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Full-wavefield Redatuming of Perturbed Fields with the Marchenko Method

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SUMMARY

Wavefield extrapolation, or redatuming, is a critical step for imaging. It is particularly challenging in areas such as subsalt or under complex overburdens. The framework of Marchenko redatuming allows for the retrieval of up- and downgoing fields at chosen locations in the subsurface that contain primary arrivals and internal multiples, while requiring relatively little knowledge of the subsurface model. In this paper, we present a new form of the Marchenko system for perturbed fields. Based on this system, we present a new iterative scheme that explicitly reconstructs only the unknown perturbations to the Marchenko focusing functions, and by consequence only the perturbed/scattered up- and downgoing Green's functions. This new scheme departs from previous versions of the method in that it requires additional inputs, which include an extra initial focusing operator and perturbations to the surface reflection data. We validate our method with numerical tests, showing that it is particularly well-suited to properly handle complex models with large/sharp constrasts such as salt boundaries. We foresee this new approach to be of use not only in general imaging applications, but also for time-lapse studies as it can directly redatum time-lapse changes.

Introduction

When imaging complex subsurface geology with waveform-based methods, accurate wavefield extrapolation or redatuming is key. The recently proposed framework of Marchenko redatuming (Broggini and Snieder, 2012; Wapenaar *et al.*, 2014; van der Neut *et al.*, 2015) provides an approach to redatuming that is capable of estimating full-waveform up- and downgoing waves in the subsurface (including internal multiples), while relying solely on one-sided surface reflection data and relatively little information about the subsurface (e.g., a conventional velocity model). While it has already achieved much progress, including validation with field data (Ravasi *et al.*, 2015), Marchenko redatuming is still subject of research. For example, when dealing with highly heterogeneous media, and in particular with models containing large contrasts (such as subsalt), Vasconcelos *et al.* (2015) proposed a modification to the Marchenko iterative scheme that makes use of inverse transmission matrices. While this modification does improve the redatuming results, it also introduces artefacts in the desired fields due to the complexity of the reference model. Here, we offer a new formulation of the Marchenko system for scattered/perturbed fields, which uses the initial conditions of Vasconcelos *et al.* (2015), while eliminating model-induced artefacts that arise under the conventional formulation.

A new Marchenko system for perturbed fields

Following van der Neut et al. (2015), to create a Marchenko system that targets wavefield perturbations, we consider the system:

$$\begin{pmatrix} -\mathbf{G}^{-} \\ \mathbf{G}^{+*} \end{pmatrix} = \begin{pmatrix} \mathbf{I} & -\mathbf{R} \\ -\mathbf{R}^{*} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{F}^{-} \\ \mathbf{F}^{+} \end{pmatrix} , \quad (1)$$

and

$$\begin{pmatrix} -\mathbf{G}_0^{-} \\ \mathbf{G}_0^{+*} \end{pmatrix} = \begin{pmatrix} \mathbf{I} & -\mathbf{R}_0 \\ -\mathbf{R}_0^{*} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{F}_0^{-} \\ \mathbf{F}_0^{+} \end{pmatrix} , \quad (2)$$

where $\mathbf{G}^{\pm}$ denotes a matrix containing the Green's functions between all points on the acquisition surface to all points on a chosen subsurface datum, with the + superscript indicating downgoing waves at the subsurface datum, and the – superscript denoting upgoing arrivals. **R** represents the medium's reflection response at the surface as given by Wapenaar *et al.* (2014), and the * superscript denotes fields that are time reversed. F are the focusing functions that characterise the Marchenko system, and here we follow the formulation of Vasconcelos et al. (2015), with $\mathbf{F}^{\pm} = \mathbf{F}_1^{\pm}$. Key to this paper is that the system in equation (1) describes fields in the real medium, and the system in equation (2), with the 0 subscripts in the all quantities, describes fields in a known reference medium (e.g., in a velocity model).

By subtracting equation (2) from (1), we obtain

$$\begin{pmatrix} -\delta\mathbf{G}^{-} \\ \delta\mathbf{G}^{+*} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & -\delta\mathbf{R} \\ -\delta\mathbf{R}^{*} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{F}_0^{-} \\ \mathbf{F}_0^{+} \end{pmatrix} + \begin{pmatrix} \mathbf{I} & -\mathbf{R} \\ -\mathbf{R}^{*} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \delta\mathbf{F}^{-} \\ \delta\mathbf{F}^{+} \end{pmatrix}, \quad (3)$$

with $\delta\mathbf{G}^{\pm} = \mathbf{G}^{\pm} - \mathbf{G}_0^{\pm}$, $\delta\mathbf{F}^{\pm} = \mathbf{F}^{\pm} - \mathbf{F}_0^{\pm}$, and $\delta\mathbf{R} = \mathbf{R} - \mathbf{R}_0$. This simple step results in an explicit system for the unknown field perturbations $\delta\mathbf{G}^{\pm}$ and focusing $\delta\mathbf{F}^{\pm}$, as a function of the known $\mathbf{R}, \mathbf{R}_0$ and $\mathbf{F}_0^{\pm}$. To perform Marchenko redatuming, we follow a similar approach to that of Wapenaar *et al.* (2014), where equation (3) is recast into a system in terms of focusing functions only:

$$\begin{pmatrix} \mathbf{0} & -\Theta\delta\mathbf{R} \\ -\Theta\delta\mathbf{R}^{*} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{F}_0^{-} \\ \mathbf{F}_0^{+} \end{pmatrix} = \begin{pmatrix} \mathbf{I} & -\Theta\mathbf{R} \\ -\Theta\mathbf{R}^{*} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \delta\mathbf{F}^{-} \\ \delta\mathbf{F}^{+} \end{pmatrix}, \quad (4)$$

where the separation operator Θ is introduced (van der Neut *et al.*, 2015; Vasconcelos *et al.*, 2015), assuming that $\Theta\mathbf{G}^{\pm} = \Theta\mathbf{G}_0^{\pm} = \Theta\delta\mathbf{G}^{\pm} = \mathbf{0}$ (same for time-reversed fields) and $\Theta\delta\mathbf{F}^{\pm} = \delta\mathbf{F}^{\pm}$. Here, we rely on first-arrival time windowing to generate Θ, the same as previous Marchenko implementations (Wapenaar *et al.*, 2014). Equation (4), in turn, allows for an iterative estimate of the focusing updates $\delta\mathbf{F}^{\pm}$, according to the Neumann expansion:

$$\begin{pmatrix} \delta\mathbf{F}^{-} \\ \delta\mathbf{F}^{+} \end{pmatrix} = \sum_{k=0}^{\infty} \begin{pmatrix} \mathbf{0} & -\Theta\mathbf{R} \\ -\Theta\mathbf{R}^{*} & \mathbf{0} \end{pmatrix}^{k} \begin{pmatrix} \mathbf{0} & -\Theta\delta\mathbf{R} \\ -\Theta\delta\mathbf{R}^{*} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{F}_0^{-} \\ \mathbf{F}_0^{+} \end{pmatrix}. \quad (5)$$

In practice, this yields the following Marchenko redatuming steps:

i. Begin with the surface reflection data $\mathbf{R}$ from the real medium, along with the reflection $\mathbf{R}_0$ and transmission $\mathbf{T}_0$ (Vasconcelos et al., 2015) modelled using a known reference medium;

ii. Based on the modelled $\mathbf{T}_0$, compute the initial focusing fields $\mathbf{F}_0^{+} = \mathbf{T}_0^{-1}$, following Vasconcelos et al. (2015), and, additionally, $\mathbf{F}_0^{-} = \mathbf{R}_0\mathbf{F}_0^{+}$;

iii. Using these initial $\mathbf{F}_0^{\pm}$, solve for the focusing updates $\delta\mathbf{F}^{\pm}$ with the system in equation (5);

iv. Substitute the estimates of $\delta\mathbf{F}^{\pm}$ back in equation (3) to yield the Marchenko fields $\delta\mathbf{G}^{\pm}$.

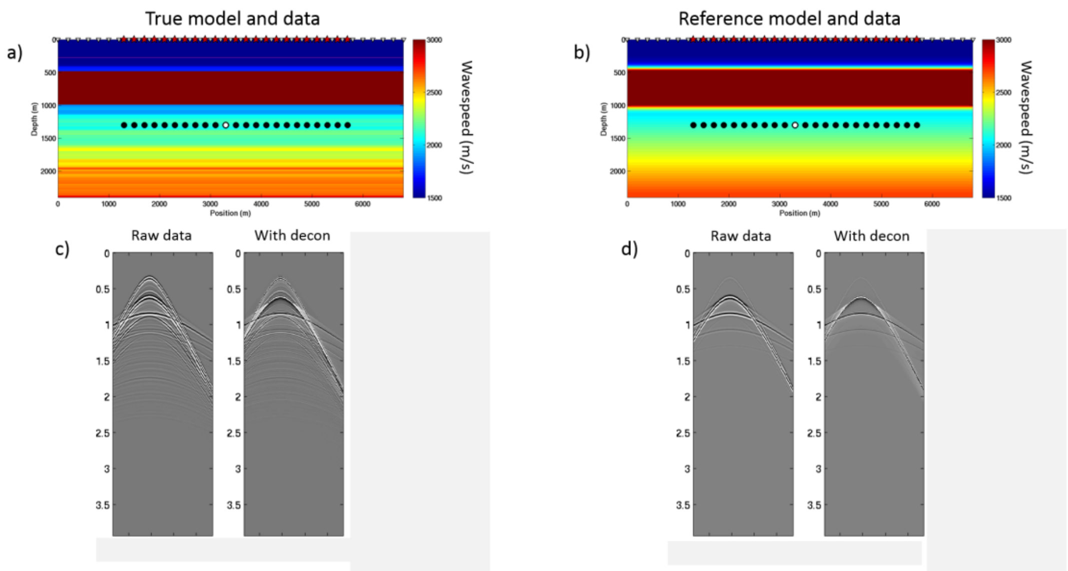

Figure 1 *Models used to generate the test data and Marchenko redatuming tests, along with the corresponding seismic data. The model in a) is the 'true' model, while b) shows the reference model used for Marchenko redatuming containing a high-speed layer with sharp interfaces. The stars and triangles on the surface represent sources and receivers, respectively, while the round dots in the subsurface are the location of redatumed fields. Panel c) shows one shot gather from the full*

reflection data $\mathbf{R}$ *in the true model (before and after source deconvolution), and d) is the corresponding shot gather from the reference* $\mathbf{R_0}$ *acquired in the model in b).*

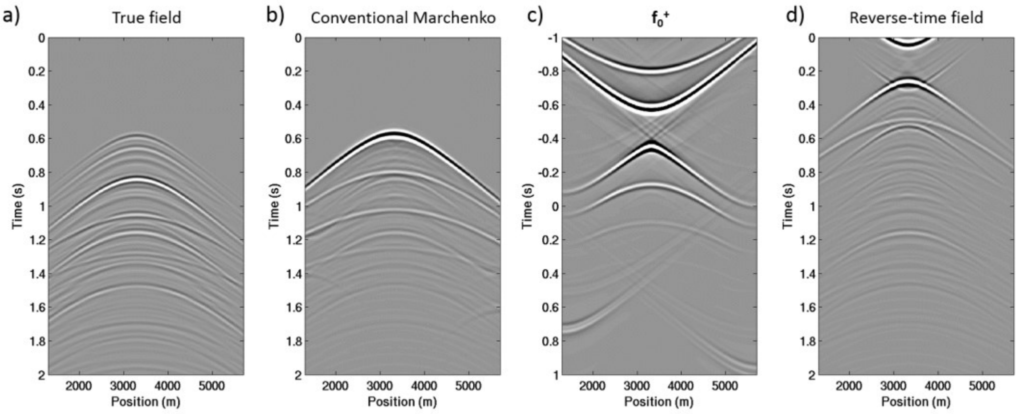

Figure 2 *Subsurface fields and initial focusing function for Marchenko redatuming. Panel a) is extracted from a column of the full* $= \delta\mathbf{G}^+ + \delta\mathbf{G}^-$ *modelled directly with the true medium in Figure 1a. This corresponds to the response of the medium at the white dot in Figure 1, due to sources at the surface. b) shows the same response from estimated by means of conventional Marchenko redatuming using the initial focusing function in panel c) (note that* $\mathbf{f}_0^+$ *corresponds to a column of* $\mathbf{F}_0^+$*). For reference, panel d) is the result of redatuming by conventional reverse-time extrapolation using the model in Figure2b.*

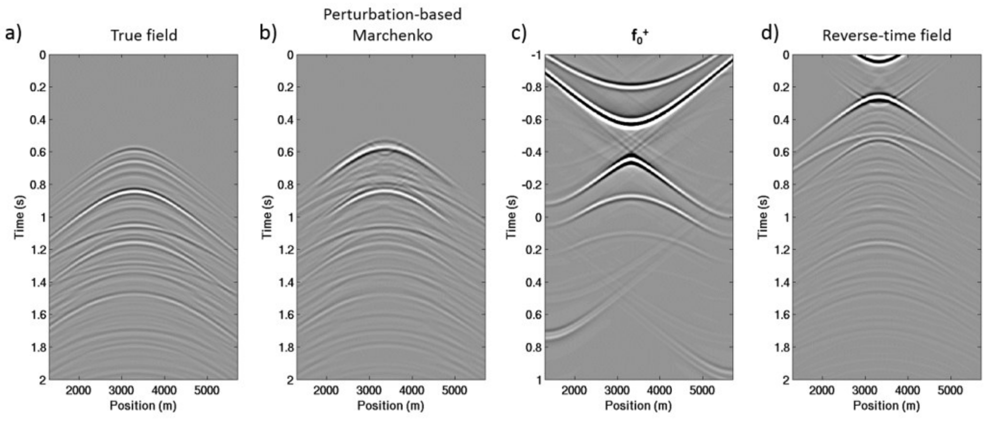

Figure 3 *As in Figure 2, panel a) is extracted from a column of the full* $\delta\mathbf{G}$ *, modelled directly with the true medium in Figure 1a. This corresponds to the response of the medium at the white dot in Figure 1, due to sources at the surface. b) shows the same response from estimated with the*

perturbation-based Marchenko redatuming, using the initial focusing function in panel c) (which is the same as that in Figure 2c) For reference, panel d) is the result of redatuming by conventional reverse-time extrapolation using the model in Figure 2b.

Layered medium example

We validate our modified Marchenko system for perturbed fields using the model shown in Figure 1. This is a layered model made from a depth profile of the Sigsbee model at a fixed horizontal location; it is ideal for our test because it contains a high-contrast salt layer that is included in the reference model in Figure 1b. The true model in Figure 1a is assumed to be unknown, whereas its reflection response is known (Figure 1c), together with that of a known reference model (Figures 1b and 1d).
As shown by Vasconcelos *et al.* (2015), in the presence of models with sharp contrasts (Figure 1b), Marchenko redatuming yields better results when the initial focusing function used in the iterative procedure consists of estimate of the inverse of the reference transmission matrix. Here, we use the same approach to provide the $\mathbf{F}_0^+$ needed for iterations based on equation (5) (Figures 2c and 3c).

For the new perturbation-based scheme we propose here, we need not only the initial $\mathbf{F}_0^+$, but also an additional focusing function $\mathbf{F}_0^-$ (accounting for reflections due to $\mathbf{F}_0^+$), along with a calculation of $\delta\mathbf{R}$ (e.g., the difference between the gathers in Figures 1c and 1d), which is explicitly used in equation (5) together with the full reflection data $\mathbf{R}$.
To illustrate the method, with Figures 2 and 3 we can compare the results of the conventional Marchenko method with those of our perturbation-based approach, both using the exact same initial focusing function $\mathbf{F}_0^+$. In Figure 2b, we observe that the redatumed perturbed field is a better estimate of that in Figure 2a than the migration-based time-reversed field in Figure 2d, in terms of reconstructing missing downgoing arrivals. However, the conventional Marchenko result in Figure 2c also contains artefacts, despite using the initial $\mathbf{F}_0^+ = \mathbf{T}_0^{-1}$, as also shown by Vasconcelos *et al.* (2015). In contrast, the perturbation-based Marchenko result in Figure 3b results in a noticeably more accurate estimate of the true perturbations (Figure 3a), retrieving downgoing events that are absent in the time-reversed field (Figure 3d), while not generating most of the artefacts seen in Figure 2b. This result is better because the new system in equation (3) properly accounts for the scattering effects due to the discontinuities in the reference model, by employing the additional focusing function $\mathbf{F}_0^-$ and by explicitly introducing the reference model reflections through $\delta\mathbf{R}$.

Conclusions

In this paper, we introduce a new, modified version of the Marchenko system of equations that aims at directly redatuming wavefield perturbations relatively to a known reference model. The new formalism demands not only one initial downgoing focusing function as in previous versions of Marchenko redatuming, but requires also a second focusing function (that is upgoing at the surface), together with accounting explicitly with backscattering due to contrasts in the reference model used for redatuming. With a numerical example, we show that not only does our new scheme produce accurate estimates of up- and downgoing wavefield perturbations in the subsurface, but it also greatly suppresses model-related artefacts that arise in the conventional Marchenko scheme, using the same input model and initial focusing function. We believe our approach will prove to be useful in complex imaging problems with large contrasts (e.g., subsalt imaging), or in time-lapse applications where the method could directly estimate time-lapse wavefield changes in the subsurface.

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