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Trajectory Tracking for Robotic Arms with Input Saturation and Only Position Measurements

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***Abstract*— This paper proposes a passivity-based control approach that addresses the trajectory tracking problem for a class of mechanical systems that comprises a broad range of robotic arms. The resulting controllers can be naturally saturated and do not require velocity measurements. Moreover, the proposed methodology does not require the implementation of observers, and the structure of the closed-loop system permits the construction of a Lyapunov function, which eases the convergence analysis. To corroborate the effectiveness of the methodology, we perform experiments with the Philips Experimental Robot Arm.**

I. Introduction

Customarily, robotic arms are modeled as mechanical systems [1], [2], [3], where energy-based modeling approaches, such as the Euler-Lagrange (EL) or the port-Hamiltonian (pH) one, have proven suitable to represent the behavior of these systems. In particular, pH models underscore the roles of the energy, the interconnection pattern, and the dissipation of the system [4], [5], which are the main components of passivity-based control (PBC) [6]. Hence, this control approach represents a suitable methodology to control complex nonlinear mechanical systems.

The literature on trajectory tracking for fully actuated mechanical systems is abundant. We refer the reader to [7], [8], [9] and the references therein contained for a detailed exposition on this topic. However, the implementation of the methodologies that address this control problem is often hampered by the necessity of high gains to ensure stability, or some practical issues like the lack of sensors to measure velocities, the necessity of considering bounded inputs to protect the actuators, and performance requirements. Concerning the requirement of control laws without velocity measurements, in [10], the authors propose a solution that ensures *global uniform exponential* convergence towards the desired trajectories, where the velocities are estimated via an *immersion and invariance* observer. Another related reference is [11], where the authors design a controller that achieves trajectory tracking with only position measurements by proposing a dynamic extension to remove the necessity of velocity measurements. On the other hand, in [12], the authors propose saturated control laws that guarantee *global uniform asymptotic* convergence towards the desired trajectories. However, to this end, velocity measurements are needed. Finally, in [13], the authors propose controllers that achieve *semi-global uniform asymptotic* convergence to the desired trajectories avoiding velocity measurements while ensuring that the inputs are bounded.

The main contribution of this work is a PBC approach that addresses the trajectory tracking problem for a class of fully actuated mechanical systems while considering saturated inputs and avoiding velocity measurements. Therefore, the proposed methodology offers an alternative to achieve trajectory tracking while considering physical limitations often neglected in the literature. To this end, we propose a pre-feedback that allows us to express the error dynamics as a pH system. Then, we devise an energy-shaping approach that guarantees saturated control laws and a dynamic extension that permits the injection of damping without measuring the velocities of the system. The proposed methodology is an extension of the controllers reported in [14]. Nevertheless, we stress that such an extension is far from trivial since the closed-loop system is nonautonomous in the trajectory tracking problem. Hence, in contrast with the problem studied in [14], La Salle's arguments are not suitable for the stability analysis. The proposed methodology differs from [10], [11], [12], [13] in the following aspects: (i) it tackles the input saturation and no velocity measurements problems simultaneously; (ii) no observers are required; (iii) the stability properties are *global*; (iv) the pH framework leads naturally to a Lyapunov function, which significantly simplifies the stability analysis.

The remainder of this paper is organized as follows: Section II contains the preliminaries and problem formulation. Section III is devoted to the design of the controllers that solve the trajectory tracking problem. In Section IV, we illustrate the applicability of the proposed methodology via its implementation in the PERA system. Finally, we present the concluding remarks and future work in Section V.

II. Preliminaries and problem formulation

This section discusses the mathematical modeling of the PERA system in the pH framework and the problem formulation. Additionally, we briefly revisit the partial linearization via change of coordinates (PLvCC) of pH mechanical systems and Barbalat's lemma.

A. pH representation of fully actuated mechanical systems

We consider mechanical systems that can be represented by the following pH model

$$\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{n\times n} & I_n\\ -I_n & \mathbf{0}_{n\times n}\end{bmatrix}\begin{bmatrix}\frac{\partial H}{\partial q}(q,p)\\ \frac{\partial H}{\partial p}(q,p)\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{n\times n}\\ I_n\end{bmatrix} u,$$
$$H(q,p) = \tfrac{1}{2}p^\top M^{-1}(q)p + V(q), \quad y = M^{-1}(q)p = \dot{q}, \tag{1}$$

where $q, p \in \mathbb{R}^n$ denote the generalized positions and momenta, respectively, $u \in \mathbb{R}^n$ is the input, $M : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the inertia matrix, which is positive definite and all its entries are assumed to be $\mathcal{C}^1$ functions, $V : \mathbb{R}^n \to \mathbb{R}_+$ denotes the potential energy of the system, $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$ is the system's Hamiltonian, and $y \in \mathbb{R}^n$ is the *passive* output.

The following assumptions characterize the class of systems for which the methodology introduced in Section III is suitable.

Assumption 1: The term $\frac{\partial V}{\partial q}(q)$ is *bounded* from above and from below.

Assumption 2: For every bounded trajectory $q(t)$ of (1), the inertia matrix $M(q(t))$ and its (element-wise) time derivative are bounded, i.e., $\|M(q(t))\| < \infty$, $\left\|\frac{dM}{dt}(q(t))\right\| < \infty$, where $\|(\cdot)\|$ denotes the spectral norm of a matrix.

Remark 1: A broad range of mechanical systems satisfies Assumptions 1 and 2. For instance, for robotic manipulators with only revolute joints, the non-constant entries of $M(q(t))$ and the elements of $\frac{\partial V}{\partial q}(q)$ are sines and cosines, which satisfy the mentioned assumptions. For further details, see Section 9.5.2 in [1].

Below, we present the definition of feasible trajectory, which is necessary to formulate the problem to be solved.

Definition 1 (Feasible trajectory): A trajectory $q = q_d(t)$ is *feasible* if there exists a control input $u = u_d(t)$ such that the pair $(q_d(t), u_d(t))$ solves (1).

Henceforth, we assume that the desired trajectories $q_d(t)$, $\dot{q}_d(t)$ are smooth and bounded.

Problem setting. Given the desired feasible trajectory $q_d(t)$, find a control law such that the trajectories of (1) converge to $q_d(t)$, and the corresponding $p_d(t)$, while ensuring that:

- **C1** The control law does not depend on p.
- **C2** The control signals satisfy $u_i(t) \in [\mathcal{U}_{\mathtt{min}_i}, \mathcal{U}_{\mathtt{max}_i}]$ for all $t \geq 0$, with $i = 1, \ldots, n$, and the constants $\mathcal{U}_{\mathtt{min}_i}, \mathcal{U}_{\mathtt{max}_i}$ satisfy $\mathcal{U}_{\mathtt{min}_i} < \mathcal{U}_{\mathtt{max}_i}$.

B. PLvCC of pH systems

Let $\Psi : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ be a factor of $M^{-1}(q)$, i.e.,

$$M^{-1}(q) = \Psi(q)\Psi^\top(q).$$

Note that, since $M^{-1}(q)$ has *full rank*, $\Psi(q)$ has full rank as well. Define the new coordinates $\mathtt{P} := \Psi^\top(q)p$. Then, (1) can be rewritten as

$$\begin{bmatrix}\dot{q}\\ \dot{\mathtt{P}}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{n\times n} & \Psi(q)\\ -\Psi^\top(q) & J(q,\mathtt{P})\end{bmatrix}\begin{bmatrix}\frac{\partial \bar{H}}{\partial q}(q,\mathtt{P})\\ \frac{\partial \bar{H}}{\partial \mathtt{P}}(q,\mathtt{P})\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{n\times n}\\ \Psi^\top(q)\end{bmatrix} u$$
$$\bar{H}(q,\mathtt{P}) = \tfrac{1}{2}\mathtt{P}^\top\mathtt{P} + V(q) \quad y = \Psi(q)\frac{\partial \bar{H}}{\partial \mathtt{P}}(q,\mathtt{P}) = \dot{q} \tag{2}$$

where $J : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is a *skew-symmetric matrix* representing the gyroscopic forces present in the system [10], and whose elements are given by

$$J_{ij}(q,\mathtt{P}) = -\mathtt{P}^\top\Psi^{-1}(q)\left[\Psi_i(q), \Psi_j(q)\right],$$

where $[\cdot,\cdot]$ denotes the standard Lie bracket [15]. For a thorough exposition on PLvCC, we refer the reader to [16].

C. Barbalat's lemma

The stability proofs contained in Section III are based on Barbalat's lemma, which we present below to ease the readability of this paper.

Lemma 1: Consider a function $f : \mathbb{R} \to \mathbb{R}$ *uniformly continuous* on the interval $[0, \infty)$. Suppose that

$$\lim_{t\to\infty}\int_0^t f(\tau)d\tau = \phi < \infty.$$

Then, $f(t) \to 0$ as $t \to \infty$.
The proof of Lemma 1 may be found in [17].

III. Control design

This section is devoted to the control design, where the main idea is to split the controller into two parts: (i) a control signal that lets us express the dynamics of the errors, between the system's trajectories and the desired ones, as a pH system. Then, following the results reported in [14], (ii) a controller that renders the origin of that pH system globally uniformly asymptotically stable while satisfying **C1** and **C2**.

A. Boundedness

When dealing with nonautonomous systems, particularly when applying Barbalat's lemma, it is fundamental to prove that the functions involved are bounded. Hence, before proceeding with the control design, we introduce the following assumption, which is instrumental for the stability proofs contained in this section.

Assumption 3: All the entries of $\Psi(q(t))$, in (2) are $\mathcal{C}^1$ functions. Moreover, for every bounded trajectory $q(t)$ of (2), the following holds:

$$\|\Psi(q(t))\| < \infty, \ \left\|\frac{d\Psi}{dt}(q(t))\right\| < \infty, \ \left\|\frac{d\Psi^{-1}}{dt}(q(t))\right\| < \infty.$$

Remark 2: Similar to Assumption 2, a large class of mechanical systems satisfy Assumption 3. Indeed, from the observation provided in Remark 1, if all the joints of a robotic arm are revolute, Assumption 3 is satisfied.

B. Desired dynamics and error system

Given the desired feasible trajectory $q_d(t)$, we can compute $\dot{q}_d(t)$ and $\ddot{q}_d(t)$. However, $q_d(t)$ is a particular solution to (2). Hence, we can define the following desired dynamics[1]

$$\begin{aligned} \dot{q}_d &= \Psi(q)\mathtt{P}_d \\ \dot{\mathtt{P}}_d &= -\Psi^\top(q)\tfrac{\partial V}{\partial q}(q_d) + J(q,\mathtt{P})\mathtt{P}_d + \Psi^\top(q)u_d(t) \end{aligned} \tag{3}$$

where $u_d(t)$ corresponds to the input that ensures the system continues tracking the trajectories $q_d, \mathtt{P}_d$ once it has reached them, i.e., $q = q_d, \mathtt{P} = \mathtt{P}_d$. Accordingly, similar to [7], [8], we compute the desired $u_d(t)$ by fixing $q = q_d, \mathtt{P} = \mathtt{P}_d$, and their corresponding time derivatives in (2). Hence,

$$\begin{aligned} u_d(t) = \Psi^{-\top}(q_d)\left[\tfrac{d}{dt}\left(\Psi^{-1}(q_d)\dot{q}_d\right) - J_d(t)\Psi^{-1}(q_d)\dot{q}_d\right] \\ + \frac{\partial V}{\partial q}(q_d), \end{aligned} \tag{4}$$

where the elements of the matrix $J_d(t)$ are given by

$$J_{d_{ij}}(t) = -\dot{q}_d^\top M(q_d)[\Psi_i(q_d), \Psi_j(q_d)].$$

Notice that q_d is bounded. Then, Assumption 3 ensures that all the eigenvalues of $\Psi(q_d)$ are bounded. Moreover, since this matrix has full rank, $\Psi^{-1}(q_d)$ is bounded as well. This, in combination with Assumption 3, guarantees that $u_d(t)$ is bounded.

The next step in the control design is to transform the tracking problem into a stabilization one. Towards this end, we define the errors

$$\tilde{q} := q - q_d, \quad \widetilde{\mathtt{P}} := \mathtt{P} - \mathtt{P}_d, \quad \tilde{u} := u - u_d(t). \tag{5}$$

Therefore, from (2), (3), and (5), we get

$$\begin{aligned} \dot{\tilde{q}} &= \Psi(q)\widetilde{\mathtt{P}} \\ \dot{\widetilde{\mathtt{P}}} &= \Psi^\top(q)\left[\tilde{u} + \tfrac{\partial V}{\partial q}(q_d) - \tfrac{\partial V}{\partial q}(q)\right] + J(q,\mathtt{P})\widetilde{\mathtt{P}} \end{aligned} \tag{6}$$

Then, to express (6) as a pH system, we fix

$$\tilde{u} = \frac{\partial V}{\partial q}(q) - \frac{\partial V}{\partial q}(q_d) + \hat{u}. \tag{7}$$

Thus, substituting (7) in (6) yields

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{q}} \\ \dot{\widetilde{\mathtt{P}}} \end{bmatrix} &= \begin{bmatrix} \mathbf{0}_{n\times n} & \Psi(q) \\ -\Psi^\top(q) & J(q,\mathtt{P}) \end{bmatrix} \begin{bmatrix} \frac{\partial \widetilde{H}}{\partial \tilde{q}}(\widetilde{\mathtt{P}}) \\ \frac{\partial \widetilde{H}}{\partial \widetilde{\mathtt{P}}}(\widetilde{\mathtt{P}}) \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{n\times n} \\ \Psi^\top(q) \end{bmatrix} \hat{u} \\ \widetilde{H}(\widetilde{\mathtt{P}}) &= \tfrac{1}{2}\widetilde{\mathtt{P}}^\top\widetilde{\mathtt{P}}, \quad \tilde{y} = \Psi(q)\tfrac{\partial \widetilde{H}}{\partial \widetilde{\mathtt{P}}}(\widetilde{\mathtt{P}}) = \dot{\tilde{q}} \end{aligned} \tag{8}$$

Note that by designing $\hat{u}$, in (8), such that the closed-loop system has a uniformly asymptotically stable equilibrium at $(\mathbf{0}_n, \mathbf{0}_n)$, we guarantee that $q \to q_d, \mathtt{P} \to \mathtt{P}_d$ as $t \to \infty$.

[1]We omit the argument t to simplify the notation.

C. Control without velocity measurements

The asymptotic stabilization problem of (8) may be addressed by performing an energy-shaping plus damping injection process. Nevertheless, the latter requires information—measurements—of $\widetilde{\mathtt{P}}$, and consequently of $\dot{q}$, which is often a nonmeasurable signal. To overcome this issue, we propose the controller state $x_c \in \mathbb{R}^n$ with dynamics, see [11], [13], [14],

$$\dot{x}_c = -R_c\left(K_I z + K_c x_c\right), \tag{9}$$

where the matrices $R_c, K_c, K_I \in \mathbb{R}^{n\times n}$ are positive definite, and $z \in \mathbb{R}^n$ is defined as

$$z(\tilde{q}, x_c) := \tilde{q} + x_c. \tag{10}$$

The following proposition provides a controller that solves the global uniform asymptotic stabilization problem for (8) without velocity measurements.

Proposition 1: Consider the augmented state vector $[\tilde{q}^\top, \widetilde{\mathtt{P}}^\top, x_c^\top]^\top$, with dynamics (6)-(9). Then, the control law

$$\hat{u} = -K_I z \tag{11}$$

ensures that $(\tilde{q}_*, \widetilde{\mathtt{P}}_*, x_{c_*}) = (\mathbf{0}_n, \mathbf{0}_n, \mathbf{0}_n)$ is a *globally uniformly asymptotically stable equilibrium* point for the closed-loop system.

Proof: Consider the function[2]

$$\widetilde{H}_d(\tilde{q}, \widetilde{\mathtt{P}}, x_c) = \frac{1}{2}z^\top K_I z + \frac{1}{2}\widetilde{\mathtt{P}}^\top\widetilde{\mathtt{P}} + \frac{1}{2}x_c^\top K_c x_c.$$

Note that $\widetilde{H}_d(\mathbf{0}_n, \mathbf{0}_n, \mathbf{0}_n) = 0$ and $\widetilde{H}_d(\tilde{q}, \widetilde{\mathtt{P}}, x_c) > 0$ for all $(\tilde{q}, \widetilde{\mathtt{P}}, x_c) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n - \{\mathbf{0}_n, \mathbf{0}_n, \mathbf{0}_n\}$. Thus, $\widetilde{H}_d(\tilde{q}, \widetilde{\mathtt{P}}, x_c)$ is *positive definite* with respect to the equilibrium. Furthermore, substituting (11) in (8), the dynamics of the augmented state vector take the form

$$\begin{bmatrix} \dot{\tilde{q}} \\ \dot{\widetilde{\mathtt{P}}} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{n\times n} & \Psi(q) & \mathbf{0}_{n\times n} \\ -\Psi^\top(q) & J(q,\mathtt{P}) & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & -R_c \end{bmatrix} \begin{bmatrix} \frac{\partial \widetilde{H}_d}{\partial \tilde{q}}(\tilde{q}, \widetilde{\mathtt{P}}, x_c) \\ \frac{\partial \widetilde{H}_d}{\partial \widetilde{\mathtt{P}}}(\tilde{q}, \widetilde{\mathtt{P}}, x_c) \\ \frac{\partial \widetilde{H}_d}{\partial x_c}(\tilde{q}, \widetilde{\mathtt{P}}, x_c) \end{bmatrix}.$$

Hence, since $J(q,\mathtt{P})$ is skew-symmetric,

$$\dot{\widetilde{H}}_d = -\left(\frac{\partial \widetilde{H}_d}{\partial x_c}(\tilde{q}, \widetilde{\mathtt{P}}, x_c)\right)^\top R_c \left(\frac{\partial \widetilde{H}_d}{\partial x_c}(\tilde{q}, \widetilde{\mathtt{P}}, x_c)\right) \leq 0. \tag{12}$$

Note that $\widetilde{H}_d(\tilde{q}, \widetilde{\mathtt{P}}, x_c)$ is *radially unbounded*, which, in combination with (12), ensures that $\tilde{q}, \widetilde{\mathtt{P}}$, and x_c are *bounded*. Thus, since we consider that $q_d(t)$ is bounded, we get that q, $\mathtt{P}$, and z are bounded. Thus, Assumption 3 guarantees that $\|\Psi(q)\| < \infty$ and $\|J(q,\mathtt{P})\| < \infty$, which together with the boundedness of the state and the pH structure of the closed-loop system, imply that $\dot{\tilde{q}}$, $\dot{\widetilde{\mathtt{P}}}, \dot{x}_c$, and $\dot{z}$ are bounded as well. Now, differentiating the dynamics of $\tilde{q}$, we get

$$\ddot{\tilde{q}} = \left(\frac{d\Psi}{dt}(q)\right)\widetilde{\mathtt{P}} + \Psi(q)\dot{\widetilde{\mathtt{P}}},$$

[2]We omit the argument $(\tilde{q}, x_c)$ from z to simplify the notation.

which, from Assumption 3, is bounded. Accordingly, $\dot{\tilde{q}}$ is *uniformly continuous*. Hence, since $\tilde{q}$ is bounded, it follows from Barbalat's lemma that $\dot{\tilde{q}} \to \mathbf{0}_n$ as $t \to \infty$. Furthermore, $\dot{\tilde{q}} \to \mathbf{0}_n$ implies $\widetilde{\mathtt{P}} \to \mathbf{0}_n$ as $t \to \infty$.

Differentiating the dynamics of $\widetilde{\mathtt{P}}$, we obtain

$$\ddot{\widetilde{\mathtt{P}}} = -\left(\frac{d\Psi^\top}{dt}(q)\right) K_I z - \Psi(q) K_I \dot{z} + J(q,\mathtt{P})\dot{\widetilde{P}} + \left(\frac{dJ}{dt}(q,\mathtt{P})\right)\widetilde{P}.$$

Therefore, given the arguments above about the boundedness of the states and their derivatives, $\ddot{\widetilde{\mathtt{P}}}$ is bounded, and consequently, $\dot{\widetilde{\mathtt{P}}}$ is uniformly continuous. Moreover, since $\widetilde{\mathtt{P}}$ tends to zero, it follows from Barbalat's lemma that $\dot{\widetilde{\mathtt{P}}} \to \mathbf{0}_n$ as $t \to \infty$. Hence, we have the following chain of implications.

$$\dot{\widetilde{\mathtt{P}}} \to \mathbf{0}_n \implies \Psi^\top(q) K_I z \to \mathbf{0} \implies z \to \mathbf{0}_n, \tag{13}$$

as $t \to \infty$. Now, note that, since $\dot{z}$ and $\dot{x}_c$ are bounded, $\dot{\widetilde{H}}_d$ is uniformly continuous. Furthermore, $\widetilde{H}_d(\tilde{q}, \widetilde{\mathtt{P}}, x_c)$ is a positive function that is decreasing. Therefore, for bounded initial conditions, $\lim_{t\to\infty} \widetilde{H}_d(\tilde{q}(t), \widetilde{\mathtt{P}}(t), x_c(t)) < \infty$.

Hence, again, invoking Barbalat's lemma we have

$$\dot{\widetilde{H}}_d \to 0 \quad \implies \quad K_c x_c + K_I z \to \mathbf{0}_n \quad \implies \quad x_c \to \mathbf{0}_n$$

as $t \to \infty$, where we used (13). Furthermore, substituting $x_c \to \mathbf{0}_n$ and (13) in the definition of z, given in (10), we get that $\tilde{q} \to \mathbf{0}_n$ as $t \to \infty$, which completes the proof. ■

The following remark provides a practical intuition interpretation of the state x_c.

Remark 3: The expressions(9), (10) can be interpreted as the state-space representation of a *dirty-derivative* filter, as is explained in [18]. Moreover, in (12), we observe that the closed-loop system exhibits damping in terms of $\frac{\partial \widetilde{H}_d}{\partial x_c}(\tilde{q}, \widetilde{\mathtt{P}}, x_c)$, which, remarkably, depends only on q and x_c.

D. Saturated control without velocity measurements

In this subsection, we modify the controller (11) to ensure that the control signals comply with the saturation imposed in **C2**, given in the problem formulation. Towards this end, consider the controller state $x_c \in \mathbb{R}^n$ with dynamics

$$\dot{x}_c = -R_c\left(\sum_{i=1}^{n} e_i \alpha_i \tanh(\beta_i z_i) + K_c x_c\right),\ i = 1, \ldots, n; \tag{14}$$

where e_i denotes an element of the canonical basis of $\mathbb{R}^n$, the constant parameters α_i, β_i are positive, the matrices $R_c, K_c \in \mathbb{R}^{n\times n}$ are positive definite, and z is defined as in (10). The following proposition provides a saturated control law that addresses the global uniform asymptotic stabilization problem of (8), which does not require velocity measurements.

Proposition 2: Consider the augmented state vector $[\tilde{q}^\top, \widetilde{\mathtt{P}}^\top, x_c^\top]^\top$, with dynamics (6)-(14). Then, the control law

$$\hat{u} = -\sum_{i=1}^{n} e_i \alpha_i \tanh(\beta_i z_i)$$

ensures that the closed-loop system has a globally uniformly asymptotically stable equilibrium at $(\tilde{q}, \widetilde{\mathtt{P}}, x_c) = (\mathbf{0}_n, \mathbf{0}_n, \mathbf{0}_n)$ with Lyapunov function

$$\widetilde{H}_{\mathtt{sat}}(\tilde{q}, \widetilde{\mathtt{P}}, x_c) = \sum_{i=1}^{n} \frac{\alpha_i}{\beta_i} \ln(\cosh(\beta_i z_i)) + \frac{1}{2}\widetilde{\mathtt{P}}^\top \widetilde{\mathtt{P}} + \frac{1}{2} x_c^\top K_c x_c.$$

Proof: Note that $\widetilde{H}_{\mathtt{sat}}(\tilde{q}, \widetilde{\mathtt{P}}, x_c)$ is positive definite with respect to the equilibrium point and is *radially unbounded*. Moreover, the closed-loop system takes the form

$$\begin{bmatrix} \dot{\tilde{q}} \\ \dot{\widetilde{\mathtt{P}}} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{n\times n} & \Psi(q) & \mathbf{0}_{n\times n} \\ -\Psi^\top(q) & J(q,\mathtt{P}) & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & -R_c \end{bmatrix} \begin{bmatrix} \frac{\partial \widetilde{H}_{\mathtt{sat}}}{\partial \tilde{q}}(\tilde{q}, \widetilde{\mathtt{P}}, x_c) \\ \frac{\partial \widetilde{H}_{\mathtt{sat}}}{\partial \widetilde{\mathtt{P}}}(\tilde{q}, \widetilde{\mathtt{P}}, x_c) \\ \frac{\partial \widetilde{H}_{\mathtt{sat}}}{\partial x_c}(\tilde{q}, \widetilde{\mathtt{P}}, x_c) \end{bmatrix},$$

and

$$\dot{\widetilde{H}}_{\mathtt{sat}} = -\left(\frac{\partial \widetilde{H}_{\mathtt{sat}}}{\partial x_c}(\tilde{q}, \widetilde{\mathtt{P}}, x_c)\right)^\top R_c \left(\frac{\partial \widetilde{H}_{\mathtt{sat}}}{\partial x_c}(\tilde{q}, \widetilde{\mathtt{P}}, x_c)\right) \leq 0.$$

The rest of the proof follows similar to the proof of Proposition 1 by noting that

$$\frac{\partial \widetilde{H}_{\mathtt{sat}}}{\partial \tilde{q}}(\tilde{q}, \widetilde{\mathtt{P}}, x_c) = \sum_{i=1}^{n} e_i \alpha_i \tanh(\beta_i z_i) = \mathbf{0}_n \iff z = \mathbf{0}_n,$$

and

$$\frac{d}{dt}\left(\frac{\partial \widetilde{H}_{\mathtt{sat}}}{\partial \tilde{q}}(\tilde{q}, \widetilde{\mathtt{P}}, x_c)\right) = \sum_{i=1}^{n} e_i e_i^\top \alpha_i \beta_i \left[\operatorname{sech}(\beta_i z_i)\right]^2 \dot{z}$$

is bounded if $\dot{z}$ is bounded. ■

E. Passivity-based trajectory tracking controller

Theorem 1 introduces the main result of this paper, namely, a control law that addresses the trajectory tracking problem for (2) while satisfying **C1** and **C2**.

Theorem 1: Consider the pH system (2) in closed-loop with

$$u = \frac{\partial V}{\partial q}(q) - \sum_{i=1}^{n} e_i \alpha_i \tanh(\beta_i z_i) + \kappa(t) \tag{15}$$

where

$$\kappa(t) := \Psi^{-\top}(q_d)\left[\frac{d}{dt}\left(\Psi^{-1}(q_d)\dot{q}_d\right) - J_d(t)\Psi^{-1}(q_d)\dot{q}_d\right]$$

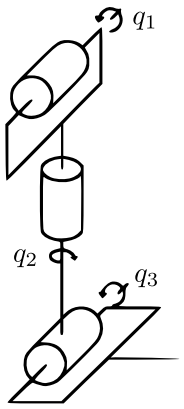
Then,

$$\lim_{t\to\infty} q(t) = q_d(t), \quad \lim_{t\to\infty} \mathtt{P}(t) = \mathtt{P}_d(t).$$

Proof: The proof follows from (4), (7), Proposition 2, noting that $\tilde{q} \to 0$ implies $q \to q_d$ and $\widetilde{\mathtt{P}} \to 0$ implies $\mathtt{P} \to \mathtt{P}_d$. ■

Remark 4: For robotic arms, the control law (15) can be physically interpreted as follows. The gradient of the potential energy compensates the gravitational forces acting on the system. The second term of the right-hand ensures that the trajectories of the system converge towards the desired ones, and the last term guarantees that the system keeps tracking such trajectories.

(a)



(b)

Fig. 1: PERA system (a) and schematic of the joints used in the experiments (b).

Remark 5: Note that the control law (15) is saturated because of Assumption 1, the fact that the desired trajectories are bounded, and the shape of the function $\tanh(\cdot)$. Moreover, the saturation limits can be adjusted by modifying the parameters α_i. In particular, $\mathcal{U}_{\min_i} = -\alpha_i + \underline{V} + \underline{\kappa}$, $\mathcal{U}_{\max_i} = \alpha_i + \overline{V} + \overline{\kappa}$, where $\underline{V}, \overline{V}$ denote the lower and upper bound of $\frac{\partial V}{\partial q}(q)$, and $\underline{\kappa}, \overline{\kappa}$ denote the lower and upper bound of $\kappa(t)$. Additionally, the parameters β_i adjust the slope of the functions $\tanh(\cdot)$, i.e., the sensitivity with respect to the error.

IV. Implementation in the PERA system

To corroborate the effectiveness of the methodology proposed in the previous section, we implement the controller (15) in the PERA system, depicted in Fig. 1a, which is a robotic arm with seven DoF that intends to emulate the motion of a human arm. To illustrate the applicability of the saturated tracking controller, we consider only three degrees of freedom, namely, the shoulder pitch q_1, shoulder yaw q_2, and elbow pitch q_3 (see Fig. 1b). Accordingly, the dynamics of the reduced system can be expressed as a pH system of the form (1), with $n = 3$, and

$$M(q) := \begin{bmatrix} \mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3 + a & 0 & \mathcal{I}_3 \cos(q_2) \\ 0 & \mathcal{I}_2 + \mathcal{I}_3 & 0 \\ \mathcal{I}_3 \cos(q_2) & 0 & \mathcal{I}_3 \end{bmatrix}$$

$$V(q) := -\left(\tfrac{1}{3}m_1 + m_2\right) g L_1 \cos(q_1) + \tfrac{1}{3} m_2 g L_2 b(q)$$

$$a := (m_1 + m_2) L_1^2$$

$$b(q) := \cos(q_2)\sin(q_1)\sin(q_3) - \cos(q_1)\cos(q_3),$$

where the constant parameters of the system are provided in Table I. Moreover, the saturation limit of the motors are

$$|u_1| \leq 18.77, \quad |u_2| \leq 3.32, \quad |u_3| \leq 7.72, \tag{16}$$

with units $[N \cdot m]$.

For further details about the PERA system, we refer the reader to [19].

TABLE I: System parameters

Parameter	Value/Units	Parameter	Value/Units
L_1	$0.32[m]$	g	$9.81[m/s^2]$
L_2	$0.48[m]$	$\mathcal{I}_1$	$0.03[kg \cdot m^2]$
m_1	$2.9[kg]$	$\mathcal{I}_2$	$0.004[kg \cdot m^2]$
m_2	$1[kg]$	$\mathcal{I}_3$	$0.02[kg \cdot m^2]$

The control objective is to track a circular trajectory with the end-effector of the system. To this end, we parameterize the desired trajectory as follows

$$q_d(t) = \begin{bmatrix} 0 \\ \arcsin\left(\frac{r}{L_2}\right) \sin\left(\frac{2\pi}{T}t\right) \\ \frac{\pi}{2} - \arcsin\left(\frac{r}{L_2}\right) \cos\left(\frac{2\pi}{T}t\right) \end{bmatrix}. \tag{17}$$

with $r \in \mathbb{R}_+$ the radius of the circle, $T \in \mathbb{R}_+$ the period of the circle trajectory and $t \in \mathbb{R}_+$ the time.

A. Experiments

To carry out the experiments, we choose values of α such that the limits provided in (16) are not reached. Then, we adjust the sensitivity of the controller by adjusting β until we get a satisfactory performance. But, alas, the criterion to fix the gains of the filter R_c and K_c is not clear and these parameters are tuned by trial an error. The chosen control parameters are $\alpha = [11\ 2\ 6]^\top$, $\beta = [400\ 100\ 120]^\top$, $K_c = \text{diag}\{30,\ 20,\ 200\}$, and $R_c = \text{diag}\{1,\ 0.1,\ 4500\} \times 10^{-4}$. Moreover, the initial conditions of the experiments are $q_0 = p_0 = \mathbf{0}_3$.

The experiments consist of two steps: (i) during the first second, we consider that the desired trajectory is a constant point, which belongs to the circle to be tracked. Then, (ii) we consider that the desired trajectory is given by (17). This procedure has two advantages: on the one hand, it is easier to physically set the robot at the origin, enabling the repeatability of the experiments under the same initial conditions. On the other hand, we improve the performance of the controller.

The experimental results are shown in Fig. 2, where it can be noticed that the system tracks the desired trajectory with a small deviation, particularly notorius in q_3. This error in the trajectory may be caused by several non-modeled phenomena that affect the behavior of the system, e.g., the friction in the joints or the anti-symmetry in the motors. Nevertheless, as depicted in the first column of Fig. 3, the absolute position error remains smaller than 0.05 $[rad]$, i.e. $|\tilde{q}| < 0.05$. Furthermore, the control signals do not exceed the limits given in (16) as it shown in the second column of Fig. 3, where the mentioned limits are plotted in black dashed lines. A video of the experiments can be found in: https://www.youtube.com/watch?v=2bW4PwwSo2s.

V. Concluding remarks and future work

This paper presents a constructive control design methodology that solves the trajectory tracking problem for a class

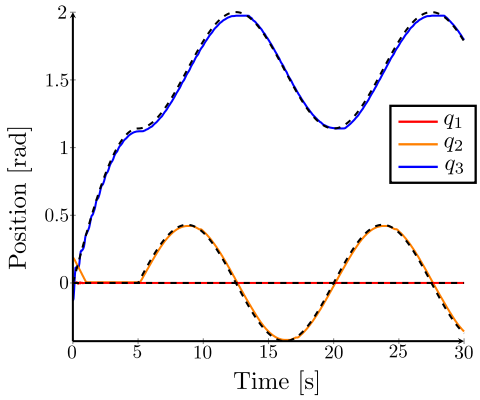


Fig. 2: Experimental results. The solid lines represent the trajectory of the joints during the experiment, while the dashed lines represent the desired trajectories.

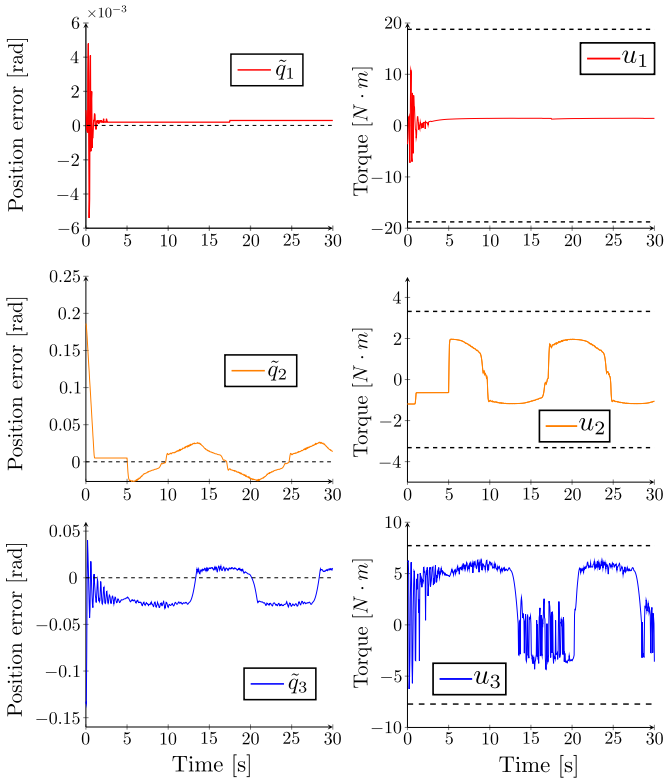


Fig. 3: Position errors (first column) and control signals (second column).

of robotic arms, where the control signals are saturated and do not require velocity measurements. Moreover, the control law uses gravity compensation based on the modeling of the gravitational force acting on the robotic arm. To prove that the trajectories of the closed-loop system *globally uniformly* converge to the desired trajectories, we conduct an analysis based on Barbalat's lemma.

The control approach was implemented in the PERA system, where the experimental results show that the trajectories of the system track the desired ones with an absolute position error of the joints that remains smaller than $0.05[rad]$. Additionally, the controller proved robustness in the presence of non-modeled phenomena such as the natural dissipation present in the joints of the system.

As future work, it is suggested further to investigate a systematic method for tuning of control gains.

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