Delft University of Technology

Exploration of Bayes Risk-Based Radar Resource Management for Multi-Target Track Maintenance

Chioccarello, S.; Driessen, H.; Yarovoy, A.

DOI
10.1109/RadarConf2559087.2025.11205111

Publication date
2025

Document Version
Final published version

Published in
Proceedings of the 2025 IEEE Radar Conference, RadarConf 2025

Citation (APA)
Chioccarello, S., Driessen, H., & Yarovoy, A. (2025). Exploration of Bayes Risk-Based Radar Resource Management for Multi-Target Track Maintenance. In M. Rupniewski, S. Blunt, J. Misiurewicz, M. S. Greco, & B. Himed (Eds.), *Proceedings of the 2025 IEEE Radar Conference, RadarConf 2025* (pp. 1128-1133). (Proceedings of the IEEE Radar Conference). IEEE. https://doi.org/10.1109/RadarConf2559087.2025.11205111

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright
Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy
Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

This work is downloaded from Delft University of Technology.
For technical reasons the number of authors shown on this cover page is limited to a maximum of 10.

Green Open Access added to TU Delft Institutional Repository
as part of the Taverne amendment.

More information about this copyright law amendment
can be found at https://www.openaccess.nl.

Otherwise as indicated in the copyright section:
the publisher is the copyright holder of this work and the
author uses the Dutch legislation to make this work public.

Exploration of Bayes Risk-Based Radar Resource Management for Multi-Target Track Maintenance

Stefano Chioccarello, Hans Driessen and Alexander Yarovoy
Microwave Sensing, Signals and Systems (MS3)
Delft University of Technology
Delft, The Netherlands
{s.chioccarello, j.n.driessen, a.yarovoy}@tudelft.nl

***Abstract*—The Radar Resource Management (RRM) problem for a multi-target tracking application is considered. The problem is defined as a stochastic optimal control problem with resource constraints. A novel Bayes risk-based formulation for the objective function, with the aim of allocating the radar resources by minimizing the risk of committing track maintenance errors, is proposed. The formulation proposed increases the interpretability in the decision-making process and conforms more with the system user's wishes. The effectiveness and benefits of active time resources management in dynamic conditions are demonstrated by means of simulations.**

***Index Terms*—Radar Resource Management, Bayes Risk, Hypothesis Testing, Track Maintenance, Lagrangian Relaxation**

I. Introduction

In long-range sensing applications, radar is the primary system due to its unique sensing capabilities. Classic radars typically operate with fixed scanning antennas and predefined waveforms. Modern radar systems use advanced technologies like active phased array antennas, digital beamforming on transmit and receive, and digital waveform generation. This offers agile beam steering and rapid reconfigurations to deal with dynamic conditions, promising superior performance. Such sensors are often called Multi-Function Radars (MFRs) because they can handle various tasks and quickly switch among them. These radars are used in several domains and for different applications, such as air traffic control, vessel traffic management, military applications and weather monitoring, among others.

Unlike classic radars, which require minimal management, an MFR demands active control to fully exploit its capabilities. The resources available to the system are limited, requiring adequate management to maintain efficiency. The problem of controlling these sensors is often called Radar Resource Management (RRM). It typically involves distributing the available resources among the tasks of interest, and adjusting system parameters dynamically based on the current situation and predictions for its development in future. Typical tasks are tracking and classification of moving targets or searching for undetected objects, but. This work is focused on the tracking task [1].

The literature on RRM presents a variety of approaches (see, for instance, overviews [2]–[4]). RRM is commonly formulated as an optimization problem, whose solution leads to the optimal sensor actions or system parameters. One critical aspect of the problem formulation is the objective function used in the optimization problem. Defining an appropriate function is crucial for RRM optimization, as it directly impacts system performance and resource allocation. Ideally, an effective objective function should optimize tasks jointly and incorporate their operational importance, while remaining understandable and leading to interpretable decisions. Ultimately, it is also desired to take into account the system user's needs for the specific application. However, balancing these requirements remains a significant challenge.

Many RRM approaches select the sensor actions using objective functions based on the track accuracy, whether it is computed from the filtering or the prediction error [5], [6]. This can lead to undesired resource allocation, requiring heuristic adjustments to compensate. As an example, optimizing for track accuracy may unintentionally prefer distant targets due to naturally higher estimation errors in Cartesian coordinates. Such cases highlight the need to reconsider whether track accuracy metrics alone are sufficient for an effective RRM.

Other management approaches select generic information-theoretic measures to guide sensor actions, such as Rényi divergence or entropy [7]–[9]. While these metrics provide a mathematically rigorous framework, they often lack intuitive interpretability for non-experts. System users may find it challenging to translate abstract information measures into actionable decisions, making it unattainable to fine-tune the resource management process according to operational needs.

However, none of these RRM approaches considers the track maintenance process and the possible mistakes that can occur. In resource management for target tracking, erroneous track maintenance can lead to undesirable resource allocation, either by continuing to track non-existent targets or prematurely terminating valid tracks [1]. These decision errors arise from factors such as false alarms and missed detections, which can be mitigated by allocating enough resources. Explicitly accounting for such errors in the RRM process can lead to more operationally relevant resource management. Driven by this criterion, we propose a novel formulation of the objective function that aims at optimizing radar resources by minimizing the risk of committing such errors when deciding upon tracked objects, and that could potentially replace track accuracy-based objective functions.

2025 IEEE Radar Conference (RadarConf25) | 979-8-3315-4433-1/25/$31.00 ©2025 IEEE | DOI: 10.1109/RADARCONF2559087.2025.11205111

Authorized licensed use limited to: TU Delft Library. Downloaded on November 18,2025 at 08:27:50 UTC from IEEE Xplore. Restrictions apply.

The remainder of the paper is organized as follows. Section II presents the formulation of the RRM problem, describing the underlying assumptions and optimization framework. In Section III, the derivation of the risk-based objective function is presented. Section IV describes the assumed sensor characteristics and the explicit computation of the risks. Section V presents simulations and relative results, analyzing the performance of the proposed approach. Finally, Section VI concludes the paper, summarizing the main findings and suggesting directions for future research.

II. RRM Problem Formulation for Multi-Target Tracking

While our ultimate goal is to develop a generically applicable approach to RRM for the tracking problem, we deliberately adopt the following simplifying assumptions in this work to focus on the core methodological aspects. We assume that there is a known number of multiple, independent, well-separated targets to be tracked. Each object is treated as a point target without any spatial extent. We allow for missed detections and false alarms, with the assumption that there is at most one correlating measurement per track.

Additionally, we consider the sensor to be stationary, and we do not optimize its movement, although the proposed approach does not exclude that possibility.

A. Motion Model

At every time step k, the current state of the n-th target can be characterized by its position x_k^n, y_k^n and velocity $\dot{x}_k^n, \dot{y}_k^n$ along x and y direction respectively on a two-dimensional Cartesian coordinate system. The state vector is defined as $\mathbf{s}_k^n = [x_k^n \quad y_k^n \quad \dot{x}_k^n \quad \dot{y}_k^n]^T$. The revisit interval with length T_n defines the time between two measurements. Therefore, the state evolution for target n can be explicitly written as

$$\mathbf{s}_{k+\Delta k}^n = f_{\Delta k}(\mathbf{s}_k^n, \mathbf{w}_k^n), \tag{1}$$

where $\mathbf{s}_{k+\Delta k}^n$ is the next state at time $k + \Delta k$, $\mathbf{w}_k^n \in \Re^4$ is the maneuverability noise for target n at time k and $f_{\Delta k}(\cdot)$ is the function characterizing the nature of the state evolution.

B. Measurement Model

It is assumed that noisy observations of the state for target n are acquired at time k through a sensor action $\mathbf{a}_k^n \in \Re^m$, where m is the number of measurement parameters, and the action vector $\mathbf{a}_k^n$ consists of the task parameters being optimized. The measurement $\mathbf{z}_k^n$ of target n at time k is expressed with the following function

$$\mathbf{z}_k^n = h(\mathbf{s}_k^n, \mathbf{v}_k^n, \mathbf{a}_k^n), \tag{2}$$

where $\mathbf{v}_k^n$ is the measurement noise for target n.

C. Tracking Algorithm

For the tracking problem considered in this paper, the chosen tracking algorithm aims at computing the posterior density in a recursive manner. For linear systems, a Kalman Filter (KF) can be adopted to achieve an exact solution. For non-linear systems, possible choices are the Extended Kalman Filter (EKF) or the Particle Filter (PF) [10].

D. RRM Problem for Multi-Target Tracking

Based on the work [6], the choice is to formulate the RRM problem as a stochastic optimal control problem with time resource constraints. The stochastic nature embedded in the RRM problem leads naturally to the assumption of an underlying Partially Observable Markov Decision Process (POMDP) to model the uncertainties in the process. Several examples that adopt this formulation can be found in the literature, e.g. [5], [11].

The resources to be optimized are the revisit intervals $\mathbf{T} = [T_1, \ldots, T_{N_k}] \in \Re^{N_k}$, and the sensor times (or dwell times) $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_{N_k}] \in \Re^{N_k}$ for a tracking problem with N_k tasks (or targets) at time k, and $B_{\max} \in [0, 1]$ is the maximum budget available to distribute among the tasks. At time k, the aim is to find the optimal set of parameters $\{\mathbf{T}, \boldsymbol{\tau}\}$ for the sensor by solving the following optimization problem:

$$\underset{\mathbf{T},\boldsymbol{\tau}}{\text{minimize}} \quad \sum_{n=1}^{N_k} \mathcal{C}(\mathbf{s}_k^n, T_n, \tau_n) \tag{3a}$$

$$\text{subject to} \quad g(\mathbf{T}, \boldsymbol{\tau}) \leq 0 \tag{3b}$$

$$\mathbf{T} \geq 0 \tag{3c}$$

$$\boldsymbol{\tau} \geq 0 \tag{3d}$$

$$\text{where} \quad g(\mathbf{T}, \boldsymbol{\tau}) = \left(\sum_{n=1}^{N_k} \frac{\tau_n}{T_n} \right) - B_{\max}. \tag{3e}$$

where $\mathcal{C}(\cdot)$ represents a generic cost function for the optimization problem.

III. Bayes Risk-based Cost Function Formulation

This section presents the steps towards a Bayes risk-based objective function for the RRM problem in (3).

A. Hypothesis Testing for Track Maintenance

In the tracking process, a decision-making framework is used to determine whether a measurement belongs to a true target and, consequently, whether to continue or terminate the associated track.

To do so, we resort to a decision-theoretic approach that involves binary statistical hypothesis testing. Through the hypothesis test we infer a decision based on the received data. We formulate a null hypothesis $\mathcal{H}_0$ and an alternative hypothesis $\mathcal{H}_1$. The null hypothesis represents the case in which the object is not present; hence, any measurement received is considered a false alarm, and the consequence is to delete the track. The alternative hypothesis is that the measurement is true and it belongs to a true target. Hence, the track will be continued. The hypotheses are then:

$$\mathcal{H}_{0,k}^n : n\text{-th object is not present at time } k,$$
$$\mathcal{H}_{1,k}^n : n\text{-th object is present at time } k,$$

with some prior probabilities $\pi_{0,k}^n, \pi_{1,k}^n$. In a Bayesian framework, the observation process can be used to update them recursively:

$$\pi_{0,k}^n = P(\mathcal{H}_{0,k}^n \mid Y_{k-1}^n), \tag{4a}$$

$$\pi_{1,k}^n = P(\mathcal{H}_{1,k}^n \mid Y_{k-1}^n), \tag{4b}$$

Authorized licensed use limited to: TU Delft Library. Downloaded on November 18,2025 at 08:27:50 UTC from IEEE Xplore. Restrictions apply.

where $Y_k^n = \{y_0^n, \ldots, y_k^n\}$ is the vector of observations up to time step k. The decision rule used to determine which hypothesis occurs is the Likelihood Ratio Test (LRT) [12].

Consider an observation of the n-th object y_k^n at time k. The LRT is then defined as the ratio of the likelihoods of y_k^n conditioned on the two hypotheses compared with a certain threshold η_k^n:

$$\Lambda(y_k^n) = \frac{p(y_k^n \mid \mathcal{H}_{1,k}^n, Y_k^n)}{p(y_k^n \mid \mathcal{H}_{0,k}^n, Y_k^n)} \underset{\mathcal{H}_{0,k}^n}{\overset{\mathcal{H}_{1,k}^n}{\gtrless}} \eta_k^n. \quad (5)$$

The explicit definition of the conditioned likelihoods depends on the modeling assumptions and may vary according to the system specifications. The LRT will determine which hypothesis is true and consequently, a decision is made:

$\mathcal{D}_{0,k}^n$: choose $\mathcal{H}_{0,k}^n$,
$\mathcal{D}_{1,k}^n$: choose $\mathcal{H}_{1,k}^n$.

B. Bayes Risk-based Function for Tracking

In hypothesis testing for track maintenance, the consequences of incorrect decisions vary depending on the task. For instance, discarding a track when a true target is present can lead to a critical failure, while maintaining a track for a non-existent object wastes radar resources. Still, such occurrences may not have the same impact. In the Bayes risk approach, costs are associated with each possible outcome of the hypothesis test $C_{ij,k}^n$, that is, the cost of deciding $\mathcal{D}_{i,k}^n$ when $\mathcal{H}_{j,k}^n$ is true, with $i, j \in \{0, 1\}$. In general, these costs might change over time and be different for each n-th object, and they can be defined according to the user's needs. Without loss of generality, and given that we are interested in the mistakes made in track maintenance, we can set $C_{00,k}^n = C_{11,k}^n = 0$, which represent the costs of taking a correct decision in the track maintenance mechanism. The expected Bayes risk for a decision rule is defined as [12]:

$$\mathcal{R}_k^n = \sum_{i=0}^{1} \sum_{j=0}^{1} C_{ij,k}^n P_i(\mathcal{H}_{j,k}^n) \pi_j^n, \quad (6)$$

where $P_i(\mathcal{H}_{j,k}^n)$ is the probability of deciding $\mathcal{D}_{i,k}^n$ when $\mathcal{H}_{j,k}^n$ is true and π_j^n, $j \in \{0, 1\}$ are the prior probabilities as in (4). The optimal decision rule minimizes $\mathcal{R}_k^n$ by comparing the expected costs associated with each hypothesis. The Minimum Bayes Risk (MBR) threshold is given by the ratio of the prior probabilities and the respective decision costs:

$$\eta_k^n = \frac{\pi_0^n C_{10,k}^n}{\pi_1^n C_{01,k}^n}. \quad (7)$$

This threshold selects the hypothesis that has the Maximum a Posteriori (MAP) probability. Hence, the MBR estimate is also a MAP estimator. The benefit of this formulation is twofold. Not only does it ensure the minimization of the Bayes risk function, but it can also be embedded in an iterative process such as a tracking task. The risk function for the n-th object at time k is given by:

$$\mathcal{R}_k^n = C_{10,k}^n P_1(\mathcal{H}_{0,k}^n) \pi_{0,k}^n + C_{01,k}^n P_0(\mathcal{H}_{1,k}^n) \pi_{1,k}^n, \quad (8)$$

where $P_1(\mathcal{H}_{0,k}^n)$ can be interpreted as the probability of keeping a track active while the n-th target is not present anymore and, vice-versa, $P_0(\mathcal{H}_{1,k}^n)$ represents the probability of deleting a track while the n-th target is still present. It should be noted that $\mathcal{R}_k^n$ is a function of the time parameters T_n and τ_n, which are omitted here for simplicity. Using this function as a base for defining the objective function in (3a) will theoretically lead to the optimal parameters that minimize the risk of committing an error in a tracking scenario.

The advantage of using this formulation is that the interpretability of the cost function is improved. In fact, a practical meaning can be assigned to the quantity in (8), that is, the risk of making the wrong decision when tracking an object. Moreover, the costs $C_{ij,k}$ can be tuned according to the specifics of the mission.

It is supposed that measurements are independent and that N_k targets are well-separated and can be considered uncorrelated with each other. Therefore, it is fair to assume that the overall risk of the system is the sum of the individual risks. Then, the objective function in (3a) to minimize becomes:

$$\mathcal{C}(\mathbf{s}_k^n, T_n, \tau_n) = \mathcal{R}_k^n. \quad (9)$$

C. Computation of the Probabilities

The probabilities in (8) necessary to determine the overall risk for target n are hereby defined. With the previous assumptions, the computation of the probabilities is directly related to the decision regions of the LRT. The probability of deleting the track for object n at time k while the object is still present is equal to the area of the likelihood function conditioned on the alternative hypothesis $\mathcal{H}_{1,k}^n$ where the LRT is below the threshold η_k^n:

$$P_1(\mathcal{H}_{0,k}^n) = \int_{\Lambda(y_k^n) < \eta_k^n} p(y_k^n \mid \mathcal{H}_{1,k}^n, Y_k^n)\, dy_k^n. \quad (10)$$

Similarly, the probability of keeping the track of object n at time k while the object is not currently present is given by the area of the likelihood function conditioned on the null hypothesis where the LRT is above the threshold η_k^n:

$$P_0(\mathcal{H}_{1,k}^n) = \int_{\Lambda(y_k^n) > \eta_k^n} p(y_k^n \mid \mathcal{H}_{0,k}^n, Y_k^n)\, dy_k^n. \quad (11)$$

In general, these integrals might not be straightforward to compute and may require numerical methods and approximations. In the case of a tractable formulation of the likelihoods, and making assumptions such as Gaussianity, they might be easier to solve, and a closed-form expression may be derived.

IV. Sensor Model and Risk Computation

A. Sensor Characteristics

It is assumed that the probability of detection for the sensor P_D follows a Swerling II model:

$$P_{D,k} = P_F^{\frac{1}{1+SNR_k}}, \quad (12)$$

Authorized licensed use limited to: TU Delft Library. Downloaded on November 18,2025 at 08:27:50 UTC from IEEE Xplore. Restrictions apply.

where P_F is the probability of false alarm. Moreover, let us assume that measurements z_k representing a false alarm are uniformly distributed over the observation volume V:

$$p_{\text{FA}}(z_k) = \frac{1}{V}, \tag{13}$$

and the number N_{FA} of false measurements is Poisson-distributed with a probability mass function:

$$p_{\text{F}}(N_{\text{FA}}) = \frac{(\beta_{\text{FA}} V)^{N_{\text{FA}}}}{N_{\text{FA}}!} \exp -(\beta_{\text{FA}} V), \tag{14}$$

with β_{FA} being the average number of false measurements per unit area.

B. Measurement Likelihood Functions

We assume that there is at most one measurement per target per scan. There are no data association issues; hence, the received measurement is either from the true target or a false alarm. The case of no measurement (e.g., a missed detection) at time k is defined as $y_k^n = \varnothing$. With these assumptions, we can define the conditioned likelihoods of an observation y_k^n at scan k as:

$$p(y_k^n \mid \mathcal{H}_{1,k}^n, Y_k^n) = \begin{cases} \frac{1}{V} p_{\text{F}}(1)(1 - P_{D,k}) + p_{\text{F}}(0) P_{D,k} \\ \times \frac{1}{\sqrt{2\pi |\mathbf{S}_k^n|}} \exp\left(-\frac{1}{2} (\tilde{\mathbf{y}}_k^n)^T (\mathbf{S}_k^n)^{-1} \tilde{\mathbf{y}}_k^n\right) & \text{if } y_k = \mathbf{z}_k \\ p_{\text{F}}(0)(1 - P_{D,k}) & \text{if } y_k = \varnothing \end{cases}, \tag{15}$$

$$p(y_k^n \mid \mathcal{H}_{0,k}^n, Y_k^n) = \begin{cases} \frac{1}{V} p_{\text{F}}(1) & \text{if } y_k = \mathbf{z}_k^n \\ p_{\text{F}}(0) & \text{if } y_k = \varnothing \end{cases}, \tag{16}$$

where the residual vector $\tilde{\mathbf{y}}_k^n = \mathbf{y}_k^n - \hat{\mathbf{s}}_n^k$ is the difference between the measured quantity and the predicted quantity as the output of the tracking filter, and $\mathbf{S}_k^n$ is the corresponding residual covariance matrix.

C. Probabilities in the Risk Function

The computation of the probabilities is directly related to the decision regions of the LRT, namely the probability that a measurement falls within a M-dimensional Gaussian ellipsoid of size G, where M is the dimension of the measurement [1]. The size of the ellipsoid can be derived by solving (7) for the non-data dependent terms:

$$G_k^n = -2 \ln \left(\frac{\beta_{\text{FA}} (\eta_k^n - 1 + P_D) \sqrt{2\pi |\mathbf{S}_k^n|}}{P_D} \right). \tag{17}$$

Using this formulation, we can explicitly express the probabilities in (10) and (11). For $M = 2$, we have:

$$P_0(\mathcal{H}_{1,k}^n) = \exp(-G_k^n / 2), \tag{18}$$

$$P_1(\mathcal{H}_{0,k}^n) = \frac{1}{V} P_{\text{F}}(1) \pi \sqrt{|\mathbf{S}_k^n|} G_k^n. \tag{19}$$

D. MBR Threshold Cost Function

This formulation allows to act directly on the decision costs in (6) based on the user's needs. The cost function[1] can be designed to reflect the operational meaning of a certain metric. For instance, the costs could decrease depending on the range of the object: this way, the risk for further targets decreases, and fewer resources shall be allocated to those. Constant values for the costs, instead, are less operationally meaningful, and the same threshold would be applied for every target at every instant, regardless of its state.

For this paper, an equal, constant cost function is compared with a sigmoid-shaped cost function:

- *Constant costs*:

$$C_{ij,k}^n = C_{ij}^n, \tag{20}$$

- *Sigmoid decaying costs*:

$$C_{ij,k}^n = \frac{C_{ij,\text{base}}^n}{1 + \exp(\sigma_s (d_k^n - d_{ref}))} + \epsilon, \tag{21}$$

where $C_{ij,\text{base}}^n$ is a default cost for target n, d_k^n represents the distance of the target from the radar, d_{ref} is the location at which the sigmoid is centered, and σ_s determines the steepness of the curve. The constant ϵ can be used to avoid the cost going to zero and encounter numerical issues.

V. Simulation Results

In this section, we present the simulation results of the risk-based approach for time budget management in a two-dimensional target tracking scenario. Some assumptions for the simulations are detailed first. Then, results for two different scenarios are presented to highlight some aspects of the proposed cost function formulation.

A. Modeling Assumptions

1) Motion Model: Targets are assumed to move according to a linear constant velocity model. Therefore, the state evolution (1) for target n can be explicitly written as

$$\mathbf{s}_{k+1}^n = \mathbf{F}_n \cdot \mathbf{s}_k^n + \mathbf{w}_k^n, \tag{22}$$

where $\mathbf{F}_n \in \Re^{4\times 4}$ is the state transition matrix defined as

$$\mathbf{F}_n = \begin{bmatrix} 1 & 0 & T_n & 0 \\ 0 & 1 & 0 & T_n \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{23}$$

and the maneuverability noise $\mathbf{w}_k^n$ follows a zero-mean multivariate Gaussian distribution with the covariance matrix

$$\mathbf{Q}_n = \begin{bmatrix} \frac{T_n^4}{4} & 0 & \frac{T_n^3}{2} & 0 \\ 0 & \frac{T_n^4}{4} & 0 & \frac{T_n^3}{2} \\ \frac{T_n^3}{2} & 0 & T_n^2 & 0 \\ 0 & \frac{T_n^3}{2} & 0 & T_n^2 \end{bmatrix} \sigma_{w,n}^2 \quad , \tag{24}$$

where $\sigma_{w,n}^2$ is the maneuverability noise variance.

[1]The term "cost function" in this context does not refer to the objective function of the optimization problem in (3a), but rather to a mathematical function that can be assigned to the decision costs to follow a certain behavior.

Authorized licensed use limited to: TU Delft Library. Downloaded on November 18,2025 at 08:27:50 UTC from IEEE Xplore. Restrictions apply.

TABLE I
PARAMETERS OF THE REFERENCE MEASUREMENT.

SNR_0	τ_0 [s]	r_0 [km]
1	1	50

TABLE II
SENSOR PARAMETERS.

	P_F	β_{FA}
Inside Area	1E-3	1E-2
Outside Area	1E-4	5E-4

2) Measurement Model: It is assumed that the radar sensor acquires noisy measurements of range r and azimuth angle θ. The relationship between the measurement vector $\mathbf{z}_k^n$ and the current object's state can be expressed as

$$\mathbf{z}_k^n = \mathbf{h}(\mathbf{s}_k^n) + \mathbf{v}_k^n, \tag{25}$$

where the function

$$\mathbf{h}(\mathbf{s}_k^n) = \left[\sqrt{(x_k^n)^2 + (y_k^n)^2}, \quad \arctan(y_k^n, x_k^n)\right]^T, \tag{26}$$

describes the non-linear transformation between the Cartesian coordinates of the state and the polar coordinates of the measurements, and $\mathbf{v}_k^n \in \Re^2$ is the zero-mean Gaussian measurement noise for target n. The range and azimuth are assumed to be independent, hence $\mathbf{v}_k^n = \left[v_k^{r,n}, \quad v_k^{\theta,n}\right]^T$ with variances $\sigma_{r,n}^2, \sigma_{\theta,n}^2$, respectively. The SNR at time k for target n is assumed to follow the model:

$$\text{SNR}_k^n(\tau_n, r_k^n) = \text{SNR}_0 \cdot \left(\frac{\tau_n}{\tau_0}\right) \cdot \left(\frac{r_k^n}{r_0}\right)^{-4}. \tag{27}$$

The noise variances $\sigma_{r,n}^2, \sigma_{\theta,n}^2$ can be defined as [13]:

$$\sigma_{\star,n}^2 = \frac{\sigma_{\star,0}^2}{\text{SNR}_k^n(\tau_n, r_k^n)}, \tag{28}$$

where $\star \in (r, \theta)$ and $\sigma_{\star,0}^2$ is the measurement noise variance for a reference target (Table I). The assumption of independent measurements leads to the following measurement covariance matrix $\mathbf{R}_k^n \in \Re^{2\times 2}$

$$\mathbf{R}_k^n = \begin{bmatrix} \sigma_{r,n}^2 & 0 \\ 0 & \sigma_{\theta,n}^2 \end{bmatrix}. \tag{29}$$

3) Tracking Algorithm: Since the assumed measurement function is non-linear, an EKF is applied for state estimation. It is assumed that separate EKFs are used to track the objects. Therefore, at this stage, no data association problem is taken into account. It is also assumed that there are no track drops or re-initializations.

B. Simulation Details

1) Solution of the Optimization Problem with Lagrangian Relaxation (LR): The constrained optimization problem in (3) is solved using the LR algorithm. The approach follows the work presented in [6] and the previous work. The reason is that the LR technique allows to relax the constraints of the problem, as well as to break down the main optimization problem into multiple sub-optimization problems, thus reducing the complexity. To find the optimal Lagrangian multipliers for the Lagrangian dual problem, the subgradient method [14] is used. It is assumed that the optimization problem is iteratively solved at fixed time intervals, and the computed time budget parameters are used until the next iteration.

2) Myopic Approximation of the POMDP Solution: For this paper, the underlying POMDP is assumed to be approximated in a myopic fashion. Hence, risk optimization takes into account only the one-step-ahead evolution of the system.

3) Measurement Modeling Assumptions: In these simulations, false alarms and missed detections were not explicitly generated as separate events within the loop. Instead, their effects were incorporated into the measurement likelihood functions, ensuring that their statistical impact was captured without directly simulating individual occurrences.

C. Tracking Scenario A

This scenario evaluates the behavior of the risk function and resource allocation by the management system when different shapes for the cost function are used. All objects start equidistant from the radar, with two moving faster than the others, and share identical features except for their initial states. Sensor parameters for this scenario are listed in the second row of Table II.

Two simulations are conducted with different cost models for the MBR threshold computation: constant costs model (20); sigmoid-decaying model (21) with parameters $\sigma_s = 0.01$ and $d_{ref} = 10000$ (10 km). The results of the time budget optimization are compared with those obtained by a default policy; namely, time resources are equally split among the objects. This comparison assesses the impact of parameter optimization on risk reduction, efficiency, and robustness.

As shown in Figure 3a, constant costs cause risk to increase as targets move away, prompting the management algorithm to allocate more resources to faster targets (1 and 5), as depicted in Figure 4a.

In contrast, the sigmoid-shaped cost function reduces the risk for distant targets beyond 10 km (Figure 3b), enabling the algorithm to reallocate resources to closer targets (2, 3, and 4), as shown in Figure 4b.

In both cases, the total system risk is significantly lower with the optimized parameters than with the naïve equal budget allocation (Figures 3a and 3b).

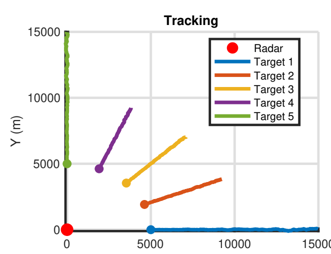


Fig. 1. Scenario A Trajectories.

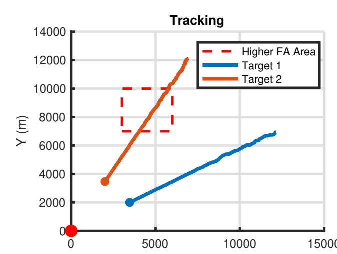
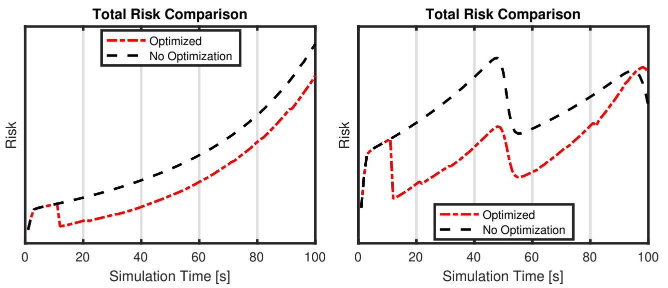


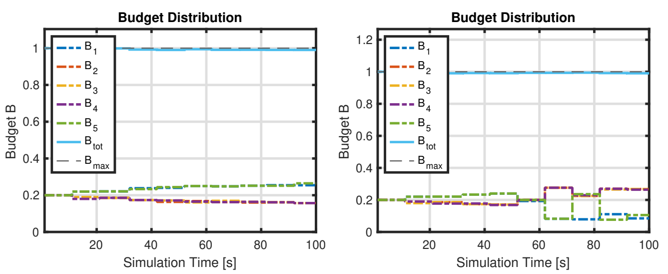
Fig. 2. Scenario B Trajectories.

Authorized licensed use limited to: TU Delft Library. Downloaded on November 18,2025 at 08:27:50 UTC from IEEE Xplore. Restrictions apply.



(a) Scenario A with equal costs. (b) Scenario A with sigmoid-decaying costs.

Fig. 3. Comparison of risk for scenario A with different cost functions.



(a) Scenario A with equal costs. (b) Scenario A with sigmoid-decaying costs.

Fig. 4. Comparison of budget allocations for scenario A with different cost functions.

D. Tracking Scenario B

Scenario B illustrates how the management algorithm adapts to changing environmental conditions during tracking. In this case, two targets are tracked, with Target 2 temporarily entering an area with higher false alarm probability and density, as specified in Table II. The cost function is uniform for all outcomes.

Results show that the risk for Target 2 rises upon entering the high false alarm area (Figure 5). In response, the management algorithm allocates more resources to Target 2 than Target 1. Once Target 2 exits the area, resources are then evenly redistributed (Figure 6). The allocation difference during this period is modest—about 48% for Target 1 and 52% for Target 2 — yet the impact on the risk function is significant.

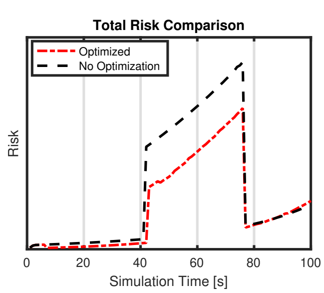


Fig. 5. Total risk for scenario B.

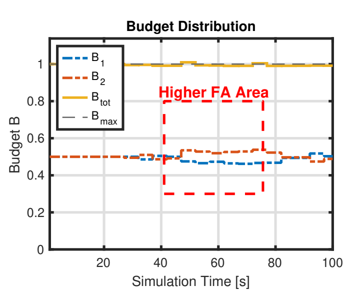


Fig. 6. Budget distribution for scenario B.

VI. Conclusions

This paper presented a novel Bayes risk-based formulation for the objective function in a RRM problem for a multi-target tracking scenario, integrating a probabilistic decision-making approach within a full optimization framework. Contrarily to track accuracy metrics and generic information-theoretic measures, the proposed approach explicitly incorporates notions of operational importance through a cost function that minimizes track maintenance errors.

Simulations demonstrated how resource allocation can be influenced by selecting a different function shape for the costs and how the risk-based approach can deal with changing conditions of the environment. The solution led to a slightly higher risk for some instances. Possible causes for that are numerical approximations, the myopic solution, and the potential non-convexity of the risk function, which remains to to be further explored. Nevertheless, the risk-based approach demonstrated effectiveness in dealing with the RRM problem.

Acknowledgment

The authors acknowledge the partial funding for this project by Thales Nederland B.V. within the TKI programme.

References

[1] S. S. Blackman, "Multiple-target tracking with radar applications," *Dedham*, 1986.

[2] A. O. Hero and D. Cochran, "Sensor management: Past, present, and future," *IEEE Sensors Journal*, vol. 11, no. 12, pp. 3064–3075, 2011.

[3] M. I. Schöpe, "Approximately optimal resource management for multi-function radar: Algorithmic solutions using a generic framework," 2021.

[4] U. S. Hashmi, S. Akbar, R. Adve, P. W. Moo, and J. Ding, "Artificial intelligence meets radar resource management: A comprehensive background and literature review," *IET Radar, Sonar & Navigation*, vol. 17, no. 2, pp. 153–178, 2023.

[5] A. Charlish and F. Hoffmann, "Anticipation in cognitive radar using stochastic control," in *2015 IEEE Radar Conference (RadarCon)*. IEEE, 2015, pp. 1692–1697.

[6] M. I. Schöpe, H. Driessen, and A. Yarovoy, "A constrained pomdp formulation and algorithmic solution for radar resource management in multi-target tracking," *Journal of Advances in Information Fusion*, vol. 16, no. 1, p. 31, 2021.

[7] C. Kreucher, A. O. Hero, K. Kastella, and D. Chang, "Efficient methods of non-myopic sensor management for multitarget tracking," in *2004 43rd IEEE Conference on Decision and Control (CDC)(IEEE Cat. No. 04CH37601)*, vol. 1. IEEE, 2004, pp. 722–727.

[8] B. La Scala, W. Moran, and R. Evans, "Optimal adaptive waveform selection for target detection," in *2003 Proceedings of the International Conference on Radar (IEEE Cat. No. 03EX695)*. IEEE, 2003, pp. 492–496.

[9] P. E. Berry and D. A. Fogg, "On the use of entropy for optimal radar resource management and control," in *2003 Proceedings of the International Conference on Radar (IEEE Cat. No. 03EX695)*. IEEE, 2003, pp. 572–577.

[10] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking," *IEEE Transactions on signal processing*, vol. 50, no. 2, pp. 174–188, 2002.

[11] A. Charlish, F. Hoffmann, C. Degen, and I. Schlangen, "The development from adaptive to cognitive radar resource management," *IEEE Aerospace and Electronic Systems Magazine*, vol. 35, no. 6, pp. 8–19, 2020.

[12] H. V. Poor, *An Introduction to Signal Detection and Estimation*. Springer, 1994.

[13] H. Meikle, *Modern Radar Systems*. Artech House, 2008.

[14] D. Bertsimas and J. N. Tsitsiklis, *Introduction to linear optimization*. Athena scientific Belmont, MA, 1997, vol. 6.

Authorized licensed use limited to: TU Delft Library. Downloaded on November 18,2025 at 08:27:50 UTC from IEEE Xplore. Restrictions apply.