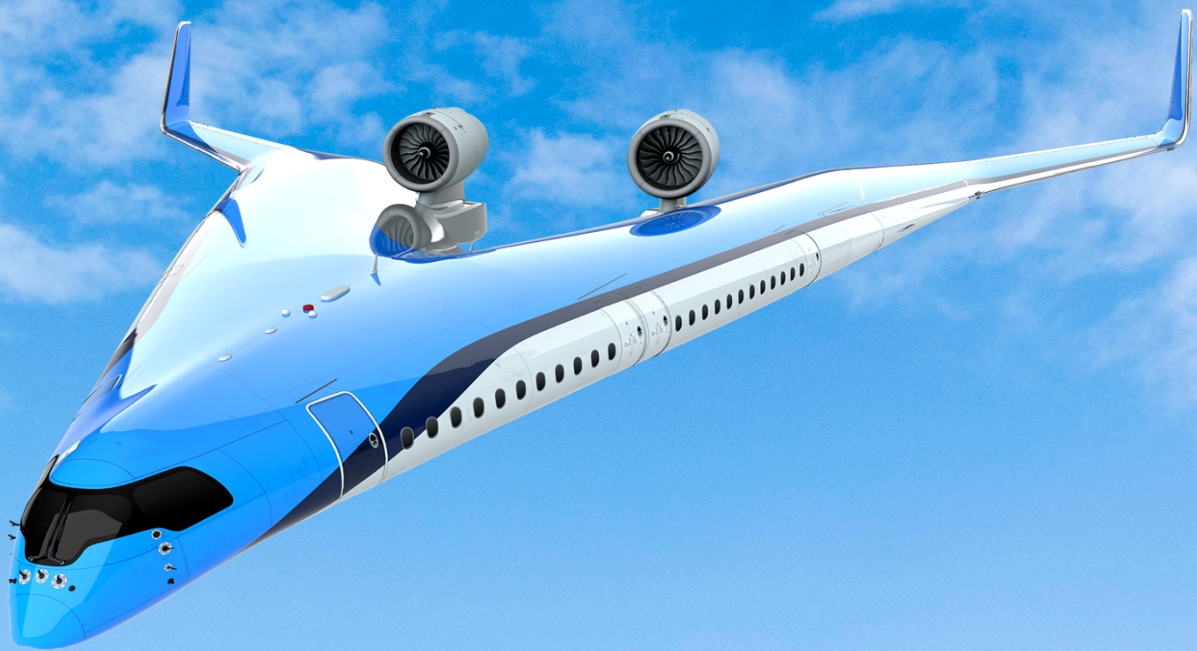


Flying V Robust Controller Design

Design of a C* Longitudinal Flight Controller using
 H_∞ - Loop Shaping Techniques

Jennifer Thornton



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Loop Shaping Techniques

Thesis report

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Abstract

The issue of the rapidly changing climate is becoming ever more present in all areas of life, and in recent decades, the subject has had influence on the aviation industry. This has led to novel aircraft designs, such as the flying wing design of the Flying V aircraft, to emerge into development. The emergence of such designs, however, pose new challenges in terms of stability, control and handling qualities. Identified uncertainties present within the Flying V aircraft's aerodynamic model, as well as minimal research into robust controller application to the Flying V aircraft indicate a raising need for the advancement of its current flight control system. Therefore, this study aims to advance the maturity of the Flying V aircraft's conceptual design by updating the current flight control system with a robustly stabilising H_∞ loop shaping C^* longitudinal controller, with the aim to ensure robust stability and robust performance under parametric aerodynamic uncertainty, as well as plant input and output disturbances, whilst aiming to achieve level 1 handling quality requirements. The study commences with establishment of the Flying V aircraft's nonlinear model into Matlab and Simulink. From there, a standard one-degree-of-freedom (1DOF) H_∞ loop shaping controller is designed using common comparable controller synthesis techniques available in Matlab. Parametric uncertainty is applied to a two-degree-of-freedom (2DOF) controller, which develops on the standard 1DOF control configuration. This 2DOF controller is applied to the nonlinear Flying V aircraft model before a controller is designed using multi-model design in order to ensure robust stability over the entire operational flight envelope. For all of the described controller designs, a thorough analysis is conducted to assess the stability and handling quality performance in time and frequency domains. Results of the 1DOF controller design conclude that a successful design is achieved through the adjustment of pre- and post compensators, affecting disturbance rejection capability and system tracking bandwidth. The design shows sufficient robust stability but insufficient robust performance. It is shown that when placed in the feedback path, the feedback controller provides superior performance when compared to being placed in the forward path. Time-domain plots show that the C^* value measured at the instantaneous centre of rotation is most ideal, which corroborates earlier frequency domain research on the demand type (normal load factor, pitch rate, C^*). The 2DOF design yielded a relatively low order controller with optimised tracking and results that met level 1 handling qualities, as well as general stability requirements. Additionally, 2DOF time-domain simulation results with applied uncertainty showed compliance with level 1 handling qualities for almost all criteria with generally healthy stability margins. Nonlinear results showed expected behaviour and the multi-model controller was able to bring all operational flight points to level 1 handling qualities whilst providing good stability margins for almost all points. Hence, H_∞ loop shaping is proven a sound method for robust controller formulation, and work to develop the technique further in this application, such as weighting filter and systune optimisation constraint refinement, as well as controller digitisation, is highly recommended.

Acknowledgements

This master thesis concludes my long but wonderful journey attending TU Delft. As though it were yesterday, I remember starting the first day of the bachelor program in September 2018, and in true aerospace fashion, I can say with ease that all of these years of attendance have flown by.

Having completed this journey, it is fair to say that it was not always a straightforward one; along with it came many ups and downs. Namely, the curveball of the Coronavirus pandemic was one of the largest lessons in adapting to drastic, sudden change, both in terms of the academic system and routines of daily life.

As well as Delft itself, the Delft University of Technology and its aerospace engineering department will always have a special place in my heart. It is here that I have had some of the most formative years of my life until now, and the memories that I have made here I know I will cherish for the rest of my life.

Not only does this thesis conclude my academic progression, but it seals this whole chapter of my life, which has surely shaped who I am as a person. Therefore, these acknowledgements are not only an ode to the master program, but to my entire TU Delft journey. It is with sadness that this chapter of my life is coming to a close, but I am ready with open arms for the next stage of life and to see what the future holds.

Kick-starting the next chapter of my life and career is something that I very much look forward to. The end of my academic journey at TU Delft marks an enormous milestone for me. If someone were to have told me years ago that I would ever achieve a master degree in aerospace engineering, I probably would not have believed them. This education has taught me that if you continue to push your boundaries, you can achieve wonderful things that you had never dreamed of, and I am proud of how far I have come.

So for this, I would also like to thank my parents, for their patience and understanding, and who have supported me since day one. It is from them that it has been instilled in me that the best things in life derive from working hard, maintaining discipline and approaching things with an open mind. They have taught me how to overcome some of my hardest challenges, which continue to be useful day-in-day-out.

After all these years, I have met some wonderful people and made some incredible friends along the way, who have continuously amazed me with their intelligence, unending kindness and understanding. I would like to thank my fellow peers, who I always felt welcome to approach if I was ever stuck with work-related conundrums. I deeply appreciate their offers of time and effort to help me resolve my issues. I would also like to thank my friends, who were always there for me when times were challenging, even when geographically far.

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Last but not least, I would like to thank my partner Martijn. I am thankful that the master program and a shared love of aviation unexpectedly lead our paths to cross, and even more grateful for the unwavering support that he has shown me ever since. It is with his day-to-day words of encouragement that I was able to continue to strive forward and overcome even my hardest challenges.

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Nomenclature

List of Abbreviations

AOA	Angle of Attack	LHP	Left Half-Plane
CA	Control Allocation	LS	Loop Shaping
CAP	Control Anticipation Parameter	LTI	Linear Time Invariant
CAS	Control Augmentation System	MIMO	Multiple Input Multiple Output
CL	Closed Loop	MS	Mixed Sensitivity
COG	Centre of Gravity	NED	North-East-Down
COM	Centre of Mass	NMP	Non-Minimum Phase
CT	Continuous Time	OL	Outer Loop
DGM	Disk Gain Margin	PID	Proportional-Integral-Derivative
DM	Disk Margin	PM	Phase Margin
DOD	Department of Defence	PRD	Pitch Rate Demand
DOF	Degree of Freedom	PS	Pilot Station
DPM	Disk Phase Margin	RHP	Right Half-Plane
EOM	Equations of Motion	RP	Robust Performance
FB	Feedback	RQ	Research Question
FCS	Flight Control System	S&T	Sensitivity & Complementary Sensitivity
GM	Gain Margin	SAS	Stability Augmentation System
GO6	Gang of Six	SGT	Small Gain Theorem
GS	Gain Scheduling	SIMO	Single Input Multiple Output
HF	High Frequency	SISO	Single Input Single Output
HQ	Handling Qualities	SM	Stability Margin
HQR	Handling Quality Rating	SP	Short Period
ICR	Instantaneous Centre of Rotation	SS	State Space
IL	Inner Loop	SV	Singular Value
IMU	Inertial Measurement Unit	TF	Transfer Function
INDI	Incremental Nonlinear Dynamic Inversion	WC	Worst Case
LF	Low Frequency	WF	Weighting Filter
		WTE	Wind Tunnel Experiments

List of Symbols			
		ω_n	Natural Frequency
α	Angle of Attack	ϕ	Roll Angle
α	Disk Margin Size	ψ	Yaw Angle
β	Angle of Sideslip	ρ	Air Density
δ	Differential Change	σ	Disk Margin Skew
δ_a	Aileron Deflection	θ	Pitch Angle
δ_e	Elevator Deflection	$\underline{u}$	Control Input Vector
δ_r	Rudder Deflection	$\underline{x}$	Dynamic State Vector
ϵ	Robustness Margin	ζ	Damping Ratio
η	Control Surface Input	q	Pitch Rate
γ	Flight Path Angle	t	Time-Step
γ	Performance Level	u	Forward Velocity component
λ	Eigenvalues	v	Sideways Velocity component
ω	Angular Velocity	w	Vertical Velocity component

Introduction

Many are aware that humanity is currently facing the effects of a warming climate. Climate change is notoriously known for worsening with the presence of greenhouse gases in the atmosphere, such as carbon dioxide, among others, which are largely caused by the combustion of fossil fuels. As this is commonly still the case for aircraft, the aviation industry is estimated to contribute 2.5% towards global energy-related CO_2 (IEA, 2025). Despite being a relatively low proportion of overall emissions, this percentage has bounced back to pre-Coronavirus levels and demand is forecast to increase rapidly throughout 2030 (IEA, 2025). Hence, a significant gap exists in improving the current top standard of aircraft fuel efficiency. The flying wing configuration is shown to improve in multiple areas upon the conventional wings and fuselage configuration: aerodynamic efficiency, lower fuel burn and associated accrued costs, as well as improved noise attenuation.

In novel aircraft design, in-flight stability and handling quality certification is essential within every operational condition, especially within commercial aviation due to stringent requirements on passenger and pilot comfort. Atypical bare airframe handling qualities, such as those pertaining to the flying wing configuration, make this process increasingly challenging. This results in relatively few examples of flying wing controllers achieving their certification and stability goal to a comprehensive degree.

Robust control techniques provide means to guarantee aircraft stability in all operating conditions. This therefore makes them essential in the process of airworthiness certification, and thus they are a standard used by industry in new aircraft development. Various methods exist, however, this research will specifically investigate the use of H_∞ -LS for the C^* controller of the FLYING V^{TM1} aircraft. Studies have shown that H_∞ -LS offers some benefits over other robust control methods such as H_∞ -MS. Of the controllers that have previously been applied to the Flying V aircraft, very little have used robust control techniques. Previous examples of applications of the H_∞ -MS technique to controllers including the C^* controller show that the implementation of robust control techniques on this controller can be successful. Despite less evidence of successful application of H_∞ -LS to the C^* controller, it perhaps offers an effective solution to bringing the Flying V aircraft closer to realisation, which merits investigation.

This document first and foremost aims to present a comprehensive piece of work which guides the reader from the beginning to the end of the rationale taken at each stage of the Flying V aircraft's C^* controller design; many influencing factors arise over the duration of the design process which each had individual impacts on the design decisions made. As well as this, it aims to outline the required stages involved in the H_∞ Loop Shaping (LS) process. H_∞ -LS as a concept within the field of robust control is still relatively new when compared to other existing techniques such as H_∞ Mixed Sensitivity (MS), and therefore fewer materials about the application of the technique exist in literature. The information gathered in this document were obtained across a variety of different sources, and therefore the aim is to produce one concise document detailing the process of loopshaping design. Combining these two aims, the overall goal of this work is to present H_∞ -LS as a feasible option that may be considered for the design of the C^* controller onboard of the Flying V aircraft. This will be supported by the numerical and graphical results that are provided as a result of this study.

¹Flying V is a trademark of Fortescue. The Flying V design is owned by Fortescue, UK & NL patents pending

Literature Review

This literature review begins by describing the Flying V aircraft model in [Section 2.1](#), giving details on the concept, design and layout, as well as the control, stability and handling qualities, followed by the aerodynamic model and aerodynamic uncertainty. Then, in [Section 2.2](#), the C* flight control law is described as well as its previous application to the Flying V aircraft. Lastly, in [Section 2.3](#), information about robust control is provided.

2.1. Flying V Aircraft Model



Figure 2.1: Flying V Aircraft: Front View
([Van Overeem, Wang, and Van Kampen, 2023](#))



Figure 2.2: Flying V Aircraft: Side View
([Van Overeem, Wang, and Van Kampen, 2023](#))

2.1.1. Concept, Design and Layout

Devised in 2015 by Justus Benad during his thesis at Airbus Hamburg, Germany, and based on the Airbus A350-900, the Flying V aircraft is the concept of a commercial, long-haul, flying wing aircraft ([Benad, 2015](#)). The name of the Flying V aircraft derives from the V-shape created by the aircraft body. Since the first appearance of the concept, extensive research has been conducted by TU Delft. The concept of the flying wing aircraft is not new, and examples of the application of the flying wing are given by the Boeing X-48B and the Northrop Grumman B-2 Spirit. A flying wing is identified by the merging of the fuselage into the wing structure. This concept was devised as a next stage aircraft design to improve on the fuel efficiency of the latest current fuselage and wing aircraft, where efficiency appears to approach an asymptotic theoretical maximum. The flying wing configuration promises improved efficiency over conventional fuselage and wing configurations, due to the increase in fuselage section. This in turn reduces the ratio of wetted area to wingspan, which reduces frictional drag. Additionally, the aircraft body has an airfoil shape, generating increased lift over this section when compared to conventional configurations. The oval shaped, aerodynamically designed wing structure of the Flying V aircraft contains the passenger cabin, fuel tanks and the cargo hold in the pressurised cabin ([Delft, 2025a](#)).

The Flying V aircraft proposes an improved lift over drag ratio of 10%, as well as a 2% reduction in empty weight. Comparing to the reference aircraft of the Airbus A350-900, the Flying V aircraft is shorter and the wing area is double that of the Airbus A350. It would be possible for the Flying V aircraft to be incorporated into current airport infrastructure, due to its comparable wingspan. The aircraft has a similar passenger capacity of 361 passengers and an identical cargo capacity of 160 m³ ([Delft, 2025a](#)). Like the Airbus A350, it has a cruise speed of 0.85 Ma and a range of 15.000 km ([Völker, 2021](#)).

The Flying V aircraft will have less drag than current wide-body aircraft such as the Boeing 787 and the Airbus A350-1000, according to preliminary calculations; the inflow surface area is reduced with respect to

the aircraft volume, when compared to the A350-1000 (Delft, 2025a). Additionally, it also has a significantly reduced structural weight compared to modern wide-body jets. Therefore the Flying V aircraft is predicted to fly very efficiently due to weight savings and a highly efficient aerodynamic shape. It is predicted that the aircraft will reduce fuel consumption compared to the A350-1000 by 20 %, which translates to a large reduction in greenhouse gas emissions and thereby contributing to counteracting the climate crisis. As well as this, it will serve as a large fuel cost saving to airlines and aircraft operators (Delft, 2025a).

The aircraft stabilisation is achieved by the location of the aerodynamic centre of the wing, as opposed to through a conventional horizontal tail-plane. The most aft COG lies in front of the aerodynamic centre, resulting in a desirable nose pitch down moment. The engines are placed with considerations for the clearance of the engine intake of the trailing edge of the wing, as well as the wing and engine aerodynamic interaction, one engine inoperative yawing moment and COG placement (Delft, 2025b). For the scaled model, the COG range is determined (1.32 m - 1.40 m from the nose of the aircraft) to guarantee trimmability of the aircraft at approach speeds and high angles of attack. A scaled prototype of the Flying V aircraft was built and successfully completed its maiden flight in 2020.

There have been many designs of the Flying V aircraft, which have been continuously iterated and improved. First of all, it can be seen that the engines are located above the wings and on the inside of the V-body, closer to the central axis of the aircraft. The reasons for doing so are so that a lower yawing moment is produced as a result of engine failure (Benad and Vos, 2022). As well as this, noise from the engines is also mitigated in the design as the engines are located behind and above the passenger cabin. Additionally, the wing partially blocks noise from the fan and exhaust noise is not reflected by the underside of the wing (Benad, 2015; Delft, 2025b).

Oosterom and Vos (2022) conducted the study of the conceptual design of a Flying V aircraft family consisting of three member aircraft designs, namely the FV-800, FV-900 and FV-1000, taking inspiration from the Airbus series. In the study, the VLM method was applied to investigate the aerodynamic properties of the aircraft. The lift to drag ratio was also estimated. An optimisation algorithm was applied with constraints on the COG excursion, wingspan and fuel tank volume, in order to minimise fuel consumption of the design. The algorithm was validated by comparing the fuel burn with official figures specified by Airbus. It concluded that the fuel burn of the -900 and -1000 models were 22% and 20% lower than the A350, respectively. Additionally, take-off mass was reduced for these aircraft by 17% and 15% respectively, compared to the A350. These two designs also exhibited lower approach speeds.

The Flying V aircraft design currently contains four control surfaces in each wing; there are three elevons on the trailing edge and one rudder. The segmented elevons, located at the trailing edge of the outboard wing, are responsible for the pitch and roll and the rudders are contained in the winglets. The design includes a low aspect ratio and large sweep angle similar to the Concorde, which means that large incidence angles are required to generate the required lift during low speed operations (Delft, 2025b). For this reason, the landing gear is relatively long in comparison to other modern commercial aircraft. It is important to note that the final control surface design is still under development, and the layout is still subject to change. The current layout of control surfaces is shown in Figure 2.3.

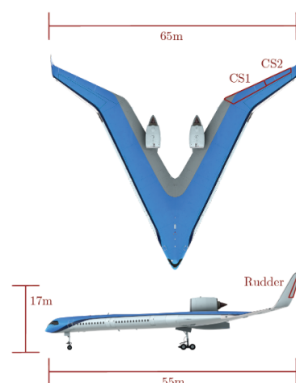


Figure 2.3: Control surface layout (Van Overeem, Wang, and Van Kampen, 2023)

2.1.2. Aerodynamic Model

Faggiano (2016) performed aerodynamic optimisation of the Flying V aircraft for a selected cruise flight point. The maximum lift-to-drag ratio optimised for cruise conditions in an aerodynamic design is estimated to be 23.7, 25% higher than the reference conventional configuration, NASA's conventional tube-and-wing 'Common Research Model' (Faggiano, 2016).

In the study of Viet (2019), three wind tunnel experiments were conducted at TU Delft. As a result, the aircraft had an untrimmed maximum lift coefficient of 1.09 at 41° AOA. Additionally, it was found that the pitching moment stability derivative was negative until 20° AOA. The neutral point was estimated to be located at 1.430 m behind the nose. Additionally, the COG location was found to be bounded between 1.345 m and 1.425 m behind the nose. Placing it at a location of 1.365 m behind the nose produced a static margin of approximately 9%.

An aerodynamic model was identified by Ruiz G. et al. (2022) using the techniques of decoupling and simplification of the aerodynamic identification problem through the use of an Iterated Extended Kalman Filter (IEKF) and ordinary least squares (OLS) regression. The study used the scaled wind tunnel tests of Palermo and Vos (2020), in which the longitudinal static stability derivatives were established. The dataset used consisted of longitudinal and lateral pilot and autopilot manoeuvres to excite the aircraft dynamic modes. The estimated lift coefficient showed "good agreement" with the wind tunnel obtained results, and less agreement with the pitching moment coefficient. The aerodynamic model and associated dynamic modes also showed accordance with pilot observations. It identified a stable SP and roll, a lightly damped Dutch roll and lightly damped/unstable phugoid and spiral modes.

Furthermore, a new aerodynamic model was developed by Siemonsma (2022) using flight data. System identification was performed using the two-step method of state estimation, followed by an IEKF. Subsequently, step-wise regression was performed due to its ability to determine polynomial models, which prove useful in early aircraft development. The current state of the Flying V aircraft's aerodynamic model at the time of writing is obtained using RANS-based CFD simulations.

2.1.3. Aerodynamic Uncertainty

Völker (2021) conducted the study of performing reinforcement learning for flight control of the Flying V aircraft. It highlighted that the model uncertainties of the Flying V aircraft are rather large, and that 'the dynamics are nonlinear and complex'. Despite the work that has previously been done to establish the aerodynamic model, it is not yet fully known with certainty, as there are only estimates of the maximum aerodynamic uncertainty within aerodynamic coefficients. The study motivates the decision to use offline learning as a solution to a model that is not known with certainty a-priori. The research used a flight simulation model of the Flying V aircraft to simulate varying amounts of aerodynamic-model uncertainty. Uncertainty factors were applied to 11 aerodynamic stability and control derivative coefficients. At the time of the study, the estimate of maximum uncertainty was found to be equal to approximately 25%. The aerodynamic uncertainty was sampled from a normal distribution with a standard deviation of 0.125, such that 95% of the uncertainty samples fell within the 25% uncertainty range. The stability and control derivatives were then multiplied by the corresponding obtained uncertainty factor. Two other scenarios were subsequently simulated with varied standard deviations, corresponding with the cases that the aerodynamic uncertainty fell above and below the estimated 25% range. For each of the scenarios, 100 simulations were run with a duration of 100 s each. As a result of the study, altitude-tracking error results were presented. The results indicated that the controller obtained was robust to 'at least the same level of aerodynamic-model uncertainty as the INDI controller', as obtained by Van Overeem, Wang, and Van Kampen (2022), although opportunities for direct comparison with this study were limited.

2.1.4. Control, Stability and Handling Qualities

The first stability and handling quality analysis was conducted by Cappuyns in 2019. Cappuyns worked with Airbus to also develop an aerodynamic model using the vortex lattice method (VLM) applied to the full scale model, with two elevons at the trailing edge (Cappuyns, 2019). The research found that classic handling quality requirements could be applied to the Flying V aircraft. Additionally, it investigated the effects of changing the size of the dihedral and vertical fin size. Most significantly, it found that the Flying V aircraft had "limited lateral-directional controllability" at low speeds, resulting in an unstable Dutch roll and unsatisfactory rudder control during one engine inoperative (OEI) condition. It also found that due to the resulting model being linear, the pitch break up tendency was not able to be modelled, and that deviation

from the linearised point could result in uncertainties being present.

Viet (2019) found that pitch break-up tendencies appear at AOAs above 20° . Furthermore, it was found through using VLM and RANS CFD methods that pitch break-up occurs during cruise conditions at around 10° of AOA.

A 1/22 half-span scale model was tested in low speed wind tunnel tests by Palermo and Vos (2020). The experiment involved investigating the effect on the lift, drag and pitching moment resulting from deflecting two control surfaces in the outboard wing. Obtained from this research were a suggested COG location, stability and control derivatives and untrimmed and trimmed maximum lift coefficients. In these tests, it was observed that the aircraft exhibited a pitch break up tendency at an angle of attack superior than 20° .

In 2021, Horwitz (2021) created a parametric definition of the winglet geometry present on the Flying V aircraft to create many different winglet geometries. The most efficient geometry was devised and an aerodynamic solver was used to obtain corresponding aerodynamic coefficient data and lateral-directional stability properties. The lateral control derivatives were obtained; however, the study found that no combination of geometry produced a sufficient yawing moment.

Adding to the model developed by Airbus, Stougie (2022) implemented a C^* control law and a rate control attitude hold roll control law, as well as load factor, roll angle and angle of attack (AOA) flight envelope protections (FEPs); the AOA FEP was implemented to prevent the pitch break up tendency and was enforced by placing commanded load factor limits. The aerodynamic models were combined with the VLM model of Cappuyns (2019) as the baseline model, with the VLM control surface layout. The lateral coefficients are taken from the VLM model and the longitudinal coefficients from both sources to include pitch break up effects from the wind tunnel experiments. Different coefficients were used based on the AOA range.

To observe the longitudinal and lateral dynamics, including pitch break up and the unstable Dutch-roll dynamics within one model, the VLM model and the wind tunnel testing results were combined by Van Overeem, Wang, and Van Kampen (2022) into a 6DOF dynamic simulation model and the HQs of the bare airframe at different COG locations for cruise and approach conditions were evaluated against relevant military HQRs, for instance, the CAP criterion. For this, the results of the wind tunnel tests needed to be scaled using Froude scaling for the differing speed regimes to be consistent with the VLM model. By doing this, uncertainty was introduced into the model. It was from the VLM model that the lateral coefficients were obtained. Resulting from the study were the damping ratio and natural frequency corresponding to each model, as well as the observation that a further aft COG produces a lower value for the CAP criterion (ie: a more sluggish response). Additionally, it was determined that the Dutch roll mode was unstable with the forward-most COG in the approach phase, and that it would require augmented stabilisation. Furthermore, it achieved borderline Level 1 HQs during approach at the forward COG and during cruise at the aft COG, and achieved Level 2 HQs for the remaining conditions. Furthermore, it was established from the study that the coefficients for the AOA should be obtained from the VLM model for angles between -5° and 15° , and from the wind tunnel experiments for angles between 15° and 30° . The model obtained as a result of this study is defined as a baseline for future research. As part of this baseline, a number of modelling assumptions are made. The first is that the aircraft is a rigid body with constant mass. Secondly, the curvature of the Earth, as well as angular velocity can be neglected. Wind is assumed to be equal to zero, and the atmosphere is assumed perfect corresponding to ISA conditions. The resultant thrust follows the direction of the aircraft symmetry plane, and gravity is assumed a constant vector.

Van Overeem, Wang, and Van Kampen (2023) continued in 2023 to implement a flight control system (FCS) of an inner loop (IL) incremental non-linear dynamic inversion (INDI) stability augmentation system (SAS) which was able to stabilise the unstable Dutch roll and phugoid such that all modes were stabilised to Level 1 HQs. An outer loop (OL) control augmentation system (CAS) was also implemented.

The lateral HQs were assessed analytically with a pilot-in-the-loop simulator assessment by Joosten et al. (2023). From the study, it was found that the bare airframe dynamics were inadequate when using conventional control allocation (CA), and insufficient control authority was found during approach conditions. The Dutch-roll was unstable during approach, but controllable during take-off and cruise conditions. Additionally, inadvertent excitation of the Dutch-roll manoeuvre was found to interfere with the performance of other manoeuvres. In the study, firstly CA was designed and applied, since CA has a large effect on aircraft commanded response. It was found that CA had a large impact on the edge trim cases

and manoeuvres. Secondly, to improve the initial inadequate bare airframe HQs, two versions of a SAS were created, one with roll-rate feedback and the other without roll-rate feedback incorporated. These were found to greatly improve the HQs, and as a result, the Dutch-roll no longer required pilot compensation to fly. Pitch control authority still persisted during approach conditions. The study recommended a reassessment of low-speed HQs between Mach 0.2-0.3. It also indicated that various aspects of the FCS were noted missing by pilots, including attitude hold, auto-throttle and turn-coordination (Joosten et al., 2023).

Traas (2024) conducted a study on the incorporation of hybrid INDI with advanced FEPs for the Flying V aircraft, with the aim to enhance SMs. The study remarked that the work of Stougie (2022) exhibited insufficient SMs due to deficient FEP structure, as well as sensor delays. The nature of the hybrid controller was to combine sensor and model based information. According to the study, the original FEPs were not challenging enough to highlight implementation deficiencies. This was evident as only the longitudinal approach condition remained stable for all test cases due to the AOA protection limiting pilot command due to the trimmed condition of the aircraft. It was concluded that despite improving on the original FCS, not all HQ requirements were achieved and the design was not fault tolerant to model uncertainties applied a posteriori.

A multi-loop μ optimal approach for designing robust, inversion based flight control laws is proposed in the work of Shahin (2024). The study proposes the design of an explicit model-following pitch rate control system corresponding to the SP dynamics of the Flying V aircraft. A structured singular value analysis was performed, in order to assess the robust stability of the control system. Given that INDI control schemes struggle in terms of achieving adequate robust stability levels, this approach promises an effective way to amend this issue. The study found that the proposed HQ criteria and SM requirements were met for the nominal plant, and WC SMs failed to meet minimum requirements.

2.2. C* Control Law

The beginnings of the C* control law originate from the fact that classical reduced order HQs are conventionally expressed in terms of the natural frequency ω_n and the damping ratio ζ of the aircraft SP response, since these parameters are enough to describe the short-term AOA and normal acceleration dynamic response. However, in the case of augmented aircraft, especially those highly augmented with advanced aerodynamic design and FCS, good SP dynamic qualities do not necessarily indicate sufficient HQs. Whilst SAS can provide dynamics at frequencies close to those of the SP, pilots may also be sensitive to cues other than normal acceleration, on which HQ requirements are often based; these include altitudes, velocities and accelerations. Therefore, time history envelopes were proposed as a method to be able to analyse these effects, and the step response chosen for its versatility and applicability to a variety of other, more common aircraft manoeuvres (Field, 1993).

C* is an example of such a time history envelope, and it is the name denoted to the parameter formulated by Tobie and Malcom (1965), introduced during the 1960's. Predecessor to the C* control law are the C* time-history envelope criteria that are used to ensure that sufficient aircraft HQs are achieved. The data of two variable stability aircraft were used in the development of these time-history envelopes; the CAL F-94, to define up and away boundaries (corresponding to approximately 400 mph and 20,000 ft), and the Boeing Model 367-80, the 707 prototype, to define the landing approach condition. Tobie and Malcom (1965) mention that the criterion can be extrapolated to different aircraft at different flight conditions, provided that normal acceleration is the predominant cue to the pilots, and if the SP response can be represented as a linear, second order system. Boeing subsequently went on to produce frequency response envelopes, as well as the pre-existing time-domain based envelopes (Field, 1993).

Following on from the time-history envelope criteria, C* is nowadays a digital fly-by-wire (FBW) control law whereby the pitch rate and normal load factor are blended, each with weighted contribution, in a response that varies with velocity. However, the particular balance between pitch rate and load factor FB has minimal importance according to the current, modern day definition of the control law (Field, 1993)]. At lower velocities, pitch dynamics dominate in the dynamic response, however at high velocities, the load factor dominates the response (Niedermeier and Lambregts, 2012). This is due to the fact that pilots are more easily able to control normal load factor at higher velocity, which aids in the handling qualities of the aircraft. The schematic block diagram of the controller is shown in Figure 2.4.

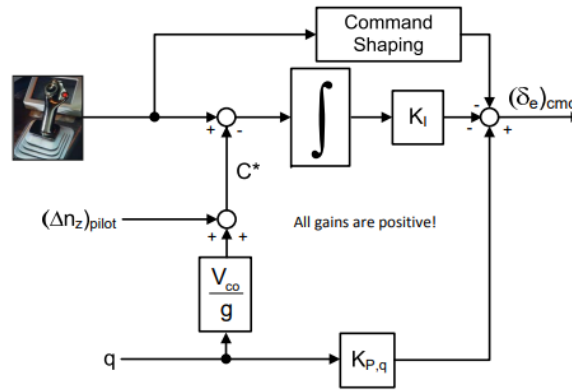


Figure 2.4: C* controller schematic block diagram (Niedermeier and Lambregts, 2012)

At the crossover speed V_{co} , the pitch rate and load factor are weighted equally, and the typical crossover speed given by Airbus is approximately equal to 240 kts. The expression for the C* output is given by Equation 4.41:

$$C^* = \Delta n_z + \frac{V_{co}q}{g} \quad (2.1)$$

In the equation, Δn_z is the incremental load factor, V_{co} the crossover airspeed, q the pitch rate and g the gravitational acceleration. The control law has been implemented in many modern day FBW transport aircraft and there are multiple examples in literature of its use. An early example is the work of Field (1993), where the C* flight control law was applied to a large civil transport aircraft. In the study, a proportional feedback C* controller was applied to a B747-100 under the landing approach configuration, and the results were assessed according to the C* criterion and MIL-STD-1797A, the US military HQ specification document. The study found that both the unaugmented and augmented models complied with MIL-STD-1797A requirements, and whilst the unaugmented model did not meet the lower limit of the C* criterion for the landing approach envelope, the augmented model did meet this requirement. It also highlighted the importance of the C* criterion on the design of novel and inherently non-well behaved aircraft, as opposed to aircraft requiring little augmentation.

Niedermeier and Lambregts (2012) is another notable example of the documentation on the C* control law. It mentions that the control law has been used by Airbus in all FBW aircraft as the primary method of pitch axis control for all flight phases except for take-off and landing flare. In the study, the C* parameter as well as the pitch rate are fed-back as part of a SAS. The report remarks that the control law is often claimed to possess flight path stability without pilot stick input as the controller stabilises the aircraft at constant flight path angle (FPA) and vertical speed, granted that airspeed is kept constant through thrust control. However, the absence of FPA integral control leads to the steady state error of the FPA which needs to be corrected periodically by pilots, and therefore FPA stability is not guaranteed by the control law.

Furthermore, the C* algorithm is said to destroy the natural speed stability of the aircraft as the airspeed is able to drift without bounds due to pilot inputs that change the vertical speed. This is due to the integral control of the vertical load factor. For this reason, it is required that the thrust is re-trimmed, since there is no associated 'trim speed'. For this reason, the C* control law does not meet Federal Aviation Regulations Part 25 in terms of static longitudinal stability. To amend this, FEPs must be applied at the edge of the aircraft flight envelope (Niedermeier and Lambregts, 2012). The solution to this issue is an adaptation of the original C* control law, named C*U, an additional Boeing modification of the existing C* control law where the 'U' term signifies airspeed. This adaptation of the control law incorporates an additional airspeed FB loop to the controller integrator. This modification can be found onboard of the B777 and B787 aircraft types (Niedermeier and Lambregts, 2012). The additional feedback is formulated according to Equation 2.2:

$$C^*U = C^* - K_V U_{err} \quad (2.2)$$

where U_{err} is the error between the aircraft calibrated airspeed V_{CAS} and the C^*U reference speed, and K_V is the speed stability gain. Despite being a fundamentally important adaptation that is applied to current state of the art FBW aircraft, this feature lays outside of the scope of the current study.

The C^* longitudinal control law was implemented as a component of manual longitudinal tracking in the work of [Lombaerts et al. \(2017\)](#). In the study, FEPs are applied with a C^* OL and an NDI IL. To do so, a commanded C^* control input is translated into a commanded load factor, which in turn is used to calculate a commanded AOA. This commanded AOA is translated into a commanded pitch rate.

[Marcos \(2017\)](#) implemented the C^* control law structure using two different approaches: the classical root-locus method and using an advanced, structured H_∞ method, to account for uncertainty within the model. It used the high fidelity, nonlinear 'RECONFIGURE' benchmark aircraft model, provided by Airbus. Provided was a set of 214 linear models derived from the benchmark, spanning the whole flight envelope, accompanied by accurate actuator and sensor models. For the root-locus approach, static gains were used. Conclusions drawn from the study were that both approaches yielded good results, where the H_∞ approach yielded very good S&T properties.

2.2.1. C* Control of the Flying V Aircraft

The C^* control law was previously applied to the Flying V aircraft model by [Stougie \(2022\)](#). The study proposed control of the IL using a FCS based on INDI control combined with FEPs, and control of the OL with a C^* longitudinal controller and RCAH lateral roll controller, with the aim to bring the Flying V aircraft's HQs to level 1. The study used the forward COG location, and cruise and approach conditions were used. The aerodynamic model used was that derived from the VLM model of [Cappuyns \(2019\)](#) and the wind tunnel experiment results obtained from [Ruiz G. et al. \(2022\)](#). The simplified C^* module block diagram designed and used by [Stougie \(2022\)](#) is shown in [Figure 2.5](#). It can be seen that the input command becomes an incremental C^* command, as opposed to what was previously an incremental load factor command. This commanded signal is turn-coordinated by the $\frac{\cos(\Theta)}{\cos(\Phi)}$ block before $\frac{V}{g}q$ is subtracted from it, formulating [Equation 4.41](#). The module of the C^* longitudinal controller contains various subsystems, including the angle of attack and load factor protections. It can be noted that [Stougie \(2022\)](#) has decided to feed back the pitch rate and load factor separately, as opposed to a combined C^* signal. The resulting signal is a pitch angular rate command, which is subsequently used by the pitch reference model.

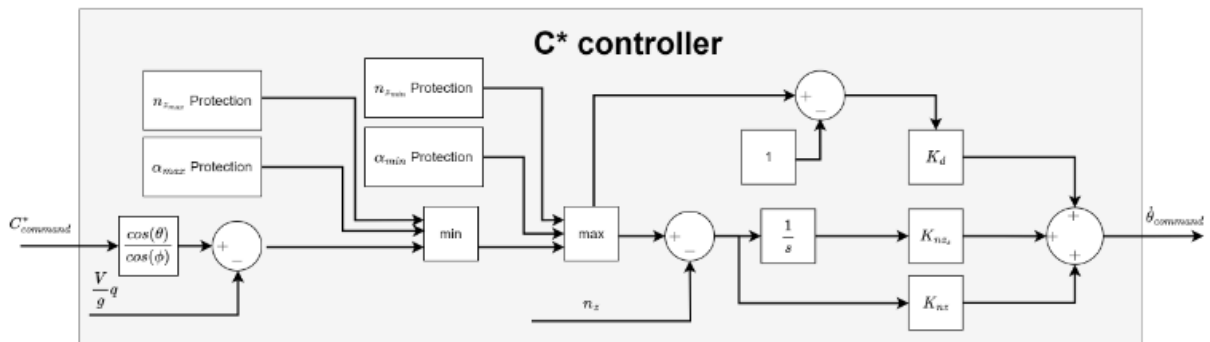


Figure 2.5: Implemented C^* controller block diagram ([Stougie, 2022](#))

The study conducted three analyses, the first incorporating no sensors, the second incorporating baseline sensors and the last incorporating sensors with adjusted time delays. The study showed that discretisation of the controller combined with the addition of the baseline sensors degraded the magnitudes of the gain and phase margins that were achieved. A large overshoot was observed in the C^* signal for all three analyses, as well as oscillatory behaviour corresponding to the baseline sensors. It was lastly concluded that the designed controller was robust to the aerodynamic uncertainty that was applied to it.

2.3. Robust Control

Robust control is quintessential to ensure that a given controller exhibits stability and desired performance when subject to uncertainty and disturbances. This section will outline the concept and history of H_∞ robust control, followed by the subdivision of H_∞ synthesis known as H_∞ -LS, the technique that will be used as part of this study.

2.3.1. H_∞ Control

Classical control methods, which include state-space equations and linear-quadratic optimisation, were widely thought of at the time as time-based methods. A pitfall of modern control methods was that system uncertainty was not appropriately recognised as a fundamental issue, which therefore often resulted in unstable and poor performing optimal controllers as a result of modelling errors (Bates and Postlethwaite, 2002). The foundation of H_∞ control was to provide a method to handle system uncertainty and improve upon the design transparency of time-domain based methods. For this reason, it is conducted predominantly in the frequency domain (Bates and Postlethwaite, 2002).

Robust control as a discipline of control theory most notably came to prominence during the 1980s, displaying a significant shift from classical and modern control methods that were widely used at the time. However, robust control can still be closely tied to the principles of classical control, and should not be considered an entirely new entity.

H_∞ optimisation is a well known robust control technique, proposed first by Zames (1981). The paper proposed a framework to define a set of constraints in terms of multiple-input, multiple-output (MIMO) CL objectives, which is known today as the H_∞ -MS approach (Bates and Postlethwaite, 2002). The objective of H_∞ synthesis is to minimise the H_∞ norm of the transfer function (TF) between disturbance input and controlled output, where the H_∞ norm represents the highest amplification of a worst case (WC) output signal vector over all frequencies. This allows open and closed loop transfer functions to be shaped to achieve specified robustness and performance goals (Skogestad and Postlethwaite, 2005; Bates and Postlethwaite, 2002). The optimisation aspect of the control technique attributes itself to the multi-objective optimisation criterion that is used to ensure stability whilst minimising the effect of these disturbances on system performance. These two aspects allow certification envelopes to be directly incorporated into the optimisation process, which makes this robust control technique very attractive for application to safety critical disciplines such as FCS design. This is in contrast to H_2 -optimal control, developed in the 1960's as the LQG control problem, which cannot provide formal guarantees, but rather only constrained safety margins against worst-case disturbances due to the norm's minimisation of the mean-square energy.

Although the term ' H_∞ ' first appeared in the 1980's, the technique was not adopted quickly by the aerospace industry. In the early days of H_∞ control development, a shortcoming was that robust control methods were renowned for their mathematical intricacy by being far more abstract than the classical control methods that were applied by control engineers prior. Additionally, the H_∞ control problem was first tackled by Doyle et al. (1989), in which it was found that the solution of two Algebraic Riccati Equations (AREs) was required. Subsequently, Gahinet and Apkarian (1994) solved the H_∞ problem by reducing it to a Linear Matrix Inequality (LMI). The issue with each of the proposed solutions until that point were that each resulted in an unstructured, otherwise known as full order, controller. The issue with obtaining an unstructured controller form is that structured PID control was vastly used in industry at the time, and therefore the application of unstructured controllers to a structured control framework was challenging. Seeing as unstructured controllers are more challenging to work with, the subsequent goal was to adapt the controller form into a structured, reduced order controller. However, this was not a straightforward task, given that the optimisation problem is non-convex and non-smooth. The term 'non-convex' indicates that standard optimisation techniques lead to local minimums as opposed to global minimums, which presents an issue during optimisation routines. This issue was subsequently solved by Apkarian and Noll (2006) through the use of bundle methods of convex, non-smooth optimisation techniques. Since then, various H_∞ methods have derived from the aforementioned principles. These include early instances of Mixed Sensitivity (MS) by Safonov, Chiang, and Flashner (1991), mixed H_2/H_∞ methods, proposed by Doyle et al. (1989) and H_∞ -LS, introduced by Glover, Sefton, and McFarlane (1990) and continued in D. McFarlane and Glover (1992).

It is usually unnecessary with H_∞ methods to calculate the optimal controller, and is usually easier to design a simpler, sub-optimal H_∞ controller using an iterative procedure. Furthermore, a sub-optimal

controller may exhibit other desirable properties such as lower bandwidth when compared to optimal controllers (Zhou and Doyle, 1998). An optimal solution for γ is found by converging the algorithm given by Glover and D. McFarlane (1989), which bisects the value of γ , until a sufficient value of γ_{min} is achieved. Some assumptions are made during the optimisation, however these are non-restrictive when the control problem is well posed (Bates and Postlethwaite, 2002).

Worth noting is that a disadvantage of H_∞ control is that it may sometimes result in the trade-off between system robustness and nominal performance. In addition, it can sometimes require the careful modelling of WC disturbances, which can prove a challenge in systems with high inherent complexity.

Prior to the 2010's, software for H_∞ -PID control was not available. However, as a result of the previously mentioned developments, time and efforts have been devoted to transforming these robust control principles into real, practical tools which have been adopted as aerospace industry standard for the design of robust controllers present in a multitude of applications, such as V/STOL aircraft, helicopters, civil transport aircraft and military aircraft (Bates and Postlethwaite, 2002). For example in Matlab, the tuning function 'hinfsyn' was available prior to 2006, whereas functions such as 'hinfstruct' and 'systune' only became available later in 2010 and 2012, respectively.

2.3.2. Application of H_∞ to C* control

H_∞ control has previously been applied to a C* controller in Marcos (2017). The study compared two approaches: the classical root-locus method and structured H_∞ control using MS. The model was the European commission's RECONFIGURE benchmark, a high fidelity non-linear aircraft model provided by Airbus. Provided with the model was a simplified benchmark linear model. 27 LTI plants were formed over a range of masses, centres of gravity, MAC, altitudes and Mach numbers at constant calibrated airspeed. The study pointed out that many options regarding objectives are provided by the function 'hinfstruct', which may be challenging to those less experienced with structured H_∞ design, as well as those with more experience. These options include tunable parameter initialisation, posing of the problem and conflicting requirement conditions as a result of multiple minima optimisations. These factors can over-constrain the optimisation process, leading to overconfidence in the software. It is also mentioned that the random nature of the algorithm requires multiple runs to produce an acceptable controller, which can pose issues regarding certification and transparency on the design process.

The H_∞ design process was restarted using a 'classical', standard H_∞ design rationale during the study due to unsatisfactory results arising from hinfstruct requirements and the standard C* control law interconnection. The S&T weights were derived ad hoc to be first order TFs, and after investigation it was concluded that despite 'very good' linear time response behaviour of the 27 plants, the frequency response plots showed potential for a lack of robustness through the lack of roll-off of the T responses, as well as clashing S&T responses in the crossover region. The hinfstruct design was formulated using an Airbus controller structure and the hinfstruct tunable parameters were initialised with classic C* gain values. As a result, the controller designed using hinfstruct yielded 'very good S&T properties' for the 27 plants as the peak in the frequency response was mitigated. Time domain simulations showed that H_∞ hinfstruct controller outperformed the classical controller, as well as the Airbus controller and the simplified Airbus benchmark at following a reference load factor input signal.

2.3.3. H_∞ Loop Shaping

The technique of H_∞ -LS emerged in the 1990's as a spin-off to the the standard H_∞ control formulation. It was based on the earlier works of Vidyasagar and Schneider (1982), Vidyasagar and Kimura (1986), Glover and D. McFarlane (1989), Glover and D. C. McFarlane (1989), Glover, Sefton, and Mcfarlane (1990), Georgiou and Smith (1990), D. McFarlane and Glover (1992), and later used by Bates and Postlethwaite (2002). The technique has since been applied to many different applications. H_∞ -LS was introduced as a concept by D. McFarlane and Glover (1992) by combining the characteristics of the H_∞ optimisation problem and the LS technique. The principle of H_∞ -LS is that it allows the designer to shape the singular values of the open loop using weighting filters, and the focus of the technique is to maximise robustness towards Normalised Coprime Factor (NCF) uncertainty.

The challenging aspect of LS is that CL stability cannot be guaranteed solely from shaping open loop singular values of a TF. Additionally, Bode (1945) proved that open-loop gain and phase near crossover have a large impact on CL stability. Moreover, stability is largely influenced by the rate of roll-off around the

crossover frequency, which should be kept to below 40 dB/dec. For these reasons, the technique is named a 'two-step' technique. The first stage is to shape according to performance objectives the open-loop singular values of the plant G with weighting filters, without considering the CL stability of the transfer. The weighting filter W_1 is often selected to provide integral action and ensure sufficient roll-off rate around the desired cross-over frequency. Subsequently, the H_∞ robust stabilisation problem is solved, producing the controller K_∞ , such that the robust stability properties of the shaped plant are maximised. It can be shown that the robustification process does not significantly degrade the performance that was specified by the plant weighting functions. A performance level greater than 4 indicates that the specified loop shapes are incompatible with robust stability, and must be redesigned (Bates and Postlethwaite, 2002). A constant prefilter can be added to the design to ensure zero steady-state tracking error, suitable for model-matching tasks.

H_∞ -LS is praised as a technique for many reasons, the first of which is that it does not require γ -iteration since the performance level is given as a result of the optimisation. This is contrary to how H_∞ problems are usually solved. Additionally, due to the maximisation of robustness to NCF uncertainty, it is able to provide robustness to unstable perturbations and uncertainty in the case of lightly damped resonant poles. The method also does not result in pole/zero cancellations, especially the cancellation of stable plant poles by controller zeros, due to the fact that the method takes into account disturbances at multiple points within the loop. Pole zero cancellation is especially a problem for systems with slow, stable poles in comparison to the system bandwidth, as they will only be cancelled for certain exogenous inputs and controlled outputs, meaning that the lightly damped poles may still appear in other CL TFs. Additionally, this can cause the system to exhibit poor robust performance, as even a small amount of uncertainty can cause movement of the damped pole locations.

Another area where H_∞ -LS performs well as a technique is the management of potentially conflicting design considerations around crossover. This is not an issue with LS, since the selection of the weighting filters shaping the open loop L simultaneously shapes the CL TFs of S and T . Additionally, LS allows the designer to set the CL bandwidth by influencing the open loop crossover frequency. Furthermore, a large benefit of H_∞ -LS is that it is able to guarantee balanced robustness and performance in the form of simultaneous gain and phase margins at both the plant input and the plant output. This is because the coprime factor singular value robustness measure γ mathematically bounds all input and output CL TFs. Additionally, since H_∞ -LS takes NCF uncertainty into consideration, the designed controller is robust to all unstable perturbations. This is highly beneficial since it helps to accurately model situations that can arise in aerospace applications, as these types of uncertainty also accept unstable TFs such as used by NCF (Bates and Postlethwaite, 2002). Drawbacks of H_∞ LS also exist, such as the fact that it can be more challenging to find optimal values of the weighting functions W_1 and W_2 , and perhaps more so for MIMO systems. Preconditioning may be required for badly aligned plants, and although likely advantageous, the state feedback/observer form differs from the easily tunable fixed-structure controller (Theodoulis, 2024). However, despite potential drawbacks, H_∞ -LS is still the chosen method of application for this study.

Comparison of H_∞ Loop Shaping to H_∞ Mixed Sensitivity

LS is not the only technique available to perform H_∞ robust control. Another notable technique is the mixed sensitivity (MS) method. Various MS methods currently exist, for instance the S/KS method, the S/T and S/KS methods and the Four-Block method. The principles of H_∞ -MS design revolve around the shaping of CL TFs S , KS and T according to design specifications (Doyle et al., 1989; Glover and D. McFarlane, 1989). Furthermore, unlike H_∞ -LS, the aim of H_∞ -MS design is to to maximise robustness towards multiplicative or additive uncertainty (Bates and Postlethwaite, 2002). Due to this reason, MS design can sometimes fail to model aerospace application situations accurately due to only accepting stable TFs (Bates and Postlethwaite, 2002).

It was previously mentioned that pole/zero cancellations were not present during LS. Unlike H_∞ -LS, pole-zero cancellations are rather common with H_∞ -MS. Whilst possible to mitigate this while using MS, it would require the use of complex weighting filters. This is one area in which the LS technique outperforms MS. However, the selection of the weighting filters used to shape the open loop L in LS can be more convoluted than those used to shape the CL sensitivity functions such as S and T in MS. This is problematic since it is necessary to enforce CL stability, and taking into consideration the plant phase may limit the open loop shape (D. McFarlane and Glover, 1992). In this case, if the MS technique is well mastered, the results obtained using H_∞ -MS design can be as equally effective as those obtained from H_∞ -LS design.

Additionally, if the plant is badly aligned in terms of singular values, it may also need to be pre-conditioned.

With MS design, the weighting filters for each of the CL sensitivity functions can be individually selected. Despite giving greater freedom in weighting filter selection during the design phase, it is usually required but seldom achieved to satisfy multiple design criteria simultaneously due to conflicts. Furthermore, where the LS method uses the open loop crossover frequency to determine the CL bandwidth, H_∞ -MS design is only able to treat LF and HF regions. Whilst LS is able to guarantee balanced robustness and performance margins at the input and the output of the plant, this is not the case with MS design, which provides guarantees at either the plant input or the plant output.

2.3.4. Applications of H_∞ Loop Shaping

H_∞ -LS has previously been applied to various studies, the first of which is a vectored thrust aircraft advanced flight control (VAAC) Harrier jet by Papageorgiou (2000). The research presented a first test flight of a Linear Parameter Variation (LPV) control law for the pitch dynamics, which was developed using the H_∞ -LS technique. The study marked an advancement on the previous successful flight test on the VAAC Harrier in 1993 through the automatic scheduling of the control laws over the flight envelope. The particular procedure followed was developed by Papageorgiou (2000) themselves. This research marked a significant stepping stone into the future of next generation FCS design.

Furthermore, Postlethwaite et al. (1999) used the H_∞ -LS technique to design and flight-test a multi-variable FCS for a NRC Bell 205 fly-by-wire helicopter. The design was based on a low order rigid-body model of the helicopter dynamics. The technique was deemed suitable due to the high complexity of the interaction between the fuselage, rotor and airflow, which resulted in a significant amount of model uncertainty. The controller structure employed is shown in Figure 2.6:

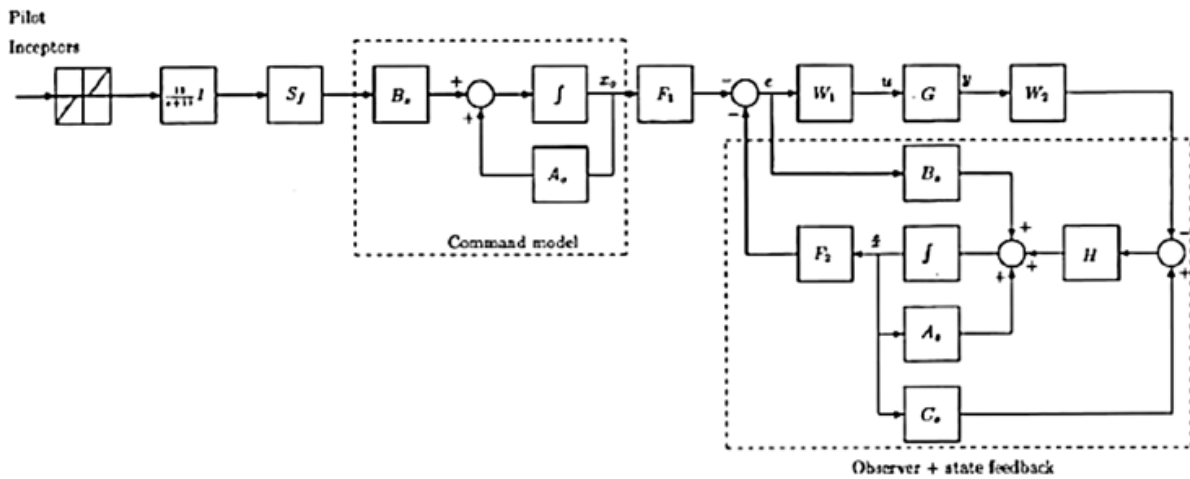


Figure 2.6: Controller scheme used onboard the Bell 205 helicopter (Postlethwaite et al., 1999)

It can be seen that the controller structure incorporates a 2DOF command prefilter for reference model tracking. Also, an observer is included in addition to state feedback. In the study, the control problem was solved using γ iteration to produce a sub-optimal controller. The research concluded successful engagement on the first flight and indicated encouraging overall performance. Desired performance was achieved given the gusty conditions and commanded pilot workload, and Level 2 HQs were established on various tasks. It is however argued how representable the coprime factor representation of uncertainty was of the discrepancies between the actual non-linear helicopter dynamics and the mathematical rotorcraft model used, and points to additionally collected flight test data for clarification. The study is believed to be the first successful test flight on a rotorcraft of an optimal H_∞ multivariable controller, undoubtedly setting an example of the capability of optimal H_∞ methods for use onboard non-conventional aircraft dynamic models.

More recently, a H_∞ -LS controller was designed by Marques (2024) for the simulator evaluation of a structured 2DOF control law for the TU Delft PH-LAB Cessna Citation I aircraft. This utilised the

corresponding high fidelity, 6DOF non-linear Delft University Aircraft Simulation Model and Analysis Tool (DASMAT). The EOMs used correspond to those established in the DASMAT model. Focus was placed on the pitch-axis longitudinal behaviour, for which a pitch capture, pitch tracking and flight path tracking controllers are relevant. The 2DOF controller structure used corresponding to the pitch rate controller is shown in Figure 2.7:

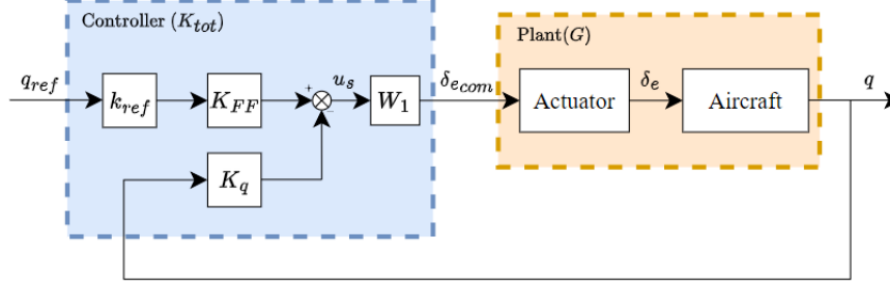


Figure 2.7: Pitch rate controller structure used by Marques (2024)

It can be seen that the post-weighting filter W_2 is negated in this control setup, and that K_{FF} and K_q are the controllers obtained through robustification. It was determined that the relevant HQ criteria to be used for assessment were the CAP, Gibson dropback and equivalent time delay criteria. Noteworthy is the incorporation of the gap metric as part of the HQ assessment, a measure of the distance of the space of potentially unstable systems. The designed reduced order controller, with an overall order of 5, met all of the specified HQ requirements and did not compromise robust stability.

A robust autopilot was also designed using the method of H_∞ -LS by Loukisas (2024). The study also implemented a 2DOF controller structure for sufficient performance reference tracking. The author points out the potential for automation of the LS two-stage design process due to its simplicity. Trimming and point linearisation is first applied over the flight envelope, generating 100 points. Proportional feedback is applied and the reference tracking output is chosen as the normal acceleration a_z , similar in definition to the normal load factor n_z . An integrator is selected for the weighting filter W_1 and the crossover frequency is selected through trial and error. The velocity based implementation and the $\dot{M}$ correction are also applied to improve reference tracking further, and a semi-dynamic TF is applied to the feedforward controller. Results show that non-linear simulations exhibited 'desirable behaviour', meeting robustness requirements, actuator rates and overshoot limits are not saturated and nonlinear simulations show a good handling of uncertainty. Settling time requirements are met by almost all linear design points.

2.3.5. Application of H_∞ Mixed Sensitivity to the Flying V Aircraft's C* Controller

A H_∞ -MS approach was recently taken by Pedroso D. (2024) to design a C* controller for the Flying V aircraft. In this work, the simulation model (including the trim and linearisation) of the Flying V aircraft was developed into a tensor-based formulation and implemented. Three design alternatives were proposed corresponding to the nature of the feedback signals, the C* measurement location and the modelling of the output disturbance. Firstly, it was decided to use the combined demand approach in feeding back the C* signal, due to good disturbance rejection in the C* signal and balanced rejection at LF for the q and n_z signals, as well as insufficient performance shown by the feedback of the individual signals apart. The C* was measured at the PS due to alignment with pilot cues and the HQ criteria definition. The feedback C* signal was selected to be measured at the Inertial Measurement Unit (IMU) location (assumed close to the ICR) as this allows for HF roll-off, an effect which is negated by the presence of the pitch rate derivative term $\dot{q}$ in the expression for the PS location. The final control structure used for the FCS is depicted in Figure 2.8:

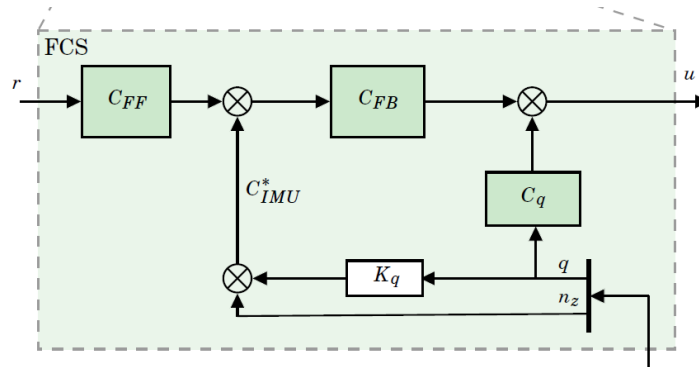


Figure 2.8: FCS structure used in the MS design of the Flying V aircraft's C^* controller (Pedroso D., 2024)

It can be seen that the feedback controller, C_{FB} , is placed in the forward path of the OL FB, rather than the backwards path; these are two design configurations that should be considered. A continuous controller, a modified continuous controller corresponding to one flight condition and a multi-modelling modified continuous controller corresponding to the whole flight envelope, were designed. Examination methods included SMs, HQs and time-domain simulations for the linear and non-linear cases. It was found that disturbances and noise were well attenuated in the input and virtual C^* output, and the continuous-time (CT) controller showed faster disturbance rejection and less overshoot than the discrete-time (DT) controller. A lack of noise attenuation at the plant input for the CT controller is compensated for in the DT design. SMs are mostly satisfied, except for the PM soft requirement for the CT plant input and the DT q signal. It is predicted that the HQs lie within level 1 for both of the designed controllers across all tested criteria.

Research Proposal

The research proposal aims to formulate comprehensive and coherent research goals that effectively provide answers to significant gaps in research that were presented in the literature review. The research gap and the significance of the subsequent research will be discussed in [Section 3.1](#), followed by a set of research objectives and questions presented in [Section 3.2](#). Lastly, the research plan is presented in [Section 3.3](#).

3.1. Research Gap and Significance

Through this literature review, the importance of the C^* control law onboard many aircraft has been made apparent; it is found to be the predominant method of pitch axis control in all flight phases except for take-off and landing flare. In addition, it has been shown that uncertainties remain in the domain of aircraft modelling, particularly under the scope of identifying accurate and precise estimates for the longitudinal and lateral aerodynamic stability derivative coefficients. Furthermore, until this point, the main types of controller that have been applied to the Flying V aircraft include classical PID control, which aimed predominantly for stability improvement, a reinforcement learning controller which did not assess resulting handling qualities, and nonlinear controller designs including sensor-based INDI approaches which are known to be highly sensitive to sensor noise, time-delays and unmodelled dynamics. A large concern arising from these studies is that no robustness can be guaranteed by the designs, especially in the presence of this uncertainty, as they are time and not frequency based approaches. However, it is important to model frequency-domain behaviour of systems, as this can sometimes highlight hidden behaviour that would may not have been noticed in the time-domain. This therefore brings attention to the gap in research that is contributing a robust controller to the body of previous controller design that is adequately able to provide adequate robustness to the present uncertainty.

The application of robust control design to the Flying V aircraft has been previously attempted, but is still in early development. It was shown by [Shahin \(2024\)](#) that a multi-loop μ optimal robust controller utilising inversion based flight control laws could be successfully applied to the Flying V aircraft, yielding sufficient HQ and SM performance for the nominal plant. However, the design failed to ensure all of the worst-case SMs met minimum requirements. In addition, the work of [Marcos \(2017\)](#) proved that the H_∞ -MS technique could be successfully applied to a C^* control law onboard a high fidelity non-linear aircraft model, providing superior S&T results compared to a classic approach, as well as high performing time-domain simulations. In addition to this, [Pedroso D. \(2024\)](#) showed that the H_∞ -MS technique could be successfully applied to the C^* control law onboard the Flying V aircraft for the stable flight points within the flight envelope, meeting level 1 HQ requirements and many of the prescribed SM requirements, as well as for WC SMs. These examples represent the successful application of H_∞ -MS methods to the C^* control law and the Flying V aircraft; however, there still exists a gap in research concerning the application of a H_∞ -LS controller to the C^* control law of the Flying V aircraft. There exist relatively little examples of the application of H_∞ -LS controllers to the C^* control law, and no examples of one applied to the Flying V aircraft in any capacity, despite compelling evidence to suggest that H_∞ -LS controllers may perhaps be able to improve on the overall results obtained using H_∞ -MS. LS controllers offer the benefits of easier implementation and increased transparency due to its structured and thus potentially low order characteristics. Robust performance and robust stability can be directly incorporated into the design due to its multi-objectivity, and a two-degree-of-freedom design allows handling quality requirements to be directly incorporated.

Therefore for the reasons presented in this chapter, this study aims to investigate this concept further, answering the question of its potential, with the purpose to bring the Flying V aircraft one step closer to airworthiness certification.

3.2. Research Objectives and Research Questions

The research gap that was identified in the previous section can be formulated into a comprehensive set of research objectives and research questions. The research objectives present the holistic aims of the research in terms of the technical goals it strives to achieve, and the two research objectives are summarised in the following:

Research Objective

Develop a robustly stabilising 2DOF H_∞ -LS controller for the C* flight control law present on the Flying V aircraft concept to guarantee closed loop stability and performance over the full operational envelope

Research Objective

Assess and validate the stability and performance of the closed loop against existing handling quality and stability requirement metrics in the presence of uncertainty and disturbances

The individual research questions that the study aims to answer are provided in the following:

Research Question 1

What are the pre-requisites in terms of the dynamic modelling of the aircraft dynamics and the posing of a framework for a robustly performing and stabilising C* controller?

Research Question 1.1

In what stage of the development process was the Flying V aircraft model that was provided at the beginning of the research?

Research Question 1.2

In what way should the model be pre-conditioned and adapted to suit the purposes of the application of robust control techniques?

Research Question 2

How can the design of a robustly stabilising H_∞ -LS C* controller benefit the current FCS of the Flying V aircraft in terms of performance and handling, and what aspects are involved in the design?

Research Question 2.1

What design choices are required are required during the FCS design?

Research Question 2.3

How can the design performance and HQ requirements effectively be incorporated into the design process?

Research Question 3

To what extent does the controller meet performance and robustness requirements that were specified a priori?

Research Question 3.1

How can the result of the designed LS controller be verified against pre-established HQ and stability margin criteria?

Research Question 3.2

To what extent does the LS controller design satisfy the specified HQ and stability margin criteria for the 1DOF and 2DOF designs?

3.3. Research Plan

The thesis plan can be divided into various phases. The first entails the development of the literature study, followed by the formation of the research objectives and research questions. The second phase encompasses the familiarisation with the Flying V aircraft model, as well as the preliminary development of the FCS. The next phase details the advancement of the design for application to the nonlinear model and the design of a multi-model controller. The research concludes with the dissemination, in which preparations for the final defence take place. The project planning of the thesis is summarised in the Gantt chart provided in [Appendix G](#).

Theoretical Background

This chapter presents the theoretical background that is required to understand prior to beginning to develop a C^* robust controller for the Flying V aircraft. It will begin with the theoretical underpinnings of H_∞ Loop Shaping design in [Section 4.1](#), followed by the introduction of disk margins in [Section 4.2](#), an important metric in the assessment of system stability. Lastly, it will discuss relevant stability analysis and HQ criteria in [Section 4.3](#).

4.1. H_∞ Loop Shaping

Before attempting to apply a H_∞ robust controller to this application, it is important to first ascertain the theoretical underpinnings behind the H_∞ -LS method. The LS procedure is divided into three distinct stages; these are firstly the shaping of the open loop, secondly robust stabilisation, and thirdly, controller implementation. This chapter will begin by explaining the theoretical background that lays behind the design procedure of the H_∞ -LS C^* controller, followed by the structure of the controller that will be implemented. Then it will explain any design decisions that were taken.

4.1.1. Generalised Feedback Configuration

In the H_∞ -LS design procedure, it is first required to define certain inputs and outputs of the generalised feedback configuration block diagram. The generalised feedback configuration block diagram shown in [Figure 4.1](#), and the relevant input and output signals are summarised in [Table 4.1](#):

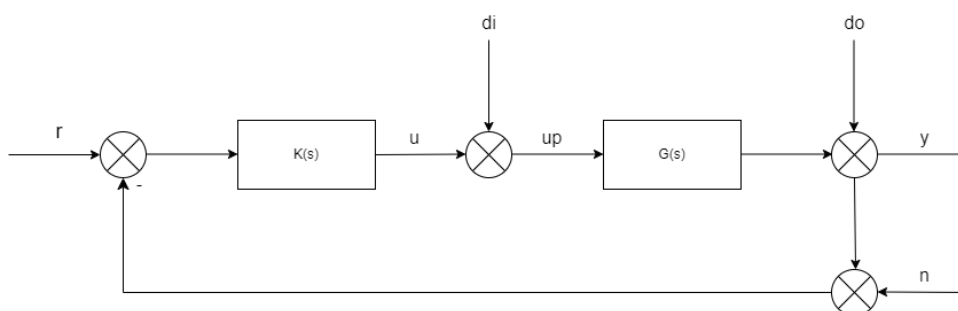


Figure 4.1: Generalised feedback configuration

4.1.2. Signal Definitions

As seen in [Table 4.1](#), the inputs and outputs obtained from the feedback configuration can be categorised into two main signal types: generalised disturbances and performance signals. Rather self-explanatorily, generalised disturbances refer to any type of disturbance input signal, such as input and output disturbances and measurement noise. The reference signal 'r' is also referred to as a generalised disturbance seeing as it is the input signal that produces a reaction from the system.

Performance signals refer to all signals that can be used to influence the system performance. The tracking error e is given as $e = r - y$, where y is the plant output, due to the fact that noise n is input into

Generalised Disturbances	r	The reference signal to be followed
	d_i, d_o	The plant input and output disturbances, respectively
	n	The measurement noise
Performance Signals	e	The tracking error ($= r - y$)
	y	The plant output
	u_p	The plant input
	u	The controller output

Table 4.1: Summary of input and output signals obtained from the generalised feedback configuration, shown in [Figure 4.1](#)

the system output y after the plant output disturbance is added, and this does not represent the tracking performance of the inherent system. The tracking error e will provide an important role in the design of the 2DOF feedforward tracking controller, as will be explained in subsequent chapters. It is also important to distinguish the difference in inputs u and u_p , which are the controller output and plant input respectively, and these are differentiated through the addition of the disturbance input d_i .

4.1.3. Gang of Six

For a successful implementation of a H_∞ -LS controller, it should aim to meet certain design requirements. These design requirements include disturbance rejection and reference tracking, noise attenuation and control system reduction. With H_∞ -LS it is possible to satisfy all of these design requirements CL TFs by shaping the open loop TFs L_i and L_o . The input-output relationships corresponding to the TFs from disturbance signals to performance signals can be summarised in matrix form as shown in [Equation 4.1](#).

$$\begin{bmatrix} y \\ u_p \\ u \\ e \end{bmatrix} = \begin{bmatrix} S_o G & S_o & -T_o & T_o \\ S_i & -K S_o & -K S_o & K S_o \\ -T_i & -K S_o & -K S_o & K S_o \\ -S_o G & -S_o & T_o & S_o \end{bmatrix} \cdot \begin{bmatrix} d_i \\ d_o \\ n \\ r \end{bmatrix} \quad (4.1)$$

Additionally the input loop gain is given by $L_i = KG$ and the output loop gain is given by $L_o = GK$, where K is the controller placed in feedback and G are the plant model dynamics. This matrix consists of CL TF components that are named sensitivity functions. The most important CL TFs are known as the 'gang of six' (GO6), and their mathematical composition and relevance is detailed in [Table 4.2](#):

CL TF	Mathematical expression
S_i	$(1 + L_i)^{-1}$
S_o	$(1 + L_o)^{-1}$
T_i	$L_i(1 + L_i)^{-1}$
T_o	$L_o(1 + L_o)^{-1}$
$S_o G$	$(1 + L_o)^{-1} G$
$K S_o$	$K(1 + L_o)^{-1}$

Table 4.2: GO6 CL TFs

S_i : Input Sensitivity

S_i represents how disturbances affect the system at the input and is denoted by the transfer from d_i to u_p .

S_o : Output Sensitivity

S_o represents how disturbances affect the system at the output and is denoted by the transfer from d_o to y .

T_i : Input Complementary Sensitivity

T_i represents how input disturbances affect the controller output, and is denoted by the transfer from d_i to u .

 T_o : Output Complementary Sensitivity

T_o represents how output disturbances or sensor measurement noise affects the output and is denoted by the transfer from n to y .

 S_oG : Output Sensitivity

S_oG represents how input disturbances affect the output.

 KS_o : Controller Sensitivity

KS_o represents how the reference signal as well as disturbances and noise at the output, affect the control input and is denoted by the transfer from d_o to u .

4.1.4. The 4-Block Problem

Considering once again the generalised feedback configuration shown in Figure 4.1 and after neglecting the presence of noise n and the reference input signal r , such that the robust controller $K(s)$ is placed in direct feedback with the plant G , the reduced generalised feedback configuration shown in Figure 4.2 is obtained:

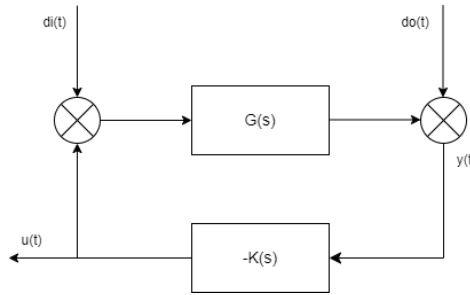


Figure 4.2: Generalised feedback configuration

As a result, the matrix representation containing the sensitivity function transfers between the performance signals and generalised disturbances can be mathematically manipulated to produce the reduced matrix known as the 4-block problem, which is expressed in Equation 4.2.

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} S_o & S_oG \\ -KS_o & -T_i \end{bmatrix} \begin{bmatrix} d_o \\ d_i \end{bmatrix} \quad (4.2)$$

This formulation provides a simpler outlook on the input-output relations, without having to account for reference tracking and noise at the sensor input.

4.1.5. Design Requirements

When designing a H_∞ -LS controller, it is important that it exhibits certain desirable characteristics. These include disturbance rejection, reference tracking, noise attenuation and control signal reduction. These characteristics can be produced in the design by enforcing certain constraining requirements which ensure that certain performance metrics are achieved. In this section, each of the characteristics and the corresponding design requirements are explained.

Disturbance Rejection

Disturbance rejection describes the ability of the system to reduce the effect of disturbances at the plant input and output. Disturbance rejection occurs at the lower frequency ranges due to the fact that disturbances generally occupy the LF range.

Reference Tracking

Good reference tracking is achieved when the effect of the reference r on the tracking error e is reduced as much as possible. Reference tracking occurs at low mid frequencies around crossover.

Noise Attenuation

Noise attenuation aims to reduce the effect of noise n on the controller output u and the plant output y . Noise is largely present at high frequencies, and therefore it is important to study the effect of sensor noise within the HF domain.

Control Signal Reduction

Control signal reduction represents the reduction in effect of the input disturbance d_i , output disturbance d_o and reference signal r on the controller output u . It should be aimed for to reduce the control signal in HF so as to avoid wearing out actuators.

4.1.6. Types of Uncertainty

Within control systems modelling there exist distinct types of uncertainty: structured and unstructured uncertainty. These can differ in their dependence on model parameters, their mathematical representation, the modelling approach taken to model them, the methods used to treat them and the amount of conservativeness accounted for in the controller design. This section will describe the properties of each uncertainty type and their differences.

Structured Uncertainty

A key characteristic of structured uncertainty is that it has a known, well-defined structure which can be mathematically represented as variations of certain parameters within a model. They can usually be represented in block-diagonal form in which each block corresponds to a specific source of uncertainty. Examples of structured uncertainty include plant parametric uncertainty, such as mass variation, aerodynamic coefficient uncertainty or uncertainty due to time delays in the system.

Analysis conducted using structured uncertainty such as structured singular value, or μ analysis, tend to be less conservative in result when compared to analysis involving unstructured uncertainty. It is also easier to finely tune controllers according to specific parameter variations. Furthermore, models that involve structured uncertainty tend to more accurately represent the variations of real world systems.

Unstructured Uncertainty

Unstructured uncertainty is a more general, less defined uncertainty source that can occur for instance due to unmodelled dynamics which may contain high frequency modes. It is often these unmodelled high frequency dynamics which prove the most troublesome in robust controller design, since the designed controller should be designed to overcome these without producing adverse effects. Only a bound on the magnitude of the uncertainty over the frequency range can be estimated using singular values. This is different from the structured uncertainty case where uncertainty sources are better known. Examples of unstructured uncertainty include additive, input multiplicative, output multiplicative and their respective inverses, and the required bounds on sensitivity functions to ensure CL robustness for each type are summarised in [Table 4.3](#):

Type of unstructured uncertainty	Sensitivity function bound
Additive	KS_o
Inverse additive	S_oG
Input multiplicative	T_i
Output multiplicative	T_o
Inverse input multiplicative	S_i
Inverse output multiplicative	S_o

Table 4.3: Types of unstructured uncertainty and required sensitivity function bounds

4.1.7. Normalised Coprime Factor Uncertainty

Normalised Coprime Factor (or NCF) uncertainty for linear systems was derived by [Vidyasagar and Schneider \(1982\)](#) and is a form of unstructured uncertainty. The normalised coprime factorisation is an example of one from an arbitrarily large set of coprime factorisations. The right coprime factorisation case also exists. It was proven that any even, unstable system G can be decomposed into two stable, coprime TFs $\tilde{M}$ and $\tilde{N}$ of the same order such that [Equation 4.3](#) holds:

$$G = \tilde{M}^{-1} \tilde{N} \quad (4.3)$$

$\tilde{M}$ and $\tilde{N}$ constitute a Left Coprime Factorisation (LCF) of the system $G(s)$ ([D. McFarlane and Glover, 1992](#)). This is the case if the following conditions hold:

- $\tilde{M}$ is a square matrix
- $\det(\tilde{M}) \neq 0$: the determinant of $\tilde{M}$ is non-zero
- $G = \tilde{M}^{-1} \tilde{N}$: the matrix product of the inverse of $\tilde{M}$ with $\tilde{N}$ is equal to the plant G
- $\exists V, U \in \mathcal{RH}_\infty$ such that $\tilde{M}V + \tilde{N}U = I$

A normalised coprime factorisation is normalised when it follows [Equation 4.4](#):

$$\tilde{M}\tilde{M}^* + \tilde{N}\tilde{N}^* = I \quad (4.4)$$

or, when the matrix $X_l = [\tilde{M} \ \tilde{N}]$ is coinner, ie: $X_l X_l^* = I$.

Coprimeness occurs when $\tilde{M}$ and $\tilde{N}$ have no common RHP zeros which would be cancelled through the multiplication of $\tilde{M}^{-1}$ and $\tilde{N}$. A result of NCF is that instability is resolved; all RHP zeros of $G(s)$ become the zeros of $\tilde{N}$ and all RHP poles of $G(s)$ become the zeros of $\tilde{M}$. To make the coprime factorisation proper, additional stable poles are added and scaled so that the TFs are normalised. The implementation of the NCF can be verified through the use of the Matlab function as follows:

```
[fact, M_tilde', N_tilde] = lncf(G)
```

where G is the plant model for which the LNCF is desired. `lncf` is the Matlab function used to calculate the LCF of a system, so it is able to decompose the system G into this factorisation. Alternative functions exist, such as 'ncfmr' (normalised coprime factor model reduction), which also reduce the model order ([lncf - Left normalized coprime factorization 2025](#)).

A model perturbed by NCF uncertainty can be visualised as according to the block diagram representing the perturbed plant, as shown in [Figure 4.3](#):

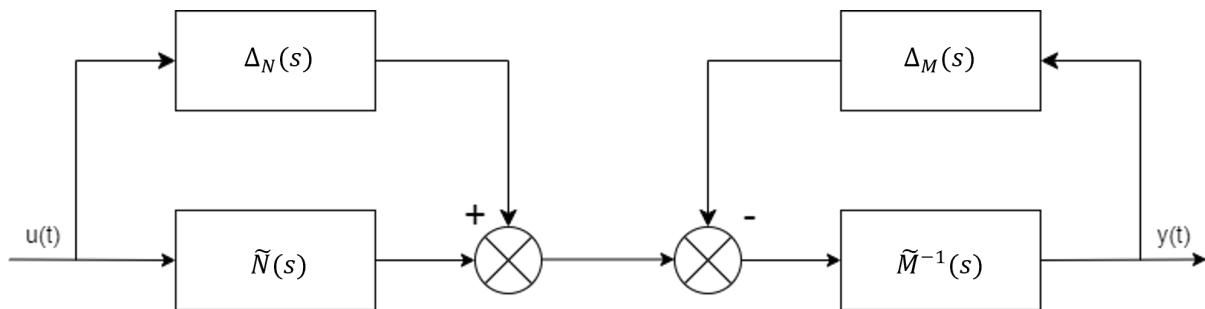


Figure 4.3: NCF perturbed plant

Evaluating the block diagram, the expression for the perturbed plant G_p is equal to [Equation 4.5](#):

$$G_p = (\tilde{M} + \Delta_M)^{-1} (\tilde{N} + \Delta_N) \quad (4.5)$$

4.1.8. NCF Uncertainty Computation

Computation of NCF uncertainty was performed by [D. McFarlane and Glover \(1992\)](#), and begins with the minimal state-space realisation of a transfer matrix G , given by [Equation 4.6](#):

$$G = C_y(sI - A)B_u + D_{yu} \quad (4.6)$$

Following this, the state-space realisation of the LNCF is given by [Equation 4.7](#):

$$[\tilde{N} \tilde{M}] \triangleq \begin{bmatrix} A + HC_y & (B_u + HD_{yu}H) \\ R^{-\frac{1}{2}}C_y & (R^{-\frac{1}{2}}D_{yu}R^{-\frac{1}{2}}) \end{bmatrix} \quad (4.7)$$

where H represents the filter gain and is given by [Equation 4.8](#):

$$H \triangleq -(YC_y^* + B_u D_{yu}^*)R^{-1} \quad (4.8)$$

Where $Y = Y^* > 0$ is the solution to the generalised filtering Riccati equation (GFARE), given in [Equation 4.9](#):

$$(A - B_u S^{-1} D_{yu}^T C_y)Y + Y(A - B_u S^{-1} D_{yu}^T C_y)^T - (YC_y^T)R^{-1}(YC_y^T)^T + B_u S^{-1} B_u^T = 0 \quad (4.9)$$

and R and S are given by [Equation 4.10](#) and [Equation 4.11](#) respectively:

$$R = I + D_{yu} D_{yu}^T \quad (4.10)$$

$$S = I + D_{yu}^T D_{yu} \quad (4.11)$$

The NCF can be computed in Matlab using the robust control toolbox function 'lncf' or using the control system toolbox function 'icare' ([icare - Implicit solver for continuous-time algebraic Riccati equations 2025](#)) to solve the GFARE equation given by [Equation 4.9](#).

Practically, to estimate the magnitude of the NCF uncertainty block in Matlab, firstly parametric uncertainty must be created and modelled. This is done by using the uncertain real parameter RCT function 'ureal' and the level of uncertainty can be specified; this may be in terms of percentage, absolute deviation or a specified range ([ureal - Uncertain real parameter 2025](#)). Then, the uncertain state space blocks are added to the Simulink model at the location of the uncertainty. The methodology required here was replicated from [Goz \(2024\)](#). In the case of this application, this is added within the aerodynamic coefficient subsystem block of the original nonlinear Flying V aircraft Simulink model. Using the function 'ulinearize' ([ulinearize - Linearize Simulink model with Uncertain State Space block 2025](#)), the uncertain state space 'uss' is created, including the modelled parametric uncertainty. Once the uss model is derived, the NCF uncertainty block is given by [Equation 4.12](#):

$$\Delta(s) = [\Delta_N \quad -\Delta_M] \quad (4.12)$$

To obtain the NCF uncertainty block, the uncertain system $G_{unc}(s)$ is sampled over the ranges of the uncertain parameters within the specified uncertainty range. This is done using the Matlab 'gridureal' ([gridureal - Grid ureal parameters uniformly over their range 2025](#)) function to obtain a set of perturbed systems $G_p(s)$. Next, the NCF for each perturbed system is calculated using [Equation 4.13](#):

$$G_p(s) = \tilde{M}_p^{-1} \tilde{N}_p \quad (4.13)$$

The components of the uncertainty block, which can be found by obtaining the difference in NCF of the perturbed model and the nominal model is calculated in [Equation 4.14](#) and [Equation 4.15](#):

$$\Delta_{M,p} = \tilde{M}_p - \tilde{M} \quad (4.14)$$

$$\Delta_{N,p} = \tilde{N}_p - \tilde{N} \quad (4.15)$$

The worst case gain bound, and thus an estimate for $\Delta(s)$, can be calculated for all NCF uncertainty blocks $\Delta_p(s) = [\Delta_{N,p} \quad -\Delta_{M,p}]$ using the 'fitmagfrd' Matlab function ([fitmagfrd - Fit frequency response magnitude data with minimum-phase state-space model using log-Chebyshev magnitude design 2025](#)). The uncertainty blocks are all real, rational and stable perturbations. The magnitude of the estimate of $\Delta(s)$ measured by the H_∞ norm gives an indication of the stability margin ϵ that is guaranteed by the robust controller that will be designed to robustly stabilise the set of NCF perturbed plants.

4.1.9. Loop Shaping Design Procedure

The basis of the LS design procedure (LSDP) lies in achieving desirable properties for the singular values of the open loop; LS aims to achieve high gain at low frequency for good disturbance rejection and tracking qualities, provided the availability of sufficient control authority, and low gain at high frequency for good noise attenuation and low control action, provided that the gain K_{max} of the controller is not too large.

Shaping of the Open Loop

The first stage in the LS process is to shape the plant $G(s)$ with the pre/post compensators W_1 and W_2 respectively, to obtain a loop gain shape $\sigma(L_i)$ and $\sigma(L_o)$ that is desired, according to earlier discussion. The selection of these compensators are not necessarily straightforward, and can require skill and experience in doing so. This is complicated further for MIMO systems compared to SISO systems, since there is not usually one selection of a combination of compensators that satisfies the LS requirements of all input-output channels ([D. McFarlane and Glover, 1992](#)). The weighting filters are chosen such that there does not need to be pole-zero cancellations, and that there are no hidden modes within the system. In general for MIMO systems, the filters are chosen to be diagonal as this corresponds to the respective input-output channels. Important to note at this stage of the design process is that the CL stability of the system is not taken into account as it does not initially account for phase requirements. CL stability is treated later during the robustification phase. The desired open loop shape is shown graphically in [Figure 4.4](#):

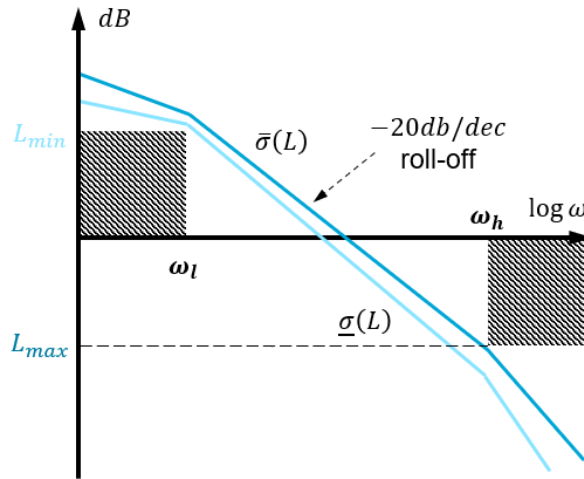


Figure 4.4: Ideal open loop shape ([Theodoulis, 2024](#))

Looking at [Figure 4.4](#), it can be noted that increasing the vertical intercept of the open loop SVs (the LF gain) in turn causes the location of the open loop crossover on the horizontal axis around MF to shift rightwards. This effect is achieved by increasing the magnitude of the pre-compensator W_1 . However, doing so is at the detriment of controller formulation ease, as will be explained later in the design process. Once the desired loop shape is obtained, the shaped plant G_s is formed according to [Equation 4.16](#), accounting for matrix in-commutativity rules, and is shown schematically in [Figure 4.5](#):

$$G_s = W_2 G W_1 \quad (4.16)$$

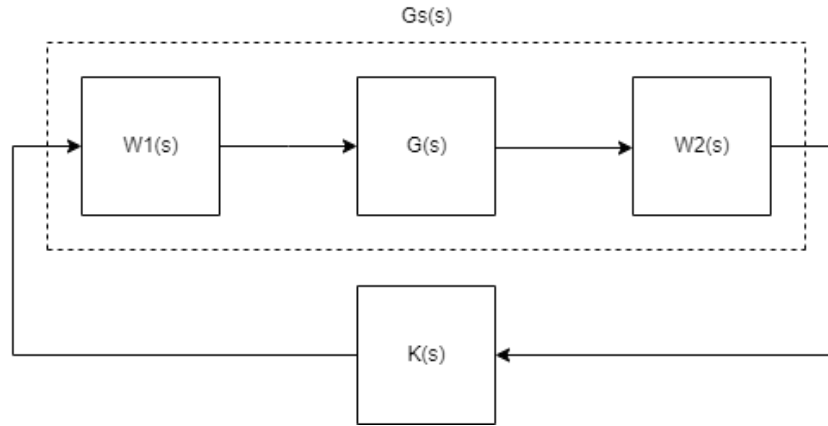


Figure 4.5: Shaped plant

where $G(s)$ is the plant that is being robustified and $K(s)$ is the robustly stabilising controller that is designed in the next stage of the H_∞ -LS process.

Robust Stabilisation

Not only should the nominal plant be stabilised, but it is also the aim of the controller to robustly stabilise the family of NCF perturbed plants. As a result of the first stage of LS, the shaped plant was not necessarily CL stable. For robust stabilisation, a controller $K(s)$ should be designed that robustly stabilises the shaped plant $G_s(s)$ under uncertainty over its LCF. The set of shaped, perturbed plants are now given by Equation 4.17:

$$G_{s,p} = (\tilde{M}_s + \Delta M_s)^{-1}(\tilde{N}_s + \Delta N_s) \quad (4.17)$$

and the size of the LCF is given as Equation 4.18:

$$\|\Delta(s)\|_\infty = \|[\Delta_N \quad -\Delta_M]\|_\infty < \epsilon \quad (4.18)$$

where ϵ represents the robustness margin which should be sufficiently large (D. McFarlane and Glover, 1992). The robustness margin is also given as the inverse of the performance level by Equation 4.19:

$$\epsilon_{max} = \gamma_{min}^{-1} \quad (4.19)$$

Glover and D. McFarlane (1989) demonstrated that a CL system is stable for a set of perturbed plants G_p when a particular generalised SM ϵ is achieved. Since in control theory, SMs are conventionally displayed using Nyquist and Nichols diagrams, Glover, Vinnicombe, and Papageorgiou (2000) derived a way to represent the SM ϵ in terms of simultaneous gain and phase variations. It is due to this that sufficient gain and phase margins are guaranteed for the H_∞ -LS technique, making it highly beneficial as a technique. The mathematical expressions pertaining to the SISO guaranteed gain and phase SMs are provided in Equation 4.20 and Equation 4.21, respectively:

$$-20\log_{10} \frac{1+\epsilon}{1-\epsilon} \text{dB} \leq \text{GM}_{cp} \leq +20\log_{10} \frac{1+\epsilon}{1-\epsilon} \text{dB} \quad (4.20)$$

$$-2\sin^{-1}\epsilon \text{degrees} \leq \text{PM}_{cp} \leq +2\sin^{-1}\epsilon \text{degrees} \quad (4.21)$$

In the MIMO case, only half of the allowed gain and phase margins are accepted at each input and output, due to reduced margins caused by channel coupling. An important result of the work of [D. McFarlane and Glover \(1992\)](#) is that robust stability is achieved with little degradation in the original performance characteristics set by LS when the robustness level ϵ is greater than 0.25. For this reason, a robustness level of $\epsilon > 0.25$ will be sought after during the design process. If this isn't achieved, the weighting filters must be redesigned.

Due to the fact that the stability margin/performance level is directly calculated in the robustification stage using functions like `systune`, LS can be advantageous over MS methods as the performance level does not need to be iteratively calculated, shortening and easing the design process.

Robust Controller Synthesis Methods

Various methods to design a robustly stabilising controller are in existence today and this section will explain the three most commonly used.

The first is based on mathematical first principles. [Skogestad and Postlethwaite \(2005\)](#) proved that for the minimal realisation of a linear time-invariant (LTI) system G with matrices A , B_u , C_y and D_{yu} , the positive feedback controller that is able to guarantee NCF robust stabilisation is given by [Equation 4.22](#):

$$\begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} = \begin{bmatrix} (A + B_u F) + \gamma^2 (L^T)^{-1} Z C_y^T \cdot (C_y + D_{yu} F) & \gamma^2 (L^T)^{-1} Z C_y^T \\ B_u^T X & -D_{yu}^T \end{bmatrix} \quad (4.22)$$

and the intermediate matrices F and L are given by [Equation 4.23](#) and [Equation 4.24](#):

$$F = -S^{-1}(D_{yu}^T C_y + B_u^T X) \quad (4.23)$$

$$L = (1 - \gamma^2)I_n + XZ \quad (4.24)$$

The matrices X and Z in [Equation 4.22](#) are the solutions to the generalised control algebraic Riccati equations, where X is the solved GCARE and Z the solved GFARE of the shaped plant respectively. These solutions can be found in Matlab using the CST function 'icare'. The solutions for X and Z are given by [Equation 4.25](#) and [Equation 4.26](#) respectively:

$$(A - B_u S^{-1} D_{yu}^T C_y)^T X + X(A - B_u S^{-1} D_{yu}^T C_y) - (X B_u) S^{-1} (X B_u)^T + C_y^T R^{-1} C_y = 0 \quad (4.25)$$

$$(A - B_u S^{-1} D_{yu}^T C_y)Z + Z(A - B_u S^{-1} D_{yu}^T C_y)^T - (Z C_y^T) R^{-1} (Z C_y^T)^T + B_u S^{-1} B_u^T = 0 \quad (4.26)$$

where R and S are equal to [Equation 4.27](#) and [Equation 4.28](#) respectively:

$$R = I_p + D_{yu} D_{yu}^T \quad (4.27)$$

$$S = I_m + D_{yu}^T D_{yu} \quad (4.28)$$

Using this method, it follows that the robustness margin can be calculated as in [Equation 4.29](#):

$$\epsilon_{max} = \sqrt{1 - \|\tilde{N}_s \tilde{M}_s\|_H^2} = (1 + \rho(XY))^{-\frac{1}{2}} \quad (4.29)$$

where $\|\cdot\|_H^2$ denotes the Hankel norm and ρ the spectral radius, known as the maximum absolute value of the eigenvalues of the system ([Skogestad and Postlethwaite, 2005](#)). This method provides an

ironclad method to synthesise a reliably accurate robust controller which is able to robustly stabilise all NCF uncertainty perturbed systems within the set.

Ncfsyn is an robust control toolbox (RCT) function in Matlab that provides an alternative way to compute the negative feedback full order robust controller ([ncfsyn - Loop shaping design using Glover-McFarlane method 2025](#)). Its syntax is shown below:

```
[K, CL, gamma, info] = ncfsyn(-G,W1,W2,tol)
```

With the function it is possible to specify the system G and the weighting filters W_1 and W_2 , and its outputs are the controller K , the CL system from disturbances to outputs, the performance level γ and within 'info', the minimum achievable performance level γ_{min} . The performance level γ is the resulting performance level given the specified tolerance level 'tol', which is typically approximately 10^{-3} and chosen so as to not give too fast dynamics, and which satisfies $|\gamma - \gamma_{min}| < \text{tol} \cdot \gamma_{min}$ ([ncfsyn - Loop shaping design using Glover-McFarlane method 2025](#)).

Next, systune is a commonly used RCT function that tunes fixed structure control systems subject to soft and hard design goals ([systune - Tune fixed-structure control systems modeled in MATLAB 2025](#)). It is able to compute an optimal robust controller of a given reduced order. It numerically optimises a set of tunable parameters to be able to meet a set of performance requirements that were earlier outlined; reference tracking is specified with `TuningGoal.Tracking`, disturbance rejection with `TuningGoal.Rejection`, gain and stability margins with `TuningGoal.Margins` and requirements on the loopshape with `TuningGoal.LoopShape`. Hard requirements are strict tuning requirements that the tuning must satisfy, whereas soft requirements usually specify performance targets. Systune tunes the parameters first according to the hard goals that are present, and if these are satisfied, it then tries to meet the soft tuning requirements. The function returns the best achieved values of the soft and hard constraints that it is subject to. The function syntax is given by:

```
[CL,fSoft,gHard,info] = systune(CLO,SoftReqs,HardReqs,options)
```

The function takes as an input the plant model, or set of plant models, CL. If parametric uncertainty is present in the model, the set of perturbed systems are contained in CL. The other inputs include the specified soft requirements `SoftReqs`, hard requirements `HardReqs` and additional optimisation options such as the maximum number of iterations and optimisation method. One output of the function is the tuned control system `CLO` interconnection, which has the same number and type of tunable blocks as the input plant model/models. Additionally, the function outputs the best achieved soft constraint values, `fSoft`, and the best achieved hard constraint values, `gHard`.

As systune allows the designer to specify the order of the obtained controller, it is possible to compare the effect of reducing or increasing the controller order on the resulting controller performance level and robustness margin. The `SoftTol` flag determines how close the achieved performance level is to the optimal performance level. Lastly, if it is desired that various parameters within a Simulink block model be tuned, the model must be reconfigured for linearisation and a `SiTuner` interface should be created ([siTuner - Interface for control system tuning of Simulink models 2025](#)).

Internal Stability

Feedback of $K(s)$ with $G(s)$ is deemed internally stable when the following criteria are met:

- All TFs are contained within the real space of the H_∞ norm, ie: $S_o, KS_o, S_oG, S_i \in \mathcal{RH}_\infty$
- The CL connection is well posed, ie: $\det(I_p + GK)(\infty) \neq 0$

It can be proven that the controller $K(s)$ stabilises the family of NCF perturbed plants $G_p(s)$ if the controller $K(s)$ internally stabilises $G(s)$ and if the condition in [Equation 4.30](#) holds true:

$$\left\| \begin{bmatrix} -K(I_p + GK)^{-1}\tilde{M}^{-1} \\ (I_p + GK)^{-1}\tilde{M}^{-1} \end{bmatrix} \right\|_\infty \leq \epsilon^{-1} \quad (4.30)$$

This can be rewritten as [Equation 4.31](#):

$$\left\| \begin{bmatrix} -K \\ I_p \end{bmatrix} (I_p + GK)^{-1} \tilde{M}^{-1} \right\|_\infty \leq \epsilon^{-1} \quad (4.31)$$

A notable characteristic of the H_∞ norm is that it does not change after right multiplication with a co-inner function.

Controller Implementation

Once the robustifying controller K_s is obtained using one of the existing available methods, it can be implemented within the control system. It is comprised of the robust controller K obtained from robust stabilisation, as well as W_1 and W_2 . Following [Glover and D. McFarlane \(1989\)](#), the total controller to be implemented is given by [Equation 4.32](#):

$$K_s = W_1 K W_2 \quad (4.32)$$

The robust controller can be implemented in different ways; the grouping of the robust controller in series with the weighting filter is recognised as the 'regulator implementation' when placed in feedback with the plant ([Theodoulis, 2024](#)). This is a similar view on the controller implementation as [Figure 4.5](#) but differing in terms of definition. The implementation is visualised in [Figure 4.6](#):

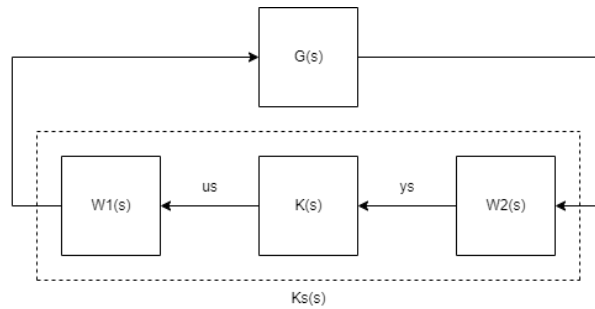


Figure 4.6: Regulator implementation

The standard regulator controller implementation form can be modified for various purposes. The tracker implementation is similar to the regulator form but contains the reference input r and a constant command pre-filter to track the reference and to ensure zero steady-state gain. For this, $W_1(s)$ or $G(s)$ should contain integral action ([Theodoulis, 2024](#)). The constant command pre-filter is given by [Equation 4.33](#):

$$K_f = K(0)W_2(0) \quad (4.33)$$

where $K(0)$ and $W_2(0)$ indicate the robust controller and post-compensator dynamics at a frequency of 0 rads^{-1} , ie: the DC gain of the feedback path. The modified tracker implementation is shown in [Figure 4.7](#).

It can be seen that negative feedback is added between the forward path and the feedback path before the pre-compensator W_1 . The robust controller can also be written in a pure state estimator and state feedback controller form, aiding robustness.

Results of Controller Implementation

Once implemented, the effects of the controller on the system CL dynamics can be investigated. Given a system containing an inner loop (IL), for instance a pitch rate damping IL, state feedback is first performed. Once the pitch rate IL provides damping to the system, the open loop TF corresponding to all of the relevant system outputs can be obtained. The open loop of the output of interest can be isolated from the SIMO system and the SVs corresponding to this damped loop can be inspected at LF and HF, as well as the bandwidth of the crossover frequency and roll-off rate. After this, it is determined whether the loop gain needs to be reshaped using the pre and post compensators.

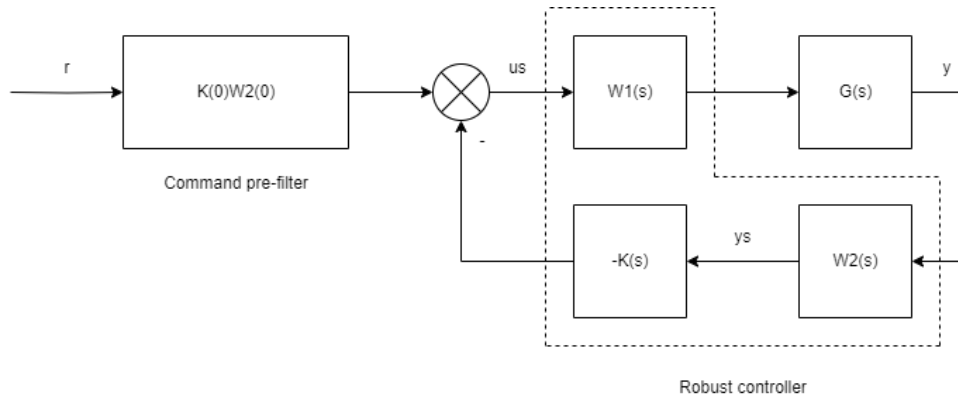


Figure 4.7: Alternative tracking form

Poles and zeros used in the weighting filters have an influence on the resulting loop shape; zeros reduce the roll-off rate, whilst poles increase the roll-off rate. A simple integrator of $\frac{1}{s}$ characteristically has a high loop gain at LF and low loop gain at HF and is used to impose a set roll-off rate of -20 dB/dec around crossover. If the selected system is first normalised with a scaling gain, this can be multiplied by the gain K_{L_t} , where L_t is the ideal target loop gain, in order to be able to adjust the exact value of the gain crossover frequency.

Forward and Feedback Implementation

It is possible to implement the controller $K(s)$ in either a forward or feedback configuration; in the forward configuration, the controller is placed after OL feedback and before the pre-compensator W_1 . In feedback, the controller is placed as depicted in Figure 4.7 in the feedback path, after the post-compensator and before the OL feedback summation.

Generally, the robust controller is first placed in the forward configuration, as with the regulator implementation. However, in this configuration, the system designed in the frequency domain may show excessive amounts of overshoot due to the reference exciting the dynamics of K . This is due to the fact that the robust controller dynamics were designed in terms of frequency to ensure robustness against NCF uncertainty and therefore do not account for satisfactory performance in the time domain (Theodoulis, 2024). Therefore, in the alternative feedback implementation, the robust controller is moved into the feedback path.

The benefits of the feedback configuration is that it generally reduces overshoot and control actuation, meaning that the actuators are pushed less to achieve certain deflection magnitudes and rates, placing less stress on the controller when aiming to achieve a certain commanded response. This can come with the additional drawback that it can result in a slower moving or sluggish system; however, this is usually not too much of an issue and this property can be improved by adding feedforward control (Theodoulis, 2024).

4.2. Disk Margins

Although not directly related to the theoretical underpinnings of H_∞ robust control theory, it is particularly relevant to introduce the frequency-domain based metrics through which the success of the H_∞ -LS design will be assessed. This is largely through the use of stability margins, including classical margins (CMs), as well as loop-at-a-time (LAAT) disk-based margins and multi-loop disk-based margins.

In control engineering, margins are introduced as safety factors to ensure that control design accounts for the potential mismatch in gain and phase between the mathematical plant model used during design and the real life system. This phenomenon is especially visible at HF where noise and unmodelled dynamics may be present (Seiler, Packard, and Gahinet, 2020). CMs are commonly used to assess system stability as it measures a system's ability to tolerate perturbations in gain and phase while retaining CL stability. Typical gain and phase margin requirements are 6 dB of gain and 45° of phase variation, which remains valid when critical frequencies lie within the range of frequencies for which the model is high fidelity

(Seiler, Packard, and Gahinet, 2020). However, CMs are generally not considered the best technique for assessing system stability; CMs are used to represent individual variations in gain and phase, however they fail to encompass simultaneous perturbations. Thus, a particular combination of a smaller amount of gain and phase variation applied together has the possibility to lead to instability within the CL system (Seiler, Packard, and Gahinet, 2020). Particular care should be taken in this regard when using automated computer-based design, which could potentially optimise CMs, sacrificing robustness to their combined variation.

Additionally, a downfall of LAAT analysis is that the effect of simultaneous perturbations across multiple channels cannot be represented. In this regard, DMs provide an effective way to investigate the effects of simultaneous perturbations across multiple channels.

4.2.1. Mathematical Definition

Simultaneous gain and phase variations are denoted by the complex valued f with nominal value equal to 1, where a perturbed open loop $L_f = fL$. According to Seiler, Packard, and Gahinet (2020), the whole set of perturbations in f is denoted by Equation 4.34:

$$f \in D(\alpha, a, b) = \left\{ \frac{1 + a\delta}{1 - b\delta} : \delta \in \mathbb{C} \text{ with } |\delta| < \alpha \right\} \quad (4.34)$$

Here, a , b and α are parameters which define the set, and α defines the size of the disk. a and b can be parametrised as $a := \frac{1}{2}(1 - \sigma)$ and $b := \frac{1}{2}(1 + \sigma)$, where σ represents the skew of the disk. Notably, α is adjusted to maintain the disk size and can be thought of as the parameter which controls the overall amount of gain and phase variation. Positive σ skews the disk rightwards to higher gain increase and can be thought of as the difference in relative gain increase and decrease, ie: a system with higher σ value will be able to tolerate more positive gain and phase variation than a system with equivalent disk size and a lower/negative σ . This is visualised in Figure 4.8. In the open set $D(\alpha_{max}, \sigma)$, CL stability is maintained for all f and the disk delimits the Nyquist exclusion region.

The effect of DM skew on the size of the DM and the location of contact points with the Nyquist trace is clearly visualised in Figure 4.9, where it shows the size of the disk (thus α) becoming smaller for increasingly positive values of σ , bounded by the superimposed trace of the Nyquist diagram.

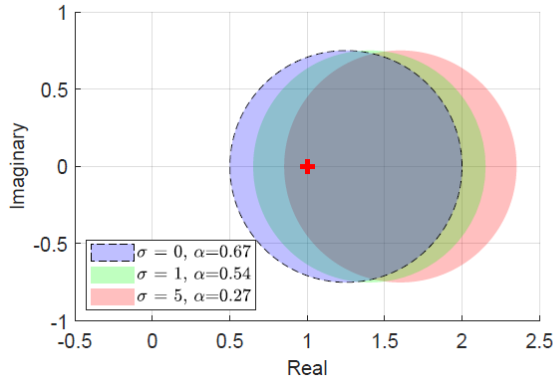


Figure 4.8: Effect of increasing DM skew σ and compensating DM size α in the complex plane, (Seiler, Packard, and Gahinet, 2020)

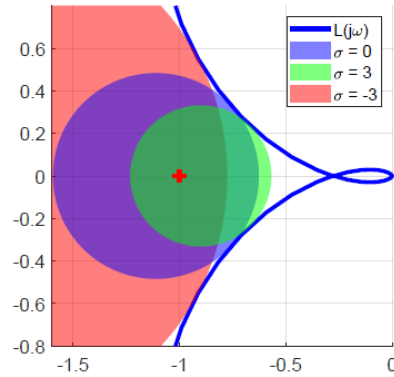


Figure 4.9: Effect of DM skew on DM size and location of contact points with superimposed Nyquist plot, (Seiler, Packard, and Gahinet, 2020)

The balanced case refers to the case where $\sigma = 0$ and the relative allowable increase and decrease in gain are equal, and it is referred to as the ($S - T$) DMs. Other special cases include $\sigma = 1$ and $\sigma = -1$, which are known as the S and T DMs, respectively. According to Seiler, Packard, and Gahinet (2020), the balanced case is an apt choice for DM formulation when little is known regarding gain variation distribution, and therefore the DMs that are calculated in this study represent the balanced case. This study will show

the map of the calculated DM regions corresponding to varied magnitudes of skew, both in the real and imaginary domains to visually assess the stability properties of the control system in detail.

Empirical formulas proposed by [Seiler, Packard, and Gahinet \(2020\)](#) for the mathematical foundations of DMs are used to manually calculate the DMs. The equations are given by [Equation 4.35](#) through to [Equation 4.40](#):

$$\gamma_{min} = \frac{2 - DM}{2 + DM} \quad (4.35)$$

$$\gamma_{max} = \frac{2 + DM}{2 - DM} \quad (4.36)$$

$$intersect_{min} = -\frac{1}{\gamma_{min}} \quad (4.37)$$

$$intersect_{max} = -\frac{1}{\gamma_{max}} \quad (4.38)$$

$$r = \left| \frac{-intersect_{min} + intersect_{max}}{2} \right| \quad (4.39)$$

$$c = -\frac{\gamma_{min} + \gamma_{max}}{2} \quad (4.40)$$

Use of these empirical formulas serves as a good way to verify that calculated DMs are correct.

4.3. Handling Qualities

The stability, control and HQs of an aircraft are a crucial aspect of aircraft design and development. Flying qualities have earlier been defined as "stability and control characteristics that have an important bearing on the safety of flight and on the pilots' impressions of the ease of flying an aircraft in steady flight and in manoeuvres" ([Phillips, 1948](#)). In other words, aircraft performance, measured against these metrics, provide vital information on how the aircraft handles, and how it reacts to pilot inputs. This comes about as a result of many factors, such as the inherent aircraft planform design, the aerodynamic characteristics of the aircraft or the control system that is developed for it. HQ metrics provide both qualitative (for instance through the Cooper-Harper rating scale) and quantifiable (for instance through the CAP and Gibson criteria) standards against which aircraft handling is assessed ([Cook, 2007](#)). Therefore, an aircraft's compliance to these metrics are thus a measure of the success or failing of the design, leading to possible design revisions by the designers.

4.3.1. Handling Qualities

According to [Cook \(2007\)](#), handling qualities (HQs) are the "properties that govern the ease and precision with which (the aircraft) responds to pilot commands in the execution of the flight task. The formal definition states that HQ requirements are used to "assure flying qualities that provide adequate mission performance and flight safety". HQs are often qualitative in nature and formulated in terms of pilot opinion surveys such as the Cooper-Harper HQ rating scale. However, HQs are strongly linked to an aircraft's quantifiable stability and control parameters, and therefore these parameters are often used to indicate flying and HQ performance.

HQ requirements are empirical, and current versions have descended from the amalgamation of years of information and pilot experience, triggered by the need to update and improve the regulation of highly augmented aircraft since the late 1960's. Newer versions of requirements comprise earlier material of classical flying HQs from previous documentation with additional content concerning command and stability system augmentation ([Cook, 2007](#)).

Many countries possess national agencies that regulate aeronautical activity in the region. For the UK this is the Civil Aviation Authority (CAA) and the Ministry of Defence (MOD) regulates military activity.

In the USA, the Federal Aviation Administration (FAA) and the Department of Defence (DOD) are the corresponding agencies. These agencies publish documentation outlining the minimally accepted standards regarding construction, operation, performance and safety for all regulated vehicles. The documents issued by these agencies essentially specify the minimum acceptable flying quality standards as well as provide advice on acceptable compliance. It is a more recent effort of international agencies to combine the standards into one common benchmark, such as the Joint Aviation Requirements (JAR) for ease of international use (Cook, 2007).

As well as being crucial to the regulation of aeronautical activities by authorities, HQ requirements also play a critical part in aircraft and flight control system design. The most stringent and comprehensive requirements are the military requirements, such as MIL-STD-1797 from Davis (1997a), and therefore a commercial aircraft designed according to military requirements will meet civil requirements by default (Cook, 2007).

4.3.2. Operational Flight Envelopes

Operational FEs define for each flight phase the boundaries imposed by Mach number, altitude and normal load factor and it is required that aircraft are able to operate within the limits of the relevant operational FE. The operational FEs from Callus (2003) are provided in Table 4.4

Flight Phase Category	A	B	C
Flight Phase	Air-to-Air Combat		
	Ground Attack		
	Weapon Delivery/Launch	Climb	
	Reconnaissance	Cruise	Takeoff
	In-Flight Refuel (Receiver)	Loiter	Approach
	Terrain Following	In-Flight Refuel (Tanker)	Overshoot
	Maritime Search	Descent	Landing
	Aerobatics	Aerial Delivery	
	Close Formation Flying		

Table 4.4: Operational FEs, (Callus, 2003)

4.3.3. C* Criterion

The C* criterion is a criterion that is widely used in the aircraft industry today, by companies such as Airbus and Boeing, and it is a criterion used to evaluate aircraft HQs. It is a time history envelope that was developed in 1965 by Boeing to prevent PIO. The criterion is shown in Figure 4.10:

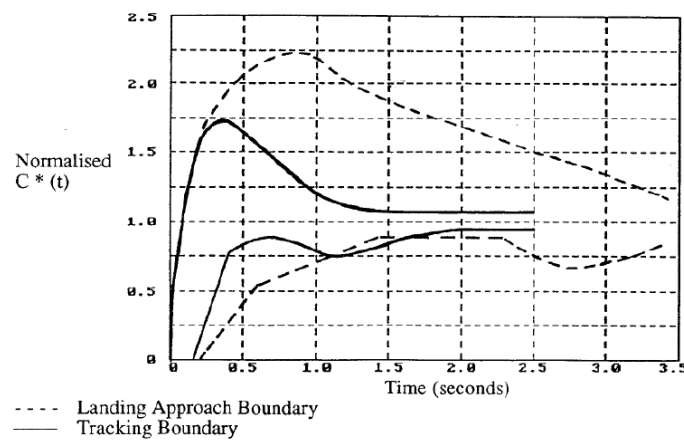


Figure 4.10: C* time history envelopes, (Paula and Paglione, 2007)

It involves the blending of pitch rate q and load factor n_z , with pitch rate being the dominant cue at low velocity and load factor being the dominant cue at high velocity (Bartley, 2001). The weighting of each varies with the operating conditions including velocity and altitude. The C^* criterion is defined as Equation 4.41:

$$C^* = K_{nz}n_z + K_qq \quad (4.41)$$

where K_{nz} is usually equal to 1 and K_q is defined as Equation 4.42:

$$K_q = \frac{V_{co}}{g} \quad (4.42)$$

where V_{co} is the crossover velocity and g is the gravitational constant. The crossover speed V_{co} is the velocity point at which the contribution to the C^* response of load factor and pitch rate are equal. This speed is generally defined as being equal to 240 kts and the gravitational constant $g = 9.80665$. The constant K_q is usually approximately equal to 12.4 (Niedermeier and Lambregts, 2012).

C^* is often used as a HQ criterion as it ensures that the aircraft responds in a predictable manner to pilot inputs, and this is especially the case for pitch control. Therefore the use of the C^* criterion ensures compliance with the Cooper-Harper Handling Qualities Rating, or HQR. Since C^* is a weighted combination of two plant output signals, the C^* signal will be a new 'output' that should meet bandwidth, phase and damping HQ and performance requirements. The C^* signal will be shaped by the outer loop controller to match a reference model possessing the desired dynamics, whilst the pitch rate inner loop will be damped, thereby also affecting the C^* outer loop.

4.3.4. Lower-Order Equivalent System

Depending on bare airframe dynamics and control system complexity, an aircraft's CL system response can often come in the form of a high order frequency response, which is typically not the most ideal form to work with in terms of mathematical manipulation. Therefore, a lower-order equivalent system (LOES) can be used instead of the full order dynamics. These provide a good enough approximation to the full order system, whilst reducing the complexity.

The longitudinal dynamics of an aircraft conventionally consist of the phugoid and SP dynamic modes. Ideally, the phugoid and SP mode are distanced far enough apart from each other in terms of frequency that they are able to be evaluated separately, and this is usually the case. This separation is usually a factor of 10, according to Davis (1997a). The LOES is an important formulation as it is commonly used to assess longitudinal and SP HQ characteristics. The TFs corresponding to the pitch rate q state of the SP mode can be formulated according to Equation 4.43:

$$\frac{q(s)}{\delta_e(s)} = \frac{K_q(s + \frac{1}{T_{\theta_2}})e^{-\tau_e s}}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4.43)$$

where K_q is a scaling gain corresponding to the q longitudinal output, τ_e the equivalent time delay and $e^{-\tau_e s}$ the system phase delay. A version of this expression is available for the AOA state α with the same structure. Explained by De Castro (2003), the system phase delay is used to estimate accumulated phase lag from high order dynamics present in the original system.

In Davis (1997a), HQ requirements are divided pertaining to both the phugoid and the SP modes, and the SP HQ requirements are rather extensive; it presents a list of six distinct requirements, although advice on the conditions in which to apply these is more convoluted. The choice of HQ requirements used in the design, as well as the motivations behind them, are explained in a subsequent subsection.

4.3.5. Predicted and Assigned Levels

One of the changes that Mitchell et al. (1994) proposes to Davis (1997a) is the inclusion of demonstration manoeuvres as part of the HQ assessment standard. It is naturally impossible for quantitative requirements such as those in Davis (1997a) to be fully comprehensive; this means that meeting the HQ requirements

does not necessarily guarantee adequate HQs, and conversely, failing to meet all HQ requirements does not necessarily signify sub-par HQs. Therefore, the proposal is to include qualitative flight test evaluation into the specification, alongside the flying qualities documentation, which allows two possibilities for the method of HQ level assessment.

The first of the two methods is the current method of predicted levels based on flying quality parameters, which compares the aircraft parameters with appropriate quantitative boundary parameters suited to the given operation. The predictive nature results from the use of earlier experiments and analysis (Mitchell et al., 1994). The second method is that of assigning levels based on flight test manoeuvres. A set of flight manoeuvres are designed and for each manoeuvre, the pilots assign HQRs for the aircraft. The average HQR becomes the HQ level for the manoeuvre and the aircraft should attain the required HQ level for each manoeuvre.

It is possible that the levels obtained from each of these two methods can conflict one another and therefore they can be compared to each other for a more accurate overall assessment of the HQ level (Mitchell et al., 1994). Since facilities to obtain assigned levels of flying quality are not available in this study, the flying quality levels used as reference in this study are only the predicted flying quality levels through the facilitation of simulations.

4.3.6. MIL-STD-1797A

The document MIL-STD-1797A is what is known as the Department of Defence Interface Standard of Flying Qualities of Piloted Aircraft and is widely used in aircraft design and development (Davis, 1997a). This military standard evolved from earlier documents such as Callus (2003), and has undergone many revisions ensuring that the HQ assurance remains current. It states that "Flying qualities encompass whatever is involved in flying the aircraft (and in piloting it on the ground) safely and in performance of operational missions from the point of view of the pilot" (Davis, 1997a). It provides the standard intended for use by fixed-wing aircraft supported primarily by aerodynamic as opposed to engine propulsive force. It was published on 30th January 1990 and serves as a vital document during aircraft design and assessment. One purpose of the document is to ensure adequate flying qualities, performance and safety despite any type of design implementation or flight control system augmentation. The document itself provides both qualitative and quantitative criteria to ensure adequate performance in terms of stability, control and HQ adherence. The main way in which MIL-STD-1797A aims to meet HQ requirements is through assessment of open-loop vehicle characteristics, including short period and phugoid dynamics, and the evaluation techniques proposed include time domain criteria, step response analysis, frequency domain analysis and metrics such as the CAP criterion. Important to note is that the HQ requirements set out by MIL-STD-1797A operate in terms of aircraft category classification and flight phases, which are outlined generally and also specifically for the Flying V aircraft in subsequent sections. Not only is MIL-STD-1797A an ideal source of HQ requirements for the Flying V aircraft, but also WL-TR-94-3162, provided by Mitchell et al. (1994); a shortcoming of the MIL-STD-1797A requirements document is that it does not provide guidance on how to use the HQ assessment metrics during application, and it is therefore that these two documents are used together as main sources of requirement compliance during the controller design.

4.3.7. Aircraft Categorisations

Various categorisations exist to classify certain aspects of an aircraft during flight. Therefore during the design process, the aircraft should be designed to meet the requirements that apply to the various categories that the aircraft flight falls under. These categorisations include the aircraft size and weight, the flight phase that the aircraft will operate in and the levels of flying quality. It is the job of the designer to achieve the highest attainable flying quality level and the job of the evaluator to ensure that the aircraft achieves the highest flying quality level in the normal operating conditions (Cook, 2007). These three groups of categorisation, as well as the categorisations within each are detailed in this section.

Aircraft Classification

Aircraft are generally classified according to size and weight by the following classes, given by Cook (2007) in Table 4.5:

Class I	Small light aeroplanes
Class II	Medium weight, low to medium manoeuvrability aeroplanes
Class III	Large, heavy, low to medium manoeuvrability aeroplanes
Class IV	High manoeuvrability aeroplanes

Table 4.5: Definition of aircraft classifications, by size and weight, (Cook, 2007)

In this regard the Flying V aircraft is classified as a class III aircraft as it is large, heavy and has low to medium manoeuvrability.

Flight Phase

Since a flight can be considered a sequence of individual flight phases, the flight phases are grouped into three distinct categories with similar flying qualities in terms of FE. They are defined by Cook (2007) according to Table 4.6:

Category A	Non-terminal flight phases that require rapid manoeuvring, precision tracking, or precise flight path control
Category B	Non-terminal flight phases that require gradual manoeuvring, less precise tracking and accurate flight path control
Category C	Terminal flight phases that require gradual manoeuvring and precision flight path control

Table 4.6: Definition of flight phases, (Cook, 2007)

Regarding the flight phase of interest, this study focuses on flight phase category B, which indicates gradual manoeuvring, less precise tracking and accurate flight path control.

Flying Quality Levels

Flying quality levels are used to quantify the degree to which an aircraft acceptably completes the mission that it was designed for. These are given by Cook (2007) in Table 4.7:

Level 1	Satisfactory	Flying qualities clearly adequate for the mission flight phase
Level 2	Acceptable	Flying qualities adequate to accomplish the mission flight phase, but with an increase in pilot workload and, or, degradation in mission effectiveness
Level 3	Controllable	Degraded flying qualities, but such that the aeroplane can be controlled, inadequate mission effectiveness and high, or, limiting, pilot workload

Table 4.7: Definition of flying quality levels, (Cook, 2007)

Level 1 flying qualities correlate with an aircraft which is totally able to complete the designed mission whilst at all times maintaining satisfactory pilot workload. It is desired where possible that the aircraft meets level 1 HQs to as far an extent as possible, and in some cases this is allowed to degrade to level 2 HQs.

4.3.8. Identification of Applicable Handling Quality Criteria

MIL-STD-1797A provides the six requirements tailored to the SP dynamic mode, but unfortunately does not give detailed description as to the rules for their application. A framework is however provided by Mitchell et al. (1994) in tabular form, its contents provided in Table 4.8. According to Table 4.8, the most ideal design criteria to oblige by for the design of the C* controller are the bandwidth criteria, including the phase delay criterion and the flight path bandwidth criterion, as well as the Gibson overshoot and dropback criterion. It is for this reason that these criteria were used in the design of a robust H_∞ -MS C* controller performed by Pedroso D. (2024). The CAP criterion can also be used, but seeing as C* is not categorised as a conventional response type, its inclusion is not necessarily required. For completeness however, it will also be considered in this study as a HQ assessment criterion.

Response-Type	Specification & Design Criteria	Criteria for Design Guidance Only	Criteria Not Applicable
Conventional	Bandwidth (or CAP) Plus Dropback	$\omega_{sp}T_{\theta_2}$, Neal-Smith Gibson Nichols-Chart Boundaries	TPR
Rate or Rate Command Attitude Hold (RCAH)	Bandwidth Plus Dropback	CAP $\omega_{sp}T_{\theta_2}$, Gibson Nichols-Chart Boundaries TPR	None
Attitude-Augmented (Including Attitude Command Attitude Hold (ACAH) & Gamma Command Gamma Hold (GCGH)	Bandwidth	None	Dropback CAP $\omega_{sp}T_{\theta_2}$, Neal-Smith Gibson Nichols-Chart Boundaries

Table 4.8: Requirement framework for short term pitch response HQ criteria, (Mitchell et al., 1994)

CAP Criterion

The Control Anticipation Parameter (CAP) criterion is an example of a HQ criterion that is used during aircraft design. It is used to evaluate aircraft HQs due to the pitch rate and load factor cues experienced during the short period longitudinal motion. In 1966, Bihrlé stated that "in order to make precise adjustments to the flight path, the pilot must be able to anticipate the ultimate response of the airplane and that angular pitching acceleration is used for this purpose" (Bihrlé, 1966). Bihrlé remarked that aircraft having high inertia or low static stability may not have perceivable angular pitching acceleration (Bihrlé, 1966). He therefore defines the CAP criterion as a measure to quantify the response's anticipatory behaviour, and it can be essentially thought of as a way to quantify desirable SP characteristics appropriate for the aircraft's operating condition and aerodynamic properties (Cook, 2007). Aircraft behaviour with a low CAP value reacts sluggishly, causing the pilot to over-steer, whereas a high CAP value reacts more quickly than the pilot anticipates. Nowadays, aircraft that have a longitudinal short-term response similar to a good second order system exhibit satisfactory anticipatory handling cues to the pilots (Cook, 2007).

The human inner ear is able to sense angular acceleration about three axes. Therefore, when the pilot is providing control inputs to the aircraft, the cue of the inner ear should be able to help the pilot to anticipate commanded normal acceleration (Stengel, 2018). The CAP criterion therefore can be used to indicate how quickly and accurately a pilot is able to control the flight path during pitch control tasks, such as a pitch step input or small doublet input to change pitch attitude, pull up manoeuvres or tracking tasks.

The CAP criterion is formally defined as "the amount of instantaneous angular pitching acceleration per unit of steady state normal acceleration" (Cook, 2007), and is therefore equal to the ratio between the initial pitch acceleration w_n^2 and the steady state normal acceleration per unit angle of attack $\frac{n}{\alpha}$ (Davis, 1997a), as given by Equation 4.44:

$$CAP = \frac{\dot{q}}{\Delta n_{z_{ss}}} \approx \frac{w_{sp}^2 W}{L_{\alpha}} \approx \frac{w_{sp}^2}{\frac{n}{\alpha}} \quad (4.44)$$

where $L_{\alpha} \approx T_{\theta_2}^{-1}$. Since the steady state normal acceleration per unit angle of attack $\frac{n}{\alpha}$ is not always readily available, it can also be expressed using the equivalent pitch time constant T_{θ_2} . T_{θ_2} represents the short-term numerator time constant in the longitudinal pitch rate response. This equivalent expression is given by Equation 4.45 (Berger, Tischler, and Horn, 2021), (McRuer, Graham, and Ashkenas, 1990):

$$\frac{n}{\alpha} = \frac{V}{g} \frac{1}{T_{\theta_2}} \quad (4.45)$$

Using the equation in Equation 4.46:

$$T_{\theta_2} = \frac{2\zeta}{\omega_n} \quad (4.46)$$

Equation 4.45 becomes:

$$\frac{n}{\alpha} = \frac{\omega_n V}{2\zeta g} \quad (4.47)$$

Through substitution of Equation 4.47 back into Equation 4.44, the CAP value can then be expressed according to Equation 4.48:

$$CAP = \frac{2\omega_n \zeta g}{V} \quad (4.48)$$

with the criterion now expressed in terms of more readily available parameters deriving from the SP mode.

For the CAP criterion, the HQ assessment is generally categorised into 3 levels: level 1, 2 or 3. Level 1 indicates the highest level of HQs and represents satisfactory performance where the aircraft is controlled easily with minimal compensation from the pilot. Level 3 indicates that the aircraft is controllable but necessitates excessive pilot workload. The CAP requirements taken from the military HQ requirements MIL-STD-1797A and graphically modified are provided in Figure 4.11:

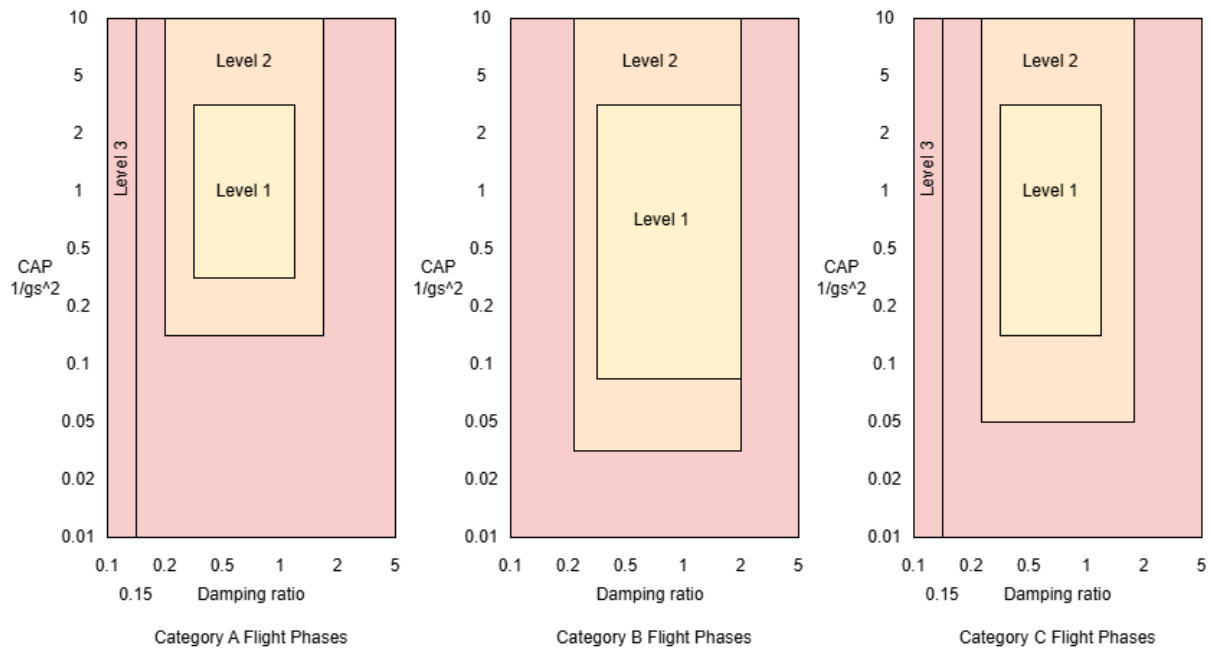


Figure 4.11: CAP requirements, graphically modified from Davis (1997a)

The levels are delimited by the different regions within the graphs. For clarity and reproducibility, the boundaries for each flight phase category and for each level are provided in Table 4.9:

	Category A	Category B	Category C
Level 1	$0.35 < \zeta < 1.3$	$0.3 < \zeta < 2$	$0.35 < \zeta < 1.3$
	$0.28 < \text{CAP} < 3.6$	$0.085 < \text{CAP} < 3.6$	$0.16 < \text{CAP} < 3.6$
Level 2	$0.25 < \zeta < 2$	$0.2 < \zeta < 2$	$0.25 < \zeta < 2$
	$0.16 < \text{CAP} < 10$	$0.038 < \text{CAP} < 10$	$0.05 < \text{CAP} < 10$

Table 4.9: Table containing the flying level quality boundaries for each flight phase category

Gibson Criterion - Pitch Rate Overshoot and Pitch Attitude Dropback

Gibson (1999) first developed the dropback criterion and this was adopted into MIL-STD-1797A in the year 1990 (Davis, 1997a). The publication of MIL-STD-1797A of 30th January 1990 was cancelled, but its contents were preserved in the guidance handbook 'Flying Qualities of Piloted Aircraft' document (Davis, 1997b) of the DOD. Gibson developed the dropback criterion with the aim to create a set of guidelines for highly-augmented fighter aircraft. His initial developments were later updated by Mitchell et al. (1994), who proposed some alterations to the criterion and were later adopted into MIL-STD-1797A. Note-worthily, the topics discussed pertain only to conventional and rate response type, ie: steady-state pitch rate criterion are not applicable to attitude-response types.

It was identified during the development of the bandwidth criteria that some aircraft configurations with high bandwidth ω_{BW} obtained level 2 Handling Qualities Ratings (HQR)s. However, in the document of Mitchell et al. (1994), it is recommended that the upper limits on the bandwidth be removed and the dropback criterion introduced as a measure instead.

According to Mitchell et al. (1994), the dropback criterion is defined as a measure of the mid-frequency response to attitude changes. With excessive pitch attitude dropback there are complaints from pilots of aircraft abruptness and lack of precision in pitch control. Complaints of the same issue also pertain to the presence of excessive pitch attitude bandwidth. A time-domain response of the pitch attitude dropback and pitch rate overshoot is shown in Figure 4.12 and Figure 4.13:

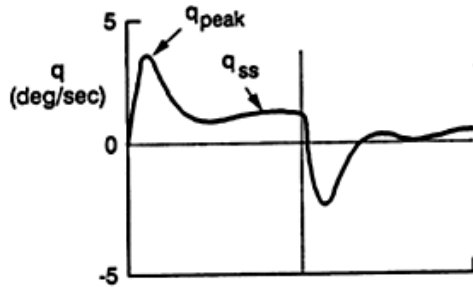


Figure 4.12: Definition of pitch rate overshoot (Mitchell et al., 1994)

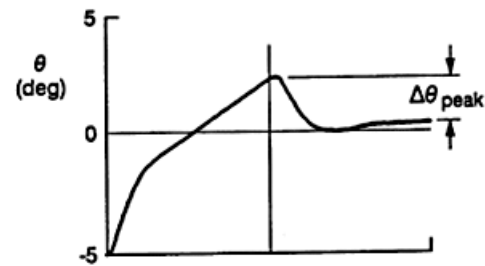


Figure 4.13: Definition of pitch attitude dropback (Mitchell et al., 1994)

It can be derived from the plots that the pitch rate overshoot is equal to the ratio of the peak pitch rate to the steady-state pitch rate, or:

$$\text{Pitch rate overshoot} = \frac{\dot{\theta}_{peak}}{\dot{\theta}_{ss}} \quad (4.49)$$

and the pitch attitude dropback is defined as:

$$\text{Pitch attitude dropback} = \theta_{peak} - \theta_{ss} \quad (4.50)$$

In applying the criteria, care must be taken as the dropback criterion is time-domain based, meaning that it is susceptible to variations in measurement; for instance, the input may vary, other aircraft dynamic modes may be present in the response or there may be difficulty in identifying a steady-state response.

A benefit of the newly proposed definition of the dropback criterion is that it is unaffected by time-delay, whilst the previous definition given by Gibson is strongly affected by it. Furthermore, the new definition does not allow negative values of dropback where Gibson's definition does allow this. Gibson's definition of negative dropback values correlates with the notion that a sluggish response arises from large time-delay, resulting in $\frac{DB}{q} < 0$.

According to [Mitchell et al. \(1994\)](#), the parameters of the dropback criterion are closely related to classical SP dynamics; without the influence of HF dynamics or time-delays, the parameters of dropback correlate directly with elements of the SP. This effect is visualised in [Figure 4.14](#):

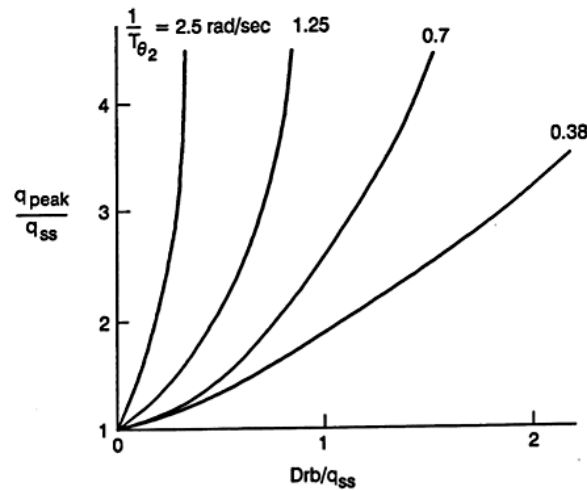


Figure 4.14: Effect of pitch attitude zero T_{θ_2} on pitch rate overshoot and dropback relation ([Mitchell et al., 1994](#))

It is visible that increasing the value of T_{θ_2} decreases the attitude dropback for a given rate overshoot. This is because dropback is inversely proportional to the pitch attitude zero T_{θ_2} , and that the pitch rate overshoot remains at a fixed value equal to $\omega_{SP}T_{\theta_2}$ corresponding to the frequency separation between the SP numerator and denominator terms. Additionally, for higher values of overshoot ratio, the dropback decreases for increasing bandwidth.

In summary, following [Gibson \(1999\)](#), key components of the Gibson criteria include:

- Pitch attitude dropback, pitch rate overshoot and time to first peak, flight path time delay and overall response time to steady-state are the SP characteristics that are the most influential on good handling performance
- Negative attitude dropback (overshoot) equates to sluggishness and unpredictable flight path control and tracking
- An attitude dropback ratio between 0 s and 0.25 s is optimal for predictable behaviour
- Higher attitude dropback leads to bobbling and is sometimes described as PIOs
- Pitch rate overshoot influences dropback behaviour - large pitch rate overshoot ratio can render dropback ratio unacceptable

Flying V Aircraft Model

This chapter will explore all aspects of the Flying V aircraft model, starting with the Simulink model structure, actuator dynamics, the used reference and aerodynamic models, the operational flight envelope and definitions of key aircraft geometry. Then, the placement of the inertial measurement unit (IMU), as well as the equations of motion (EOMs) are discussed, followed by a description of the trimming and linearisation process and the method of inclusion of parametric uncertainty.

5.1. Flying V Aircraft Model

The existing MATLAB and Simulink model project files of the Flying V aircraft were provided at the start of this project. The model received was rather large and complex, involving many feedback connections and many discrete time function blocks. This was partly due to the fact that the model was created and used primarily for the purpose of designing a non-linear INDI controller for the Flying V aircraft by [Stougie \(2022\)](#). Therefore in this form, the model was not linearisable and suitable for the purpose of performing robust control. It was therefore necessary that the model be remodelled into a more suitable version. This was done concurrently to this work by [Pedroso D. \(2024\)](#). As part of their thesis, the Flying V aircraft model was remodelled into a tensor based formulation according to the existing 6 degree of freedom (DOF) F-16 aircraft model, used by [Naäman \(2024\)](#). The top level architecture is depicted in [Figure 5.1](#). It consists of four global inputs of thrust T , aileron deflection δ_a , elevator deflection δ_e and rudder deflection δ_r , as well as the airframe and sensor blocks. All desired outputs are contained in the 'Outputs' block.

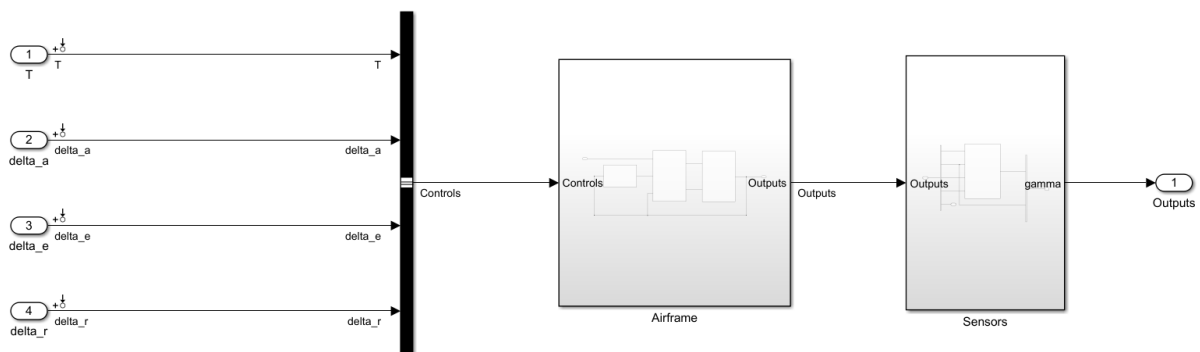


Figure 5.1: Flying V aircraft top level architecture

The sensor subsystem block contains the block dynamics corresponding to the sensors that would be onboard the Flying V aircraft, which by including a factor for noise and measurement errors, translates the inherent calculated system outputs to an estimate of the actually measured aircraft outputs. The airframe is expanded in a lower level view in [Figure 5.2](#):

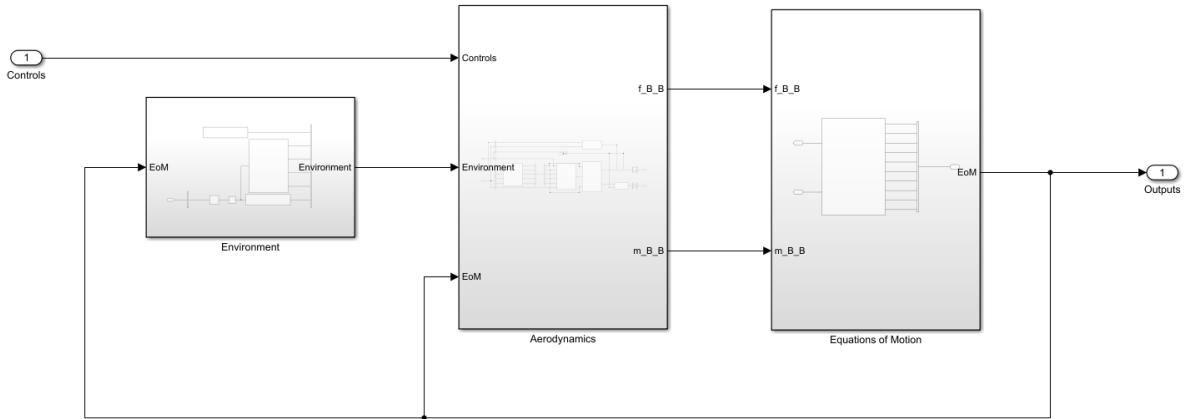


Figure 5.2: Flying V airframe

The one stage lower level view consists of the 'Environment', 'Aerodynamics' and 'Equations of Motion' subsystem blocks. The environment block takes inputs from the aircraft behaviour through the equations of motion to update the surrounding environment. The updated environment parameters are given as input to the aerodynamics subsystem block, as well as the control inputs and once again the equations of motion. In the aerodynamic subsystem, aerodynamic parameters are updated to continuously rebuild a relevant aerodynamic model, which is done by updating a set of global aerodynamic coefficients. From the aerodynamic subsystem block, the aircraft body forces and moments are parsed to the equation of motion (EOM) subsystem, which translates these forces and moments into the aircraft's relevant translational and rotational outputs.

5.2. Actuator Dynamics

Playing an crucial role in the behaviour of the bare airframe response are the dynamics of the actuator. The actuator Simulink block diagram is provided in [Figure 5.3](#):

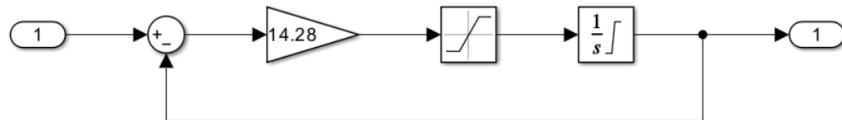


Figure 5.3: Actuator dynamics in Simulink

Shown are first-order actuator dynamics with a time constant equal to 0.07 s, thus operating at a frequency of 14.286 rad/s. Also present are position and rate saturation blocks to impose limits on these. These elevator deflection and rate limits are provided in [Table 5.1](#):

	δ_a	δ_e	δ_r
δ_{min} [deg]	-25	-25	-30
δ_{max} [deg]	+14	+14	+30
$\dot{\delta}_{max}$ [deg/s]	± 60	± 60	± 60

Table 5.1: Actuator position and rate limits, provided by [Bérard, Biannic, and Saussie \(2012\)](#)

The actuator dynamics are expressed in TF form as [Equation 5.1](#):

$$G_{actuator} = \frac{\delta_e}{\delta_{ecom}} = \frac{14.286}{s + 14.286} \quad (5.1)$$

This TF will be included in block form within the Flying V aircraft's linear Simulink models. These actuator dynamics are taken from [Bérard, Biannic, and Saussie \(2012\)](#) and the values correspond to the typical values of a commercial jet. At the current stage of the design, the Flying V aircraft will possess several trailing edge elevons. However, the Flying V aircraft's aerodynamic model currently only simulates a single trailing edge control surface, playing the combined role of the aileron and elevator. This has the function of being able to control both pitch and roll motions simultaneously, depending on the manner in which the two are deflected. A rudder is also located on the vertical panel of the wing-tip and the contribution of both rudders are considered together within the aerodynamic data. Lastly, the engine dynamics are modelled by a first-order system possessing a time constant of 0.2 seconds.

5.3. Reference Model

It was subsequently required to choose a reference model that the Flying V aircraft's controlled C^* response would aim to emulate, in order to ensure sufficient HQ requirements are achieved. For this, it was decided to use a second-order reference model of the format according to [Equation 5.2](#):

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (5.2)$$

as this structure is considered suitable for an aircraft response. The second-order system is parametrised with natural frequency ω_n and damping ratio ζ . Seeing that there are no clearly defined hard requirements on the natural frequency and damping ratio of the reference model, these were somewhat up to choice. In the decision of the reference model, the step response, singular values, CAP and Gibson criteria of the transfer function were evaluated based on a variety of different ω_n and ζ to identify a suitable combination of the two parameters that would satisfy the required HQ characteristics. These are shown in [Figure 5.4](#), [Figure 5.5](#), [Figure 5.6](#) and [Figure 5.7](#) respectively:

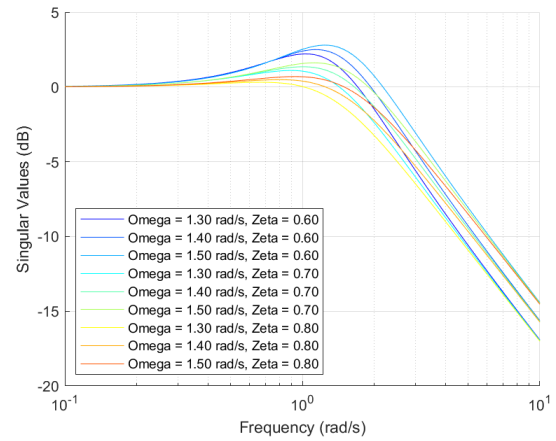
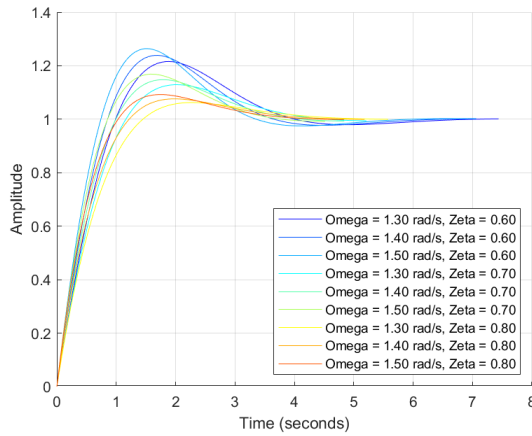


Figure 5.4: Reference step response for a variety of ω_n and ζ **Figure 5.5:** Reference SV plot for a variety of ω_n and ζ

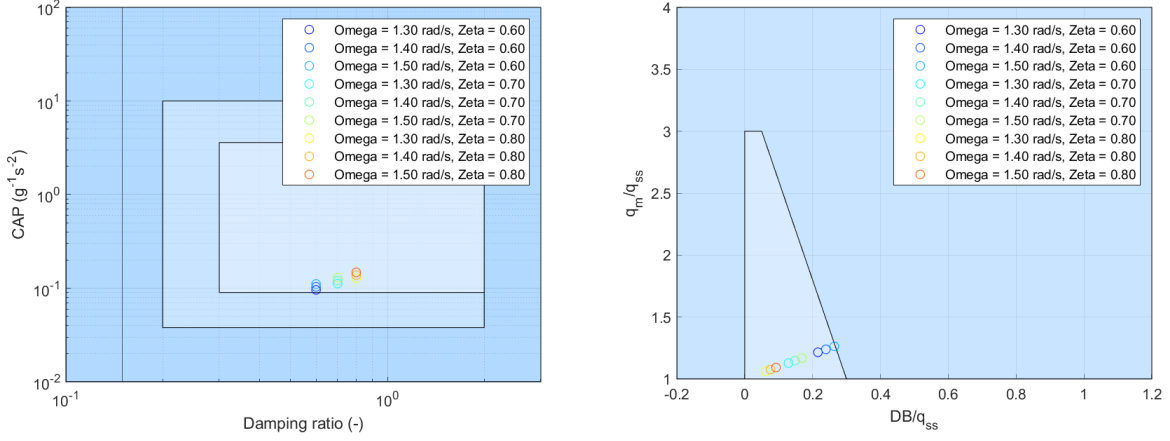


Figure 5.6: Reference CAP criterion for a variety of ω_n and ζ

Figure 5.7: Reference Gibson criterion for a variety of ω_n and ζ

Upon inspection, it was deemed that the parameters resulting in the most ideal response were as according to Table 5.2, and the second-order transfer function is augmented to include a set zero location, as shown in Equation 5.3:

Natural frequency ω_n	1.4 (rad/s)
Damping ratio ζ	0.7 (-)

Table 5.2: Reference model parameters

$$\frac{\frac{\omega_n^2}{z_{ref}}s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (5.3)$$

such that it is suitably far from the imaginary axis, mitigating any NMP effects. For these purposes, the zero is placed at a location of -1.2 rad/s. The reference model transfer function then becomes according to Equation 5.4, and the pole-zero map is given by Figure 5.8:

$$T_{ref} = \frac{1.6333(s + 1.2)}{s^2 + 1.96s + 1.96} \quad (5.4)$$

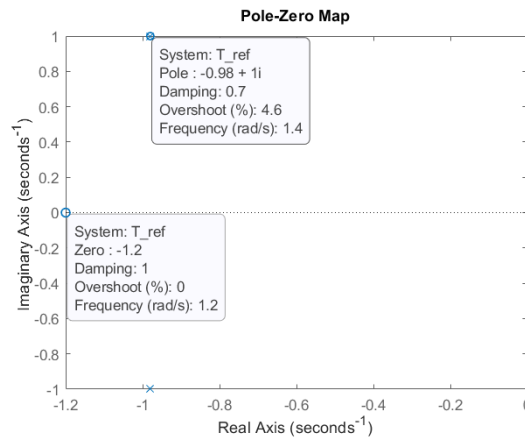


Figure 5.8: Reference model pole-zero map

These parameter values can be considered typical for a second-order reference model. These parameters were selected as final as they produced a step response with a suitable settling time of 3.09 s, a peak value of 1.15 and therefore an overshoot of 14.8 %. It can be calculated that the parameter T_{θ_2} is equal to 1.4. Furthermore, the SV plot indicates that the -3 dB point occurs at a frequency of 2.45 rad/s, which is a reasonable value for the controlled aircraft system to track the reference model accurately. It can be further seen that the chosen reference model falls comfortably within level 1 HQ requirement for both the CAP and Gibson criteria, with some space for deviation from this reference point. The final reference model characteristics are shown in Figure 5.9, Figure 5.10, Figure 5.11 and Figure 5.12:

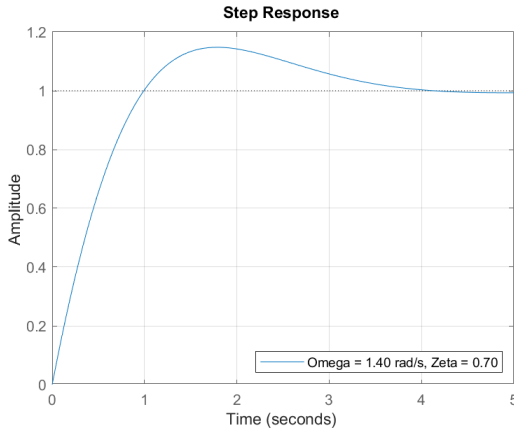


Figure 5.9: Unit step response of the reference model, T_{ref}

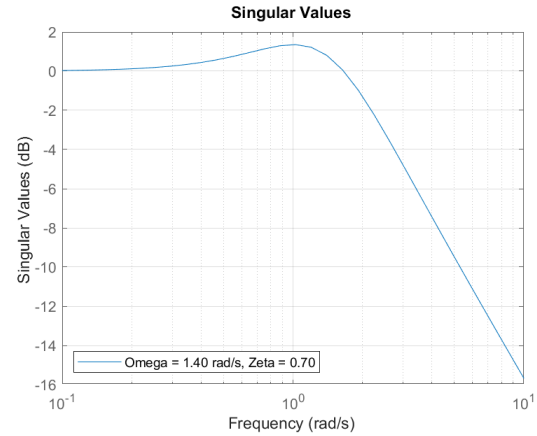


Figure 5.10: SV plot of the reference model, T_{ref}

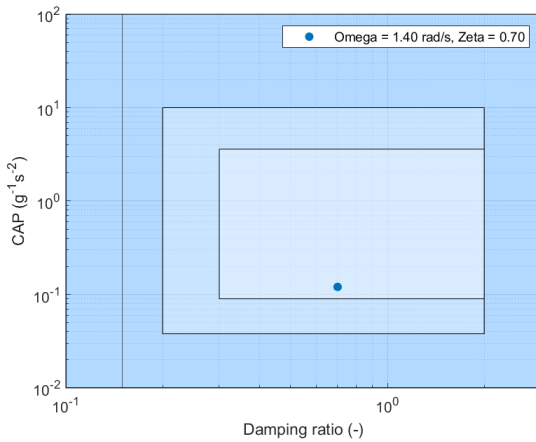


Figure 5.11: CAP criterion of the reference model, T_{ref}

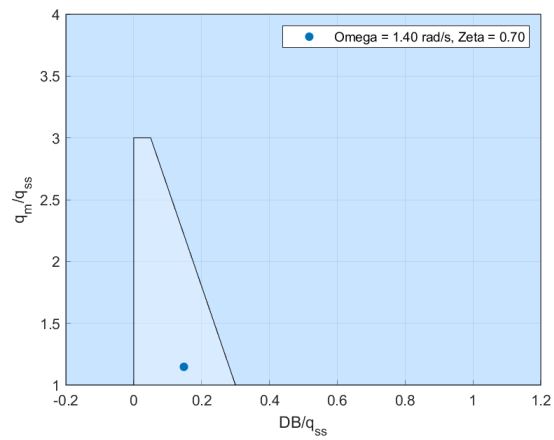


Figure 5.12: Gibson criterion of the reference model, T_{ref}

5.4. Aerodynamic Model

The aerodynamic data that is available was generated by the Aerodynamics department of TU Delft using RANS-based CFD simulations. All of the aerodynamic coefficients were provided for 16 individual flight points that varied within the flight envelope (FE) by Mach number and altitude. It was discovered that the initial flight points used in obtaining the aerodynamic data were not particularly extensive, and did not closely correspond to that of a conventional commercial aircraft. Therefore, the flight points of a conventional FE of a Boeing 747 was taken as an example to use in the case of the Flying V aircraft. This included a number of additional flight points which required further intensive computations of the aerodynamic coefficients from the aerodynamics department. Expanding the known flight points of the

Flying V aircraft thereby made it more feasible to design a set of flight controllers that are valid over a typical commercial aircraft operational window. This data made it possible to calculate the forces and moments acting on the aircraft, which are used in the calculation of the EOMs. The 16 flight points are shown in [Table 5.3](#) for the relevant combinations of Mach number and altitude:

Altitude [m] Mach [-]	0	3250	5450	7650	9750
0.2	x				
0.25	x				
0.3	x	x			
0.4		x	x		
0.5		x	x	x	x
0.6		x	x	x	x
0.7				x	x

Table 5.3: Set of operational flight points for the Flying V aircraft

The FE is defined as the region of airspeed and altitude in which an aircraft is designed to operate, and therefore all combinations of airspeed, altitude and load factor are contained within the FE. The boundaries of the FE are set by the limitations on the aircraft performance, such as structural loads, available engine power and stalling ([Ruijgrok, 2009](#)). In this study, the FE was defined using the aerodynamic coefficient data provided by the aerodynamics department of the Flying V project. The three-dimensional operating envelope of the aircraft was defined using Mach number, AOA and altitude, and is shown in [Figure 5.13](#). This is otherwise depicted in the 2D FE by omitting the AOA, as shown in [Figure 5.14](#):

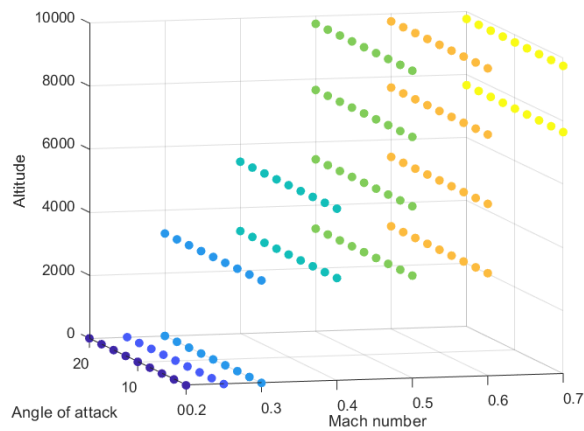


Figure 5.13: Three-dimensional FE consisting of Mach number, altitude and angle of attack

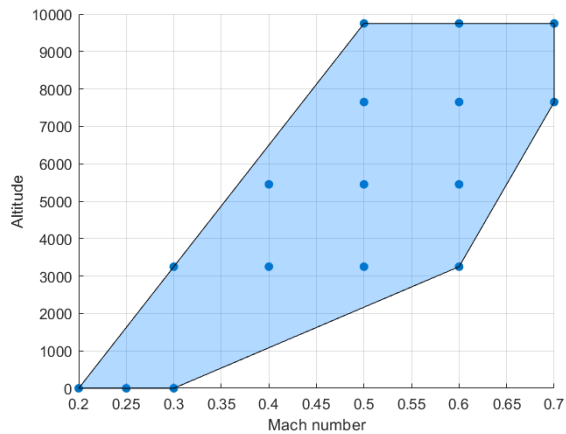


Figure 5.14: Two-dimensional FE consisting of Mach number and altitude

Global aerodynamic coefficients are expressed in terms of partial coefficients defined through other variables, including the angle of attack α , the angle of sideslip β , dimensionless angular accelerations $\hat{p}$, $\hat{q}$, $\hat{r}$, control surface deflections δ_a , δ_e , δ_r , and the location of the COG. They are expressed according to [Equation 5.5](#):

$$\begin{aligned}
C_x &= C_{x_\alpha}(M_\alpha, h, \alpha) + C_{x_{\delta_e}}(M_\alpha, h, \delta_e) \\
C_y &= C_{y_\beta}(M_\alpha, h, \beta) + C_{y_r}(M_\alpha, h, r, COG) + C_{y_{\delta_\alpha}}(M_\alpha, h, r, COG) + C_{y_{\delta_\alpha}}(M_\alpha, h, \delta_\alpha) + C_{y_{\delta_r}}(M_\alpha, h, \delta_r) \\
C_z &= C_{z_\alpha}(M_\alpha, h, \alpha) + C_{z_q}(M_\alpha, h, q) + C_{z_{\delta_e}}(M_\alpha, h, \delta_e) \\
C_l &= C_{l_\beta}(M_\alpha, h, \beta) + C_{l_p}(M_\alpha, h, p) + C_{l_r}(M_\alpha, h, r, COG) + C_{l_{\delta_\alpha}} + C_{l_{\delta_r}}(M_\alpha, h, \delta_r) \\
C_m &= C_{m_\alpha}(M_\alpha, h, \alpha, COG) + C_{m_q}(M_\alpha, h, q, COG) + C_{m_{\delta_e}}(M_\alpha, h, \delta_e, COG) \\
C_n &= C_{n_\beta}(M_\alpha, h, \beta, COG) + C_{n_p}(M_\alpha, h, p, COG) + C_{n_r}(M_\alpha, h, r, COG) + C_{n_{\delta_\alpha}}(M_\alpha, h, \delta_\alpha, COG) + C_{n_{\delta_r}}(M_\alpha, h, \delta_r, COG)
\end{aligned} \tag{5.5}$$

It should be noted that it is required to transform the coordinate system from the local-level coordinate system to the body coordinate system to yield $[v_B^A]^B$. The aerodynamic coefficients obtained from the aerodynamic department were dimensional, and therefore given in pre-multiplied form with the respective contribution towards the global coefficient. This meant that the global aerodynamic coefficients were then calculated by simply summing the individual dimensional contributions.

To update the aerodynamic coefficients, it is first necessary to obtain as outputs of the Air Data Computer (ADC) V , α , β , dynamic pressure and Mach number. To do this, the sensor measured velocity $[v_B^A]^L$ corresponding to the aircraft velocity w.r.t surrounding air, is obtained. This velocity is defined in terms of the local-level coordinate system. Noteworthy of this coordinate system is that it is referred often to as the North-East-Down (NED) coordinate system, and that fixed, level orientation is maintained. This coordinate system is a form of the geographic coordinate system, and it is used when the actual location on the globe is not of importance. Through obtaining this, the airspeed and aerodynamic angles can be subsequently calculated, as shown in [Equation 5.6](#):

$$\begin{aligned}
[v_B^A]^B &= \begin{bmatrix} u_A \\ v_A \\ w_A \end{bmatrix} \\
V &= ||[v_B^A]^B||_2 \\
\alpha &= \arctan\left(\frac{w_A}{u_A}\right) \\
\beta &= \arcsin\left(\frac{v_A}{V}\right)
\end{aligned} \tag{5.6}$$

Once the aircraft's aerodynamic velocity and angles are known, these can be used to interpolate the given aerodynamic coefficients. The determination of the aerodynamic coefficients depending on AOA was performed in a two-stage interpolation process, due to the fact that aerodynamic coefficients are dependent on both AOA and Mach number. The first computation was to calculate the aerodynamic coefficients with respect to the AOA by linearly interpolating and extrapolating the α -dependent longitudinal coefficients over the AOA ranges specified for the given Mach number. Then the dimensional aerodynamic coefficients were determined by interpolating/extrapolating over the given Mach range for the chosen Mach number. As this study concerns the design of a longitudinal controller, this was performed only for the coefficients that affect longitudinal behaviour, namely C_x , C_z and C_m . This resulted in the dimensional coefficients C_{x_α} , C_{z_α} and C_{m_α} . Subsequently, the remaining aerodynamic coefficients that are not dependent on AOA were calculated using standard interpolation of the Mach number dependent aerodynamic coefficients with respect to the selected Mach number. As a result, the total aerodynamic forces and moments are calculated according to [Table 5.4](#):

$$\begin{aligned}
F_{x_\alpha} &= \bar{q}SC_x & M_{x_\alpha} &= \bar{q}SbC_l \\
F_{y_\alpha} &= \bar{q}SC_y & M_{y_\alpha} &= \bar{q}ScC_m \\
F_{z_\alpha} &= \bar{q}SC_z & M_{z_\alpha} &= \bar{q}SbC_n
\end{aligned}$$

Table 5.4: Aerodynamic forces and moments

In these expressions, $\bar{q}$ represents the dynamic pressure, given by $\bar{q} = \frac{1}{2}\rho V^2$, S represents the wing area, b represents the wingspan and c represents the chord length. Added to these force and moment equations is the force due to thrust, as the distances of the engine from the x , y and z axes are known (Pascual, 2018). As only the longitudinal aircraft behaviour is investigated in this study, only the effects of the displacement in the z -axis are included, as engine thrust at a given displacement from the z -axis affects the moment around the y -axis, or the pitching behaviour of the aircraft.

The coefficient with the largest impact on stability is C_m , as the derivative of this coefficient with respect to α , known as the pitching moment coefficient, is what determines whether the aircraft is able to longitudinally recover from an upset disturbance and avoid the stall condition. As this coefficient carries most significance, the interpolated values with respect to Mach and AOA are depicted in Figure 5.15:

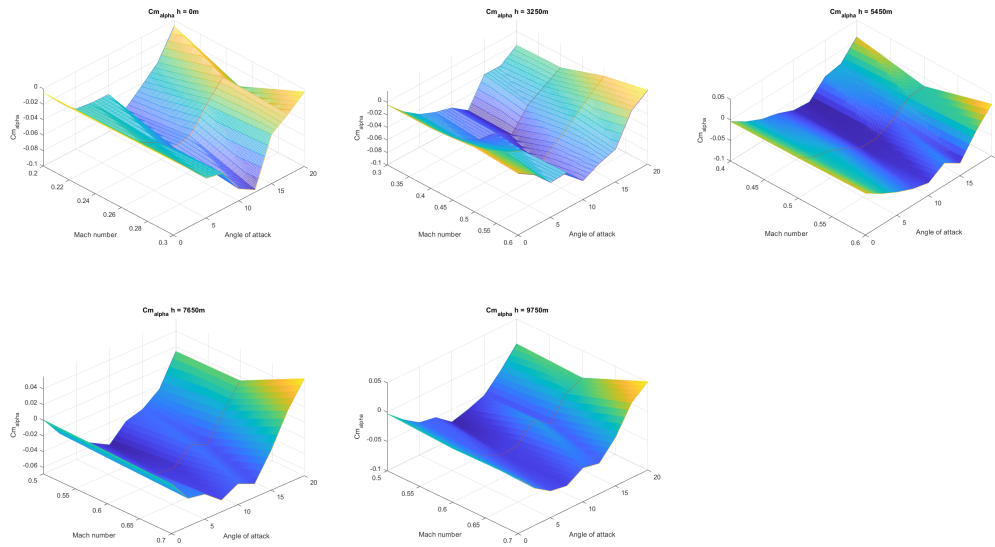


Figure 5.15: Aerodynamic coefficient C_m with angle of attack at various flight points within the FE

This data is perhaps more clearly visualised in Figure 5.16, where C_m is provided two-dimensionally as a function of AOA for each relevant Mach number. It can be noted that the derivative of C_m w.r.t the AOA is not constant along the range of AOA, and that the value of the derivative is negative in some areas, whereas it is positive in others. The sign of the derivative, ie: whether the derivative is positive or negative, has consequences for the aircraft's longitudinal stability; an aircraft with a negative C_{m_α} when disturbed from equilibrium with a positive input on α , will produce a negative pitching moment, countering the initial upset, and return the aircraft to longitudinal static stability. However, an aircraft with a positive value of the pitching moment derivative will diverge from equilibrium once disturbed with an α input, therefore rendering the point unstable.

The set of stable and unstable flight points within the data of the Flying V aircraft are provided in Figure 5.17. It is visible that using the current model of the Flying V aircraft, not all flight points are inherently stable for all combinations of Mach number, AOA and altitude. This phenomenon requires the addition of aircraft design features to the Flying V aircraft to assist in inherent bare airframe stability across the entire FE. However, it is decided, using the current version of the Flying V aircraft model, to proceed with the choice of a stable flight point, given by Table 5.5:

Mach number	0.5 (-)
Altitude	5450 (m)

Table 5.5: Selected stable flight point

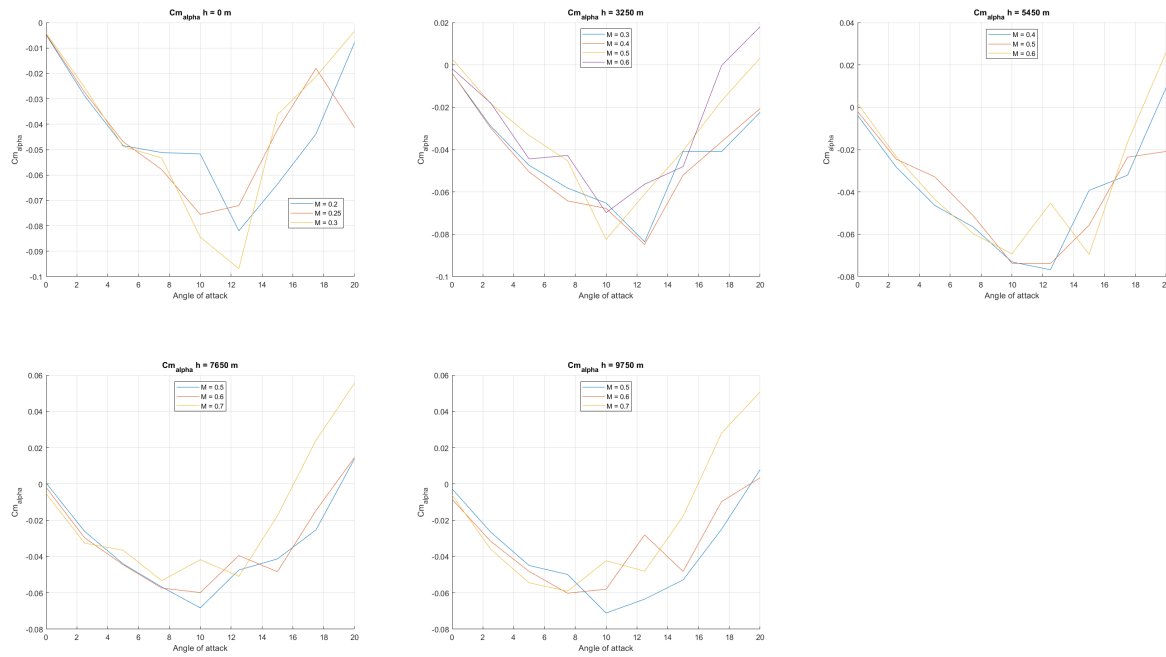


Figure 5.16: Aerodynamic coefficient C_m with angle of attack for various flight points and Mach number within the FE

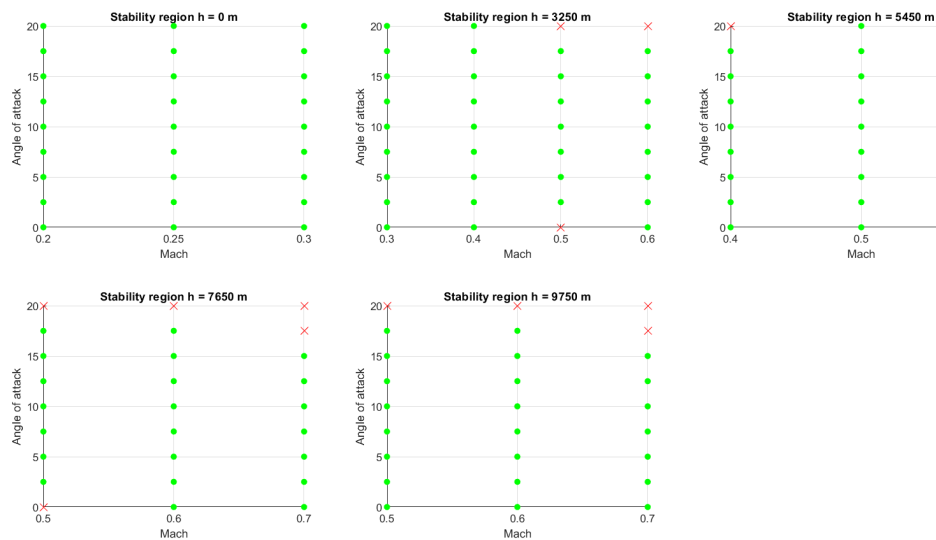


Figure 5.17: Set of stable flight points within the FE

The reason for selecting this flight point is that all flight points for $M = 0.5$ are stable for every AOA at the altitude of 5450 m, and therefore this is a flight point close to the cruise condition for which the aircraft is stable to an adequate degree of certainty.

5.5. Definitions of Significant Locations

There are a number of significant points located along the longitudinal axis which prove to be crucial towards the stability and HQs of an aircraft. These include the locations of the centre of gravity (COG) and the instantaneous centre of rotation (ICR). This section will explain the significance of each, and how the location of each point in turn affects aircraft handling characteristics.

5.5.1. Centre of Gravity

The COG is a measurement that is important when dealing with the weight and balance calculations of an aircraft. According to the FAA, the COG is identified as the "point at which the aircraft would balance if it were suspended at that point" (FAA, n.d.). NASA identifies it as the "average location of the weight of the aircraft", equivalent to the simplification of the "mass weighted average of the component locations" (Center of Gravity n.d.). The location at which this point occurs depends on the distribution of weight along the longitudinal axis of the aircraft, and therefore the location of this point can change by shifting the weight distribution. The forward and aft COG limits represent the furthest forward and aft extremes of the allowable COG location as prescribed by the aircraft manufacturer, and operating outside of these operational limits can have adverse effects on control and result in the loss of stability (FAA, n.d.). The Flying V aircraft is identified as having two locations corresponding to the fore and aft COG, where the forward COG is at a location of $x_{COG} = 29.372m$ and the aft COG is at a location of $x_{COG} = 31.714m$ from the nose of the aircraft (Van Overeem, Wang, and Van Kampen, 2022).

5.5.2. Instantaneous Centre of Rotation

The instantaneous centre of rotation (ICR), also known as the instantaneous centre of zero velocity (ICZV), is defined as the point on a body along the instantaneous axis of zero velocity where there is zero velocity when undergoing general planar movement (Hibbeler, 2016). In aircraft pitch rotation, the ICR is the point around which the vehicle rotates and its location has an effect on the aircraft's pitch dynamics and HQs. Following its definition, the location of the ICR can be identified by determining the point at which the net normal acceleration in the z -direction is equal to zero directly after a pitch control input. The ICR can be located either fore or aft of the COG, and its location is able to vary w.r.t the location of the COG.

The ICR gives valuable insight into the non-minimum phase behaviour of the aircraft; an ICR located further fore can give rise to increased NMP behaviour. This effect can be seen by looking at the transient step response of the short-period system in the very initial time stages by identifying the distance separation of the IMU from the COG that no longer results in NMP behaviour. This can also be visualised by the translation of a RHP zero to the LHP in a pole-zero map of the transfer. The aircraft IMU is often placed at the aircraft's ICR as by definition, it would measure zero normal acceleration due to rotation after a pitch control input. This can be advantageous in terms of HQs, for instance when using the load factor measurement taken from the PS.

5.6. Key Aircraft Geometry

An important aspect of the controller design is the inherent aircraft geometry. The geometry to be used is provided by Pluijm (2021). In their report, they parametrise the key measurements, including the nose, cockpit and fairings, which can be seen in a top view of the Flying V aircraft in Figure 5.18.

It can be seen from the figure that the distances are defined in a coordinate system with the origin located the distance x_{nose} in front of the foremost nose location of the aircraft. This is determined to be a distance of 1.5 m from the origin of the coordinate system. The COG location is defined as the distance from the origin to the COG and this is once again defined as being the most forward COG location ($x_{COG} = 29.372m$) due to the increased sluggish aircraft response at this location. In this context, the location of the PS is defined as the distance from the nose to the PS, which is measured to be 6.7 m (Pluijm, 2021). The distance x_a corresponds to the distance between the COG and the PS, and using Figure 5.18, can be derived as equating to Equation 5.7:

$$x_a = x_{COG} - x_{nose \rightarrow PS} = x_{COG} - x_{PS} + x_{nose} = 29.4 - 6.7 + 1.5 = 24.1720 \text{ m} \quad (5.7)$$

The distance x_a is important as it is widely used in relation to the assessment of the C* HQ criterion, as the load factor obtained at the PS and the load factor obtained at the location of the IMU are mathematically

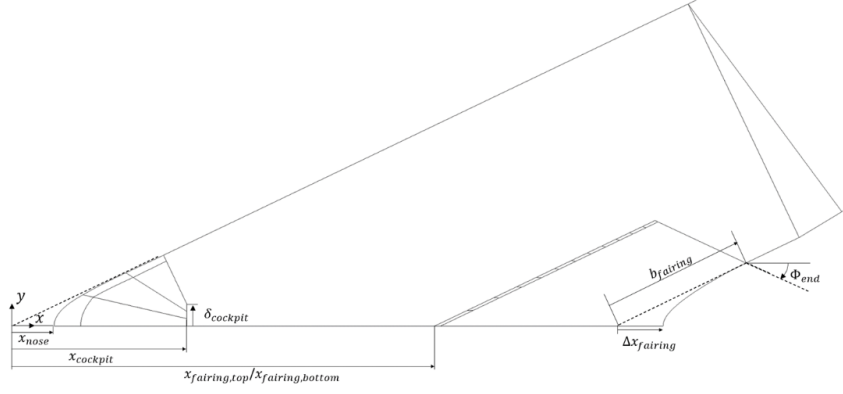


Figure 5.18: Planform geometry of Flying V aircraft with key measurements indicated

related to one another according to [Equation 5.8](#):

$$n_{z_{PS}} = n_{z_{IMU}} + \frac{\dot{q}(x_{IMU \rightarrow COG} + x_a)}{g_0} \quad (5.8)$$

through the use of the one-dimensional Grubin transformation, which is explained subsequently. In [Equation 5.8](#), $n_{z_{PS}}$ represents the load factor measured at the PS and $n_{z_{IMU}}$ is the load factor measured at the IMU. The distance $x_{IMU \rightarrow COG}$ is the distance between the placement of the IMU and the COG, which was earlier determined to be equal to -7.2 m.

5.7. Grubin Transformation

When the aircraft centre of mass (COM) is located far behind the pilot station (PS), the forces experienced at the PS will be different from those experienced at the COM. In the case that the IMU/accelerometer is placed in the PS, or in any location displaced from the COM, it is necessary to apply the Grubin transformation to obtain a new reference point ([Zipfel, 2007](#)). The setup for the mathematical foundations of the Grubin transformation is depicted in [Figure 5.19](#):

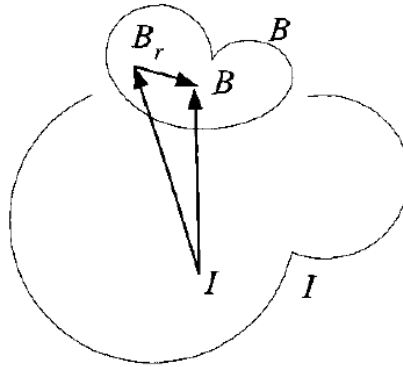


Figure 5.19: Reference point B_r

The transformation proposed by Grubin writes Newton's law w.r.t an arbitrary reference point B_r of a body B , and correction terms are moved to the RHS of the equation, as in [Equation 5.9](#):

$$m^B D^I v_{B_r}^I = f + \text{correction terms} \quad (5.9)$$

Looking at the diagram shown in [Figure 5.19](#), the vector triangle can be derived as being:

$$s_{BI} = s_{BB_r} + s_{B_rI} \quad (5.10)$$

where s_{BB_r} is the separation between the point B_r and point B . This is equivalent to saying that the position vector of a point in the body B w.r.t a point in the inertial frame I can be derived as being equal to the sum of position vectors between the sensor B_r to the COM and between the inertial point to the reference sensor. By taking the second derivative of Equation 5.9 and substituting this into the expression for Newton's Second Law, Equation 5.11 is obtained:

$$m^B D^I D^I s_{BB_r} + m^B D^I D^I s_{B_rI} = f \quad (5.11)$$

The second term does not need additional modification, as it is already in the correct form, while the first term contains the apparent forces and will be elaborated further. First, it is assumed that $D^B s_{BB_r} = 0$ as both B and B_r belong to the body frame B . Next, the chain rule is applied in the following stages:

$$\begin{aligned} m^B D^I D^I s_{BB_r} &= m^B D^I (D^B s_{BB_r} + \Omega^{BI} s_{BB_r}) = m^B D^I (\Omega^{BI} s_{BB_r}) \\ &= m^B D^I \Omega^{BI} s_{BB_r} + m^B \Omega^{BI} D^I s_{BB_r} \\ &= m^B D^I \Omega^{BI} s_{BB_r} + m^B \Omega^{BI} (D^B s_{BB_r} + \Omega^{BI} s_{BB_r}) \\ &= m^B D^I \Omega^{BI} s_{BB_r} + m^B \Omega^{BI} \Omega^{BI} s_{BB_r} \end{aligned} \quad (5.12)$$

substituting this result back into Equation 5.11 leads to Equation 5.13:

$$m^B D^I v_{B_r}^I = f - m^B \left\{ \begin{array}{l} \Omega^{BI} \Omega^{BI} s_{BB_r} \quad \text{Centrifugal acceleration} \\ +(D^I \Omega^{BI}) s_{BB_r} \quad \text{Angular acceleration} \end{array} \right\} \quad (5.13)$$

or equivalently,

$$a_{B_r}^I = a_B^I - \left\{ \begin{array}{l} \Omega^{BI} \Omega^{BI} s_{BB_r} \quad \text{Centrifugal acceleration} \\ +(D^I \Omega^{BI}) s_{BB_r} \quad \text{Angular acceleration} \end{array} \right\} \quad (5.14)$$

This form contains a component of centrifugal acceleration and a component of angular acceleration. In the context of the Flying V aircraft, B_r corresponds to the placement of the accelerometer, and B corresponds to the location of the PS or the ICR. This means that a pilot sitting at the PS would experience the additional forces caused by these two accelerations, where they would not if the displacement vector B_r tended to zero, ie: the pilot is located at the COM. Later in this study, it is discussed whether it is most beneficial that the load factor measurement is taken at the ICR or the PS, both in terms of control feedback design and HQ assessment. Lastly, it is worthwhile to note that the conventional sign of the load factor is opposite to that of the normal acceleration such that positive normal acceleration according to the NED reference frame in steady, wings-level flight corresponds to 1 g.

5.8. Placement of the IMU

As discussed in Section 5.5, there are various locations of importance within the aircraft: the COG, the ICR and the PS. The effect of accelerometer displacement from the COM of the vehicle can have significant implications on the achieved stability and HQs experienced by the pilot. The IMU accelerometer can be placed at any location which is convenient in terms of placement, and can also be placed strategically such that rotational acceleration due to offset from the centre of rotation is minimised. Furthermore, the IMU is also able to detect higher frequency oscillation present due to resonance and structural flexible modes of the aircraft in addition to motion of the rigid body, which could cause adverse destabilising coupling effects within the FCS (Kim et al., 2024). This section will discuss how the load factor measurement location impacts various decisions made during the design process, and which locations should ultimately be selected under certain circumstances.

5.8.1. Inertial Measurement Unit

The inertial measurement unit (IMU), also referred to as an accelerometer, is a crucial device that measures aircraft motion, orientation and gravitational forces. They play a very important role in aircraft stability, navigation and control. An IMU is composed of accelerometers, gyroscopes and processors to provide data fusion for improved measurement. Having access to these properties allows for a complete picture of the aircraft behaviour in three-dimensional space. IMUs are especially useful in the sensor measurement of normal acceleration, for instance, in the measurement of n_z ([A Complete Guide to Inertial Measurement Unit \(IMU\) 2023](#); [Inertial Measurement Unit \(IMU\) – An Introduction 2023](#); [IMU - Inertial Measurement Unit n.d.](#)).

5.8.2. Effect of IMU Placement on Normal Acceleration

One of the first stages in the FCS design is to determine the placement of the IMU relative to the COG, thus to find the distance $x_{IMU \rightarrow COG}$. The first placement of interest is just forward of the most forward location of the ICR. Aside from this point being known for exhibiting minimised sensitivity to structural and HF sources, according to definition, the aircraft would experience no translational velocity during a pitch manoeuvre around this point. Therefore, this would not incur pitch acceleration as a component of measured normal acceleration. The placement of the accelerometer at the ICR thereby ensures that the normal acceleration in this point is equal to zero.

5.8.3. Non-Minimum Phase Systems

For conventional aircraft, the by-product of a pitch up manoeuvre (thus a negative elevator deflection input) is an initial loss of lift on the tailplane. This causes a brief drop in altitude due to the total reduced lift of the aircraft before it reacts to the input and pitches up into climb. This causes the aircraft's normal acceleration to act initially in the opposite direction to that expected during climb. A system which exhibits this effect, responding initially in the opposite direction to the steady-state response, is known as non-minimum phase (NMP), and the loss of tailplane lift is a well known example within aerospace control engineering. This effect is clearly visualised in the frequency domain through the presence of a zero in the right half-plane (RHP). The effect is nullified once the zero enters the left half-plane (LHP).

5.8.4. Calculation of ICR Location

To ascertain the offset of the IMU from the COG, the distance, given by the x-component of the parameter s_{SB_B} , was adjusted incrementally and the corresponding SP response was obtained. It is also possible to calculate the location of the ICR for each flight condition depending on Mach number and altitude according to equations from [McRuer, Graham, and Ashkenas \(1990\)](#), given by [Equation 5.15](#):

$$l_{x_{ICR}} = \frac{Z_{\delta_e}}{M_{\delta_e}} \quad (5.15)$$

in which values are taken from the B matrix of the ss from the SP motion. The ICR locations for each trimmed flight point are identified using this method in [Pedroso D. \(2024\)](#). However, the graphical approach is used in this study due to simplicity, clarity and to act as a method of verification. [Figure 5.20](#) shows the unit step response of the load factor output with variations in the placement of the IMU, as well as the zoomed in transient response.

It can be seen from [Figure 5.20](#) that for some placements of the IMU, the step response initially moves positively before becoming negative, whereas for other placements, the response immediately reacts in the negative direction. The location of the ICR can be determined as the location where there is no longer a NMP response in the channel of elevator command to the normal load factor n_z . The window of locations where the transition between NMP and non-NMP can best be seen corresponds to an IMU placement of between 6.8 m and 7.2 m in front of the COG. Here, a rather small resolution is used between measurements to establish the exact crossover point. [Figure 5.21](#) shows the pole-zero map corresponding to this transfer function varying with IMU placement location:

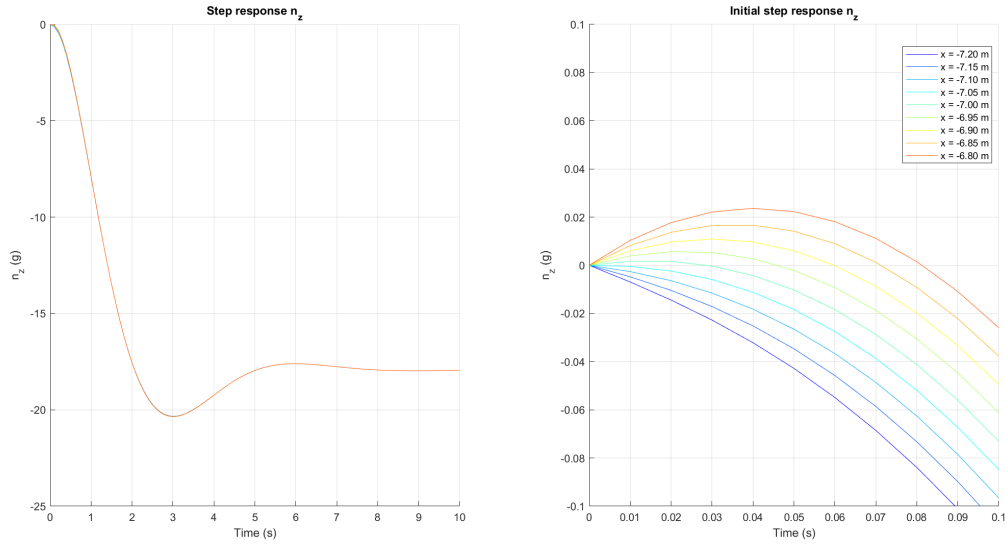


Figure 5.20: SP unit step response from elevator input δe to load factor output n_z

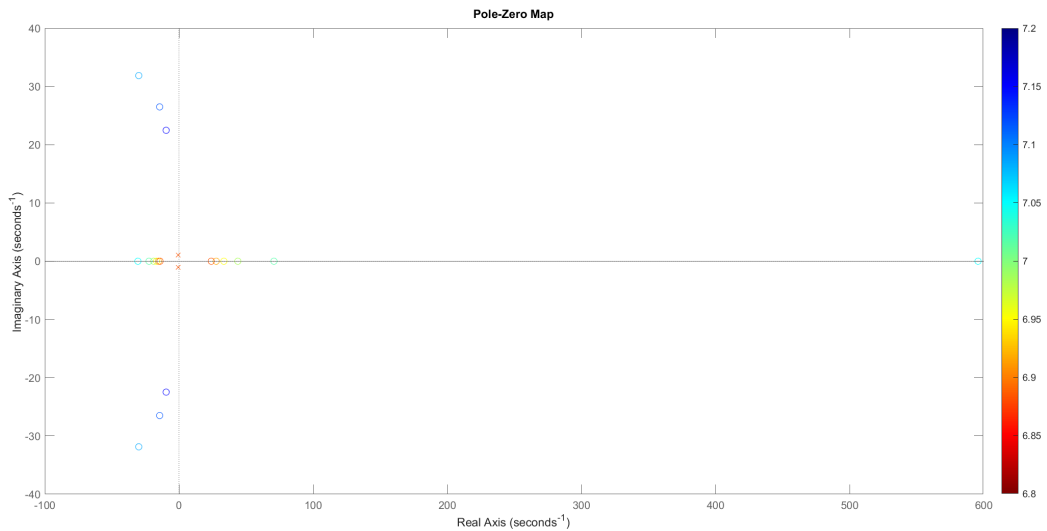


Figure 5.21: Pole-zero map of transfer between elevator input d_e to load factor output n_z and effect of IMU placement on RHP zero

It can be seen that the variation of distance has an influence on the location of the pair of zeros of the transfer function. The most dominant NMP response is associated with smaller proximity of a RHP zero to the imaginary axis, which corresponds here to smaller distances in placement of the IMU. It is visible from [Figure 5.21](#) that at 6.8 m, the RHP zero is located very close to the origin and it moves further away as distance is increased. The associated pair zero is located in the LHP along the real axis. At distances greater than 7.05 m, the RHP zero is shifted to the LHP, where the pair of zeros become complex in value. It is therefore at distances greater than 7.05 m fore of the COG that the non-minimum phase effect disappears for this transfer function. However, a point slightly further fore of this is selected as the ideal IMU placement to allow for a small confidence margin and corroboration with the study conducted by [Pedroso D. \(2024\)](#), and thus for the selected flight point, the IMU is selected to be located at a distance of 7.2 m fore of the COG. For additional detail, pole-zero locations corresponding to each placement option

(ICR, COG) for every the trimmed flight point are provided by [Pedroso D. \(2024\)](#).

5.9. Equations of Motion

The nomenclature used in the description of the EOMs is taken from [Zipfel and Schiehlen \(2001\)](#). The full configuration for aircraft simulation is the six degree of freedom model (6DOF) and the equations of motion (EOM) model all of the three translational and three attitude degrees of freedom ([Zipfel and Schiehlen, 2001](#)). Despite the time and resource consuming task of generating a 6DOF model, it is the most reliable in terms of testing and failure analysis. The 6DOF EOM can be applied to various aircraft and categorised by various reference frames. The EOM applied in this study correspond to the general translational equations assuming flat Earth where Earth becomes the inertial frame. In this section, an overview of the modelling of the relevant EOMs will be provided for insight into the setup of the Flying V aircraft's model simulation.

5.9.1. Attitude Dynamics

The attitude dynamic equations are derived from Newton's law. The law states that the rate of change in time of the angular momentum equates to the externally applied moments, m_b ([Zipfel and Schiehlen, 2001](#)). This is shown in [Equation 5.16](#):

$$D^E(I_B^B \omega^{BE}) = m_b \quad (5.16)$$

In this equation, I_B^B is the moment of inertia (MOI) of the aircraft body frame B relative to the COM, and ω^{BE} corresponds to the angular rates of the body frame B w.r.t the Earth frame E. Subsequently, Euler's transformation is applied to this expression. Choosing the body frame as the reference frame, gives the transformation as according to [Equation 5.17](#):

$$D^E l_B^{BE} + \Omega^{BE} l_B^{BE} = m_B \quad (5.17)$$

where the angular momentum vector $D^E l_B^{BE}$ can be expanded using the chain rule according to [Equation 5.18](#):

$$D^B l_B^{BE} = D^B I_B^B \omega^{BE} + I_B^B D^B \omega^{BE} = I_B^B D^B \omega^{BE} \quad (5.18)$$

In this expression, the rigid body assumption is made which makes the time rate of change of the MOI $D^B I_B^B$ equal to zero. Next, the body coordinate system is once again chosen for simplified time derivative and constant tensor MOI, so that the rotational EOMs are according to [Equation 5.19](#):

$$\left[\frac{d\omega^{BE}}{dt} \right]^B = ([I_B^B]^B)^{-1} (-[\Omega^{BE}]^B [I_B^B]^B [\omega^{BE}]^B + [m_B]^B) \quad (5.19)$$

where

$$[\omega^{BE}]^B = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (5.20)$$

and

$$[\Omega^{BE}]^B = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \quad (5.21)$$

5.9.2. Translational Dynamics

The translational dynamics are derived from Newton's second law (Zipfel and Schiehlen, 2001), which with the flat Earth assumption, is expressed in Equation 5.22:

$$mD^E v_B^E = f_{a,p} + mg \quad (5.22)$$

where m is the vehicle mass, D^E is the rotational time derivative with respect to the inertial frame E , v_B^E is the velocity vector of the COM of the vehicle with respect to the inertial frame E , $f_{a,p}$ is the vector which contains aerodynamic and propulsive forces and g is the gravitational constant. In this expression, the inertial frame I is assumed equal to the Earth frame E . Next, Zipfel and Schiehlen (2001) converts the rotational derivative to the Body frame B for simplicity and coordination, making the formulation valid in any coordinate system. This is given by Equation 5.23:

$$m(D^E v_B^E + \Omega^{BE} v_B^E) = f_{a,p} + mg \quad (5.23)$$

However, it was decided by Pedroso D. (2024) to instead choose the wind frame as the coordinate system through findings of Naäman (2024), rather than the body coordinate system as given by Zipfel and Schiehlen (2001), as this was deemed more suitable due to the fact that the states V and α could be used directly during linearisation, rather than it leading to approximations of these states through using the body frame. Therefore, Equation 5.23 expressed in the wind frame results in Equation 5.24:

$$m(D^W v_B^E + \Omega^{WB} v_B^E) + m\Omega^{BE} v_B^E = f_{a,p} + mg \quad (5.24)$$

and given that

$$\left[\frac{dv_B^E}{dt} \right] = -[\Omega^{BE}]^B [v_B^E]^B + \frac{1}{m} [f_{a,p}]^B + [T]^{BL} [g]^L \quad (5.25)$$

Equation 5.24 in expanded form becomes Equation 5.26:

$$m \left[\frac{dv_B^E}{dt} \right]^W + m[\Omega^{WB}]^W [v_B^E]^W + m[\Omega^{BE}]^W [v_B^E]^W = [f_{a,p}]^W + m[g]^W \quad (5.26)$$

where

$$[\Omega^{BE}]^W = [T]^{WB} [\Omega^{BE}]^B [\bar{T}]^{WB} \quad (5.27)$$

and

$$[f_{a,p}]^W = [T]^{WB} [f_{a,p}]^B \quad (5.28)$$

Gravitational acceleration is most conveniently expressed in the local level coordinate system as $[\bar{g}]^L = \begin{bmatrix} 0 & 0 & g \end{bmatrix}$. Through using a transformation matrix called the Direction Cosine Matrix (DCM), given as $[T]^{BL}$ in Equation 5.29:

$$\begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}^{BL} \quad (5.29)$$

the gravitational component then becomes according to Equation 5.30:

$$[g]^W = [T]^{BL} [g]^L \quad (5.30)$$

5.9.3. Attitude Kinematics

The next part of the model that will be treated is the attitude kinematic relations. The derivative of the Euler angles can be found by transforming the body angular velocities in the body reference frame $[p \ q \ r]$ with a rotational transformation matrix, given by [Equation 5.31](#):

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (5.31)$$

This is a relatively simple method of obtaining the Euler angles, as opposed to other methods involving the use of quaternions. A disadvantage to this approach for calculating Euler angles is that the terms contained in the transformation matrix are non-linear, meaning that during linearisation some non-linear effects are lost. This could lead to difficulties when implementing a controller designed using the linearised model once again into the full non-linear model.

Whilst beneficial that the Euler rates can be directly calculated, singularities occur at $\theta = \pm \frac{\pi}{2}$. This is known as rotation (or gimbal) lock and it occurs when the pitch angle increases until roll is no longer discernable from yaw. Since a commercial aircraft would never normally encounter such conditions, this method is a suitable way to calculate the Euler angles. This phenomenon should be addressed more carefully in the case of more agile manoeuvring aircraft design.

5.9.4. Translational Kinematics

The translational kinematics represent the physical location of the aircraft's COM in the body frame w.r.t an Earth frame reference point. These can be found by twice integrating the velocity vector differential equations, provided by the translational dynamics subsection ([Zipfel and Schiehlen, 2001](#)). These derivative equations are shown in [Equation 5.32](#):

$$\begin{aligned} \dot{u} &= \cos(\alpha) \cos(\beta) \dot{V} - \cos(\beta) \sin(\alpha) V \dot{\alpha} - \cos(\alpha) \sin(\beta) V \dot{\beta} \\ \dot{v} &= \sin(\beta) \dot{V} + \cos(\beta) V \dot{\beta} \\ \dot{w} &= \cos(\beta) \sin(\alpha) \dot{V} + \cos(\alpha) \cos(\beta) V \dot{\alpha} - \sin(\alpha) \sin(\beta) V \dot{\beta} \end{aligned} \quad (5.32)$$

and given that

$$[D^E s_{BE}] = [v_B^E] \quad (5.33)$$

and that the integration was conducted in the local-level coordinate system, then

$$\left[\frac{ds_{BE}}{dt} \right]^L = [\bar{T}]^{BL} [v_B^E]^B \quad (5.34)$$

Now, six first order differential equations have been produced that are able to describe the motion of the aircraft in terms of the Earth as the Inertial reference frame.

5.10. Model Trim and Linearisation

Due to the majority of previous works contributing to the incorporation of non-linear control techniques, the linearised state space system of the aircraft aerodynamic model and equations of motion was not initially present. However, [Pedroso D. \(2024\)](#) recently conducted the extended trimming and linearisation routine, of which a summary will be provided in this section.

5.10.1. Trimming Routine

In order to perform robust control on an aircraft model, it is first required that it is trimmed for stable, steady, level flight, and subsequently linearised into a linear system. Firstly, the equilibrium point of the

model must be found around which controller design can be conducted for the given flight point. Since the act of trimming and linearisation depend on very small perturbations around a given operating point, it is important that the trimming and linearisation routine are accurate. In the identification of the equilibrium point, the total forces and moments arising from the body and control surfaces is by definition equal to zero. The aircraft is in equilibrium if the derivative of the state vector $\dot{\mathbf{x}}(t) \triangleq 0$, meaning that none of the state parameters are changing in time. To solve this, there should be an equal number of algebraic equations as unknown variables. The state vector is given by [Equation 5.35](#):

$$\mathbf{x} = \begin{bmatrix} p & q & r & \phi & \theta & \psi & V & \alpha & \beta & x_L & y_L & z_L \end{bmatrix}^T \quad (5.35)$$

the input vector is given by [Equation 5.36](#):

$$\mathbf{u} = \begin{bmatrix} T & \delta_a & \delta_e & \delta_r \end{bmatrix}^T \quad (5.36)$$

and the output vector is given by [Equation 5.37](#):

$$\mathbf{y} = \begin{bmatrix} n_x & n_y & n_z & p & q & r \end{bmatrix}^T \quad (5.37)$$

For steady, wings-level flight, the roll angle ϕ and angular rates of roll, pitch and yaw p, q, r , are assumed zero. The yaw angle ψ is free and the sideslip angle β is determined by the algorithm. Of the remaining states from the state vector, the velocity V is deemed known and the AOA deemed unknown, as the two are inherently coupled through dynamic pressure and the expression for the lift coefficient. Since more unknown variables than equations are established, an extra constraint is imposed on the flight path angle γ , enforcing it to be zero, and thus providing another equation relating the pitch angle to the AOA through $\gamma = \theta - \alpha$. Since the number of equations to variables is satisfied, the trimming routine can be run. The trimming algorithm is responsible for adjusting the values of the control variables, including the thrust T and the control surface deflections of the aileron δ_a , the elevator δ_e and the rudder δ_r . The routine was conducted a numerical method due to its ease and time-saving nature. This involves iteratively adjusting the values of the independent variables such that requirements are met. The final trimmed values for the control variables obtained for the flight point $M = 0.5$, $h = 5450m$, are provided in [Table 5.6](#):

Variable	Value
T (N)	114420
δ_a (deg)	0
δ_e (deg)	-1.75
δ_r (deg)	0

Table 5.6: Trimmed control variable values, obtained as a result of the trimming routine

Extended detail about the setup of the trimming routine, as well as trimming control variable values for adjacent flight points can be found in [Pedroso D. \(2024\)](#). This report also provides time-domain simulations obtained using the nonlinear Flying V aircraft model, indicating the success of the trimming routine by showing that each of the state variables are kept stable in response to the trimmed control inputs. As well as this, certain trends could be identified from the trimming results; this included the values for the pitch angle θ , which were identified as being equal to the AOA for the given flight point, resulting from the additional constraint imposed on the flight path angle γ . Additionally, at lower velocities, a higher value of AOA was obtained to compensate the reduction in lift generated as a result. Lastly, the AOA was increased for a given velocity and higher altitude in order to compensate the lift coefficient for the reduction in corresponding air density.

Important to note is that the trimmed value of the thrust T includes the contribution of both engines. Additionally, as a result of the trimming and linearisation routine, it was decided to only proceed with 8 of

the 16 data points for which aerodynamic data was provided. This is due to the fact that the aircraft design and resulting aerodynamic data are still in the early stages of the design, and require further development before relevant controllers can be designed to operate over the entire desired FE. Furthermore, flight points below $M = 0.3$ were not considered in the trimming routine due to limitations imposed by the thrust as a result of the current Flying V aircraft's aerodynamic design. It was also observed that the aerodynamic model struggled at data points including and exceeding $M = 0.7$, as the flight points would become unstable at AOA values greater than 2.5° . For this reason, the further development of the current Flying V aircraft (aerodynamic) design is highly recommended, especially from the perspective of control and stability design.

5.10.2. Linearisation

The process of linearisation concerns the approximation of nonlinear behaviour given in linear form, taken around the trim points that are obtained from the trimming routine. For the purposes of this study, linearisation is conducted using a numerical method, as for the trimming routine. The linearised system resulting from the nonlinear approximation can be expressed in state-space form in terms of deviations from the trimmed condition.

The trimmed nonlinear dynamics are subsequently represented in this state-space form, including all lateral and longitudinal states, inputs and outputs. The various aircraft modes represented by the combination of certain states and outputs can be obtained from the full-order model by mathematically isolating the states of interest from the full-order matrix (for instance, α and q for the SP mode, V and θ for the phugoid mode, or all four corresponding to the total longitudinal motion).

5.11. Incorporating Parametric Uncertainty

To be able to perform H_∞ -LS control, parametric uncertainty must be incorporated into the model. In this case, the main parameters that are considered to be uncertain are the aerodynamic coefficients and the nature of the uncertainty is unstructured NCF uncertainty.

The aerodynamic coefficients are assumed to vary by 30% above and below their nominal value. This was incorporated using the Matlab's 'ureal' function, which allows the user to specify a nominal value and the magnitude of the deviation due to uncertainty. Due to the aerodynamic coefficients being largely varied in value between each other, it was decided that it was programmatically simpler to assume an average value of 1, plus and minus 30% (0.7 - 1.3), with which the nominal coefficient value was multiplied. The ureal variables were added to the aircraft initialisation Matlab file 'FV_init_airframe', as shown in the following:

```

1 Uncertainty_AeroCoeff.Cz_alpha_unc = ureal('Cz_alpha_unc',1,'Percentage',30)
  ;

1 Uncertainty_AeroCoeff.Cm_alpha_unc = ureal('Cm_alpha_unc',1,'Percentage',30)
  ;
2 Uncertainty_AeroCoeff.Cm_q_unc = ureal('Cm_q_unc',1,'Percentage',30);
3 Uncertainty_AeroCoeff.Cm_delta_e_unc = ureal('Cm_delta_e_unc',1,'Percentage'
  ,30);

```

Additionally, uncertainty blocks were added to the linearisable model Simulink file 'FV_Linearize_Vab_Simulink' in the 'Aerodynamic Coefficients' subsystem within the Aerodynamics subsystem. This was done by incorporating uncertain state space blocks for each aerodynamic coefficient and specifying within the block the name of the respective ureal variable that was created.

The last significant modification that was made in order to create the uncertain model was regarding the linearisation method used. The function 'ulinearize' was used in place of the previous linearisation Matlab function 'linearize'. This new linearisation function is used in the creation of uncertain 'uss' models when uncertainty is present in a model. The resulting uss systems are similar to the ss system that would be obtained using regular linearisation in that the nominal model can be extracted. In addition, uss systems provide details about the amount of uncertainty that was applied to each of the relevant variables that are present. The uncertainty blocks can be enabled and disabled easily, allowing different combinations

and magnitudes of parametric uncertainty to be considered. Noteworthy of using 'ulinearize' is that it is necessary to manually verify whether the resulting state ordering is correct such that the correct flight mode is obtained through the isolation of specific states.

The amount of parametric uncertainty that is present in the model will have a direct consequence on the ease of formulation of a robustly stabilising controller; generally, the larger the parametric uncertainty present, the less accurate the model and the wider the range of plant variations that the controller must robustly stabilise. As a result, the controller design is more conservative. A larger uncertainty region means that it is more difficult to obtain a controller that is able to satisfy requirements such as satisfactory gain and phase margins, and therefore the likelihood of obtaining such a controller is reduced. It also limits the bandwidth of the controller to account for the more extreme model variations, which can have a compromising effect on the performance of the controller. The magnitude of the uncertainty present is decided based on realistic variations that are expected to be present in real-world scenarios; if the magnitude of uncertainty is under-estimated, the controller may fail to robustly stabilise the plant under the effect of all real-world parameter variations, and therefore for this reason, a large percentage of uncertainty of 30% was selected. After model trim and linearisation are successfully completed, a longitudinal state-space system corresponding to the bare airframe is obtained, which is further reduced to a short period (SP) system.

Longitudinal Bare Airframe Analysis

Trim and linearisation of the Flying V aircraft model was conducted previously in [Chapter 5](#), which resulted in a bare aircraft longitudinal state-space system. This chapter will perform preliminary analysis of the obtained SP system, followed by the addition of the C^* signal to the system output, as well as pitch rate damping to improve the system steady-state response. Then, scaling gain is applied and to conclude, the pre-conditioned airframe model will be compared to the reference model.

6.1. Static and Dynamic Stability

The static stability of an aircraft is known as the tendency to converge to the initial equilibrium condition after being slightly disturbed from trim. Dynamic stability is the term used to denote the transient motion in which it recovers equilibrium after disturbance ([Cook, 2007](#)). Static and dynamic stability, exhibited in the system A matrix, are ultimately phenomena that are tightly coupled.

In the frequency domain, the criteria for overall stability is such that all eigenvalues λ of the A matrix (and thus all poles of the system) lay in the LHP, or in other words, that all eigenvalues have a negative real component. This holds for both kinds of stability, and is expressed mathematically according to [Equation 6.1](#):

$$\text{Re}(\lambda) < 0 \quad (6.1)$$

6.2. Full-Order Longitudinal Aircraft Model

The full-order longitudinal aircraft model is commonly defined according to [Equation 6.2](#) and [Equation 6.3](#):

$$\dot{x}_{FO}(t) = A_{FO}x_{FO}(t) + B_{FO}u_{FO}(t), A_{FO} \in R^{4 \times 4}, B_{FO} \in R^{4 \times 1} \quad (6.2)$$

where:

$$\begin{aligned} x_{FO}(t) &= \begin{bmatrix} \alpha & q & V_{TAS} & \theta \end{bmatrix}^T \\ u_{FO}(t) &= \begin{bmatrix} \delta_e & T \end{bmatrix}^T \end{aligned} \quad (6.3)$$

consisting of the 4 states V_{TAS}, α, q and θ and the two control inputs δ_e and T . LTI systems can otherwise be expressed as TFs, denoted as H , which can be obtained in terms of the Laplace operator s in the following form:

$$H(s) = C(sI - A)^{-1}B \quad (6.4)$$

where C is the system output matrix and B is the system input matrix.

6.3. Reduced-Order Short Period Approximation

Normal longitudinal, full-order aircraft dynamics are usually composed of two dynamic modes: these are the phugoid mode and the short period (SP) mode. The phugoid mode is a slower and less damped mode compared to the SP. The SP mode operates at higher frequencies, very close to the frequencies of pilot handling, meaning that pilots are more likely to encounter flying effects relating to the SP motion than the phugoid dynamic mode. For this reason, the SP mode is very often used as the primary dynamic mode in the assessment of pilot experienced HQs. The SP system is the primary focus of this study as the states α and q most clearly exhibit the SP mode, and the SP is the motion that most clearly exhibits the effects of load factor and pitch rate, the factors of highest importance to a C* controller.

Deriving from the longitudinal FO model, the SP mode is obtained through the short-period approximation, a concept explored previously by [Etkin and Reid \(1959\)](#). Where the FO longitudinal model contains the states V_{TAS}, α, q and θ , the SP approximation omits V_{TAS} and θ to leave the states α and q . Also, provided the relevant desired output states of n_z and q , this results in the model formulation given in [Equation 6.5](#), and [Equation 6.6](#):

$$\dot{x}_{SP}(t) = A_{SP}x_{SP}(t) + B_{SP}u_{SP}(t), A_{SP} \in R^{2 \times 2}, B_{SP} \in R^{2 \times 1} \quad (6.5)$$

$$\begin{aligned} x_{SP}(t) &= \begin{bmatrix} \alpha & q \end{bmatrix}^T \\ u_{SP}(t) &= \begin{bmatrix} d_e \end{bmatrix}^T \end{aligned} \quad (6.6)$$

The outputs of n_z and q are extracted using [Equation 6.7](#):

$$y_{SP}(t) = \begin{bmatrix} n_z & q \end{bmatrix}^T \quad (6.7)$$

6.4. Aircraft State-Space System

Given this model approximation, the SP state-space system rates and outputs obtained after model trim and linearisation are presented in [Equation 6.8](#) and [Equation 6.9](#) respectively:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -0.6013 & 0.9742 \\ -1.154 & -0.7482 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -0.1413 \\ -3.198 \end{bmatrix} \delta_e \quad (6.8)$$

$$\begin{bmatrix} n_z \\ q \end{bmatrix} = \begin{bmatrix} 8.808 & -0.1271 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -0.04736 \\ 0 \end{bmatrix} \delta_e \quad (6.9)$$

6.5. Bare Airframe Model Response

The bare airframe refers to an aircraft without any stability augmentation system (SAS), in which the pilot controls the actuators directly using control inceptors such as the sidestick, throttle and control column. This is expressed schematically by placing the actuator dynamics in series with the aircraft dynamics, mathematically computing the matrix product $G_{actuator} \cdot G_{aircraft}$. The resulting state-space system is provided in [Equation 6.10](#) and [Equation 6.11](#):

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\delta_e} \end{bmatrix} = \begin{bmatrix} -0.6013 & 0.9742 & -0.1413 \\ -1.154 & -0.7482 & -3.198 \\ 0 & 0 & -14.29 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 14.29 \end{bmatrix} \delta_e \quad (6.10)$$

$$\begin{bmatrix} n_z \\ q \end{bmatrix} = \begin{bmatrix} 8.808 & -0.1271 & -0.04736 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \delta_e \quad (6.11)$$

From Equation 6.10 it is clear to see that the state and input matrices A and B have been expanded to account for the additional elevator state δ_e that has been contributed through the actuator dynamics. The step responses corresponding to this state-space system are shown in Figure 6.1 and for the uncertain case in Figure 6.2 respectively:

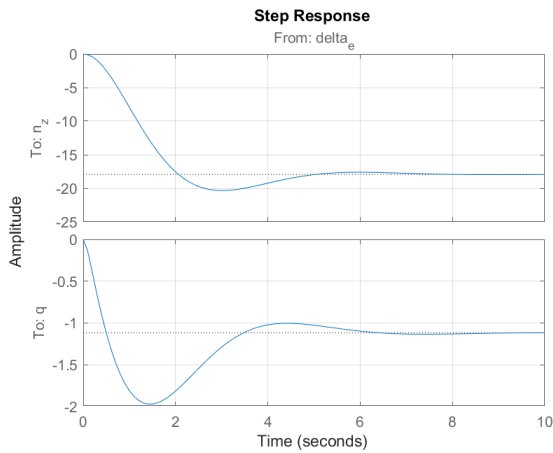


Figure 6.1: Step response of airframe dynamics $G_{airframe}$

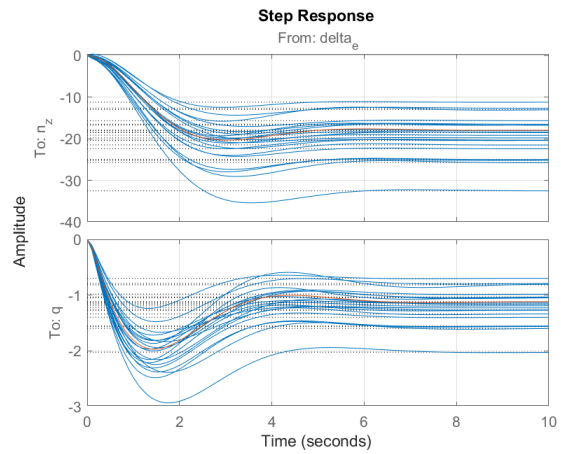


Figure 6.2: Step response of airframe dynamics with applied uncertainty $G_{airframe_{unc}}$, as well as nominal system

It is visible that a step input in the positive direction of the elevator δ_e is responsible for a negative response for both n_z and q . This is to be expected as a positive elevator deflection pitches the aircraft downwards, therefore with negative pitch rate. The aircraft begins to experience a loss in load factor due to the previous definition that an upwards input is responsible for an increase in load factor.

Additionally, it is visible that the system is stable but under-damped due to its rather long settling time. Next, the pole-zero map of the bare airframe dynamics is provided in Figure 6.3 and these poles and zeros are summarised again in Table 6.1:

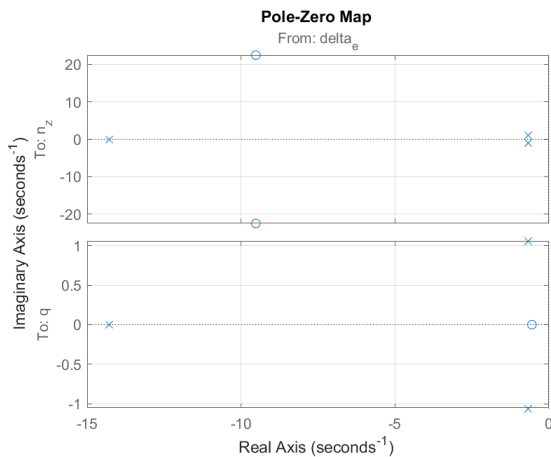


Figure 6.3: Pole-zero map of airframe dynamics $G_{airframe}$

	Poles	Zeros
n_z	$-0.6747 + 1.0576i$	$-9.5201 + 22.4760i$
	$-0.6747 - 1.0576i$	$-9.5201 - 22.4760i$
	-14.286	
q	$-0.6747 + 1.0576i$	-0.5503
	$-0.6747 - 1.0576i$	
	-14.286	

Table 6.1: Bare airframe poles and zeros

Beneficial to note is that the poles of the aircraft and the actuator are sufficiently separated, and therefore no issues concerning control authority should be encountered. Furthermore, the poles and corresponding damping information are provided in Table 6.2:

Pole	Damping	Frequency (rad/s)	Time constant (s)
-0.6747 + 1.0576i	0.538	1.25	1.48
-0.6747 - 1.0576i	0.538	1.25	1.48
-14.3	1	14.3	0.07

Table 6.2: Bare airframe damping information

It can be concluded that the damping ratio of 0.538 associated with the aircraft poles is relatively low. A typical value of 0.7 is assumed for good settling performance and therefore pitch rate damping will be applied. Furthermore, it is notable that the natural frequency of the system is low at 1.25 rad/s. This can have negative consequences for the SMs of the system as it could become prone to oscillatory behaviour and even instability, which is supported by the pole-zero map, showing that the poles lie not too far from the imaginary axis. Additionally, due to the first order nature of the actuator pole, the damping ratio is equal to 1 through critical damping.

6.6. Addition of C* Output

As previously discussed, the C* signal is a combination of weighted load factor and pitch rate outputs. In order to investigate the C* signal, it must first be created from the existing outputs. The state and input matrices A and B remain unchanged, and are equal to those provided earlier in Equation 6.10. Using Equation 4.41, the extra output is added to the output state-space systems C and D as in Equation 6.12 to produce the updated output matrices given by Equation 6.13:

$$\begin{bmatrix} n_z \\ q \\ C^* \end{bmatrix} = \begin{bmatrix} 8.808 & -0.1271 & -0.04736 \\ 0 & 1 & 0 \\ 8.808 + \frac{V_{co}}{g} * 0 & -0.1271 + \frac{V_{co}}{g} * 1 & -0.04736 + \frac{V_{co}}{g} * 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 + \frac{V_{co}}{g} * 0 \end{bmatrix} \delta_e \quad (6.12)$$

$$\begin{bmatrix} n_z \\ q \\ C^* \end{bmatrix} = \begin{bmatrix} 8.808 & -0.1271 & -0.04736 \\ 0 & 1 & 0 \\ 8.808 & 12.46 & -0.04736 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \delta_e \quad (6.13)$$

The only significant difference between the n_z and C^* output row entries is the significantly higher and now positive contribution of the pitch rate (12.46 instead of -0.1271). The step response corresponding to the new C* output is shown in Figure 6.4 and for the system with applied uncertainty in Figure 6.5:

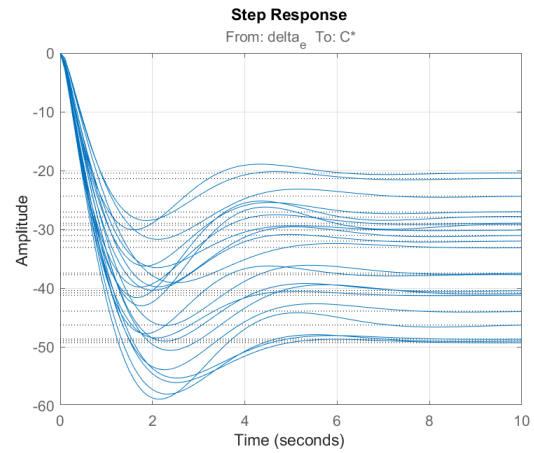
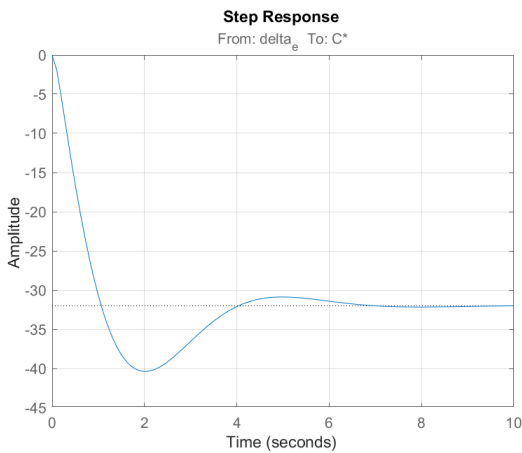
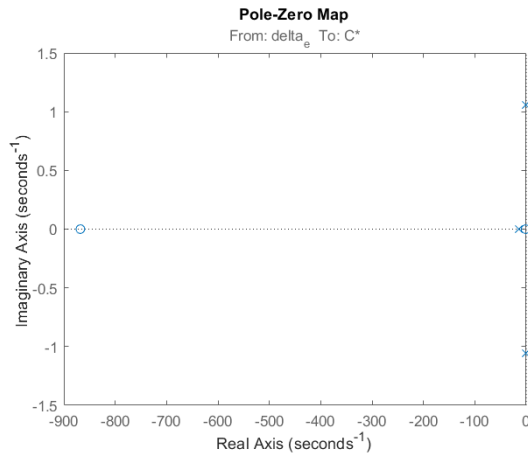


Figure 6.4: Step response of airframe C* dynamics, $G_{airframeC^*}$ **Figure 6.5:** Step response of airframe C* dynamics with applied uncertainty, $G_{airframeuncC^*}$

The magnitude of the peak value of the C^* response is rather large, even when compared to that of the n_z response. This is due to the very large step input of magnitude 1, as well as the additional contribution of the pitch rate response in the signal. The corresponding pole-zero map is shown in Figure 6.6, and these poles and zeros of the C^* output transfer are summarised in Table 6.3:



	Poles	Zeros
C^*	$-0.6747 + 1.0576i$	-867.8311
	$-0.6747 - 1.0576i$	-1.2256
	-14.3	

Table 6.3: Poles and zeros of C^* signal

Figure 6.6: C^* pole zero map

The zeros of the C^* signal are modified with respect to the existing outputs; the C^* signal has two zeros, one located relatively close to the origin at -1.2256 rad/s and one located further from the origin at -867.8311 rad/s. Additionally, the very fast zero located at -867.8311 rad/s will produce a phase lead around this frequency, which is far outside of the bandwidth of most practical controllers; controller design concerns much lower frequency dynamics, and therefore the zero will have a negligible effect.

6.7. Pitch Rate Damping

Next, pitch rate feedback (FB) should be applied to improve the damping response, as it is initially still under-damped. Pitch rate FB presents itself in block form as a rate FB IL. The pitch rate response is isolated from the other signals by selecting the second row of the airframe ss. Then, the sisotool UI provides the pole-zero map corresponding to the transfer. A design requirement of $\zeta = 0.7$ is added, and moving the aircraft's oscillatory poles to these locations produces the gain that achieves the desired damping ratio. The required compensator gain K_q associated with the selected flight point model is equal to -0.1735 . Once found, this gain is placed in the IL, and the updated step response of the airframe with stabilising pitch rate FB applied is shown in Figure 6.7 and with uncertainty in Figure 6.8:

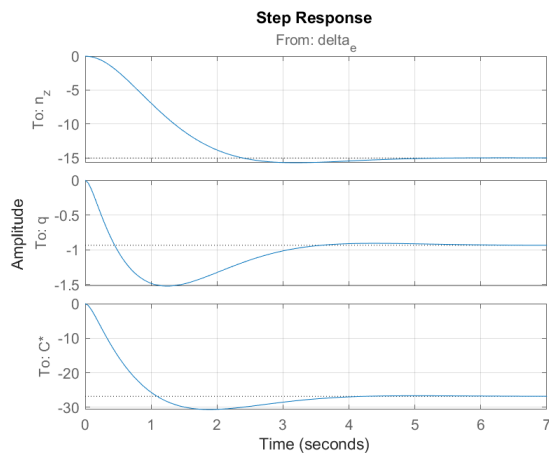


Figure 6.7: Step response of airframe with pitch rate damping, G_{Cq}

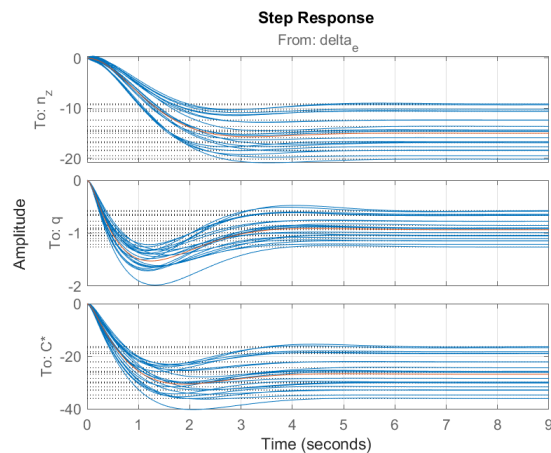


Figure 6.8: Step response of airframe with applied uncertainty and pitch rate damping, $G_{Cq_{unc}}$

Once again, the airframe pole-zero map is shown in [Figure 6.9](#) and the modified poles and zeros in [Table 6.4](#):

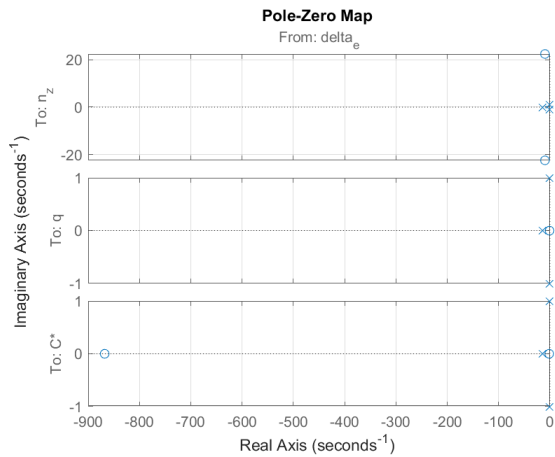


Figure 6.9: Pole-zero map of airframe with pitch rate FB applied

	Poles	Zeros
n_z	$-0.9805 + 1.0009i$	$-9.5201 + 22.4760i$
	$-0.9805 - 1.0009i$	$-9.5201 - 22.4760i$
	-13.6742	
q	$-0.9805 + 1.0009i$	-0.5503
	$-0.9805 - 1.0009i$	
	-13.6742	
q	$-0.9805 + 1.0009i$	-867.8311
	$-0.9805 - 1.0009i$	-1.2256
	-13.6742	

Table 6.4: Airframe poles and zeros with pitch rate FB applied

By comparing [Table 6.4](#) with [Table 6.1](#) and [Table 6.3](#), it is evident that the pitch rate FB has shifted the location of the poles from their previous location of $-0.6747 \pm 1.0576i$ and -14.3 to their new locations of $-0.9805 \pm 1.0009i$ and -13.6742 , thus moving them further from the imaginary axis and moving them closer to the real axis. Naturally, the zero locations stay intact. The success of the pitch rate damping is shown in [Table 6.5](#):

Pole	Damping	Frequency (rad/s)	Time constant (s)
$-0.981 + 1i$	0.7	1.4	1.02
$-0.981 - 1i$	0.7	1.4	1.02
-13.7	1	13.7	0.0731

Table 6.5: Airframe damping information with pitch rate FB applied

Where it is seen that the damping ratio of the airframe poles is now equal to 0.7. Furthermore, the frequency of the actuator pole is reduced slightly and thus the time constant has increased from 0.07 to 0.0731.

6.8. Comparison of Bare Airframe C* Response with Reference Model

Once the signal corresponding to the scaled, pitch rate damped C* output is obtained, it can be compared to the reference model for a preliminary assessment of the response behaviour and associated HQs. [Figure 6.10](#) shows the step response of the signal compared to that of the reference model:

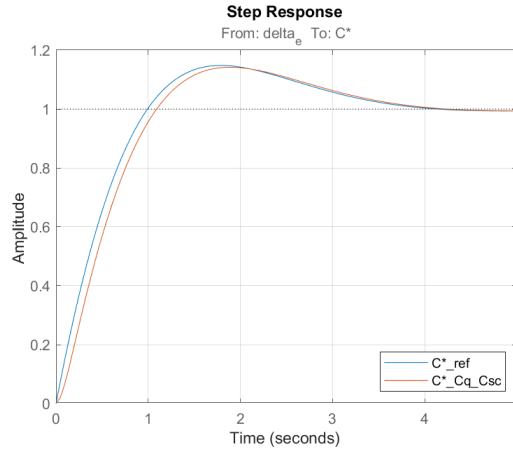


Figure 6.10: Step response comparison of C* signal to reference model

It is evident that the scaled and damped C* signal very closely approximates the reference model in terms of step response characteristics; there is a visible yet minimal initial delay, however the signal still matches the reference closely after approximately 2 s in terms of peak value, damping ratio and settling time. The peak value is equal to 1.14 yielding an overshoot of 14.2 %. This is minimally less than that of the reference, with a peak value of 1.15 with a corresponding overshoot of 14.8 %. Additionally, the settling time t_s for the C* signal is equal to 3.64 s, slightly higher than the reference with a settling time t_s of 3.56 s. The signal is also assessed in terms of the CAP and Gibson HQ criteria in [Figure 6.11](#) and [Figure 6.12](#) respectively:

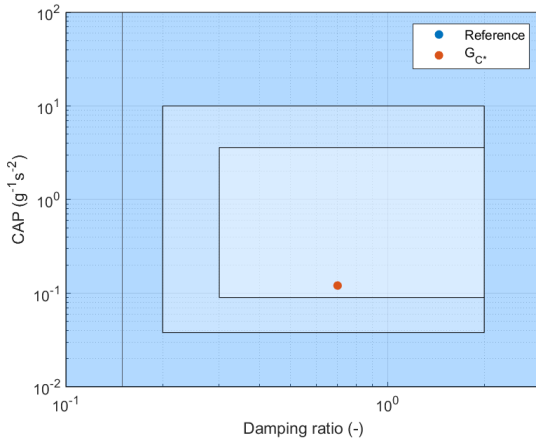


Figure 6.11: CAP criterion of scaled airframe C* signal with pitch rate damping, $G_{CqCscC*}$

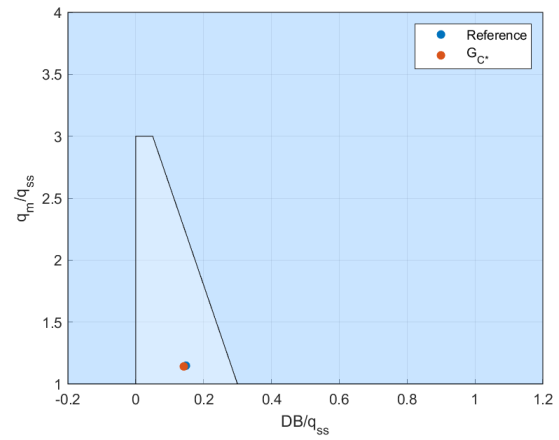


Figure 6.12: Gibson criterion of scaled airframe C* signal with pitch rate damping, $G_{CqCscC*}$

Firstly, [Figure 6.11](#) shows that the scaled, damped C* response matches the reference model almost perfectly with the CAP criterion. The damping ratio is almost exactly correct ($\zeta = 0.6998$) through having moved the pole locations using 'sisotool'. Furthermore, it also shows an identical CAP value, as the value for the natural frequency ω_n is also identical to that of the reference model at 1.4 rad/s. The Gibson criterion in [Figure 6.12](#) shows that the response also almost perfectly matches in terms of OS and DB. These results would indicate that the signal already accurately represents the reference model. However, there is yet C* outer loop (OL) FB to be applied, and a robust controller must still be formulated in order to ensure that the system is robustly stable to all cases of NCF uncertainty that may be present. Therefore, this robust controller will be developed in subsequent chapters, and its affect on the HQs of the FCS will be reanalysed.

Flight Control System Design

This chapter starts with outlining the design considerations that should be taken for the FCS, followed by the discussion of NCF uncertainty before beginning the 1DOF and 2DOF design processes. These two designs are formulated and assessed according to stability and HQ criteria, and lastly, the 2DOF design is re-assessed in the presence of uncertainty.

7.1. Controller Design Considerations

There are a number of design considerations to address before the design of a robust C^* longitudinal controller. The first of these, crucially, is the type of the signal that will be fed back by the controller. As previously mentioned, the output signals obtained from the SP model include the pitch rate q , load factor n_z and C^* . It is not apparently obvious which of these signals would be most beneficial to be placed in feedback, and therefore this is first investigated. Secondly, the measurement location of the n_z signal will have a large effect on the feedback nature of the n_z or C^* signal, and will also be discussed. Lastly, the measurement locations used in the performance signals to be compared against in the controller performance assessment will be discussed. Since the aircraft dynamic mode investigated in this study concerns the SP, only the constant speed, transient effects relating to the 'immediate short term' will be taken into consideration.

7.1.1. Feedback Signal Configuration

Expectedly, varied handling quality characteristics arise depending on the type of signal that is placed in feedback within a feedback configuration. Gibson (1999) addressed the SP behaviour resulting from the different feedback types, including that of pitch rate and load factor. It is the SP mode that has lead to the most unpredicted misunderstandings in this regard during the design process.

Pitch Rate Demand

Generic pitch rate demand (PRD) is a controller configuration which constitutes feedback of the pitch rate signal. It is well known for its excellent capabilities of disturbance rejection, especially during turbulence. Furthermore, it gives pilots a higher sense of control, especially nearer to the ground, since the attitude is less randomly disrupted by external effects. A shortfall of the configuration is that heave motions may increase as the conventional response of pitching into gusts for gust alleviation is prevented. Additionally, despite excellent pitch rate control with excellent frequency and damping, sometimes unacceptable sluggishness in the flight path response can be observed. Despite these drawbacks, it is generally still considered a reasonable compromise thanks to its benefits (Gibson, 1999).

Normal Acceleration Demand

The normal acceleration demand is a conventional control response which commands the parameter that the flight path directly follows; an almost constant g is maintained through the constant adjustment of the AOA. For this reason, the flight path is less affected during turbulence than a PRD. As a consequence, attitude disturbances are more prevalent which could lead to higher pilot workload, especially at lower altitudes, thereby making HQ performance comparable to or less satisfactory than the PRD (Gibson, 1999).

C* Demand

C* demand is an example of a combined demand feedback configuration which has the potential to reap the benefits and lessen the drawbacks of both the PRD and normal acceleration demand configurations on their own. This is possible since C* provides pitch rate cues to the pilots at low velocity and gradually transitions to normal load factor cues at higher velocities, in line with pilots' expectations (Gibson, 1999).

The most suitable approaches of the three outlined configurations for this application are those of the n_z feedback and C* feedback configurations, due to relevance to existing literature in this field. To determine which of the two possibilities would produce the most successful outcome, Pedroso D. (2024) previously investigated the possible feedback configurations by synthesising a controller through the use of signal-based H_∞ -MS design and evaluated the SVs of the open loops for each of the relevant channels. The investigation found that for each of the feedback signals (n_z , C*), the open loop SVs at HF are low as desired, and the LF SVs for each of the feedback signals are sufficiently high, indicating satisfactory DR. However most relevantly, it identified a trade-off of the LF SVs between the q and n_{zCOG} channels; feeding back n_{zPS} boosted the LF gain of the n_{zPS} open loop due to the presence of integral action (thus improving DR on that channel) at the cost of significantly lowering the LF gain of the q channel. Selecting C*_{PS} as the feedback signal resulted in medium gain for both of the n_{zCOG} and q channels, as well as high LF gain in each of the other channels. With better DR capabilities overall, the investigation concluded that the C* signal ought to be used as the controller feedback configuration.

7.1.2. C* Measurement Location - Robust Stability

It is established that the C* signal should be used as feedback to the controller. However, an important discussion to be had is that of the measurement location of the load factor n_z . The C* signal is composed of a weighted combination of the vertical load factor n_z and the pitch rate q , therefore the C* parameter value will be highly influenced by the measurement location of n_z .

A logical placement for the load factor measurement location is the ICR. As previously mentioned, load factor measurement at this location ensures that no component of the load factor is due to rotational pitch acceleration $\dot{q}$. This has the potential to eliminate any unwanted transient effects in the load factor (thus also C* responses), thereby potentially improving the behaviour of the response and additionally reducing the amount of control action required from the controller to maintain the desired trajectory.

The equation relating n_z at the PS to n_z at the IMU is provided in Equation 7.1:

$$n_{zPS} = n_{zIMU} + \frac{\dot{q}(x_{IMU \rightarrow COG} + x_a)}{g_0} \quad (7.1)$$

The distance $x_{IMU \rightarrow COG}$ corresponds to the distance between the IMU and the COG locations. From the equation it is clearly visible that the load factor at the PS increases with increasing distance between the IMU and the COG.

Pedroso D. (2024) synthesises two controllers and examines the open loop SVs across different channels for the two IMU placements (PS and IMU). The study concluded that $\dot{q}$ dynamics have little contribution at LF as there is little discrepancy between n_z and C* signals, and high contribution at HF since all signals have higher gain. The study also concludes that the C* signal at the IMU will be fed back by the controller due to the lack of a pitch rate derivative term in the load factor measurement, improving roll-off characteristics at HF, as well as the reduced control action necessary to minimise the small bump present in the load factor step response.

7.1.3. C* Measurement Location - HQ Assessment

The C* parameter was devised as a HQ criteria used to assess the performance of an aircraft's C* response. Logically, as the HQ requirements relate to how the pilot perceives the aircraft behaviour, the C* parameter is usually calculated using n_z measurement at the PS due to the fact that here it is more aligned with how the pilot perceives the aircraft's behaviour and the required stick input command. The C* signal with measurement at the PS can be calculated as an additional system output and used as criteria for the controlled system to follow a reference performance signal, ie: the reference model that is being used for the design. A difference in load factor measurement location between the feedback signal and HQ performance signal is allowable since it is not strictly necessary that the same signal be used in feedback as used for HQ assessment.

7.1.4. Controller Structure

The controller will be a two degree-of-freedom (2DOF) controller for improved reference tracking compared to a one degree-of-freedom (1DOF) controller. The controller structure will correspond to a SISO system with the key output being the C^* parameter, used for both OL FB and performance evaluation. The implementation configuration of the FB controller will be considered and compared.

7.2. NCF Uncertainty

Using normalised coprime factorisation, the resulting TF for $\tilde{N}$ for the nominal model is given by Equation 7.2:

$$\tilde{N} = \frac{-0.67663(s + 867.8)(s + 1.226)}{(s + 1.227)(s^2 + 36.88s + 587)} \quad (7.2)$$

and $\tilde{M}^{-1}$ is found by calculating the inverse of $\tilde{M}$, and is given by Equation 7.3:

$$\tilde{M}^{-1} = \frac{(s + 1.227)(s^2 + 36.88s + 587)}{(s + 13.67)(s^2 + 1.961s + 1.963)} \quad (7.3)$$

The SVs of the original TF, as well as $\tilde{M}^{-1}$ and $\tilde{N}$ and their product are shown in Figure 7.1.

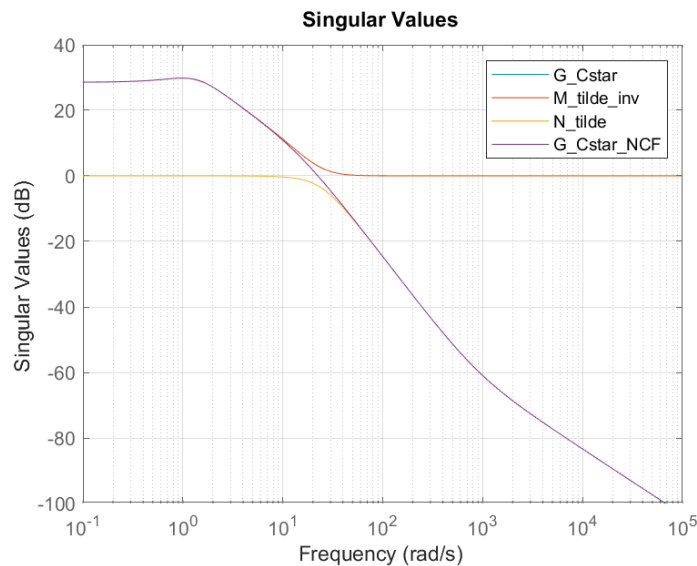


Figure 7.1: NCF decomposition of G_{C^*}

It can be seen from the SV plot that the multiplication of the two components $\tilde{M}^{-1}$ and $\tilde{N}$ result in the original TF again, proving that the decomposition is correctly implemented. Furthermore, both of the component TFs are stable and proper. Lastly, the high magnitude of the system is clear to see through the very high SVs at the LFs. The derivation of the normalised coprime factors is an important factor of H_∞ loopshaping as it forms the basis for the estimate of NCF uncertainty computation.

7.2.1. NCF Uncertainty Computation

For each global stability coefficient, 30% parametric uncertainty has been applied to each partial stability coefficient, and a value of 3 samples per coefficient have been taken from the uncertain system per global coefficient. The following Matlab command:

```
[G_p, sample_values] = gridureal(G_unc, ...
```

provides the set of perturbed systems as well as the sampled coefficient values, where the ellipsis indicates the name of the uncertainty followed by the dimension of the sampled coefficient. Important to note is that the coefficient sample size has an impact on the cases that are accounted for; the samples generated by gridureal are equally spaced over the uncertainty range. Therefore, a sample size of three is the minimum needed per coefficient to account for the maximum and minimum uncertainty edge cases as well as the nominal case without uncertainty. To account for the nominal system case, it is important that the sample size specified for each coefficient be an odd value.

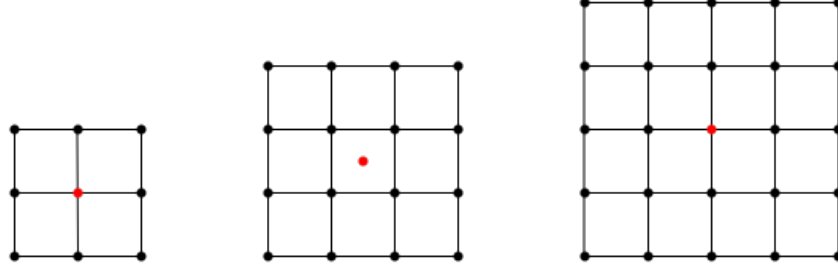


Figure 7.2: Increasing sample size and effect on inclusion of edge and nominal cases

The SVs are calculated for each of the uncertainty blocks, as well as the H_∞ norm. After this, the maximum SV for all perturbed systems over all frequencies is identified. Lastly, a line of SVs is fit to the SVs plot with an ordered estimate using the Matlab function 'fitmagfrd' and specifying the order of the line that best fits the plot. Since the contribution of the stability derivative C_m to the system response is the largest of the three longitudinal stability derivatives, the SVs of the uncertainty blocks and the ordered estimate of the uncertainty blocks for C_m is shown in Figure 7.3:

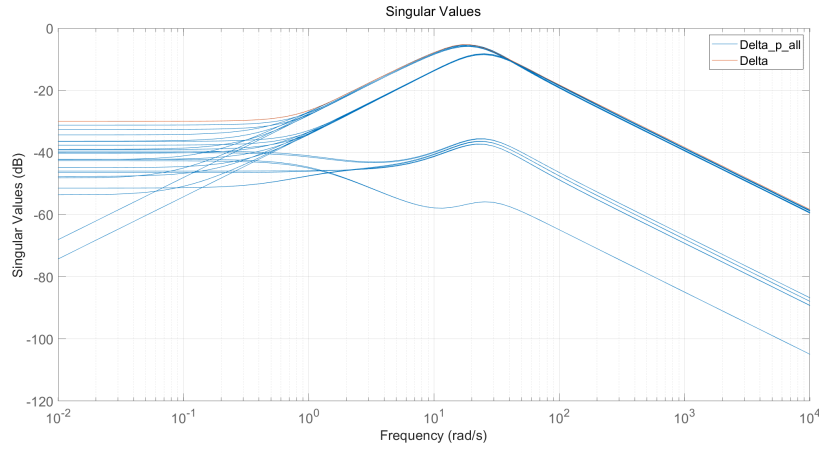


Figure 7.3: SVs of NCF uncertainty blocks of C_m

7.2.2. Significance of NCF Uncertainty Magnitude

The zpk TF that has been fit to the largest magnitude of SVs present for the uncertainty blocks of coefficient C_m is given in Equation 7.4:

$$\Delta(s) = \frac{12.154(s + 10.69)(s^2 + 1.639s + 0.8515)}{(s + 8.417)(s + 1.218)(s^2 + 23.33s + 339.9)} \quad (7.4)$$

The peak value on the NCF uncertainty block plot represent the magnitude of the maximum robustness margin ϵ , as measured by the H_∞ norm, that is guaranteed by the later designed NCF robust controller. The maximum SV present in the uncertainty blocks of coefficient C_m is given by Equation 7.5:

$$\|H_\infty\|_\infty < \epsilon = 0.5445 = -5.28 \text{ dB} \quad (7.5)$$

7.3. H_∞ Loop Shaping Design: 1DOF

The first stage of the FCS design process will entail the formulation of a 1DOF controller. This is done as a preliminary attempt to formulate a suitable robustifying controller before placing demands on reference model following as in the 2DOF case. This will provide a solid foundation in terms of the identification of suitable weighting filters (WFs), as well as familiarisation with the robust controller formulation techniques available in Matlab. This section will detail the WF selection process, followed by a comparison of the performance of various Matlab functions, including `ncfsyn` and `ncftune`, to the direct solving of the AREs.

7.3.1. Weighting Filter Optimisation

Firstly the plant G to robustify is selected, which is the airframe after pitch rate damping has been applied, and the TF from elevator input δ_e to C^* output. It is common practice to begin with filters as simple as possible as these are the easiest to implement and usually already yield satisfactory results. Therefore, the preliminary choice of WFs was according to Equation 7.6:

$$W_1 = \frac{C}{s} \quad W_2 = 1 \quad (7.6)$$

Where C is a scalar constant. Where filter W_1 is used to set the desired open loop gain and bandwidth, W_2 shapes the output complementary sensitivity and is usually used to penalise HF gain in the shaped plant. This has the largest influence with respect to reducing sensor noise and unmodelled HF dynamics. As sensor noise will not be treated, W_2 is set equal to 1 for design simplicity reasons. The choice of the integrator constant in W_1 has a direct influence on the crossover-frequency ω_{CO} and also LF gain, as well as the associated robustness level ϵ .

For the nominal plant model corresponding to the unshaped C^* channel, the crossover frequency is equal to 3.462 rad/s. The effect of varying the WF integrator constant on the performance level, robustness level and crossover frequency are shown in Figure 7.4, Figure 7.5 and Figure 7.6 respectively. By implementing an integrator as WF for W_1 , the SVs of the OL are increased at LFs and lowered at HFs. This is visible as G_s , the shaped plant, in Figure 7.7. It can be seen that increasing the integrator constant increases the crossover frequency, and also decreases the robustness level ϵ . It can also be noted that for this plant, increasing the bandwidth of the controller does not have a significant impact on ϵ .

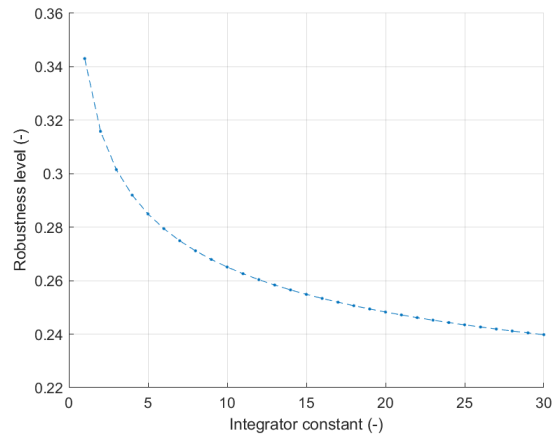
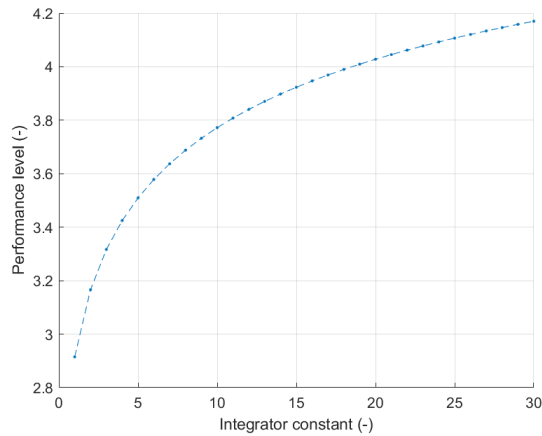


Figure 7.4: Effect of increasing integrator constant on performance level γ **Figure 7.5:** Effect of increasing integrator constant on robustness level ϵ

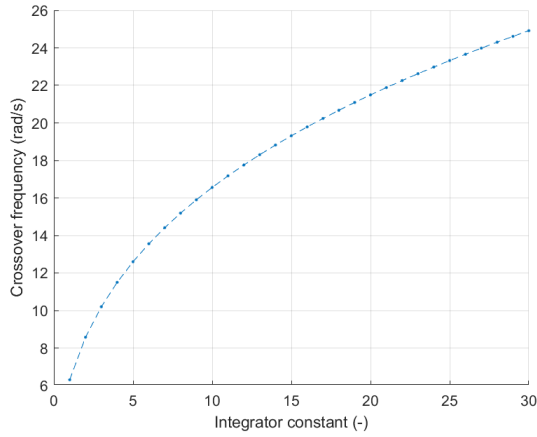


Figure 7.6: Effect of increasing integrator constant on crossover frequency

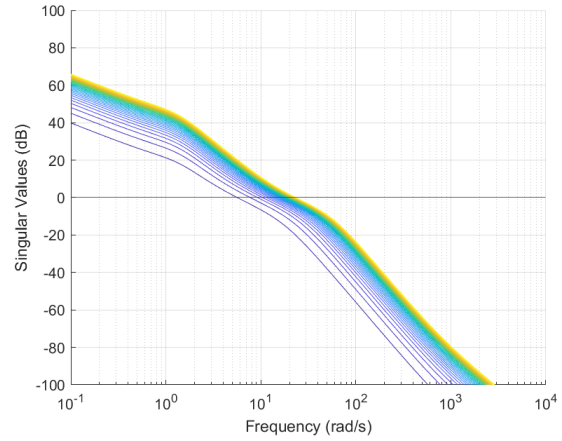


Figure 7.7: Effect of increasing integrator constant on the shaped OL SVs

With the LS technique, the crossover frequency can essentially be chosen. To do so, the -3 dB threshold of the reference model is obtained, as it is generally known as the gain required for satisfactory reference tracking. This frequency is found to be equal to 2.457 rad/s. The crossover frequency of the shaped plant is set to be just above the -3 dB by setting C equal to 0.3 , which produces a shaped plant crossover frequency of 3.612 rad/s. Although greater than 1 rad/s higher, the crossover of the robustified shaped plant will be reduced further after robustification.

7.3.2. Robustifying Controller

After the WFs are decided, a robustifying controller is designed that will stabilise the set of perturbed plants. The robustifying controller can be formulated in a number of different ways, including solving the AREs directly, using built-in Matlab functions such as 'ncfsyn' or by using externally developed functions such as 'ncftune', which facilitates the Matlab built-in function 'systune'.

Solving the AREs

The robustifying controller can be obtained by solving the set of AREs directly in Matlab. Although requiring slightly more effort to set up than using built-in Matlab functions such as 'ncfsyn' and 'systune', it is a good way to verify that function implementations are executed successfully. The full order zpk of the controller obtained by solving the AREs directly is given by [Equation 7.7](#):

$$K_{ARE} = \frac{-3763.3(s + 13.67)(s^2 + 2.908s + 2.475)}{(s + 1656)(s + 1.229)(s^2 + 21.76s + 142.2)} \quad (7.7)$$

The robustness level associated with this controller is provided in [Table 7.1](#):

Maximum achievable robustness level ϵ_{max}	0.4032
Achieved robustness level ϵ	0.4028

Table 7.1: Maximum achievable and achieved robustness levels of controller obtained from solving AREs

The corresponding SV plot is shown in [Figure 7.8](#):

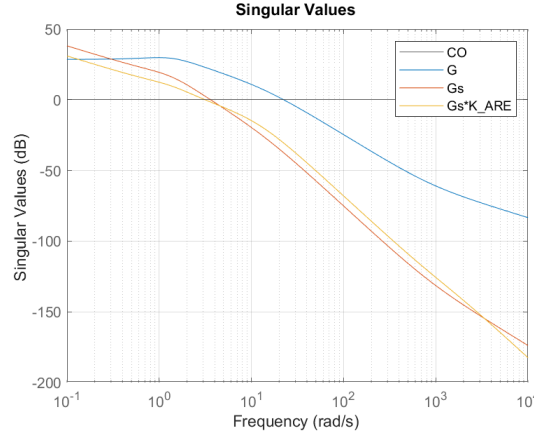


Figure 7.8: SV plot of C* open-loopgain with ARE derived robustifying controller

It is notable that there is a very large disparity between the original crossover frequency of the plant G and the low value of the -3 dB frequency of the reference model. This may lead to a significant trade-off between robust stability and robust performance later on in the robust controller design process. Additionally, the gradient of the SV plot around crossover for the robustified shaped plant is shallower than the gradient of the shaped plant itself. As highlighted in an earlier chapter, despite the reduction in gradient after robustification, the H_∞ robustification process ensures that the roll-off of the open-loopgain around crossover remains within acceptable limits of approximately -20 dB/dec. Lastly, the crossover of the robustified shaped plant occurs at a lower frequency than the crossover of the shaped plant. This is because the process must trade-off performance in terms of controller bandwidth for robust stability.

Ncfsyn

The 'ncfsyn' function is called using the following code:

```
[K_ncfsyn, CL_ncfsyn, gamma_ncfsyn, info_ncfsyn] = ncfsyn(-Gs, [], [],  
    tol_ncfsyn);
```

The shaped plant is input as negative as the function assumes positive feedback. 'tol_ncfsyn' is the specified tolerance on the performance level γ and how closely this approximates the optimal achievable performance level of the shaped system. The smaller this tolerance, the closer the final controller achieves its optimal performance. The output 'K_ncfsyn' is the robustifying controller obtained including the effect of the WFs W_1 and W_2 . The zpk system of the full order robust controller obtained from using ncfsyn is equal to [Equation 7.8](#):

$$K_{ncfsyn} = \frac{-3763.3(s + 13.67)(s^2 + 2.908s + 2.475)}{(s + 1656)(s + 1.229)(s^2 + 21.76s + 142.2)} \quad (7.8)$$

and the achieved robustness level, as well as the maximum achievable robustness level associated with the controller obtained using the function 'ncfsyn' are given in [Table 7.2](#):

Maximum achievable robustness level ϵ_{max}	0.4032
Achieved robustness level ϵ	0.4028

Table 7.2: Maximum achievable and achieved robustness levels of controller obtained from function 'ncfsyn'

It can be seen that this TF is identical to the FO controller obtained by solving the AREs, and therefore the implementation of the function 'ncfsyn' as well as the construction of the AREs have been cross-verified.

The achieved robustness level falls short of the calculated theoretical maximum achievable robustness level by 0.0004, and therefore it is clear that the value achieved by the full order ncfsyn controller is suboptimal. Furthermore, the value of the robustness level will naturally degrade further with reduced controller order, since the controller will no longer have direct influence on some frequencies constituting the controller bandwidth.

One disadvantage of using the function 'ncfsyn' to synthesise a robust controller is that it does not allow the user to specify a desired controller order. The function instead automatically constructs a controller of order equal to that of G_s . Furthermore, the controller that is obtained using 'ncfsyn' may result in a very high order controller that could be challenging to implement in practice on physical hardware. Therefore, it is interesting to consider the balanced reduction of the obtained controller order.

Model Reducer

The full order robust controller can be order reduced using balanced truncation using the function 'modelReducer' for instance. The reduction of the number of states of a TF involves the removal of poles and zeros which have the least contribution to the TF, and adjusting the DC gain accordingly. The contribution of each state to the TF is visualised by the Hankel norm and the bode diagrams of the system within the model_Reducer interface. With the model_Reducer options, a limiting bandwidth can be set up to which the reduced order systems should aim to match the full order system as closely as possible. As the bandwidth of the designed controller is not required to be very high, the upper bandwidth of the reduction is set to be a low value just above the required bandwidth of the controller, such that the model reducer does not unnecessarily struggle to match the full order controller dynamics at higher frequencies. In real world terms, this is also done to match the bandwidth of the actuators such that the actuators have sufficient control authority of the controlled aircraft plant.

The disadvantage to using model reducer is that it does not directly provide the performance level associated with the reduced order controller, as is the case with ncfsyn. Therefore, if one wishes to know the performance levels of the reduced order controllers, these must be manually calculated by placing the reduced controller in closed loop and obtaining the H_∞ norm of the configuration. This makes the use of this technique less functional than alternative methods which are directly able to do so.

Nonetheless, the bode diagrams corresponding to the original full order controller as well as the reduced controller orders are shown in Figure 7.9, and the SV plot of the shaped and robustified C^* open-loopgains corresponding to the ncfsyn and reduced ncfsyn controllers is shown in Figure 7.10:

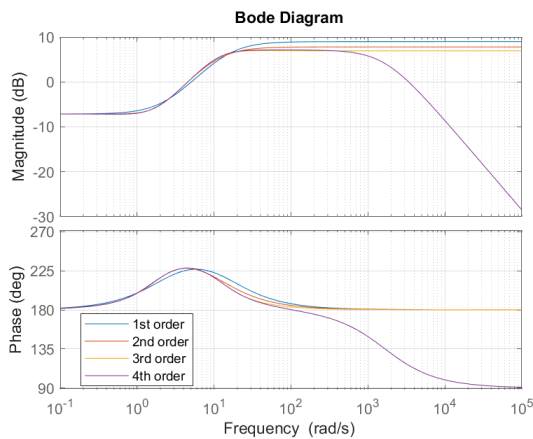


Figure 7.9: Bode diagram of full order and reduced order controllers obtained using the function 'ncfsyn'

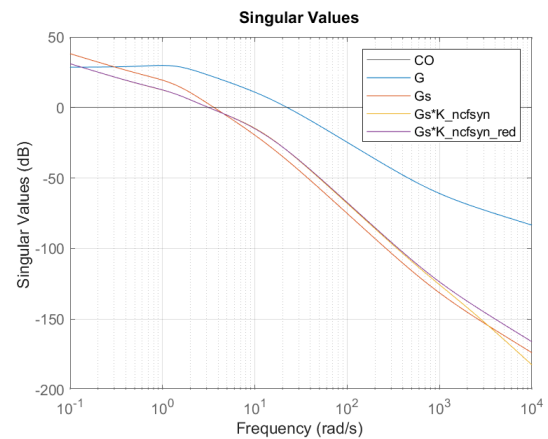


Figure 7.10: SV plot of C^* open-loopgain corresponding to full and reduced order ncfsyn controller

When looking at Figure 7.9, from the magnitude plot, there doesn't appear to be much discrepancy between controller orders up to a frequency of approximately 400 rad/s; the only apparent change between controller orders is the difference in magnitude around the mid to HFs. Until approximately 20 rad/s, it is

only the first order controller that deviates from the higher controller orders. When looking at the phase plot, the 1st order controller deviates the most significantly compared to higher controller orders. This is visible through a shift in the phase plot slightly towards the higher frequencies. It can be deduced that the first order controller is unsatisfactory in terms of modelling the true controller dynamics within the operating bandwidth. Subsequently, the higher order controllers are approximately equal in terms of bode appearance, aside for the 4th order controller at higher frequencies. It is evident that the 4th order controller contains a higher frequency pole which is negated for its minimal contribution to the reduced order controllers during balanced truncation. This pole is located at 1656 rad/s, as indicated in Equation 7.8. This pole is responsible for the tapering-off of the bode plot (in terms of magnitude and phase) around this frequency. To conclude, a second order controller is deemed the most fitting.

Figure 7.10 shows the SV plot of the original plant G , the shaped plant G_s , the robustified shaped plant $G_s \cdot K_{ncfsyn}$ and the reduced 2nd order robustified shaped plant $G_s \cdot K_{ncfsyn_{red}}$. The shape of the open loop gain of the reduced robustified plant evidently very closely approximates that of the full order controller, again deviating at a frequency of 1656 rad/s with a slight increase in magnitude after this frequency for the reduced controller.

Ncftune

The 'ncftune' function utilises systune and is able to obtain an optimal reduced order robust controller. In addition, it can provide the corresponding robustness levels pertaining to each of the controller orders specified, the first of the presented methods able to do this.

Firstly, a tunable parameter state-space block in the form of a 'tunableSS' is defined, representing the robustifying controller. Furthermore, the controller order and the number of controller inputs and outputs, as well as the optimisation tolerance of the performance level are defined. In this case the tolerance is set to 10^{-4} as this is deemed an adequately fine resolution. The 1st, 2nd, 3rd and 4th order controllers and associated performance levels are obtained and provided in Equation 7.9, Equation 7.10, Equation 7.11, Equation 7.12 and Table 7.3:

$$K_{ncftune_{O1}} = \frac{-2.4351(s + 1.915)}{(s + 11.36)} \quad (7.9)$$

$$K_{ncftune_{O2}} = \frac{-2.3069(s^2 + 3.413s + 3.903)}{(s + 2.202)(s + 9.433)} \quad (7.10)$$

$$K_{ncftune_{O3}} = \frac{-2.2694(s + 13.67)(s^2 + 2.909s + 2.476)}{(s + 1.229)(s^2 + 21.76s + 141.9)} \quad (7.11)$$

$$K_{ncftune_{O4}} = \frac{-2.2694(s + 13.67)(s + 8.749)(s^2 + 2.909s + 2.476)}{(s + 1.229)(s + 8.748)(s^2 + 21.76s + 141.9)} \quad (7.12)$$

	1st order	2nd order	3rd order	4th order
Robustness level ϵ	0.3800	0.3977	0.4032	0.4032

Table 7.3: Robustness levels associated with different orders of controllers obtained using the 'ncftune' function

Increasing the order of the controller increases the robustness level achieved from the robustification. Additionally, the correct implementation of ncftune is validated as the robustness level corresponding to the 3rd and 4th order controllers is equal to the maximum achievable robustness level ϵ_{max} as calculated using 'ncfsyn' in Table 7.2. This further proves that systune produces an optimal controller in the sense that it achieves the optimal theoretical maximum value of robustness level in the space of a relatively low controller order.

Figure 7.11 shows the bode diagram corresponding to each of the controller orders obtained using ncftune, and Figure 7.12 shows the C* open loop gains of each of the corresponding controller orders:

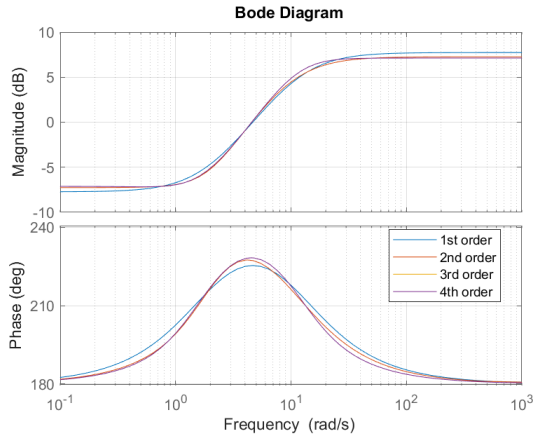


Figure 7.11: Bode diagram of ncfune obtained controller with varying order

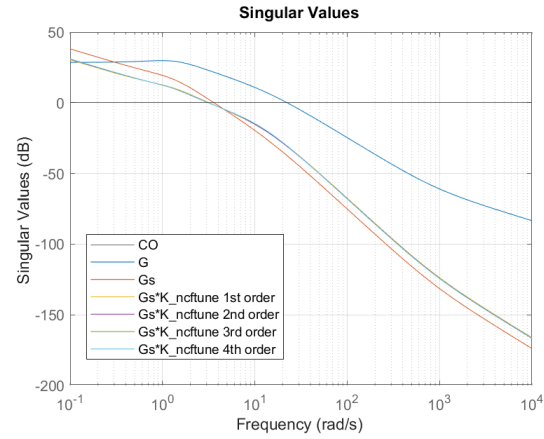


Figure 7.12: Bode diagram of C^* open-loop gains corresponding to ncfune controllers with varying order

Figure 7.11 shows that each of the controller orders closely approximate one another in terms of the shape of the magnitude and phase plots, that being except for a marginal deviation of the first order controller. This implies that it does not reflect the optimal controller as closely as higher controllers, which was earlier reflected in Table 7.3 by the robustness levels converging to the theoretical maximum with increasing order. However, it would be sufficient in this case to proceed with a first order controller for simplicity reasons, as the deviation in robustness level is incredibly small in the range of 10^{-2} . The minute difference in controller order is observed further in Figure 7.12, where it can be seen that there is almost no observable difference in the C^* open loop gain SVs over the entire frequency range.

To conclude, since the reduced ncfsyn controller and the ncfune controller are very similar in terms of frequency response, the ncfune controller which uses systune is chosen as the controller synthesis method to proceed with, due to the additional synthesis information that it provides.

7.3.3. Handling Quality Assessment

CAP Criteria

After implementation of the 1DOF ncfune controller into the controller architecture, its HQ performance was assessed. This started with the CAP criterion, shown in Figure 7.13. It can be noted that the CAP criterion falls comfortably within the level 1 HQ criteria region, meaning that its load factor response is very good and even better than that associated with the reference model. Both the damping ratio and the CAP value are slightly increased in relation to the reference model, meaning that the 1DOF controlled system commands a higher load factor for pitch rate changes, and behaves more crisply and less sluggishly to pilot inputs than the reference model.

Gibson Criteria

Figure 7.14 shows the Gibson HQ criteria of the 1DOF controller design relative to the reference model. The 1DOF design exhibits substantially less dropback, as well as a slightly reduced maximum pitch rate response compared to the peak response. This is reflected by the slightly increased damping ratio shown in the CAP criterion diagram in Figure 7.13. It is fair to say however that it doesn't sufficiently meet the HQ requirements specified by the reference model, and for this, it is required that the design configuration be augmented with a feedforward controller to aid in reference model matching.

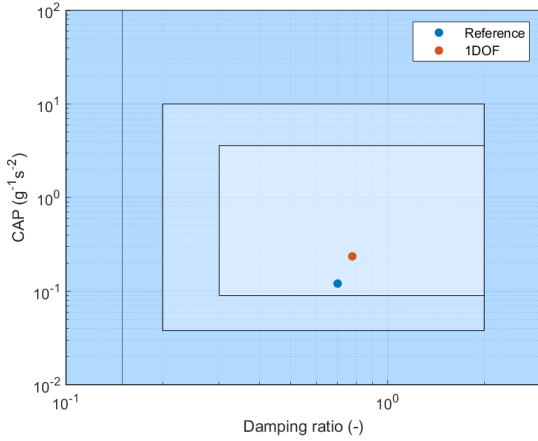


Figure 7.13: CAP criterion of 1DOF feedback controller configuration

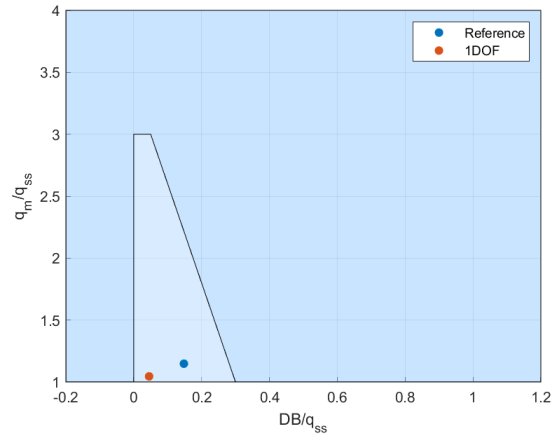


Figure 7.14: Gibson criterion of 1DOF feedback controller configuration

Attitude Bandwidth Criteria

In the calculation of the attitude bandwidth criterion, it is first required that the classical gain margin (GM) and phase margin (PM) of the response be found; this is indicated in [Figure 7.15](#):

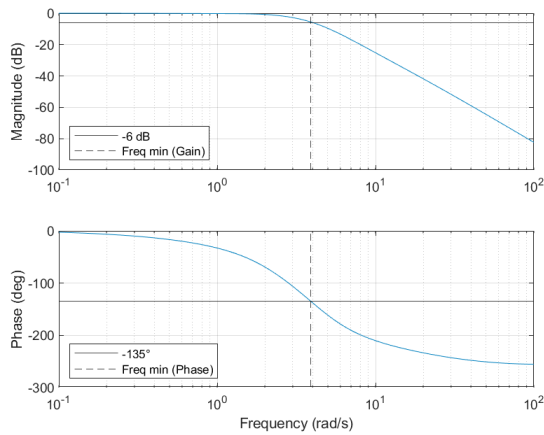


Figure 7.15: Bode diagram magnitude and phase plots of the $C*P_S$ channel, with GM = 6 dB and PM = 45 deg indicated

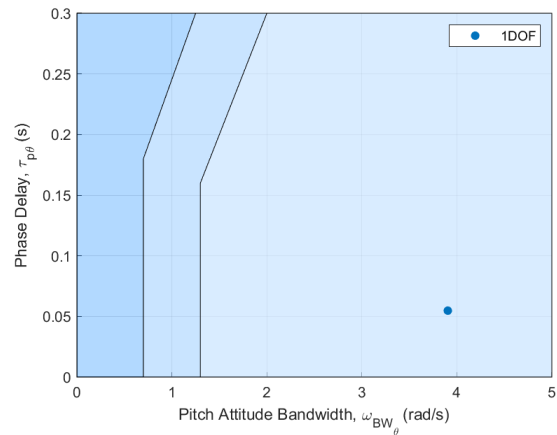


Figure 7.16: Phase delay HQ criterion corresponding to 1DOF controller design configuration

The frequencies at which these occur are found to be $\omega_{GM} = 4.0421$ rad/s and $\omega_{PM} = 3.9047$ rad/s respectively. The lower of the two, thus $\omega_{PM} = 3.9047$ rad/s, is then identified as the pitch attitude bandwidth, ω_{BW_θ} for use in the attitude bandwidth criteria. The associated phase delay is calculated using [Equation 7.13](#)

$$\tau_p = \frac{\Delta\Phi 2\omega_{180}}{57.3(2\omega_{180})} \quad (7.13)$$

where ω_{180} is equal to the frequency at -180° of phase. The resulting criteria given by [Figure 7.16](#) shows that the phase delay of the 1DOF design configuration lays comfortably within the level 1 HQ region, with ample margin before reaching level 2 HQ performance. The pitch attitude bandwidth is relatively high at 3.9047 rad/s; however, the low value for the phase delay indicates that there is minimal lag between the pilot input and the aircraft response.

Flight Path Bandwidth Criteria

The first stage in the assessment of the flight path angle (FPA) bandwidth criteria is the isolation of the flight path response from the aircraft dynamics. This was done by adding the γ output to the aircraft model state-space dynamics and linearising between the C^* commanded input and the flight path, γ , output. The step response a result of the 1DOF controller design configuration is shown in Figure 7.17.

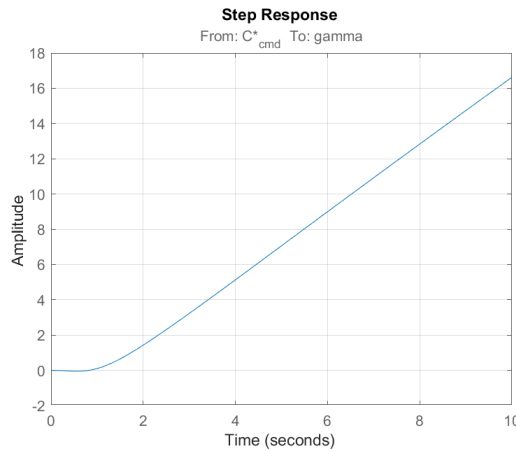


Figure 7.17: Step response of the flight path angle in response to a C^* commanded input of magnitude 1

It can be seen that the FPA diverges to a continuously greater value. This makes sense, seeing as a commanded C^* input in the linear controller of any magnitude greater than 0, can be effectively seen as a constant load factor command, causing the aircraft to continue to pitch up, aiming to maintain the commanded normal load factor. This in turn continuously increases the flight path angle, until in practical cases, the aircraft reaches stall.

The bode diagram corresponding to the transfer, as well as the flight path bandwidth criterion are shown in Figure 7.18 and Figure 7.19, respectively:

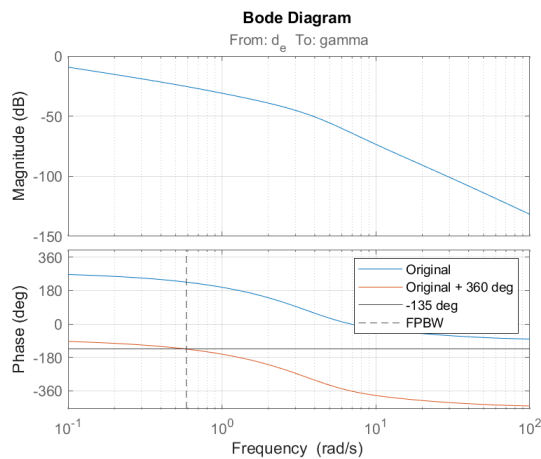


Figure 7.18: Bode diagram corresponding to the FPA with $PM = 45^\circ$ indicated

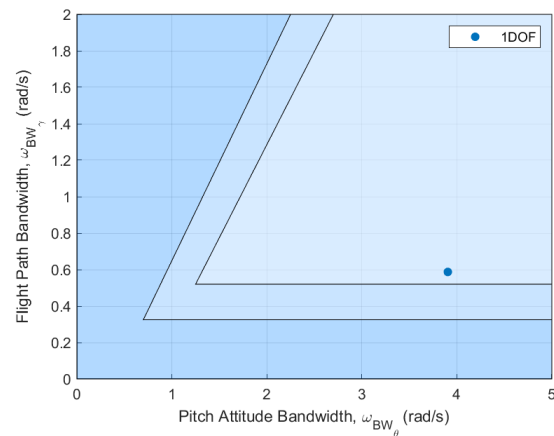


Figure 7.19: Flight path bandwidth criteria corresponding to the 1DOF controller

From Figure 7.18, it can be seen that the phase plot is shifted upwards such that the phase crosses 0° at a very high bandwidth. Since this is not realistic, the phase plot is shifted downwards by one full cycle of 360° such that it crosses 0° at a far lower frequency. This value is equal to the flight path bandwidth ω_{BW_γ} , and has a value of 0.5872 rad/s .

The location of the point on Figure 7.19 indicates that the pitch attitude response is a lot faster than the flight path response. However, the ratio between the two bandwidth is still satisfactory since the point lays

just within the level 1 HQ limits for this criterion. Further still, this point can even degrade to level 2 HQ performance in this criteria, due to the fact that the designed controller is a C^* controller, and therefore, the FPA is not the primary means of control.

Concluding the HQ performance of the 1DOF controller, it is fair to say that it is a decent controller in that it meets level 1 HQ requirements for all relevant criteria. However, seeing that the aim of the study is to produce a controller that more closely approximates the selected reference model, it will be necessary to add a tracking controller to the design.

7.3.4. Frequency Domain Analysis

This section will entail the frequency domain aspect of the 1DOF controller, to assess how it performs in terms of classical gain and phase margins, as well as disk gain, phase and multiloop margins, the appearance of the regions of stability and frequency-domain based plots such as Nyquist and Nichols plots.

The airframe is connected with the designed controller in negative feedback in order to assess its frequency domain performance. A controller placed in feedback with the airframe takes the form of [Equation 7.14](#):

$$C = \begin{bmatrix} W_1 \cdot K \cdot W_2 \\ C_q \end{bmatrix} \quad (7.14)$$

The second component of the controller C corresponding to the controller of the pitch rate loop, C_q is the previously found gain that stabilises the pitch rate inner loop.

There are a number of ways to calculate classical margins (CMs), be it using Matlab or Simulink. Using Matlab, this can be done using the function 'Loopsens'. For a plant P with a controller C placed in negative feedback, loopsens is used to calculate the multivariable sensitivity, complementary sensitivity and open loop TFs of the closed loop. It is calculated with 'loops = loopsens(P,C)', and it essentially opens the loop at a particular point within the control loop.

For the input open loop TF L_i , the Matlab function effectively opens the loop at the plant input. Conventionally, the input loopgain L_i is given by CP . Conversely, the open loop TF L_o is given by PC and the loop is effectively opened at the plant output. It is important to note that for SIMO systems, loopsens opens all loops concurrently, meaning that if one output loopgain is desired in isolation, then the other loops need to be closed using feedback. This is illustrated graphically in [Figure 7.20](#):

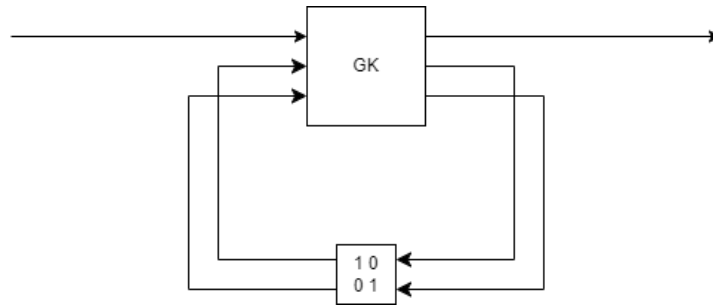


Figure 7.20: Closed loop feedback configuration

Closing the loops using feedback is shown in [Equation 7.15](#) and [Equation 7.16](#):

$$\text{Lo1_temp} = \text{feedback}(G^*C, 1, 2, 2, -1) \quad (7.15)$$

$$\text{Lo2_temp} = \text{feedback}(G^*C, 1, 1, 1, -1) \quad (7.16)$$

The resulting loopgains contain the output loopgains corresponding to all of the relevant plant outputs. Therefore, the desired output loopgain still needs to be selected, and this is done for both output channels in Equation 7.18 and Equation 7.19:

$$Li1 = \text{zpk}(\text{loops}.Li) \quad (7.17)$$

$$Lo1 = \text{zpk}(Lo1_temp(1,1)) \quad (7.18)$$

$$Lo2 = \text{zpk}(Lo2_temp(2,2)) \quad (7.19)$$

From the 'loops' output of the function 'loopsens', it is possible to directly derive the sensitivity functions and complimentary sensitivity functions.

There is however a limitation to using this approach for the application of the C^* controller. This stems from the fact that the function 'loopsens' assumes that the outputs of the plant G and therefore the product ' GK ' will be placed in direct feedback, as depicted by Figure 7.20. However, the outputs of the airframe are n_z and q , and not the channels desired for feedback including C^* . A work around is to augment the plant with the C^* output. However, it was decided not to do this to preserve the SP model structure and allow more flexibility between the conversions between load factor measurements at the PS or the ICR, which can be easily implemented in a Simulink subsystem block.

Gang of Six

The gang of six (GO6) is used to study each of the important signal transfers such as the sensitivity and complementary sensitivity. It is shown for the 1DOF design in Figure 7.21:

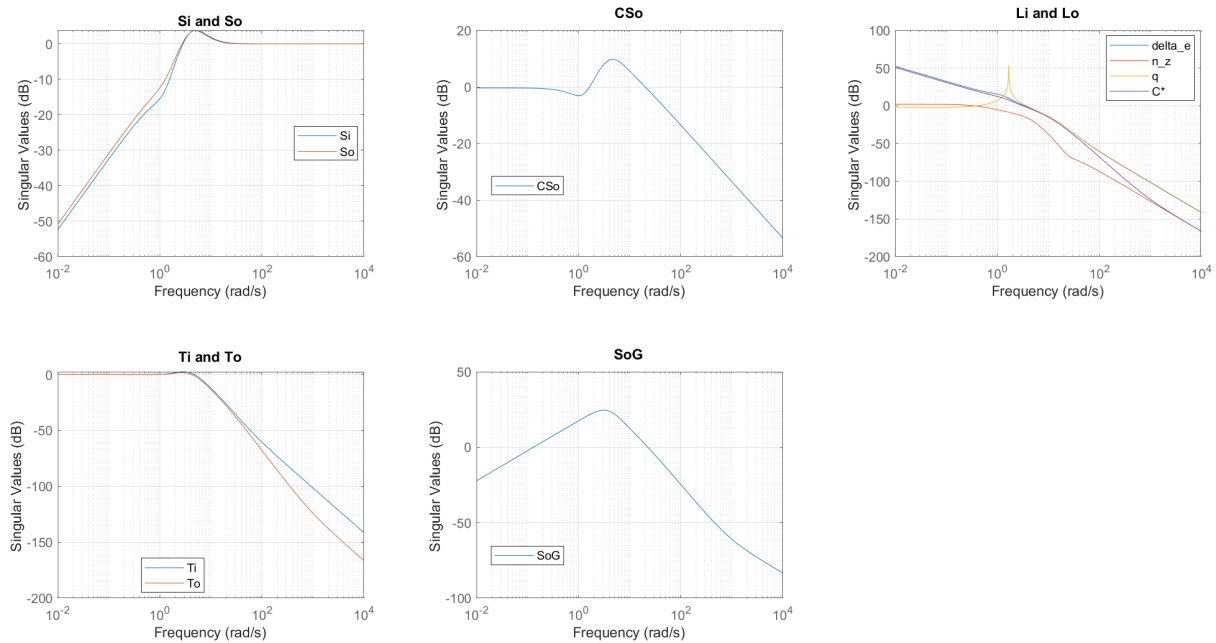


Figure 7.21: GO6 TF SV plots corresponding to 1DOF design

First of all, the input and output sensitivity functions S_i and S_o show an expected shape of low gain at LF and unit gain at HF, stabilising at 0 dB and indicating good disturbance rejection characteristics. The peak in the signal occurring near the crossover frequency is not excessive in magnitude, which shows reasonable robustness. It can be concluded that input and output disturbances are attenuated at both the plant input and output, respectively.

This is mirrored to the complementary sensitivity functions T_i and T_o , which show the opposite behaviour. This is expected due to the fact that $S(s) + T(s) = 1$. The steep roll-off of the complementary sensitivity indicates that the controlled system is aptly able to attenuate sensor noise; the control signal is attenuated from input disturbances, T_i , and noise is attenuated at the plant output T_o .

CS_o , being an indicator of controller effort, shows a peak in magnitude around the crossover frequency. This peak is closely tied with the LS WF applied, W_1 , since increasing the crossover frequency (and thus the bandwidth) with W_1 commands greater actuator effort at these higher frequencies. The steep roll-off after crossover indicates the controller's lack of bandwidth and therefore incapability to match higher frequency signals without exhibiting actuator rate and position saturation.

The TF S_oG shows low magnitude SVs at LF, as it is approximately equal to the inverse SV of the feedback controller. This is due to the large control feedback gains of the controller which are introduced through the application of the integrator.

Looking at the input and output OL gain plot, it is visible that there is high gain at LF and low gain at HF for the C* signal, which is known to benefit disturbance rejection properties. The n_z and q signals do not show high gain at LF, as they do not contain integral action, as only the C* channel was robustified using LS.

As well as visually analysing the GO6 SV performance, some transfers can be used to provide crucial performance information. The first of this is the maximum actuator rate allowed for the elevator. This is expressed according to Equation 7.20:

$$\dot{\delta}_{e_{max}} = \max(\text{step}(CS_{o\delta_e} \cdot G_{act} \cdot s \cdot \frac{180}{\pi})) \quad (7.20)$$

where $CS_{o\delta_e}$ is the unscaled TF from the output disturbance d_o to the controller input u , G_{act} is the TF corresponding to the elevator actuator and s is the Laplace derivative operator. This expression uses the peak of the control actuation SVs to calculate the theoretical maximum elevator rate achieved by the controlled system. Doing so for the 1DOF case, the maximum rate becomes $\dot{\delta}_{e_{max}} = 18.6018 \text{ deg/s}$. This shows that control under the 1DOF controller produces controller effort that lays comfortably within the defined allowable limits, prescribed by Table 5.1.

The output complementary sensitivity function is also able to provide the settling time of the C* step response to within 5% of final value, which is found to be equal to $t_{st} = 1.3019 \text{ s}$. This is a very low value, especially when compared to the reference model settling time of $t_{s_{ref}} = 3.09 \text{ s}$, which prematurely indicates before time-domain simulation analysis that the 1DOF controlled C* response settles far more quickly than desired.

Classical Margins

This CMs were calculated for all of the relevant channels, including the input channel δ_e and the output channels n_z , q , C*. The results are as tabulated in Table 7.4. Each of the channels have healthy classical GMs and PMs. Recalling once again that the recommended classical GMs should exceed 6 dB GM, all of channels surpass this with values ranging from 16.73 dB to 20.75 dB, far higher than typically required for stability. The PMs come closer to the recommended value of 45° PM, with the lowest PM corresponding to the input δ_e channel. It is allowable for the achieved CMs to slightly undershoot the recommended guidelines, so therefore, this marginal difference is tolerated. It is clear that the output n_z channel exhibits the largest region of stability in terms of CMs due to the very large PM of 138.09° , rendering it very stable for individual perturbations.

	Channel			
	δ_e	n_z	q	C*
Gain Margin (dB)	19.9202	20.0932	20.7512	16.7250
Phase Margin (deg)	45.6191	138.0929	48.991	49.6204

Table 7.4: Tabular results of the classical gain and phase margins of each of the relevant channels under 1DOF control

Disk Margins

The achieved disk margins will almost certainly be lower than CMs, due to systems' general tendency to be more sensitive to simultaneous variation. The LAAT DMs for the 1DOF control design are given by [Table 7.5](#):

	Channel			
	δ_e	n_z	q	C*
Disk Gain Margin (dB)	7.5120	16.4814	8.2049	7.9652
Disk Phase Margin (deg)	44.3267	72.9447	47.5055	46.4264
Disk Margin (dB)	0.8147	1.4784	0.8801	0.8577

Table 7.5: LAAT DGM, DPM and DM for all relevant channels corresponding to the 1DOF design

The DGMs and DPMs are noticeably reduced compared to the CMs. The GM is reduced across the board, for each of the channels. This is mostly the case for the q signal, reducing from 20.75 dB to 8.20 dB. All GMs however remain above the 6 dB recommended threshold by a suitable margin. The difference in PM is most noticeable in the n_z channel, where the PM has reduced from 138.09° to 72.94° , highlighting the magnitude of sensitivity of the n_z channel to simultaneous variation.

Multiloop Disk Margins

DMs fail to encompass simultaneous variation of gain and phase in all channels at once, and this is achieved by the multiloop DM. Beginning with the control design configuration, loop transfer points are added to the input δ_e and the plant outputs n_z and q . These are the channels that the variation will be applied to. A loop transfer point is not added to the C* output channel, since C* is a direct amalgamation of n_z and q signals, and therefore the effects of gain and phase variation in these two channels will directly influence the outcome of the C* channel. The multiloop margin is then found in the following manner:

```
io = getlinio mdl
MM_sim = linearize(mdl, io)
[DM, MM] = diskmargin(MM_sim)
```

The multiloop margin is contained in the MM parameter. The results are summarised in [Table 7.6](#):

Multiloop Gain Margin (dB)	0.6859	1.4579
Multiloop Phase Margin (deg)	-21.1052	21.1052
Multiloop Disk Margin (dB)	0.3726	

Table 7.6: Multiloop margin representing simultaneous gain and phase variations over all relevant channels at once for the 1DOF design configuration

Once again, the multiloop margin is significantly lower than the DMs and CMs previously obtained. This is unsurprising, since the system would need to remain stable whilst under a large influence of many combined perturbations. Additionally, there are relatively little existing guidelines that concern case appropriate multiloop margins, as is the case for CMs and DMs. Therefore, for the purpose of this study, the resulting multiloop margins are deemed satisfactory; however, this could warrant further investigation.

Nyquist Diagrams

In addition to looking at DMs, it is useful to look at the Nyquist plot; the Nyquist plot shows the locus of the loop TF $L(s)$ of a system, and can be useful to understand the proximity of this system to instability, especially in combination with the allowable DM. The Nyquist plot for all channels is shown in [Figure 7.22](#) and the Nyquist plot of the C* channel is shown in [Figure 7.23](#):

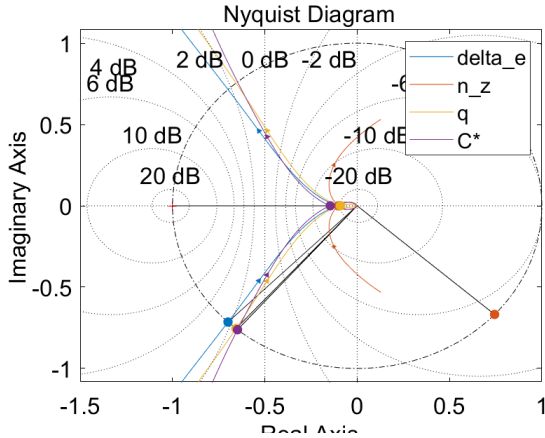


Figure 7.22: Nyquist diagram of all channels superimposed with one another under the 1DOF control design

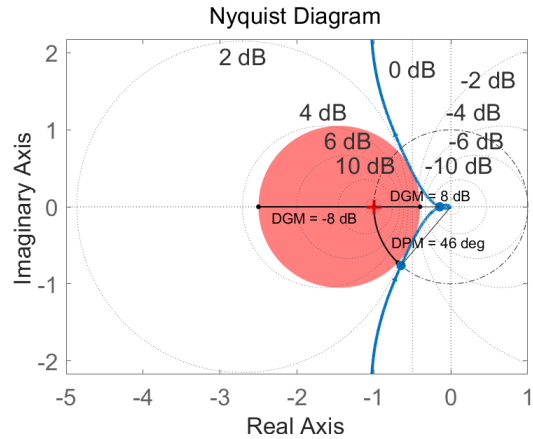


Figure 7.23: Nyquist diagram of C^* channel under the 1DOF control design

Studying Figure 7.22 shows the similarity of all of the channels to one another, except for the n_z channel. The n_z channel shows extremely high stability margins, reflected both in its high DM properties and also through the tendency of its Nyquist plot to diverge quickly away from the critical instability point. Figure 7.23 shows the Nyquist diagram of the C^* loop alone, with the corresponding DM superimposed. The diagram shows a healthy DM that satisfies the set stability requirements, albeit with not too great of a disk phase margin.

Nichols Diagrams

The Nichols diagram corresponding to the C^* response is shown in Figure 7.24:

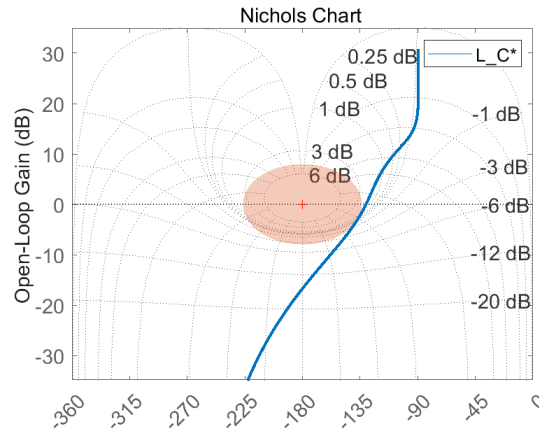


Figure 7.24: Nichols diagram corresponding to the C^* channel for the 1DOF design configuration

It shows the variation of the C^* open loop gain with the phase. The critical point to avoid is the combination of 0 dB gain with -180° of phase. The DM indicates the proximity of the open-loop gain to this critical point in terms of the gain and phase. It can be seen in Figure 7.24 that the correct calculation of the DM is further verified by the matching of the disk size with the contact location of the loopgain of the C^* channel. The C^* loopgain is well behaved as it follows a relatively nominal profile and never approaches the critical point.

7.3.5. Time-Domain Analysis

In robust control, it is usually frequency domain analysis that more importantly proceeds time-domain analysis. However, time-domain performance should also be sufficient for a controller to be adopted into application.

Comparison of Controller Forward and Feedback Configurations

The obtained controller can be placed both in the forward path and the feedback path. [Figure 7.25](#) shows the comparison between the two configurations for the C^* signal:

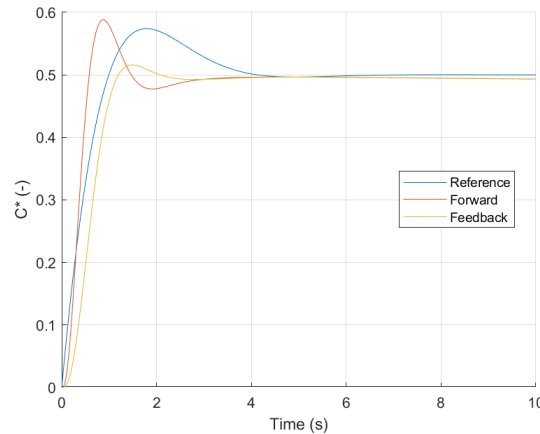


Figure 7.25: Time-domain C^* step response comparing the 1DOF controller in the forward and the feedback configuration

Looking at [Figure 7.25](#), it can be seen that the controller placed in the forward configuration exhibits far greater overshoot and a greater peak value than in the feedback configuration. Although the peak response of the forward configuration is comparable with the reference model, the signal very quickly damped and is more oscillatory, thus it shows more aggressive handling. The feedback configuration response on the other hand greatly undershoots the reference model and is damped far more. It does however show a much more smooth response, despite the two responses having similar settling times, which is attractive in terms of handling quality. It is in the end decided to opt for the feedback configuration, as this produces a smoother response that more closely represents that of the reference model. However, the disparity between the peak value and dropback between the feedback response and the reference model will need to be adjusted with a feedforward tracking controller.

Comparison of Accelerometer Positions

It is also beneficial to analyse the difference in time-domain responses caused by different accelerometer placements to further consolidate the decision. [Figure 7.26](#) and [Figure 7.27](#) show the time-domain simulations corresponding to the n_z and C^* responses respectively, comparing the feedback of C^* with the accelerometer placed at the ICR and at the PS.

The difference between the ICR and the PS response is very clear in [Figure 7.26](#); the PS response shows a noticeable bump in the transient response while reaching the maximum value before meeting the ICR response, whereas the ICR response is very smooth throughout, which confirms the findings presented in [Section 7.1.2](#). It is therefore clear that in terms of feedback, the signal as measured at the ICR should be used for feedback of n_z .

The difference between the time responses at the ICR and the PS are less distinguishable from one another for the C^* signal. When looking at [Figure 7.27](#), the ICR response is slightly delayed compared to the PS response, and also shows a little higher overshoot. Although the difference in time response in this case is minimal, the additional delay of the ICR implies that the PS response will be more fitting for C^* HQ performance analysis. This conclusion follows from [Section 7.1.3](#), where it was decided that the C^* measurement at the PS will be used for HQ assessment.

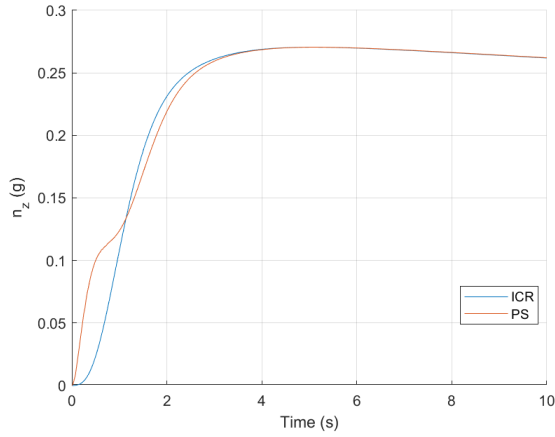


Figure 7.26: Time-domain response comparing the n_z step response for the accelerometer position located in the PS and in the ICR for the 1DOF design configuration

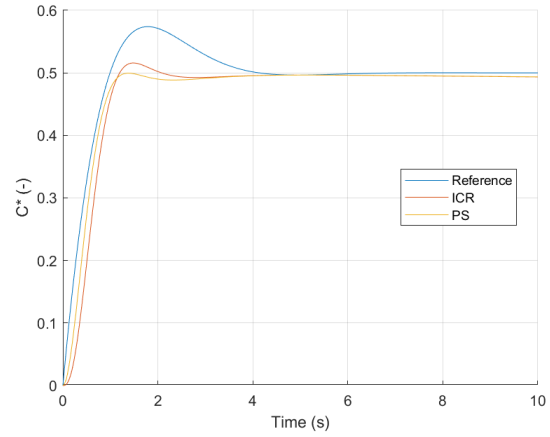


Figure 7.27: Time-domain response comparing the C^* step response for the accelerometer position located in the PS and in the ICR for the 1DOF design configuration

7.4. H_∞ Loop Shaping Design: 2DOF

After completing the 1DOF design, it was decided that a feedforward controller must be added to provide better performance in terms of tracking. This is done through a two degree-of-freedom (2DOF) controller. In this scenario, the 1DOF design configuration is augmented with a tracking requirement, and a feedforward controller is synthesised. This section will outline all of the steps that were executed in order to proceed with the 2DOF controller design, and present and analyse the results that were achieved.

7.4.1. Preliminary Weighting Filter Selection

It was decided that since the 1DOF design produced good frequency domain results such as the shaping of the open-loop gain, that the WFs chosen would be a good preliminary choice for the 2DOF design. Therefore once again, the chosen WFs for the 2DOF design are given by Equation 7.21:

$$W_1 = \frac{0.3}{s} \quad W_2 = 1 \quad (7.21)$$

Depending on resulting time and frequency based characteristics, these WFs can be subject to change later in the design process in order to amend the design. With these WFs, the crossover frequency of the shaped loop once again becomes $\omega_{Gs} = 3.612 \text{ rad/s}$.

7.4.2. Augmentation of Standard 1DOF LSDP Setup

To augment the standard 1DOF LSDP configuration into the 2DOF configuration, an additional performance signal e_M is added. Additionally, a feedforward controller K_{ffd} is introduced on the path of the reference signal, thus the commanded input that the controller receives. This joins the pre-existing pitch rate feedback IL gain and the feedback controller in the OL. The latter is denoted in NCF robust stability as ϕ . The augmented 1DOF design configuration is depicted in Figure 7.28.

Here, $G_{CqCstar}$ is the system corresponding to the C^* channel with IL pitch rate feedback already applied, W_1 is the pre-WF and T_{ref} is the reference model. Since W_2 is kept to 1, its presence in the setup is neglected. It can be seen that now the output of the plant $G_{CqCstar}$ is subtracted from the output of the reference model T_{ref} . This represents the model matching error signal, denoted at the exogenous output as z_{et} . The parameter FF_{Rho} , is a WF that is applied both before and after the reference model.

7.4.3. Optimisation of FF_{Rho}

The parameter FF_{Rho} , or FF_ρ , is introduced as a design technique which is used to scale the importance of the model matching constraint by minimising the model matching error output signal z_{et} . This allows

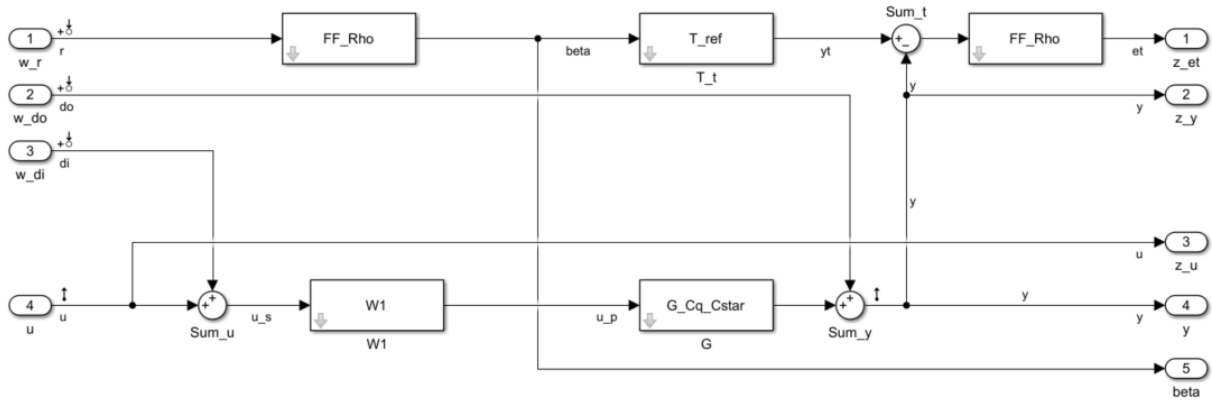


Figure 7.28: 1DOF standard design configuration augmented with additional performance signal e_M for tracking in 2DOF controller design

the designer to adjust between robust stability and robust performance. For instance, when FF_{Rho} is set equal to zero, the configuration models the standard NCF robust stabilisation problem. The WF can adopt different structures; it can simply be a scalar gain, thus a static filter, or it could be a TF. This is what is known as a dynamic filter, and it can be used in cases where the model matching performance requires more fine tuning. For the purpose of this study and for simplicity, a preliminary scalar gain WF will be used here, however, the value that it assumes will be investigated.

In order to begin to study the effects of the WF, it was required that the TF corresponding to the transfer between the reference input and the exogenous performance output z_{et} be obtained. Firstly, the configuration shown in Figure 7.28 was linearised. This produced the linearised set of TFs corresponding to each of the input/output transfers. Then, the set of tunable feedforward and feedback controllers was created using the Matlab 'tunableTF' objects. The tunableTF function allows the designer to manually select the number of input and outputs to the TF as arguments, which thus determines the TF order. Once these were defined, the tunable controller was constructed and the tunable closed loop was obtained using the Matlab function 'lft', where the linearised system and the set of tunable controllers were provided as arguments. Subsequently, the TF corresponding to the model matching error was isolated. This was repeated for varying values of the FF_{Rho} parameter, since its effect would be visible in the linearised system. The loss in robustness level with increasing ρ is shown clearly in Figure 7.29:

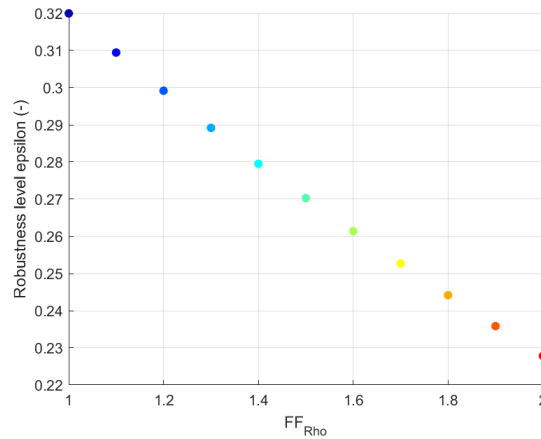


Figure 7.29: Robustness levels corresponding to the varied values of the RS/RP tuning parameter ρ

The reduction in robustness level ϵ shows an almost exact negative linear trend. However, seeing as a requirement for H_∞ -LS design is that the robustness level associated with a robust controller should stay above a value of 0.25, not all values of ρ are available to the designer. Values of ρ of 1.8 and greater

cannot be used in this design. Depending on the value, this means also that the signal may not very tightly approximate the reference, and this trade-off could become a limiting factor in the design. For these reasons, it is also helpful to look at the time-domain response in order to select an appropriate value for the WF ρ . Figure 7.30 shows the step response of the C* signal for different values of ρ , and Figure 7.31 shows a zoomed in portion of the step response to show the tracking accuracy associated with each value:

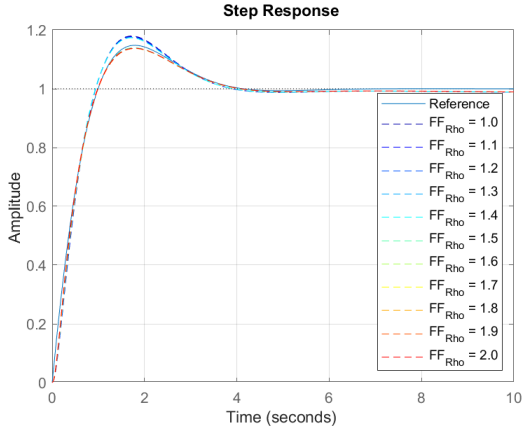


Figure 7.30: C* step response showing the effect of increasing the RP parameter ρ compared to the reference model

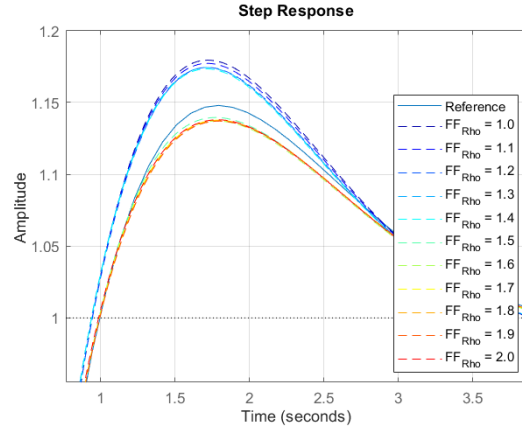


Figure 7.31: Zoomed in C* step response showing the effect of increasing the RP parameter ρ compared to the reference model

Looking at Figure 7.31, it can be seen that too low values of ρ cause the controlled response to overshoot the reference by some margin; the peak response is approximately 1.175, compared to that of reference of approximately 1.15. As the parameter ρ is increased, the peak response shifts downwards, below but very close to the reference. It appears that increasing ρ further continues to shift the response marginally downwards. Therefore, it was decided to set $\rho = 1.4$ in the weighting factor, as this produces a peak response of approximately 1.14, the difference to the reference being equal to 0.87%. Once this was decided, the controller was regenerated by re-calling systune. This produced various controllers of orders between 1 and 4, since these were deemed the most appropriate for easy application.

Frequency-Domain Based Comparison of Controller Orders

The bode diagrams corresponding to the feedforward and feedback controllers C_{ffd} and C_{fbk} are shown in Figure 7.32 and Figure 7.33 respectively:

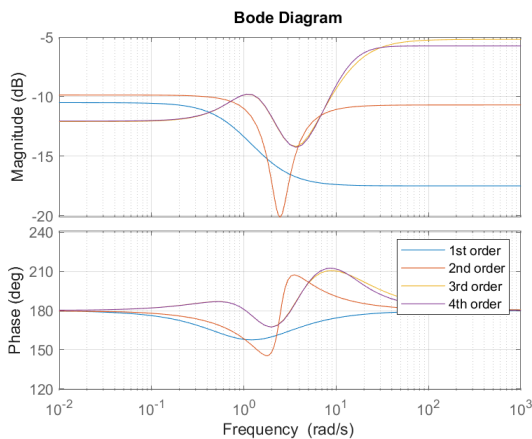


Figure 7.32: Bode diagram of different orders of feedforward controller C_{ffd} with $\rho = 1.4$, using systune

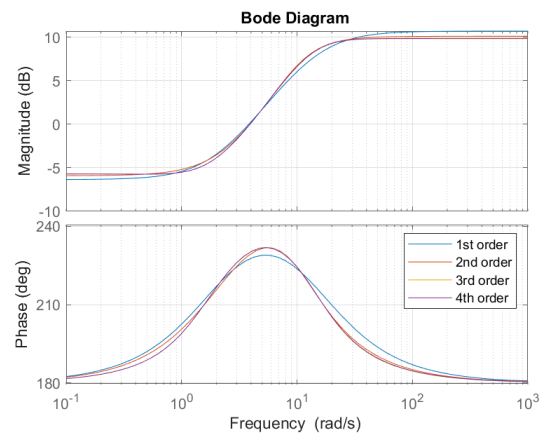


Figure 7.33: Bode diagram of different orders of feedback controller C_{fbk} with $\rho = 1.4$, using systune

Figure 7.32 shows that the bode diagram of each of the orders of controller are rather different to each other until the order reaches 3rd order. This indicates that the 1st and 2nd order TFs have Hankel norms that are too small to adequately represent the higher-order system, and therefore, these two should be discarded. There is little noticeable difference between 3rd and 4th order until the higher frequencies, around 10 rad/s, where the lines begin to diverge; on the magnitude plot, this is seen by a slightly lower magnitude before this frequency and a slightly higher magnitude after this frequency. On the phase plot, this is seen by a slightly increased phase after this frequency. However, since only lower frequencies below 10 rad/s are being considered during the controller design, the controller behaviour above 10 rad/s does not have very much impact. Therefore, it is decided that a 3rd order controller will be used for the feedforward controller C_{ffd} as this order is sufficient and increasing the order further has minimal impact. Looking at Figure 7.33 however, the bode diagrams for each order of controller C_{fbk} very closely approximate each other, with the small exception of the 1st order controller. Therefore, seeing as the higher order TFs appear uniform in relation to one another, a 2nd order controller is selected for the feedback controller C_{fbk} , since this is approximately equal to the higher orders in terms of bode appearance and is still easily implementable.

Given that the orders of controllers used have been selected, the resulting feedforward and feedback controllers are given by Equation 7.22 and Equation 7.23 respectively:

$$C_{ffd} = \frac{-0.55027(s + 0.9325)(s^2 + 4.498s + 11.96)}{(s + 12.48)(s^2 + 1.985s + 1.979)} \quad (7.22)$$

$$C_{fbk} = \frac{-3.2078(s + 7.925)(s + 2.424)}{(s^2 + 19.24s + 121.6)} \quad (7.23)$$

The pole and zero locations for both controllers are summarised in Table 7.7:

	C_{ffd}	C_{fbk}
Zero	-2.2489 + 2.6265i	-7.9251
	-2.2489 - 2.6265i	-2.4243
	-0.9325	
Pole	-0.9926 + 0.9966i	-9.6187 + 5.3961i
	-0.9926 - 0.9966i	-9.6187 - 5.3961i
	-12.4790	

Table 7.7: Summary of poles and zeros obtained for feedforward and feedback controllers from 2DOF systune design

Time-Domain Based Comparison of Controller Orders

The decision of controller orders made in the previous section is consolidated with the generation of the linear time-domain plots that show the step response of the system to different combinations of controller order; Figure 7.34 shows the linear design step response to all variations of controller order, both for the feedforward controller C_{ffd} and the feedback controller C_{fbk} :

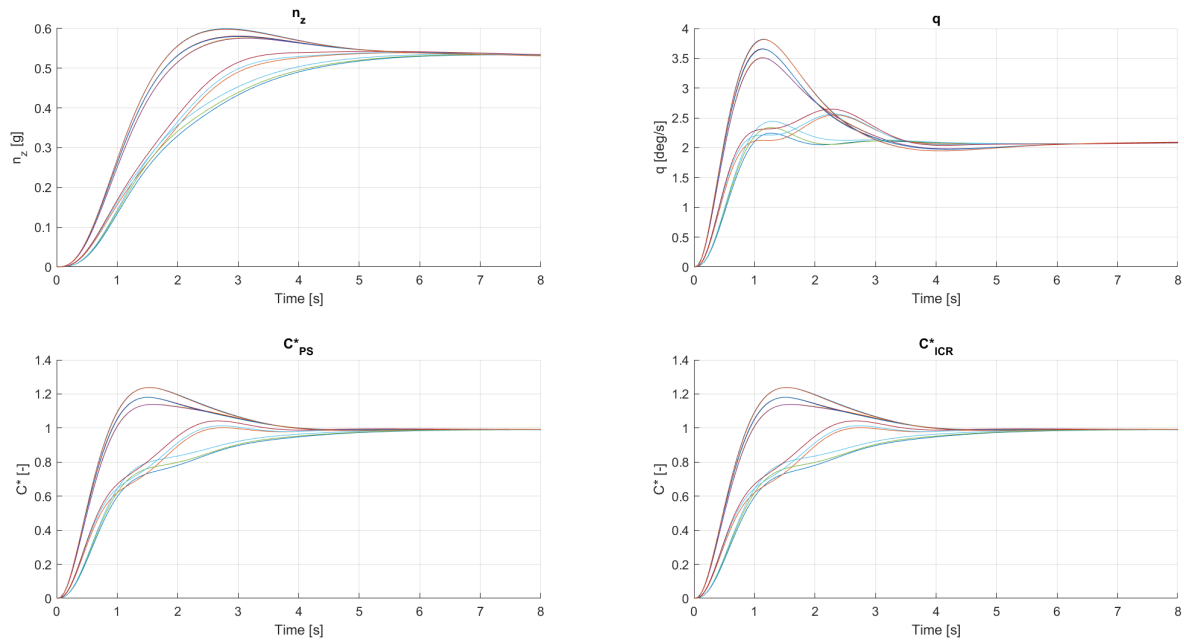


Figure 7.34: Linear design configuration step response to all variations in controller order for both controllers, C_{ffd} and C_{fbk}

Looking at Figure 7.34, it can be seen that the feedforward controller has the largest influence on the shape of the response. This is logical seeing that the feedback controller is responsible for ensuring robust stability, while the feedforward controller is responsible for robust tracking performance. The effect of the variation of the feedforward controller order is shown in Figure 7.35:

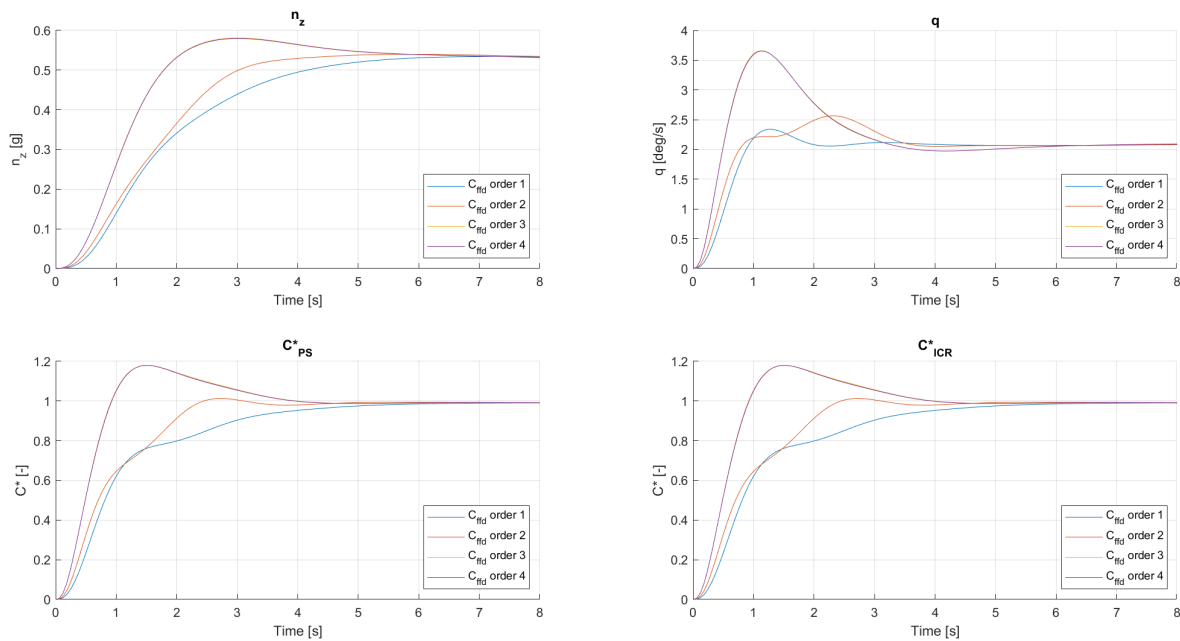


Figure 7.35: Effect of variation of feedforward controller order for the linear 2DOF configuration, with feedback controller order set constant to 2

In this scenario, the FB controller is set to 2nd order. A 3rd order feedforward controller is required for the step response to resemble the reference model, since a lower order controller does not exhibit a similar maximum value and dropback. The decision of a 3rd order feedforward controller is motivated by the shape of the step response.

7.4.4. Handling Quality Assessment Comparison of Controller Orders

The effect of controller order on the final response has been evaluated in the time-domain by looking at the step response plots of the system. It is also useful to assess the differences through the HQ metrics that are being used in this study to determine where each of the controllers rank in terms of HQ performance.

CAP Criteria

The first metric of interest is the CAP criterion. Figure 7.36 shows how each of the orders of feedforward controller fare in terms of CAP performance. Whilst all points lay within level 1 HQ performance boundary limits, the points corresponding to order 1 and 2 still lay very far from the reference model, highlighting the under-damping associated with these orders of controller. The 3rd and 4th order controllers however, lay perfectly on the point of the reference model, indicating an exact match of the damping ratio and CAP value of these two controlled systems with the reference model.

Gibson Criteria

Moving on to the Gibson criteria, it is clear that the 1st and 2nd order controllers are unsatisfactory; the points lay exactly on and almost at the lower left corner of the allowable Gibson region, indicating that the dropback and the maximum response value as a proportion of the steady state value are both too little. This means that the response shows little to no overshoot like the reference model, and this can be seen in Figure 7.35. As with the CAP criteria previously, the performance improves for the 3rd and 4th order controllers concurrently, resulting in a dropback and maximum pitch rate ratio of slightly above that of the reference model. This point still lays within acceptable Gibson HQ criteria limits and is relatively close to the reference model, thus the chosen controllers can be considered sufficient for the application.

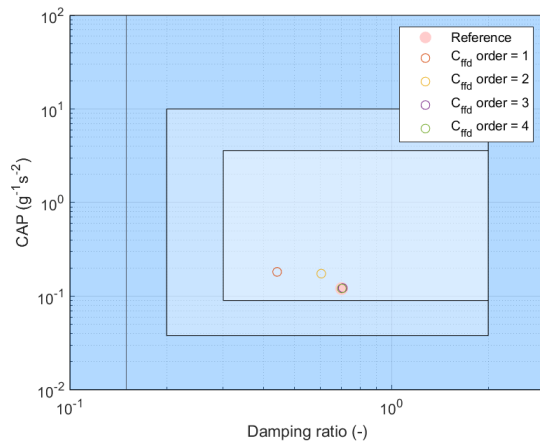


Figure 7.36: CAP criteria corresponding to different orders of feedforward controller

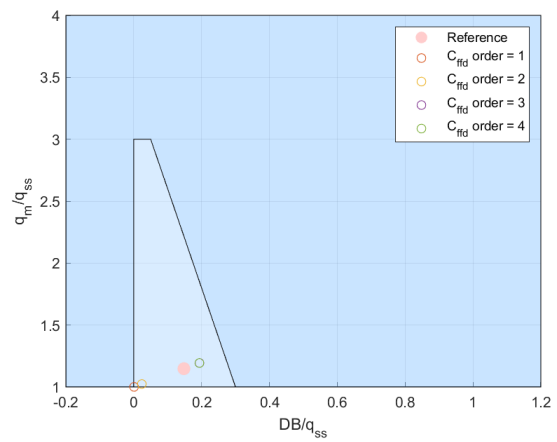


Figure 7.37: Gibson criteria corresponding to different orders of feedforward controller

Attitude Bandwidth Criteria

It can be seen from Figure 7.38 that changing the order of the controller has a larger effect on the pitch attitude bandwidth than on the phase delay itself. This can be seen by the points corresponding to the different orders shifting rightward but not too far upward. All of the points lay within the level 1 HQ boundary, rendering them all satisfactory in terms of performance. From the plot it can be seen that the third and fourth order controller once again very closely approximate one another, having a pitch attitude bandwidth of approximately 4.1 rad/s.

Flight Path Bandwidth Criteria

The same can be said for the flight path bandwidth criteria, where it can be seen from Figure 7.39 that the first and second order controllers lie in the level 2 HQ region, and the third and fourth order controllers lie within the level 1 HQ region.

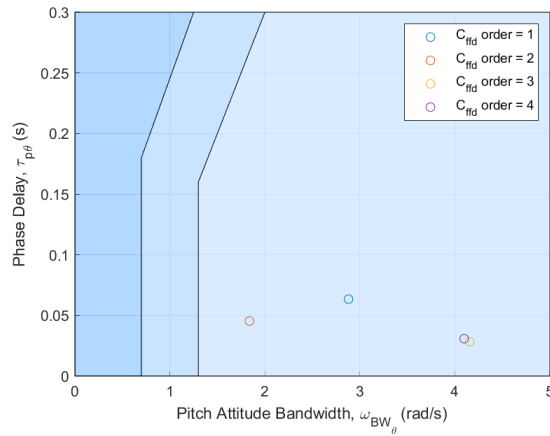


Figure 7.38: Phase delay criteria corresponding to different orders of feedforward controller

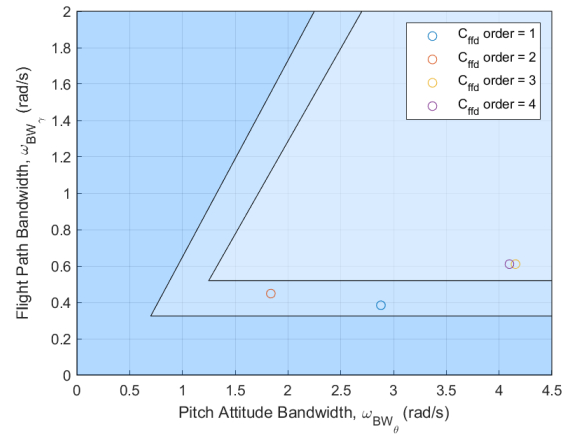


Figure 7.39: Flight path bandwidth criteria corresponding to different orders of feedforward controller

7.4.5. Frequency-Domain Analysis

The frequency domain analysis can once again be conducted for the nominal 2DOF design, starting with the GO6 results.

Gang of Six

The GO6 sensitivity functions are shown once again for the 2DOF design in Figure 7.40:

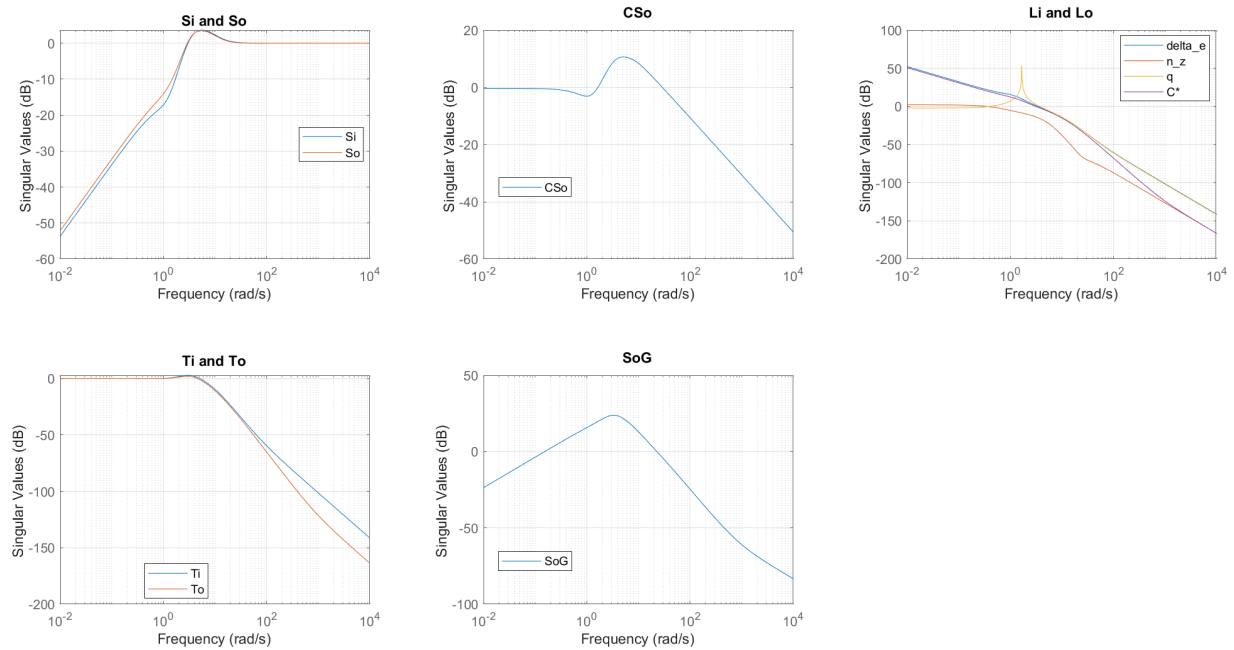


Figure 7.40: GO6 sensitivity functions corresponding to the 2DOF design configuration

A normal profile can be seen once again for each of the CL TFs, and little difference can be observed with the 1DOF results obtained in [Section 7.3.4](#). Therefore, it can be concluded that the augmented design possesses all of the desirable qualities in terms of robustness that were provided by the 1DOF design, such as LF disturbance rejection, control signal and noise attenuation and unsaturated actuator rate and position. The latter is supported through the calculation of the maximum actuator rate and settling time, as previously calculated for the 1DOF case; this results in $\dot{\delta}_{e_{max}} = 24.3166$ deg/s and $t_s = 1.9889$ s. The actuator rate falls within prescribed limits and the settling time indicates well behaved signal behaviour.

Classical Margins

The calculated CMs are tabulated in [Table 7.8](#):

	Channel			
	δ_e	n_z	q	C*
Gain Margin (dB)	18.162	21.5341	18.8163	16.431
Phase Margin (deg)	44.7615	138.8330	48.3516	47.9587

Table 7.8: Classical gain and phase margins corresponding to the 2DOF design configuration

It can be straight away seen that all channels satisfy the 6 dB GM requirement by a large margin, as with the 1DOF case. Furthermore, the PM requirement of 45° is satisfied by most of the channels, except for the δ_e channel, which misses it by 0.2385° . This deviation from the target is minimal, and still considered acceptable in the design.

Disk Margins

The DMs are considered for the 2DOF nominal design in [Table 7.9](#):

	Channel			
	δ_e	n_z	q	C*
Disk Gain Margin (dB)	7.5305	16.7799	8.2925	8.0162
Disk Phase Margin (deg)	44.4140	73.5127	47.8942	46.6575
Disk Margin (dB)	0.8165	1.4938	0.8882	0.8625

Table 7.9: Disk gain margin, disk phase margin and DM for all relevant channels corresponding to the 2DOF design

Comparing the results of [Table 7.9](#) to the 1DOF results of [Table 7.5](#), there is relatively very little difference between the results of the two configurations. What can be seen is a very slight increase in both magnitude and phase across all channels. The greatest increase in magnitude and phase DM is seen in the pitch rate channel, q , with a gain margin increase of 0.2985 dB and a phase margin increase of 0.568° .

Multiloop Margin

The multiloop margins calculated for the 2DOF nominal design are tabulated in [Table 7.10](#):

Multiloop Gain Margin (dB)	0.6871	1.4555
Multiloop Phase Margin (deg)	-21.0170	21.0170
Multiloop Disk Margin (dB)	0.3710	

Table 7.10: Multiloop margin representing simultaneous gain and phase variations over all relevant channels at once for the 2DOF design configuration

Relative to [Table 7.6](#), the lower and upper bounds of the GM have shifted compared to the 1DOF case; the lower margin raises by 0.0012 dB, from 0.6859 dB to 0.6871 dB, whereas the upper bound shifts

by 0.0024 dB, from 1.4579 dB to 1.4555 dB. Additionally, the PMs have slightly decreased relative to the 1DOF case, implying an increased multiloop coupling SM sensitivity in the 2DOF case. It can however be concluded that the multiloop DMs are satisfactory for the implementation of the 2DOF design.

Stability Regions

The stability regions corresponding to the C^* channel for the 2DOF design configuration are depicted in the complex and real gain and phase plane in Figure 7.41 and Figure 7.42 respectively:

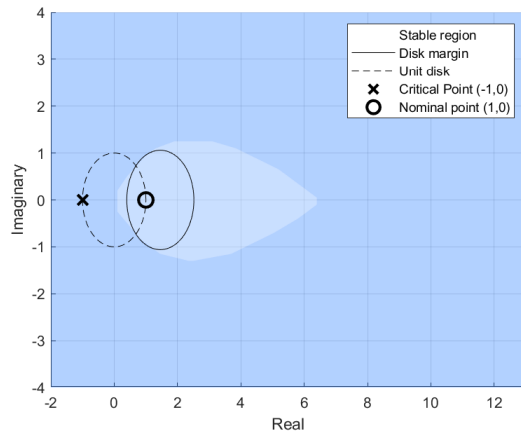


Figure 7.41: Stability region of the C^* channel with corresponding DM for the 2DOF design configuration, shown in the complex plane

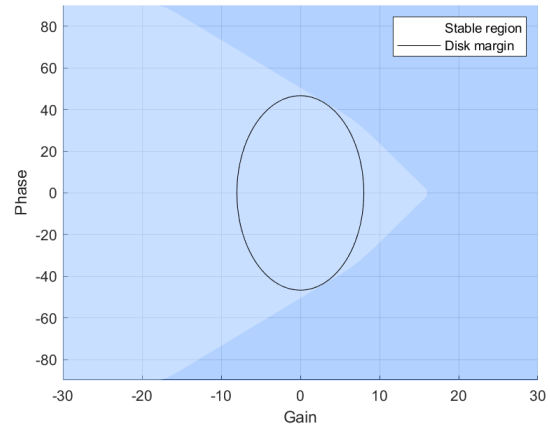


Figure 7.42: Stability region of the C^* channel with corresponding DM for the 2DOF design configuration, shown in the real gain-phase plane

It can be seen from both plots that the obtained DMs fit accurately into the computed stability region in both planes, and also avoid the critical instability point, shown in Figure 7.41. The increase in DMs indicates that the area delimited by the disk is slightly greater than that of the 1DOF case, which is reinforced by the updated tabulated DM results.

Disk Margin Plots

The DM plots are once again depicted but for the 2DOF design configuration. These are given for all channels in Figure 7.43 and for the C^* channel singularly in Figure 7.44:

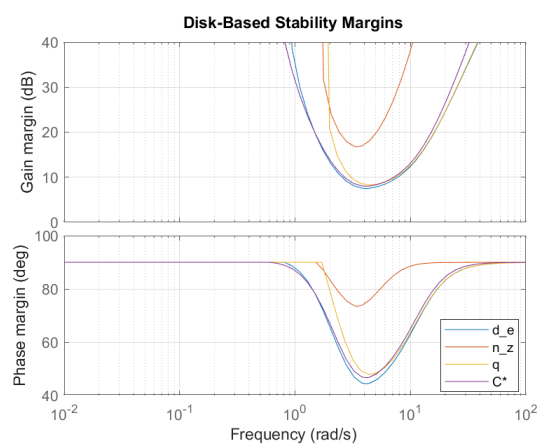


Figure 7.43: DM plots for all channels with nominal skew under the 2DOF design configuration

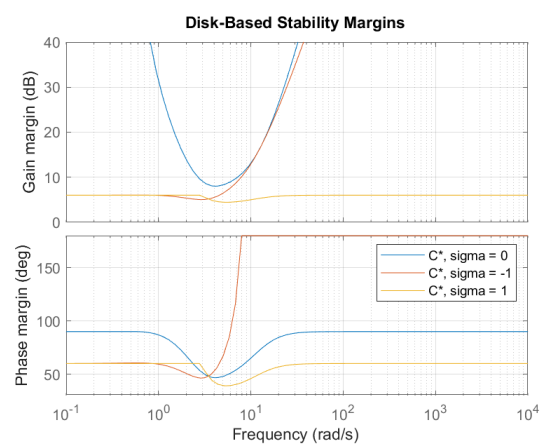


Figure 7.44: DM plots for the C^* channel with varied skew under the 2DOF design configuration

The DM plots show very much the same shape as the 1DOF case, and this is expected with the very

similar obtained DMs. This also holds true for Figure 7.44, where it can be seen that the effect of DM skew is similar for the 2DOF case to the 1DOF case.

Nyquist Diagrams

The Nyquist diagrams corresponding to the 2DOF design are provided in Figure 7.45 and Figure 7.46:

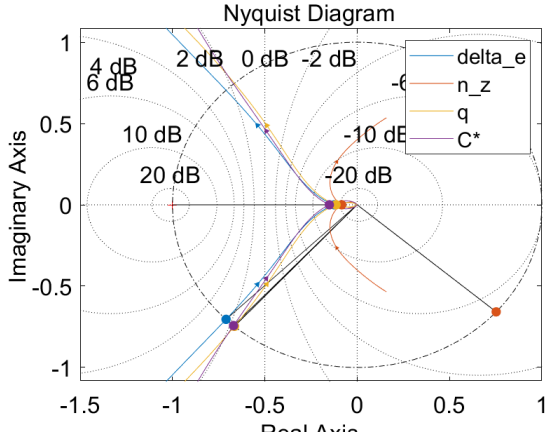


Figure 7.45: Nyquist diagram of all channels superimposed with one another under the 2DOF control design

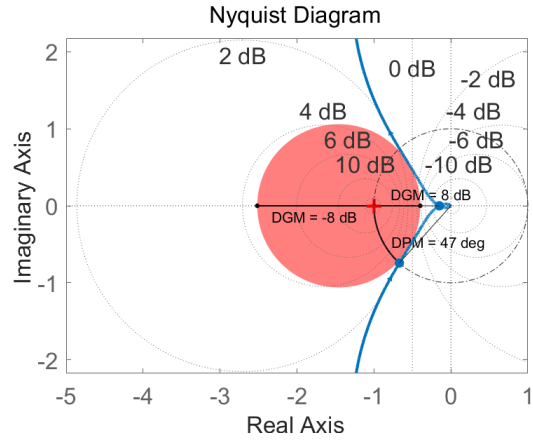


Figure 7.46: Nyquist diagram of the C* channel under the 2DOF control design configuration

The Nyquist plot results for every signal in Figure 7.45 further confirm the system CL stability, with the loci for every signal avoiding the critical instability point at $(-1, 0)$. Additionally, the increased DM is visualised in Figure 7.46, with the disk phase margin being equal to 47° , rather than 46° .

Nichols Diagrams

The Nichols diagram corresponding to the 2DOF design is provided in Figure 7.47:

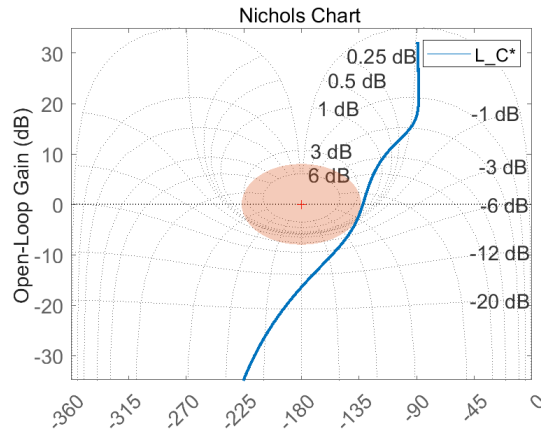


Figure 7.47: Nichols diagram of the C* channel under the 2DOF control design configuration

The sufficient DM is visualised once again through the disk occupying greater space than the region delimited for 6 dB of gain variation across the entire frequency range. The DM region is not violated, and therefore a sufficient DM can be concluded for the 2DOF design.

7.4.6. Time-Domain Analysis

The following will contain the time-domain responses of the nominal 2DOF system, comparing the 2DOF to the 1DOF design for different inputs and visualising the effects of disturbance rejection (DR).

Step Response

The first simulated scenario was the step response, with a magnitude of $C^* = 0.5$. As this is the linear case, the response is initialised at 0, and the final value corresponds to the deviation from the trimmed condition. The simulated step response for both accelerometer positions is shown in Figure 7.48. The response measured at the PS tracks the reference model incredibly accurately, and the difference in the response measured at the ICR and the PS can be seen. The response exhibits almost identical overshoot, damping ratio and settling time. From this result, it can be concluded that the controller fares well in terms of step time-domain performance.

Comparison of 1DOF and 2DOF Design Configurations

To visualise the apparent advantages of the 2DOF design configuration over the 1DOF design configuration designed in Section 7.3, the linear step responses for each case were overlaid together in one plot. This is depicted in Figure 7.49. With this side-by-side visualisation of both responses, the effect of the addition of the feedforward controller becomes even more apparent; the 2DOF design approximates the reference model far more closely than the 1DOF, performing almost identically in terms of classical, time-domain based requirements such as overshoot/undershoot, damping and settling time, highlighting the necessity of the feedforward element in the design of a tracking controller.

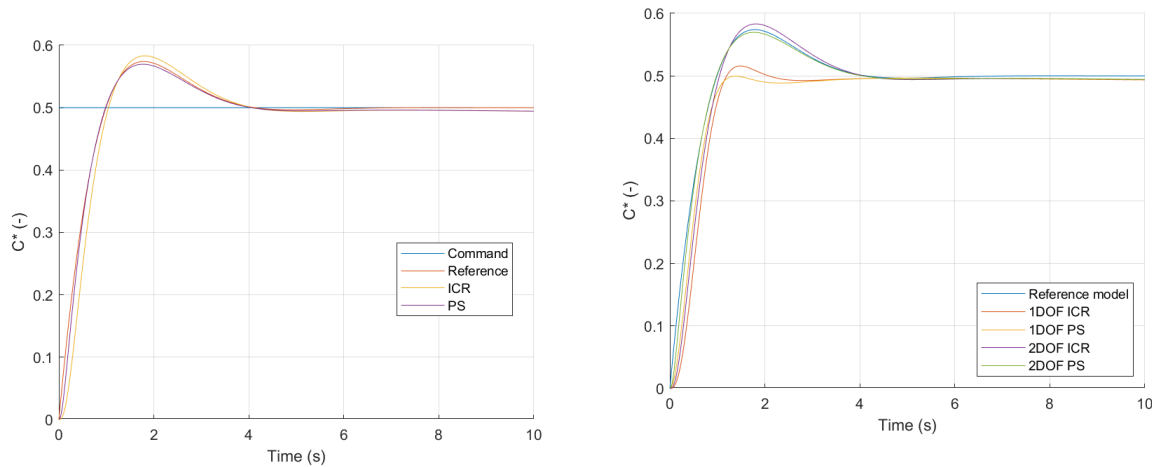


Figure 7.48: Step response of 2DOF linear controller

Figure 7.49: Step response comparison of linear 1DOF and 2DOF controllers, for both C^* measurement locations

Staircase Response

In addition to the step input, a staircase input was manually constructed using the signal generator in Matlab. This input was generated in order to test the range of the controller, and ensure that accurate tracking was maintained over the course of the simulation time window. To do so, the staircase input was designed such that it would incorporate steps of large magnitude variation and in varying directions, making it more difficult for the controller to track. The staircase signal is incremented in steps of 8 seconds to allow the tracking signal to have time to be able to settle before receiving a new input magnitude. Since the linear model will be simulated, this input corresponds to an actual input that is shifted by 1 upwards for the C^* response, since the input here represents the deviation from the C^* trim point of 1. The controlled response to the designed staircase input is depicted in Figure 7.50:

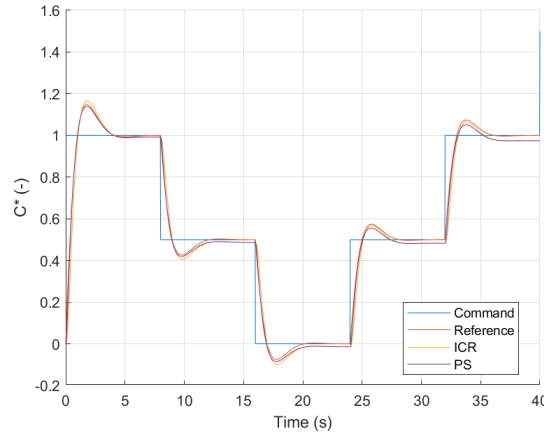


Figure 7.50: Staircase response of 2DOF linear controller

Once again, as for the step response in Figure 7.48, the controller is able to track the reference signal very well, both at the ICR and PS outputs. This is especially the case at the start of the simulation. It can be seen later however that the controller struggles a little to maintain the commanded C^* value. This starts to become visible at approximately 12 seconds, and the gap between the commanded input, as well as the steady state reference model, and the tracking response becomes greater as time progresses. However, the purpose of the staircase input was to test the boundaries of tracking and robustness capability of the controller. As mentioned earlier, the response shown in Figure 7.50 in practice should be shifted upwards by 1. Therefore, a commanded maximum C^* value of 2 is bound to put a large amount of stress on the controller, since this is a very large commanded C^* value which the aircraft may struggle to achieve. For this reason, the C^* tracking ability degrades over time.

Step Response with Disturbance Rejection

The next aspect of the controller design to test is the DR capabilities. Two types of DR were tested on the same step response that is shown in Figure 7.48; the first disturbance was applied on the plant input δ_e , and the second was applied was on the plant output C^* . The input disturbance has a magnitude of $-\frac{\pi}{360}$ rad (approx. -0.0087 rad), and is applied at a time of $t = 4$ s. The output disturbance has a magnitude of 0.2 rad and is applied at a time of $t = 6$ s. The step response including DR is shown in Figure 7.51:

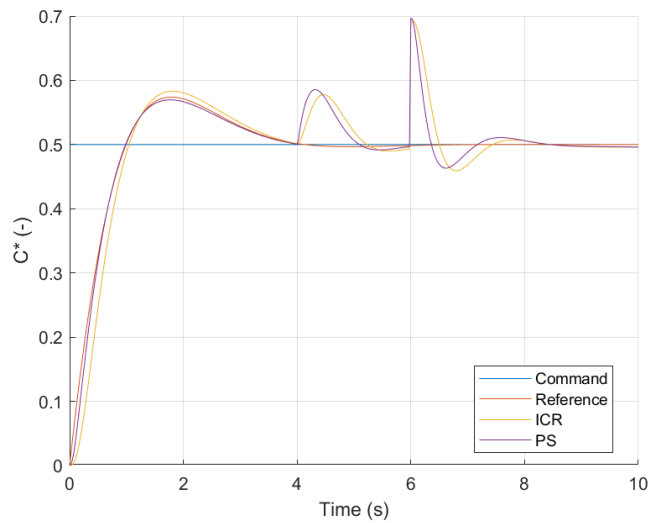


Figure 7.51: Step response of 2DOF linear controller with applied input and output disturbances

The plot shows the DR capabilities for both the ICR and PS measured C^* signals. The response shows the same step response initially as previously in Figure 7.48, and the first deviation is indeed immediately seen at $t = 4s$. It is noticeable that the ICR measured response shows a slight delay relative to the PS response, indicating that the PS signal deals with and suppresses the disturbance more promptly. However, the disturbance is rejected rather quickly as it returns to the target value within 2 seconds of the disturbance. Furthermore, for the given output disturbance, there is a rather large peak of 0.2 in C^* value relative to the target value. The feedback controller does however suppress this very quickly as well, with the steady-state value returning to the target hold value in a little over 2 seconds after the disturbance. It can be concluded that the DR properties of the robust feedback controller are deemed suitable for controller implementation.

7.5. H_∞ Loop Shaping Design: 2DOF with Applied Uncertainty

Whilst the previous results showed the frequency-domain and time-domain based criteria of the nominal 2DOF model, this section will display the results of the 2DOF design with uncertainty applied.

7.5.1. Handling Quality Assessment with Applied Uncertainty

This subsection will encompass the HQ criteria and the performance of the uncertain models in each. This will be executed for the previously covered HQ criteria, where any modifications to the criteria will be detailed.

Gibson Criteria

The Gibson criterion is shown in Figure 7.53. A straight trajectory is formed by the set of uncertain models from within the Gibson level 1 HQ boundary near the reference point to leaving this region. A higher density of points is visible closer to the region centre, closer to the reference model, with only 8 of the uncertain models laying on or outside the boundary. It appears that the applied uncertainty has a linearly translatable effect on the maximum pitch rate and dropback characteristics of the response. Since a robust controller was designed, it was required that this controller was able to stabilise the plant in the face of all types of uncertainty. This is the case, however, the controller has failed to achieve level 1 HQs in the Gibson criteria for all uncertain models within the set. This can in part be due to the large magnitude of uncertainty that was chosen to be applied in the design. It is likely that a smaller magnitude of applied uncertainty would result in fewer perturbed models laying outside of the acceptable region, since worst case (WC) uncertainty is usually responsible for performance outlier cases. An implication of this is that the controller may not behave well over time in terms of PIOs in a nonlinear simulation. A recommendation for a future redesign would be to impose stronger constraints in systune regarding tracking and maximum gain to bound the maximum pitch rate and dropback.

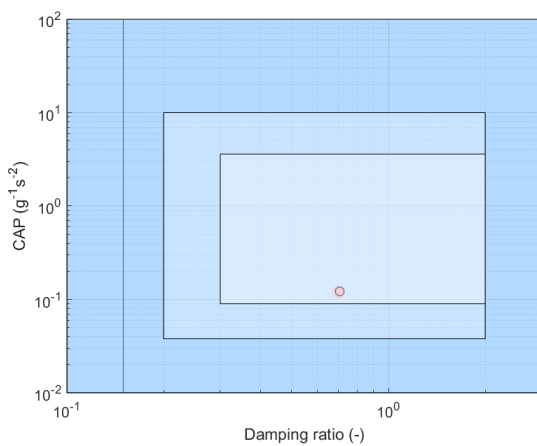


Figure 7.52: CAP criteria for 2DOF design with all uncertain models

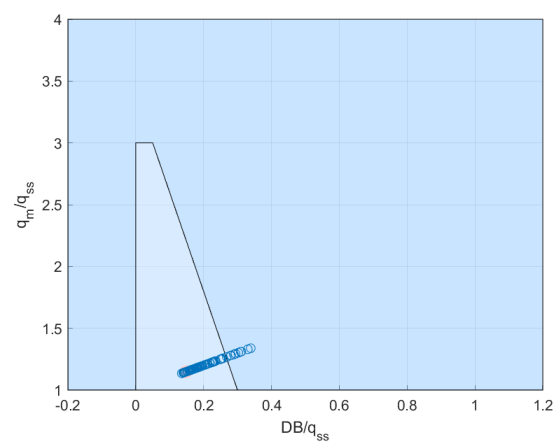


Figure 7.53: Gibson criteria for 2DOF design with all uncertain models

Attitude Bandwidth Criteria

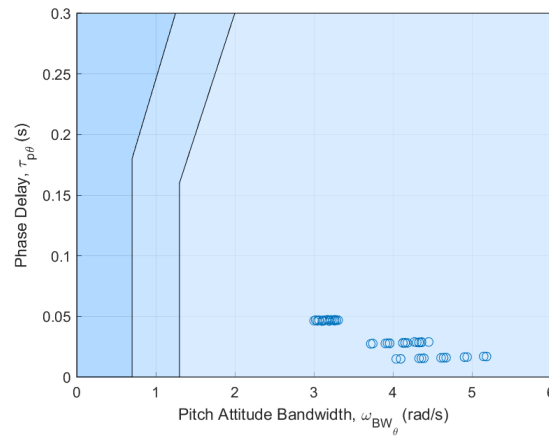


Figure 7.54: Attitude bandwidth criteria for 2DOF design with all uncertain models

From Figure 7.54, it can be seen that all points lay safely within the level 1 HQ region, and therefore this is of no concern to the 2DOF design over the entire uncertainty range. Relative to the nominal point, the pitch attitude bandwidth is both increased and decreased, as well as the phase delay. This variation is to be expected given the nature of the uncertainty.

Flight Path Bandwidth Criteria

Firstly, the bode diagram corresponding to the γ transfer is shown in Figure 7.55. Looking closer at the phase plot, the path followed by all models in HF appear to approach an asymptote at -90° . It is only at much higher frequency that the bode plot crosses the -135° threshold. This is problematic for the calculation of the FPBW since this is equal to the highest frequency at which a 45° phase margin is obtained, or in other words, the frequency at which the bode phase plot crosses -135° . For this reason, the bode phase plot is shifted negatively by 360° , as shown in Figure 7.56. As this is equivalent to one full rotation, this is a valid approach in obtaining a more realistic reading of FPBW.

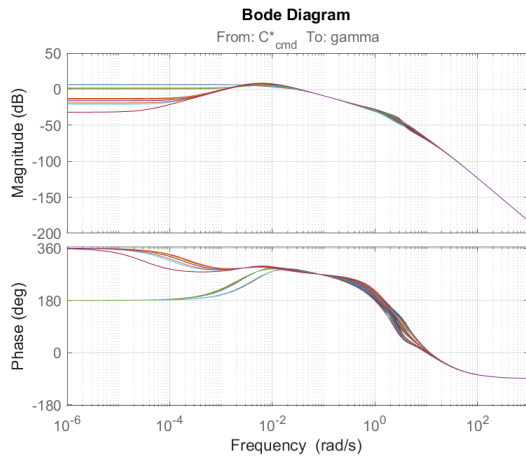


Figure 7.55: Original bode diagram of gamma output for 2DOF design with all uncertain models

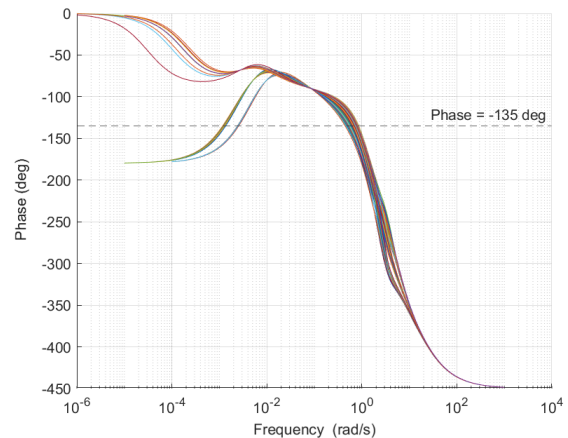


Figure 7.56: Shifted phase diagram of gamma output for 2DOF design with all uncertain models

It is visible that some of the uncertain models cross the 45° phase margin threshold at very LFs in the range of 10^{-3} to 10^{-2} . This is however outside of the operating frequency range of the controller and pilot controlling bandwidth, and therefore does not contribute to the final FPBW. The actual FPBW can be read from the graph at the point where all of the models cross the -135° line. The pitch attitude bandwidth

ω_{BW_θ} for each model remain equal to the values calculated for the attitude bandwidth criteria. The FPBW criteria results are shown for each uncertain model in Figure 7.57:

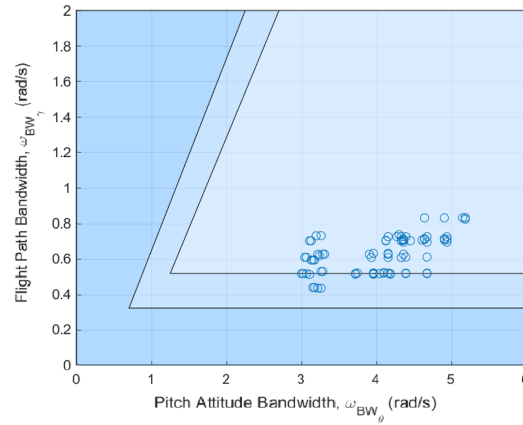


Figure 7.57: Flight path bandwidth criteria for 2DOF design with all uncertain models

The plot shows that most uncertain systems lay within level 1, followed by a number of systems laying on the boundary of level 1 and level 2. Lastly, four systems lay definitively within level 2. None of the points lay within level 3. This indicates that the controller is successful in terms of FPBW performance, since the performance for system whose primary means of control is not the pitch angle (ie: C* controller), is allowed to degrade to level 2.

7.5.2. Frequency-Domain Analysis

Once again, the frequency based metrics will be assessed for the 2DOF design with uncertainty.

Gang of Six

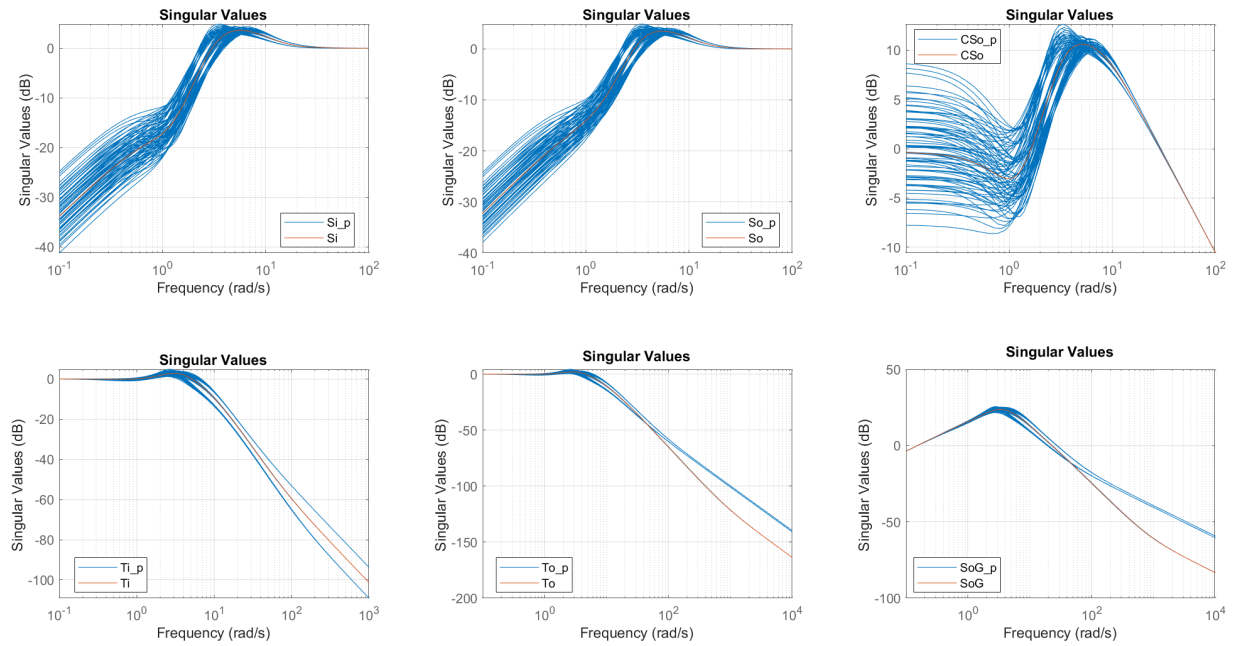


Figure 7.58: GO6 TFs SV plots for 2DOF configuration with applied parametric uncertainty

The GO6 in Figure 7.58 shows the normal profiles associated with each of the transfers, such as the S and T transfers. The model uncertainty appears to have a greater effect on the sensitivity functions compared to the complementary sensitivity functions, as indicated by the spread of the SV plot lines around the nominal model shown by the red line. The input and output sensitivity functions show the normal profile of low SVs at LF, rising smoothly to 0 dB, highlighting good DR capability. Looking at the input and output complementary sensitivity functions, uncertainty causes the perturbed systems to roll off at different rates compared to the nominal model, which affects the degree of noise attenuation at HF. In this case, all models with uncertainty still display good noise attenuation profiles.

For the control sensitivity function CSO, the spread due to uncertainty at LF is quite high, meaning that the control sensitivity gain for some models at these frequencies is rather high relative to the resonant peak that is once again observed surrounding the nominal model around the operating bandwidth of the controller (between 1-10 rad/s). Around this point, there is less variation in SVs relative to the nominal model, indicating that minimal extra stress is placed on the actuators within the operating frequency range due to uncertainty.

Loop Gains

Next to be covered within the frequency-domain assessment are the open loop gains relating to each of the channels, and particularly the C^* channel. These are shown in Figure 7.59 and Figure 7.60, respectively:

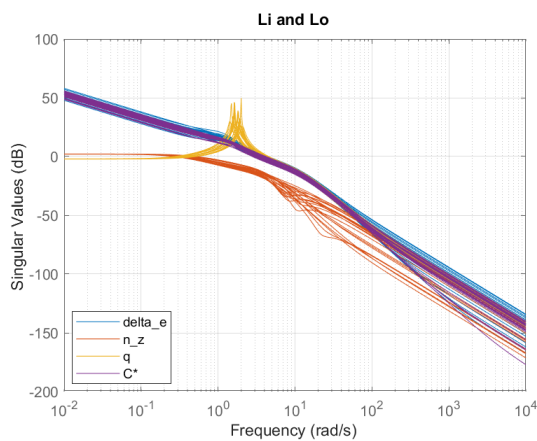


Figure 7.59: OL gains of all channels with applied parametric uncertainty for the 2DOF design configuration

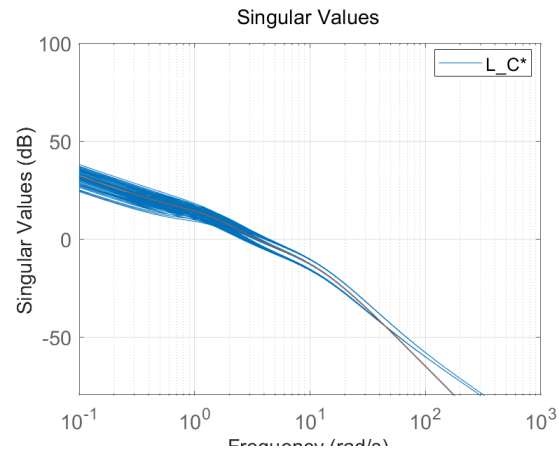


Figure 7.60: OL gains of C^* channel with applied parametric uncertainty for the 2DOF design configuration

The loopgains with the applied uncertainty are very uniform for the δ_e and C^* channels, especially in the low to mid frequency ranges. Deviation begins to appear from approximately 20 rad/s, which lies outside the most important controller design frequency range. The n_z channel remains uniform until approximately 4 rad/s, where some of the perturbed models begin to roll-off in SV more drastically than others. After the initial roll-off, the rate of roll-off remains fairly constant across uncertain models, but the magnitudes of SV remain rather varied at HF. The peak shown in the q channel changes slightly in magnitude and location of peak frequency, with the lowest SV being approximately equal to 25 dB and the highest SV equal to approximately 50 dB. The location of the peak is within the frequency range of 1.5 rad/s - 2 rad/s, thus it does not vary considerably. Figure 7.60 shows the uncertain open loop gains, with the nominal open loop gain overlaid in red. At LF to MF, the uncertain models appear evenly distributed around the nominal model. However, at HF, the nominal model's loopgain rolls-off more quickly and becomes the path of lower SVs compared to the uncertain models. This does not matter however, since the LF gain is high and the HF gain is also still low for all models with uncertainty applied.

Worst-Case Disk Margins

The WC DGMs and DPMs are tabulated in Table 7.11:

	Channel			
	δ_e	n_z	q	C*
Disk Gain Margin (dB)	5.6797	10.7707	7.1407	5.8745
Disk Phase Margin (deg)	35.0496	57.7215	42.5484	36.0949
Disk Margin (dB)	0.6316	1.1023	0.7787	0.6517

Table 7.11: Worst case DMs for the 2DOF design configuration

It can be seen that the worst case margins are significantly reduced when compared to the nominal model, that was provided in [Table 7.9](#). This is evident especially through the δ_e and C* channels. The WC DGM of the δ_e channel is equal to 5.68 dB and the WC DPM equal to 35.05°. For the q channel, the WC DPM is equal to 42.55°. Lastly, the C* channel has a WC DGM of 5.87 dB and a WC DPM of 36.09°. All of these mentioned margins are below the recommended DM guidelines of 6 dB and 45°. However, they are still considered healthy margins, especially seeing as the DGMs are not too far below 6 dB and all of the DPMs are still above 30°. The most important loop, the C* loop, has a DGM just below 6 dB, so this is considered acceptable to proceed with the design. A large amount of effort was placed in trying to raise these DMs, however it proved challenging to raise them further than the achieved results. Thus, a revision of the controller design could aim to increase these DMs to ensure that all of the WC margins are above the guided threshold values.

Nyquist Diagram

The WC Nyquist diagram for the 2DOF design with applied uncertainty is shown in [Figure 7.61](#). As shown in the figure, each of the uncertain perturbed models follow a roughly similar trajectory, but with varying magnitudes of SM. The trajectories appear to be rather uniformly distributed due to the presence of the uncertainty. It is visible from the plot by the line delimiting the closest approach to the critical point, that the WC scenario falls just within the 6 dB region of the plot, as indicated previously by [Table 7.11](#). The Nyquist plot otherwise indicates healthy SMs which diverge from the critical point (-1,0).

Nichols Diagram

The Nichols diagram corresponding to the C* signal with various amounts of uncertainty applied, is shown in [Figure 7.62](#). In the plot, the WC Nichols line is indicated in red and the corresponding WC DM is shown. It can be read from the plot that the WC DM is just below 6 dB, which corresponds to the WC DM obtained in [Table 7.11](#). The plot shows a relatively uniform distribution of the uncertain loops in a diverging direction from the critical stability point at the centre of the DM, indicating the robust stability of the 2DOF design for the C* loop, even in the case of large magnitude applied uncertainty.

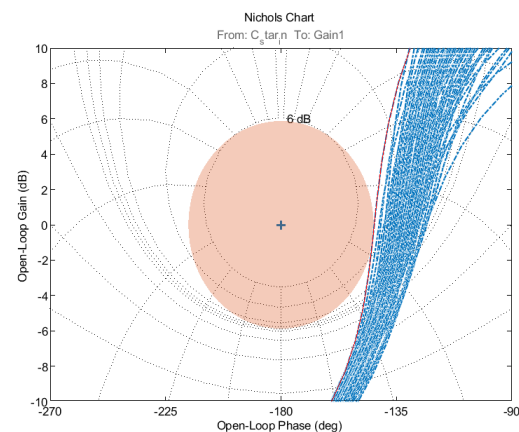
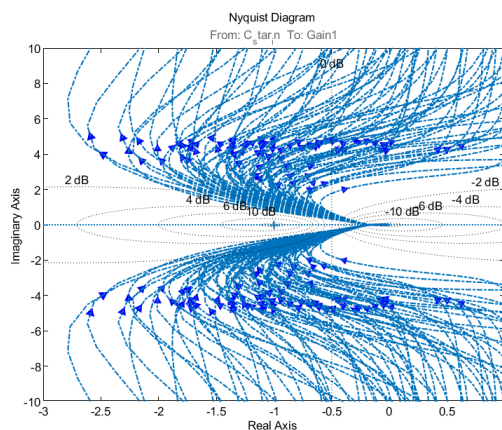


Figure 7.61: Nyquist diagram of 2DOF design with **Figure 7.62:** Nichols diagram of 2DOF design with various amounts of applied uncertainty, with the WC scenario highlighted various amounts of applied uncertainty, with the WC DM highlighted

7.5.3. Time-Domain Analysis

To conduct the corresponding time-domain analysis, a simple step input on the C^* signal is conducted with all uncertain systems, as shown in [Figure 7.63](#):

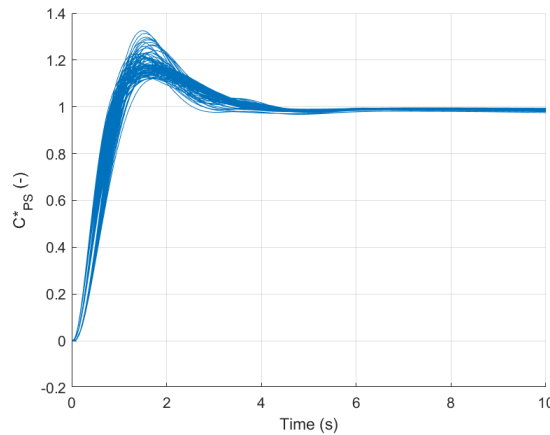


Figure 7.63: C^* step response of all uncertain systems pertaining to the 2DOF design configuration

The step response shows that the set of perturbed systems react rather uniformly to a given step input, since there is not too much deviation in response between the maximum and minimum responses. The most noticeable difference is in the overshoot and dropback of the response within the first 3 seconds, which was highlighted earlier through the Gibson criteria. This indicates that certain combinations of uncertainty could produce a response that edges on being overly-abrupt. All of the responses return to steady-state in approximately 4 s, however. Therefore, to overcome this, more stringent constraints on the maximum gain and tightness on tracking could be imposed during the systune design process to produce a more uniform step response. Considering that the rather high magnitude of the applied uncertainty of 30% was applied on the selected coefficients, the results are not unexpected nor poor for a preliminary design.

7.6. H_∞ Loop Shaping Design: 2DOF Nonlinear Model

Once a working 2DOF linear controller is established, it can be integrated into the full Flying V aircraft nonlinear model to test the functioning of the controller within the nonlinear scope. This chapter will detail how the controller was reintegrated, giving details of model initialisation in [Section 7.6.1](#), before conducting a time-domain analysis in [Section 7.6.2](#).

7.6.1. Model Initialisation

The modifications made to the controlled system between the linear and nonlinear designs have implications on the behaviour of the nonlinear system. Firstly, by initialising the initial condition of the integrator as the trimmed elevator deflection during trim, the integrator enforces the trimmed input as input to the plant.

An issue that can be experienced during nonlinear simulation is the presence of an undesired transient response for the first few seconds of the simulation, before the commanded step input on the C^* signal is applied. This is due to the fact that both the feedforward and feedback paths received initial inputs to the feedforward and feedback controllers of the trimmed C^* condition, which is equal to approximately 1. Due to the controllers being designed simultaneously using systune with minimal constraints imposed on their state-space representation, this causes the feedforward and feedback paths to produce differing initial responses to the trimmed input, leading to a non-zero difference after the OL is fed to the integrator. Subsequently, the integrator integrates this difference, and causes an initial, pre-emptive response of the actuators, which in turn causes the plant to react. This eventually leads to the measured outputs deviating from the trimmed state, and these deviating conditions feeding back into the controller, exacerbating the effect.

When directly implemented, the controller exhibits undesired undershoot in the C^* response during

the trimmed holding window. This is due to controller induced transients at the trim condition, and this is amended with correct model initialisation, involving DC gain matching of the steady state gains of the feedforward and feedback controllers. Firstly, the initial controller outputs are set to match their steady state outputs at trim. For the feedforward controller, this is equal to Equation 7.24:

$$y_{ffd_{trim}} = C_{ffd}(0) \cdot u_{trim} \quad (7.24)$$

In the nonlinear model, the trimmed input presents as the commanded C^* input to the controller, which during trim is equal to the trimmed C^* value. For the feedback controller, this is equal to Equation 7.25:

$$y_{fbk_{trim}} = C_{fbk}(0) \cdot u_{trim} \quad (7.25)$$

The trimmed input to the feedback controller is the fed-back total C^* value during trim hold, which is also equal to the trimmed C^* value. The controllers are subsequently represented as TF blocks where the designer is permitted to set the initial input and output of the controller. For each controller, the input is set as the trimmed C^* value and the output is set as the steady state output of the controller at trim. This causes both controllers to operate around their equilibrium condition and therefore no transient response arises as a result. There is however still a mismatch between the gains of the controller outputs, and therefore a gain in the forward path takes the value given by Equation 7.26:

$$\frac{C_{fbk}(0)}{C_{ffd}(0)} \quad (7.26)$$

This matches the forward path DC gain to that of the FB path, such that their difference equals zero after OL FB. This effectively removes the transient response as both controllers are held at their equilibrium, and a zero signal is fed to the integrator, which ensures to maintain the trimmed elevator command initially during the trim hold window.

7.6.2. Time-Domain Analysis

The time-domain responses will be obtained for different signal types to test the controller capability. The first of these will be the step response.

Step Response with D-implementation (Velocity-Based Implementation)

One way in which the setup of the nonlinear model differs from the linear setup is through having the 'Derivative Implementation', otherwise known as the 'D-Implementation' or the 'Velocity-Based Implementation'. This tactic can be employed to reduce the impact of transient effects in situations where a system should be able to maintain a specific value. For various reasons, this is not always possible, and therefore this technique could be necessary.

Kaminer introduced the method for implementing gain scheduled controllers in nonlinear plants, which is able to preserve the input-output properties of the linear CL around the given equilibrium point. The foundation of the method he proposes lies in the fact that linear controllers operate according to perturbations at the plant inputs and outputs. The method involves providing integral action at the plant input to preserve the internal input and output characteristics of the CL. As a result, a derivative must in turn be applied to the measured outputs before being fed back to the controller. Beneficially, the method does not amplify noise in the FB system since the original properties of the CL transfers are preserved.

The method itself is very simple and applies generally to the class of tracking controllers, and it has been applied to a considerable number of flight control situations successfully. Rather commonly, linear gain-scheduled controllers are implemented directly on the nonlinear plant. This however can lead to adverse effects such as the reduction in performance and further, even the loss of stability of the feedback system designed for the given operating point. The D-implementation aims to amend this clearly undesirable outcome.

After applying the D-implementation, the two corresponding step responses are shown in Figure 7.64:

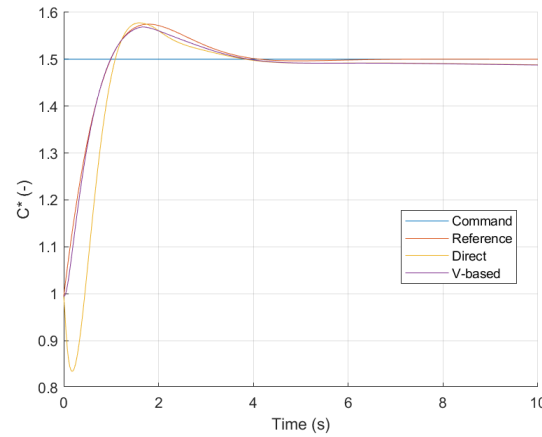


Figure 7.64: Step response of 2DOF controlled nonlinear system with direct and V-based implementations

It can be seen in [Figure 7.64](#) that the direct implementation without the D-implementation applied causes the signal to initially drop from trim with a peak negative magnitude of approximately 0.15, before climbing again steeply to the target value. The direct implementation shows a slightly higher peak value and drops to the steady-state value more abruptly than the reference model, exhibiting greater overshoot and dropback. Comparing the signal lines together after the peak, the direct implementation also appears to have a two-staged return to steady-state, which results in slightly more abrupt and bumpy handling. The V-based implementation on the other hand does not exhibit any undershoot initially after trim. Additionally, the signal tracks the reference very well in the initial rise phase, with the signals matching almost exactly.

A few additional things can be noticed about the system's step response in general. Around the peak of the response, the nonlinear model diverges slightly. This could be attributed to unknown, modelled nonlinear dynamics, and this is not necessarily an indication of the controller performing poorly. Next to note is that after the signal re-meets the reference signal at approximately 4 seconds, it begins to drop away gradually from the commanded steady-state value of 1.5. This could be due to physical phenomenon that the controller struggles to maintain the commanded C^* , since the aircraft in this time continues to pitch upwards. It is evident through comparing the direct implementation and the V-based implementation that the V-based approach is necessary in the nonlinear implementation for this case scenario.

Step Response with Disturbance Rejection

Once again, as with the nominal 2DOF linear design case, the same input and output disturbances are applied on the input δ_e and output C^* . The results are shown in [Figure 7.65](#):

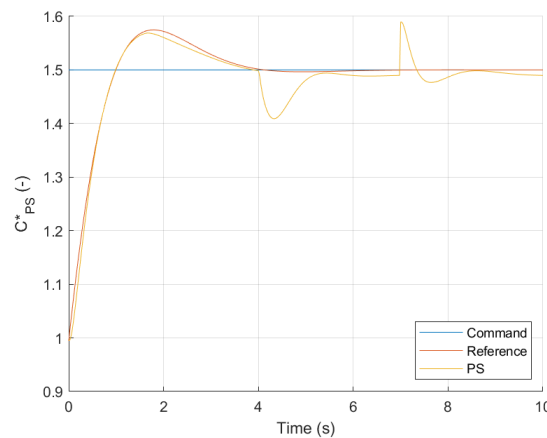


Figure 7.65: Step response of 2DOF controlled nonlinear system with input and output disturbances applied

First to note is that the response tracks the initial rise phase of the reference model very well, as in the case of the V-based response, and the same deviation from the reference model after the peak is seen. It can further be seen that even still in the nonlinear case, the robust controller exhibits very good DR properties, where the response doesn't deviate very highly in magnitude from the undisturbed response. Furthermore, the disturbed response appears to resettle rather quickly for both disturbance types, with a settling time within 3 seconds for both. It can be concluded that the suitable DR characteristics are successfully translated into the nonlinear design.

Comparison Between Linear and Nonlinear Responses

To validate the implementation of the linear controller design in the nonlinear model, the linear and nonlinear responses are simulated and compared. Between the design of the linear controller model and the nonlinear controller model, some aspects in the design process differ to each other. The main notable difference between them is the nature of the signal placed in FB. Due to the fact that the trim point is not known along the trajectory of a given manoeuvre that differs significantly from the trimmed point (such as a substantial C^* command input), the values that need to be fed back in the nonlinear controlled model are no longer the deviation of the C^* measured value from the trimmed point, as was the case with the linear controlled model, but instead the actual total measured value. In this case, this would be the actual C^* value that is measured during the manoeuvre. This is necessary as the controller was designed to operate around a specific flight point and any large deviations from the trimmed point could cause the controller to fail. In turn, the commanded input to the controller also becomes the total C^* command rather than the deviation in input command, as in the linear case. Seeing that the linear model reflects the deviation from the chosen equilibrium point, the initial value of the linear simulation is equal to 0.

To amend this, the linear response should be adjusted. This is achieved in practice by adding a switch to the linear simulation model, which subtracts the C^* trimmed value from the input signal. During plotting, the trimmed value is re-added to the total signal such that the response is correctly translated. This method ensures that both models, linear and nonlinear, receive the correct form of input and FB signals to produce responses that reflect reality. The step responses of the linear and nonlinear systems are shown in [Figure 7.66](#):

Comparing the nonlinear step response to the linear step shows that the nonlinear model almost exactly approximates the linear response up until the response peak. After this point, it shows the same shallower deviation as explained previously. The two responses appear to almost re-meet at approximately 4 seconds, before diverging from one another as time progresses. It is interesting to note that both simulations suffer from the same diverging steady-state 'hold' effect, but at different rates. In reality, such a high C^* value would not be sustained for the simulated amount of time, and therefore the effect is not of concern.

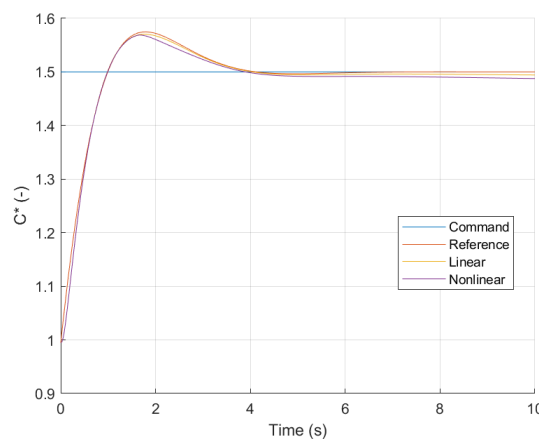


Figure 7.66: Step response of 2DOF controlled linear and nonlinear systems

Staircase Response

After investigating the step response characteristics of the nonlinear model, the staircase response was also tested to assess how the controller within the nonlinear model would behave in response to an

extended and varying input. The staircase input that was used was identical to that used in [Section 7.4.6](#). The nonlinear staircase response compared to its linear counterpart is given by [Figure 7.67](#):

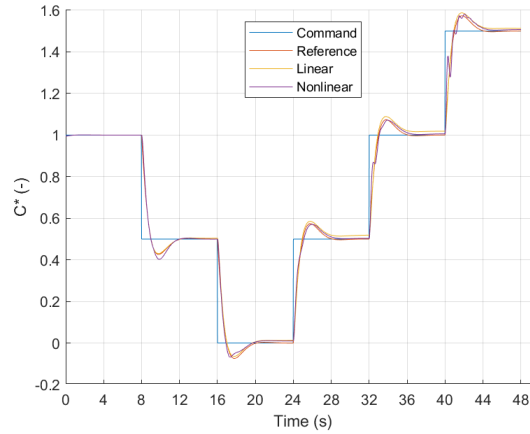


Figure 7.67: Staircase response of 2DOF controlled linear and nonlinear systems

Starting at the initial response, the commanded C^* trim condition, it can be seen that the nonlinear signal is well held at the trim point for the duration of the trim hold, a result of successful initialisation. Moving along the signal trajectory, it can be seen that the nonlinear response tracks the reference and follows the linear signal almost perfectly, but overshoots slightly on the first step input at around $t = 9$ s. On the second step input to $C^* = 0$ between $t = 16$ s and $t = 24$ s, the nonlinear signal appears to show some dropback rather early on in the response peak. Like the linear response, it returns to a steady-state value that is slightly positively offset from the commanded signal value. On the next step rise, the nonlinear model appears to approximate the reference signal even more closely than the linear model, and the steady state value is equal to the commanded input. This is not the case for the linear signal.

The nonlinear model begins to show signs of struggle on the step input to $C^* = 1$, where the signal begins to inflect upon itself during the rise and peak phases. The degradation of the nonlinear signal especially begins to show on the final rise step to $C^* = 1.5$, with greater oscillatory behaviour, both during the rise and peak phases. The signal does not become unstable, however.

This negative effect, not necessarily of great impact in a simple step input, can be exacerbated by the repeated large inputs of a staircase input, which can cause delay and decrease corresponding PMs, leading to small oscillations.

Investigating the reason for this behaviour, the first viable possibility is that the rate limit could be repeatedly hit for this design. However, despite a high maximum actuation rate as calculated using CS_o of 24.3166 deg/s, this does not violate the actuator rate limit of 25 deg/s that was established prior to the study. Furthermore, to check for certain whether rate limits have been hit, the deflection and deflection rate plots are shown in [Figure 7.68](#) and [Figure 7.69](#), respectively:

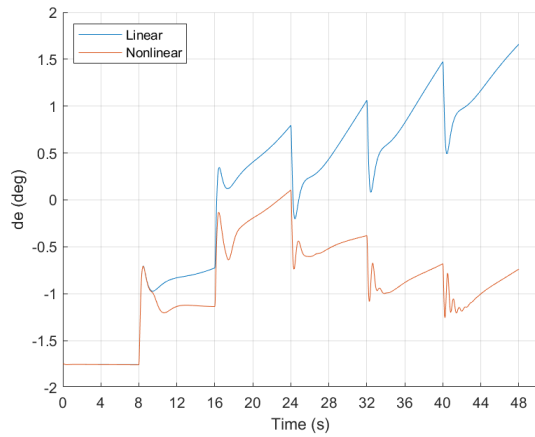


Figure 7.68: Staircase elevator deflection response of 2DOF controlled linear and nonlinear systems

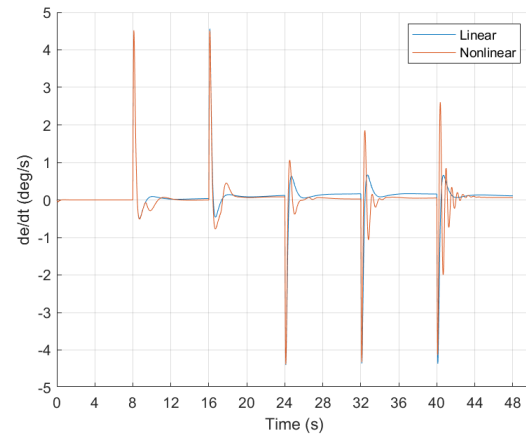


Figure 7.69: Staircase elevator rate response of 2DOF controlled linear and nonlinear systems

Figure 7.68 shows that the deflection of the linear and nonlinear models match exactly initially, but very soon begin to drastically diverge from one another. This can occur because the controller is operating in a regime that the linearisation did not capture. This effect was not initially visible in the linear design as it did not include nonlinear effects, rate limitation and saturation. This is a phenomenon that should be identified in the nonlinear design phase and corrected by controller design revision. Another cause that is in close conjunction with this is integrator windup. Due to the presence of the integrator after the pitch rate IL, it is possible for integrators to continue to accumulate error and pass large and oscillatory elevator inputs to the aircraft. If windup is present, anti-windup techniques can be applied. However, the tell-tale sign that the actuators have been saturated is through clipping of the signal, where a maximum value is reached and maintained. This would also be present in the rate signal, shown in Figure 7.69, but this is also not the case. Therefore, it can be concluded that actuator saturation is not the cause.

The more likely explanation for the oscillation present at the end of the nonlinear time simulation, as well as the drifting of the deflection from the linear case, can also be explained by the aircraft behaviour drifting from the trimmed operating point. The trimmed operating point is selected as $C^* = 1$ for steady, wings level flight, and it is this point around which the controller was designed. Very large magnitude inputs far from this operating point, as is the case with a staircase input with each subsequent value having its own particular trim characteristics, can cause the aircraft to operate in a regime (altered airspeed) for which the controller was not designed, especially if these large magnitude inputs are applied repeatedly in short succession.

Multi-Modelling

Multi-modelling (MM) is the concept within aerospace engineering whereby multiple, distinct mathematical models are used to represent vehicle dynamics over a range of different flight conditions, rather than using one global model. The flight condition can vary in terms of altitude, Mach number, AOA and COG for instance. By considering a set of systems during the design, rather than just one corresponding to a single flight point, aircraft designers are able to produce a controller that works more effectively across the whole operational FE, withdrawing the possible arduous work to produce a gain scheduled controller to work for each flight point, which can aid in relieving effort during the design process. It is a way for flight control systems to achieve a high performance across a wide flight envelope.

The first stage in design using MM is the obtaining of the systems corresponding to each flight point within the FE. This is done by running the 'FV_Main_Modelling' script for each combination of Mach number and altitude. The SP model is then obtained in the same fashion as earlier detailed in [Chapter 5](#). The step plots corresponding to the C^* response for each of the models within the FE are shown in [Figure 8.1](#):

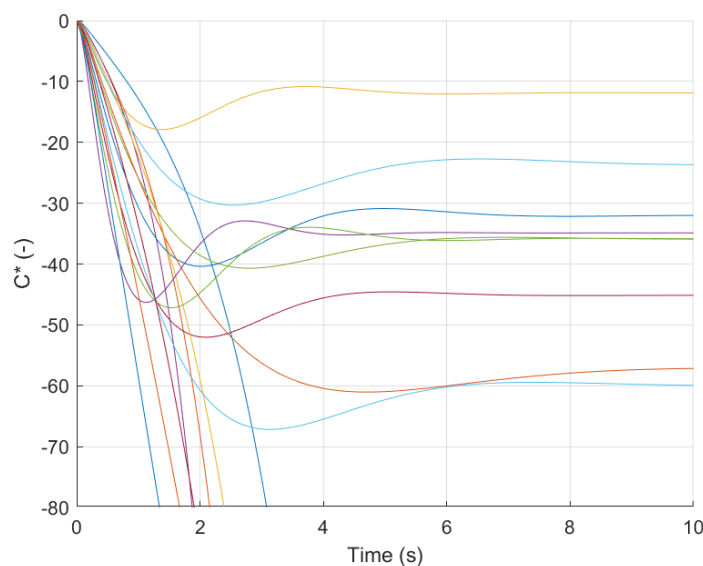


Figure 8.1: Step response of all original SP systems corresponding to varying flight points

It can be seen from the plot that not all of the models are stable, since some exhibit the SP mode while other models diverge towards instability, which was earlier outlined in [Chapter 5](#). Therefore, only the stable selection of flight points are considered during the MM design. The flight point of $M = 0.2$ and $h = 0$ m is also a stable flight point; however, due to the large difference in response compared to the other set of stable flight points, it is left out of the design. The remaining set of flight points are in close proximity to one another in terms of Mach and altitude, and are provided in [Table 8.1](#):

Mach	Altitude [m]
0.4	3250
0.4	5450
0.5	3250
0.5	5450
0.5	7650
0.6	3250
0.6	5450
0.6	7650

Table 8.1: Set of stable flight points used in the MM design

After the stable flight points are obtained, pitch rate damping is applied using gain scheduling to each of the systems using Matlab's 'sisotool' function, to obtain gains that would ensure a pitch rate damping of 0.7 for each system. It is done in this way to mirror the earlier 1DOF and 2DOF design processes, however, having to gain schedule any FB loop is not the most ideal approach, as mentioned before. This was done for the sake of contingency, but alternative options such as obtaining the gain using systune concurrently with the feedforward and feedback controllers, should be explored. Each of the systems with pitch rate damping and scaling gain applied are shown in [Figure 8.2](#):

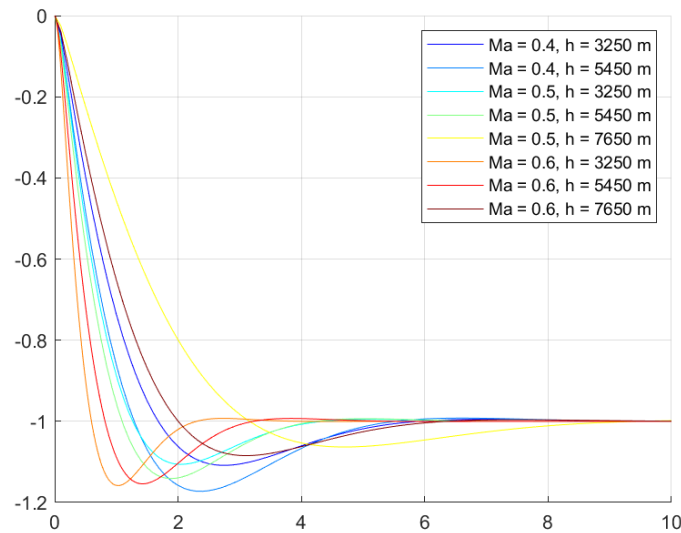


Figure 8.2: Step response of set of pitch rate damped and scaled aircraft systems to be used in the MM design

8.1. H_∞ Controller Design

The first stage in the H_∞ -LS MM design is the LS process, which was done previously in the 1DOF and 2DOF designs using the prior weighting filter W_1 and took the form of a simple scaled integrator. The plant

that was used previously in the design was the aircraft system corresponding to the flight point $M = 0.5$ and $h = 5450m$. In the systune aspect of the design, the plant that was used was this aircraft system, with only pitch rate damping applied, so that it represented the inner loop dynamics. The scaling of the plant dynamics did not matter too much since this could be incorporated into the outer loop weighting filter W_1 .

In the MM case however, all of the plants corresponding to the set of flight points have different magnitudes of DC gain, and therefore, each of the inner pitch rate damped loops are first scaled by an appropriate gain C_{sc} , such that all of the plants are normalised. In this way, the selection of the weighting filter W_1 which is applied to all plants, will have a more consistent effect across all models.

Firstly, the pole-zero map corresponding to the C^* transfer of each of the pitch rate damped and scaled flight point models is shown in Figure 8.3:

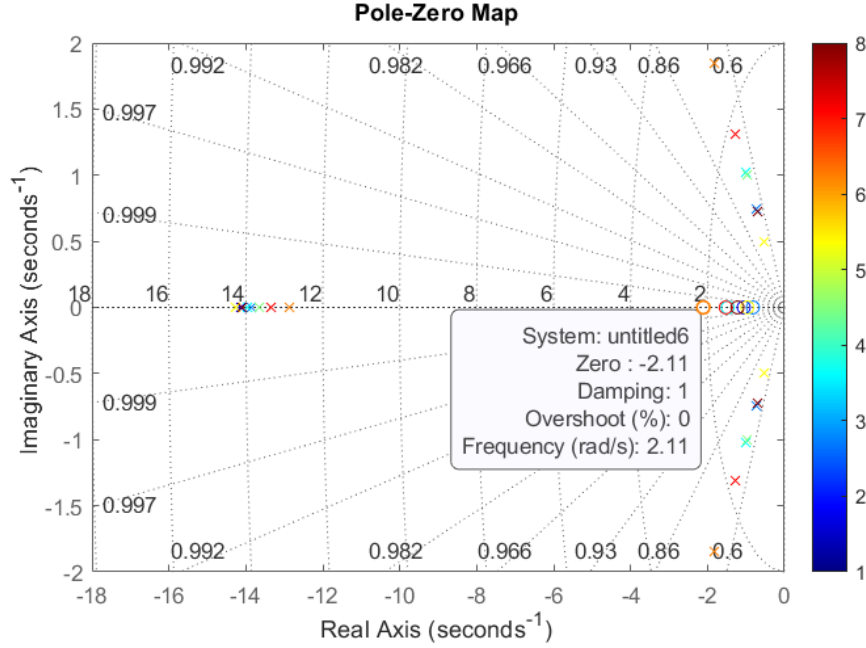


Figure 8.3: Pole-zero map of C^* open loop gain for all pitch rate damped and DC gain scaled flight point models

From the pole-zero map, it can be seen that the 6th model, corresponding to the flight point of $M = 0.6$ and $h = 3250m$, has a zero at a location of 2.11 rad/s. It was earlier mentioned in Chapter 5, that the reference model T_{ref} had a -3 dB frequency of 2.4 rad/s. The zero in this location means that the phase associated with this model is severely affected at the frequency where it is needed, near crossover, as zeros work to reduce phase. In the previous 1DOF and 2DOF design, a simple integrator as weighting filter W_1 proved to be satisfactory. However, it was established that for the MM design, this would no longer be the case; it was found that in order to provide sufficient tracking time-domain performance, as well as provide sufficient gain and phase margins, a more complex weighting filter in the form of a lead filter would have to be introduced into W_1 to promote higher CMs and DMs. A standard lead filter is given by Equation 8.1:

$$W_{lead} = \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \quad (8.1)$$

where ω_z is the frequency location of the filter zero, and ω_p is the frequency location of the filter pole, respectively. The main goal of the lead filter was to boost phase around the crossover and reference model -3 dB location. To do so, ω_z should be selected as being below the crossover frequency to initially raise phase, and ω_p should be selected as being above the crossover frequency, to decrease the phase

once again afterwards. Therefore the zero was chosen as being at 1.6 rad/s and the pole was chosen as being at 5.8 rad/s, as the pole and zero locations surrounded the crossover frequency. As well as this filter, another lead filter was introduced in series with the first, at slightly higher frequencies as part of a two staged lead, to aid in providing a moderate phase boost. The two-staged lead takes the form given in Equation 8.2:

$$W_{lead} = \frac{1 + \frac{s}{\omega_{z1}}}{1 + \frac{s}{\omega_{p1}}} \cdot \frac{1 + \frac{s}{\omega_{z2}}}{1 + \frac{s}{\omega_{p2}}} \quad (8.2)$$

and for the second stage of the filter, a zero location of 3.5 rad/s and a pole location of 14 rad/s was selected. Lastly, a low-pass filter is added to roll-off at high frequency, which has the form of Equation 8.3:

$$\frac{1}{1 + \frac{s}{\omega_{HF}}} \quad (8.3)$$

The frequency location of the roll-off was chosen as being equal to 10 times the frequency of the highest lead filter pole, thus 140 rad/s. Lastly, the existing integrator filter was tuned to adjust the overall crossover bandwidth of the controller. Therefore, the final weighting filter W_1 is given as Equation 8.4:

$$W_1 = \frac{6.65}{s} \cdot \frac{1 + \frac{s}{1.6}}{1 + \frac{s}{5.8}} \cdot \frac{1 + \frac{s}{3.5}}{1 + \frac{s}{14}} \cdot \frac{1}{1 + \frac{s}{140}} \quad (8.4)$$

The bode plot of the C^* open loop gain for all pitch rate damped and scaled models is given by Figure 8.4:

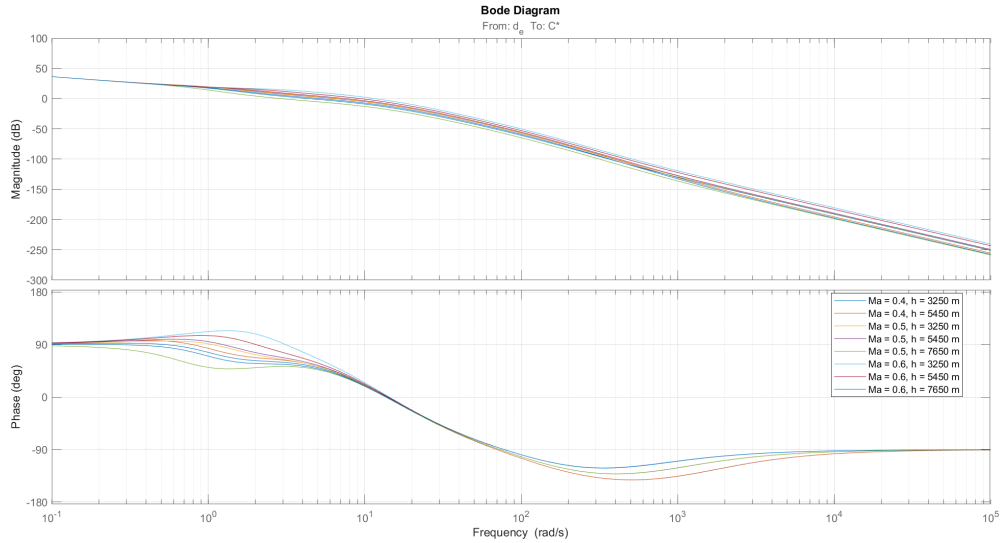


Figure 8.4: Bode diagram of C^* open loop gain for all pitch rate damped and DC gain scaled flight point models

The effect of first scaling all of the models is visible in Figure 8.4, as it can be seen from the magnitude plot that all of the models have the same magnitude response in the LF range, before beginning to diverge in the MF range. This highlights the benefits of scaling the plants first, before applying W_1 .

Additional differences in the systune design setup include setting the tracking weighting filter ρ equal to 1.75, to aid in reference model tracking, and setting the maximum gain constraint as a soft constraint, rather than a hard constraint. This was done in order to reduce the effort taken by systune to create a feasible controller that stayed within robustness level limits. Lastly, the order of the feedback controller

was increased by 1 to be 3rd order, and the feedforward controller was increased by a couple of orders to 5th order, as the previously obtained 2nd and 3rd order controllers, respectively, were no longer sufficient in mapping the signal to the reference model. The resulting controllers obtained by systune are given by Equation 8.5 and Equation 8.6 respectively:

$$\frac{-2.2401(s + 1.204)(s^2 + 25.25s + 223.4)}{(s + 1.495)(s^2 + 37.16s + 579.2)} \quad (8.5)$$

$$\frac{-0.47593(s + 4.692)(s + 1.091)(s + 8.225e - 05)(s^2 + 27.59s + 238.5)}{(s + 8.225e - 05)(s^2 + 1.959s + 1.929)(s^2 + 38.94s + 669.7)} \quad (8.6)$$

These controllers had a corresponding robustness level provided by systune as $\epsilon = 0.3264$, therefore there is some healthy margin in the RS of the design.

8.2. Handling Quality Criteria

After obtaining the controller using systune that would be able to robustly stabilise all of the plant models, a HQ assessment was performed once again.

8.2.1. CAP Criterion

Looking firstly at Figure 8.5, it can be seen that all of the controlled models obtain the same CAP criterion value within level 1 and almost exactly equal to the reference CAP value.

8.2.2. Gibson Criterion

Subsequently, looking at Figure 8.6, it can be seen that all of the controlled flight point models satisfy the Gibson criterion to varying degrees. It appears that the models that lay furthest from the reference model point are the centre models, including $M = 0.5, h = 7650m$ and $M = 0.6, h = 3250m$. This phenomenon could be explained by the fact that the controller is trying to robustly stabilise all plant models, including the edge cases, and therefore loses some tracking performance around the centre models. The latter model corresponds to the earlier mentioned model which initially had a zero location at 2.11 rad/s, and therefore of all the models, this was the one which was the most challenging to ensure RS and RP.

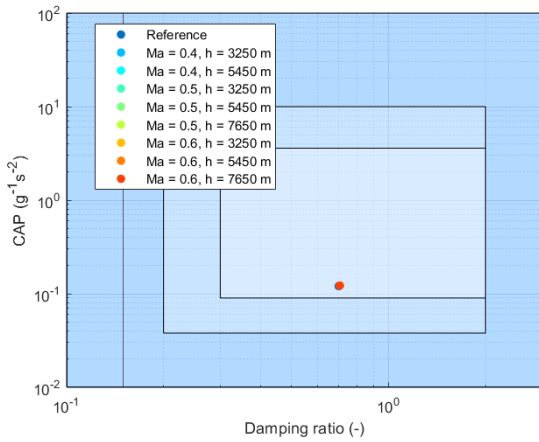


Figure 8.5: CAP criterion corresponding to the MM design

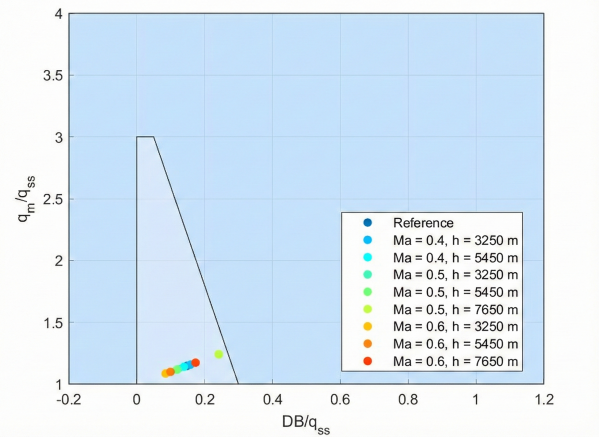


Figure 8.6: Gibson criterion corresponding to the MM design

8.2.3. Attitude Bandwidth Criterion

The attitude bandwidth criterion for the MM design can be seen in Figure 8.7. It can be seen that all models lay within the level 1 HQ region, each with very similar results for the magnitude of the phase delay and with varying magnitudes of pitch attitude bandwidth.

8.2.4. Flight Path Bandwidth Criterion

Figure 8.8 shows the results of the MM design for the FPBW criterion. It can be seen that all points lay comfortably within the level 1 HQ region of the criteria, despite being allowed to drop to level 2. This indicates that the controller design exhibits sufficient FPBW performance.

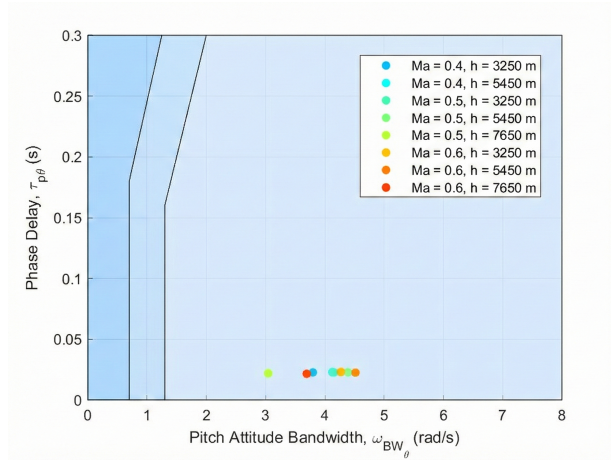


Figure 8.7: Attitude bandwidth criterion corresponding to the MM design

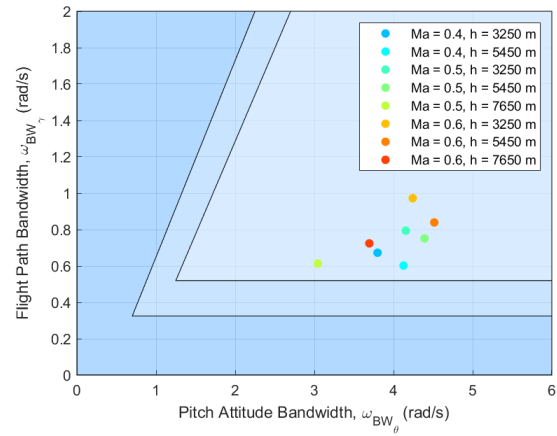


Figure 8.8: Flight path bandwidth criterion corresponding to the MM design

8.2.5. Original, Shaped and Robustified C* Loop Gains

The bode diagram of the reference model super imposed with the original C* open loop gains, shaped C* open loop gains and robustified C* open loop gains for each of the flight point models in the MM design, is shown in Figure 8.9:

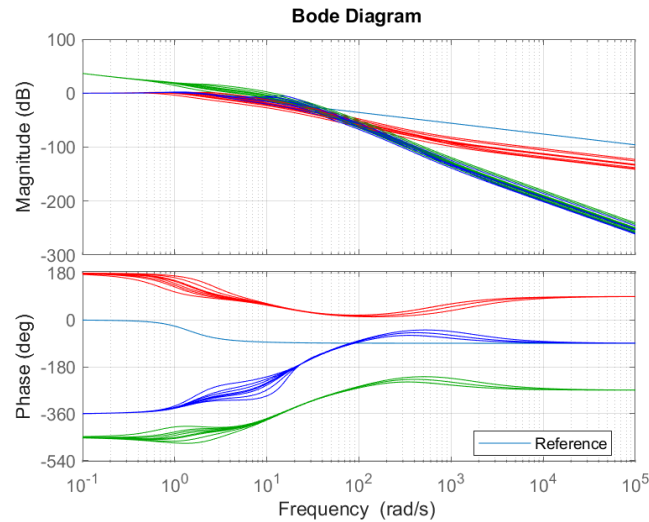


Figure 8.9: Bode plot of reference model, original C* open loop, shaped C* open loop and robustified C* open loop resulting from MM

with the reference model in cyan, the original loop in red, the shaped loop in green, and the robustified loop in blue. From the plot, it can be read that the crossover frequency of the shaped loop gain corresponding to the model with the lowest BW is set to just above the -3 dB frequency of the reference model, equal to 2.4 rad/s, with a value of 2.96 rad/s. All of the other models in the set have a higher shaped loop crossover frequency. This ensures that all of the models should in principle have the BW to be able to sufficiently track the reference model. After robustification, the crossover frequency of the robustified loop is reduced

such that the crossover of the lowest BW model is equal to 1.37 rad/s, which is approximately equal to the reference model BW of 1.4 rad/s.

8.3. Frequency-Domain Analysis

Since the HQ performance has been assessed for the MM design, now the stability can be assessed in terms of frequency-domain performance. This once again takes the form of CMs, DMs, Nyquist and Nichols diagrams.

GM and PM guidelines prescribed an absolute GM of 6 dB and a PM of 45° , although this can be allowed to drop to a value that is at least above 30° . Whether each particular flight point satisfies these stability guidelines is indicated by colour coordination, as in Table 8.2:

	Satisfies condition
	$30^\circ \leq \phi \leq 45^\circ$
	Does not satisfy condition

Table 8.2: Colour coordination for stability criteria satisfaction

8.3.1. Classical Gain Margins

The classical GMs obtained for the design are given by Table 8.3:

Flight point		Classical Gain Margin [dB]			
Mach [-]	Altitude [m]	δ_e	n_z	q	C*
0.4	3250	16.8754	34.3465	17.1019	16.8859
0.4	5450	14.8510	37.6238	14.9793	14.8757
0.5	3250	14.4766	29.6338	14.7404	14.4866
0.5	5450	13.3149	32.3100	13.4845	13.3338
0.5	7650	22.4902	35.5052	22.5997	22.4931
0.6	3250	8.0708	23.6463	8.3380	8.0680
0.6	5450	10.9469	30.4604	11.0712	10.9564
0.6	7650	18.8097	32.6473	18.8599	18.8242

Table 8.3: Classical GMs obtained for the MM design

Looking at the table, it can be seen that all of the channels corresponding to all flight points satisfy the classical GM requirement of $GM \geq 6\text{dB}$. This is especially the case for the n_z channel, which always exhibited the greatest margins out of all of the loops. The flight point which comes closest to violating the GM requirements is $M = 0.6, h = 3250\text{m}$ with GMs of approximately 8 dB across all channels except n_z , but this still allows a healthy margin of approximately 2 dB above the guideline, and it remains that all classical GMs are sufficient in this design.

8.3.2. Classical Phase Margins

Next, the obtained classical PMs are tabulated in Table 8.4. Once again, it is the flight point of $M = 0.6, h = 3250\text{m}$ which suffers the most in terms of classical SMs. All classical PM guidelines are satisfied with at least a few degrees margin, except for this flight point, where the δ_e , q and C* channels achieve classical PMs where $PM \leq 45^\circ$. It is however still the case that $PM \geq 30^\circ$, and therefore the classical PM performance is not considered poor. In the case of a redesign, it is desired to boost the phase at this flight point further such that $PM \geq 45^\circ$ for all flight points.

Flight point		Classical Phase Margin [deg]			
Mach [-]	Altitude [m]	δ_e	n_z	q	C*
0.4	3250	66.9543	162.6312	71.1431	68.4061
0.4	5450	61.7098	171.5610	63.5134	65.3600
0.5	3250	60.6987	138.5534	64.0761	63.2355
0.5	5450	56.3841	141.3589	58.3304	60.4871
0.5	7650	64.0849	140.9711	74.5006	64.2128
0.6	3250	36.7742	132.0713	37.6943	39.9838
0.6	5450	47.9309	130.9866	49.1999	51.9404
0.6	7650	66.2204	129.1791	73.0833	68.1813

Table 8.4: Classical PMs obtained for the MM design

8.3.3. Disk Gain Margins

Table 8.5 shows the tabulated results for the DGMs for all channels and flight points:

Flight point		Disk Gain Margin [dB]			
Mach [-]	Altitude [m]	δ_e	n_z	q	C*
0.4	3250	11.1073	24.8507	11.5506	11.2735
0.4	5450	9.5052	28.6443	9.7398	9.8524
0.5	3250	9.2412	23.2688	9.6736	9.4797
0.5	5450	8.3257	25.6004	8.6056	8.6891
0.5	7650	12.3093	19.1402	16.0366	12.3652
0.6	3250	4.8013	21.2320	4.9964	5.0429
0.6	5450	6.6669	24.4150	6.8810	6.9928
0.6	7650	12.1065	20.6957	12.7686	12.4191

Table 8.5: DGMs obtained for the MM design

It can be seen that for the troublesome flight point, the obtained DGM falls short of the 6 dB requirement for all channels except for n_z , once again. Whilst values of 4.8 dB, 5 dB and 5.04 dB are not poor, further consideration during the design phase is required to boost these results above the desired threshold of 6 dB.

8.3.4. Disk Phase Margins

The last area of interest in terms of SMs of the MM design are the DPMs, provided in Table 8.6. In this case, it is no longer only the flight point of $M = 0.6, h = 3250m$ that shows insufficient margins, but also the adjacent flight point $M = 0.6, h = 5450m$. This point only just falls short of the guidelines however, with PMs approximately equal to 41° . Additionally, the PMs of the previous flight point are once again damaged, with values only just above the threshold of 30° . For both of these flight points, the DPMs remain in the amber zone, and should not decrease further than this. It is therefore recommended once again to alter the design to accommodate for more generous SMs across all flight points.

Flight point		Disk Phase Margin [deg]			
Mach [-]	Altitude [m]	δ_e	n_z	q	C^*
0.4	3250	58.8880	83.4515	60.3663	59.4500
0.4	5450	52.9827	85.7661	53.9043	54.3397
0.5	3250	51.9211	82.1470	53.6463	52.8811
0.5	5450	48.0409	83.9919	49.2607	49.6188
0.5	7650	62.7484	77.3995	72.0628	62.9168
0.6	3250	30.1718	80.0811	31.2782	31.5401
0.6	5450	40.2037	83.1154	41.2735	41.8256
0.6	7650	62.1295	79.4527	64.1036	63.0783

Table 8.6: DPMs obtained for the MM design

8.3.5. Nyquist and Nichols Diagrams

The Nyquist and Nichols plots for all models are shown in Figure 8.10 and Figure 8.11, respectively:

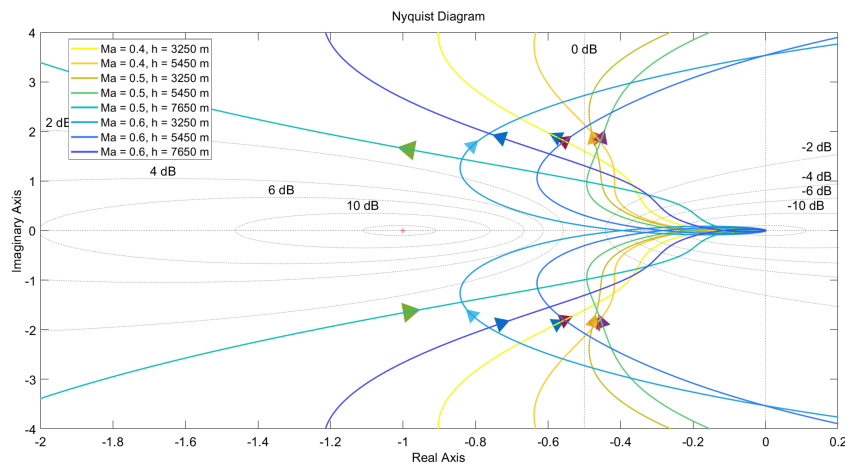


Figure 8.10: Nyquist plot of all flight point models after applying robustifying MM controller

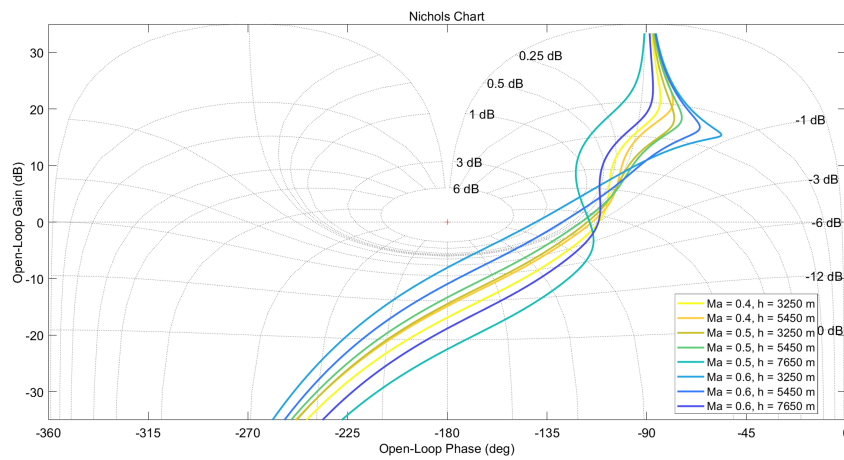


Figure 8.11: Nichols plot of all flight point models after applying robustifying MM controller

It can be seen that all models are robustly stabilised by the controller with DMs correlating to the tabulated results.

8.4. Time-Domain Analysis

The next stage of the MM design is the time-domain performance assessment, in which comments will be made about the ability of each of the controlled flight point systems to track the reference model signal. The step response of each of the systems in response to a step input of magnitude 0.5 is shown in Figure 8.12:

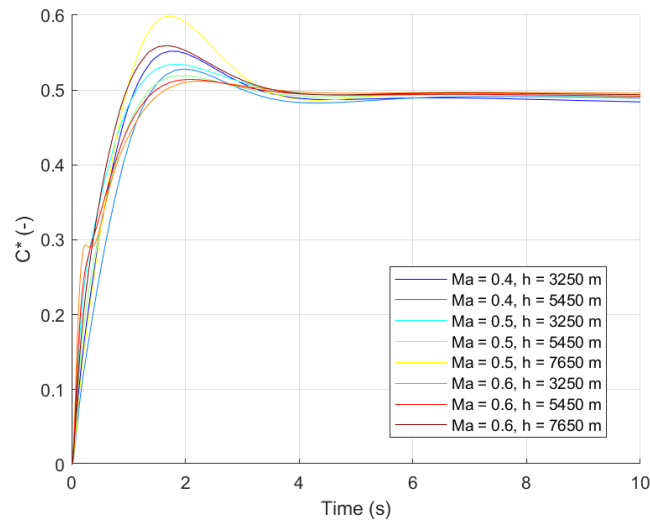


Figure 8.12: Step response of each of the flight point systems under the MM controller

It can be seen from the time-domain response that each of the models respond rather well to the step input in that they all emulate the shape of the reference signal, but with the caveat of having varying amounts of overshoot. This is especially the case for the model $M = 0.5, h = 7650m$, which shows an overshoot of approximately 0.5 relative to the reference model. This was earlier indicated on the Gibson plot in Figure 8.6, but this showed that even this response laid within acceptable Gibson HQ limits. This overshoot can once again be attributed to this model having the lowest frequency pair of SP poles out of all of the models, which was visible in Figure 8.3, and thus the slowest SP response out of all of the models, visible in Figure 8.2. This fact thereby made the choice of the magnitude of the filter W_1 more crucial in ensuring that the resulting BW of this controlled system was sufficient to be able to track the reference. To circumvent this overshoot, the controller bandwidth could be increased further through the increase in W_1 to impose tighter tracking in the time-domain; however, upon attempting this it was found that this would have violated the CMs and DMs too significantly.

Looking further at Figure 8.12, it can be seen that a small dip is present during the signal rise for the model $M = 0.6, h = 3250m$. This is once again due to the presence of the zero at 2.11 rad/s in the original flight point model, which as a result, reduced overall GMs and PMs corresponding to that system, and hence the tracking performance of the signal is slightly reduced. The remaining signals track the reference model relatively well, with only minor deviation in terms of overshoot. It is however recommended for future work that a more intuitive weighting filter is selected for W_1 that enables tighter tracking of the reference model without compromising the SMs across all of the flight points.

Conclusions & Recommendations

This research marked a first attempt at using the H_∞ -LS technique to robustly stabilise the given plant, the Flying V aircraft model. This work aims to open up the discussion and the possibility of revising this technique and design for the consideration of its future implementation within this aircraft. As well as this, it contributes to the growing body of work on the subject of the H_∞ -LS method. It can be concluded that a well functioning and robustly stable 2DOF controller was successfully designed using systune, which to a large degree, was able to satisfy stability and HQ assessment metrics that were identified prior to the work's undertaking.

This concluding chapter will firstly focus on answering the research questions that were proposed at the beginning of this study, followed by proposing some recommendations for future work within this scope.

9.1. Revisiting the Research Objectives and Research Questions

In order to reflect on the success of the research, first of all, the research objectives as first stated in [Chapter 3](#) are reiterated, and the degree to which each of these research objectives are achieved is reflected upon.

Research Objective

Develop a robustly stabilising 2DOF H_∞ -LS controller for the C* flight control law present on the Flying V aircraft concept to guarantee closed loop stability and performance over the full operational envelope

Research Objective

Assess and validate the stability and performance of the closed loop against existing handling quality and stability requirement metrics in the presence of uncertainty and disturbances

It can be concluded that the study was able to successfully design a 2DOF controller using H_∞ -LS, and its performance has been comprehensively assessed against many applicable HQ and stability assessment criteria under a large variety of assessment scenarios (disturbance rejection, with applied uncertainty, nonlinear simulation). Stability and HQ assessment showed that stability was ensured for all flight points under all test cases and HQ criteria were met for a large number of flight points and test scenarios.

Research Question 1

What are the pre-requisites in terms of the dynamic modelling of the aircraft dynamics and the posing of a framework for a robustly performing and stabilising C* controller?

This research question poses the following sub-questions:

Research Question 1.1

In what stage of the development process was the Flying V aircraft model that was provided at the beginning of the research?

The Flying V aircraft model provided prior to this research was adapted by [Pedroso D. \(2024\)](#) into a properly trimmed and linearised tensor based formulation based on the construction of the F-16 model developed by [Naäman \(2024\)](#). The original condition of the model prior to the reorganisation was deemed insufficient for the purposes of robust control due to modelling errors which could lead to invalidity of results, as mentioned by [Traas \(2024\)](#). This reorganisation was essential as a properly trimmed and linearised model is highly important when working in robust control, since small deviations from the accurate trimmed point can result in largely deviating results in the presence of uncertainty. Furthermore, the flight envelope and the aerodynamic dynamic properties were investigated in order to determine the current state of the aerodynamic design of the Flying V aircraft. It was ascertained and supported that the aircraft aerodynamic design of the Flying V aircraft requires additional development in regard to the operational flight envelope.

Research Question 1.2

In what way should the model be pre-conditioned and adapted to suit the purposes of the application of robust control techniques?

The provided linear model obtained from the nonlinear Flying V aircraft model allowed the user to obtain reduced order state-space dynamic systems representing the aircraft lateral and longitudinal dynamics. The longitudinal dynamics could be represented in full or divided into those corresponding to phugoid or short period dynamics. These reduced order system did not initially contain applied aerodynamic uncertainty and it therefore needed to be incorporated into the modelling process. It was done so by introducing uncertain state-space system blocks into the Flying V aircraft's Simulink linearisation model. Parametric uncertainty was applied to the aerodynamic coefficients present in the aerodynamic model which were the most impactful on the longitudinal dynamics, C_{m_α} , C_{m_q} , $C_{m_{\delta_e}}$ and C_{z_α} . Uncertainty was defined in both Matlab and Simulink environments using the function 'ureal' within the Simulink model linearisation code in Matlab and a degree of uncertainty of 30% was set, resulting in uncertain SS systems. As a result of these changes, the linearisation method was changed from the function 'linearize', to the function 'ulinearize'. With this came some difficulties such as the lack of clarity in the state ordering of the resulting uncertain state-space system. From this, nominal systems for the longitudinal aircraft model could be obtained.

The next way in which the provided Flying V aircraft model needed to be altered for the purposes of the robust controller design was through the addition of an IL pitch rate feedback controller to obtain the desired pitch rate damping ratio, as well as the addition of the C^* signal to the dynamic system. As a result, the airframe system displayed desirable HQ characteristics prior to initiating the main robust controller design. The concept of NCF uncertainty was subsequently introduced to give indication of the largest magnitude of uncertainty that would need to be guaranteed by the robust controller.

Research Question 2

How can the design of a robustly stabilising H_∞ -LS C^* controller benefit the current FCS of the Flying V aircraft in terms of performance and handling, and what aspects are involved in the design?

This research question poses the following sub-questions:

Research Question 2.1

What design choices are required are required during the FCS design?

During the H_∞ -LS controller design, a number of design choices were considered. Preliminary stages in the research included the identification of key aircraft geometry, as well as the location of the ICR. Once the basic control structure was identified through literature, the relevant outputs became apparent: this were predominantly the pitch rate q , the normal load factor n_z and the C^* signals. The first decision required was concerned which of these signals to be used in OL FB. It was concluded that improved performance could be achieved by the combined demand approach of C^* OL FB (Gibson, 1999), and proven subsequently by Pedroso D. (2024) through the demonstration of balanced LF DR properties in the signals q and n_z , which provided the most confidence in favour of this setup. Additionally, it was decided to measure the C^* signal used in OL FB at the IMU location (assumed near the ICR), to minimise pitch acceleration contributions to the n_z derivative, beneficially resulting in improved HF roll-off characteristics. For tracking performance and HQ assessment however, it was decided to measure the load factor n_z used in the C^* signal at the PS, since the behaviour obtained at this location was more closely tied to cues inherently felt by pilots.

Consideration needed to be taken as to whether the design would facilitate a 1DOF or a 2DOF control structure. Looking first at the 1DOF setup, it was identified that placement of the OL FB controller in the feedback path, as opposed to the forward path of the OL, resulted in reduced overshoot, settling times and general sluggishness in the time-domain response to a commanded C^* input. It was also observed that despite this trade-off, the 1DOF control configuration still could not achieve suitable tracking performance in line with prescribed HQs corresponding to the reference model used (including CAP, Gibson and bandwidth criteria), and for this reason a feedforward controller was incorporated as part of a 2DOF design structure, which in this regard showed significant improvement compared to the original 1DOF design.

Research Question 2.3

How can the design performance and HQ requirements effectively be incorporated into the design process?

During the study, the required HQ criteria prescribed by military HQ requirements MIL-STD-1797A pertaining to the designed C^* controller for the Flying V aircraft were identified according to Mitchell et al. (1994) as the CAP, Gibson, attitude bandwidth and flight path angle bandwidth criteria. Each of the criteria are accompanied by a formulation computed based on parameters that could be derived through manipulation of the original Flying V airframe longitudinal dynamic model. In the same fashion, a generic, second order reference model was derived that satisfied all of the mentioned criteria to which the controlled system could be compared. By extension, if the controlled system were to approximate the reference model when assessed according to these HQ metrics, then the controller would exhibit adequate HQ performance, thereby providing an effective way to include HQ requirements into the design process.

In addition to HQ requirements, the LS process inherently possesses mathematical metrics used to assess the quality of the shaped open loop as part of the LS design. These are denoted by the robustness level ϵ , or its inverse value the performance level γ . These values obtained from systune after robustification of a shaped open loop provide a reliable indication as to whether the design performs well enough in terms of RS and RP, or whether the weighting filter design needs to be revisited.

Good tracking performance by the designed controller is ensured in different ways, the first of which is through the decision of weighting filters to shape the open loop dynamics. The weighting filter W_1 is chosen such that the crossover bandwidth of the shaped open loop is slightly higher than the -3 dB frequency of the reference model, since this gain value is determined to be the minimum required to follow a reference signal to an accurate degree. Furthermore, the addition of the exogenous error output signal e_M through the 2DOF (augmented 1DOF) control structure provides systune with a constraint that allows the program to minimise the error to the reference model, thereby improving tracking quality.

Research Question 3

To what extent does the controller meet performance and robustness requirements that were specified a priori?

This research question poses the following sub-questions:

Research Question 3.1

How can the result of the designed LS controller be verified against pre-established HQ and stability margin criteria?

As previously mentioned, HQ requirements can be incorporated into the design through the isolation of the relevant parameters from the dynamic models that are required for the assessment of each HQ criteria. This places the controlled system within each of the HQ requirements, assigning a particular HQ level achieved by the design for that criterion, and the proximity of the achieved point to that of the reference model and the HQ level boundaries indicates the degree of success of the design.

Through translation of the relevant dynamics into the frequency-domain rather than the time domain, it also allows for assessment of stability margins achieved by the design. Stability margins are predominantly assessed through the calculation of classical margins and disk margins, as well as multiloop margins. Tabulated results can be directly compared to pre-existing requirements on classical and disk-based gain and phase margins. Furthermore, the generation of plots such as Nyquist and Nichols diagrams can visually aid in analysis of the systems' stability properties, highlighting again the achieved disk margins and thus proximity to instability. Lastly, time-domain simulations are used as a visual, comprehensible check of the controller behaviour when compared directly to the desired reference model dynamics.

Research Question 3.2

To what extent does the LS controller design satisfy the specified HQ and stability margin criteria for the 1DOF and 2DOF designs?

Regarding the controller design, firstly, a 1DOF controller was designed. This was beneficial to become familiar with the loopshaping process and robust controller synthesis techniques (such as derivation from first principles from the AREs and Matlab's `ncfsyn`, `ncftune` and `systune`). Within this, an investigation into the effects of weighting filters was made, as well as a discussion of the approach to frequency domain analysis. It could be concluded from the 1DOF design that the controller stabilised the plant, but despite displaying good stability margins, did not track the reference model and thus could not enforce robust performance as prescribed by the HQ criteria presented in MIL-STD-1797A. To amend this, the 2DOF controller, consisting of a feedforward controller as well as a feedback controller, was designed by augmenting the standard 1DOF design case with an additional tracking performance signal, e_M . The optimal scalar magnitude for the weighting filter FF_ρ was identified, despite attempts at the incorporation of a dynamic weighting filter in this place. Subsequently, the effect of the controller order of both the feedforward and feedback controllers was analysed, both in terms of time-domain and HQ performance, to identify suitably sized controllers. Overall analysis of the 2DOF design concluded that the controller was robustly stable, showing good stability margins (classical and disk-based) as well as good tracking qualities in the time-domain, including to a varied staircase input and with disturbances applied on the input and the output.

Analysis of the 2DOF design with applied uncertainty showed that some of the systems did not achieve level 1 HQs in the Gibson criterion, and some systems fell into level 2 HQs for the flight path bandwidth criterion, although this is allowed for this criterion. Furthermore, worst-case stability analysis highlighted that whilst still showing decent disk-margin properties, these did not meet prescribed requirements of $DGM \geq 6$ dB and $DPM \geq 45^\circ$. For the nonlinear model, the simple yet effective V-based approach of [Kaminer et al. \(1995\)](#) was implemented, which resolved implementation issues. The nonlinear results showed fast disturbance rejection and little deviation from the linear results in the C^* time-domain step response. A small tremor was identified in the staircase results due to the compounding of controller effort over time across largely deviating inputs. Lastly, multi-model design produced a controller to stabilise all plants, displaying good CAP, Gibson, attitude bandwidth and flight path bandwidth characteristics, as well as largely sufficient disk-based stability margins.

9.2. Recommendations

Due to the nature of H_∞ -LS, the recommendations that will be made are largely related to the selection/identification of weighting filters. Notorious for the skill, understanding and experience required behind developing appropriate filters, the shortcomings of the research could potentially be radically improved in this area.

A first recommendation regarding the 2DOF design would be to investigate the effects of a dynamic filter for the scaling weighting factor FF_ρ . In the study, it was first attempted to use a scaling gain in this place. This appeared to reach a threshold of effectiveness as tracking performance did not see a significant improvement, however increasing the value of the parameter drove down further the robustness level ϵ as indicated by systune. A scalar value appeared to overly constrain the SVs associated with the performance signal TF, and therefore a dynamic filter was investigated. However, insufficient expertise in dynamic filter design meant that the designed filters did not achieve the intended goal, and therefore this could be researched further.

Furthermore, the selection of the weighting filter W_1 , and perhaps W_2 , could be looked into further. In this study, a simple scaled integrator weighting filter was used for W_1 , which achieved the aim of boosting the LF SVs of the C^* open loop. However, the 2DOF controller eventually displayed less than desired tracking performance for all combinations of uncertainty. It would be advised to engineer a more intuitive composition for W_1 , such that robust performance is improved but at minimal expense of the robustness level.

The same can be said for the two-staged lead-lag filter applied in the MM design. This was included to try to combat the effects of a zero which was un-ideally located, however expertise in this area may have provided a cleaner and more savvy approach to boosting the stability margins for this and surrounding flight points.

Another suggestion would be to re-conduct the systune optimisation using other initialisation settings and experimenting with varying design objectives. The design objectives included in this study regarded only maximum gain constraints, as well as the augmentation of the tracking performance signal in the 2DOF standard design configuration. The function systune is rather expansive in functionality, offering many objectives and settings to tailor to the design process. Deeper knowledge and understanding of the systune process could potentially result in less constrained and more advantageous designs.

Recommendation for future research would include the investigation into the design of a digitised controller and the effect of time delays on the performance and stability margins achieved by the design. This was previously conducted by [Pedroso D. \(2024\)](#), and would offer a nice comparison between the MS and LS techniques for the same application under digitisation.

Lastly, a key recommendation regarding the design of the Flying V aircraft would be to continue investment and research into the identification of the aerodynamic model. The less than fully mature current status of the design provided a challenging operational flight window. For this reason, a large proportion of aerodynamic uncertainty (30%) was applied to aerodynamic coefficients, which may have partially contributed to adverse effects observed under uncertainty. Solidification of the aerodynamic model will pay dividends when it comes to robust control design applied to the Flying V aircraft in the future.

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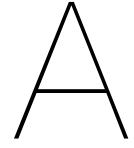
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Appendix A: 1DOF Results

A.1. Frequency-Domain Analysis

A.1.1. Disk Margins

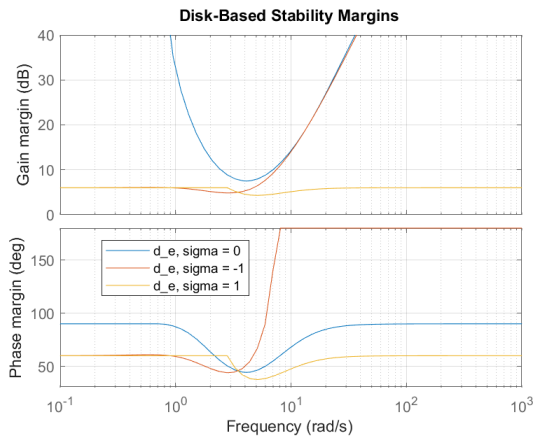


Figure A.1: 1DOF: Disk margin plot corresponding to input δ_e with varying skew

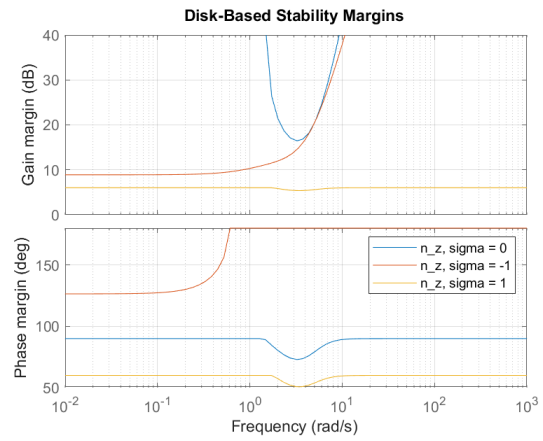


Figure A.2: 1DOF: Disk margin plot corresponding to output n_z with varying skew

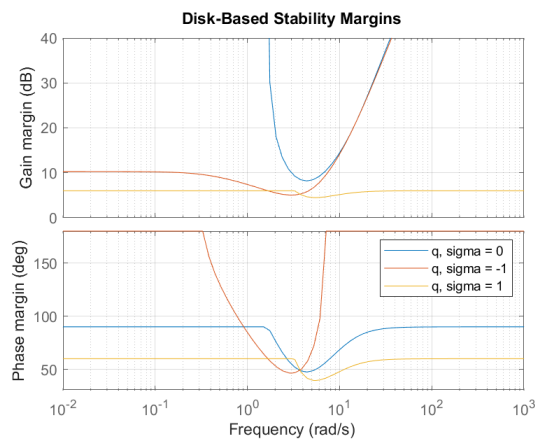


Figure A.3: 1DOF: Disk margin plot corresponding to output q with varying skew

A.1.2. Stability Regions

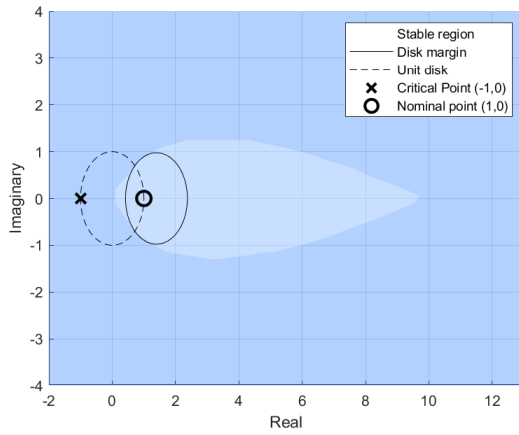


Figure A.4: 1DOF: Stability region corresponding to input δ_e , shown in the complex plane

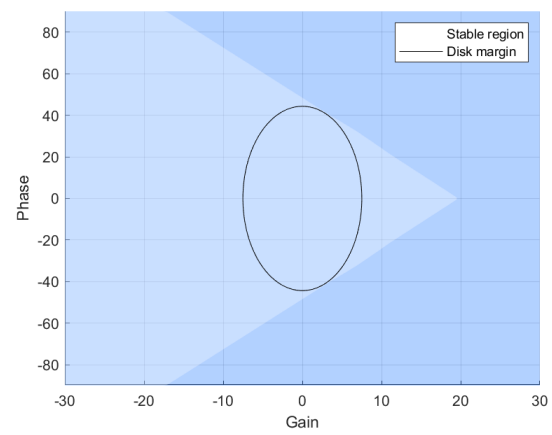


Figure A.5: 1DOF: Stability region corresponding to input δ_e , shown in the real gain and phase plane

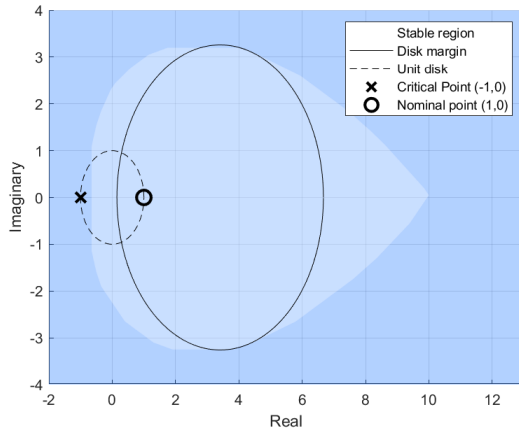


Figure A.6: 1DOF: Stability region corresponding to output n_z , shown in the complex plane

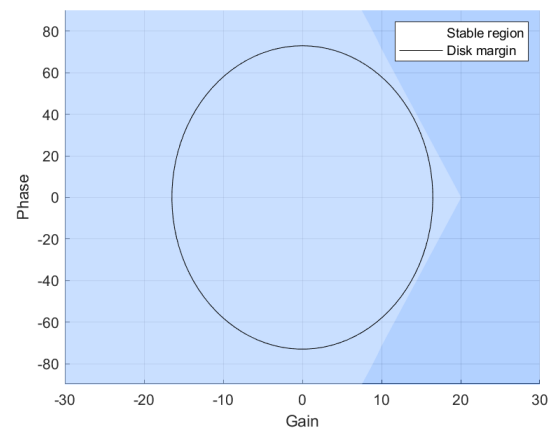


Figure A.7: 1DOF: Stability region corresponding to output n_z , shown in the real gain and phase plane

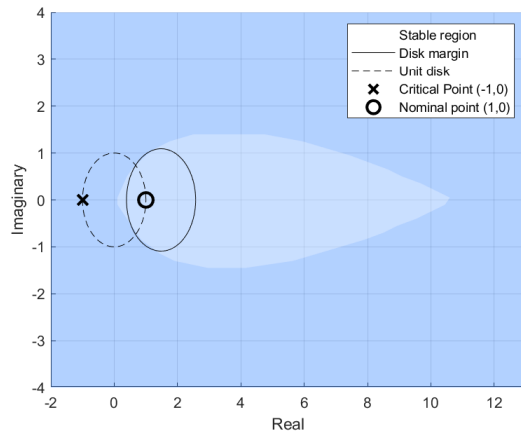


Figure A.8: 1DOF: Stability region corresponding to output q , shown in the complex plane

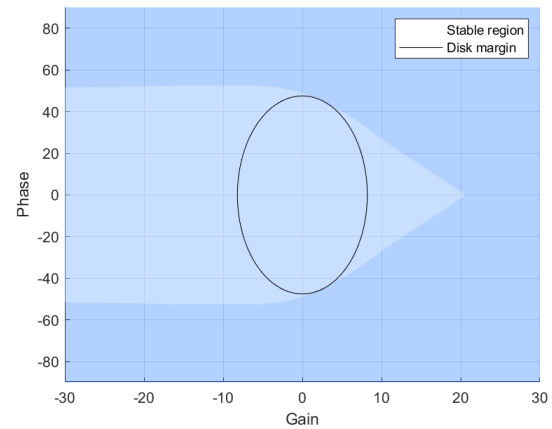


Figure A.9: 1DOF: Stability region corresponding to output q , shown in the real gain and phase plane

A.1.3. Nyquist Diagrams

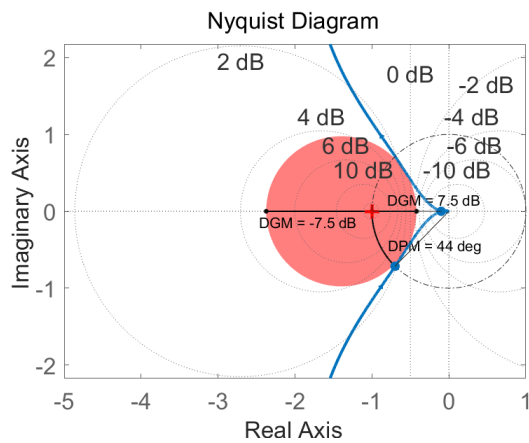


Figure A.10: 1DOF: Nyquist diagram corresponding to input δ_e

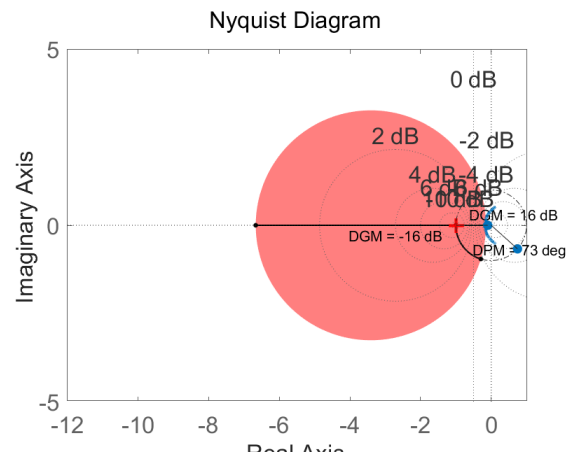


Figure A.11: 1DOF: Nyquist diagram corresponding to output n_z

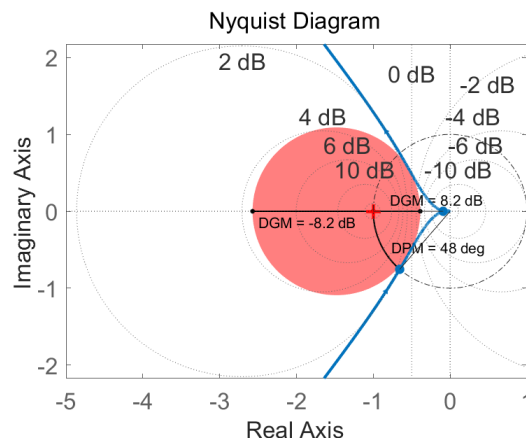


Figure A.12: 1DOF: Nyquist diagram corresponding to output q

A.1.4. Nichols Diagrams

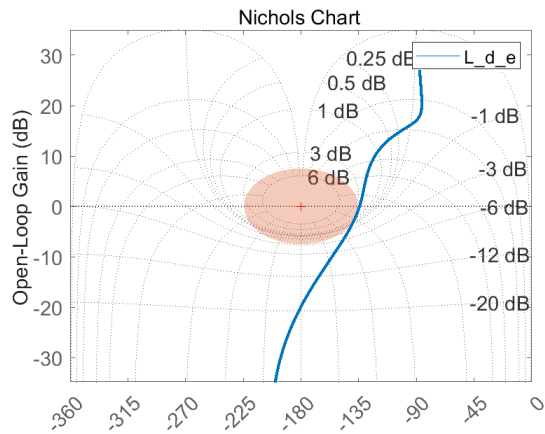


Figure A.13: 1DOF: Nichols diagram corresponding to input δ_e

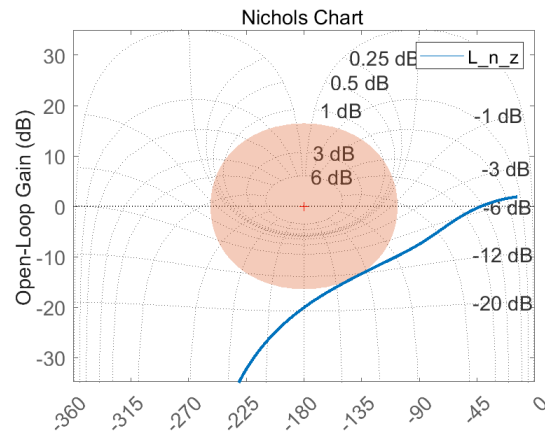


Figure A.14: 1DOF: Nichols diagram corresponding to output n_z

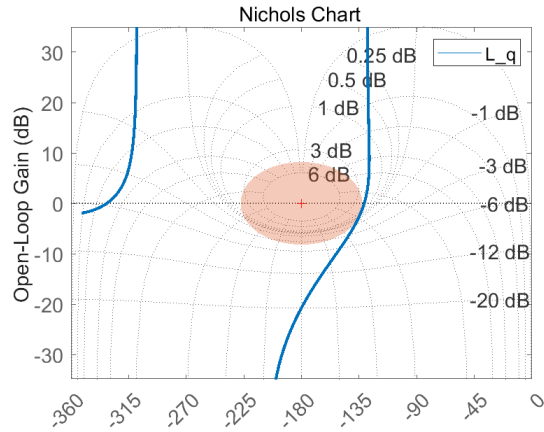


Figure A.15: 1DOF: Nichols diagram corresponding to output q

A.2. Time-Domain Analysis

A.2.1. Step Response: Forward and Feedback Configurations

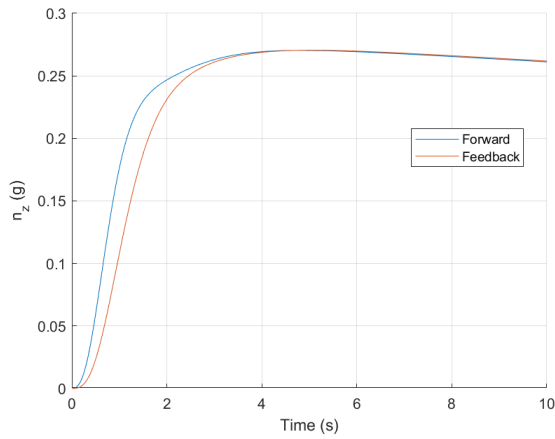


Figure A.16: 1DOF: Comparison of forward and feedback controller implementations for output n_z

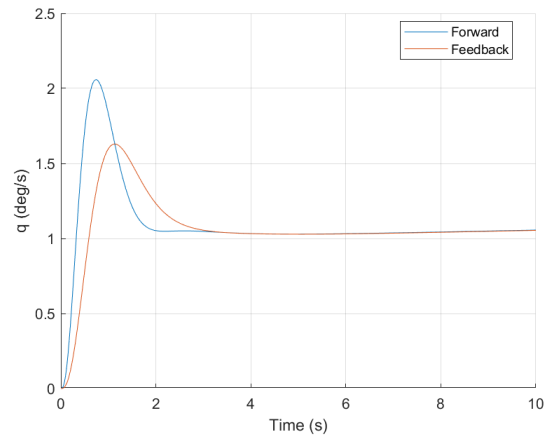


Figure A.17: 1DOF: Comparison of forward and feedback controller implementations for output q

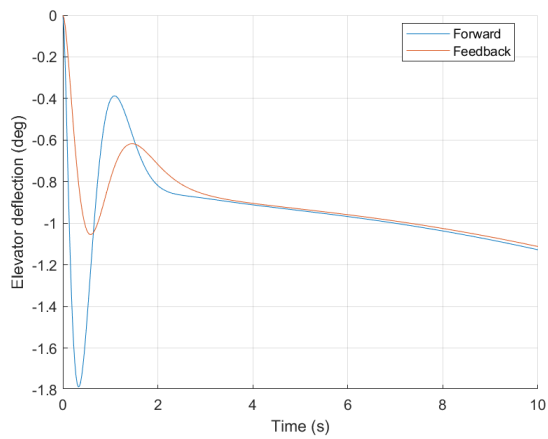


Figure A.18: 1DOF: Comparison of forward and feedback controller implementations for elevator deflection δ_e

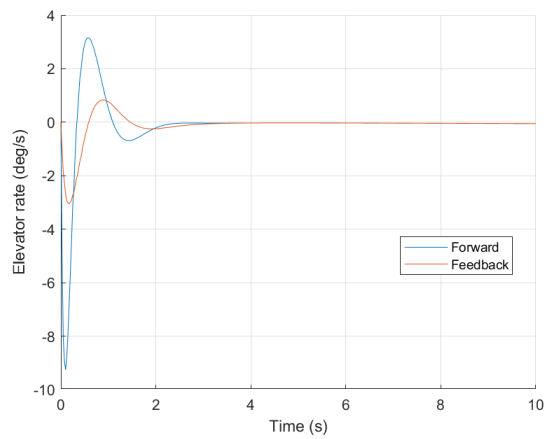


Figure A.19: 1DOF: Comparison of forward and feedback controller implementations for elevator rate $\dot{\delta}_e$

A.2.2. Step Response: ICR and PS Placements

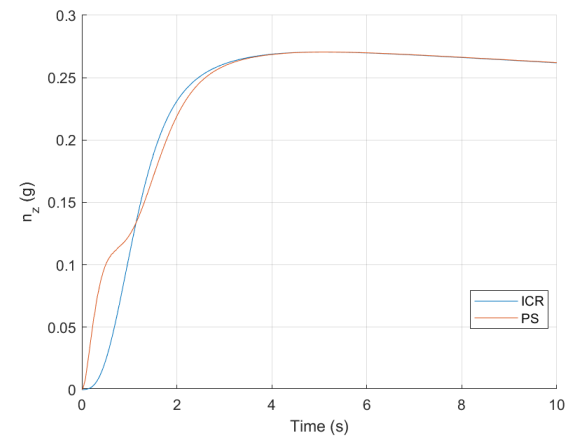


Figure A.20: 1DOF: Comparison of ICR and PS results for output n_z

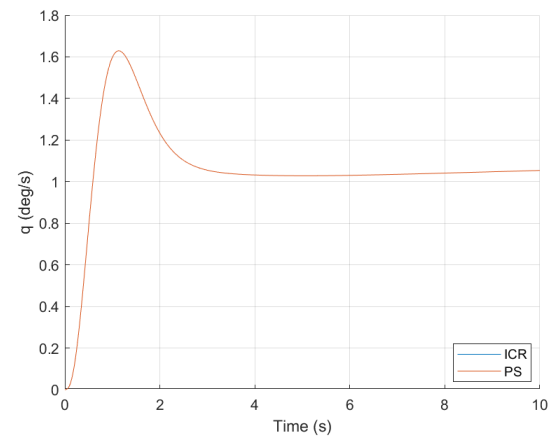


Figure A.21: 1DOF: Comparison of ICR and PS results for output q

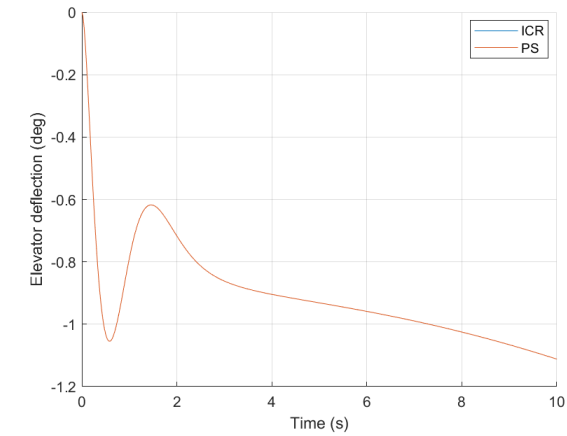


Figure A.22: 1DOF: Comparison of ICR and PS results for elevator deflection δ_e

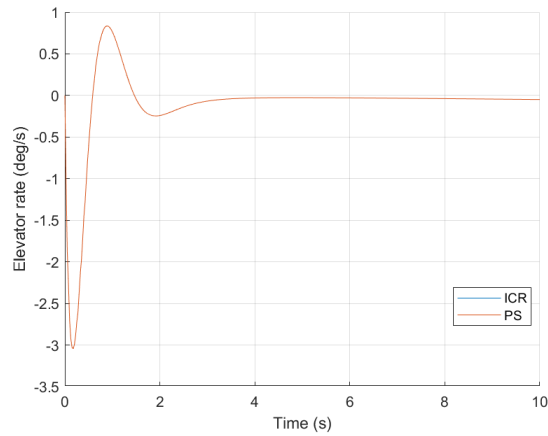


Figure A.23: 1DOF: Comparison of ICR and PS results for elevator rate $\dot{\delta}_e$

Appendix B: 2DOF Results

B.1. Frequency-Domain Analysis

B.1.1. Disk Margins

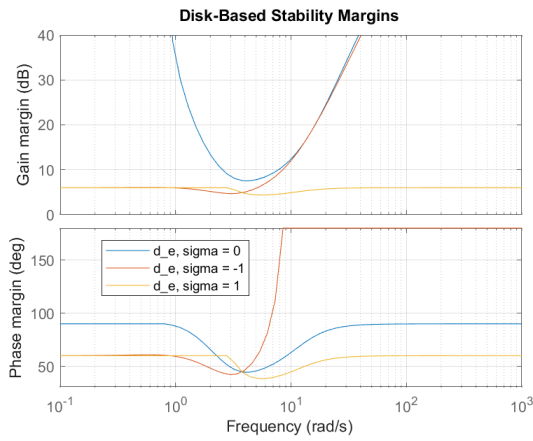


Figure B.1: 2DOF: Disk margin plot corresponding to input δ_e with varying skew

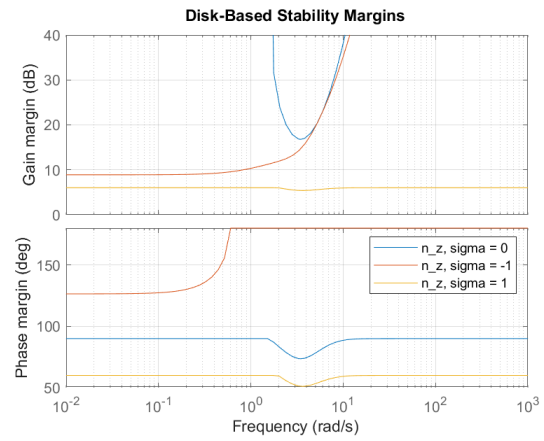


Figure B.2: 2DOF: Disk margin plot corresponding to output n_z with varying skew

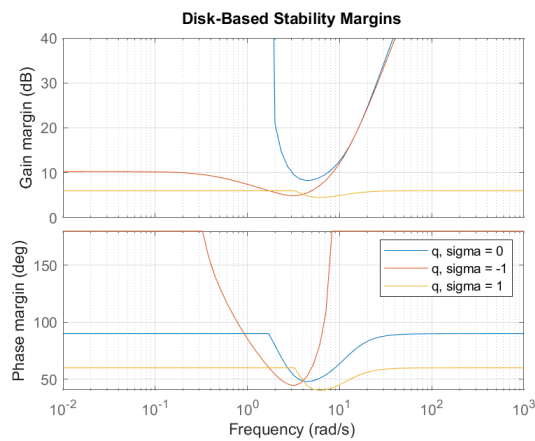


Figure B.3: 2DOF: Disk margin plot corresponding to output q with varying skew

B.1.2. Stability Regions

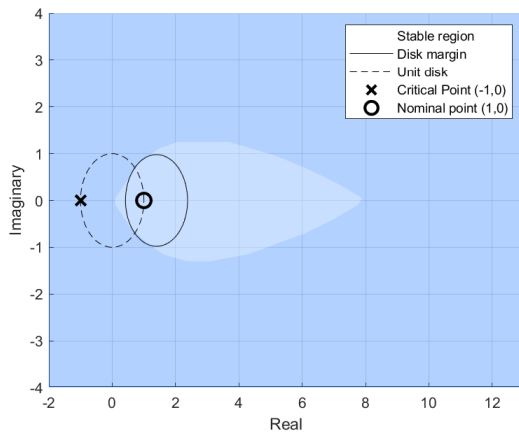


Figure B.4: 2DOF: Stability region corresponding to input δ_e , shown in the complex plane

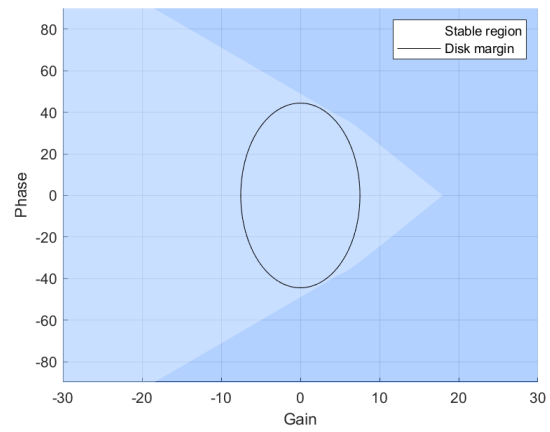


Figure B.5: 2DOF: Stability region corresponding to input δ_e , shown in the real gain and phase plane

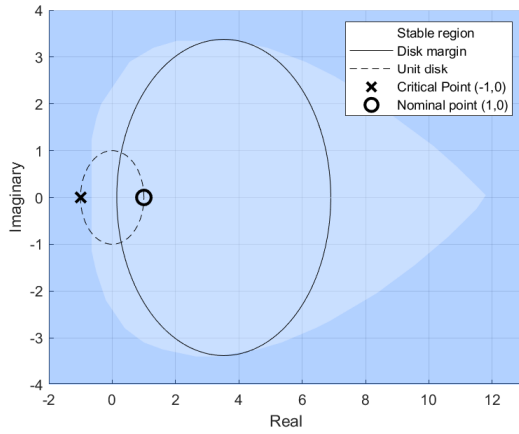


Figure B.6: 2DOF: Stability region corresponding to output n_z , shown in the complex plane

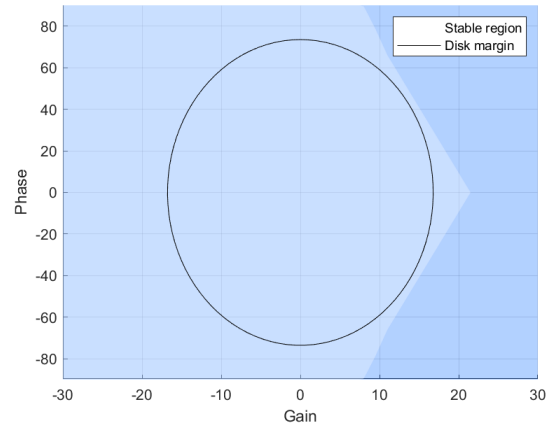


Figure B.7: 2DOF: Stability region corresponding to output n_z , shown in the real gain and phase plane

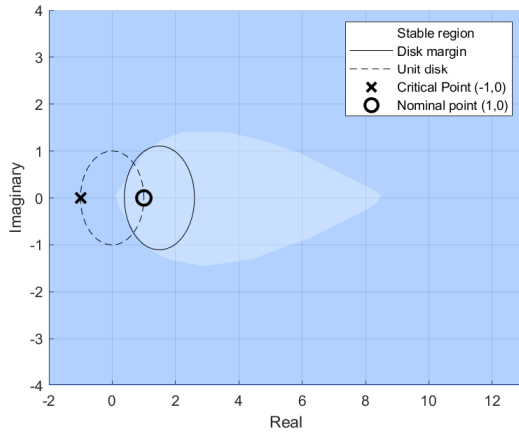


Figure B.8: 2DOF: Stability region corresponding to output q , shown in the complex plane

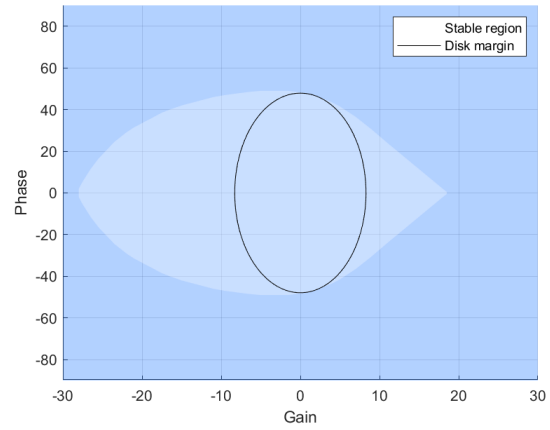


Figure B.9: 2DOF: Stability region corresponding to output q , shown in the real gain and phase plane

B.1.3. Nyquist Diagrams

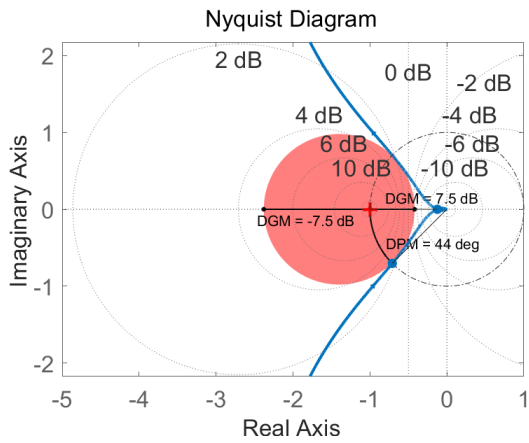


Figure B.10: 2DOF: Nyquist diagram corresponding to input δ_e

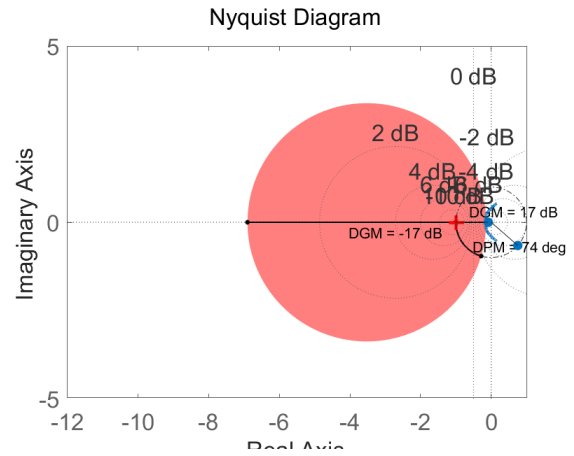


Figure B.11: 2DOF: Nyquist diagram corresponding to output n_z

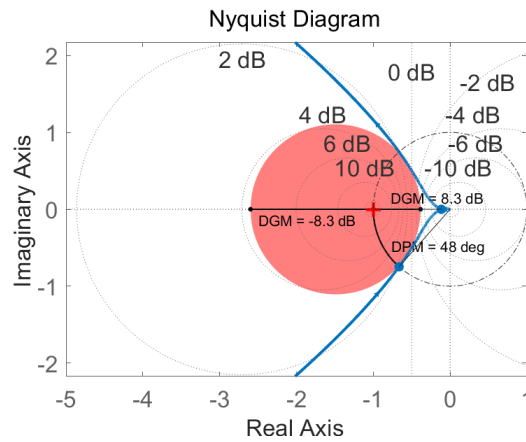


Figure B.12: 2DOF: Nyquist diagram corresponding to output q

B.1.4. Nichols Diagrams

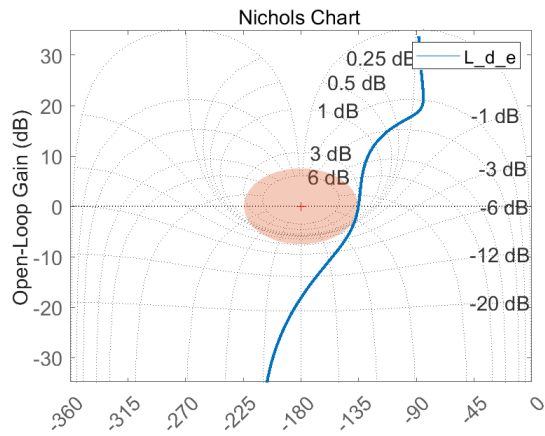


Figure B.13: 2DOF: Nichols diagram corresponding to input δ_e

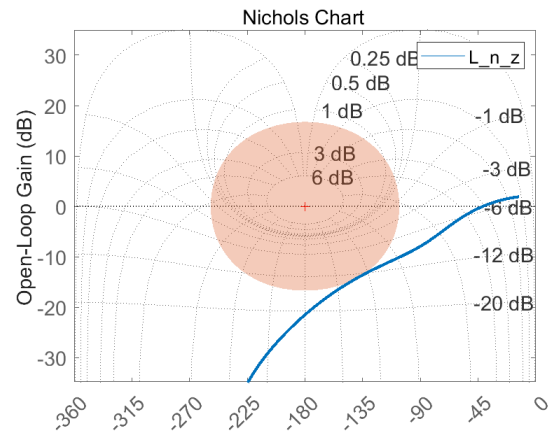


Figure B.14: 2DOF: Nichols diagram corresponding to output n_z

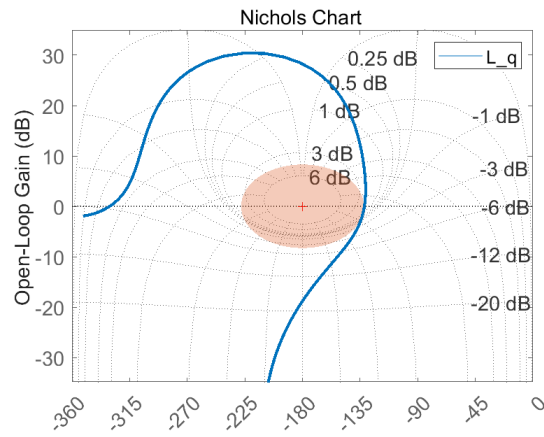


Figure B.15: 2DOF: Nichols diagram corresponding to output q

B.2. Time-Domain Analysis

B.2.1. Step Response: Nominal

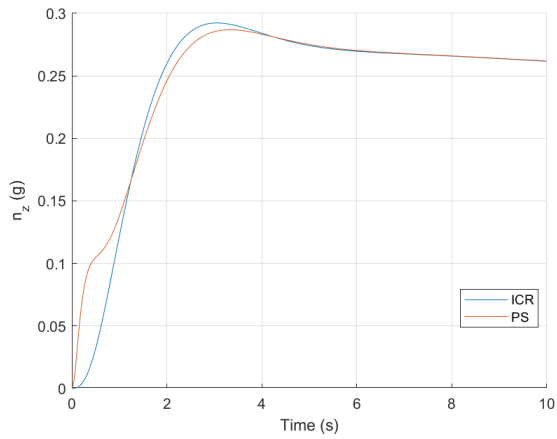


Figure B.16: 2DOF: Comparison of ICR and PS results for output n_z

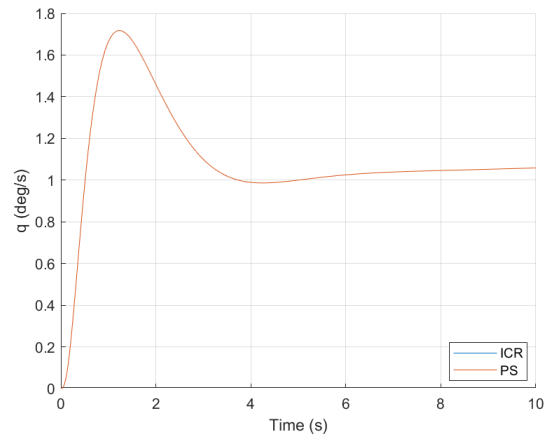


Figure B.17: 2DOF: Comparison of ICR and PS results for output q

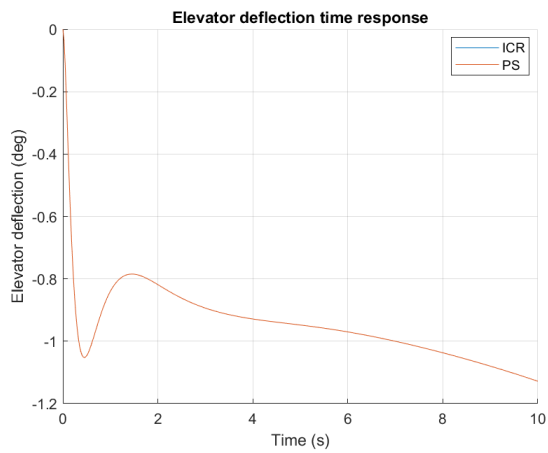


Figure B.18: 2DOF: Comparison of ICR and PS results for elevator deflection δ_e

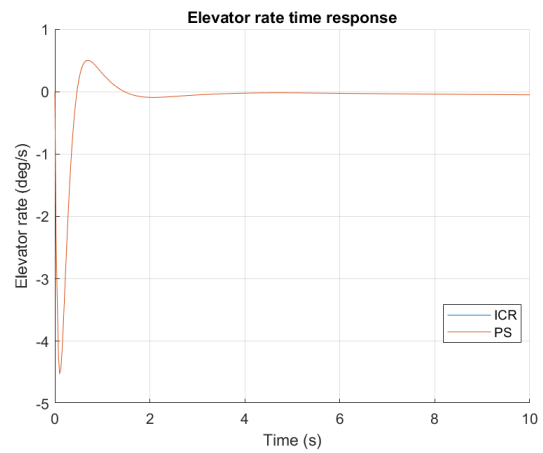


Figure B.19: 2DOF: Comparison of ICR and PS results for elevator rate $\dot{\delta}_e$

B.2.2. Step Response: Disturbance Rejection

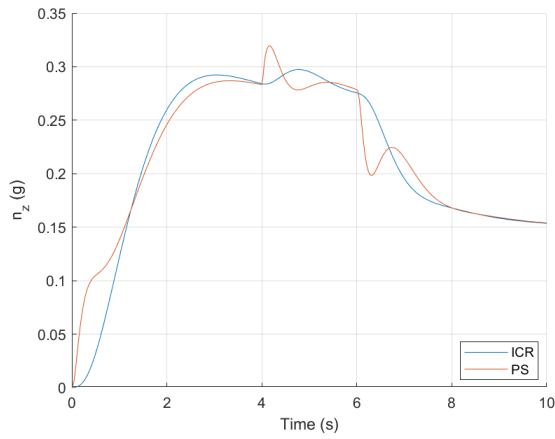


Figure B.20: 2DOF: DR comparison of ICR and PS results for output n_z

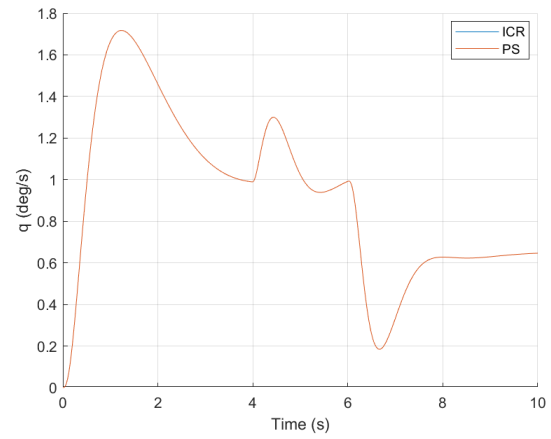


Figure B.21: 2DOF: DR comparison of ICR and PS results for output q

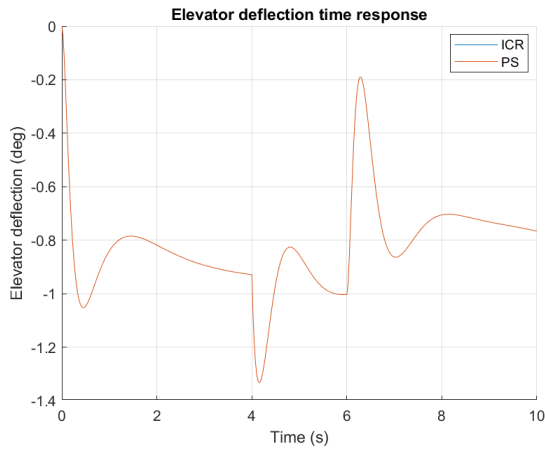


Figure B.22: 2DOF: DR comparison of ICR and PS results for elevator deflection δ_e

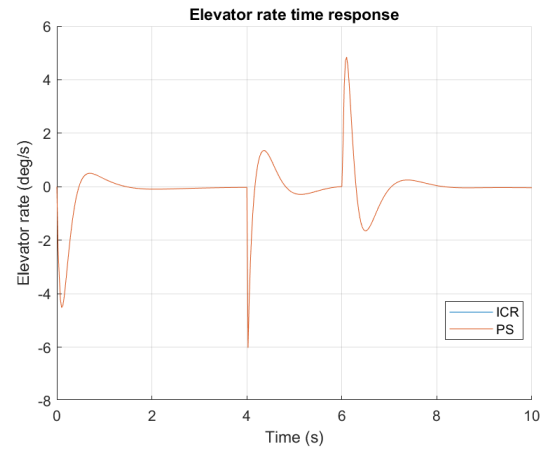


Figure B.23: 2DOF: DR comparison of ICR and PS results for elevator rate $\dot{\delta}_e$

Appendix C: 2DOF Uncertain Results

C.1. Frequency Domain Analysis

C.1.1. Loop Gains

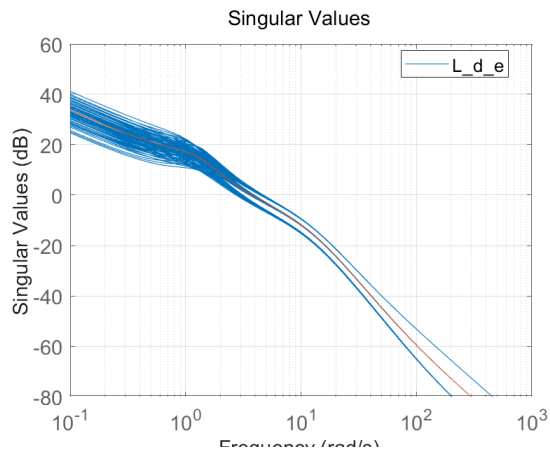


Figure C.1: 2DOF with uncertainty: Loopgain SVs corresponding to input δ_e

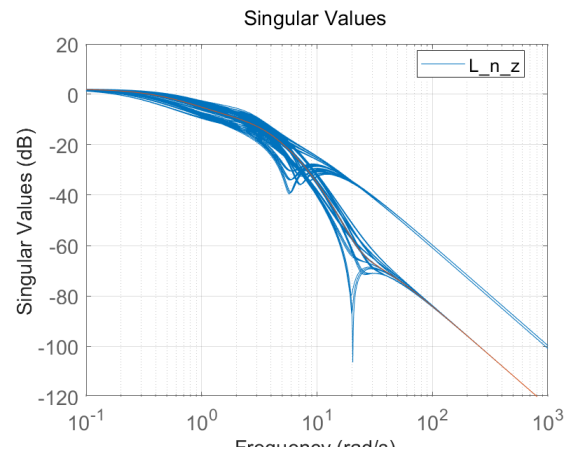


Figure C.2: 2DOF with uncertainty: Loopgain SVs corresponding to output n_z

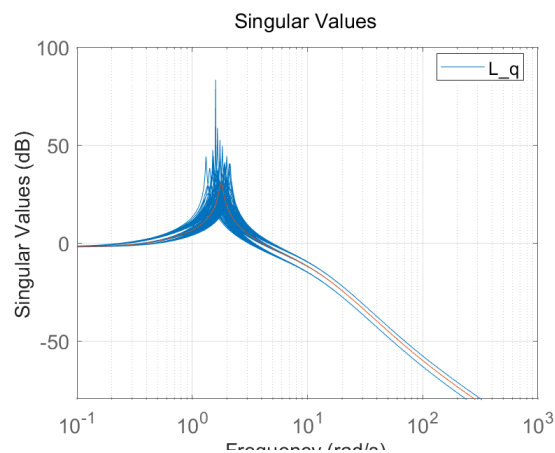


Figure C.3: 2DOF with uncertainty: Loopgain SVs corresponding to output q

C.1.2. Nyquist Diagrams

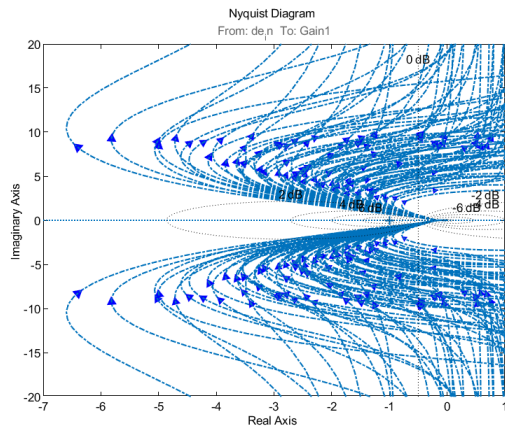


Figure C.4: 2DOF with uncertainty: Nyquist diagram corresponding to input δ_e

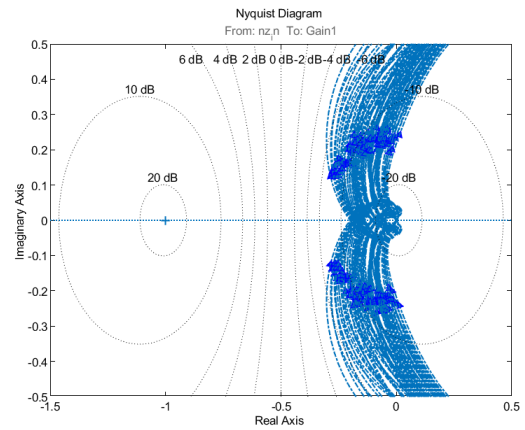


Figure C.5: 2DOF with uncertainty: Nyquist diagram corresponding to output n_z

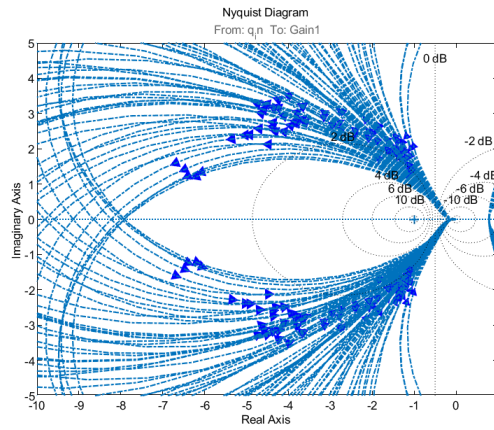


Figure C.6: 2DOF with uncertainty: Nyquist diagram corresponding to output q

C.1.3. Nichols Diagrams

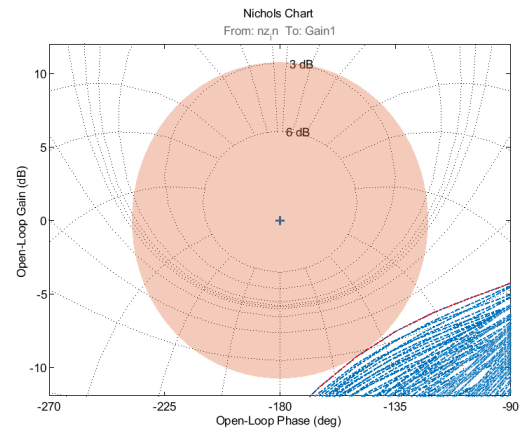
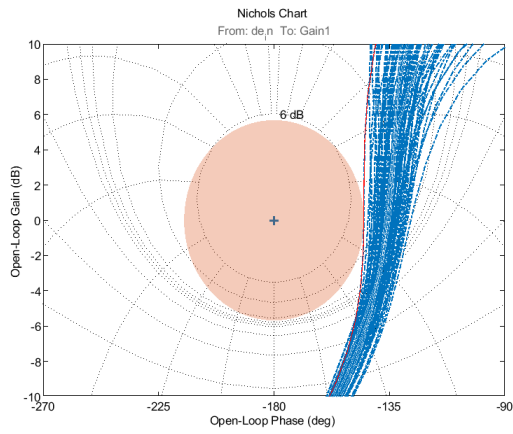


Figure C.7: 2DOF with uncertainty: Nichols diagram corresponding to input δ_e with WCDM indicated **Figure C.8:** 2DOF with uncertainty: Nichols diagram corresponding to output n_z with WCDM indicated

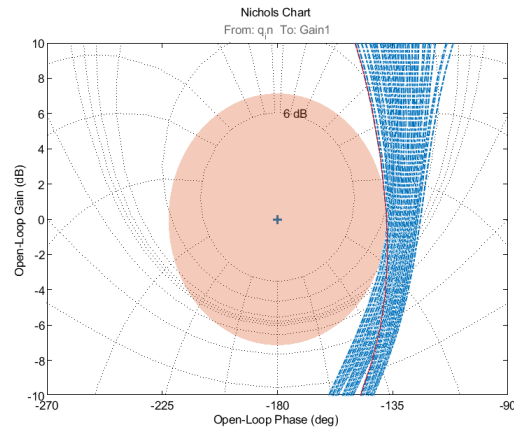


Figure C.9: 2DOF with uncertainty: Nichols diagram corresponding to output q with WCDM indicated

C.2. Time-Domain Analysis

C.2.1. Step Response

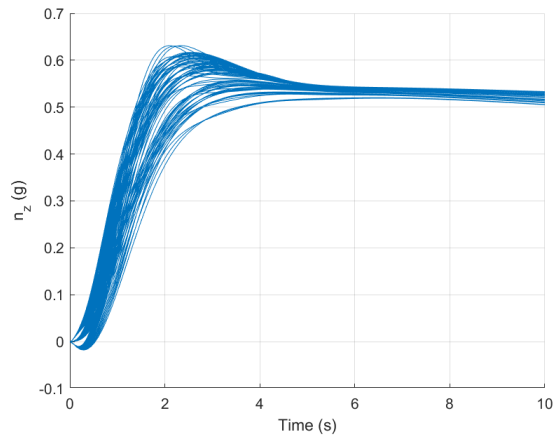


Figure C.10: 2DOF with uncertainty: Time-domain response corresponding to output n_z

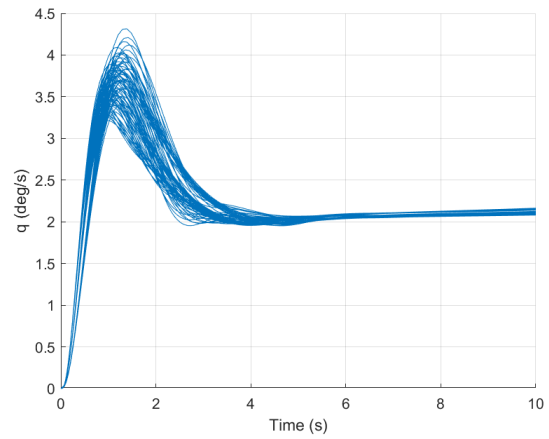


Figure C.11: 2DOF with uncertainty: Time-domain response corresponding to output q

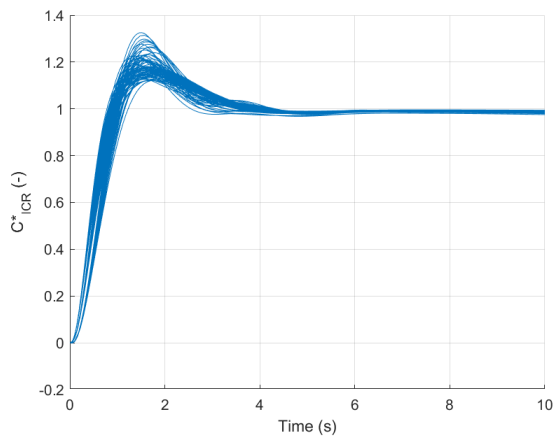


Figure C.12: 2DOF with uncertainty: Time-domain response corresponding to C^* at the ICR (C^*_{ICR})

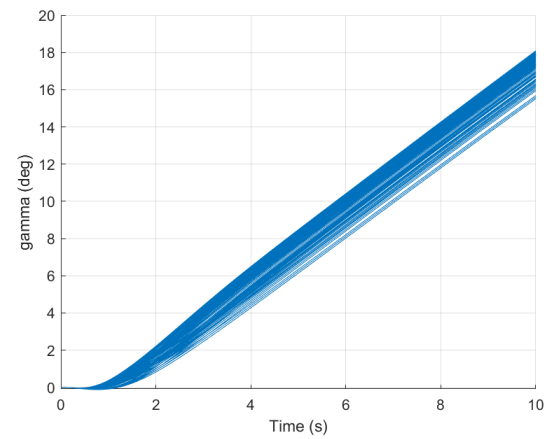
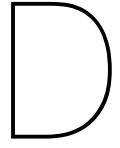


Figure C.13: 2DOF with uncertainty: Time-domain response corresponding to flight path angle γ



Appendix D: 1DOF vs 2DOF Results

D.1. Time-Domain Analysis

D.1.1. Step Response

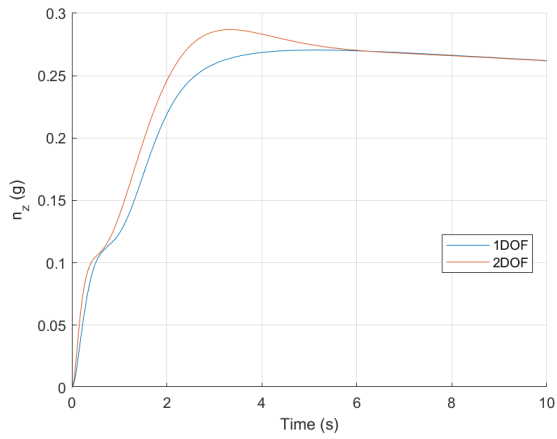


Figure D.1: 1DOF & 2DOF: Comparison of time-domain responses for the output n_z

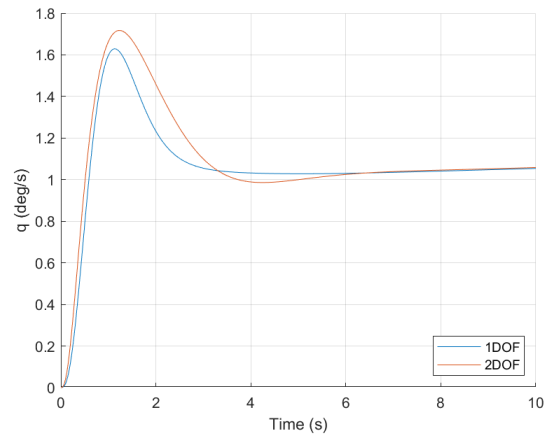


Figure D.2: 1DOF & 2DOF: Comparison of time-domain responses for the output q

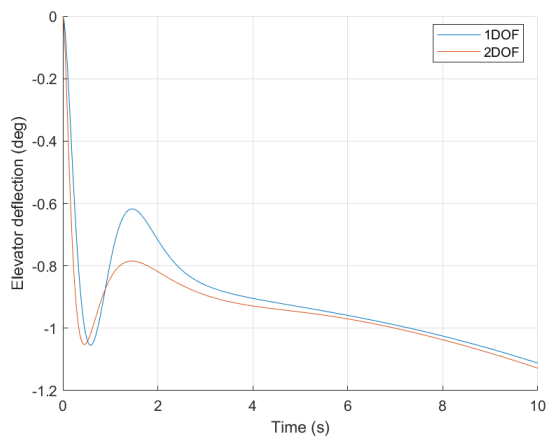


Figure D.3: 1DOF & 2DOF: Comparison of time-domain responses for the elevator deflection δ_e

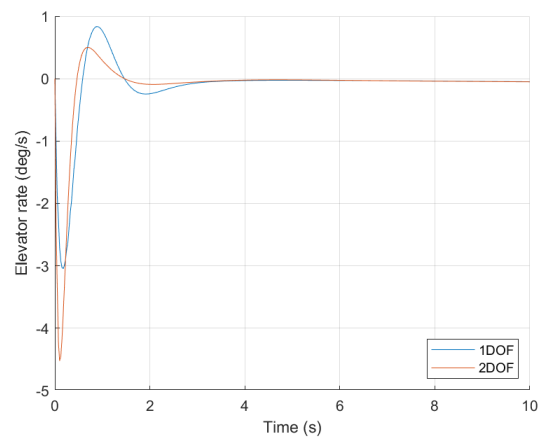
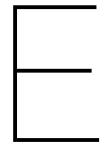


Figure D.4: 1DOF & 2DOF: Comparison of time-domain responses for the elevator rate $\dot{\delta}_e$



Appendix E: Nonlinear Results

E.1. Time-Domain Analysis

E.1.1. Step Response

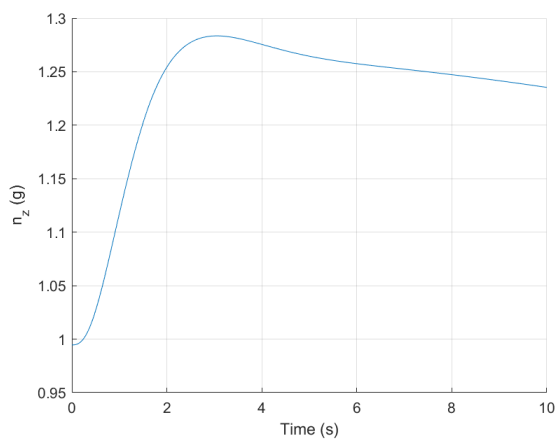


Figure E.1: 2DOF Nonlinear: Step response corresponding to output n_z

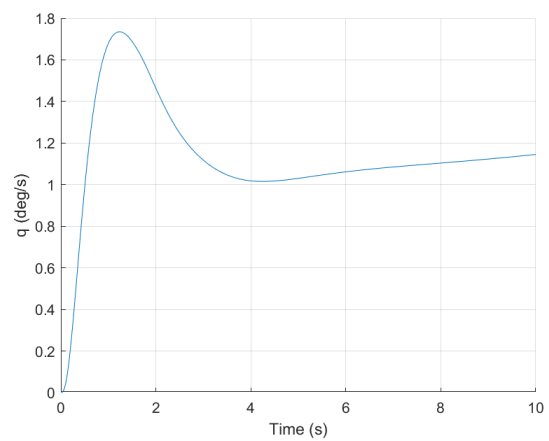


Figure E.2: 2DOF Nonlinear: Step response corresponding to output q

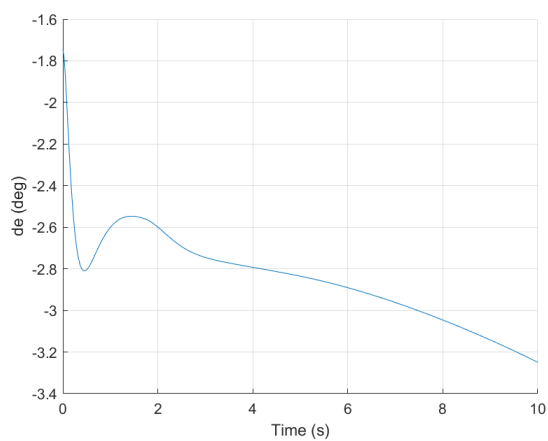


Figure E.3: 2DOF Nonlinear: Step response corresponding to elevator deflection δ_e

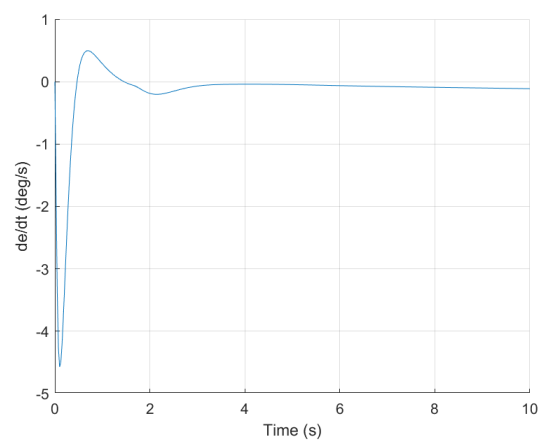


Figure E.4: 2DOF Nonlinear: Step response corresponding to elevator rate $\dot{\delta}_e$

E.1.2. Step Response: Linear & Nonlinear

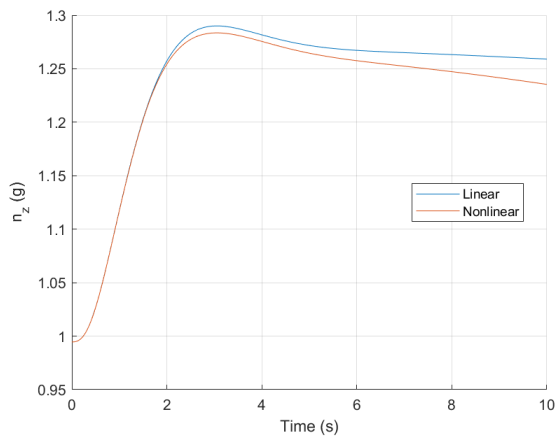


Figure E.5: 2DOF Linear & Nonlinear: Comparison of step responses for output n_z

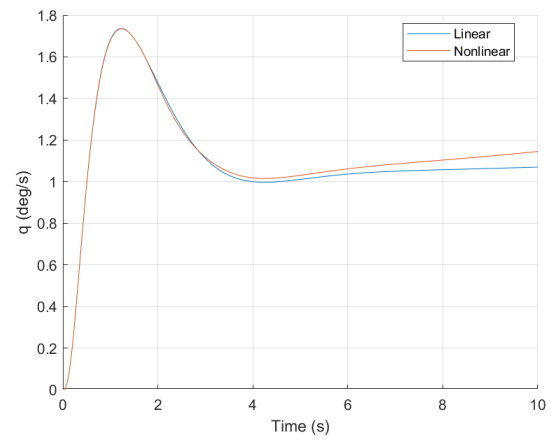


Figure E.6: 2DOF Linear & Nonlinear: Comparison of step responses for output q

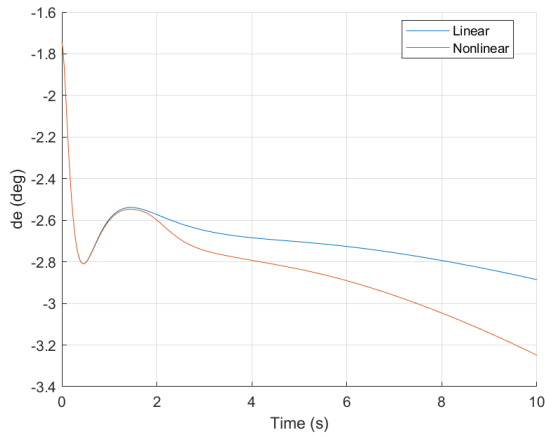


Figure E.7: 2DOF Linear & Nonlinear: Comparison of step responses for elevator deflection δ_e

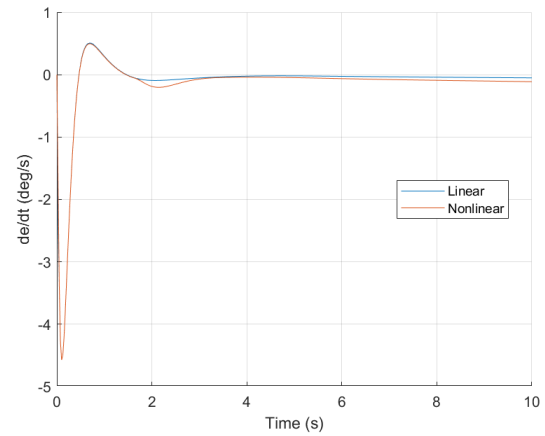


Figure E.8: 2DOF Linear & Nonlinear: Comparison of step responses for elevator rate $\dot{\delta}_e$

E.1.3. Step Response: Disturbance Rejection

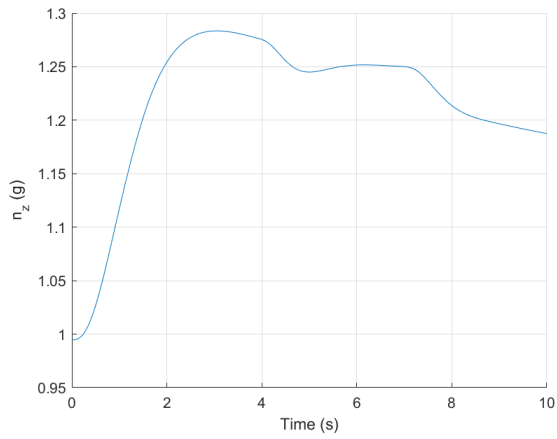


Figure E.9: 2DOF Nonlinear: Step response of output n_z with DR

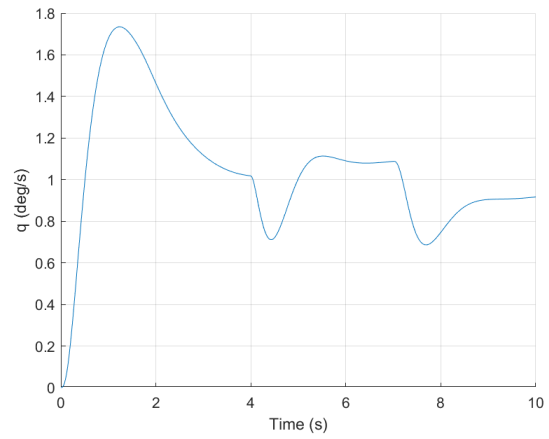


Figure E.10: 2DOF Nonlinear: Step response of output q with DR

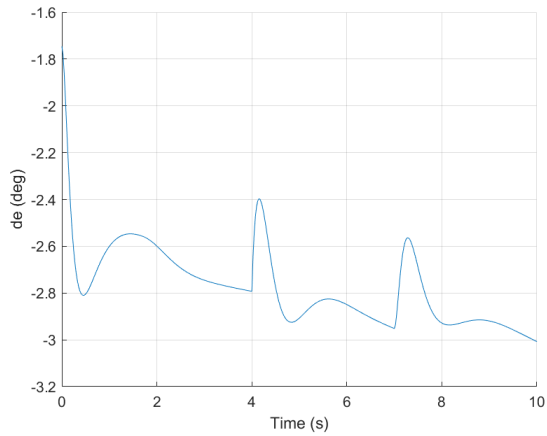


Figure E.11: 2DOF Nonlinear: Step response of the elevator deflection δ_e with DR

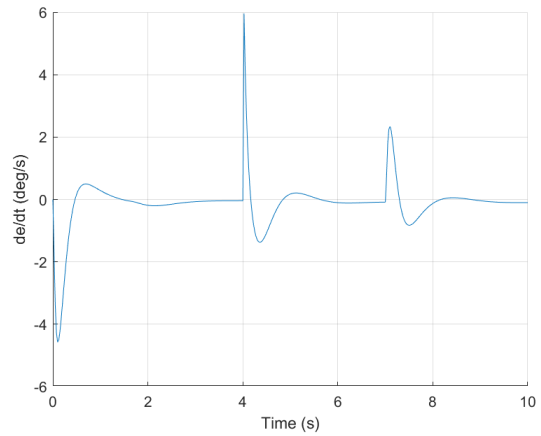


Figure E.12: 2DOF Nonlinear: Step response of the elevator rate $\dot{\delta}_e$ with DR

E.1.4. Step Response: Direct & V-based Implementation

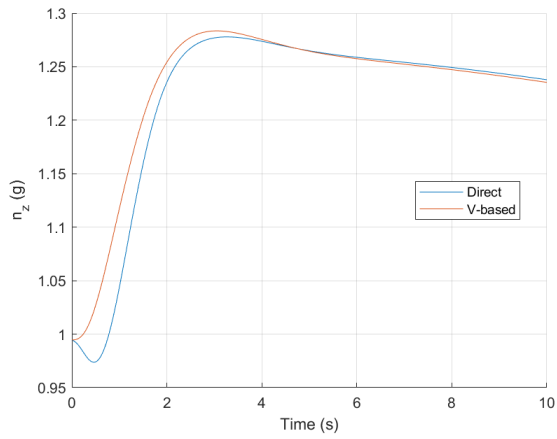


Figure E.13: 2DOF Nonlinear: Step response comparison between direct and V-based implementation for output n_z

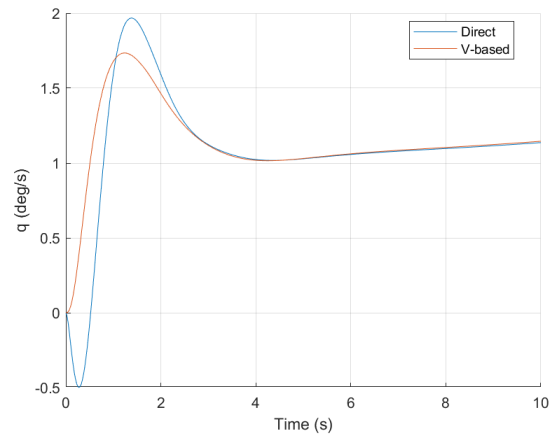


Figure E.14: 2DOF Nonlinear: Step response comparison between direct and V-based implementation for output q

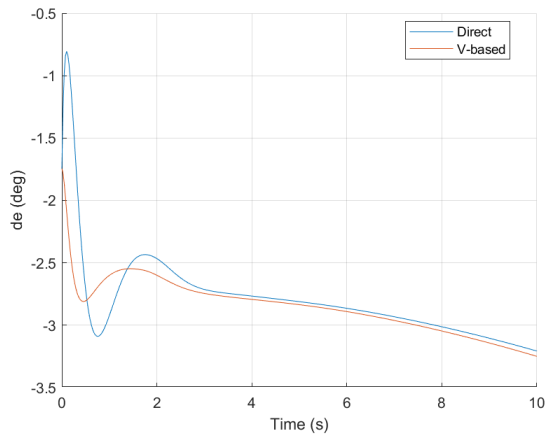


Figure E.15: 2DOF Nonlinear: Step response comparison between direct and V-based implementation for elevator deflection δ_e

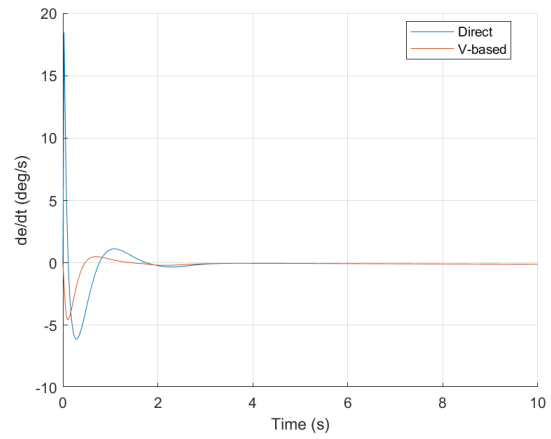


Figure E.16: 2DOF Nonlinear: Step response comparison between direct and V-based implementation for elevator rate $\dot{\delta}_e$

E.1.5. Staircase Response

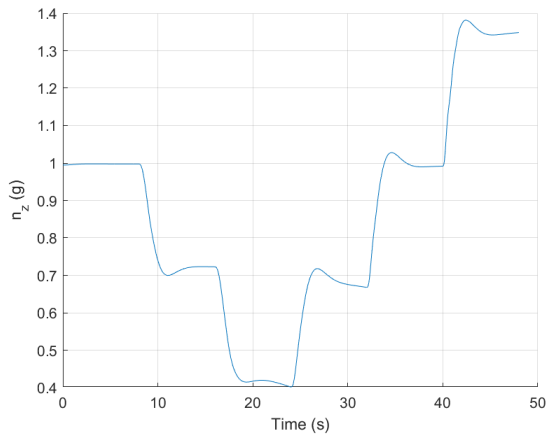


Figure E.17: 2DOF Nonlinear: Staircase response for output n_z

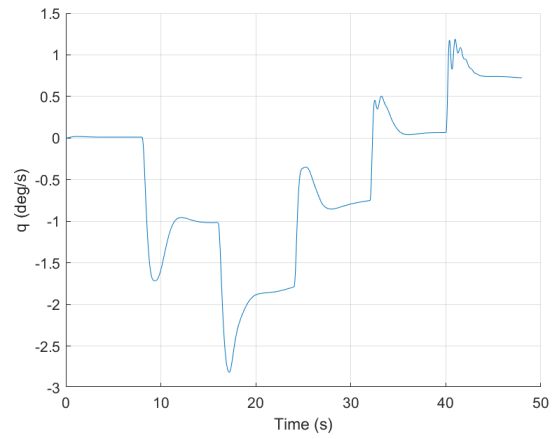


Figure E.18: 2DOF Nonlinear: Staircase response for output q

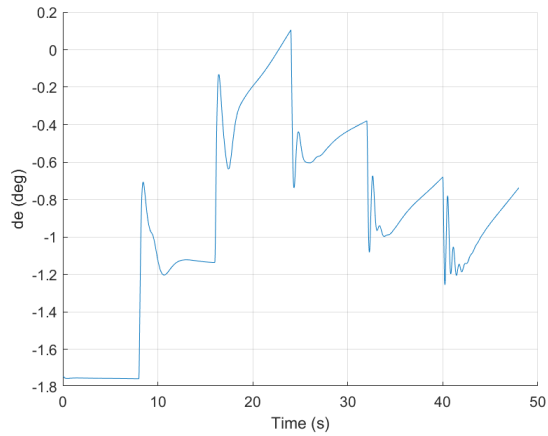


Figure E.19: 2DOF Nonlinear: Staircase response for elevator deflection δ_e

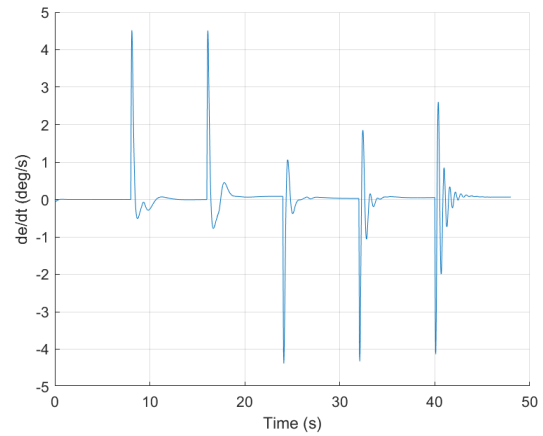


Figure E.20: 2DOF Nonlinear: Staircase response for elevator deflection $\dot{\delta}_e$

E.1.6. Staircase Response: Linear & Nonlinear

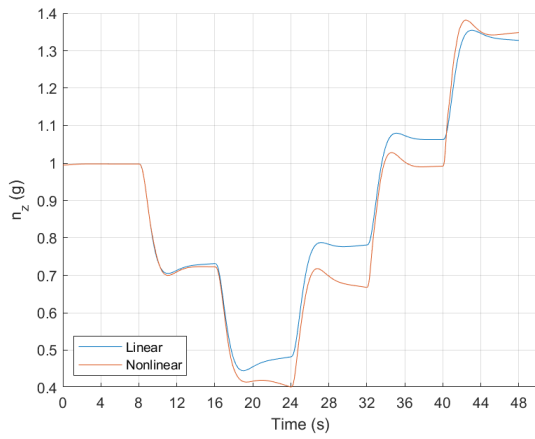


Figure E.21: 2DOF Linear & Nonlinear: Comparison of staircase responses for output n_z

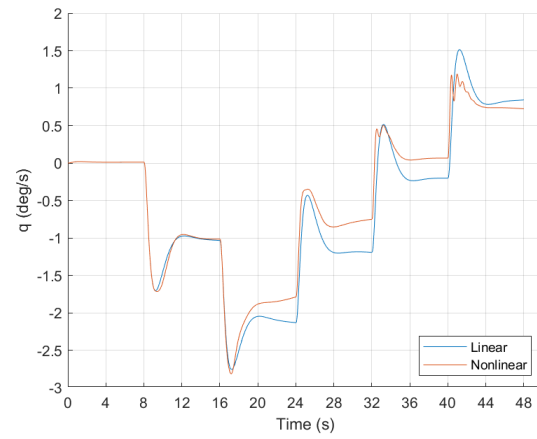


Figure E.22: 2DOF Linear & Nonlinear: Comparison of staircase responses for output q

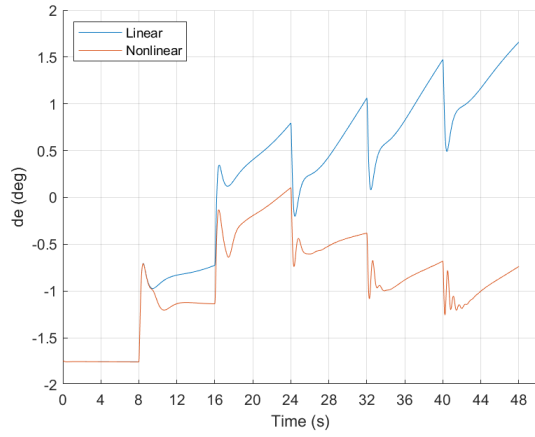


Figure E.23: 2DOF Linear & Nonlinear: Comparison of staircase responses for elevator deflection δ_e

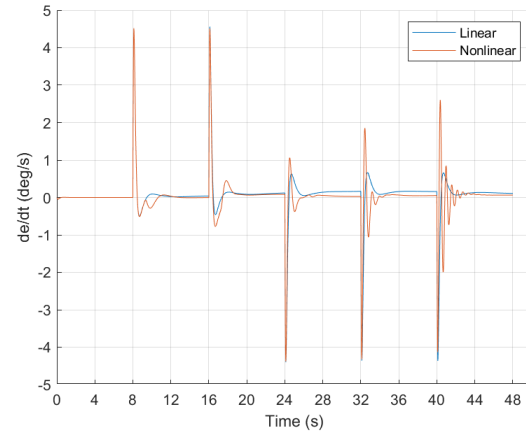


Figure E.24: 2DOF Linear & Nonlinear: Comparison of staircase responses for elevator rate $\dot{\delta}_e$

Appendix F: Multi-Modelling

F.1. Frequency-Domain Analysis

F.1.1. Nyquist Diagrams

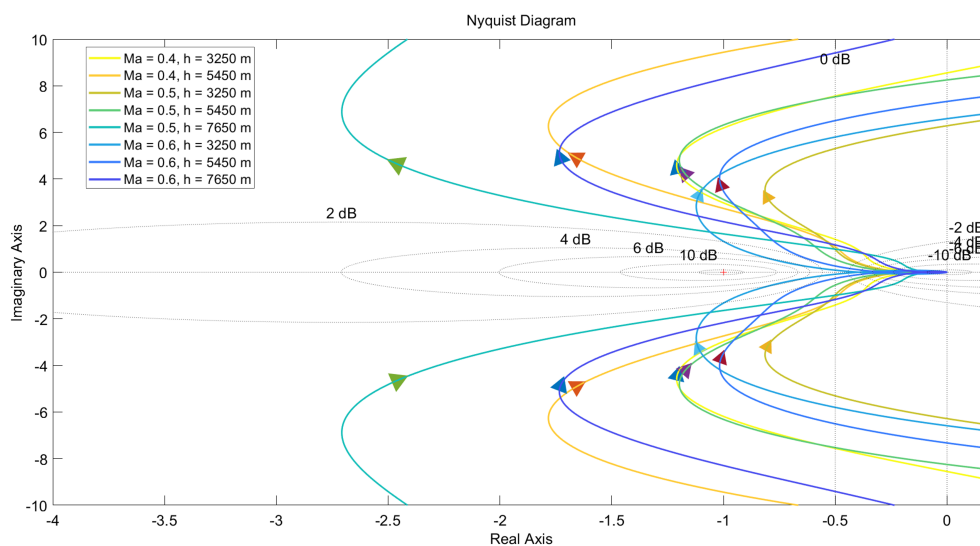


Figure F.1: Multi-modelling: Nyquist diagram for input δ_e

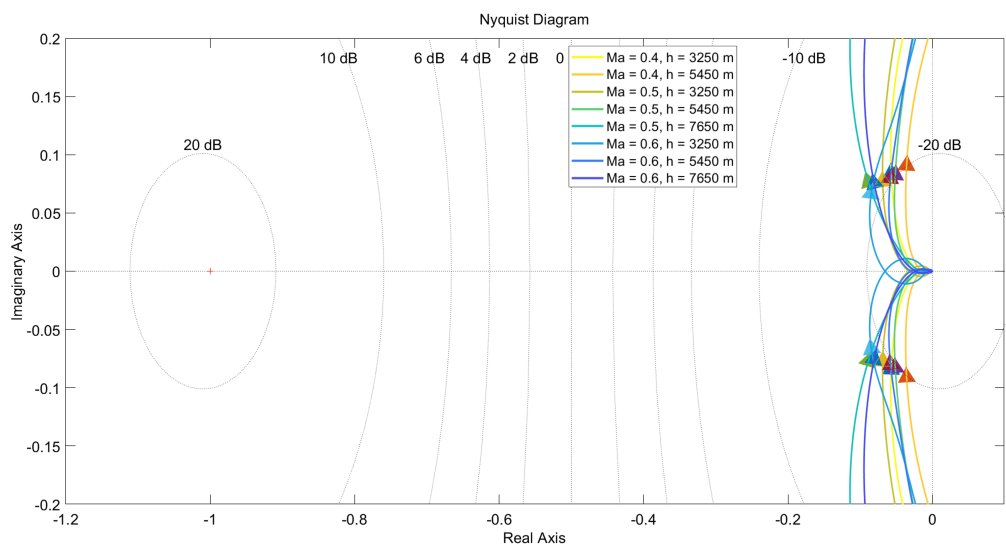


Figure F.2: Multi-modelling: Nyquist diagram for output n_z

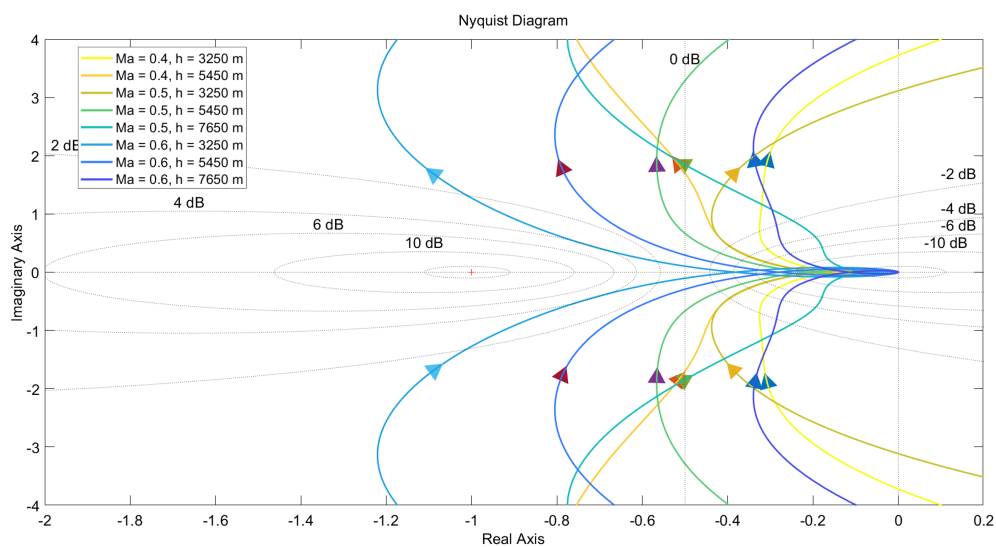


Figure F.3: Multi-modelling: Nyquist diagram for output q

F.1.2. Nichols Diagrams

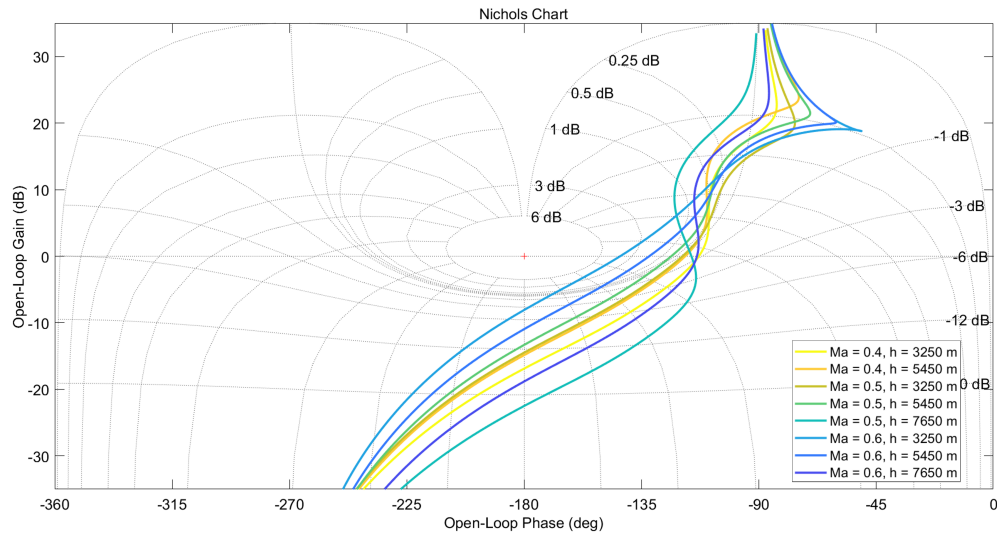


Figure F.4: Multi-modelling: Nichols diagram for input δ_e

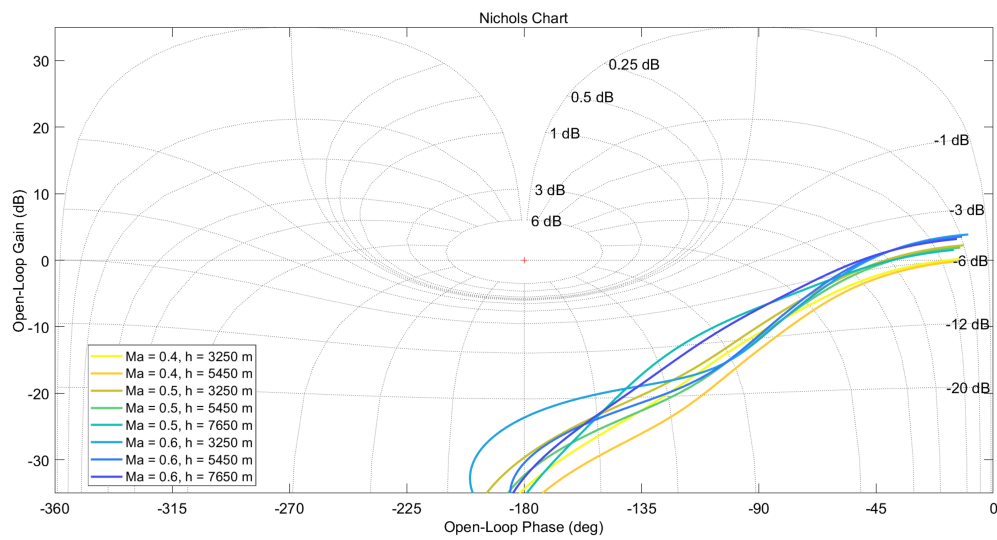


Figure F.5: Multi-modelling: Nichols diagram for output n_z

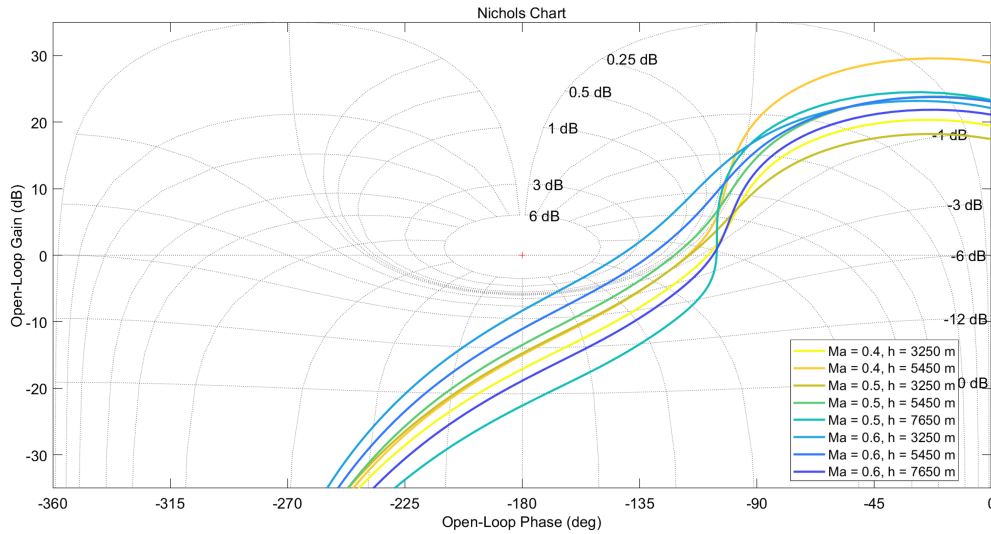


Figure F.6: Multi-modelling: Nichols diagram for output q

F.2. Time-Domain Analysis

F.2.1. Step Response

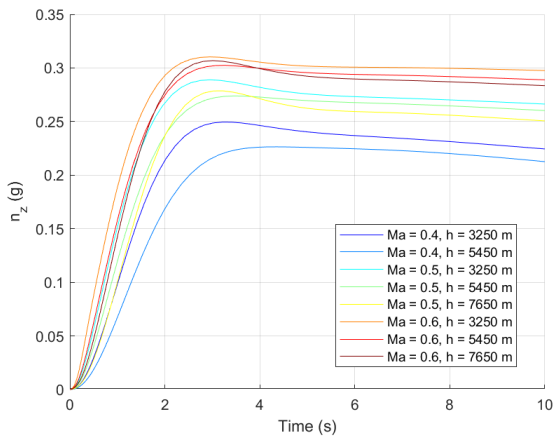


Figure F.7: Multi-modelling: Step response for output n_z

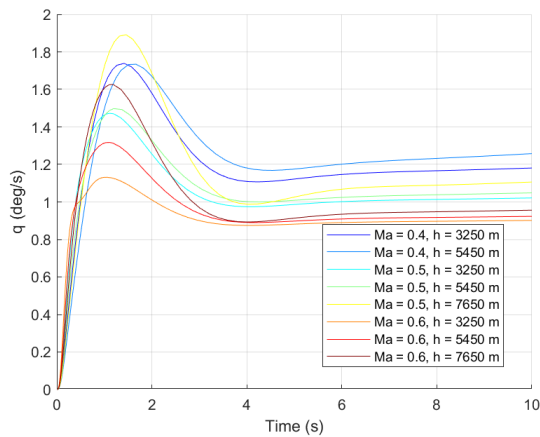


Figure F.8: Multi-modelling: Step response for output q

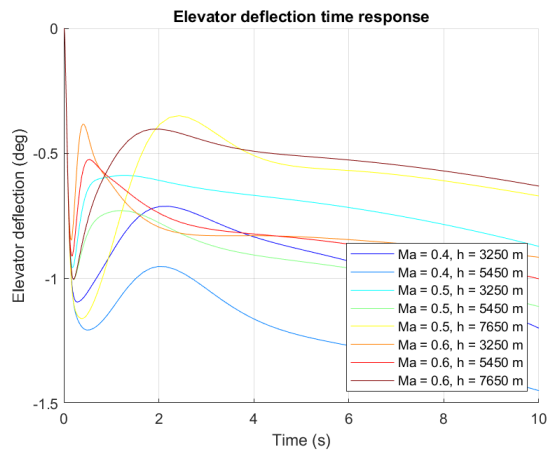


Figure F.9: Multi-modelling: Step response for elevator deflection δ_e

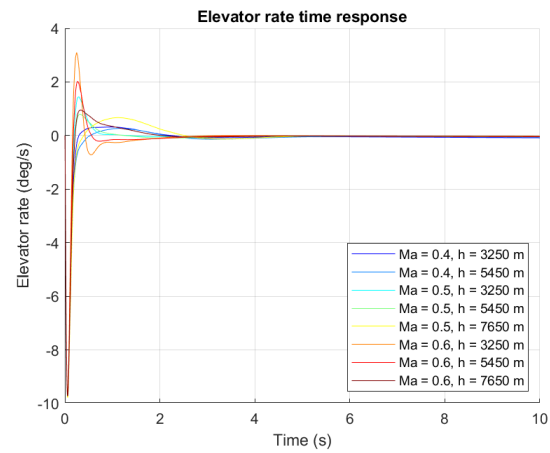
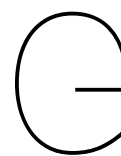


Figure F.10: Multi-modelling: Step response for elevator deflection $\dot{\delta}_e$



Appendix G: Gantt Chart

Gantt chart flow of activities completed during the thesis

Holiday & Required Leave

ACTIVITY	PLAN START	PLAN DURATION	ACTUAL START	ACTUAL DURATION	WEEK	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
					March		April		May		June		July		August		September		October																		
Literature Review																																					
1. Topic definition	1	2	1	2																																	
2. Flying-V	2	2	2	2																																	
3. C* Control Law	3	2	3	2																																	
4. Hinf Loop Shaping	4	3	4	3																																	
5. Stability Analysis	6	2	6	2																																	
6. HQ Assessment	7	2	7	2																																	
7. Research Proposal	8	1	8	1																																	
8. Reporting	9	3	9	3																																	
Research Phase 1 - Modelling & Dynamics																																					
9. Previous Modelling Understanding	11	3	11	3																																	
10. Modelling	13	8	13	8																																	
10.1 Aerodynamic Modelling	13	2	13	2																																	
10.2 Equations of Motion	15	2	15	2																																	
10.3 Actuator Dynamics	17	1	17	1																																	
10.4 Trimming	18	1	18	1																																	
10.5 Linearisation	19	2	19	2																																	
11. Bare Airframe Analysis	21	3	21	3																																	
Research Phase 2 - 1DOF - Preliminary Controller Design																																					
12. NCF Uncertainty	24	2	24	2																																	
13. FCS Design Considerations	26	2	26	2																																	
14. Weighting Filter Optimisation	28	3	28	3																																	
15. Controller Robustification Method	31	2	31	2																																	
16. Stability & HQ assessment	33	4	33	4																																	
16.1 HQ Assessment	33	2	33	2																																	
16.2 Frequency-Domain Analysis	34	2	34	2																																	
16.3 Time-Domain Analysis	35	2	35	2																																	
Research Phase 3 - 2DOF - Advanced Controller Design																																					
17. Weighting Filter Selection	37	3	37	3																																	
18. 1DOF Control Configuration Augmentation	42	3	42	3																																	
19. Feedforward Parameter Optimisation	45	2	45	2																																	
20. Controller Order Selection	46	3	46	3																																	
21. Stability & HQ assessment	53	3	53	3																																	
21.1 HQ Assessment	53	1	53	1																																	
21.2 Frequency-Domain Analysis	54	1	54	1																																	
21.3 Time-Domain Analysis	55	1	55	1																																	
Research Phase 4 - 2DOF - Nonlinear Model Integration																																					
22. Integration with the Nonlinear Model	56	3	56	3																																	
23. Design Adjustments	59	4	59	4																																	
Research Phase 5 - 2DOF - Multi/Modelling Design																																					
24. Flight Envelope Definition	63	1	63	1																																	
25. Weighting Filter Selection	64	3	64	3																																	
26. Stability & HQ assessment	67	6	67	6																																	
26.1 HQ Assessment	67	2	67	2																																	
26.2 Frequency-Domain Analysis	69	2	69	2																																	
26.3 Time-Domain Analysis	71	2	71	2																																	
Research Dissemination																																					
27. Project Re-Organisation	73	4	73	4																																	
28. Report Writing	79	7	79	7																																	
29. Gantt Chart & Proofing	86	2	86	2																																	
30. Report Submission	90	1	90	1																																	
31. Preparation for Defence	90	2	90	2																																	
32. Defence	92	2	92	2																																	

33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96

November December January February March April May June July August September October November December January February

