1}^{Q} \lambda_i^q \begin{bmatrix} \mathbf{w}_0^q \\ \mathbf{w}_1^q \end{bmatrix} \begin{bmatrix} \mathbf{w}_0^{q*} & \mathbf{w}_1^{q*} \end{bmatrix} \;=\; \mathbf{W}\mathbf{\Lambda}_i\mathbf{W}^*\,, \qquad i = 1,\cdots,Q$$

where

$$\mathbf{W} := \begin{bmatrix} \mathbf{w}_0^1 & \cdots & \mathbf{w}_0^Q \\ \mathbf{w}_1^1 & \cdots & \mathbf{w}_1^Q \end{bmatrix}, \qquad \mathbf{\Lambda}_i := \mathrm{diag}[\lambda_i^1,\cdots,\lambda_i^Q]$$

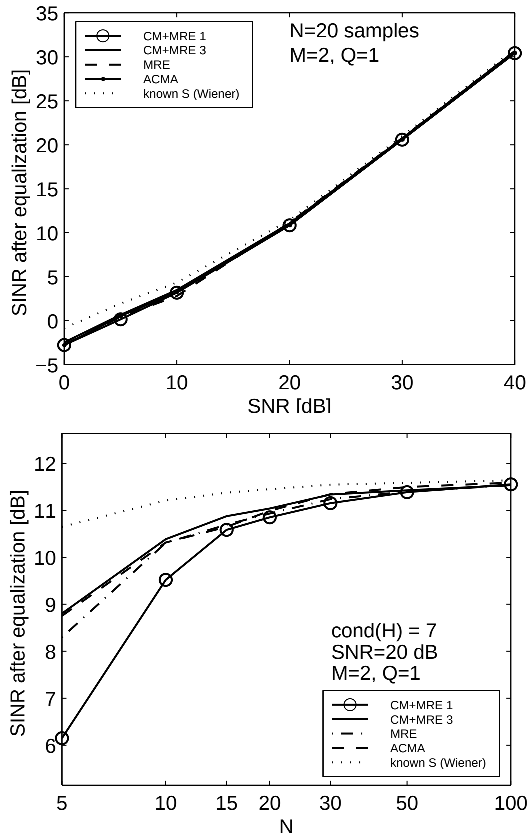

Figure 2. Single user, SINR after equalization

$\mathbf{W}$ is obtained as the joint diagonalizer of the $\mathbf{Y}_i$. The joint diagonalization problem is a bit different than before, because the $\mathbf{Y}_i$ have size $2Q \times 2Q$ but only rank Q. A Gauss-Newton optimization procedure for problems of this type is considered in [9].

4. Simulations

We first illustrate the performance of the algorithms for the single user case, with $MP = 2$ antennas/oversampling. We used a random channel of length $L = 2$ and a conditioning of about 7. The second path was about 3 dB weaker than the first. We compare to the MRE followed by ACMA to separate the users (section 3.1), and to ACMA followed by a combinatorial search to relate the equalizers (section 3.2). We also show the performance of the Wiener equalizer computed from known symbols.

Figure 2 shows the resulting SNR at the output after equalization, for the best equalizer among $(\mathbf{w}_1, \mathbf{w}_2)$, for varying number of samples N and input SNR. The results indicate that for $N \geq 20$ all algorithms perform about equally well and converge towards the Wiener equalizer in samples or SNR. Usually, the performance of CM+MRE version 1 is a bit worse than the others, and ACMA can be somewhat better, especially if the second path is much weaker than the first.

Figure 3 shows the results for $Q = 2$ users and $MP = 4$

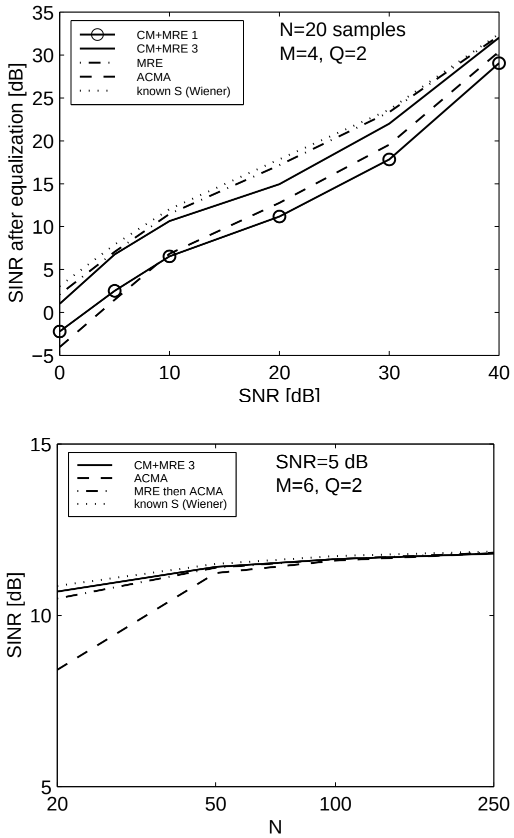

Figure 3. Two users, worst user SINR after equalization

antennas or oversampling. The channel was selected randomly, but the second paths of each user were a factor 10 weaker than the first paths. The resulting conditioning of $\mathbf{H}$ was about 65. We plot the SINR performance of the best equalizer of the worst user. In most cases, the performance of MRE is the best and of ACMA is worst, especially for small number of samples. CM+MRE version 1 is usually close to ACMA, whereas the version 3 is similar to that of MRE.

Similar conclusions are obtained for a higher user load. Figure 4 shows a case with $Q = 4$, $MP = 12$, and random channels. A moderate performance gain over ACMA (which asymptotically converges to the Wiener solution) is observed for low SNR and for small number of samples, and similar performance to the combination of MRE and ACMA.

5. Conclusions

Combination of blind equalization and source separation in a single stage is possible, and we have derived three versions of an algorithm to do so. The simplest version is the most elegant, has a complexity similar to ACMA (but omits the combinatorial search to find equalizer pairs at the end), and also similar performance unless the second path is much weaker than the first. The other two versions are significantly more complex, and their performance for the

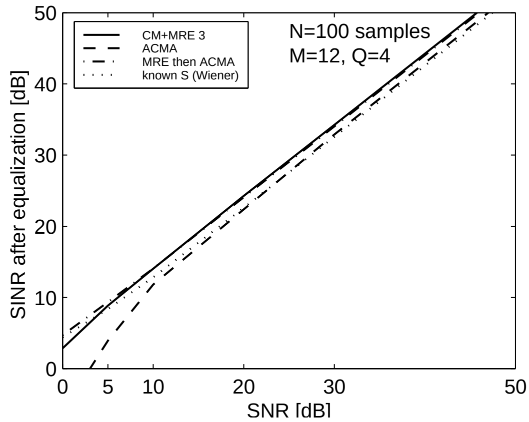

Figure 4. Four users, worst user SINR after equalization

multi-user case is almost never better than that of MRE followed by ACMA. We observed that all algorithms converge asymptotically to the Wiener solution. Further research is needed to assess the performance under non-ideal channel conditions.

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