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# A practical case study of slope stability analysis using the random finite element method

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**ABSTRACT:** The Random Finite Element Method (RFEM) has been shown in many theoretical publications to offer advantages in the quantification of the probability of failure. However, it has rarely been applied in real situations (geometry, material properties, soil layers) and seldom, if at all, to a well instrumented geotechnical failure. This paper reports a case study of a full-scale controlled dyke failure, where the heterogeneity was previously measured via CPTs (Cone Penetration Tests), and the dyke itself was highly instrumented. This offers the opportunity to compare and apply various techniques previously developed (e.g. random field conditioning) with field data, rather than to computer generated data. The RFEM analyses presented are compared with deterministic analyses, demonstrating the relative performance of the methods.

## 1 INTRODUCTION

A large number of numerical benchmark tests have shown the veracity of the Random Finite Element Method (RFEM) in simulating the probability of failure. However, limited data are available of field tests that can be compared with an RFEM analysis. In particular, the spatial variability (heterogeneity) of material properties has seldom been investigated in field tests. In this paper, the instrumented failure of a dyke, coupled with a site characterisation focused on identifying the site's material variability and heterogeneity in the vertical and horizontal planes, is used to compare deterministic FEM and RFEM analyses.

Several sources of uncertainty may be identified in dyke stability analysis, the two main ones being: (1) natural variability, either temporal or spatial; (2) knowledge uncertainty, relating to model uncertainties and corresponding uncertainties in material properties. In this paper, model uncertainties have not been taken into account, and the boundaries between soil layers have been taken as deterministic. The uncertainties in natural variability have been modelled using RFEM analyses, with the variation in the subsurface within layers being modelled using unconditional random fields or random fields conditioned on local CPT measurements.

In this paper, four analyses are compared for the particular conditions recorded at failure: (1) Unconditional RFEM (UC-RFEM), where the point and spatial statistics are based on local CPTu data; (2) Conditional RFEM (C-RFEM), with random fields conditioned to actual CPTu

measurements; (3) FEM using the average value per layer; (4) FEM using the average value minus one standard deviation per layer. Using these approaches, a range of responses corresponding to the moment of failure are computed and compared.

## 2 FAILURE TEST

In the dyke failure test, over the period of a month the dyke was saturated with water, and soil in front of the toe was excavated in steps and replaced by water, effectively increasing the height of the dyke. In the final stage, the water in the excavation was removed and the dyke failed under its own weight. This, in combination with an extensive site investigation and laboratory testing programme, provided detailed information that was used to investigate the dyke failure both deterministically and stochastically. In the days before the controlled failure, the slopes of the ditch were steepened to 1:1 and the ditch was excavated to the bottom of the peat layer at 2.5 m depth.

In [Figure 1\(a\)](#), the data recorded during the pumping period until failure are shown at the centre cross-section of the excavation. The figure indicates the differential displacements (bold arrows) and excess pore water pressures (vertical arrows). During the experiment, the main failure occurred just south of the centre. Large differential displacements were measured in the toe, below the peat and organic clay boundary, and in the organic clay layer. Measured differential displacements below the crest of the

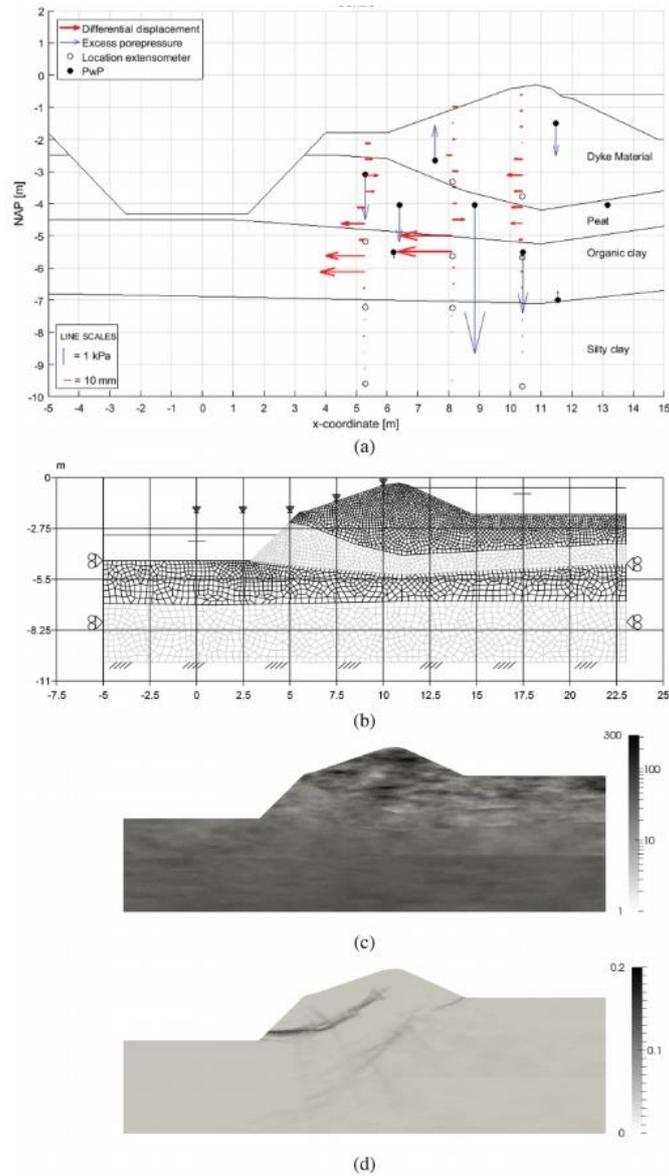


Figure 1. Analysis of the embankment: (a) measured differential displacements and excess pore pressures at the moment of failure; (b) finite element mesh used for analysis, with triangles indicating CPT locations, water level in ditch at NAP -3.15 m and waterlevel upstream at NAP -0.6 m; (c) example of an RFEM analysis, with shear strength illustrated on a logarithmic scale in kPa (d) the calculated shearbands as contours of deviatoric strain.

dyke were distributed near the boundary between the peat and dyke material. The failure occurred between a drawdown of 1.5 m and 2.0 m; the precise water level at failure could not be determined, but was estimated to be 1.6 m, i.e. NAP -3.5 m.

### 3 MATERIAL PARAMETERS

A dataset of CPT and laboratory test data had been collected to investigate and determine the vertical and horizontal heterogeneity. This dataset was taken over an area of  $50 \times 15 \text{ m}^2$  in the immediate vicinity of the dyke failure, and included 100 CPTu tests that were obtained over a two week period. De Gast et al. (2017) evaluated and showed that the vertical heterogeneity under the dyke was influenced by compression of the material.

From the data at the cross-section of the failure location, both the vertical and horizontal scales of fluctuation were determined using the CPTs. The

average horizontal interval of the CPTs was 2.5 m (perpendicular to the dyke). Due to the deposition history, it is generally assumed that the horizontal scale of fluctuation is larger than the vertical scale of fluctuation, so that the spacing of data in the horizontal plane was anticipated to be acceptable. The scale of fluctuation was estimated using the method elaborated in Gast et al. (2017).

The shear strength  $s_u$  in kPa was determined using

$$s_u = \frac{q_t - \sigma_v}{N_{kt}} \quad (1)$$

Where  $q_t$  is the total cone resistance (kPa),  $\sigma_v$  is the total vertical stress (kPa) and  $N_{kt}$  is an empirical correction factor (-). Values for  $N_{kt}$  suggested by Robertson (2009) range from 10–20 and, by comparing the CPT and laboratory data (20 consolidated undrained triaxial tests and four direct simple shear tests), the  $N_{kt}$  values for the different

materials were refined. The values of  $N_{kt}$  for the four materials are: (1) dyke material,  $N_{kt} = 20$ ; (2) peat,  $N_{kt} = 15$ ; (3) organic clay,  $N_{kt} = 10$ ; and (4) silty clay,  $N_{kt} = 10$ .

Table 1 presents the results of the analysis of the CPT data. These comprise the mean shear strength, the standard deviation of shear strength, and estimates of the vertical and horizontal scales of fluctuation,  $\theta_v$  and  $\theta_h$ , respectively. Note that the mean and standard deviation of the shear strength were obtained from the dataset before de-trending.

#### 4 RFEM VS FEM

To account for the spatial variability of the soil parameters, FEM has been combined with random field theory within a stochastic (Monte Carlo) process. This involves multiple simulations (i.e. realisations) of the same problem, a procedure often referred to as RFEM. In each realisation of an RFEM analysis, a random field of material properties is generated, based on the point and spatial statistics of the material properties. The method has proven to be an efficient approach for conducting stochastic slope stability analyses (e.g. Hicks & Samy Hicks & Samy 2002).

Spatial variation has been modelled by random fields generated using covariance matrix decomposition, with local averaging for unstructured meshes (van den Eijnden & Hicks 2017). This method starts by generating a field with a standard normal distribution, in which the spatial variation of property values is related to a correlation function incorporating the scales of fluctuation. The standard normal field is then transformed to the appropriate distribution based on the mean and standard deviation of the variable being modelled.

In this paper, only the undrained shear strength is spatially random, while other parameters are assumed to be constant. In the first RFEM analysis the spatially random undrained shear strength has been generated only from the input statics ( $\mu$ ,  $\sigma$ ,  $\theta_v$ ,  $\theta_h$ ); in the other RFEM analysis the uncertainty has been reduced by conditioning the spatially random undrained shear strength to CPT data (Li et

al. 2016). Both the conditional and unconditional RFEM analyses assume a lognormal strength distribution. Moreover, for each analysis four different material layers, with each layer having its own random field, were discretised using a mesh of 4250 eight-node elements, with each element using  $2 \times 2$  Gaussian integration points. Nearer to the top of the mesh (where most of the failure mechanism was expected) a finer mesh was used. On each side of the dyke a load was applied representing the waterload at the time of failure; on the right side, the waterlevel was at NAP  $-0.6$  m and on the left side (the excavation) the waterlevel was at NAP  $-3.5$  m, i.e. equal to a drawdown of 1.6 m. Figure 1(b) shows the mesh used in the analyses, indicating the waterlevels at both sides of the dyke. Figure 1(c) shows one of the RFEM realisations, indicating the shear strength, and Figure 1(d) indicates the calculated shear strains at failure of the same realisation.

Figure 2 presents the results of the analyses in terms of safety factor (SF). Specifically, the solid curve is the unconditional RFEM analysis and the broken curve is the conditional RFEM analysis, with each curve based on the results of 400 realisations; the dotted line with crosses is the FEM analysis using the mean strength value and the solid line with circles is the FEM analysis using a strength estimate one standard deviation below the mean.

The unconditional RFEM analysis shows the largest range of solutions for SF, from 0.47 to 1.15, with an average SF of 0.83. The conditional RFEM analysis shows a reduced range of SF, from 0.76 to 1.09, with a higher average SF of 0.93. The deterministic FEM analysis based on mean strength values gives SF = 1.16 and the FEM analysis using strengths of one standard deviation lower than the mean gives SF = 0.73.

Table 1. Material parameters based on CPT data at the failure location.

	$S_{\mu,\mu}$ [kPa]	$S_{\mu,\sigma}$ [kPa]	$\theta_v$ [m]	$\theta_h$ [m]
Dyke material	19.5	16.4	0.4	2.13
Peat	10.4	5.7	0.76	2.84
Organic clay	14.9	3.9	0.76	2.84
Silty clay	22.2	3.8	0.26	2.1

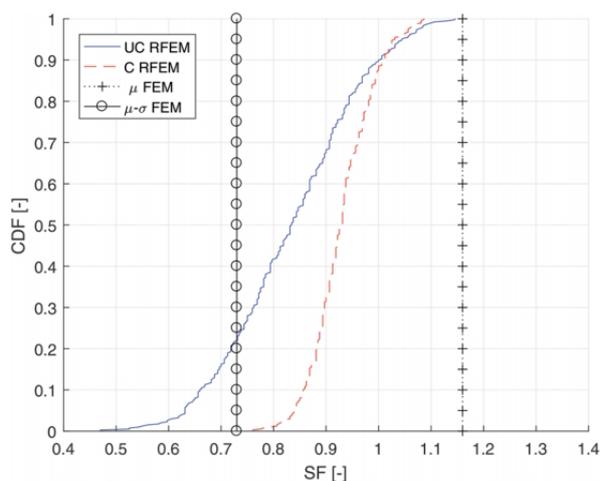


Figure 2. Analyses results for safety factor (SF).

## 5 DISCUSSION

As previously discussed, the analyses are undertaken for the conditions when the dyke failed; therefore, SF would be expected to be approximately 1. In the deterministic FEM analysis the SF using the mean shear strength is 1.16, overestimating the safety of the dyke. Comparing this result to the RFEM analyses, the mean SFs predicted (at CDF = 0.5) are 0.83 and 0.93 for the unconditional and conditional RFEM analyses, respectively. This means that, for this example, the conditional RFEM analysis has led to a calculated SF almost indistinguishable from reality and therefore gives confidence in using this method to calculate the slope reliability.

Both the conditional and unconditional RFEM analyses compute lower SF values than the deterministic FEM result based on the mean. This is as expected, due to the failure passing through weaker zones of the materials, as has been previously reported by Hicks & Samy (2002) and Hicks & Spencer (2010). The distribution of SF calculated by the conditional RFEM is significantly narrower than for the unconditional RFEM, also as expected, due to the variation in the spatial distribution of the material parameters being smaller (while the variation in the point statistics is the same).

Note that at the 95% confidence level, the unconditional RFEM analysis has an SF of 0.63, whereas for the conditional RFEM analysis it is 0.84. This has a significant implication for the assessment of dykes, as it could make the difference between a dyke being assessed as reliably safe or requiring costly improvement.

Further work is needed to investigate the impact of this work on 3D failures; so far only 2D stochastic analyses have been carried out. The methodology to incorporate CPT data in 3D slope stability assessments, including the possible impact on slope reliability, has been theoretically investigated by Li et al. (2016). Moving from 2D to 3D simulations has been investigated deterministically as part of this research (details not presented here), and from 2D to 3D incorporating spatial variability by Li et al. (2015) and Varkey et al. (2017). In the deterministic, case the SF increases by  $\approx 15\%$  when moving from 2D to 3D, due to the impact of the sides of the failure surface; however, in cases where the spatial variability has been incorporated, it may even be possible for the SF to decrease by several percent, depending on the scale of fluctuation in the longitudinal direction of the dyke (relative to the dyke length).

Additional information from measurements can also be incorporated into analyses, further reducing the variation in the calculated SF, especially on the hydro-mechanical behaviour and impact of a variable phreatic surface, as demonstrated by Vardon et al. (2016). However, it is noted that with all additional measurements and analyses, there

are financial and time implications. Therefore, a cost benefit judgement must be made.

## 6 CONCLUSION

A real dyke failure has been induced and monitored, with the conditions of the dyke at failure being used in a series of comparative numerical analyses using deterministic FEM and RFEM approaches. The observed and calculated failure modes are similar, although the computed safety factors differ significantly depending on the adopted approach (deterministic FEM versus RFEM) and on the relative use of data (i.e. conditional versus unconditional analysis). By incorporating spatial variability the confidence in the stability can be calculated, and by incorporating additional measurements the confidence can be increased. This leads to a higher calculated reliability and more efficient design.

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