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THE EFFECTS OF BEAM ON THE HYDRODYNAMIC CHARACTERISTICS OF SHIP HULLS

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ABSTRACT

Forced oscillation experiments have been carried out with a systematic ship model family of which the length-beam ratio was ranging from 4 to 20. The experiments also included a thin plate to simulate the case of an infinite length-beam ratio. Vertical and horizontal harmonic motions in calm water have been considered and the corresponding hydrodynamic coefficients have been determined. Moreover the vertical motions and added resistance in waves have been measured. The results are presented in grafical form and are compared with some existing calculation methods.

NOMENCLATURE

A,B,C,D,E,G a,b,c,d,e,g	hydrodynamic coefficients of the equations of pitch and heave respectively
B	ship's beam
C_B	block coefficient
C_P	prismatic coefficient
C_s	horizontal sectional added mass coefficient
c	wave celerity
F	total vertical wave force
F'	sectional hydromechanic force
F_n	Froude number
g	acceleration owing to gravity
I_{yy}	vertical longitudinal moment of inertia
I'_{zz}	dimensionless horizontal moment of inertia
K'	coefficient of accession to moment of inertia
K	empirical coefficient in the low aspect ratio lift formula
$K_{1,2}$	coefficients of accession (long., lat.)
k	wave number
k_{yy}	vertical longitudinal radius of inertia of ship
k_{zz}	horizontal longitudinal radius of inertia of ship
L	ship's length
M	total vertical wave moment; mass of ship
M'	dimensionless mass of ship
m'	vertical sectional added mass
N'	vertical sectional damping coefficient
$N'_{\dot v}, N'_v, N'_{\dot r}, N'_r$ $Y'_{\dot v}, Y'_v, Y'_{\dot r}, Y'_r$	hydrodynamic coefficients of the equations of yaw and sway respectively
r'	dimensionless yaw velocity
$\dot r'$	dimensionless yaw acceleration
T	ship's draught
T^*	effective draught
T_e	period of encounter
V	forward velocity of ship
V_z	vertical relative velocity with respect to the water
v'	dimensionless sway velocity
$\dot v'$	dimensionless sway acceleration
$X'_{\dot u}$	dimensionless longitudinal added mass
x'	dimensionless length coordinate in a right hand body fixed coordinate system with centre of gravity in the origin and the starboard side positive
x_b, y_b, z_b	righthand coordinate system fixed to ship with the origin situated in the ship's waterline and the portside positive
x'_{p1}	dimensionless centre connected with the first moment of viscous force distribution
x'_{p2}	dimensionless centre connected with the second moment of viscous force distribution
x'_r	point of application of total yaw force
x'_v	point of application of total sway force
Y'	dimensionless hydrodynamic lateral force
y'_o	dimensionless motion amplitude
y_w	half width of waterline (z=0)
z	heave displacement
ε	phase angle
λ	wave length
∇	volume of ship's displacement
ω	circular wave frequency
ω'	dimensionless PMM frequency
ω_e	circular frequency of encounter
ρ	density of water
σ'_1	dimensionless stability root
σ'_2	dimensionless stability root
θ	pitch angle
ζ	instantaneous wave elevation

Subscripts :

a	amplitude of denoted parameter
$F\zeta$	wave force with respect to wave elevation
$M\zeta$	wave moment with respect to wave elevation

Superscripts :

'	sectional values or dimensionless values according to SNAME-nomenclature

1. INTRODUCTION

The calculation of the vertical hydrodynamic forces and moments acting on a ship in seawaves, according to the strip theory, has proved to be a valuable tool. This is also true to a limited extent for horizontal motions, but the experimental verification for low frequency motions, which are of interest for manoeuvring and steering problems, is rather scarce.
The detailed comparisons of calculation and experiment for pitch and heave are for the greater part restricted to more or less average hull dimensions, for instance a length-beam ratio of approximately 6 to 8 and block coefficients around .70. Although predictions of vertical motions of extreme ship forms have been quite succesful, it has not been known to what extent the strip theory is valid when more extreme hull dimensions are considered. Intuitively one may imagine, that the thinner the ship form, the more the application of the strip method is justified.
For manoeuvring and steering purposes the hydrodynamic coefficients of the equations of motion depend to a larger extent on viscous effects introducing lift phenomena, when compared with vertical motions of a ship in waves. Existing methods to approximate these hydrodynamic forces have a more empirical character. Apart from the length-draught ratio in both cases the length-beam ratio may be regarded as a useful parameter in a comparison of theory and experiment.
The main objective of this paper is to provide extensive experimental data respecting the influence of the length-beam ratio of a systematic ship model family on the hydrodynamic forces on the hull for vertical oscillatory motions in the wave frequency range, as well as for low frequency horizontal motions of interest for steering and manoeuvring.
The experiments cover a large range of length-beam ratio's which includes a very thick shipform (L/B=4) and a very thin ship with L/B=20. In addition a thin plate has been tested in horizontal motion to simulate an infinitely large length-beam ratio. All of the models have been derived from the standard Sixty Series hull form with L/B=7 and C_B= .70 [1], by multiplying the width by constant factors, to arrive at L/B=4, 5.5, 7, 10 and 20. All models have been made from glass reinforced polyester and have a length of 10 feet. For main particulars see table 1.

2. EXPERIMENTAL PROGRAM AND RESULTS

With a vertical Planar Motion Mechanism (PMM) the hydrodynamic coefficients of the heave and pitch equations according to equations (1) of appendix 1 have been measured for Froude numbers F_n=.20 and F_n=.30.
The latter speed is high for all models and large wave making has been observed during the experiments.
Excellent linearity has been found for the considered heave amplitudes which go to 1% of the model length and pitch amplitudes up to 3.5 degrees.
For the wave tests wave heights of 2.5 % of the model length have been considered.
The linearity has been proved to be good with L/B=4.
The non-dimensional mass and damping coefficients as well as the mass and damping cross coupling coefficients are given in figures 1 to 8 in non-dimensional form as a function of the Froude number, the frequency of oscillation and the length-beam ratio.
Figures 9 and 10 give the dimensionless motion amplitudes of heave and pitch and figure 11 gives the added resistance in regular head waves.
The motions and the added resistance in waves could not be measured for the L/B=20 model owing to experimental difficulties.
The hydrodynamic coefficients for yaw and sway according to equations (13) of appendix 3 have been measured for three velocities : F_n=.15, .20 and .30.
A large amplitude PMM has been used; the model frequency range has been between ω=.2(.1) 1.0. Strutamplitudes for both modes of motion were respectively 5, 10, 15, 20 and 25 cm, the horizontal distance between the struts being 1 m. A relatively small wave making was observed for the lowest of the three velocities considered, and therefore the experimental results for F_n=.15 have been used for comparing with some calculation methods.
Figure 12, 13 and 14 show the coefficients, derived from the force and moment measurements as a function of L/B-ratio for the three considered forward speeds. Table 2 gives the numerical values of the various hydrodynamic coefficients.
In figure 15 and 16 the results of the swaying force and swaying moment are presented as a function of speed, frequency, L/B-ratio and amplitude.

3. DISCUSSION OF THE RESULTS

3.1. Vertical Motions

First of all the heaving and pitching motions have been calculated with as a basis a formulation of the strip theory as given in appendix 1 and [2]. This formulation has been derived using earlier work by Shintani [3], Söding [4], Semenof-Tjan-Tsansky et al [5], Tasai [6] and affords the same results as given by Salvesen et al [7]. Afterwards the method has been used, which has been formulated principally by Korvin-Kroukovsky and Jacobs [8] and modified by the authors [9].
The results of both methods have been compared with the experimental results.

The added resistance in waves owing to the pitching and heaving motions has been calculated by the method described in appendix 2. The added resistance is determined by calculating the work done by the radiated damping waves, which result from the vertical motions of the ship relative to the water. In [10] this method has been confirmed by experimental results derived from model tests with a fast cargo ship hull form. Further experience included blunt tanker forms, although in some of these cases the agreement has been somewhat less satisfactory at high frequency of encounter.

In the figures 1 to 11 the experimental values are compared with corresponding calculations according to the modified Korvin-Kroukovsky formulation [9] and according to equations (6) and (7). For convenience we will call these the old and the new method respectively.
With regard to the coefficients of the equations of motion for heave and pitch the two calculation methods give almost identical results, except for the pitch damping coefficient at low frequencies and for the added mass cross coupling coefficient D for pitch.
The differences between the measured added mass and the calculated value are small, even for the very low L/B ratio's. For the added moment of inertia the correlation is still satisfactory, with only few differences for the highest speed nd the lowest L/B ratio.
The heave damping coefficient is reasonably predicted except for high frequencies where viscous effects, for instance separation of flow, may be important.
Both the new and the old method predict the pitch damping rather poorly, particularly at low frequencies. The experimental data do not show a clear preference for one of the two methods. For practical purposes the overestimation of the pitch damping at low frequencies, according to the new method is not too important in the motion prediction.
Considering the absolute magnitude of the damping cross coupling terms the coefficients e and E are very well predicted by both theories for the two considered forward speeds, as well as for all length-beam ratio's.
Also the added mass cross coupling coefficient d for heave is reasonably well predicted by both methods, but in the case of the mass cross coupling coefficient D for pitch the experimental points for low frequencies lie between the two predicted curves. For low frequencies the experimental values favour the prediction according to the new method.
leave amplitudes in waves are somewhat overestimated by the new method. Earlier experience with both methods has shown us a slight preference for the modified Korvin-Kroukovsky and Jacobs method although a desired symmetry in the mass cross coupling coefficients is not fulfilled in their presentation. Moreover added resistance is overestimated by the new method and in this respect it should be remembered that added resistance varies as the squared motion amplitudes.
For F_n=.20 the predicted added resistance agrees very well with the measured values, with only minor differences at high frequencies. Even for the very low length-beam ratio's the agreement is satisfactory, considering the more or less extreme hull form and the relatively high forward speed in those cases. For F_n=.30 the correlation between theory and experiment is less. However for all length-beam ratio's, except for L/B=7 this speed is very high, with corresponding high ship waves.
Especially for L/B=4 the added resistance at high frequencies is under estimated by the theory.

3.2. Horizontal Motions

The coefficients have been determined in a standard graphical way from the in phase and quadrature components of forces and moments measured with the PMM. The accuracy of the coefficients which are displayed in fig. 12, 13 and 14, is probably not high since the relevant forces and moments are small in magnitude. The coefficients indicate a trend in the results and do not pretend to be highly accurate.
In table 2 the numerical values of the coefficients are summarized using the dynamic modes of motions. The figures 12, 13 and 14 clearly show the effect of beam, which is not very pronounced for a low Froude number. As could be expected the forward speed affects the results to a certain extent : the thicker the model the more the model generated wave system plays a decisive role in the creation of the resulting hydrodynamic forces and moments. Hu [11] predicted the effect of speed upon the hydrodynamic coefficients, applying sources and doublets in the ship's centerplane and wake and taking into account the boundary conditions on the surface. Comparing the trend of the experimental results and the predicted values with regard to the forward velocity according to Hu, it can be said, that his prediction gives a more pronounced effect of speed.
It is interesting to note, that Van Leeuwen's results of his PMM tests [12] with an 8 feet model of the L/B=7 are practically the same as the results presented in this paper, taking a reasonable margin of accuracy into account. In figures 12, 13 and 14 some evidence is produced, that the values of the static and dynamic sway coefficients are approaching each other closely.
The condition for straight line stability (this word is used rather than controls fixed stability, since no rudder, propeller nor other hull appendices have been fitted) yields :

$$\frac{x'_v}{x'_r} < 1$$

When x'_v and x'_r both are positive this condition postulates, that the point of application of the total yaw force is located before the point of application of the sway force. In figure 12, 13 and 14 it may be observed, that for a L/B-ratio exceeding 8 this condition is fulfilled. Since at a L/B-ratio of approximately 20 Y'_r equals the mass M', x'_r will change sign and becomes extremely negative. In this case the aforementioned criterion is still satisfied, since it is obvious that x'_v remains positive. In table 2 the stability roots are calculated; the smaller roots are positive for the smaller L/B-ratio's and they are becoming negative for the larger L/B-ratio's. Noteworthy is the difference between the two last columns indicating, that the actually used plate for the experiments has a stable behaviour, but that an imaginary massless plate has an oscillatory stable behaviour. This fact is also found in stability analysis of ships which have large fins or deep keels, like sailing yachts and is caused by the small inertia forces relative to the lift forces [13] .

Jacobs [14,15] published a brief account of a simple theory for the calculation of the linear coefficients of the horizontal motion based upon simple hydrodynamic concepts. Apart from an ideal fluid treatment of a wing shaped body in an unbounded flow, resulting in hydrodynamic added masses and added moments of inertia with cross coupling coefficients, a viscous part is included representing the generation of a lift. Therefore, as an example the Jones' low aspect ratio lift formula has been applied. Lift generation depends upon the flow conditions near the trailing edge. As these conditions vary, it seems appropriate to introduce an empirical constant K to take these variations into account, as was suggested by Inoue [16]. This K-constant turns out to be nearly .75 as an average. In appendix 3 a brief account is given of Jacobs' method, which has been chosen for a comparison with the measured results. The total lift, as a result of an inertia distribution and a viscous distribution along the ship length is generated for the greater part in the forebody, which means that the viscous part counter-balances nearly the inertia part in the afterbody [15,17]. The centre of the viscous force distribution therefore lies well aft of the centre of gravity (x'_{p1})The second moment of the viscous force distribution is characterized by x'_{p2} and obviously this quantity is negative.
From the measurements of the relevant quantities the values of K, x'_{p1} and x'_{p2} are calculated and they are displayed in figure 17. They coincide remarkably well with empirical values presented by Inoue and Albring [18]. The coefficient Y'_r can also be used to check the validity of the empirical constants K_1, x'_{p1}. In figure 12 it may be seen, that there is a satisfactory agreement for the lower Froude number. Apart from considerations regarding the damping coefficients it is obvious, that the added mass, added moment of inertia and the mass cross coupling coefficients are accurately predicted by the simple stripwise integration of sectional values of added mass depending on local fullness and local B/T-values. So called three dimensional corrections have been applied as indicated by Jacobs and others. In order to compare the measured results with other methods available in literature, it has been decided to use the results of Inoue which are principally based upon Bollay's low aspect ratio theory and a number of empirical allowances. Appendix 3 gives a brief account of the used formulae according to Inoue. As can be seen in figure 12 the calculation agrees with the measured results with the exception of Y'_r. Norrbin [19] analysed statistical material and derived regression formulae on the basis of the so called "bis" system of reference. In appendix 3 these regression formulae are "translated" into the nomenclature adopted in this paper. Inspecting the formulae a small effect of the L/B-ratio can be demonstrated, while generally speaking the calculated results using these regression formulae are in close agreement in the normal range of L/B-ratio's, as shown in figure 12. Since lift generation is of primary importance in manoeuvring problems and since experimental material about this subject is not extensively published in literature, it has been decided to give the transverse force and moment in the sway motion for two speeds : F_n=.15 and F_n=.30, as a function of reduced frequency and amplitude in figures 15 and 16.
In a very restricted range full linearity in frequency and amplitude exists. For the higher frequencies linearity is lost to some extent especially in the transverse force and to a smaller extent in the moment. A number of effects are obscuring the results, for instance nonlinearity owing to the cross flow. Also frequency- and amplitude effects are interferring when one tries to interpret the experimental results.

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5. APPENDIX 1

The equations of motion of heave and pitch

The equations of motion of heave and pitch and their solution are given by :

$$\left.\begin{array}{ll}(\rho\nabla+a)\ddot{z}+b\dot{z}+cz-d\ddot{\theta}-c\dot{\theta}-g\theta)=F & \text{(heave)}\\ (I_{yy}+A)\ddot{\theta}+B\dot{\theta}+C\theta-D\ddot{z}-E\dot{z}-Gz=M & \text{(pitch)}\\ z=z_a\cos(\omega_e t+\varepsilon_{z\zeta}) \quad , \quad \theta=\theta_a\cos(\omega_e t+\varepsilon_{\theta\zeta}) & \end{array}\right\} \quad (1)$$

The various coefficients a-g and A-G are derived from :

$$\rho\nabla\ddot{z}=\int_L F'dx_b$$

$$I_{yy}\ddot{\theta}=-\int_L F'x_b dx_b \qquad (2)$$

where F' is the hydromechanical force acting on a cross-section of the ship.

It can be found that :

$$F'=-2\rho g y_w(z-x_b\theta)-\left(\frac{\partial}{\partial t}-V\frac{\partial}{\partial x}\right)(\dot{z}-x_b\dot{\theta}+V\theta-\dot{\zeta}^*)\left(m'-\frac{iN'}{\omega_e}\right) \quad (3)$$

The effective wave elevation ζ^* is defined as :

$$\zeta^*=\zeta e^{-kT^*} \quad \text{, where :}$$

$$T^*=-\frac{1}{k}\ln\left(1-\frac{k}{y_w}\int_{-T}^{0} y_w e^{kz_b}dz_b\right) \qquad (4)$$

This expression follows from the integration of the vertical component of the undisturbed incident wave pressure on a cross section contour. The time derivatives of ζ^* are used in the calculation of the damping and added mass correction to the "Froude-Kriloff" wave force and moment.
Because harmonic motions only are considered, equation (3) can be written as :

$$F'=-2\rho g y_w(z-x_b\theta-\zeta^*)-m'(\ddot{z}-x_b\ddot{\theta}+2V\dot{\theta}-\ddot{\zeta}^*)+$$
$$+V\frac{dm'}{dx_b}(\dot{z}-x_b\dot{\theta}+V\theta-\dot{\zeta}^*)-N'(\dot{z}-x_b\dot{\theta}+2V\theta-\frac{\omega}{\omega_e}\dot{\zeta}^*)+$$
$$+V\frac{dN'}{dx_b}(z-x_b\theta-\frac{V\dot{\theta}}{\omega_e^2}-\frac{\omega}{\omega_e}\zeta^*) \qquad (5)$$

Combining equations (2) and (5) one finds :

$$a=\int_L m'dx_b+\left[\frac{V}{\omega_e^2}\int\frac{dN'}{dx_b}dx_b\right]$$

$$b=\int_L\left(N'-V\frac{dm'}{dx_b}\right)dx_b$$

$$c=2\rho g\int_L y_w dx_b$$

$$d=\int_L m'x_b dx_b+2\frac{V}{\omega_e^2}\int_L N'dx_b-\frac{V^2}{\omega_e^2}\int_L\frac{dm'}{dx_b}dx_b+$$
$$+\left[\frac{V}{\omega_e^2}\int_L\frac{dN'}{dx_b}x_b dx_b\right]$$

$$e=\int_L N'x_b dx_b-2V\int_L m'dx_b-V\int_L\frac{dm'}{dx_b}x_b dx_b+$$
$$-\left[\frac{V^2}{\omega_e^2}\int_L\frac{dN'}{dx_b}dx_b\right]$$

$$g=2\rho g\int_L y_w x_b dx_b$$

(6a)

$$A=\int_L m'x_b^2dx_b+2\frac{V}{\omega_e^2}\int_L N'x_bdx_b-\frac{V^2}{\omega_e^2}\int_L\frac{dm'}{dx_b}x_bdx_b+\left[\frac{V}{\omega_e^2}\int_L\frac{dN'}{dx_b}x_b^2dx_b\right]$$

$$B=\int_L N'x_b^2dx_b-2V\int_L m'x_bdx_b-V\int_L\frac{dm'}{dx_b}x_b^2dx_b+-\left[\frac{V^2}{\omega_e^2}\int_L\frac{dN'}{dx_b}x_bdx_b\right]$$

$$C=2\rho g\int_L y_wx_b^2dx_b$$

$$D=\int_L m'x_bdx_b+\left[\frac{V}{\omega_e^2}\int_L\frac{dN'}{dx_b}x_bdx_b\right]$$

$$E=\int_L N'x_bdx_b-V\int_L\frac{dm'}{dx_b}x_bdx_b$$

$$G=2\rho g\int_L y_wx_bdx_b \qquad (6b)$$

If $F=F_a\cos(\omega_et+\epsilon_{F\zeta})$ and $M=M_a\cos(\omega_et+\epsilon_{M\zeta})$ then:

$$\frac{F_a}{\zeta_a}\substack{\cos\\ \sin}\,\epsilon_{F\zeta}=2\rho g\int_L y_we^{-kT^*}\substack{\cos\\ \sin}\,kx_bdx_b+$$
$$\mp\omega\int_L\left(\frac{\omega}{\omega_e}N'-V\frac{dm'}{dx_b}\right)e^{-kT^*}\substack{\sin\\ \cos}\,kx_bdx_b+$$
$$-\omega^2\int_L\left(m'+\left[\frac{V}{\omega\omega_e}\frac{dN'}{dx_b}\right]\right)e^{-kT^*}\substack{\cos\\ \sin}\,kx_bdx_b \qquad (7a)$$

$$\frac{M_a}{\zeta_a}\substack{\cos\\ \sin}\,\epsilon_{M\zeta}=-2\rho g\int_L y_wx_be^{-kT^*}\substack{\cos\\ \sin}\,kx_bdx_b+$$
$$\pm\omega\int_L\left(\frac{\omega}{\omega_e}N'-V\frac{dm'}{dx_b}\right)x_be^{-kT^*}\substack{\sin\\ \cos}\,kx_bdx_b+$$
$$+\omega^2\int_L\left(m'+\left[\frac{V}{\omega\omega_e}\frac{dN'}{dx_b}\right]\right)x_be^{-kT^*}\substack{\cos\\ \sin}\,kx_bdx_b \qquad (7b)$$

For ships where N' and m' are zero at the stem and stern the expressions (6) and (7) can be simplified, but this has not been carried through in the corresponding computerprogram.

When the terms between the brackets are left out from equations (6) and (7) and when $\frac{\omega}{\omega_e}=1$ in the coefficients of N' in (7) the resulting equations of motion are equal to those derived by the modified Korvin-Kroukovsky and Jacobs' results [9].

APPENDIX 2

The Added Resistance in Waves

The added resistance of a ship in waves is a result of the radiated damping waves created by the motions of the ship relative to the water. Joosen [20] showed that for the mean added resistance can be written :

$$R_{AW}=\frac{\omega^3}{2g}(bz_a^2+B\theta_a^2) \qquad (8)$$

This expression was derived by expanding Maruo's expression [21] into an asymptotic series with respect to a slenderness parameter and taking into account only first order terms. His simplified treatment results in an added resistance which is independent of the forward speed. This latter fact is roughly confirmed by experiments [10].
Equation (8) is equivalent to Havelock's equation [22]. Although not consistent with the theory, the frequency of encounter is used by Joosen in (8) when a ship with forward speed is considered. In equation (8) uncoupled motions are considered. In the present work the following procedure is adopted for the calculation of the radiated damping energy P of the oscillating ship during one period of encounter :

$$P=\int_L\int_0^{T_e} b'V_z^2\,dtdx_b \qquad (9)$$

where $b'=N'-V\frac{dm'}{dx_b}$, the sectional damping coefficient for ship at speed and :

$V_z=\dot{z}-x_b\dot{\theta}+V\theta-\dot{\zeta}^*$, the vertical relative watervelocity at a cross section of the ship.
As V_z is a harmonic function with amplitude V_{za} and a frequency equal to the frequency of encounter ω_e we find :

$$P=\frac{\pi}{\omega_e}\int_L b'V_{za}^2\,dx_b \qquad (10)$$

Following the reasoning given by Maruo in [21] the work being done by the towing force R_{AW} is given by :

$$P=R_{AW}(V+c)T_e=R_{AW}\cdot\lambda \qquad (11)$$

From (10) and (11) it follows that :

$$R_{AW}=\frac{k}{2\omega_e}\int_L b'V_{za}^2dx_b \qquad (12)$$

This expression is almost equal to (8) when the wave elevation ζ is small compared with the vertical motions of the ship in addition to a very low forward speed and fore and aft symmetry.

APPENDIX 3

The Equations of Motion of Yaw and Sway

Principally the following account is based upon work by Jacobs [14,15].
The equationsof motion for the bare hull condition are given by :

$$M'(\dot{v}'+r') = Y'_{\dot{v}}\dot{v}' + Y'_v v' + Y'_{\dot{r}}\dot{r}' + Y'_r r' \qquad \text{(sway)}$$
$$I'_{zz}\dot{r}' = N'_{\dot{v}}\dot{v}' + N'_v v' + N'_{\dot{r}}\dot{r}' + N'_r r' \qquad \text{(yaw)} \tag{13}$$

The hydrodynamic coefficients in (13) can be calculated by assuming a division between an inertia force distribution and a viscous force distribution along the ship's hull. The distribution of the hydrodynamic inertia forces can be found by well-known methods in hydrodynamics of which brief accounts can be found, among others in [19, 23]. Confining ourselves to horizontal motions at a constant forward velocity in an ideal fluid the following expressions for the right-hand sides of (13) are derived :

$$Y'_{id} = Y'_{\dot{v}}\dot{v}' + X'_{\dot{u}}r' + Y'_{\dot{r}}\dot{r}'$$
$$N'_{id} = N'_{\dot{r}}\dot{r}' + (Y'_{\dot{v}} - X'_{\dot{u}})v' + Y'_{\dot{r}}(\dot{v}'+r') \tag{14}$$

The coefficients appearing in (14) are calculated by the following expressions, assuming that the strip method is applicable together with Lamb's correction coefficients of accession :

$$Y'_{\dot{v}} = -\frac{\pi K_2 T^2}{L^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} C_s dx'$$
$$N'_{\dot{v}} = -\frac{\pi K_2 T^2}{L^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} C_s x' dx'$$
$$Y'_{\dot{r}} = -\frac{\pi K' T^2}{L^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} C_s x' dx' = \frac{K'}{K_2} N'_{\dot{v}} \tag{15}$$
$$N'_{\dot{r}} = -\frac{\pi K' T^2}{L^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} C_s x'^2 dx'$$
$$X'_{\dot{u}} = K_1 M'$$

From (14) it is obvious, that for the damping coefficients the following expressions exist in an inviscid fluid :

$$Y'_{v\ id} = 0$$
$$Y'_{r\ id} = X'_{\dot{u}}$$
$$N'_{v\ id} = Y'_{\dot{v}} - X'_{\dot{u}} \tag{16}$$
$$N'_{r\ id} = Y'_{\dot{r}}$$

A ship-shaped low aspect ratio wing in a real fluid develops a circulation around the profile generating a lift owing to the viscosity. This lift can be approximated for moderate speeds by the corrected Jones' low aspect ratio formula, taking into account the action of the water surface by doubling the draught. This formula can also be considered as the integral of the viscous force distribution along the hull. The first and second moments of this distribution yields the remaining damping derivatives :

$$Y'_{v\ visc} = -K\pi\frac{2T}{L}\cdot\frac{T}{L}$$
$$N'_{v\ visc} = Y'_{r\ visc} = -x'_{p1}2\pi\frac{KT^2}{L^2} \tag{17}$$
$$N'_{r\ visc} = -x'^2_{p2}\,2\pi\frac{KT^2}{L^2}$$

Numerical values of the empirical constants K_1, x'_{p1} and x'_{p2} are displayed in figure 17. Combining equations (16,17) the total damping coefficients can be listed as follows, assuming that mutual interference between inertia and viscous forces can be neglected :

$$Y'_v = -2K\pi\frac{T^2}{L^2}$$
$$N'_v = Y'_{\dot{v}} - X'_{\dot{u}} - x'_{p1}2K\pi\frac{T^2}{L^2}$$
$$Y'_r = X'_{\dot{u}} - x'_{p1}2K\pi\frac{T^2}{L^2} \tag{18}$$
$$N'_r = -x'^2_{p2}\,2K\pi\frac{T^2}{L^2}$$

For the purpose of comparing the results of the experimental coefficients with some existing formulae concerning damping coefficients, the following expressions are appropriate for the even keel condition, following Inoue [16]:

$$Y'_v = -2K\pi\frac{T^2}{L^2}$$
$$N'_v = -2\frac{T^2}{L^2}$$
$$Y'_r = 2K\pi\frac{T^2}{L^2}(.367+.42\frac{T}{L}) \tag{19}$$
$$N'_r = -1.08\frac{T^2}{L^2}$$

Norrbin [19] published data respecting the damping derivatives. His results are given in the form of regression formulae in his non dimensional so called 'bis' system. In the nomenclature adopted in this paper the expressions are given preceded by the corresponding formulae in the 'bis' system.

$$Y''_{uv} = -1.69\frac{\pi}{2}\frac{LT^2}{\nabla} - 0.04\ ;\ Y'_v = -1.69\pi\frac{T^2}{L^2} - 0.08\frac{B}{L}\frac{C_B T}{L}$$
$$N''_{uv} = -1.28\frac{\pi}{4}\frac{LT^2}{\nabla} + 0.02\ ;\ N'_v = -1.28\frac{\pi}{2}\frac{T^2}{L^2} + 0.04\frac{B}{L}\frac{C_B T}{L}$$
$$Y''_{ur} = 1.29\frac{\pi}{4}\frac{LT^2}{\nabla} - 0.18\ ;\ Y'_r = 1.29\frac{\pi}{2}\frac{T^2}{L^2} - 0.36\frac{B}{L}\frac{C_B T}{L}$$
$$N''_{ur} = -1.88\frac{\pi}{8}\frac{LT^2}{\nabla} + 0.09\ ;\ N'_r = -1.88\frac{\pi}{4}\frac{T^2}{L^2} + 0.18\frac{B}{L}\frac{C_B T}{L} \tag{20}$$

TABLE 1

		L/B=4.0	L/B=5.5	L/B=7.0	L/B=10.0	L/B=20.0	L/B= ∞
L_{PP}	m	3.048	3.048	3.048	3.048	3.048	3.048
LWL	m	3.099	3.099	3.099	3.099	3.099	3.099
B	m	.7620	.5542	.4354	.3048	.1524	.006
T	m	.1742	.1742	.1742	.1742	.1742	.1742
∇	m^3	.2832	.2060	.1618	.1133	.0566	.0032
A_W	m^2	1.8267	1.3342	1.0435	.7331	.3652	-
I_L	m^4	.9737	.7117	.5566	.3909	.1947	-
C_B		.70	.70	.70	.70	.70	-
C_P		.71	.71	.71	.71	.71	-
LCB before $L_{PP/2}$		.014	.014	.014	.014	.014	-
LCF before $L_{PP/2}$		-.063	-.063	-.063	-.063	-.063	-
k_{yy}/L_{PP}		.25	.25	.25	.25	.25	-
M	kgf sec^2/m	28.859	20.988	16.491	11.544	5.772	7.513
k_{zz}/L_{PP}		.267	.268	.230	.229	.229	.275

TABLE 2

$F_n = .15$

L/B		4	5.5	7	10	20	∞	∞*
M'		1978	1433	1122	779	379	521	0
I'_{zz}		142	103	59	41	20	39	0
Y'_v		-1800	-1700	-1600	-1450	-1400	-1500	-1500
N'_v		- 610	- 670	- 730	-780	- 700	- 500	- 500
$Y'_{\dot v}-M'$	$\cdot 10^5$	-3198	-2703	-2352	-1899	-1559	-1601	-1080
$N'_{\dot v}$		- 120	- 50	- 40	0	0	+ 20	+ 20
Y'_r-M'		-1858	-1243	- 872	- 479	0	0	+ 521
N'_r		- 265	- 295	- 290	- 280	- 240	- 260	- 260
$Y'_{\dot r}$		- 110	- 90	- 60	0	0	0	0
$N'_{\dot r}-I'_{zz}$		- 190	- 165	- 125	- 105	- 88	- 95	- 56
σ'_1		.538	.304	.200	-.048	-.901	-.935	Re=-2.930
σ'_2		-2.051	-2.468	-2.955	-3.382	-2.724	-2.739	Im=±1.471

Table 2 to be continued

$F_n = .20$

L/B		4	5.5	7	10	20	∿	∿*
Y'_v	$\cdot 10^5$	-1850	-1760	-1750	-1500	-1400	-1600	-1600
N'_v	$\cdot 10^5$	- 650	- 720	- 790	- 800	- 700	- 450	- 450
$Y'_{\dot v}-M'$	$\cdot 10^5$	-3198	-2543	-2442	-1919	-1559	-1601	-1080
$N'_{\dot v}$	$\cdot 10^5$	- 180	- 70	- 50	0	0	0	0
Y'_r-M'	$\cdot 10^5$	-1748	-1283	- 892	- 499	0	0	- 521
N'_r	$\cdot 10^5$	- 270	- 300	- 310	- 310	- 250	- 240	- 240
$Y'_{\dot r}$	$\cdot 10^5$	- 120	- 60	- 60	0	- 50	0	0
$N'_{\dot r}-I'_{zz}$	$\cdot 10^5$	- 195	- 165	- 135	- 112	- 97	- 120	- 81
σ'_1		.548	.369	.170	-.088	-1.064	-.997	Re=-2.222
σ'_2		-1.929	-2.584	-2.928	-3.461	-2.180	-2.002	Im=±1.458

$F_n = .30$

L/B		4	5.5	7	10	20	∿	∿*
Y'_v	$\cdot 10^5$	-2450	-2300	-2070	-1760	-1450	-1600	-1600
N'_v	$\cdot 10^5$	- 700	- 840	- 900	- 980	- 860	- 500	- 500
$Y'_{\dot v}-M'$	$\cdot 10^5$	-3078	-2603	-2652	-2189	-1599	-1621	-1100
$N'_{\dot v}$	$\cdot 10^5$	- 160	- 100	- 20	0	- 50	0	0
Y'_r-M'	$\cdot 10^5$	-1878	-1303	-1042	- 559	- 29	0	+ 521
N'_r	$\cdot 10^5$	- 330	- 360	- 400	- 340	- 310	- 230	- 230
$Y'_{\dot r}$	$\cdot 10^5$	- 180	- 100	- 100	0	- 50	0	0
$N'_{\dot r}-I'_{zz}$	$\cdot 10^5$	- 200	- 160	- 120	- 115	- 95	- 90	- 51
σ'_1		.387	.225	.090	-.054	-.955	-.985	Re=-2.982
σ'_2		-2.227	-2.909	-3.879	-3.706	-2.878	-2.558	Im=±1.517

∿* plate without mass

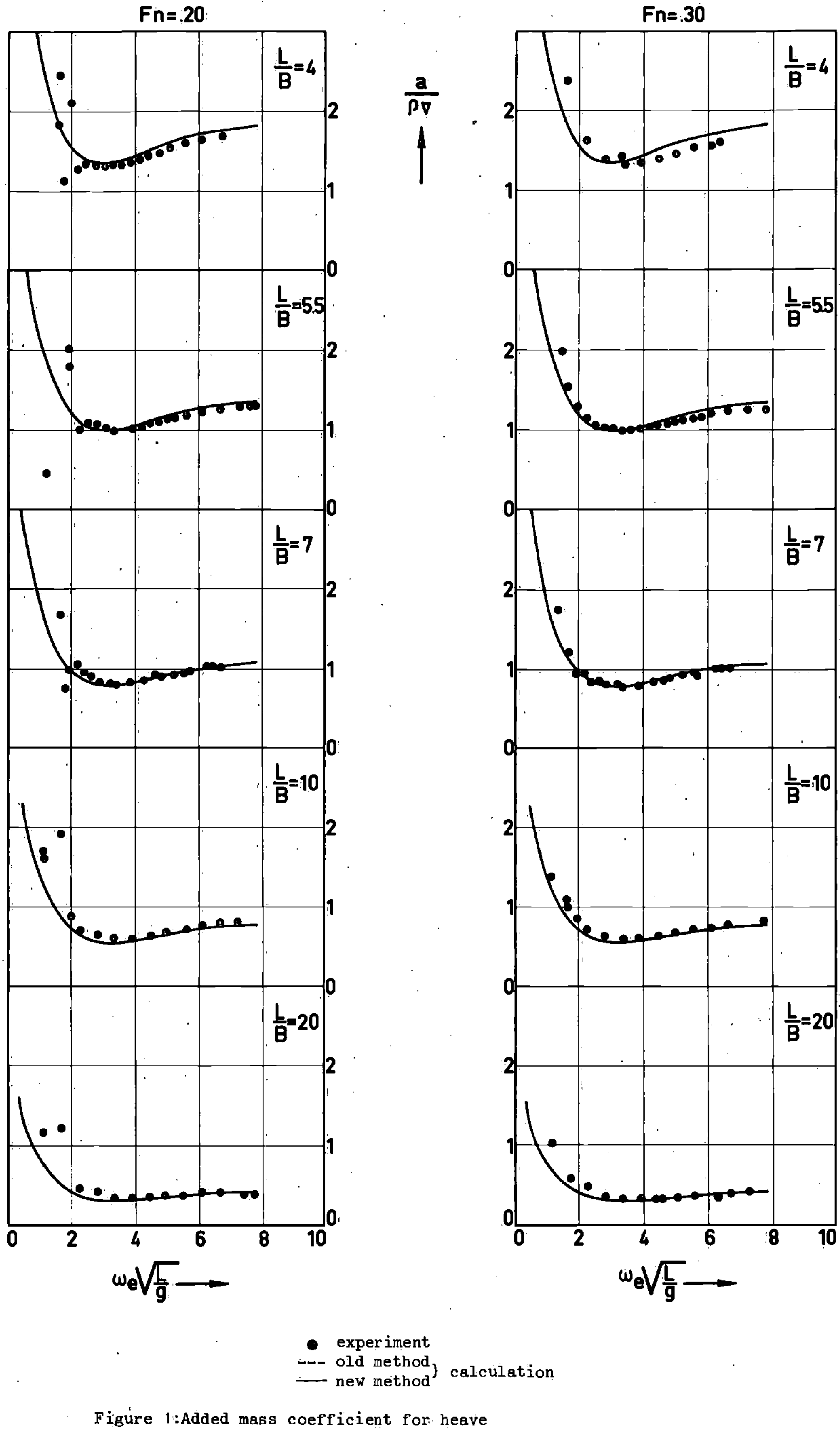
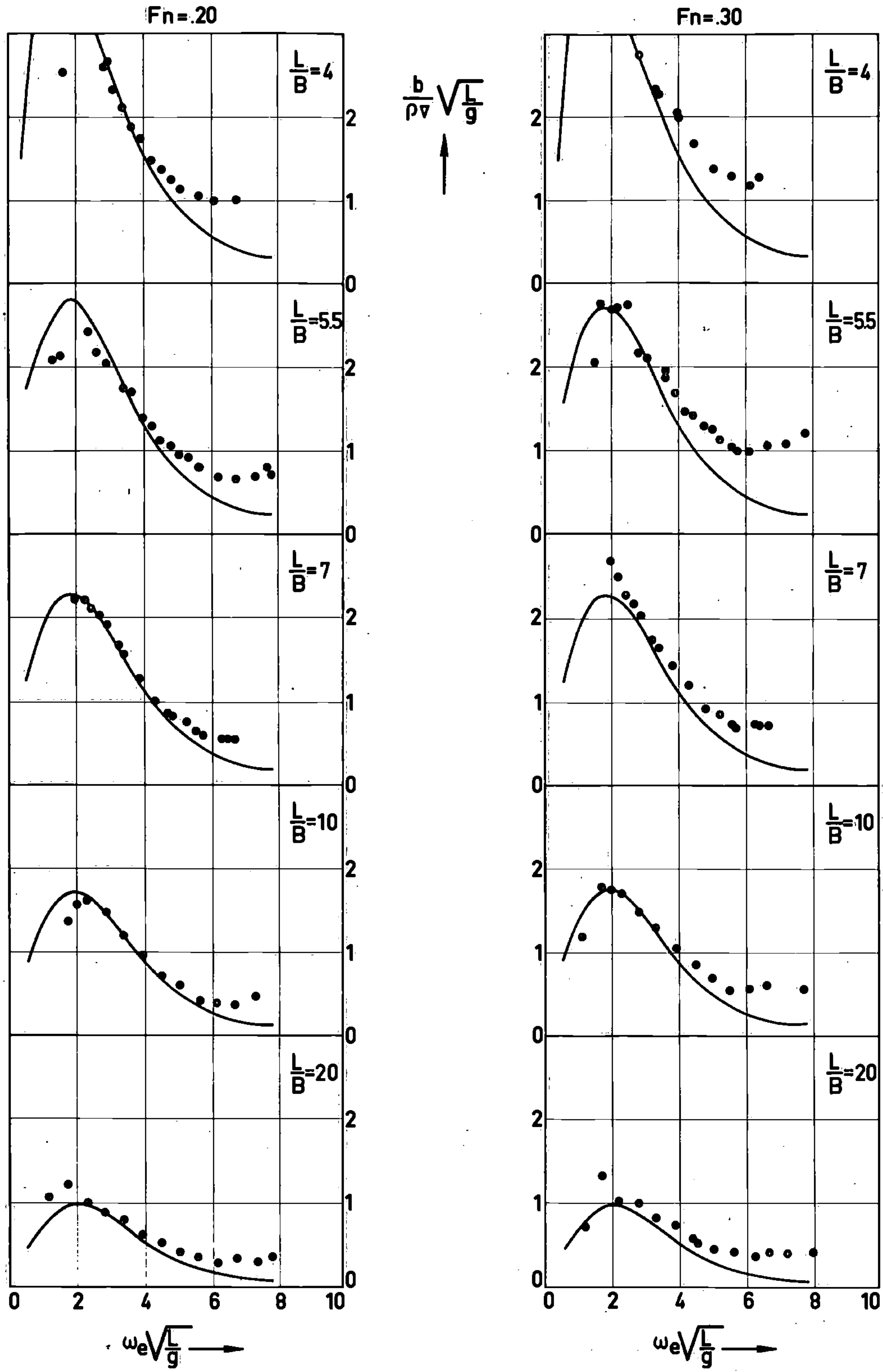
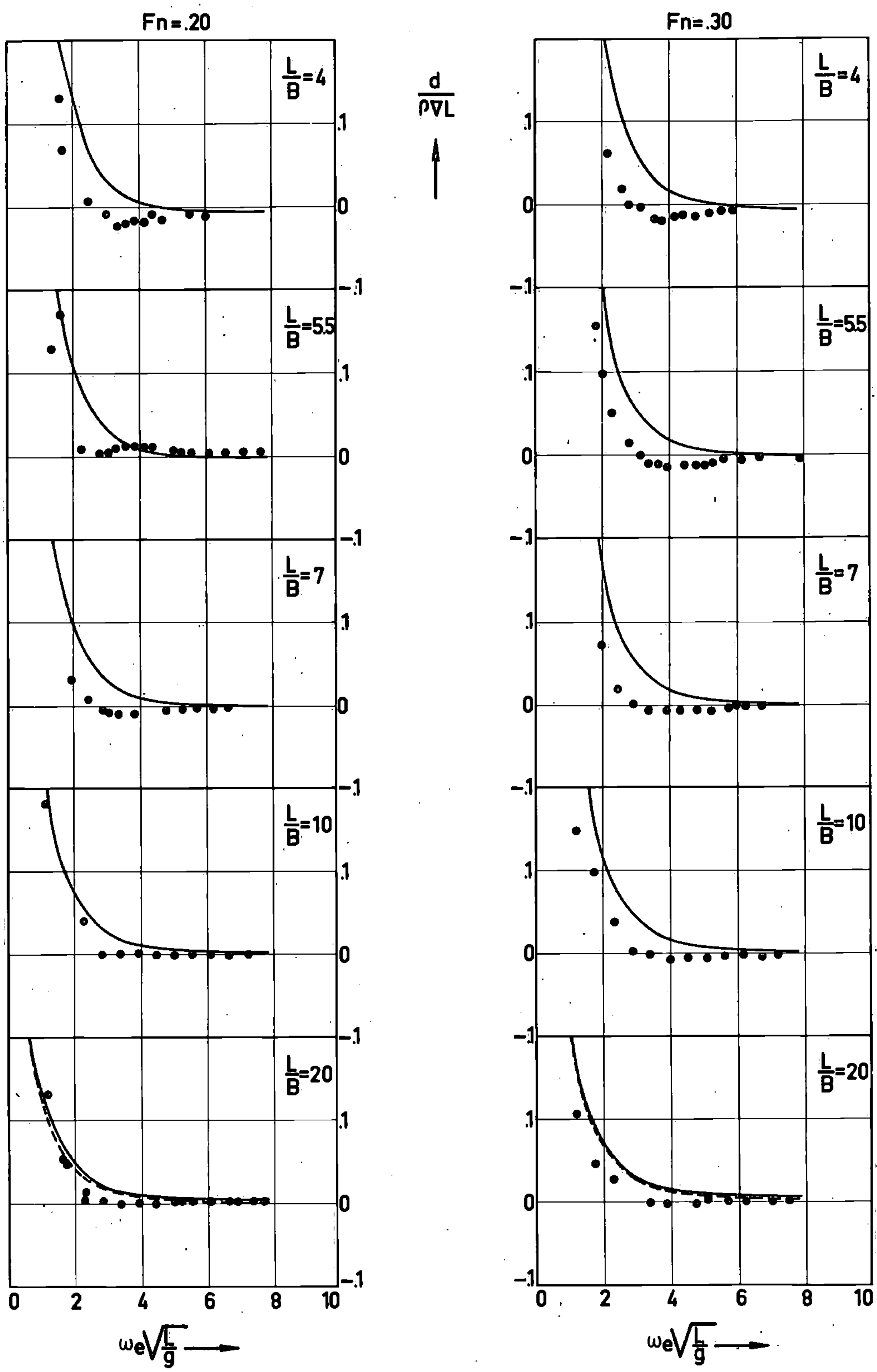


Figure 1:Added mass coefficient for heave



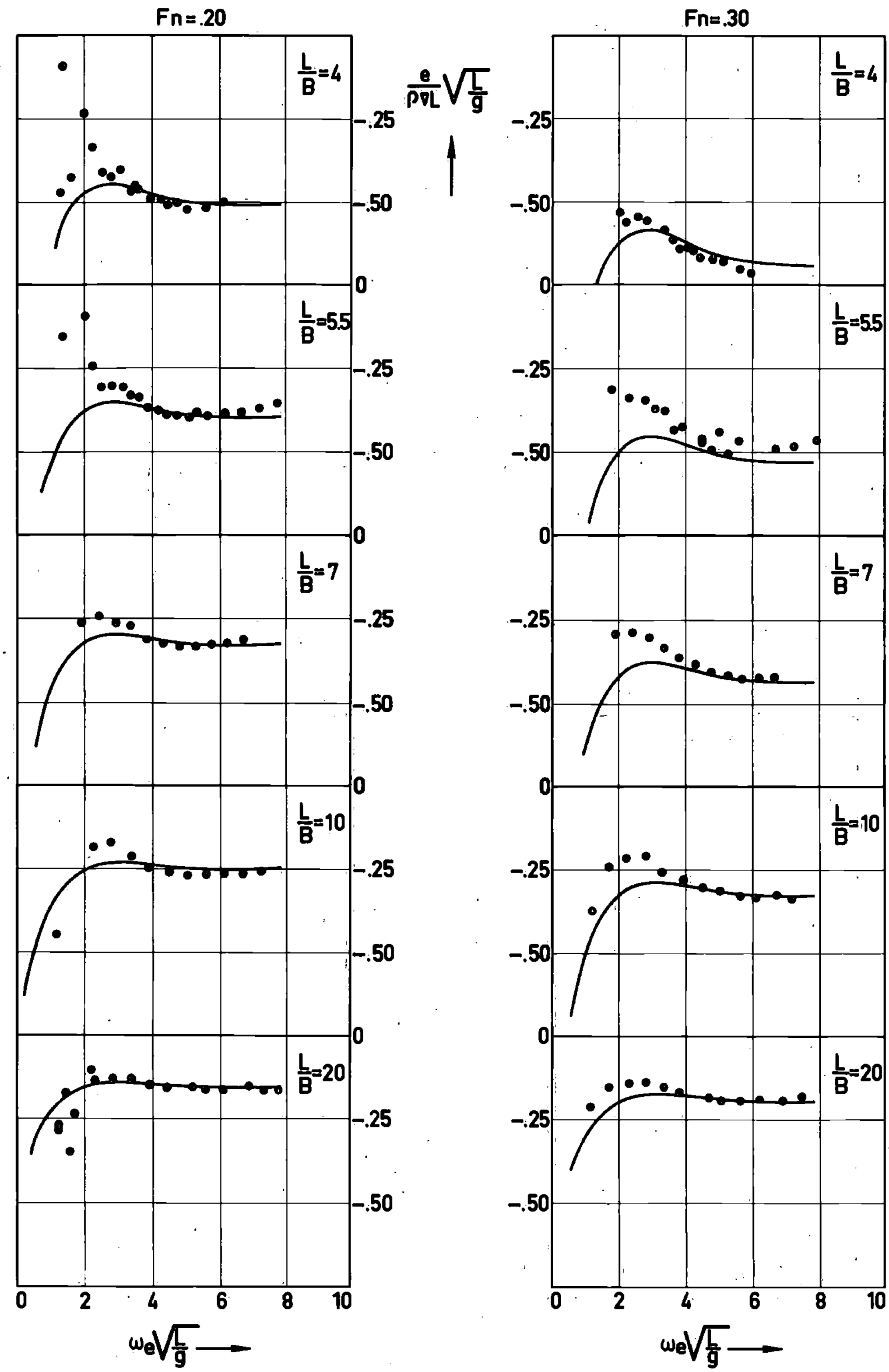
● experiment
--- old method } calculation
—— new method

Figure 2 : Heave damping coefficient



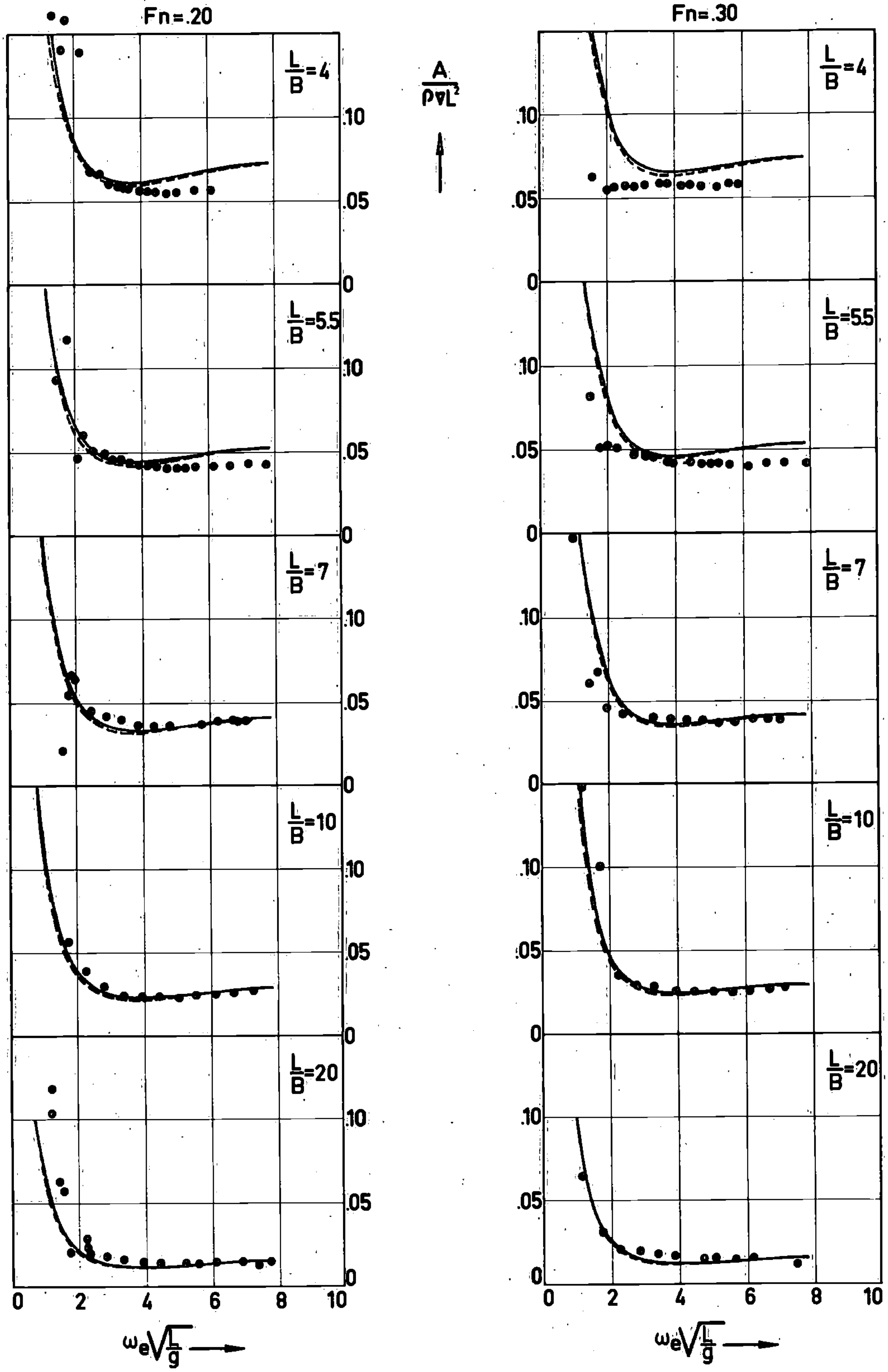
● experiment
--- old method } calculation
—— new method

Figure 3 : Added mass cross coupling coefficient for heave



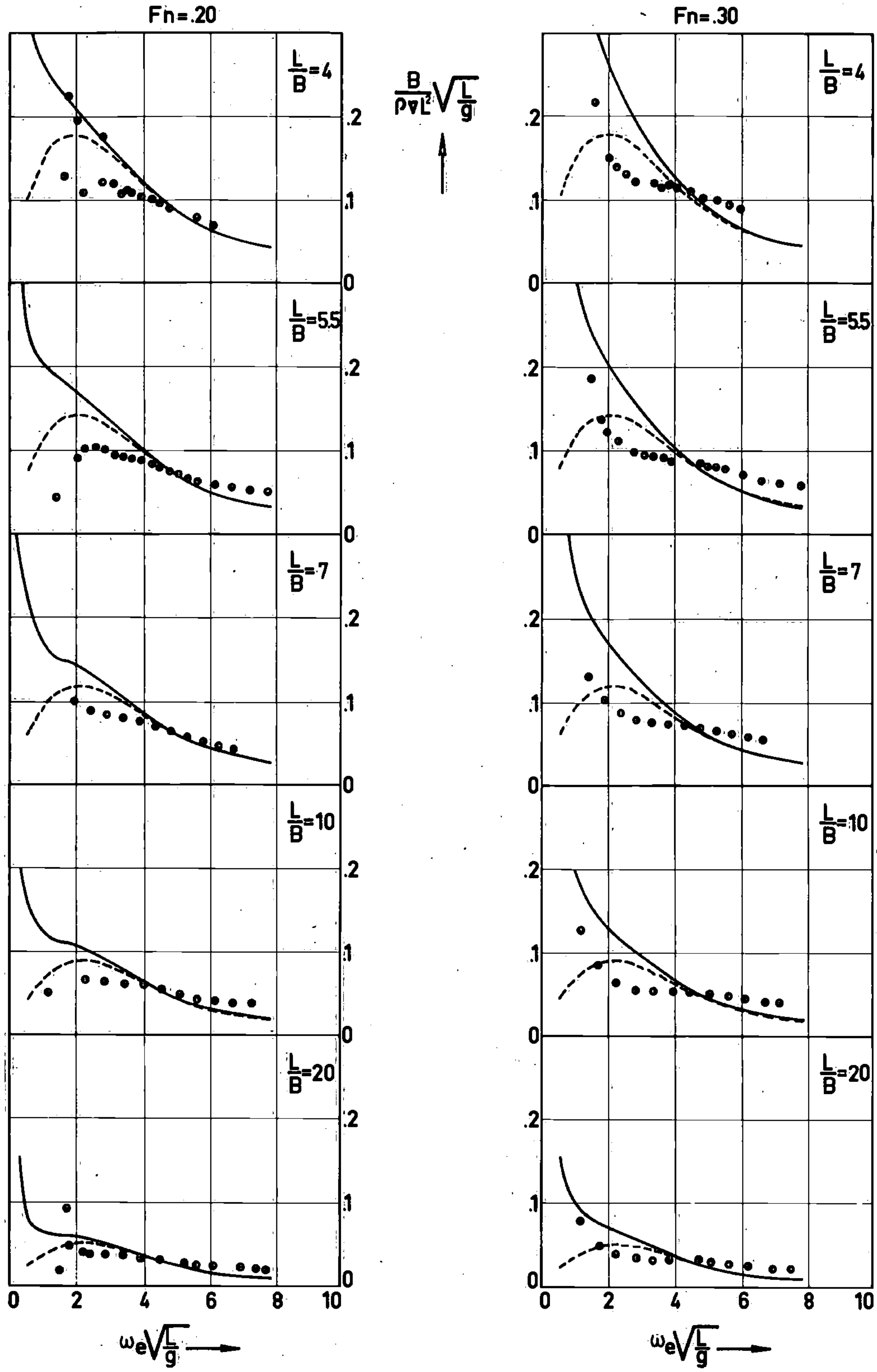
● experiment
--- old method } calculation
—— new method

Figure 4 : Damping cross coupling coefficient for heave



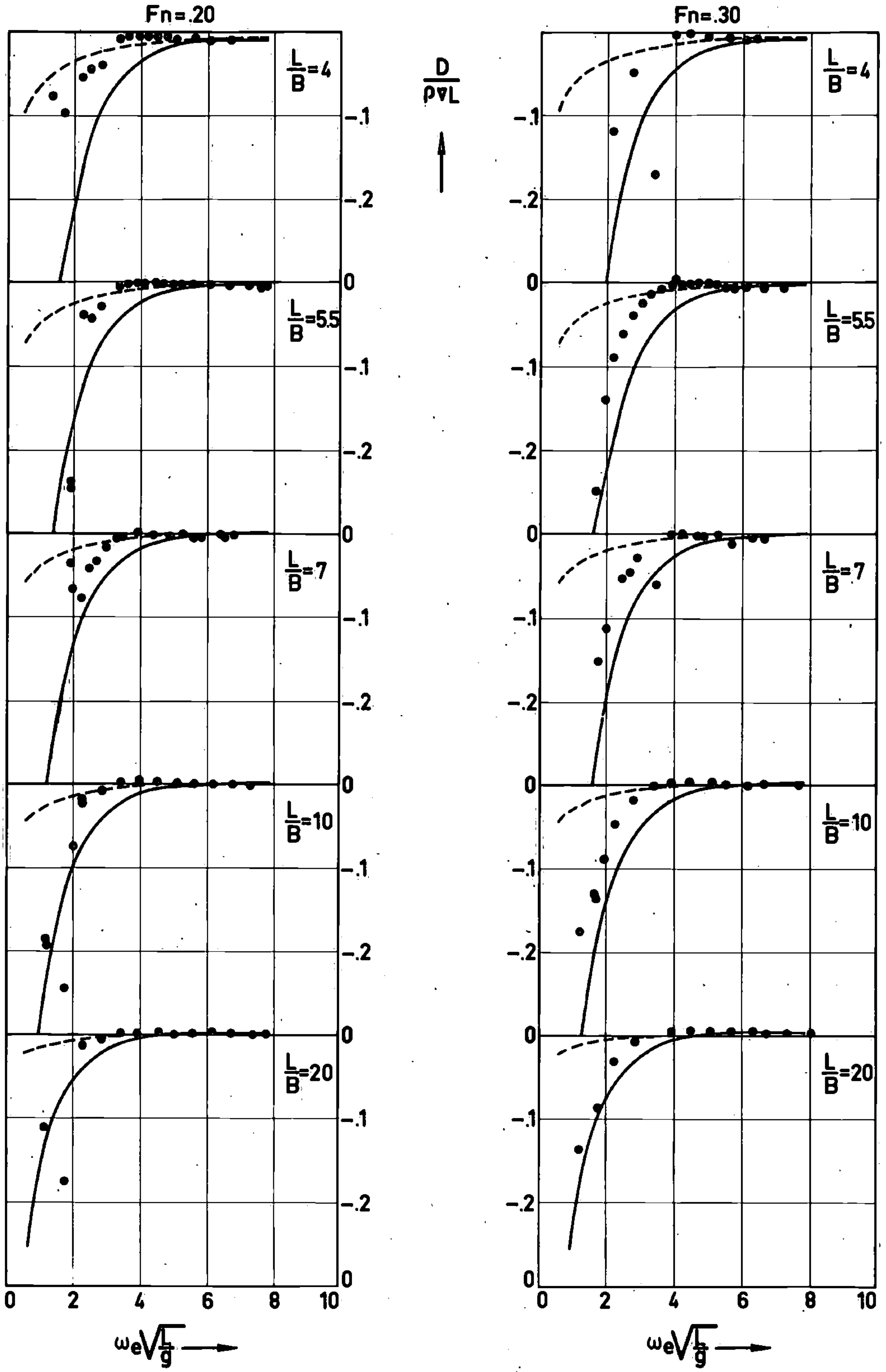
● experiment
--- old method } calculation
—— new method

Figure 5 : Coefficient of added mass moment of inertia for pitch



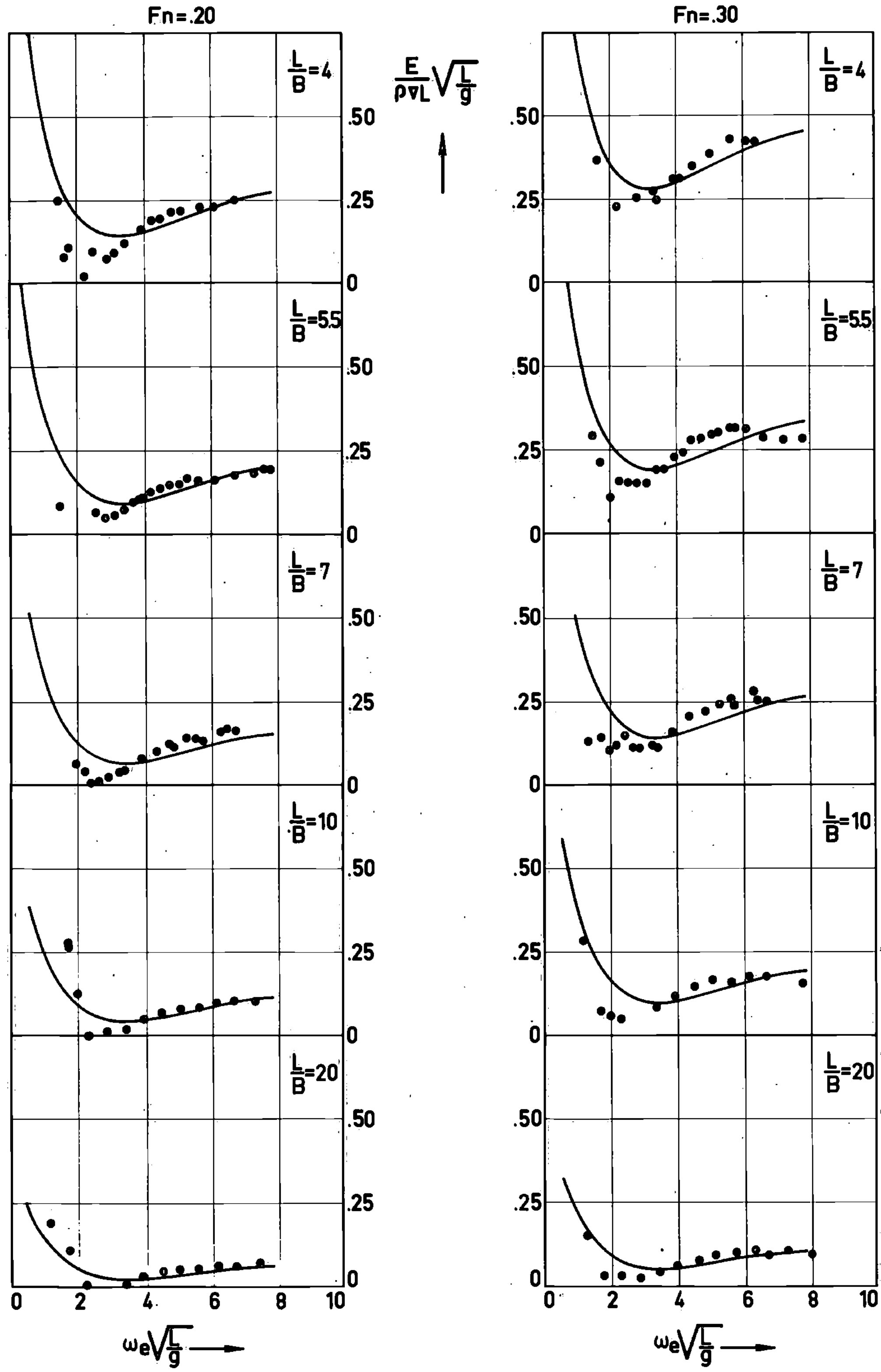
● experiment
--- old method } calculation
—— new method

Figure 6 : Pitch damping coefficient



● experiment
--- old method } calculation
— new method

Figure 7 : Added mass cross coupling coefficient for pitch



● experiment
--- old method } calculation
—— new method

Figure 8 : Pitch damping cross coupling coefficient

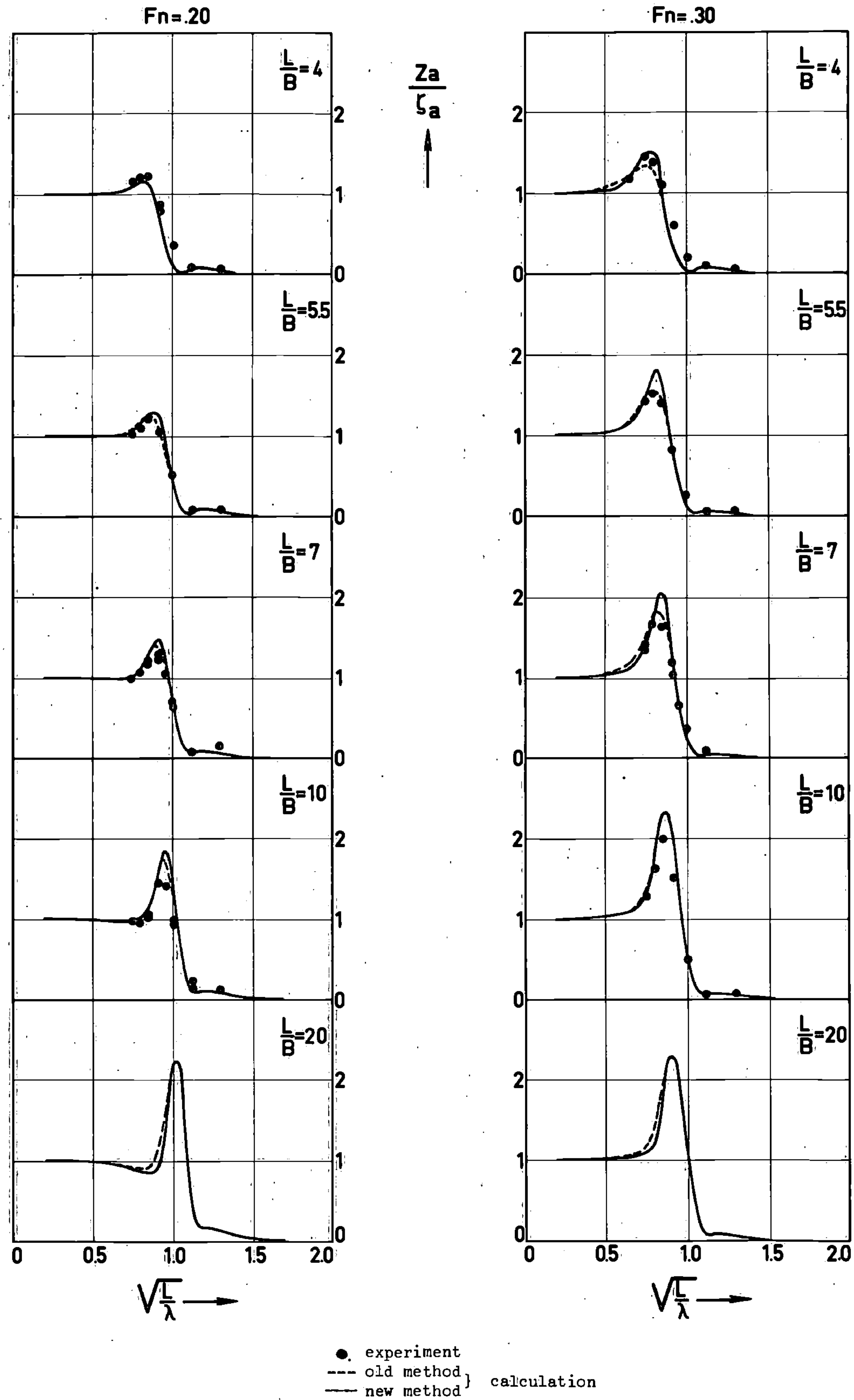
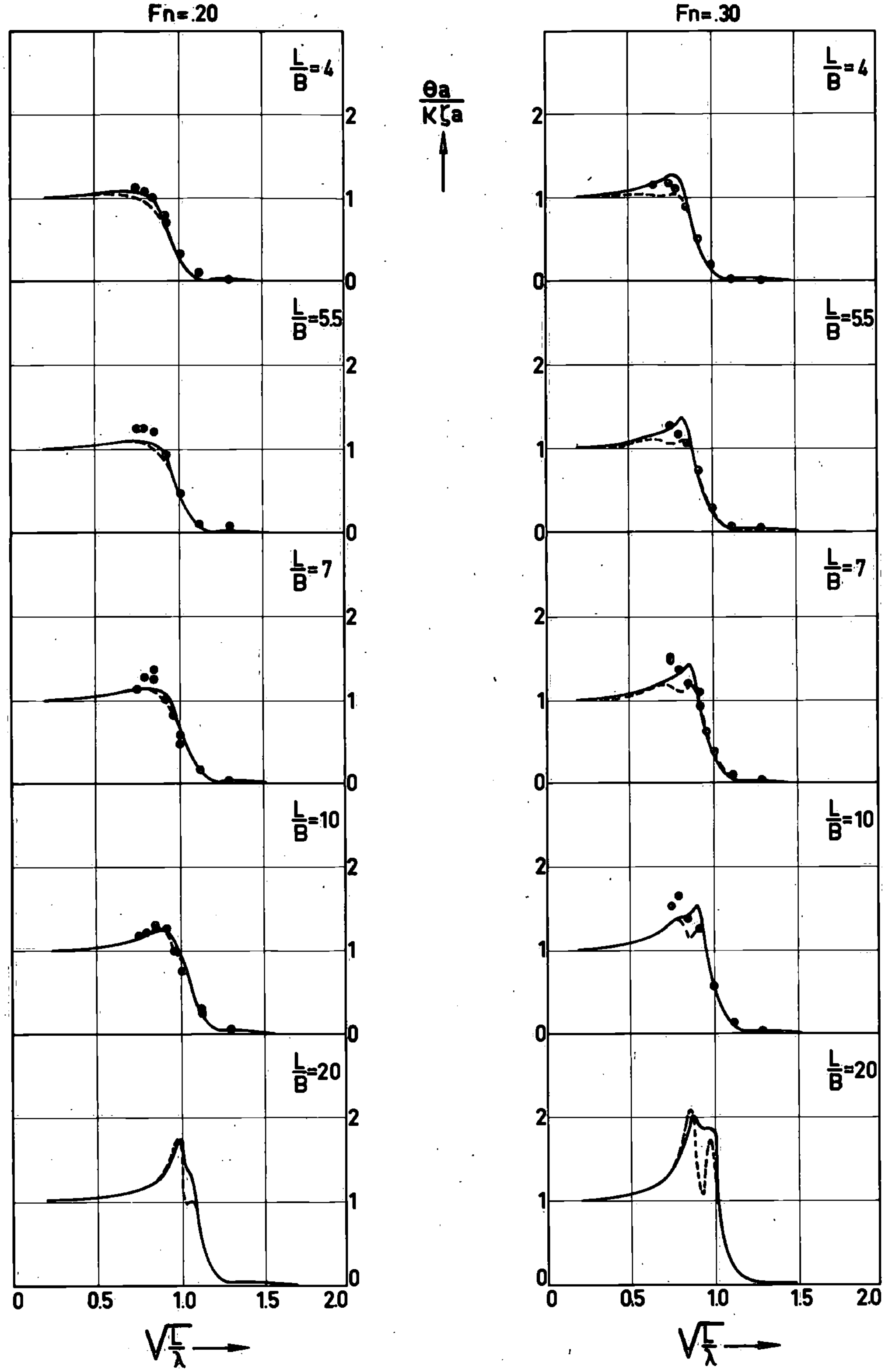


Figure 9 : Heave amplitude in waves



● experiment
--- old method } calculation
— new method

Figure 10 : Pitch amplitude in waves

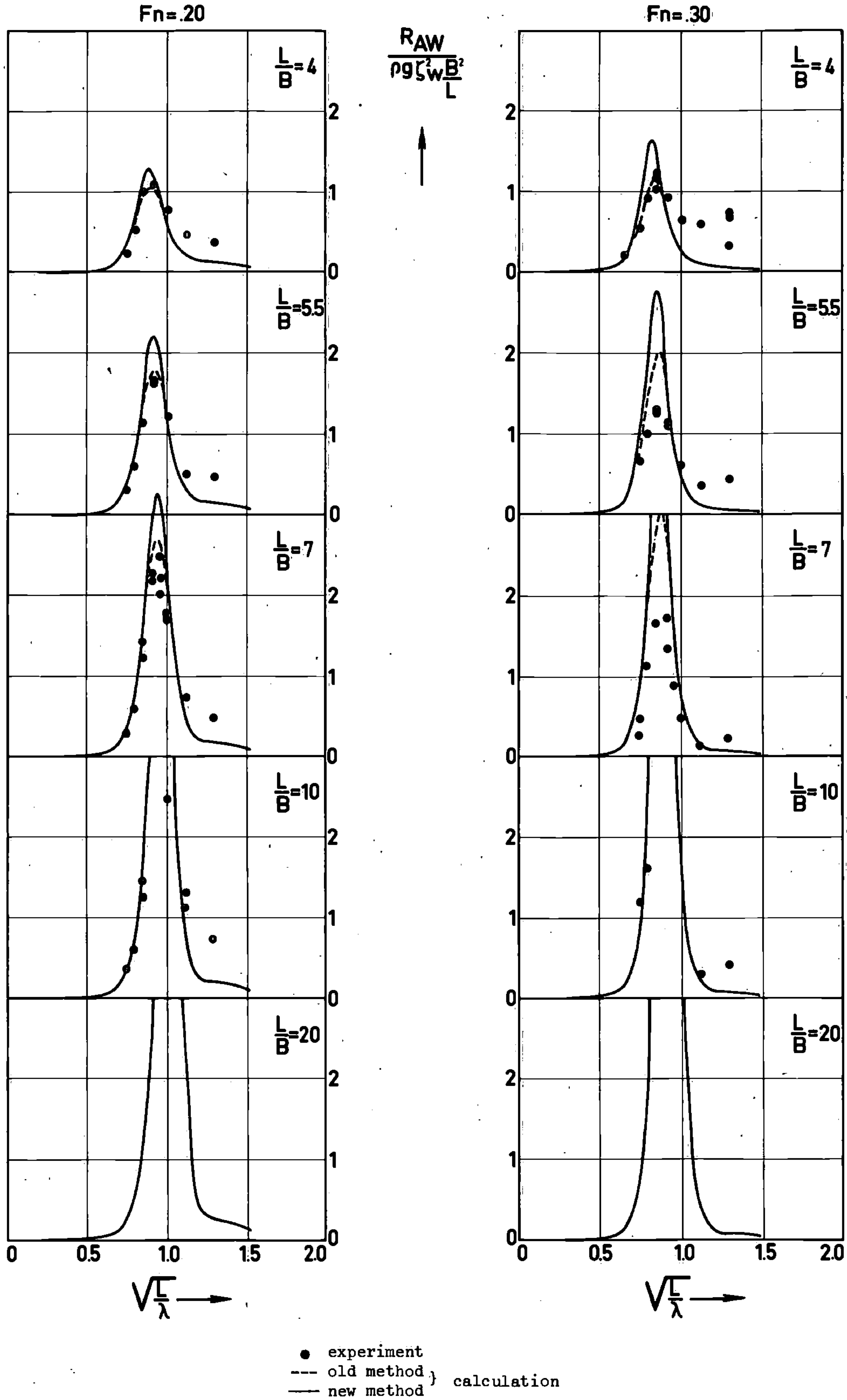


Figure 11 : Added resistance in waves

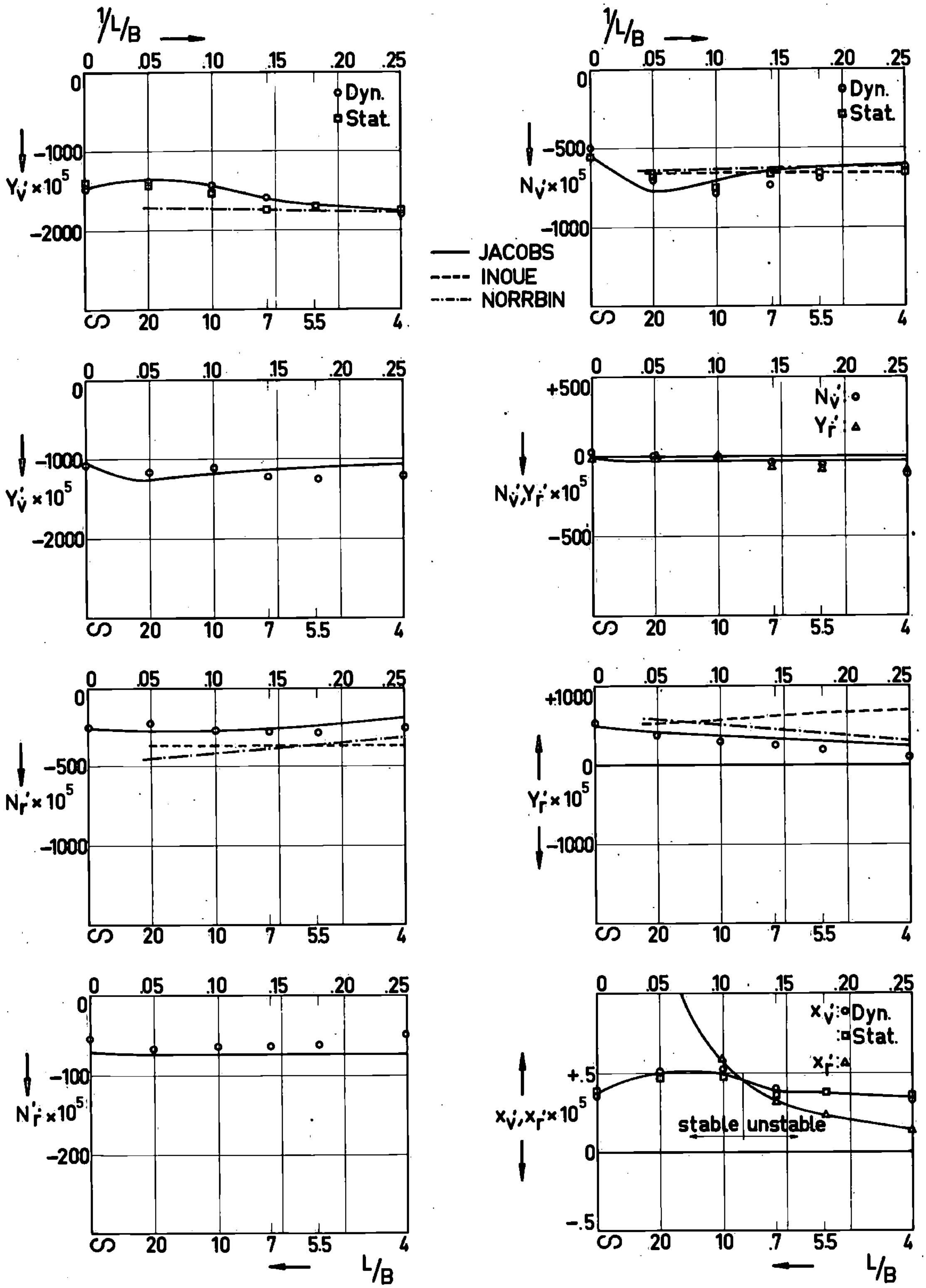


Figure 12 : Hydrodynamic coefficients for F_n=.15

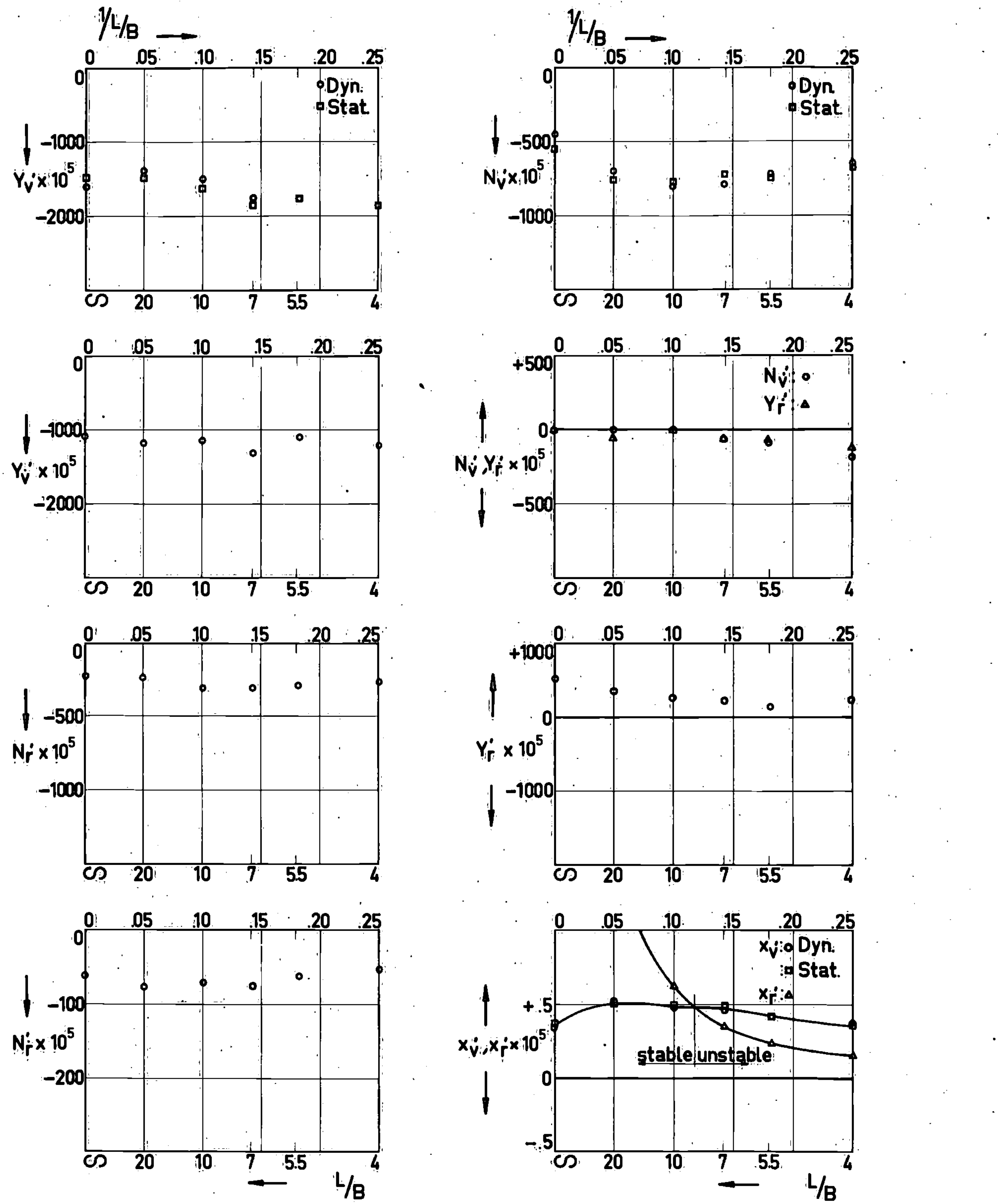


Figure 13 : Hydrodynamic coefficients for $F_n = .20$

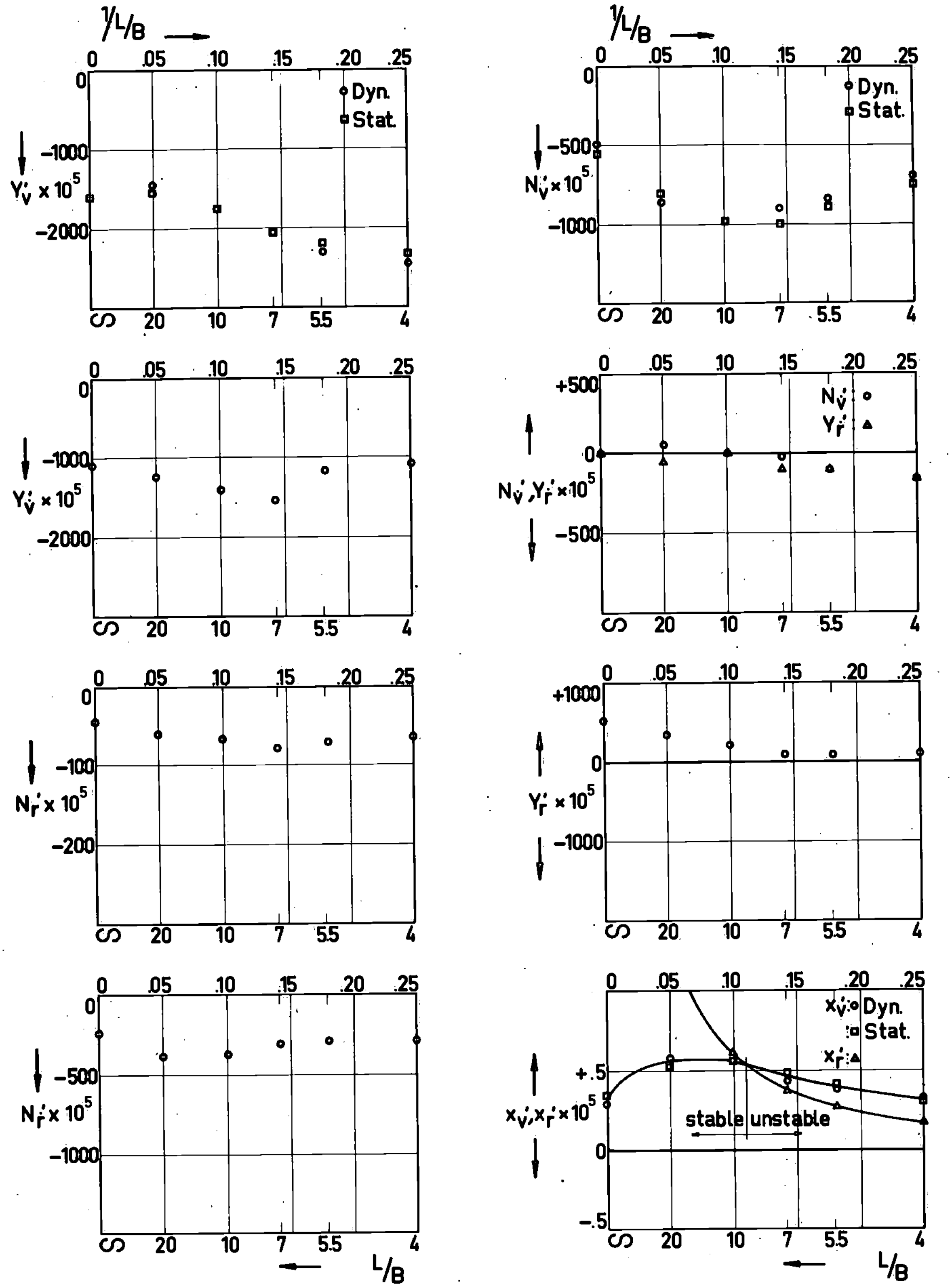


Figure 14 : Hydrodynamic coefficients for F_n=.30

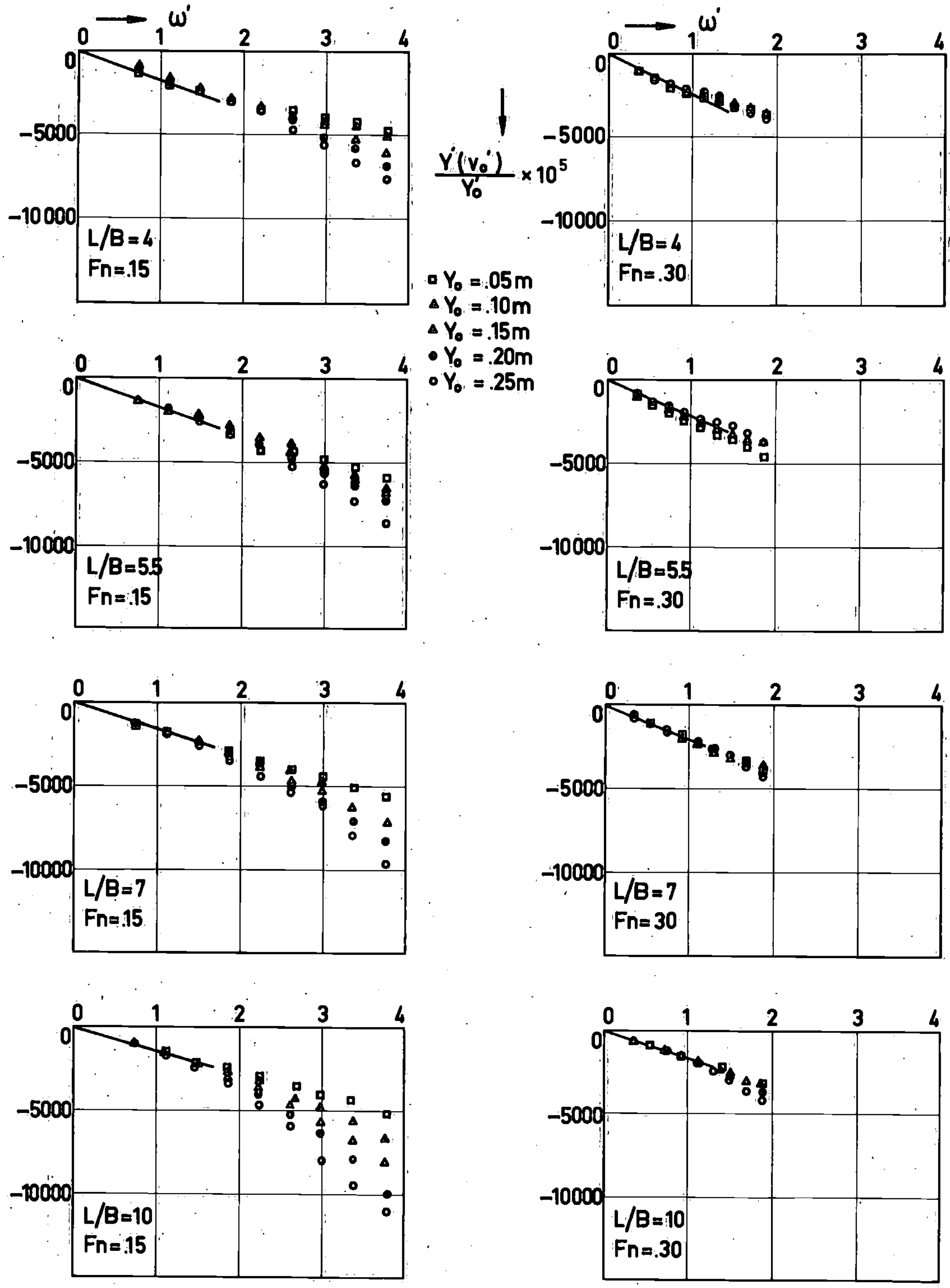


Figure 15 : to be continued

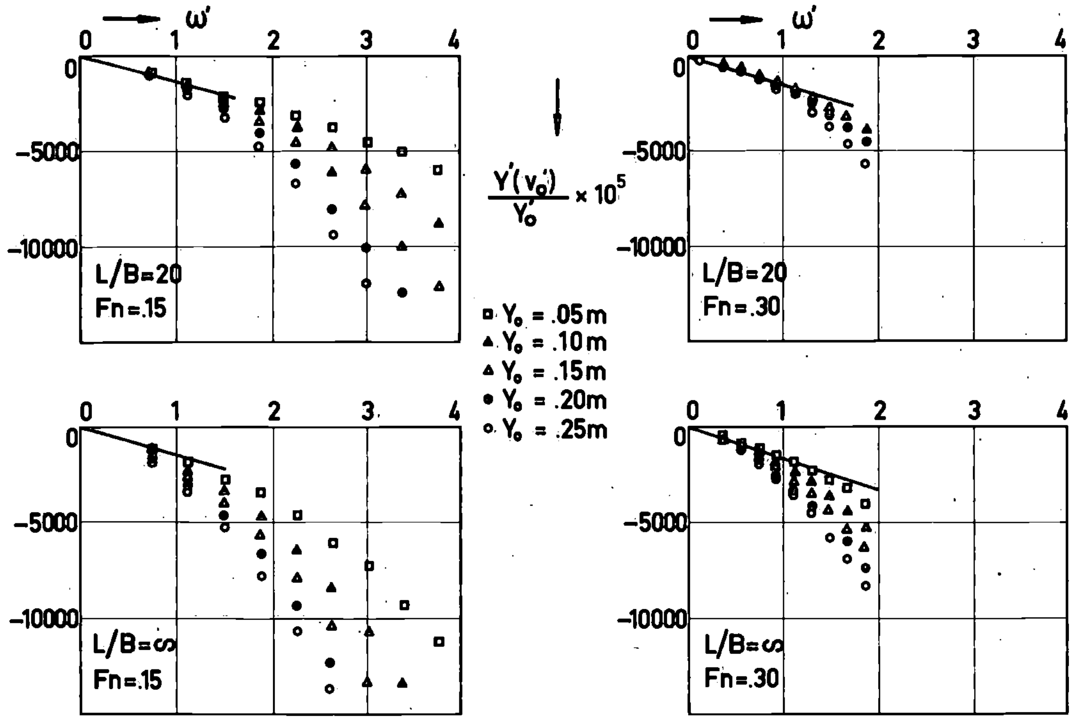


Figure 15 : Sway force for two Froude numbers as function of L/B-ratio

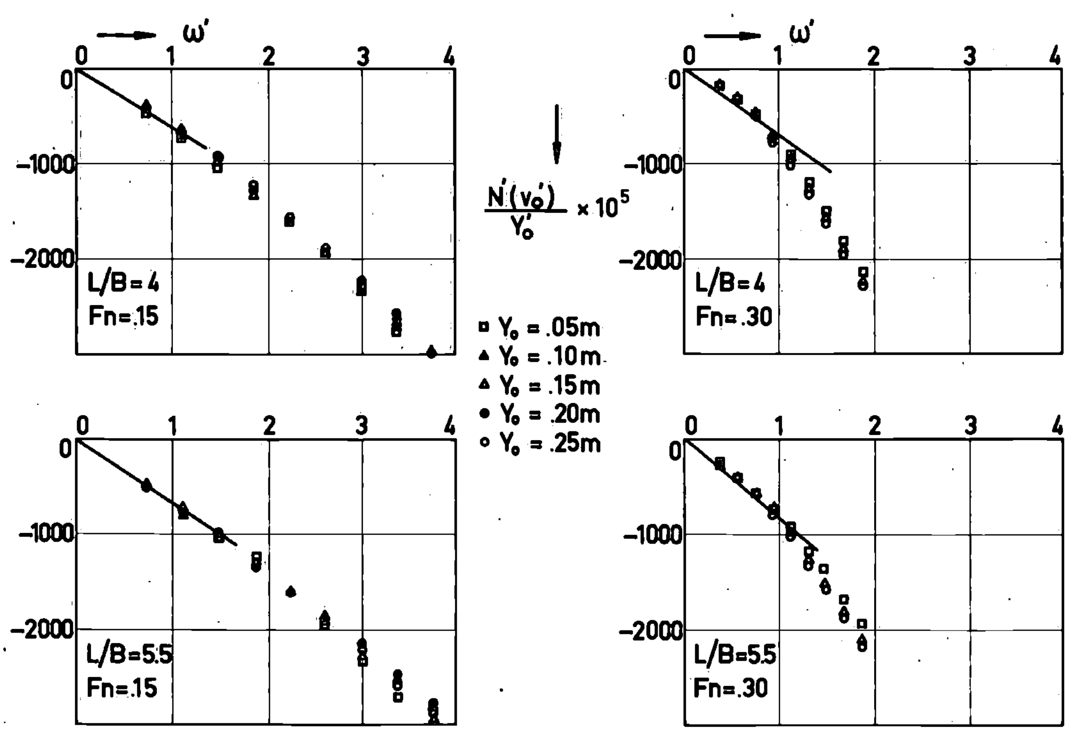


Figure 16 : to be continued

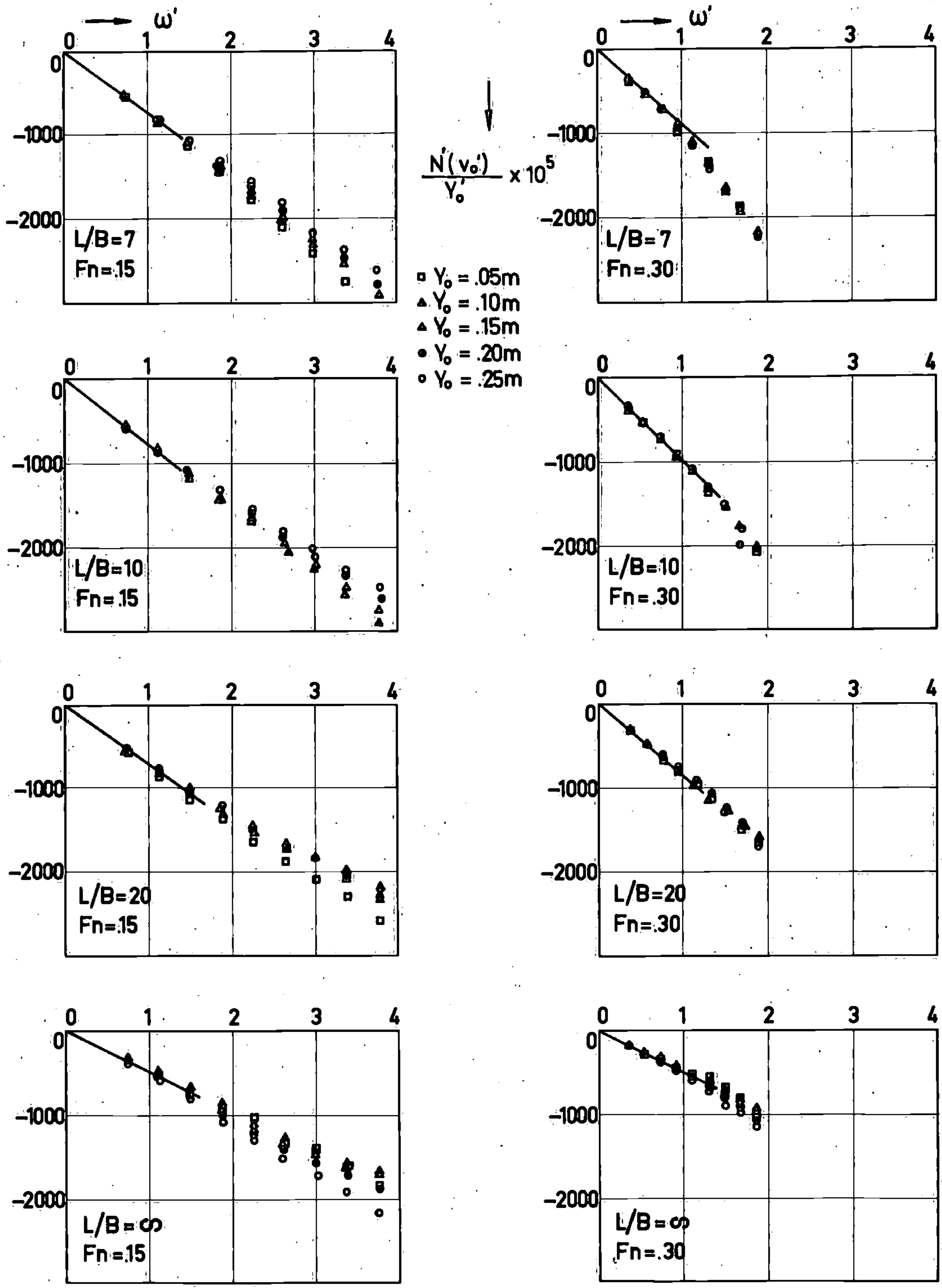


Figure 16 : Sway moment for two Froude numbers as function of L/B-ratio

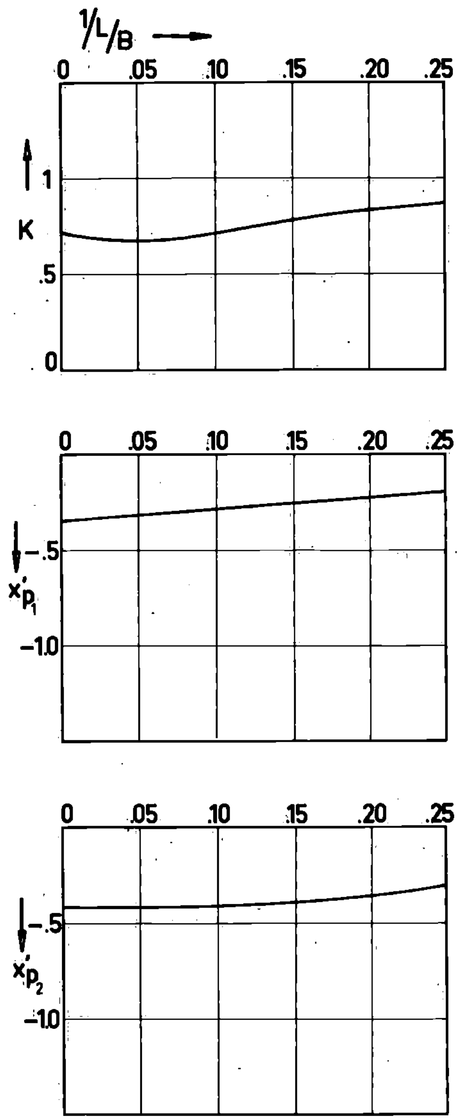


Figure 17 : Empirical coefficients derived from the experiments

DISCUSSION

W.R. JACOBS

I appreciate greatly your asking for my comments on your well-reasoned and informative paper. I am gratified, moreover, to see that the Korvin-Kroukovsky and Jacobs method for predicting heaving and pitching motions in regular head seas and the Jacobs method for estimating the linear hydrodynamic coefficients of the horizontal motions still hold up so well at Froude numbers no greater than 0.20. At such speeds, of course, the effects of wave-making can be neglected. (The experimental values in Fig. 13 for $F_n = 0.20$ are almost identical with those of Fig. 12 for $F_n = 0.15$ and therefore agreement between calculation and experiment should be as good.)

In your introduction, you state that, in the case of horizontal motions, "apart from the length-draft ratio..... the length-beam ratio may be regarded as a useful parameter in a comparison of theory and experiment." The length-beam ratio does not appear explicitly in my calculation method (Appendix 3). I wish to make clear that length-beam ratio is implicit in the ship mass coefficient M' which is identically equal to $2\ C_B\ B/L$.

K. NOMOTO

It is a great pleasure to take part in the discussion on this interesting paper. Certainly the effect of length-beam ratio on the hydrodynamic damping in directional control of a ship is of great interest with special reference to the ease of control of giant tankers of the present day, whose length-beam ratio is lessening as low as 5.

In this connexion a look into Table 2 is highly suggestive. The damping in yaw and sway, and consequently the directional stability is governed by

$$Y_v'\ N_r' - (Y_r' - M')\ N_v'$$

Among these derivatives, what is most sensitive to the length-beam ratio is definitely $(Y_r' - M')$, and this comes largely from the drastic decrease in the nondimensional mass M' with increasing length-beam ratio. Compared with this, the purely hydrodynamic derivatives Y_v', N_v' and N_r' are much less sensitive.

Since M' represents the contribution of the centrifugal force upon directional stability, this result suggests that the effect of length-beam ratio upon directional stability is more of the matter of mechanics rather than of hydrodynamics. This might sound a bit reluctant to hydrodynamicists, yet one thing worth noting.

Incidentally one can guess the effect of the block coefficient on the directional stability along the same line; the change in M' largely governs the fact.

As another remark, the frequency in PMM experiments should be adequately low so that $(WL/V) < 2 \sim 2.5$ in order to obtain the derivatives that are free from the frequency effect, in the discusser's view. That means in the present case $W < 0.7$ for $F_n = 0.15$ and < 1.4 for $F_n = 0.3$) and accordingly most of these experiments are apparently within this limit.

EDWARD V. LEWIS

This paper represents the type of well conceived and well executed experimental research that we expect from Delft University of Technology.

My brief comments refer only to the first part of the paper dealing with vertical motions. The experimental determination of coefficients for pitch and heave for an unusually wide range of L/B ratios shows encouraging results. Even at such extreme proportions as $L/B = 4$, the agreement between experiment and theory (Figures 1-8) is as good, or almost as good, as for narrower hulls. The so-called "new" theoretical method appears to give better agreement in some cases but not in others.

It is not surprising then that excellent agreement is obtained in Figures 9 and 10 between calculated and experimental motions over this wide range of L/B. In general, the "new method" shows somewhat better results. Of particular interest is the excellent agreement shown in Figure 11 for added resistance in waves. All in all, the paper shows clearly the tremendous value of the "vigorous", though perhaps not entirely "rigorous", strip theory approach to ship motions. The high degree of practical usefulness of the method is due in large part to work such as reported in this paper, covering both refinements in the theory and experimental verification of various aspects.

C.M. LEE

Prof. Gerritsma and his co-authors, as always, have shown us again a valuable work which will greatly contribute to the advancement of knowledge in ship hydrodynamics.

The following is my opinion on a minor point which I would like to take this occasion to present to the authors for their comments.

The equations of motion for ships in waves which are derived under an assumption of linear frequency response, are usually given in the form of the second order differential equations with frequency-dependent coefficients. As Dr. Cummins* rightly pointed out, the physical meaning of these coefficients can be often misleading depending on how one arranges the coefficients in the equations. To be more specific, there is always a possibility of interchanging the coefficients between the inertia terms and restoring terms with only change in the factor $(-W_e^2)$. For instance, the coefficient A and C are given in Equation (6b) as

$$A = \int_L m' x_b^2\, dx_b + 2\frac{V}{\omega_e^2}\int_L N' x_b\, dx_b - \frac{V^2}{\omega_e^2}\int_L \frac{dm'}{dx_b} x_b\, dx_b + \frac{V}{\omega_e^2}\int_L \frac{dN'}{dx_b} x_b^2\, dx_b\,, \qquad C = 2\rho g \int_L y_w x_b^2\, dx_b$$

We can transfer the terms containing $1/W_e^2$ in A to C by multiplying the terms by $(-W_e^2)$ without impairing the solutions of the equations. If this is done for a ship without abrupt ends, we have

*Cummins, W.E., "The Impulse Response Function and Ship Motions," Schiffstechnik, Vol. 9, 1962

$$A' = \int_L m' x_b^2 \, dx_b$$

$$C' = 2\rho g \int_L y_w x_b^2 \, dx_b - V^2 a$$

The second term, $-V^2a$, in C' is often called "Munk's Moment" and it is always a destabilizing moment due to its negative sign. A difference resulting from interchanging this Munk's moment term is in the determination of natural frequencies, especially for pitch. The natural frequency for uncoupled pitch mode can be estimated by

$$\omega_p = [C/A]^{\frac{1}{2}}$$

If we use A' and C' instead, then we have

$$\omega_p' = [C'/A']^{\frac{1}{2}}$$

The difference between W_p and W_p' is usually small for conventional ships for low speeds. However, the difference can be large for high speeds and particularly, for small waterplane area ships with a high cruising speed.

For a ship with very small waterplane area the vertical-plane stability can become a problem for high speeds. Depending on where the Munk moment term is placed, the estimation of vertical-plane stability can significantly change. There is no question that for a stability study the Munk moment should be placed in the restoring term.

For determining the natural frequencies and the vertical-plane stability, it appears physically more adequate to use A' and C' than to use A and C. I would like to know if the authors have some comments on this point.

MAX HONKANEN

At first I would like to express my gratitude for this very useful paper presented here as the first one today. I was very pleased to read it, because the first part, of which some details were published at ITTC in 1972, has already been used by me in checking the validity of my own calculations. There is one question regarding the lateral motions and forces associated with them that is bothering me and I would appreciate if the authors could throw some light on it.

As we all know, the theoretical treatment of the rotative modes of motions is based on the assumption of fixed axes of rotation. This, however, needs not necessarily be the truth, and in fact, there exists an apparent center of rotation, which usually differs slightly from the intersection point of the waterline and the symmetry plane of the ship. I have formulated a strip theory that makes allowance for an arbitrary center of rotation, and preliminary calculations show that the location of this virtual center of rotation may have a significant effect on the hydrodynamic coefficients of the lateral motions. It should be understood that the PMM test results may very well be in a perfect agreement with the theoretical results, since the tests are actually run on the same assumption of a fixed center of rotation as the theory has been derived.

I would simply like to ask the authors if they have any experience on the effect of the virtual center of rotation on the hydrodynamic coefficients of the lateral motions and what order of magnitude they think that such an effect would be.

NILS H. NORRBIN

In this summary of my oral discussion I will once more bear witness to the benefit the reader may derive from results of careful systematic studies of this kind. I will restrict my comments to the analysis of the dynamic stability in the horizontal plane. Within the particular bare hull Series 60 family tested dynamic stability is inherent for L/B ratios above 8. With stern appendices stability will be realized for wider forms.

The analytical stability criterion compares the magnitude of two force levers, in the authors' notation x_r' and x_v'. In particular, $x_v' = 1_v/L$ is the relative center-of-pressure-in-sideslip, or the quotient N_v'/Y_v'. For a model family this quotient will be given by the slope of the radius vector to the locus $N_v'(Y_v')$. In Fig. 1 this locus is shown by the arc shape to the right. In the same diagram but to another scale the corresponding locus is also drawn as given in the "bis" system $N''_{uv}(Y''_{uv})$: the locus now illustrates a moment and a force, which both uniquely increase with increasing L/B. The radius vecter slope is shown for L/B = 7, for the bare hull as well as for a configuration with screw and rudder. (The finite increments of Y''_{uv} and N''_{uv} have been taken from model test results by van Leeuwen in authors' ref. [12].)

The diagram may be completed by adding the locus of $x_g'' - N''_{ur}$ to a base of $1 - Y''_{ur}$. (Again the use of the "bis" system will arrange the test data in a unique form.) The stability criterion and the way it is affected by modifications to the stern is easily appreciated from a comparison of vector slopes.

It would be of great value if, in the future, the authors could find an opportunity to include some results for hulls with screw and rudder, say for the cases of L/B = 5.5, 7 and 10.

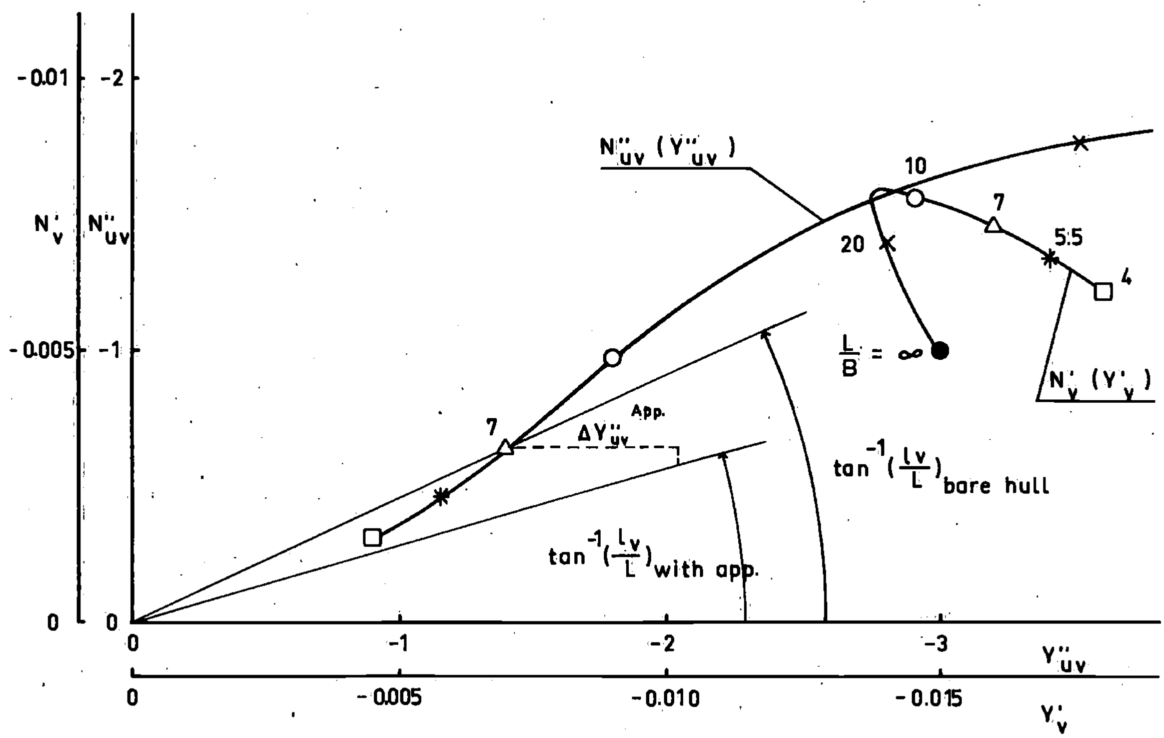


Figure 1: Loci of drift force derivatives for model family, also illustrating initial force c.p. position

AUTHOR'S REPLY

Referring to the kind remarks of Miss Jacobs, we agree that the differences in the experimental results for the Froude numbers .15 and .20 are so small that the effect of wave making in the development of simple theories can safely be ignored. Since the wing analogy, primarily represented by the length-draught ratio, is playing an important role in these theories for assessing the lateral maneuvering derivatives, it was though that the length-beam ratio would provide some correction factors respecting the distribution of viscous forces along the length.

Prof. Nomoto points out that the derivative $Y_r' - N'$ is the most sensitive one, since the dimensionless mass appeared, which shows the largest changes with varying length-beam ratio's, see table 2. He concludes that the straight-line stability is more a matter of mechanics than of hydrodynamics. We agree with this conclusion. However we want to put emphasis upon the fact that the experiments were executed with a modelseries having a block coefficient C_B = .70 and a length-beam ratio L/B = 7 as a parent hull. It is therefore dangerous to extrapolate the information contained in this paper to blunt tanker forms with different block coefficients and different length-beam ratios. Furthermore one should bear in mind that the models tested were bare hulls. A rudder and propeller fitted to the models will improve straight line stability. Since changes in the form of the body and the distribution of displacement along the length sometimes might induce drastic changes in the hydrodynamic coefficients, we do not fully agree with Prof. Nomoto's remarks respecting the effect of block coefficient. The last remark refers to the maximum permissible frequency in horizontal PMM-tests to avoid frequency effects. In the in this paper presented results there seems to be some evidence, Figs. 15 and 16, to conclude that the dimensionless frequency W′ should be lower than 1 or at the most 1.5. Nevertheless not in all cases higher frequencies could be avoided in order to obtain measurable results.

Prof. Lewis confirms our point of view with regard to the usefulness of strip theory calculations. From the practical point of view we do not favor one of the two theories for the calculation of vertical motions. This is also based on further incidental comparisons for theory and experiment for slender ship hull forms at high speeds of advance. Of particular interest is the agreement between the two theories with regard to phase angles and the more or less overestimation of the heave amplitudes at resonance by the new theory. Up to now we use the old method for the prediction of heave, pitch and resistance increase in waves for design purposes.

Mr. Lee makes some valuable remarks about the determination of the natural frequency. In our formulation of the strip theory the restoring term is considered to be speed-independent and consequently the speed dependent part has been transferred to the added mass term. For the solution of the motion equations it is irrelevant where the speed dependent parts are situated. However for the determination of the natural frequency this may be important especially for high forward speeds. It is probably not correct to keep the restoring term speed-independent and the "Munk's moment" might be one significant addition for high speeds. However, there is another influence of the speed on the restoring term and this is due to the change of trim and the wave formation. This effect should also be taken into account for the determination of the natural frequency. Experimentally we did not investigate the influence of "Munk's moment" but we will certainly take into account Mr. Lee's remarks in this respect.

According to Mr. Honkanen the situation of the centre of rotation may influence the hydrodynamic coefficients of the lateral motions. Unfortunately no experimental values of this influence are available. To the opinion of the authors the effect will not be so rigorous as suggested by the discusser. This effect can be determined by means of PMM test considering different positions of the rotation axis. However, up to now these tests have not been carried out by the authors.

Dr. Norrbin points out that the representation according to the "bis" system of reference is much more illustrative respecting the straight line stability as can be seen in Fig. 1 of his discussion. Nevertheless the SNAME-nomenclature is very widespread and used in a number of countries and the authors prefer to stick to this nomenclature. The authors agree with Dr. Norrbin's remark respecting the availability of results including propeller and rudder. Some results, however, have been published in [1] in case of full tankermodels and probably it is possible to extrapolate some information of these tests to the length-beam series.

[1] Glansdorp, C.C. Pijfers, J.G.L.
"Effect of Design Modifications on the natural course stability of full tanker models"
The Institution of Mechanical Engineers
17-21 April 1972; London.

REPORT 403-P

The effects of beam on the hydrodynamic characteristics of ship hulls

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Forced oscillation experiments have been carried out with a systematic ship model family of which the length-beam ratio was ranging from 4 to 20. The experiments also included a thin plate to simulate the case of an infinite length-beam ratio. Vertical and horizontal harmonic motions in calm water have been considered and the corresponding hydrodynamic coefficients have been determined. Moreover the vertical motions and added resistance in waves have been measured. The results are presented in grafical form and are compared with some existing calculation methods.

Nomenclature

A, B, C, D, E, G / a, b, c, d, e, g	hydrodynamic coefficients of the equations of pitch and heave respectively
B	ship's beam
C_B	block coefficient
C_P	prismatic coefficient
C_s	horizontal sectional added mass coefficient
F	total vertical wave force
F'	sectional hydromechanic force
F_n	Froude number
g	acceleration owing to gravity
I_{yy}	vertical longitudinal moment of inertia
I'_{zz}	dimensionless horizontal moment of inertia
K'	coefficient of accession to moment of inertia
K	empirical coefficient in the low aspect ratio lift formula
$K_{1,2}$	coefficients of accession (long., lat.)
k	wave number
k_{yy}	vertical longitudinal radius of inertia of ship
k_{zz}	horizontal longitudinal radius of inertia of ship
L	ship's length
M	total vertical wave moment; mass of ship
M'	dimensionless mass of ship
m'	vertical sectional added mass
N'	vertical sectional damping coefficient
$N'_{\dot v}, N'_v, N'_{\dot r}, N'_r$ / $Y'_{\dot v}, Y'_v, Y'_{\dot r}, Y'_r$	hydrodynamic coefficients of the equations of yaw and sway respectively
r'	dimensionless yaw velocity
$\dot r'$	dimensionless yaw acceleration
T	ship's draught
T^*	effective draught
T_e	period of encounter
V	forward velocity of ship
V_z	vertical relative velocity with respect to the water
v'	dimensionless sway velocity
$\dot v'$	dimensionless sway acceleration
$X'_{\dot u}$	dimensionless longitudinal added mass
x'	dimensionless length coordinate in a right hand body fixed coordinate system with centre of gravity in the origin and the starboard side positive
x_b, y_b, z_b	righthand coordinate system fixed to ship with the origin situated in the ship's waterline and the portside positive
x'_{p1}	dimensionless centre connected with the first moment of viscous force distribution
x'_{p2}	dimensionless centre connected with the second moment of viscous force distribution
x'_r	point of application of total yaw force
x'_v	point of application of total sway force
Y'	dimensionless hydrodynamic lateral force
y'_0	dimensionless motion amplitude
y_w	half width of waterline ($z = 0$)
z	heave displacement
ε	phase angle
λ	wave length
∇	volume of ship's displacement
ω	circular wave frequency
ω'	dimensionless PMM frequency
ω_e	circular frequency of encounter
ρ	density of water
σ'_1	dimensionless stability root
σ'_2	dimensionless stability root
θ	pitch angle
ζ	instaneous wave elevation

Subscripts

a	amplitude of denoted parameter
$F\zeta$	wave force with respect to wave elevation
$M\zeta$	wave moment with respect to wave elevation

Superscripts

	sectional values or dimensionless values according to SNAME-nomenclature

1 Introduction

The calculation of the vertical hydrodynamic forces and moments acting on a ship in sea waves, according to the strip theory, has proved to be a valuable tool. To a limited extent this is also true of horizontal motions, but little experimental verification is available with regard to low frequency motions, which are of interest for manoeuvring and steering problems. Detailed comparisons of calculations and experiments relating to pitch and heave are mainly restricted to more or less average hull dimensions – for instance, a length/beam ratio of approximately 6 to 8 and block coefficients of around 0.70. Although predictions regarding the vertical motions of extreme ship forms have been quite successful, the extent of the validity of the strip theory with respect to more extreme hull dimensions has remained unknown.

Intuitively one may imagine that the more slender the ship form, the greater the justification for applying the strip method. For manoeuvring and steering purposes, the hydrodynamic coefficients of the equations of motion depend to a greater extent on viscous effects introducing lift phenomena than they do when considering the vertical motions of a ship in waves.

Existing methods for approximating these hydrodynamic forces are more empirical in nature.

Apart from the length/draught ratio in both cases, the length/beam ratio may be regarded as a useful parameter in a comparison of theory and experiment.

The main objective of this paper is to provide extensive experimental data relating to the influence of the length/beam ratio of a systematic ship model family on the hydrodynamic forces on the hull for vertical oscillatory motions in the wave frequency range, as well as for low frequency horizontal motions of interest for steering and manoeuvring.

The experiments cover a large range of length/beam ratios, including a very broad ship form ($L/B = 4$) and a very slender ship $L/B = 20$. In addition, a thin plate has been tested in horizontal motion to simulate an infinitely large length/beam ratio. All of the models have been derived from the standard Sixty Series hull form with $L/B = 7$ and $C_B = 0.70$[1], by multiplying the width by constant factors, to arrive at $L/B = 4$, 5.5, 7, 10 and 20. All models were made from glass-reinforced polyester and were 10 feet long. For the main particulars see Table I.

2 Experimental programme and results

Using a vertical Planar Motion Mechanism (PMM) the hydrodynamic coefficients of the heave and pitch equations according to equations (1) in Appendix 1 were measured for Froude numbers $F_n = 0.20$ and $F_n = 0.30$.

The latter speed is high for all models and large wave formation was observed during the experiments.

Excellent linearity was found for the heav amplitudes considered. These go to 1% of the model length and achieve pitch amplitudes of up to 3.5 degrees. For the wave tests, wave heights of 2.5% of the model length were considered. Linearity proved to be good with $L/B = 4$.

The non-dimensional mass and damping coefficients, as well as the mass and damping cross coupling coefficients, are given in Figs. 1 to 8 in non-dimensional form as a function of the Froude number, the frequency of oscillation and the length/beam ratio. Figures 9 and 10 give the dimensionless motion amplitudes of heave and pitch and Fig. 11 gives the added resistance in regular head waves. The motions and the added resistance in waves could not be measured for the $L/B = 20$ model owing to experimental difficulties. The hydrodynamic coefficients for yaw and sway according to equations (13) of Appendix 3 were measured for three velocities: $F_n = 0.15$, 0.20 and 0.30.

A large amplitude PMM was used; the model frequency range was between $\omega = 0.2$ (0.1) and 1.0.

Strut amplitudes for both modes of motion were 5, 10, 15, 20 and 25 cm respectively, the horizontal distance between the struts being 1 m. Relatively small wave formation was observed for the lowest of the three velocities considered, and the experimental results for $F_n = 0.15$ were therefore used for comparison with some calculation methods.

Figures 12, 13 and 14 show the coefficients derived from the force and moment measurements as a function of the L/B ratio for the three forward speeds considered. Table II gives the numerical values of the various hydrodynamic coefficients.

In Figs. 15 and 16 the results of the swaying force and swaying moment are presented as a function of speed, frequency, L/B ratio and amplitude.

Table I

		$L/B = 4.0$	$L/B = 5.5$	$L/B = 7.0$	$L/B = 10.0$	$L/B = 20.0$	$L/B = \sim$
L_{PP}	m	3.048	3.048	3.048	3.048	3.048	3.048
LWL	m	3.099	3.099	3.099	3.099	3.099	3.099
B	m	0.7620	0.5542	0.4354	0.3048	0.1524	0.006
T	m	0.1742	0.1742	0.1742	0.1742	0.1742	0.1742
∇	m^3	0.2832	0.2060	0.1618	0.1133	0.0566	0.0032
A_W	m^2	1.8267	1.3342	1.0435	0.7331	0.3652	–
I_L	m^4	0.9737	0.7117	0.5566	0.3909	0.1947	–
C_B		0.70	0.70	0.70	0.70	0.70	–
C_P		0.71	0.71	0.71	0.71	0.71	–
LCB before $L_{PP/2}$		0.014	0.014	0.014	0.014	0.014	–
LCF before $L_{PP/2}$		−0.063	−0.063	−0.063	−0.063	−0.063	–
k_{yy}/L_{PP}		0.25	0.25	0.25	0.25	0.25	–
M kgf	s^2/m	28.859	20.988	16.491	11.544	5.772	7.513
k_{zz}/L_{PP}		0.267	0.268	0.230	0.229	0.229	0.275

3 Discussion of the results

3.1 Vertical motions

First of all, the heaving and pitching motions were calculated on the basis of a formulation of the strip theory as given in Appendix 1 and Ref. 2. This formulation was derived from earlier work by Shintani[3], Söding[4], Semenov-Tjan-Tsansky et al.[5] and Tasai[6], and yields the same results as given by Salvesen et al. . Next, we used the method formulated principally by Korvin-Kroukovsky and Jacobs[8] as modified by the authors[9]. The results of both methods were compared with the experimental results.

The added resistance in waves resulting from the pitching and heaving motions was calculated by the method described in Appendix 2. The added resistance is determined by calculating the work done by the radiated damping waves, which result from the vertical motions of the ship relative to the water. In[10] this method has been confirmed by experimental results derived from model tests with a fast cargo ship hull form. Further experience included blunt tanker forms, although in some of these cases the agreement was somewhat less satisfactory at high frequency of encounter.

In Figs. 1 to 11 the experimental values are compared with corresponding calculations according to the modified Korvin-Kroukovsky formulation[9] and equations (6) and (7). For convenience we will call these the old and the new method respectively.

With regard to the coefficients of the equations of motion for heave and pitch, the two calculation methods give almost identical results, except for the pitch damping coefficient at low frequencies and the added mass cross coupling coefficient D for pitch.

The differences between the measured added mass and the calculated value are small, even for the very low L/B ratios. The correlation is still satisfactory for the added moment of inertia, with only a few differences for the highest speed and the lowest L/B ratio. The heave damping coefficient is reasonably predicted except for high frequencies where viscous effects such as separation of flow may be important.

Both the new and the old method predict the pitch damping rather poorly, particularly at low frequencies. The experimental data do not show a clear preference for either of the two methods. For practical purposes, the overestimation of the pitch damping at low frequencies according to the new method is not very important in the motion prediction.

Considering the absolute magnitude of the damping cross coupling terms, the coefficients e and E are very well predicted by both theories for the two forward speeds considered, as well as for all length/beam ratios.

The added mass cross coupling coefficient d for heave is also reasonably well predicted by both methods, but in the case of the mass cross coupling coefficient D for pitch the experimental points for low frequencies lie between the two predicted curves. For low frequencies the experimental values favour the prediction according to the new method.

Heave amplitudes in waves are somewhat overestimated by the new method. Earlier experience with both methods has shown that there is a slight preference for the modified Korvin-Kroukovsky and Jacobs method, although the desired symmetry in the mass cross coupling coefficients is not fulfilled in their presentation. Moreover, added resistance is overestimated by the new method and in this respect it should be remembered that added resistance varies as the squared motion amplitudes.

For $F_n = 0.20$ the predicted added resistance agrees very well with the measured values, only minor differences being observed at high frequencies. Satisfactory agreement is even found

Table II

L/B		4	5.5	7	10	20	∞	∞*
$F_n = 0.15$								
M'		1978	1433	1122	779	379	521	0
I'_{zz}		142	103	59	41	20	39	0
Y'_v		−1800	−1700	−1600	−1450	−1400	−1500	−1500
N'_v		− 610	− 670	− 730	− 780	− 700	− 500	− 500
$Y'_{\dot{v}} - M'$		−3198	−2703	−2352	−1899	−1559	−1601	−1080
$N'_{\dot{v}}$	$\times 10^5$	− 120	− 50	− 40	0	0	+ 20	+ 20
$Y'_r - M'$		−1858	−1243	− 872	− 479	0	0	+ 521
$Y'_{\dot{r}}$		− 265	− 295	− 290	− 280	− 240	− 260	− 260
$N'_{\dot{r}} - I'_{zz}$		− 110	− 90	− 60	0	0	0	0
$N'_r - I'_{zz}$		− 190	− 165	− 125	− 105	− 88	− 95	− 56
σ'_1		0.538	0.304	0.200	−0.048	−0.901	−0.935	Re = −2.930
σ'_2		−2.051	−2.468	−2.955	−3.382	−2.724	−2.739	Im = ±1.471
$F_n = 0.20$								
Y'_v		−1850	−1760	−1750	−1500	−1400	−1600	−1600
N'_v		− 650	− 720	− 790	− 800	− 700	− 450	− 450
$Y'_{\dot{v}} - M'$		−3198	−2543	−2442	−1919	−1559	−1601	−1080
$N'_{\dot{v}}$	$\times 10^5$	− 180	− 70	− 50	0	0	0	0
$Y'_r - M'$		−1748	−1283	− 892	− 499	0	0	− 521
N'_r		− 270	− 300	− 310	− 310	− 250	− 240	− 240
$Y'_{\dot{r}} - I'_{zz}$		− 120	− 60	− 60	0	− 50	0	0
$N'_{\dot{r}} - I'_{zz}$		− 195	− 165	− 135	− 112	− 97	− 120	− 81
σ'_1		0.548	0.369	0.170	−0.088	−1.064	−0.997	Re = −2.222
σ'_2		−1.929	−2.584	−2.928	−3.461	−2.180	−2.002	Im = ±1.458
$F_n = 0.30$								
Y'_v		−2450	−2300	−2070	−1760	−1450	−1600	−1600
N'_v		− 700	− 840	− 900	− 980	− 860	− 500	− 500
$Y'_{\dot{v}} - M'$		−3078	−2603	−2652	−2189	−1599	−1621	−1100
$N'_{\dot{v}}$	$\times 10^5$	− 160	− 100	− 20	0	− 50	0	0
$Y'_r - M'$		−1878	−1303	−1042	− 559	− 29	0	+ 521
N'_r		− 330	− 360	− 400	− 340	− 310	− 230	− 230
$Y'_{\dot{r}}$		− 180	− 100	− 100	0	− 50	0	0
$N'_{\dot{r}} - I'_{zz}$		− 200	− 160	− 120	− 115	− 95	− 90	− 51
σ'_1		0.387	0.225	0.090	−0.054	−0.955	−0.985	Re = −2.982
σ'_2		−2.227	−2.909	−3.879	−3.706	−2.878	−2.558	Im = ±1.517

* plate without mass

for the very low length/beam ratios, considering the more or less extreme hull shape and the relatively high forward speed in those cases. For $F_n = 0.30$ the correlation between theory and experiment is less. However, for all length/beam ratios, except for $L/B = 7$ this speed is very high, with correspondingly high ship waves. The added resistance at high frequencies is underestimated by the theory, particularly in the case of $L/B = 4$.

3.2 *Horizontal motions*

The coefficients have been determined in a standard graphical way from the in-phase and quadrature components of forces and moments measured with the PMM. The accuracy of the coefficients shown in Figs. 12, 13 and 14, is probably not high since the relevant forces and moments are small in magnitude. The coefficients indicate a trend in the results and do not pretend to be highly accurate.

In Table II the numerical values of the coefficients are summarized using the dynamic mode of motions. Figures 12, 13 and 14 clearly show the effect of beam, which is not very pronounced for a low Froude number. As could be expected, the forward speed affects the results to a certain extent: the broader the model, the more decisive the part played by the model-generated wave system in the creation of the resulting hydrodynamic forces and moments. Hu[11] predicted the effect of speed upon the hydrodynamic coefficients, applying sources and doublets in the ship's centre plane and wake, and taking the boundary conditions on the surface into account. Comparing the trend of the experimental results and the predicted values with regard to the forward velocity according to Hu, it can be said that his prediction gives a more pronounced effect of speed. It is interesting to note that the results obtained in van Leeuwen's PMM tests[12] with an 8-foot model of the

$L/B = 7$ are practically the same as the results presented in this paper, taking a reasonable margin of accuracy into account. In Figs. 12, 13 and 14 some evidence is produced to show that the values of the static and dynamic sway coefficients approach each other closely. The condition for straight line stability (this term is used rather than controls fixed stability since no rudder, propeller or other hull appendices have been fitted) yields:

$$\frac{x'_v}{x'_r} < 1$$

When x'_v and x'_r are both positive, this condition postulates that the point of application of the total yaw force is located before the point of application of the sway force. In Figs. 12, 13 and 14 it may be observed that this condition is fulfilled for an L/B ratio exceeding 8. Since at an L/B ratio of approximately 20, Y'_r equals the mass M', x'_r will change sign and becomes extremely negative. In this case the aforementioned criterion is still satisfied, since it is obvious that x'_v remains positive. The stability roots are calculated in Table II; the smaller roots are positive for the smaller L/B ratios and they become negative for the larger L/B ratios. There is a noteworthy difference between the last two columns, indicating that the plate actually used for the experiments behaves stably, but that an imaginary massless plate has an oscillatory stable behaviour. This fact is also found in the stability analysis of ships such as sailing yachts which have large fins on deep keels, and is caused by the smallness of the inertia forces relative to the lift forces[13].

Jacobs[14,15] published a brief account of a simple theory for the calculation of the linear coefficients of the horizontal motion based upon simple hydrodynamic concepts. Apart from an ideal fluid treatment of a wing-shaped body in an unbounded flow, resulting in hydrodynamic added masses and added moments of inertia with cross coupling coefficients, a viscous part is included to represent the generation of a lift. The Jones' low aspect ratio lift formula has therefore been used as an example. Lift generation depends upon the flow conditions near the trailing edge. As these conditions vary, it seems appropriate to introduce an empirical constant K to take these variations into account, as suggested by Inoue[16].

The average value of this K-constant turns out to be nearly 0.75. Appendix 3 contains a brief account of Jacobs' method, which has been chosen for comparison with the measured results.

As a result of an inertia distribution and a viscous distribution along the ship's length, the total lift is generated for the greater part in the forebody, which means that the viscous part almost counter-balances the inertia part in the afterbody[15,17]. The centre of the viscous force distribution therefore lies well aft of the centre of gravity (x'_{p1}). The second moment of the viscous force distribution is characterized by x'_{p2} and this quantity is obviously negative.

The values of K, x'_{p1} and x'_{p2} are calculated from the measurements of the relevant quantities and they are displayed in Fig. 17. They coincide remarkably well with empirical values presented by Inoue and Albring[18]. The coefficient Y'_r can also be used to check the validity of the empirical constants K_1, x'_{p1}. In Fig. 12 it may be seen that there is a satisfactory agreement for the lower Froude number. Apart from considerations relating to the damping coefficients, it is obvious that the added mass, added moment of inertia and the mass cross coupling coefficients are accurately predicted by the simple stripwise integration of sectional values of added mass, depending on local fullness and local B/T values. Three-dimensional corrections have been applied as indicated by Jacobs and others. In order to compare the measured results with those obtained by other methods available in the literature, it was decided to use the results of Inoue which are principally based upon Bollay's low aspect ratio theory and a number of empirical allowances. Appendix 3 gives a brief account of the formulae used, according to Inoue. As can be seen in Fig. 12, the calculation agrees with the measured results with the exception of Y'_r. Norrbin[19] analysed statistical material and derived regression formulae on the basis of the 'bis' system of reference. In Appendix 3 these regression formulae are 'translated' into the nomenclature adopted in this paper. Inspecting the formulae, it can be demonstrated that the L/B ratio has a slight effect, while the results calculated by using these regression formulae are generally in close agreement in the normal range of L/B ratios, as shown in Fig. 12.

Since lift generation is of primary importance in manoeuvring problems, and since experimental material about this subject is not extensively published in the literature, it has been decided to give the transverse force and moment in the sway motion for two speeds: $F_n = 0.15$ and $F_n = 0.30$, as a function of reduced frequency and amplitude in Figs. 15 and 16.

Full linearity in frequency and amplitude exists in a very restricted range. For the higher frequencies, linearity is lost to some extent, especially in the transverse force, and to a smaller extent in the moment. The results are obscured by a number of effects – for instance, nonlinearity owing to the cross flow. Frequency and amplitude effects also interfere with the interpretation of the experimental results.

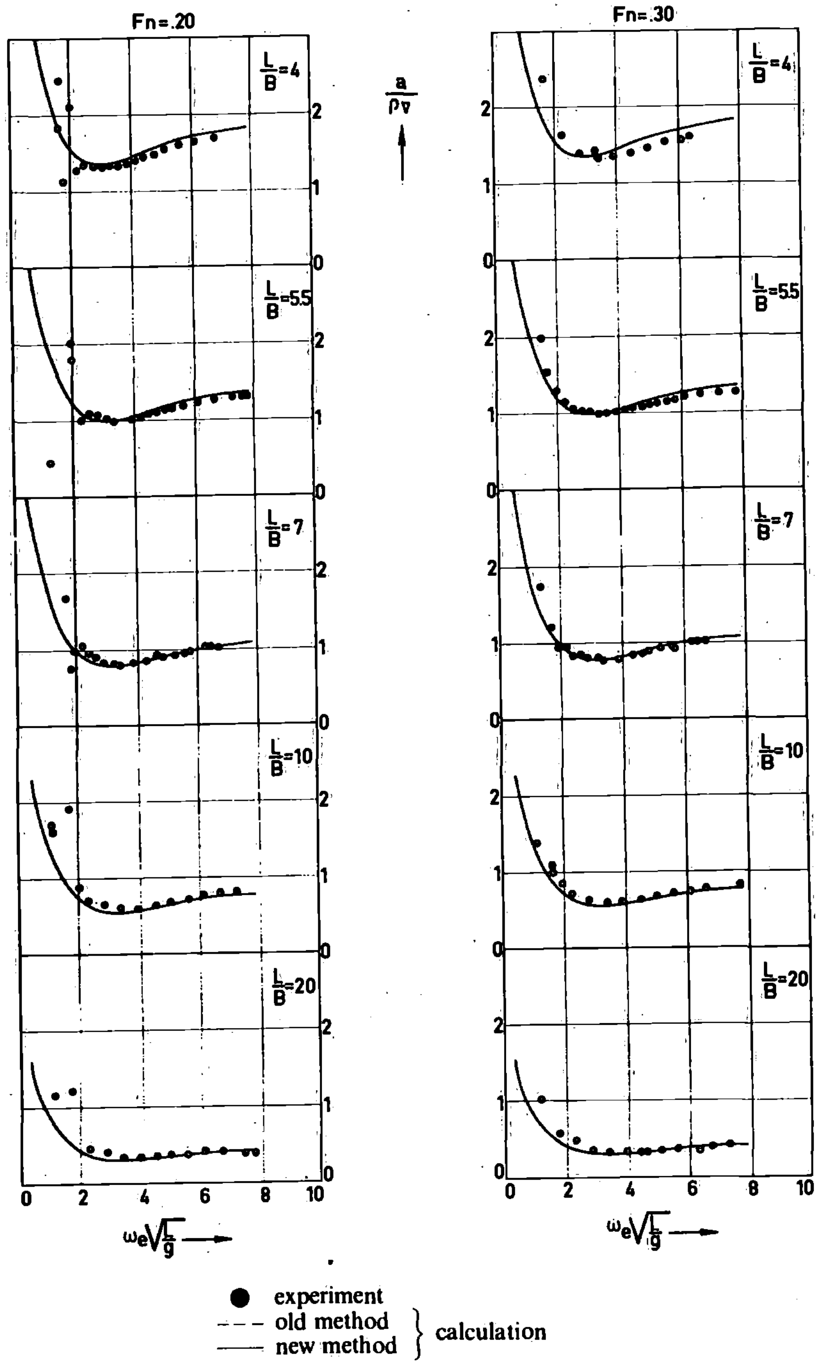


Fig. 1. Added mass coefficient for heave.

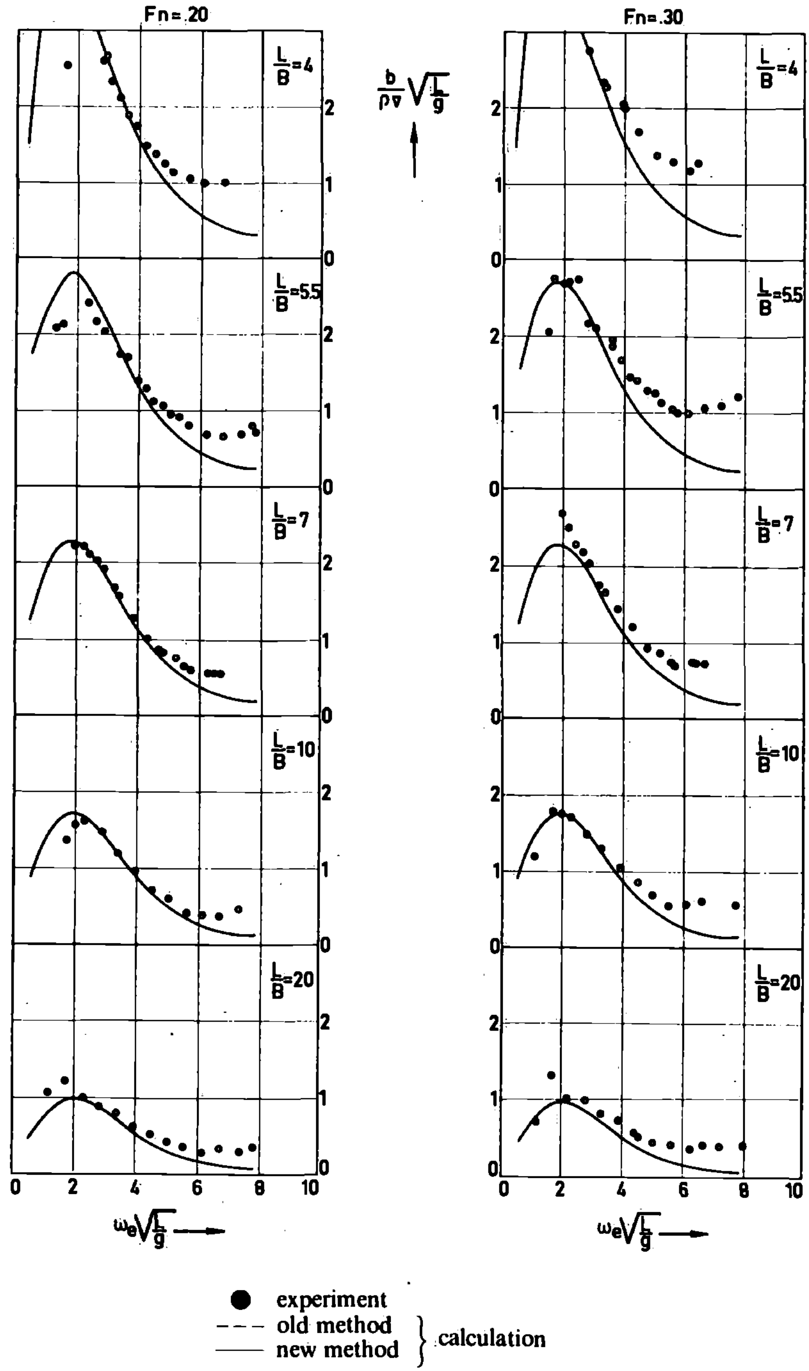
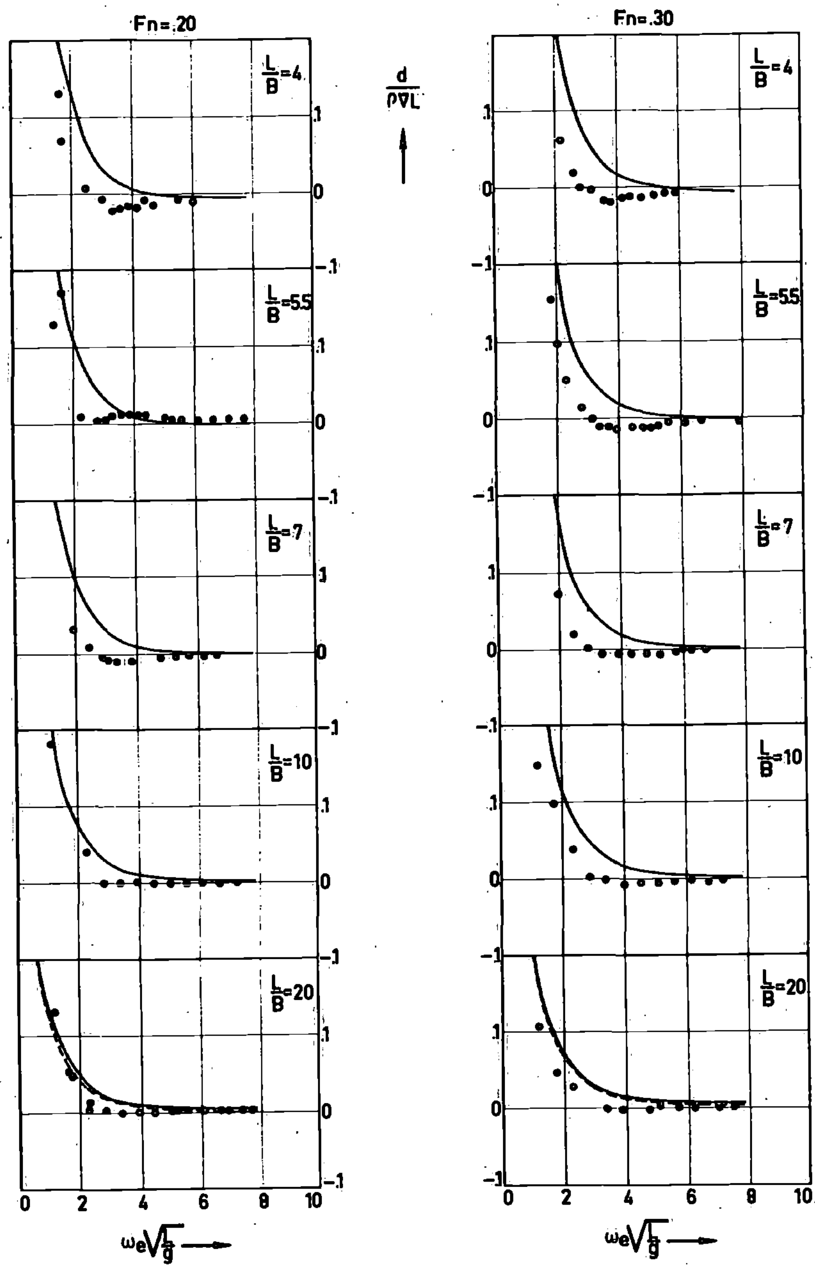


Fig. 2. Heave damping coefficient.



● experiment
--- old method } calculation
— new method }

Fig. 3. Added mass cross coupling coefficient for heave.

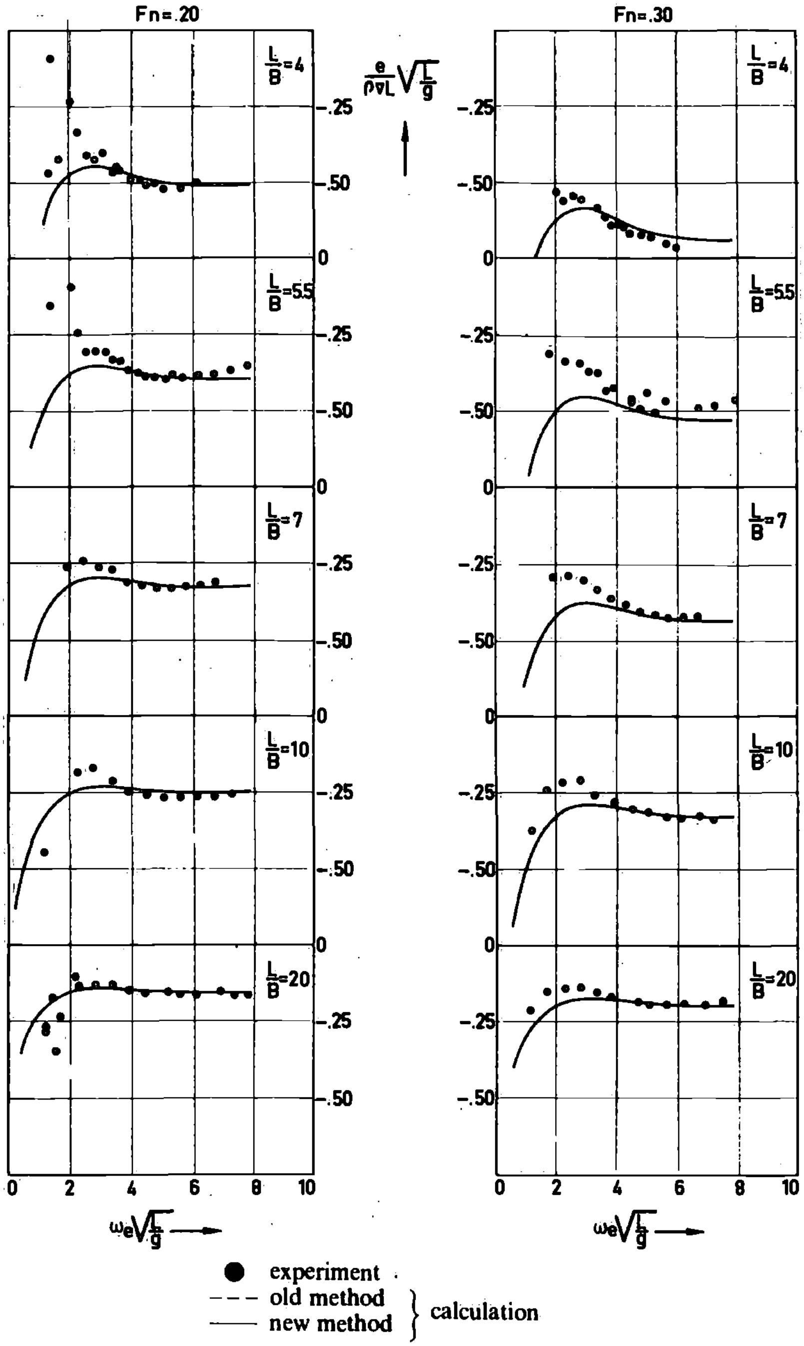
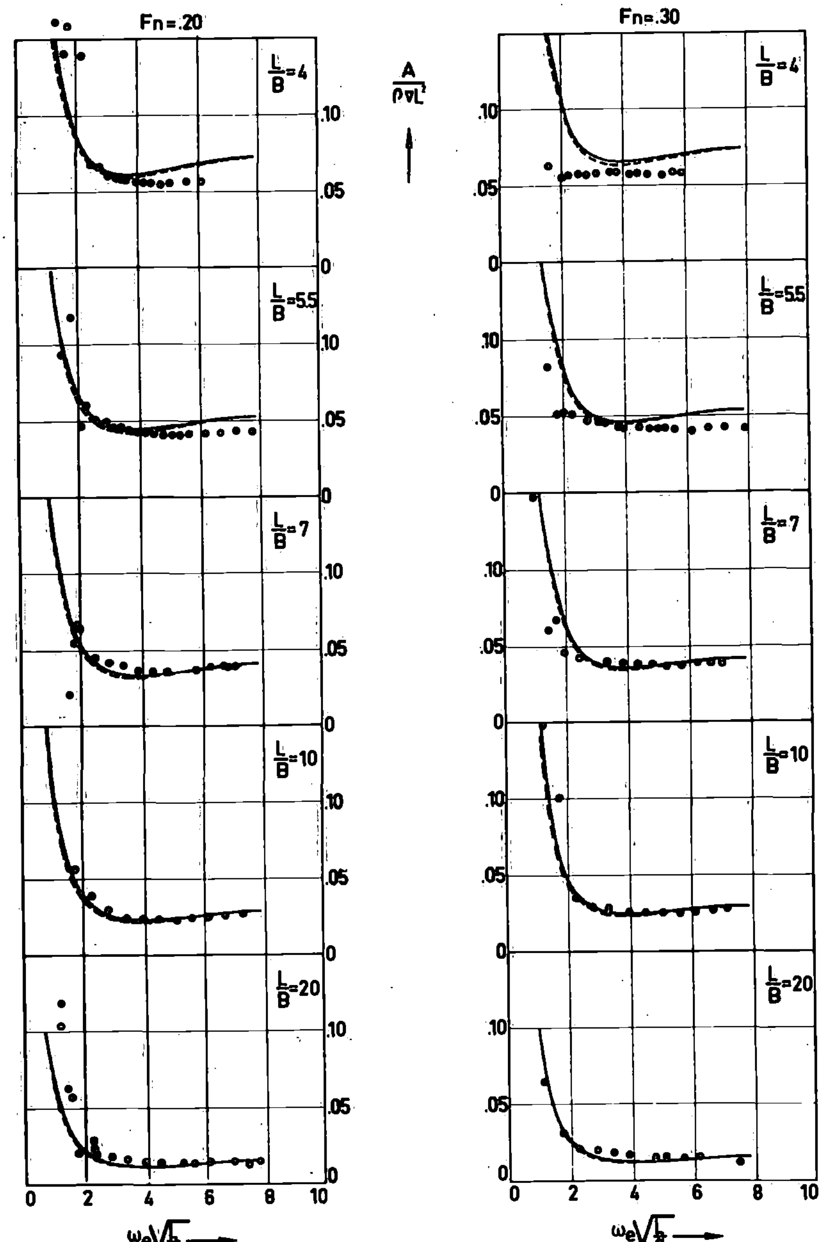


Fig. 4. Damping cross coupling coefficient for heave.



● experiment
--- old method } calculation
—— new method }

Fig. 5. Coefficient of added mass moment of inertia for pitch.

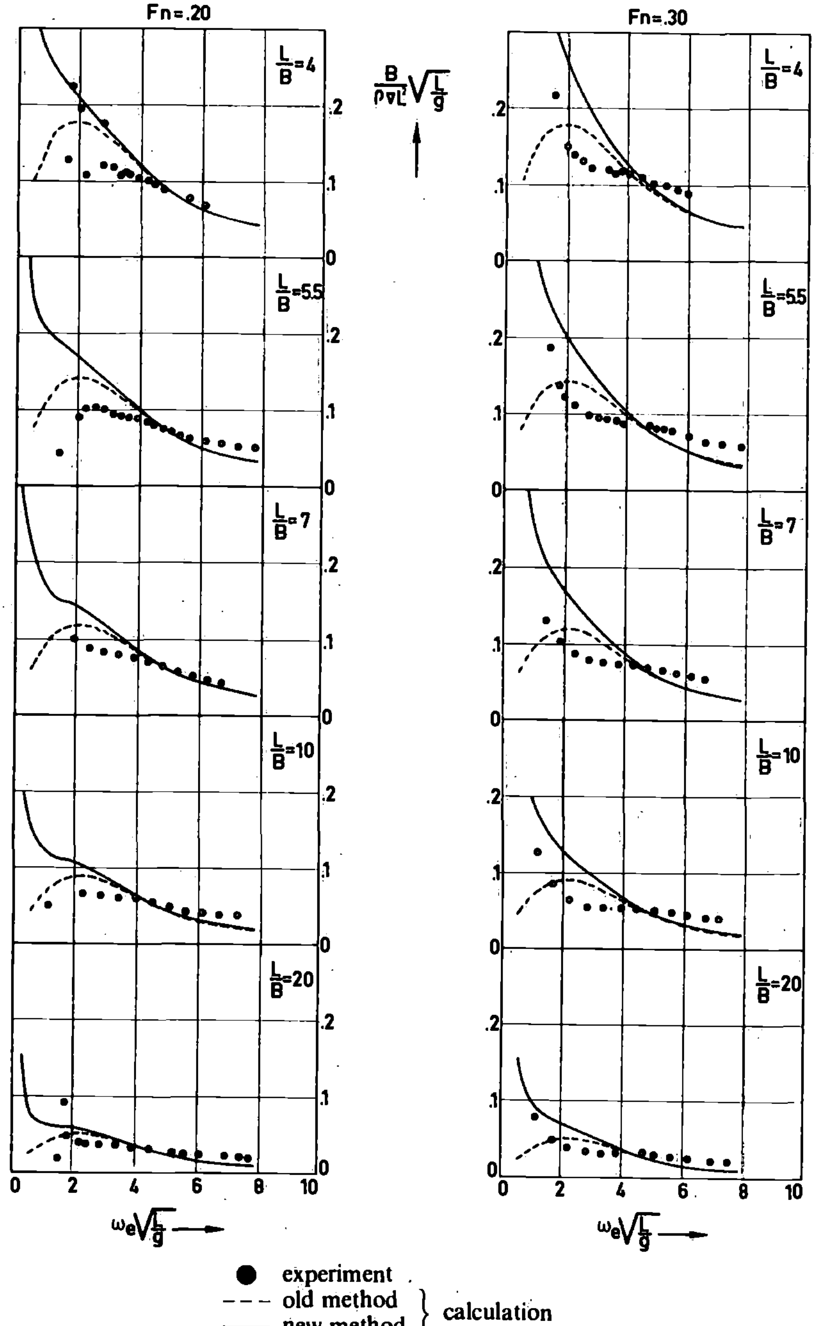


Fig. 6. Pitch damping coefficient.

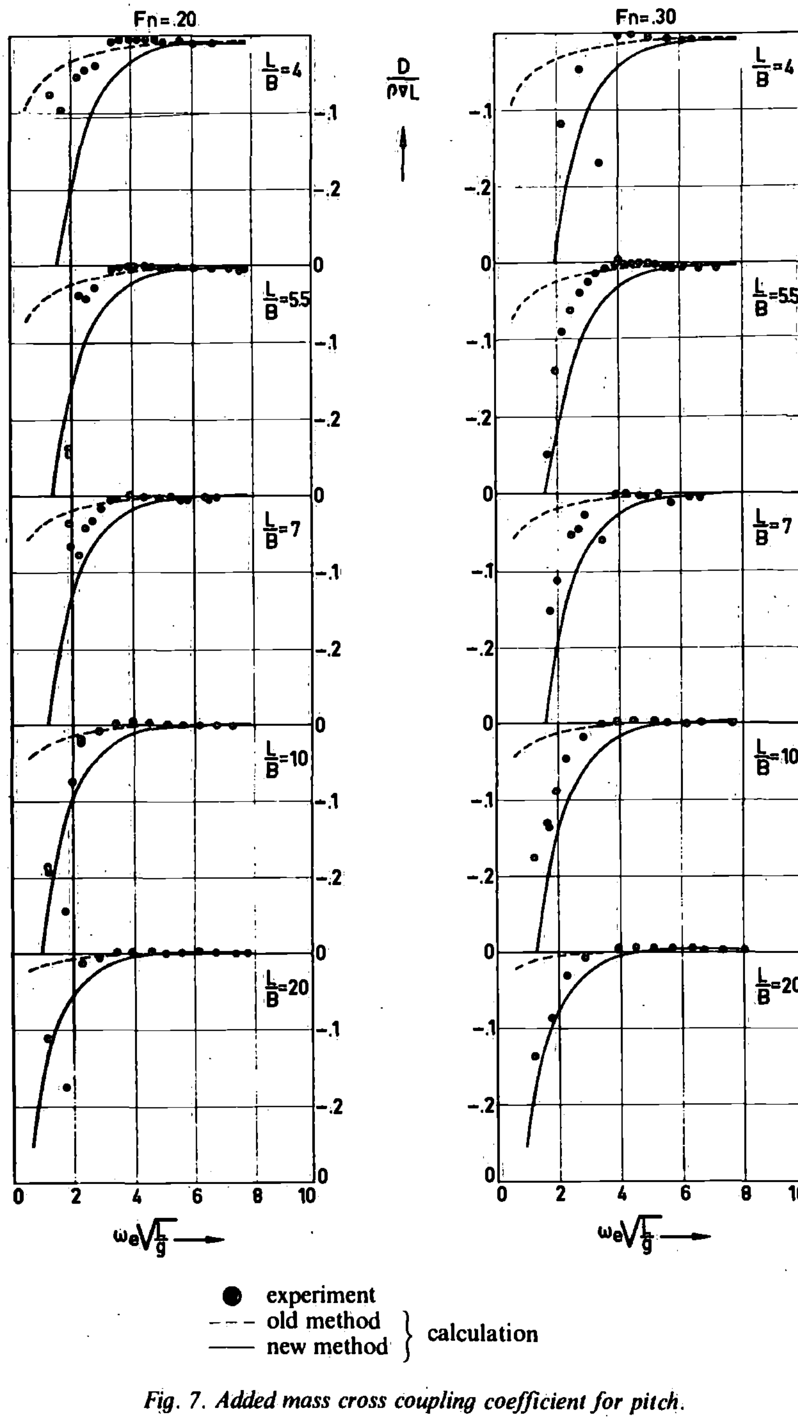
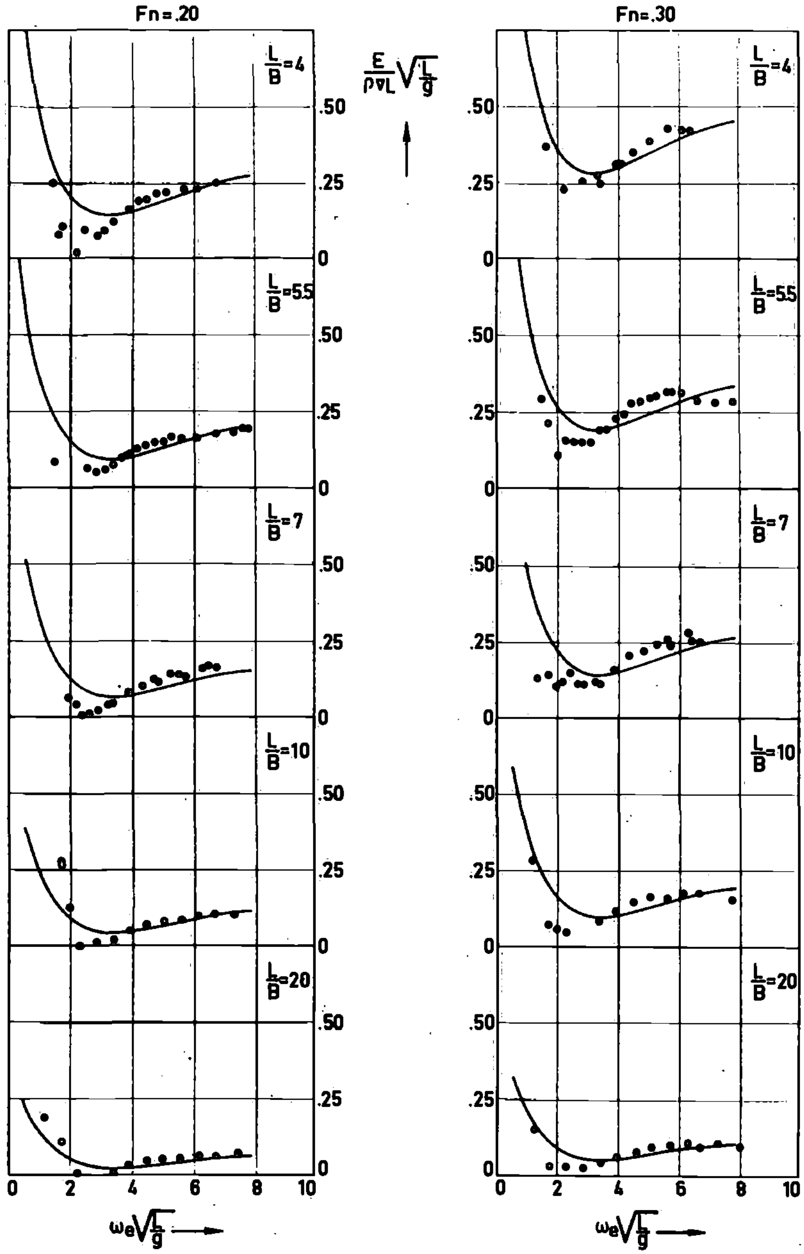
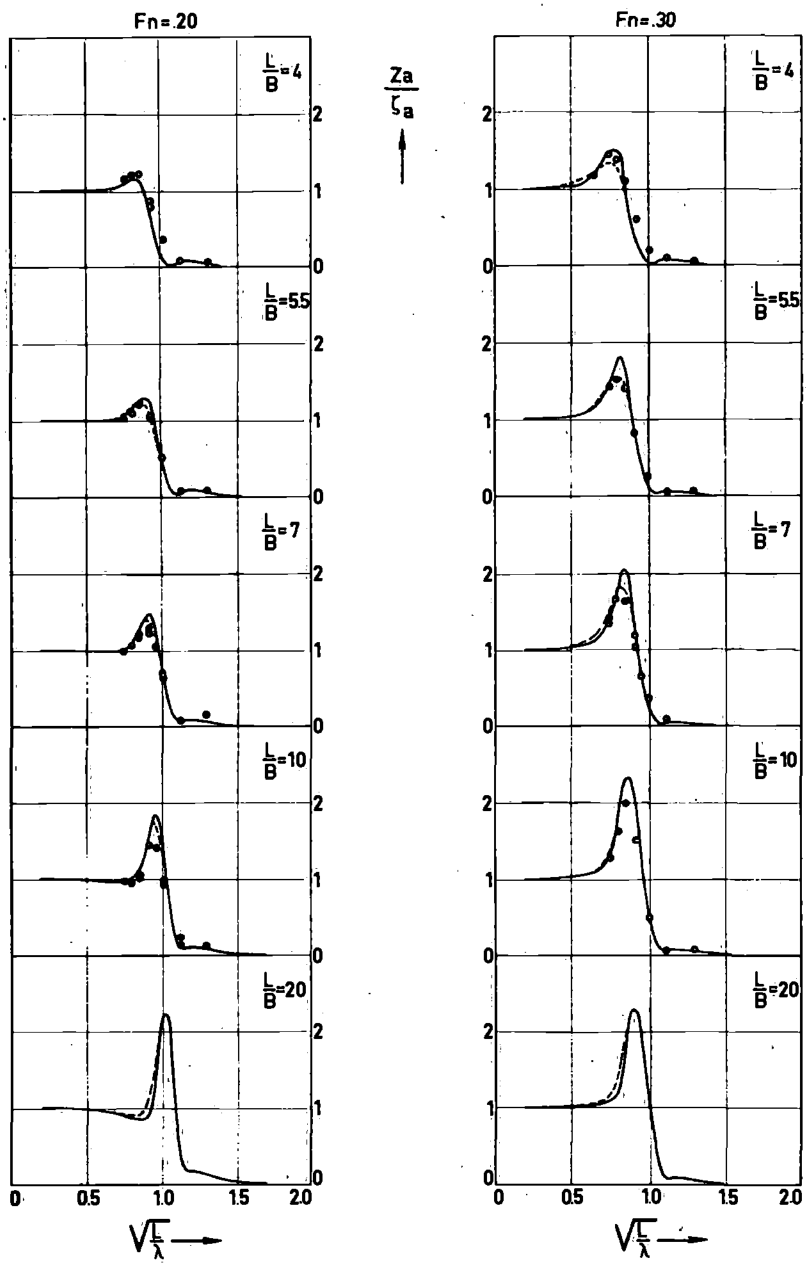


Fig. 7. Added mass cross coupling coefficient for pitch.



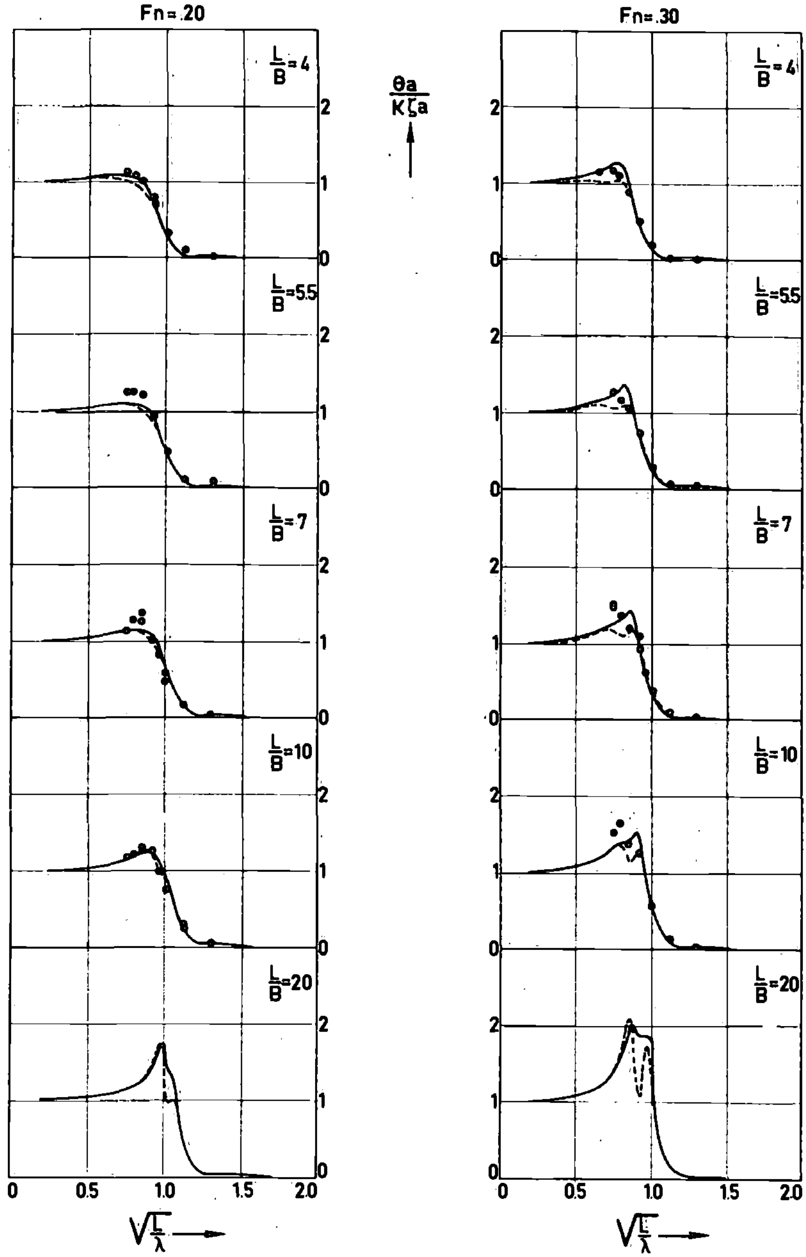
● experiment
--- old method } calculation
—— new method }

Fig. 8. Pitch damping cross coupling coefficient.



● experiment
--- old method } calculation
—— new method }

Fig. 9. Heave amplitude in waves.



● experiment
– – – old method } calculation
—— new method }

Fig. 10. Pitch amplitude in waves.

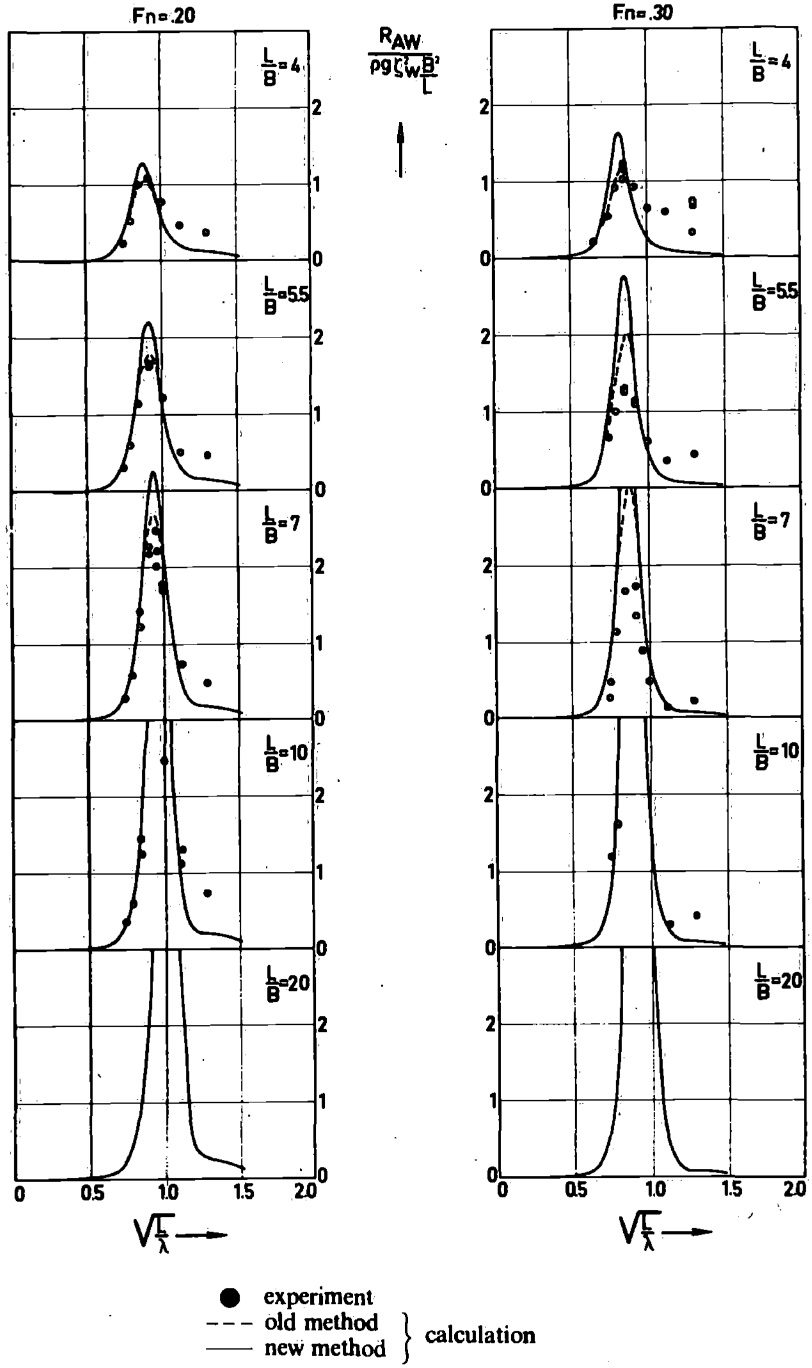


Fig. 11. Added resistance in waves.

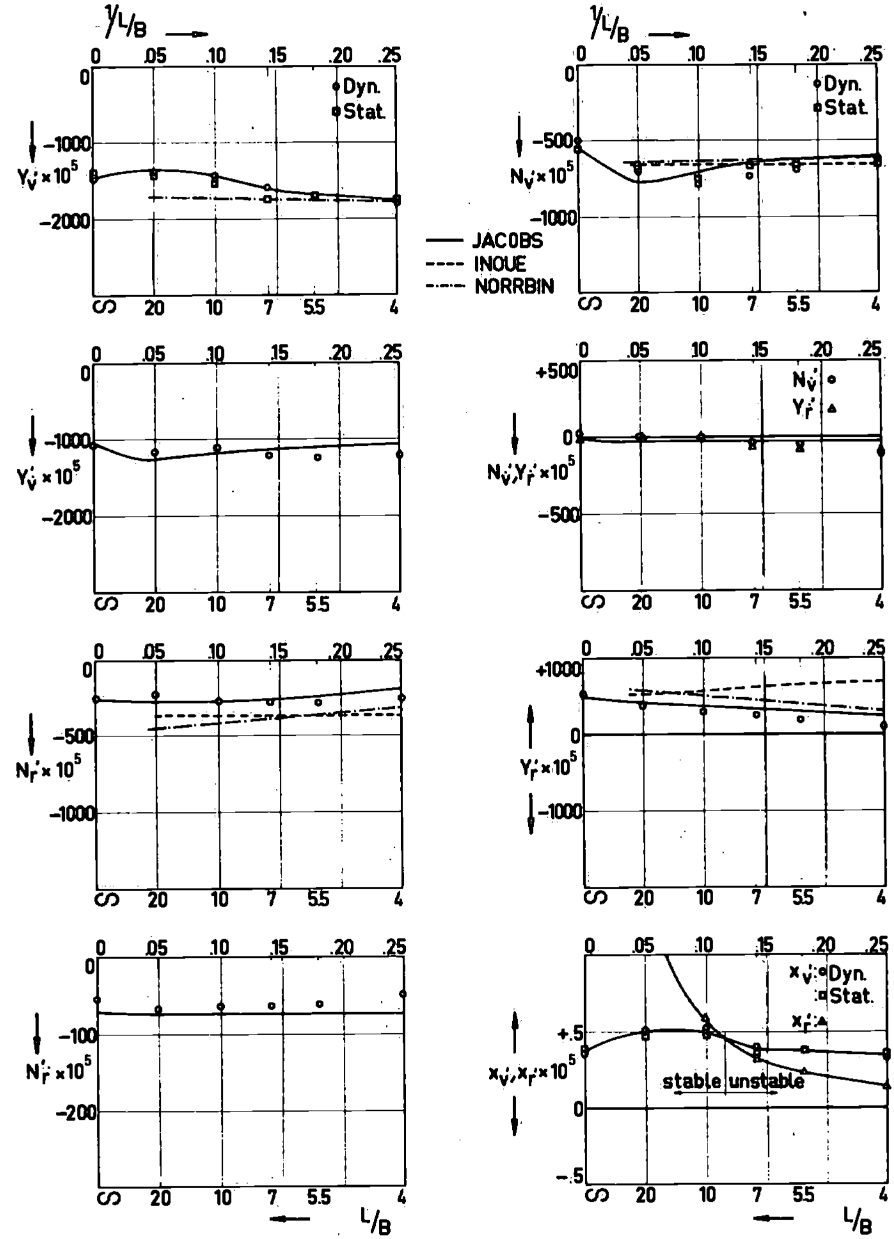


Fig. 12. Hydrodynamic coefficients for $F_n = 0.15$.

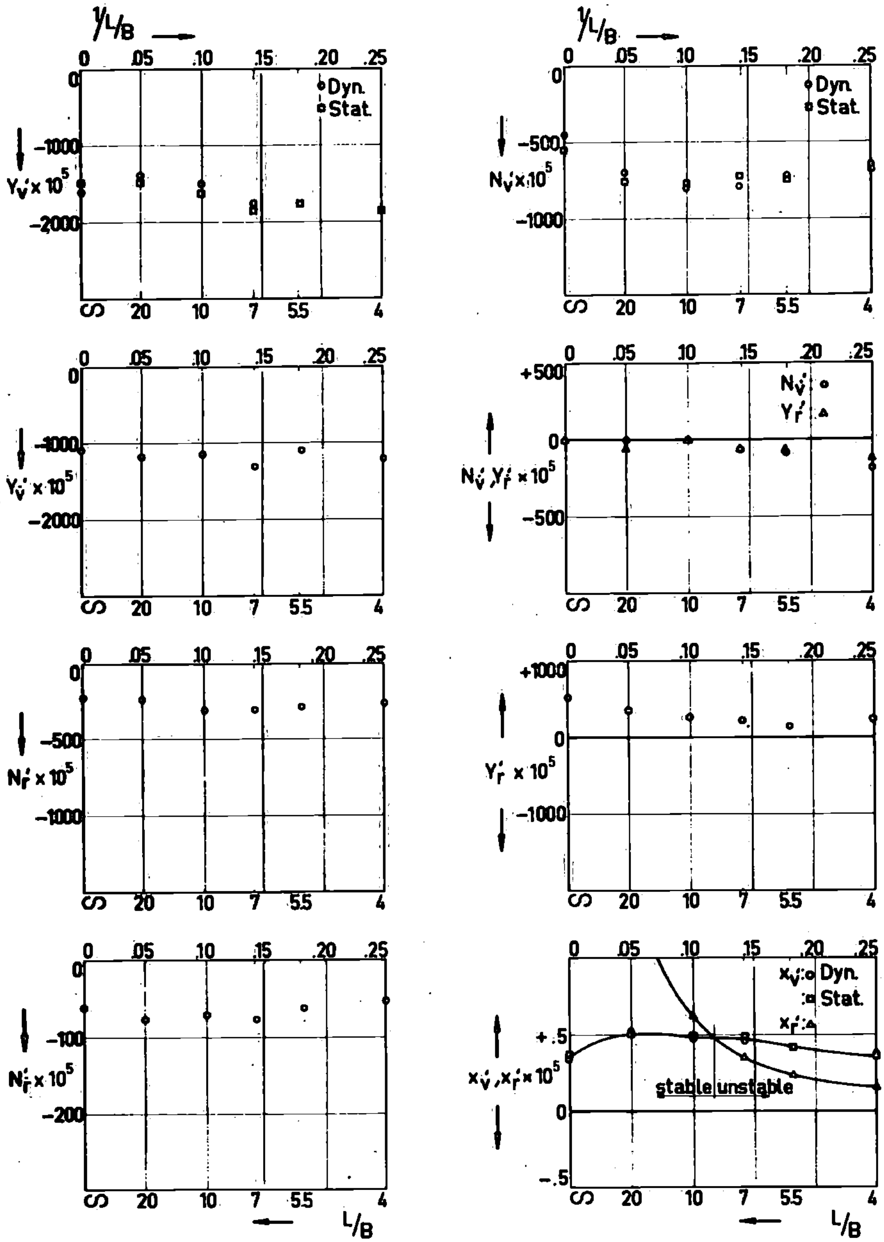


Fig. 13. Hydrodynamic coefficients for $F_n = 0.20$.

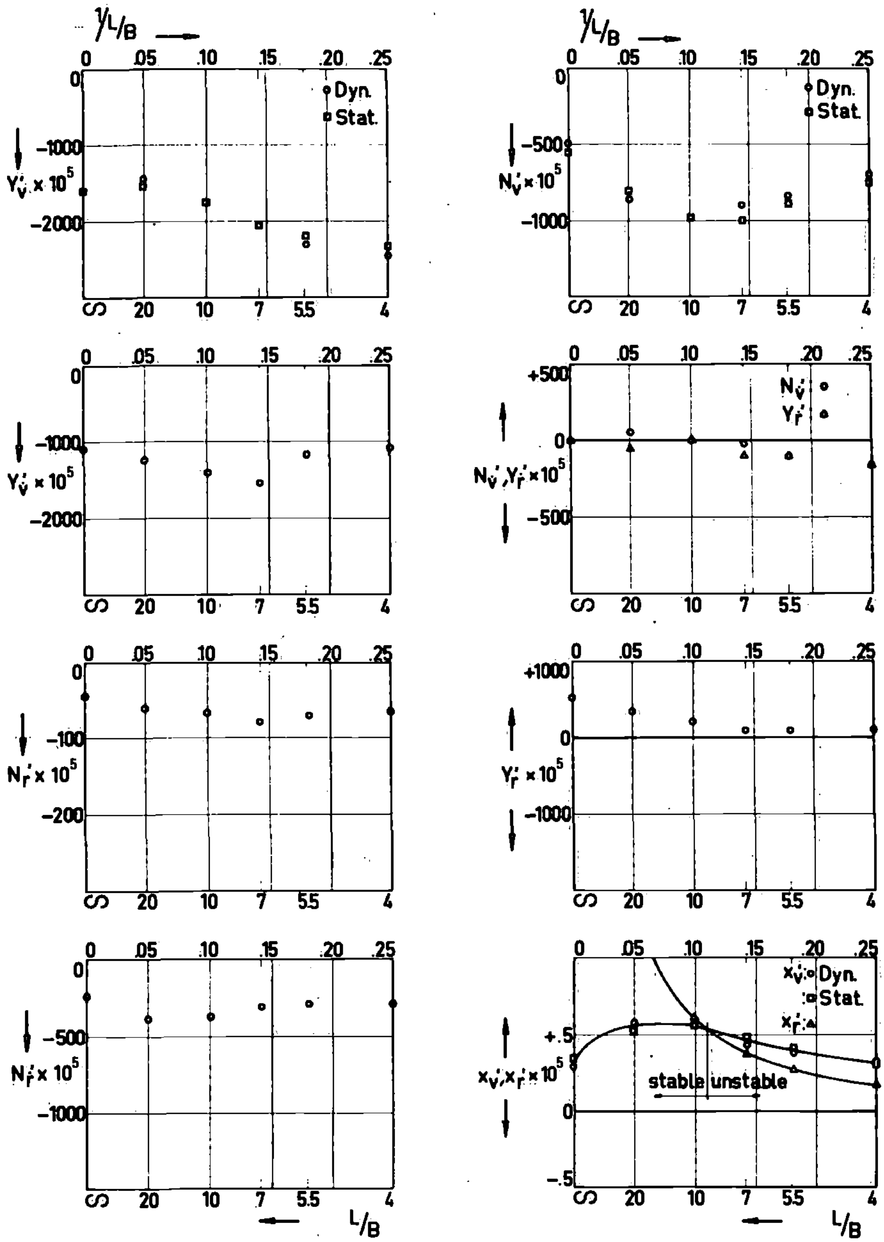


Fig. 14. Hydrodynamic coefficients for $F_n = 0.30$.

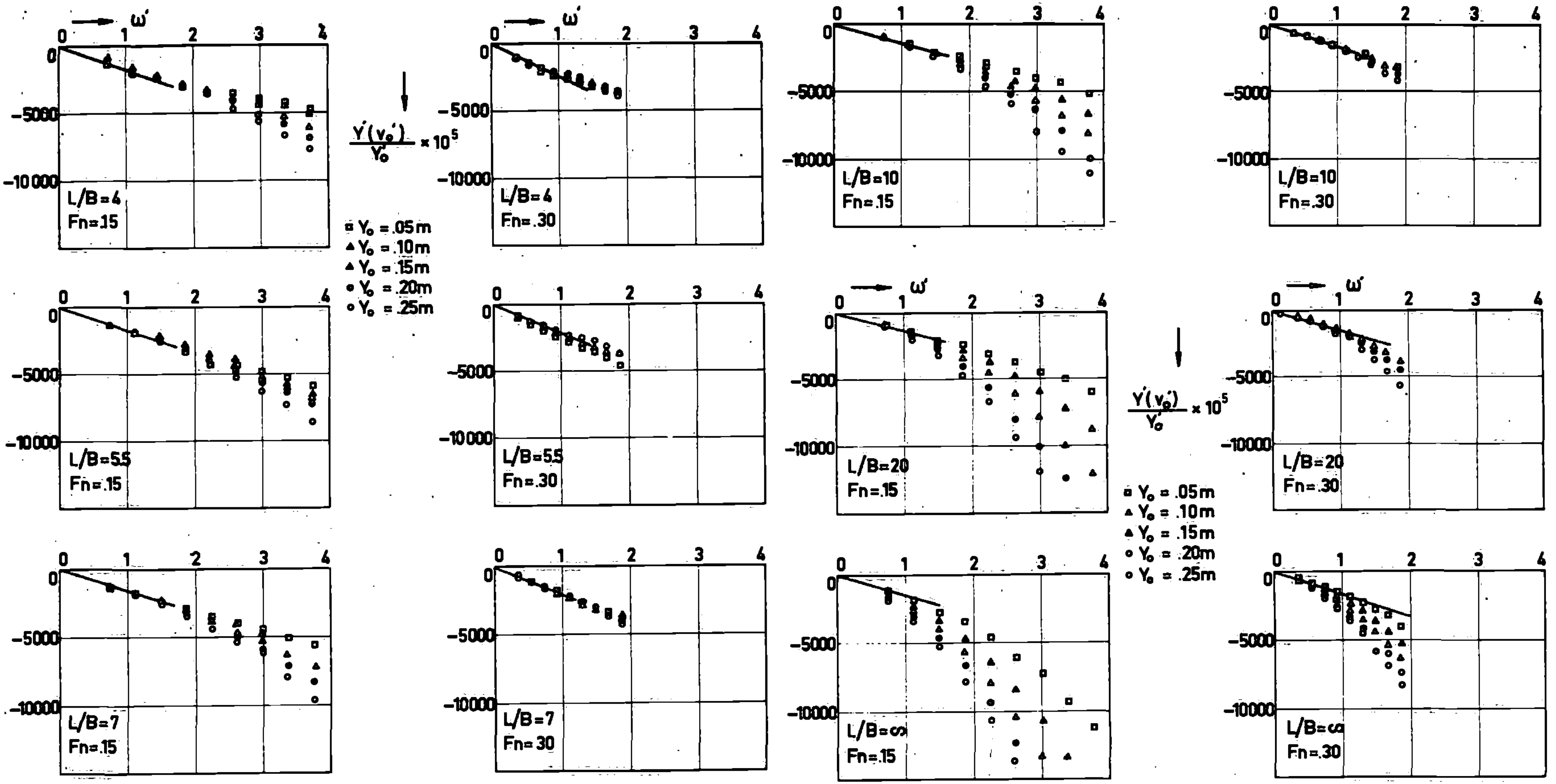


Fig. '˜ Sway Force for two Froude numbers as a function of the ' 'B ratio.

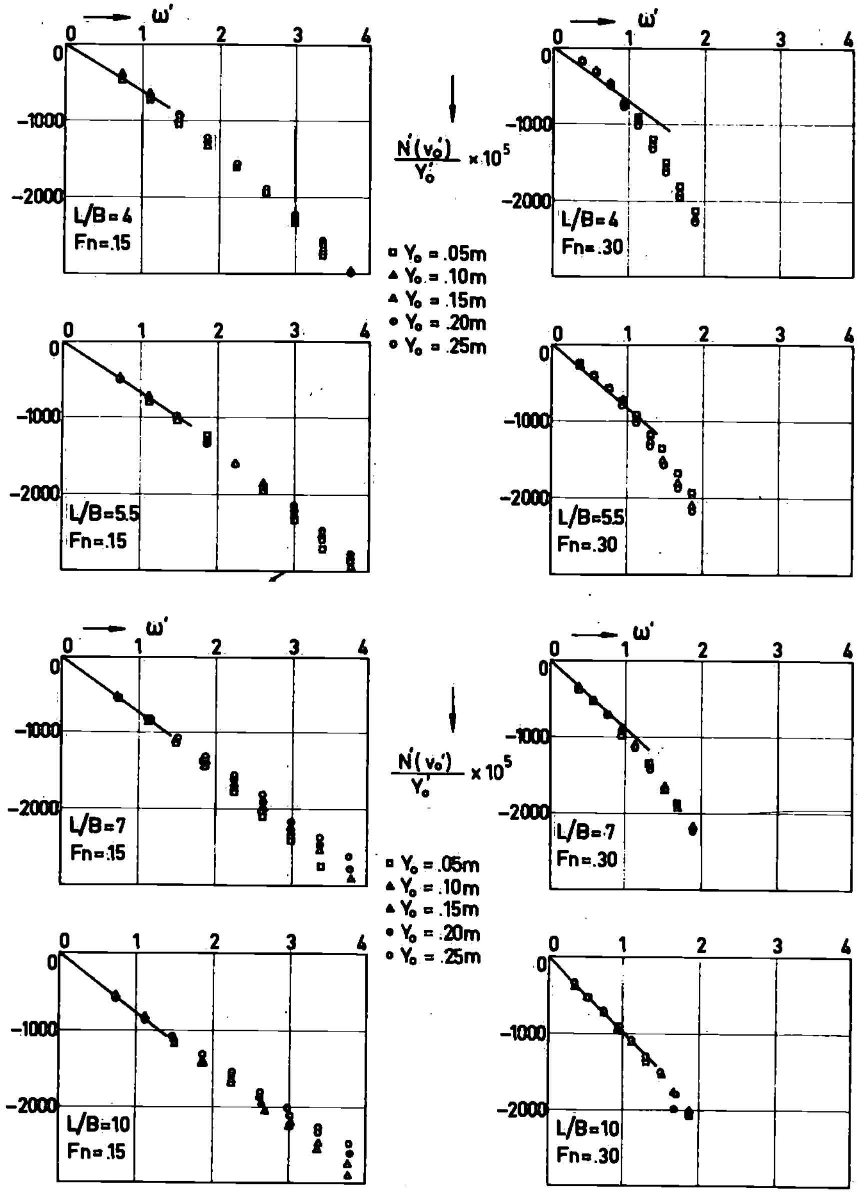


Fig. 16. to be continued.

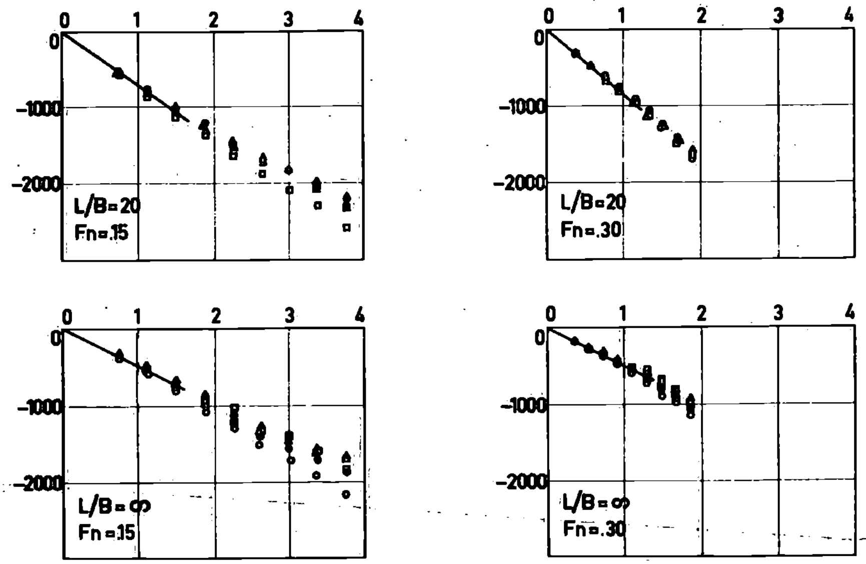


Fig. 16. Sway moment for two Froude numbers as a function of the L/B ratio.

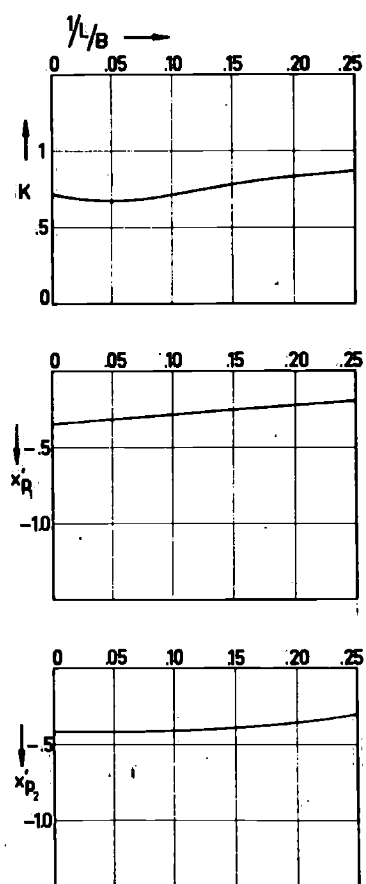


Fig. 17. Empirical coefficients derived from the experiments.

Appendix 1

The equations of motion of heave and pitch

The equations of motion of heave and pitch and their solution are given by:

$$(\rho\nabla + a)\ddot{z} + b\dot{z} + cz - d\ddot{\theta} - c\dot{\theta} - g\theta = F \quad \text{(heave)}$$

$$(I_{yy} + A)\ddot{\theta} + B\dot{\theta} + C\theta - D\ddot{z} - E\dot{z} - Gz = M \quad \text{(pitch)}$$

$$z = z_a \cos(\omega_e t + \varepsilon_{z\zeta}),$$
$$\theta = \theta_a \cos(\omega_e t + \varepsilon_{\theta\zeta}) \qquad (1)$$

The various coefficients $a-g$ and $A-G$ are derived from:

$$\rho\nabla\ddot{z} = \int_L F' dx_b$$

$$I_{yy}\ddot{\theta} = -\int_L F' x_b dx_b \qquad (2)$$

where F' is the hydromechanical force acting on a cross-section of the ship.
It can be found that:

$$F' = -2\rho g y_w (z - x_b\theta) - \left(\frac{\partial}{\partial t} - V\frac{\partial}{\partial x}\right) \times$$
$$(\dot{z} - x_b\dot{\theta} + V\theta - \dot{\zeta}^*)\left(m' - \frac{iN'}{\omega_e}\right) \qquad (3)$$

The effective wave elevation ζ^* is defined as:

$$\zeta^* = \zeta e^{-kT^*}$$

$$T^* = -\frac{1}{k}\ln\left(1 - \frac{k}{y_w}\int_{-T}^{0} y_w e^{kz_b} dz_b\right) \qquad (4)$$

This expression follows from the integration of the vertical component of the undisturbed incident wave pressure on a cross section contour. The time derivatives of ζ^* are used in the calculation of the damping and added mass correction to the 'Froude-Kriloff' wave force and moment.
Because harmonic motions only are considered, equation (3) can be written as:

$$F' = -2\rho g y_w (z - x_b\theta - \zeta^*) +$$
$$- m'(\ddot{z} - x_b\ddot{\theta} + 2V\dot{\theta} - \ddot{\zeta}^*) +$$
$$+ V\frac{dm'}{dx_b}(\dot{z} - x_b\dot{\theta} + V\theta - \dot{\zeta}^*) +$$
$$- N'\left(\dot{z} - x_b\dot{\theta} + 2V\theta - \frac{\omega}{\omega_e}\dot{\zeta}^*\right) +$$
$$+ V\frac{dN'}{dx_b}\left(z - x_b\theta - \frac{V\dot{\theta}}{\omega_e^2} - \frac{\omega_a}{\omega_e}\zeta^*\right) \qquad (5)$$

Combining equations (2) and (5) one finds:

$$a = \int_L m' dx_b + \left[\frac{V}{\omega_e^2}\int \frac{dN'}{dx_b} dx_b\right]$$

$$b = \int_L \left(N' - V\frac{dm'}{dx_b}\right) dx_b$$

$$c = 2\rho g \int_L y_w dx_b$$

$$d = \int_L m' x_b dx_b + 2\frac{V}{\omega_e^2}\int_L N' dx_b +$$
$$- \frac{V^2}{\omega_e^2}\int_L \frac{dm'}{dx_b} dx_b + \left[\frac{V}{\omega_e^2}\int_L \frac{dN'}{dx_b} x_b dx_b\right]$$

$$e = \int_L N' x_b dx_b - 2V\int_L m' dx_b +$$
$$- V\int_L \frac{dm'}{dx_b} x_b dx_b - \left[\frac{V^2}{\omega_e^2}\int_L \frac{dN'}{dx_b} dx_b\right]$$

$$g = 2\rho g \int_L y_w x_b dx_b \qquad (6a)$$

$$A = \int_L m' x_b^2 dx_b + 2\frac{V}{\omega_e^2}\int_L N' x_b dx_b +$$
$$- \frac{V^2}{\omega_e^2}\int_L \frac{dm'}{dx_b} x_b dx_b +$$
$$+ \left[\frac{V}{\omega_e^2}\int_L \frac{dN'}{dx_b} x_b^2 dx_b\right]$$

$$B = \int_L N' x_b^2 dx_b - 2V\int_L m' x_b dx_b +$$
$$- V\int_L \frac{dm'}{dx_b} x_b^2 dx_b +$$
$$- \left[\frac{V^2}{\omega_e^2}\int_L \frac{dN'}{dx_b} x_b dx_b\right]$$

$$C = 2\rho g \int_L y_w x_b^2 dx_b$$

$$D = \int_L m' x_b dx_b + \left[\frac{V}{\omega_e^2}\int_L \frac{dN'}{dx_b} x_b dx_b\right]$$

$$E = \int_L N' x_b dx_b - V\int_L \frac{dm'}{dx_b} x_b dx_b$$

$$G = 2\rho g \int_L y_w x_b dx_b \tag{6b}$$

If $F = F_a \cos(\omega_e t + \varepsilon_{F\zeta})$ and $M = M_a \cos(\omega_e t + \varepsilon_{M\zeta})$ then:

$$\frac{F_a}{\zeta_a}\begin{matrix}\cos\\ \sin\end{matrix}\,\varepsilon_{F\zeta} = 2\rho g \int_L y_w e^{-kT*}\begin{matrix}\cos\\ \sin\end{matrix}\, kx_b dx_b +$$

$$\mp\omega \int_L \left(\frac{\omega}{\omega_e} N' - V\frac{dm'}{dx_b}\right) e^{-kT*}\begin{matrix}\sin\\ \cos\end{matrix}\, kx_b dx_b +$$

$$-\omega^2 \int_L \left(m' + \left[\frac{V}{\omega\omega_e}\frac{dN'}{dx_b}\right]\right) e^{-kT*}\begin{matrix}\cos\\ \sin\end{matrix}\, kx_b dx_b \tag{7a}$$

$$\frac{M_a}{\zeta_a}\begin{matrix}\cos\\ \sin\end{matrix}\,\varepsilon_{M\zeta} =$$

$$= -2\rho g \int_L y_w x_b e^{-kT*}\begin{matrix}\cos\\ \sin\end{matrix}\, kx_b dx_b +$$

$$\pm\omega \int_L \left(\frac{\omega}{\omega_e} N' - V\frac{dm'}{dx_b}\right) x_b e^{-kT*}\begin{matrix}\sin\\ \cos\end{matrix}\, kx_b dx_b$$

$$+\omega^2 \int_L \left(m' + \left[\frac{V}{\omega\omega_e}\frac{dN'}{dx_b}\right]\right) \times$$

$$\times x_b e^{-kT*}\begin{matrix}\cos\\ \sin\end{matrix}\, kx_b dx_b \tag{7b}$$

For ships where N' and m' are zero at the stem and stern the expressions (6) and (7) can be simplified, but this has not been carried through in the corresponding computer-program.
When the terms between the brackets are omitted out from equations (6) and (7), and when $\omega/\omega_e = 1$ in the coefficients of N' in (7), the resulting equations of motion are equal to those derived by the modified Korvin-Kroukovsky and Jacobs' results[9].

Appendix 2

The added resistance in waves
The added resistance of a ship in waves is a result of the radiated damping waves created by the motions of the ship relative to the water. Joosen[20] showed that for the mean added resistance it is possible to write:

$$R_{AW} = \frac{\omega_e^3}{2g}(bz_a^2 + B\theta_a^2) \tag{8}$$

This expression was derived by expanding Maruo's expression[21] into an asymptotic series with respect to a slenderness parameter and taking into account only first order terms. His simplified treatment results in an added resistance which is independent of the forward speed. This latter fact is roughly confirmed by experiments[10].
Equation (8) is equivalent to Havelock's equation[22]. Although not consistent with the theory. the frequency of encounter is used by Joosen i (8) when a ship with forward speed is considered. Uncoupled motions are considered in equation (8). In the present work, the following procedure is adopted for the calculation of the radiated damping energy P of the oscillating ship during one period of encounter:

$$P = \int_L \int_0^{T_e} b' V_z^2 dt dx_b \tag{9}$$

where $b' = N' - V(dm'/dx_b)$, the sectional damping coefficient for a ship at speed and:

$$V_z = \dot{z} - x_b\dot{\theta} + V\theta - \dot{\zeta}^*,$$

the vertical relative water velocity at a cross section of the ship. As V_z is a harmonic function with amplitude V_{za} and a frequency equal to the frequency of encounter ω_e we find:

$$P = \frac{\pi}{\omega_e}\int_L b' V_{za}^2 dx_b \tag{10}$$

Following the reasoning given by Maruo[21] the work being done by the towing force R_{AW} is given by:

$$P = R_{AW}(V + c)T_e = R_{AW}\cdot\lambda \tag{11}$$

From (10) and (11) it follows that:

$$R_{AW} = \frac{k}{2\omega_e}\int_L b' V_{za}^2 dx_b \tag{12}$$

This expression is almost equal to (8) when the wave elevation is small compared with the vertical motions of the ship, in addition to the ship having a very low forward speed and fore and aft symmetry.

Appendix 3

The equations of motion of yaw and sway

The following account is mainly based upon work by Jacobs[14, 15]

The equations of motion for the bare hull condition are given by:

$$M'(\dot{v}'+r') = Y'_{\dot{v}}\dot{v}' + Y'_{v}v' + Y'_{\dot{r}}\dot{r}' + Y'_{r}r' \quad (sway)$$

$$I'_{zz}\dot{r}' = N'_{\dot{v}}\dot{v}' + N'_{v}v' + N'_{\dot{r}}\dot{r}' + N'_{r}r'. \quad (yaw) \tag{13}$$

The hydrodynamic coefficients in (13) can be calculated by assuming a division between an inertia force distribution and a viscous force distribution along the ship's hull. The distribution of the hydrodynamic inertia forces can be found by well-known methods in hydrodynamics of which brief accounts can be found. For example in[19, 23]. Confining ourselves to horizontal motions at a constant forward velocity in an ideal fluid the following expressions for the right-hand sides of (13) are derived:

$$Y'_{id} = Y'_{\dot{v}}\dot{v}' + X'_{\dot{u}}r' + Y'_{\dot{r}}\dot{r}'$$

$$N'_{id} = N'_{\dot{r}}\dot{r}' + (Y'_{\dot{v}} - X'_{\dot{u}})v' + Y'_{\dot{r}}(\dot{v}' + r') \tag{14}$$

The coefficients appearing in (14) are calculated by the following expressions, assuming that the strip method is applicable together with Lamb's correction coefficients of accession:

$$Y'_{\dot{v}} = -\frac{\pi K_2 T^2}{L^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} C_s dx'$$

$$N'_{\dot{v}} = -\frac{\pi K_2 T^2}{L^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} C_s x' dx'$$

$$Y'_{\dot{r}} = -\frac{\pi K' T^2}{L^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} C_s x' dx' = \frac{K'}{K_2}N'_{\dot{v}}$$

$$N'_{\dot{r}} = -\frac{\pi K' T^2}{L^2}\int_{-\frac{1}{2}}^{\frac{1}{2}} C_s x'^2 dx'$$

$$X'_{\dot{u}} = K_1 M'. \tag{15}$$

From (14) it is obvious, that for the damping coefficients the following expressions exist in an inviscid fluid:

$$Y'_{v\,id} = 0$$

$$Y'_{r\,id} = X'_{\dot{u}}$$

$$N'_{v\,id} = Y'_{\dot{v}} - X'_{\dot{u}}$$

$$N'_{r\,id} = Y'_{\dot{r}}. \tag{16}$$

A ship-shaped low aspect ratio wing in a real fluid develops a circulation around the profile which generates a lift owing to the viscosity. This lift can be approximated for moderate speeds by the corrected Jones' low aspect ratio formula, taking account of the action of the water surface by doubling the draught. This formula can also be considered as the integral of the viscous force distribution along the hull. The first and second moments of this distribution yield the remaining damping derivatives:

$$Y'_{v\,visc} = -K\pi\frac{2T}{L}\frac{T}{L}$$

$$N'_{v\,visc} = Y'_{r\,visc} = -x'_{p1}2\pi\frac{KT^2}{L^2}$$

$$N'_{r\,visc} = -x'^2_{p2}2\pi\frac{KT^2}{L^2}. \tag{17}$$

Numerical values of the empirical constants K_1, x'_{p1} and x'_{p2} are displayed in Fig. 17.

Combining equations (16, 17) the total damping coefficients can be listed as follows, assuming that mutual interference between inertia and viscous forces can be neglected:

$$Y'_v = -2K\pi\frac{T^2}{L^2}$$

$$N'_v = Y'_{\dot{v}} - X'_{\dot{u}} - x'_{p1}2K\pi\frac{T^2}{L^2}$$

$$Y'_r = X'_{\dot{u}} - x'_{p1}2K\pi\frac{T^2}{L^2}$$

$$N'_r = -x'^2_{p2}2K\pi\frac{T^2}{L^2}. \tag{18}$$

For the purpose of comparing the results of the experimental coefficients with some existing formulae relating to damping coefficients, the following expressions derived from Inoue[16] are appropriate for the even keel condition:

$$Y'_v = -2K\pi\frac{T^2}{L^2}$$

$$N'_v = -2\frac{T^2}{L^2}$$

$$Y'_r = 2K\pi\frac{T^2}{L^2}\left(.367 + .42\frac{T}{L}\right)$$

$$N'_r = -1.08\frac{T^2}{L^2}. \tag{19}$$

Norrbin[19] published data on the damping derivatives. His results are given in the form of regression formulae in his nondimensional so called 'bis' system. In the nomenclature adopted in this paper the expressions are preceded by the corresponding formulae in the 'bis' system.

$$Y''_{uv} = -1.69\frac{\pi}{2}\frac{LT^2}{\nabla} - 0.04;$$

$$Y'_{v} = -1.69\pi\frac{T^2}{L^2} - 0.08\frac{B}{L}\frac{C_BT}{L}$$

$$N''_{uv} = -1.28\frac{\pi}{4}\frac{LT^2}{\nabla} + 0.02;$$

$$N'_{v} = -1.28\frac{\pi}{2}\frac{T^2}{L^2} + 0.04\frac{B}{L}\frac{C_BT}{L}$$

$$Y''_{ur} = 1.29\frac{\pi}{4}\frac{LT^2}{\nabla} - 0.18;$$

$$Y'_{r} = 1.29\frac{\pi}{2}\frac{T^2}{L^2} - 0.36\frac{B}{L}\frac{C_BT}{L}$$

$$N''_{ur} = -1.88\frac{\pi}{8}\frac{LT^2}{\nabla} + 0.09;$$

$$N'_{r} = -1.88\frac{\pi}{4}\frac{T^2}{L^2} + 0.18\frac{B}{L}\frac{C_BT}{L}. \qquad (20)$$

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