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Detecting a risk mismatch between actual investment portfolio and its strategic asset allocation

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"Detecting a risk mismatch between actual investment portfolio and its strategic asset allocation"

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Preface

This thesis is the "pièce de résistance" of my Master in Applied Mathematics at TU Delft. Nine months ago I started this thesis project at the Private Wealth Management department of Ortec Finance, a company that amongst other things is a supplier of risk management software for private wealth firms. The existing software helps the portfolio manager providing investment advice based on the risk tolerance of the client. However, due to all kinds of factors, the risk tolerance and the risk of the actual investment portfolio may in time drift apart. The subject of this thesis is to construct or find a methodology that can detect this drift.

During this thesis project I have learned a lot, both about the theory and the practice of risk management. On that account, I am very grateful to Ortec Finance and specifically Bert Kramer and Ronald Janssen who have guided me through this process. Also I would like to thank Lennart Wijnand for helping to harvest the enormous amount of data. Then of course I would like to thank Kees Oosterlee for the supervision on behalf of TU Delft and Robbert Fokkink and Frank van der Meulen for joining the thesis committee.

Finally I would like to thank my family and friends for their support and ideas, sometimes very practical hints can solve a big problem.

Zoë Lagerweij
December 10, 2012

Contents

Introduction

When an investor wants to invest his funds, he needs to construct a portfolio and manage the portfolio or let it be constructed and managed by a portfolio manager. Let us first, before continuing, introduce and define the process of portfolio construction as will be used throughout this thesis report. After defining the portfolio construction process, different strategies for portfolio management will be introduced shortly and finally, the goal of this thesis project will be introduced. The portfolio manager considered in this thesis, is a private wealth portfolio manager, which means that the managed portfolios are owned by private individuals who own more than €500.000.

Portfolio Construction

The process of portfolio construction is in this report defined as follows: An investor goes to a portfolio manager to invest his funds. When the investor is investing his money he will take on risk, but the amount of risk that is taken depends on the securities that are contained in his portfolio. Therefore the risk profile of the investor should be determined. The risk profile of the investor is determined by the risk the investor is willing to take, often identified by an inquiry, and the risk the investor is able to take. The risk profile should be identified by the portfolio manager. The risk profile can range from for example very defensive, which implies very low risk, to very aggressive, which implies a very high risk.

Once the portfolio manager knows the risk profile of the investor, the can put together a *Strategic Asset Allocation* (SAA). This SAA is a specific (theoretic) diversification of funds to asset classes such that the risk of the SAA matches the risk profile. Examples of an asset class are shares, bonds, commodities and cash. An example of a SAA is shown in table 1. To illustrate the risk of an SAA: the risk of having cash is low, therefore a low risk SAA might contain a lot of cash. The risk of investing in equity is higher, so a risky SAA might contain a lot of equity.

Asset class	**percentage**
Equity	80%
Govt. Bonds	5%
Corp. Bonds	5%
Cash	10%

Table 1: Strategic asset allocation.

The risk of investing an asset class can be measured by using a benchmark, this is a predefined norm, that shows behaviour that is assumed to be representative for the asset class. Examples of a benchmark are the AEX index, Dow Jones index and the S&P 500.

To complete the portfolio construction, the portfolio manager will choose securities from the asset classes to the portfolio according to the proportions of the SAA.
The process of creating an investment portfolio is illustrated by figure 1.

Figure 1: Process of creating a investment portfolio.

Portfolio Management

After the actual portfolio is constructed, the portfolio should be managed. There are several strategies to manage the portfolio: the first strategy is called the 'buy-and-hold' strategy, which means that the portfolio will not be adjusted after construction. This is a passive management strategy.

A second strategy is the opposite of the buy-and-hold strategy and is called active management. It means that after the portfolio construction, the portfolio manager actively adjusts the portfolio such that the returns will be higher due to right timing and selection.

A third strategy is called rebalance-to-risk, which means that the portfolio manager monitors the portfolio and when the risk of the portfolio does not match the risk profile of the investor anymore, the portfolio manager will adjust the portfolio such that the portfolio will match the risk profile again. Another rebalance strategy is to rebalance-to-plan: the SAA allocates a specific weight to each asset class, however, due to changes in the market, it is possible that after a while the weights of the portfolio do not match the weights if the SAA anymore, the strategy rebalance to weights is to evaluate the weights of the portfolio after a predetermined period and rebalance the portfolio according to the weights of the SAA.

Goal

Because securities do not follow the exact same 'path' as the benchmark, and the portfolio manager determines what securities within an asset class are chosen, the risk of the actual portfolio might deviate from the risk of the SAA. As long as this deviation falls within the boundaries of the risk profile, there is no problem or reason to interfere. However, when the risk does not fall within the boundaries anymore, there is a risk mismatch between the actual portfolio and the SAA.

The goal of this thesis project is to find or construct a methodology to identify the mismatch between portfolio risk and the SAA risk, identify the causes of this mismatch and investigate the possibilities to create a computerized monitoring system to prevent future SAA mismatch. This process will ensure that an investor is not exposed to risk that he is not able or willing to take and still be able to reach his investment objectives.

Structure

In order to reach the goals that were formulated, the thesis project will consist of the following three parts:

(i) theoretic analysis

(ii) practical application

(iii) Implementation, Results and Conclusion

In the theoretic part, first part of the problem will be analyzed. Main questions to be answered are: 'What is risk?', 'What is Strategic Asset Allocation?', 'How should risk be measured?' (i.e. ex-post versus ex-ante and VaR vs. cVaR vs. standard deviation vs. ...), 'What methodology is already available?' and 'What statistical tests can be used to detect a deviation?'.

After answering these questions a clear methodology of identifying risk mismatch is formulated in different steps in the practical application.

In the final part, the results from the practical application will be implemented and evaluated, the analysis will be applied to investigate the possibilities of implementing a risk mismatch warning system.

Part I

Theoretic Analysis

Chapter 1

Risk and the strategic asset allocation

In this introductory chapter two basic notions will be introduced: risk and the strategic asset allocation. In the first section the concept of risk is explained and in the second section the strategic asset allocation is (further) introduced and some widely used basic models are introduced.

1.1 Risk

Before we can dive into the subject of risk management, we should take a look at the meaning of risk and the different types of risk. Risk is the potential of a negative side-effect or result of some event. This event could be anything: painting a house, skydiving, going for a walk in the park, investing in the company of your sister or buying some stocks. The negative effects of these events can differ per person. For example when someone, let him call John, decides to go for a walk in the park, but he knows that his annoying neighbour, who he really hates to talk to, is home and there is a possibility that he runs into him when going to the park, he is taking a risk. This example illustrates that it is not easy to quantify risk. John just does not like talking to his neighbour and his mood might worsen, but no actual value can be appointed to this risk. Another risk of going outside is that while walking to the park, John might be run over by a bus, since the probability of being run over by a bus is very small, even smaller when John is careful, he will face this risk, however, since it is much more likely that John runs into his neighbour, he might decide to stay in.

The example above illustrates that both the probability of occurrence of the side-effect and the impact of the side effect plays an important role in the assessment of the willingness to take the risk. Since everyone values risk in a different way, it is difficult to assign an objective measure to it and so it is not easily quantified.

Because risk is a general concept different types of risk are specified, examples are health risk, safety risk and financial risk. In this thesis financial risk is considered. When talking of financial risk, is is easier to give a value to the risk. In [Bowman, 1980], Edward Bowman introduces (financial) risk as follows: “Risk is the concept which captures the uncertainty (...) associated with the outcome of resource commitments.” Formulated differently, financial risk is the risk of a financial loss as a result of some financial investment, this loss can be quantified by money. In chapter 2 different measures for risk are mentioned and explained.

Within financial risk there are also many different types of financial risk. A financial institution mainly faces the following risks [McNeil et al., 2005]:

Systemic risk is the risk that the entire financial system collapses. Cause of this collapse are the interdependency of banks. If a healthy bank has lend to an unhealthy bank, and this bank collapses, the healthy bank might not be able to absorb this shock and as a result go into default.

Default risk is the risk that a borrower of money cannot fulfill his repayment obligation, it is also called credit risk. This borrower could be an individual, but also a firm, country or a bank.

Market risk is also called systematic risk and should not be confused with systemic risk. Market risk is the risk that the value of a portfolio will decrease due to changes in the market. Market risk can be subdivided into different risk factors: equity risk, interest rate risk, currency (or exchange rate) risk and commodity risk. Market risk contains the risk that stock prices, commodity prices, currencies and interest rates or their implied volatilities change.

Operational risk encompasses the risks of operating a firm. This includes the possibility of a shortcoming in the performance of the personnel, systems and processes. Jérôme Kerviel is an example of an employee that caused damage. He was a banker at Société Générale and performed illicit transactions in futures, that caused a loss of €4,9 billion [NRC Handelsblad, 2008]. Also the risk of failure in systems, a blackout or earthquakes are operational risk.

Liquidity risk is related to the speed of the finalization of transactions. When a transaction is not completed in time to make a profit, the transaction might even result in a loss. This risk is called liquidity risk.

Reputational risk is related to the degree of trustworthiness of a bank. When a bank is blamed for a certain negative event, clients might, as a result, withdraw their funds resulting in a downward spiral.

Not all of the types of risk listed are relevant for this thesis. A portfolio manager cannot influence, for example, systemic, operational and liquidity risk. The goal of the thesis is to detect a mismatch in risk of a portfolio and its strategic asset allocation. The risk of a portfolio, which is a collection of single securities, is mainly affected by the conditions of the market and the main task of the portfolio manager is therefore to ensure that the market risk of the portfolio matches with the risk profile of the owner of the portfolio. So, the risk considered in this thesis is mainly market risk.

Finally, another type of risk that finance-related firms are facing is model risk. Model risk is risk on a different level than the types of risks listed above, but certainly worth mentioning. Model risk is the risk that models that are used for trading strategies or risk management are incorrect. This risk factor will always exist since it is not possible to design a model which can exactly reflect reality. If this would be the case, a market would not exist.

1.2 Strategic asset allocation

Strategic Asset Allocation (SAA) is the long term optimal portfolio allocation to different asset classes with respect to the risk profile of an investor. The SAA is the result of a risk-return trade-off and is adjusted to the risk appetite or risk profile of the investor. The risk profiles range from extremely defensive, which implies a low risk tolerance, to extremely aggressive, which implies a high risk tolerance. Examples of asset classes are commodities, cash, equities and bonds. Other factors that may be taken into account in the SAA are possible (periodical) withdrawals and deposits. An example of a strategic asset allocation was given in table 1 in the introduction.

To determine the optimal choice of allocation of funds to different asset classes (e.g. stocks, bonds, cash) portfolio managers use mathematical models. Many different models are available, but in this chapter we will consider Markowitz's modern portfolio theory (MPT) the capital asset pricing model (CAPM) and arbitrage pricing theory (APT).

Modern portfolio theory

Markowitz introduced in the 1950s Modern Portfolio Theory (MPT). Markowitz wrote in 1952, that at that moment the common way of creating and managing a portfolio was to "maximize the discounted (or capitalized) value of future returns" [Markowitz, 1952]. As a result, diversified portfolios would never be preferred over a non-diversified portfolio (i.e. a portfolio containing only one security). In the same article Markowitz introduced a new framework: mean-variance optimization.

The framework of mean-variance optimization was defined in [Markowitz, 1952] as follows: Let R_i be a normally distributed random variable, which represents the return of the i^{th} security, $E[R_i] = \mu_i$ is the expected return of security i, σ_{ij} is the covariance between R_i and R_j and ω_i portfolio weight of security i with $\sum \omega_i = 1$. Now the return of the portfolio containing n securities is given by:

$$R = \sum_{i=1}^{n} \omega_i R_i. \tag{1.1}$$

The weights ω_i can be determined by the portfolio manager, although the condition $\omega_i \geq 0$ should hold.
Now, the expected return and variance of return, which is here considered as the measure for risk, are given by:

$$E := E[R] = \sum_{i=1}^{n} \omega_i \mu_i \qquad V := \text{Var}(R) = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \sigma_{ij} \tag{1.2}$$

The feasible combinations of the ω_i's (i.e. $\omega_i \geq 0$ for all $i \in \{1, ..., n\}$ and $\sum_{i=1}^{n} \omega_i = 1$ are satisfied), which are all possible portfolios, give a range of all possible values for E and V. For these values, Markowitz introduced the mean-variance rule. The rule states that "the investor would (or should) want to select one of those portfolios [...] with minimum V for given E or more and maximum E for given V or less" [Markowitz, 1952], in other words: the portfolio which has minimum risk for given return or more, or maximum return for given risk

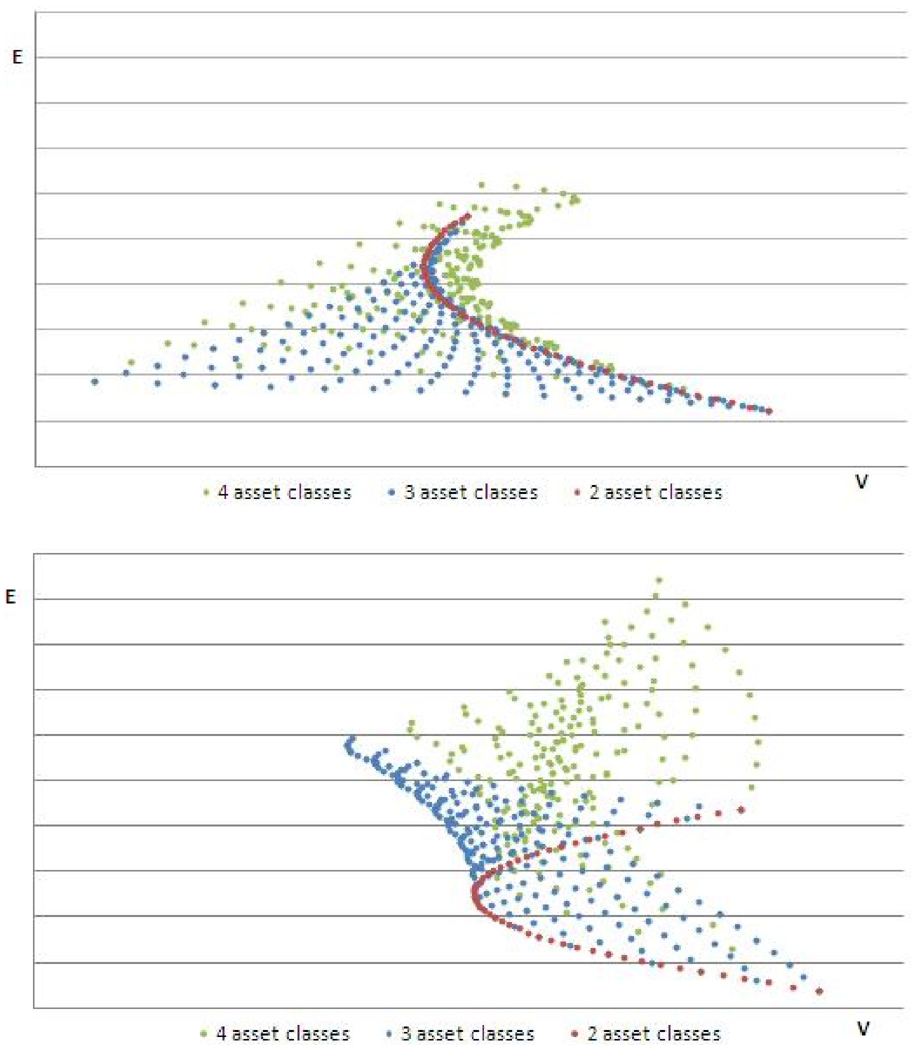


Figure 1.1: Expected return and variance of return of possible portfolios for two (red), three (blue) and four (green) asset classes.

or less should be chosen. The portfolios that satisfy this rule are called efficient portfolios. When the mean and variance of these portfolios are plotted in a graph with the variance of return and expected return on the axes, the efficient portfolios form what is called an efficient frontier.

A practical example of mean-variance analysis is shown in figure 1.1. In the two graphs in figure 1.1, every dot represents a portfolio with specific weights assigned to the assets. The red dots represent portfolios containing two assets, the blue dots represent three-asset portfolios and the green dots represent four-asset portfolios.
The two different graphs represent a different set of asset returns, but within the graphs the returns remain constant and the position is solely determined by the weights assigned to the different assets. The horizontal position denotes the variance of the portfolio, the vertical position denotes the expected value of the portfolio.
The efficient frontier of for example the two-asset portfolio in the lower graph in figure 1.1 is given by the upper half of the red arc, since for all portfolios the values of the variance of return, the maximum expected return is higher on the upper arc, than on the lower arc.

The implications of MPT are that a higher return goes hand in hand with a higher risk. Although this model is an obvious improvement of the in that time existing theories, it also has its flaws. One important limitation is the (exclusive) use of standard deviation as a risk measure.

The Capital Asset Pricing Model (CAPM) is built on the ideas of Markowitz described above. Among others, Sharpe and Treynor, some of their efforts are mentioned in section 2.4, have independently introduced the CAPM [Sharpe, 1966],[French, 2002]. As its name suggests, the CAPM is an asset pricing model.
The model is defined as follows:

$$
\begin{aligned}
(1.3) \qquad && E[R_i] &= R_f + \beta_i \left(E[R_m] - R_f\right), \\
& \text{where} & R_i &= \text{required return on asset } i; \\
&& R_f &= \text{risk-free return;} \\
&& R_m &= \text{return of the market;} \\
&& \beta_i &= \text{measure of risk, see equation (2.36) in section 2.4.}
\end{aligned}
$$

The model gives a required return that an asset should have, taking the risk β into account, to perform –in terms of return– as well as the market does.

As mentioned earlier, β gives the volatility of an asset or portfolio compared to the volatility of the market, so when an asset has $\beta = 1$, the asset is expected to behave as the market, and so the asset should fulfill the equation $E[R_i] = E[R_m]$.
However, when the $\beta = 2$, the return of the market is 10% and the risk-free return is 2.5%, to compensate for the excess risk taken, the required return of the asset should be $E[R_i] = 2.5\% + 2 \cdot (10\% - 2.5\%) = 17.5\%$. After the required return is calculated, it can be compared with the estimated rate of return to determine whether the asset is worth investing in.

Arbitrage pricing theory

The CAPM is a so-called single-factor model. Factor models are asset pricing models where the price of the asset depends on different factors. In the case of the CAPM it is one factor. The factor is the excess return on the market, also called the market factor. Compared to a single-factor model like the CAPM, multi-factor models take multiple factors into account. The arbitrage price theory (APT), introduced by Ross in 1976 [Ross, 1976], induces that a multi-factor model determines the right rate of return. If the asset price deviates from the expected end of period price discounted at the rate implied by the model, the asset is over or under priced. When this happens arbitrageurs can make a riskless profit and by their interventions the asset price will return to the right price.

Multi-factor models have the following form:

$$R_i - R_f = \alpha_i + \sum_{j=1}^{n} \beta_{ij} F_j + \varepsilon_i, \qquad (1.4)$$

where α_i is a constant, β_{ij} is the sensitivity of factor j to asset i, F_j is factor j and ε_i is asset i's individual shock with $E[\varepsilon_i] = 0$. These factors should affect expected returns, and should not be captured by the market factor. Examples of factors are risk factors and macroeconomic factors like unemployment and GDP growth.

The advantage of the APT compared to the CAPM is that APT has less restrictive assumptions.

Chapter 2

Risk measures

In the previous chapter, the concept of risk was introduced. In this chapter several measures to quantify risk will be introduced. The goal of this chapter is to find a measure, or a combination of measures which to be able to assign a number to the risk level of a portfolio. In order to find this measure or these measures, first the difference between ex-post and ex-ante measures is explained, then several risk measures and their properties will be discussed, and finally a framework to determine the 'quality' of the measures will be introduced.

Before getting started, it should be established which variable (e.g. profit, loss, return) is used to denote the value of a portfolio or security. The risk of a security or portfolio is based on the values of this variable.
The reason for this is that when, for example, the variable considered denotes loss, a big, positive number is not good, as a negative number denotes profit, on the other hand when talking about a return, a positive number denotes profit and a negative number is a loss. Therefore, to be consistent and clear, throughout this thesis risk will be determined using a return distribution, which means that negative values are losses.

Let us define variables concerning portfolios and returns. When a portfolio contains n securities, and at time $t = 0$, an amount of $X_{0,i}$ euro (or dollars, Swiss francs, etc.) is invested in asset i, then the total return of security i at time t is

$$R_{t,i} = \frac{X_{t,i}}{X_{t-1,i}}, \tag{2.1}$$

which is the payoff of the security per euro invested. The rate of return is then defined as: $r_i = \frac{X_{1i}-X_{0i}}{X_{0i}}$. So, the following will hold: $X_{1i} = (1 + r_i)X_{0i}$. The log return of a security i at time t is defined as:

$$\text{LR}_{ti} = \ln(X_{t,i}) - \ln(X_{t-1,i}) = \ln\left(\frac{X_{t,i}}{X_{t-1,i}}\right). \tag{2.2}$$

The log return is often used to analyze financial data because it takes the ratio of two data points into account, instead of absolute difference. Therefore, data series of, for example, different currencies, can easily be analyzed without having to exchange one of the series to the other currency.

The amount invested in the portfolio at time t is $X_t = X_{t,1} + X_{t,2} + \cdots + X_{t,n}$. The portfolio weight, the percentage of the portfolio invested in security i, is defined as:

$$\omega_{t,i} = \frac{X_{t,i}}{X_t} \qquad \text{and} \qquad \sum_{i=1}^{n} \omega_{t,i} = 1. \tag{2.3}$$

2.1 Ex-ante vs. ex-post

The risk of a portfolio or security can be assessed both in advance and by looking back. Risk which is assessed by looking back is called ex-post risk assessment and it means that risk is measured based on historical data of a portfolio and assumes that the past is representative for future results. When risk is assessed beforehand, which is called ex-ante risk assessment, the risk is assessed by looking forward by using for example scenario analysis.

Scenario analysis is a kind of Monte Carlo simulation. Based on several economic variables such as inflation, employment, etc. a thousand economic scenarios are created. The idea is that these scenarios represent all possible paths and outcomes of, for example, a benchmark or a portfolio. Based on these scenarios, some measures which are discussed below, can be used as an ex-ante measure. The variance (and standard deviation, semi-standard deviation and semivariance), is one of these measures. In section 2.3 a Monte Carlo method for calculating the Value-at-Risk (VaR) is described, so the VaR can also be seen as an ex-ante risk measure, the Conditional value-at-risk (CVaR) can also be calculated using scenarios.

All measures that are listed in this chapter can both be used as an ex-post and an ex-ante measure. To calculate the ex-post measures, only historical data are needed to compute the risk measures, however, one has to keep in mind that ex-post measures do not necessarily provide a good insight into the future. When using ex-ante measures, one has to keep in mind that when the risk of single securities in portfolios is measured, every single security should be modeled on its own, which could be a lot of work.

2.2 Standard deviation & variance

The first measure that is introduced is very basic, very common and important in probability theory: the standard deviation. The standard deviation indicates how far the elements of a dataset lie from the mean of the dataset. Directly related to the standard deviation is variance. The variance of a dataset is the squared standard deviation and is sometimes also called volatility. The variance and standard deviation of random variable X are defined as:

$$\begin{aligned} \mathrm{Var}(X) = \sigma^2 &= E\left[(X - E[X])^2\right] \qquad (2.4) \\ &= E[X^2] - E[X]^2 \\ \sigma &= \sqrt{E[X^2] - E[X]^2} \qquad (2.5) \end{aligned}$$

When an $(n \times 1)$ vector $x_1, x_2, ..., x_n$ is a sample of random variable X, the sample variance and sample standard deviation are represented by:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2 \tag{2.6}$$

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2}. \tag{2.7}$$

The expected rate of return of a portfolio as defined above is given by: $E[r] = \sum_{i=1}^{n} \omega_i \, E[r_i]$. Now, the variance of the rate of return is:

$$\begin{aligned} \sigma^2 = \mathrm{Var}(r) &= \mathrm{Var}(\omega_1 r_1 + \omega_2 r_2 + ... + \omega_n r_n) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \mathrm{Cov}(r_i, r_j) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \left(E[r_i r_j] - E[r_i] E[r_j] \right) \end{aligned} \tag{2.8}$$

The spread of the return can be seen as a measure for risk, when the spread is concentrated around the expected return, it is more likely that the return will not deviate much from the expected return, which can be interpreted as low risk. When the spread is not at all concentrated around the expected return, chances are high that the return deviates significantly from the expected return, so the risk can be considered as high.

Equation (2.8) also shows that the variance of a portfolio can decrease, when the portfolio is well diversified. Take for example a portfolio A containing two securities, with equal weights (i.e. $\omega_1 = \omega_2 = \frac{1}{2}$) and let us assume that $\mathrm{Var}(r_1) = \mathrm{Var}(r_2) = C > 0$ and that the securities are independent, so $\mathrm{Cov}(r_1, r_2) = 0$, and on the other hand a portfolio B containing only one security, security 1. Now, we have $\mathrm{Var}(r^A) = \omega_1^2 \, \mathrm{Var}(r_1) + \omega_2^2 \, \mathrm{Var}(r_2) + 2 \, \omega_1 \omega_2 \, \mathrm{Cov}(r_1, r_2) = \frac{1}{4}\mathrm{Var}(r_1) + \frac{1}{4}\mathrm{Var}(r_2) = \frac{1}{2}C < C = \mathrm{Var}(r_1) = \mathrm{Var}(r^B)$. The variance of the second portfolio is twice as high as the variance of the first, diversified portfolio. The assets in this example were independent, however, if the covariance would have been negative, which means that the returns of the assets move in the opposite direction, the variance would have been even lower. This is a first illustration of the concept of hedging.

A downside of the variance is that it measures the spread of the data, but it does not say anything about extreme events. Another downside of the variance is that it is a symmetric measure: it depends both on positive and negative deviations. So, when the distribution of a random variable is asymmetric, the variance may not be a correct reflection of the actual risk, as it only gives accurate results when the distribution of the random variable is symmetric. [Lleo, 2009]

However, there is a remedy for this problem which is called the semi-deviation and accompanying semi-variance. These measures only take into account the negative (or positive) deviations from the mean, but this measure still does not take extreme risk into account.

2.3 Extreme value theory & tail risk measures

The tail of a distribution is the end part of a distribution of a random variable, characterized by low probabilities but high deviation from the mean. The concept is easily understood when visualized: in figure 2.1 both left and right ends can be called the tails.

Since the probability of a random variable ending up in the tail is (very) small it is tempting to ignore these values. However, when ignoring the occurrence of the values in the tail, the results of models using this random variable are often too optimistic. This is easy to understand when we are considering a random variable that represents the return of a portfolio: although the probability of the return of the portfolio being -2.5 is very small, the implications can be significant. To measure this kind of risk, the concept of tail risk was introduced. [Yamai & Yoshiba, 2002]

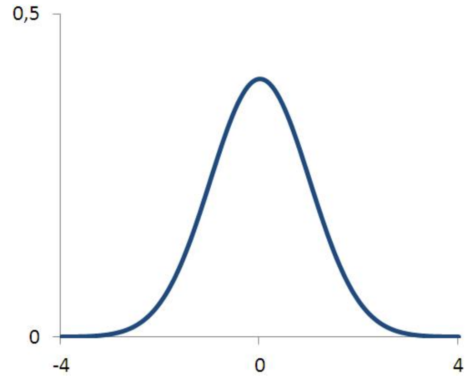


Figure 2.1: Probability density function of a $N(0, 1)$-distribution.

There are different ways of defining the tail risk, a practical example of the definition is the risk that the random variable lies at least three times the standard deviation from the mean. Another more theoretical description is: “tail risk is the risk that a large move in a portfolio is greater than what is implied by traditional risk management”[DB, 2010].

Extreme value theory (EVT) provides the framework for the analysis of the tail. From this framework different measures for risk, or specifically tail risk, can be derived. These measures, also called extreme risk measures, are the value-at-risk (VaR) and the Conditional value at risk (CVaR) or expected shortfall (ES).

Extreme value theory

In [McNeil, 1999] McNeil defines extreme value theory (EVT) as: “Extreme value theory is a method for modeling and measuring extreme risks”. EVT is a method to estimate the probability distribution of the tail, based on available data. A challenge in estimating this distribution is the choice of the bound of which values are considered ‘extreme’. On the one hand a threshold that is too low gives a scarce amount of data points, on the other hand, a threshold that is too high will not produce a valid distribution.

In EVT there are basically two approaches to determine the tail distribution. These methods are distinguished by the manner of selecting the extreme value. The *peaks-over-threshold*-model selects all data points that exceed a given threshold. The *block maxima* model selects from every ‘block’ of data points the maximum value. Both methods can be applied when it is assumed that the random variable comes from a common distribution. In the next sections both methods will be introduced.

The models described below estimate the upper tail of a distribution, but can be analogously used for lower tails. Both models are discussed elaborately in many different articles, the description of the models below is based on the articles of McNeil and Gilli & Këllezi [McNeil, 1999, Gilli & Këllezi, 2006].

Block maxima model

When $(X_1, X_2, \ldots, X_n)$ are the observations of some event, the observations are grouped in blocks (unordered). For example, when one has quarterly data of some statistic, the data may be subdivided into blocks, each of which represents one year. Every block will contain four occurrences. In formula form this is written as follows: when there are n blocks, with m occurrences each, i.e. $((X_{1,1}, \ldots, X_{1,m}, X_{2,1}, \ldots, X_{2,m}, \ldots, X_{n,1}, \ldots, X_{n,m})$. Of each block, the maximum is:

$$M_j = \max \{X_{j,1}, X_{j,2}, \ldots, X_{j,m}\} \tag{2.9}$$

As a result a dataset of n occurrences is formed: $(M_1, M_2, \ldots, M_n)$. Now the Fisher-Tipett theorem, which states that on a dataset of extreme values, an extreme value distribution $H(x)$ can be fitted for sufficiently large m, can be applied [McNeil, 1996]. The fitting distribution is given by:

$$H(x) = H_\xi\left(\frac{x-\mu}{\sigma}\right), \tag{2.10}$$

where $H_\xi(x)$, which is called the generalized extreme value distribution (GEV), is given by:

$$H_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-1/\xi}} & \xi \neq 0; \\ e^{-e^{-x}} & \xi = 0. \end{cases} \tag{2.11}$$

The maximum likelihood estimation (MLE) method is used to find $\hat{\xi}$, $\hat{\mu}$ and $\hat{\sigma}$. To give an idea of the interpretation of this distribution: the value $H^{-1}_{\hat{\xi},\hat{\mu},\hat{\sigma}}(1 - \frac{1}{k})$ is the return of the portfolio that will be expected to be (negatively) exceeded once every k years. [McNeil, 1999]

Peaks-over-threshold model

The general idea of the peaks-over-threshold (POT) model is equivalent to the block maxima model. However, instead of using the maximal occurrences of every 'block', the POT model uses all occurrences that exceed a certain value u. We call n the number of realizations of the random variable and N_u is the number of realizations that exceed u. With the distribution function of some random variable given by $F(x) = P(X_i \leq x)$, the distribution of its tail, which is defined as all values of the random variable exceeding u, is given by:

$$\begin{aligned} F_u(y) &= P(X - u \leq y \,|\, X > u) \\ &= \frac{F(y+u) - F(u)}{1 - F(u)}. \end{aligned} \tag{2.12}$$

This is called the excess distribution. As a result of the theorem of Pickands, Balkema and De Haan [Gilli & Këllezi, 2006], the generalized Pareto distribution (GPD) can be fitted to this tail and, for some threshold value u, the excess distribution, may be exactly given by the GPD:

$$F_u(y) = G_{\xi,\zeta}(y), \tag{2.13}$$

where the GPD is defined as:

$$G_{\xi,\zeta}(x) = \begin{cases} 1-(1+\frac{\xi x}{\zeta})^{-\frac{1}{\xi}} & \text{when } \xi < 0, \quad \text{where } 0 \le x \le -\frac{\zeta}{\xi}, \\ 1-(1+\frac{\xi x}{\zeta})^{-\frac{1}{\xi}} & \text{when } \xi > 0, \quad \text{where } x \ge 0, \\ 1-e^{\frac{x}{\zeta}} & \text{when } \xi = 0, \quad \text{where } x \ge 0, \end{cases} \tag{2.14}$$

with $\zeta > 0$. When we substitute (2.13) in equation (2.12) and set $x = u + y$, such that we get:

$$\begin{aligned} F_u(x-u) &= \frac{F(x)-F(u)}{1-F(u)} \\ F_u(x-u)\cdot(1-F(u)) &= F(x)-F(u) \\ F(x) &= (1-F(u))\,G_{\xi,\zeta}(x-u)+F(u), \end{aligned} \tag{2.15}$$

for $x > u$. To make an estimate $\hat{F}(x)$, we replace $F(u)$ by its estimator $\frac{n-N_u}{n}$. The values $\hat{\xi}$ and $\hat{\zeta}$ are estimated by the MLE method. We have the following tail estimator, when $\hat{\xi} \neq 0$ and $x > u$:

$$\begin{aligned} \hat{F}(x) &= (1-F(u))\,G_{\xi,\zeta}(x-u)+F(u) \\ &= \left(1-\frac{n-N_u}{n}\right)\left(1-\left(1+\frac{\hat{\xi}(x-u)}{\hat{\zeta}}\right)^{-\frac{1}{\hat{\xi}}}\right)+\frac{n-N_u}{n} \\ &= 1-\frac{N_u}{n}\left(1+\frac{\hat{\xi}(x-u)}{\hat{\zeta}}\right)^{-\frac{1}{\hat{\xi}}}. \end{aligned} \tag{2.16}$$

When $\hat{\xi} = 0$ and $x > u$, the tail estimator takes the following form:

$$\begin{aligned} \hat{F}(x) &= (1-F(u))G_{\xi,\zeta}(x-u)+F(u) \\ &= \left(1-\frac{n-N_u}{n}\right)\left(1-e^{\frac{x-u}{\hat{\zeta}}}\right)+\frac{n-N_u}{n} \\ &= 1-\frac{n-N_u}{n}-e^{\frac{x-u}{\hat{\zeta}}}+\frac{n-N_u}{n}e^{\frac{x-u}{\hat{\zeta}}}+\frac{n-N_u}{n} \\ &= 1-\frac{N_u}{n}e^{\frac{x-u}{\hat{\zeta}}}. \end{aligned} \tag{2.17}$$

Function $\hat{F}(x)$, for $x > u$, gives the distribution of the tail. We can use this tail distribution or the tail distribution from the block maxima model to measure risk. There are two common tail risk measures the value-at-risk and conditional value at risk. These risk measures will be discussed in the next sections.

Value at risk

When one has a probability distribution of the return of a security, portfolio or any other item, the $(100-\gamma\%)$ value at risk (VaR) is the value at the boundary of the confidence interval at confidence level γ. This is a measure of exposure to risk. To illustrate this concept, we take a look at figure 2.2. We assume that there is an asset and its return is $N(0,1)$ distributed.

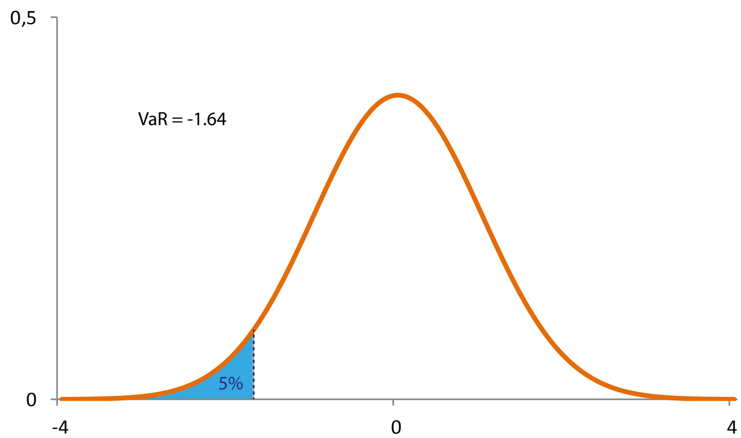


Figure 2.2: 95% Value at Risk (VaR) with a N(0,1) distribution.

The $100 - 95\%$ one-day VaR is -1.64, so the probability that tomorrow's value of the asset is below -1.64 is 5%. The VaR is in principal a measure of downside risk, but as opposed to the negative VaR, there is also a positive VaR, which lies at the other side of the mean.

The VaR is easy to calculate when the distribution of the return of the security or portfolio, is known. With the cumulative distribution function given by $F(x) = P(X \leq x)$, the value at risk at risk level γ is defined as:

$$\text{VaR}_\gamma(X) = F^{-1}(\gamma). \tag{2.18}$$

When the distribution of the tail is known from the POT model or block maxima model, for probability $\gamma > F(u)$ a VaR estimate is given by the inverse of the distribution function. For the POT model this means:

$$\gamma = \hat{F}(q) \iff q = \hat{F}^{-1}(\gamma) = \widehat{\text{VaR}}_\gamma(X), \tag{2.19}$$

$$\widehat{\text{VaR}}_{\gamma,\hat{\xi}\neq 0}(X) = u + \frac{\hat{\zeta}}{\hat{\xi}}\left(\frac{n}{N_u}(1-\gamma)^{-\hat{\xi}} - 1\right), \tag{2.20}$$

$$\widehat{\text{VaR}}_{\gamma,\hat{\xi}=0}(X) = u - \hat{\zeta}\ln\left(1 - \frac{n}{N_u}(1-\gamma)\right). \tag{2.21}$$

However, when the actual distribution of the random variable is not known. There are three methods to compute the VaR, firstly the variance-covariance method, secondly historical simulation methods and thirdly Monte Carlo simulation methods [Aniūnas et al., 2009].

The variance-covariance method assumes that the random variable, in this case the return of a security, has a normal distribution and that therefore, the 95% VaR is given by $\mu - 2\,\sigma$ and the 99% by $\mu - 2.33\,\sigma$. In order to calculate the VaR, one needs to find σ and plug it in the formula. One calculates the variance according to formula (2.8), the accompanying covariance matrix is calculated based on historical data. [Aniūnas et al., 2009].

The method based on historical simulation is rather basic. To calculate the VaR of the return of a security, the time series of the return of the security is used. From the time series a dataset is created which contains the day-to-day changes in returns, and the dataset is ordered. The 99% positive VaR is the 99% precentile of the dataset and the 99% negative VaR is the 1% precentile. To give recent events more influence on the VaR, it is possible to assign weights in the dataset prior to ordering. [Aniūnas et al., 2009].

The method based on Monte Carlo simulation is very similar to the historical simulation, however instead of having a dataset based on historical data, the dataset is created by simulation of future asset prices [Aniūnas et al., 2009].

All three methods are based on historical data, the historical simulation fully by definition and the Monte Carlo simulation to some extent. The main disadvantage of the historical simulation method, because it implies that the history will repeat itself. The use of historical data is also a drawback of the variance-covariance method, and the assumption that the returns are normally distributed cannot generally be accepted, since there are many more outliers than the normal distribution accounts for. The computation of the covariance matrix is responsible for the accuracy of this method, and depends on historical data. The main drawback of the Monte Carlo method is that in order to get an accurate simulation, many paths have to be computed and this can be costly. In general, the Monte Carlo simulation is more difficult. However, of the three methods, the Monte Carlo method seems the most flexible when choosing a distribution for the returns. It also relies the least on historical data.

These methods to compute the VaR are not too difficult, however, the methods all have their downsides, which makes the computation of the VaR unreliable. Also, when the VaR is used as a risk measure it should be taken into account that the estimated VaR is often too optimistic. The VaR mainly indicates where the tail of the distribution begins, information about the thickness of the tail is not given. In a normal distribution outliers are very unlikely, while in practice outliers are not that unlikely. As a result, when a firm only considers VaR as a risk measure, the firm may actually be exposed to much higher risk.

The main disadvantage of the VaR is that it does not take the distribution of the tail into consideration, the possible distributions of the tail with equal VaR can be very different. Another negative feature of the VaR is that since the VaR only returns a boundary, which can can be easily adjusted by adjusting the portfolio. VaR can be more easily manipulated than other measures. Portfolio managers might use this to take a higher risk than is allowed and mask this risk by influencing the VaR such that the risk criterion is met [Aniūnas et al., 2009]. In other words, the VaR is not a coherent risk measure.

Manipulating the VaR is rather easy, Daníelsson explains it as follows [Daníelsson, 2000]: assume that a portfolio contains one asset and that the actual VaR of the portfolio is given by VaR_a. The desired VaR level is given by VaR_d, the cumulative return distribution of the portfolio is sketched in figure 2.3.

Now when a portfolio manager buys a put option with strike price $K_p = \text{VaR}_d + \epsilon$, $\epsilon > 0$, the return of the portfolio will be represented by the dashed vertical blue line, until it crosses the distribution, from that point on, the return will again be given by the solid orange line. The reasoning is as follows: when the return of the portfolio is less than the strike price, the manager will exercise the option, sell the portfolio and get a return of K_p.

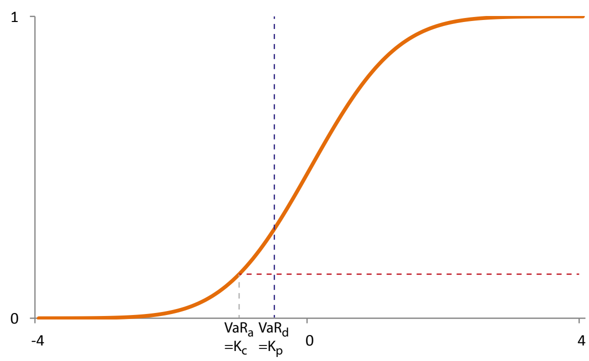


Figure 2.3: Graph of manipulation of the VaR.

With the put option, the manager should write a call option with strike price $K_c = \text{VaR}_a - \epsilon < K_p$, $\epsilon > 0$. As a result, when the return of the portfolio is less than the strike price K_c, the holder of the option will not exercise the option, however, when the return is higher, the option holder will exercise the option and buy the asset for K_c.

Finally, the manager should buy a call option with strike price K_p. Now the return distribution is a combination of four pieces of distribution, first, until K_c, the orange solid line, then until K_p the dashed red line. From the point that the dashed red and the dashed blue line cross, the distribution follows the dashed blue line and finally from the point the dashed blue line and solid orange line cross, the distribution will follow the solid orange line again.
This strategy is easy to implement and will mask the actual VaR. This example illustrates that the VaR is easily manipulated.

There are some problems, both in computing the VaR and manipulating the VaR. The VaR may also cause problems when interpreting the measure: the VaR only returns the value at which certain boundary is crossed. This means that two random variables with equal VaR can have completely different (tail) distributions. A variable which has a fat, short tail can be considered less risky than a variable with a long thin tail, which may give rise to a very big loss.

In spite of all negative features of the VaR, it is a very popular measure. This might be because it reflects the risk in a very clear manner and, although the methods to calculate the VaR are not too accurate, the VaR is easy to calculate.

Conditional value at risk and other measures related to VaR

Because the VaR has pitfalls, an alternative for it was derived: the conditional value at risk (CVaR). The general idea is that the CVaR measures the conditional mean of the tail which is cut off at the γ-level VaR. Or, as formulated in [Acerbi, 2002], the "average of the $\gamma \cdot 100\%$ worst losses".

There is a lot of literature available on the subject of the CVaR. Some articles are quite technical, and some are less technical. Because of the variety of the literature the CVaR has many names: expected shortfall (ES), worst conditional expectation (WCoE), average value at risk (AVaR), expected tail loss (ETL), tail mean (TM) tail conditional expectation (TCE) and, of course, conditional value at risk (CVaR).

In some literature the definitions that accompany this name are the same ([Mazaheri, 2008], [Föllmer & Schied, 2008]), but in other articles different definitions have different names ([Artzner et al., 1999], [Acerbi & Tasche, 2002], [Lleo, 2009]). The, sometimes very subtle, differences in definitions are mostly caused by the assumptions on the distribution of the random variable. For example, in [Artzner et al., 1999] five different definitions are given, these are the CVaR, TCE, WCE, ES and TM. These different definitions, however, coincide when the distribution of the random variable is continuous. When the distribution of the random variable is discontinuous, differences may occur, although they are quite subtle.

An example of this difference when distributions are discontinuous is given in [Lleo, 2009], where a distinction between two definitions is made. The different measures are the expected shortfall and the conditional value at risk. Their definitions are as follows:

$$\text{ES}_\gamma(X) := \frac{1}{\gamma}\int_0^\gamma F_X^{-1}(p)\,dp, \tag{2.22}$$

$$\text{CVaR}_\gamma(X) := E[X|X \le F_X^{-1}(\gamma)] = E[X|X \le \text{VaR}_\gamma(X)]. \tag{2.23}$$

When the tail distribution is defined through either the block maxima model, or the POT model, the CVaR is defined as:

$$\text{CVaR}_\gamma(X) = \text{VaR}_\gamma(X) + E[X - \text{VaR}_\gamma(X)|X > \text{VaR}_\gamma(X)], \tag{2.24}$$

where the second term of the right-hand side is the mean of the excess distribution $F_{\text{VaR}_\gamma(X)}(y)$. The mean of the $G_{\xi,\zeta}(y)$ distribution is $\frac{\zeta}{1-\xi}$. As we have introduced $F_u(y)$ as the tail distribution of $F(x)$, we can introduce a distribution of the tail $F_{\text{VaR}_\gamma(X)}(y)$, when $\text{VaR}_\gamma(X) > u$, by replacing $\zeta = \zeta + \xi(\text{Var}_\gamma(X) - u)$. Now the mean of this distribution is $\frac{\zeta+\xi(\text{VaR}_\gamma(X)-u)}{1-\xi}$. When all estimates are substituted, we get the following formulas for the CVaR:

$$\widehat{\text{CVaR}_\gamma}(X) = \widehat{\text{VaR}_\gamma}(X) + E[X - \widehat{\text{VaR}_\gamma}(X)|X > \widehat{\text{VaR}_\gamma}(X)] \tag{2.25}$$

$$\widehat{\text{CVaR}_\gamma}(X) = \widehat{\text{VaR}_\gamma}(X) + \frac{\hat\zeta + \hat\xi(\widehat{\text{VaR}_\gamma}(X) - u)}{1-\hat\xi} \tag{2.26}$$

$$\widehat{\text{CVaR}_\gamma}(X) = \frac{\widehat{\text{VaR}_\gamma(X)}}{1-\hat\xi} + \frac{\hat\zeta - \hat\xi u}{1-\hat\xi}. \tag{2.27}$$

Depending on the choice of variable and distribution, the CVaR can also be defined as $\text{CVaR}_\gamma(X) = E[X|X > \text{VaR}_\gamma(X)]$. For convenience we will use the generic name CVaR for the concept of the measure that is defined by "average of the $\gamma \cdot 100\%$ worst losses".

Another risk measure that is closely related to the CVaR, but actually is not really identical to the CVaR, is the Worst Case Expectation (WCaE). The WCaE can be used when the actual distribution of the random variable is not known, but the family to which it belongs is. When we call the family of distributions $\mathbb{P}$, with the i^{th} distribution, $i \in 1, ..., n$ given by P_i then the worst case expectation at confidence level γ is defined by:

$$\text{WCaE}_\gamma(X) = \sup_{P_i \in \mathbb{P}} \text{CVaR}_\gamma(X). \tag{2.28}$$

The CVaR and its related measures give better representations of the risk than the VaR, as we will see in section 2.6, but they also have their downsides. A disadvantage of the CVaR is that only the tail is considered. CVaR does not say anything about the rest of the distribution: it will not be possible to choose the best security from two securities with equal CVaR but otherwise completely different distributions. This argument is also valid for the VaR. Contrary to the VaR, the CVaR is not easily manipulated, because the CVaR accounts for the whole tail. Another issue is that the CVaR depends on the VaR, since it is the expected value, given that the value does not exceed the VaR [Uryasev, 2000].

2.4 Relative measures of risk

The standard deviation, variance and tail risk measures all measure the risk of one specific security or portfolio, stand alone, these measures do not give complete information. Only when these measures are also known for other securities or portfolios, a comparison can be made.
Another group of risk measures are the relative measures of risk. In these measures a 'comparison' is already processed, and therefore these measures are often applied to a portfolio instead of a single security. There are many different relative measures of risk and a few of them will be introduced below.

Tracking error

The main property of the tracking error (TE) is that it measures the deviation of the actual portfolio from a benchmark portfolio. As a result, many portfolio optimizing models are based on minimizing the tracking error or tracking error volatility. The tracking error volatility or tracking error variance is the variability of the deviation of the managed portfolio return and the benchmark return [Roll, 1992].

There are different methods to measure the tracking error. Five different tracking errors are defined in [Rudolf et al., 1999]. To illustrate the relevance of the existence of multiple definitions and the properties of the different definitions, below various different TE's are stated. TE_1 is the most elementary tracking error, TE_2-TE_6 are from [Rudolf et al., 1999], finally TE_7 is an example of how also the variance can be used for the tracking error.

Let $\mathbf{R_B}$ be a $(T \times 1)$ vector of benchmark returns for times $t = 1, 2, ..., T$, and let $\mathbf{R_P}$ be a $(T \times n)$ matrix of n asset returns for times $t = 1, 2, ..., T$. Let $\boldsymbol{\omega}$ be an $(n \times 1)$-vector with portfolio weights of the assets, an important condition for this vector is that $\sum_{i=1}^{n} \omega_i = 1$ so that the portfolio sums up to 100%. Finally, let matrices $\overline{\mathbf{R}}_\mathbf{P}$ and $\overline{\mathbf{R}}_\mathbf{B}$ be matrices that contain only the rows τ for which $R_{P_\tau}\omega < R_{B_\tau}$, which are all points at time τ at which the deviations of the portfolio from the benchmark are negative. So,

$$(2.29) \quad \text{TE}_1 := \mathbf{1}'(\mathbf{R_B} - \mathbf{R_P}\boldsymbol{\omega}) = \sum_{i=1}^{T} R_{B_i} - R_{P_{i1}}\omega_1 - R_{P_{i2}}\omega_2 - ... - R_{P_{in}}\omega_n;$$

$$(2.30) \quad \text{TE}_2 := \mathbf{1}'(|\mathbf{R_B} - \mathbf{R_P}\boldsymbol{\omega}|) = \sum_{i=1}^{T} |R_{B_i} - R_{P_{i1}}\omega_1 - R_{P_{i2}}\omega_2 - ... - R_{P_{in}}\omega_n|;$$

$$(2.31) \quad \text{TE}_3 := \mathbf{1}'(|\overline{\mathbf{R}}_\mathbf{B} - \overline{\mathbf{R}}_\mathbf{P}\boldsymbol{\omega}|);$$

$$(2.32) \quad \text{TE}_4 := (\mathbf{R_B} - \mathbf{R_P}\boldsymbol{\omega})'(\mathbf{R_B} - \mathbf{R_P}\boldsymbol{\omega}) = \sum_{i=1}^{T} (R_{B_i} - R_{P_{i1}}\omega_1 - ... - R_{P_{in}}\omega_n)^2;$$

$$\text{(2.33)} \qquad \mathrm{TE}_5 := \max_i \{|\mathbf{R_B} - \mathbf{R_P}\boldsymbol{\omega}|\};$$

$$\text{(2.34)} \qquad \mathrm{TE}_6 := \max_i \{|\overline{\mathbf{R}}_\mathbf{B} - \overline{\mathbf{R}}_\mathbf{P}\boldsymbol{\omega}|\};$$

$$\text{(2.35)} \qquad \mathrm{TE}_7 := \mathrm{Var}(\mathbf{R_B} - \mathbf{R_P}\boldsymbol{\omega}) = E[(\mathbf{R_B} - \mathbf{R_P}\boldsymbol{\omega})^2] - (E[(\mathbf{R_B} - \mathbf{R_P}\boldsymbol{\omega})])^2,$$

where $\mathbf{1}'$ denotes the transpose of a vector of ones. TE_1 is a very basic tracking error, it is just the sum of the differences between the portfolio return and the benchmark return for every t. TE_1 assigns a positive value to a positive deviation of portfolio return compared to the benchmark return, but it assigns a negative value when the return of the portfolio is lower than the benchmark. A result of this is that equal (positive and negative) values cancel each other out. In theory this means that a portfolio that has very high deviation, but where the positive and negative deviations are of equal value, can have tracking error 0.

In tracking error TE_2, the problem of outliers which cancel each other out is resolved by assigning a positive value to both positive and negative deviations by the absolute value. This ensures that in portfolio optimizing models which use the tracking error as a minimizer, both positive and negative outliers are penalized.
TE_3 has the same linear form, but only takes into account those rows in the equation for which the condition $R_{P_m}\omega < R_{B_m}$, this means that a deviation relative to the benchmark is not taken into account when it is an advantageous deviation. This implies that the model is not penalized for positive outliers, but they also do not cancel out negative outliers.

TE_4 is a simple quadratic tracking error, the properties of a quadratic tracking error are that larger outliers have a larger influence on the tracking error. A second property of the quadratic tracking error is that the square ensures that all deviations are positive and thus, negative values do not cancel out positive values.

TE_5 just addresses the highest (absolute) deviation of all deviations of assets. This tracking error does not take into account the second largest and all other deviations. TE_6 is a combination of TE_3 and TE_5 and gives the maximum deviation that is disadvantageous for the return of the portfolio.

Finally, TE_7 is the variance of the deviation of the portfolio from the benchmark. One can argue that the tracking error already is some sort of volatility measure, because is measures the deviation of a variable relative to another variable. So TE_7 can be interpreted as the variance of the volatility.

In this section we have seen several definitions for the tracking error, but there are many more definitions of the tracking error. These seven examples give an insight in the usefulness.

Parameter β

The β of a portfolio is the volatility of a single security compared to the volatility of a whole portfolio or the whole market. In formula form it is defined as:

$$\text{(2.36)} \qquad \beta = \frac{\mathrm{Cov}(R_p, R_m)}{\mathrm{Var}(R_m)},$$

$$\text{where} \quad R_p = \text{return of the portfolio,}$$
$$R_m = \text{return of the market.}$$

Parameter β represents the relation between the volatility of the portfolio in relation to the volatility of the market. When the value of β equals 1, an increase of the market return of 10% should imply an equal increase of the portfolio return. However, when the value of β equals 2, a 10% increase of the market should lead to a $2 \cdot 10\% = 20\%$ increase of the portfolio. The β can be considered to be a measure for market risk. [Hübner, 2005]

Sharpe ratio

The Sharpe ratio is defined by the excess return of a portfolio relative to the risk free rate per unit of risk, when the risk is considered to be measured by the variance. In formula form it is given by:

$$S = \frac{R_p - R_f}{\sigma}, \tag{2.37}$$

$$\begin{aligned} \text{where} \quad R_p &= \text{return of the portfolio,} \\ R_f &= \text{risk free return,} \\ \sigma^2 &= \mathrm{Var}(R_p - R_f) = \text{variance of the difference.} \end{aligned}$$

Instead of the risk free return, R_f, sometimes the value 0 is used. A higher Sharpe ratio implies a better performing portfolio. The Sharpe ratio gives a good insight in the trade-off risk versus return, a high return is good, but not if the risk taken to get this return is too high. The Sharpe ratio gives a clear insight in how well a portfolio has performed. [Hübner, 2005]

Treynor ratio

The Treynor ratio gives the excess return compared to the risk free return per unit of β:

$$T = \frac{R_p - R_f}{\beta}, \tag{2.38}$$

$$\begin{aligned} \text{where} \quad R_p &= \text{return of the portfolio,} \\ R_f &= \text{risk free return,} \\ \beta &= \beta \text{ of the portfolio.} \end{aligned}$$

The Sharpe ratio measures the excess return per unit of risk, whereas the Treynor ratio measures the excess return per unit of market risk. [Hübner, 2005]

Information ratio

The information ratio is similar to the Sharpe ratio, but it does not measure return compared to the risk-free rate, but compared to the benchmark return :

$$\mathrm{IR} = \frac{R_p - R_b}{\sigma}, \tag{2.39}$$

$$\begin{aligned} \text{where} \quad R_p &= \text{return of the portfolio,} \\ R_b &= \text{benchmark return,} \\ \sigma^2 &= \mathrm{Var}(R_p - R_b) = \text{variance of the difference.} \end{aligned}$$

(See [Hübner, 2005])

Jensen's α

Jensen's alpha is a measure of the performance. Jensen [Jensen, 1967] formulates it as follows: "it represents the average incremental rate of return on the portfolio per unit time which is due solely to the managers ability to forecast future security prices" . In formula form it is given by :

$$
\begin{aligned}
\alpha &= (R_p - R_f) - \beta\,[R_m - R_f] + \varepsilon \qquad (2.40)\\
\text{where} \quad R_p &= \text{return of the portfolio,}\\
R_f &= \text{risk free return,}\\
R_m &= \text{return of the market portfolio,}\\
\varepsilon &= \text{error term with } E[\varepsilon] = 0.
\end{aligned}
$$

When $\alpha > 0$, the portfolio manager is performing well, when $\alpha < 0$ the portfolio manager performed worse than the market. [Hübner, 2005]

2.5 Greeks

There is another group of risk measures, called the Greeks, which are risk measures mainly applicable to options. These measures will not be needed in the remainder of this thesis, but to be complete they will be discussed very briefly in this section.

The Greeks represent the sensitivities of prices of options to changes in for example the underlying asset price, volatility or time. The price of an option can be determined by the Black-Scholes model. There are several different variables and parameters on which the Black-Scholes solution depends, and therefore there are also several Greeks. We will address the three most important ones, given by:

$$
\Delta = \frac{\partial V}{\partial S}; \qquad \nu = \frac{\partial V}{\partial \sigma}; \qquad \Theta = \frac{\partial V}{\partial \tau}; \qquad (2.41)
$$

where V is the value of the option, S is the price of the underlying asset, σ is the volatility and τ is time. These sensitivities of the price of an option when the variables change, can be considered to be risk measures.

2.6 Which measure(s) to use

In the previous sections various measures for risk have been discussed, with their advantages and disadvantages. However, these (dis)advantages are not easily comparable. In [Lleo, 2009] classes of risk measures are named, each with its own properties. The classes are monetary risk measures, coherent risk measures, convex risk measures and spectral risk measures. In the next sections, these classes will be defined, as in [Lleo, 2009].

Monetary measures of risk

Monetary risk measures are in [Lleo, 2009] defined as the group risk measures, that return the value of the amount of a risk-free investment r (so r could be an amount of cash) that needs to be added to the portfolio in order to make the risk of the underlying investment (i.e. single

security, portfolio, etc.) acceptable. To illustrate: when one owns a portfolio with a monetary profit and loss (P & L) X, the risk measure is determined by the minimal riskless investment r that should be added to the portfolio, so that the risk of the investment is acceptable.
Or in formula form:

$$\rho(X) = \min_{r \in \mathbb{R}} \{\text{investment in a position } [X + r] \text{ is acceptable}\} \tag{2.42}$$

In [Föllmer & Schied, 2008], a monetary measure of risk satisfies on top of this condition the following conditions:

Translation invariance or cash invariance Adding a risk-free instrument r, for example cash, to an existing position, decreases the risk by an equal amount:

$$\rho(X + r) = \rho(X) - r. \tag{2.43}$$

Monotonicity If the return of asset X is less or equal to the return of asset Y, then the risk of asset X must be greater or equal:

$$X \leq Y \qquad \Longrightarrow \qquad \rho(X) \geq \rho(Y). \tag{2.44}$$

The monotonicity is best explained when ρ is considered as capital requirement. If the profit of Y is always higher than or equal to the profit of X, i.e. $Y \geq X$, then the minimal amount of cash that needs to be added to Y, to make the risk of it acceptable, is less than the amount that needs to be added to X to make the risk acceptable, i.e. $\rho(X) \geq \rho(Y)$.
The translation invariance can be similarly explained: if an amount of cash is added to a portfolio, the additional capital needed to satisfy the capital requirements of the portfolio is obviously lowered.

Convex measures of risk

A convex risk measure is a special case of a monetary risk measure, thus it satisfies the properties of a monetary risk measure. Next to that, it also satisfies the following property:

Convexity When a portfolio is diversified, the risk of the total diversified portfolio is smaller than or equal to the risk of the separate assets:

$$\rho(\lambda X + (1 - \lambda)Y) \leq \lambda\rho(X) + (1 - \lambda)\rho(Y), \qquad \text{for } 0 \leq \lambda \leq 1. \tag{2.45}$$

In section 2.2 this principle has been illustrated for the variance.

Coherent measures of Risk

Artzner, Delbaen, Eber and Heath proposed in [Artzner et al., 1999] the notion of coherent risk measures. A coherent risk measure is a special case of a convex risk measure. In [Lleo, 2009] the properties of a coherent risk measure are stated as follows:

Subadditivity For assets X and Y, the risk of the portfolio of the combined assets is smaller than the risk of the separate assets:

$$\rho(X + Y) \leq \rho(X) + \rho(Y). \tag{2.46}$$

Positive homogeneity When the position of asset X is increased by some rate k , then the risk of the investment increases by the same proportion k:

$$k \cdot \rho(X) = \rho(k \cdot X). \tag{2.47}$$

Since a coherent measure of risk also is a monetary measure of risk and a convex measure of risk, a coherent risk measure satisfies the monotone, convex and translation invariance properties. However, the property of subadditivity is equivalent to the property of convexity when $\lambda = \frac{1}{2}$. So even without defining convexity, a coherent risk measure automatically is a convex risk measure.

The subadditivity ensures, just as convexity, that portfolio diversification cannot increase the risk of the portfolio. The positive homogeneity ensures that the risk of having k times position X is equal to the risk of having position $k \cdot X$.

In [Acerbi, 2002], Acerbi derives the following statement: "a measure is coherent if it assigns bigger weights to worse cases".

Spectral measures of risk

Spectral measures of risk are coherent measures of risk which are 'customized' by multiplication by a risk-aversion formula.

In [Acerbi, 2002], the spectral risk measure is constructed as follows: Acerbi argues that if ρ is a one parameter family of coherent risk measures, $\gamma \in [a, b]$, then, for any measure $d\mu(\gamma)$ in $[a, b]$ with $\int_a^b \gamma \, d\mu(\gamma) = 1$, the statistic defined as $\rho = \int_a^b d\mu(\gamma)\, \rho_\gamma$ is a coherent risk measure. (proposition 2.2 in [Acerbi, 2002].)

When $F_X(x)$ is a cumulative distribution function of some P&L, X, and $F_X^{-1}(p)$ is its inverse distribution function, we define the expected shortfall as:

$$ES_\gamma(X) = -\frac{1}{\gamma} \int_0^\gamma F_X^{-1}(p) dp. \tag{2.48}$$

When we assume that the expected shortfall is coherent, we can define

$$\begin{aligned}
M_\mu(X) &= \int_0^1 d\mu(\gamma)\, \gamma\, ES_\gamma(X) \\
&= -\int_0^1 d\mu(\gamma) \int_0^\gamma F_X^{-1}(p)\, dp \\
&= -\int_0^1 F_X^{-1}(p)\, dp \int_p^1 d\mu(\gamma) \\
&= -\int_0^1 F_X^{-1}(p)\, \phi(p)\, dp, \\
&= M_\phi(X)
\end{aligned}$$

where $\phi(p) = \int_p^1 d\mu(\gamma)$. Now, according to the proposition of Acerbi, $M_\mu(X)$ is coherent if $\int_0^1 \gamma \, d\mu(\gamma) = 1$. The following holds:

$$\int_0^1 \phi(p)\, dp = \int_0^1 dp \int_p^1 d\mu(\gamma) = \int_0^1 d\mu(\gamma) \int_0^\gamma dp = \int_0^1 d\mu(\gamma)\, \gamma = 1.$$

To ensure that $M_\phi(X)$ satisfies the properties of a coherent risk measure, $\phi(p)$ should be an "*admissible*" risk spectrum, as it is called in [Acerbi, 2002]. An element $\phi \in \mathcal{L}^1[a,b]$ is an "*admissible*" risk spectrum if:

(i) ϕ is positive, i.e. if for all $I \subset [a,b]$, $\int_I \phi(p)\, dp \geq 0$,

(ii) ϕ is decreasing, i.e. if $\forall q \in [a,b]$ and $\forall \varepsilon > 0$ such that $[q-\varepsilon, q+\varepsilon] \subset [a,b]$, $\int_{q-\varepsilon}^q \phi(p)\, dp \geq \int_q^{q+\varepsilon} \phi(p)\, dp$,

(iii) $\|\phi\| = 1$.

Let $M_\phi(X)$ be defined by $M_\phi(X) = -\int_0^1 F_X^{-1}(p)\, \phi(p)\, dp$ with $\phi \in \mathcal{L}^1([0,1])$. If $\phi(p)$ is an admissible risk spectrum, then $M_\phi(X)$ is a risk measure. The admissible risk spectrum $\phi \in \mathcal{L}^1([0,1])$ is called the "*risk aversion function*"; the risk measure $M_\phi(X)$ will be called the "*spectral risk measure*" generated by ϕ.

The risk aversion function can be seen as a function that gives weights to the area that is integrated. Since the risk aversion function is decreasing it assigns higher weights to worse values.

Comparison of classes

Now that the classes of risk measures are defined, we can decide to which class the measure that we will use to measure the risk, has to belong to. First, measures of risk that do not satisfy the subadditivity condition might not properly measure the risk of a diversified portfolio and we therefore conclude that a risk measure must at least be convex. When a measure is also coherent or even a spectral risk measure it would be helpful, since the measure is then adjustable to different risk aversion functions. However, a convex risk measure has satisfactory properties itself. From the measures that are discussed above, the only spectral measures of risk are the CVaR or ES, when the distribution of the random variable is continuous. VaR only is a monetary risk measure.

2.7 Use of risk measures in practice

In the previous section a variety of risk measures has been introduced, however their use in practice has not yet been discussed. In the article of Amenc et al. [Amenc et al., 2011] the results of a questionnaire for portfolio managers was published. In the questionnaire handed out to portfolio managers of both small and large firms, the portfolio managers indicated how they measure the risk of a portfolio. The results of this article are discussed below.

When portfolio managers were asked whether they set absolute risk objectives in portfolio optimization, most reported that they use tail risk and average risk. For portfolio optimization the measures that are mainly used are VaR, CVaR and variance. Firms also use relative risk objectives for portfolio optimization, however, to a lesser extent than absolute risk measures. Most firms indicated that they use the tracking error to measure relative risk.

Although this is interesting, the information that is really relevant is how the firms measure risk when the portfolio is implemented. When the firms were asked how they calculate extreme risk when implementing portfolio optimization, almost a quarter of the respondents answered that they do not calculate extreme risk measures. Most respondents use VaR and some use CVaR. Another notable result is that most firms that use VaR, measure VaR by assuming the normal distribution, which is not a sophisticated manner to measure.

To measure the absolute performance of a portfolio, almost three-quarters of the respondents answered that they use the Sharpe ratio. Also the average return in excess of the risk free rate was a popular measure. Amenc et al. argue that these measures are popular because of their simplicity.

The results of the survey of Amenc et al. imply that most firms assess risk by easy-to-use risk measures and often do not consider extreme risk. Based on the questionnaires, Amenc et al. gained a more general insight: large firms are more likely to use sophisticated methods than small firms.

2.8 Conclusion

An important conclusion is that every risk measure that was named, has its merits and its demerits. Therefore, it may make sense to use more than one measure to determine the risk of a portfolio. Here a combination of risk measures is suggested.

The CVaR appears to have valuable properties, it is a spectral measure of risk when the distribution of the random variable is continuous and a convex risk measure when it is discontinuous [Lleo, 2009]. The downside of the CVaR is that it only considers the tail, a measure that would complement the CVaR, should take the behaviour of the other part of the distribution into account. As a second risk measure the standard deviation will therefore be chosen, which also is a convex risk measure and is easily calculated. The standard deviation and CVaR give complementary information about the risk of a security: the CVaR gives information on the size of the tail and the standard deviation on the body of the return distribution. Together they may give a more complete report on the risk.

Chapter 3

Available methodology

In order to develop new methodology to assess the existence of a mismatch between risk profile and strategic asset allocation, insight in the existing methodology and tools is necessary. In this chapter the relevant tools of Ortec Finance and other important suppliers of investment performance systems will be evaluated. Of every tool two aspects are discussed:

- In what way does the tool provide insight in the portfolio?
- How does the tool monitor risk?

3.1 Ortec Finance

Ortec Finance is a Dutch firm that supplies technology and advice on risk and return management. Ortec Finance has several finance related specialties, one of them is private wealth management.

OPAL Wealth Planner

OPAL is the abbreviation of Optimized Personal Asset and Liability management. OPAL Wealth Planner is the tool which is developed by Ortec Finance for private banks. The main goal of the tool is to list the (private) investment objectives and preferences of a client, determine the client's risk profile and create an optimal asset allocation. The optimal asset allocation is determined with the help of a scenario set, which is a set of 500 possible economic scenarios. Based on these scenarios, a scenario analysis can be performed, which is a kind of Monte Carlo analysis.

In figure 3.1 an example of the results of a scenario analysis is shown. Each light blue line represents the development through time of one scenario. The horizontal red line represents the target capital, the scenarios that are at the end of the period –which is in this case January 2032– below this line, do not reach the target capital. The orange line represents the 10^{th} percentile scenario, i.e. of all scenarios, 10% lies below this line at the end of the period. The green line represents the 90^{th} percentile scenario, so 90% of the scenario lie at the end of the period below the green line. The blue line represents the expected scenario, or the 50^{th} percentile.
With the help of this graph it can be assessed how realistic the objective of the investment, in this case €180.000, is, with the current allocation.

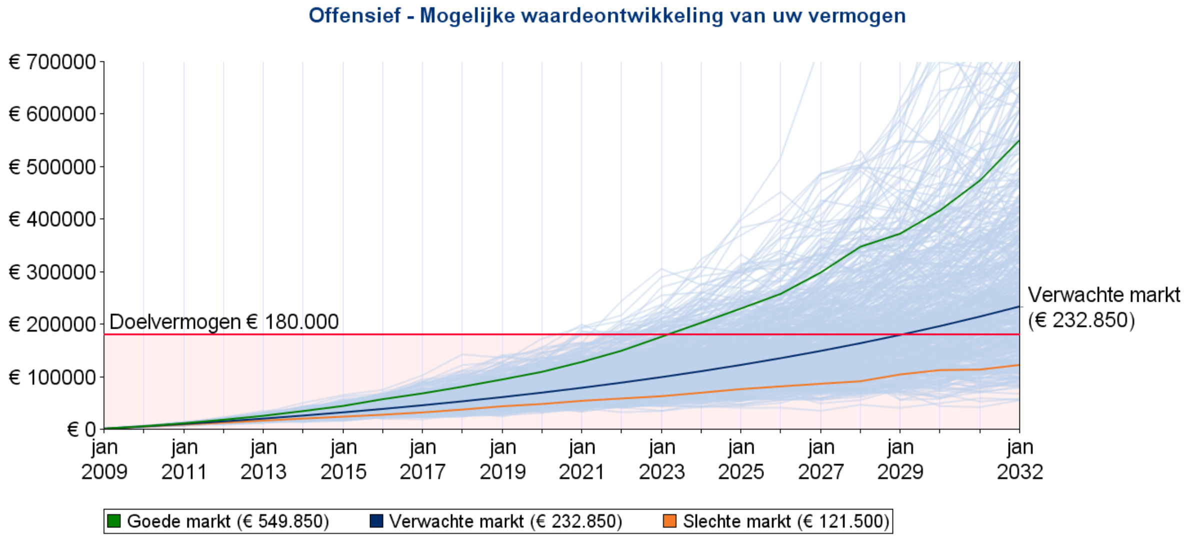


Figure 3.1: Example of the use of scenario analysis.

In OPAL Wealth Planner Professional, the advisor can create a custom-made portfolio based on a mean-variance method. The portfolio can be determined up to asset class level, the actual allocation on security level is determined by the advisor. The investment strategy of the portfolio can be set to rebalance-to-plan and buy and hold.

In OPAL Wealth Planner Professional the composition of the portfolio can be viewed on asset level. The percentages of the portfolio assigned to each asset class can be viewed in a graph and in a table. Also the composition of different portfolios from different risk profiles can be compared.

The risk of the portfolio can be assessed by the scenario analysis which was discussed above. To see whether or not a portfolio is feasible can also be determined in a portfolio graph. In figure 3.2 such a graph is shown. On the horizontal axis the risk is defined by the standard deviation (see section 2.2 for the details), the vertical axis represents the feasibility of the objectives. The squares represent an income objective (over a certain range of time a certain amount of money is withdrawn from the portfolio periodically), the circles represent a capital objective (at a certain point of time a fixed amount of money should be available). Each (vertical) pair of one square and one circle represents a risk profile, the pair at the most left-hand side is the least risky pair, so it is the most defensive risk profile. The pair on the right hand side is the most risky pair, so it is the most aggressive risk profile. The white circle and square pair is, in this case, the optimal fit for the client.

The orange areas represent combinations that either do not fit the risk attitude or have a feasibility below 50%. The red area represents combinations that both do not fit the risk attitude and have feasibility below 50%. The remaining areas are feasible with an increasing desirability.

PEARL

PEARL is a performance evaluation system which consists of two parts: a calculation manager and a web portal. The calculation manager represents the engine of the system; it performs

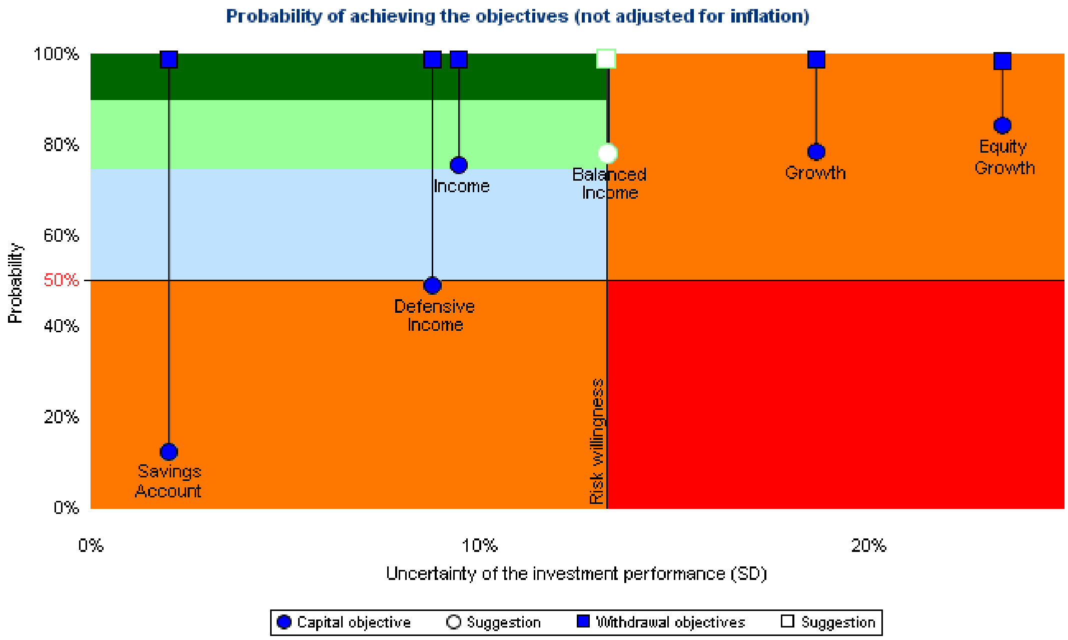


Figure 3.2: Portfolio graph in OPAL.

the calculations and manages the data. The web portal is the user interface which data analysts can use for their analyses. In this section the PEARL Web Portal will be briefly discussed.

In PEARL the portfolio is represented by a tree. On the highest level the portfolio is shown and the portfolio is subdivided into asset classes, sub-asset classes and finally the portfolio can be viewed on single security level. In figure 3.3 an example of such a tree is presented.

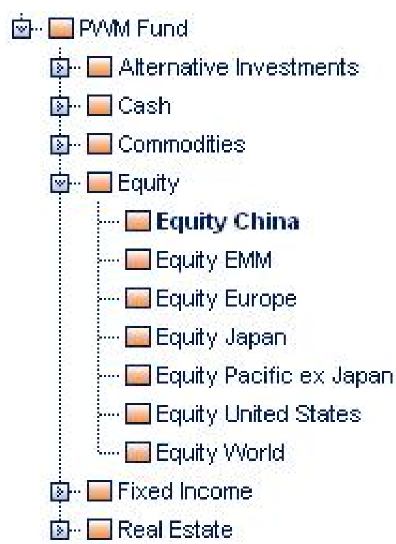


Figure 3.3: Overview of a portfolio in a tree.

At each level, a report table and several graphs can be viewed. Different reports can be put together in PEARL. For example, a basic report which contains of each level the benchmark and portfolio weights and the benchmark and portfolio returns, a currency overlay report, where the risk exposure of the portfolio to the change in exchange rates is shown, and an attribution report, where the allocation and selection effects of the assets in a portfolio are listed. Graphs are also available in the reports.

PEARL also offers the opportunity to make a risk report, as the most common risk measures are available, such as the standard deviation, tracking error, Treynor ratio and Sharpe ratio. The VaR and CVaR, however, are not available. The variables can, again, be viewed at the portfolio level, asset class level, sub-asset class level or security level.

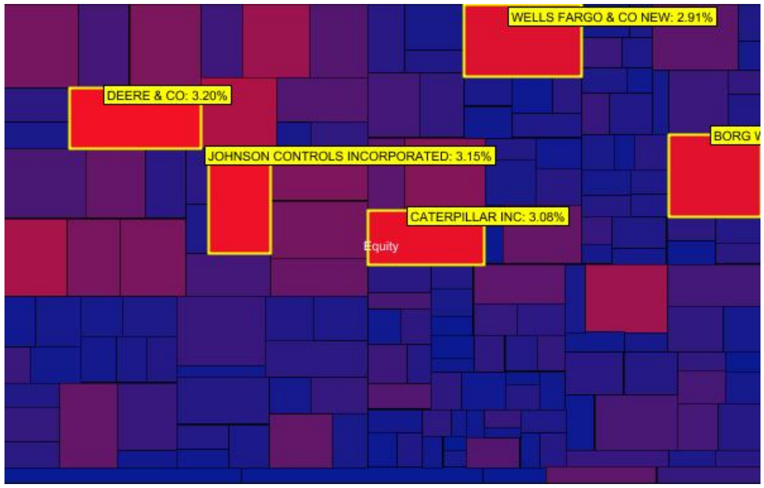


Figure 3.4: Risk map report of a portfolio in RiskMetrics WealthBench. The red shaded areas represent risky assets.

3.2 MSCI

MSCI is a world leading firm in investment decision support tools. MSCI is divested from Morgan Stanley and has acquired among others RiskMetrics, a spin-off of J.P. Morgan's risk department, and Barra Inc. MSCI provides two tools that have relevant features: RiskMetrics WealthBench, RiskMetrics RiskManager and Barra Portfolio Manager. These will be discussed below.

RiskMetrics WealthBench

RiskMetrics WealthBench is MSCI's private wealth management tool. It assists advisors to construct a clients' portfolio, based on the desires and objectives of the client. WealthBench takes the risk appetite of the client into account and determines an optimal asset allocation. Based on the data in the system, the advisor can choose the appropriate securities. WealthBench focuses on market risk.

The portfolio of a client can be analyzed both at asset class level and security level. The system can measure the risk under different market conditions, such as historical events like Black Monday (October 19, 1987) in a present day environment and model the impact of changes in a portfolio.

When additional services from MSCI are acquired, advisors can also use WealthBench to manage the portfolio of the client pro-actively, [MSCI, 2011]. These additional services appear to be the monitoring of the gap between the actual portfolio and the (strategic) asset allocation, but this is not explicitly mentioned.

WealthBench has several reporting possibilities, an example of a risk report component of a portfolio is shown in figure 3.4[1]. Figure 3.4 presents a risk map, all assets are represented by squares where the surface area represents the portfolio weights and the colour represents the

[1]Picture from: `http://www.msci.com/resources/pdfs/Product_Documentation_WealthBench.pdf`

riskiness of an asset. The reports that are available are of presentation quality and can be custom built.

The measures for risk that are available in WealthBench are:

- Standard deviation,
- RiskGrades,
- Value at Risk (VaR),
- Expected Shortfall,
- Risk Impact (marginal contribution),
- Diversification Benefit,
- Max Drawdown.

RiskGrades (RG) is a measure of volatility developed by RiskMetrics, ranging from 0 to 1000. 100 is the average RG value of major equity market indices during normal market conditions from 1995 to 1999 [Kim & Mina, 2001], a portfolio with RG 150 is twice as risky as a portfolio with RG 75.
The risk impact is the unit of change in risk per unit of change in the position of the asset, or in formula form: $\frac{\partial r}{\partial x_i}$, where x_i is the position in asset i and r is the risk of the portfolio which can be represented by, for example, the tracking error.
The maximum drawdown is the biggest decrease that the return of a portfolio, given by $X(t)$, has experienced in a given time frame $[0, T]$, formally given by:

$$\text{MDD}(T) = \max_{\tau \in [0,T]} \left\{ \max_{t \in [0,\tau]} X(t) - X(\tau) \right\}. \tag{3.1}$$

The relative risk measures available in WealthBench are:

- α, see section 2.4,
- β, see section 2.4,
- R-Squared (R^2),
- Tracking error, see section 2.4,
- Information ratio, see section 2.4,
- Capture ratios,

R-squared or R^2, ranging from 0 to 100, is a measure to see to which extent the performance of the portfolio is determined by the benchmark index. A value of 100 represents a portfolio of which the performance is entirely determined by the benchmark.
The capture ratio measures the performance of a portfolio manager. The capture ratio is given by:

$$\text{Market Capture Ratio} = \frac{\text{Manager's Returns}}{\text{Market's Returns}} \cdot 100. \tag{3.2}$$

RiskMetrics RiskManager

RiskMetrics RiskManager is a tool to manage risk of a portfolio. RiskManager contains a market data viewer, a portfolio viewer, a report builder, stress testing and what-if analysis. RiskManager assesses both market risk and default risk. Contrary to WealthBench, RiskManager is not a solution for private banks, where the reports are constructed to inform a client, RiskManager is meant for a broader use. The process of RiskManager is represented by figure 3.5[2].

*

[2]Picture from: http://www.msci.com/resources/factsheets/RiskMetrics_RiskManager.pdf

Insight in the portfolio

In the portfolio viewer, clients can analyze their portfolio. Next to the portfolio, the market data viewer provides 10 years of daily historical data for more than 750,000 time series across 85 markets.

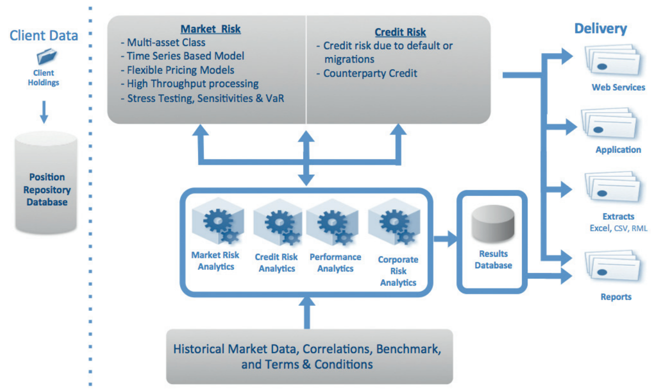


Figure 3.5: RiskManager process.

RiskMetrics RiskManager gives a great variety of possibilities to assess the risk of a portfolio and report the risk. RiskManager offers the possibilities to create a customized report with drag-and-drop. A what-if analysis of the portfolio can be executed, which is an analysis of the portfolio under circumstances assigned by the portfolio manager. Also a stress test of the portfolio can be executed. These properties can provide insight in the risk to which the portfolio holder is exposed. RiskManager uses multiple VaR methodologies to provide market exposures and sensitivities.

Barra Portfolio Manager is a relatively new tool –launched December 13, 2010– for portfolio managers. The tool can be used within a firm by different departments like research, sales and marketing and portfolio management, with each department having its own type of access.

Barra Portfolio Manager uses Barra Optimizer to create portfolios. Also, Barra Optimizer can be used to set up a rebalance cycle for portfolio management, as well as a pre-trade 'what if' analysis, which gives insight in the possible performance of potential changes in the portfolio.

Barra Portfolio Manager can identify sources of risk specified to industry, style, market, or specific risk sources and determine which factors are the largest contributors to the portfolios risk and return. The change of a risk profile of the portfolio can be forecasted by time series of risk.

3.3 Morningstar

Morningstar provides investment research, and developed both a web platform, freely accessible for individuals, as well as software for institutions and advisors. The software has a modular set-up. It is more focused on portfolio construction and management.

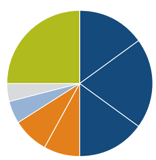
In Morningstar's Portfolio Builder, the process of building a portfolio is performed in a stepwise fashion. One of these steps consists of determining the risk tolerance of the client by a questionnaire.

In the Advisor Workstation, the so-called Portfolio X-Ray function provides a detailed overview of the portfolio composition. In figure 3.6[3] a part of the Portfolio X-Ray is shown. The

[3] Picture from:
`http://corporate.morningstar.com/nl/documents/SampleReports/AWS/UK_AWS_SampleReport_XRay.pdf`.

Portfolio X-Ray™

Asset Allocation



Asset Allocation	Portfolio %
Country (Example: UK) Equity	17.00
European Equity	18.00
US Equity	15.00
Bond 1	10.00
Bond 2	10.00
Cash	25.00
Other	3.00
Not Classified	2.00
	100.00

Top Ten Countries

Country	Equity %	Country	Equity %
Great Britain	16.91	France	1.59
Netherlands	11.19	Poland	1.56
Switzerland	6.34	Belgium	1.29
Italy	4.19	Portugal	1.25
Germany	2.80	Greece	1.16

World Regions

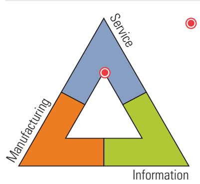
	Portfolio %
Greater Asia	**0.00**
Japan	0.00
Australasia	0.00
Asia-4 Tigers-Emrg	0.00
Asia-ex 4 Tigers-Emrg	0.00

	Portfolio %
Americas	**99.67**
North America	98.16
Cent & Latin Am-Emrg	1.51

	Portfolio %
Greater Europe	**0.31**
United Kingdom	0.31
West Europe-ex UK	0.00
Emerging Europe	0.00
Africa	0.00

	Portfolio %
Market Maturity	
Developed Markets	0.31
Emerging Markets	0.00
Not Available	0.00

Stock Sectors



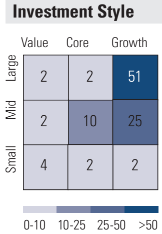
Portfolio

	Equity %
Manufacturing	**29.06**
Consumer Goods	9.37
Materials	10.88
Energy	5.95
Utilities	2.86

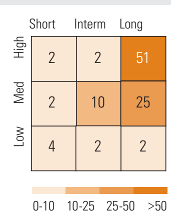
	Equity %
Services	**48.24**
Health Care	14.60
Consumer Services	8.99
Business Services	4.09
Financial	20.56

	Equity %
Information	**22.66**
Software	4.71
Hardware	9.86
Media	4.20
Telecom	3.89

Investment Style



Equity Style	Portfolio
Market Cap ($Mil)	00,000.00
Price/Earnings	00.00
Price/Cash Flow	00.00
Price/Book	0.00
Price/Sales	0.00



Fixed-Income Style	Portfolio
Avg Eff Duration (Yrs)	0.00
Avg Eff Maturity (Yrs)	0.00
Avg Credit Quality	AAA

Figure 3.6: Components of the X-Ray of Morningstar.

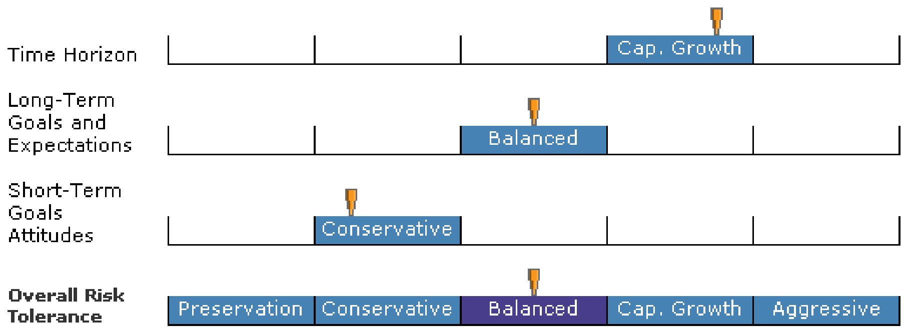


Figure 3.7: The risk tolerance structure of Morningstar's Advisor Workstation.

★★★★★	top 10% in the category
★★★★	the next 22.5% in the category
★★★	the middle 35% in the category
★★	the next 22.5% in the category
★	the last 10 % in the category

Table 3.1: Distribution of stars is the Morningstar Rating.

portfolio is decomposed in four different ways: according to asset classes, world regions, stock sectors and, finally, investment style. The Portfolio X-Ray also contains a list of the portfolio contents with the annualized 3 months, 1 year, 3 years and 5 years return.

Morningstar does not provide any insight in the risk of the portfolio. A risk assessment report is available, but it only provides information about the risk tolerance of the client, and does not relate this tolerance level to the risk level of the portfolio. In figure 3.7[4] the composition of a client's risk tolerance is illustrated. In the planning module, investment goals of the client can be managed and reviewed.

Morningstar also developed a rank measure called Morningstar Rating which employs a scale of one to five stars. The rating gives a qualitative assessment of a funds' past risk and return performance. The funds are categorized and within the categories, a risk adjusted return measure is used to rank the funds. The stars are appointed to the funds according to the guidelines stated in table 3.1.

It is not clear which risk measures are available in the Morningstar software.

3.4 SimCorp Dimension

SimCorp is a company which develops asset management solutions and is based in Denmark. The main product of SimCorp is SimCorp Dimension, which is a modular system. Different modules can be acquired, that are merged into one tailor-made system. The complete system provides solutions for the front, middle and back office of financial companies.

Because SimCorp Dimension is a complete system, it focuses mainly on mitigating operational risk and is not specifically developed for portfolio optimization and management. The

[4]Picture from: `http://corporate.morningstar.com/nl/asp/imageloader.aspx?image=../images/ADV_AWO_SP2D_1b_w.gif&title=Morningstar.`

mitigation of operational risk is achieved via the completeness of the system: the system can both execute analyses of portfolios and strategies and actually execute trades. The fact that this is one system, prevent errors and makes it possible to detect errors and their origin.

However, the completeness of the system prevents it from being one of the specialized niche systems, which focus on developing private wealth management systems and looking into more detail. Since SimCorp Dimension does not focus on market risk, the specifics of portfolio and (market) risk monitoring are not clear.

Chapter 4

Hypothesis tests

When looking at two datasets, just comparing the values of the means, standard deviations and distributions, absolute or relative, is not sufficient to judge whether or not, the means, standard deviations or distributions differ sufficiently to state that the difference is significant. The application of statistics can help to make this distinction, take several properties of the datasets into account and judge whether a hypothesis should be accepted or rejected.

A hypothesis test can be used when, in this case, the risk measures of different datasets are compared, or to determine whether two datasets originate from the same distribution. Therefore, in this chapter several statistical tests for different applications, with their advantages and disadvantages will be introduced.

The statistical tests are introduced to answer the following three questions:

1. Do two data samples come from the same distribution?
2. Do the standard deviations of two data samples deviate significantly?
3. Do the CVaRs of two data samples deviate significantly?

These questions will be discussed in the second part of this thesis.

In the first section distribution tests, to test whether two data samples originate from the same distribution, are discussed. In the second section some tests to detect statistical differences of means and standard deviations are discussed.

4.1 Goodness-of-fit and two-sample tests

Other than significance tests, goodness-of-fit tests check whether a certain dataset can be fitted to a pre-specified distribution. A related test is a two-sample test, testing whether two data samples originate from the same distribution, which is not necessarily predefined.

In [Darling, 1957], the goodness-of-fit test and the two-sample test are formally introduced as follows: let $X_1, X_2, ..., X_n$ be independent variables drawn from a distribution with cumulative distribution function $G(x) = P(X_i < x)$. Then the empirical distribution function, the distribution of the actual draws, $F_n(x)$, can be defined as follows: $F_n(x) = \frac{k}{n}$, if k observations are less or equal than x, for $k = 0, 1, ..., n$. The empirical distribution function represents the observed distribution [Anderson & Darling, 1952].

When $F(x)$ is a given distribution, the null and alternative hypotheses of goodness-of-fit tests are:

$$H_0 : G(x) = F(x), \text{ for all } x, \quad H_1 : G(x) \neq F(x). \tag{4.1}$$

When we have the same sample from continuous distribution, $G(x)$, as in the goodness-of-fit test, and when $Y_1, Y_2, ..., Y_n$ represents a sample from distribution $H(x) = P(Y_i < x)$, we can formulate the hypotheses of the two-sample problem:

$$H_0' : G(x) = H(x), \text{ for all } x, \quad H_1' : G(x) \neq H(x). \tag{4.2}$$

The specifics of well-known goodness-of-fit and two-sample tests are discussed next: first Pearson's Chi-squared (χ^2) test will be addressed briefly then the Kolmogorov-Smirnov test, Cramér-Von Mises test and finally the Anderson-Darling test are discussed.

Chi-squared goodness-of-fit test

The (Pearson) Chi-squared goodness-of-fit test was first formulated in 1900 by Karl Pearson. When $X_1', X_2', ..., X_n'$ are observed frequencies from a distribution, which has theoretical frequencies $X_1, X_2, ..., X_n$, then the statistic is given by:

$$X^2 = \sum_{i=1}^{n} \frac{(X_i' - X_i)^2}{X_i}. \tag{4.3}$$

The statistic represents a "standard deviation of the standard deviation" [Plackett, 1983]. To determine whether the null-hypothesis should be accepted or rejected, the p-value $p = P(X^2 > \chi_d)$ is determined. The number of degrees of freedom d is $n - p$ where p is the reduction in degrees of freedom.

Kolmogorov-Smirnov test

The two-sided Kolmogorov-Smirnov test statistic is a result of a series of theorems that are listed in an article by Kolmogorov [Kolmogorov, 1941]), which in turn is based on the work of Smirnov. The first building blocks of the method are the following theorems from [Kolmogorov, 1941].

> **Theorem 1** *If a function $F(x)$ is continuous, then the distribution law of the quantities*
>
> $$D_n = \sup |F(x) - F_n(x)| \sqrt{n} \tag{4.4}$$
>
> *does not depend on $F(x)$,where $F_n(x)$ is the empirical distribution function.*
>
> **Theorem 2** *Whatever be the distribution function of $F(x)$, the probability*
>
> $$P(D_n \leq \lambda) \geq \Phi_n(\lambda), \tag{4.5}$$

where $\Phi_n(\lambda)$ tends to $\Phi(\lambda)$ uniformly as $n \to \infty$, and

$$\Phi(\lambda) = P(K \leq \lambda) = \sum_{i=-\infty}^{\infty} (-1)^i e^{-2i^2\lambda^2}. \tag{4.6}$$

These theorems lead to the Kolmogorov-Smirnov goodness-of-fit test. Kolmogorov extended the results also to the two-sample Kolmogorov-Smirnov test. The accompanying theorem from [Kolmogorov, 1941] is stated below.

Theorem 3 *If the probability law $F(x)$ is continuous, then the probability,*

$$P\left\{\sup|F_n(x) - G_m(x)| \leq \lambda\sqrt{\frac{n+m}{nm}}\right\} = \Phi_{n,m}(\lambda), \tag{4.7}$$

is independent of the function $F(x)$. If n and m are indefinitely increased, subject to the restriction that the ratio n/m remains between two fixed numbers $0 < a_1 < \frac{n}{m} < a_2 < \infty$, then

$$\Phi_{n,m}(\lambda) \to \Phi(\lambda). \tag{4.8}$$

In general, in the cases where the probability law $F(x)$ is absolutely arbitrary, we have

$$P\left\{\sup|F_n(x) - G_m(x)| \leq \lambda\sqrt{\frac{n+m}{nm}}\right\} \leq \Phi_{n,m}(\lambda). \tag{4.9}$$

This leads to a distribution-free statistic for the two-sided Kolmogorov-Smirnov test:

$$D_{n,m} = \sqrt{\frac{nm}{n+m}} \sup_{x\in[-\infty,\infty]} \left|F_n(x) - G_m(x)\right|. \tag{4.10}$$

This statistic is just the largest distance between two datasets that are ordered from small to large values. In the one-sample case the statistic is the largest distance between the empirical distribution and the chosen distribution. In figure 4.1 a graph of two (hypothetical) empirical distributions and their difference at $x = 0.03$ is shown. The fact that the statistic is distribution-free is the main advantage of the test. When the distributions are continuous, the null hypothesis is rejected when $D_{n,m}$ is sufficiently large, or, more specifically when $D_{n,m} > K_\alpha$, where $P(K \leq K_\alpha) = 1 - \alpha$ and $\Phi(\lambda) = P(K \leq \lambda)$.

Cramér-Von Mises test

The (one-sided) Cramér-Von Mises test was at first designed to test the hypothesis formulated in formula (4.1): a goodness of fit test. Anderson extended in [Anderson, 1962] the test to a two-sample test, used to test the hypothesis from formula (4.2). The idea behind the test is that when data originate from the same distribution, every order of r_i's and s_i's is equally likely to occur.
The Cramér-Von Mises criterion is derived and given by:

$$\omega^2 = \int_{-\infty}^{\infty} \left[F_n(x) - F(x)\right]^2 dF(x), \tag{4.11}$$

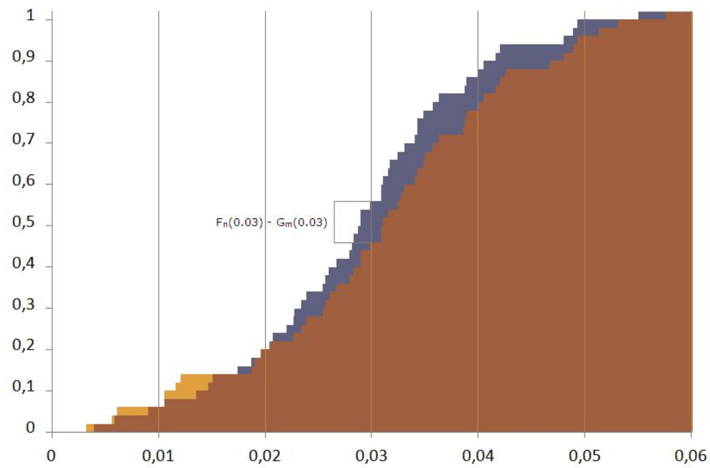


Figure 4.1: Empirical distributions of $F_n(x)$ and $G_m(x)$ and their difference at $x = 0.03$.

where $F_n(x)$ is the empirical distribution function as defined above. From this criterion the one-sample test statistic is given by:

$$T = n\omega^2 = \frac{1}{12n} \sum_{i=1}^{n} \frac{2i-1}{2n} - F(x_i). \tag{4.12}$$

The two-sample alternative of the Cramér-Von Mises criterion is given by Anderson in [Anderson, 1962]:

$$T = \frac{nm}{n+m} \int_{-\infty}^{\infty} \left[F_n(x) - G_m(x)\right]^2 dH_{n+m}(x), \tag{4.13}$$

where $G_m(x)$ is the empirical distribution function of the second data sample and $(n+m)\,H_{n+m}(x) = nF_n(x) + mG_m(x)$ is the empirical distribution function of both samples. The Lebesgue-Stieltjes integral of equation (4.13) is the following sum:

$$T = \frac{nm}{(n+m)^2} \left\{ \sum_{i=1}^{n} \left[F_n(x_i) - G_m(x_i)\right]^2 + \sum_{j=1}^{m} \left[F_n(x_j) - G_m(x_j)\right]^2 \right\}. \tag{4.14}$$

In [Anderson, 1962], Anderson simplifies this equation to the following test statistic:

$$T = \frac{U}{nm(n+m)} - \frac{4mn-1}{6(m+n)}, \quad \text{where} \tag{4.15}$$

$$U = n \sum_{i=1}^{n} (r_i - i)^2 + m \sum_{j=1}^{m} (s_i - i)^2, \tag{4.16}$$

where $r_i, i = 1, ..., n$ and $s_j, j = 1, ..., m$ are the rankings of the joint dataset.

The Cramér-Von Mises test rejects the null hypothesis when the value for T is too large. The exact critical values are listed in [Anderson, 1962].

Anderson-Darling test

In [Anderson & Darling, 1952], Anderson and Darling consider the following statistic:

$$\omega^2 = \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \psi[F(x)] dF(x)$$

with weight function $\psi(t), 0 \leq t \leq 1, \psi \geq 0$. When $\psi = 1$, the formula leads to the Cramér-Von Mises test, introduced in the previous section. According to [Anderson & Darling, 1954] the advantage of assigning a weight function is that the statistic becomes more flexible.

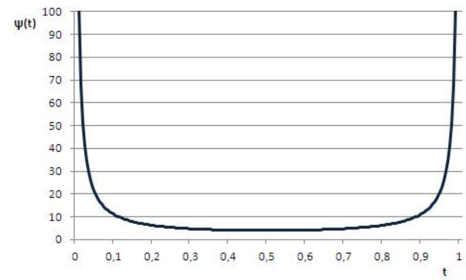


Figure 4.2: Graph of weight function $\psi(t) = [t(1-t)]^{-1}$.

In [Anderson & Darling, 1954], Anderson and Darling introduce what is known as the Anderson-Darling test. This test uses the specific weight function $\psi(t) = [t(1-t)]^{-1}$, which "has the effect of weighting the tails heavily since this function is large near $t = 0$ and $t = 1$" [Anderson & Darling, 1954]. In figure 4.2 a graph to illustrate this effect is shown. In [Anderson & Darling, 1954] the test statistic for the Anderson-Darling test is finally derived.

When $u_i = F(x_i)$, the Anderson-Darling statistic is given by:

$$W_n^2 = -n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \left[\ln u_i + \ln(1 - u_{n-i+1}) \right]. \tag{4.17}$$

4.2 Significant difference of mean and standard deviation

The literature on significance tests for the mean and standard deviation is rich. Below five tests will be introduced: the Student's F-test, T-test, ANOVA, Wilcoxon signed rank test, Sign test and the sampling distribution method.

F-test

The F-test can be used to test the hypothesis that the sample standard deviations of two data samples that originate from normal distributions have different standard deviations.

$$H_0 : \sigma_X = \sigma_Y \qquad H_1 : \sigma_X \neq \sigma_Y, \tag{4.18}$$

where $X_1, X_2, ..., X_n$ and $Y_1, Y_2, ..., Y_m$ are two samples that are assumed to come from the normal distributions with mean μ_X and μ_Y and standard deviations σ_X and σ_Y. The F-test statistic, is given by:

$$F = \frac{S_X^2}{S_Y^2}, \tag{4.19}$$

where S_X and S_Y are the sample standard deviations. When the ratio is close to one, the standard deviations can be considered equal. To find the exact boundary for at significance level α, the F-distribution is considered: test statistic F has an F-distribution with $n-1$ and $m-1$ degrees of freedom.

The F-test is easy to execute, but the normality assumption narrows the tests' possible field of application enormously. [Freund & Wilson, 2003]

Student's T-test

The student's T-test is a very common used statistical test. The T-test can, among other applications, be used to compare the means of two data samples.

When $X_1, X_2, ..., X_n$ are samples from a normal distribution with unknown mean, the null and alternative hypotheses to test whether μ_0 is the mean of the distribution are formulated as follows:

$$H_0 : \mu = \mu_0 \qquad H_1 : \mu \neq \mu_0, \tag{4.20}$$

The difference between μ and μ_0 under H_0 is 0. When the actual deviation is for example smaller than 0.1, it seems small, however, when the standard deviation is also small, the difference is not actually small [Dekking et al, 2007]. Therefore the T-test statistic corrects for the standard deviation. The T-test statistic is given by:

$$T = \frac{\bar{X}_n - \mu_0}{S_n \sqrt{n}}, \tag{4.21}$$

where n is the sample size, $\bar{X}_n$ is the sample mean and S_n is the sample standard deviation. When the data sample comes from the normal distribution, T has a T-distribution with $n-1$ degrees of freedom, so the p-value can be looked up in a table. When the data do not come from a normal distribution, but the sample size is large ($n \geq 30$), the distribution of T can be approximated by a $N(0,1)$ distribution. [Dekking et al, 2007]

To see whether the means of two data samples, X_1 and X_2, are equal, the T-test can also be executed, only with a slightly different statistic:

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_1^2/n + S_2^2/m}}, \tag{4.22}$$

where $\bar{X}_1$ and $\bar{X}_2$ are the sample means, n and m are the sample sizes and S_1 and S_2 are the sample standard deviations.

The advantage of the Student's T-test is that the statistic is calculated easily and the conclusions are clear, but a major disadvantage is that it is assumed that the data come from a normal distribution and unless the sample size is large, the test is not applicable to non-normal data samples.

ANOVA

ANOVA, or Analysis of Variance, is a collection of statistical techniques to test, among other things, whether the means of two or more datasets are different or not. When applied to two datasets, the results of the T-test and ANOVA give similar results.

The idea behind the ANOVA techniques, specifically the one-way ANOVA, is to test whether the deviation from the means is due to an error:

(4.23) $$X_{ij} = \mu + \alpha_i + \varepsilon_{ij}, \qquad i = 1, ..., n, \qquad j = 1, ..., m,$$

or if it is structural. In this formula, n represents the number of different groups, each group representing a data sample of size m. The null hypothesis will be that the 'group factor' does not play a roll or:

(4.24) $$H_0 \; : \; \alpha_1 = \alpha_2 = ... = \alpha_n = 0$$

Compared to the T-test, ANOVA can easily compare means of multiple datasets, where with the T-test only two series are compared, on the other hand the same disadvantage as the T-test, its normality requirement, unfortunately also holds. [Freund & Wilson, 2003]

Wilcoxon signed rank test

When the means of two samples are compared, one can use a T-test, however, the T-test cannot be generally applied when the data do not come from a normal distribution. The Wilcoxon signed rank test can be applied to non-normal, paired data. The test was introduced by and named after Frank Wilcoxon. When two data samples are considered, $X_i, Y_i, i = 1, ..., n$, the following steps are followed to calculate test statistic T [Wilcoxon, 1945]:

1. The differences between the two samples are calculated: $d_i = X_i - Y_i$.
2. The absolute values $|d_i|$ are ranked, the smallest difference getting rank 1, the largest difference rank $n - k$ where k is the number of d_i's with value 0.
3. The original signs (+ and -) are added to the ranks. All positive ranks are summed to T_+ and all negative signs are summed to T_-:

(4.25) $$T_- = \sum_{i=1}^{n-k} \{\text{Rank}(|d_i|) | d_i < 0\},$$

(4.26) $$T_+ = \sum_{i=1}^{n-k} \{\text{Rank}(d_i) | d_i > 0\}.$$

4. The final statistic is:

(4.27) $$T = \min \{T_+, T_-\}.$$

The values for which the null hypothesis should be accepted (or rejected) are available in a table[5]. In [Wilcoxon, 1945], Wilcoxon also introduces a similar test for unpaired data, which later became known as the Mann-Whitney test.

Although the Wilcoxon signed rank test does not make any assumption on the distribution of the data samples, it is necessary that the data come from a symmetric distribution. Wilcoxon mentions in [Wilcoxon, 1945] also a simpler test, for which such an assumption is unnecessary. In the next section this test, the sign test, will be introduced.

[5] Pearson E.S. and Hartley, H.O., ed. (1972). Biometrika Tables for Statisticians. 2. Cambridge University Press. pp. 117123, Tables 54, 55.

Sign test

The idea behind the sign test is that when the means of two data samples are the same, the differences between the (paired) data samples, are as many times bigger than 0 as they are smaller. So when two data samples $X_1, X_2, ..., X_n$ and $Y_1, Y_2, ..., Y_n$ are considered, the number of X_i's that are smaller than Y_i's is as high as the number of X_i's that are larger than Y_i's. The data points for which $X_i = Y_i$ are not considered, this reduces the number of data points $d_i = X_i - Y_i$ to $m = n - k$ where k is the number of d_i's for which $d_i = 0$.

The test statistic W is the number of d_i's for which $d_i = X_i - Y_i > 0$. The hypothesis that is tested is:

$$\begin{aligned} H_0 &: \ p = P(X > Y) = 0.5, \\ H_1 &: \ p = P(X > Y) \neq 0.5, \end{aligned} \tag{4.28}$$

Under H_0, the test statistic W will follow a binomial distribution with $p = 0.5$: $W \sim B(m, 0.5)$. Assuming that $W > m/2$, the p-value is calculated by:

$$p = P(W \geq w) = \sum_{i=w}^{m} \binom{m}{i} p^i (1-p)^{m-i}. \tag{4.29}$$

The fact that this test can be applied to non-normal, non-symmetric data samples of course comes with disadvantages: the power of this test is not particularly strong. In [Cochran, 1937] the efficiency of the test is calculated to be only is 63%. Efficiency is in the Encyclopedia of Mathematics defined as follows: "a concept used to compare statistical procedures in a given class with an optimal one." [Encyclopedia of Mathematics].

Sampling distribution method

The sampling distribution is the distribution of a statistic, in this case the mean. To illustrate the concept: let us say that from a standard normal distribution 1000 different data samples are drawn. Each data sample has a sample mean: m_i is the sample mean of the i^{th} sample. All these sample means can be considered as a data sample on its own: $m_1, m_2, ..., m_{1000}$. The sampling distribution is the distribution from which these realizations originate.

When the initial distribution is distributed normally with mean μ and standard deviation σ, then the mean is also distributed normally with mean μ and standard deviation $\sigma/\sqrt{n}$, where n is the number of samples that are available. Due to the central limit theorem, when n is large, the sampling distribution of the mean, coming from any distribution, can be approximated by a $N(\mu, \sigma/\sqrt{n})$-distribution. [Freund & Wilson, 2003]

When the sampling distribution of the mean of the first data sample is known, a α-confidence interval can be constructed in which the mean of the data sample should fall to be able to say that the means of both samples do not deviate.

When only one data sample is available, with the help of an empirical or parametric bootstrap, additional samples can be generated. The empirical bootstrap uses the available data set to determine the empirical distribution function. Subsequently, from the empirical distribution a certain number of samples are drawn. [Dekking et al, 2007]

The parametric bootstrap assumes that the distribution of the data sample is known and uses the available dataset to estimate the parameters of the distribution with for example maximum likelihood estimation. Now the random samples are drawn from the estimated distribution.
The parametric bootstrap often gives a better approximation than the empirical bootstrap, but the fact that the type of distribution of the data sample has to be known is a downside. [Dekking et al, 2007]

To summarize the sampling distribution method: first a large number of data samples, based on the first data sample, are drawn with the empirical bootstrap method. Then, from these data samples, the sampling distribution and the accompanying confidence interval of the mean are determined. When the mean of the second data sample falls within the α-level confidence interval, the mean is not considered to deviate significantly, otherwise it will.

4.3 Conclusion

In tables 4.2 and 4.1, the different tests and their properties are summarized. As mentioned before, these tests are used to answer three questions:

1. Do two data samples originate from the same distribution?
2. Do the standard deviations of two data samples deviate significantly?
3. Do the CVaRs of two data samples deviate significantly?

The first question can be answered using two-sample tests listed in table 4.1. The test that will be used to answer the first question is the Kolmogorov-Smirnov test. The test is the easiest to implement and does not give worse results than the other tests.

The second question can be answered using the F-test. The test requires normally distributed data or a large ($n \geq 30$) data sample. The samples that are going to be analyzed have sufficient data points, since daily data of more than one month will be used, so the F-test can be applied.

The third question is most difficult question to answer, because no ready-to-use tests for CVaR are available. However, finding the 5% CVaR of a data series is the same as finding the mean of the 5% lowest values of the data series. Therefore a test to find the significance of the difference in means of two data series will help us here. The difficulty is that the size of resulting data samples is only $1/20^{\text{th}}$ of the original data sample size, which means that the sample might not contain more than 30 data points. Due to this restriction all tests except for the sampling distribution method and sign test are eliminated.

Name	**Advantages**	**Disadvantages**
Kolmogorov-Smirnov	easy to perform	
Cramér-Von Mises		time consuming
Anderson-Darling	gives weights to the tails	time consuming
Chi-squared	easy to perform	basic

Table 4.1: Summary of all two-sided and goodness-of-fit tests and their properties

Name	**Advantages**	**Disadvantages**
F-test	easy to perform	normality or large sample needed
T-test	easy to perform	normality or large sample needed
ANOVA	available for multiple data samples	normality or large sample needed
Wilcoxon signed rank test	has not many conditions	symmetric distribution needed
Sign test	has no conditions	not very efficient
Sampling dist. method	works with little data	many different steps

Table 4.2: Summary of all tests for deviation in means and variances and their properties

Part II

Practical application

Chapter 5

Research method

After the theoretical analysis, the next step in the process is to determine how to model the mismatch between the actual portfolio and the Strategic Asset Allocation (SAA). In figure 5.1, a schematic representation of the model is shown. The upper block represents the input of the model, the lower block represents the output which are the results. The white block in the middle is the method that is to be developed. The input can be subdivided into two groups: the case specific information such as the state, which is different per client and contains the SAA, the single securities and their weights in a portfolio. The other input group contains the data of securities and benchmarks.

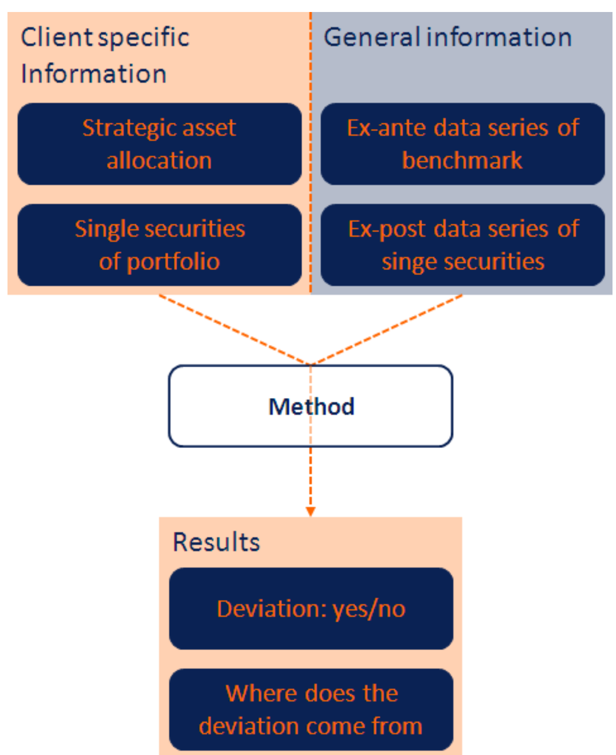


Figure 5.1: Schematic representation of the model.

As was concluded in the previous part, ex-ante risk information is beneficial over ex-post risk information, since ex-post information does not give any indication of the future. So it is preferred to use ex-ante scenarios to determine the risk of the portfolio and the SAA. However, it will be impossible to model the ex-ante risk of each asset in a portfolio separately, since there are many different securities and it would take too much time to determine how each security should be modeled. Also the computing time would be too long.

Thus, the assets of the portfolio have to be divided into a limited number of different classes which have similar properties. When a proper distinction of classes is determined, the portfolio can be projected or mapped to these classes. Since then the number of classes will be limited, it will be possible to determine the ex-ante risk for each class. When the classes are properly defined, the risk of the collection of these classes should indeed be representative for the risk of the actual portfolio.

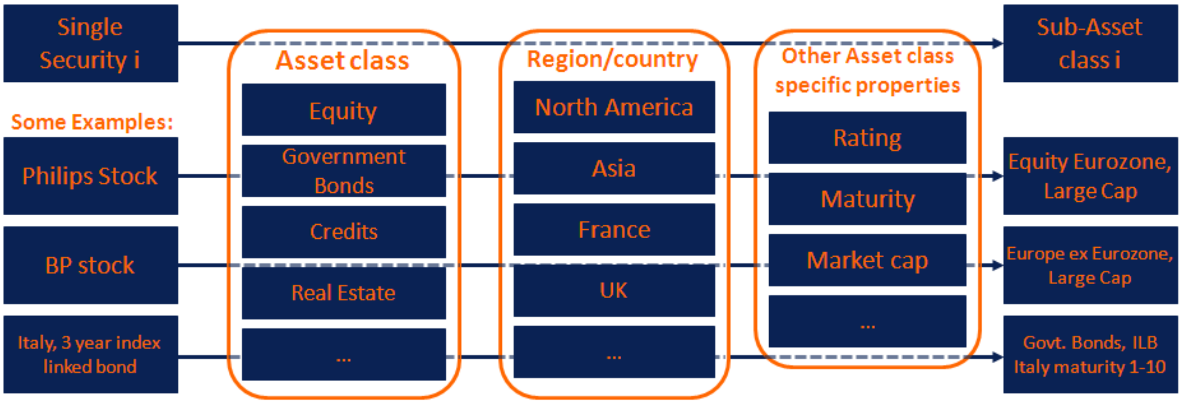


Figure 5.2: Schematic representation of mapping and some examples.

The process of appointing single securities to classes will throughout this thesis be called the *mapping*. The different securities will be first sorted by asset class (e.g. equity, government bonds etc.), followed by other different properties that this asset class has. The final classes to which the securities are appointed are called sub-asset classes (SAC). In section 6.1 all asset classes and their sub-asset classes are introduced separately. In figure 5.2 the process of mapping a security to a sub-asset class is shown. In this figure also three examples of securities that are mapped to a class are shown.

The method to find the deviation between SAA and actual portfolio can be subdivided into the following steps.

1. Find a starting point for the mapping;
2. Check whether the classes can be represented by their securities (ex-post);
3. Determine the mapping;
4. Determine ways to find the 'bandwidth' of the allowed deviation;
5. Test the mapping (ex-post);
6. Compare the portfolio and SAA (ex-ante).

First, a starting point to appoint every security to a class should be found. This process is described in section 6.1. When this basic mapping is determined it should be verified that a small number of securities indeed can be represented by a class, this process is described in section 6.2. When this is verified, obsolete classes should be identified and excluded from the structure in order to optimize the number of classes. This is a trade-off between calculation time and accuracy of the mapping. This process is described in section 6.3. A technique to detect a deviation in risk should be found, which is described in section 7.1. After the final mapping and a way to calculate risk, the bandwidths are determined. The mapping is evaluated as described in section 7.2. Finally, the risk of the portfolio and SAA can be compared, the details of this step are discussed in section 7.3.

Chapter 6

Create the mapping

In this chapter the details of the first three steps of the method will be elaborately introduced. In the first section, the starting point of the mapping is discussed, in the second section, the validation of using the mapping is discussed and in the third section, the manner to determine the final structure is introduced. Also the implementation of the second and third steps are discussed. The first step does not have an implementation.

6.1 Starting point of the mapping

In this section the starting point of the mapping structure will be discussed. Below, of every asset class several possible classifications to subdivide the asset class are introduced and explained. The final basic structure of each asset class is also shown.

Finding a proper mapping of assets into classes is the main pitfall of this process. The foundation of the classification lies in the different asset classes:

- Equity,
- Government Bonds,
- Corporate Bonds,
- Real Estate,
- Hedge Funds,
- Commodities,
- Private Equity,
- Cash,
- Convertibles,
- Infrastructures.

The SAA is defined on the asset class level. Per asset class an elaborate structure of possible sub-asset classes (SAC) should be created, which can be considered as the starting point for the mapping. This structure can consist of different levels.

Determining the structure of the tree, requires financial insight, and has been done by Ortec previously.

When putting the structure together, several practical issues are considered:

1. For all potential sub-asset classes a benchmark should be available. The absence of a suitable benchmark will make it impossible to compare the sub-asset class with other classes and it will not be possible to model the sub-asset class with the Monte Carlo scenario generator.

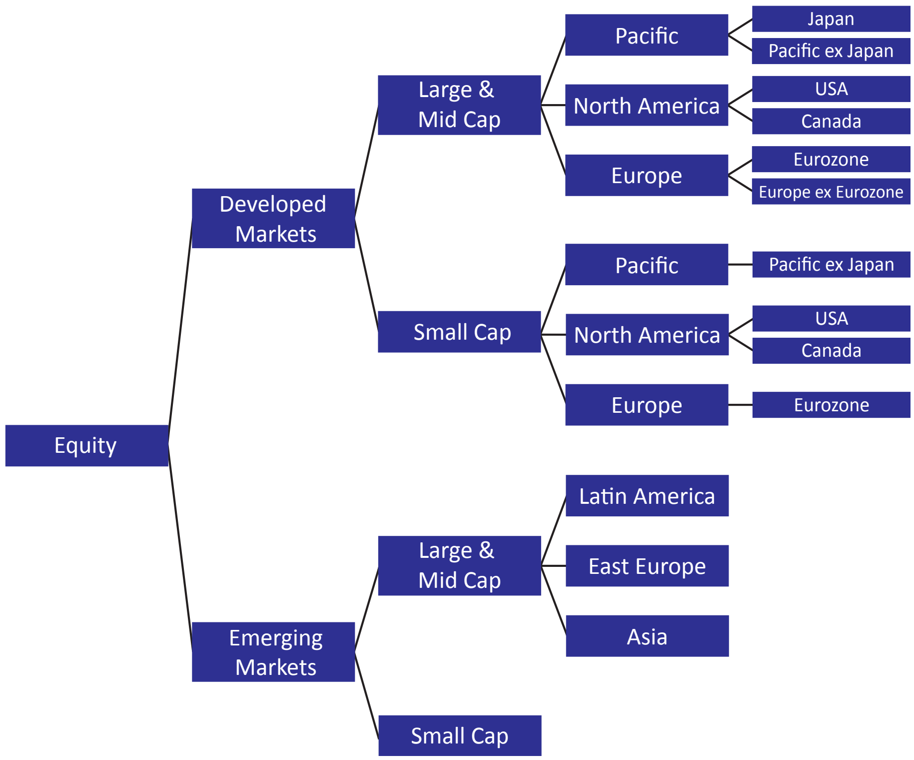


Figure 6.1: Structure of asset class *equity*.

2. This study is performed in cooperation with Ortec Finance, which is a Dutch based firm. The clients of Ortec Finance are mostly from The Netherlands, therefore, the structure is created from a Dutch point of view. This implies a focus on Europe and therefore more detailed European branches. When portfolio holders are based in the USA a different structure would have to be created.

Considering these issues in the next sections, the different asset classes are discussed separately.

Equity

To subdivide stocks into sub-asset classes, different kinds of classifications can be used. First of all, they can be arranged in size, where as a measure of size the market capitalization is taken, which is the stock price multiplied by the number of shares outstanding. Firms that have a market capitalization between 10 and 200 billion dollars are defined as large cap. Mid cap firms have a market capitalization that ranges from 2 to 10 billion dollars and small cap firms have a market capitalization between 250 million and 2 billion dollars.
Equity can also be subdivided into region and sector (i.e. energy, consumer products, etc.). In the selected structure, the sector classification is not admitted. The structure is represented by figure 6.1.

Government Bonds

Government bonds are instruments of governments to raise money to fund the activities of the government. Each country issues its own bonds (for now we ignore the fact that in the near

future in the eurozone, eurobonds may be issued), so we have immediately a set of possible sub-asset classes, namely, continents, regions and countries.
Governments issue bonds with different maturities, which also results in a set of possible sub-asset classes. Another possible classification of the government bonds is according to their rating. Credit rating agencies like Standard & Poor's, Moody's and Fitch study the creditworthiness of the governments and based on their studies, they give the rating. A less specific rating-linked classification is the distinction in Investment Grade (IG) and High Yield (HY), all bonds with rating BBB or higher (i.e. AAA, AA+, AA, AA-, A+, A, A-, BBB+, BBB and BBB-, or Moody's' equivalent) are defined as IG and all bonds with rating lower than BBB are called HY.
Finally, there are two types of government bonds: nominal and index-linked bonds. Index-linked bonds provide coupon payments that are linked to inflation, so there is no inflation risk anymore, nominal bonds pay just the nominal coupon rate.

Government bonds can thus be classified in four ways: to region, maturity, rating and inflation-linked or nominal coupon rates. With these possible classifications kept in mind, the structure, with a focus on maturity and region, represented by figure 6.2, is proposed. Not every possible region is specified in this structure, however, not of every region a benchmark was available or is interesting from a European point-of-view, therefore, for example emerging markets are automatically represented by the *nominal world* or *indexed-linked world* sub-asset classes.

Corporate Bonds

Corporate bonds, also called credits, are bonds issued by firms and have the same purpose as government bonds. Therefore, corporate bonds could be classified in the same way, however, these bonds can also be subdivided into sector related classes (corporate excl. financials, government related, etc.). The mapping is created from a European point-of-view, therefore Asia and the Pacific are excluded from the structure. The structure as in figure 6.3 is proposed.

Real Estate

There are different ways to invest in real estate. First of all, we have direct investments, which means that funds are actually directly investing in buildings. Direct properties can be subdivided into sectors[6], e.g. residential, industrial, etc. The second way to invest in real estate is through indirect investments. An indirect investment is an investment in a fund which in turn (directly) invests in buildings. Indirect investments can be both listed and unlisted, where listed means that the fund is listed on a stock exchange. Indirect unlisted property investments can be subdivided in the different management styles: Core, value added and opportunistic investments. A core fund is a low risk fund, which invests in stable property, a value added fund is a higher risk fund, the properties in a value added fund need some refurbishments, and an opportunistic fund is an even higher risk fund, which needs large enhancements[7]. Of course, real estate, in each form, can also be subdivided into regions.

[6] Information from:
`http://www.propertyfunds.org.au/index.php?option=com_content\&view=article\&id=65\&Itemid=119`.

[7] As defined by the European Association for Investors in Non-listed Real Estate Vehicles (INREV) on:
`http://inrev.org/index.php?option=com_coredef\&Itemid=5`.

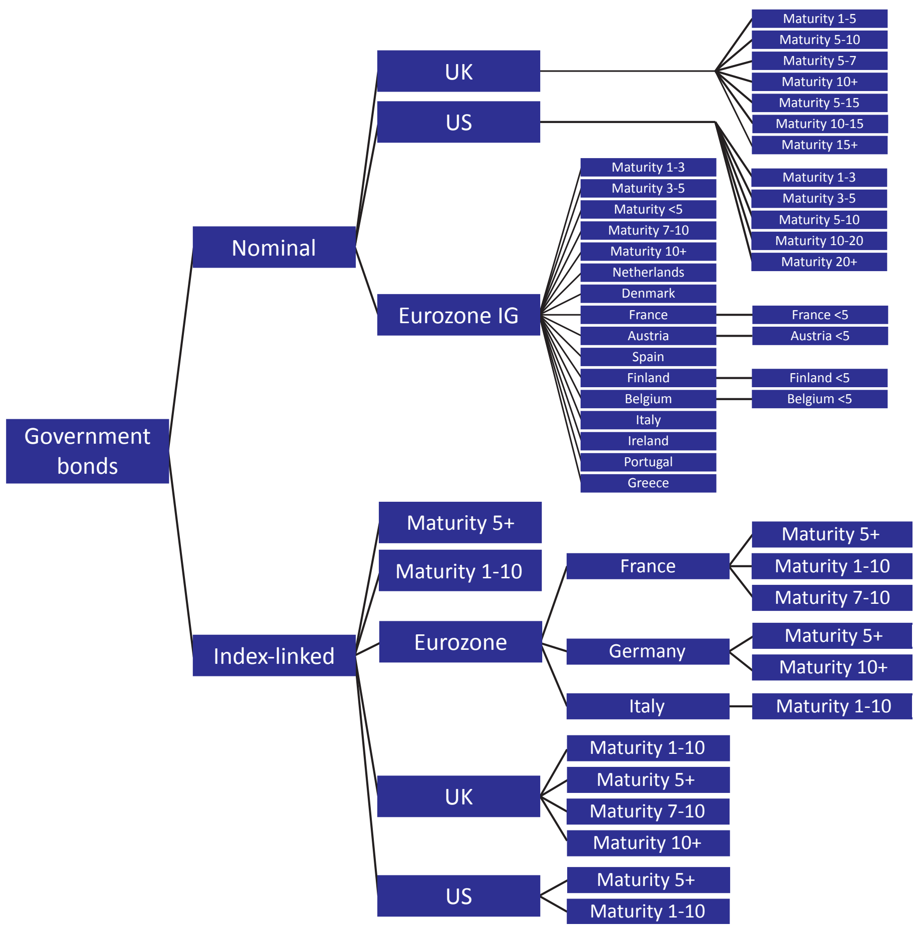


Figure 6.2: Structure of asset class *government bonds*.

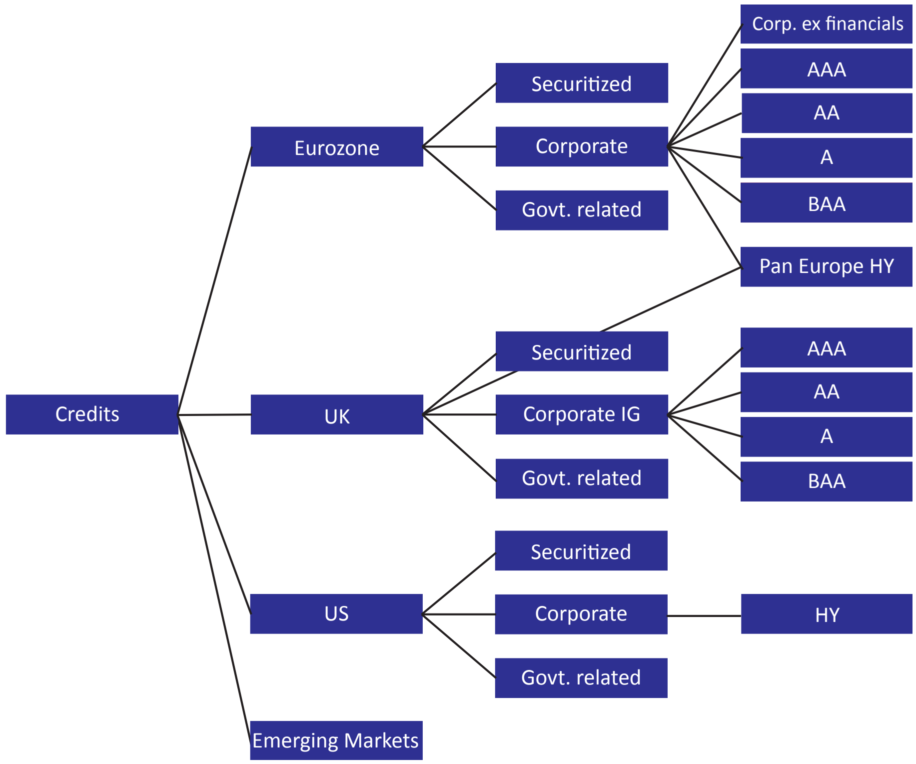


Figure 6.3: Structure of asset class *corporate bonds*.

The structure for the asset class real estate is shown in figure 6.4. In the structures of previous asset classes the Pacific was considered as a region and in this asset class the Pacific and Middle-East are combined in the region named *Far East*, this is because the provider of the benchmark data has chosen this divisions into regions.

Hedge Funds

Hedge funds are complicated investment vehicles and are not open for all investors. The measure of performance of hedge fund managers is the absolute return. To maximize this return, the hedge fund manager can use a broad set of available techniques and instruments available to absolute returns [Connor & Woo, 2003].

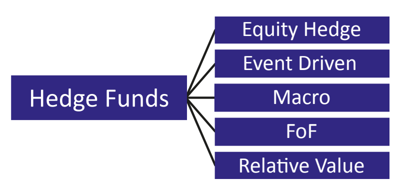


Figure 6.5: Structure of asset class *hedge funds*.

The firm Hedge Fund Research, Inc. (HFR) is specialized in the analysis of hedge funds. HFR distinguishes four different hedge fund strategies: Equity Hedge, Event driven, Macro and Relative value. These strategies are defined on the website of HFR[8], as well as in the paper of Connor & Woo [Connor & Woo, 2003]. The definitions below are based on both sources. An equity hedge strategy uses short and long positions of securities to decrease market risk, when focusing on the selection of securities in order to maximize absolute returns. Event driven strategies focus on positions in firms that are involved in events like mergers and bankruptcies. The macro strategy employs

[8] Information from `http://www.hedgefundresearch.com/index.php?fuse=indices-str\#2703`.

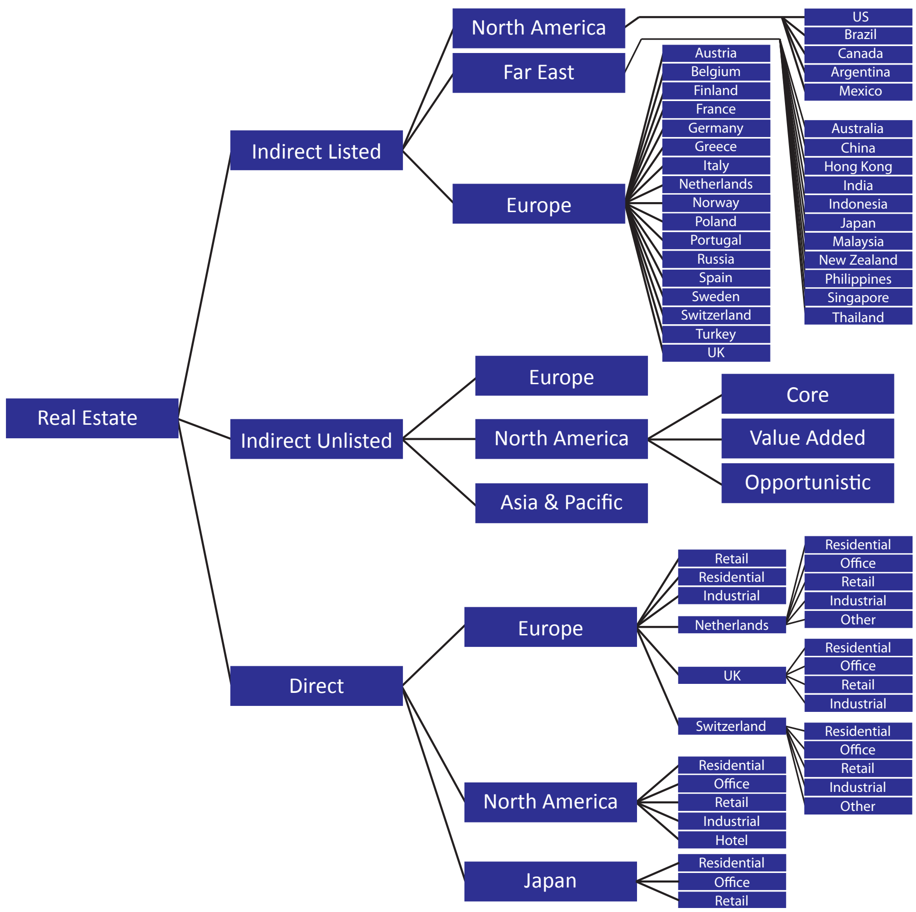


Figure 6.4: Structure of asset class *real estate*.

macroeconomic variables and their impact on securities to make a profit. The relative value strategy uses the discrepancy between two related securities to make a profit.
Besides these different strategies, HFR also distinguishes 'fund of funds', which is also called fund of hedge funds and is defined by Connor & Woo as a "managed portfolio of other hedge funds" [Connor & Woo, 2003].

These strategies all have further specifications, one of them region, but the level of detail becomes too high to also include these in the structure. The structure for the hedge funds can be found in figure 6.5.

Private Equity

The European Private Equity and Venture Capital Association (EVCA) defines in [EVCA, 2007] private equity as follows: "Private equity is the provision of equity capital by financial investors – over the medium or long term – to non-quoted companies with high-growth potential". Private equity comes in two main styles: venture capital, which encompasses the investment in starting firms, and buyouts, which encompasses a fund buying stocks of a company in order to restructure and improve the company to sell it a few years later with a profit.

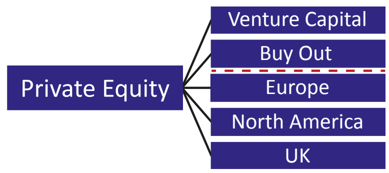


Figure 6.6: Structure of asset class *private equity*.

As with Hedge Funds, the styles venture capital and buyout can be subdivided into more sub-asset classes, but considering the level of detail, it is chosen not to specify the sub-asset classes further. Another way of classifying private equity is through region. In the end the specification to region and to type cannot co-exist, since they overlap completely, therefore one of these specification needs to be chosen. The structure of the asset class private equity is represented by figure 6.6.

Commodities

Commodities are raw materials, used in the production of goods. There are different ways to invest in commodities: directly and indirectly in commodities futures. A commodity future is a contract to buy (or sell) a predetermined quantity of a particular commodity for a predetermined price at a specific moment in the future. Bloomberg, a provider of financial information has specified five categories of commodities futures, namely[9]: energy, precious metals, agriculture, industrial metals and livestock. The proposed tree is shown in figure 6.7.

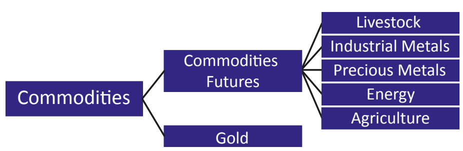


Figure 6.7: Structure of asset class *commodities*.

Cash, Convertibles and Infrastructures

The potential sub-asset classes of currency are quite straightforward, the main currencies (euro, Great British pound, U.S. dollar, Swiss franc and Japanese yen) are the main possible sub-asset classes.

[9]From: `http://www.bloomberg.com/markets/commodities/futures/`.

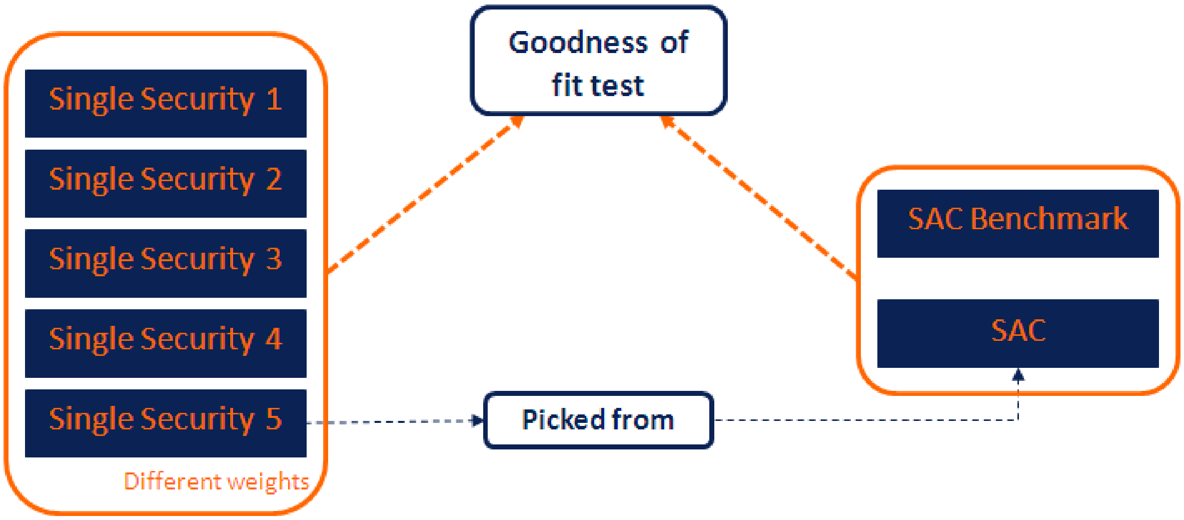


Figure 6.8: Schematic representation of goodness of fit single securities to benchmark.

Convertibles are a type of bonds that can be converted to a predefined number of stocks. The convertible holder has the option to exercise the right to convert the bond into stocks. An advantage of a convertible over normal bonds for a holder of the convertible is that when the stock price increases, it might become profitable to convert the bond. On the other hand, the buyers of convertibles will accept lower interest rates, because of the possibility to convert the bonds.
Convertibles are common investment instruments and proper indices exist. However, it is not possible to get the relevant data, so the branch convertibles is ignored for now.

Infrastructures can also be considered as an asset class, as Idzorek and Armstrong have concluded in their article [Idzorek & Armstrong, 2009]. In the asset class infrastructures, two main sub-asset classes can be identified, namely utilities and infrastructures excluding utilities.

6.2 Validation and robustness of mapping

When a portfolio is subdivided into multiple sub-asset classes (SAC), to each SAC only a handful of single securities will be appointed. The mapping makes use of the assumption that a few single securities indeed can be represented by the benchmark of the SAC. This assumption needs to be tested and moreover, a minimum number of securities needed for this assumption to hold is needed. In other words, the goal of this step is to test the robustness of the mapping: if, for example, five single securities can not be represented by the SAC benchmark, the mapping should be adapted to a higher level so that more securities are contained in the SAC. In figure 6.8 the schematic representation of this step is presented.

First the data series of the single securities are combined with different weights to form one data series, which we call, in this section, the ‘portfolio’. The portfolio is represented by the left-hand side of figure 6.8. The benchmark of the SAC to which the single securities belong, is the second data series and it is represented by the right-hand side of the figure. To check whether the portfolio can be represented by the benchmark, we test whether these two series, the portfolio and the benchmark, originate from the same distribution. In chapter 4 statistical tests were introduced which can be used for this matter. The test that was decided upon was the two-sample Kolmogorov-Smirnov test. This test was chosen because of its simplicity.

The test can be performed with different portfolios and with different confidence levels. Various portfolios will be put together with different weights and thus different levels of diversification. It is expected that the most diversified portfolios, which are the portfolios with the most single securities, will fit the benchmark best.

Several interpretations of the results are possible: when in a portfolio securities are chosen which perform as the benchmark, we conclude that the series will come from the same distribution. When most of the portfolios do not originate from the same distribution, it can be concluded that the mapping is not valid. However, another conclusion can be that the securities that were chosen in the portfolio, are too risky and have a negative effect on the portfolio. This does not necessarily mean that the mapping is not valid, but simply that the wrong securities are chosen within the SAC. Therefore, this test can also be used to monitor this type of risk.

Implementation

Because it is difficult and very time-consuming to get data series of securities of all asset classes, it was decided that the steps which are based on ex-post data of single securities are only pursued for the asset class *equity*. This implies that besides this step, also the fifth step, which is discussed in section 7.2 is only pursued for the asset class *equity*. Once these steps are modeled for one asset class, it is, when the proper data are available, not difficult to also use the model for the other asset classes.
The reason that the *equity* asset class is selected is that this asset class has a very elaborate structure and data on stocks (contrary to *corporate bonds*, *government bonds*, etc.) are widely available.

For the implementation, first representative stocks for each SAC in the tree should be found. This is possible with the help of GoogleFinance. With a simple formula, daily price data of a range of shares are available. When selecting the single securities from a SAC, the following guidelines should be taken into account:

- well-known as well as less known firms should be represented;
- two firms should not come from the same sector;
- at least 3 years of daily data should be available;
- (when SAC consists of multiple countries) firms from as much different countries as possible should be represented.

These guidelines are set, because otherwise the portfolio would not be representative for both a real portfolio and the benchmark.

The securities in the lower level SAC, will also ‘fit’ in the higher level SAC. For example when selecting BAM, a Dutch constructor, it will be appointed to the lowest level SAC *LC Eurozone*, but it also fits in the parent SAC, *LC Europe*. Therefore, the study for those higher level SACs can be performed with the same single securities.

The exact number of securities per SAC on the lowest level of the tree is chosen to be five. This number is chosen, because fewer stocks per SAC would not give a wide variety in portfolios, but more than five stocks per SAC would be too many to fulfill the guidelines in the paragraph above. The single securities that are selected are listed in appendix A.

The portfolios can be defined by assigning their weights with a random number generator, but this will not ensure that each type of portfolio is represented. Therefore in the portfolios containing 5 securities, all combinations of weights $\omega_i = \{0, 0.2, 0.4, 0.6, 0.8, 1\}$ are assigned to the securities. The portfolios that are selected are those portfolios for which the following equation holds:

$$\sum_{i=1}^{5} \omega_i = 1 \tag{6.1}$$

This results in 126 unique portfolios, with different properties. For example, some of the portfolios are well-diversified (for example those with weights $\omega_1 = ... = \omega_5 = 0.2$ and some of the portfolios are poorly diversified (for example those with weights $\omega_1 = ... = \omega_4 = 0, \omega_5 = 1$. Of course, the portfolio that has weights that are representative for the market capitalization, is expected to fit the benchmark best. The higher level SACs also have 126 portfolios, since then the model could without adjustments be used for these SACs.

The data series of the benchmarks of the SACs are available on the website of MSCI[10]. The SACs for which the test is performed are on the lowest level in the tree, and three SACs have a higher level in the tree, such that we can observe whether more than five single securities give better results. These SACs are listed below.

- Japan large cap (LC),
- Pacific ex. Japan LC,
- USA LC,
- Canada LC,
- Eurozone LC,
- Europe ex Eurozone LC,
- Latin America LC,
- East Europe LC,
- Asia LC,
- Japan small cap (SC),
- Pacific ex. Japan SC,
- USA SC,
- Canada SC,
- Eurozone SC,
- Emerging markets SC,
- Developed markets LC,
- North America LC,
- Europe LC.

Now for each SAC, 126 pairs of data series are available, each pair contains a portfolio and the SAC benchmark data series. The single securities series contain closing prices and the benchmark series, indexed returns. The tests are performed with log returns of the data series. The properties of log returns are discussed in chapter 2 and the formula to calculate the log return is given by formula (2.2).

For each of the pairs the Kolmogorov-Smirnov test is applied at different significance levels: $\alpha \in \{0.1, 0.05, 0.025, 0.010, 0.005, 0.0025\}$. The results of this step are discussed in section 8.1.

[10]URL: `http://www.msci.com/products/indices/size/small_cap/performance.html`

The results will either confirm the use of the mapping as defined before, or give an indication that the mapping should be determined differently. When indeed more than five securities are needed, perhaps a dynamic mapping should be created. The number of SACs in the mapping structure then depends on the number of securities that are mapped to the SACs.

6.3 Ranking the SACs

With the starting point for the mapping proposed in section 6.1, we now determine whether each sub-asset class (SAC) is indeed needed. Too many SACs will unnecessarily complicate the process, since of each SAC a time-consuming ex-ante analysis should be performed. In this section a method to eliminate obsolete SACs will be proposed.

Below, first the criteria to eliminate SACs that are not necessary are formulated. With the help of these criteria the SACs will be ranked with respect to to their importance. This process and its implementation will also be discussed below.

Criteria to eliminate obsolete SACs

An asset class is defined by Greer in [Greer, 1997] as follows: "An asset class is a set of assets that bear some fundamental economic similarities to each other, and that have characteristics that make them distinct from other assets that are not part of that class".
This same definition can be used to define SACs. The question is: what are the properties that make SACs distinct? No articles could be found that answered exactly this question. However, there are different questions that lead to the same answer: What are the factors in factor models, or what properties are used to define different asset classes, or what properties should be looked at when putting together a diversified portfolio. These questions are answered in many articles, such as [Sharpe, 1992], [Stephan et al., 2000], [DeLisle, 2002] and [Considine, 2008] and [Kitces, 2012].
The properties that come forward are the following:

1. **Market size:** This criterion is best explained by an example: When we consider the asset class *equity*, we have the SAC *LC North America*, it could be possible to go into more detail, namely specify *LC Canada* and the *LC USA*, however, when equity in the USA takes credit for 95% of the North American market (determined by the market capitalization, it is not interesting to actually split up the SAC *LC North America*.

2. **Correlation:** The correlation coefficient "measures the strength of the linear relationship between two [...] variables" [Freund & Wilson, 2003]. When this strength is high, the two variables are highly likely to perform alike. When the benchmarks of the SACs have strong correlation it is a clear sign that the SACs are not too different and therefore, one of them might be obsolete.

3. **Risk characteristics:** Since the main goal of this thesis is to determine deviation in risk, the risk characteristics of the SACs are very important. When two benchmarks have similar risk characteristics, why make a distinction between them when we are looking for a difference in risk?

4. **Credit default spread:** The credit default spread is another possible criterion. To explain what a credit default spread is, first the concept of a credit default *swap* (CDS) should be explained. One generally buys a CDS together with a security. The CDS acts as an insurance against the event that the issuer of the security defaults. The credit default *spread* is the fee that is payed to obtain a CDS. This fee can be considered as a measure for default risk. However, since this spread is not available for all asset classes, it is not possible to use this criterion in general.

These criteria give each on their own not enough information to decide on the elimination of a SAC, but when the information of the first three criteria are considered together, it should be possible to create a proper mapping structure. In the next section, the process of eliminating SACs is discussed.

Ranking the SACs

To study which of the SACs should be eliminated it is possible to determine which asset classes would be eliminated when the target number of SACs is for example 5, and subsequently consider which SACs remain when the target is increased to 10, 15, 20, etc. Another, more efficient, way to determine which SACs to remove is to make, within each asset class, a ranking from most deviating SAC to least deviating SAC.

To summarize, the criteria that were established to rank the SACs are the following: market capitalization, correlation and risk characteristics. The first two of these are quite specific, however, the risk characteristics of the SACs can be measured and compared in different ways. In chapter 2, we have chosen the CVaR and standard deviation to be the risk measures. These will be used to detect the mismatch and we will also use these measures in this process.

Since the risk characteristics are subdivided into two risk measures, we have four criteria to rank the SACs. For the ranking, the SACs are first subdivided into different groups. The characteristics of the different groups are listed in table 6.2. An important criterion is the market cap, because this was discussed in all articles. The second criterion is the relative deviation of the standard deviation of the SAC compared to the standard deviation of the SAC that is one level higher in the tree. Three levels of relative deviation in standard deviation are defined, as in equations (6.2), (6.3) and (6.4) below.

To explain equation (6.2): when the standard deviation of higher-level SAC is given by σ_{M}, the SACs for which the standard deviation deviates over 50% from σ_{M} are grouped in the highest deviating group. The mid-level deviation is 25%, and the lowest-level deviation is 10%. When $\sigma_{\mathrm{M}} < 2$ or $\sigma_{\mathrm{M}} > 10$ the deviation is determined absolute, since the intervals otherwise become, respectively, too small and too large.

$$A = \begin{cases} \text{if} \quad \sigma_{\mathrm{M}} \leq 2 & [\sigma_{\mathrm{M}} - \min(1, \sigma_{\mathrm{M}}); \sigma_{\mathrm{M}} + 1], \\ \text{if} \quad 2 < \sigma_{\mathrm{M}} < 10 & [0.5 \cdot \sigma_{\mathrm{M}}; 1.5 \cdot \sigma_{\mathrm{M}}], \\ \text{if} \quad \sigma_{\mathrm{M}} \geq 10 & [\sigma_{\mathrm{M}} - 5; \sigma_{\mathrm{M}} + 5]. \end{cases} \tag{6.2}$$

$$(6.3) \qquad B = \begin{cases} \text{if} \quad \sigma_{\mathrm{M}} \leq 2 & [\sigma_{\mathrm{M}} - \min(0.5, \sigma_{\mathrm{M}}); \sigma_{\mathrm{M}} + 0.5]\,, \\ \text{if} \quad 2 < \sigma_{\mathrm{M}} < 10 & [0.75 \cdot \sigma_{\mathrm{M}}; 1.25 \cdot \sigma_{\mathrm{M}}]\,, \\ \text{if} \quad \sigma_{\mathrm{M}} \geq 10 & [\sigma_{\mathrm{M}} - 2.5; \sigma_{\mathrm{M}} + 2.5]\,. \end{cases}$$

$$(6.4) \qquad C = \begin{cases} \text{if} \quad \sigma_{\mathrm{M}} \leq 2 & [-\infty; \infty]\,, \\ \text{if} \quad 2 < \sigma_{\mathrm{M}} < 10 & [0.9 \cdot \sigma_{\mathrm{M}}; 1.1 \cdot \sigma_{\mathrm{M}}]\,, \\ \text{if} \quad \sigma_{\mathrm{M}} \geq 10 & [\sigma_{\mathrm{M}} - 1; \sigma_{\mathrm{M}} + 1]\,. \end{cases}$$

The next criterion to subdivide the SACs into groups, is the correlation coefficient. When the correlation between two samples is 1, they are dependent. We want to know for what correlation, two samples are not dependent. The null hypothesis for this problem is:

$$(6.5) \qquad \begin{aligned} H_0 &: \rho = 1 \\ H_1 &: \rho < 1. \end{aligned}$$

The test statistic for this test is:

$$(6.6) \qquad t = (r - \rho)\,\frac{\sqrt{n-2}}{\sqrt{1-r^2}},$$

where r is the observed correlation coefficient and n is the sample size. Statistic t has a T-distribution with $n-2$ degrees of freedom. If $|t| > T_{\alpha,n-2}$ then the null hypothesis is rejected. When $n = 36$, which is the number of data points when considering 3 years of monthly data, and $\alpha = 0.001$ then the correlation for which the hypothesis is rejected is 0.8. For a higher n this correlation is higher and for a lower n this correlation is lower. To keep the requirements as simple as possible, we will use this boundary for all datasets. This is possible since the number of data points remains constant for each time horizon. When the correlation is lower than 0.8, this is an indication that the SACs are different. A final way to distinguish the SACs is to study whether the standard deviation is statistically different, which is done using an F-test, introduced in chapter 4.

To illustrate the grouping of the SACs we will now give an example. In table 6.1 the (made-up) properties of six SACs are shown, it is assumed, for simplicity, that they have the same higher level SAC, namely BM. The relevant properties of BM are also shown in the table. The standard deviation of BM is 5, which means that we have the following standard deviation intervals: $A = [2.5, 7.5]$, $B = [3.75, 6.25]$ and $C = [4.5, 5.5]$. Also, the SACs have all market cap > 10, so they will all fall in one of the following groups from table 6.2: 1, 2, 5, 6, 15 and 21. The first criterion is whether the standard deviation will fall outside interval $A = [2.5, 7.5]$, the only SAC for which this is valid, is SAC 4, therefore SAC 4 will be placed in group 1. The second group has the criterium that the standard deviation falls outside interval B, therefore SAC 1 will fall in group 2. Although the correlation coefficient is smaller than 0.8, this SAC will not be placed into group 6 because the criterium of interval B is classified as more important. Now SAC 2 will belong to in group 5, because $4 \notin C$. Then SAC 6 will fall in group 5, SAC 4 in group 15 and SAC 6 in group 21. The groups from table 6.2, in which the made-up SACs will be placed, are listed in the sixth column of table 6.1.

Within the groups, the SACs should also be ranked. This is done according to the conditions listed in table 6.3. First the groups are sorted according to the first listed condition, which is whether the F-test is significant or not. All SACs with a significant F-test are ranked higher

	market cap	standard deviation (σ_X)	correlation	different F-test	group №
BM	-	5	-	-	-
SAC 1	15	6.5	0.7	no	2
SAC 2	15	4	0.7	yes	5
SAC 3	15	10	0.9	no	1
SAC 4	15	4.8	0.9	yes	15
SAC 5	15	4.8	0.7	no	6
SAC 6	15	5.3	0.9	no	21

Table 6.1: Example of grouping the SACs

№	first property	second property
1	market cap $> 10\%$	$\sigma_{\mathrm{X}} \notin A$
2	market cap $> 10\%$	$\sigma_{\mathrm{X}} \notin B$
3	$5\% <$ market cap $\leq 10\%$	$\sigma_{\mathrm{X}} \notin A$
4	$5\% <$ market cap $\leq 10\%$	$\sigma_{\mathrm{X}} \notin B$
5	market cap $> 10\%$	$\sigma_{\mathrm{X}} \notin C$
6	market cap $> 10\%$	$\rho_{\mathrm{XM}} < 0.8$
7	$5\% <$ market cap $\leq 10\%$	$\sigma_{\mathrm{X}} \notin C$
8	$5\% <$ market cap $\leq 10\%$	$\rho_{\mathrm{XM}} < 0.8$
9	$1\% <$ market cap $\leq 5\%$	$\sigma_{\mathrm{X}} \notin A$
10	$1\% <$ market cap $\leq 5\%$	$\sigma_{\mathrm{X}} \notin B$
11	$0 <$ market cap $\leq 1\%$	$\sigma_{\mathrm{X}} \notin A$
12	$0 <$ market cap $\leq 1\%$	$\sigma_{\mathrm{X}} \notin B$
13	$1\% <$ market cap $\leq 5\%$	$\sigma_{\mathrm{X}} \notin C$
14	$1\% <$ market cap $\leq 5\%$	$\rho_{\mathrm{XM}} < 0.8$
15	market cap $> 10\%$	σ deviates significantly
16	$5\% <$ market cap $\leq 10\%$	σ deviates significantly
17	$1\% <$ market cap $\leq 5\%$	σ deviates significantly
18	$0 <$ market cap $\leq 1\%$	$\sigma_{\mathrm{X}} \notin C$
19	$0 <$ market cap $\leq 1\%$	$\rho_{\mathrm{XM}} < 0.8$
20	$0 <$ market cap $\leq 1\%$	σ deviates significantly
21	market cap $> 10\%$	
22	$5\% <$ market cap $\leq 10\%$	
23	$1\% <$ market cap $\leq 5\%$	
24	$0 <$ market cap $\leq 1\%$	
25	market cap $= 0$	

Table 6.2: In the first column the order of ranking of the groups is listed, in the second column the size of the market cap is listed and in the third column a second property to belong to the group is listed.

№	sort on	order if multiple classes fulfill the property
1	significant deviation σ_X	classes with significant F-test have a higher ranking than classes without significant F-test
2	correlation	X is higher than Y if $\rho_{\mathrm{XM}} < \rho_{\mathrm{YM}} < 0.8$ and $\lvert\rho_{\mathrm{XM}} - \rho_{\mathrm{YM}}\rvert > 0.1$
3	deviation σ_{X} vs. σ_{M}	X is higher than Y if $\lvert\sigma_{\mathrm{X}} - \sigma_M\rvert > \lvert\sigma_{\mathrm{Y}} - \sigma_M\rvert$ and $\lvert\lvert\sigma_{\mathrm{X}} - \sigma_M\rvert - \lvert\sigma_{\mathrm{Y}} - \sigma_M\rvert\rvert > 0.2$
4	market cap	X is higher than Y if $\mathrm{MarketCap}_{\mathrm{X}} > \mathrm{MarketCap}_{\mathrm{Y}} >$ 10% and $\lvert\mathrm{MarketCap}_{\mathrm{X}} - \mathrm{MarketCap}_{\mathrm{Y}}\rvert > 10\%$
5	highest deviating CVaR	X is higher than Y if $\lvert\mathrm{CVaR}_{\mathrm{X}} - \mathrm{CVaR}_{\mathrm{M}}\rvert > \lvert\mathrm{CVaR}_{\mathrm{Y}} - \mathrm{CVaR}_{\mathrm{M}}\rvert$ and $\lvert\lvert\mathrm{CVaR}_{\mathrm{X}} - \mathrm{CVaR}_{\mathrm{M}}\rvert - \lvert\mathrm{CVaR}_{\mathrm{Y}} - \mathrm{CVaR}_{\mathrm{M}}\rvert\rvert > 0.3$
6	highest market cap	

Table 6.3: The ranking of SACs within a group is determined by these properties in the order listed in the second column. In the third column conditions to order the SACs are stated.

№	Condition
1	$\sigma \notin A$
2	$\sigma \notin B$
3	$\sigma \notin C$
4	$\rho_{\mathrm{XM}} < 0.8$
5	significant deviation σ
6	highest deviating σ
7	highest deviating CVaR
8	lowest correlation

Table 6.4: Sorting conditions for asset classes for which the market cap is not available.

than the SACs without. The correlation is the next condition: the SACs which fulfill the condition $\rho_{\mathrm{XM}} < 0.8$ are sorted, with the lowest correlation having the highest rank, since a lower correlation means that the benchmark M and SAC X are less dependent. However, when two SACs, let us name them X and Y, both have a correlation coefficient that is lower than 0.8 but differ less than 0.1 from each other, i.e. $\lvert\rho_{\mathrm{XM}} - \rho_{\mathrm{YM}}\rvert < 0.1$, then this difference is not sufficiently large to decide that the SAC with the lowest correlation should be ranked higher and then the next condition in table 6.3 is considered. The remaining conditions in the table have similar requirements. When all factors we have named above are considered, but none of them gives a definite answer on the ordering of the SACs, the size market cap gives the final decision, no matter the size of the difference.

From some of the dataseries it is not possible to get the market capitalization. This is dealt with by using the list in table 6.4, which is a combination of the tables 6.2 and 6.3. The additional requirements that are listed in table 6.3 are also transferred, although they are not specifically named.

When the SACs are all ranked, the next step is to determine the size of the final tree. Then the lowest ranked SACs can be eliminated. This will be further discussed in section 7.2

This exact classification system is not based on any literature, because no specific information could be found. Also, it is unlikely that there is just one suitable system, every bank has its own preferences, and this list of criteria can easily be adjusted. This list of criteria is just a way to order the SACs.

Implementation

In theory, each asset class should have one ranking, however, in some asset classes a clear distinction in type of securities could be made. For example in the asset class *Government Bonds*, these bonds can be subdivided into nominal and index-linked bonds. To prevent that, all index-linked bonds get a high ranking and the nominal bonds a low ranking, and therefore are possibly eliminated, the index-linked and nominal bonds are given two separate rankings. Not all SACs are included in the ranking. This is explained using the same example, the SACs'*nominal world* and *index-linked world* bonds are not taken up in the ranking, because if they end up at a low ranking, there is a chance that they are eliminated. When they are eliminated, there would be no representation for the bonds from those SACs.

Of each SAC a historical benchmark data series is collected. These data series come from different data providers such as MSCI (equity), Barclays (government bonds, credits) and IPD (real estate). Of each series the necessary measures are calculated. These measures are: standard deviation, CVaR and correlation and the outcome of the F-test (both with respect to the 'parent' SAC). Of all benchmarks except for the direct real estate benchmarks, monthly data are available. For the direct real estate SACs only quarterly or yearly data are available. For the ranking a minimum of 10 data points should be available, otherwise the information is not trustworthy.

The measures are calculated for two different time windows: 2009-2012 and 2002-2011. For both windows the rankings are determined. The difference in these rankings gives information about the time-sensitivity of the mapping: when the rankings differ significantly, it is important to update the mapping periodically. When the mapping barely changes, there is no acute need for this. Another positive result of a non time-sensitive mapping is that the use of the ex-post determined mapping does not conflict with the ex-ante use of the mapping, since the mapping is more likely to be constant over time.

The final rankings are included in appendix C and the results are discussed in section 8.2.

Chapter 7

Risk budget, testing and the final step

In this chapter the last three steps of the method, which were defined in chapter 5, are explained in detail. In the first section the method to determine what risk is acceptable is introduced. In the second section the mapping is tested with ex-post data. In the final section, the last and most important step is discussed: the comparison of the SAA with the actual portfolio with ex-ante data.

7.1 Risk measures and risk budget

After the process to determine a proper mapping is defined. A method to find a bandwidth in which the acceptable values for the risk measures fall, from now on called risk budget, should first be determined. Once the method to determine a risk budget is defined, it can be applied to each sub-asset class (SAC) individually. This risk budget can be used for two purposes. First to determine the optimal size of the mapping structure by comparing the risk of mappings with different sizes, based on ex-post data. The other purpose is to compare the mapping and the SAA based on ex-ante (and possibly also ex-post data, since the ex-post and ex-ante data series contain similar data). An overview of the possible use of ex-post and ex-ante data is presented in figure 7.1.

In this section the method to determine the risk budget will be discussed. Since for both comparisons the same method will be applied, for the remainder of this chapter we consider a comparison of the risk measures, the CVaR and standard deviation, between two data series: series X and Y, where series X is the 'benchmark' series and Y is the series of which the deviation from X should be determined. When a reference method is established, it will be applied to both the ex-post and the ex-ante comparisons in sections 7.2 and 7.3, respectively.

In chapter 2, several risk measures were introduced and explained. The risk measures that we will use are the CVaR and the standard deviation. When these measures are used to determine the risk of series X and Y, there are several ways to determine for which values it should be concluded that Y deviates too much from X:

1. absolute values,

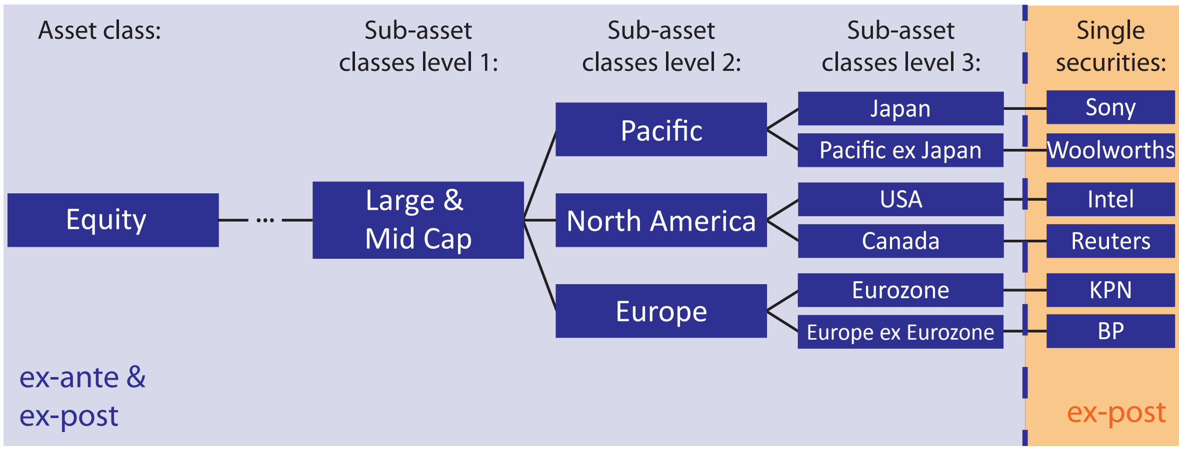


Figure 7.1: Overview of comparisons.

2. relative values,
3. statistical deviation.

An absolute bandwidth means that constants c_{CVaR} and c_σ are chosen, and the risk measures of series Y, $\mathrm{CVaR}(Y)$ and $\sigma(Y)$, should stay within a bandwidth of c_{CVaR} and c_σ from $\mathrm{CVaR}(X)$ and $\sigma(X)$ respectively. Formally this is written as:

$$(7.1) \qquad |\mathrm{CVaR}(X) - \mathrm{CVaR}(Y)| < c_{\mathrm{CVaR}} \quad \text{and} \quad |\sigma(X) - \sigma(Y)| < c_\sigma.$$

A disadvantage is that when data series change over time, and for example increase significantly over time the bandwidth will not correct itself. Therefore a relative bandwidth of p percent, formally written as in formula (7.2) would seem more convenient. An example of the properties of relative and absolute bandwidths is shown in figure 7.2. The blue shaded area represents a relative bandwidth and the orange shaded area represents an absolute bandwidth.

$$(7.2) \quad |\mathrm{CVaR}(X) - \mathrm{CVaR}(Y)| < p_{\mathrm{CVaR}} \cdot \mathrm{CVaR}(X) \quad \text{and} \quad |\sigma(X) - \sigma(Y)| < p_\sigma \cdot \sigma(X)$$

This manner of defining a risk budget seems basic, but to be able to set an appropriate bound, knowledge on the subject is necessary and even when this knowledge is present, determining the bounds will still be subjective. The solution to this problem is to turn to statistical tests.

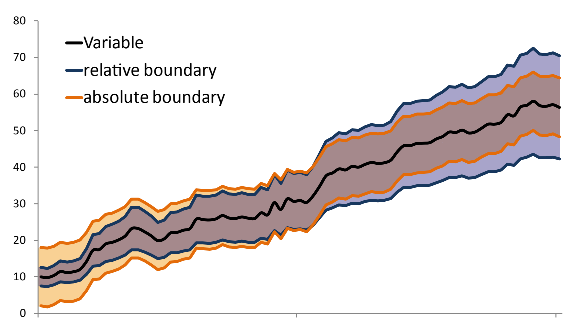


Figure 7.2: A data series with both a relative (blue) and absolute (orange) bandwidth.

In chapter 4 different types of statistical tests were introduced and the possibility of using statistical tests was also already discussed in section 6.3. A simple F-test was used to determine whether the standard deviation of a benchmark and its 'parent' benchmark were significantly different or not. These statistical tests can also be used when a risk budget has to be determined.

BM	-1,22	-1,13	-1,89	0,22	-0,95	1,37	1,3	0,52	-0,89	1,2
	-0,85	-0,47	0,74	-0,89	0,54	-0,52	-0,61	0,52	1,53	-1,41
SS	0,58	0,04	0,11	0,67	0,29	-0,12	0,13	1,04	-0,76	1,23
	0,84	-0,06	0,34	-0,51	1,52	1,34	2,36	0,02	-0,48	-0,33

Table 7.1: Data samples with data drawn from normal distributions ($N(0,1)$ and $N(0.2,0.8)$, rounded to two decimals

When the value of the risk measure of the 'benchmark series' X is taken as the 'true' value $\text{Risk}(X)$, where the function $\text{Risk}(X)$ is the risk defined as:

$$\text{Risk}(X) = \begin{cases} \text{CVaR(X)}, & \text{when the CVaR is taken as risk measure;} \\ \sigma(X), & \text{when the standard deviation is taken as risk measure,} \end{cases} \tag{7.3}$$

then the null and alternative hypothesis are formulated as follows:

$$H_0 \ : \ \text{Risk}(Y) = \text{Risk}(X) \qquad H_1 \ : \ \text{Risk}(Y) \neq \text{Risk}(X). \tag{7.4}$$

When a proper statistical test is chosen, the critical values at a certain significance level α of the test can act as the risk budget for benchmark series X. Let us illustrate this by an example. Consider the normally distributed data samples listed in table 7.1, where BM is the benchmark series and SS the single security series. When we take the standard deviation as a risk measure then we have sample standard deviations $S_{BM} = 1.045$ and $S_{SS} = 0.788$, the hypotheses will be:

$$H_0 \ : \ \sigma_{SS} = S_{BM} = 1.045 \qquad H_1 \ : \ \sigma_{SS} \neq S_{BM}. \tag{7.5}$$

Since the data are distributed normally, we can use an F-test to see whether H_0 can be accepted. This test was introduced and explained in chapter 4. The F-statistic is calculated, as

$$F = \frac{S_X^2}{S_Y^2} = \frac{1.045^2}{0.788^2} = 1.761, \tag{7.6}$$

since both samples have size $n = 20$, we have to consider the F-distribution with $n-1 = 19$ and $n-1 = 19$ degrees of freedom. When we take significance level $\alpha = 0.05$, the upper level, from which H_0 will be rejected is $f_{0.975,19,19} = 2.526$, the lower level, until which H_0 will be rejected is $f_{0.025,19,19} = 0.396$. Since $0.396 < F = 1.761 < 2.168$, H_0 will not be rejected.

Now it is assumed that the two sample standard deviations are not statistically different. However, we wish to know the risk budget of BM for significance level $\alpha = 0.05$ and we know that $S_{BM} = 1.045$. We wish to find the values of S_{SS} for which H_0 gets rejected, or:

$$\begin{cases} F & > & 2.526 \\ \frac{S_{BM}^2}{S_{SS}^2} & > & 2.526 \\ \frac{1.045^2}{S_{SS}^2} & > & 2.526 \\ S_{SS} & < & 0.658 \end{cases} \quad \text{and} \quad \begin{cases} F & < & 0.396 \\ \frac{S_{BM}^2}{S_{SS}^2} & < & 0.396 \\ \frac{1.045^2}{S_{SS}^2} & < & 0.396 \\ S_{SS} & > & 1.661. \end{cases} \tag{7.7}$$

So we have a risk budget for the BM series with significance level $\alpha = 0.05$, namely $[0.658, 1.661]$. This example was just to illustrate the concept of creating a risk budget based on a significance test. In the next sections this concept will be worked out further for the risk measures standard deviation and CVaR.

Standard Deviation

In the example a test for the standard deviation was already used. The difference with the real world case is that the data will not be drawn from a normal distribution and, as was mentioned before, for financial data it is not generally true that the data are normally distributed, which would imply that the F-test cannot be used. However, as in section 4.2 is mentioned, when the data samples are large, the F-test can be used.

Because of the necessity of a large dataset, the data samples that are going to be used should contain daily data, from at least 2 months year, this implies datasets of at least around 60 data points, large enough to pursue an F-test.

CVaR

In chapter 4 the test to be used to detect statistically different CVaR's was presented. The result was that the sampling distribution method should be used.

First the empirical bootstrap is used to find the sampling distribution of the mean of the 5% lowest values. Then the $\alpha\%$ confidence interval of this sampling distribution is created. This confidence interval is the risk budget of the CVaR.

7.2 Test the mapping at different levels

As a result of the previous steps in the process a mapping with a priority ranking of all sub-asset classes is available and for each benchmark a standard deviation and CVaR risk budget can be constructed.
The next matter in defining a proper mapping structure is the decision on the number of SACs that should be present up in the final structure. The final number of SACs that the structures should contain is a trade-off between calculation time and accuracy of the mapping. The tree should be sufficiently detailed to capture the development of each single security. However, the idea behind the mapping is to reduce the number of items that should be modeled, so the number of SACs should be restricted in some way. Although the number of asset classes should be restricted, one must keep in mind that *each* security should be mapped to a proper SAC. An important requirement of this tree is thus completeness.
Keeping this in mind, there are several strategies to determine the optimal structure:

1. Determine an optimal tree, without any restriction the in number of SACs.
2. Impose a restriction on the total number of SACs: find the optimal tree and cross off the least distinctive SACs that fall outside the restricted amount.
3. Impose a restriction on the total number of SACs per asset class (i.e. government bonds, equity, etc.): find the optimal tree per asset class and remove within this class the least distinctive SACs. The restricted number of SACs can be different for the different asset classes.

Of these three strategies the first one is not desirable, since it ignores the fact that the number of SACs should be restricted to keep the model manageable. The third strategy will take more effort compared to the second strategy, because each asset class must be studied to understand

Figure 7.3: Paths the portfolio containing the data series will walk through.

what a reasonable number of asset classes is, while the quality of the result may not differ that much. The second strategy will be used.

Because it will be too much work to generate results of the mapping for each number of SACs in a tree, 5 different mapping levels are introduced. The different levels of mappings are 0%, 25%, 50%, 75% and 100%. The 0% mapping takes only the minimum number of sub-asset classes into account, the 100% mapping takes all sub-asset classes into account, the 25% mapping takes the 25% sub-asset classes into account that have the highest ranking with the criteria formulated in section 6.3, and so on.

This step assesses which level of mapping is sufficient to accurately represent a portfolio.

In order to compare the mapping on different levels with the portfolios, of each SAC the data series of several single securities as well as the data series of the benchmarks of the SACs are obtained. First several portfolios will be put together with these single securities. The composition of these portfolios can be done both randomly and according to different strategies, such that for example both well-diversified but also poorly diversified portfolios are obtained. An example of such a strategy is to give the portfolio weights that are representative for the market cap.

Each of these portfolios will pass two paths, represented in figure 7.3. The upper path represents the path where the risk and risk budgets (of both risk measures, CVaR and standard deviation) will be determined on a single security (SS) level, the lower path represents the path where the risk will be determined by the mapping, so on a sub-asset class (SAC) level. Finally, the paths connect and the question whether the risk of the mapping falls within the risk budget of the portfolio is answered.

As a result of each portfolio it is determined which levels of mapping fall into the risk budgets. The final level of the mapping, which can be determined per asset class, will be the lowest level that includes for the market cap portfolio within the risk budget. If for this portfolio the risk of the 75% and 100% mapping falls within the risk budget, the 75% level will be chosen, since the computation time will be lower with the 75% mapping.

Since of single securities only ex-post data are available, this whole process is executed with ex-post data.

Implementation

First the implementation details of the upper path of figure 7.3 are discussed, followed by the implementation details of the lower path. As mentioned in section 6.2, this step is only executed for the asset class *equity*. Also, the data of the same single securities are used as in the second step, so the data series of 78 stocks are available.

Since it is not possible to get data on real investment portfolios of private banks, portfolios have to be put together. An advantage of putting the portfolios together, is that certain properties can be assigned to the portfolios, such as degree of diversification. Another advantage is that as many portfolios as desired can be constructed.

Apart from one portfolio, which will be representative for the market capitalization of the market, the portfolios are put together randomly. The weight of a security is determined as follows: a random draw from the $Ber(0.5)$-distribution is multiplied by the absolute value of a random draw of the $N(0,1)$-distribution. When the weights of all securities are determined, they are normalized, such that the weights sum up to 1. The Bernoulli draw will ensure that not every security is contained in each portfolio and the $N(0,1)$-draw determines the weight and ensures that more securities have a small contribution in the portfolio than a large contribution. Of course, the parameters of the distributions and even the method to put together the portfolios can be chosen differently, but this is not relevant at the moment.

The number of portfolios that are put together is not fixed. For the time being the number of portfolios is set to 25, since this number gives a good variety of portfolios, and also ensures a reasonable calculation time. The final part of the upper path consists of determining the standard deviation and CVaR risk budget of each with the help of respectively the F-test and the sampling distribution method, which are explained in chapter 4. These risk budgets can be determined with different significance levels $\alpha \in \{0.1, 0.05, 0.025, 0.010, 0.005, 0.0025\}$.

The path of the mapping in figure 7.3 is implemented as follows. Of each portfolio the mapping is determined. This mapping is determined on different levels, to compare the results and see what level is necessary for a proper mapping. Which SACs are excluded from the mapping, is determined by the ranking. When an asset class has 12 ranked SACs, in the 75%-mapping the SACs with ranking 1 through 9 are admitted and SACs with ranking 10 through 12 are excluded. Of each mapping the CVaR and standard deviation are determined.

The final step is to check whether the risk of the mapping falls into the risk budget of the accompanying portfolio. The decision what level of mapping is optimal, is made by looking at the lowest percentage mapping that will fall into the risk budget of the actual portfolio. The market cap portfolio is the most important portfolio for this check, since that portfolio is the most representative, the other portfolios will give additional information. The results of this step are listed in appendix D and discussed in section 8.3.

7.3 Mapping vs. SAA

All previous steps are preparatory for this last step. These steps are processed only once and produce required input for the last step. The last step will be executed by a portfolio manager for every client, while the previous steps are executed by a model implementator. This step can be executed with both ex-ante and ex-post data, however the strength of the process is that with the help of the mapping, ex-ante scenarios from a Monte Carlo scenario generator can be used to determine the ex-ante deviation. Therefore all data used in this step are ex-ante data, produced by the Monte Carlo scenario generator.

Figure 7.4: Paths the SAA, portfolio and mapping will walk through.

The goal of this step is to determine whether the risk of a portfolio of a client, deviates significantly from the risk of the strategic asset allocation (SAA). For this step the input is three-fold: first of all the strategic asset allocation (SAA) of the client, second the current portfolio of the client, and finally the mapping. A schematic representation and examples of the mapping are shown in figure 5.2, which was introduced and explained in chapter 5. The mapping is fixed for each client, while the SAA and the current portfolio vary per client. When the three components are given, the final step can be represented by figure 7.4.

As is illustrated by the upper SAA-path in figure 7.4, first the risk and risk budget of the SAA are determined, the concept of risk budget was introduced in section 7.1. In the lower portfolio-path, first, the current portfolio will be mapped with the help of the mapping, and then the risk of the mapped portfolio will be determined. The final check is whether the risk of the current portfolio falls within the risk budget of the SAA. If it does, a positive signal will be transmitted, if it does not a warning will be transmitted.

This procedure will be repeated for both CVaR and standard deviation, to give a complete insight in the status of the portfolio.

Implementation

First the implementation of the SAA-path of figure 7.4 will be discussed, the implementation of the portfolio-path will be discussed. The SAA is defined on asset class level and, as explained in section 1.2, the SAA represents the risk appetite of the client. An example of an SAA is listed in table 7.2.

First the risk budget of the SAA needs to be determined This is done with the help of the Monte Carlo scenario generator. Of every asset class the benchmark is modeled, which leads to 1000 scenarios per asset class. The scenarios have a horizon of over 32 years and the data have a monthly frequency. The standard deviation and CVaR risk budget of an asset class are determined with the help of the scenarios and will be determined for different time intervals: 6 months, 1 year, 3 years, 5 years, 10 years, 20 years and 30 years. In order to do this, each scenario of the SAA should be calculated separately, with the weights of the SAA. Then, at each point in time, the standard deviation and CVaR and their risk budgets of the log returns can be determined. The properties of the log return are discussed in chapter 2. This can be done on several different significance levels: $\alpha \in \{0.1, 0.05, 0.025, 0.010, 0.005, 0.0025\}$.

The portfolio-path is implemented as follows. First the mapping of the portfolio will be determined and of all SACs which occur in the selected mapping the benchmarks are modeled with the Monte Carlo scenario generator. Then of each SAC the i^{tinyth} scenario is summed according to its weight in the mapping. Since there are 1000 scenarios generated, this is repeated for each $i = 1, \ldots, 1000$. For example, when the securities from figure 5.2 are contained in a portfolio with the following weights: 50% Philips stock, 30% BP stock and 20% Italian index-linked bonds with 3-year maturity. Then each scenario of their SACs – equity Eurozone large cap, equity Europe ex. Eurozone large cap and govt. bonds, ILB, Italy maturity 1-10 years– are summed with these weights. This results in 1000 scenarios which are representative for the mapping of the portfolio.

Finally, at each point in time the CVaR and standard deviation of the log returns of the scenarios are determined. The measures should be calculated with the same time horizon as the risk budgets. The final step is to check whether the mapping falls within the risk budget.

For this thesis this step is executed with one random selected portfolio, which are put together by randomly selecting SACs, in principle, securities should be selected, but since the single securities are immediately mapped to the SACs, We immediately generate SACs. The probability of selecting a SAC from a certain asset class is fixed, but within the asset class the probability of selecting the SACs is equal. The results are discussed in section 8.4.

Asset class	**percentage**
Equity	40%
Govt. Bonds	30%
Corp. Bonds	20%
Real estate	5%
Hedge funds	2%
Commodities	0%
Private equity	2%
Alternatives	0%
Cash	1%

Table 7.2: Strategic asset allocation.

Part III

Results and Conclusion

Chapter 8

Results

In this section the results of the different steps are discussed.

8.1 Validation and robustness of mapping

In this section the results of the test to check whether the distribution of portfolios containing single securities from one sub-asset class (SAC) originates from the same distribution as the benchmark of that SAC. The implementation of this step was discussed in section 6.2. As was discussed, this test is only pursued for the asset class *equity*. The test is based on ex-post data, since there is not ex-ante data available on single securities.
The null and alternative hypothesis of the Kolmogorov-Smirnov test that is executed are:

$$\begin{aligned} H_0 &: BM_i(x) = PF_{i,j}(x) \qquad \text{for all } x, \\ H_1 &: BM_i(x) \neq PF_{i,j}(x) \qquad \text{for all } x, \end{aligned} \tag{8.1}$$

where $BM_i(x)$ is the distribution function of the benchmark of SAC i and $PF_{i,j}(x)$ is the distribution function of portfolio j of SAC i. When the null hypothesis is rejected, portfolio j of SAC i and the benchmark of SAC i do not originate from the same distribution.

The Kolmogorov-Smirnov test is executed for different significance levels and for different time horizons. We have chosen to show the results for three different periods, namely, 6 months, 3 years and 7 years, because more periods do not give necessarily more information. In appendix B, for all SACs and different periods, the percentage of portfolios for which the null hypothesis was *rejected*, is listed (100% = 126 portfolios).

6 months When we take a glance at the data of time horizon 6 months, we see that for all except 3 SACs the null hypothesis is accepted at every significance level. When executing this test for 6 month intervals in different years, we see similar results. The SAC *LC Eastern Europe* has the highest percentage portfolios for which the null hypothesis is rejected, as at the 0.1 significance level, 48% of the portfolios do not originate from the SAC benchmark distribution. The cause of this result can be retrieved from table A.1. For this SAC only three valid data series of single securities could be found, all three of them from the same country (Russia). Therefore, the portfolio is not sufficiently diversified to originate from the same distribution as the benchmark.

At significance level 0.1, the distribution of 16% of the portfolios of the SAC *SC Canada* do not originate from the same distribution as the SAC benchmark. From the detailed results it can be observed that in all portfolios for which the null hypothesis is rejected, 80% or 100% of the portfolio is 'invested' in the same two single securities, which deviate relatively strong from the benchmark. Because only weights which are a multiple of 0.2 are assigned to the single securities, the rejected portfolios all contain a maximum of three single securities. For the portfolios containing more single securities (four or five), the null hypothesis was not rejected. We might say that the influence of two relatively strong deviating single securities is canceled out when the portfolio is well diversified, i.e. contains more than three single securities.
In SAC *LC Eurozone*, for one portfolio, the null hypothesis is rejected at level $\alpha = 0.1$. This is the portfolio, only containing one single security: KPN.

The higher level SACs, *LC Emerging Markets*, *LC North America* and *LC Europe*, all have 0% rejected portfolios. This is not a surprise, as these portfolios contain at least 10 single securities, which is more than the lower level SACs, which contain a minimum of 5 single securities. The influence of the single securities from Eastern Europe (contained in *LC EM*) is canceled out, because of better diversification.

3 years The results of the time horizon of 3 years show more rejections. At the 0.1 significance level, for on average for 18% of the portfolios, the null hypothesis is rejected. The distributions of these portfolios do not originate from the same distribution as the benchmark. At the 0.0025 level, for on average for 8% of the portfolios, the null hypothesis is rejected.
The fact that the general level of rejection is higher could be explained by the increase in the sample size. The Kolmogorov-Smirnov test statistic is the largest absolute distance between the ordered datasets. The 6 month datasets contain 132 data points, the 3 year datasets contain 783 data points and the 7 year dataset contains 1827 data points. In 3 years, more information is available than in 6 months, therefore it is more likely that a large difference occurs, with a longer time horizon. On the other hand, the statistic is corrected for the sample size, by multiplying the statistic with the term $\sqrt{\frac{2n}{n^2}}$. So, the fact that the data sample is larger is corrected, however, not sufficiently.

It is rather difficult to find a pattern in the portfolios for which the null hypothesis is rejected, these portfolios are both well-diversified and poorly-diversified portfolios. However, the SACs for which the percentage of rejections lie above the average, are all small cap or emerging market SACs, or both. Of the small caps and emerging markets few data were available, so the few data series that were available needed to be selected, which possibly lead to biased data. Also, within the small cap and emerging markets SACs the diversity of the various securities is larger. Therefore, the SACs need to contain more different securities in order for them to originate from the same distribution as the benchmark.

The higher level SACs performed much better, in all higher level SACs, none of the portfolios were rejected.

7 years The 7 years time horizon gives even more different results. On average, per SAC, for 56% of the portfolios, the null hypothesis is rejected at the 0.1 significance level. At the 0.0025-level, the average is 33%. A rather surprising result is that for SAC *LC Eurozone*, for

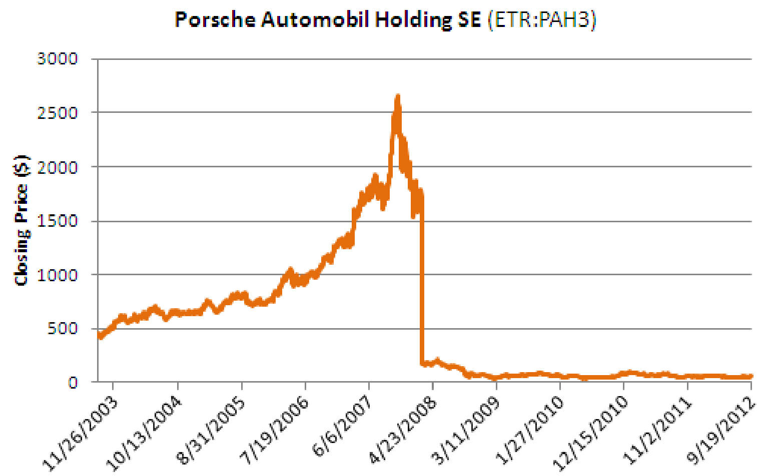


Figure 8.1: Closing prices of Porsche Automobile Holding shares in USD.

all portfolios, the null hypothesis is rejected at the highest significance level. When looking at a lower significance level, all portfolios that are accepted do not contain the single securities Porsche Automobile Holding and Koninklijke BAM Groep. When taking a closer look at the data series of these two securities, it can be seen that both series have a large drop in stock price at some point in time more than 3 years ago. In figure 8.1, the development of the stock price of the Porsche share is shown.

Also with a longer time horizon, the higher level SACs perform better. In the SAC *North America*, the null hypothesis is still not rejected for any portfolio. In the SAC *LC Emerging Markets* for some portfolios the hypothesis is rejected, but on a lower level than the average SAC, also, there are no data available for the SAC *LC Eastern Europe*, therefore this SAC is less diversified.
The SAC *LC Europe* is an interesting case. For 93 % of the portfolios, the null hypothsis is rejected, however, in these portfolios, the securities Porsche and BAM are contained. The 7 % portfolios for which the null hypothesis is accepted are those portfolios in which the single security Porsche was not contained. So the drop in the value of Porsche shares, shown in figure 8.1, is too significant to be compensated by the other securities, however, the drop in the value of the share BAM is absorbed.

When a different interval is chosen for the 3 year horizon, the average rejection level is 34% at the 0.1 significance level, this is higher, but considering that in this interval major drops in prices of shares Porsche and BAM occur, this is not a strange thing. For the 6 month horizon 2 alternative intervals were tested, but these test gave equivalent results as the first interval: for almost all SACs for 0% of the portfolios the null hypothesis is rejected.

Conclusion The general results are that when looking at a shorter time horizon, 5 single securities can be represented by the SAC benchmark. But when looking at a longer time horizon, the percentage of portfolios containing 5 securities for which the null hypothesis is rejected is much higher. When the number of securities in a portfolio is increased to 10, also with a longer time horizon, the rejection level is lower, because the portfolio is more diversified. A lower rejection level means that the benchmark represents the securities properly. Having more securities in each SAC will therefore improve the quality of the mapping.

For this specific example this results in the conclusion that although a SAC is distinct from its parent benchmark (i.e. has a high ranking), when looking at a longer time horizon, one is still forced to use the parent SAC. Since in that SAC more securities are contained, so that the distributions of the portfolio and the benchmark originate from the same distribution.

For a portfolio with more than 10 single securities, one single security with a high risk can still cause the null hypothesis to be rejected. This might imply that the mapping will not be representative for the portfolio, on the other hand, a single security that has such a large influence, is too risky for a portfolio, or the portfolio is not sufficiently diversified. Therefore this test can also serve as a method to identify single securities in a SAC that are too risky, or to detect a portfolio which is not sufficiently diversified.

8.2 Ranking the SACs

In section 6.3, the method to determine the ranking of the SACs is described. In appendix C the results are listed: for every asset class two rankings are shown, one for the period 2002-2011 and one for the period 2009-2012. As mentioned before, the highest level SACs are not admitted in the ranking, because these SACs are required in order to be able to map every single security to a SAC. These SACs are in the appendix marked by 'NR'.

The ranking of the SACs do not remain completely constant over time, but the top and bottom parts of the list, mostly do. The ranking for the asset class corporate bonds, remains exactly constant. We will address the separate asset classes shortly.

Equity When looking at developed markets, no clear pattern can be discovered. The top of the list remains the same for the different periods, but the SACs *LC Europe* and *SC Europe* move –together– from the bottom to the top of the list. In period 2002-2011 they differ from the parent benchmark, but in period 2009-2012 they do not deviate that much anymore, so the parent SAC is more important. On average every SAC moves (due to these sub-asset classes) 2.5 places in the ranking. The emerging markets ranking does not change at all.

Government Bonds As mentioned in section 6.3, the rankings of the nominal and index-linked bonds are separated. In the ranking listed in table C.2 it can be seen that for Europe, the SACs which are differentiated by maturity, are generally higher ranked, than the region SACs. To illustrate: Eurozone maturities > 10 years, 1-3 years, 7-10 years and < 5 years all have for period 2002-2011 higher ranking than all separate Eurozone countries. This effect also holds for 2009-2012, but not as clear as for the period 2002-2011. Considering the current sovereign debt crisis, this is not a surprising result. Since the maturity-related and geographical-related SACs overlap completely, either the maturity-related *or* the geographical-related SACs should be represented in the final mapping structure. Because in both rankings, the maturity effect dominates, the separate European countries are not represented in the final mapping structure.

For index-linked bonds, some SACs also overlap, for example, for the UK, data on the following SACs are available: maturities $1 - 10$ years, > 10 years, > 5 years and $7 - 10$ years. Since maturities $1 - 10$ years and > 10 years have a higher ranking than the others (for both periods), and these SACs do not overlap, the SACs with maturities > 5 and $7 - 10$ are not

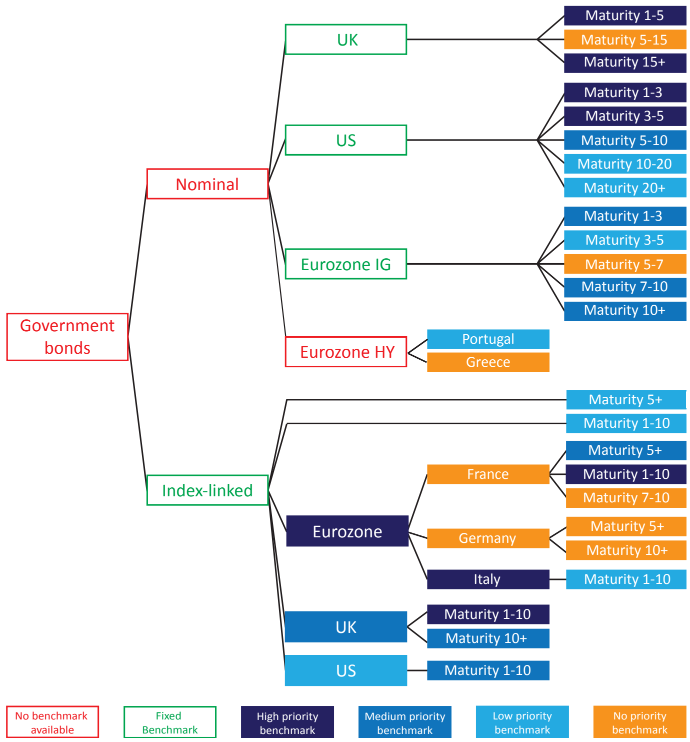


Figure 8.2: Structure of asset class equity, where the orange shaded SACs have the lowest priority and the dark blue SACs have the highest priority, the unshaded SACs do not have a ranking.

admitted in the final structure. Sometimes, there is overlap between two or more SACs, but these SACs both have high rankings, for example World index-linked bonds with maturities > 5 years and 1-10 years. In principle, the choice should be made between the maturity sets 1-10 years & > 10 years and < 5 years & > 5 years, since both of these sets contain the whole spectrum of available bonds. However, for maturities > 10 years and > 5 years no data were available, therefore the choice to admit both maturities 1-10 years and > 5 is made. The reason for this choice is that when for example only > 5 years is chosen, the bonds with maturity 3 years are mapped to the general index-linked bond SAC and information will be lost. When a bond fits in both SACs a choice between these SACs could be made with the help of the ranking. The final structure for government bonds for the period 2002-2011 is represented by figure 8.2.

Corporate Bonds The corporate bonds structure does not change when different periods are considered. Apart from the SAC *HY PanEurope*, also the benchmark *HY Eurozone* was available, however, these SACs did not differ significantly and no HY SAC for the UK was available. Therefore the *HY PanEurope* SAC was chosen, so that more securities are covered.

Real Estate The indirect listed real estate mapping differs when taking different periods into account, between the two periods the ranking indirect listed differs 4.8 positions in the ranking per SAC. This amount is mainly due to the change in position of the SACs *UK*, *Germany* and *Thailand*. The change of the position of *Thailand* is not relevant, since we are looking at the tree from a European, and more specifically Dutch, point-of-view and Dutch investors will not invest a lot in Thailand. When these three SACs are not taken into account, the average position change between the rankings is only 2.5. The origin of these changes lies in the standard deviations. For *Mexico* and *Argentina*, no recent data are available, so these SACs are excluded from the 2009-2012 ranking.

For the direct real estate a mapping for 2009-2012 is not created, since the data have a quarterly or yearly frequency instead of a monthly frequency. For the 2009-2012 mapping the direct real estate SACs would be ranked, based on only 3 data points. However, the fact that the data are only available on such a low frequency may already be an indication that this asset class might not be very appropriate to map. On the other hand, private wealth clients will not invest on a large scale in direct real estate, so the influence of the ranking of this part of the asset class is negligible.

For the indirect unlisted asset class, the data were not available, so no rankings were put together.

Alternatives It is difficult to say something about the changes in the alternative asset classes, since there are very few sub-asset classes. Because of this, there is no need to discard any SACs. The asset class *private equity* forms an exception: since the different SACs overlap, a choice between type of private equity (venture capital or buy out) or regional differentiation should be made. Based on the 2002-2011 ranking, region is chosen, but the period 2009-2012 gives different information. However, since the UK has the highest ranking in the 2009-2012 period and the regions have higher rankings in the 2002-2011 ranking, the SACs venture capital and buy out are not admitted in the final mapping.

8.3 Test the mapping

In order to know how detailed the mapping should be, the risk portfolios based on real ex-post are compared to the risk of its mapping on different levels. In section 7.2 the implementation of the test is discussed. In this section the results of the implementation are discussed. The results are listed in appendix D. In tables D.1 through D.8 the standard deviations and CVaR risk budgets and the standard deviations and CVaRs of the single security portfolio and mappings at different levels are listed, these numbers are calculated with ex-post data of the single securities and benchmarks.

In tables D.1 through D.4, the standard deviation results for four different time horizons (1 month, 6 months, 3 years and 7 years of data) are listed. In tables D.5 through D.8 the CVaR results for the four different time horizons are listed. All values in the tables are multiplied by 1000 to get a clear overview.

Of each table in the second column the standard deviation or CVaR values, calculated as was described in section 7.2, of the actual portfolio are listed. In the third and fourth column, the lower and upper bounds of the risk budgets are listed, the calculation of the risk budget is explained in section 7.1. In the next five columns the standard deviations or CVaRs of the different level mappings (100%-0%) are listed. Finally, in the last column, the standard deviation or CVaR of the SAA is listed, which is in this case the standard deviation or CVaR of the benchmark of asset class equity only. The values which are green, fall into the risk budget. The results in table D.1 through D.8 are calculated at significance level $\alpha = 0.05$.

The first portfolio, *MC PF* is the portfolio for which the weights are representative for the market capitalization of the shares. In figures D.1 through D.4 the risk budgets and the standard deviations or CVaRs of the mappings and SAA are graphically represented.

In table D.9 at each significance level and for each time horizon, the percentage of the 25 calculated portfolios which fall into the risk budget is listed. First the results at the 0.05 significance level of the standard deviation will be discussed, then the 0.05 level results of the CVaR are discussed and, finally, the comparison with the other significance levels is discussed.

Standard deviation When quickly comparing the results for the standard deviation at different horizons, one observes that at shorter time horizons, fewer values fall outside the risk budget. This can be related to the results of the Kolmogorov-Smirnov test in section 8.1. These results implicate that for longer time horizons more than 5 single securities should be contained in each SAC in order for the benchmark to be representative for the single securities. With portfolios that contain on average around 40 securities, this will not the the case. In more detailed mappings (i.e. 100%, 75%, 50% mapping) more SACs are admitted, therefore the SACs represent much less than 5 single securities. According to the results of section 8.1 these mappings should therefore perform worse than the less detailed mappings when a longer horizon is considered. However, this is not confirmed by the results.

When looking at the one month table (table D.1) for all but one portfolio (PF 15) the mappings fall into the risk budget. Only for portfolio 15 the 0% mapping and the SAA do not fall into the risk budget. This can also be observed in the lower graph in figure D.1. In this graph it can also be seen that the 100% mapping does not necessarily perform better than the SAA. However, this result is only based on 23 data points, which is a rather small amount to draw conclusions from.

	1M	**6M**	**3Y**	**7Y**
100%	4	9	8	25
75%	1	2	2	0
50%	1	4	4	0
25%	9	7	6	0
0%	2	3	1	0
SAA	8	0	4	0

Table 8.1: Number of portfolios for which σ of the mapping deviates the least from the actual σ of the portfolio.

The 6 month results in table D.2 also do not give much information. For most portfolios either all mappings fall within the risk budget or all mappings fall outside the risk budget. However, portfolio 18 and 23 exhibit the expected results. The more detailed mappings fall into the risk budget and the less detailed mappings and the SAA outside the risk budget. In the upper graph of figure D.1 it can be seen that the standard deviations of the more detailed mappings are, in general, higher than the standard deviation of the SAA, while the standard deviation of the 50% mapping is mostly lower than the standard deviation of the SAA.

For every portfolio the standard deviation of one of the mappings or the SAA lies closest to the actual standard deviation of the portfolio. In table 8.1 the number of times that each mapping lies closest to the actual standard deviation is listed for each time horizon. In this table we can see that for 9 out of 25 portfolios, the 100% mapping fits best to the actual value of the portfolio when looking at a time horizon of 6 months, the SAA not once. This is the first indication that the 100% mapping performs better than the SAA.

The 3 year results in table D.3 show, again, varying results. The more detailed mappings conform more frequently to the risk budgets, and the 75% mapping performs best: for 11 of the 25 portfolios, the standard deviation of the 75% mapping falls into the risk budget. For 8 of the 25 portfolios, the 100% mapping comes closest to the actual standard deviation of the portfolio.

The 7 year results give a clearer pattern, as the standard deviation of the 100% mapping always is the closest mapping to the true value of the standard deviation. In table D.4 it can be clearly seen that the 100% mapping performs best. The 75% mapping performs second best for 21 of the 25 portfolios. In the top graph of figure D.2, this result is confirmed. It can also be seen that the standard deviations of the more detailed mappings differ more from each other. This is not an unexpected outcome, since different factors (i.e. benchmarks) influence the standard deviation in the 100% mapping than in the 0% mapping.

At least 30 data points are needed to be able to set the risk budget with the help of the F-test and sampling distribution method. Therefore, when monthly data are available, only a 3, 5 and 7 year time horizon are relevant. The results of the 3 year time horizon are not at all clear and the results of the 5 and 7 year results are not as clear as the results for the daily data, but they are pointing out that the 100% mapping performs, again, best. Because the 3 year monthly data do not give a result and the 5 year monthly results do, a requirement of at least 6 months daily data or 60 data points with a lower frequency are needed to be able to give quality statements about the mapping.

We can conclude that the mappings only show evident results for a larger time horizon and the mapping that performs best is the 100% mapping. This mapping contains all SACs that were proposed in section 6.1 and is presented in figure 6.1. From section 8.1 it was concluded that for a larger time horizon, more single securities should be contained in an SAC to give a proper representation of the SAC benchmark. These results conflict. However, the fact that the mapping performs well might indicate that the results of the Kolmogorov-Smirnov test do not influence the mapping, or that the Kologorov-Smirnov test did not give accurate results for longer time horizons. However, we can only test portfolios which have a maximum of 78 securities. Probably, when the portfolios would contain more single securities, the 100% mappings would perform even better, since then the benchmark is an appropriate representation of the single securities.

Another result worth mentioning is that the five portfolios with the highest standard deviations (i.e. portfolios 3, 4, 15, 20 and 22) all have at least 2.5% of the portfolio invested in Porsche.

CVaR First of all, it should be noted that for the CVaR graphs, the absolute values of the CVaRs were taken, so that the right-hand side of the graph indicates a high risk. The tables contain the original values, multiplied by 1000.

The CVaR risk budgets are much smaller than the standard deviation risk budgets, so much less mappings fall into the risk budget. Therefore we mainly take a look at the distance of the CVaR of the mapping to the CVaR of the actual portfolio. In table 8.2 the number of portfolios for which the CVaR of the mapping lies closest to the actual CVaR for all mappings and time horizons, is shown. From that table, it can be seen that for the one month time horizon the 25% mapping performs best. However, for longer time horizons, the 100% mapping performs best. In the graphs in figures D.3 and D.4 it can be seen that the results of the CVaR are equivalent to the results of the standard deviation: with short time horizons, no clear logic can be spotted, but as the time horizon increases, the higher differentiated mappings perform better. Again, the portfolios containing a significant amount of Porsche shares have the highest risk.

The narrow risk budget of the CVaR could be caused by the significance level. In table D.9 the percentage portfolios that fall into the risk budget is listed for all significance levels and all time horizons. It can be seen that, indeed, when the significance level is lower, more mappings fall into the risk budget. However, for the CVaR the change is minimal.
The reason that CVaR risk budgets are narrow is that the CVaR distribution it is calculated with historical simulation and is therefore only based on the 5% lowest values of the datasets. Because the number of different realizations is rather small, the CVaRs that are generated contain the same values. As a result, the standard deviation of the sampling distribution of the CVaR is rather small.

	1M	**6M**	**3Y**	**7Y**
100%	3	10	8	20
75%	3	4	1	3
50%	2	1	6	0
25%	13	7	6	0
0%	3	1	3	0
SAA	1	2	1	2

Table 8.2: Number of portfolios for which the CVaR of the mapping deviates the least from the actual CVaR of the portfolio.

Conclusion For both risk measures with short time horizon, not much can be said about the quality of the mapping. When the horizon increases, the 100% mapping, stated in figure 6.1, clearly outperforms the other mappings and the SAA. This step was modeled such that the computation time was not influenced by the number of SACs. Now computation time only plays a role in generating the scenarios. Since the generation of scenarios is executed each month, quarter or year, depending on the preferences of the customer, the computation time does not influence the problem any more.
This result holds for the asset class *equity* and could hold for the other asset classes, but this must be verified in order to be certain.

For the 7 year time horizon, the 100% mapping performs best, however, the mapping does not fall into the risk budget often. This gap occurs because in each SAC, a maximum of 5 securities are contained. We have seen in section 8.1 that for a longer time horizon more than 5 securities need to be contained in each SAC to create a proper mapping. The 100% mapping does perform best, but might perform better when the portfolio contains more securities. A dynamic mapping which adjusts for the number of securities in each SAC might be a solution.

8.4 Mapping vs. SAA

In this final section the results of the comparison of the mapping of a portfolio and the strategic asset allocation are shown. This step is only executed for one portfolio, since this step only shows the end product of the process and has little implications for the rest of the process. The data that are used in this step are ex-ante scenarios generated by the Monte Carlo scenario generator.

In figure 8.3 the two result graphs of a portfolio that is put together randomly. The graph on the left-hand side shows the standard deviation risk budget. The light blue area shows the risk budget of the strategic asset allocation belonging to the significance level 0.0025, the dark blue area shows the risk budget for the 0.1-level. The green arrow represents the standard deviation of the mapping of the portfolio. The different bars represent the different periods for which the risk budget holds. The graph on the right-hand side gives the same information, but this time, the Subject is the CVaR.

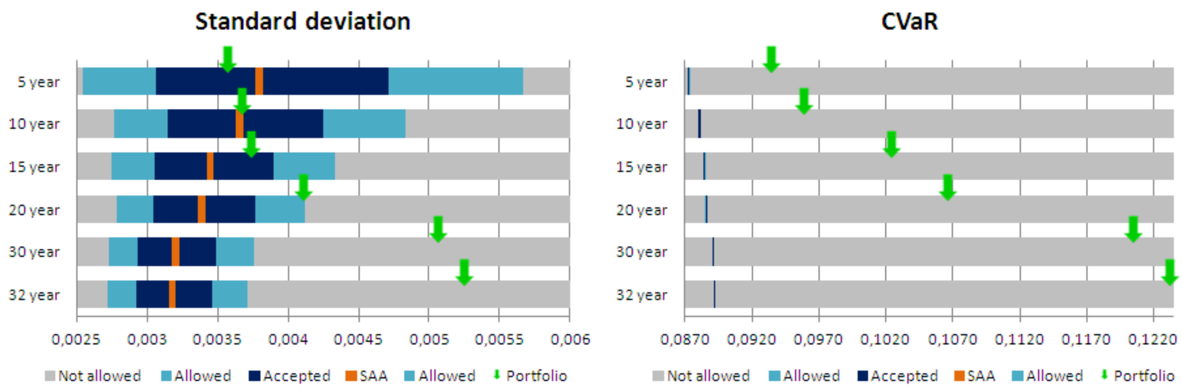


Figure 8.3: Results of the comparison of strategic asset allocation and the mapping of a randomly selected portfolio.

Chapter 9

Conclusion and recommendations

The first and an important conclusion at the end of the first part of this thesis was that both standard deviation and CVaR should be used for a more complete overview of the risk. The standard deviation gives information on the 'body' of the distribution, whereas the CVaR gives information on the tail. This gives a more complete overview on the risk of a security or portfolio.

To calculate the risk budget of the standard deviation and CVaR, two methods were proposed. For the standard deviation the F-test was used. This test worked properly and when analyzing the final results, as it gives a risk budget which can easily be adjusted by using different significance levels. The risk budget of the CVaR was calculated with the sampling distribution method.

A general problem of the use of CVaR is that the distribution of only the tail is considered. When looking at the 5% CVaR, only $1/20^{\text{th}}$ of the datapoints is used and therefore the number of data points is limited. As a result, from a limited number of data points, conclusions should be drawn. The empirical bootstrap is used to generate more data points, but these points are reproductions of the original data. As a result, when calculating the CVaR of these generated data series the CVaRs may resemble each other and the confidence interval will be rather narrow.

The use of a parametric bootstrap, which employs the distribution of the tail would be a solution to this problem. This distribution could be found with the help of extreme value theory, which was introduced in section 2.3. Another solution is to account for a larger standard error in the sampling distribution of the CVaR. This means that the standard deviation of the CVaR distribution, which is given by $\sigma/\sqrt{n}$, is increased with a factor ε, so that the CVaR has a $N(\mu, \frac{\sigma}{\sqrt{n}} + \varepsilon)$ distribution. This standard deviation will increase the risk budget of the CVaR. At the 0.05 significance level, the risk budget of the CVaR was given by $\left[\text{CVaR} - 1.96\frac{\sigma}{\sqrt{n}}, \text{CVaR} + 1.96\frac{\sigma}{\sqrt{n}}\right]$. The adapted interval would on each side be extended by $1.96 \cdot \varepsilon$, to

$$\left[\text{CVaR} - 1.96\left(\frac{\sigma}{\sqrt{n}} + \varepsilon\right), \text{CVaR} + 1.96\left(\frac{\sigma}{\sqrt{n}} + \varepsilon\right)\right]. \tag{9.1}$$

When interpreting the results, one should look at different time horizons. The case of the stock price of a Porsche stock (represented in figure 8.1), is a very helpful example. When

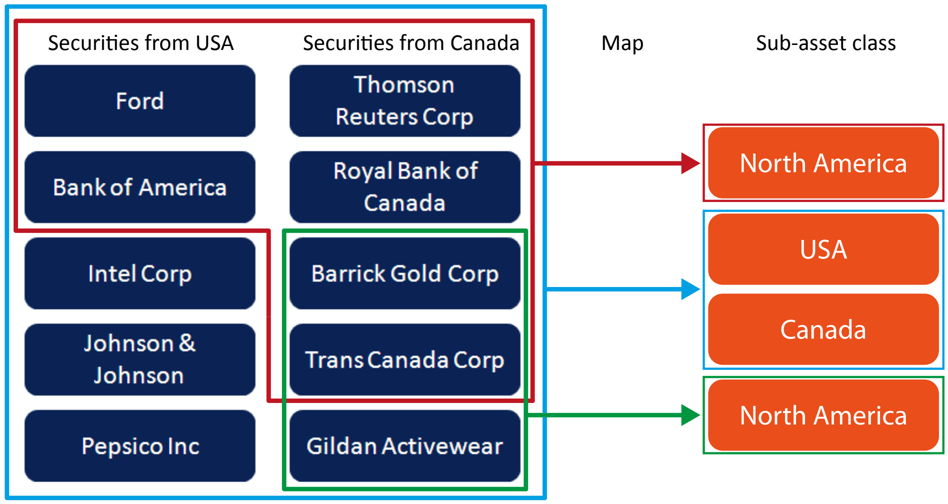


Figure 9.1: Example of a dynamic mapping for SAC North America, Canada and USA.

looking at the last three years, the stock price was fairly constant, so when looking at this period, the stock's performance is not unusual. When looking at the data of more than three years ago, the drop in value is significant, which causes the standard deviation and CVaR to be very high. Risk in the different periods do not give representative information, but together, they give more complete information.

The mapping was created from a Dutch private banker point-of-view, but for different clients, different mappings could be relevant. Also, an important result was that 10 single securities are needed for the mapping to be representative. When not enough securities are contained in each SAC, the deviation of a mapping may be caused by the specific risk of a few single securities. This required number can be higher for SACs like emerging market and small cap SACs or lower for more stable SACs. When the required number of securities in a SAC is determined for each SAC individually, the quality of the mapping can be improved by switching to a higher level SAC when less than the required number of securities are contained in a SAC. This required number of securities can be different when a different time horizon is considered. This dynamic mapping might give better results than a fixed mapping.

An example of the behaviour of a dynamic mapping is show in figure 9.1 for the SACs *North America*, *Canada* and *USA*. Let us assume for the example that five securities per SAC are sufficient for a proper mapping. Each blue square represents a single security. The securities on the left are from the USA, on the right from Canada. The red, blue and green boxes around the securities each represent a portfolio. The securities in each box are the securities that are contained in the portfolio. The blue portfolio contains 10 securities, 5 of each sub-asset class. Therefore, the portfolio is mapped to both the SACs Canada and USA. The red portfolio contains in total 6 securities, which is enough for one SAC, but since the SAC Canada would contain 4 securities and USA only 2, the securities are mapped to *North America*, the parent SAC. The green portfolio contains 3 single securities, all from Canada. The securities are mapped to *Canada* since no securities from the USA are contained in the portfolio and therefore a minimum of 5 never will be reached. Because there are too little securities in the portfolio, a signal should be given that the portfolio is not sufficiently diversified.

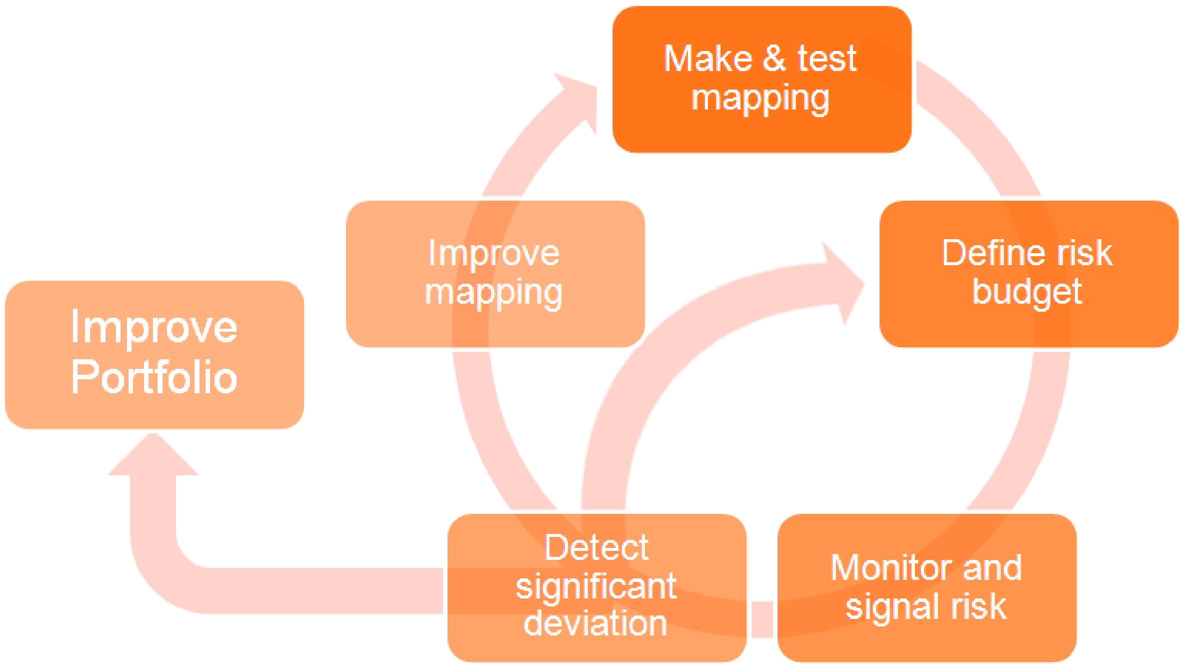


Figure 9.2: Schematic representation of monitoring process.

Also in SACs containing more than 10 single securities, one security (in this case Porsche) which has a very deviant data series, can influence the distribution of the portfolio so that it does not originate from the same distribution as the benchmark. This property can be used to detect specific risk within a portfolio. When the influence of such a security is this large, the portfolio is not sufficiently diversified. However, this can only be identified with ex-post data and therefore after an extreme event has occurred. Because it is not possible to model the single securities, the behaviour of the single securities cannot be predicted. The damage of a large loss because of a drop in the price of one single securities can therefore only be prevented by good diversification and not by a better mapping.

For the asset class *equity*, the 100% mapping gives with ex-post data the most representative information. Therefore this mapping structure, presented by figure 6.1, is chosen to map the single securities to SACs for the ex-ante analysis of the portfolio. The time horizon over which the risk is determined should at least contain 6 months of daily data or, when the data frequency is lower, 60 data points for the mapping to give clear results. The mapping is based on ex-post (historical) data from 10 years and is conform the 3 years of ex-post data. The ex-ante scenarios range over 32 years, which is a long time. Although the mapping is fairly robust over a period of 10 years, during such a long period, the optimal mapping might change. Therefore one must be cautious when drawing conclusions on ex-ante results for more than 10 years.

A possible monitoring process for a dynamic mapping is shown in figure 9.2. First, the mapping is created and tested by the implementator. Then the risk budget is determined. The next step is to monitor and signal the risk. When a large deviation is detected, the process can be adapted in three ways, the mapping can be adjusted, for example such that more single securities fall in each SAC. Also the risk budget can be extended, by adjusting

the significance level. A third possibility is to take a more detailed look at the portfolio and observe whether individual securities cause a deviation in the portfolio.
The outer circle in this process is pursued by the implementator only he can adjust the mapping. An important task for the implementator is to evaluate the mapping regularly. The inner circle, which moves from the detection of the significant deviation to the adjustment of the risk budget can be pursued by the portfolio manager, he can also improve the portfolio.

In the whole thesis, portfolios containing only stocks were considered. However, one can also invest in different asset classes and in funds. When investing in a fund, one already invests in a variety of single securities. Therefore, a portfolio containing one fund, which often tracks a benchmark, should originate from the same distribution as the benchmark. Its mapping should be accurate without having a minimum number of securities in the SAC.

In the introduction, the goal for this thesis was formulated as follows:

> *To find or construct a methodology to identify the mismatch between portfolio risk and the SAA risk, identify the causes of this mismatch and investigate the possibilities to create a computerized monitoring system to prevent future SAA mismatch.*

A possible methodology was described in this thesis, some of the steps in this method were only executed for the asset class equity. We have not been able to take a closer look at the other asset classes, since proper data were not available. Before the methodology can be applied in the real world, for true portfolios, the different steps should also be executed for the other asset classes. Once the data are available, this should not be too much work, since the models were all created while keeping in mind that other data could be plugged in.

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Appendix A

Single securities equity

	Sector	Security name	Country	symbol
LC Canada	Communications	Thomson Reuters Corp	Canada	TRI
	Financial	Royal Bank of Canada	Canada	RY
	Precious metals	Barrick Gold Corp	Canada	ABX
	Pipelines	Trans Canada Corp	Canada	TRP
	Consumer, cyclical	Gildan Activewear Inc	Canada	GIL
LC Japan	Technology	Canon	Japan	7751
	Automotive	Toyota	Japan	7203
	Chemicals	Fujifilm Holdings Corp	Japan	4901
	Rubber products	Bridgestone	Japan	5108
	Electric Machinery	Sony	Japan	6758
LC USA	Pharmaceuticals	Johnson & Johnson	USA	NYSE:JNJ
	Financial	Bank of America	USA	BAC
	IT	Intel Corp	USA	INTC
	Consumer, non-cyclical	Pepsico Inc	USA	NYSE:PEP
	Automotive	Ford	USA	NYSE:F
LC Eurozone	Telecommunication	Koninklijke KPN NV	Netherlands	KPN
	Industrial	BAM	Netherlands	BAMNB
	Consumer, cyclical	Porsche	Germany	PAH3
	Building & Construction	Bouygues SA	France	EN
	Consumer, non-cyclical	L'Oreal	France	EPA:OR
LC Europe ex Eurozone	Oil	BP	UK	LON:BP
	Consumer	Sainsbury	UK	LON:SBRY
	Financial	Credit Suisse	Switzerland	NYSE:CS
	Pharmaceuticals	Glaxosmithkline	UK	LON:GSK
	Consumer, non-cyclical	Nestlé	Switzerland	NESR

LC Pacific ex Japan	Oil	Woodside Petroleum Ltd	Australia	WPL
	Consumer, non-cyclical	Woolworths Ltd	Australia	ASX:WOW
	Consumer, cyclical	Pumpkin Patch	New Zealand	NZE:PPL
	Industrial	Cheung Kong Infra-structure Holdings Ltd	Hong Kong	1038
	Financial	BOC Hong Kong Holdings Ltd	Hong Kong	2388
LC EM Latin America	Energy	Petroleo Brasileiro SA	Brazil	PBR
	Basic Materials	Sociedad Quimica y Minera	Chili	SQM
	Financial	Credicorp Ltd	Chili	BAP
	Financial	Bancolombia SA	Colombia	CIB
	Communications	America Movil SAB de CV	Mexico	AMX
LC Asia EM	Communications	HTC corp	Taiwan	2498
	Consumer, non-cyclical	Charoen Pokphand Foods PCL	Thailand	CPOKY
	Pharmaceuticals	Cadila Healthcare Ltd	India	532321
	Technology	Neusoft Corporation	China	SHA:600718
	Consumer, non-cyclical	Jollibee Foods Corp	Phillipines	JBFCF
LC East Europe	Financial	Sberbank Rossii OAO	Russia	SBER
	Oil & Gas	Gazprom OAO	Russia	GAZP
	Financial	Severo-Zapadnoye Parokhodstvo OAO	Russia	SZPR
SC USA	Oil & Gas	Newpark Resources Inc	USA	NR
	water	Middlesex Water Co	USA	MSEX
	technology	iGate	USA	IGTE
	Consumer, non-cyclical	BJ's restaurants	USA	BJRI
	financial	First Merchants Corp	USA	FRME
SC Canada	Consumer, non-cyclical	Canada Bread Co Ltd	Canada	CBY
	Commercial Services	MacDonald Dettwiler & Associates Ltd	Canada	MDA
	Gold mining	Gabriel Resources Ltd	Canada	GBU
	Industrial	Intertape Polymer Group	Canada	ITP
	Financial	Equitable Group Inc	Canada	ETC
SC Europe ex Eurozone	Consumer, cyclical	Laura Ashley plc	UK	LON:ALY
	Industrial	Keller Group	UK	LON:KLR
	Consumer, non-cyclical	Nobel Biocare Holding AG	Switzerland	NBHGF
	Internet gambling	Betsson AB	Sweden	BETS-B
	Industrial	Dfds A/S	Denmark	DFDS

SC Eurozone	Industrial	Somfy SA	France	EPA:SO
	Consumer, non-cyclical	Duvel Moortgat SA	Belgium	DUV
	Consumer, Cyclical	Juventus FC	Italy	JUVE
	Industrial	Pfeiffer Vacuum Technology AG	Germany	PFV
	Software	Exact Holding NV	Netherlands	EXACT
SC Emerging markets	Hand/Machine Tools	Fag Bearings India Ltd	India	FAGBEA
	Pharmaceuticals	Gansu Duyiwei Bio Pharmaceutical Co	China	SHE:002219
	Financial	Gafisa SA	Brazil	GFA
	Industrial	VSMPO-AVISMA Corporation OAO	Russia	VSMO
	Consumer, non-cyclical	Vina Concha y Toro S.A.	Chile	VCO
SC Pacific ex Japan	Mining	Kingsgate Consolidated Ltd	Australia	ASX:KCN
	Consumer, cyclical	Michael Hill International Ltd	New Zealand	NZE:MHI
	Consumer, cyclical	Wing On Company International Ltd	Hong Kong	0289
	Media	Fairfax Media Ltd	Australia	FXJ
	Industrial	Harbin Power Equipment Company Ltd	Hong Kong	1133

Table A.1: All selected stocks with sector, countries and GoogleFinance symbol.

Appendix B

Test the mapping

Period	SAC name	$\alpha =$ 0.1	$\alpha =$ 0.05	$\alpha =$ 0.025	$\alpha =$ 0.01	$\alpha =$ 0.005	$\alpha =$ 0.0025
6 month	LC Japan	0%	0%	0%	0%	0%	0%
	LC Pacific ex Japan	0%	0%	0%	0%	0%	0%
	LC USA	0%	0%	0%	0%	0%	0%
	LC Canada	0%	0%	0%	0%	0%	0%
	LC Eurozone	1%	0%	0%	0%	0%	0%
	LC Europe ex Eurozone	0%	0%	0%	0%	0%	0%
	SC Pacific ex Japan	0%	0%	0%	0%	0%	0%
	SC USA	0%	0%	0%	0%	0%	0%
	SC Canada	16%	10%	7%	3%	1%	0%
	SC Europe	0%	0%	0%	0%	0%	0%
	SC Eurozone	0%	0%	0%	0%	0%	0%
	LC Eastern Europe	48%	48%	48%	43%	38%	33%
	LC Latin America	0%	0%	0%	0%	0%	0%
	LC Asia	0%	0%	0%	0%	0%	0%
	SC Emerging markets	0%	0%	0%	0%	0%	0%
	LC North America	0%	0%	0%	0%	0%	0%
	lC Emerging markets	0%	0%	0%	0%	0%	0%
	LC Europe	0%	0%	0%	0%	0%	0%
3 year	LC Japan	0%	0%	0%	0%	0%	0%
	LC Pacific ex Japan	1%	1%	1%	1%	0%	0%
	LC USA	5%	4%	3%	2%	2%	2%
	LC Canada	13%	12%	12%	10%	7%	7%
	LC Eurozone	13%	9%	7%	6%	4%	2%
	LC Europe ex Eurozone	6%	2%	1%	0%	0%	0%
	SC Pacific ex Japan	24%	15%	6%	1%	0%	0%
	SC USA	30%	25%	20%	17%	16%	9%
	SC Canada	42%	39%	37%	35%	33%	29%
	SC Europe	2%	2%	2%	2%	2%	2%
	SC Eurozone	1%	1%	1%	1%	1%	1%

	LC Eastern Europe	62%	57%	57%	52%	52%	48%
	LC Latin America	14%	10%	9%	6%	6%	2%
	LC Asia	24%	18%	16%	13%	11%	6%
	SC Emerging markets	66%	61%	57%	52%	45%	33%
	LC North America	0%	0%	0%	0%	0%	0%
	LC Emerging Markets	0%	0%	0%	0%	0%	0%
	LC Europe	0%	0%	0%	0%	0%	0%
7 year	LC Japan	30%	17%	15%	2%	0%	0%
	LC Pacific ex Japan	88%	86%	86%	83%	79%	75%
	LC USA	23%	21%	20%	18%	18%	17%
	LC Canada	65%	58%	45%	36%	31%	19%
	LC Eurozone	100%	100%	100%	93%	89%	84%
	LC Europe ex Eurozone	52%	47%	44%	40%	37%	34%
	SC Pacific ex Japan	67%	63%	54%	50%	47%	39%
	SC USA	67%	64%	61%	54%	51%	39%
	SC Canada	61%	45%	36%	28%	28%	25%
	SC Europe	60%	52%	46%	37%	33%	24%
	SC Eurozone	14%	13%	10%	8%	6%	4%
	LC Eastern Europe	Not enough data					
	LC Latin America	70%	60%	55%	48%	48%	39%
	LC Asia	80%	77%	75%	71%	71%	71%
	SC Emerging markets	77%	71%	70%	68%	64%	57%
	LC North America	0%	0%	0%	0%	0%	0%
	LC Emerging Markets	40%	31%	22%	17%	12%	5%
	LC Europe	93%	93%	93%	93%	93%	92%

Table B.1: percentage of portfolios for which the benchmark and the portfolio do not originate from the same distribution.

Appendix C

Ranking the SACs

Equity	**ranking 2002-2011**	**ranking 2009-2012**
Developed markets	NR	NR
DM Large cap	NR	NR
DM Small cap	NR	NR
SC Pacific ex Japan	1	1
SC Pacific	2	2
SC Canada	3	3
LC Eurozone	4	8
LC Europe ex Eurozone	5	9
LC Japan	6	6
SC North America	7	4
LC North America	8	10
LC USA	9	11
LC Pacific ex Japan	10	12
LC Canada	11	13
SC USA	12	16
SC Europe	13	7
LC Europe	14	5
SC Eurozone	15	15
LC Pacific	16	14
Emerging markets	NR	NR
EM Large cap	NR	NR
EM Small cap	NR	NR
LC Eastern Europe	1	1
LC Latin America	2	2
LC Asia	3	3

Table C.1: Ranking of asset class *Equity*.

Government bonds	**ranking 2002-2011**	**ranking 2009-2012**
Nominal	NR	NR
Eurozone IG	NR	NR
USA	NR	NR
UK	NR	NR
USA maturity 1-3	1	1
USA maturity 3-5	2	2
UK maturity 1-5	3	3
UK maturity >15	4	5
UK maturity >10	5	6
Eurozone maturity >10	6	7
Eurozone maturity 1-3	7	10
Eurozone maturity 7-10	8	22
USA maturity 5-10	9	4
Eurozone maturity <5	10	9
Italy	11	8
Eurozone maturity 3-5	12	23
Spain	13	12
USA maturity >20	14	13
USA maturity 10-20	15	11
Portugal	16	17
Ireland	17	18
Greece	18	19
UK maturity 5-15	19	24
UK maturity 5-10	20	21
France	21	25
Denmark	22	15
Eurozone maturity 5-7	23	26
France maturity <5	24	14
Belgium	25	27
Netherlands	26	16
Finland	27	20
UK maturity 10-15	28	29
Austria	29	28
Belgium maturity <5	30	30
Austria maturity <5	31	31
Finland <5	32	32
Index-linked	NR	NR
Eurozone	1	6
Italy	2	4
UK maturity 1-10	3	3
France maturity 1-10	4	11
France maturity >5	5	19
UK	6	5

UK maturity >10	7	2
USA maturity 1-10	8	10
USA	9	1
World maturity >5	10	14
World maturity 1-10	11	15
Italy maturity 1-10	12	12
Germany maturity >10	13	13
Germany	14	7
UK maturity >5	15	16
France	16	17
USA maturity 7-10	17	18
Germany maturity >5	18	9
UK maturity 7-10	19	8
France maturity 7-10	20	20

Table C.2: Ranking of asset class *Government Bonds*.

Corporate bonds	**ranking 2002-2011**	**ranking 2009-2012**
Euro Government-related	NR	NR
Euro Securitized	NR	NR
Euro Corporate	NR	NR
UK corporate IG	NR	NR
UK Government-related	NR	NR
UK Securitized	NR	NR
USA	NR	NR
USA Corporate	NR	NR
USA Government-related	NR	NR
USA Securitized	NR	NR
Euro HY PanEurope	1	1
USA HY	2	2
UK AAA	3	3
Euro AAA	4	5
Euro BAA	5	4
UK A	6	6
Euro A	7	7
UK Bbb	8	8
Euro Corporate excl. financials	9	9
UK AA	10	10
Euro AA	11	11

Table C.3: Ranking of asset class *Corporate Bonds*.

Real estate	**ranking 2002-2011**	**ranking 2009-2012**
Indirect Listed	NR	NR
North America	NR	NR
Europe	NR	NR
Far east	NR	NR
Japan	1	1
United Kingdom	2	30
Canada	3	2
Austria	4	6
Germany	5	24
China	6	8
Brazil	7	4
Turkey	8	5
Switzerland	9	10
Belgium	10	9
Singapore	11	12
Hong Kong	12	7
Sweden	13	11
Indonesia	14	22
Philippines	15	17
Spain	16	15
Greece	17	19
Russia	18	14
Poland	19	16
Italy	20	18
Norway	21	23
Finland	22	21
Malaysia	23	26
Thailand	24	3
New Zealand	25	28
Australia	26	25
India	27	20
United States	28	29
Netherlands	29	13
France	30	27
Mexico	31	
Argentina	32	
Portugal	33	31
Indirect unlisted	NR	NR
Europe	NR	NR
Core	NR	NR
Value Added	NR	NR
Opportunistic	NR	NR
North America	NR	NR

Core	NR	NR
Value Added	NR	NR
Opportunistic	NR	NR
Asia & Pacific	NR	NR
Core	NR	NR
Value Added	NR	NR
Opportunistic	NR	NR
Direct	NR	NR
Europe	NR	NR
North America	NR	NR
Japan	NR	NR
UK	1	NR
Europe Residential	2	NR
Japan Residential	3	NR
Europe Retail	4	NR
Switzerland	5	NR
Europe Industrial	6	NR
Netherlands	7	NR
Japan Office	8	NR
North America Retail	9	NR
North America hotel	10	NR
Japan Retail	11	NR
UK Retail	12	10
Switzerland Industrial	13	NR
UK Residential	14	NR
Netherlands Other	15	NR
UK Industrial	16	NR
Switzerland Retail	17	NR
Switzerland Other	18	NR
Netherlands Industrial	19	NR
North America Office	20	NR
North America Residential	21	NR
North America Industrial	22	NR
UK Office	23	NR
Netherlands Residential	24	NR
Netherlands Retail	25	NR
Switzerland Office	26	NR
Switzerland Residential	27	NR
Netherlands Office	28	NR

Table C.4: Ranking of asset class *Real estate*.

Alternatives	**ranking 2002-2011**	**ranking 2009-2012**
Hedge funds	NR	NR
Macro	1	3
Equity Hedge	2	1
Relative Value	3	2
FoF	4	4
Event driven	5	5
Private equity	NR	NR
USA	1	5
UK	2	1
Europe	3	4
VC		3
BO		2
Commodities	NR	NR
Gold	1	3
Energy	2	2
Agriculture	3	1
Industrial Metals	4	4
Livestock	5	5
Precious Metals	6	6
Infrastructures	NR	NR
incl utilities	1	1
ex utilities	2	2

Table C.5: Ranking of asset class *Alternatives*.

Appendix D

Test the mapping

1 month standard deviation

	σ **PF**	**lower bound**	**upper bound**	σ **100%**	σ **75%**	σ **50%**	σ **25%**	σ **0%**	σ **SAA**
MC PF	6.9	4.5	10.7	8.3	7.7	7.4	6.7	6.6	6.7
PF 1	5.6	3.6	8.5	7.3	7.2	6.9	7.5	6.8	6.7
PF 2	7.7	5	11.8	7.5	7.3	7	7.8	6.7	6.7
PF 3	8.9	5.8	13.7	7.4	7.4	7	7.6	6.8	6.7
PF 4	8.6	5.6	13.2	7.9	7.6	7.3	7.9	7	6.7
PF 5	6	3.9	9.2	7	6.9	6.9	7.7	6.8	6.7
PF 6	5.9	3.9	9.1	7.6	7.3	7.1	7.7	6.8	6.7
PF 7	6.3	4.1	9.6	7.7	7.5	7	7.6	6.6	6.7
PF 8	8.9	5.8	13.7	7.3	7.2	7.3	7.8	6.9	6.7
PF 9	7.8	5.1	12	7.3	7.2	6.7	7.5	7	6.7
PF 10	7.7	5	11.8	7.4	7	6.9	7.5	6.9	6.7
PF 11	6.7	4.4	10.3	7.6	7.5	6.9	7.7	7.2	6.7
PF 12	6.5	4.2	9.9	7.5	7.2	6.6	7.1	6.7	6.7
PF 13	5.3	3.4	8.1	7.3	7.1	7	7.8	6.8	6.7
PF 14	6.1	4	9.4	7.2	7.1	6.8	7.3	6.8	6.7
PF 15	10.5	6.8	16.1	7.6	7.6	7.3	7.8	6.7	6.7
PF 16	6.8	4.4	10.5	7.8	7.7	7.4	8	6.9	6.7
PF 17	8.1	5.2	12.4	7.2	7	6.9	7.1	7.1	6.7
PF 18	9.8	6.4	15	8	7.9	7.6	8	6.7	6.7
PF 19	6.4	4.2	9.8	7.2	7	6.8	7.4	6.8	6.7
PF 20	8	5.2	12.3	7.6	7.3	7	7.9	6.8	6.7
PF 21	6.6	4.3	10.2	7.6	7.4	6.9	7.3	6.7	6.7
PF 22	9	5.9	13.8	7	6.7	6.6	7.2	6.7	6.7
PF 23	9.6	6.2	14.7	7.9	7.8	7.1	7.8	7	6.7
PF 24	7.2	4.7	11	7.2	7.2	6.9	7.5	6.9	6.7

Table D.1: Standard deviation of the actual portfolio, lower and upper bound of the risk budgets and the standard deviations of the different mappings at significance level $\alpha = 0.05$. All values are multiplied by 1000. The standard deviations of the mappings are marked green when they fall within the risk budget.

6 months Standard deviation

	σ PF SS's	lower bound	upper bound	σ 100%	σ 75%	σ 50%	σ 25%	σ 0%	σ SAA
MC PF	9.5	8	11.2	10.6	9.9	9.6	9.1	8.9	8.9
PF 1	8.4	7.1	10	9.4	9.3	8.4	9.5	8.9	8.9
PF 2	9.3	7.8	11.1	9.7	9.5	8.6	10	8.9	8.9
PF 3	11.4	9.6	13.5	9.3	9.2	8.3	9.5	8.9	8.9
PF 4	13.6	11.5	16.2	10.1	9.8	9.2	9.8	8.9	8.9
PF 5	8.1	6.8	9.6	9	8.9	8.5	9.7	8.9	8.9
PF 6	8.7	7.3	10.3	9.2	8.9	8.1	9.8	8.9	8.9
PF 7	8.2	6.9	9.7	9.4	9.3	8.6	9.6	8.9	8.9
PF 8	12.8	10.8	15.2	9.7	9.5	8.4	9.7	8.9	8.9
PF 9	11.4	9.6	13.5	9.3	9.1	8.2	9.3	8.9	8.9
PF 10	12.5	10.5	14.8	10.1	9.7	8.8	9.4	8.9	8.9
PF 11	10.4	8.8	12.4	10.2	10.1	8.6	9.4	8.9	8.9
PF 12	8.8	7.5	10.5	9.4	9.2	8.7	9.2	8.9	8.9
PF 13	9.5	8	11.3	9.4	9.1	8.5	9.7	8.9	8.9
PF 14	9.2	7.8	10.9	9	8.9	7.8	9.2	8.9	8.9
PF 15	10.4	8.8	12.4	8.9	8.9	8.3	9.7	8.9	8.9
PF 16	9.4	7.9	11.2	10.5	10.4	8.7	9.7	8.9	8.9
PF 17	10.2	8.5	12.1	9.1	8.9	8.2	8.9	8.9	8.9
PF 18	12.3	10.4	14.7	10.6	10.4	9.5	9.7	8.9	8.9
PF 19	9	7.5	10.6	9.5	9.3	8.3	9.4	8.9	8.9
PF 20	9.5	8	11.3	10	9.6	8.6	9.9	8.9	8.9
PF 21	9.2	7.7	10.9	9	8.7	8.2	9.5	8.9	8.9
PF 22	12.4	10.4	14.7	9.2	8.9	8.2	9.3	9	8.9
PF 23	11.2	9.4	13.3	10.7	10.6	8.8	9.5	8.9	8.9
PF 24	10.2	8.6	12.1	8.9	8.9	7.9	9.4	8.9	8.9

Table D.2: Standard deviation of the actual portfolio, lower and upper bound of the risk budgets and the standard deviations of the different mappings at significance level $\alpha = 0.05$. All values are multiplied by 1000. The standard deviations of the mappings are marked green when they fall within the risk budget.

3 year standard deviation

	σ **PF SS's**	**lower bound**	**upper bound**	σ **100%**	σ **75%**	σ **50%**	σ **25%**	σ **0%**	σ **SAA**
MC PF	11.9	11	12.7	12.2	11.7	11.4	11.1	11	10.9
PF 1	10.5	9.8	11.3	11.3	11.2	10	11.4	10.8	10.9
PF 2	11.6	10.8	12.4	11.6	11.4	10.4	11.9	10.9	10.9
PF 3	14.2	13.2	15.2	11.3	11.2	9.9	11.4	10.8	10.9
PF 4	17.8	16.6	19.1	12.2	11.9	11.2	11.6	10.8	10.9
PF 5	9.7	9.1	10.4	10.9	10.9	10.3	11.7	10.9	10.9
PF 6	9.9	9.3	10.7	10.7	10.5	9.6	11.7	10.8	10.9
PF 7	10.4	9.7	11.2	11.4	11.2	10.3	11.5	10.9	10.9
PF 8	15.9	14.9	17.1	11.6	11.5	9.9	11.5	10.8	10.9
PF 9	14.8	13.8	15.9	11.1	11	9.8	11.1	10.8	10.9
PF 10	16	14.9	17.2	12.2	12	10.7	11.3	10.9	10.9
PF 11	13	12.1	13.9	12.4	12.4	10.4	11.2	10.7	10.9
PF 12	10.2	9.5	10.9	11.6	11.4	10.6	11.1	10.8	10.9
PF 13	12.6	11.8	13.5	11.2	11.1	10.1	11.6	10.8	10.9
PF 14	11.8	11	12.6	10.7	10.6	9.3	11.1	10.8	10.9
PF 15	11.9	11.1	12.7	10.6	10.5	9.8	11.6	10.9	10.9
PF 16	11.8	11	12.6	12.5	12.4	10.3	11.5	10.7	10.9
PF 17	11.9	11.1	12.8	11	10.9	9.7	10.7	10.7	10.9
PF 18	14.9	13.9	16	12.7	12.6	11.4	11.6	10.9	10.9
PF 19	10.9	10.2	11.7	11.4	11.3	10	11.3	10.9	10.9
PF 20	10.9	10.1	11.7	11.8	11.6	10.4	11.9	10.8	10.9
PF 21	11.1	10.3	11.9	10.7	10.5	9.7	11.4	10.8	10.9
PF 22	16	14.9	17.2	11.3	11	9.9	11.2	11	10.9
PF 23	12.8	11.9	13.7	12.9	12.8	10.6	11.4	10.7	10.9
PF 24	11	10.3	11.8	10.7	10.6	9.4	11.3	10.8	10.9

Table D.3: Standard deviation of the actual portfolio, lower and upper bound of the risk budgets and the standard deviations of the different mappings at significance level $\alpha = 0.05$. All values are multiplied by 1000. The standard deviations of the mappings are marked green when they fall within the risk budget.

7 year standard deviation

	σ PF SS's	lower bound	upper bound	σ 100%	σ 75%	σ 50%	σ 25%	σ 0%	σ SAA
MC PF	14.7	14	15.3	13.9	11.9	11.6	11.3	11.2	11.6
PF 1	13.7	13.1	14.4	13.6	12	10.5	11.5	11.1	11.6
PF 2	16	15.3	16.7	13.6	12	10.7	11.8	11.2	11.6
PF 3	26.3	25.1	27.6	13.7	12.1	10.5	11.5	11.2	11.6
PF 4	21.4	20.4	22.4	14.1	12.3	11.7	11.7	11.2	11.6
PF 5	12.4	11.9	13	12.7	11.2	10.7	11.7	11.2	11.6
PF 6	17.4	16.6	18.2	12.8	11.4	10.4	11.7	11.1	11.6
PF 7	13.1	12.5	13.7	13.5	12	10.7	11.6	11.2	11.6
PF 8	19.3	18.4	20.2	14.1	12.4	10.6	11.6	11.1	11.6
PF 9	18.7	17.8	19.6	13.4	11.8	10.5	11.4	11.2	11.6
PF 10	19.5	18.6	20.4	14.6	12.8	11.1	11.5	11.2	11.6
PF 11	16.4	15.7	17.2	15.5	13.5	11.1	11.5	11.2	11.6
PF 12	12.3	11.8	12.9	13.7	12.1	11.1	11.4	11.2	11.6
PF 13	15.6	14.9	16.3	13.3	11.7	10.6	11.6	11.1	11.6
PF 14	15.5	14.8	16.2	13.2	11.8	10.2	11.3	11.1	11.6
PF 15	26.4	25.2	27.6	12.7	11.4	10.4	11.6	11.2	11.6
PF 16	14	13.4	14.7	15.4	13.5	10.8	11.7	11.2	11.6
PF 17	15.3	14.6	16	13.6	12	10.5	11.2	11.2	11.6
PF 18	16.9	16.1	17.7	15.3	13.5	11.8	11.8	11.2	11.6
PF 19	13.1	12.5	13.7	13.7	12	10.4	11.4	11.2	11.6
PF 20	20.5	19.5	21.4	13.9	12.4	11	12.1	11.3	11.6
PF 21	13	12.4	13.6	12.7	11.3	10.4	11.5	11.1	11.6
PF 22	21	20	22	13.4	11.8	10.3	11.3	11.2	11.6
PF 23	16.8	16.1	17.6	16.1	14.1	11.2	11.7	11.2	11.6
PF 24	14.9	14.2	15.6	13.1	11.7	10.3	11.5	11.2	11.6

Table D.4: Standard deviation of the actual portfolio, lower and upper bound of the risk budgets and the standard deviations of the different mappings at significance level $\alpha = 0.05$. All values are multiplied by 1000. The standard deviations of the mappings are marked green when they fall within the risk budget.

1 month CVaR

	CVaR PF	lower bound	upper bound	CVaR 100%	CVaR 75%	CVaR 50%	CVaR 25%	CVaR 0%	CVaR SAA
MC PF	-14.6	-15	-14.3	-8.8	-8.6	-8.1	-9.1	-8.8	-8.9
PF 1	-9.1	-9.3	-9	-9.2	-9.2	-9.3	-10.1	-8.9	-8.9
PF 2	-15.1	-15.3	-14.9	-9.4	-9.5	-9.5	-10.4	-8.9	-8.9
PF 3	-10.9	-10.9	-10.8	-9.4	-9.5	-9.2	-10.1	-8.9	-8.9
PF 4	-11.4	-11.6	-11.2	-10.8	-11	-10.1	-10.6	-8.9	-8.9
PF 5	-10.6	-10.7	-10.5	-9.6	-9.6	-9.8	-10.8	-8.9	-8.9
PF 6	-7.2	-7.2	-7.1	-8.7	-8.8	-8.5	-9.9	-8.9	-8.9
PF 7	-11.4	-11.5	-11.2	-10.8	-10.8	-10.1	-10.7	-8.9	-8.9
PF 8	-12.5	-12.5	-12.4	-7.8	-7.9	-8.9	-9.9	-8.9	-8.9
PF 9	-10.4	-10.6	-10.2	-9.3	-9.3	-8.8	-9.6	-8.9	-8.9
PF 10	-9	-9.2	-8.9	-8.3	-8.5	-9.3	-9.7	-8.9	-8.9
PF 11	-7.5	-7.6	-7.4	-8.8	-8.8	-8.9	-9.4	-8.9	-8.9
PF 12	-5	-5	-5	-10.7	-10.8	-9.4	-9.8	-8.8	-8.9
PF 13	-9.4	-9.5	-9.3	-9.3	-9.4	-9.6	-10.5	-8.9	-8.9
PF 14	-8.4	-8.5	-8.2	-8.4	-8.5	-8.5	-9.7	-8.9	-8.9
PF 15	-9.9	-9.9	-9.9	-10.3	-10.3	-9.9	-11.2	-8.9	-8.9
PF 16	-13.7	-14	-13.4	-8.6	-8.2	-9.3	-10.1	-8.9	-8.9
PF 17	-13.8	-13.8	-13.7	-8	-8	-8.2	-8.9	-8.9	-8.9
PF 18	-20.6	-20.7	-20.5	-10.4	-10.4	-11	-11.3	-8.9	-8.9
PF 19	-10.9	-11	-10.7	-8.5	-8.6	-9.1	-9.8	-8.9	-8.9
PF 20	-8.7	-8.9	-8.5	-9.1	-9.1	-9.4	-10.4	-8.8	-8.9
PF 21	-9.2	-9.3	-9.1	-9.6	-9.7	-8.8	-9.9	-8.9	-8.9
PF 22	-11.1	-11.3	-10.9	-8.4	-8.6	-9.1	-9.8	-9	-8.9
PF 23	-15.9	-16	-15.9	-9.1	-9.1	-9.8	-10.5	-8.8	-8.9
PF 24	-11.4	-11.5	-11.3	-9	-9	-9	-10.2	-8.9	-8.9

Table D.5: CVaR of the actual portfolio, lower and upper bound of the risk budgets and the CVaR of the different mappings at significance level $\alpha = 0.05$. All values are multiplied by 1000. The CVaRs of the mappings are marked green when they fall within the risk budget.

6 months CVaR

	CVaR PF	**lower bound**	**upper bound**	**CVaR 100%**	**CVaR 75%**	**CVaR 50%**	**CVaR 25%**	**CVaR 0%**	**CVaR SAA**
MC PF	-20.2	-20.3	-20.1	-21.7	-19.9	-19.4	-18	-17.7	-17.7
PF 1	-17	-17.1	-17	-18.9	-18.5	-16.6	-18.7	-17.8	-17.7
PF 2	-20.7	-20.8	-20.6	-19.4	-18.9	-17	-19.6	-17.9	-17.7
PF 3	-21.3	-21.4	-21.1	-18.5	-18.3	-16.3	-18.7	-17.7	-17.7
PF 4	-26.1	-26.3	-25.9	-20	-19.4	-18.5	-19.4	-17.8	-17.7
PF 5	-17.7	-17.8	-17.7	-17.9	-17.7	-17	-19.3	-17.9	-17.7
PF 6	-17.1	-17.2	-17.1	-18.2	-17.6	-16.1	-19.1	-17.7	-17.7
PF 7	-18.4	-18.5	-18.3	-18.4	-18.1	-16.9	-18.9	-17.9	-17.7
PF 8	-24.8	-25.1	-24.6	-19.7	-19.4	-16.5	-19.1	-17.7	-17.7
PF 9	-21.1	-21.3	-20.9	-18.4	-18	-16.2	-18.3	-17.7	-17.7
PF 10	-24.9	-25	-24.8	-20.4	-19.4	-17.7	-18.7	-18	-17.7
PF 11	-19.5	-19.6	-19.3	-20.4	-20.2	-17.1	-18.7	-17.7	-17.7
PF 12	-19.3	-19.5	-19	-18.3	-17.9	-17.4	-18.3	-17.8	-17.7
PF 13	-18.8	-18.9	-18.7	-18.7	-18.2	-16.7	-19.1	-17.8	-17.7
PF 14	-18.4	-18.5	-18.3	-18	-17.6	-15.5	-18.2	-17.8	-17.7
PF 15	-19.9	-20	-19.7	-17.2	-17.2	-16.1	-19	-17.9	-17.7
PF 16	-19	-19.1	-18.9	-21.4	-21.1	-17.1	-19.2	-17.7	-17.7
PF 17	-20.4	-20.5	-20.3	-18.4	-17.9	-16.5	-17.7	-17.6	-17.7
PF 18	-25.9	-26	-25.8	-20.9	-20.7	-18.8	-19.2	-17.9	-17.7
PF 19	-18.4	-18.5	-18.3	-19.1	-18.5	-16.6	-18.6	-17.9	-17.7
PF 20	-18	-18.1	-17.9	-20	-19.3	-17.2	-19.8	-17.7	-17.7
PF 21	-18.2	-18.3	-18.1	-17.7	-17.1	-16.2	-18.7	-17.7	-17.7
PF 22	-23.2	-23.3	-23	-18.5	-17.8	-16.3	-18.4	-18.1	-17.7
PF 23	-22.8	-23	-22.7	-21.2	-20.9	-17.5	-18.8	-17.6	-17.7
PF 24	-21.6	-21.7	-21.5	-17.6	-17.5	-15.6	-18.5	-17.8	-17.7

Table D.6: CVaR of the actual portfolio, lower and upper bound of the risk budgets and the CVaR of the different mappings at significance level $\alpha = 0.05$. All values are multiplied by 1000. The CVaRs of the mappings are marked green when they fall within the risk budget.

3 year CVaR

	CVaR PF	**lower bound**	**upper bound**	**CVaR 100%**	**CVaR 75%**	**CVaR 50%**	**CVaR 25%**	**CVaR 0%**	**CVaR SAA**
MC PF	-27.1	-27.2	-27.1	-28.8	-27.6	-27	-26.7	-26.5	-26.2
PF 1	-24.9	-24.9	-24.8	-27	-26.7	-23.9	-26.9	-25.9	-26.2
PF 2	-26.6	-26.6	-26.5	-27.6	-27.3	-24.7	-28.2	-26.2	-26.2
PF 3	-34.3	-34.5	-34.1	-26.9	-26.7	-23.4	-27	-25.8	-26.2
PF 4	-42.6	-42.9	-42.3	-28.9	-28.4	-26.6	-27.5	-25.8	-26.2
PF 5	-22.6	-22.7	-22.5	-26.2	-26	-24.5	-27.5	-26.1	-26.2
PF 6	-23.4	-23.5	-23.3	-25.3	-24.9	-22.7	-27.8	-26	-26.2
PF 7	-24.3	-24.4	-24.3	-26.9	-26.6	-24.6	-27.1	-26.3	-26.2
PF 8	-38.4	-38.6	-38.2	-27.7	-27.4	-23.4	-27.3	-25.8	-26.2
PF 9	-35.7	-35.9	-35.5	-26.2	-26	-23.4	-26.5	-25.7	-26.2
PF 10	-38.7	-38.9	-38.5	-29.5	-28.8	-25.6	-26.9	-26	-26.2
PF 11	-31.5	-31.6	-31.3	-29.8	-29.6	-24.9	-26.6	-25.5	-26.2
PF 12	-24.3	-24.4	-24.3	-27.4	-27	-25.5	-26.4	-26	-26.2
PF 13	-30.3	-30.5	-30.2	-26.8	-26.4	-24	-27.3	-25.9	-26.2
PF 14	-27.9	-28	-27.7	-25.4	-25.2	-21.9	-26.3	-25.9	-26.2
PF 15	-28.6	-28.7	-28.5	-24.9	-24.8	-22.9	-27.3	-26.2	-26.2
PF 16	-27.1	-27.2	-27	-30	-29.7	-24.4	-27.2	-25.7	-26.2
PF 17	-27.8	-27.9	-27.7	-26.3	-25.9	-23	-25.7	-25.7	-26.2
PF 18	-34.3	-34.4	-34.1	-30.3	-30.1	-26.9	-27.3	-26.1	-26.2
PF 19	-25.2	-25.2	-25.1	-27.3	-26.9	-23.9	-26.8	-26.1	-26.2
PF 20	-25.9	-26	-25.9	-28.3	-27.8	-24.7	-28	-25.7	-26.2
PF 21	-25.8	-25.9	-25.7	-25.2	-24.7	-23.1	-27.1	-26	-26.2
PF 22	-38.2	-38.5	-37.9	-27	-26.5	-23.8	-26.7	-26.4	-26.2
PF 23	-29.5	-29.6	-29.5	-30.9	-30.7	-25.1	-26.9	-25.6	-26.2
PF 24	-25.5	-25.5	-25.4	-25.2	-25.1	-22.2	-26.6	-25.9	-26.2

Table D.7: CVaR of the actual portfolio, lower and upper bound of the risk budgets and the CVaR of the different mappings at significance level $\alpha = 0.05$. All values are multiplied by 1000. The CVaRs of the mappings are marked green when they fall within the risk budget.

7 year CVaR

	CVaR PF	**lower bound**	**upper bound**	**CVaR 100%**	**CVaR 75%**	**CVaR 50%**	**CVaR 25%**	**CVaR 0%**	**CVaR SAA**
MC PF	-34.6	-34.7	-34.5	-34.6	-29.4	-28.6	-28	-27.7	-29.6
PF 1	-34.1	-34.2	-34	-34.2	-29.7	-25.8	-28.6	-27.6	-29.6
PF 2	-37	-37.2	-36.8	-34.2	-29.7	-26.3	-29.5	-27.7	-29.6
PF 3	-53.4	-54	-52.9	-34.3	-30.1	-25.6	-28.6	-27.6	-29.6
PF 4	-53	-53.3	-52.8	-36.2	-31.1	-29.1	-29.4	-27.7	-29.6
PF 5	-29.6	-29.8	-29.4	-32.2	-28	-26.4	-29.2	-27.7	-29.6
PF 6	-35.6	-36	-35.2	-31.7	-27.9	-25.3	-29	-27.6	-29.6
PF 7	-31.6	-31.7	-31.5	-33.9	-29.4	-26.3	-28.8	-27.6	-29.6
PF 8	-46.8	-47	-46.6	-35.2	-30.9	-25.8	-28.9	-27.6	-29.6
PF 9	-45.9	-46	-45.8	-33.6	-29.4	-25.7	-28.3	-27.6	-29.6
PF 10	-47.8	-48	-47.6	-37	-31.9	-27.3	-28.5	-27.7	-29.6
PF 11	-40	-40.2	-39.9	-39	-34	-27.2	-28.6	-27.8	-29.6
PF 12	-30.7	-30.8	-30.5	-34.7	-29.9	-27.4	-28.2	-27.6	-29.6
PF 13	-38.5	-38.6	-38.4	-33.6	-29.1	-26.1	-29	-27.6	-29.6
PF 14	-38.4	-38.5	-38.3	-32.8	-29	-24.8	-28.2	-27.6	-29.6
PF 15	-51.6	-52.2	-51	-31.4	-27.8	-25.4	-29.1	-27.6	-29.6
PF 16	-33.9	-34.1	-33.8	-38.6	-33.7	-26.5	-29	-27.6	-29.6
PF 17	-37.1	-37.2	-37	-34	-29.7	-25.7	-27.7	-27.7	-29.6
PF 18	-40.2	-40.4	-40.1	-38.9	-33.7	-29.3	-29.2	-27.6	-29.6
PF 19	-31.4	-31.5	-31.3	-34.5	-30	-25.7	-28.4	-27.6	-29.6
PF 20	-42.9	-43.3	-42.5	-35.2	-30.5	-26.8	-30	-27.7	-29.6
PF 21	-31.5	-31.6	-31.4	-31.5	-27.5	-25.3	-28.6	-27.6	-29.6
PF 22	-50.2	-50.5	-50	-33.9	-29.4	-25.5	-28.1	-27.7	-29.6
PF 23	-40.6	-40.7	-40.4	-40.7	-35.3	-27.6	-29	-27.6	-29.6
PF 24	-35.9	-36.1	-35.7	-32.4	-28.7	-25	-28.5	-27.6	-29.6

Table D.8: CVaR of the actual portfolio, lower and upper bound of the risk budgets and the CVaR of the different mappings at significance level $\alpha = 0.05$. All values are multiplied by 1000. The CVaRs of the mappings are marked green when they fall within the risk budget.

			1M	3M	6M	1Y	3Y	7Y		1M	3M	6M	1Y	3Y	7Y
σ	**100%**	$\alpha = 0.5$	48	36	32	20	8	4	$\alpha = 0.25$	84	52	40	32	20	12
	75%		60	40	44	24	12	0		84	64	56	32	20	4
	50%		48	40	20	20	4	0		84	52	40	32	16	0
	25%		52	24	32	8	8	0		80	48	44	32	24	0
	0%		60	48	32	20	16	0		72	60	48	32	20	0
	SAA		52	48	28	24	16	0		72	60	52	32	20	0
CVaR	**100%**	$\alpha = 0.5$	8	0	4	0	0	8	$\alpha = 0.25$	8	0	4	4	0	12
	75%		0	0	4	0	0	0		4	0	4	4	0	0
	50%		0	4	0	0	0	0		0	4	0	0	0	0
	25%		0	0	0	0	0	0		0	0	0	0	0	0
	0%		0	0	0	0	0	0		4	0	0	0	0	0
	SAA		0	4	4	0	0	4		0	4	4	0	0	4
σ	**100%**	$\alpha = 0.1$	100	76	68	32	28	16	$\alpha = 0.05$	100	88	76	36	32	24
	75%		100	72	68	36	36	8		100	88	76	40	44	8
	50%		96	64	48	40	28	0		100	72	52	40	28	0
	25%		96	76	60	32	28	0		100	88	68	48	32	0
	0%		92	64	60	52	28	0		96	72	68	56	32	0
	SAA		88	64	60	52	24	0		96	68	68	56	32	0
CVaR	**100%**	$\alpha = 0.1$	8	0	8	4	0	16	$\alpha = 0.05$	8	0	8	4	0	16
	75%		12	0	4	4	0	0		12	0	4	4	0	0
	50%		8	8	0	4	0	0		8	8	0	4	0	0
	25%		0	0	0	0	0	0		0	0	0	0	0	0
	0%		4	0	0	0	0	0		8	0	0	0	0	0
	SAA		0	8	4	0	0	4		4	8	4	0	0	4
σ	**100%**	$\alpha = 0.025$	100	100	88	80	56	24	$\alpha = 0.01$	100	100	88	60	56	32
	75%		100	100	88	76	56	8		100	96	84	64	52	8
	50%		100	92	72	72	32	0		100	84	60	64	32	0
	25%		100	100	84	72	56	0		100	100	80	60	48	0
	0%		100	96	80	68	56	0		100	92	68	64	52	0
	SAA		100	96	80	72	56	0		100	88	68	64	56	4
CVaR	**100%**	$\alpha = 0.025$	8	0	8	8	0	16	$\alpha = 0.01$	8	0	8	8	0	16
	75%		12	0	4	4	0	4		12	0	4	4	0	0
	50%		8	8	0	4	4	0		8	8	0	4	4	0
	25%		0	0	0	0	4	0		0	0	0	0	0	0
	0%		12	0	0	8	0	0		12	0	0	4	0	0
	SAA		12	8	4	4	0	4		12	8	4	0	0	4
σ	**100%**	$\alpha = 0.005$	100	100	88	64	56	32	$\alpha = 0.0025$	100	100	88	80	56	32
	75%		100	100	84	68	56	8		100	100	88	76	56	8
	50%		100	88	68	72	32	0		100	92	72	72	32	0
	25%		100	100	84	64	52	4		100	100	84	72	56	4
	0%		100	92	76	68	52	0		100	96	80	68	56	0
	SAA		100	92	72	68	56	4		100	96	80	72	56	8
CVaR	**100%**	$\alpha = 0.005$	8	0	8	8	0	16	$\alpha = 0.0025$	8	0	8	8	0	16
	75%		12	0	4	8	0	0		12	0	4	4	0	4
	50%		8	8	0	4	4	0		8	8	0	4	4	0
	25%		0	0	0	0	4	0		0	0	0	0	4	0
	0%		12	0	0	4	0	0		12	0	0	8	0	0
	SAA		12	8	4	4	0	4		12	8	4	4	0	4

Table D.9: Percentage of the 25 portfolios which fall into the risk budget of the single security portfolio for each significance level and time horizon.

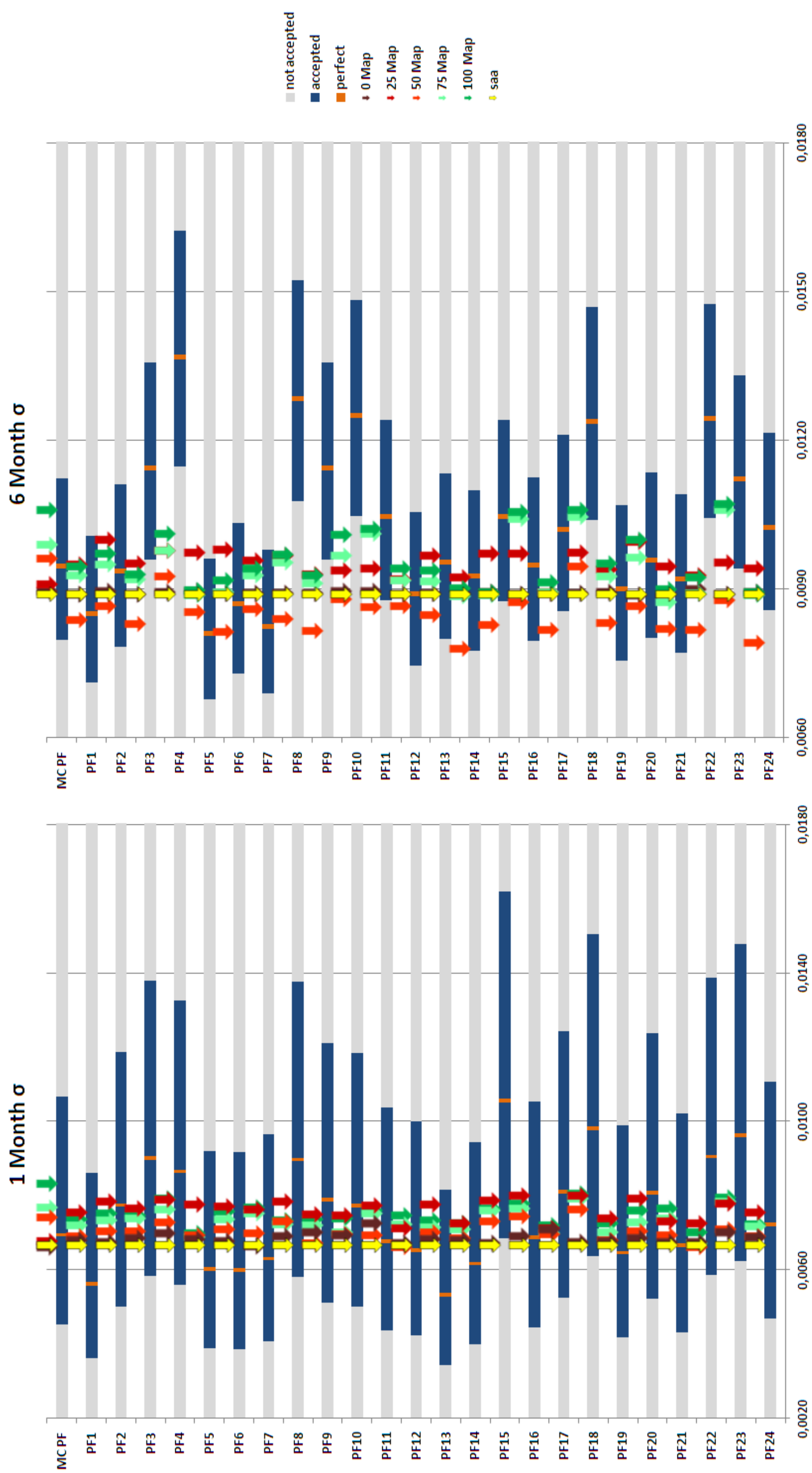

Figure D.1: Graphs standard deviation risk budgets with time horizons 1 and 6 months.

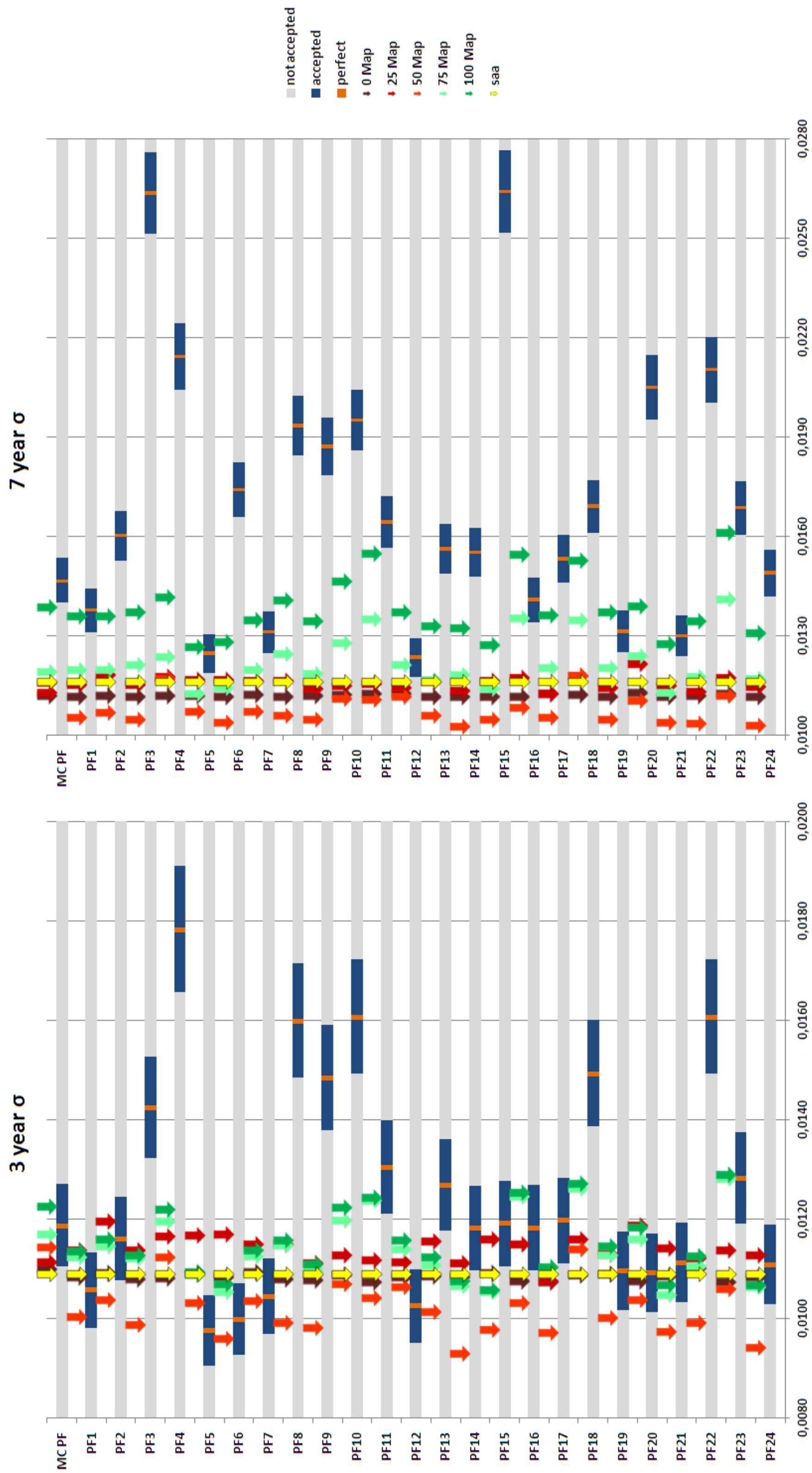

Figure D.2: Graphs standard deviation risk budgets with time horizons 3 and 7 years.

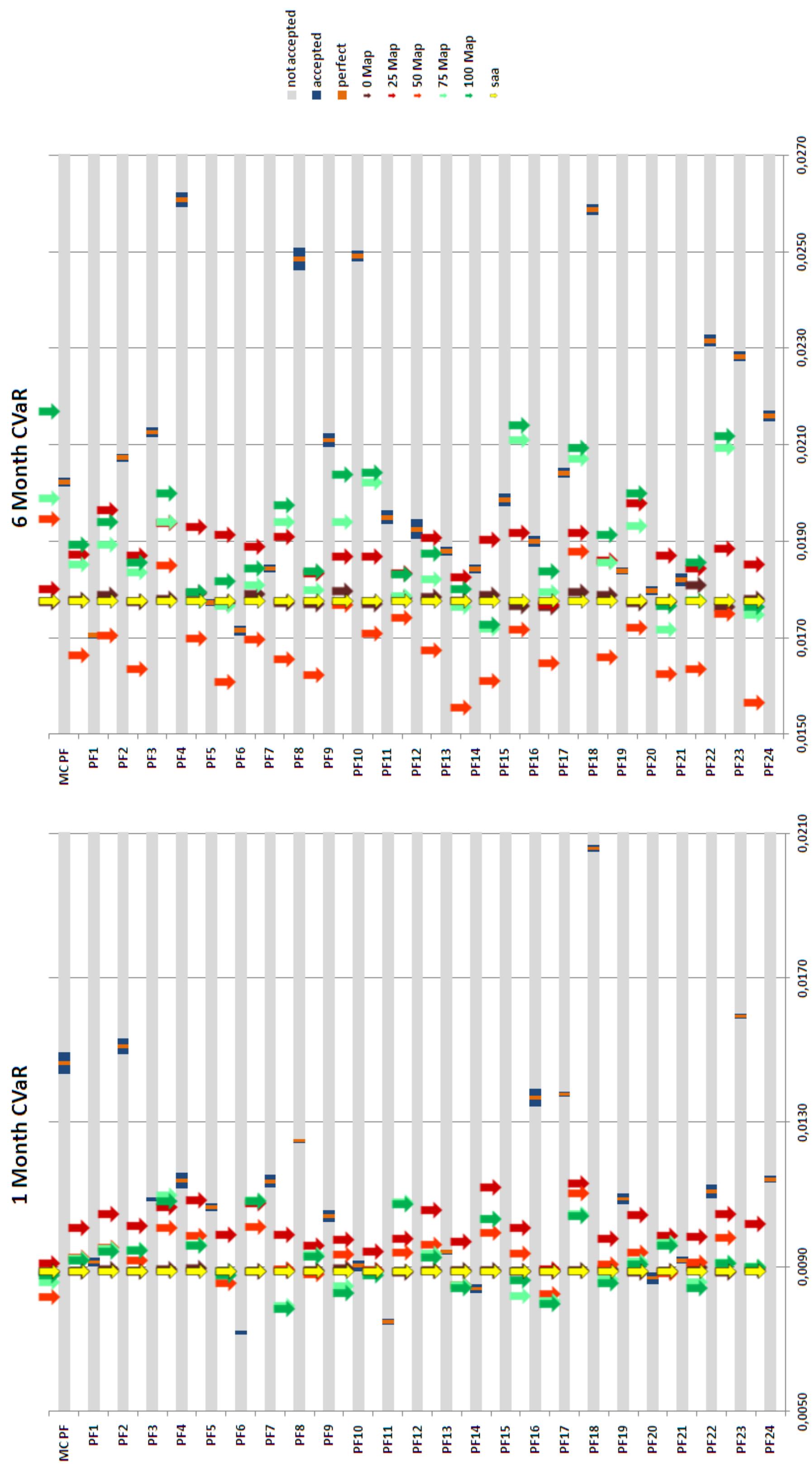

Figure D.3: Graphs CVaR risk budgets with time horizons 1 and 6 months.

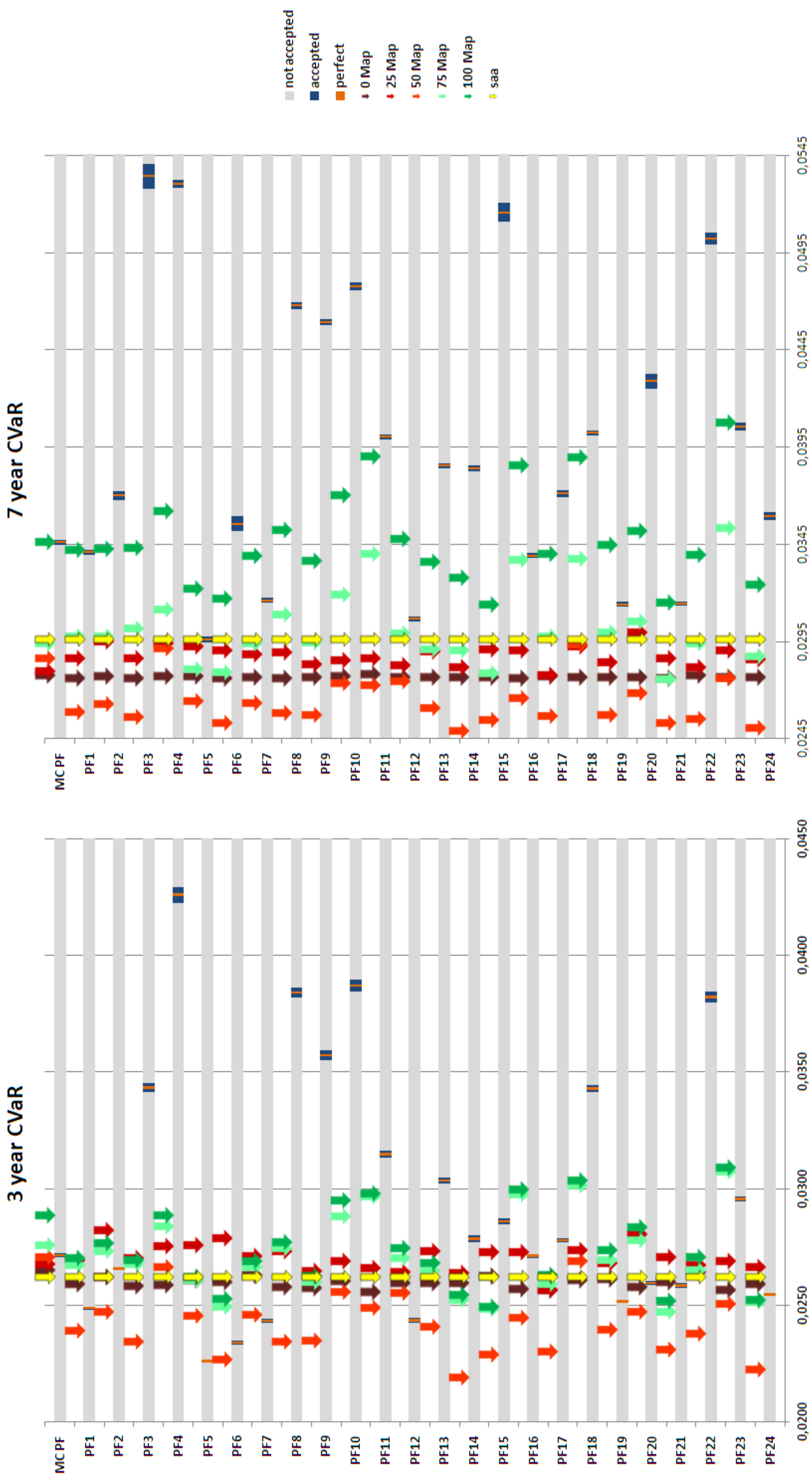

Figure D.4: Graphs of CVaR risk budgets with time horizons 3 and 7 years.