february 1986

PUSHOVER ANALYSIS OF A FIXED STEEL OFFSHORE PLATFORM

part II

PILES AND PILE GROUPS UNDER EXTREME LOADS

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PUSHOVER ANALYSIS OF A FIXED STEEL OFFSHORE PLATFORM

part 1: BEAM-COLUMN AND STRUT ELEMENTS
IN THE INTRA PROGRAM

part 2: PILES AND PILEGROUPS UNDER EXTREME LOADS

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ACKNOWLEDGEMENT

More than a year ago, Jan Kerstens offered me the chance to participate in a research project at KSEPL to fulfill the work for my final thesis, and prof. van Douwen and prof. Blaauwendraad from TH Delft agreed to supervise the work. I would like to thank them for this special opportunity, and for their critical advice during the progress of the work.

The year I spent at KSEPL has been an extremely pleasant and instructive time. I want to thank especially Klaas van Ommen and Mike Efthymiou for their large contribution to this satisfying period.

The work I did for part one of this thesis could not have been performed without the great help of Marianne Hordijk who spent many days on unravelling the mysteries of the INTRA computer program.

With regard to part two, I am indebted to Gerald Schotman and especially to Cor Kenter who reacted with so much enthusiasm at my first steps in the field of offshore soil mechanics.

SUMMARY

As a preparation to pushover analysis of a complete fixed steel offshore structure, failure of the foundation has been analyzed in a qualitative way.

Structure-foundation interaction is considered and some methods are treated to describe load-deformation behaviour of piles loaded up to failure. Main attention is paid to methods in conjunction with the computer program INTRA.

Computational rules to achieve load settlement curves for pile groups under extreme loads have been derived with the aid of modified t-z curves.

Alternatives are presented to model the foundation in both extensive and more simplified ways.

CONTENTS

NOMENCLATURE

a	constant	-
A	area	$[L^2]$
c_u	undrained shear strength	$[F/L^2]$
C_1	constant	-
C_2	constant	-
d	pile diameter	[L]
D	group centre diameter	[L]
D'	group outer diameter	[L]
E	Youngs modulus	$[F/L^2]$
F	force	[F]
G	shear modulus	$[F/L^2]$
k	Darcy constant	[L/T]
M	bending moment	[FL]
M_{pl}	plastic moment	[FL]
n	number of piles in a group	-
n_{crit}	critical number of piles	-
n^*	number of piles in a filled up circular group	-
p	lateral pressure per unit pile length	[F/L]
p_{max}	maximum lateral pressure per unit pile length	[F/L]
q	point pressure	$[F/L^2]$
q_{max}	unit bearing capacity	$[F/L^2]$
Q	ultimate bearing capacity	[F]
Q	charge per unit well length	$[L^2]$
r	coordinate	-
r^*	radius (fixed value)	[L]
r_o	pile radius	[L]
r_m	"magical" radius	[L]
R	group centre radius	[L]
R'	group outer radius	[L]
s	centre spacing	[L]
S_{ij}	coefficient stiffness matrix	-

t	shaft friction per unit pile length	[F/L]
t_{max}	soil shear strength times pile diameter	[F/L]
t_{res}	residual soil shear strength times pile diameter	[F/L]
T	moment	[FL]
v	superficial velocity	[L/T]
x	coordinate	-
y	coordinate	-
y	lateral displacement	[L]
y_{max}	displacement according to p_{max}	[L]
z	coordinate	-
z	axial displacement	[L]
z_0	depth of plastic area	[L]
z_{max}	displacement according to t_{max}, τ_{max} or q_{max}	[L]
z_{res}	displacement according to t_{res}	[L]

α	reduction factor	-
α_{ij}	interaction factor	-
γ	shear	-
δ	displacement	[L]
η	efficiency	-
θ	angle	-
λ	constant	-
υ	Poissons ratio	-
ξ	settlement ratio	-
ρ	ratio of G modulus at depth 1/2 and at pile tip	-
τ	shear stress	[F/L^2]
τ_0	unit shaft friction	[F/L^2]
τ_{corr}	corrected soil shear strength	[F/L^2]
τ_{max}	soil shear strength	[F/L^2]
τ_{red}	reduced soil shear strength	[F/L^2]
ϕ	rotation	-
ϕ	ground water head	[L]
ϕ	friction angle	-
ψ	constant	[L]

subscripts:

a	annulus
eq	equivalent
g	group
i	initial
p	pile
p	point
s	soil
sp	single pile
u	unit

1. INTRODUCTION

Schematizations of piles and pile groups as required for structural analysis of offshore platforms can be realized in several ways: Refined computer methods represent each pile and the surrounding soil with the aid of numerous 'finite elements'. Simple techniques replace each pile or pile group by a set of 'equivalent' springs. This report pays main attention to methods of foundation modelling as required for pushover analysis using the computer program INTRA. This program, originally developped for earthquake analysis, requires modelling of the piles with the aid of beam-column elements, representing the soil with the aid of nonlinear springs.

This report starts with a general description of structure-pile-soil interaction in chapter 2.

Chapter 3 describes some existing techniques to generate the characteristics of springs that represent the soil around a single pile.

Chapter 4 gives a short review of methods to describe pile group behavior. It appears that no simple method exists to generate the spring characteristics that represent the soil around pile groups loaded upto failure.

Because of this reason some computational rules are developped in chapter 5 to generate NOnlinear Group Affected (NOGA) spring characteristics.

Finally, chapter 6 treats some different ways to schematize the foundation in order to minimize the number of pile elements and soil springs.

2. STRUCTURE-PILE-SOIL INTERACTION

Traditional jacket piled foundation consists of single piles driven through the legs of the jacket. Piles and superstructure are connected above the waterline, so the deck-loads are directly transmitted to the piles. The wave forces are transmitted to the piles by the jacket space frame. More recently designed platforms are supported by groups of piles arranged around the corner legs. All loads are transmitted by the superstructure to the piles through grouted connections just above the sea bottom, see figure 1.

Waves wind and current result in a horizontal force at the platform and in an overturning moment, see figure 2. The overturning moment is mainly transferred through axial pile forces, in a so-called push-pull mode, see figure 2. Actual tension may occur. The overall horizontal force is mainly transmitted as lateral loads to the pile heads. Important bending moments are introduced in the piles by imposed lateral displacements from the superstructure. Similarly, rotation of the jacket round a vertical axis introduces lateral forces and bending moments in the piles.

Consider the foundation of one leg detached from the superstructure: The relationship between forces and deformations at the pile head is expressed with the help of a stiffness matrix. In the case of a single pile, choosing a coordinate system as in figure 3, the stiffness matrix may be written as:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} S_{11} & 0 & 0 & 0 & S_{15} & 0 \\ 0 & S_{22} & 0 & S_{24} & 0 & 0 \\ 0 & 0 & S_{33} & 0 & 0 & 0 \\ 0 & S_{42} & 0 & S_{44} & 0 & 0 \\ S_{51} & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \phi_x \\ \phi_y \\ \phi_z \end{bmatrix}$$

The coefficients S_{ij} are constants if soil and pile behave linear elastic. However, their value is dependent at the momentary state of forces and displacements in the actual case of nonlinear behaviour. Dependency occurs between lateral displacement and rotation because an imposed lateral force will not only produce lateral displacement but also rotation of the pile head. The off-diagonal terms describing this dependency are symmetric in the case of linear elastic behavior. An asymmetric matrix may occur in the nonlinear elasto-plastic situation where Maxwell's law is not valid anymore.

Consider a group of piles. The rotational resistance of the group is much larger then of a single pile because the piles, acting together, can take up bending moments in a push-pull mode, see figure 4. Similarly, torsional resistance increases. Besides, the stiffness matrix is more dense because extra dependencies occur. For example, a lateral load applied at a pile group with a non-symmetric configuration causes axial displacements and axial torsion. And an additional dependency exists in each pile group, because a moment at the group-head introduces an axial displacement of the entire group, due to the nonlinear load-displacement relationship from axially loaded piles. Note that the latter effect is caused by material properties of the soil, while the former is a result of geometry of the group.

Another form of geometric interaction may play a role for both single piles and groups. Axial forces in a pile will increase the lateral displacements and the bending moments, resulting in a geometric nonlinear effect.

Axially overloaded offshore piles collapse due to failure of the soil supporting the pile along shaft and tip. If an increasing lateral load is applied to a pile, a zone of soil loaded up to failure will extend gradually towards the tip. However collapse is a result of failure - plastic bending - of the pile itself. Rotation of the entire pile through the ground as may happen with short onshore piles does not happen with offshore piles due to their larger length.
Interaction now occurs because the development of a plastic hinge is influenced by the normal force in the pile.

Interactions also play a role in the soil itself. This phenomenon will be treated in the next chapters.

Several authors (ref.[15],[34] and [39]) describe some of the mentioned interaction effects but no indication is given about the relative importance of those effects. However, it should be clear that overloading of a pile group may result in a complicated pattern of collapsing piles, especially when ultimate axial and lateral capacities are reached at about the same time.

3. SINGLE PILES

This chapter treats some soil-mechanics aspects of single offshore piles, especially those aspects that are useful for foundation modelling with INTRA. Sometimes it will anticipate on the next chapter, that treats pile groups. Only static loading will be considered.

Establishment of the axial load-displacement behavior of piles usually consists of two parts:

- Determination of the ultimate bearing capacity (strength); traditionally the most important quantity.
- Determination of displacements going with gradual loading (stiffness). This results in a pile head load-settlement (F_z - δ_z) curve.

The bearing capacity of a conventional closed ended foundation pile consists of two contributions: shaft bearing and end bearing. However, the bearing capacity of an offshore pile - an open ended steel tube - can be developped in two different ways, either in a 'plugged' or in a 'non-plugged' situation, see figure 5. If the inner shaft friction exceeds the end bearing, the ultimate capacity is calculated from outer shaft friction plus end bearing. On the contrary, if end bearing exceeds the inner shaft friction, the ultimate capacity consists of the sum of inner and outer friction. A large number of methods exists to calculate those quantities, see e.g. ref.[37].

The above determination of axial capacity suggests that the maximum values of skin friction and end bearing will be reached at the same moment. However, in reality the ultimate skin friction is reached at a much smaller displacement than the ultimate end bearing. In addition the ultimate skin friction itself is not mobilized at the same moment at each point along the shaft, because of difference in displacements caused by compressibility of the pile. To predict those effects the load-displacement characteristics of the soil along the shaft and under the tip should be known.
To that purpose several methods exist, as will be described in the next paragraphs.

Poulos and Davis (ref.[29]) present a solution based on theory of elasticity. The incompressible pile is divided into segments, and at each boundary the vertical displacements are defined by means of "Mindlins equation" - an expression for displacements of a point in an infinite half

space. The pile head settlement is established with the aid of a numerical technique based on this schematization. In ref.[30] Poulos and Davis extend the theory by including the compressibility of the piles. This elastic method will be useful for pile groups as will be shown in the next chapter. However, a more simple elastic schematization for single piles is developped by Randolph and Wroth, see also appendix A.

Randolph and Wroth (ref.[32]) state that under axial loads the deformations of the soil along the shaft mainly consist of shear. The soil can be represented as horizontal layers with different G modulus. Starting from this assumption a simple expression can be derived for the settlement in a soil layer around a pile, z, as a function of the shaft friction per unit pile length, t: A so called t-z curve. Besides, an ultimate shear strength can be described for each soil layer, and in this way bilinear elasto-plastic t-z curves can be established, see figure 6a. In reality the soil around a pile does not collapse suddenly, but gradually looses its stiffness under increased loading. A better description of this actual soil behaviour can be found in the following:

Kraft, Ray and Kagawa (ref.[14]) present a formulation starting from a nonlinear stress dependent G-modulus. Figure 6b shows a typical shape of the resulting nonlinear t-z curve. Up to point A the theory of Kraft et al. is valid. After this point "softening" of the soil may occur. Only little knowledge is available about this post failure behaviour. For offshore piles z_{max} is of the order of 0.005 to 0.02 times the pile diameter. The value of z_{res} may be 1 to 5 times as much, while t_{res} may be 70 % to 100 % of t_{max}.

To evaluate the settlement under axial loads not only t-z curves must be known, but also the local load settlement relationship under the tip: the so-called q-z curve. The displacement z_{max} according to the maximum tip load can be estimated with the aid of simple methods, see appendix B.

Several special nonlinear computer programs for pile foundations exist that use t-z and q-z curves as input, and also with INTRA a pile can be modelled in this way.

The analysis of piles under lateral loading is usually based on the work of Matlock, (ref.[1] and [18]). It is supposed that horizontal soil layers act independently at the pile and that they can be represented by springs. The nonlinear spring coefficients are derived with the help of formulas based on experiments combined with analytical methods. The relationship

between lateral soil pressure at the pile, p, and displacement, y, is represented by a so-called p-y curve. Again special foundation programs exist that use those curves as input. Similarly INTRA can calculate laterally loaded piles. Standard formulas are available to define modified "cyclic" p-y curves, based on cyclic loading experiments of Matlock, see figure 7.

Interactions between vertical and horizontal stresses in the soil are not included. Little knowledge is available on this subject, and the effect is not mentioned in the references dealing with pile group behaviour. It is likely that cyclic loading has a dominant influence on axial and lateral capacity. Remoulding of the soil may drastically reduce its ultimate strength, while lateral movements reduce the horizontal soil pressure or even may cause "gapping" between pile and soil. As mentioned before those effects will not be considered.

To obtain some understanding of the failure mechanism of piles and to obtain a feel for the importance of the parameters involved, some INTRA analyses have been performed for a 1.22 m diameter pile. The soil profile consisted of medium dense sand with a thin layer of medium stiff clay, see figure 8. Ultimate skin and tip resistance were evaluated according to both API rules and t-z curves based on the work of Kraft et al; the p-y data were readily available. Figure 10 shows the results of some alternative computational methods for t-z curves. It should be clear that the shape of a t-z curve is not uniquely defined. Figure 11a shows the pile head F_z-δ_z curve resulting from the INTRA analysis. If the pile is rather stiff in axial direction and if the value of the quake z_{max} does not change strongly along the shaft it appears that the shape of the F_z- δ_z curve is strongly influenced by those of the local t-z curves, see Tideman (ref.[37]). Actual piles are very stiff indeed and actual quake seems to be a rather constant value for a certain type of soil, see Verruijt (ref.[41]). Consequently, failure of the soil along the shaft can be observed almost at the same time at top and toe, which is the reason for the mentioned influence. This means that if local soil failure is abrupt also the axial collapse of the pile will be abrupt.

Quite a different case is the development of the end bearing. Figure 11b shows that failure of the soil along the shaft occurs at a displacement of ± 15 mm. End bearing is hardly of any importance at that moment. Although

different methods to the establishment of the q-z curves show a substantial scatter (figure 12) it is clear that the maximum end bearing is mobilized at a displacement of ± 10 times the mentioned value for the shaft friction. Consequently end bearing is often neglected for calculation of the bearing capacity under working loads. However, it may play an important role for loads up to failure. Although the unit shaft friction will reduce to t_{res} under extreme loads, the total axial resistance may still increase to almost two times the value at "first failure".

A totally different picture will exist when an offshore pile is loaded up to failure in lateral direction: An area of yielding soil expands from the top downwards until the pile collapses because of plastic bending of the steel tube. Verruijt (ref.[41]) mentions the important influence at this behaviour of the p_{max} values in the p-y curves (figure 7). To the contrary the y_{max} values are of very little influence. This is clearly illustrated in figure 13, which shows the effect of doubled and halved values of p_{max} and y_{max} at the shape of the pile head $F_y - \delta_y$ curve. In addition figure 14 shows the obvious change in shape of the pile head $F_z - \delta_z$ curve to changes in the local t-z values, an effect which is already mentioned. A possible reason for the insensitivity of the lateral pile head load deflection curves to changes in y_{max} is the relative bending flexibility of the pile, see appendix G.

4. PILE GROUPS

This chapter considers the interaction between offshore piles placed in a group. As in the previous chapter it will concentrate on methods which can be used in conjunction with the INTRA program. Because little literature is available on groups loaded up to failure it was felt necessary to establish some simple computational rules, which is the subject of the next chapter.

O' Neill (ref.[22]) gives an extensive review on pile group behaviour. He distinguishes two kinds of interaction effects:

- Installation effects: During pile driving the soil around a pile is pushed aside, and a substantial increase in density of the soil between the piles may occur, which may result in a higher shear strength and a higher shear modulus.
- Loading effects: Piles in a group "work together" because the interjacent soil transmits forces from one pile to the others. This "pile-soil-pile" interaction may result in a bearing capacity of the group lower than the sum of the individual values, while displacements may be drastically larger.

Installation effects for a group are very difficult to predict, and therefore presumably no computational methods exist that take this influence into account. The techniques described hereafter only consider the loading effects.

The group effect on the ultimate capacity is defined as efficiency η ,

$$\eta = \frac{\text{(ultimate load for the group)}}{\text{(ult.load single pile) * (number of piles)}}$$

To predict this effect for axial loads, Terzaghi and Peck (ref.[36]) suggest the "equivalent pier concept" which assumes that the group behaves as one solid unit. De Ruiter and Beringen (ref.[34]) give a variant for offshore pile groups. In both cases the ultimate axial capacity is the lowest of two values: Either the sum of the capacities of the individual piles ($\eta = 1$), or the capacity of the equivalent pier ($\eta < 1$), see appendix C. Actual efficiency may exceed unity ($\eta > 1$) because of installation effects, but as already mentioned no methods exist that include those effects, see Matlock et al., ref.[19] and O'Neill, ref.[22] and [23].

The efficiency for laterally loaded pile groups at failure is equal to one or even more: The strength of a steel pile is not influenced by the surrounding group, whereas the group can even take up an extra moment because of the push-pull working of the piles.

Settlements of pile groups are conveniently expressed in terms of the settlement ratio ξ, where

$$\xi = \frac{\text{group settlement}}{\text{settlement of a single pile under } \underline{\text{average}} \text{ group pile load}}$$

A similar expression exists for lateral deflections.
Generally, a pile in a group shows a weaker load-deformation behavior than a single pile under the same load, both axial and lateral, ($\xi > 1$).
The most simple method to predict the settlement of pile groups is again the equivalent pier concept. Although simple it can take in to account nonlinear behaviour.

Poulos (ref.[26] and [30]) proposes a method that establishes pile-soil-pile interaction based on the theory of elasticity, see also appendix D. As for single piles (chapter 4) the settlement of a pile in a group has been evaluated from the 'Mindlin' expressions for the settlement of points along the shaft, see ref.[26]. Such a calculation is too complicated to serve as an engineering tool, and therefore Poulos presents his results in a more usable way: The amplification of the settlement of a pile due to the influences of the neighbouring piles is represented with the aid of so called interaction factors α, see appendix D. Poulos prepared charts and formulas to compute values of α for different soil stiffness, pile geometries and spacings, (ref.[30] and [31]). However these factors obscure several effects like local scatter in elasticity modulus of the soil, or the influence of end bearing.
Reference [27] presents a comparable way of evaluation of group action for laterally loaded piles. For this case results will be of an even more approximate nature due to the complex rotational and lateral behaviour of groups, see appendix D.

Focht and Koch (ref.[8]) suggest to combine the use of elastic interaction factors with nonlinear elasto-plastic single pile solutions. To justify this hybrid approach they assume that because of the distance

between neighbouring piles only elastic interaction occurs. Although this seems a questionable statement and superposition is applied illegitimate, acceptable results are reported, see O'Neill (ref.[24]). At present this "hybrid" interaction factor method is probably the most widely used technique to analyse offshore pile groups.

The above interaction factor methods use Mindlins equation in an indirect way because of the computational effort. Indeed extensive computer programs exist that perform this kind of calculations directly, i.e. they apply Mindlins equation at several points along the shaft of each pile (see ref.[6]).

However, direct computation of displacements along the shaft of a single pile can be performed comparatively easy by means of t-z, q-z and p-y curves, see chapter 4. This approach is also useful to describe group behaviour. The problem is how to modify the curves to take the pile-soil-pile interaction into account.
The use of such "group affected" curves can give several advantages, especially if they are established using nonlinear analyses:
- Avoidance of the use of the rather inaccurate, elastic interaction factors.
- No hybrid linear / nonlinear calculations.
- The possibility of analysing complex configurations under combined axial and lateral extreme loads.

Randolph and Wroth (ref.[33]) present a description of bilinear group affected t-z curves, which is a logic extension of their single pile theory, based on superposition, see appendix A. They give similar rules for q-z curves, also for a bilinear elasto-plastic soil behaviour, see appendix B. Bilinear group affected p-y curves are derived by Hariharan and Kumarasamy, in accordance with the single pile approach of Matlock, see ref.[9].
All those superposition techniques use a constant Youngs or shear modulus, whereas in reality a gradual loss of stiffness of the soil can be observed during increased loading. Because of this simplification the above techniques do not give sufficiently accurate results for displacements near ultimate loading. Ref.[74] suggests to solve this problem with the help of an incremental procedure applying a subsequent updating of the stiffness parameters.

However, an alternative approach is possible, making use of an additional datum: It seems reasonable to assume that a totally "filled up" circular group behaves as one equivalent pile, see figure 17. The equivalent pile diameter may be somewhere between D and D'. Furthermore it seems likely that the displacements of an actual, not totally filled up, circular group lay somewhere between the following two extremes: they will exceed those of a single pile, but they will be smaller than those of the equivalent pile. The two extremes can be easily evaluated with the aid of the nonlinear t-z, and p-y techniques described in the previous chapter.

Bogard and Matlock (ref.[2]) start from this idea to compute lateral deflections. Eventually they present a somewhat misty computational trick, see chapter 5, but they report good agreement with test results.

A more straightforward way of interpolation is possible and yields tractable formulas. Chapter 5 extensively describes the derivation of those NOnlinear Group Affected (NOGA) curves.

Figure 15 and 16 show the results of some INTRA computations performed at a 6 pile test group. The data of the individual piles are equal to those of the single test pile from chapter 4, (figure 8). The group has a diameter D = 6,10 m, hence the piles have a centre spacing of 3.05 m = 2.5 d. Curve I in figure 15 depicts the single pile solution achieved from bilinear Randolph and Wroth (ref.[32]) t-z curves, and curve II depicts the single pile settlement derived with the aid of a chart of Poulos (ref.[30]). Curve III in Figure 15 shows the result for a pile in a group under the same load as the single pile, derived with the Poulos interaction factor method. It appears that the group settlement is almost twice the single pile value, i.e. the settlement ratio ξ is about equal to two. An alternative way of establishing interaction factors (ref.[31]) results in a settlement ratio $\xi \simeq 1.5$ and consequently a steeper curve is obtained, see curve IV. Superposition according to Randolph and Wroth (ref.[33]) also yields a somewhat steeper curve, see curve V, and the same applies for the use of "group affected" t-z curves (chapter 5) resulting in curve VI.
In Figure 16 the single pile curve is achieved with the help of nonlinear t-z curves from Kraft et al. (ref.[14]). Use of interaction factors now results in "hybrid" calculation of curves VII and VIII, with $\xi = 2$ and $\xi = 1.5$ respectively (ref.[8]). Superposition is not allowed because of the

nonlinear nature of the t-z curves. NOnlinear Group Affected (NOGA) curves (chapter 5) result in curve IX, which shows an apparently weaker behavior in the upper region. Indeed, methods to predict group behaviour are usually valid for loads up to half the failure load.

Note that in this example:
- Soil is fairly homogenous
- Installation effects are neglected
- Point bearing is not taken into account

Several authors on group behaviour are not mentioned in this chapter because their work can not directly be applied to collapse analyses with INTRA. Frequently quoted are the following authors.
Butterfield and Banerjee (ref.[5]) present a refined version of the elastic Mindlin approach.
Meyerhof (ref.[21]) did a lot of research on eccentric loaded pile groups, much shorter than offshore piles.
In a recent article Nogami (ref.[25]) presents a simple elastic computational method that may be useful to predict settlements under working loads.

5. NOGA CURVES

NOnlinear Group Affected curves are variants on single pile t-z and p-y curves. If the latter describe the load-deformation behavior of the soil near a single pile, NOGA curves do the same for the soil around a pile in a group.
The following shows the derivation for circular (offshore) groups:

The sequence of thoughts is as follows: As stated in the previous chapter it seems reasonable to assume that deformations from a group lay somewhere between two extremes: they will exceed those of a single pile, but they will be smaller than those of a totally "filled up" circle, see figure 17. The values for the two extremes can easily be established with the aid of the usual single pile t-z or p-y techniques. To this purpose the filled up group is treated as one equivalent pile with a diameter equal to the group diameter. Intermediate values for an actual, not completely filled up group, can be achieved by an interpolation between the two extremes.
The next paragraphs treat the derivation of a suitable interpolation rule for axially loaded groups. Thereafter some attention will be paid to laterally loaded groups.

5.1 NOGA t-z curves

A brief summary of this paragraph reads as follows:

a) To achieve an interpolation formula for axially loaded groups, it is not sufficient to consider only the two extremes (single pile and filled up group) because not-completely filled up circles may also behave as one equivalent pile. Therefore it will be necessary to determine the "critical" number of piles, n_{crit}, that separates groups with individual behaving piles from groups with piles acting together.

b) Once this n_{crit} has been achieved an interpolation rule for an n-pile group ($1 < n < n_{crit}$) can be derived, and with this rule "group affected" t-z curves can be constructed both for bilinear and for more complicated nonlinear representations of the soil.

c) A difficulty occurs when the soil immediately near the pile shaft has a strength different from the soil at a distance from the pile. However, an approximate value of n_{crit} can be established in this case.

d) Subsequently the t-z curves obtained by interpolation will be compared with the results from a different computational method.

e) Finally some conclusions will be presented.

a) critical numbers

Consider a widely spaced circular pile group of n piles, see Figure 18. The efficiency η can be calculated with the aid of efficiency formulas based on failure mechanisms, see appendix C. According to those formulas the bearing capacity of the group is the lowest of two values, either the sum of the individual capacities, or the capacity of an equivalent pier with a diameter equal to the group diameter. This assumption results in a sharply defined critical number of piles. From this n_{crit}, the piles behave as a solid unit and their displacement can be predicted with an equivalent pier approach. However it is doubtful whether such an n_{crit} actually occurs.

To obtain an answer to this question, first consider the settlement around a single pile computed with the linear τ-z formula from Randolph and Wroth (ref.[32]), see appendix A:

$$z(r) = \frac{\tau_o \, r_o}{G} \ln \left(\frac{r_m}{r}\right) \qquad (1)$$

z = settlement

r = radius (coordinate)

r_o = pile radius

τ_o = shear stress at shaft, $r = r_o$

G = shear modulus

r_m = "magical" radius at which shear stress is negligible

(Instead of shaft friction per unit pile length t in kN/m, as applied in t-z curves, the formula uses unit shaft friction τ_o in kN/m^2 equal to t divided by the pile circumference.)

Figure 20 shows the resulting logarithmic lines that represent the settlement around a single pile. Figure 23b shows the settlement around two identically loaded piles according to the superposition variant of (1), (ref.[33]):

$$z(r_1,r_2) = \frac{\tau_o\, r_o}{G} \{ \ln \frac{r_m}{r_1} + \frac{r_m}{r_2} \} \qquad (2)$$

r_1, r_2 = distances from pile centres

It is clear that in between the piles the slope of the settlement line, $\frac{\partial z}{\partial r}$, is more flat than at the outside. This means, while $\frac{\partial z}{\partial r} = \tau$, that shear stresses tend to concentrate at the outside.
(Formula (2) abusively suggests a constant τ_o around the circumference of each pile. However one can write $\tau_o r_o = \frac{t}{2\pi}$, thus avoiding the abusive character without violating the superposition result.)
Appendix E treats an analogy between the settlement around a pile and the groundwater head around a sink in a confined aquifer. Figure 22 shows a flow net of a group of sinks. The net of squares consists of stream lines and potential lines. Considering the analogue situation of a group of axially loaded piles, the lines represent stress trajectories and lines of equal settlements respectively.
In fact the relatively very high shear moduli of the piles disturb this pattern: settlements at the circumference of a pile are almost equal. However this does not change the tendency that shear stresses concentrate at the outside of the pile, and that the major part of the forces is transmitted in that area, (little distance between the "stream lines").
For an increasing load plastic deformations will develop first at spots with the highest stress concentrations. A further increase in loading makes the 'stream line' pattern change, and new plastic deformations will occur at those places where the distance between the lines becomes too small. Subsequently, plastic zones will extend until they meet and form either an annulus around each pile, or one circular area around the whole group. Consequently, indeed a critical number of piles exists that separates individual from block failure. Theory of plasticity learns that such a failure mechanism will take place that yields the lowest collapse load.

Returning to efficiency formulas, consider the widely spaced pile group of n piles again, (Figure 18). Terzaghi and Peck (ref.[36]) compute η with an equivalent pier expression, see also appendix C. It reads (considering only shaft friction):

$$\eta = 1 \quad \text{if} \quad n\, 2\, \pi\, r_o\, \tau_{max} < 2\, \pi\, R'\, \tau_{max}$$
$$\eta < 1 \quad \text{if} \quad n\, 2\, \pi\, r_o\, \tau_{max} > 2\, \pi\, R'\, \tau_{max} \qquad (3)$$

τ_{max} = soil shear strength
r_o = single pile radius
R' = outer radius of the group, (see Figure 18 and Figure 19a)

For n_{crit} follows:

$$n_{crit} = \frac{R'}{r_o} \qquad (4)$$

Note that n_{crit} is not necessarily an integer.
For groups with $n > n_{crit}$ the linear t-z or τ-z curves can be estimated with the aid of formula (1), using $r = R_{eq}$, where R_{eq} = radius of the group. Some questions may arise about R_{eq}, which may probably lay somewhere between R and R', see figure 18. In the following it is assumed that $R_{eq} = R'$.

b) Interpolation

In order to illustrate the derivation of the interpolation rule, a soil layer around an arbitrary test pile group is considered. Figure 23 shows the soil and group data.
Expression (4) gives n_{crit}, and with the aid of formula (1) the bilinear τ-z curves of the single pile and the equivalent pile are established for an ideal elasto-plastic soil behaviour, see figure 25. The demanded group affected τ-z curves will lay somewhere in between those two extremes. Note that only the maximum settlements of the piles differ, whereas the maximum shear stresses around the pile remain unchanged, i.e. remain equal to the soil shear strength.
Figure 24 shows the calculated maximum elastic settlements, z_{max} as a function of the number of piles in the group. Point A represents the

settlement for n = 1, i.e. the single pile settlement z_{si}. Defining n^* as the number of piles in a completely filled up circular group, the line between the points B and C represents the settlement for $n = n_{crit}$ upto $n = n^*$, i.e. the equivalent pile settlement z_{eq}. A linear interpolation between A and B yields:

$$z = z_{si} + \frac{n - 1}{n_{crit} - 1} * (z_{eq} - z_{si}) \qquad (6)$$

With the aid of this formula the bilinear group affected τ-z curves from figure 26a are derived for the above mentioned soil layer around an arbitrary test group. Figure 26b shows t-z curves for the same configurations; the values of t now are divided by the number of piles relating to the curve. Both in figure 26a and in figure 26b the efficiency decrease can be recognized for groups with more than the critical number of piles.

Finally, NOnlinear Group Affected (NOGA) τ-z curves can be established in the same fashion as the above derived bilinear curves. If the "extreme" τ-z curves from the single and equivalent piles are established in a nonlinear way, e.g. with Kraft et al. (ref.[14]), instead of bilinear with (1), the same interpolation formula (5) yields the z-values of the NOGA curves in between the nonlinear extreme τ-z curves, see figure 31.

c) Reduced soil strength

A volume of strongly inhomogenous soil around the piles in a pile group may result from pile driving, due to remoulding or compression.
In that case the efficiency formula of de Ruiter and Beringen (ref.[40]) may be useful (figure 19b), see also appendix C. It is assumed that the shear strength of the disturbed soil can be expressed as a constant fraction of the shear strength of the undisturbed soil at some distance from the pile:

$$\tau_{red} = \alpha \, \tau_{max}$$

τ_{red} = reduced soil shear strength
τ_{max} = soil shear strength
α = reduction factor

The efficiency formula now reads:

$$\eta = 1 \quad \text{if} \quad n\ 2\ \pi\ r_o\ \tau_{max} < n\ \pi\ r_o\ \tau_{red} + (2\ \pi\ R - n\ 2\ r_o)\ \tau_{max}$$

$$\eta < 1 \quad \text{if} \quad n\ 2\ \pi\ r_o\ \tau_{max} > n\ \pi\ r_o\ \tau_{red} + (2\ \pi\ R - n\ 2\ r_o)\ \tau_{max} \qquad (7)$$

Which yields:

$$n_{crit} = \frac{2\ \pi\ R}{a\ \pi\ r_o + 2\ r_o} \qquad (8)$$

More refined failure mechanisms may result in slightly lower values of n_{crit}, but this refinement would introduce only seeming accuracy.

d) Comparision with superposition

In the above section b) the derivation of NOGA curves was introduced with the description of an interpolation rule for bilinear τ-z curves. The results of this interpolation rule can be compared with the results of the superposition formula from Randolph and Wroth for bilinear τ-z curves. The formula reads for n piles:

$$z = \frac{\tau_o\ r_o}{G}\ \ln \left\{ \frac{r_m^{\,n}}{r_o \cdot \prod_{i=1}^{n-1} s_i} \right\} \qquad (8)$$

s_i = centre spacing between pile 1 and pile i+1

Figure 27 presents values of z established with (8), using $\tau_o = \tau_{max}$. The values are calculated for several numbers of piles in the above mentioned arbitrary test group. In the same figure the interpolation results of figure 24 are repeated. Note that the points derived with (8) are not necessarily on a straight line.

It appears that the interpolation results agree reasonably well with the superposition values up to n_{crit}. From n_{crit} up to n^* the results essentially differ because the superposition does not automatically reckon with efficiency decrease.

e) Conclusions

- A t-z curve for a pile in a group can be obtained by interpolation between single pile and equivalent pile t-z curves
- For each circular pile group a critical number of piles, n_{crit}, can be defined. If the group contains less piles than n_{crit}, the piles individually reach their ultimate capacity. If the group contains more piles, the piles act together in such a way that block failure occurs.
- For groups with less piles than n_{crit}, group affected t-z curves obtained by interpolation, starting from (bi)linear soil behaviour, show a good agreement with t-z curves derived by means of superposition according to Randolph and Wroth. For groups with more piles than n_{crit}, t-z curves can be obtained with an equivalent pile approach; superposition now gives incorrect results.
- Group affected t-z curves, obtained by interpolation, starting from nonlinear soil behaviour, (NOGA curves) are not necessarily well predicted. However their values do not exceed those of the extreme situations (i.e. single pile behavior and equivalent pile behavior).
- NOGA curves can be derived in a simple but consistent way, avoiding hybrid combinations of elastic and elasto-plastic theories as appear in existing analysis methods for nonlinear pile group behavior.

5.2 NOGA q-z curves

Ultimate point bearing capacity Q_p is usually defined as:

$$Q_p = A_p \, q_{max}$$

A_p = gross end area (plugged pile)
q_{max} = unit bearing capacity

This expression seems to indicate that block failure of a group will never occur, but that the piles punch individually.
However equivalent pile behaviour as described for side friction contradicts this statement.

It must be emphasized that the description of single pile q-z behaviour is subject to a large number of uncertainties.
For the moment no further attempt is made to derive NOGA q-z curves.

5.3 NOGA p-y curves

Bogard and Matlock (ref.[2]) derive NOGA p-y curves starting from the usual Matlock p-y method.
To a single pile p-y curve an influence part is added. The latter is derived from the p-y curve of an equivalent pile. The p-values are divided by the number of piles in the group, n. The y-values are divided by the centre spacing of the piles in the group, s. Adding the two components of deflection y at each value of soil pressure p yields the eventual NOGA curve, see figure 32.
Apart from this they state that the maximum value of p is given by the highest value of either the single pile or the equivalent pile.

As mentioned in chapter 5, y-values are of little importance to pile head load-deflection behaviour, compared with the p-values.
This suggests that these values can be derived in a more simple way, e.g. by linear interpolation between single pile and equivalent pile y-values. This would result in a correct description of the two limiting values which is not the case for the Bogard and Matlock method.

6. SCHEMATIZATION

The standard schematization of piled foundations in the computer program INTRA, as used in earthquake analysis, consists of beam-column elements representing the piles, and of nonlinear springs representing the surrounding soil, see figure 33.

The following treats several alternatives to schematize the foundation, in order to minimize the number of pile elements and soil springs.

A description of the available INTRA spring elements can be found in appendix F.

A standard extensive modeling of a single pile is shown in figure 33. (For simplicity a plane frame is considered.) The soil is schematized by horizontal layers, acting independently at the pile, represented by axial and lateral nonlinear springs. The pile is modeled by beam-column elements, which can simulate both material and geometrical nonlinearity, see appendix F.

The length of the pile elements, i.e. the thickness of the soil layers, depends on factors such as homogenity of the soil and required precision of the analysis. Lateral and axial behaviour ask for a different division of the pile in elements, because lateral displacements are almost completely governed by the soil around the upper part of the pile, while axial displacements depend on the properties of the soil along the entire pile. Consequently the latter require a regular element length while the former ask for a more dense element division at the upper part of the pile and only few elements at the lower part.

In the following paragraphs a different approach will be adopted. It is tried to replace either the whole pile or a part of it by "equivalent" springs.

Figure 34 depicts some variants of an equivalent modeling of a single pile. Neglecting the dependency between lateral displacements and rotations (see chapter 2), the schematization is as shown in figure 34a, whereas taking into account this dependency yields figure 34b, 34c or 34d.

To determine the characteristics of the equivalent springs an extensively modeled pile isolated from the superstructure has to be analysed. In an elastic situation, unit displacements directly give spring

coefficients. However for the nonlinear situation every combination of an increasing moment and an increasing lateral force imposed at the pile head would yield two new load displacement curves for the representing springs, because superposition is not valid. Therefore the nonlinear axial load-deformation curve can be obtained without much difficulty. As for the lateral and rotational springs, their curves can only be established exactly for a known combination of imposed forces and/or displacements at the pile head.

An intermediate modeling of a single pile may combine the advantages from the extended and the equivalent model. As mentioned above, lateral behavior is almost completely governed by the soil around the upper part of the pile. Also failure of the pile occurs near the pile top. Figure 35 now shows a solution that allows application of any desired loading combination. Nevertheless, it has much less elements compared with the extensive modeling. Appendix G gives some suggestions to determine the length of the upper part of the pile that still needs to be modeled extensively.

Schematization of a group of piles can be carried out in more than the three above mentioned ways:

An extensive modeling of a group of n piles consists of n single pile schematizations. The characteristics of the soil can be represented by NOGA curves, see chapter 5. All the following group problems are included: Excentricities, increased rotational "push-pull" resistance, geometrical and physical interaction under extreme loads, etc. Only interactions in the soil itself are not taken into account.

In a more simple schematization the n-pile group is replaced by one "equivalent pile", which should not be confused with the equivalent modeling. The soil can again be modelled with NOGA curves, (the force values multiplied by n, the displacement values unchanged). To approximate the increased rotational resistance an additional spring may be added to the pile head. Accurate simulation of collapse under eccentric loading can not be expected. The same applies for simultaneous axial and lateral failure.

An intermediate modeling of a group is possible in the same way as described for the single pile. This also applies for an equivalent pile.

The equivalent representation of a group is almost identical to that of a single pile model, see figure 36. Not all the subtle interactions are

included, but probably the effect of this shortcoming may be neglected compared with the uncertainties in the spring curves.
As in the case of a single pile the nonlinear dependencies require previous knowledge of the pattern of forces and/or displacements at the pile head. However, at collapse of the superstructure those values are difficult to predict. Nevertheless, an equivalent model may be useful for preliminary computations if several assumptions are introduced. The spring characteristics have to be derived from more or less extensive modeled groups under a given load or displacement.

Litterature: [15], [16].

7. CONCLUSIONS

Considering offshore piles loaded up to failure (both axial and lateral) the following can be concluded:

- Prediction of the load-deformation behaviour (quantitative):
  - Soil mechanics knowledge and experience are required: Values of parameters describing strength and stiffness of the soil strongly influence the results.
  - Full scale loading tests are lacking. Consequently poor knowledge about behavior around failure is available, especially about the mobilization of end bearing.
  - Cyclic loading effects are not considered but may play an important role.

- Description of load-deformation behaviour (qualitative):
  - A good description of is possible with nonlinear t-z and p-y curves. Single pile curves can be obtained from: Kraft, Ray and Kagawa (ref.[14]) for t-z, and Matlock (ref.[1]) for p-y curves.
  - Group collapse may be complicated if ultimate axial and lateral capacities are reached at the same time.
  - This complicated behaviour can be analyzed with the aid of of the NOnlinear Group Affected (NOGA) t-z and p-y curves derived in this paper.
  - A different approximate description of group behaviour can be derived with a "hybrid" interaction factor method.

- Modelling of the foundation with INTRA is possible in several ways:
  - The rate of admissible simplification will depend on the sensitivity of the superstructure to its foundation behavior under extreme loads.
  - Preliminary pushover analysis can be performed with a simple "equivalent" modelling, consisting of one axial and two lateral nonlinear springs at each jacket leg.
  - If only axial collapse can be expected, also extensive pushover analysis is possible with an 'equivalent' schematization. However when lateral collapse plays a role, a more refined modelling is required.

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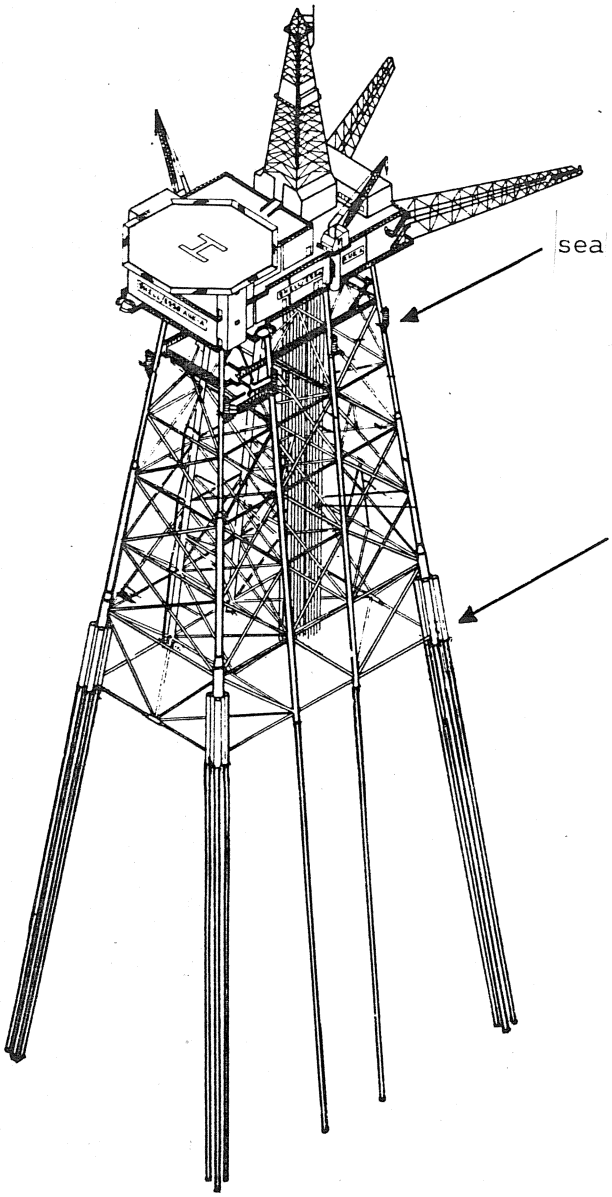


Figure 1
Fixed offshore structure
with single piles and pilegroups

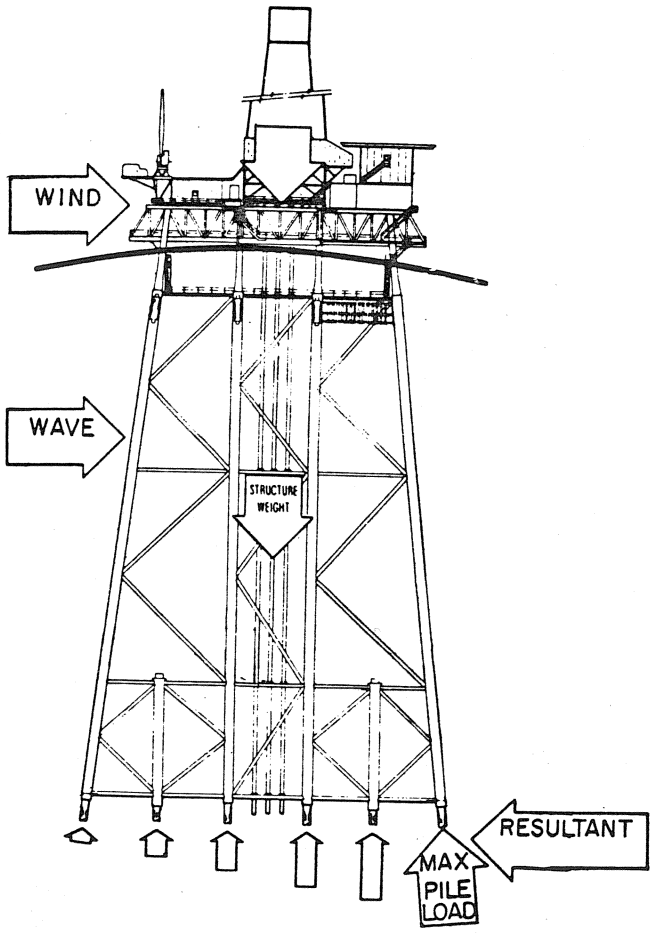


Figure 2
Forces at a jacket structure

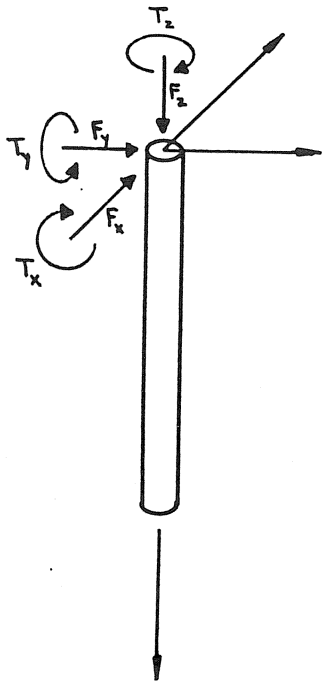


Figure 3
Coordinate system

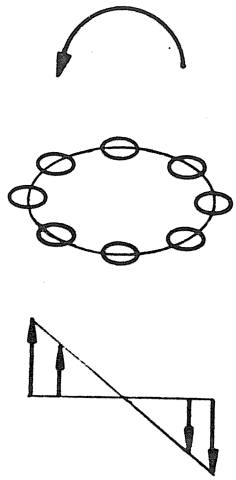
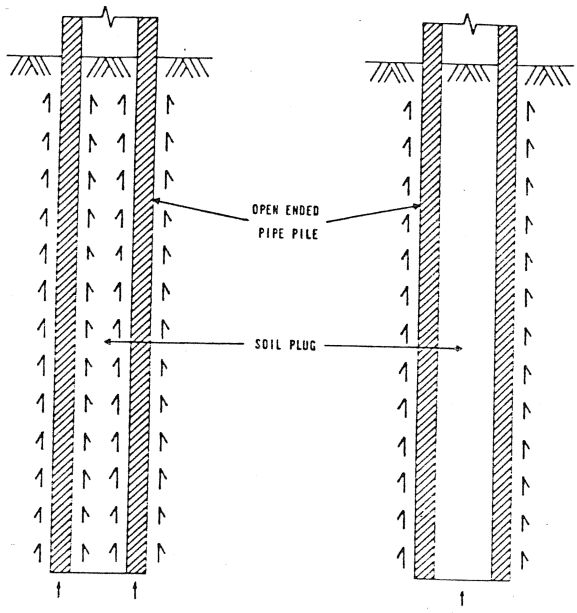


Figure 4
Adaption of a bending moment on a pile group, by normal forces in a push-pull mode



plugged: $Q = q_{max}A_p + \Sigma\tau_{max}A_o$

non plugged: $Q = q_{max}A_a + \Sigma\tau_{max}(A_o + A_i)$

Q = ultimate pile bearing capacity

q_{max} = unit soil bearing capacity

τ_{max} = soil shear strength

A_p = gross end area

A_a = annulus area

A_o = outside shaft area

A_i = inside shaft area

Figure 5
'Plugged' and 'non-plugged' open ended tubular pile (ref.[34])

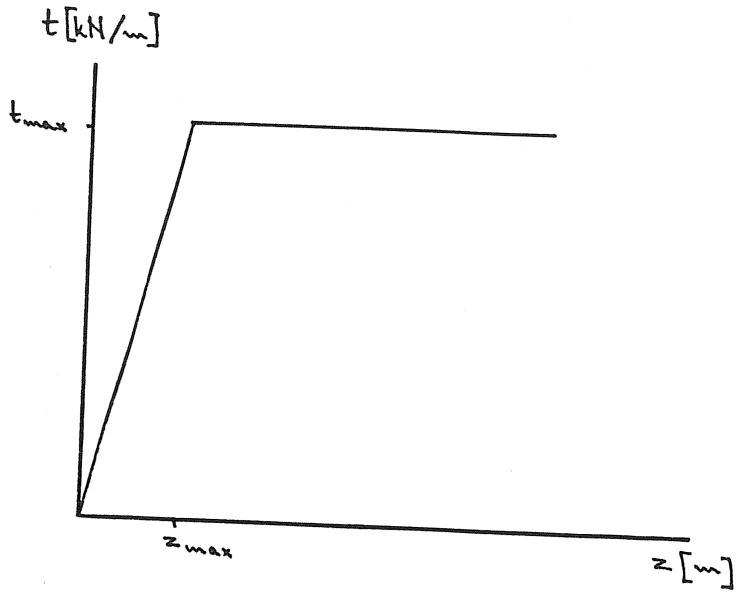


Figure 6a
Bi-linear t-z curve

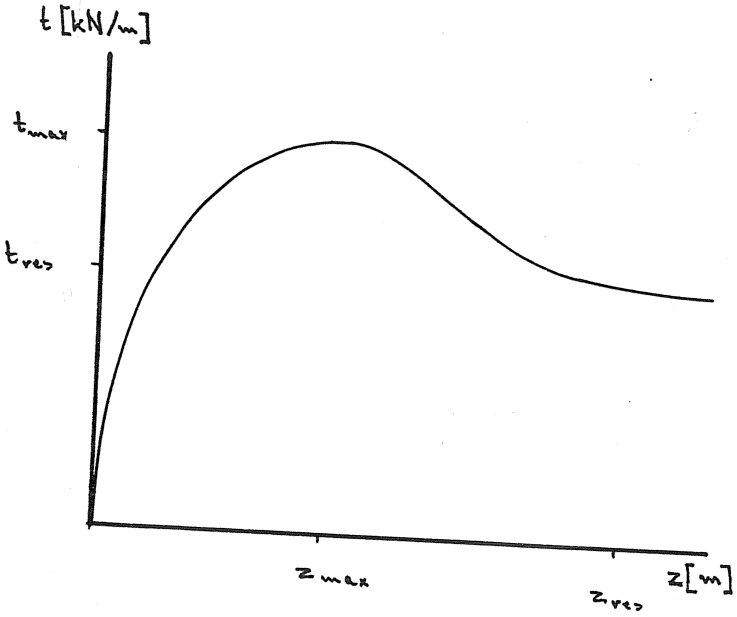


Figure 6b
Nonlinear t-z curve

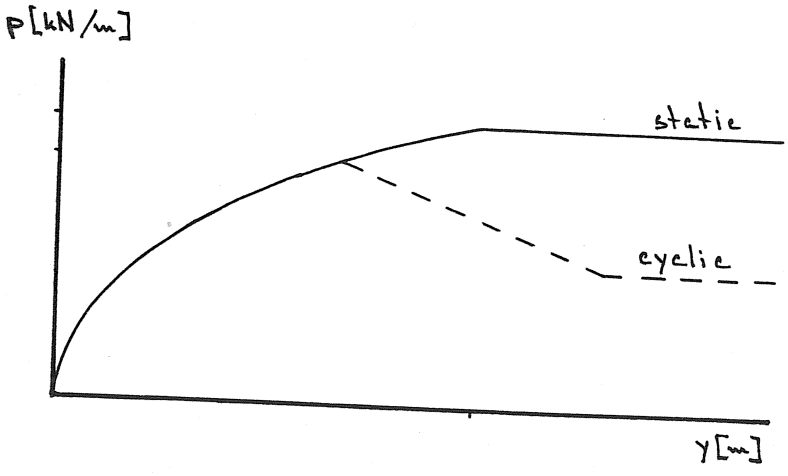


Figure 7
General shape of a p-y curve

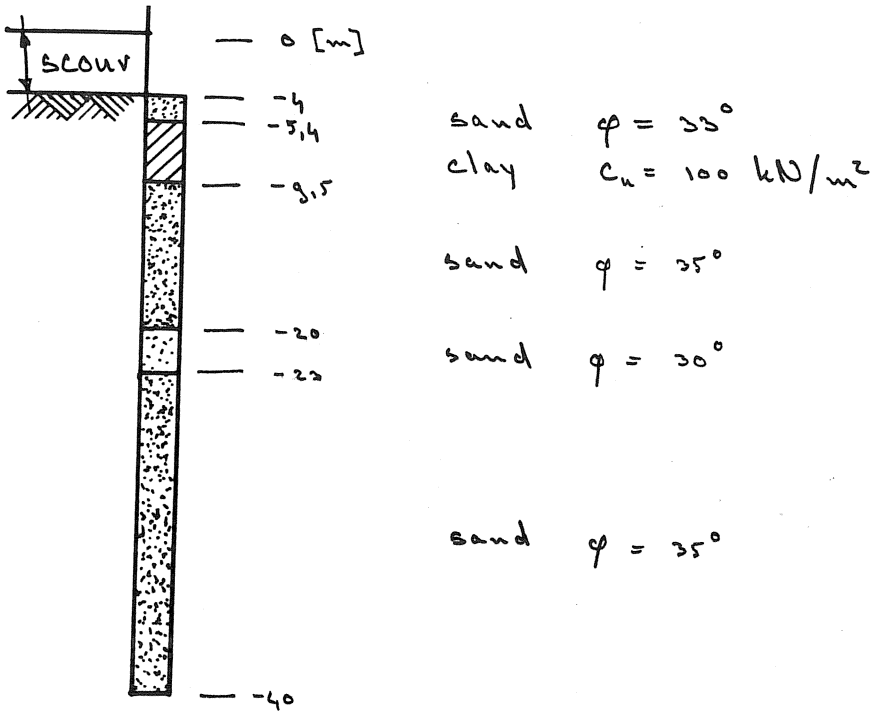


Figure 8
Soil profile

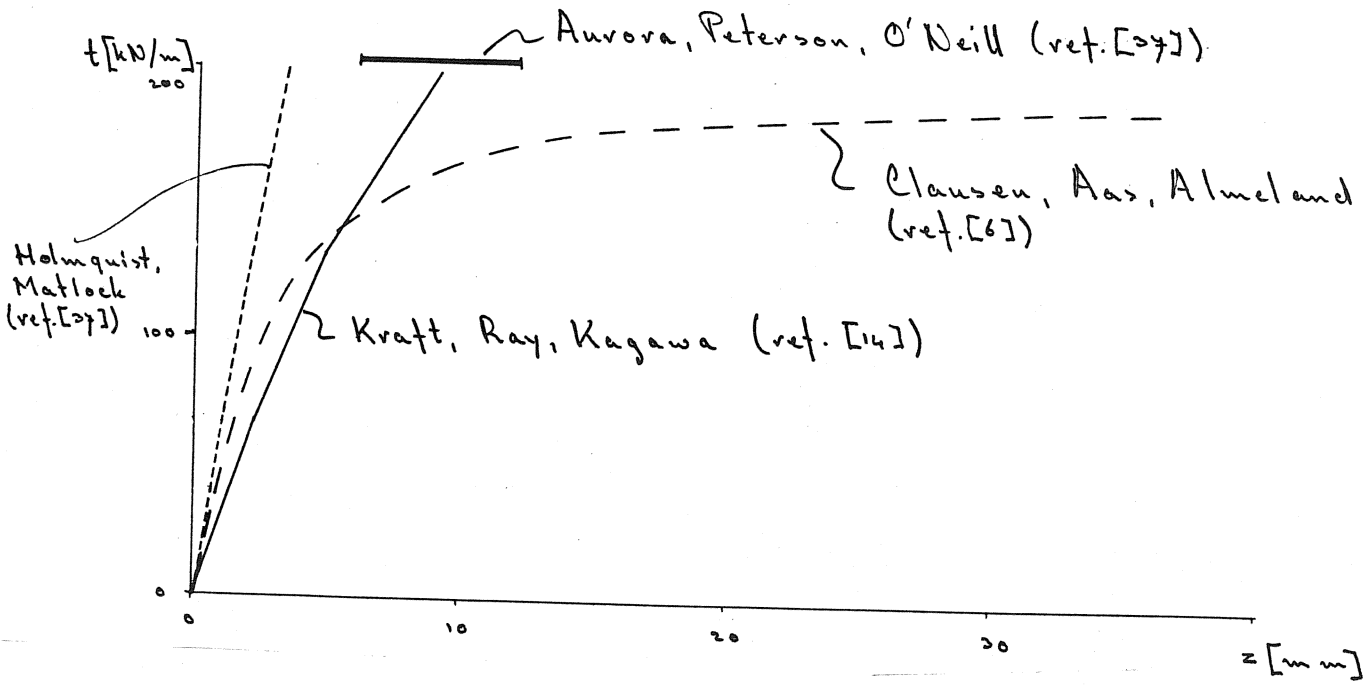


Figure 10
Different results of t-z curve predictions

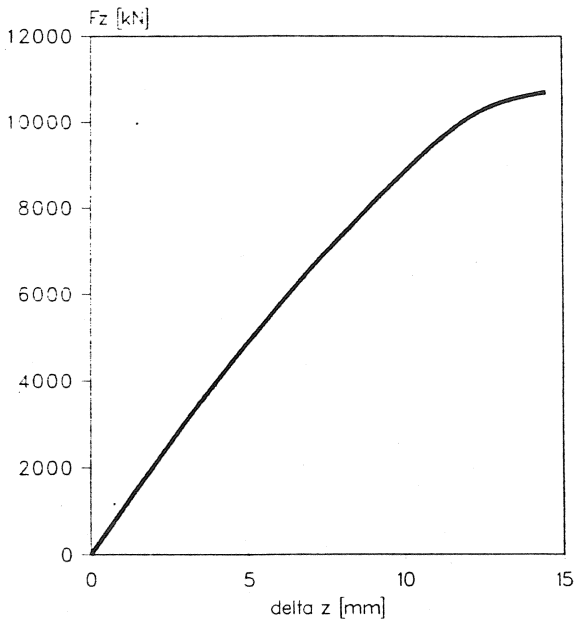


Figure 11a
Pile head F-δ curve
no end bearing

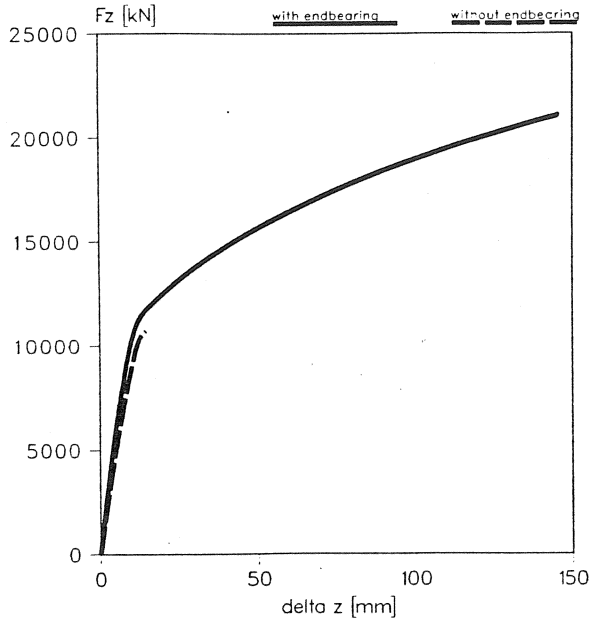


Figure 11b
Pile head F-δ curve

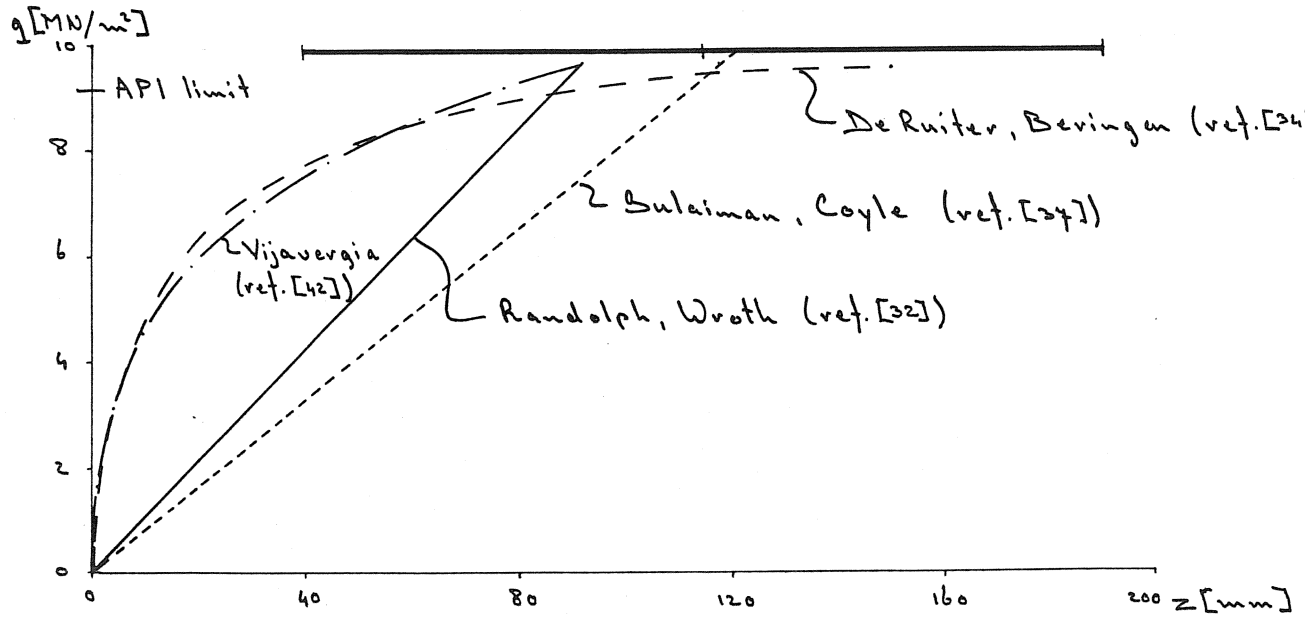


Figure 12
Different results of q-z predictions

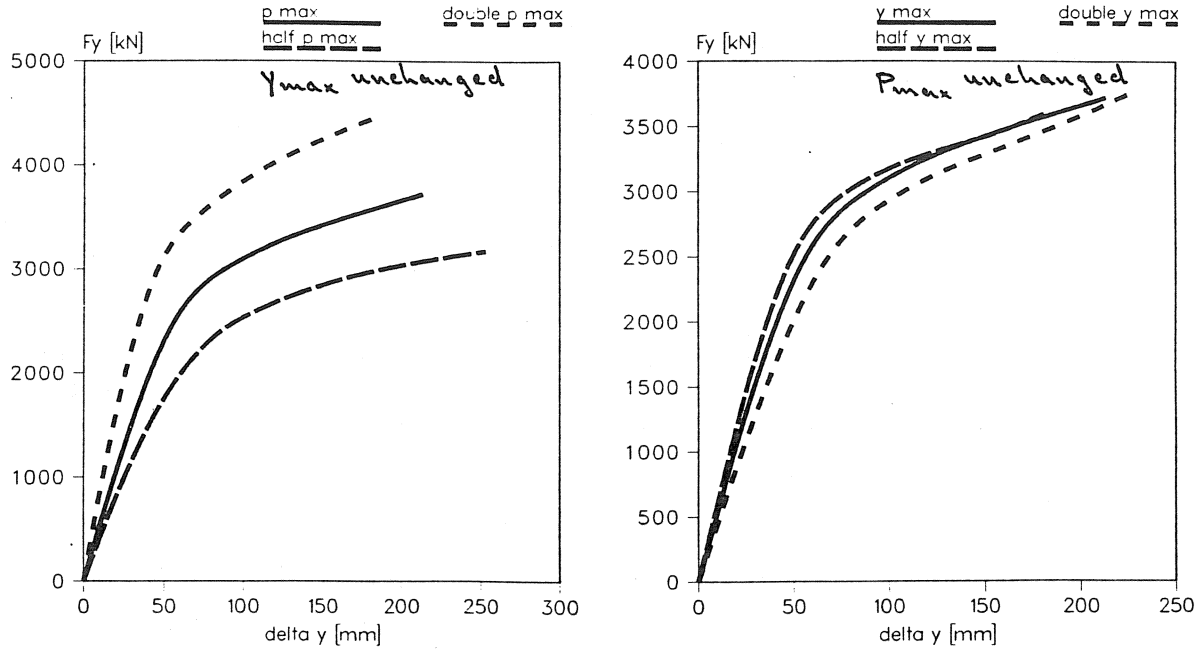


Figure 13

Influence of the local p_{max} and y_{max} values

at the shape of the pile head F-δ curves

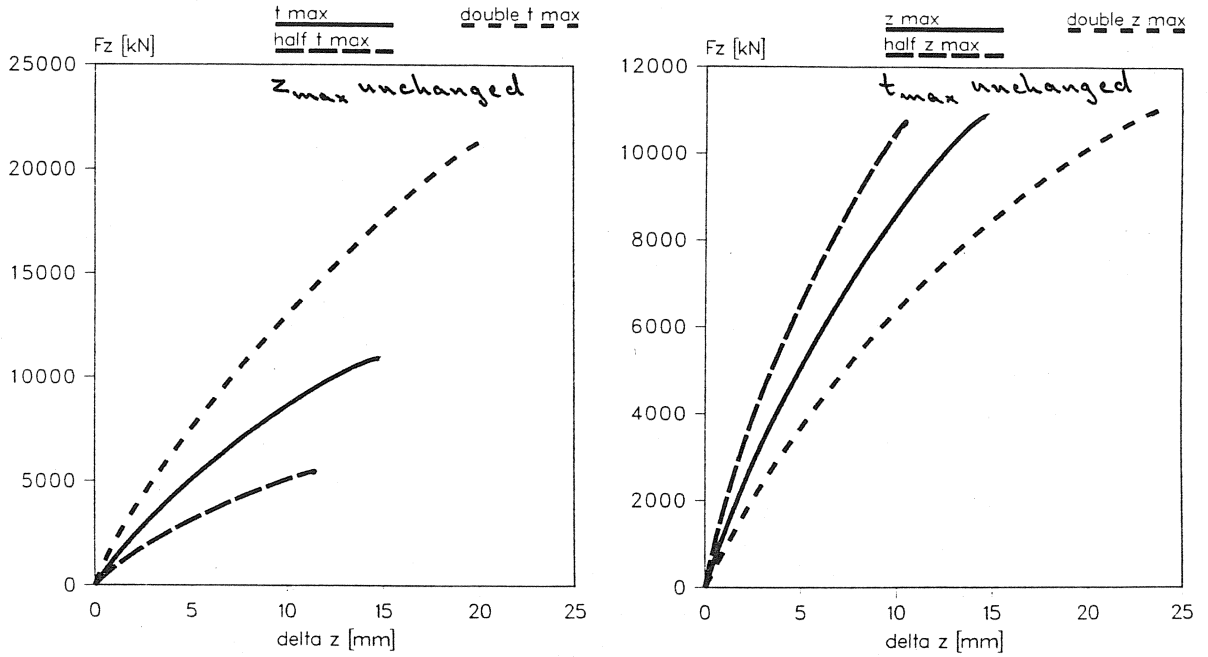


Figure 14

Influence of the local t_{max} and z_{max} values

at the shape of the pile head F-δ curves

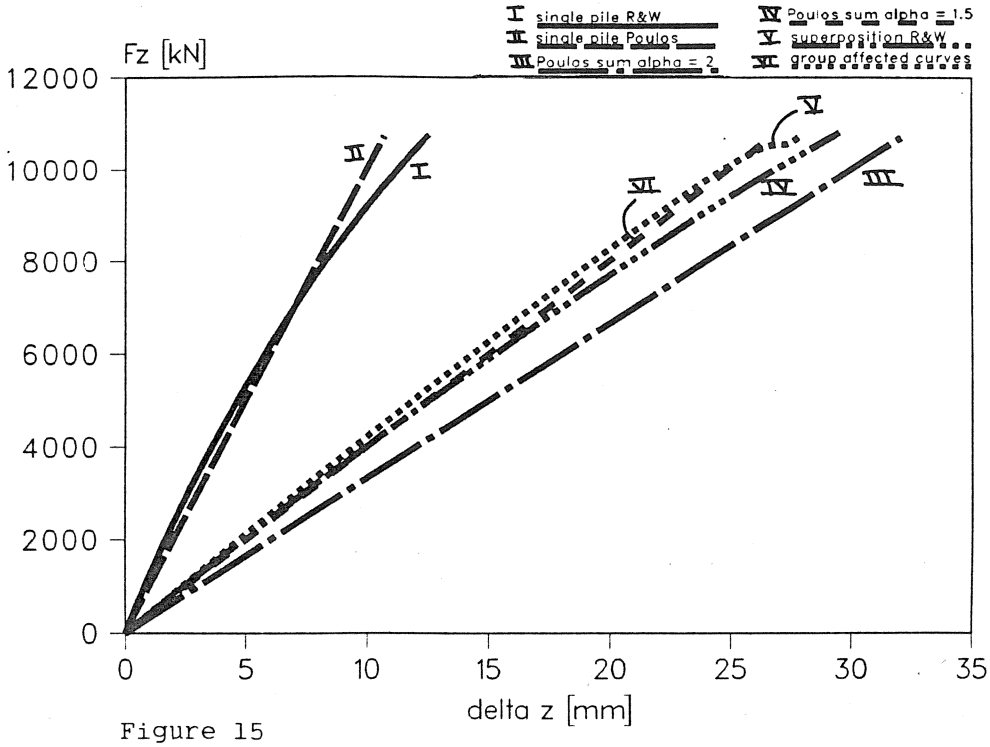


Figure 15

Axial pile head F-δ curves (linear G modulus)

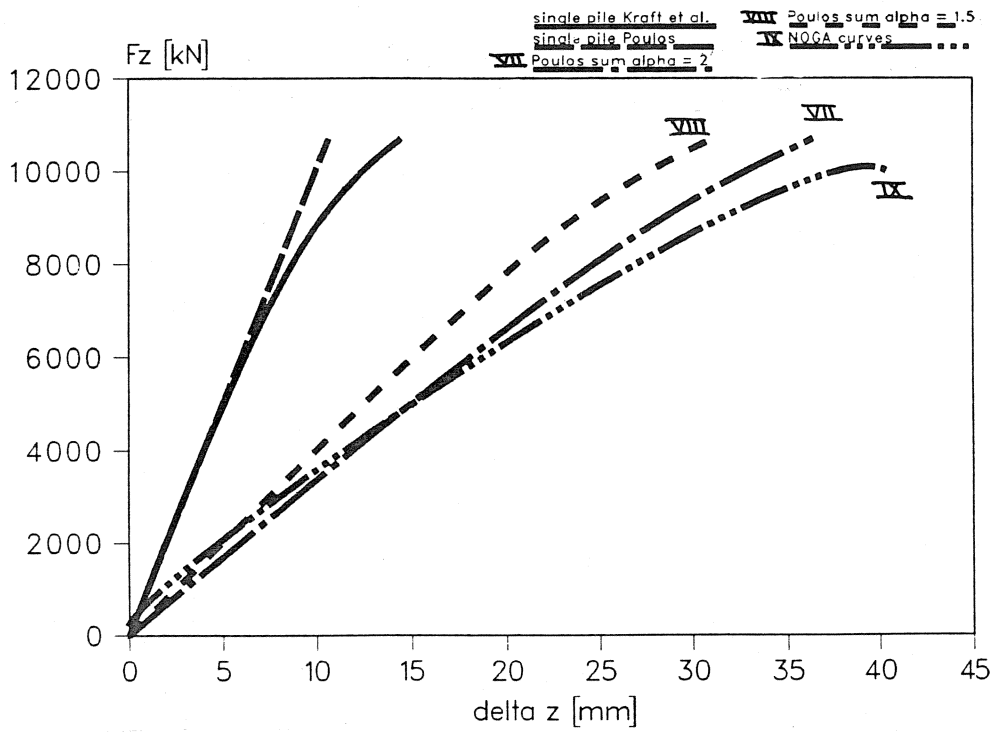


Figure 16

Axial pile head F-δ curves (nonlinear G modulus)

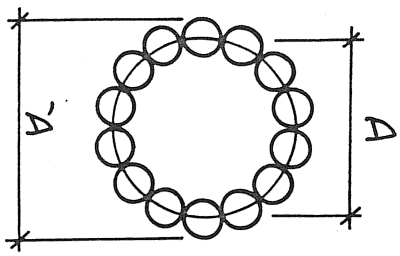


Figure 17
Filled up circular group

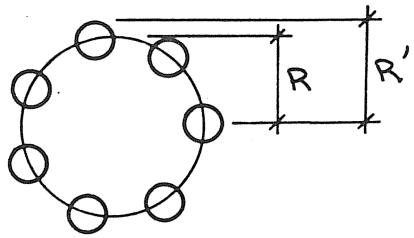


Figure 18
Widely spaced group

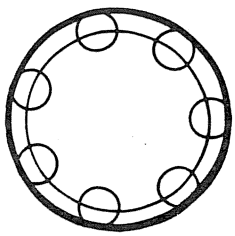


Figure 19a
Failure mechanism
Terzaghi and Peck (ref.[36])

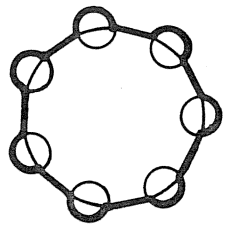


Figure 19b
Failure mecchanism
De Ruiter and Beringen (ref.[34])

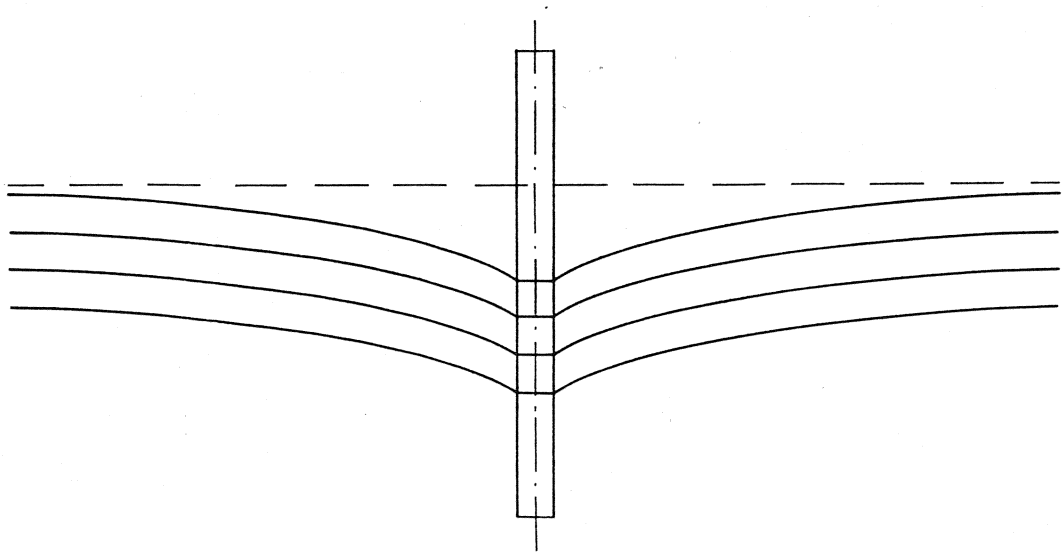


Figure 20
Settlement of soil layers around an axially loaded pile

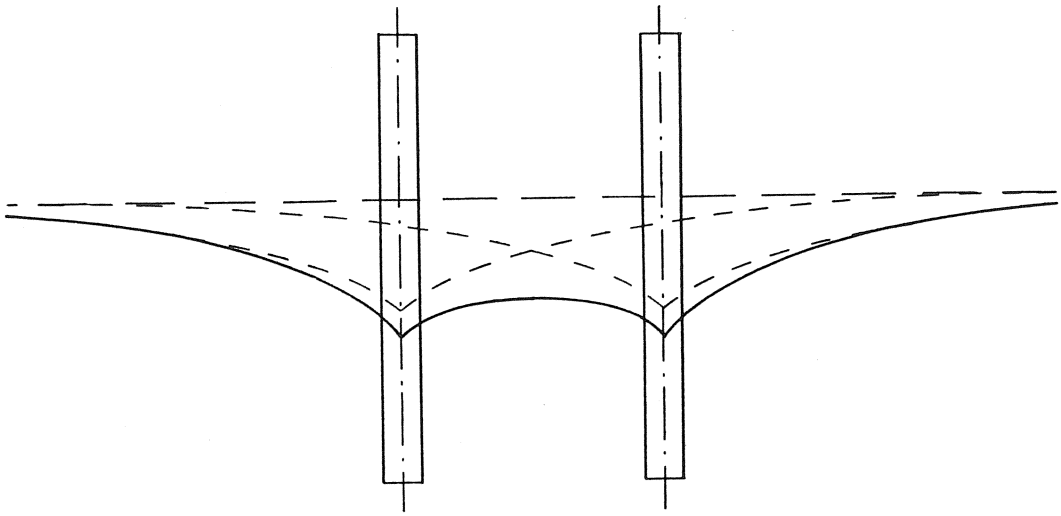


Figure 21
Settlement of soil layers around two equally axially loaded piles

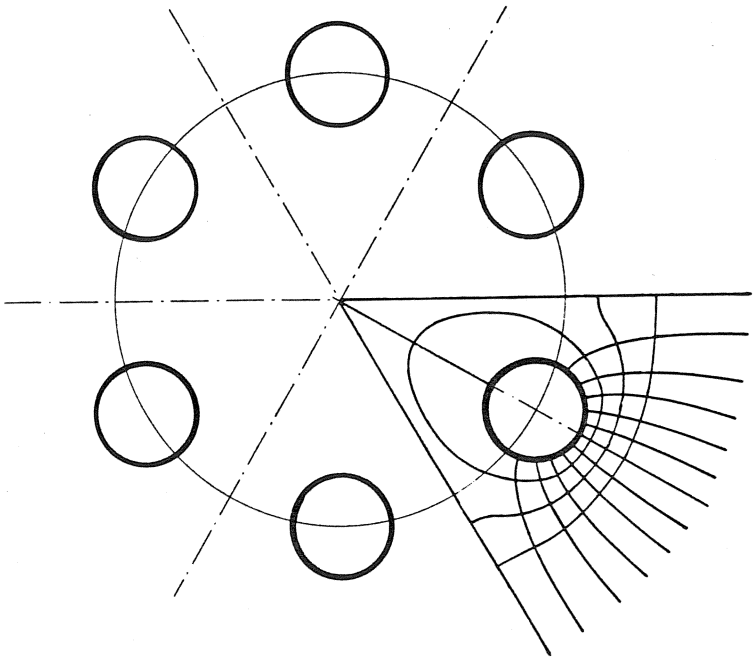


Figure 22
Schematic flow net around a group of sinks

n^* = number of piles in filled up group
= 24

$r = 1{,}5$ m

$$\Rightarrow R = \frac{r}{\sin\frac{\pi}{n^*}} = 11{,}49 \text{ m}$$

$\tau_{max} = 80$ kN/m²

$G = 40$ MN/m²

all data arbitrary

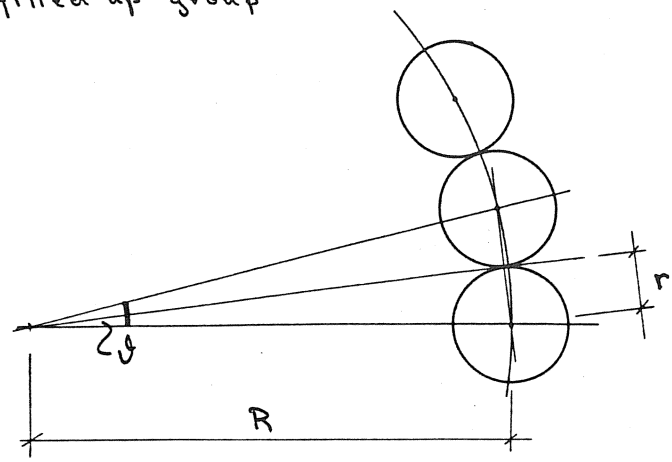


Figure 23

Pile group data

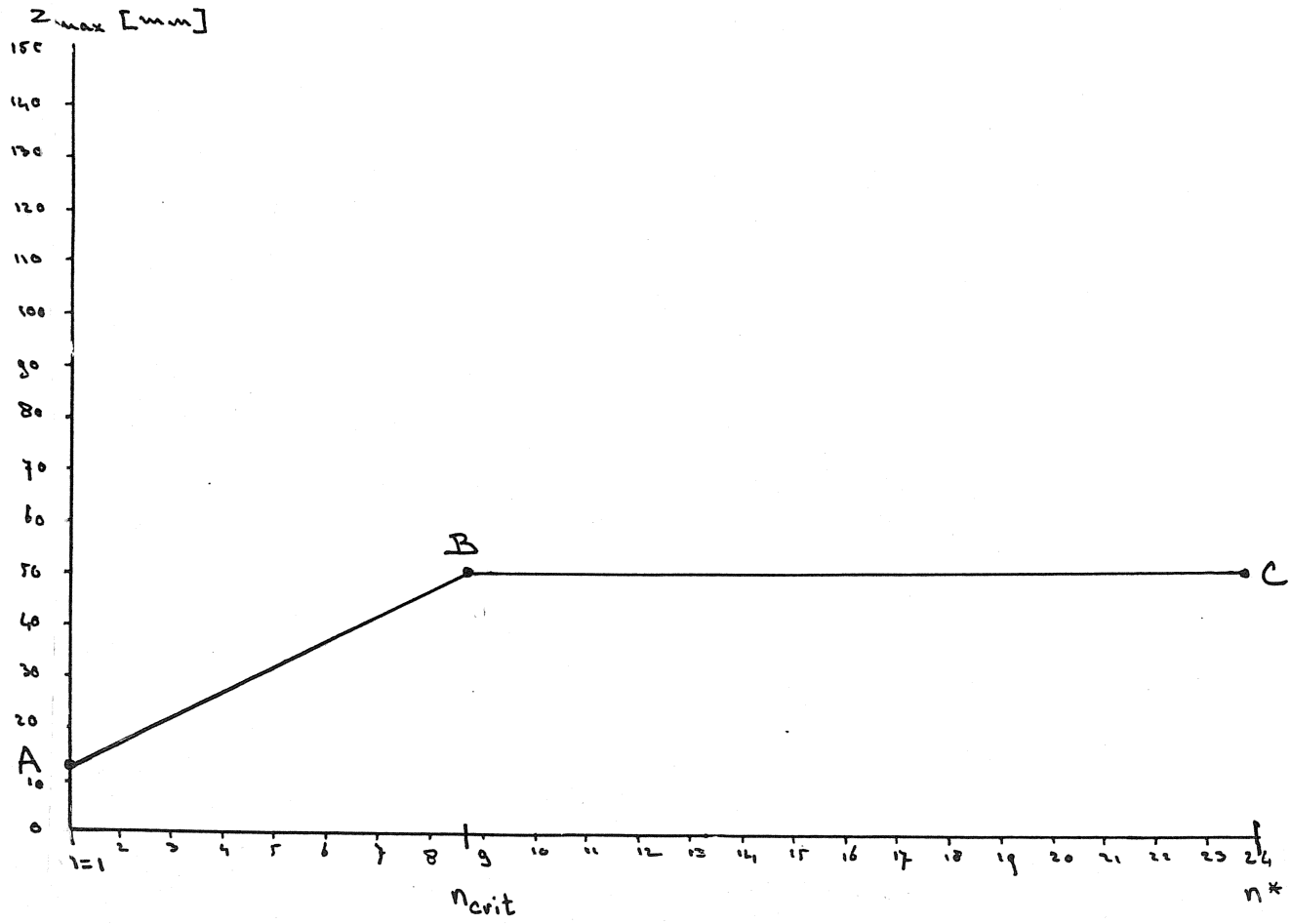


Figure 24

Maximum elastic settlement versus number of piles

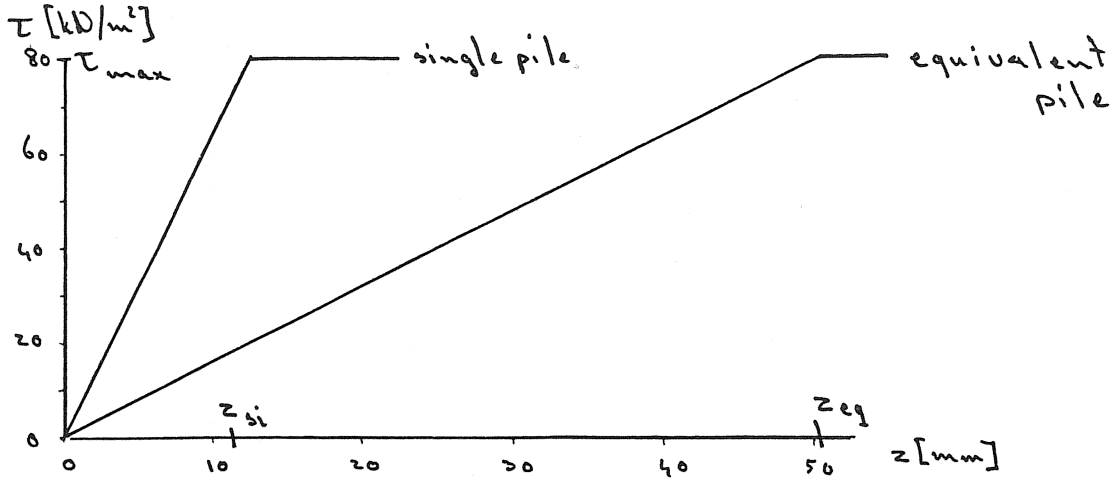
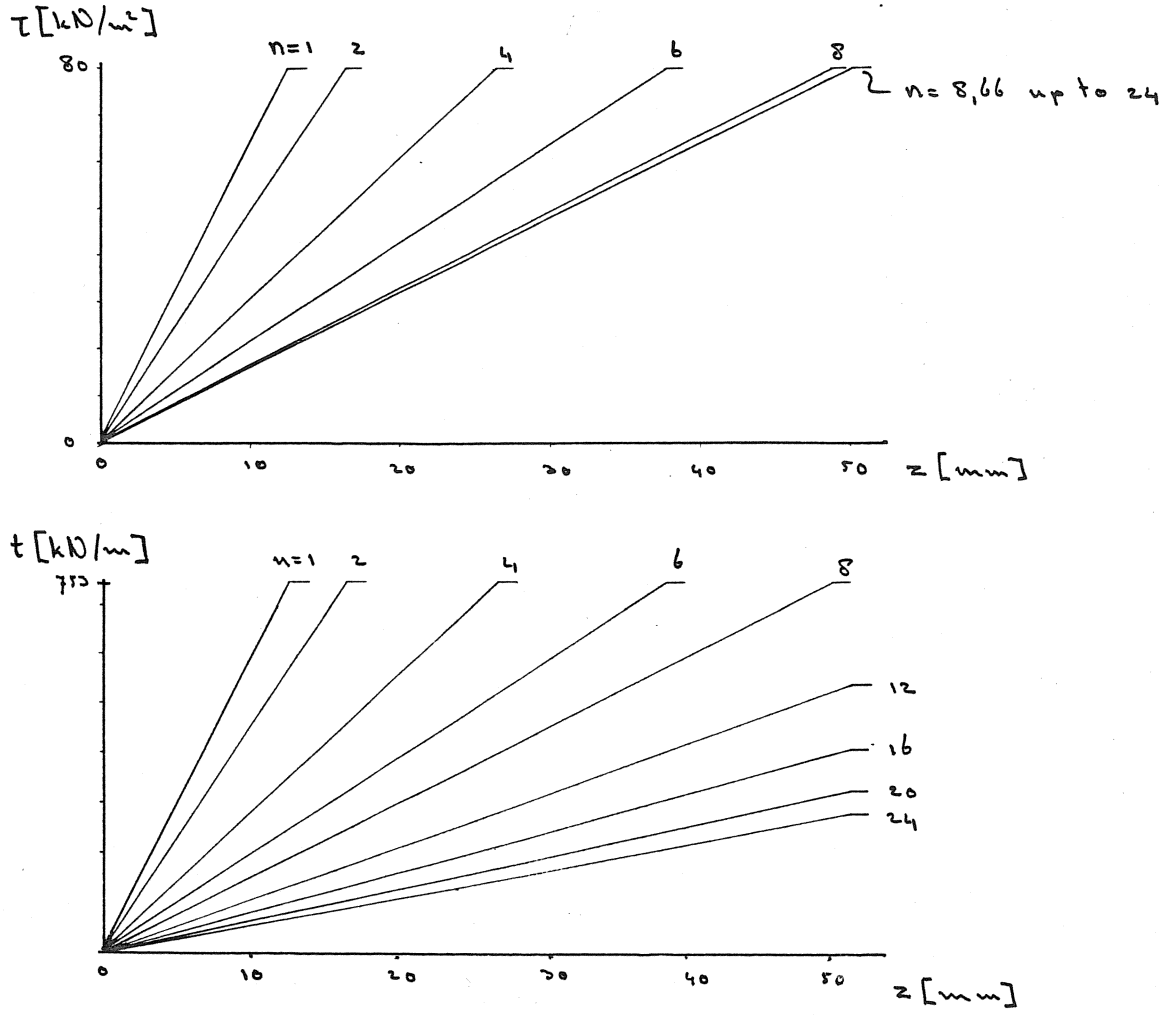


Figure 25
Bi-linear τ-z curve for a single pile
and for an equivalent pile



Figures 26a and 26b
Bi-linear τ-z and t-z curves for several
numbers of piles in a circular group

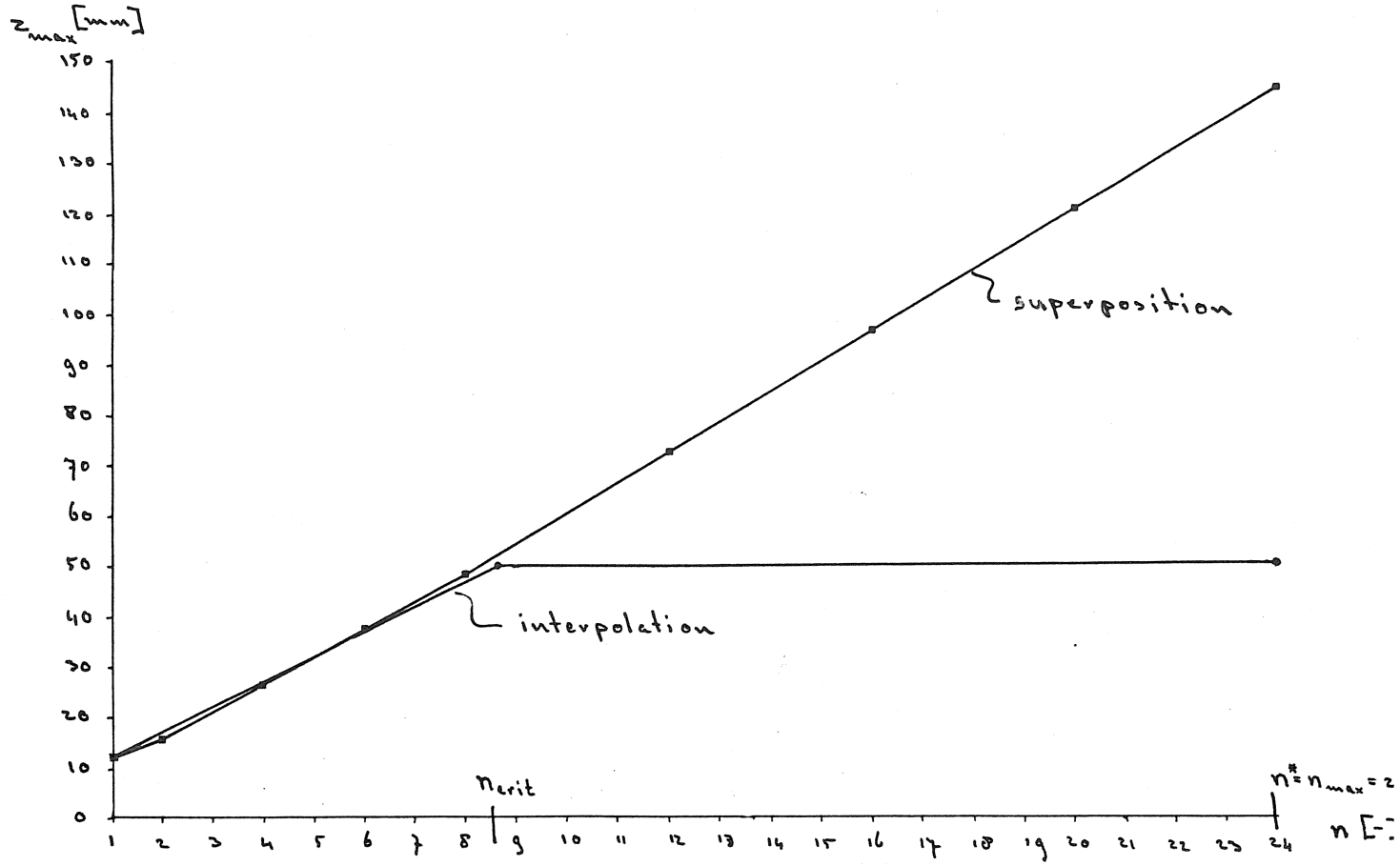


Figure 27

Maximum elastic settlement versus number of piles

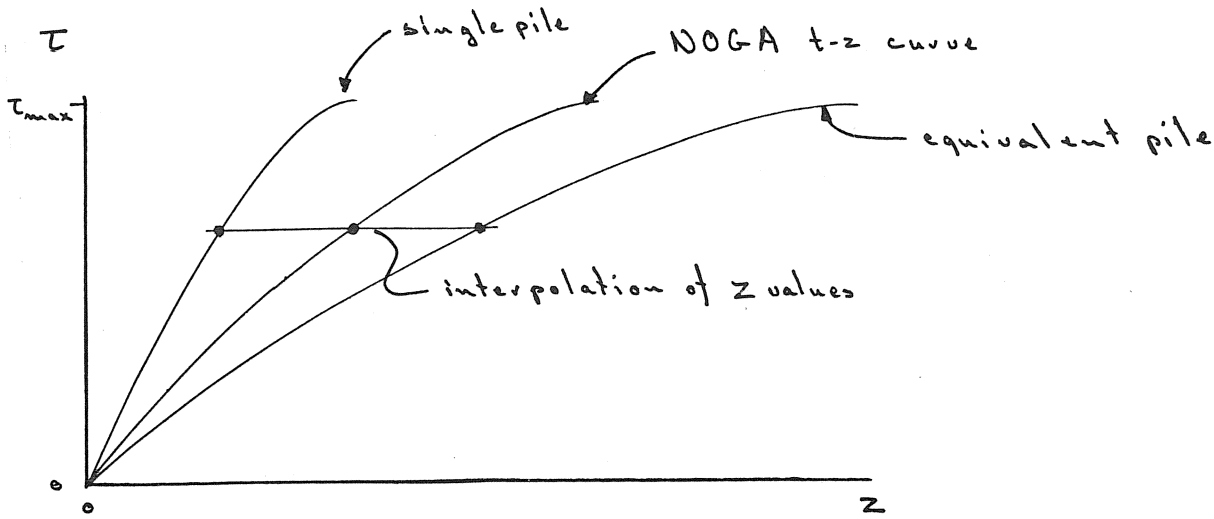


Figure 31
Schematic construction of NOGA τ-z curve

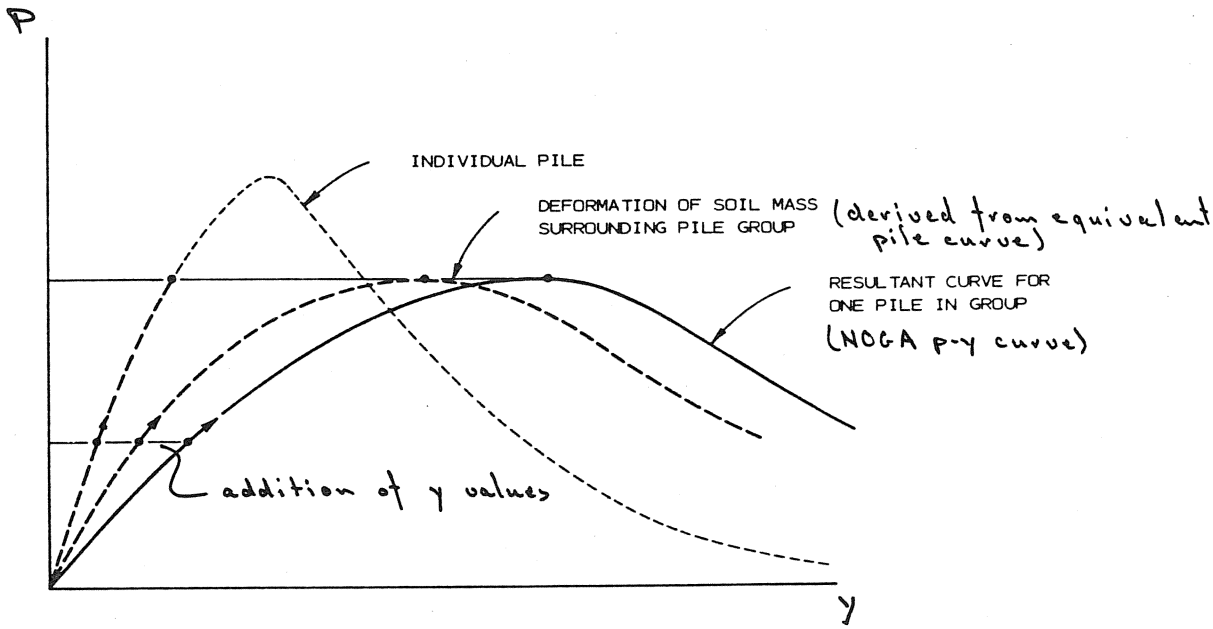


Figure 32
Schematic construction of NOGA p-y curve
according to Bogard and Matlock (ref.[2])

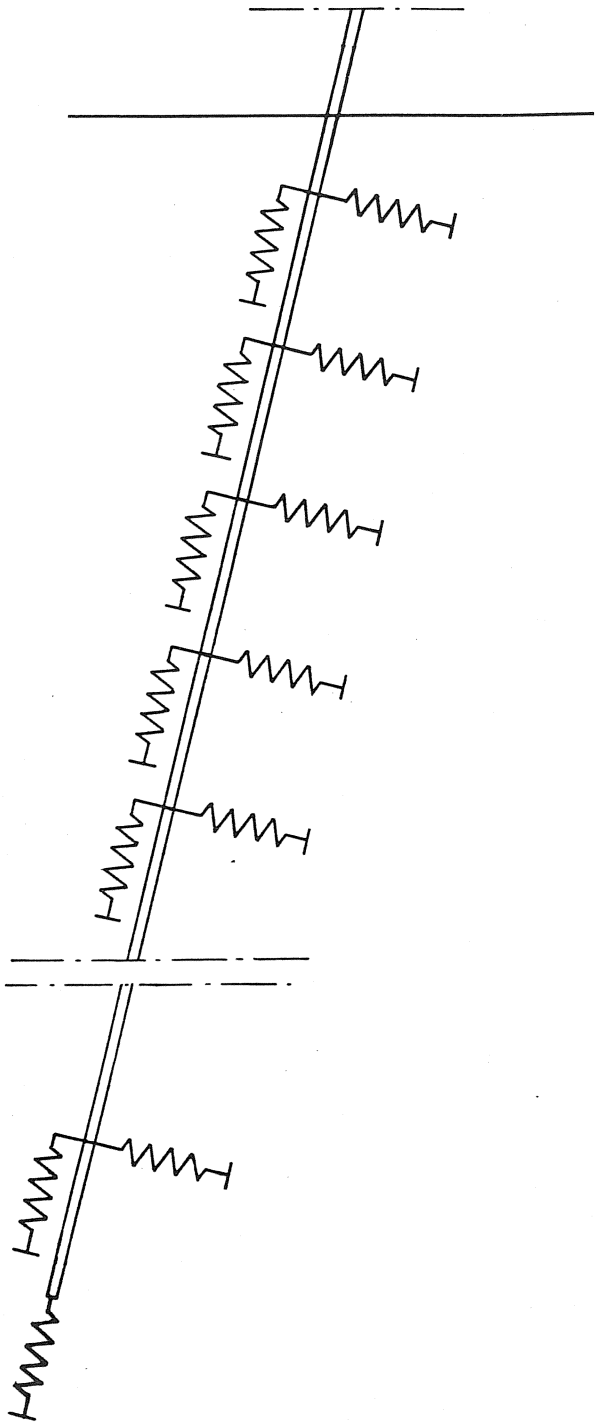


Figure 33
Extensive modelling,
with the aid of nonlinear springs

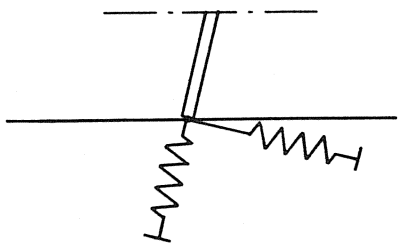
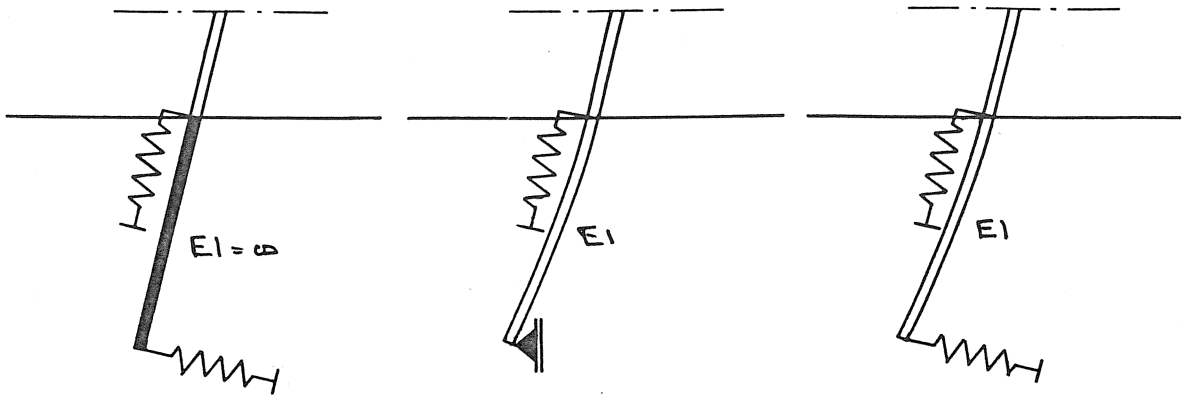


Figure 34a
Equivalent modelling,
neglecting dependency between
lateral displacements and rotations



Figures 34b, 34c and 34d
Equivalent modelling, taking into account dependency between lateral displacements and rotations

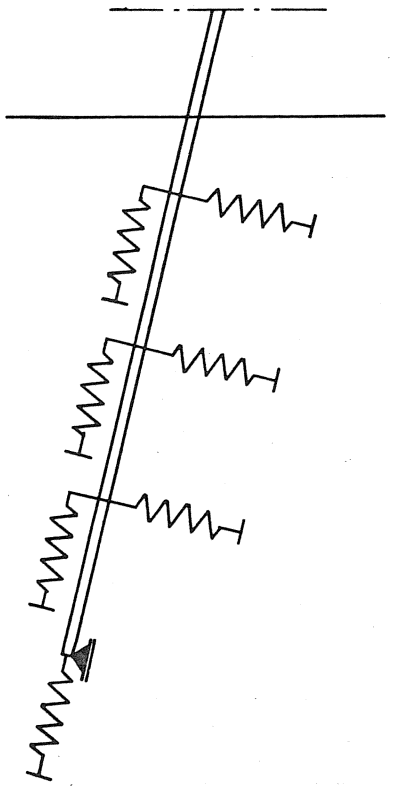


Figure 35
Intermediate modelling

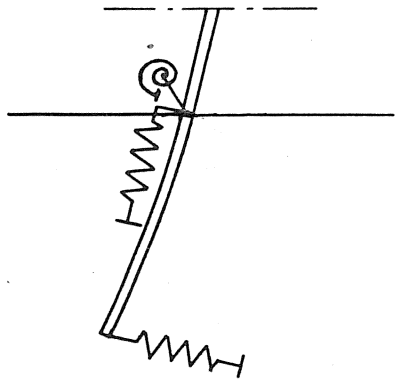


Figure 36
Equivalent modelling of a group including increased rotational resistance

APPENDIX A

T-Z CURVES

Randolph and Wroth (ref.[32]) present a way to compute settlements around a pile, partly quoting previous publications. The deformation of the soil around a pile shaft is idealised as shearing of concentric circles, see Figure A1. Consider small horizontal layers, see Figure A2. Ignoring horizontal displacements the kinematic equation reads:

$$\frac{\partial z}{\partial r} = \gamma$$

z = settlement

r = radius (coordinate)

γ = shear

The constitutive equation:

$$\gamma = \frac{\tau}{G}$$

τ = shear stress

G = shear modulus

The geometry of concentric circles yields a simple equilibrium relationship:

$$\tau = \frac{\tau_o \, r_o}{r}$$

r_o = pile radius

τ_o = stress at shaft, $r = r_o$

Substitution and integration yields:

$$z(r) = \frac{\tau_o \, r_o}{G} \int \frac{1}{r} \, dr$$

Integration from r_o to ∞, results in a value of z equal to infinity. To avoid this unacceptable solution, a "magical radius" r_m is introduced at which the shear stress becomes neglectible. Now one finds:

$$z(r) = z(r^*) = \frac{\tau_o \, r_o}{G} \int_{r^*}^{r_m} \frac{1}{r} \, dr = \frac{\tau_o \, r_o}{G} \ln \left(\frac{r_m}{r^*} \right)$$

This yields a pile settlement of :

$$z = \frac{\tau_o \, r_o}{G} \ln \left(\frac{r_m}{r_o} \right)$$

An expression for r_m reads:

$$r_m = 2.5 \, l\rho(1-\upsilon)$$

l = pile length

r = radius (coordinate)

r^* = radius (fixed value)

ρ = ratio of G modulus at depth l/2 and pile tip

υ = Poissons ratio of soil.

Kraft, Ray and Kagawa (ref.[14]) introduce a nonlinear G modulus in the t-z formula:

$$G = G_i \left\{ 1 - \left(\frac{\tau \, \lambda}{\tau_{max}} \right) \right\}$$

G_i = initial G modulus at small strains

λ = stress-strain curve fitting constant, usually taken as 0.9

With

$$\psi = \frac{\tau_o \, \lambda}{\tau_{max}}$$

follows:

$$\tau_o \, r_o \int_{r^*}^{r_m} \frac{dr}{G_i \left\{ 1 - \left(\frac{\tau \, \lambda}{\tau_{max}} \right) \right\} r} = \frac{\tau_o \, r_o}{G_i} \int_{r^*}^{r_m} \frac{dr}{r - r_o \psi} =$$

$$= \frac{\tau_o \, r_o}{G_i} \ln \frac{(r_m - r_o \psi)}{(r^* - r_o \psi)}$$

With $r^* = r_o$ this becomes:

$$z = \frac{\tau_o \, r_o}{G_i} \ln \left\{ \frac{\frac{r_m}{r_o} - \psi}{1 - \psi} \right\}$$

Superposition is possible using the expression with a linear G modulus. Randolph and Wroth in reference [33] present a formula for two piles. Each pile has the same load and shows the same settlement:

$$z = \frac{\tau_o \, r_o}{G} \left\{ \ln \left(\frac{r_m}{r_o} \right) + \ln \left(\frac{r_m}{s} \right) \right\}$$

s = centre spacing of the two piles

Extension to a group of n piles yields:

$$z = \frac{\tau_o \, r_o}{G} \left\{ \ln \left(\frac{r_m}{r_o} \right) + \sum_{i=1}^{n-1} \ln \left(\frac{r_m}{s_i} \right) \right\} =$$

$$= \frac{\tau_o \, r_o}{G} \ln \left\{ \frac{r_m^{\,n}}{r_o \cdot \prod_{i=1}^{n-1} s_i} \right\}$$

s_i = centre spacing between pile 1 and pile i+1

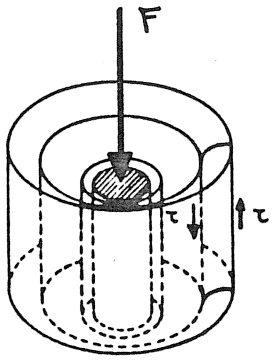


Figure A1
Shear of concentric circles

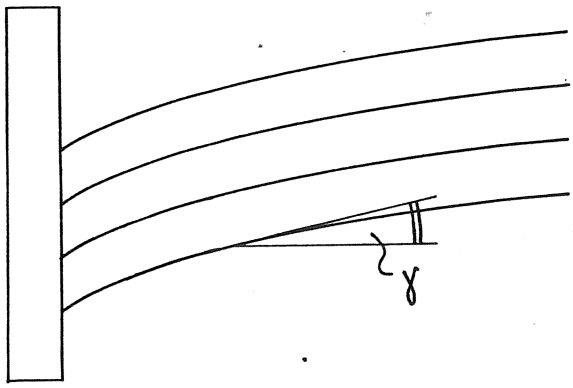


Figure A2
Horizontal layers around a pile

APPENDIX B

Q-Z CURVES

Little literature on q-z behaviour is available.
A linear curve can be derived with the help of Timoshenko and Goodier, (ref.[38]). Their solution for a rigid cylinder pressed against the plane boundary of an elastic half space reads:

$$z = \frac{F\ (1-\upsilon^2)}{2\ r_o\ E}$$

z = settlement
F = load
r_o = radius of cylinder
υ = Poissons ratio of soil
E = elastic modulus of soil

Randolph and Wroth, (ref.[32]) mention some modifications for this expression to take in account the depth below the surface of the pile point. At the same time they use the shear modulus G instead of E:

$$z = \frac{F\ (1-\upsilon)}{4\ r_o\ G}\ C_1 \qquad \text{with } 0.5 < C_1 < 0.85$$

For failure loads holds C_1 equal to 0.85.

Kraft, Ray and Kagawa, (ref.[14]) give the equivalent, using E again:

$$z = \frac{2\ q\ r_o\ (1-\upsilon^2)}{E}\ C_2 \qquad \text{with } 0.39 < C_2 < 0.67$$

q = point pressure

By expressing F in q, they suggest a uniformly distributed pressure under the pile point which is not valid for loads up to failure. Besides, sand and clay show totally different patterns. For collapse however the formula in this way agrees with the usual ultimate end bearing as defined by e.g. Terzaghi and Peck, (ref.[36]) or the API recommendations, (ref.[1]):

$$Q_p = q_{max} \cdot A_p$$

Q_p= ultimate point bearing capacity

A_p= gross end area (plugged pile)

q_{max}= unit bearing capacity

The value of q_{max} is a function of soil properties and depth.

To obtain nonlinear q-z curves the formulas mentioned should be modified with the aid of nonlinear E or G modulus, or they should be used in an incremental fashion with continually updated values.

Apart from those rigid punch formulas, empirical load-deformation relationships exist for end bearing, e.g. relating settlement to pile diameter: De Ruiter and Beringen (ref.[34]) present a curve that shows a settlement at failure of about 0.12 times the pile diameter.
Several ways exist to describe a q-z curve up to failure. E.g. an expression from Vijavergia (ref.[42]) reads:

$$\frac{q}{q_{max}} = \left\{ \frac{z}{z_{max}} \right\}^{1/3}$$

In reference [33] Randolph and Wroth introduce superposition of end bearing:
At some distance of the pile base the deformations will be almost equivalent to deformations caused by an imaginary point load in the centre of the pile base.

$$z(r) = \frac{F\,(1-\upsilon)}{2\,\pi\,r\,G}$$

Superposition of settlements caused by n equally loaded piles yields for every pile:

$$z = \frac{F\,(1-\upsilon)}{4\,r_o\,G}\,C_1\,\left\{ 1 + \frac{2\,r_o}{\pi} \sum_{i=1}^{n-1} \left(\frac{1}{s_i} \right) \right\}$$

s_i= centre spacing

APPENDIX C

EFFICIENCY FORMULAS

Terzaghi and Peck (ref.[36]) state that the bearing capacity of a group is the lowest of two values. Either the sum of the individual capacities or the capacity of the group behaving as equivalent pier, see Figure C1. The latter value is computed from:

$$Q_g = \pi D' l \tau_{max} + \tfrac{1}{4} \pi D'^2 q_{max}$$

Q_g = ultimate bearing capacity of the group
D' = outer diameter of the group
l = pile length
τ_{max} = soil shear strength
q_{max} = unit bearing capacity

De Ruiter and Beringen (ref.[34]) use a slightly different failure mechanism, see Figure C2. The expression according to this variant reads:

$$Q_g = l \left\{ \frac{n \pi d}{2} \tau_{red} + n (s-1) \tau_{max} \right\} + \tfrac{1}{4} \pi D'^2 q_{max}$$

$$= l \left\{ \frac{n \pi d}{2} \tau_{red} + (\pi D - n d) \tau_{max} \right\} + \tfrac{1}{4} \pi D'^2 q_{max}$$

n = number of piles in the group
d = pile diameter
$\tau_{red} = \alpha \tau_{max}$ = reduced soil shear strength
s = pile spacing (expressed in pile diameters)
D = centre diameter of the group
α = reduction factor

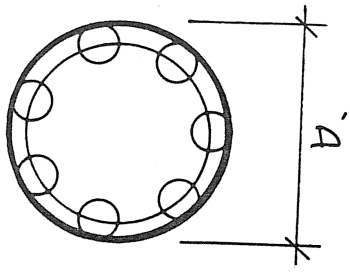


Figure C1
Failure mechanism
Terzaghi and Peck (ref.[36])

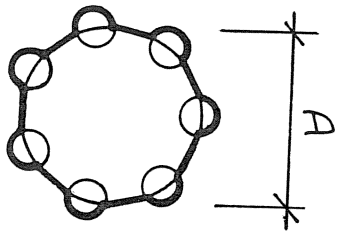


Figure C2
Failure mechanism
De Ruiter and Beringen (ref.[34])

APPENDIX D

INTERACTION FACTOR METHODS

Poulos (ref.[26],[28],[30],[31]) defines group settlement δ_g as

$$\delta_{g_i} = \delta_{sp_i} + \delta_u \sum_{j=1}^{n} \alpha_{ij} F_j \quad \text{with } \alpha_{ii} = 0$$

δ_{g_i} = settlement pile i (group solution)
δ_{sp_i} = settlement pile i (single pile solution)
δ_u = unit settlement single pile (single pile solution for unit load)
α_{ij} = interaction factor (influence pile j at pile i)
F_j = load at pile j

A definition for the interaction factors α_{ij} reads:

$$\alpha_{ij} = \frac{\text{additional settlement due to adjacent pile j}}{\text{settlement of pile i under its own load}}$$

Poulos used a computer program that calculates pile settlements out of the local displacements and the local influence of a neighbouring pile at several points along the shaft. On the basis of this, charts with interaction factors for different soil stiffness and pile spacings can be established, see references [30] and [31]. Although the factors are derived for two-pile groups, they can also be used for groups with more than two piles because the linear elastic soil representation allows superposition of the two-pile group results.

Another expression for δ_{g_i} reads:

$$\delta_{g_i} = \delta_u \sum_{j=1}^{n} \alpha_{ij} F_j \quad \text{with } \alpha_{ii} = 1$$

In the case of a pile group with a fixed head the individual settlements are all the same and thus equal to the group settlement δ_g.

$$\delta_g = \delta_{g_j} \quad \text{for } j = 1...n$$

Unknown are the n forces F_j and the group settlement δ_g. Known are n equations for δ_{g_i}. Together with the knowledge that the total force acting at the group, F_g, is equal to the sum of the n pile forces ($F_g = \sum_{j=1}^{n} F_j$) the n+1 unknowns can be solved. One of the results is that the piles in the centre of the group are less loaded than the piles at the outside.

Poulos (ref.[27]) formulates an equivalent way of computation of group action for laterally loaded piles. In this case the interaction factors are dependent on the angle between the line of connection of two piles and the direction of the applied force. Also the connection of the pile heads to the superstructure, fixed or pinned, now plays a role.

As in the case of interaction factors for axial behaviour, Poulos derived the factors for laterally loaded groups only for two-pile groups. However, application of those factors to groups with more than two piles does not provide a good representation of the increased rotational resistance that occurs because of push-pull working. Hence an approximate approach to describe the moment-rotation response of large groups is required, see reference [31].

For laterally loaded piles, on the contrary to the case of axial loading, it is not sufficient to achieve only the forces at the pile head as a result, because the lateral ultimate capacity depends on the bending moments in the pile. An approximate curve of the bending moments of a pile in a group is obtained as follows.
Consider a hypothetical single pile with the same pile head conditions as a pile in a group, i.e. with the same degree of fixity and the same lateral load. With the aid of a trick also the lateral pile head deflection is set equal to that of the pile in the group. This is achieved by multiplying the deflection scales of the p-y curves by so-called y-modifiers, thus generating a softer soil response. It is now assumed that the moment distribution and the deflection line of the hypothetical single pile are the same as for the pile in the group, see ref.[34].

Focht and Koch (ref.[8]). To understand their 'hybrid' approach, consider again the Poulos definition of δ_g :

$$\delta_{g_i} = \delta_{sp_i} + \delta_u \sum_{j=1}^{n} a_{ij} F_j \quad \text{with } a_{ii} = 0$$

The expression can be modified by replacing the understood value of unity preceding the "single pile deflection" δ_{sp_i} by a relative stiffness factor R, defined as $R = \delta'_{sp_i} / \delta_{sp_i}$ with δ'_{sp_i} = "single pile settlement" derived from a nonlinear pile head t-z curve. This factor R however is changing with increasing δ_{sp_i}. Therefore an iterative procedure for the solution of the set of n=j equations is required. Fortunately offshore pile groups are often placed in a circle. Because of symmetry now yields for the axial case that all F_j are equal: $F_j = F_g / n$ for j = 1...n. This means that only one equation has to be solved iteratively, which can be done by hand. For non-symmetrical pile configurations or for eccentric loads numerical solutions are needed. An imposed rotation at the pile group head which in the elastic case can be taken into account without difficulty, will lead here to more extensive calculation.

Although it does not follow from symmetry, the lateral loading case for a circular configuration will also yield equal forces for the different piles. See Bogard and Matlock, reference [2].

APPENDIX E

GROUND WATER FLOW ANALOGY

Settlement around a pile in a circular area (Figure E1)	Groundwater head around a sink in a circular island (Figure E2)
See Randolph and Wroth (ref.[32])	See Verruijt (ref.[40])
Consider a layer with unit thickness	Idem
kinematic equation: $\gamma = \frac{dz}{dr}$ (1)	
constitutive eq.: $\tau = G\gamma$ (2)	
(1) + (2) → $\tau = G \frac{dz}{dr}$ (3)	Darcy: $v = -k \frac{d\phi}{dr}$ (4)
equilibrium eq.: $\tau \cdot 2\pi r = t$ (5)	continuity eq.: $v \cdot 2\pi r = -Q$ (6)
(3) + (5) + integration →	(4) + (6) + integration →
$z(r) = \frac{1}{G} \frac{t}{2\pi} \int \frac{1}{r} dr$	$\phi(r) = \frac{1}{k} \frac{Q}{2\pi} \int \frac{1}{r} dr$
boundary value: $z(r_m) = 0$ →	boundary value: $\phi(r_m) = 0$ →
$z(r) = \frac{1}{G} \frac{t}{2\pi} \ln \left(\frac{r_m}{r} \right)$	$\phi(r) = \frac{1}{k} \frac{Q}{2\pi} \ln \left(\frac{r_m}{r} \right)$
r = radius (coordinate)	
z = settlement	ϕ = groundwater head
γ = shear	
τ = shear stress	v = superficial velocity
G = shear modulus	k = Darcy constant
t = shaft friction per unit pile length	Q = charge per unit well length

APPENDIX F

MODELLING WITH INTRA

The finite component program "INelastic Tower Response Analysis" has been developped to simulate earthquake loadings on piled offshore structures.
From the set of twelve available elements (components), there are two elements suitable for describing soil behaviour: The "Free Field Soils Element" is only to be used in dynamic analyses. The "Near Field Soils Element" is suitable for the static case. See ref.[11].

Extensive modelling: Consider Figure F1. The piles itself are represented by beam-column elements. At each node a NEAR element is connected, describing:
- a bi-linear axial t-z curve
- a multi-linear (up to 9 steps) lateral p-y curve.

It is supposed that the soil behaves isotropically in lateral direction, therefore only one p-y curve has to be used. A third curve can be prescribed at the tip, to model q-z behaviour (bi-linear).
A more refined shape of t-z curves can be achieved by "misusing" the NEAR elements by a 90-degrees turn which can be realized by changing the position of reference nodes A and B. Even combination from "turned" and standard NEAR elements in one node is possible to describe both t-z and p-y curves exactly. However output now becomes rather disorganized.

Equivalent modelling is possible by means of combination of two NEAR elements as described above.
Leung (ref.[16]) mentioned another way: parallel use of bi-linear elements yields a multi-linear load-deformation curve, see Figure F2.
Probably several other combinations of INTRA elements are possible, e.g. using column-buckling elements (such as the STRT element) to represent softening behaviour.

NOTE: Recently a new INTRA soil element, PSAS, has been devellopped. The main difference with the NEAR element is the possibility to generate default values for t-z and p-y curves based on experimental data. Both t-z and p-y relationships can be described with multi-linear curves.

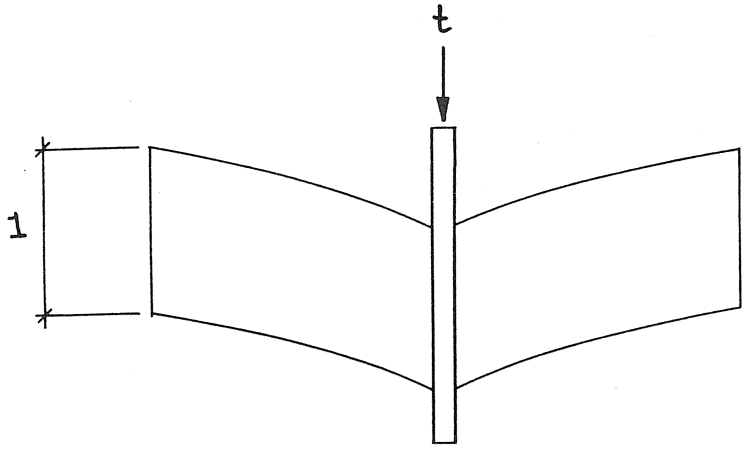


Figure E1
Settlement around a pile
in a circular area

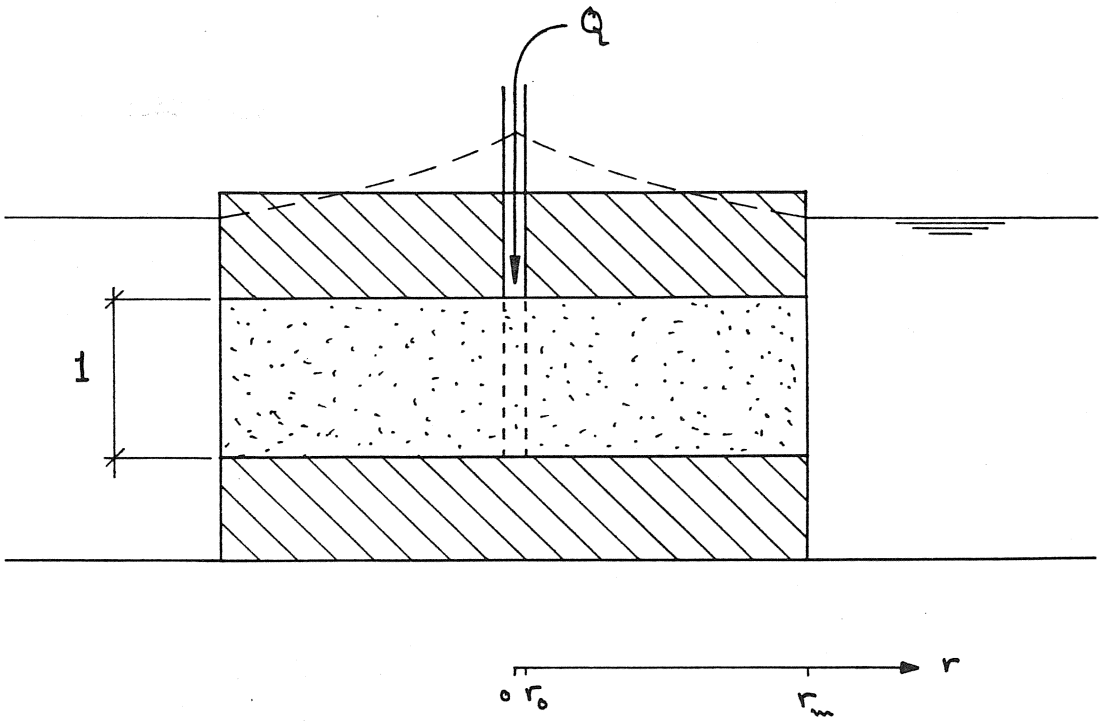


Figure E2
Groundwater head around a sink
in a circular island

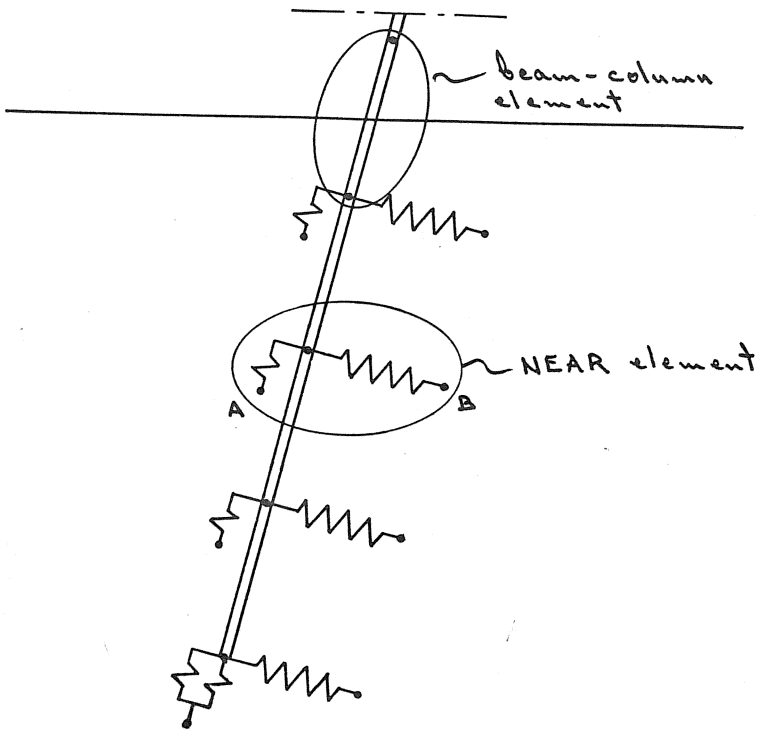
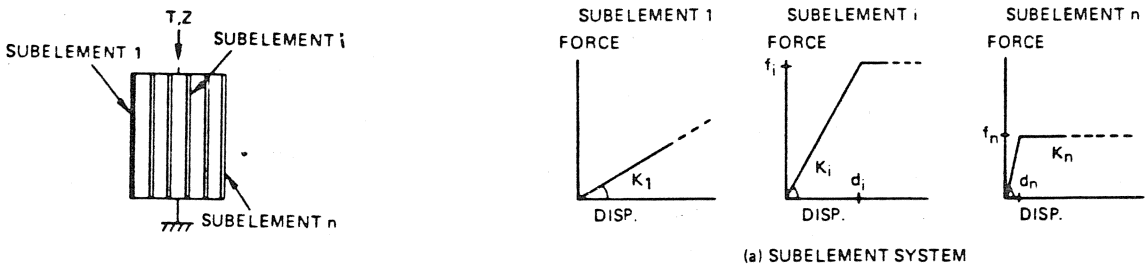


Figure F1

Extensive modelling with INTRA



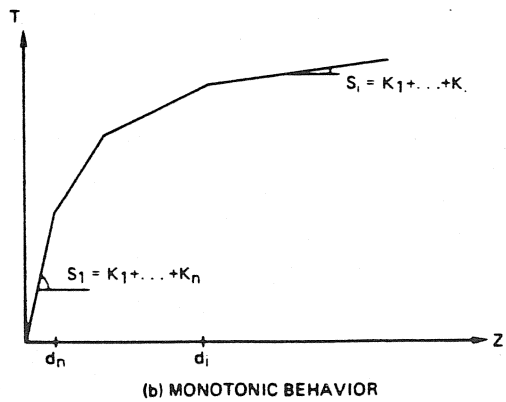


Figure F2

Parallel bilinear elements

(ref.[16])

APPENDIX G

CRITICAL LENGTH FOR INTERMEDIATE MODELING

An offshore pile under a small lateral loading behaves like a beam on elastic foundation. As the loading increases, plastic deformations will occur in the soil around at the top region of the pile, after which an area of yielding soil extends downwards until the foundation collapses because of the devellopment of a plastic hinge in the pile itself.

In order to perform a numerical analysis of the foundation up to failure, it is nescessary to model at least the region where plastic deformation of the soil can be expected. To determine this minimum length consider the following: If soil yields in a certain region, from pile top up to z_0 the resulting horizontal forces are known, see Figure G1.
Scott (ref.[35]) computes pile head deflections by combining this knowledge with an elastic approach for the lower part. However, in this way it is not possible to achieve an explicit value of z_0.

Verruijt (ref.[41]) mentions that the elastic soil properties are of little importance to the establishment of pile head load-deflection curves. He presents an approximate method to compute such curves for a pinned head pile, starting from only plastic behaviour of the soil: The pile below z_0 is neglected. As boundary conditions in that point he suggests zero displacement and zero rotation. At the same time he assumes a bending moment equal to zero. Figure G2 shows the moment distribution according to this assumption. (The yield strength of the soil increases proportional with the depth.)
Verruijt states that the lower part of the pile - a beam on elastic foundation - is stiff with respect to deflection but relatively weak with respect to bending. This means that it attracts few moments, so the assumed boundary values seem reasonable. In addition the support from numerical calculations is mentioned.
Equilibrium now requires:

$$F_y = \frac{1}{6} a z_0^2$$

z_0= depth of plastic area
F_y= lateral load at pile head
a = parameter describing increase of soil strength with depth

The maximum of z_0 is found at time of collapse of the pile:

$$M = \tfrac{1}{6}\, a\, (z^3 - z_0{}^2 \cdot z)$$

$$\frac{dM}{dz} = \tfrac{1}{2}\, a\, z^2 - \tfrac{1}{6}\, a\, z_0{}^2 = 0$$

$$\rightarrow \quad z = \tfrac{1}{3}\sqrt{3}\; z_0$$

$$\rightarrow \quad M_{max} = -\frac{1}{27}\, a\, \sqrt{3}\, z_0{}^3 = M_{pl}$$

$$\rightarrow \quad z_0 = \sqrt[3]{\frac{27\, M_{pl}}{a\, \sqrt{3}}} \simeq \sqrt[3]{16\, \frac{M_{pl}}{a}}$$

M = bending moment in pile

M_{pl} = plastic moment of pile

A comparable computation for the fixed head situation, see Figure G3, yields a slightly lower value for z_0:

$$z_0 \simeq \sqrt[3]{12\, \frac{M_{pl}}{a}}$$

However this value is certainly too low because an actual pile head is never absolutely fixed.

Actual offshore soil profiles are often rather inhomogenous and therefore the soil strength can not be represented by a simple triangular shaped graph.
In this case the minimum length that has to be modelled to achieve acceptable results has to be determined by trial and error.
De Ruiter and Beringen (ref.[34]) state that only the upper ten pile diameters of soil are significant.

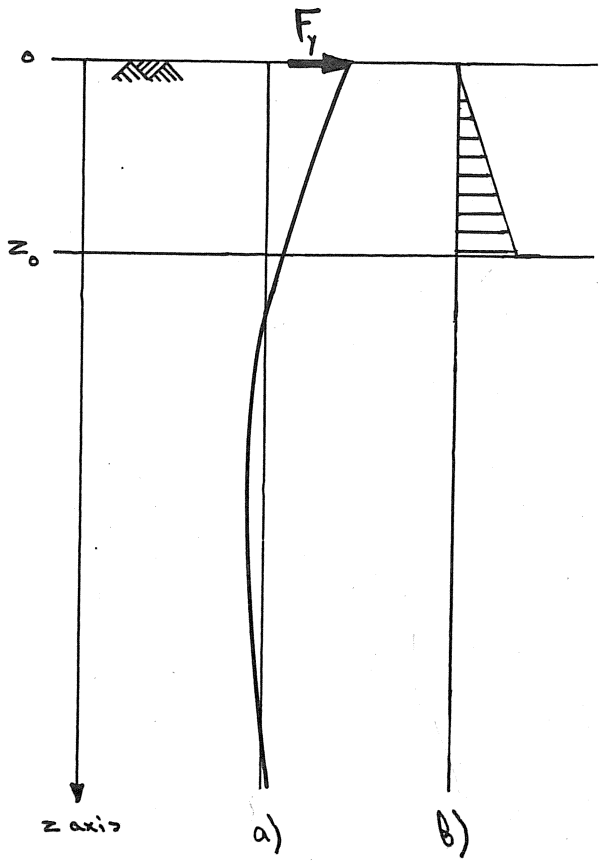


Figure G1

a) Deformation of pile under lateral load

b) Known horizontal forces from top upto z_0

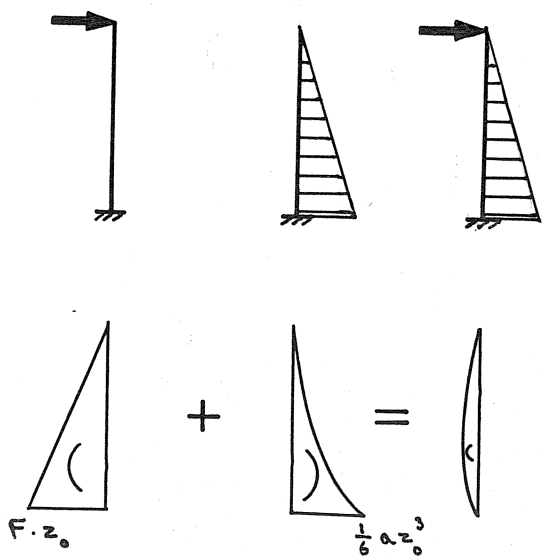


Figure G2

Moment distribution according to assumed boundary conditions (pinned head pile)

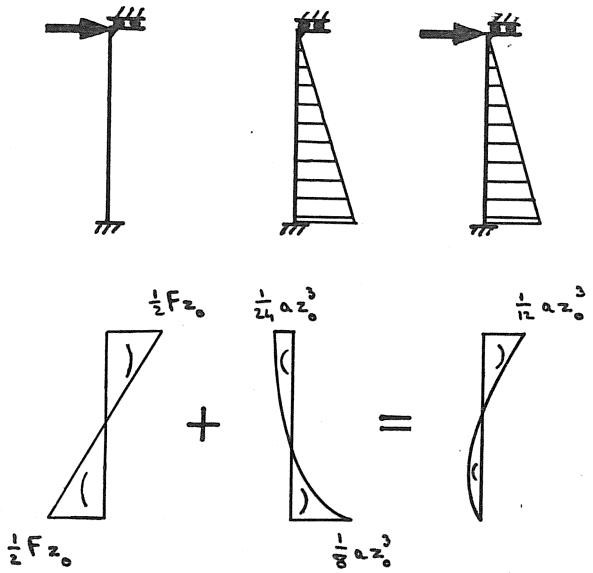


Figure G3
Moment distribution according to assumed boundary conditions (fixed head pile)