INTERNATIONAL COURSE IN HYDRAULIC ENGINEERING

LECTURE NOTES ON

SEDIMENT TRANSPORT 1

BY

H.N.C. BREUSERS

DELFT

1974 – 1975

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CONTENTS

1. INTRODUCTION

A study of the sediment transport by water is of importance in several aspects of hydraulic engineering:

- fluvial hydraulics: knowledge of sediment transport forms the basis for the design of river training works, navigation improvement, flood control.
- irrigation: design of stable channels, intakes, settling bassins.
- coastal engineering: prediction of littoral drift, design of coastal protection works and harbours.
- dredging: the suction, transport and deposition of material has many aspects related to the transport of sediments.

The main objective of sediment transport hydraulics is to predict whether an equilibrium condition, erosion (scour) or deposition (silting) will occur and to determine the quantities involved. The rate of sediment transport, expressed as mass, weight or volume per unit time can be determined from measurements or from calculations. Both methods only have a low degree of accuracy so that the sensitivity of the design to possible variations in the calculated transport rates has to be considered.

The main reason for the empirical character of sediment transport knowledge is the complexity of the transport process. The interaction of a turbulent flow, the characteristics of which are only known by empirism, and a boundary consisting of loose sediments can not be described by simple equations. Most of our knowledge is based therefore on experiments and measurements both in the field and in laboratories.

The following subjects will be discussed:

- the flow characteristics of the water.
- the characteristics of the sediments.
- their mutual interaction:
    - initiation of motion.
    - transport mechanisms.
    - bed forms, roughness.
    - stable channels.
    - bed material transport - bed load.
      - suspended load.
    - siltation and scour.
- sediment transport measurements.
- applications.

These lecture notes should be considered as a introduction to the subject. The following general references may be used for further studies.

S. LELIAVSKY	1955	An introduction to fluvial hydraulics. Constable, London.
T. BLENCH	1957	Regime behaviour of canals and rivers. Butterworths, London.
F.M. HENDERSON	1966	Open channel flow (Ch. 4 - Sediment transport) Mac Millan, New York.
F. ENGELUND E. HANSEN	1967	A monograph on sediment transport in alluvial streams. Teknisk forlag, Copenhagen.
A.J. RAUDKIVI	1967	Loose boundary hydraulics. Pergamon, Oxford (good introduction)
W.H. GRAF	1971	Hydraulics of sediment transport. Mc Graw Hill, New York (very complete treatment)
H.W. SHEN (ed.)	1971	River mechanics I. Ft. Collins, Colorado.
M. SELIM YALIN	1972	Mechanics of sediment transport. Pergamon Press, New York.

ASCE, Progress Reports of the Task Committee on the preparation of a Sedimentation Manual. Proc. ASCE Hydr. Div.

1962	88	(HY 4)	Introduction and properties of sediment
1963	89	(HY 5)	Suspension of sediments
1965	91	(HY 2)	Wind erosion and transportation
1966	92	(HY 2)	Initiation of motion
1968	94	(HY 4)	Erosion of cohesive sediment
1969	95	(HY 5)	Sediment measuring techniques: lab. procedures
1969	95	(HY 5)	Sediment measuring techniques: fluvial sediments
1970	96	(HY 6)	Sediment sources and sediment yield
1970	96	(HY 7)	Transport of sediment in pipes
1971	97	(HY 1)	Hydraulic relations for alluvial streams
1971	97	(HY 4)	Sediment discharge formulas
1971	97	(HY 12)	Fundamentals of sediment transportation

2. PROPERTIES OF THE WATER

Some of the relevant properties of water are:

Property	symbol	dimension	remarks
density (specific weight) specific gravity	ρ_w γ_w s	(kg/m^3) (kg/m^2s^2) or (N/m^3) (1) (ratio)	– $\gamma_w = \rho_w \cdot g$ $s = \gamma/\gamma_w$ or ρ/ρ_w
dynamic viscosity kinematic viscosity	η υ	$(kg/m\,s)$ or $(N\,s/m^2)$ (m^2/s)	$\tau = \eta \frac{\partial u}{\delta z}$ $\upsilon = \eta/\rho_w$
surface tension	σ	(kg/s^2) or (N/m)	

The following S.I. units are used:

mass	(kg)	(kilogram)
length	(m)	(meter)
time	(s)	(second)
force	(kgm/s^2) or (N)	(Newton)
energy	(kgm^2/s^2) or (Nm) or (J)	(Joule)
power	(kgm^2/s^3) or (Nm/s) or (J/s) or (W)	(Watt)
pressure, stress	(kg/ms^2) or (N/m^2)	

2.1. Density (kg/m^3)

The density of water varies with temperature T

T:	0	4	12	16	21	32	(° C)
ρ_w:	999.87	1000.0	999.5	999.0	998.0	995.0	(kg/m^3)

Conversion: $1\ lb/ft^3 = 16.02\ kg/m^3$

$1\ slug/ft^3 = 514\ kg/m^3$

The variation of the density may be neglected in most sediment transport calculations:

	kg/m^3	lb/ft^3	$slug/ft^3$
ρ_w fresh water	1000	62.4	1.94
ρ_w sea water	1026	63.9	1.99

2.2. Viscosity

Dynamic viscosity (Ns/m^2)

Defined as the factor of proportionality in:

$$\tau = \eta \frac{\partial U}{\partial z}$$

which is valid for laminar flow.

$\partial U / \partial z$ = velocity gradient (1/s)

Unit in the c g s (centimeter, gram, second) system: poise

1 poise = 1 dyne.s/cm^2 = 0.1 Ns/m^2

Conversion: 1 pdl . s/ft^2 = 1.488 Ns/m^2

(1 pdl = 0.138 N)

1 lbf . s/ft^2 = 47.87 Ns/m^2

(1 lbf = 4.45 N)

Kinematic viscosity (m^2/s)

Defined by $\upsilon = \eta/\rho_w$

Unit in the c g s system: Stokes

1 stokes = 1 cm^2/s = 10^{-4} m^2/s

Conversion: 1 ft^2/s = 0,093 m^2/s

η and υ are a function of temperature. The influence of temperature is significant.

T	0	5	10	15	20	25	30	35	40	(° C)
υ	1.79	1.52	1.31	1.14	1.01	0.90	0.80	0.72	0.65	(10^{-6} m^2/s)

2.3. Surface tension

For the surface water/air: σ = 0.074 N/m' at atmospheric pressure. The variation with temperature can be neglected.

Conversion 1 pdl/ft = 0.454 N/m'

1 lbf/ft = 14.59 N/m'

2.4. Uniform flow in open channels

The equation of steady, uniform flow is reduced to:

$$\frac{\partial \tau}{\partial z} = \frac{\partial p}{\partial x}$$

or $\tau(z) = \rho_w\, g\,(h - z)\,.\,I$

h = water depth

z = distance from the bed

I = hydraulic gradient or slope

The difficulty is now the relation between shear stress and velocity distribution which is necessary to predict this distribution.

For laminar flow the relation is:

$$\tau(z) = \eta \cdot \frac{\partial U(z)}{\partial z}$$

which leads to the parabolic velocity distribution:

$$U(z) = \frac{g\,I}{2\,\upsilon}\,(h^2 - (h-z)^2)$$

and a mean velocity $\overline{U} = \frac{g\,I}{3\,\upsilon} \cdot h^2$

For turbulent flow Prandtl gave the following empirical mixing-length expression:

$$\tau(z) = \rho_w\, l^2\, (\partial U(z)/\partial z)^2$$

Near the bed $\tau(z) \approx \tau(0)$, the bed shear stress

$\tau_o = \rho_w\, g\, h\, I$

and $l = \varkappa z$

$\varkappa$ = kappa, von Kármán's constant ≈ 0.4

This leads to the logarithmic velocity distribution:

$$U(z) = 1/\varkappa\,\sqrt{g h I}\,.\,\ln(z/z_o)$$

Define $u^* = \sqrt{g h I}$ = shear velocity = $\sqrt{\tau_o/\rho_w}$

and take: $\varkappa = 0.4$

then: $U(z) = 2.5\, u^*\, \ln\,(z/z_o)$

z_o = the point where $U = 0$ according to the logarithmic profile.

$U(z)$ is equal to the mean velocity at $z \approx 0.4\, h$

or $\overline{U} = 2{,}5\, u^*\, \ln\,(0.4\, h/z_o)$

or $\overline{U} = 5.75\, u^*\, \log\,(0.4\, h/z_o)$ (ln $\rightarrow$ log gives factor 2.303)

Although the logarithmic velocity distribution was derived for the area near the bed, it appears from measurements that the logarithmic velocity profile is a good approximation for the full depth of the flow due to a simultaneous decrease in shear stress and mixing-length with z.
Values of z_o are found from experiments on smooth and rough boundaries. For smooth boundaries a viscous sublayer exists in which viscous effects predominate. The approximate thickness of this layer is $\delta = 10\, \upsilon/u^*$ (see below) and $z_o \approx 0.01\, \delta \approx 0.1\, \upsilon/u^*$. For boundaries with uniform roughness Nikuradse has found:

$$z_o \approx 0.03\ k_s$$

in which k_s was the size of the sand grains used as roughness. This k_s is used as a standard roughness for other types of roughness.

<u>Smooth boundary</u>

$z_o \approx 0{,}01\ \delta$

$U(z) = 5.75\ u^* \log 100\ z/\delta$

$\bar{U} \approx 5.75\ u^* \log\ 40\ h/\delta$

<u>Rough boundary</u>

$z_o \approx 0.03\ k_s$

$U(z) = 5.75\ u^* \log 33\ z/k_s$

$\bar{U}\cancel{(z)} = 5.75\ u^* \log 12\ h/k_s$

$$\bar{U} = 5.75\ u^* \log\left(\frac{12\,h}{k + 0.3\delta}\right)$$

or $$\bar{U} = (5.75\sqrt{g}) \cdot \sqrt{h\,I} \cdot \log\left(\frac{12\,h}{k + 0.3\delta}\right)$$

or $$\bar{U} = 18\sqrt{h\,I} \cdot \log \frac{12\ h}{k + 0.3\delta}$$

which is the well-known Chezy equation: (see figure 2.1)

$$\bar{U} = C\sqrt{h\,I}$$

A bed is defined as hydraulically rough for $k \overset{>}{\cancel{<}} 0.3\,\delta$

hydraulically smooth for $k\ \cancel{>}^{<}\ 6\,\delta$

The transition laminar – turbulent flow is generally given as

$Re = \bar{w} \cdot h/\upsilon \approx 600$ for open channels.

The value of u^* is related to the velocity distribution by

$$u^* = \frac{1}{5.75} \cdot \frac{\partial\, u(z)}{\partial(\log z)}$$

but this method gives generally inaccurate results.

<u>Viscous sublayer δ</u>

In the viscous sublayer viscosity predominates. The velocity distribution therefore follows from: $\tau(z) = \rho\, \partial u(z)/\partial z$

$$\tau(z) = \tau_o = \rho_w\, g\, h\, I = \rho_w\, u_*^2$$

or $$\frac{u(z)}{u_*} = \frac{u^* z}{\upsilon}$$

Intersection with the logarithmic velocity distribution gives a "theoretical" value for δ:

$$\delta = 11.6\ \upsilon/u^*$$

In fact there is a transition zone from the linear to the logarithmic profile extending from:

COËFFICIËNT VAN DE CHÉZY

IN $m^{1/2}/s$

$$C = 18 \log \frac{12\,R}{k + 2/7\,\delta}$$

$$Re = \frac{\bar{u}\,R}{\nu}$$

$$\bar{u} = C\sqrt{RI}$$

FOR PIPES:
$C' = C + 1$ to 2

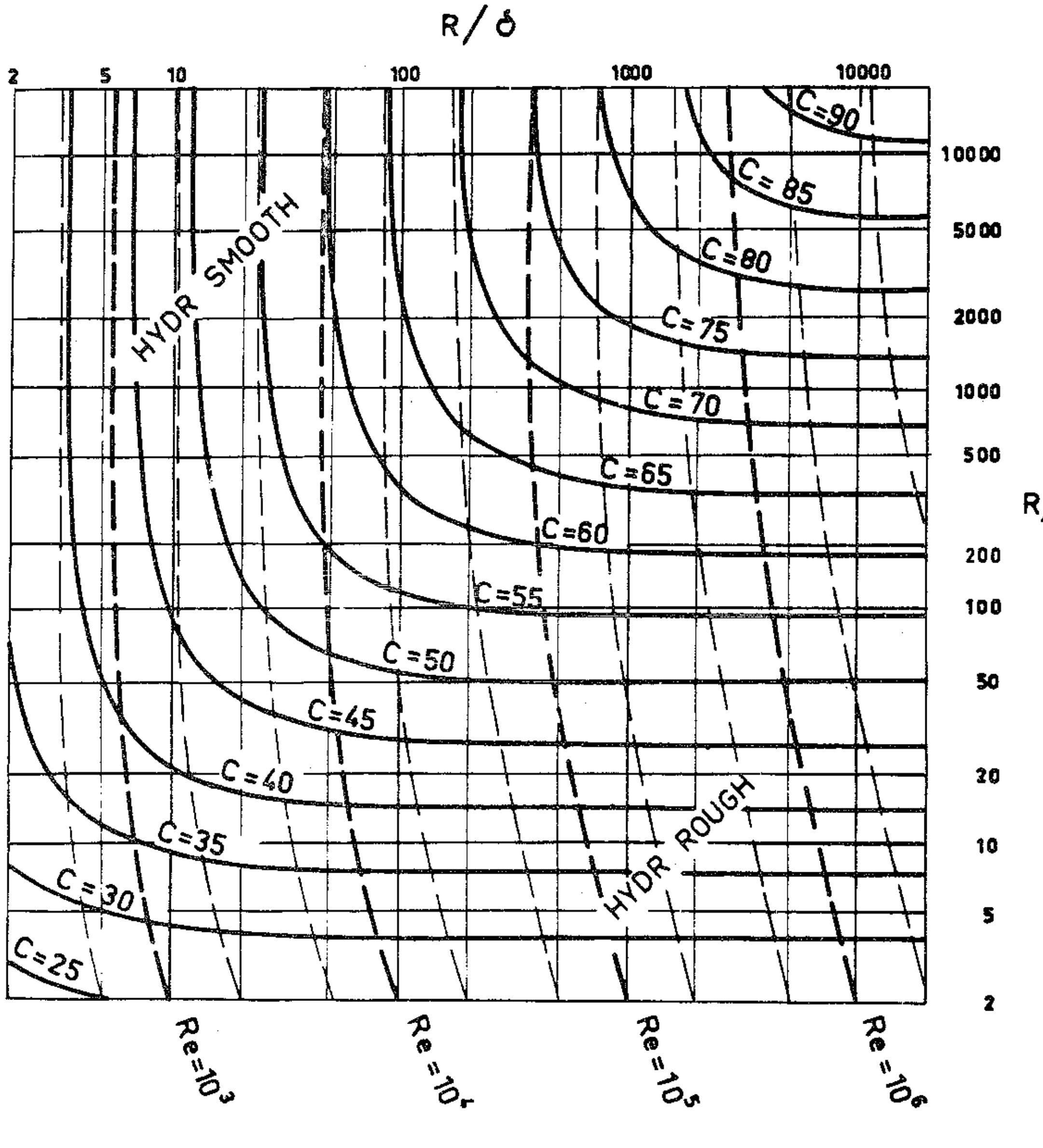
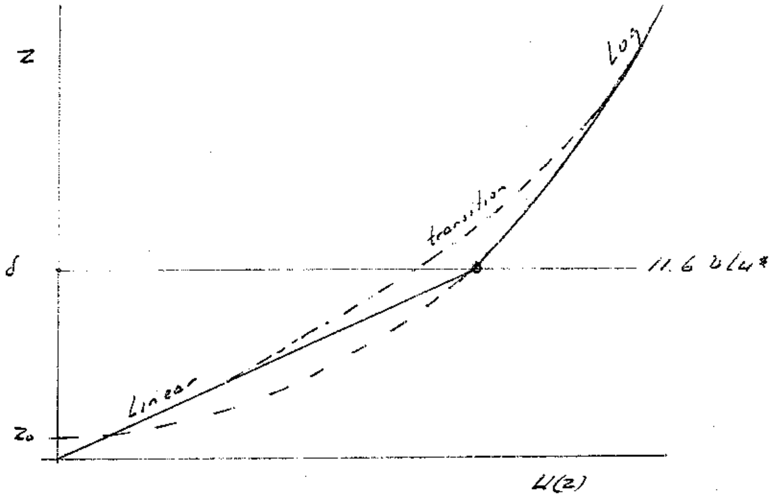


FIGURE 2.1

$z = 5\ \upsilon/u^*$ to $30\ \upsilon/u^*$



Roughness value k_s

For uniform sediment $k_s = D$
For graded sediment $k_s = D_{65} - D_{90}$
For ripples $k_s = 0.5$ à $1\ h_{ripple}$
Errors in k_s give the following errors in C:

$\frac{k_{actual}}{k_{estimated}}$	1	2	5	10
$C_{est-act}$	0	5,5	12,5	18

2.5. Turbulence

Turbulence is a random fluctuating velocity field which interacts with and derives its energy from the mean flow field. A turbulent velocity field can only be described by statistical quantities such as r.m.s. values, amplitude distribution, correlations and spectra. The amplitudes are generally normally distributed so that the root-mean-square deviation gives a good idea of the fluctuations. $\sigma_u = \overline{(U-\bar{U})^2}$ where U = the instantaneous velocity and $\bar{U}$ the time-averaged value.

A turbulent field has a diffusive character. Gradients of momentum and scalar quantities are rapidly diminished by this diffusive action.

The analogy of turbulent motion with the movements of molecules leads to the analogy given by Boussinesq and the introduction of a eddy-viscosity concept for the apparent turbulent shear stress $-\rho_w\overline{u'w'}$

$$-\rho_w\overline{u'w'} = \rho_w\varepsilon_m\ \partial U/\partial z$$

so that the total shear stress becomes

$$\tau = {}^{\partial U}/_{\partial z} - \rho_w\ \overline{u'w'} = \rho_w\ (\upsilon + \varepsilon_m)\ {}^{\partial U}/_{\partial z}$$

ε_m = eddy viscosity

The logarithmic velocity distribution

$$U(z)/u^* = \frac{1}{\varkappa} \ln (z/zo)$$

and the linear shear stress distribution

$$\tau(z) = \tau(o)\ (h-z)\ / h$$

gives the following distribution for $\varepsilon(z)$

$$\varepsilon(z) = \varkappa\ u^*\ z\ (1 - z/h)$$

The average value of $\varepsilon(z)$ (averaging over the depth) is therefore

$$\bar{\varepsilon} = \frac{1}{6} u^* h$$

2.6 Diffusion

The diffusion of scalair quantities (concentration, heat) is described by analogy with the diffusion of momentum by:

$$N = (D + \varepsilon_c)\ \partial C/\partial z$$

in which:

N = lateral flux of scalair quantity

D = molecular diffusivity

ε_c = turbulent diffusion coefficient

C = concentration

The value of D depends on the properties of the scalair

heat in water $D \approx 0.2.10^{-6}\ m^2/s$

salt in water $D \approx 2.10^{-3}\ m^2/s$

The ratio of ε_c to ε_m depends also on the properties of the scalair but the value of this ratio is generally of the order one.

2.7 Literature

H. Rouse, 1950 — Fundamental principles of flow, Engineering Hydraulics

V.L. Streeter, 1966 — Handbook of Fluid Dynamics , chapter 9 and 10, Mc. Graw Hill New York

J.O. Hinze, 1959 — Turbulence, Mc Graw Hill New York

3. PROPERTIES OF THE TRANSPORT MATERIAL

Some of the properties of sediment which are often used are :

size
shape
density
fall velocity
porosity

3.1 Size

A classification of particles according to size is given in table 3.1. This table gives the classification by the American Geophysical Union for clay, silt, sand, gravel, cobbles and boulders.

Various definitions of "diameter" are possible :

sieve diameter D = diameter of square mesh sieve which will just pass the particle.

sedimentation diameter Ds = diameter of sphere with same density and same settling velocity in same fluid at same temperature

nominal diameter Dn = diameter of sphere with equal volume

triaxial dimension a,b,c (a = largest c = smallest axis)

Size determination

boulders, cobbles and gravel	:	direct measurement
gravel, sand		sieving
fine sand, silt		sedimentation or microscope analysis

3.1.1 Sieving

Sieving can be applied for particles down to 44 μm but gives good results down to 74 μm. Sieve sizes (openings) are made in a geometric series with every sieve being $\sqrt[4]{2}$ larger in size than the preceding. Taking every other size gives a $\sqrt{2}$ series . For most sands a $\sqrt{2}$ series gives sufficient results but a $\sqrt[4]{2}$ series may be necessary for very uniform sands. Some general rules for sieving can be given.

1. Do not overload the sieves to avoid clogging. The following maximum residues on individual 8-inch sieves are recommended (after Shergold 1946)

Major classification of sediment size

acc. to H.A. Einstein

Size	Designation	Remark
$d < 0.5\ \mu$	Colloids	Always flocculated
$0.5\ \mu < d < 5\mu$	Clay	Sometimes or partially flocculated
$5\ \mu < d < 64\ \mu$	Silt	Nonflocculating-individual crystals
$64\ \mu < d < 2$ mm	Sand	Rock fragments
2 mm $< d$	Gravel, boulders	Rock fragments

American Geophysical Union (AGU) grade scale for particle sizes

Size			Class
Millimeters	Microns	Inches	
4,000-2,000		160-80	Very large boulders
2,000-1,000		80-40	Large boulders
1,000-500		40-20	Medium boulders
500-250		20-10	Small boulders
250-130		10-5	Large cobbles
130-64		5-2.5	Small cobbles
64-32		2.5-1.3	Very coarse gravel
32-16		1.3-0.6	Coarse gravel
16- 8		0.6-0.3	Medium gravel
8-4		0.3-0.16	Fine gravel
4-2		0.16-0.08	Very fine gravel
2.00 -1.00	2,000-1,000		Very coarse sand
1.00-0.50	1,000-500		Coarse sand
0.50-0.25	500-250		Medium sand
0.25-0.125	250-125		Fine sand
0.125-0.062	125-62		Very fine sand
0.062-0.031	62-31		Coarse silt
0.031-0.016	31-16		Medium silt
0.016-0.008	16-8		Fine silt
0.008-0.004	8-4		Very fine silt
0.004-0.0020	4-2		Coarse clay
0.0020-0.0010	2-1		Medium clay
0.0010-0.0005	1-0.5		Fine clay
0.0005-0.00024	0.5-0.24		Very fine clay

Sieve opening mm.	U.S. Sieve no.	Maximum residue in grams		
		2-series	$\sqrt{2}$ series	$\sqrt[4]{2}$ series
2.4	8	150	75	38
1.2	16	100	50	25
0.6	30	70	35	18
0.295	50	50	25	12
0.15	100	35	18	9
0.076	200	25	12	6

The total sample size should be about 20- 50 grams for 8"-inch sieves and fine sand.

2. A sieving time of 10 minutes with a mechanical sieving apparatus should be used.
3. For coarse sands and gravel the following minimum size is recommended to obtain a sufficient number of grains in each fraction (see De Vries 1971).
   Sample size (gram) $> 20 \,.\, D85^3$ D85 in mm.

Sieve types and series are different in various countries, but are generally based on a $\sqrt[4]{2}$ series.

3.1.2 Sedimentation

For fine sand and silt a size distribution can be determined by sedimentation. For particles $< 50 \mu m$ the Stokes law for the settling velocity is

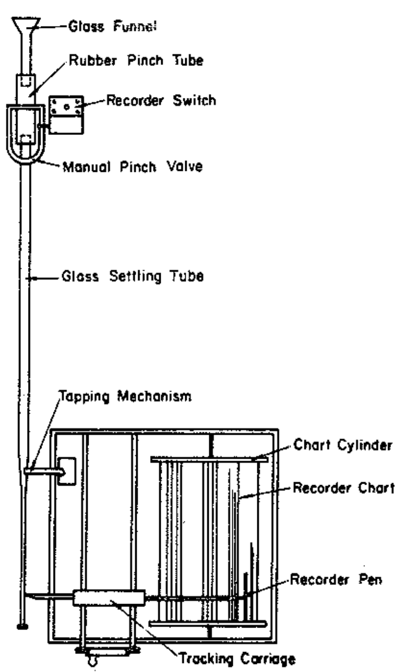


FIG. 3.1 .–SKETCH OF VISUAL ACCUMULATION TUBE AND RECORDING MECHANISM

valid ; for coarser particles empirical relations have to be used. Various principles are used : sedimentation balances, pipette analysis, visual accumulation tube (fig. 3.1) (for a review see ASCE 1969)

3.1.3 Size distribution

By sieving or sedimentation a size distribution can be obtained which is generally expressed as a "percent by weight" vs "grain size" distribution. The cumulative size distribution of most sediments can be approximated by a log-normal distribution. A log-normal distribution will give a straight line if logarithmic probability paper is used (fig. 3.2)

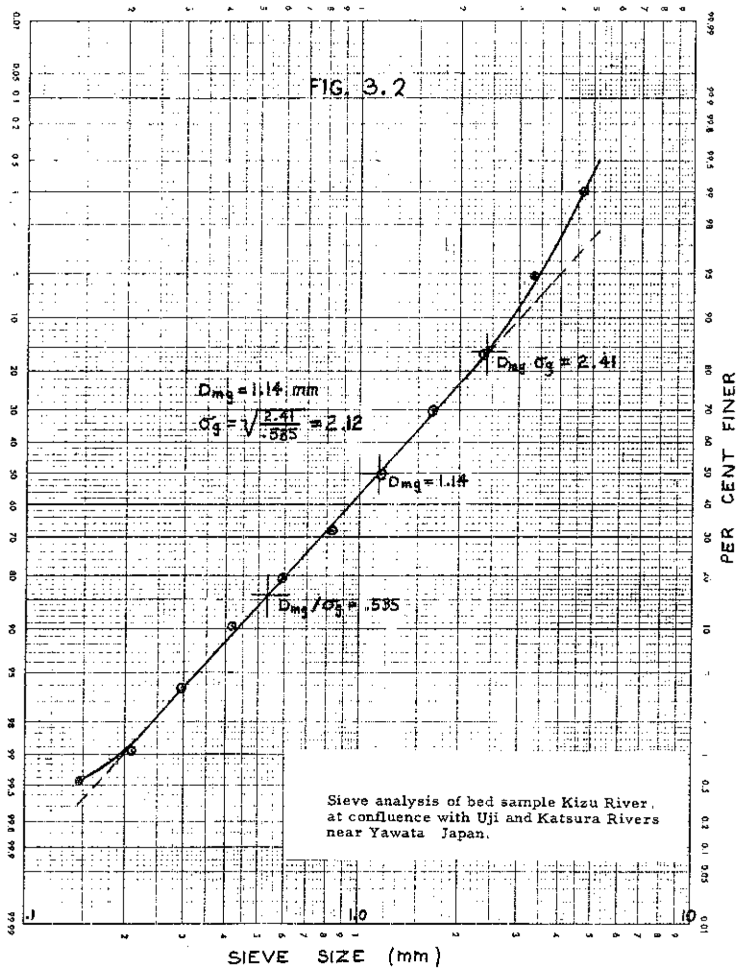


Fig. 3.2 Example of cumulative distribution of sieve diameter on logarithmic probability paper

From the cumulative size distribution the <u>mean diameter</u> can be defined :

$$\bar{D} \text{ or } Dm = \sum^{i} p_i D_i \Big/ \sum^{i} p_i$$

in which p_i : fraction with diameter D_i.

D_i is the geometric mean of the size fraction limits.

Also the notation Dp is used which denotes the diameter in a mixture of which p % is smaller than Dp. D50 is also called the median diameter

For a log-normal distribution we can define the geometric mean diameter

$$Dg = (D84 \cdot D16)^{\frac{1}{2}}$$

and the geometric standard deviation :

$$\sigma_g = \frac{1}{2}\left[\frac{D84}{Dg} + \frac{Dg}{D16}\right]$$

In geological literature also ϕ - units are used

$$\phi = -{}^{2}\log D50 \quad (D50 \text{ in mm})$$

$$\phi\ (1\text{ mm}) = 0,\ \phi\ (0{,}5\text{ mm}) = 1 \quad \text{etc.}$$

σ_g becomes in ϕ -units : $\sigma_\phi = \frac{1}{2}(\phi 16 - \phi 84)$

3.2 Shape

Besides the grain-diameter also the shape is of importance. A flat particle will have a smaller fall velocity and will be more difficult to transport as a rounded particle.

Several definitions may be used to characterise the shape :

Sphericity = ratio of the surface areaof a sphere and surface area of the particle at equal volume

Roundness = ratio of the average radius of curvature of the edges and the radius of circle inscribed in the maximum projected area of the particle

Shape factor = S.f = $c/\sqrt{a\,b}$ in which a,b,c are three mutually perpendicular axes, from which a is major, b is intermediate and c is minor axis.

For spheres s.f = 1, for natural sands s.f $\approx$ 0.7

Roundness and sphericity are not suited for practice whereas the simple shape factor gives sufficient results.

3.3 Density

Most sediments originate from disintegration or decomposition of rock.

clay	:	fragments of feldspars and micas
silt	:	silicas
sand	:	quartz
gravel	:	fragments of original rock
boulders	:	all components of original rock

The density of most sediment particles (< 4 mm) varies between narrow limits. Since quartz is predominant in natural sediments the average density can be assumed to be 2650 kg/m^3 (specific gravity 2.65). Sometimes heavy minerals are present which can be segregrated during ripple formation or other modes of transport. Clay minerals range from 2500-2700 kg/m^3.

3.4 Fall velocity

The fall velocity of a sediment is an important parameter in studies on suspension and sedimentation of sediments. The fall velocity is defined by the equation giving equilibrium between gravity force and flow resistance :

$$\underbrace{\frac{\pi}{6} \cdot D^3 (\rho_s - \rho_w)\, g}_{\text{gravity}} = \underbrace{C_D \cdot \tfrac{1}{2}\rho_w W^2 \cdot \frac{\pi}{4} D^2}_{\text{resistance}}$$

in which C_D = drag coefficient

W = fall velocity

From this relation follows :

$$W = \left(\frac{4}{3} \cdot \frac{gD}{C_D} \cdot \Delta \right)^{\frac{1}{2}}$$

in which

$$\Delta = (\rho_s - \rho_w) / \rho_w$$

Values of C_D depend on a Reynolds number $W.D/\upsilon$ and the shape of the particle (expressed by s.f = $c/\sqrt{ab}$)

For spherical particles and low Reynolds number (Re < 1), C_D can be given by $C_D = 24/Re$ so that:

$$W = \frac{\rho_s - \rho_w}{18\ \upsilon}\, g\, D^2 \qquad \text{(\underline{Stokes} law)}$$

For large Reynolds numbers C_D becomes a constant so that W varies as :

$$(\Delta g D)^{\frac{1}{2}}$$

Therefore W varies with $D^{\frac{1}{2} - 2}$.

Relations between C_D, Re and s.f are given by Albertson (1953) (see fig. 3.3) For natural sands s.f $\approx$ 0.7. From these relations graphs for W as a function of grain size and temperature can be obtained (see fig. 3.4)

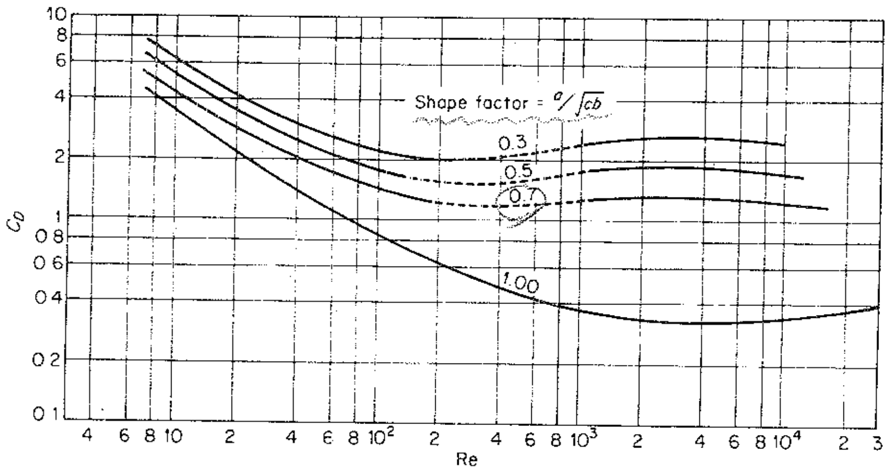


Fig.3.3 Drag coefficient vs. Reynolds number for different shape factors. [*After* ALBERTSON (*1953*)]

The presence of a large number of other particles will decrease the fall velocity of a single particle. A cluster of particles will have a greater velocity however. Care must be taken therefore with experiments on the fall velocity to avoid currents in the fluid which will increase the fall velocity of the particle and the influence of concentration should be considered.

There are many expressions giving the influence of concentration on the fall velocity. Based on systematic experiments, Richardson and Zaki (1954) give a useful expression :

$$W(c) \,/\, W(o) = (1-c)^{\alpha} \qquad 0 \leq c < 0{,}3$$

W(c) is the fall velocity of a grain in a suspension with concentration by volume c

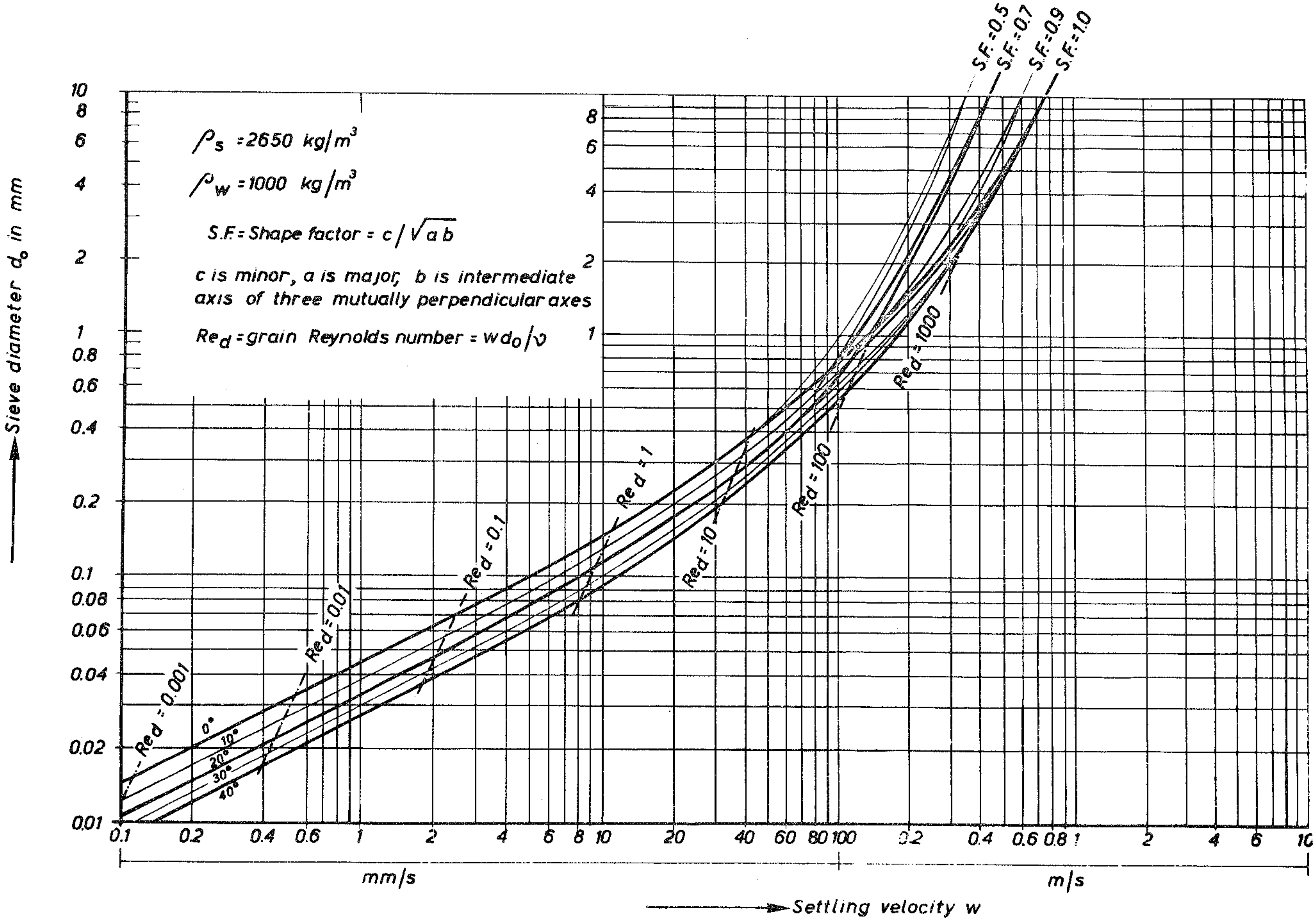
W(o) is the fall velocity for a single grain

α is a function of Reynolds number $W.D/\nu$

Re < 0,2	$\alpha = 4.65$
0,2 < Re < 1	$\alpha = 4.35 \cdot Re^{-0.03}$
1 < Re < 200	$\alpha = 4.45\ Re^{-0.1}$
Re > 500	$\alpha = 2.39$

The coefficient is slightly dependent on particle shape but this can be neglected. For fine sediments this means that a concentration of 1% gives a reduction in fall velocity of 5%.

The fall velocity of a particle in a turbulent fluid can be different from that in a quiescent fluid (see chapter 7.2)



ρ_s = 2650 kg/m³
ρ_w = 1000 kg/m³
S.F. = Shape factor = $c/\sqrt{ab}$
c is minor, a is major, b is intermediate axis of three mutually perpendicular axes
Re_d = grain Reynolds number = $w d_o / \nu$
S.F. = 0.5
S.F. = 0.7
S.F. = 0.9
S.F. = 1.0
Red = 0.001
Red = 0.01
Red = 0.1
Red = 1
Red = 10
Red = 100
Red = 1000
0°
10°
20°
30°
40°
Sieve diameter d_o in mm
Settling velocity w
mm/s
m/s

FIG. 3.4

3.5 Bulk density and porosity

In estimating the life of a reservoir and similar cases the calculated weight of the sediment transported to the reservoir has to be converted into volume. For this the dry weight per unit volume of sediment in place, bulk density, ρ_b , has to be estimated.

For instance for air-dried fine sediments 1200–2000 kg/m^3 (75- 125 pounds/ft^3) applies. The same material deposited under continuously submerged conditions may range from 300 - 1000 kg/m^3 (20 - 60 pounds/ft^3). The density will also depend on the grainsize and silt content.

Bulk density , ρ_b = the mass of dry sedimentary material within a unit of volume (kg/m^3). (Also : "dry density" or "unit dry weight"). The volume taken by the sediment depends on the conditions of settling and may be a function of time due to consolidation. An empirical relation is presented by Lane and Koelzer (1953) for estimating the bulk density of deposits in reservoirs :

$$\rho_{b_T} = \rho_{b_1} + B \log T$$

T = time in years

ρ_{b_1} = initial bulk density taken to be the value after one year of consolidation

B = consolidation coefficient

Reservoir operations	sand		silt		clay	
	ρ_{b_1}	B	ρ_{b_1}	B	ρ_{b_1}	B
sediment always submerged or nearly submerged	1500	0	1050	90	500	250
normally a moderate reservoir drawdown	1500	0	1185	45	750	170
normally considerable reservoir drawdown	1500	0	1275	15	950	100
reservoir normally empty	1500	0	1320	0	1250	0

For sediments containing material of more than one size class proportional addition of the fractions can be made. A compilation of initial bulk densities from various observations given by Trask shows appreciable deviations between the values. Data given in table are based on normal conditions and applicable for a first estimate.

3.6 Literature

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4. INITIATION OF PARTICLE MOTION

4.1 Introduction

The equilibrium of a particle on the bed of a stream is disturbed if the resultant effect of the disturbing forces (drag force, lift force, viscous forces on the particle surface) become greater than the stabilising forces as gravity and cohesion. Cohesion is only important for sediments in the clay and silt range or fine sands with an appreciable silt content. The acting forces have to be expressed in known quantities such as velocities or bottom shear stress. They will have a strongly fluctuating character so-that the initiation of motion also has a statistical aspect.

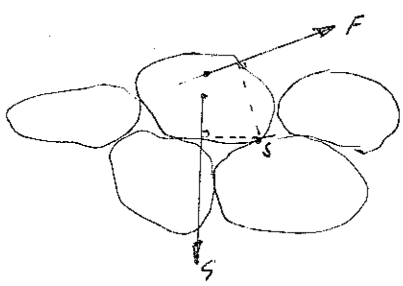
Theoretical work on the initiation of a motion has started with work by Brahms (1753) who gave a sixth power relation between flow velocity and the necessary weight of a stone and by Dubuat (1779, 1786) who introduced the concept of bottom shear stress and did some experiments on particle movement. Most of the older relations have the form:

$$U_{\text{bottom, crit}} = (4 - 5)\sqrt{D} \qquad (D \text{ in m}, U \text{ in m/s})$$

As the"bottom" is not well defined the use of this type of formula is limited.

4.2 Theory

White (1940) gave a thorough discussion on the equilibrium of a grain on the bed of a stream.



The disturbing force F (resultant of drag and lift forces) will be proportional to the bottom shear stress τ_o and the particle surface area (D^2).
The stabilising gravity force is proportional to $(\rho_s - \rho_w) g\, D^3$.
Taking the moment with respect to the turning point S gives the equation

$$\alpha_1\, \tau_o\, D^2 \geqslant \alpha_2\, (\rho_s - \rho_w) g\, D^2$$

$$\text{or } \tau_o \geqslant C\, (\rho_s - \rho_w) g\, D$$

The factor C will depend on the flow condition near the bed, particle shape, the position of the particle relative to other particles etc.

The flow condition near the bed can be described by the ratio of grainsize to thickness of the viscous sublayer which ratio is proportional to $U^* D/\upsilon = Re^*$, a Reynoldsnumber based on grainsize and shear velocity.

4.3 Experiments

The relation :

$$\psi_{cr} = \frac{\tau_{cr}}{(\rho_s - \rho_w)gD} = \frac{U^{*2}_{cr}}{\Delta gD} = f\ (Re^*) = f\ \left(\frac{U^* D}{\upsilon}\right)$$

has been investigated by many authors especially by Shields (1936) who did systematic tests and compared his results with results from other investigations (see fig. 4.1 and 4.2) . The difficulty in all tests is the definition of "initiation" of motion. Is it the movement of the first particle or of a large number of grains ? Shields correlated the rate of sediment transport with τ_o and defined τ_{cr} by extrapolating to zero material transport.

All other theoretical considerations based for example on drag force due to velocity will give the same result that

$$\psi_{cr} = U^{*2}_{cr} / \Delta gd = f(Re^*)$$

For large Re^* (rough bed) it can be seen that U^* varies with $\sqrt{D}$ (fig. 4.3) . For equal values of h/D and therefore equal values of $\bar{U}/U^*$ it follows that $\bar{U} \sim \sqrt{D}$ and that the critical velocity of a stone is proportional to the 1/6 power of the weight of the stone (or stone weight proportional to $\bar{U}^6$ (Brahms)!).

4.4 Influence of various factors

4.4.1 Effect of criterion

It is clear that the critical value of τ_o will depend on the criterion for initiation of motion. To get an objective criterion Neill (1968, 69) proposed the parameter

$$N = n\ D^3/U^*$$

in which n is the number of grains displaced per unit area and unit time. Shields graph corresponds roughly with a N-value of 15.10^{-6} for coarse material. For designs of bottom protections etc. a much lower criterion should be used (for instance $N = 10^{-6}$).

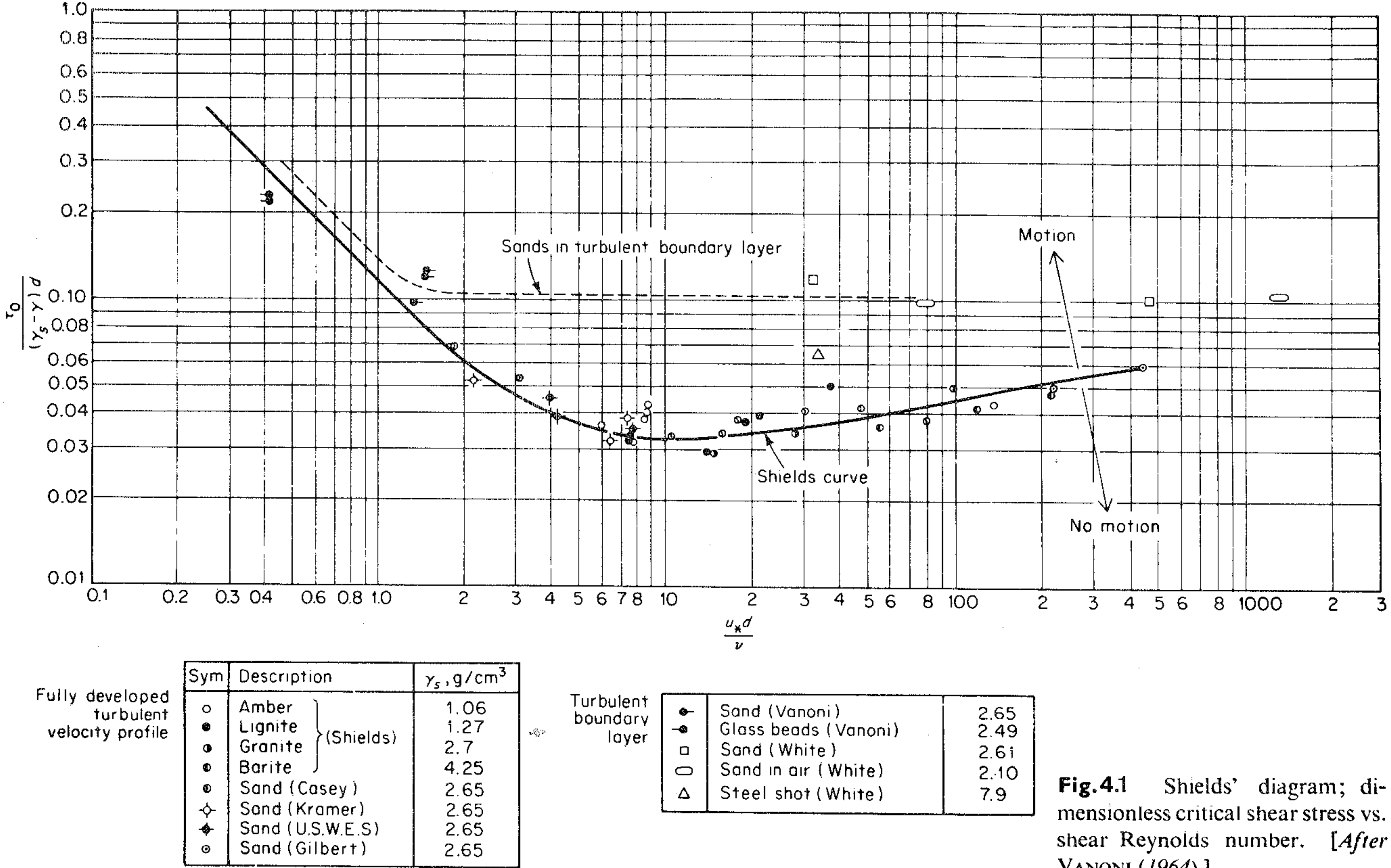
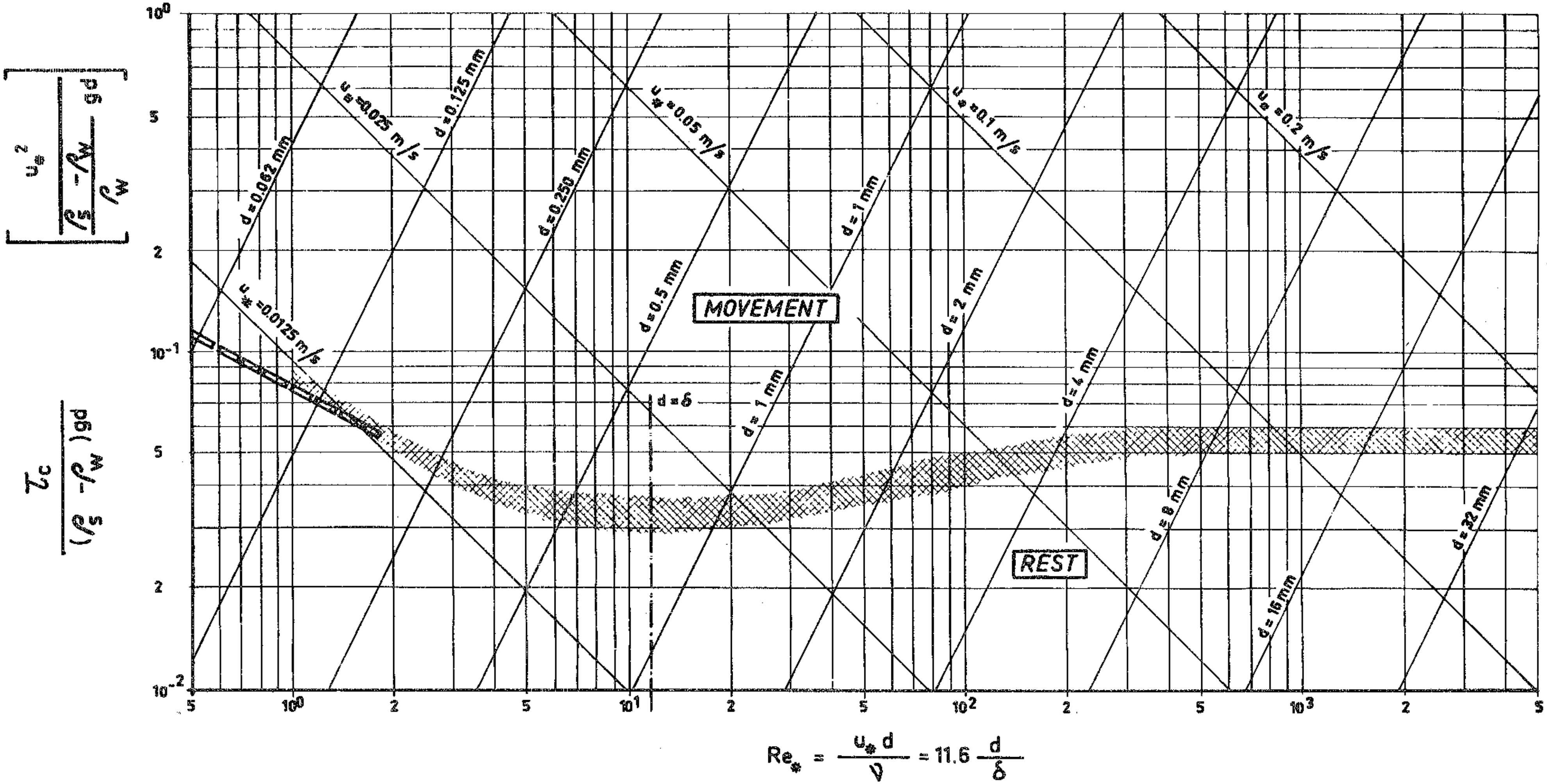


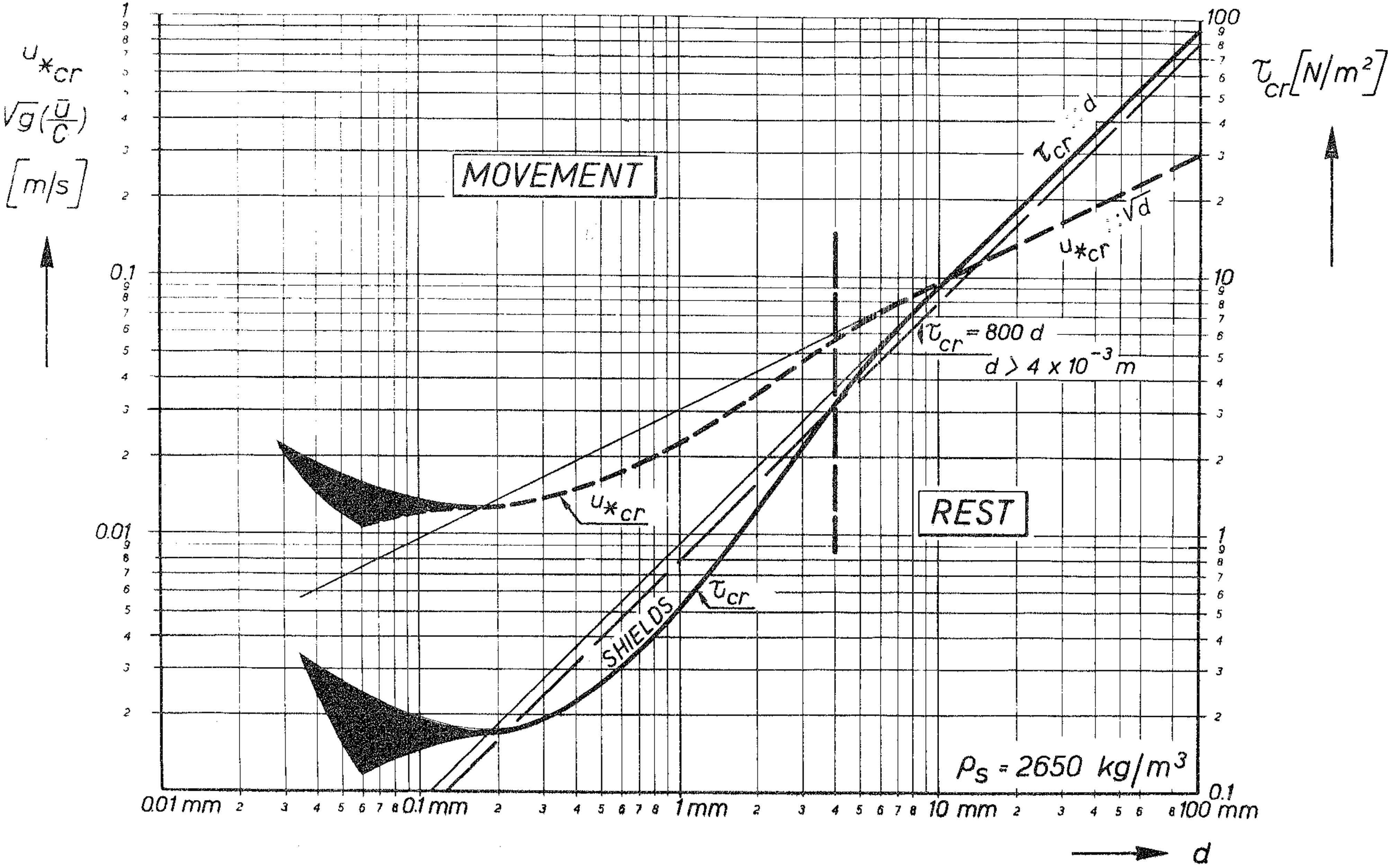
Fig. 4.1 Shields' diagram; dimensionless critical shear stress vs. shear Reynolds number. [*After* VANONI (*1964*).]



LINES OF EQUAL u_* AND d BASED ON ρ_s = 2650 kg/m^3 AND ν = 1.25 × 10^{-6} m^2/s (12 °C)

RELATIONSHIP OF CRITICAL SHEAR STRESS AND DIAMETER FOR A BED OF UNIFORM GRAINS. ACC. TO SHIELDS (1936)

FIGURE 4.2



CRITICAL SHEAR STRESS AND CRITICAL SHEAR VELOCITY
AS FUNCTION OF GRAIN SIZE FOR $\rho_s = 2650\ kg/m^3$ (SAND)

FIGURE 4.3

4.4.2 Effect of particle shape

Shields experiments were done with several types of material and systematic influence of shape could be observed. Tests at the Delft Hydraulics Laboratory with coarse material showed that the critical value of ψ is the same for various shapes (spheres, cubes, broken stones etc) if the nominal diameter is used for comparison.

4.4.3 Effect of gradation

It will be clear that a wide gradation will have an influence on τ_{cr}. In practice however the gradation has an influence for $D_{90} / D_{10} > 5$ only, (Knoroz, 1962) because the larger grains are more exposed and smaller grains are shielded by the larger ones. Therefore D_{50} is a good measure for most samples.

For a wide particle gradation the effect of armoring will occur which means that fine particles are eroded and an armor layer of coarse particles is formed, which prevents the bed from further scour. This effect is very important in degradation downstream of dams (Livesey, 1963, Gessler 1965). In that case D_{95} can be taken as a representative value for the mixture.

4.4.4 Effect of h/D

For small values of h/D (waterdepth /particle diameter) a deviation from Shields graph is possible because τ_o is not representative in that case for the turbulent flow structure. The turbulence structure near the bed in an infinite fluid is completely defined by bed shear stress (τ_o) and roughness (K_s) but for small values of h/D also the waterdepth gives a limitation on size and frequency of the large eddies. Also the ratio of eddy duration and the time necessary to accelerate a particle becomes small sothat an influence of h/D may be expected (more stability with smaller h/D).

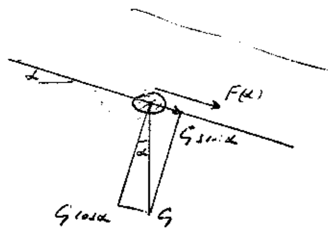
4.4.5 Influence of bed slope

For a particle on a large longitudinal slope the value of τ_{cr} will be reduced. For a horizontal bed the relation

$F/G > \tan \varphi$ is valid

in which φ is an angle characteristic for the stability of a particle. For a bed slope of α the critical condition becomes

$$\frac{F(\alpha) + G \sin\alpha}{G \cos\alpha} > \tan \varphi$$



so that F (α) becomes:

$$F(\alpha) = G\cos\alpha\tan\varphi - G\sin\alpha$$

and

$$K(\alpha) = \frac{\varphi\,cr(\alpha)}{\varphi\,cr(o)} = \frac{F(\alpha)}{F(o)} = \frac{G\cos\alpha\,tg\varphi - G\sin\alpha}{G\,tg\,\alpha} =$$

$$\frac{\sin(\varphi - \alpha)}{\sin\varphi}$$

(given by Schoklitsch in 1914)

For a cross-slope β a similar reduction in $\varphi\,cr$ can be computed

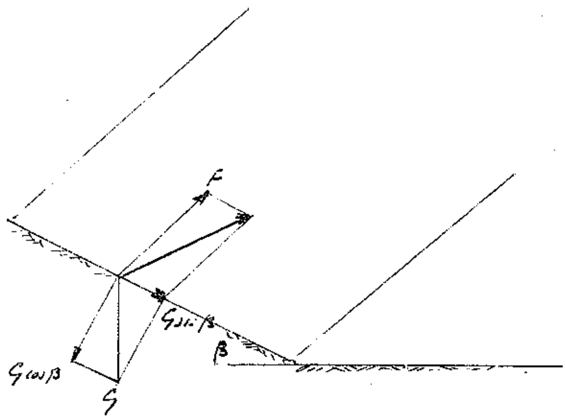
$$\frac{\sqrt{F^2(\beta) + G^2\sin^2\beta}}{G\cos\beta} > \tan\varphi$$

or

$$F(\beta) = \sqrt{G^2\cos^2\beta\tan^2\varphi - G^2\sin^2\beta}$$

sothat:

$$K(\beta) = \frac{\varphi cr(\beta)}{\varphi\,cr(o)} = \cos\beta\sqrt{1 - \left(\frac{\tan\beta}{\tan\varphi}\right)^2}$$



(given by Leiner 1912)

For a combination of slopes (α, β) the reduction factor

K (α,β) becomes $K(\alpha)$, $K(\beta)$.

4.5 Cohesive sediments

A cohesive character of a soil will increase the resistance against erosion. Emperical data on critical velocities are given by Lane 1953.

material	loose	moderately compact	compact
sandy clay	0.45 m/s	0.9 m/s	1.25 m/s
clay	0.35 m/s	0.8 m/s	1.2 m/s
lean clayey soil	0.30 m/s	0.7 m/s	1.05 m/s

Several authors have tried to correlate critical shear stress with mechanical properties of the soil (siltcontent, plasticity index, vane shear strength) (see Smerdon and Beasly (1959), Carlson and Enger (1960), Partheniades (1965,1970). From the data given it appears that for cohesive

soils with D_{50} = 10 - 100 μ a critical shear velocity U^*_{cr} of 3 - 4,5 cm/s is possible.
There is some tendency for an increase of U^*_{cr} with vane shear strength and plasticity index.

For very recently deposited sediments (silt in estuaries) Migniot (1968) and Partheniades (1970) give relations between U^*_{cr}, vane shear strength and dry weight of the sediments. Minimum values are in the order of U^*_{cr} = 1.0 cm/s (consolidation period of some days) to 3.0 cm/s for consolidation periods of some weeks.

For an exact determination of a critical shear stress of a cohesive soil a special test for each soil will be necessary.

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5. TRANSPORT MECHANISM, BED FORMS, ALLUVIAL ROUGHNESS

5.1 Introduction

For turbulent flow over a rigid bed a description of the flow structure could be given only by empirical methods. Bottom shear stress, waterdepth and bed roughness were the most important parameters. Description of particle motion under the action of the flow is also largely empirical sothat it is not difficult to understand why there is only a limited theoreticalbasis for the relation between flow and sediment transport.

Most of the existing knowledge is obtained from experiments and general physical arguments. For the initiation of motion a reasonable picture was obtained in this way. At greater values of the bed-shear stress sediment transport will increase and deformation of the bed will occur. As the deformation is also time-dependent and nature is always unsteady, an equilibrium situation will be hardly found in practice.

5.2 Transport mechanism

According to the mechanism of transport two major modes may be distinguished :

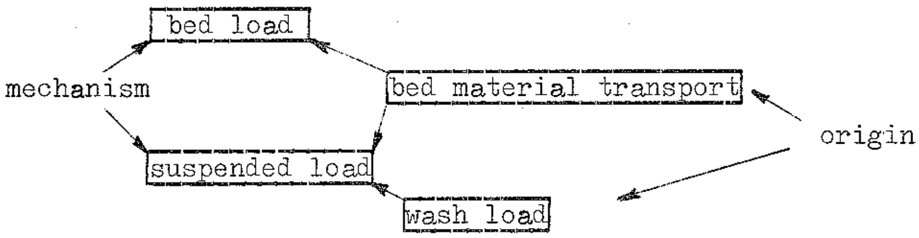
1. Bed load - movement of particles in contact with the bed by rolling, sliding and jumping
2. Suspended load- movement of particles in the flow. The settling tendency of the particle is continuously compensated by the diffusive action of the turbulent flow field.

A sharp distinction is not possible. A general criterion for the beginning of suspended load is a ratio of shear velocity and fall velocity $u^{*}/w \approx 2$. Sometimes also saltation load is mentioned. This is the mode where particles bounce from one position to another. This is only important for particle movement in air. The maximum particle elevation of a particle moving in water is in the order of 2-3 times the diameter sothat this mode of transport can be considered as bed load.

According to the origin of the transported material a distinction is made as follows :

A. Bed-material transport - This transport has its origin in the bed, which means that the transport is determined by the bed and flow conditions (can consist of bed load and suspended load)

B. Wash load - Transport of particlesnot or in small quantities in the bed. The material is supplied by external sources (erosion) and no direct relationship with the local conditions exist (can only be transported as suspended load, generally fine material $< 50 \mu m$)



Wash load is not important for changes in the bed of a river but only for sedimentation in reservoirs etc.

5.3 Bed Forms

Much literature exists on the classification and dimensions of bedforms, mainly in the form of empirical relations. Bed forms are of interest in practice for several reasons.

- Bed forms determine the roughness of a stream. A change in bed form can give changes in friction factor of 4 and more.
- Navigation is limited by the maximum bed level and depends therefore on the height of the bed deformation.
- Bed forms can influence sediment transport.

A generally accepted classification is the following :

A. Lower flow regime (Froude number $Fr = \bar{U}/\sqrt{gh} < 0{,}4\text{-}1$)

A.1 flat bed At values of the bed shear stress just above the critical, sediment transport without deformation of the bed is possible. Grains are transported by rolling and bouncing.

A.2 ripples For sediment sizes $<$ 2mm and an increasing bed shear stress small regular waves appear with wavelenghts in the order of 5-10 cm and heights in the order of cm. They become gradually irregular and three-dimensional in character.

A.3 dunes For all sediment sizes and increasing shear stress dunes are developed. Dunes are more two-dimensional than ripples and have much greater wavelengths and heights. The crests of the wave are perpendicular to the flow, the form is more or less triangular with a gentle slope along which the particles are transported and a steep downstream slope where particles are deposited. The angle of this slope is roughly the angle of repose of the material.

B. Upperflow regime Fr > 0,4-1,0

B.1 flat bed As the velocity is further increased, the dunes are flattened, gradually disappear and the bed becomes flat. Sediment transport rates are high.

B.2 antidunes A further increase in velocity and Froude numbers around 1,0 causes the water surface to become instable. Interaction of surface waves and the bed (sediment transport is maximum under the troughs of the waves)gives a bed form called antidunes.
They can travel upstream and occur in trains of 4 to twenty. Antidunes and surface waves grow in amplitude and often break in a way similar to ocean waves.

B.3 chute and pools At still higher velocities chute and pools are formed. For an illustration of the bed forms see fig. 5.1 (Simons and Richardson 1968).

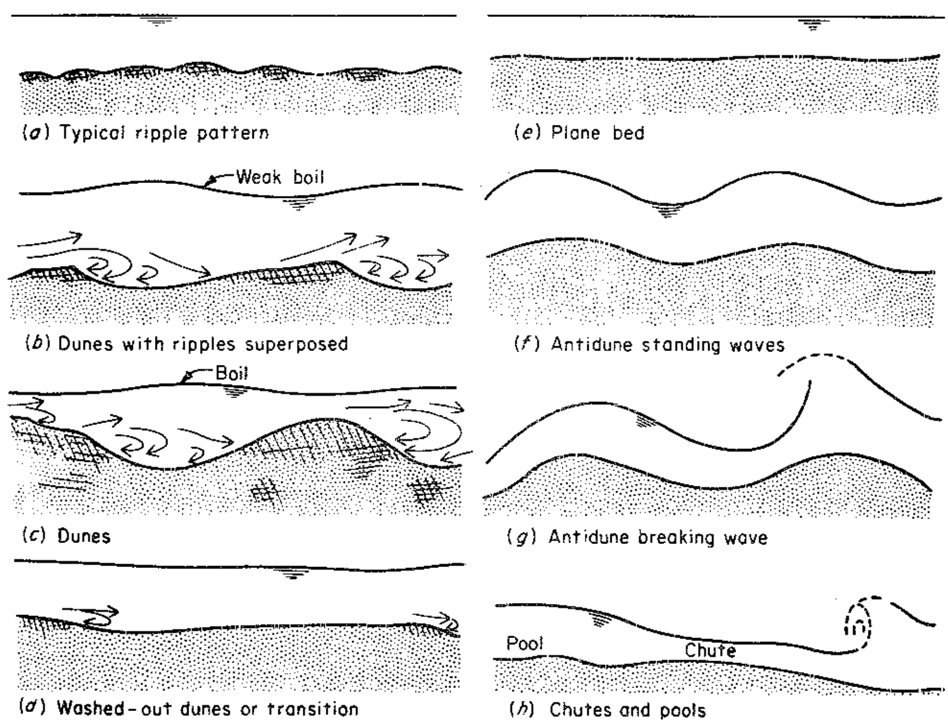


Fig. 5.1 Idealized bedforms in alluvial channels. [*After* SIMONS *et al.* (*1961*)]

5.4 Classification Criteria

Several authors have tried to develop theoretical explanations for the origin of ripples and dunes (see for example Exner (1925) who discusses the growth of an initial instability on a sand bed.

Other authors have assumed potential flow to predict the reaction of the main-flow on variations in bed level. (Kennedy 1963, Engelund 1966). The result of Kennedy's work is a relation between the wavelength λ of the bed deformation and the Froude number (see fig. 5.2)

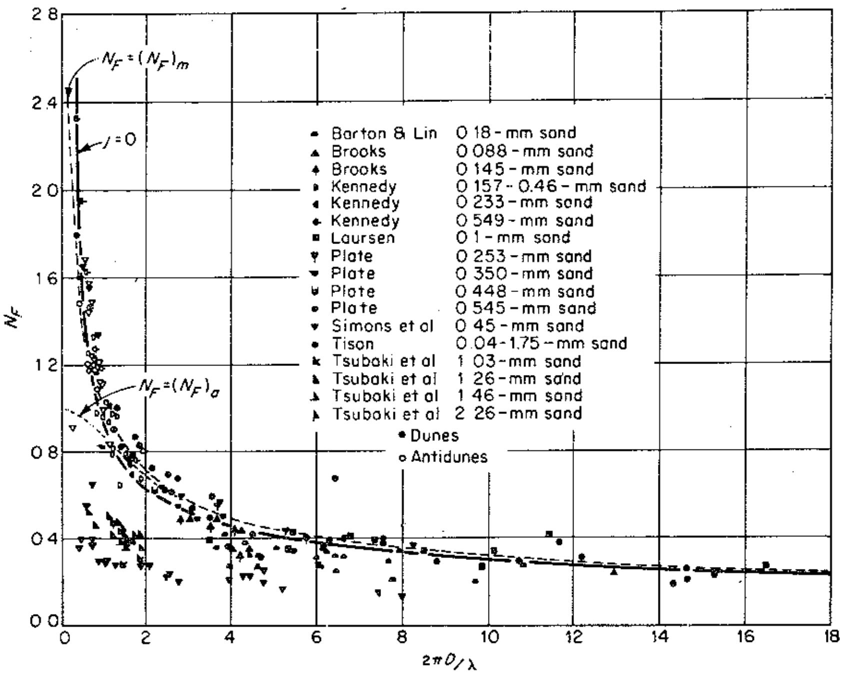


Fig.5.2 Comparison of predicted and observed bedform regions. [*After* KENNEDY (*1963*).]

Results of the theoretical models are not very convincing tothat we have to rely again on empirical correlations.
The first classification was given by Liu (1957) who proposed u^*/w vs $U^* D/\upsilon$ as a criterion for ripple formation. This diagram was extended by Simons (1966) for other bed forms (see fig. 5.3).

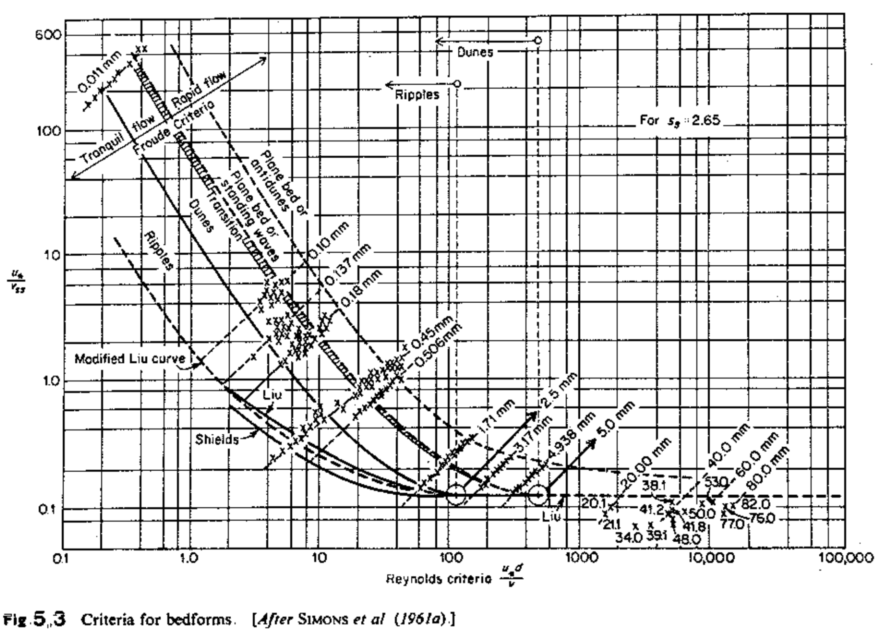


Fig.5.3 Criteria for bedforms. [*After* SIMONS *et al* (*1961a*).]

Simons et al 1963 gave a diagram based on grainsize and streampower ($\tau_0 \cdot \bar{U}$) see fig. 5.4.

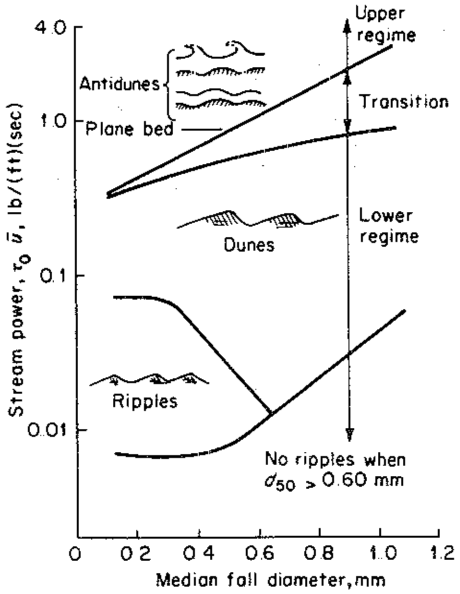


Fig. 5.4 Relation of bedforms to stream power and grain diameter. [*After* SIMONS *et al.* (*1963a*).]

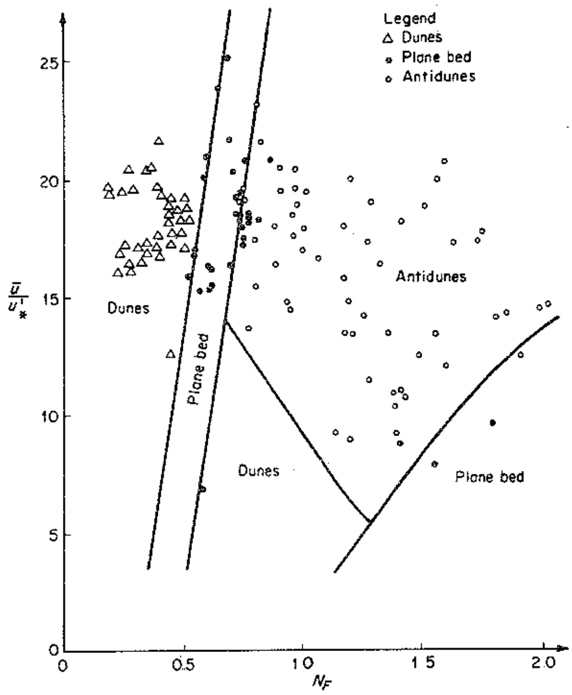


Fig. 5.5 Stability diagram; the bedforms are indicated. [*After* ENGELUND *et al.* (1966).]

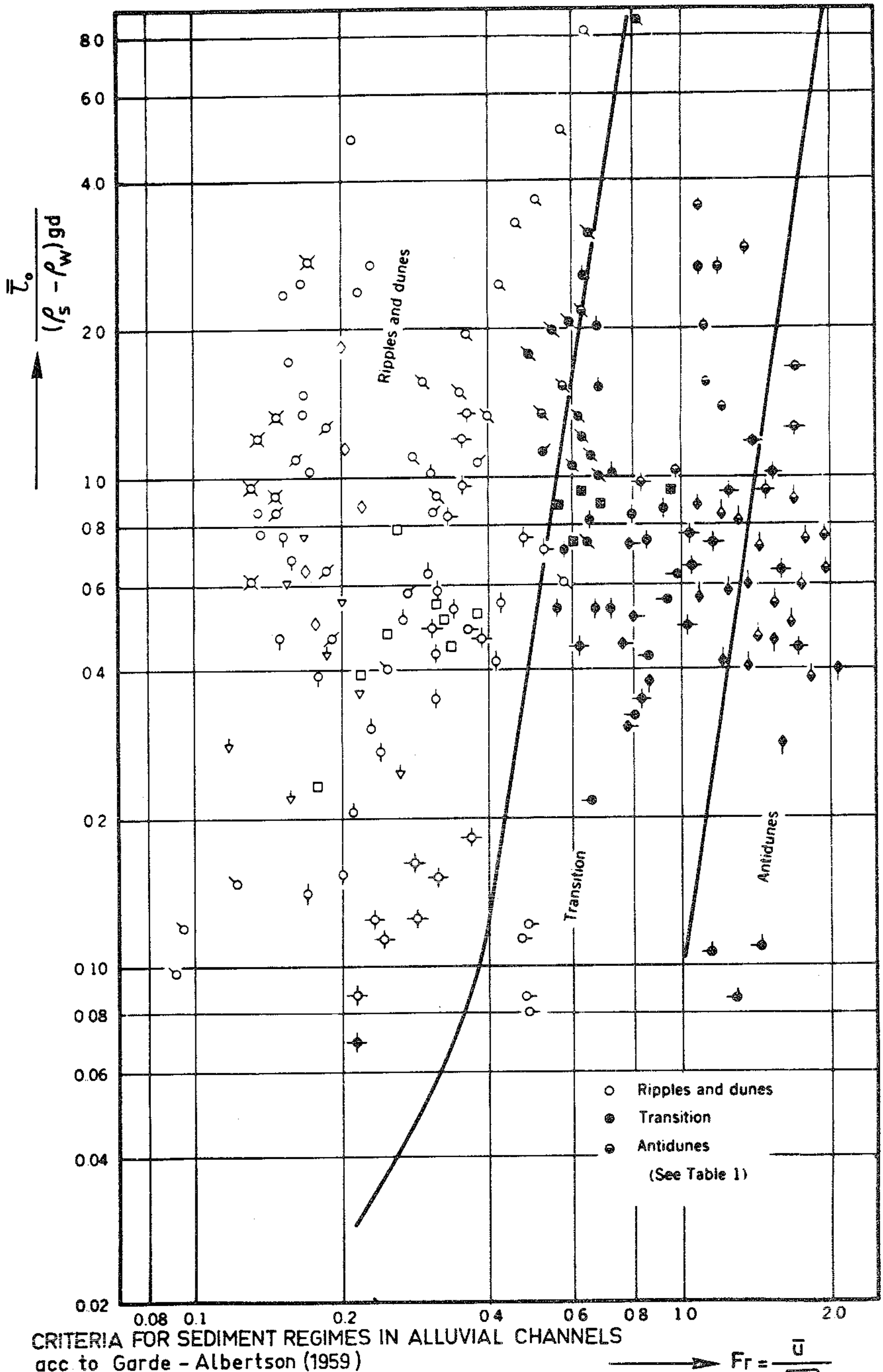
The Froude number has to be considered as an important parameter as well. This was done by several authors. (Garde-Albertson fig. 5.6) and by Engelund (fig. 5.5). $U^{*'}$ is the value of U^* computed from $\bar{U}$ by assuming the grainsize as bed roughness.

It must be borne in mind that the transition in practical conditions from one bed form to another may show an important phase lag with changes in flow condition.

Raudkivi (1967) has measured the shear stress distribution on a dune profile (see fig. 5.7). The maximum shear stress on the upperpart of the ripple had about the same value as for a horizontal bed with the same mean velocity and grain roughness. Behind the steep downstream face of the dune an eddy develops. Around the reattachmentpoint the flow is very turbulent sothat particles are transported in bursts.

5.5 Alluvial roughness

The bed forms discussed in par. 5.3 all have their specific roughness. For a flat bed without transport it can be assumed that the roughness is in the order of the grainsize (for example D_{65} or D_{90}). For flows over ripples and dunes the total resistance consists of two parts :
the roughness of the grains and the form drag of the bed forms. The roughness of a dune bed is much greater than that of a flat bed and the corresponding friction factor is also much larger. Dunes generally give the



CRITERIA FOR SEDIMENT REGIMES IN ALLUVIAL CHANNELS
acc to Garde - Albertson (1959)
Proc. IHAR - Montreal

FIGURE 5.6

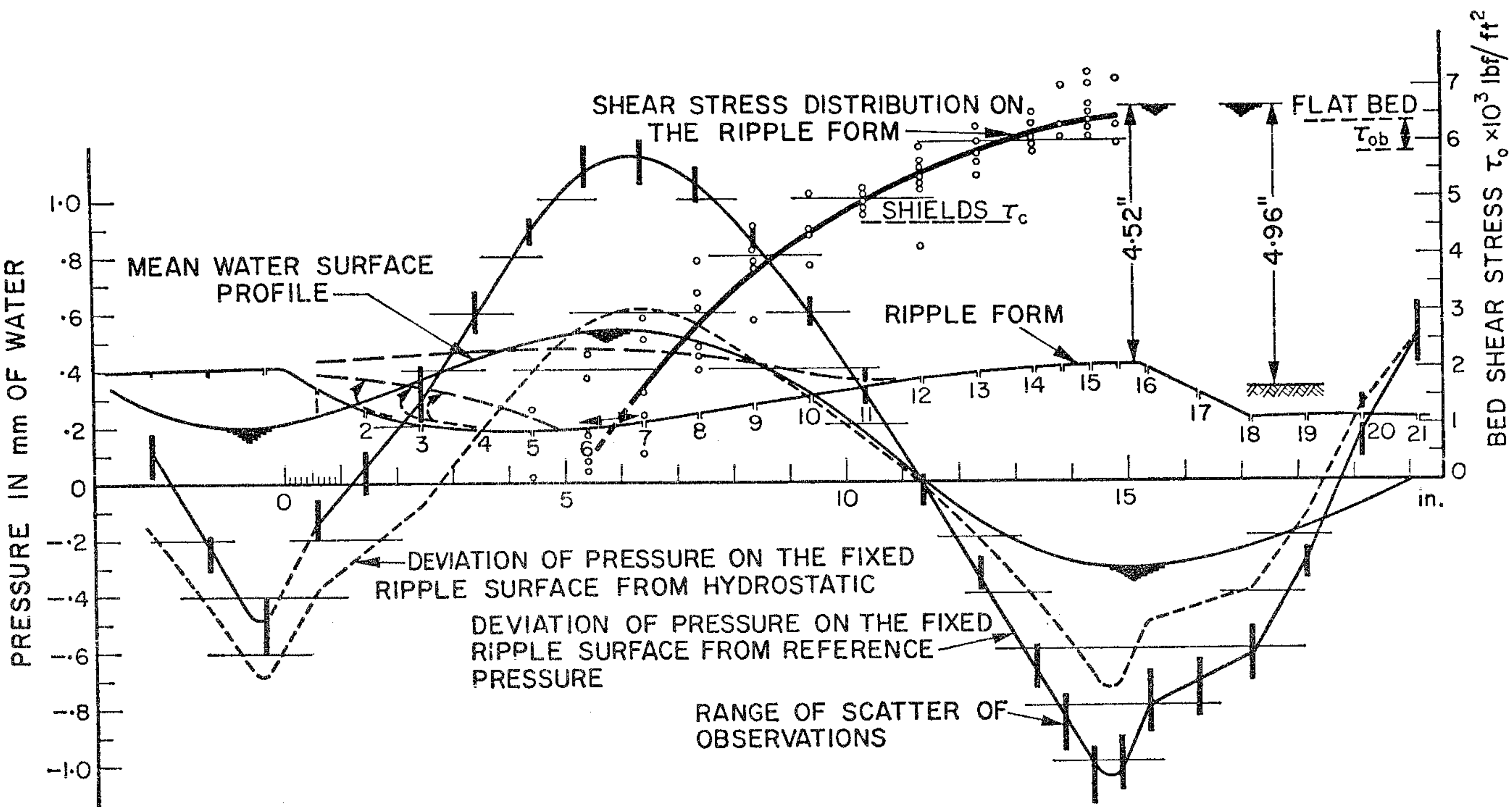


FIG. 5.7 Ripple form. Water surface profile, and distribution of pressure deviations and bed shear stress. (From ref. 34, p. 23, by permission of the American Society of Civil Engineers.)

maximum roughness of a flow.

A flat bed with sediment transport (B.1) can have a friction factor slightly different from that of a flat bed without transport. The presence of antidunes does not appreciably change the magnitude of the effective roughness of the bed if compared with a flat bed. If the waves break however, the friction factor will be increased due to the energy dissipation in wave breaking.

It cannot be expected in general that the friction factor of an alluvial channel is constant. Experiments have shown that the friction factor can vary by a factor 5 or more. This is demonstrated in fig. 5.8 and 5.9 where changes in bed form give a great difference in bed roughness.

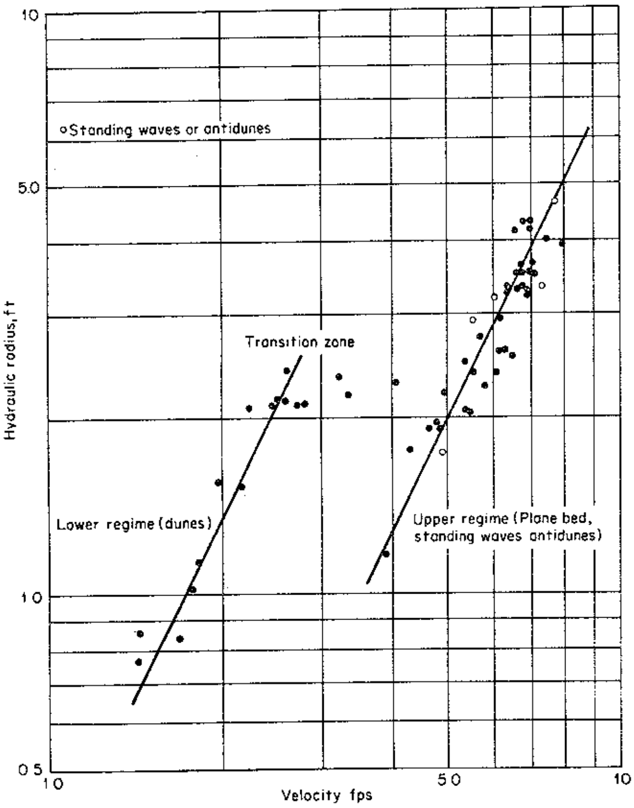


Fig. 5.8 Relation of hydraulic radius to velocity for Rio Grande near Bernalillo. [*After* NORDIN (*1964*).]

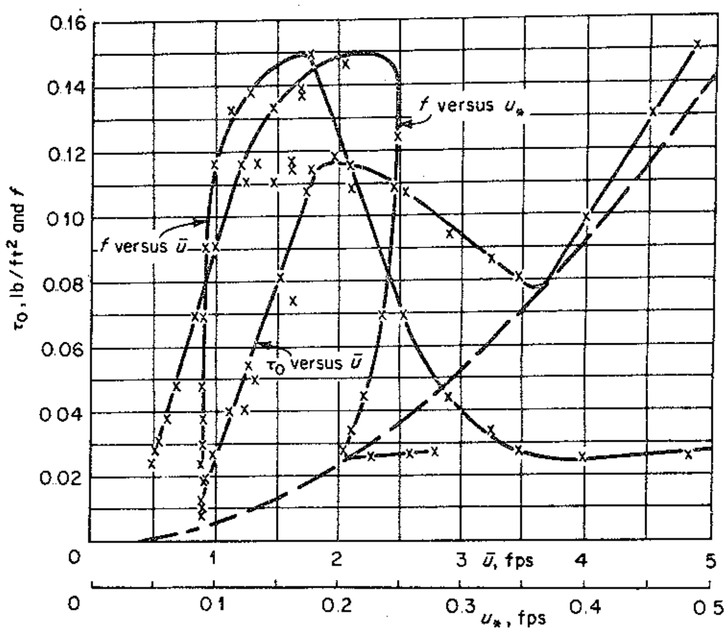


Fig. 5.9 Flow resistance due to bedforms [*After* RAUDKIVI (*1967*)]

Figure 5.9 shows that the same value of τ_o can occur for different values of $\bar{U}$ (take for example τ_o = 0,1 lbs/ft^2) . Due to phase lags between bed form (and roughness) and flow condition rivers very often exhibit hysteresis effects in discharge-stage relations.

Prediction methods for the roughness of an alluvial stream generally divide the total shear τ_o or friction factor (C or f) into two parts, one for the grain roughness (surface drag) denoted by τ_o' or C' or f' and one for the form drag (τ_o'', C" or f").

By definition:

$$\tau_o = \tau_o' + \tau_o'' \qquad C^{-2} = C'^{-2} + C''^{-2} \qquad f = f' + f''$$

f is defined by $I = \text{slope} = f \cdot \frac{1}{4\,R_h} \cdot \frac{U^2}{2\,g} \qquad f = \frac{8\,g}{C^2}$

Several procedures are given in literature.

1. Einstein - Barbarossa (1952)

E.B. divide the hydraulic radius R_h in two parts: R_h' and R_h'' where $R_h' + R_h'' = R_h$ and $R_h'/R_h'' = \tau_o'/\tau_o''$. $U^{*}{}''$ is computed by taking $k_s = D_{65}$ and Ψ_{35} by taking:

$$\Psi_{35} = \frac{\Delta\, D_{35}}{R_h' . I}$$

With the diagram given in figure 5.10 the value of $\bar{U}/U_x''$ is found by trial and error For larger values of Ψ_{35} (> 5) deviations are suggested by other authors (see figure 5.11).

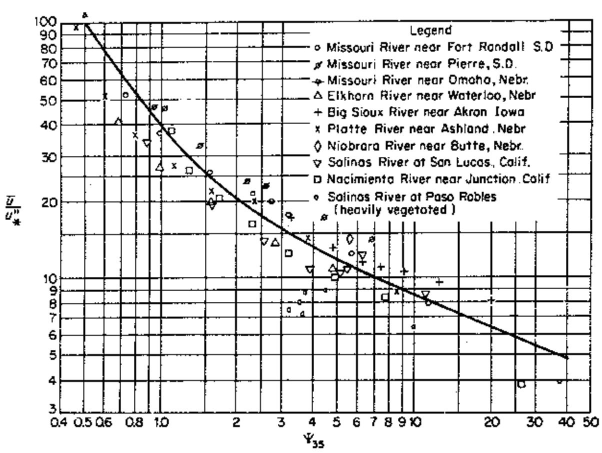


Fig 5.10 Flow resistance due to bedforms [*After* EINSTEIN *et al.* (*1952*)]

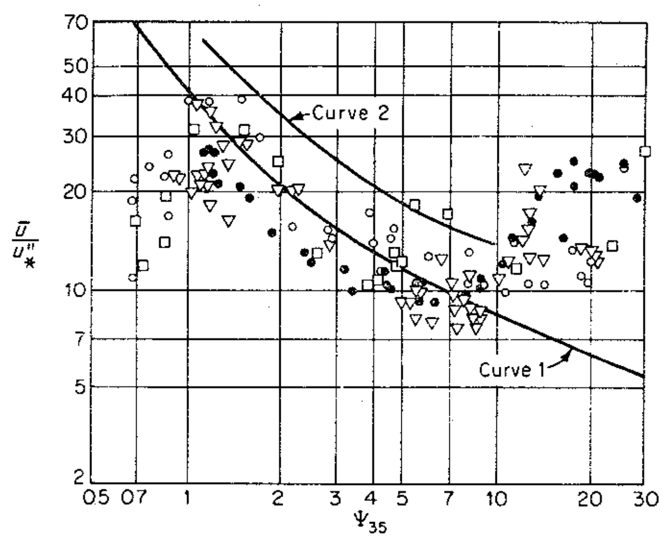


Fig. 5.11 Flow resistance due to bedforms; curve 1—river data, curve 2—flume data. [*After* SIMONS *et al.* (*1966*).]

2. Engelund and Hansen (1967)

E. and H. give an expression for f'' of the form:

$$f'' = \alpha\, H^2/h.\lambda \qquad H = \text{dune height} \quad \lambda = \text{dune length} \quad h = \text{water depth}$$

and introduce the dimensionless parameters

$\tau_* = \tau/\rho\, g\, \Delta\, D \qquad \tau_*' = \tau'/\rho\, g\, \Delta\, D \qquad \tau_*'' = \alpha\, F^2/H\, \Delta\, D\, \lambda \qquad F$ = Froude no.

Engelund concludes that τ_* is a function of τ_*' only (see figure 5.12) in which $\Theta = \tau_*$

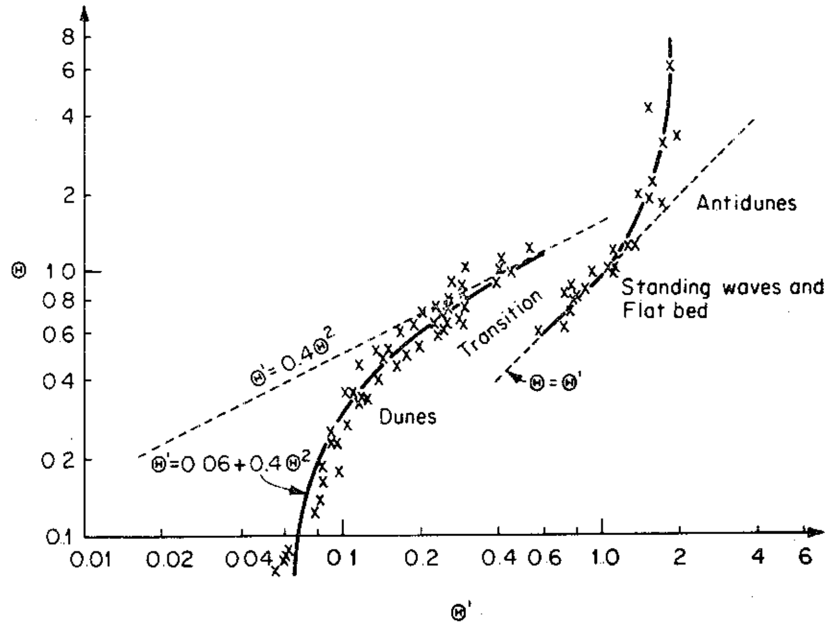


Fig. 5.12 Flow resistance given by Eq. (11.83). [*After* ENGELUND *et al. (1967)*]

3. Alam and Kennedy (1969); Lovera and Kennedy (1969)

A., L. and K divide the friction factor f into two parts: for f' a graph is given based on experiments with flat beds with transport (see figure 5.13) and f'' is given as a function of $\overline{U}/\sqrt{g\, D_{50}}$ and h/D_{50} (figure 5.14). Most of the observations are for $D_{50} < 0.5$ mm.

The accuracy of all these predictions is limited but may be used as a first estimate. If accurate data are needed, observations in the field for the specific situation are necessary.

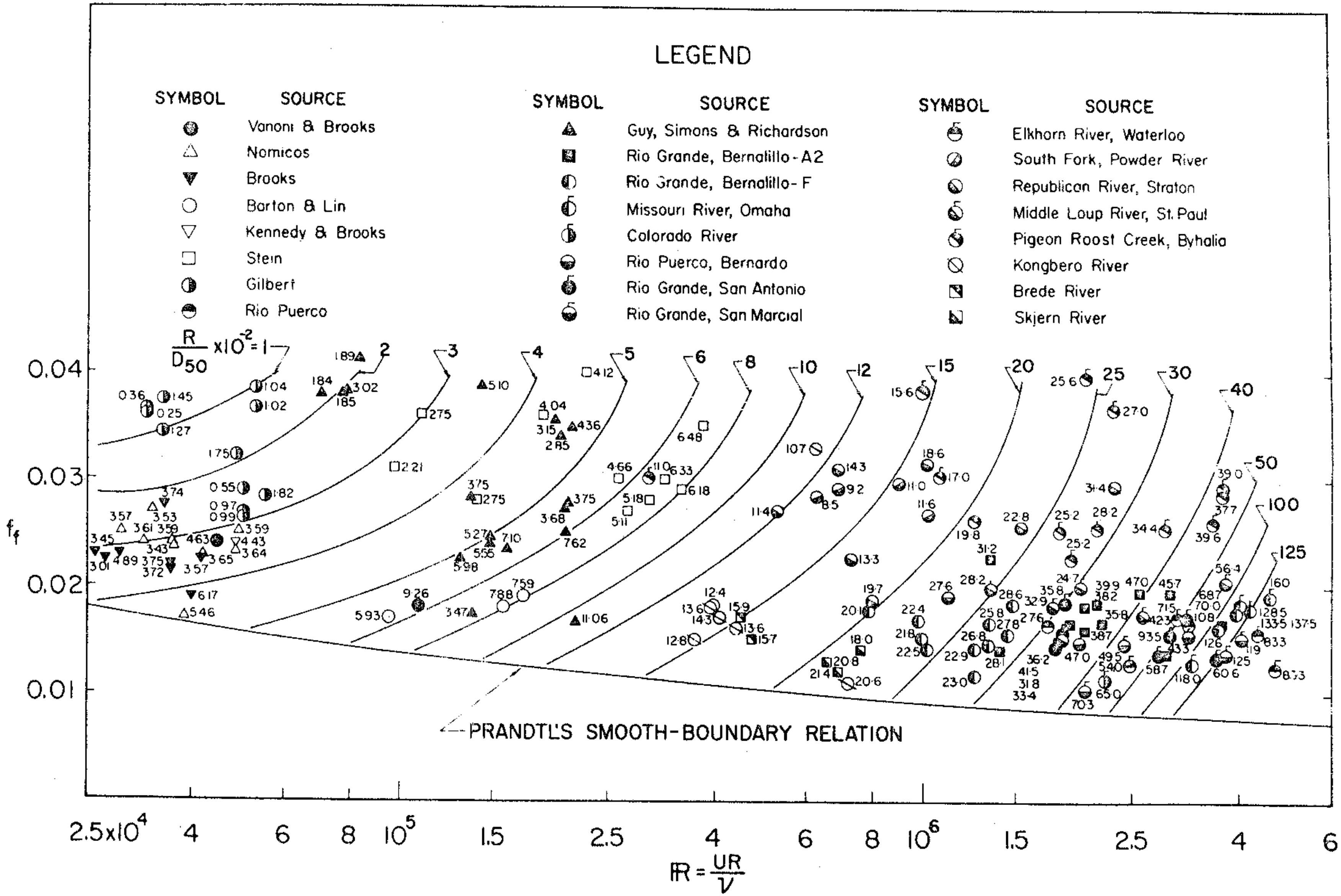


FIG.5.13 FRICTION-FACTOR PREDICTOR FOR FLAT-BED FLOWS IN ALLUVIAL CHANNELS (THE NUMBER BY EACH POINT IS $R/D_{50} \times 10^{-2}$)

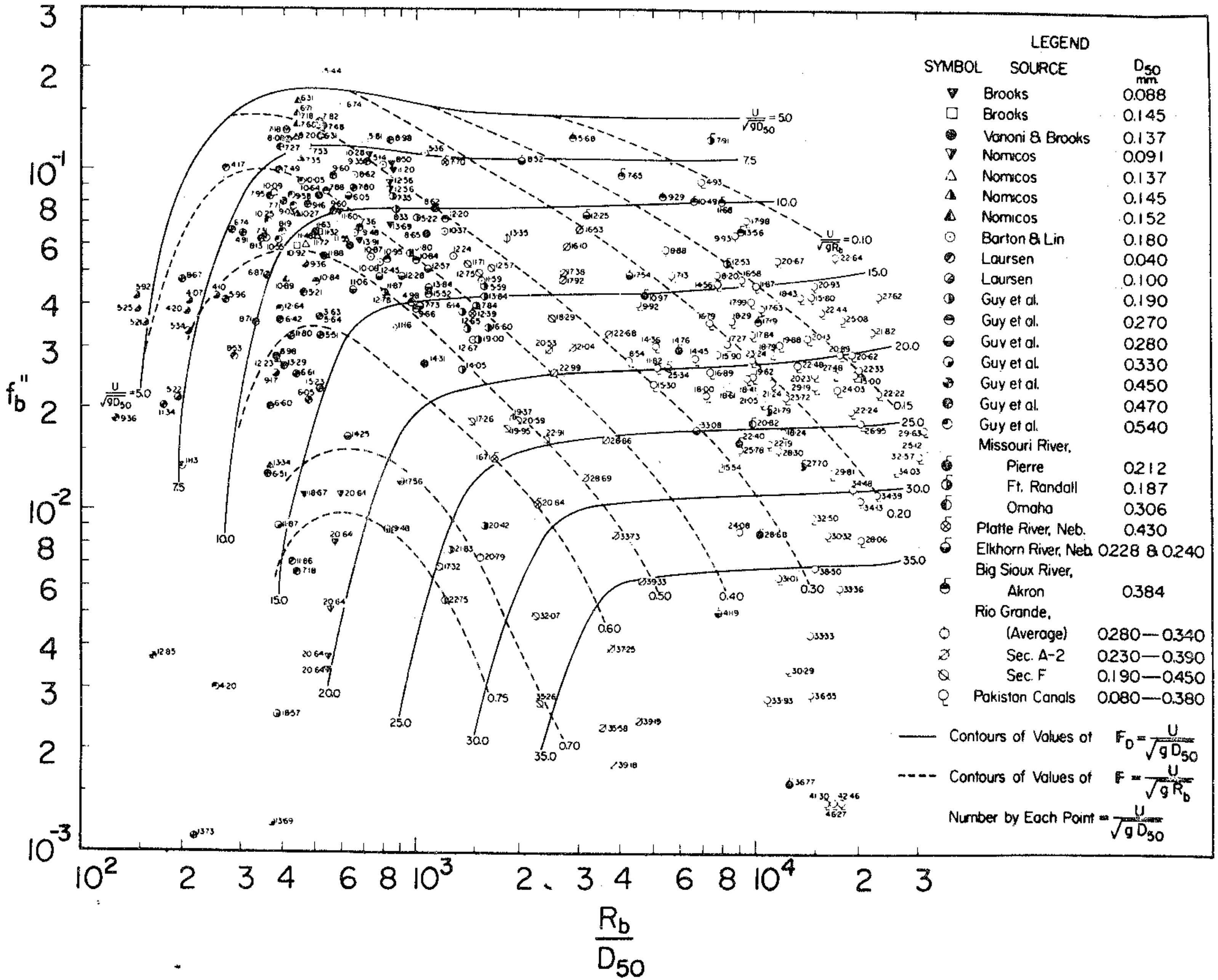


FIG.5.14 GRAPHICAL PREDICTOR FOR THE BED-FORM FRICTION FACTOR

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6 BED-MATERIAL TRANSPORT

Bed material transport can be divided in bed load and suspended load. Both modes of transport have an influence on processes of erosion and deposition. Many relations between sediment transport and flow conditions are based on the bed shear stress. It has been shown that the bed-shear stress may be divided in a form drag and a roughness drag. It will be clear that the form drag does not contribute to the transport but that only the roughness drag will be of importance. Measurements of water depth and slope give the total bed shear stress, so that most transport relations require a reduction of the total bed shear stress to a value which is relevant for the transport.

This reduction factor is called the ripple factor μ. Theoretically one should expect: $\mu = \frac{\lambda'}{\lambda} = (\frac{C}{C'})^2$

Many authors use μ as a closing term, however, so that various expressions are given. This manipulation with the bed shear stress has led several authors to use the mean velocity $\overline{U}$ instead of τ_o as the important factor for the sediment transport. The problem then is that the same value of $\overline{U}$ in different water depths will give different sediment transport rates, so that again some correction is necessary.

6.1 BED LOAD

Because several authors use some type of a physical model to predict a sediment transport relation it is not surprising that most formulas may be expressed as relations between dimensionless groups. The most common are a group related to the transport:

$\phi = S/[D^{3/2} (g\Delta)^{\frac{1}{2}}]$

S = transport in $m^3/m^1.s.$ transport = volume of grains

For conversion to total volume, S has to be divided by $(1-\varepsilon)$ in which ε = porosity.

$\Delta = (\rho_s - \rho_w)/\rho_w$

D = grainsize

and a group related to the flow:

$\psi = U^{*2}/\Delta g D$

(the parameter used by Shields for the initiation of motion).

Some of the relations given in literature are the following:

1. Du Boys (1879)

Du Boys gave a simple model in which layers of sediment move relative to each other. The number of layers was proportional to τ_o/τ_{cr}

The resulting expression is of the form:

$$S = \text{const. } \tau_o \, (\tau_o - \tau_{cr})$$

Although the physical model is not very convincing, it has been found that the form of the relation can be used to describe experiments in a reasonable way.

2. Kalinske (1947)

Kalinske assumed that grains are transported in a layer with thickness D with an instantaneous grain velocity U_g equal to:

$U_g = b\,(U_o - U_{cr})$

U_o = instantaneous fluid velocity at grain level

U_{cr} = critical fluid velocity to start grain movement

For U_o a normal distribution is assumed:

$$f\,(U_o) = \frac{1}{\sigma\sqrt{2\pi}} \text{ exp. } \left[- \frac{(U_o - \bar{U}_o)^2}{2\sigma^2} \right]$$

σ = r.m.s. value of velocity fluctuations

Taking the number of grains per unit area $p/(\pi/4\,D^2)$ and using $\bar{\bar{U}}_g$ then the mean rate of particle movement, by dry weight per unit width and time is:

$$T_b = \frac{2}{3}\rho_s\, g\, D \cdot \bar{U}_g$$

$$\text{where } \bar{U}_g = b \int_{U_c}^{\infty} (U_o - U_{cr}) \; f\,(U_o)\, dU_o$$

The resulting expression may be made dimensionless with the parameters ϕ and ψ with the result:

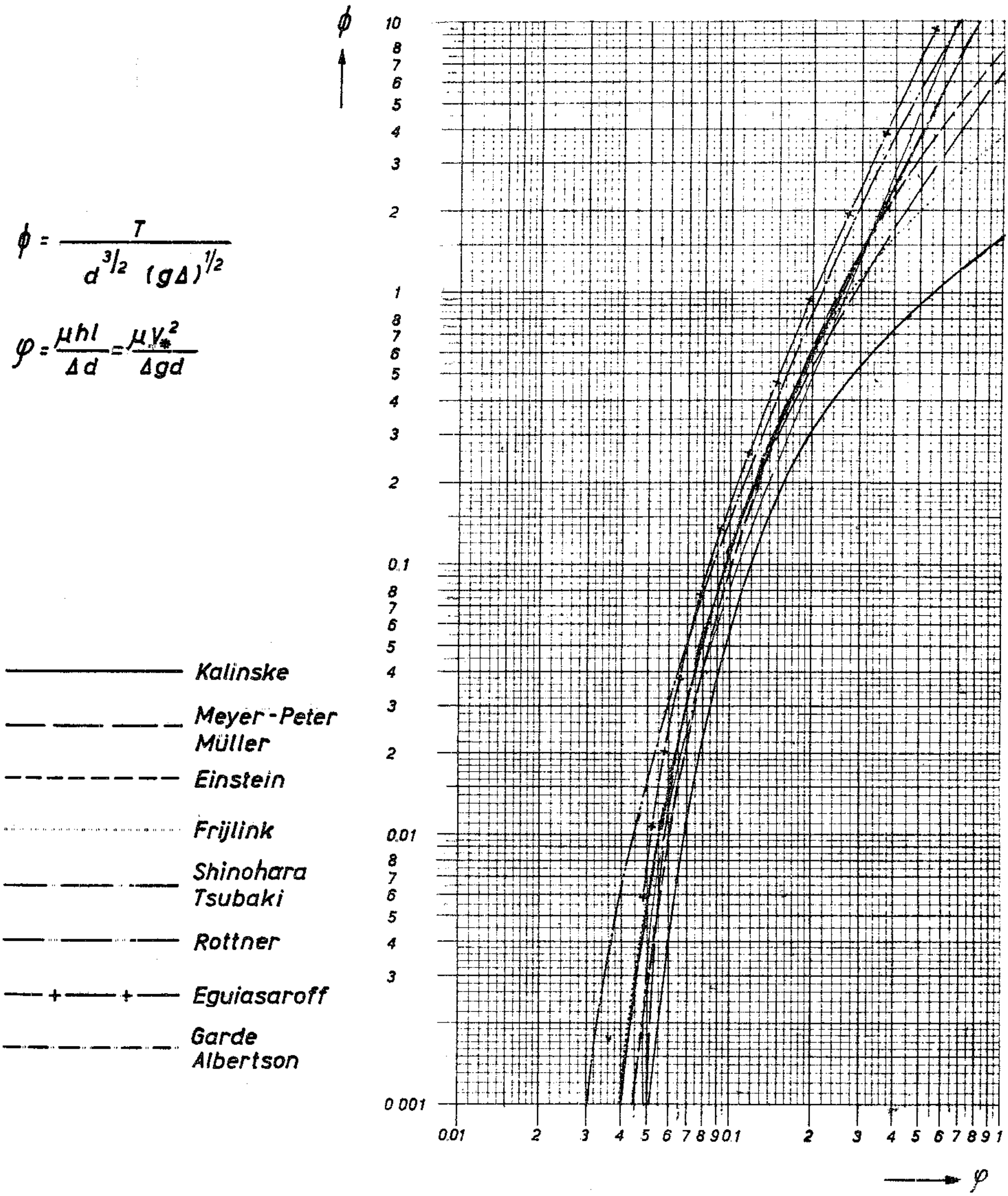
$$\phi = 2.5\,\psi^{\frac{1}{2}} \left\{ \frac{r}{\sqrt{2\pi}} \text{ exp.} \left[- \frac{1}{2r^2} \left(\sqrt{\frac{0.12}{\psi}} - 1 \right)^2 \right] - \left(\sqrt{\frac{0.12}{\psi}} - 1 \right) \frac{1}{2\sqrt{\pi}} \text{ erf} \left[\left[\frac{1}{r\sqrt{2}} \left(\sqrt{\frac{0.12}{\psi}} - 1 \right) \right] \right\}$$

in which $r = \sigma/\bar{U}_o$ (see figure 6.1)

Kalinske did not reduce the bed shear stress, so the relation is valid for plane beds only.

$$\phi = \frac{T}{d^{3/2}\ (g\Delta)^{1/2}}$$

$$\varphi = \frac{\mu h I}{\Delta d} = \frac{\mu V_*^2}{\Delta g d}$$



COMPARISON OF BED-LOAD TRANSPORT EQUATIONS

FIGURE 6.1

3. Meyer - Peter and Müller

M.P.M. have performed a large number of experiments in a wide flume with coarse sands. The resulting empirical expression may be written in ϕ and ψ units as:

$$\phi = (4\psi - 0.188)^{3/2} \quad \text{(figure 6.1)}$$

By comparison of results with flat beds and dune beds the ripple factor is found:

$$\mu = (C/C')^{3/2} \quad \text{(theoretical exponent 2)}$$

For a mixture M.P.M. take:

$Dm = \overline{D} = \Sigma p \cdot D/\Sigma p$ as the relevant parameter and for the grain roughness $D = D_{90}$.

4. Einstein (1950)

Einstein gave a complicated statistical description of the grain transport process in which the exchange probability of a grain is related to flow conditions. The resulting expression is given in figure 6.1 in a graphical form.

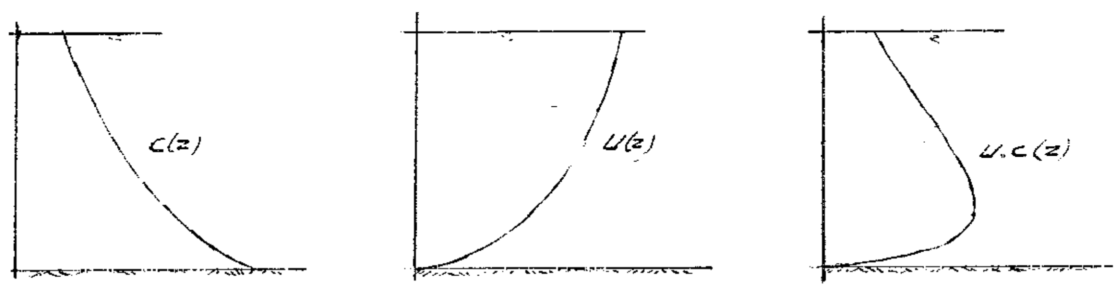
For the determination of the ripple factor μ a graphical procedure is given by Einstein. He used $D = D_{35}$ as the relevant parameter for the transport and $D = D_{65}$ for the roughness. The correlation is not valid for large rates of transport because there the transport varies with the first power of velocity $\overline{U}$ only.

The relations given were for bed-load. In most conditions a predominant contribution of suspended load will be present. The final accuracy of the bed-material discharge will depend therefore mostly on the accuracy of the suspended load determination.

6.2 SUSPENDED LOAD

Suspended load can be determined from measurements of U(z) and c(z) and integration of:

$$S = \int_0^h C(z)\, U(z)\, dz$$



In most cases estimates based on theoretical expressions will be necessary. The basic equation describing the concentration distribution in uniform steady flow is:

$$W \cdot C + \varepsilon_s \cdot \frac{\partial c}{\partial y} = 0$$

The first term W . C (W = fall velocity ; C = volume concentration of sediments) represents the settling tendency of the flow. The second term represents the diffusive action of the turbulence. ε_s is the turbulent diffusion coëfficient. An explanation for this term is the following. Water packets moving upward carry a larger amount of grains as packets moving downward because there is a concentration gradient. Although there is no net transport of water there will be a net vertical transport due to this exchange of water packets, which will be proportional to the local value of the concentration gradient.

If it is assumed that the diffusion coefficient for sediment is equal to the coefficient to the exchange of momentum, then:

$$\varepsilon_s = \varepsilon_m = \varkappa u^* y\ (1 - y/h)$$

The resulting equation may be integrated and gives:

$$\frac{C(y)}{C(a)} = \left(\frac{h-y}{y} \quad \frac{a}{h-a}\right)^z$$

with $z = W/\varkappa u^*$. a is a reference level where C = C(a).
For a graphical presentation see figure 6.2

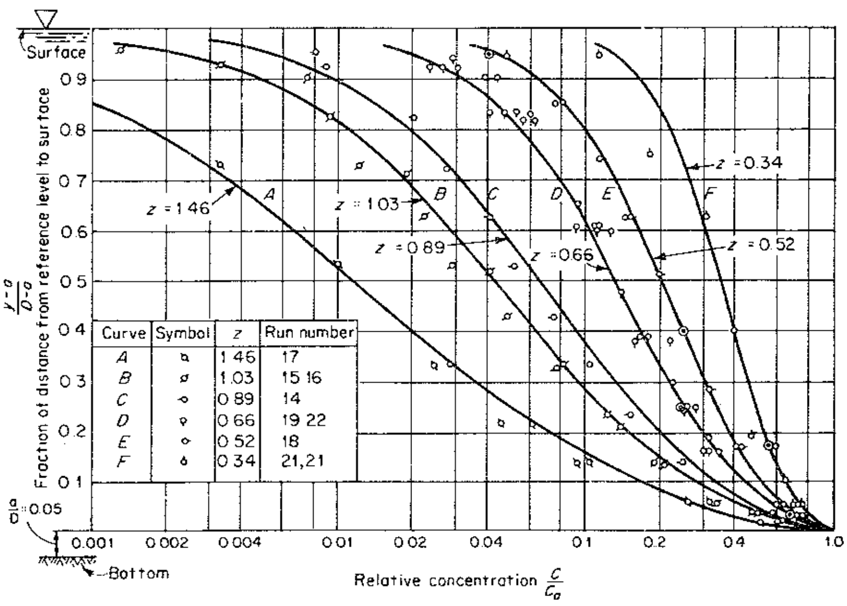


Fig. 6.2 Distribution of suspended sediment; comparison of experimental data with Eq (8.35) [*After* VANONI (*1946*).]

From this figure the following rough critiria may be given

$W/x u^*$	u^*/W	description
2	1.2	some suspension
0.8	3	concentration at surface > 0
0.25	10	fully developed suspension
0.06	40	almost uniform concentration

The last criterion shows that particles < 50 μ ($W < 0.2$ cm/s) are uniformly distributed for $u^* > 8$ cm/s or $\overline{U} > 1 - 1.5$ m/s.

Although the basic equation is very simple, some critical remarks have to be made:

1. The term W.C. should be $(1 - C).C.W$ to account for the presence of the particles (see Hunt 1954). This correction is not important for $C \ll 1$.
2. The fall velocity is changed by the presence of other particles (see chapter 3) and by the turbulent movements of the water. Symmetric vertical velocity fluctuations give a-symmetrical drag forces for non-Stokes particles. Therefore, although the mean value of the vertical velocity is zero, there will be a resultant vertical force which will reduce the settling velocity.
3. The expression for ε_s gives $\varepsilon_s = 0$ for $y = 0$ or $\partial C/\partial y = \infty$ at $y = 0$ which is not very real.
4. The value of $C = C(a)$ is not given. Several assumptions are made in the literature. Einstein (1950) divides the computed bed-load by a layer with thickness 2 D and by the velocity in this layer.
   The value of C(a) is one the problems to be solved in sediment transport.
5. The velocity distribution is influenced by the presence of the particles. The weight of the particles suppresses the vertical velocity fluctuations and gives a decrease in the momentum diffusion coefficient. This is similar to a decrease in the value of **x**. In fact several expressions have been given in which **x** decreases with the power to keep the sediment in suspension:
   $C.W.\Delta/U.I$ (see figure 6.3)
   Velocity profiles become less "full" by this effect. Care should be taken in the application of this correlation because the determination of **x** from velocity profiles or concentration profiles is not very accurate. For literature see Einstein and Ning Chien (1954) and Ippen (1971).

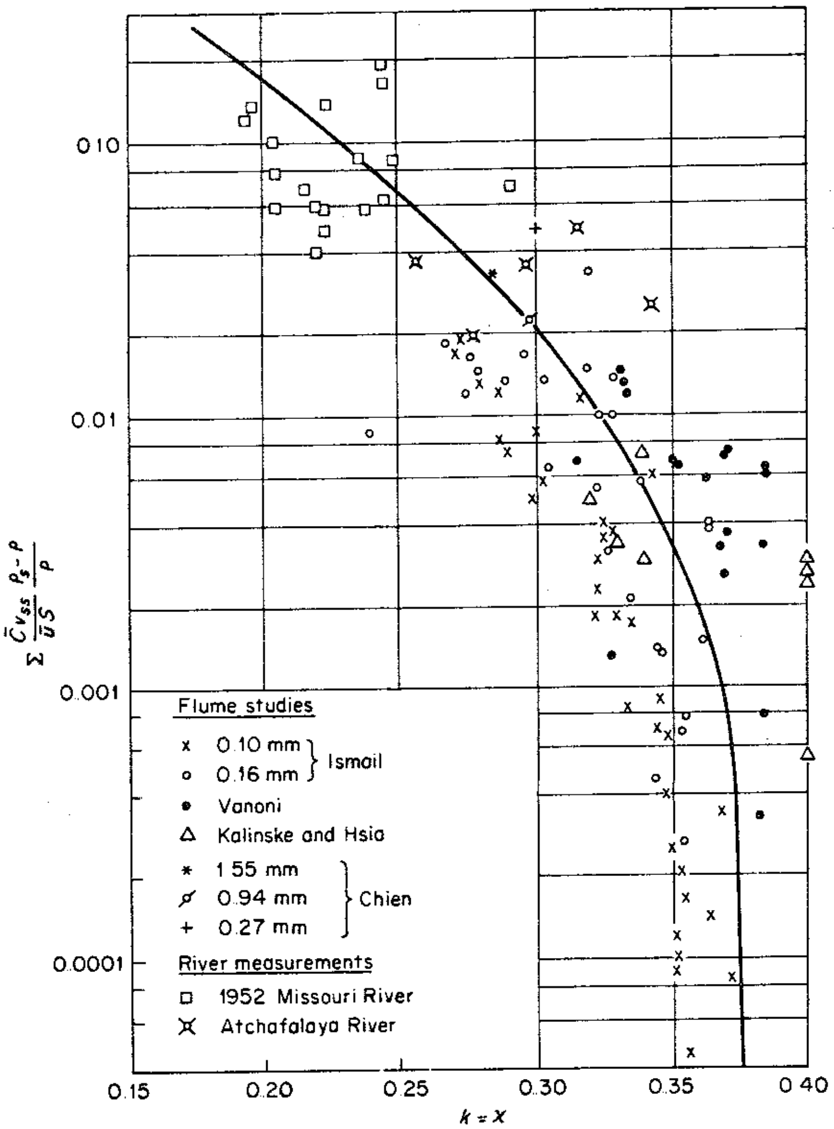


Fig. 6.3 Effect of suspended load on the *k* value. [*After* EINSTEIN et al. (*1954*).]

V_{ss} ⇒ fall velocity W

S ⇒ slope I

$\frac{\rho_s - \rho}{\rho}$ ⇒ relative density Δ

6. The assumption $\varepsilon_s = \varepsilon_m$ has also some objections. It is not necessary that the diffusion of particles is not equal to that of momentum. Measurements by Coleman (1970) show indeed that ε_s - values derived from concentration profiles give some differences with values of ε_s obtained from

$$\varepsilon_s = \varepsilon_m = \kappa U^* y (1 - y/h)$$

It is a reasonable assumption, however. Differences between ε_s and ε_m are generally put in κ which is often used as a closing factor.
If U(y) and C(y) are known, integration will give the suspended load. The integration cannot be performed analytically. Graphs are presented by Einstein (see Graf 1971 p. 189 - 195).

6.3 TOTAL LOAD

The total sediment load of a stream can be determined by adding the bed-load and the suspended load. This is done in the Einstein (1950) procedure. This procedure was modified by Colby (1955, 1961). Also Toffaletti (1969) gives a procedure which is especially adopted for computer programming.

Besides these "adding" procedures several direct empirical relations are proposed in literature:

1. Shinohara and Tsubaki (1959) gave an empirical relation:

   $\phi = 25\,\psi^{1.3}\,(\psi - 0.038)$ (see figure 6.1)

   The corresponding ripple factor $\mu = (C/C')$

2. Garde and Albertson (1961) gave a graphical relation of $\phi/\sqrt{\psi}$ with $\overline{U}/U^*$ as the third variable. The resulting $\phi - \psi$ relation is almost identical with Shinohara (figure 6.1)

3. Colby (1964) has given a graphical relation between total load, mean velocity $\overline{U}$, flow depths and grain-size with correlationfactors for temperature and silt content (see figure 6.4 and 6.5)

4. Engelund and Hansen (1967) gave an emperical relation of the form:

   $\phi = 0.05\ \psi^{2.5}$ with: $\mu = (C^2/g)^{0.4}$

   ($\mu > 1$ in this case)

   The formula is based on measurements with $D_{50} < 1$ mm and gave good results in comparison with sediment transport measurements in rivers. At all large values of ψ, the sediment rate increases with the fifth power of the velocity.

It must be noted that due to the strong variation of sediment transport with velocity, predictions of total sediment lead will not be very accurate. Differences of a factor 10 between various formulas or between computations and measurements are no exception (see figure 6.6 and 6.7).

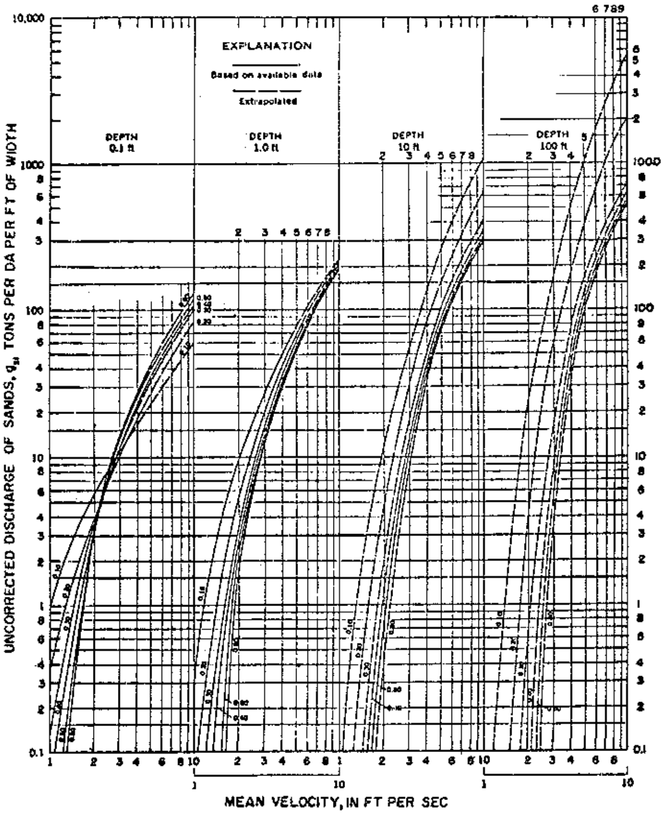


FIG. 6.4 —COLBY'S (2-H.14) RELATIONSHIP FOR DISCHARGE OF SANDS IN TERMS OF MEAN VELOCITY FOR 6 MEDIAN SIZES OF BED SANDS, 4 DEPTHS OF FLOW, AND WATER TEMPERATURE OF 60° F

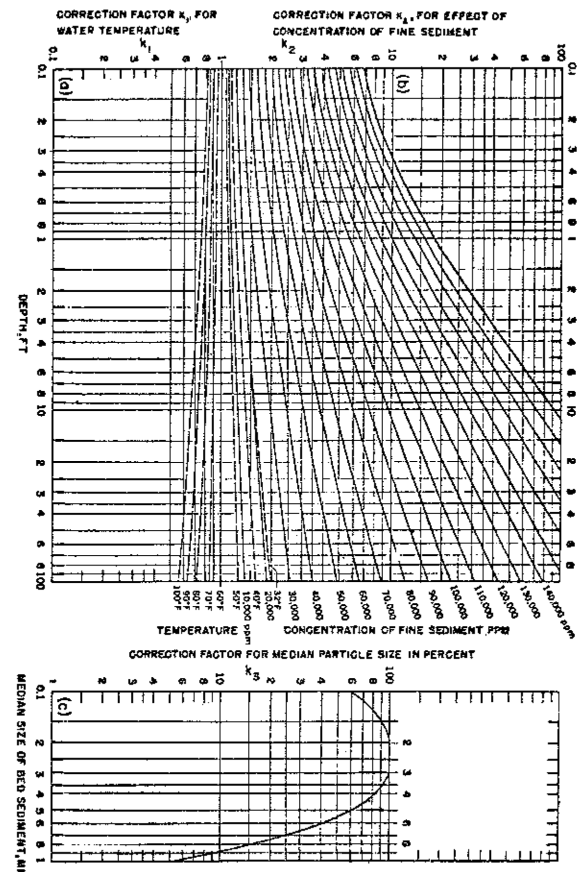


FIG. 6.5 —COLBY'S (2-H.14) CORRECTION FACTORS FOR EFFECT OF WATER TEMPERATURE, CONCENTRATION OF FINE SEDIMENT AND SEDIMENT SIZE TO BE APPLIED TO UNCORRECTED DISCHARGE OF SAND GIVEN BY FIG. 2-H.10

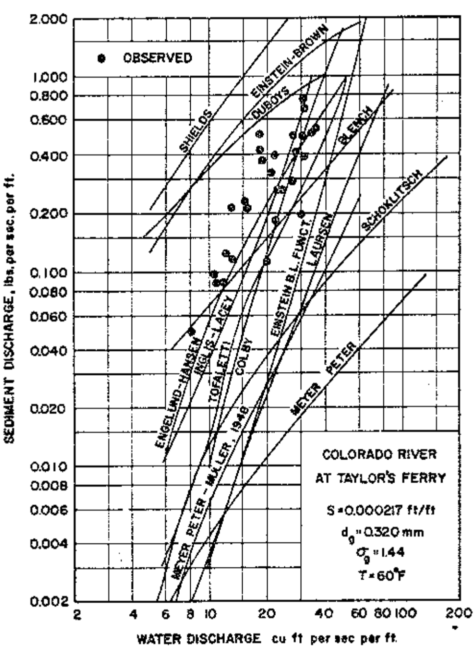


FIG 6.6 —SEDIMENT DISCHARGE AS FUNCTION OF WATER DISCHARGE FOR COLORADO RIVER AT TAYLOR'S FERRY OBTAINED FROM OBSERVATIONS AND CALCULATIONS BY SEVERAL FORMULAS

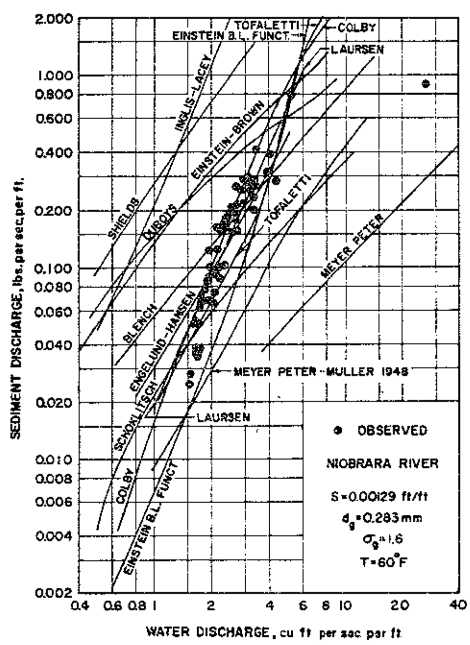


FIG. 6.7 —SEDIMENT DISCHARGE AS FUNCTION OF WATER DISCHARGE FOR NIOBRARA RIVER NEAR CODY, NEB. OBTAINED FROM OBSERVATIONS AND CALCULATIONS BY SEVERAL FORMULAS

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7. STABLE CHANNELS

7.1 Introduction

For the design of stable channels two approaches can be distinguished:

1. The tractive force (Lane)
2. The regime theory (originating in India. Developed by Kennedy, Lindley, Lacey, Iglis, Blench).

Both methods are used to design stable channels. A stable channel in alluvial material is one in which scour of banks and changes in alignment do not occur. Deposition on or scour of the bed is not objectionable in general, provide there is equilibrium over a long period.

The stability of a channel depends on the properties of the excavated material, (grain,size, cohesion) of the flow (discharge, silt content, transported material) and the design variables such as profile, shape, slope. Sediments introduced in the channel must be conveyed in view of the definition given above.

7.2 Tractive force theory

This approach is specially suited if the flow transports very little or no sediments. The design is then based on a limiting velocity or critical shear stress of the bed material.

For uniform cohesionless material, Shields graph may be used to compute τ_{cr}. In practice materials will have a wide gradation and will have some cohesion due to the silt content. For these materials, Lane's (1953) design curves are recommended (see figure 7.1)
It must be noted, however, that the large values of τ_{cr} as compared with Shields values are due to the fact that τ_{cr}-values are based on actual channel roughness, including irregularities, bedforms, whereas Shields graph is based on a flat bed. When there is some bed load, the problem is more complicated and calculations should be made to check the transport capacity of the channels.

Bank stability will depend on the characteristics of the bed material and the side slopes of the channel. From experiments and calculations it appeared that for trapezoidal channels with side slopes 1 : 1 to 1 : 2. The following values can be given for the shear stress :

τ_o, horizontal part of the profile $\approx \rho g h I$

τ_o, side slopes $\approx 3/4 \, \rho g h I$

For non-cohesive materials the reduction of τ_{cr} due to the side slope was given in chapter 4:

$$\frac{\tau_{cr}(\beta)}{\tau_{cr}(0)} = \cos\beta \sqrt{1 - \frac{tg\,\beta}{tg\,\varphi}^{2}} \qquad \text{see figure 7.2}$$

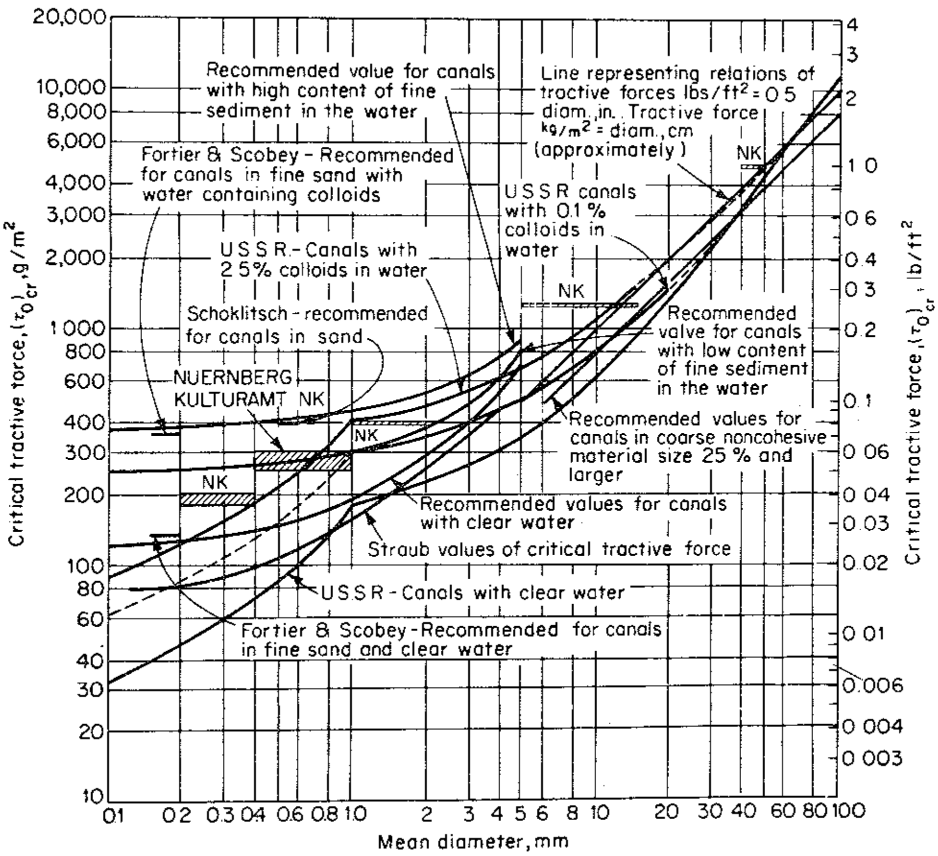


Fig. 7.1 Critical shear stress as function of grain diameter. [*After* LANE (*1953*).]

Assuming $\tau_{side} / \tau_{horizontal} = 3/4$ and $\varphi = 30 - 40^{\circ}$ it can be seen that a side slope of 1 : 2 to 1 : 3 is necessary. For practical values of φ see figure 7.3.

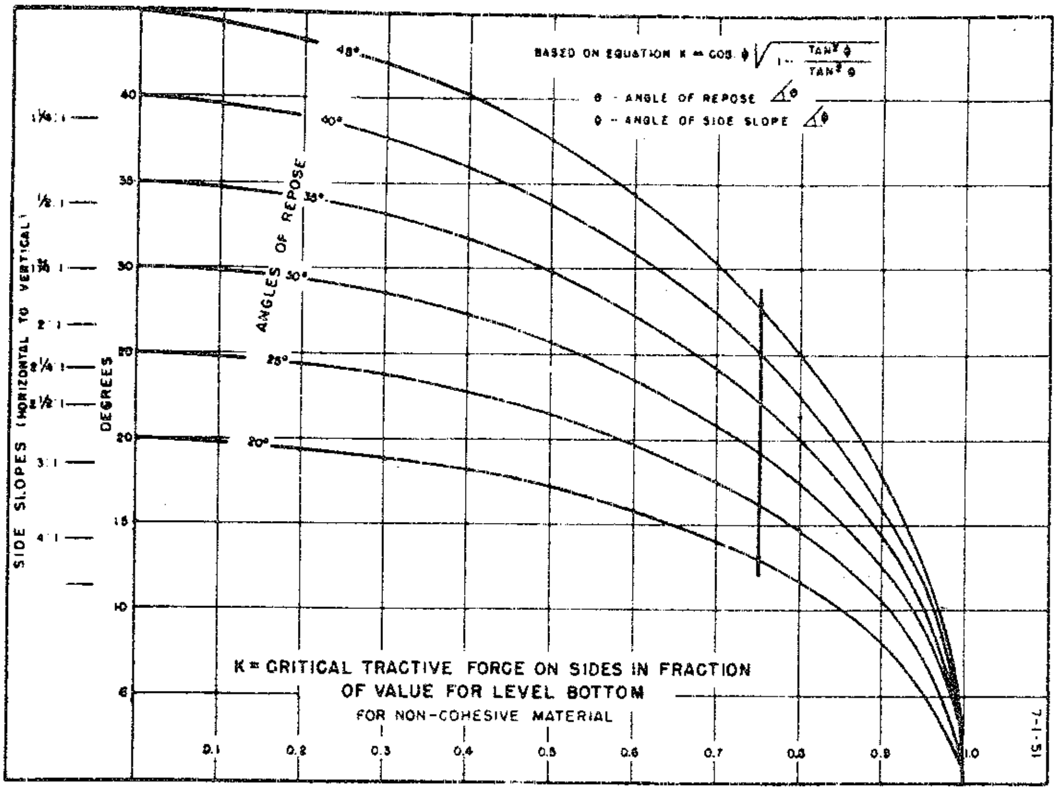
A theoretical stable profile for which at all points the same critical conditions occurs is found in the following way. Assume that the local value of the shear-stress $\tau(y)$ is proportional to the local water depth $h(y)$ and acts on a length of $1/\cos\beta(y)$.

Then $\tau(y) = \tau_{max} \cdot h(y) \cdot h_{max}^{-1} \cos\beta(y)$

in which h_{max} is the maximum depth with corresponding τ_{max}.

With the reduction formula for $\tau_{cr}(\beta)$ the following theoretical sinus profile results:

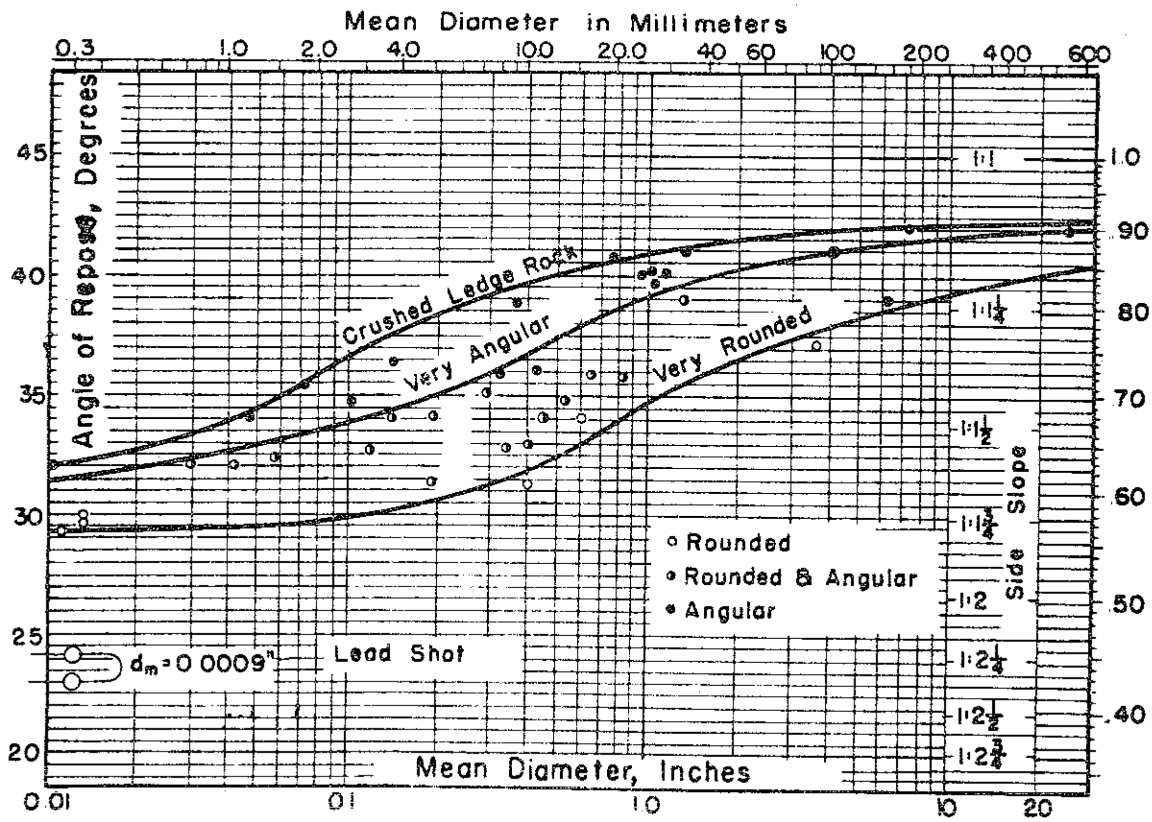
$$h/h_{max} = \sin(y/h_{max} \cdot tg\,\varphi)$$



- K = Critical tractive force on sides in fraction of value for level bottom for non-cohesive material.

Acc to Lane (1955)

FIG. 7.2



ANGLE OF REPOSE OF NON-COHESIVE MATERIAL

Acc. to Simons and Albertson (1960)

FIG. 7.3

The width of the channel is then $B = \pi \cdot h_{max} \cdot / \operatorname{tg}\varphi$ but may be extended of course with a horizontal section.
It is clear that the sinusoidal profile is difficult to construct and will be approximated by a trapezoidal profile in practice.

Lane (1953) gives some reduction factors to account for the sinuosity of channels.

Channel type	τ_{cr}/τ_{cr} straight	U_{cr}/U_{cr} straight
straight	1.00	1.00
slightly sinuous	0.90	0.95
moderately sinuous	0.75	0.87
very sinuous	0.60	0.78

Also local effects of contractions, bridges etc. should be considered.
In applying Shields graph for coarse material ($\psi_{cr} \approx 0.06$) for the stability of stones on revetments and banks care should be taken with the criterion (general movements). For a safe design without movement of the stones a value $\psi_{cr} = 0.03$ is recommended.

7.3 Regime theory

With the tractive force theory designs of channels can be made. Another approach to the problem is to study succesfull alluvial channels. Numerous studies of man-made and natural alluvial channels have given empirical relations between depth, width, velocity, discharge, sediment transport and material characteristics. These techniques are referred to as "regime theory". Usually three equations are presented: (1) a flow formula which gives the required slope (2,3) formulas for channel depth and width. Regime theory originated in India where extensive canal systems were built. One of the disadvantages of the regime theory is that results are related to a specific area, so that application to other areas can giveerrors.

"Regime" can be defined as a situation in which a channel will not change on a long-term average. Short term changes will occur with charges in in discharge on sediment transport.

Important contributions were given by Kennedy, Lindley, Lacey and Blench (1957). Some of the results of Blench are given here. Blench gave three equations

(1) $F_B = U^2/h$ F_b = bedfactor (ft. s - units)

(2) $F_s = U^3 . h/A$ F_s = side factor A = area of cross section

(3) $$I = \frac{F_b^{5/6}\, F_s^{1/12} \cdot \upsilon^{1/4}}{3.63\, Q^{1/6} \cdot g(1 + c/2330)}$$

I = slope

υ = viscosity of water-sand mixture

c = bed load concentration in p.p.m. by weight

From these equotion the following equotions are derived:

$$\overline{B} = A/h = \sqrt{(F_B/F_s) \cdot Q}$$

$$h = \sqrt[3]{(F_s/F_B^2) \cdot Q}$$

$$\overline{U} = \sqrt[6]{F_B \cdot F_s \cdot Q}$$

Blench suggests: $F_B = 1.9\sqrt{D}\,(1 + 0.012\ C)$
with D in mm, sand range only and F_s = 0.1 , 0.2 , 0.3 for loam with very slight, medium and high cohesion. For practical applications tests in similar channels seem necessary.

Simons and Albertson (1960) have analysed a large number of Indian and American canals. The results presented are valid for sediment concentrations < 500 p.p.m. and grain sizes $0.1 < D < 7.5$ mm. From figure 7.4 to 7.8 width and depth can be selected. Curves E should be used for channels with high sediment load only. From the graphs $A = P . R$ can be computed and also $\overline{U} = Q/A$. Values of depth, average width and top width can be adjusted as required to maintain these values of hydraulic radius and wetted perimeter. If the bank is non-cohesive, the side slope must not exceed the value for the angle of repose given in figure 7.3. For a good design values $5 - 10^{\circ}$ lower than the angles given should be taken. Figures 7.9 to 7.11 can be used to estimate three values of the slope S (depth = D in Simons notation, W = width, R = hydraulicradius). The designer must now invoke his engineering judgement, guided by these slopes to arrive at the design slope.

It will be clear that with the regime theory only rough estimates of channel dimensions will be obtained. Experience in a specific area will be of equal importance.

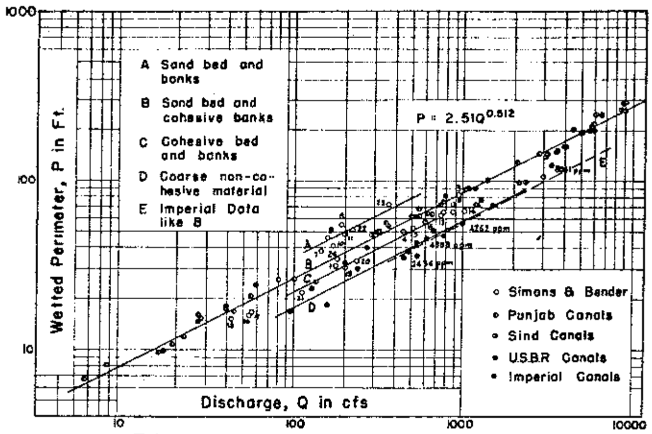


Fig. 7.4 Variation of wetted perimeter with discharge for regime channels.

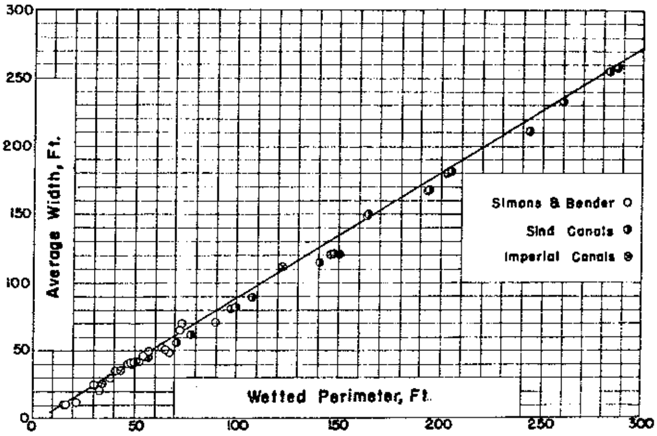


Fig 7.5 Variation of average width with wetted perimeter for regime channels.

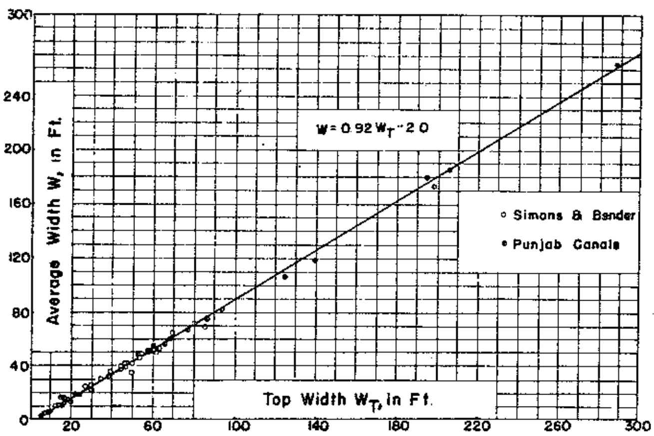


Fig 7.6 Variation of top width with average width for regime channels

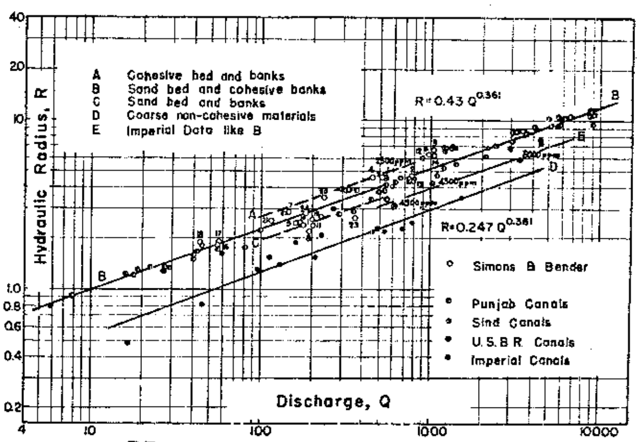


Fig 7.7 Variation of hydraulic radius with discharge for regime channels.

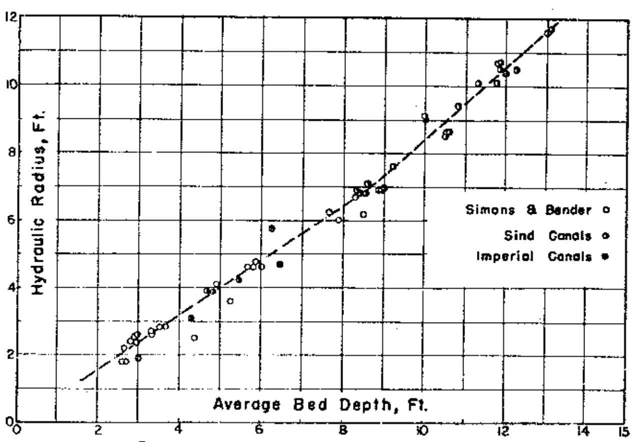


Fig 7.8 Variation of average bed depth with hydraulic radius for regime channels.

74 b

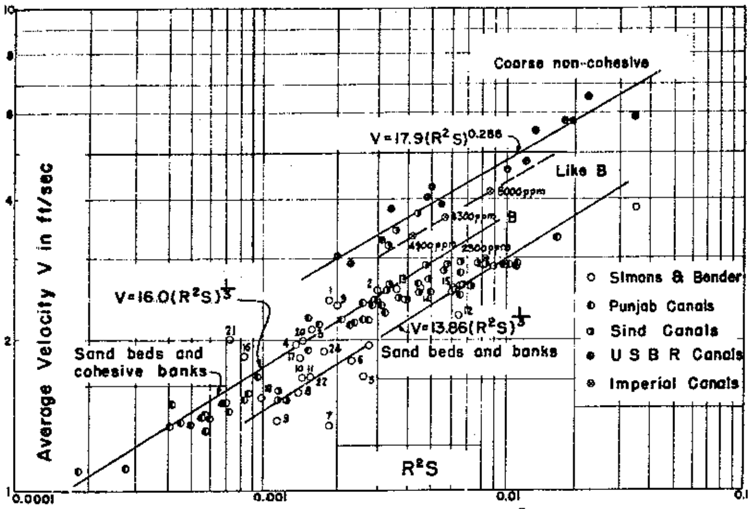


Fig 7.9 Variation of mean velocity with R^2S for regime channels (Lacey type slope relation)

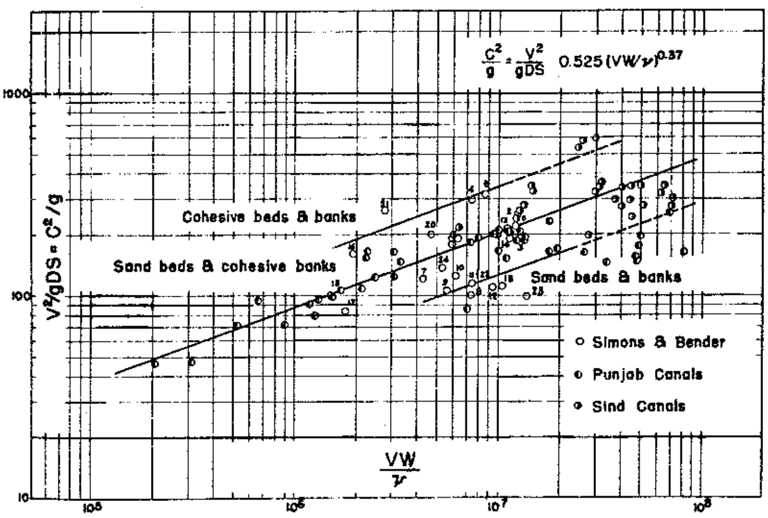


Fig 7.10 Variation of $\frac{V^2}{g\,DS}$ with $\frac{VW}{\nu}$ for regime channels (Blench-King type slope relation.)

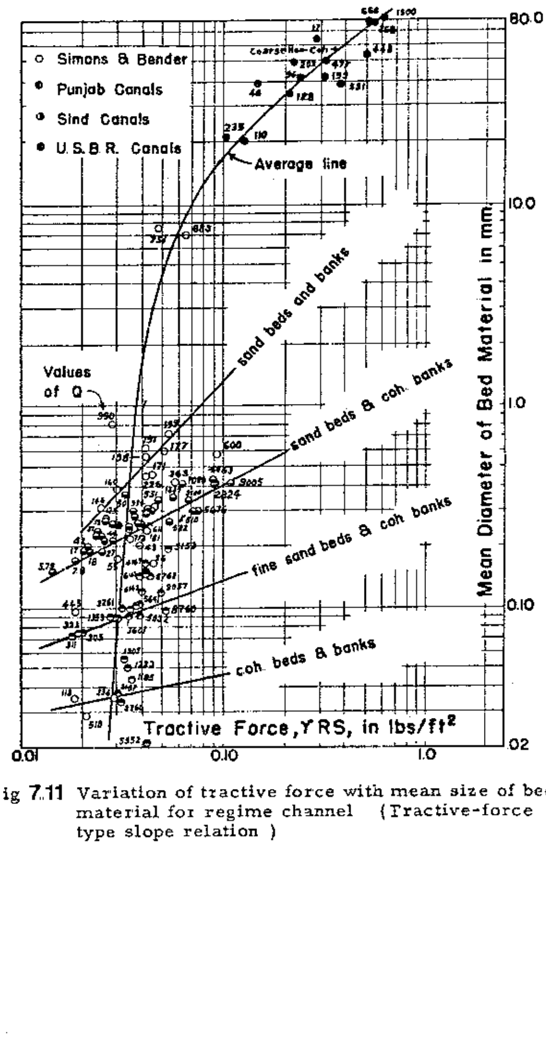


Fig 7.11 Variation of tractive force with mean size of bed material for regime channel (Tractive-force type slope relation)

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8. RIVER BED VARIATIONS, AGGRADATION AND DEGRADATION

A natural river will never be in an exact equilibrium condition. Variations in discharge can give variations in bed level, roughness etc. Also changes in the regime of a river may give deviations from an equilibrium state. If the sediment discharge S entering a river reach is greater than the equilibrium value Se aggradation will occur untill a new equilibrium is approached. Some examples:

- aggradation upstream from a reservoir,
- tributary channel bringing heavy sediment load to a main channel giving local aggradation,
- river regulations eliminating floods which formerly periodically cleared the channel of accumulated sediment.

Some examples of degradation ($S < Se$):

- degradation downstream of dams,
- canals in fine material carrying clear water,
- realigned channels with increased slope.

For calculations on non-steady or non-uniform conditions it is necessary to introduce a sediment transport relation of the form $s = a \cdot v^b$
For the transport relations given in ch. 6 values in the order of 3 - 7 are found. (high values for low transport rates), with b = 4 to 5 for high sediment rates and fine material. (Engelund b = 5, Shinohara b = 4.6).

Application is shown in the following example.
What will be the reaction of a river to a local decrease in width?
Suppose $Q_1 = Q_o$ (continuity) $S_o = S_1$ (after some time continuity of sediment transport) $S = s \cdot B$ B = width.
$C_1 = C_o$ (Chezy value, index 1 = new situation; index 0 = old situation)
From $S = B \cdot s = B \cdot a\, v^b = B^{1-b} \cdot a Q^b \cdot h^{-b}$ $(v = Q\ B^{-1}\ h^{-1})$
it follows that:

$$\frac{h_1}{h_o} = \left(\frac{B_o}{B_1}\right)^{\frac{b-1}{b}}$$

From the Chezy formula it follows that:

$$B_1 \cdot h_1^{3/2}\ I_1^{1/2} = B_o\ h_o^{3/2}\ I_o^{1/2}$$

With the relation for h_1/h_o it follows that:

$$\frac{I_1}{I_o} = \left(\frac{B_1}{B_o}\right)^{1-\frac{3}{b}}$$

It can be seen from the equation for h_1/h_o that for a large value of b and values of B_o/B_1 not far from 1 , that :

$$h_1/h_o \approx B_o/B_1$$

or the decrease in width is compensated by an increase in depth. The slope will always decrease.

This rough approximation only gives a first estimate. For accurate values detailed computations or model studies will be necessary.

Computation on river-bed variations

The reaction of a river to a change in its regime (meander cut-off, dam) can be computed with the equations of motion and continuity of water and sediment. In most cases non-steadiness of the flow may be neglected so that the following equations are valid:

(1) $$U\frac{\partial U}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial z}{\partial x} = -g\frac{U.(U)}{C^2R}$$

(2) $$u \cdot h = q$$

(3) $$s = f(u)$$

(4) $$\frac{\partial z}{\partial t} + \frac{\partial s}{\partial x} = 0$$

h = waterdepth
z = bed level from reference datum
C = Chezy value
R = hydraulic radius
q = river discharge / m'
s = sediment discharge /m'

Solving these equations requires numerical techniques (see Vreugdenhil and De Vries 1973, De Vries 1973).

As a first approximation it may be assumed that the flow is uniform and two-dimensional so that equation (1) reduces to:

(5) $$\frac{\partial z}{\partial x} = \frac{U^2}{C^2h} = \frac{U^3}{C^2q} \qquad (q = u \cdot h)$$

or $$\frac{\partial^2 z}{\partial x^2} = -3\frac{U^2}{C^2q} \cdot \frac{\partial U}{\partial x}$$

Combination with (3) and (4) gives:

(6) $$\frac{\partial z}{\partial t} - k\frac{\partial^2 z}{\partial x^2} = 0$$

in which: $k = \frac{1}{3}\,\frac{C^2q.(ds/dU)}{U^2} = \frac{1}{3}\,\frac{U.(ds/dU)}{I_o} \cdot (\frac{U_o}{U})^3$

in which I = slope and index o refers to the original, uniform situation.

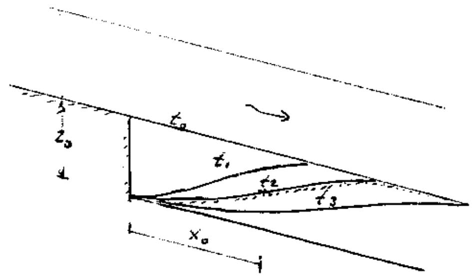
After linearination (possible for $U/U_o \approx 1$)

$$k = \frac{1}{3}\,\frac{U_o\,(ds/dU)}{I_o}$$

or for $s = a\,u^b$: $k = \frac{1}{3}\,b\,.\,\frac{s}{I_o}$

The equation (6) is a parabolic one (diffusion equation) for which solutions are known. This will be applied to compute the reaction of a river to a sudden decrease in sediment discharge s which will give a decrease in bed and water level of z_o after a long time.
Introduce $z' = z\,(x,\,0) - z$

with the boundary conditions:



$$z'\,(x\,,0) = 0$$
$$z'\,(0,\,t) = z_o$$

The solution of (6) is given by:

$$\frac{z'}{z_o} = \mathrm{erf\_c}\,(x/\,2\sqrt{kt})$$

in which erf_c is the function: $\mathrm{erf\_c}(\alpha) = \frac{2}{\sqrt{\pi}}\int_{\alpha}^{\infty} e^{-\xi^2}\,d\xi$

From this solution one can compute for which time t_{50} at $x = x_o$, 50% of the final lowering of the bed (z_o) has been reached.
This is the case for $\mathrm{erf\_c}(x_o/2\sqrt{k\,.\,t_{50}}) \approx 0.5$

or $\quad x_o \approx 1.0\,.\,\sqrt{k\,t_{50}}$ (see tables for erf c(α))

or $\quad t_{50} \approx x_o^{\;2}/k$

From comparison with the solution of the full equation it appears that this solution is valid for: $x_o > 2\,I_o\,.\,h$

Suppose $s = 10^4\ m^3/year$ (river with B = 100 m $S = 10^6\ m^3/year$)

$I = 2\ 10^{-4}$

$b = 5$ m

$h = 3$ m

Then the solution is valid for $x > 30$ km

$$k = \frac{5}{3}\,.\,\frac{10^4}{2.10^{-4}} = 0.83\ .\ 10^8$$

or $\quad t_{50} = \frac{(30\,.\,10^3)^2}{83\ 10^6} \approx 10$ years

This means that at 10 km from the discontinuity 50% of the expected change in bed level will be reached in 10 years.
This means that the reaction of this river to change in regime is relatively slow.

For a move complete treatment of these problems see the references cited above.

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9. MEASUREMENT TECHNIQUES

9.1 Introduction

Besides computations on sediment transport also measurements are necessary for a good description of a river and of the consequences of changes in regime. The existing techniques can be divided into two groups:

- Measurements with samplers
- Tracer techniques

9.2 Measurements with samplers

Due to the difference in mechanism of bed-load and suspended load there are different samplers for each type of transport.

1. Bed-load samplers

Variations in bed-load transport and the influence of bed forms will give large variations in results with samplers. For a significant value of the bed load a large number of observations has to be taken. Bedforms can give variations in local transport rate of $0 - 2\,\bar{s}$ (zero in the trough of a sand wave and $2\,\bar{s}$ at the crest of the wave). It is therefore advised to take many samples at various locations instead of taking long sampling times.

Further problems with bed-load samplers are:

1. It is difficult to give the equipment a correct vertical and horizontal alignment with the bed.
2. The meter should be calibrated to determine the efficiency which is also a function of the amount of material catched.
3. It should be avoided that the sampler collects bed material during the lowering of the instrument.
4. A sampler disturbs the flow field. Scour and a decrease of velocity in the sampler can occur.

Most samplers are of the box or basket type and consist of a pervious container. Water and sediment enter the sampler; the sediment is catched. As an example figure 9.1 is given. This sampler is used extensively on Dutch and other rivers.

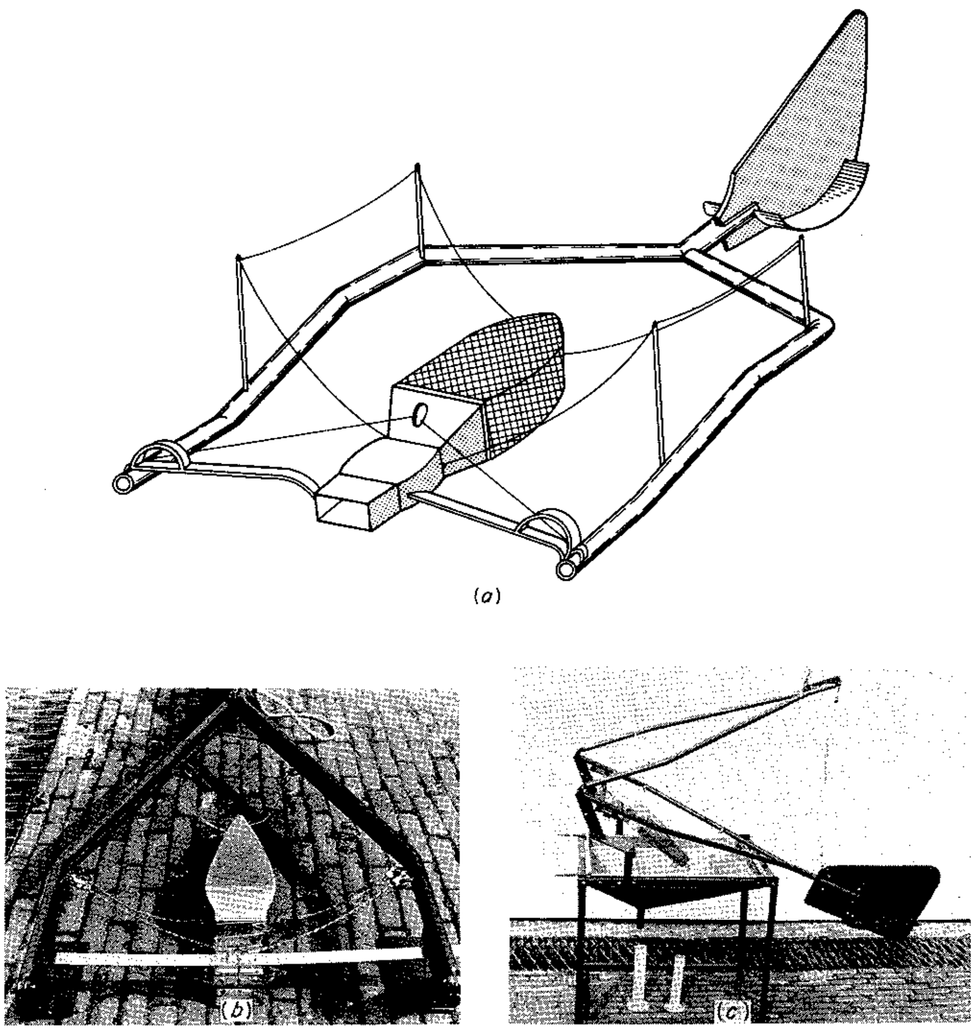


Fig. 9.1 Arnhem sampler (BTMA). (*a*) The Arnhem sampler [*after* HUBBELL (*1964*)]. (*b*) The new Arnhem sampler with an improved frame construction. (*c*) The emptying of the instrument; the catch is measured volumetrically.

2. <u>Suspended load samplers</u>

The amount of material catched depends on the hydraulic coefficient (U inside / U outside) and the efficiency (% of material catched) of the instrument. Both factors have to be determined by calibration. The following points should be considered:

1. Suspended load shows large fluctuations so that repeated sampling is necessary.
2. Suspended sediment sampling also includes part of the wash load.
3. If only concentration is measured also velocity profiles have to be measured.

There are two types of samplers:

a) concentration samplers: a value of water is sampled at a certain level or as an average over the depth
(Nansen bottle, mouse trap, depth integrating samplers)

b) suspended-load samplers - suspended-load(U.C.) is measured at a point or by integration over the depth.
For an example see figure 9.2 (Delft bottle). The bottle acts as a sand trap so that most of the material is catched.

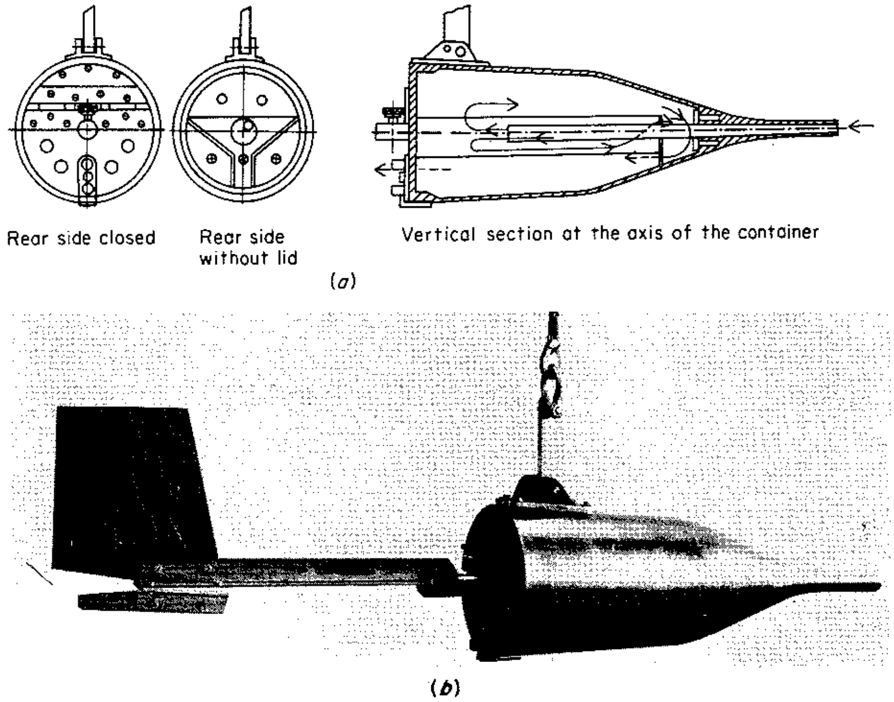


Fig. 9.2 The Delft bottle. (*a*) Schematic sketch, (*b*) suspended on a cable

3. Bed-material samplers

Numerous devices are described in literature to collect bed-samples. Several types may be distinguisted: grabs, corers etc.

9.3. Tracer techniques

For sediment transport measurements also tracers can be used. Grains are marked so that their transport characteristics are not changed, are added to the flow in small quantities and their displacements are determined. From their displacements the transport can be computed.

Several types of tracers are used:

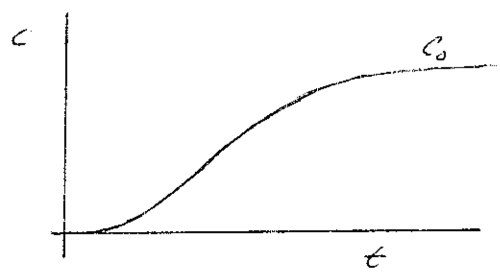
1. Fluorescent (luminofores). Marked grains can be detected after sampling under U.V. light. Different types can be used simultaneously.
2. Radio active. Natural sand is provided with a coating with radioactive material. Disadvantage: public health is important because relatively large quantities are necessary to remain above the back-ground level of radio-activity. Advantage: detection in situ.

3. Activation analysis. Particles are marked and radiated after sampling. Difficult procedure; applied for silt.

Several techniques for the interpretation are used:

1. Constant injection method. A constant amount of tracer material (rate τ) is distributed over the profile and injected during a long time-interval. At a downstream cross section samplers are collected and concentration as a function of time is determined. After some time the concentration becomes constant = C_o. Then the rate of transport can be computed from the relation:

$$s = \tau / C_o$$



2. Point-injection method. At a certain time an amount of tracer material is injected. At several downstream locations concentration is determined as a function of distance. From the displacement of the centre of gravity of the concentration-distance curves the average transport velocity can be computed. Multiplication with the effective depth of transportation δ gives the rate of transport. The effective depth is of the order of half the height of the bed forms and can also be determined by sampling in the bed. Both methods are relatively inaccurate.

Problems with these techniques is the length of the measuring interval, the fact that the external conditions have to be constant and the large number of observations. Some of these restrictions can be diminished by applying "dispersion methods". The data are compared with a theoretical dispersion model. (see De VRIES 1966).

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10. SEDIMENT TRANSPORT IN PIPES

10.1 Introduction

Sediment transport in pipes is of importance in the field of sewage transport and dredging. In both cases the purpose of the system is to transport solids without deposits at minimum head losses. Important aspects of this way of transport is the prediction of head losses and of minimum (or critical) velocities to avoid deposition.

In pipe transport several modes of transport can be distinguished:

1. for fine sediments and high velocities a pseudo-homogeneous suspension is formed. The criterion for a fully developed suspension was $U^*/W > 10$. For $\overline{U}/U^* \approx 20$ it follows that $\overline{U}/W > 200$. This means in practice ($\overline{U} = 3-4$ m/s) a value of $W < 2$ cm/s or $D < 150$ µm.
2. heterogeneous suspension. For smaller velocities and coarser material a heterogeous suspension is formed with a strongly non-uniform concentration distribution.
3. Sliding bed regime. For very coarse material all sediments will be transported sliding along the pipe wall. The criterion for beginning of suspension was $U^*/W > 1.5$. It follows then for $\overline{U}/W > 30$ or $W > 10$ cm/s, $D > 1$ mm.

Thus:

	$D < 150$ µm	pseudo-homogeneous suspension
150 µm <	$D < 1$ mm	heterogeous suspension
	$D > 1$ mm	sliding bed

In practice transition zones between the various regimes will be found.

Typical head-loss curves for pipe-line transport of sediments are given in figure 10.1. For large velocities losses approach the loss for clear water, but at the critical velocity where deposition starts head-losses strongly increase. The difference between head-losses for the mixture and clear water increase linearly with concentration.

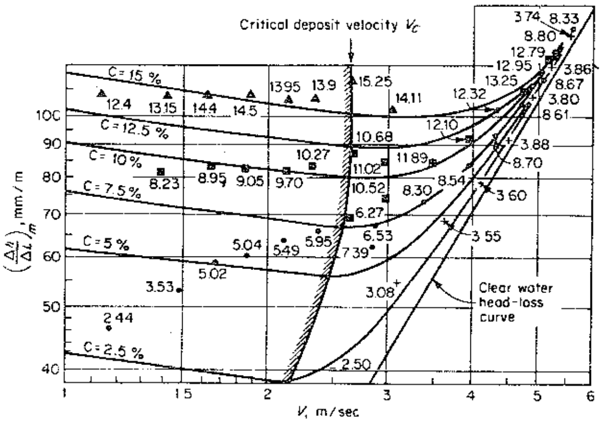

Fig.10.1 Head loss vs. velocity relationship with equiconcentration lines. for sand graded to 0.44 mm [*After* CONDOLIOS *et al* (*1963*)]

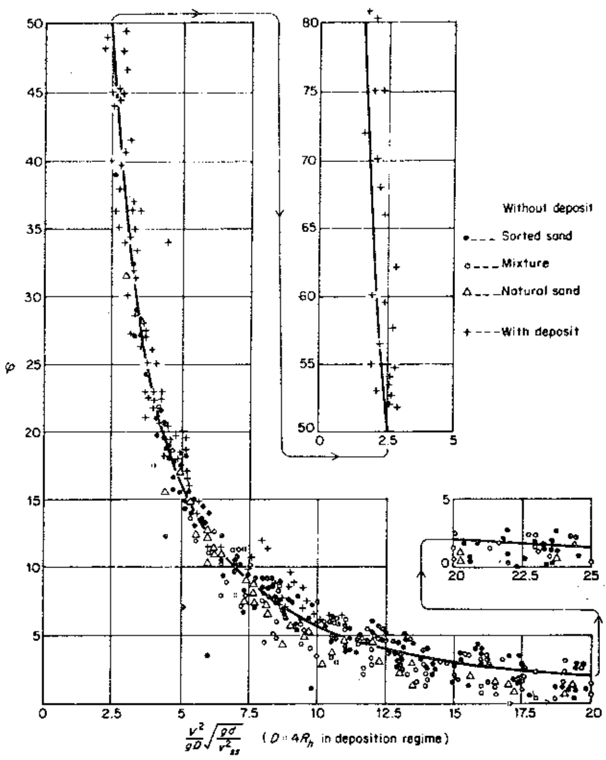

Fig. 10.2 Head loss as given by the Durand-Condolios relation for sand ($s_s = 2.65$); SOGREAH data are used. [*After* GIBERT (*1960*)]

10.2 Critical velocity

The critical velocity is defined as the velocity where stationary deposits are formed at the bottom of the pipe, or by deposition from the suspension or because part of the sliding material comes to rest. This condition is very critical for the operation of the pipe line because around the critical velocity head losses are at a minimum and a decrease in velocity will given an increase in head loss.

A large number of data and formulas are given in literature. The best known are the results by Durand (1953) (see figure 10.3 , where $F_L = V_{crit}/\sqrt{2\,g\,\Delta\,(2\,a)}$ and a = pipe radius).

From experiments it follows that the critical velocity U_{crit} increases with $(2\,a)^{\frac{1}{2}}$, slightly increases with transportconcentration C (up to volume concentrations of 15%) and increases with grain size D up to D $\approx$ 1 mm. The values given by Durand are somewhat pessimistic for fine sand in large diameters;

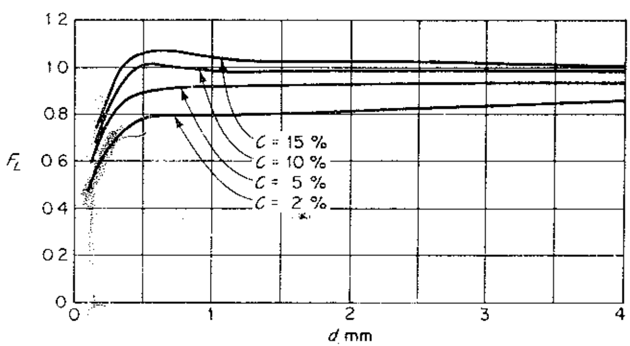

Fig 10.3 F_L value vs. particle diameter, concentration as parameter. [*After* DURAND *et al* (*1956*)]

here an increase with a smaller exponent (1/4 to 1/3 instead of 1/2) gives better results. For more information see the literature.

10.3 Head losses

Most (empirical) relations for the head losses in sand-water mixtures have the form:

$I_m = I_w + C \cdot f$ (u, 2 a , grain characteristics)

I_m = hydraulic gradient for the sand-water-mixture

I_w = hydraulic gradient for clear water

$$I_w = \frac{\lambda}{2a} \cdot \frac{U^2}{2g}$$

For λ smooth-pipe values can be used, because the sand polishes the surface of the pipe. For I_m various relations are proposed for example:

Durand (1953):

$$\frac{I_m - I_w}{C \cdot I_w} = 176 \left[\frac{g.2\,a}{U^2} \cdot \frac{W}{\sqrt{g\,D}} \right]^{3/2}$$

(see figure 10.2, $v_{ss} \Rightarrow W$ $D \Rightarrow 2\,a$ $d \Rightarrow D$ = grainsize)

Führböter (1961):

$$\frac{I_m - I_w}{C} = \frac{S_{kt}}{U_m}$$

in which S_{kt} is a parameter depending on grain size (see figure 10.4, d $\Rightarrow$ grain size, D $\Rightarrow$ pipe diameter)

From experience with dredge pipe lines it appeared that the relation given by Führböter is suited for fine sand and large pipe diameter and that Durand's equation is good for smaller pipe diameters and coarse sand.

For further information see the literature (par. 10.4.)

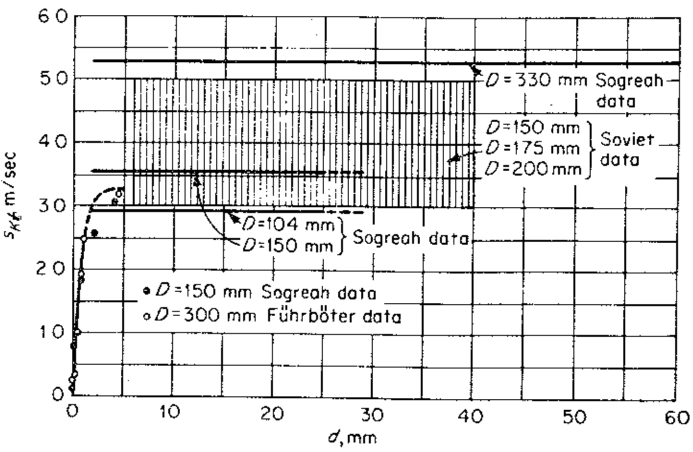

Fig.10.4 Relationship of the sediment constant s_k. [*After* FÜHRBÖTER (*1961*).]

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