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Citation (APA)

Tian, R., Zheng, S., Zhao, W., Antonini, A., & Iglesias, G. (2026). Wave power absorption of semi-circular oscillating water column arrays mounted on a straight coastal structure. *Renewable Energy*, 266, Article 125712. <https://doi.org/10.1016/j.renene.2026.125712>

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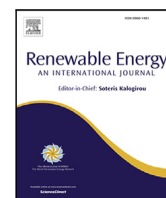
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Wave power absorption of semi-circular oscillating water column arrays mounted on a straight coastal structure

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ARTICLE INFO

Keywords:

Oscillating water columns
Linear potential flow theory
Hydrodynamic response
Wave power absorption
Wave forces
Coastal structure

ABSTRACT

Grouping wave energy converters to form an array, also known as a wave-farm, enhances their energy absorption performance. A cost-effective strategy is to install multiple wave energy converters along a coastal structure, such as a breakwater or seawall. Not only does this reduce construction costs, but it also improves energy absorption, as waves are reflected towards the wave energy converters by the structure. In this paper, the hydrodynamic response of an array of semi-circular oscillating water columns mounted on a straight coastal structure is investigated. By applying linear potential flow theory and an eigenfunction-matching formulation, an analytical solution is derived to evaluate the wave excitation, wave radiation coefficients, wave power absorption, and wave forces on the oscillating water column chambers. Having confirmed the reliability of the model through comparison with previous research, parametric analyses are conducted. The overall hydrodynamic performance and wave forces depend significantly on the array configuration and the incident wave direction. It is important to note that significant interference effects are observed between the oscillating water column chambers of the array, which can either enhance or reduce power output. These findings provide some insight into the design and optimization of multi-chamber oscillating water column systems.

1. Introduction

Wave energy in the ocean is widely acknowledged as a substantial renewable resource, whose development is aligned with a broad spectrum of environmental, social, and economic policy objectives [1]. However, converting this resource into reliable and economically competitive electricity remains challenging, as improvements in efficiency, survivability, and cost-effectiveness must be achieved concurrently [2]. In practice, wave energy costs are mainly driven by deployment and maintenance, and economies of scale will help reduce these costs. Consequently, the key engineering question is which design concepts can achieve robust performance while remaining deployable, maintainable, and scalable in complex sea conditions.

A wide range of wave energy converters (WECs) has been proposed to address this challenge, including point absorbers, overtopping devices, attenuators, and oscillating water columns (OWCs), which have

emerged as key technologies for harnessing wave energy. Point absorbers capture energy through vertical motion using hydraulic pistons or generators [3]; overtopping devices allow waves to spill over into a reservoir, driving turbines to generate power [4]; and attenuators can convert wave energy through the motion of hinged segments. Among these concepts, OWCs represent one of the most mature wave energy concepts and remain particularly attractive for nearshore deployment, as their power take-off (PTO) systems can be situated above the waterline, isolating critical components from direct seawater contact [5] (as shown in Fig. 1). Another advantage is that they are easier for combination with other types of energy systems, forming hybrid energy systems [6], such as floating multi-use platforms integrating OWC arrays [7].

OWC performance depends on the coupled dynamics of the internal free surface, the compressible air chamber, and the PTO system. Falcão

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<https://doi.org/10.1016/j.renene.2026.125712>

Received 12 January 2026; Received in revised form 30 March 2026; Accepted 30 March 2026

Available online 31 March 2026

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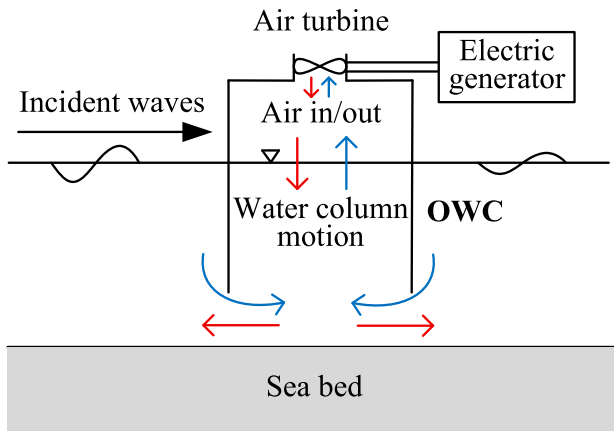


Fig. 1. Sketch of the principle of OWC.

and Henriques [8] clarified the way OWC performance depends on the coupled dynamics of the internal free surface and PTO characteristics, while highlighting the practical significance of turbine selection and control strategies. The performance also depends on: the coupling between the chamber and the air turbine, which should be optimized for different wave climates; the effects of air compressibility, which have been investigated theoretically [9], experimentally [10] and numerically [11]. In addition, the design and geometry of the multi-chamber OWCs have been the object of much attention recently, using experimental [12], numerical [13] and theoretical [14] methods.

For hydrodynamic design and early-stage optimization, analytical and semi-analytical models remain indispensable, as they offer physical transparency and enable extensive parametric sweeps that are impractical with high-fidelity Computational Fluid Dynamics (CFD) [15]. A foundational analytical approach framed the OWC as a wave-structure interaction problem driven by oscillatory pressure and boundary condition at the opening of the chamber under linear potential-flow theory [16]. Based on such frameworks, analytical solutions for OWC-type devices were derived to obtain hydrodynamic coefficients and capture frequency-dependent behaviours in a form suitable for optimization and control [17]. This method was later applied to develop optimal algorithms for the control of the rotational speed of an OWC energy plant equipped with Wells turbines, where efficient coupling between the PTO damping and hydrodynamic response across broad frequency ranges is necessary [18].

A major pathway towards practical deployment is the integration of OWCs with coastal infrastructure. Mustapa et al. [19] noted that breakwater-integrated WECs can reduce installation costs, simplify grid connections, afford harbour protection, and simultaneously provide a viable pathway to large-scale deployment. Analytical studies demonstrated that a rigid coastline or vertical wall can fundamentally alter the wave field, thereby modifying both the achievable maximum efficiency and the optimal location for device placement [20]. Within this context, Boccotti [21] developed a theoretical model of an OWC embedded in a caisson breakwater, and corresponding experiments confirmed the feasibility of such concepts and demonstrated that hydrodynamics-chamber coupling can be exploited through design choices [22]. For a cylindrical OWC embedded into a straight coast/breakwater, linear theory demonstrated that coastline reflection and chamber resonance interact to create strong direction and frequency dependent behaviour [23]; for an OWC positioned at the tip of a breakwater, the same theoretical perspective indicated that geometric asymmetry and directional incidence can be leveraged to produce highly non-trivial absorption characteristics [24]. In a recent theoretical study, Tian et al. [25] proposed attaching a semi-circular OWC to a breakwater instead of embedding it within the structure. Their analysis showed that both

the wave excitation volume flux and the energy absorption of this configuration are unaffected by the direction of the incoming waves.

To achieve practical energy output, WECs are typically deployed in arrays rather than as isolated units. The theoretical basis for array analysis is commonly formulated in terms of hydrodynamic interaction effects, where optimal absorbed energy depends on an interaction impedance matrix that couples radiation and diffraction between individual devices [26]. Early analytical studies developed models for arrays of three-dimensional wave absorbers within linear potential-flow theory by explicitly accounting for mutual diffraction and radiation effects between devices [27], and a major methodological advancement was the development of algebraic multiple-scattering formulations for interacting three-dimensional bodies, in which the array problem is assembled from single-body diffraction properties [28]. Subsequent studies clarified key hydrodynamic aspects of wave energy arrays and emphasized that the interaction can be either beneficial or detrimental, depending on the spacing between devices and wave conditions [29]. More recently, for fixed nearshore devices, analytical models were established for multiple cylindrical [30] and rectangular [31] OWCs, respectively, arranged along a vertical straight coast, enabling systematic evaluation of excitation volume flux, wave power absorption characteristics and array interaction behaviour under oblique wave incidence. For rectangular OWC arrays, recent investigations have further revealed that wave resonance phenomena, particularly along-shore sloshing modes, can significantly influence both hydrodynamic efficiency and wave loading, with strong dependence on the incident wave direction and chamber geometry [32,33]. Moreover, the performance of multi-chamber OWC caisson arrays under oblique waves has been shown to involve complex interactions between coastal reflection and array coupling, leading to constructive or destructive effects that vary with frequency and incident angle [34]. Concise performance metrics are essential for engineering assessment and layout optimization. Key considerations include: the high sensitivity of array power to spatial configuration [35]; interaction factors extended to account for wave directionality in realistic seas [36]; and parametric studies at the farm scale, which identify device spacing, wave incidence directions, and weather scenarios as critical factors for optimal layout and energy yield [37].

The present work extended our recent work regarding a single breakwater-attached semi-circular OWC [25] into an array. Despite these advances as mentioned above, the optimal design and performance of an array of semi-circular OWCs installed on breakwaters, seawalls, or similar coastal structures without compromising the integrity of the original structure remain insufficiently explored. This configuration is attractive not only for the avoidance of caisson cutting in coastal structures, which simplifies installation and maintenance procedures, but also because array deployment offers a practical route to enhance wave power absorption; nevertheless, most existing studies still focus on isolated devices and circular OWCs, and only limited attention has been given to semi-circular arrays coupled with coastal structures where additional diffraction, radiation, and interaction mechanisms may arise. Similar to the motion response differences observed in single and multi-body floating photovoltaic systems [38], these interaction mechanisms can significantly influence the performance and loading of OWC arrays. The present study addresses this gap by providing a detailed theoretical investigation into semi-circular OWC arrays mounted on coastal structures, focusing on hydrodynamic coefficients, wave power capture factor, and wave forces on each OWC chamber. Furthermore, the wave power capture factors of multiple semi-circular and circular OWCs [30] are compared, and wave forces on semi-circular OWC arrays versus a single semi-circular OWC [25] are analysed. By proposing a more efficient arrangement and analysing its interaction with coastal structures, this research will contribute to the development of more efficient and adaptable designs for nearshore wave energy farms. The remainder of this paper is organized as follows: Section 2 presents the mathematical formulation and analytical solution

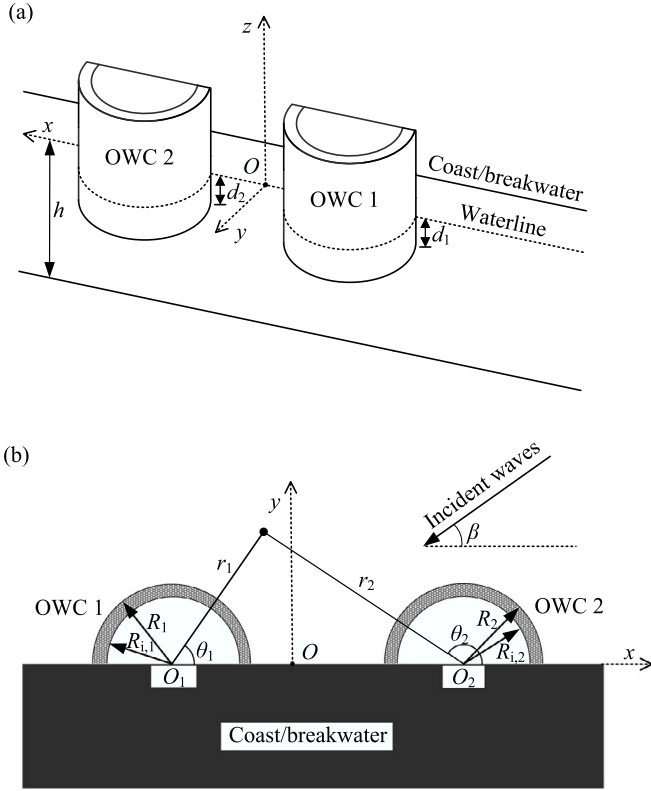


Fig. 2. Definition sketch: (a) general layout of an array of semi-circular OWCs; (b) plan section with key dimensions.

procedure for semi-circular OWC arrays mounted on straight coastal structures, considering the wave forces on the OWC system; Section 3 reports verification and convergence checks; comparisons with existing studies and a multi-parameter investigation of the semi-circular OWC array are provided in Section 4, followed by the conclusions in Section 5.

2. Mathematical model

In the present configuration, an array of N semi-circular OWCs is considered along a straight coast or breakwater in water of finite depth h (see Fig. 2, where $N = 2$ is shown for illustration). The study extends our previous work on an isolated coast-attached semi-circular OWC [25] to the array case, which is more relevant to wave-farm applications. A global Cartesian coordinate system $Oxyz$ is introduced, with the Oxy -plane coincident with the mean free surface and the Oxz -plane aligned with the coastal sidewall. These N vertical semi-circular OWC chambers are numbered sequentially in ascending order along the positive direction of the Ox -axis, and a local cylindrical coordinate system $O_n r_n \theta_n z$ is defined for each OWC to precisely characterize its geometry. The origin O_n of each local coordinate system is situated on the central vertical axis of the n th OWC, with coordinates $(x_n, 0, 0)$ in the global Cartesian system $Oxyz$. Additionally, key geometric parameters of the n th OWC chamber are defined as follows: R_n and $R_{i,n}$ represent the outer and inner radii of the chamber, respectively; d_n denotes the draught of the chamber; and D_n refers to the interval between the origin of the n th and $(n+1)$ th OWCs, $D_n = |O_n O_{n+1}|$ (i.e., the interval between the central vertical axes of adjacent units).

For coast-attached semi-circular OWC arrays excited by low-steepness regular waves incident at an angle β relative to the coastal alignment (see Fig. 2), the flow in the water domain is described within linear potential-flow theory. The velocity potential is expressed as

$$\Phi(x, y, z, t) = \text{Re} \{ \phi(x, y, z) e^{-i\omega t} \}, \quad (1)$$

where ϕ is the complex (time-independent) spatial velocity potential, ω is the angular frequency of the incident waves, i denotes the imaginary unit, and t is time.

Within linear water-wave theory, the complex spatial potential ϕ can be expressed as the combination of the scattering (diffraction) and radiation components, i.e.,

$$\phi = \phi_0 + \sum_{n=1}^N p_n \phi_n, \quad (2)$$

in which ϕ_0 denotes the scattered spatial velocity potential, which describes the wave field when the semi-circular OWCs are subjected to incident waves with their chamber tops fully open to the atmosphere (i.e., no dynamic air pressure is present). The variable p_n denotes the complex amplitude of the air pressure inside the n th OWC chamber, while ϕ_n denotes the spatial velocity potential induced by a unit air pressure oscillation within the n th OWC chamber, with all other OWCs at rest.

The ϕ_0 can be expressed as the superposition

$$\phi_0 = \phi_I + \phi_D, \quad (3)$$

in which ϕ_I represents the wave-field potential generated by the incident waves together with their reflection from the straight coastline in the absence of the OWCs, whereas ϕ_D corresponds to the diffraction potential produced by the OWCs.

With respect to the n th local cylindrical coordinates $O_n r_n \theta_n z$, ϕ_I can be expressed as

$$\phi_I(r_n, \theta_n, z) = -\frac{2igA}{\omega} \frac{Z_0(z)}{Z_0(0)} e^{-ikx_n \cos \beta} \sum_{m=0}^{\infty} \epsilon_m (-i)^m J_m(kr_n) \cos(m\beta) \cos(m\theta_n), \quad (4)$$

where $\epsilon_m = 1$ for $m = 0$, while $\epsilon_m = 2$ for $m \neq 0$, representing the axisymmetric and non-axisymmetric modes, respectively, A is the amplitude of incident waves, g is the gravitational acceleration, k is the wave number, J_m is the Bessel function, and

$$Z_0(z) = N_0^{-0.5} \cosh[k(z+h)], \quad N_0 = \frac{1}{2} \left[1 + \frac{\sinh(2kh)}{2kh} \right]. \quad (5)$$

2.1. Boundary conditions

The potentials ϕ_j for $j = 0, 1, 2, \dots, N$ are required to satisfy the following governing equation together with the corresponding boundary conditions:

$$\nabla^2 \phi_j = 0, \quad \text{in water}, \quad (6)$$

$$\frac{\partial \phi_j}{\partial z} - \frac{\omega^2}{g} \phi_j = 0, \quad z = 0, \quad \text{on the external free surface (outside all chambers)}, \quad (7)$$

$$\frac{\partial \phi_j}{\partial z} - \frac{\omega^2}{g} \phi_j = \frac{i\omega}{\rho g} \delta_{n,j}, \quad z = 0, \quad \text{on the internal free surface of the } n\text{th OWC chamber}, \quad (8)$$

$$\frac{\partial \phi_j}{\partial n} = 0, \quad \text{on all solid boundaries}, \quad (9)$$

where $\partial \phi_j / \partial n$ is the derivative along the unit normal, i.e. $\partial \phi_j / \partial n = \vec{n} \cdot \nabla \phi_j$, and $\vec{n}$ is taken on the solid boundaries pointing into the fluid domain; ρ is the water density. Furthermore, the standard far-field radiation condition applies to ϕ_D and to ϕ_j for $j = 1, 2, \dots, N$.

2.2. Analytical forms of ϕ_j in different regions

In each region, ϕ_j can be expanded in orthogonal eigenfunction series as follows:

1. Inner region enclosed by the n th OWC ($r_n \in [0, R_{i,n}]$, $\theta_n \in [0, \pi]$, $z \in [-h, 0]$)

$$\phi_j(r_n, \theta_n, z) = \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} A_{m,l}^{j,n} J_m(k_l r_n) \cos(m\theta_n) Z_l(z) - \frac{i\delta_{j,n}}{\rho\omega}, \quad (10)$$

where $A_{m,l}^{j,n}$ are undetermined expansion coefficients to be solved for, $k_0 = k$, and for $l = 1, 2, 3, \dots$, k_l consists of the single positive real root and the infinitely many purely imaginary roots of the dispersion relation $\omega^2 = gk \tanh(kh)$. $Z_l(z)$ is an eigenfunction given by

$$Z_l(z) = N_l^{-1/2} \cosh[k_l(z+h)], \quad N_l = \frac{1}{2} \left[1 + \frac{\sinh(2k_l h)}{2k_l h} \right]. \quad (11)$$

It is a standard result that the functions Z_l form a complete, orthogonal basis on the interval $-h < z < 0$:

$$\int_{-h}^0 Z_l(z) Z_m(z) dz = h\delta_{l,m}, \quad l, m = 0, 1, 2, 3, \dots \quad (12)$$

2. Ring region beneath the n th OWC ($r_n \in [R_{i,n}, R_n]$, $\theta_n \in [0, \pi]$, $z \in [-h, -d_n]$)

$$\phi_j(r_n, \theta_n, z) = \sum_{m=0}^{\infty} \left[B_{m,0}^{j,n}(r_n) + \sum_{l=1}^{\infty} \left(C_{m,l}^{j,n} \frac{I_m(\beta_{l,n} r_n)}{I_m(\beta_{l,n} R_n)} + D_{m,l}^{j,n} \frac{K_m(\beta_{l,n} r_n)}{K_m(\beta_{l,n} R_n)} \right) \right] \cos(\beta_{l,n}(z+h)) \cos(m\theta_n), \quad (13)$$

in which

$$B_{m,0}^{j,n}(r_n) = \begin{cases} C_{m,0}^{j,n} + D_{m,0}^{j,n} \left[1 + \ln\left(\frac{r_n}{R_n}\right) \right], & m = 0, \\ C_{m,0}^{j,n} \left(\frac{r_n}{R_n}\right)^{|m|} + D_{m,0}^{j,n} \left(\frac{r_n}{R_n}\right)^{-|m|}, & m \neq 0, \end{cases} \quad (14)$$

in which $C_{m,l}^{j,n}$ and $D_{m,l}^{j,n}$ are undetermined coefficients, I_m and K_m denote the modified Bessel functions of the first and second kinds, respectively, and $\beta_{l,n}$ denotes the l th eigenvalue defined as

$$\beta_{l,n} = \frac{l\pi}{h-d_n} \quad l = 0, 1, 2, 3, \dots \quad (15)$$

3. Outside region (i.e., the exterior region seaward of the coast and outside the OWC chambers, extending to infinity in the horizontal direction) ($r_n \in [R_n, \infty)$, $\theta_n \in [0, \pi]$, $z \in [-h, 0]$)

$$\phi_j = \delta_{j,0} \phi_I + \sum_{p=1}^N \phi_{j,p}(r_p, \theta_p, z), \quad (16)$$

where $\phi_{j,p}$ represents the diffracted/radiated potential due to the p th OWC. Written in the local cylindrical coordinates $O_p r_p \theta_p z$ attached to that chamber, $\phi_{j,p}$ takes the form

$$\phi_{j,p}(r_p, \theta_p, z) = \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} E_{m,l}^{j,p} H_m(k_l r_p) \cos(m\theta_p) Z_l(z), \quad (17)$$

where $E_{m,l}^{j,p}$ are undetermined coefficients, and H_m denotes the Hankel function of the first kind.

Applying Graf's addition theorem for Bessel functions [30,39],

$$\begin{aligned} & H_m(k_l r_p) \cos(m\theta_p) \\ &= \sum_{m'=-\infty}^{\infty} H_{m+m'}(k_l R_{pn}) J_{m'}(k_l r_n) e^{i(m\alpha_{pn} + m'\alpha_{np})} \cos(m'\theta_n), \quad r_n \leq R_{pn}, \end{aligned} \quad (18)$$

where R_{pn} and α_{pn} denote, respectively, the norm and angle of the vector $\vec{O_p O_n}$. This enables the representation of $\phi_{j,p}$ in the n th local cylindrical coordinate system. Accordingly, Eq. (16) can be written in

$O_n r_n \theta_n z$ as

$$\begin{aligned} \phi_j &= \delta_{j,0} \phi_I + \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} E_{m,l}^{j,n} H_m(k_l r_n) \cos(m\theta_n) Z_l(z) \\ &+ \sum_{p=1}^N \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} E_{m,l}^{j,p} Z_l(z) \\ &\sum_{m'=-\infty}^{\infty} H_{m+m'}(k_l R_{pn}) J_{m'}(k_l r_n) e^{i(m\alpha_{pn} + m'\alpha_{np})} \cos(m'\theta_n), \quad r_n \leq \min_{p=1, \dots, N; p \neq n} R_{pn}. \end{aligned} \quad (19)$$

2.3. Computation of unknown coefficients

Across each interface between two adjacent regions, ϕ_j must fulfil the continuity requirements of radial flux and pressure.

(1) Normal-velocity continuity across the interface $r_n = R_{i,n}$ and $\theta_n \in [0, \pi]$:

$$\frac{\partial \phi_j}{\partial r_n} \Big|_{r_n=R_{i,n}^-} = \begin{cases} 0, & -d_n < z < 0, \\ \frac{\partial \phi_j}{\partial r_n} \Big|_{r_n=R_{i,n}^+}, & -h < z < -d_n. \end{cases} \quad (20)$$

(2) Normal-velocity continuity across the interface $r_n = R_n$ and $\theta_n \in [0, \pi]$:

$$\frac{\partial \phi_j}{\partial r_n} \Big|_{r_n=R_n^+} = \begin{cases} 0, & -d_n < z < 0, \\ \frac{\partial \phi_j}{\partial r_n} \Big|_{r_n=R_n^-}, & -h < z < -d_n. \end{cases} \quad (21)$$

(3) Pressure continuity across the interface $r = R_{i,n}$ and $\theta_n \in [0, \pi]$:

$$\phi_j|_{r=R_{i,n}^+} = \phi_j|_{r=R_{i,n}^-}, \quad -h < z < -d_n. \quad (22)$$

(4) Pressure continuity across the interface $r = R_n$ and $\theta_n \in [0, \pi]$:

$$\phi_j|_{r=R_n^+} = \phi_j|_{r=R_n^-}, \quad -h < z < -d_n. \quad (23)$$

Combining the regional series for ϕ_j (Section 2.2) with the interface conditions (Eqs. (20)–(23)) and applying orthogonality relations leads to a truncated linear system, from which $A_{m,l}^{j,n}$, $C_{m,l}^{j,n}$, $D_{m,l}^{j,n}$, and $E_{m,l}^{j,n}$ are determined. For brevity, the calculation steps are provided in Appendix A. While visually similar to our prior work regarding a single breakwater-integrated semi-circular OWC [25], the present model considers an array of semi-circular OWCs on a straight coast. This fundamental shift accounts for wave interaction between units and requires new mathematical formulations not only for the external velocity potential, i.e., Eq. (19), but also for the dynamic equation and wave excitation forces etc., which will be reported in the following subsections.

2.4. Wave excitation volume flux and hydrodynamic coefficients

The wave excitation volume flux of the n th OWC produced by the oscillating internal water surface inside the n th OWC chamber due to the scattering waves can be written as

$$Q_e^{(n)} = \int_0^\pi \int_0^{R_{i,n}} \frac{\partial \phi_0}{\partial z} \Big|_{z=0} r_n dr_n d\theta_n = \frac{\pi \omega^2 R_{i,n}}{g} \sum_{l=0}^{\infty} A_{0,l}^{j,n} \frac{1}{k_l} J_1(k_l R_{i,n}) Z_l(0). \quad (24)$$

Following the same procedure, the radiation-induced volume flux of the n th OWC, produced by imposing a unit oscillatory air pressure in the j th chamber, is given by

$$Q_{r,j}^{(n)} = \int_0^\pi \int_0^{R_{i,n}} \frac{\partial \phi_j}{\partial z} \Big|_{z=0} r_n dr_n d\theta_n = \frac{\pi \omega^2 R_{i,n}}{g} \sum_{l=0}^{\infty} A_{0,l}^{j,n} \frac{1}{k_l} J_1(k_l R_{i,n}) Z_l(0)$$

$$= -(c_j^{(n)} - i a_j^{(n)}), \quad (25)$$

where the third equality introduces the coefficients $c_j^{(n)}$ and $a_j^{(n)}$, which represent the radiation damping and added mass terms, respectively.

2.5. Wave power absorption in regular and irregular waves

2.5.1. Regular waves

In the present idealized model, each PTO is approximated by a lossless linear Wells turbine, therefore the mass flow rate through the turbine is taken to vary linearly with the oscillatory air pressure in the chamber. $\mathbf{p}$ denotes the vector of complex air pressure amplitudes, with components p_n for the n th OWC ($n = 1, 2, \dots, N$). Furthermore, based on the scattered and radiated potentials, $\mathbf{p}$ can be derived from

$$[-i(\mathbf{A}_{\text{PTO}} + \mathbf{A}) + (\mathbf{C}_{\text{PTO}} + \mathbf{C})] \mathbf{p} = \mathbf{Q}_e, \quad (26)$$

where $\mathbf{A}_{\text{PTO}}$ is an $N \times N$ diagonal matrix that accounts for air-compressibility effects. Its n th diagonal entry is $a_{\text{PTO}}^{(n)} = \omega V_n / (v^2 \rho_0)$, where V_n is the air-chamber volume of the n th OWC, v denotes the speed of sound in air, and ρ_0 is the ambient (static) air density. The matrix $\mathbf{C}_{\text{PTO}}$ is likewise diagonal of size $N \times N$ and represents the PTO damping; it is governed by the turbine rotational speed, rotor geometry, turbine design parameters, and ρ_0 . The matrices $\mathbf{A}$ and $\mathbf{C}$ (both of dimension $N \times N$) collect the hydrodynamic coefficients $a_j^{(n)}$ and $c_j^{(n)}$, respectively. The vector $\mathbf{Q}_e$ is an N -component column vector comprising the complex wave excitation volume flux $Q_e^{(n)}$ acting on the n th OWC ($n = 1, 2, \dots, N$). In the present work, the optimal PTO damping for each semi-circular OWC is taken to be the same as that for an isolated coast-attached semi-circular OWC. Formally, the array-optimal PTO settings would be obtained by maximizing the absorbed power with respect to the N PTO damping variables, which yields N coupled optimality conditions. While such optimization is straightforward for a single device and still manageable for very small arrays (e.g., $N = 2$), it becomes increasingly complex for large N and typically requires numerical optimization at each wave frequency, therefore we adopt the isolated optimum as a simplified, suboptimal baseline in the present analytical study. It also remains a reasonable approximation when device spacing is large relative to the wavelength, as hydrodynamic coupling weakens under those conditions.

The time-averaged power output P absorbed by the PTO system of the coast-attached semi-circular OWC array is expressed as:

$$P = \frac{1}{2} \mathbf{p}^* \mathbf{C}_{\text{PTO}} \mathbf{p}, \quad (27)$$

in which the superscript $*$ indicates the complex-conjugate transpose.

Wave power absorption is conveniently quantified using the relative capture width, also known as the wave power capture factor:

$$\eta = \frac{kP}{P_{\text{in}}} = \frac{2kP}{\rho g A^2 c_g}, \quad (28)$$

where P_{in} is the incident wave power per unit width of wave front, and c_g is the group velocity of the incident wave defined as $c_g = d\omega/dk = (\omega/2k)(1 + 2kh/\sinh 2kh)$.

Unless otherwise specified, we restrict attention to a uniform array in which all OWCs share identical geometry and are equally spaced, and the same PTO system is employed for every unit. Furthermore, we adopt the simplified notation $R_n = R$, $R_{i,n} = R_i$, $d_n = d$, $D_n = D$, $c_{\text{PTO}}^{(n)} = c_{\text{PTO}}$, and $a_{\text{PTO}}^{(n)} = a_{\text{PTO}}$. The air-compressibility term is evaluated as $a_{\text{PTO}} = \omega V_0 / (v^2 \rho_0)$, where $\rho/\rho_0 = 1000$, $v = 340$ m/s, $h = 10$ m, and $V_0 = \pi R_i^2 h/2$. We denote the wave power capture factor attained by the n th OWC by η_n .

In addition to the overall wave power capture factor η , the influence of hydrodynamic interactions within an array of coast-attached semi-circular OWCs on wave power absorption performance can be evaluated by introducing the q -factor, defined as:

$$q = \frac{\eta}{N\eta_0}, \quad (29)$$

where η_0 represents the wave power capture factor attained by an isolated coast-attached semi-circular OWC under the same PTO setting. When $q > 1$, deploying multiple OWCs along the coast enhances energy absorption (constructive interaction); in contrast, when $q < 1$, the interactions between the coast-attached OWCs impair energy absorption (destructive interaction).

Hereinafter, the dimensionless coefficients of $Q_e^{(n)}$, $c_j^{(n)}$ and $a_j^{(n)}$, together with the PTO-related parameters, are redefined as follows:

$$\bar{Q}_e^{(n)} = \frac{\sqrt{g/h}}{Ahg} Q_e^{(n)}, \quad (\bar{c}_j^{(n)}, \bar{a}_j^{(n)}, \bar{c}_{\text{PTO}}^{(n)}, \bar{a}_{\text{PTO}}^{(n)}) = \frac{\rho\sqrt{g/h}}{h} (c_j^{(n)}, a_j^{(n)}, c_{\text{PTO}}^{(n)}, a_{\text{PTO}}^{(n)}). \quad (30)$$

2.5.2. Irregular waves

Although the above derivations are presented for regular waves, the present frequency-domain linear framework can also be extended to irregular seas using a standard spectral approach. Specifically, following widely used formulations (e.g., Cui et al. [6], Goda [40]), the sea state is represented by the JONSWAP spectrum of free-surface elevation,

$$S_\eta(\omega_p, \omega) = \alpha_\eta \frac{H_s^2}{\omega} \left(\frac{\omega_p}{\omega} \right)^4 \exp \left[-1.25 \left(\frac{\omega_p}{\omega} \right)^4 \right] \gamma_\eta \exp \left[-\frac{(\omega/\omega_p - 1)^2}{2\sigma^2} \right], \quad (31)$$

where H_s denotes the significant wave height (also referred to as H_{m0}), ω_p is the spectral peak angular frequency, and

$$\alpha_\eta = \frac{0.0624 (1.094 - 0.01915 \ln \gamma_\eta)}{0.23 + 0.0336 \gamma_\eta - 0.185 (1.9 + \gamma_\eta)^{-1}}, \quad \sigma = \begin{cases} 0.07, & \omega \leq \omega_p, \\ 0.09, & \omega > \omega_p, \end{cases} \quad \gamma_\eta = 3.3. \quad (32)$$

To evaluate the mean absorbed power in irregular waves, we define the unit-amplitude pressure response $\mathbf{p}_1(\omega)$ as the solution of the governing system (i.e., Eq. (26)) for a regular incident wave with amplitude $A = 1$ m at frequency ω . For a given sea state, a frequency-independent PTO damping matrix $\mathbf{C}_{\text{PTO}}^{\text{irr}}$ can be adopted for the irregular-wave evaluation. The time-averaged power absorbed by the PTO system is then obtained through spectral integration,

$$P_{\text{irr}}(\omega_p) = \int_0^\infty S_\eta(\omega_p, \omega) \mathbf{p}_1^*(\omega) \mathbf{C}_{\text{PTO}}^{\text{irr}} \mathbf{p}_1(\omega) d\omega. \quad (33)$$

The corresponding incident wave power per unit crest width is

$$P_{\text{in,irr}}(\omega_p) = \int_0^\infty \rho g c_g(\omega) S_\eta(\omega_p, \omega) d\omega, \quad (34)$$

where $c_g(\omega)$ is the group velocity defined previously. Accordingly, the irregular-wave capture factor is defined as

$$\eta_{\text{irr}}(\omega_p) = \frac{k_p P_{\text{irr}}(\omega_p)}{P_{\text{in,irr}}(\omega_p)}, \quad (35)$$

with k_p determined from the peak-frequency dispersion relation $\omega_p^2 = g k_p \tanh(k_p h)$.

For consistency, the regular-wave capture factor can also be evaluated under the same fixed (frequency-independent) PTO damping used for irregular waves, i.e.,

$$\eta^*(\omega) = \frac{2k(\omega) P^*(\omega)}{\rho g A^2 c_g(\omega)}, \quad P^*(\omega) = \frac{1}{2} \mathbf{p}^*(\omega) \mathbf{C}_{\text{PTO}}^{\text{irr}} \mathbf{p}(\omega), \quad (36)$$

where $\mathbf{p}(\omega)$ is obtained from Eq. (26) with $\mathbf{C}_{\text{PTO}} = \mathbf{C}_{\text{PTO}}^{\text{irr}}$. The superscript “*” indicates that η^* is computed with a fixed frequency-independent PTO setting and is therefore directly comparable with $\eta_{\text{irr}}(\omega_p)$.

2.6. Wave excitation and radiation forces

By applying a unit oscillatory air pressure in the j th chamber, the resulting wave excitation and radiation forces on the n th semi-circular

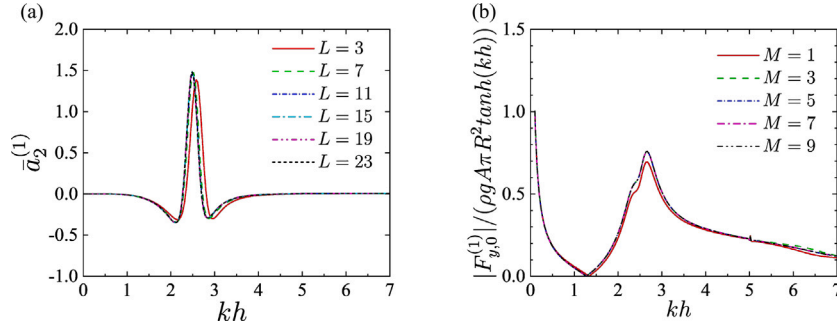


Fig. 3. Convergence results: (a) Effect of the vertical truncation number L on the frequency response of $\bar{a}_2^{(1)}$ for $M = 7$; (b) Effect of the angular truncation number M on the frequency response of $|F_{y,0}^{(1)}|/(\rho g A \pi R^2 \tanh(kh))$ for $L = 19$.

OWC along the x - and y -axes are given by

$$F_{x,j}^{(n)} = F_{cyl,x,j}^{in(n)} + F_{cyl,x,j}^{out(n)}, \quad (37)$$

and

$$F_{y,j}^{(n)} = F_{cyl,y,j}^{in(n)} + F_{cyl,y,j}^{out(n)}, \quad (38)$$

respectively, in which $F_{cyl,x,j}^{in(n)}$ and $F_{cyl,y,j}^{in(n)}$ ($j = 0$ and $j = 1, 2, \dots, N$) are, respectively, the x - and y -components of the forces on the inner semi-cylindrical wall of the n th OWC arising from a unit air pressure oscillation in the j th chamber, including the wave excitation contribution when $j = 0$ and the wave radiation contributions when $j = 1, 2, \dots, N$. Similarly, $F_{cyl,x,j}^{out(n)}$ and $F_{cyl,y,j}^{out(n)}$ ($j = 0$ and $j = 1, 2, \dots, N$) refer to the corresponding force components on the outer semi-cylindrical wall. The calculation details are provided in [Appendix B](#).

In addition, the maximum resultant wave forces $F_{max}^{(n)}$ over one wave period are computed,

$$F_{max}^{(n)} = \max \left(\sqrt{|F_x^{(n)}|^2 \cos^2(\varphi_1^{(n)} + \omega t) + |F_y^{(n)}|^2 \cos^2(\varphi_2^{(n)} + \omega t)} \right) \\ = \sqrt{\frac{|F_x^{(n)}|^2 + |F_y^{(n)}|^2 + \sqrt{(|F_x^{(n)}|^2 + |F_y^{(n)}|^2)^2 - 4|F_x^{(n)}|^2 |F_y^{(n)}|^2 \sin^2(\varphi_1^{(n)} - \varphi_2^{(n)})}}{2}}, \quad (39)$$

where

$$F_x^{(n)} = F_{x,0}^{(n)} + \sum_{j=1}^N p_j F_{x,j}^{(n)}, \quad (40)$$

and

$$F_y^{(n)} = F_{y,0}^{(n)} + \sum_{j=1}^N p_j F_{y,j}^{(n)}, \quad (41)$$

the phase angles of $F_x^{(n)}$ and $F_y^{(n)}$ are denoted by $\varphi_1^{(n)}$ and $\varphi_2^{(n)}$, respectively.

3. Convergence analysis and model verification

Taking a semi-circular OWC system with $N = 2$, $R/h = 0.5$, $(R - R_i)/h = 0.1$, $D/h = 1.5$ and $d/h = 0.2$ as a representative case, the convergence of the series truncations is examined. Firstly, the vertical truncation number L is increased while the angular truncation is fixed at $M = 7$. The response $\bar{a}_2^{(1)}$ becomes insensitive to further increases in L over the investigated frequency range once $L \geq 19$ ([Fig. 3\(a\)](#)). Next, with $L = 19$ held constant, the angular truncation M is varied, showing that $|F_{y,0}^{(1)}|/(\rho g A \pi R^2 \tanh(kh))$ attains convergence for $M \geq 7$ ([Fig. 3\(b\)](#)). Hereinafter, $L = 19$ and $M = 7$ are adopted in the remainder of the study as conservative truncations to ensure stable results for the subsequent parametric investigations involving β , R/h , d/h , D/h , N and R_1/R_2 .

The present theoretical model focuses on an array of coast-attached semi-circular OWCs ($N \geq 2$). When the OWCs are spaced sufficiently far apart, their hydrodynamic interactions become negligible, and the performance of each device in the array is equivalent to a single coast-attached semi-circular OWC, a case previously investigated by Tian et al. [25]. [Fig. 4](#) presents the frequency responses of $\bar{Q}_e^{(1)}$, $\bar{c}_1^{(1)}$ and $\bar{a}_1^{(1)}$ for the semi-circular OWC(s) with $R/h = 0.5$, $(R - R_i)/h = 0.1$ and $d/h = 0.2$. The present results for $N = 2$, $D/h = 500$ and $\beta = \pi/3$ show good agreement with those for a single semi-circular OWC as expected.

Moreover, through applying image theory, the scattering configuration considered herein can be mapped to an equivalent problem in which bidirectional incident waves interact with offshore full-circular OWCs. Under this equivalence, the flux for multiple coast-attached semi-circular OWCs equals one half of the flux obtained for the same number of offshore full-circular OWCs under bidirectional wave incidence [41]. In the same manner, the radiation problems associated with multiple coast-attached semi-circular OWCs can be recast as those for an offshore array of full-circular OWCs, implying that the corresponding hydrodynamic coefficients should be reduced to one half of the offshore full-circular values. To demonstrate this property, [Fig. 5](#) compares the dimensionless coefficients ($\bar{Q}_e^{(1)}$, $\bar{Q}_e^{(2)}$, $\bar{c}_1^{(1)}$, $\bar{c}_2^{(1)}$, $\bar{a}_1^{(1)}$ and $\bar{a}_2^{(1)}$) from the present formulation with those tested for offshore circular OWCs, using the same geometric and wave parameters ($N = 2$, $R/h = 0.5$, $(R - R_i)/h = 0.1$, $D/h = 1.5$, $d/h = 0.2$ and $\beta = \pi/3$). The close agreement observed confirms that the present model accurately captures both diffraction and radiation effects for coast-attached semi-circular OWC arrays.

4. Results and discussion

4.1. Effect of incident wave direction

[Fig. 6](#) illustrates the wave excitation volume flux of each OWC, the η attained by the OWC system, q -factor and both together with the dimensionless maximum value of the total resultant force of each OWC for different incident wave directions β . As shown in [Fig. 6\(a\)](#), the first peak of the wave excitation of the down-wave semi-circular OWC ($|\bar{Q}_e^{(1)}|$) diminishes and shifts towards lower frequencies with β increasing from $\pi/6$ to $\pi/3$, and shifts rapidly to the same value as $|\bar{Q}_e^{(2)}|$ for $\beta = \pi/2$. It is worth noting that, for $\beta = \pi/3$, an additional peak of $|\bar{Q}_e^{(1)}|$ - kh curve can be observed at $kh = 2.6$, which is attributed to the frequency being close to the resonance of the water bodies between the two semi-circular OWCs, and this resonant frequency is believed to be dependent on the relative size of the interval between the OWCs and the incident wavelength. In contrast, as β increases, the main peaks of the $|\bar{Q}_e^{(1)}|$ - kh curves (see [Fig. 6\(b\)](#)) intensify and shift towards larger kh . As shown in [Fig. 6\(c\)](#), at $kh \approx 2.5$ and 5.2 , two peaks of η - kh curves are observed, which correspond to the first- and second-mode resonances of the internal water column of the semi-circular OWC chambers for the

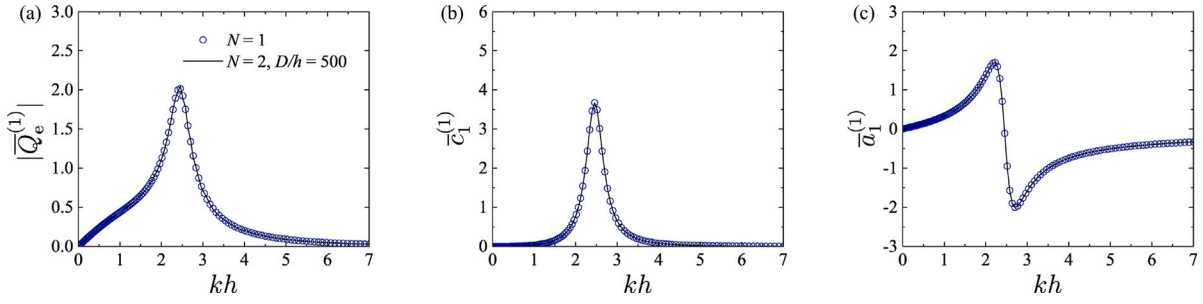


Fig. 4. Frequency-response comparison for the dimensionless coefficients $\bar{Q}_e^{(1)}$, $\bar{c}_1^{(1)}$, and $\bar{a}_1^{(1)}$ between the present results for two semi-circular OWCs ($N = 2$) far away from each other ($D/h = 500$) and the theoretical results for a single semi-circular OWC ($N = 1$) [25] for $R/h = 0.5$, $(R - R_1)/h = 0.1$, $d/h = 0.2$ and $\beta = \pi/3$: (a) $|\bar{Q}_e^{(1)}|$; (b) $\bar{c}_1^{(1)}$; and (c) $\bar{a}_1^{(1)}$.

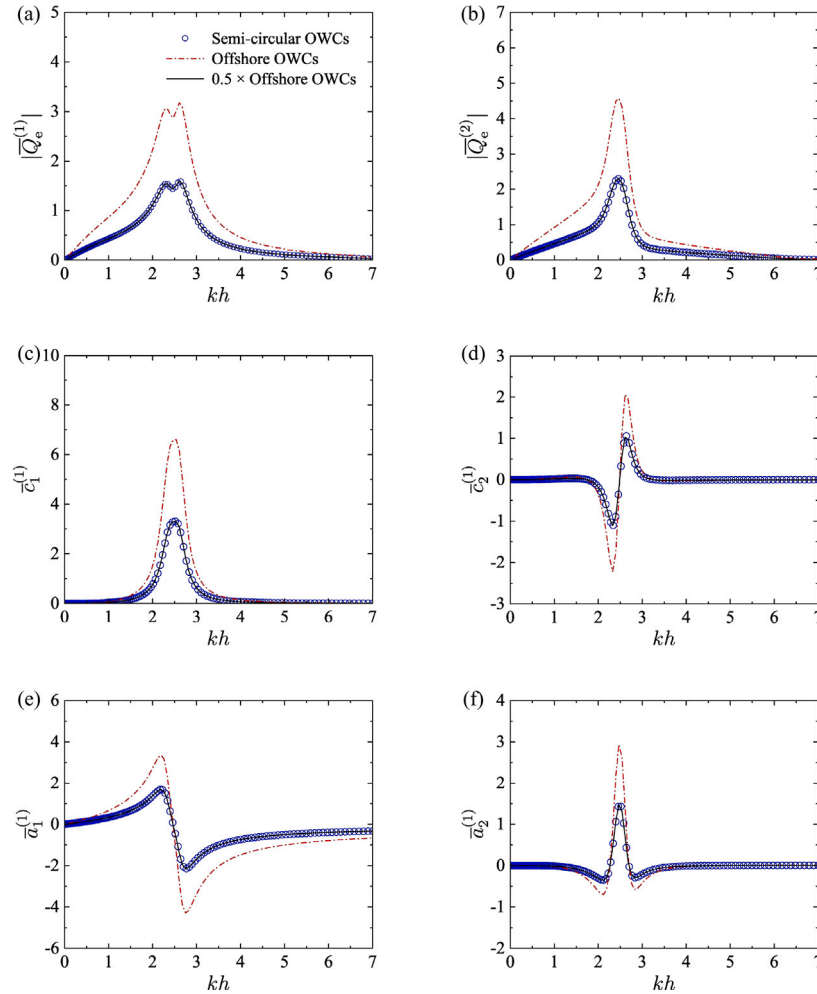


Fig. 5. Frequency-response comparison for the dimensionless coefficients $\bar{Q}_e^{(1)}$, $\bar{Q}_e^{(2)}$, $\bar{c}_1^{(1)}$, $\bar{c}_2^{(1)}$, $\bar{a}_1^{(1)}$ and $\bar{a}_2^{(1)}$ between the present results for a two-unit semi-circular OWC configuration and the corresponding theoretical solutions for two offshore circular OWCs [41]: (a) $|\bar{Q}_e^{(1)}|$; (b) $|\bar{Q}_e^{(2)}|$; (c) $\bar{c}_1^{(1)}$; (d) $\bar{c}_2^{(1)}$; (e) $\bar{a}_1^{(1)}$; and (f) $\bar{a}_2^{(1)}$.

present configuration. These resonant frequencies are identified as the wave frequencies where $a_n^{(n)} + a_{PTO}^{(n)} = 0$ is achieved as reported in Tian et al. [25] for a single coast-attached semi-circular OWC. For $kh \in (1.5, 2.4)$, the wave power capture factor η increases with increasing β from $\pi/6$ to $\pi/2$. For $kh \in (3.5, 6.0)$, the values of η corresponding to $\beta = \pi/4$ and $\pi/3$ are larger than $\beta = \pi/6$ and $\pi/2$ — the reason being the array effect, as shown in Fig. 6(d), where constructive effects ($q > 1.0$) and destructive effects ($q < 1.0$) are indicated for $\beta = \pi/4$, $\pi/3$ and $\beta = \pi/6$, $\pi/2$, respectively, for $kh \in (3.5, 6.0)$. In Fig. 6(e), at $kh \approx 5.2$, sharp increases in the $k|F_{max}^{(1)}|/(\rho g A R \tanh(kh))$ - kh curves

are observed for $\pi/6$, $\pi/4$ and $\pi/3$; similar behaviours are apparent in the $k|F_{max}^{(2)}|/(\rho g A R \tanh(kh))$ - kh curves for $\pi/6$ and $\pi/4$ (Fig. 6(f)). When kh is close to 5.2, corresponding to the sloshing mode resonance of the inner semi-circular OWC water area, these behaviours become evident. With β increasing from $\pi/6$ to $\pi/2$, the main peaks of the $k|F_{max}^{(2)}|/(\rho g A R \tanh(kh))$ - kh curves lose intensity gradually. The value of $k|F_{max}^{(1)}|/(\rho g A R \tanh(kh))$ for $\beta = \pi/4$ and $kh \approx 5.2$, over 18, is larger than any value for all the other incident wave directions, and this may be attributed to resonance in the sloshing modes of the water bodies

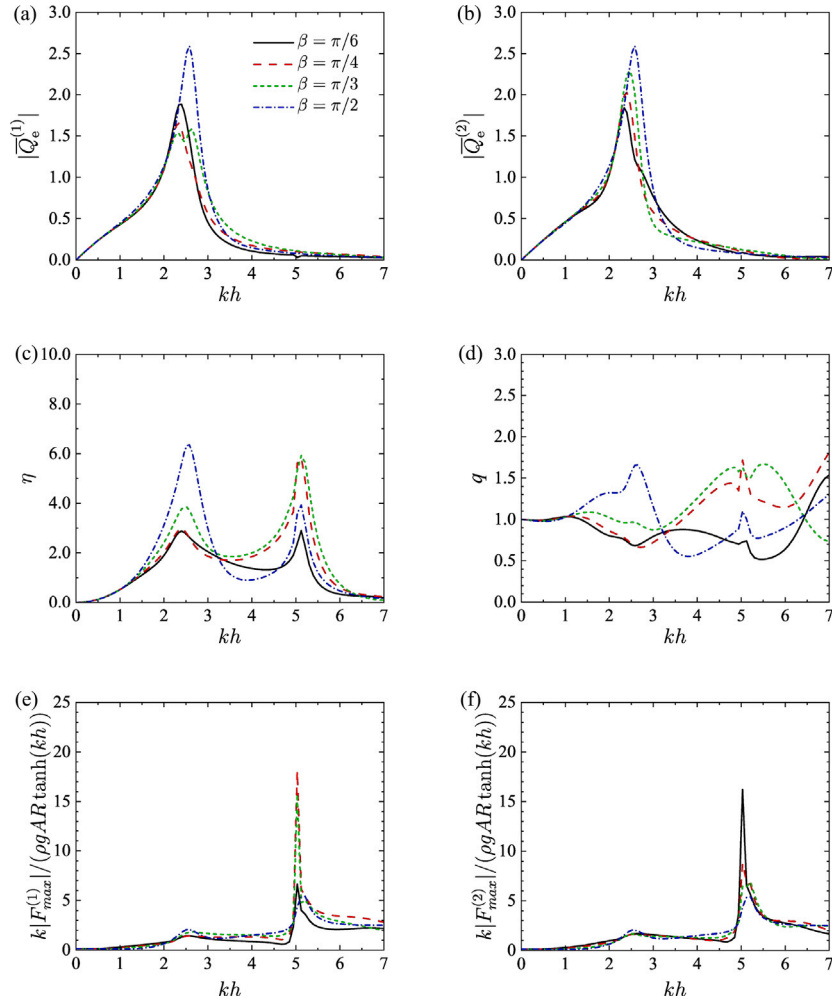


Fig. 6. Comparison for different incident angles β with $N = 2$, $R/h = 0.5$, $(R - R_1)/h = 0.1$, $d/h = 0.2$, $V_0 = \pi R_1^2 h/2$ and $D/h = 1.5$. (a) $|\bar{Q}_e^{(1)}|$; (b) $|\bar{Q}_e^{(2)}|$; (c) η ; (d) q -factor; (e) dimensionless maximum value of the total resultant force of the first semi-circular OWC; (f) dimensionless maximum value of the total resultant force of the second semi-circular OWC.

confined between the two OWCs (see Fig. 7(b)). In practical engineering, the possibility of resonance in the water areas between an array of semi-circular OWCs should be assessed to ensure structural safety. The strong dependence on the incident direction β observed here is consistent with trends reported for rectangular OWC arrays under oblique waves. Specifically, theoretical studies show that the occurrence and intensity of the along-shore sloshing resonance in rectangular chambers are highly angle-dependent, and the hydrodynamic characteristics can vary substantially as the incident angle changes [32,33]. Moreover, related caisson-array configurations embedding multiple rectangular OWC chambers indicate that oblique incidence can narrow the effective capture bandwidth and that constructive and destructive interactions may both occur due to the combined effects of coastal reflection and array coupling [34]. These established findings provide a useful reference frame for interpreting the present semi-circular OWC array results and further highlight that β should be treated as a primary design variable for coastal OWC arrays.

As illustrated in Fig. 7, we focus on the resonance state of the water column confined within the inner semi-circular region of the OWC at $kh = 5.2$, which corresponds to the second peak of η (see Fig. 6(c)), and plot the free-surface patterns $\text{Re}(\omega \phi e^{-i\omega t}/g)/A$ around a two-unit semi-circular OWC array ($N = 2$) for four representative incident angles, namely $\beta = \pi/6, \pi/4, \pi/3$ and $\pi/2$. For $\beta = \pi/6$, Fig. 7(a) shows that the wave motion in the up-wave chamber is mainly characterized by a sloshing mode, leading to pronounced free-surface amplification. A

comparable sloshing-dominated response is observed in the down-wave chamber when $\beta = \pi/3$ (Fig. 7(c)). Due to hydrodynamic interactions between the two devices, the oscillation patterns within the two chambers differ under oblique incidence ($\beta = \pi/6, \pi/4$ and $\pi/3$), as shown in Figs. 7(a)–7(c). When $\beta = \pi/2$, the combined symmetry of the semi-circular chambers and the normally incident wave field leads to a symmetric free-surface pattern both within and around the OWCs (Fig. 7(d)), and the chamber water motion is primarily governed by a piston-type mode.

Fig. 8 presents the difference in performance between the present array of straight coast-attached semi-circular OWCs and an array of straight coast-embedded circular OWCs [30] with the η evaluated for various incident wave angles β . It is worth noting that, $V_0 = \pi R^2 h$ and $D/h = 2.0$ are set to be consistent with the configuration of Zheng et al. [30]. In the low-frequency range, $kh \in (0.5, 2.0)$, the η of two coast-embedded circular OWCs is greater than that of two coast-attached semi-circular OWCs under all the incident wave directions tested. Nevertheless, in the operationally relevant intermediate-frequency band $kh \in (3.2, 4.7)$, the semi-circular OWCs achieve larger η values than the corresponding embedded circular OWCs. While the coast-embedded circular OWCs exhibit a prominent peak at $kh = 4.9$, which is due to the sloshing mode-dominated resonance as reported in Zheng et al. [30], this performance enhancement is restricted to an extremely narrow frequency band, rendering it of limited utility from an engineering standpoint. From structural and economic perspectives, an array of

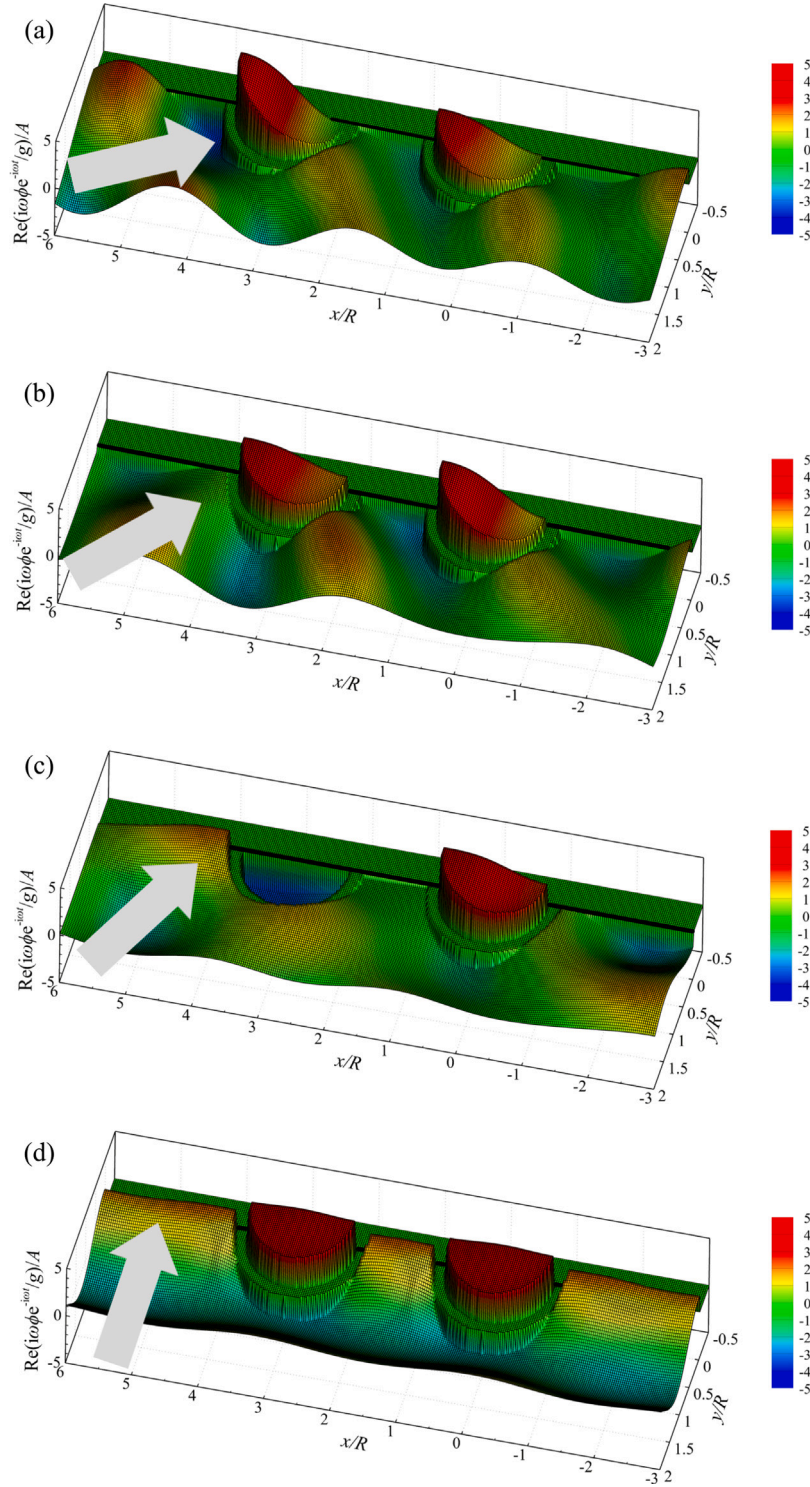


Fig. 7. Free-surface elevation inside and around an array of semi-circular OWCs mounted on a straight coastal structure, $N = 2$, $R/h = 0.5$, $(R - R_i)/h = 0.1$, $d/h = 0.2$, $V_0 = \pi R_i^2 h/2$, $kh = 5.2$, $D/h = 1.5$ and $t = 2\pi/\omega$: (a) $\beta = \pi/6$; (b) $\beta = \pi/4$; (c) $\beta = \pi/3$; (d) $\beta = \pi/2$.

semi-circular OWCs installed along straight coastal structures offer distinct advantages over embedded circular OWCs. For a given radius, the semi-circular configuration requires approximately half the chamber perimeter, directly reducing material usage and construction costs. These semi-circular units can be externally mounted on existing straight coastal infrastructures, eliminating the need for cutting semi-circular recesses into the primary structure. This simplification of the installation process reduces the risk of construction damage.

The array effect between an array of coast-attached semi-circular OWCs not only intensifies the wave power absorption of every OWC across a broad range of frequencies, but also changes the wave-induced forces on each OWC, thereby affecting the structural safety of the system. Fig. 9 compares the resultant wave forces acting on an array of coast-attached semi-circular OWCs and a single coast-attached semi-circular OWC [25] under different incident wave directions β . For $\beta = \pi/6$ (Fig. 9(a)), at the resonant frequency $kh \approx 5.2$, the resultant wave

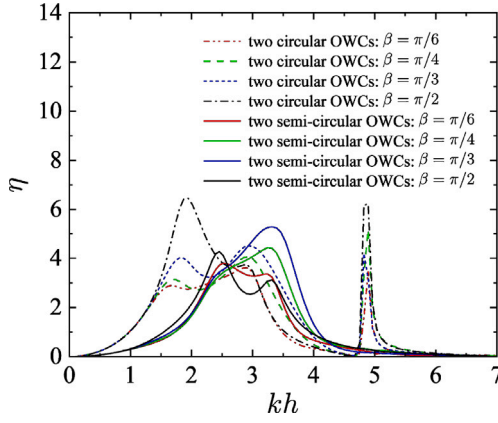


Fig. 8. Comparison of η between the coast-attached semi-circular OWC arrays and the coast-embedded circular OWCs [30] for varying incident angles β , with $N = 2$, $R/h = 0.5$, $(R - R_1)/h = 0.1$, $d/h = 0.2$, $V_0 = \pi R^2 h$ and $D/h = 2.0$.

forces on both the up-wave and down-wave semi-circular OWCs are smaller than that on the single semi-circular OWC. In contrast, for $\beta = \pi/4$ and $\beta = \pi/3$, the down-wave device bears a larger resultant force than the single semi-circular OWC at the same resonant frequency (see Figs. 9(b) and 9(c)). Engineering designs should consider the impact of such a sharp increase in the resultant wave force on the structural safety of an array of coast-attached semi-circular OWCs. As shown in Fig. 9(d), owing to the symmetry of the OWC geometry and the normally incident waves on the straight coastal structure, the two semi-circular OWCs experience identical total resultant wave forces. The first peak values of $k|F_{\max}|/(\rho g A R \tanh(kh))$ - kh curves of the array of semi-circular OWCs are higher than that of the single semi-circular OWC, while the second peak values of these three curves are almost the same.

4.2. Radius of the OWC chambers

Fig. 10 illustrates how the chamber radius influences the wave excitation volume flux $|\bar{Q}_e^{(n)}|$, hydrodynamic coefficients $\bar{c}_1^{(1)}$, $\bar{c}_2^{(1)}$, $\bar{a}_1^{(1)}$ and $\bar{a}_2^{(1)}$, wave power capture factor η , q -factor and the dimensionless peak total resultant force on each semi-circular OWC $k|F_{\max}|/(\rho g A R \tanh(kh))$ over the R/h ratio range of 0.2 to 0.6. As R/h increases from 0.2 to 0.6, the main peaks of the $|\bar{Q}_e^{(n)}|$ - kh curves shift towards the lower frequency; similar trends are also evident in the $\bar{c}_1^{(1)}$ - kh , $\bar{c}_2^{(1)}$ - kh , $\bar{a}_1^{(1)}$ - kh , $\bar{a}_2^{(1)}$ - kh , and η - kh curves (see Figs. 10(b)–10(f)). As illustrated in Figs. 10(a), 10(b), 10(d) and 10(f), within the low-frequency range ($kh \in (0, 2.1)$), $|\bar{Q}_e^{(n)}|$, $\bar{c}_1^{(1)}$, $\bar{a}_1^{(1)}$ and η increase with larger R/h . As R/h increases, the first peak value of the η - kh curve becomes larger, reaching 6.4 at $kh = 2.5$ (see Fig. 10(f)). In contrast, for small R/h values, such as $R/h = 0.2$, the η curve exhibits only a single peak within the computed wave frequency range. In contrast, for larger R/h values ($R/h = 0.3, 0.4$ and 0.5), an additional peak becomes evident, and three peaks can be observed for $R/h = 0.6$. This phenomenon arises because the resonant frequencies of the inner water region within the semi-circular OWC are associated with the relative dimensions of the OWC chamber to the incident wave wavelength. Fig. 10(g) demonstrates that for $R/h = 0.6$, constructive array effects ($q > 1.0$) can be attained over a broad wave frequency range ($kh \in (1.1, 3.2)$). Concurrently, the η remains notably high within this frequency range (see Fig. 10(f)). From an engineering perspective, this specific OWC configuration can be fully leveraged to enhance η within the target mid-frequency band. As shown in Fig. 10(h), the first-mode resonant peak of the $k|F_{\max}|/(\rho g A R \tanh(kh))$ - kh curve shifts towards lower frequencies as R/h increases from 0.2 to 0.6. This can be attributed to the pronounced attenuation of the first-mode resonant peak in the low-frequency range, caused by the increased

radiation damping and added-mass contributions of each semi-circular OWC (see Figs. 10(b) and 10(d)).

4.3. Draught of the OWC chambers

Fig. 11 contrasts the semi-circular OWC array responses obtained with different chamber draughts d . As d/h increases, the first peaks of the $|\bar{Q}_e^{(n)}|$ - kh curves shift towards lower frequencies (see Fig. 11(a)). A similar behaviour appears in the $\bar{c}_1^{(1)}$, $\bar{c}_2^{(1)}$, $\bar{a}_1^{(1)}$, $\bar{a}_2^{(1)}$ and η curves (see Figs. 11(b)–11(f)), and the peak values of the $\bar{c}_1^{(1)}$ - kh and $\bar{a}_1^{(1)}$ - kh curves increase in magnitude as d/h increases. Figs. 11(a)–11(e) demonstrate that the curves of hydrodynamic coefficients become sharper, and the high-frequency cut-off (where the responses decay to near zero) shifts to lower kh as d/h increases. For wave diffraction problem, this behaviour stems from the tendency of wave power to concentrate in the near-surface region at high frequencies. From a wave radiation perspective, increasing the draught of an OWC chamber reduces both wave generation and, consequently, the hydrodynamic interaction between such chambers is also diminished. From an engineering standpoint, this implies that the draught d of the OWCs must be customized to match the dominant wave frequencies to optimize the overall η . Fig. 11(g) shows that constructive array effects ($q > 1.0$) can be achieved for $kh \in (1.7, 2.6)$. Moreover, the array effect q for $d/h = 0.1$ is greater than any other condition in $kh \in (3.1, 3.9)$. Therefore, to optimize the array effect and ultimately achieve high η in the mid-frequency band, the draught of OWC chambers should be minimized, while ensuring that the OWC openings remain fully submerged even under extreme sea conditions. As illustrated in Fig. 11(h), increasing d/h causes the first resonant peaks of the $k|F_{\max}|/(\rho g A R \tanh(kh))$ - kh curves to shift towards lower frequencies while increasing in magnitude. These peaks reach their maximum value, exceeding 14.0, for $d/h = 0.5$ and $kh = 2.9$. This trend underscores the importance of minimizing the draught of OWC chambers to safeguard the structural load-bearing capacity of OWC systems.

4.4. Interval between OWCs

Fig. 12 illustrates the performance characteristics of a semi-circular OWC array with different intervals between OWCs. As depicted in Fig. 12(a), as D/h increases from 1.5 to 3.0, the main peaks of the $|\bar{Q}_e^{(n)}|$ - kh curves shift towards lower frequencies and decrease in magnitude; an analogous trend is evident in the $\bar{a}_2^{(1)}$ curves (see Fig. 12(e)). The peak value of $|\bar{Q}_e^{(n)}|$ for $D/h = 1.0$ is lower than that observed for all other values of D/h . The reduction in wave excitation flux is likely attributable to a weakening of the standing waves that form between the OWCs when their spacing is insufficient. Figs. 12(c) and 12(e) demonstrate that the $\bar{c}_2^{(1)}$ and $\bar{a}_2^{(1)}$ curves generally become flatter with increasing D/h . This behaviour aligns with the physical expectation that the radiation damping and added mass induced by the other OWC tend towards zero as the inter-OWC interval becomes sufficiently large ($D/h \rightarrow +\infty$). Furthermore, the influence of D/h on the curves $\bar{c}_1^{(1)}$ - kh and $\bar{a}_1^{(1)}$ - kh is relatively minor within the frequency range $kh \in (2.0, 3.5)$ (see Figs. 12(b) and 12(d)). This is because the radiation damping and added mass inherent to each individual semi-circular OWC are less dependent on the presence of the other OWC, albeit with a non-negligible mutual influence. Fig. 12(f) demonstrates that within the frequency range $kh \in (1.8, 2.8)$, the wave power capture factor η for $D/h = 1.5$ is greater than that observed under all other interval conditions, with the highest peak value exceeding 6.0 occurring at $kh = 2.6$. However, for $kh \in (2.8, 4.9)$, η is maximized when $D/h = 1.0$. Within this latter frequency range, the closely spaced OWCs ($D/h = 1.0$) exhibit constructive hydrodynamic interactions by array effect ($q > 1.0$) (see Fig. 12(g)), accompanied by the largest dimensionless maximum total resultant force on each semi-circular OWC (as shown in Fig. 12(h)). In practical engineering applications, the design of an

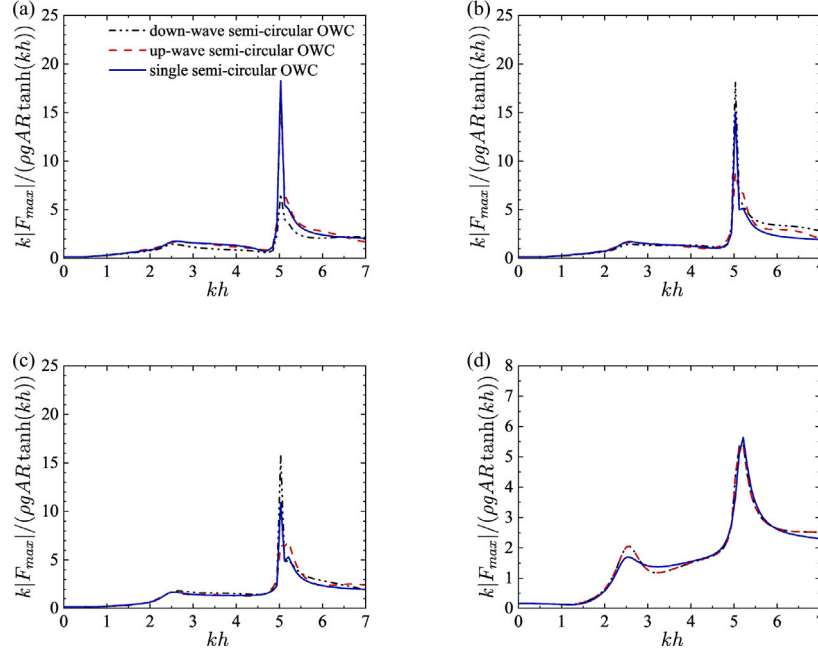


Fig. 9. Comparison of the dimensionless maximum value of the total resultant force acting on the OWCs between an array of the semi-circular OWCs ($N = 2$ and $D/h = 1.5$) and single semi-circular OWC [25] as the incident wave angle β varies, with $R/h = 0.5$, $(R - R_1)/h = 0.1$, $d/h = 0.2$ and $V_0 = \pi R_1^2 h/2$: (a) $\beta = \pi/6$; (b) $\beta = \pi/4$; (c) $\beta = \pi/3$; (d) $\beta = \pi/2$.

array of semi-circular OWCs mounted on a straight coastal structure must take into account both the η attained by the OWC system and the structural safety of individual OWCs. Therefore, the inter-OWC interval should be appropriately optimized to maximize wave power absorption while ensuring the structural safety performance of the OWC system. To guide engineers in selecting an optimal D/h interval, we recommend considering the more compact spacing of $D/h = 1.0$ for semi-circular OWC layouts, particularly in the frequency range $kh \in (2.8, 4.9)$, where the wave power capture factor η reaches its maximum. This configuration is especially efficient when the available coastal length is limited. However, it is important to consider that the closer spacing increases the resonant forces acting on the OWC system, which can lead to structural safety concerns. Consequently, engineers should reinforce the OWC structures to withstand these amplified wave forces, ensuring the OWC array design remains both efficient and safe.

4.5. Number of OWCs

Fig. 13 shows the frequency-dependent variations of the wave power capture factor for individual OWCs, the overall OWC system and the q -factor (for $N = 2, 3, 4$, and 5). For comparative purposes, the wave power capture factor attained by an isolated coast-attached semi-circular OWC (η_0) is also included. For semi-circular OWCs arranged uniformly along a straight coast and exposed to head-on incident waves ($\beta = \pi/2$), each OWC device exhibits symmetric performance characteristics relative to the central plane of the array. For brevity, only the first half of the OWCs are reported; when N is odd, the central device is also included. As shown in Figs. 13(a)–13(d), the main peak values of the η_n - kh curves for semi-circular OWC arrays exceed those of the isolated OWC. Additionally, the peak energy capture factor attained by a semi-circular OWC increases as its position approaches the midpoint of the array. Conversely, within the range $kh \in (3.5, 4.5)$, each device of OWC system exhibits reduced wave power absorption relative to the isolated OWC. This reduction coincides with destructive interaction levels ($q < 1$) indicated by the corresponding q -factor trend (see Fig. 13(f)). As

illustrated in Fig. 13(e), for all calculated wave frequencies except $kh \in (3.6, 4.0)$, the system-level η increases with N . This aligns with the intuitive expectation that a larger array (comprising more semi-circular OWCs) can harvest more wave power in most wave frequency ranges. Typically, for $kh = 2.5$, the wave power capture factor for a five-unit array ($N = 5$) is 18.0, which is even more than nine times that of the isolated case. Fig. 13(f) demonstrates that the q -factor increases with N for $kh \in (2.6, 3.2)$. In addition, for $kh \in (1.0, 3.0)$, all the OWC array configurations considered achieve $q > 1.0$, indicating constructive array effects. Under these wave conditions, therefore, semi-circular OWC arrays comprising more devices are recommended to fully exploit the constructive array effect and extract wave power with higher capture factor.

4.6. Radius difference

As depicted in Fig. 14, we investigate the influence of varying radius ratios (R_1/R_2) on the wave power capture factor for a two-unit semi-circular OWC configuration, where the sum of their cross-sectional areas is kept constant ($R_1^2 + R_2^2 = h^2/2$). This constraint on R_1 and R_2 is imposed to maximize wave power absorption while adhering to fixed structural material limitations in engineering designs. As illustrated in Fig. 14(a), with an increase in R_1/R_2 , the primary peaks of the η_1 - kh curves shift towards lower frequencies and decrease in magnitude. This downshift is consistent with a reduction in the first-mode resonant frequency of the first OWC when its radius increases. For $kh \in (2.3, 2.5)$, η_1 decreases as R_1/R_2 increases; conversely, an opposite trend is observed for $kh \in (3.6, 4.2)$. As shown in Fig. 14(b), increasing R_1/R_2 shifts the first peaks of η_2 to higher frequencies while reducing their magnitude. This loss of peak intensity is due to a narrower effective wave power absorption band when the OWC chamber becomes smaller. Fig. 14(c) demonstrates that as R_1/R_2 increases from 1 to 5, the first peaks of the η - kh curves shift towards lower frequencies and diminish in intensity. For $kh \in (1.5, 2.8) \cup (4.7, 5.8)$, η decreases with increasing R_1/R_2 , and for $kh = 2.5$, η for $R_1/R_2 = 1$ is over twice those for the mixed radii;

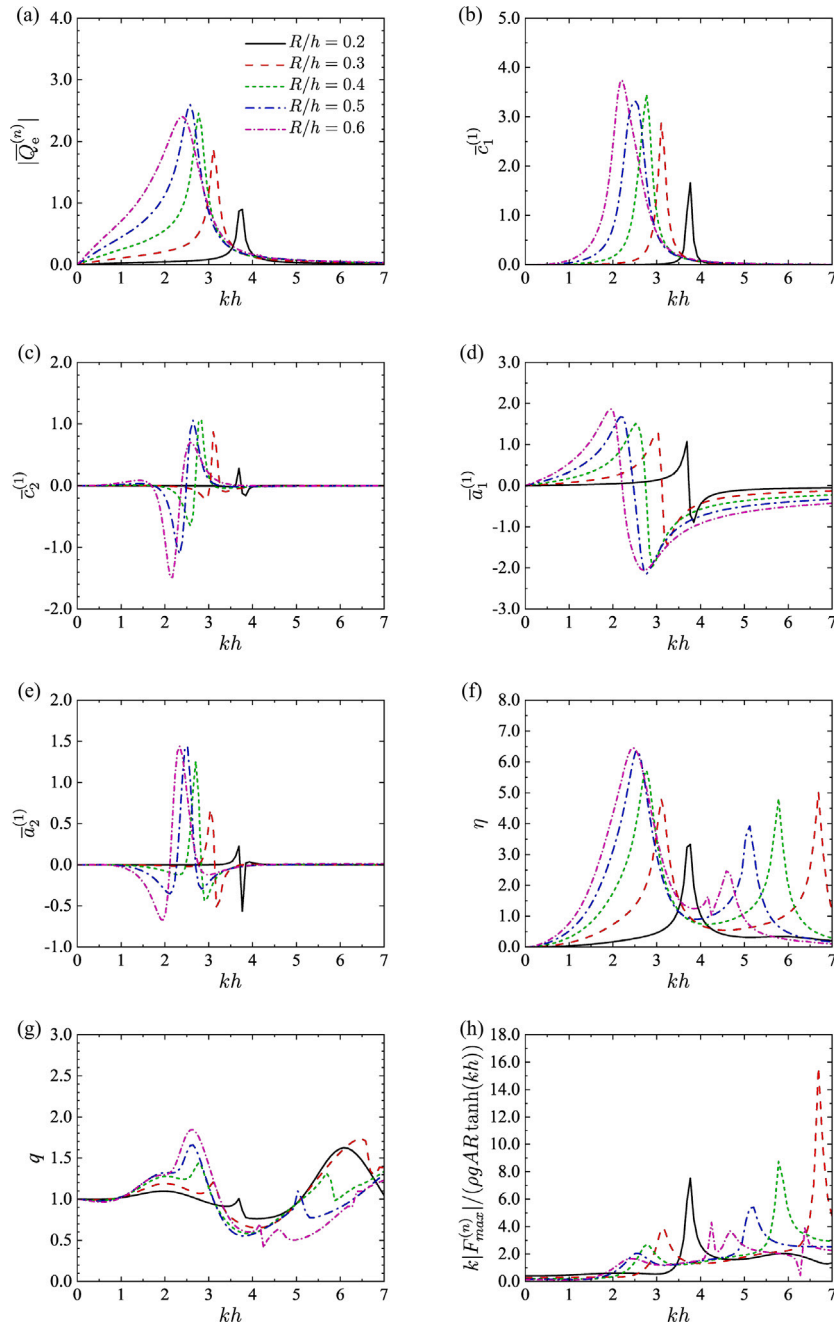


Fig. 10. Comparison for different radii of the semi-circular OWC chambers, R , with $\beta = \pi/2$, $N = 2$, $(R - R_i)/h = 0.1$, $d/h = 0.2$, $V_0 = \pi R_i^2 h/2$ and $D/h = 1.5$. (a) $|Q_e^{(n)}|$; (b) $c_1^{(1)}$; (c) $c_2^{(1)}$; (d) $a_1^{(1)}$; (e) $a_2^{(1)}$; (f) η ; (g) q -factor; (h) dimensionless maximum value of the total resultant force of each semi-circular OWC.

while for $kh \in (3.5, 4.5)$, an OWC array of chambers with different radii absorbs more wave power than an array with two identical chambers. This occurs because increasing the OWCs' radius ratio separates and shifts their resonant frequencies towards opposite directions. In the mid-frequency range, where the wave power absorption response typically has a valley, this separation causes the larger chamber's second absorption peak and the smaller chamber's first absorption peak to shift towards these frequencies. This dual shift enhances overall energy absorption in the previously underperforming band. Therefore, for efficient wave power absorption in engineering applications: in the low-frequency (i.e. $kh \in (1.5, 2.8)$) and high-frequency (i.e. $kh \in (4.7, 5.8)$)

ranges, the two semi-circular OWCs should be configured with identical dimensions; in the mid-frequency range (say $kh \in (3.5, 4.5)$), the wave power capture factor is improved by changing two identical OWC units into those with different chamber radii.

4.7. Comparison of wave power absorption in regular and irregular waves

In practice, tuning the PTO damping of a semi-circular OWC system (C_{PTO}) continuously over a wide frequency range is challenging. Therefore, we consider a frequency-independent PTO damping, selected as the optimum at a representative wave condition $kh = 2.5$ for the present

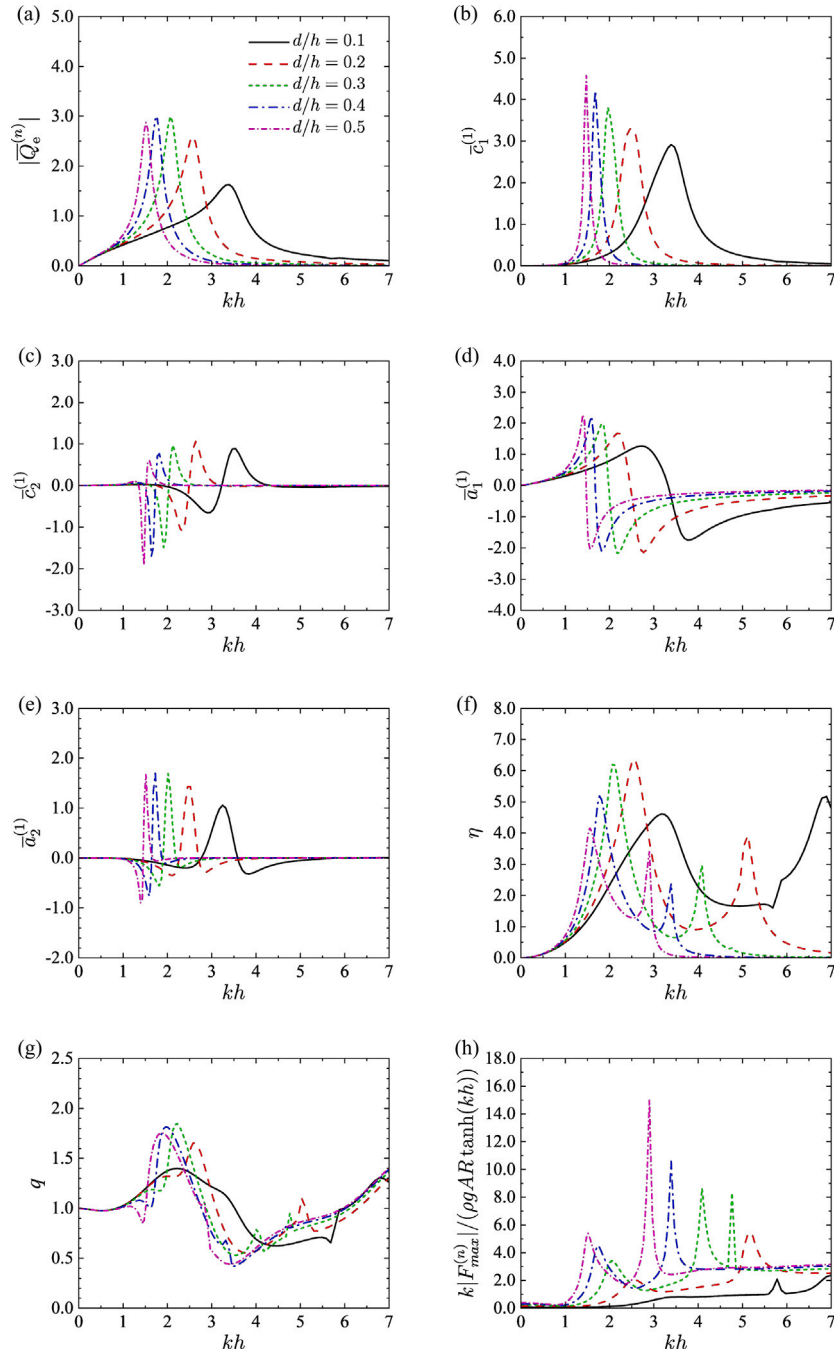


Fig. 11. Comparison for different draughts of the semi-circular OWC chambers, d , with $\beta = \pi/2$, $N = 2$, $R/h = 0.5$, $(R - R_i)/h = 0.1$, $V_0 = \pi R_i^2 h/2$ and $D/h = 1.5$. (a) $|Q_e^{(n)}|$; (b) $c_1^{(1)}$; (c) $c_2^{(1)}$; (d) $a_1^{(1)}$; (e) $a_2^{(1)}$; (f) η ; (g) q -factor; (h) dimensionless maximum value of the total resultant force of each semi-circular OWC.

configuration ($N = 2$, $\beta = \pi/2$, $R/h = 0.5$, $(R - R_i)/h = 0.1$, $d/h = 0.2$, $V_0 = \pi R_i^2 h/2$ and $D/h = 1.5$). Fig. 15 compares the wave power capture factors of the semi-circular OWC system under regular waves $\eta^*(\omega)$ and irregular waves $\eta_{irr}(\omega_p)$. It is worth noting that the irregular-wave capture factor $\eta_{irr}(\omega_p)$ is evaluated as a function of the spectral peak frequency ω_p . Consistent with expectations, spectral averaging smooths the sharp resonance features of the regular-wave response, and the peak of the irregular-wave curve is near the dominant regular-wave resonance, $kh(k_p h) = 2.5$, but with the peak magnitude of the wave power

capture factor reducing from $\eta^* = 6.5$ to $\eta_{irr} = 3.2$. Meanwhile, although $\eta_{irr}(\omega_p) < \eta^*(\omega)$ in the vicinity of the main resonant frequencies, the irregular-wave performance can be comparatively more efficient away from resonance due to the broader effective bandwidth induced by the spectrum.

5. Conclusions

This study examines the hydrodynamic response of coast-attached semi-circular OWC arrays within linear potential-flow theory using

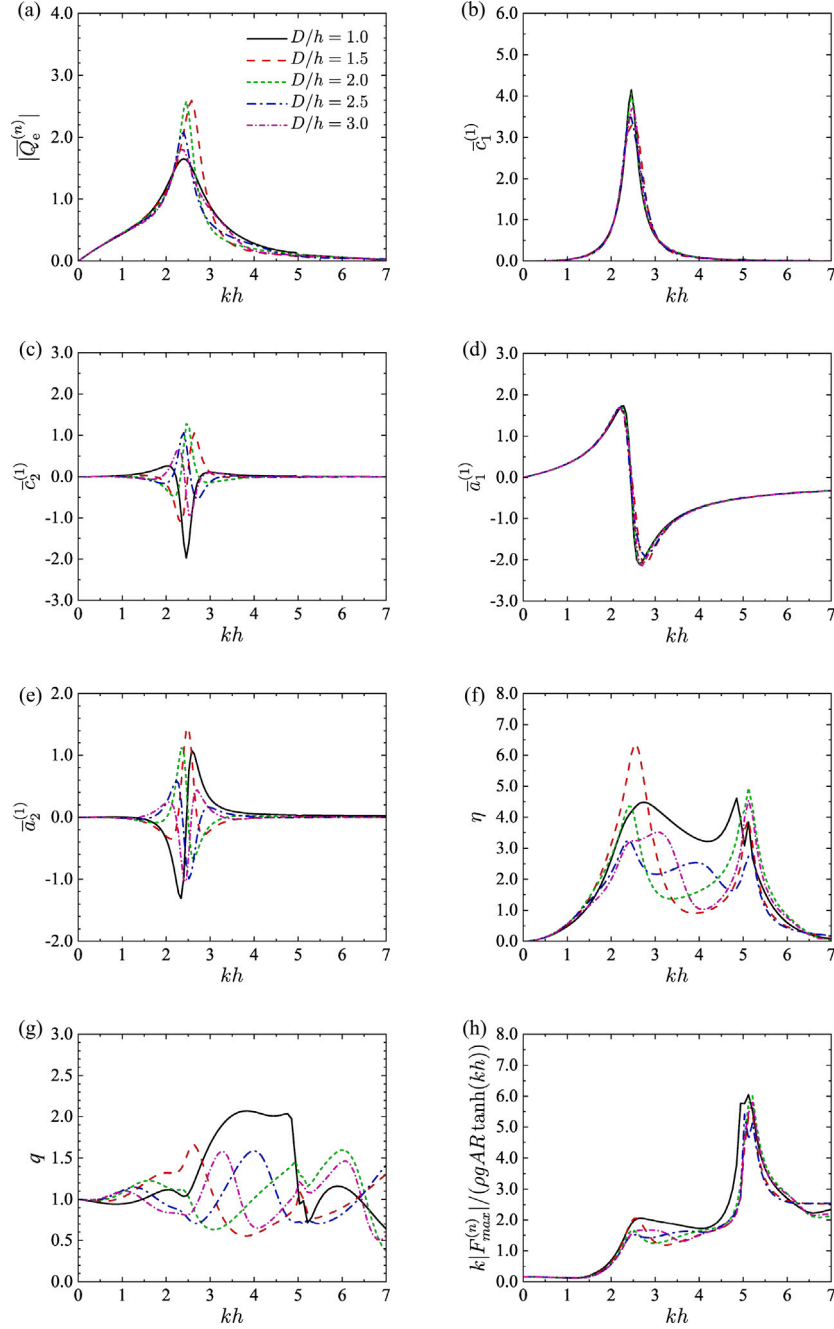


Fig. 12. Comparison for different intervals between the semi-circular OWCs, D , with $\beta = \pi/2$, $N = 2$, $R/h = 0.5$, $(R - R_1)/h = 0.1$, $d/h = 0.2$ and $V_0 = \pi R_1^2 h/2$. (a) $|\overline{Q}_e^{(n)}|$; (b) $\overline{c}_1^{(1)}$; (c) $\overline{c}_2^{(1)}$; (d) $\overline{a}_1^{(1)}$; (e) $\overline{a}_2^{(1)}$; (f) η ; (g) q -factor; (h) dimensionless maximum value of the total resultant force of each semi-circular OWC.

an eigenfunction-matching approach. The wave power capture factors of multiple coast-attached semi-circular and coast-embedded circular OWCs are compared, while wave forces on those semi-circular OWC arrays versus a single coast-attached semi-circular OWC are analysed to assess the array effect on the structural safety of OWC systems. Based on the validated model, an extensive parametric investigation is carried out to quantify the influences of wave conditions, chamber size, chamber draught, interval between adjacent OWCs, the number of OWCs, and the radius ratio between OWCs on the hydrodynamic performance. Key findings are summarized as follows:

Oblique incident waves can excite resonance in the water area between adjacent coast-attached semi-circular OWCs, resulting in significant force amplification at specific frequencies. Compared to coast-embedded circular OWCs, semi-circular arrays exhibit lower wave power absorption at low frequencies but attain a higher wave power capture factor across a practical mid-frequency band. Relative to a single coast-attached semi-circular OWC, the array may either reduce or enhance wave forces under different wave conditions, highlighting the necessity for designers to account for oblique incident wave directions

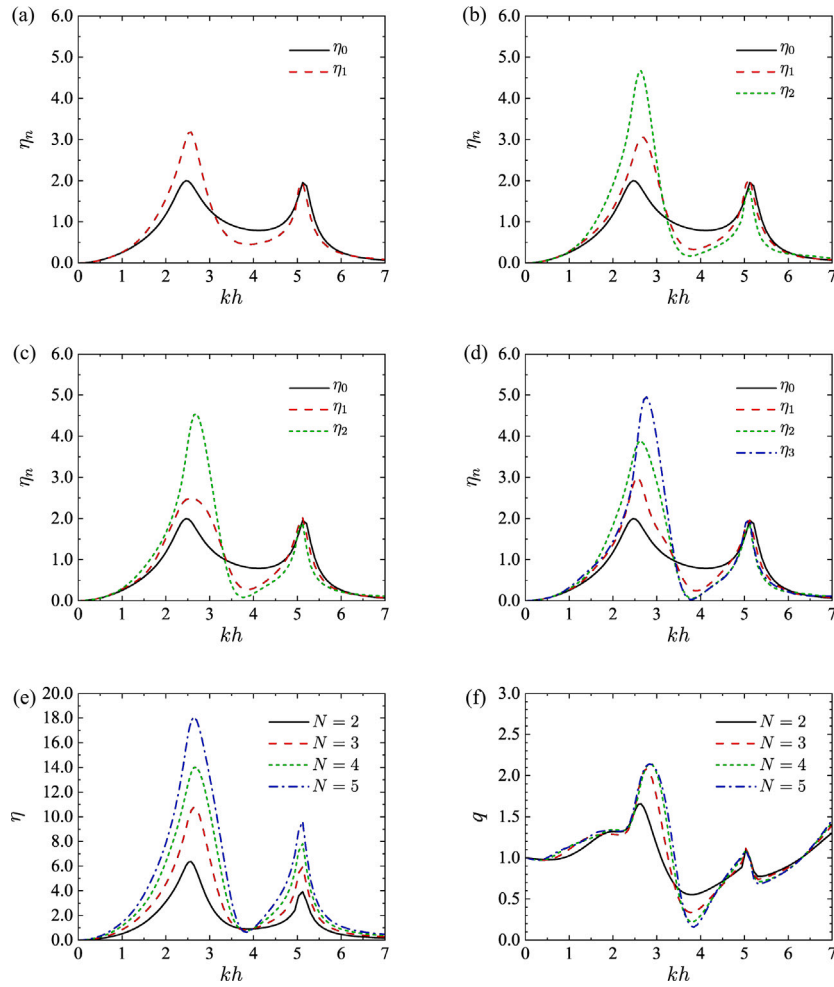


Fig. 13. Effects of different numbers of semi-circular OWCs, N , with $\beta = \pi/2$, $R/h = 0.5$, $(R - R_i)/h = 0.1$, $d/h = 0.2$, $V_0 = \pi R_i^2 h/2$ and $D/h = 1.5$. (a) η_n with $N = 2$; (b) η_n with $N = 3$; (c) η_n with $N = 4$; (d) η_n with $N = 5$; (e) η ; (f) q -factor. η_0 is the wave power capture factor attained by the isolated coast-attached semi-circular OWC.

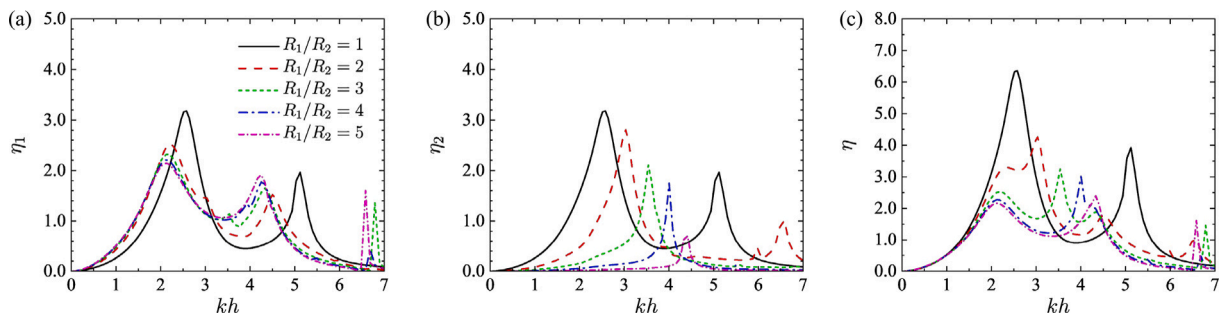


Fig. 14. Influence of the chamber-radius ratio, R_1/R_2 , with $\beta = \pi/2$, $N = 2$, $R_1^2 + R_2^2 = h^2/2$, $(R_1 - R_{i,1})/h = 0.1$, $(R_2 - R_{i,2})/h = 0.1$, $d/h = 0.2$, $V_1 = \pi R_{i,1}^2 h/2$, $V_2 = \pi R_{i,2}^2 h/2$, and $D/h = 1.5$. (a) η_1 ; (b) η_2 ; (c) η .

and explicitly evaluate resonance between adjacent units. As the chamber radius increases, the dominant peaks in both the hydrodynamic coefficients and the capture factor attained by each semi-circular OWC move towards lower frequencies. For sufficiently large radii, the first resonance in the total resultant force is attenuated, implying that

enhanced damping can effectively reduce wave forces in the low-frequency range. Increasing the chamber draught shifts the primary responses of hydrodynamic coefficients towards lower frequencies and intensifies their frequency dependence, while significantly increasing resonant forces on the semi-circular OWC system. Therefore, a deeper

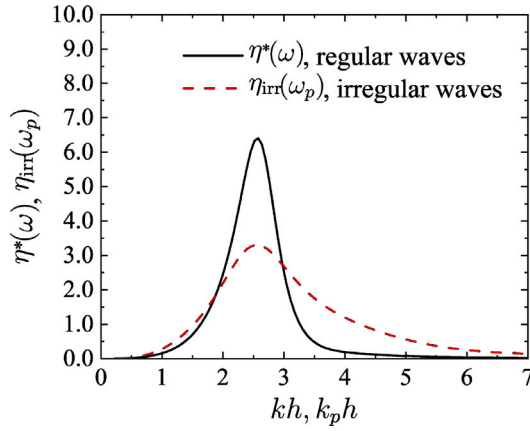


Fig. 15. Comparison of $\eta^*(\omega)$ ($\eta_{irr}(\omega_p)$) with $kh(k_p h)$ for $C_{PTO}^{irr} = C_{PTO}|_{kh=2.5}$, with $N = 2$, $\beta = \pi/2$, $R/h = 0.5$, $(R - R_1)/h = 0.1$, $d/h = 0.2$, $V_0 = \pi R_1^2 h/2$ and $D/h = 1.5$.

draught enhances low-frequency wave power absorption capability but comes at the cost of increased resonance-induced wave forces on the semi-circular OWCs.

Compact layouts of semi-circular OWCs can yield the highest wave power capture factors in the mid-frequency range. However, the interval that optimizes wave power capture factor also induces larger peaks in the resultant force. This emphasizes that wave power absorption optimization must be coupled with wave force assessments to ensure structural safety of the OWC system. Increasing the number of semi-circular OWC devices enhances the system wave power capture factor in most computed wave frequencies, and generates constructive array effects across a broad low-to-mid frequency range. Therefore, semi-circular OWC arrays should incorporate more devices to fully exploit array effects and improve wave power absorption. Under a fixed construction material constraint, semi-circular OWCs with equal radii are preferable for wave power absorption at low and high wave frequencies, whereas an OWC array of chambers with different radii absorbs more wave power than an array with two identical chambers within the mid-frequency band. In engineering practice, it is recommended that semi-circular OWCs with an optimal radius ratio be installed under different wave conditions to maximize wave power absorption.

Although this study presents a theoretical model for coast-attached semi-circular OWC arrays, it does not consider viscous effects and nonlinear wave dynamics, nor does it evaluate extreme wave conditions (e.g., wave breaking and other extreme wave conditions). The optimal PTO damping selected for each OWC in the array is assumed to be equivalent to that of an isolated OWC, leading to a conservative prediction of wave power absorption. Higher array performance is likely achievable with advanced control strategies. Despite these limitations, the present linear calculations remain valuable for identifying the optimal frequency ranges for wave power absorption of OWC systems. Future work will incorporate these complex effects into the analysis of semi-circular OWC systems using CFD simulations.

CRediT authorship contribution statement

Renjie Tian: Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Siming Zheng:** Writing – review & editing, Validation, Supervision, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Wenhua Zhao:** Writing – review & editing, Investigation, Formal analysis, Conceptualization. **Alessandro Antonini:** Writing – review & editing, Conceptualization. **Gregorio Iglesias:** Writing – review & editing, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was partially supported by the National Key R&D Program of China (grant no. 2024YFB4207000) and the National Natural Science Foundation of China (No. U25A20372).

Appendix A. Analytical derivation of the coefficients $A_{m,l}^{j,n}$, $C_{m,l}^{j,n}$, $D_{m,l}^{j,n}$ and $E_{m,l}^{j,n}$

After substituting Eqs. (10) and (13) into Eq. (20), both sides are multiplied by $Z_\zeta(z) \cos(\tau\theta_n)$ and integrated over $z \in [-h, 0]$ and $\theta_n \in [0, \pi]$. For any nonnegative integer pair (τ, ζ) , this yields

$$hk_\zeta J'_\tau(k_\zeta R_{1,n}) A_{\tau,\zeta}^{j,n} - \sum_{l=0}^{\infty} (X_{\tau,l}^{(1,n)} C_{\tau,l}^{j,n} + Y_{\tau,l}^{(1,n)} D_{\tau,l}^{j,n}) L_{l,\zeta}^{(n)} = 0, \quad (42)$$

where

$$L_{l,\zeta}^{(n)} = \int_{-h}^{-d_n} \cos[\beta_{l,n}(z+h)] Z_\zeta(z) dz = \frac{(-1)^l (h-d_n)^2 k_\zeta Z_\zeta(0) \sinh[k_\zeta(h-d_n)]}{[(h-d_n)^2 k_\zeta^2 + l^2 \pi^2] \cosh(k_\zeta h)}, \quad (43)$$

$$X_{\tau,l}^{(1,n)} = \begin{cases} \frac{\tau}{R_n} \left(\frac{R_{1,n}}{R_n} \right)^{\tau-1}, & l = 0, \\ \beta_{l,n} \frac{I'_\tau(\beta_{l,n} R_{1,n})}{I_\tau(\beta_{l,n} R_n)}, & l \neq 0, \end{cases} \quad (44)$$

$$Y_{\tau,l}^{(1,n)} = \begin{cases} \frac{1}{R_{1,n}}, & \tau = 0, l = 0, \\ -\frac{\tau}{R_n} \left(\frac{R_{1,n}}{R_n} \right)^{-\tau-1}, & \tau \neq 0, l = 0, \\ \beta_{l,n} \frac{K'_\tau(\beta_{l,n} R_{1,n})}{K_\tau(\beta_{l,n} R_n)}, & l \neq 0. \end{cases} \quad (45)$$

After substituting Eqs. (13) and (19) into Eq. (21), both sides are multiplied by $Z_\zeta(z) \cos(\tau\theta_n)$ and integrated over $z \in [-h, 0]$ and $\theta_n \in [0, \pi]$. For any nonnegative integer pair (τ, ζ) , this yields

$$\sum_{l=0}^{\infty} (X_{\tau,l}^{(2,n)} C_{\tau,l}^{j,n} + Y_{\tau,l}^{(2,n)} D_{\tau,l}^{j,n}) L_{l,\zeta}^{(n)} - hk_\zeta H'_\tau(k_\zeta R_n) E_{\tau,\zeta}^{j,n} - \sum_{\substack{p=1, \\ p \neq n}}^N \sum_{m=0}^{\infty} T_{m,\tau,\zeta}^{p,n(1)} E_{m,\zeta}^{j,p} = -\frac{2igAk}{\omega} \delta_{j,0} \frac{1}{Z_0(0)} h \delta_{\zeta,0} e_\tau (-i)^\tau J'_\tau(k R_n) \cos(\tau\beta) e^{-ikx_n \cos \beta}, \quad (46)$$

where

$$T_{m,\tau,\zeta}^{p,n(1)} = \frac{\epsilon_\tau}{2} [H_{m+\tau}(k_\zeta R_{pn}) + (-1)^\tau H_{m-\tau}(k_\zeta R_{pn})] hk_\zeta J'_\tau(k_\zeta R_n) e^{i(m\alpha_{pn} + \tau\alpha_{np})}, \quad (47)$$

$$X_{\tau,l}^{(2,n)} = \begin{cases} \frac{\tau}{R_n}, & l = 0, \\ \beta_{l,n} \frac{I'_\tau(\beta_{l,n} R_n)}{I_\tau(\beta_{l,n} R_n)}, & l \neq 0, \end{cases} \quad (48)$$

$$Y_{\tau,l}^{(2,n)} = \begin{cases} \frac{1}{R_n}, & \tau = 0, l = 0, \\ -\frac{\tau}{R_n}, & \tau \neq 0, l = 0, \\ \beta_{l,n} \frac{K'_\tau(\beta_{l,n} R_n)}{K_\tau(\beta_{l,n} R_n)}, & l \neq 0. \end{cases} \quad (49)$$

After substituting Eqs. (10) and (13) into Eq. (22), both sides are multiplied by $\cos[\beta_{\zeta,n}(z+h)]\cos(\tau\theta_n)$ and integrated over $z \in [-h, -d_n]$ and $\theta_n \in [0, \pi]$. For any nonnegative integer pair (τ, ζ) , this yields

$$\sum_{l=0}^{\infty} J_{\tau}(k_l R_{i,n}) L_{\zeta,l}^{(n)} A_{\tau,l}^{j,n} - (h-d_n) \left(X_{\tau,\zeta}^{(3,n)} C_{\tau,\zeta}^{j,n} + Y_{\tau,\zeta}^{(3,n)} D_{\tau,\zeta}^{j,n} \right) = \frac{i\delta_{j,n}}{\rho\omega} \delta_{\zeta,0} \delta_{\tau,0} (h-d_n), \quad (50)$$

where

$$X_{\tau,\zeta}^{(3,n)} = \begin{cases} \left(\frac{R_{i,n}}{R_n} \right)^{\tau}, & \zeta = 0, \\ \frac{I_{\tau}(\beta_{\zeta,n} R_{i,n})}{2I_{\tau}(\beta_{\zeta,n} R_n)}, & \zeta \neq 0, \end{cases} \quad (51)$$

$$Y_{\tau,\zeta}^{(3,n)} = \begin{cases} 1 + \ln\left(\frac{R_{i,n}}{R_n}\right), & \tau = 0, \zeta = 0, \\ \left(\frac{R_{i,n}}{R_n} \right)^{-\tau}, & \tau \neq 0, \zeta = 0, \\ \frac{K_{\tau}(\beta_{\zeta,n} R_{i,n})}{2K_{\tau}(\beta_{\zeta,n} R_n)}, & \zeta \neq 0. \end{cases} \quad (52)$$

After substituting Eqs. (13) and (19) into Eq. (23), both sides are multiplied by $\cos[\beta_{\zeta,n}(z+h)]\cos(\tau\theta_n)$ and integrated over $z \in [-h, -d_n]$ and $\theta_n \in [0, \pi]$. For any nonnegative integer pair (τ, ζ) , this yields

$$\frac{h-d_n}{\epsilon_{\zeta}} \left(C_{\tau,\zeta}^{j,n} + D_{\tau,\zeta}^{j,n} \right) - \sum_{l=0}^{\infty} H_{\tau}(k_l R_n) L_{\zeta,l}^{(n)} E_{\tau,l}^{j,n} - \sum_{\substack{p=1, \\ p \neq n}}^N \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} T_{m,\tau,l}^{p,n(2)} L_{\zeta,l}^{(n)} E_{m,l}^{j,p} = -\frac{2igA}{\omega} \delta_{j,0} \frac{1}{Z_0(0)} L_{\zeta,0}^{(n)} \epsilon_{\tau} (-i)^{\tau} J_{\tau}(k R_n) \cos(\tau\beta) e^{-ikx_n \cos \beta}, \quad (53)$$

where

$$T_{m,\tau,l}^{p,n(2)} = \frac{\epsilon_{\tau}}{2} \left[H_{m+\tau}(k_l R_{pn}) + (-1)^{\tau} H_{m-\tau}(k_l R_{pn}) \right] J_{\tau}(k_l R_n) e^{i(m\alpha_{pn} + \tau\alpha_{np})}. \quad (54)$$

A truncated linear algebraic system is obtained from Eqs. (42), (46), (50) and (53) to determine the unknown coefficients $A_{m,l}^{j,n}$, $C_{m,l}^{j,n}$, $D_{m,l}^{j,n}$ and $E_{m,l}^{j,n}$ numerically. Specifically, the infinite series in $\cos(m\theta)$, $Z_l(z)$, and $\cos[\beta_l(z+h)]$ are truncated at $m = M$ and $l = L$. This yields a total of $4N(M+1)(L+1)$ unknown coefficients, namely $A_{m,l}^{j,n}$, $C_{m,l}^{j,n}$, $D_{m,l}^{j,n}$ and $E_{m,l}^{j,n}$, which are solved using an equal number of equations provided by Eqs. (42), (46), (50) and (53).

Appendix B. Analytical expressions of wave forces acting on each semi-circular OWC

$$F_{cyl,x,j}^{in(n)} = i\omega\rho \int_{-d_n}^0 \int_0^{\pi} \phi_{j,n}|_{r=R_{i,n}^-} R_{i,n} \cos \theta_n d\theta_n dz = \frac{\pi}{2} i\omega\rho R_{i,n} \sum_{l=0}^{\infty} A_{1,l}^{j,n} J_1(k_l R_{i,n}) K_l^{(n)}, \quad (55)$$

and

$$\begin{aligned} F_{cyl,y,j}^{in(n)} &= i\omega\rho \int_{-d_n}^0 \int_0^{\pi} \phi_{j,n}|_{r=R_{i,n}^-} R_{i,n} \sin \theta_n d\theta_n dz \\ &= i\omega\rho R_{i,n} \left(\sum_{m=0}^{\infty} \sum_{l=0}^{\infty} A_{m,l}^{j,n} J_m(k_l R_{i,n}) F_m^{(n)} K_l^{(n)} - 2 \frac{i\delta_{j,n}}{\rho\omega} d_n \right), \end{aligned} \quad (56)$$

where

$$F_m^{(n)} = \int_0^{\pi} \cos(m\theta_n) \sin \theta_n d\theta_n = \begin{cases} 0, & m \text{ is odd,} \\ \frac{2}{1-m^2}, & m \text{ is even,} \end{cases} \quad (57)$$

and

$$K_l^{(n)} = N_l^{-1/2} \frac{1}{k_l} \left(\sinh(k_l h) - \sinh[k_l(h-d_n)] \right). \quad (58)$$

Furthermore,

$$\begin{aligned} F_{cyl,x,j}^{out(n)} &= -i\omega\rho \int_{-d_n}^0 \int_0^{\pi} \phi_{j,n}|_{r=R_n^+} R_n \cos \theta_n d\theta_n dz \\ &= -\frac{\pi}{2} i\omega\rho R_n \left(\sum_{l=0}^{\infty} E_{1,l}^{j,n} H_1(k_l R_n) K_l^{(n)} + \sum_{\substack{p=1, \\ p \neq n}}^N \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} E_{m,l}^{j,p} K_l^{(n)} T_{m,1,l}^{p,n(2)} \right. \\ &\quad \left. - \frac{4\delta_{j,0}gA}{\omega} \frac{N_0^{1/2} K_0^{(n)}}{\cosh(kh)} J_1(k R_n) e^{-ikx_n \cos \beta} \cos \beta \right), \end{aligned} \quad (59)$$

and

$$\begin{aligned} F_{cyl,y,j}^{out(n)} &= -i\omega\rho \int_{-d_n}^0 \int_0^{\pi} \phi_{j,n}|_{r=R_n^+} R_n \sin \theta_n d\theta_n dz \\ &= -i\omega\rho R_n \left(\sum_{m=0}^{\infty} \sum_{l=0}^{\infty} E_{m,l}^{j,n} H_m(k_l R_n) F_m^{(n)} K_l^{(n)} \right. \\ &\quad \left. + \sum_{\substack{p=1, \\ p \neq n}}^N \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} E_{m,l}^{j,p} K_l^{(n)} \sum_{m'=-\infty}^{\infty} F_{m'}^{(n)} H_{m+m'}(k_l R_{pn}) J_{m'}(k_l R_n) e^{i(m\alpha_{pn} + m'\alpha_{np})} \right. \\ &\quad \left. - \delta_{j,0} \frac{2igA}{\omega} \frac{N_0^{1/2} K_0^{(n)}}{\cosh(kh)} e^{-ikx_n \cos \beta} \sum_{m=0}^{\infty} \epsilon_m (-i)^m J_m(k R_n) \cos(m\beta) F_m^{(n)} \right). \end{aligned} \quad (60)$$

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