

Evaluating EMF Emissions in Submarine HVAC Cables

Innovative Approaches for Modeling Current Distribution and Environmental Impact

MSc Thesis Sustainable Energy Technology

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Evaluating EMF Emissions in Submarine HVAC Cables

Innovative Approaches for Modeling Current
Distribution and Environmental Impact

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Preface

This thesis represents the end of my MSc journey and thereby as a student at the Delft University of Technology, a time filled with both challenges and growth. I am grateful to those who have supported me throughout this process.

I would like to extend my sincere thanks to my supervisors, Mohamad, Frank, and Peter. Their guidance, support, and feedback have been invaluable in shaping this thesis. I am appreciative of their time and expertise.

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Thank you all.

*Walter van der Klugt
Delft, August 2024*

Summary

This thesis investigates the impact of geometric, material, and operational parameters on the electromagnetic fields (EMFs) emitted by submarine power cables, particularly those used for offshore wind power transmission. The study is essential due to the growing deployment of offshore wind farms and the corresponding need for efficient submarine power transmission systems, combined with ecological concerns. The primary focus is on high-voltage alternating current (HVAC) cables, commonly used to connect offshore wind farms to the onshore grid.

The rapid expansion of offshore wind power has highlighted significant ecological and environmental concerns, especially the effects of EMFs on marine life. Species such as elasmobranchs (sharks, rays, and skates) are highly sensitive to EMFs, making this an important area of research. The objective of this thesis is to develop guidelines for modeling EMFs from submarine power cables to aid ecological research, focusing on establishing effective parameters for EMF emission models.

A literature review has shown that EMFs can impact marine animals' navigation, predator-prey relationships, and embryonic development. Elasmobranchs, in particular, are vulnerable due to their sensitivity to EMFs. The review highlights the necessary accuracy levels for EMF models, focusing on HVAC cables.

Starting with the basic physics including Maxwell's equations, the Biot-Savart law, and the Lorentz force, followed by an outline of HVAC cable and transmission system design parameters, an understanding was created of the electromagnetic phenomena occurring in a submarine power cable. It was concluded that modeling EMFs requires considering current distribution along the conductor and metallic sheath, and the local interactions between cable components that result in shielding of the EMFs.

A method from literature was used for predicting the longitudinal distribution of current along the conductor. The current distribution was affected by voltage and current transmission requirements, impedance, and the capacitive and inductive properties of the phases as well as reactive power compensation. Analyzing these parameters showed that all are crucial for accurate EMF modeling, as parameter changes within realistic ranges could result in differences over 10%.

A significant part of this thesis examines the intensity of the metallic sheath currents. Modern HVAC cable designs use conductive polyethylene layers around the metallic sheaths, creating an electrical interface between them. This was conventionally assumed to dissipate circulating currents in the sheath, but this thesis questioned this belief. It was proven that some circulating currents remain and that induced currents were even unaffected by the conductive interface. The induced currents in the metallic sheath are shown to only be influenced by the conduction current and its design parameters, inductance, and resistance per unit length. Testing on the Borssele Alpha cable showed that sheath current for all standard operations was more than 12% of the conduction current.

An analysis of the local impact of various design and operational parameters on EMF emissions was conducted using COMSOL Multiphysics. The Borssele Alpha 1 cable design served as a standard test case. The analysis highlighted the importance of the metallic sheath and even more so, the armor layer. The steel armor layer created a path of low reluctance, with geometric parameters and permeability playing significant roles.

The effect of twist in the phases and armor wires on field emissions was also shown. The lay-length of the phases played a large role due to its effect on destructive interference between emissions of different phases. The armor lay-length had a considerable impact as well, possibly affecting the reluctance of the armor layer.

The parameter analysis also confirmed the validity of an ultra-shortened section length in COMSOL, greatly reducing simulation time without impacting results.

This research provides insights for developing accurate EMF emission predictions, aiding biologists and ecologists in evaluating the environmental impacts of offshore wind power infrastructure. This can guide the development of mitigation strategies and support sustainable expansion of renewable energy sources.

Recommendations for future research include experimental testing of the sheath current model, further development of the transmission line method for the metallic sheath and developing better mitigation strategies. Unrelated to the main question is that in thermal analysis of comparable submarine power cable designs, the induced current must be incorporated, as it was unclear if this is the case. The findings of this thesis can significantly contribute to the sustainable growth of offshore wind power by addressing ecological concerns and improving the understanding of EMF effects on marine life.

Contents

Preface	i
Summary	ii
1 Introduction	1
1.1 The Ecological Need for This Research	1
1.2 Transmission Using High Voltage AC Cables	2
1.3 Thesis Objective and Approach	3
1.4 Potential Impact of this Thesis	4
1.5 Thesis Structure	4
2 Literature Review: Ecological Effects of Electromagnetic Fields	5
2.1 Ecological Effects of Electromagnetic Fields Emitted by Submarine Power Cables	5
2.1.1 The Interactions Between Marine Animals and Electromagnetic Fields	5
2.1.2 Potential Ecological Impact of Electromagnetic Fields from Submarine Power Cables	6
2.2 Requirements of a Electromagnetic Field model for Ecological Research	6
3 Review of Electromagnetism and Modeling	7
3.1 Maxwell's Equations	7
3.1.1 Gauss' Law of Electricity	7
3.1.2 Gauss' Law of Magnetism	8
3.1.3 Faraday's Law of Induction	8
3.1.4 Ampère's Law	9
3.2 Lorentz Force and Induced Electric Fields	9
3.3 Biot-Savart Law	10
3.4 Electric Power	10
3.5 Conclusions from Electromagnetic Field Theory	11
4 Electromagnetism Within Submarine HVAC Cables	12
4.1 Review of Submarine Three-Phase AC Cable Design	12
4.1.1 The Conductor of an SPC	13
4.1.2 Electrical Insulation of SPCs	14
4.1.3 Use of Metallic Sheaths in SPCs	14
4.1.4 Modern Design of Polyethylene Outer Sheaths	15
4.1.5 The Armor Layer and Short Twisted Periodicity	15
4.2 Brief Overview of Power Transmission System Design	16
4.2.1 Active and Reactive Power	16
4.2.2 Grounding Method	19
4.3 Electromagnetism in HVAC Submarine Power Cables	20
4.3.1 Grounding and Interface of Conductors and Sheaths	20
4.3.2 Induced Currents In The Metallic Sheath	21
4.3.3 Induced Eddy Currents	23
4.3.4 Thermal Effect on Conductivity	23
4.3.5 The Skin Effect	23
4.3.6 Capacitance and Charging Currents	24
4.3.7 Capacitance of Transmission Lines	26
4.3.8 EMFs in the Environment	27
4.4 Calculating the Conductance at Sheath Interfaces	27
4.4.1 Calculating the Conductance for Separated Sheaths Using COMSOL	28
4.4.2 Calculating the Effective Conductance From Electric Phase-Shift	29
4.5 Conclusion on Relevant Modeling Parameters	30
5 Current Flow Prediction in SPCs	32
5.1 Conductive Outer Sheath and Remaining Currents	32

5.2	Conductor Voltage and Current Distribution Using General Wave Equations	35
5.2.1	General Transmission Line Equations	35
5.2.2	Mathematical Definitions Used For Solving Wave Equations	36
5.2.3	Solution To Transmission Line Equations: Conductor	37
5.3	Verification of Transmission Line Solution: Borssele Alpha	38
5.3.1	Conclusions on Verification of the Conductor Transmission Formulas	43
5.4	Analytical Derivation of Charging Currents in the Sheath	43
5.4.1	Application of Wave Equations to Metallic Sheaths	43
5.5	Junction-Series Approximation of the Sheath Current and Voltage	46
5.5.1	Representation of the Metallic Sheath Using Junctions	46
5.5.2	Matrix Representation Using KCL	46
5.6	Validation of Zero Sheath Current Assumption	48
5.7	Conclusion on the Current Distribution Analysis	50
6	Parameter Impact Analysis for EMF Modeling	51
6.1	Methodology of the Parameter Impact Analysis	51
6.1.1	Simulation Tools and Software	51
6.1.2	Construction of a Standard COMSOL Model	51
6.2	Parameter Impact Analysis on Total Phase Current	56
6.2.1	Impact Analysis of Design Parameters on Phase Current	57
6.2.2	Impact Analysis of Operational Parameters on Phase Current	61
6.3	The Impact of Design and Operational Parameters on EMF Emissions	62
6.3.1	Impact of Conductor Design Parameters on EMF Emissions	63
6.3.2	Impact of Metallic Sheath Design Parameters	64
6.3.3	Sensitivity to Armor Design Parameters	66
6.3.4	The Impact of Lay-Length	70
6.3.5	Results and Limitations of Design Parameter Impact Analysis	72
6.4	The Impact of Animal Movement on Perceived E-Field	73
6.5	Conclusion of The Parameter Analysis	75
7	Conclusion	76
7.1	Summary of Findings	76
7.1.1	Requirements for an EMF-emission model	76
7.1.2	Prediction of Net Current	76
7.1.3	Important Parameters in COMSOL Multiphysics Models	77
7.1.4	EMF Prediction of the Borssele System	77
7.2	Comprehensive Conclusion	77
8	Recommendations for Further Research	78
	References	79
A	Appendix A: The Transmission Line Equations	83
A.0.1	General Transmission Line Equations	83
A.0.2	Lossless Propagation of Sinusoidal Waves	85
A.0.3	Complex Analysis of Sinusoidal Waves	85
A.0.4	Transmission Line Equations and Their Solutions in Phasor Form	86
A.0.5	Low-Loss Propagation in the Conductor	88
A.0.6	Solution of the Transmission Line Equations for an Uncompensated Power Line	89
A.0.7	Application of Wave Equations to Metallic Sheaths	90
A.0.8	Application of the Specialized Wave Equations	92
B	Appendix B: Matlab Code	95
B.1	Calculation of Current and Voltage Distribution (MATLAB)	95
B.2	Sheath Current and Voltage Distribution: Transmission Line (MATLAB)	98

Introduction

1.1. The Ecological Need for This Research

The rapid expansion of offshore wind power shows a shift towards renewable energy, using the large potential of the wind in the open sea. By 2022, the global offshore wind energy capacity reached 64.3 GW, with projections indicating a rise to 444.3 GW by 2032 [1]. In Europe, wind energy is expected to become the predominant energy source by the mid-2000s. Offshore wind, in particular, will have the largest share in the global energy mix, reaching a total of 22% by 2024 [2]. However, the deployment of offshore wind farms requires extensive submarine power transmission systems to connect these energy sources to the onshore grid, raising both technological challenges and environmental concerns.

The integration of renewable sources such as wind power increases the intermittency of the energy supply, compelling a shift toward more interconnected and resilient power grids. The North Sea, rich in wind resources, will play an essential role in European energy connectivity. According to the European Commission [3], the North Sea Energy Cooperation has agreed to develop 193 GW of installed power by 2040. For the energy transition, this is positive news; however, a system of this scale also increases the environmental concerns associated with it. While we improve our renewable infrastructure, particularly through submarine power cables, we must also address the potential impacts on marine ecosystems. This is because the transmission of large amounts of electricity through these cables results in the emission of electromagnetic fields, which can create physiological problems for animals that swim through[4][5][6].

Among the marine species affected, Elasmobranchs, including sharks, rays, and skates, stand out due to their electromagnetic sensitivity, crucial for navigation and survival[7][8][9]. Their ability to detect magnetic and electric fields through specialized organs makes them vulnerable to electromagnetic fields (EMFs) produced by subsea power cables. Understanding and mitigating these impacts is vital for the sustainable expansion of offshore wind power.

In response to these challenges, grid operators and research initiatives in Europe are increasingly focused on assessing the effects of EMF on marine life. The collaboration with the Dutch transmission system operator (TSO) TenneT and the insights gained from the ElasmoPower project [10] (outline of activities depicted in Figure 1.1) provide a unique opportunity to leverage both proprietary system data and comprehensive EMF measurements. This combination of resources is crucial for validating models of EMF emissions, which sets this research apart in its potential to bridge the gaps between theoretical models and empirical observations.

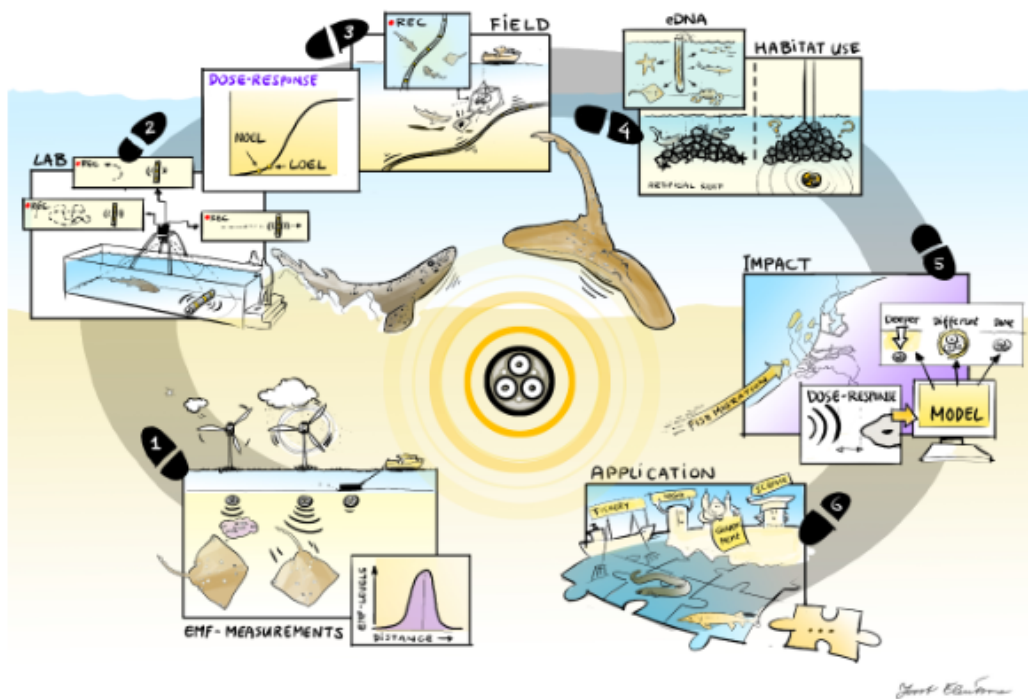


Figure 1.1: Outline of the activities performed by the Elasmopower project [11]

Despite ongoing research, current EMF models often fail to accurately represent real-world conditions, such as unpredictable burial depth, complex electromagnetic mechanisms, most notably within high-voltage alternating-current (HVAC) cables. and changing operating conditions, leading to discrepancies between predicted and measured models. This has made it difficult for environmental research to make progress. For this reason, the Dutch government now requires TenneT to aid in this research, which resulted in large sensing projects and also research into the development of a better approach for EMF modeling.

Borssele Alpha[12], being the oldest existing Dutch wind farm and connected to the grid by TenneT, is the perfect test case for this research. Due to the collaboration with TenneT, historic transmission data are available for Borssele as well as cable design; and long-term onshore and some underwater transect measurements of EMF emissions have been performed by the Elasmopower project[13].

1.2. Transmission Using High Voltage AC Cables

There are two types of high voltage transmission: high voltage alternating current (HVAC) and high voltage direct current (HVDC), both with different properties that make them desirable for their particular implementation. Currently, HVAC cables are the most common, due to their historical advantage in voltage conversion [14]. HVAC is always implemented with three phases, electrically phase-shifted by 120° , as this offers the most constant and efficient power. Most wind farms are installed at a distance of less than 60 km from the shore [15], where the HVAC cables are effective. Due to the lower cost of converter stations, HVAC will be the main source of connection of offshore wind to the grid for the coming years.

The main implementation of HVAC cables for submarine applications is by combining the three cores into a single package, to greatly reduce laying costs. A combined package, with multiple different electrical components and the alternating electric current in the conductors, results in more complex current distributions within the cable and thereby creates a more interesting electromagnetic profile. The transmission of alternating current requires charging currents to polarize its insulation layer. Alternating magnetic fields can also induce currents in the metallic sheath of the cable. To be able to calculate the exact fields emitted requires the analysis of these currents, which should be superimposed on the known conduction current.

The complexity of HVAC cables has resulted in a larger knowledge gap compared to the EMFs emitted by HVDC cables. Because of this, the thesis will focus solely on the three-core HVAC cables, which we will refer to as submarine power cables (SPCs).

1.3. Thesis Objective and Approach

This thesis aims to develop guidelines on how to model EMF for submarine power cable systems such that it can be used for in-depth research into the ecological effects, leading to the main research question of this thesis:

What are the most effective parameters to include in electromagnetic field emission models to aid research into the ecological effects of submarine power transmission?

This is a broad question to answer, but it can be broken down into more specific subquestions that together provide a comprehensive answer.

- What are the requirements for an EMF-emission model that can be utilized in ecological research?
- How can the current at a location along the length of a submarine power cable be predicted based on its design- and usage parameters?
- What are the most important parameters to include in a submarine power cable model in COMSOL Multiphysics, by means of their impact on the electromagnetic field emissions?

Before we can start improving on current modeling methods, it is essential to know what the requirements are of EMF-emission models. These requirements will be examined using expert opinions and a review of the literature on ecological effects measured. The main requirements will be taken from literature following the ElasmPower project, as this is the most relevant for the North Sea and can be combined with knowledge, as well as system usage and design data from TenneT. Requirements are therefore mainly set by the Elasmobranchs, but other marine animals are also considered.

Analysis of the current distribution along the length of an SPC is essential for prediction of the EMF emitted, as the intensity of the EMF is directly proportional to the conducted current. However, for the SPCs of interest, this current distribution is far from trivial, so one cannot simply use the current that is supplied to the grid as being constant along its length. SPCs in general have to deal with a different kind of resistance, called reactance, which affects alignment of voltage and current and thereby the efficiency of transmission. To do this, the TSO has to balance the capacitive and inductive properties of the cable system so that the desired current is delivered at the receiving end. This can be done using shunt reactors at the sending and/or receiving end, resulting in different distributions along the length that have to be determined.

The power lost to the reactance (reactive power) of the cable is exchanged with other metallic components of the cable through capacitive and inductive mechanisms. Therefore, to find the 'effective' current at a point along the cable, we have to find the superposition of both the conductive current (affected by the reactance) and the current in other metallic components. This analysis will be done with a combination of existing and newly developed analytical methods, using a specialized case of the 'General Wave Equations', that are validated using the COMSOL Multiphysics[16]. For this part of the research the Magnetic Fields module[17] will be used to analyze inductive effects, and the Electric Currents module[18] will be used for analyzing the capacitive (charging) current.

Developing a computationally efficient but effective structure for creating COMSOL models of SPCs requires the analysis of the parameters that are used as inputs. A simpler model is more computationally efficient, so eliminating parameters and complex geometric components that have no significant effect can increase efficiency. The input parameters for a model consist of cable design parameters, parameters describing the environment of the cable, and the usage parameters, including the voltage, current, and reactive power compensation. A parameter analysis will be done to see the impact of each individual parameter, such that a selection can be made of essential and non-essential parameters to be used in modeling. Based on the literature review and the rules of physics, many parameters can already be disregarded from the list of options. Parameter analyses will be performed by implementing a realistic cable model, based on the Borssele Alpha[12] cable system in COMSOL and running a parametric sweep on the parameters that are deemed interesting based on the foregoing research.

The focus will be on modeling specifically the Borssele Alpha (HVAC) cable, due to the availability of magnetic field (MF) measurement data, cable load, and design specifics from TenneT. This system will be used as a test case for checking if the modeling methodology developed in the previous steps can produce an accurate representation of the measurements. For this task, a 3D model of the cable will be created that aligns with the conditions at the measurement

sites. Then, on the basis of the cable loading data supplied by TenneT, a prediction of the local magnetic and electric field will be made. Verification is done by comparing this with the magnetic field data from the measurements.

1.4. Potential Impact of this Thesis

By achieving a more accurate prediction of EMF emissions, this research will provide biologists and ecologists with a more reliable method to evaluate the environmental impacts of offshore wind power infrastructure. An improved understanding of the effects of EMFs on marine ecosystems will guide the development of mitigation strategies, supporting the sustainable growth of offshore wind power and aiding the transition to renewable energy sources.

1.5. Thesis Structure

The thesis is organized as follows:

- **Literature Review** (chapter 2): This chapter will provide a comprehensive review of the literature to determine the expected levels of exposure to electromagnetic fields that can impact marine animal species. It will cover previous studies, existing models, and the biological effects of EMFs on marine life, focusing on Elasmobranchs and other sensitive species.
- **Review of Electromagnetism** (chapter 3): This section will review the underlying physics of electromagnetism, covering fundamental concepts and equations that are essential for understanding the behavior of electromagnetic fields in various contexts. The understanding is linked to ideas about modeling the electromagnetic fields in COMSOL
- **Electromagnetism in Submarine HVAC Cables** (chapter 4): This section will focus on the technical aspects of submarine power cables (SPCs), including the mechanisms affecting currents and the electromagnetic fields emitted. It will provide detailed descriptions of HVAC cable structures and their electromagnetic properties.
- **Current Flow Prediction in SPCs** (chapter 5): This chapter will describe the specific approach to predicting currents in SPCs. It will present the analytical and computational methods used to model current distribution along the cables, and verify these models with COMSOL simulations and empirical data.
- **Parameter Analysis for EMF Modeling** (chapter 6): This chapter will explain the approach for selecting and verifying the essential parameters for modeling electromagnetic fields in COMSOL. Given the computational limitations of a typical laptop (32GB RAM), it will discuss how to optimize the model for efficiency without sacrificing accuracy.
- **Model Verification and Application**: A verification step will be done after new statements are made. This thesis is based in theory and modeling, with no physical means of verification within its scope. However, the potential methods of verification will be discussed where necessary.
- **Conclusion** (chapter 7): This chapter will summarize the key findings of the thesis, reflect on the implications for environmental impact assessments of offshore wind power, and discuss the broader significance of the research.
- **Recommendations for Further Research** (chapter 8): This final chapter will provide recommendations for future studies. It will suggest areas where further research is needed to refine EMF models and enhance our understanding of the ecological impacts of submarine power cables.

Literature Review: Ecological Effects of Electromagnetic Fields

The research in this thesis is based on the need expressed by ecologists to better understand the electromagnetic fields emitted by submarine power cables. Their reasoning is that many marine animal species use some form of electromagnetic sensing to trigger behavioral cues and navigate oceans and that it could hinder their embryogenic development. A review of the literature on this topic will be done in section 2.1. To improve on their research, it is necessary to know what electromagnetic fields are emitted by current submarine cable systems or are expected to be emitted by future systems. There are multiple methods for calculating electromagnetic field emissions that have different levels of accuracy and computational cost.

2.1. Ecological Effects of Electromagnetic Fields Emitted by Submarine Power Cables

Research has shown that electromagnetic fields from submarine cables can have a physiological impact on marine animal species [6], [7], [9], [19], [20], [21], [22]. This is of concern to ecologists and has raised the question with the Dutch government about the impact and sustainability of submarine power cable systems [23], [24]. In addition to direct concern for animal safety, this is an important topic for the goals of achieving a high share of renewable energy. The development of offshore renewable energy infrastructure depends on the ease with which cable laying projects can be permitted. Therefore, related companies and universities have started to collaborate for more insight into electromagnetic fields emitted by submarine power cables and their potential impact. Contribution to this research requires an understanding of the mechanisms behind these interactions, which will be outlined in this section.

2.1.1. The Interactions Between Marine Animals and Electromagnetic Fields

There are multiple types of interaction between marine animals and electromagnetic fields. Most are said to operate through the detection of a magnetic field and/or an induced electric field (although these can not really be decoupled) [7].

The most well-known interaction is through navigation. It has been scientifically proven that some animals can navigate using the magnetic field of the Earth (called the geomagnetic field) [25]. This effect has been prevalent specifically in marine animals [26], [27], [8], probably due to the lack of other locational references when navigating vast oceans. Migratory behavior is often an important aspect of the survival of a species. For example, some species of salmon breed only once, at the end of their lives, at their exact spawning grounds [28].

There are species that use electric fields as triggers for their predator-prey behaviors [8], [20], [7]. Electrosensitive species are capable of detecting weak electric fields that are used to detect prey and predators, which makes it important sensory information for survival.

In addition to the impact on behavioral cues electromagnetic fields can affect embryogenic development, through their effect on gene expression [7], [29].

2.1.2. Potential Ecological Impact of Electromagnetic Fields from Submarine Power Cables

The dependency on magnetic fields for migration is expected to be mainly problematic when interacting with static magnetic fields that are emitted by direct-current (DC) cables. DC submarine power cables are used mainly for long-distance high-power transmission such as interconnectors between countries [30], [31] and are likely not to be spread densely throughout coastal waters. However, it should still be concerning to see if DC proves to have a significant impact on the migrational behavior of animals. The **no observed effect level (NOEL)** for migratory behavior is **0.05 μT** [7].

Affecting the predator-prey relationships of marine animals is also concerning, as it could damage already endangered species and possibly the balance of marine ecosystems [7]. The main species of interest are the Elasmobranchii as they are both highly electrosensitive and magnetosensitive. In addition, they are a bottom-dwelling species, meaning they could be highly exposed to electromagnetic fields emitted by submarine cables. Elasmobranchii have been shown to be able to sense **electric fields as low as 5 nV/cm** [7] and could therefore be sensitive to the changing magnetic field emitted from alternating currents (AC). For behavioral changes during local dwelling (related to predator-prey behavior), the **NOEL is 0.8 μT for AC**. **Low effects** have been observed at (**LOEL**) **8 μT for AC** and **0.03 μT for DC** [7].

Embryogenic development is also affected by the varying magnetic fields. Electromagnetic fields, including standard conduction frequencies, of 50 to 100 Hz have been shown to have degrading effects on their survival by affecting gene expression [29] and potentially the magnetic imprinting of natal sites [7]. An enhancement of this interaction is that submarine cable sites seem to be attractive as egg-laying sites, resulting in greater exposure. The **NOEL** during embryogenic development is **0.4 μT for AC and DC**.

2.2. Requirements of a Electromagnetic Field model for Ecological Research

It is clear that electromagnetic fields can have negative effects on marine animals. However, in order to predict whether submarine power cable systems damage marine ecosystems, it is still unknown. In the previous section, we have discussed the possible impact that electromagnetic fields can have on marine animals. Elasmobranchii in particular are very sensitive to these fields, but that does not directly mean that submarine power cables would have an effect on their survival. To say more on the topic it is required to know the extent, type, and intensity of electromagnetic field emissions. These questions are the basis of this research. Because the extent of emissions is an important aspect in defining the impact that submarine power cable systems can have, combined with the much larger knowledge gap, this thesis focuses on high-voltage AC power cables. Based on the choice of AC power cables and the focus on Elasmobranch species, there are four important electromagnetic field emission thresholds.

- *Induced electric fields $\geq 500 \text{ nV/m}$ (5 nV/cm).*
- *Alternating magnetic fields of $\geq 0.8 \mu\text{T}$ (NOEL).*
- *Alternating magnetic fields of $\geq 8 \mu\text{T}$ (LOEL).*
- *Alternating magnetic fields of $\geq 0.4 \mu\text{T}$ (NOEL).*

This leads to a minimal magnetic field emission model accuracy of 0.4 μT and a minimal induced electric field emission model accuracy of 0.5 $\mu\text{V/m}$.

Review of Electromagnetism and Modeling

When defining important parameters for modeling EMF emissions by submarine HVAC power cables, a deep understanding of electromagnetic fields is required. Describing electromagnetic fields is most comprehensively done using Maxwell's equations, from which all important phenomena in this field of research can be derived. Modern electromagnetism is based on Maxwell's equations, with specific cases such as the Biot-Savart law and the Lorentz force. Section 3.1 will explain the meaning and application of Maxwell's equations. In section 3.3, the Biot-Savart law, a simplified form of Amperes law, is discussed, which creates an intuitive visualization of magnetic field emissions. Section 3.2 gives an explanation of how magnetic fields are perceived as electric fields for moving charges, showing the duality of electromagnetism. Because of its importance in the analysis of submarine cables, an introduction to the concept of electric power and power losses was made in section 3.4. The chapter is concluded by stating the findings and takeaways for modeling EMFs in section 3.5.

3.1. Maxwell's Equations

When attempting to understand the complex electromagnetic phenomena within submarine cables, Maxwell's equations are invaluable. These fundamental laws of electromagnetism, unify electricity and magnetism into a coherent theoretical framework. Maxwell's equations have become indispensable for analyzing and predicting EMF behavior, as they are comprehensive and generally applicable.

3.1.1. Gauss' Law of Electricity

The first equation is Gauss' law of electricity, which relates electric flux through a surface to the amount of charge enclosed by it. This is most clearly described in its integral form:

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \quad (3.1)$$

Where ϕ is the electric flux, $\vec{E}$ the vector of the electric field, $d\vec{A}$ the orthogonal and infinitesimal surface area of the enclosure, Q the amount of charge and ϵ_0 the permittivity of free space. The right side of the equation states that the flux through the surface is independent of its shape.

There is also the differential form of Gauss's law, which states that the divergence of the electric displacement field $\mathbf{D}$ is equal to the electric charge density ρ at any point in space:

$$\nabla \cdot \vec{D} = \rho \quad (3.2)$$

Here, the $\mathbf{D}$ field is the effective local electric field, which is defined as the $\mathbf{E}$ field multiplied by the permittivity ϵ plus the polarization $\mathbf{P}$ and ρ is the charge density. Because of this relation, the equation can also be written as

$$\nabla \cdot (\epsilon \vec{E} + \vec{P}) = \rho \quad (3.3)$$

The displacement current can often be simplified to $\vec{D} = \epsilon \vec{E}$. However, polarization occurs within a dielectric material such as the insulation of a submarine power cable and should therefore be considered in this research.

3.1.2. Gauss' Law of Magnetism

Building on the analogy to Gauss' Law of Electricity, Gauss' Law of Magnetism emphasizes the absence of magnetic monopoles, asserting that for any magnet, the north and south poles are inseparable, ensuring magnetic field lines form closed loops. This concept is encapsulated by the equation:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 \quad (3.4)$$

Here, $\vec{B}$ represents the magnetic field vector, and $d\vec{S}$ denotes the orthogonal infinitesimal surface area, highlighting that the net magnetic flux through any closed surface is zero. Analogously to the electric case, the magnetic flux's independence from the surface's geometry underscores the non-existence of isolated magnetic charges.

In its differential form, Gauss' Law of Magnetism is presented as:

$$\nabla \cdot \vec{B} = 0 \quad (3.5)$$

Indicating that the magnetic field's divergence is zero, the concept of magnetic field continuity is reinforced. This principle, when considering $\vec{B} = \mu\vec{H}$, also leads to:

$$\nabla \cdot \mu\vec{H} = 0 \quad (3.6)$$

These expressions describe the nonexistence of magnetic monopoles and thereby states that magnetic field lines always form closed paths. In Equation 3.6 the magnetic permeability μ can be identified, which describes the relation between the magnetic field and the flux.

A further note on magnetic field lines is that they will concentrate themselves in paths of low magnetic reluctance $\mathfrak{R}$. This is analogous to preferring the path of lowest resistance, but for magnetism. Reluctance is calculated by

$$\mathfrak{R} = \frac{d}{\mu S} \quad (3.7)$$

where d is the length of the path and S is the cross-sectional area of that path. The magnetic permeability can also be written as the product of the vacuum permeability $\mu_0 = 4\pi 10^{-7}$ times the relative (to vacuum) permeability μ_r . As this relative permeability thereby dictates the preference of a path relative to other paths, it will be used in further analysis.

3.1.3. Faraday's Law of Induction

Faraday's Law of Induction is a cornerstone in the study of electromagnetism, as it describes how a changing magnetic environment within a loop induces an electromotive force (EMF). This law can be formally stated as:

$$\varepsilon = -\frac{d\Phi_B}{dt} \quad (3.8)$$

Where:

- ε denotes the induced EMF in volts (V).
- Φ_B represents the magnetic flux through the loop in webers (Wb).
- The negative sign indicates the direction of the induced EMF and current, according to Lenz's Law, which opposes the change in magnetic flux.

The law highlights the interplay between electric and magnetic fields, indicating that a time-varying magnetic field can generate an electric field. This fundamental principle underpins the operation of transformers, electric motors, and generators, showcasing the ability of magnetic fields to convert mechanical energy into electrical energy and vice versa. Faraday's Law also extends to the concept of electromagnetic induction in coils, where the induced EMF in a coil of N turns is given by:

$$\varepsilon = -N \frac{d\Phi_B}{dt} \quad (3.9)$$

Here, the total EMF is influenced by the number of turns in the coil, amplifying the effect of the changing magnetic flux. For a coaxial cable, the amount of coils will be equal to one, as both are continuous.

A different mathematical derivation allows us to come up with a more general form of the differential Faraday law. As described in Equation 3.4, $\Phi_B = \oint \vec{B} \cdot d\vec{A}$

Magnetic flux, described as the integral of the magnetic field $\vec{B}$ over a vector area $\vec{A}$ through a closed surface, relates to how an electromotive force, or EMF, arises when there is a flow of current. The mathematical definition of the EMF involves an integral over a closed loop, expressed as $\varepsilon = \oint \vec{E} \cdot d\vec{l}$.

Faraday's law can be rephrased using the time derivative of the magnetic field within an area, leading to a reformulation of the law: $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{A}$. Stokes' theorem allows for simplification, translating the loop integral to a surface integral $\oint \vec{E} \cdot d\vec{l} = \int (\nabla \times \vec{E}) \cdot d\vec{A}$, resulting in the following equation $\int (\nabla \times \vec{E}) \cdot d\vec{A} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{A}$. Removing integral terms leaves us with the general differential form:

$$\nabla \times \vec{E} = -(\partial \vec{B} / \partial t) \quad (3.10)$$

In practice, Faraday's law can aid in calculating the induced electric field due to a changing magnetic field when the changing magnetic field is known.

3.1.4. Ampère's Law

Ampère's law, enhanced by Maxwell's addition, provides a framework for understanding how magnetic fields are influenced not just by electric currents but also by changing electric fields. This expanded law is captured by:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad (3.11)$$

In this expression, $d\vec{l}$ represents the differential length vector along the closed loop and μ_0 signifies the permeability of free space. The term $\mu_0 I_{\text{enclosed}}$ accounts for the magnetic field due to the conventional electric current, while $\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ introduces the displacement current, representing the rate of change of electric flux contributing to the magnetic field. Transforming the left side of the equation using Stokes' theorem considering the continuity of magnetic fields, and writing the right side in integral form, leads to:

$$\int_S (\nabla \times \vec{B}) d\vec{A} = \int_S (\mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}) d\vec{A} \quad (3.12)$$

Here, $\vec{J}$ is the current density and $\vec{A}$ denotes the area vector on the surface S . Applying the principle of $\vec{B} = \mu_0 \vec{H}$ and recognizing $\varepsilon_0 \vec{E} = \vec{D}$, the local, point-wise relationship between these fields can be extracted.

$$\nabla \times \vec{H} = \vec{J} + \partial \vec{D} / \partial t \quad (3.13)$$

This equation, known as the differential form of Ampère's law with Maxwell's addition, is essential in the exploration of electromagnetic phenomena, notably in the environment of submarine power cables where varying electric and magnetic fields are of paramount concern.

The nature of the cross product in Ampère's results in magnetic fields forming as circular vector fields around the current being conducted. This gave rise to the Biot-Savart law (section 3.3), but also results in two phenomena that have a positive (negating) effect on the electromagnetic fields emitted by submarine cables.

In particular, a submarine power cable is often a three-core cable, in which the three phases are shifted electrically by 120° . This phase shift means that the sum of currents flowing in the longitudinal direction of the cable is always zero. If the three cores were concentric, this would mean that there would be no electromagnetic field emissions at all. Although this is clearly not the case, the intensity of the emissions quickly drops with distance to the cable.

Another important effect that limits field emissions is the twist of a cable. The twist of the phases also increases the interference between the electromagnetic fields of the three phases and thereby decreases their intensity [4].

3.2. Lorentz Force and Induced Electric Fields

Another potentially important physical phenomenon in this investigation is the Lorentz force. Specifically, the movement-related component of the Lorentz force. Lorentz described how the force $\vec{F}$ on a test charge is related to its charge q and velocity $\vec{v}$, when subject to an electric field $\vec{E}$ and magnetic field $\vec{B}$:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (3.14)$$

This means that a charge will not only experience the local electric field, but that a moving charge will also experience a stationary B-field as an E-field. This might be of interest in a dynamic environment such as the sea floor.

With the definition of an electric field: $\vec{E} = \frac{\vec{F}}{q}$ combined with Equation 3.14, we can calculate the **motion-induced electric field** (or MIF):

$$\vec{E}_{IF} = \vec{v} \times \vec{B} \quad (3.15)$$

Induced electric fields in moving objects could be an important aspect to consider when modeling to explain effects on marine animals. The induced electric field could be modeled simply using Equation 3.15, assuming a velocity and taking the cross product of the velocity with the magnetic field that is already calculated.

3.3. Biot-Savart Law

For practical purposes, there is another law that is often used for the analysis of EMFs, the Biot-Savart law. The equation offers a way of calculating magnetic fields for steady electric currents. Here, starting with the differential form:

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2} \quad (3.16)$$

This differential formula describes the infinitesimal contribution to the magnetic $d\vec{B}$ of a current I through an infinitesimal wire element $d\vec{l}$ to the magnetic field at a point p a distance r away. The unit vector $\hat{r}$ points from the coordinate $d\vec{l}$ in the direction of p . The field's strength at this location depends on μ_0 , the permeability of free space.

In its integral form, Biot-Savart can be used to find the total field at a particular location:

$$\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{r}}{r^2} \quad (3.17)$$

Due to its simplicity, the Biot-Savart law is an essential tool for modeling magnetic fields. However, it has clear limitations:

- First, it is inherently limited to steady-state currents and becomes invalid for time-varying EMFs.
- It assumes a cable without a diameter, meaning it is incorrect at short distances. However, from $10 \times D_{cab}$, it is practically the same.
- It is not suitable for complex geometries.

Although the Biot-Savart law helps calculate magnetic fields from steady electric currents and offers simplicity in modeling, it has too many limitations for a comprehensive analysis of submarine power cables. Specifically, it does not account for time-varying currents, crucial in AC applications, nor does it consider complex inductive and capacitive effects (see subsection 4.3.6 and subsection 4.3.2) resulting from interactions between different cable phases and their metallic sheaths. Furthermore, the assumption of an infinitely small wire diameter can lead to inaccuracies in scenarios where the physical dimensions of the cable are significant.

3.4. Electric Power

This thesis is about analyzing submarine power cables, so an explanation of electrical power and power losses should be provided. By the definition of its components, electrical power P (in watts) is calculated as follows:

$$P = V \cdot I \quad (3.18)$$

Where V is the root-mean-squared (rms) voltage and I the (rms) current flowing through a conductor, the power is calculated using the phase-to-phase voltage and current for transmission with three-phase alternating current. Due to the electrical phase shift between the conductors, the voltage difference between one phase and the other two is always equal to $\sqrt{3}$ times the phase-to-ground voltage. From this, the three-phase electrical power equation can be written.

$$P = \sqrt{3} \cdot V \cdot I \quad (3.19)$$

V represents the root-mean-squared (rms) voltage, and I represents the (rms) current flowing through a conductor. To maximize power transmission, both voltage and current should be increased. However, there are limits to this because increasing power transmission also leads to greater power losses. These losses occur in two main ways:

- Ohmic Losses: According to Ohm's law, when current flows through a conductor, it causes a voltage drop and generates heat, leading to energy loss.
- Reactive Power Losses: These losses occur due to a phase shift (ϕ) between voltage and current. This phase shift can be caused by capacitive effects resulting in a positive ϕ , and inductive effects resulting in a negative ϕ .

This phase shift reduces the power factor, which is $\cos \phi$, making the power transmission less efficient. More details on reactive power can be found in subsection 4.2.1.

Ohm's law states that the voltage drop over a conductor is equal to

$$V = I \cdot R \quad (3.20)$$

where R is the resistance over the length of the conductor, such that ohmic losses are calculated by $P_{loss} = I^2 \cdot R$, which directly increases the temperature of the cable. High cable temperature is not desirable, as it increases resistance and

can degrade components [32] and is therefore limiting the cable's **ampacity** (maximum steady state transmission current $I_{\max}$). From this, it can be deduced that a high voltage is more desirable than a high current, as it increases the power transmitted without directly increasing the losses. However, due to voltage limits of cable components (section 4.1) and the increase in reactive power (subsection 4.3.6, subsection 4.2.1), this also has limitations.

Reactive power losses will be explained in subsection 4.2.1, which requires a more in-depth understanding of the cable and transmission system design. However, to understand further analysis, it is good to remember that reactive power is the power associated with a mismatch in electrical phase of the current and voltage sinusoids, due to capacitive and inductive components in the system. Reactive power is generally undesirable, as it produces extra losses in the cable without adding to the useful power transmitted. However, a transmission system operator can choose to produce or absorb extra reactive power in order to end up with a better power factor ϕ at the receiving end.

3.5. Conclusions from Electromagnetic Field Theory

This chapter has created an outline of the physical laws of electromagnetism. Starting with the fundamental Maxwell equations, their simplified derivatives (Biot-Savart law and Lorentz force) were discussed. Maxwell's equations comprehensively analyze electromagnetic fields, and solving them in conjunction will result in the most accurate prediction of reality.

An introduction to the effect of field mitigation due to the nature of a three-phase system and its twist has been made. This will be explained in more detail later in this thesis. Still, it can be concluded that the distance between the phases and the level of twist of the conductors are essential parameters to consider in modeling. As discussed in section 3.3, a relatively easy method to model magnetic fields is based on the intuitive Biot-Savart law. However, it is limited to analyzing simple geometries with constant current and does not incorporate capacitive and inductive effects.

An introduction to Ohm's law and electrical power was made to aid in further understanding of cable- and transmission-system design and the flow of currents within the system. A transmission system design is based on achieving a high power factor. This involves balancing capacitive and inductive effects within the system.

Given their general applicability, it is more accurate to solve Maxwell's equations directly, which will be done using COM-SOL Multiphysics[16]. This approach allows for a more accurate and flexible analysis of dynamic electromagnetic fields in submarine cables, taking into account complex interactions and configurations.

4

Electromagnetism Within Submarine HVAC Cables

This chapter provides a detailed understanding of the key electromagnetic components in submarine high-voltage alternating current (HVAC) cables. It explores the capacitive and inductive phenomena that occur along and within transmission lines, and gives an overview of the overarching power transmission system. The purpose of this design and physics review is to support the subsequent analysis of the flow of current and the EMF emissions associated with it.

The design and structure of three-phase alternating current cables (SPCs) is discussed in section 4.1, detailing the roles of essential components such as the conductor, insulation, metallic screens, anti-corrosion sheath, and armor. Understanding these components is crucial for analyzing the electromagnetic interactions within the cables.

Next, an overview of transmission system design is given in section 4.2, introducing the concept of reactive power, reactive power compensation and grounding of the cable. This is the basis for analyzing capacitive and inductive effects in submarine cables. This is followed up on in section 4.3 by a more in-depth analysis of the inductive and capacitive phenomena within the cable, as well as some smaller concepts that are

Finally, we identify key parameters for modeling electromagnetic fields and summarize their impact on cable design and performance. This chapter lays the groundwork for understanding the complex electromagnetic phenomena in submarine HVAC cables, which is essential for optimizing their design and mitigating their environmental impact.

4.1. Review of Submarine Three-Phase AC Cable Design

When attempting to analyze electromagnetics within three-phase alternating current cables (SPCs from now on), we should first define what a typical SPC looks like. Figure 4.1 shows us the cross-section of an SPC, which is the basic blueprint for all concentric 3PH AC submarine cables.

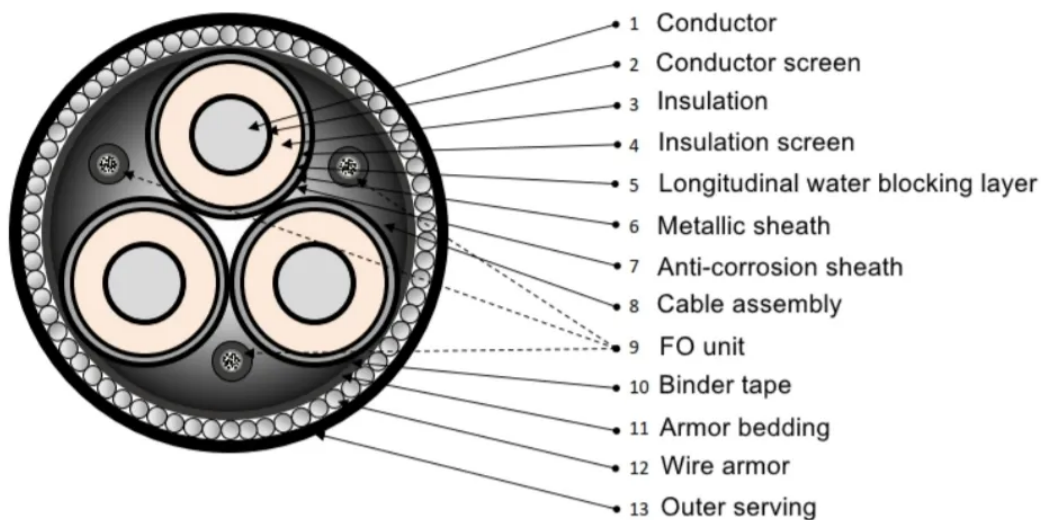


Figure 4.1: Cross section of a standard 3PH AC submarine power cable [33]

The components displayed in the drawing are always present within this type of cable, but the materials and dimensions differ, depending on the specific application. It must be noted that modern cable design intentionally has seawater within the filler layer, which is left out in the figure. Table 4.1 summarizes the most important components for this research and their specific use. For the purpose of this research, the conductor (see subsection 4.1.1), insulation (together with the semiconducting layers, see subsection 4.1.2), metallic screens/sheaths (see subsection 4.1.3), the anti-corrosion (polyethylene) sheath (subsection 4.1.4) and armor (see subsection 4.1.5) need to be discussed more in depth.

Table 4.1: Components of a Submarine Power Cable

Component	Description
Conductor	Used for conducting the current, which is the main goal of a SPC.
Insulation	Essential in the conduction of current, to prevent short circuiting with the ground. Also protects living organisms from the electricity in the cable.
Metallic Screen/Sheath	Provides a path for fault currents, a return path for induced capacitive charge currents, earth potential for human safety, and a moisture barrier for the cable insulation.
Conductor/Insulation screen	Placed on the interfaces of the insulation with the conductor and the metallic sheath. Used to smoothen the connection between the two such that the chance of electric breakdown of the insulation is lower.
Anti-corrosion (polyethylene) sheath	Conventionally used to insulate the metallic sheath from the seawater to protection it from corrosion. New cable designs, however, use a semi-conducting outer sheath to suppress circulating currents in the metallic sheath due to capacitive and inductive effects.
Fiber optic (FO) unit	Used for communication and thermal monitoring of the cable during use.
Cable assembly (fillers)	The open sections within the cable contain fillers, which are non-metallic rods that protect the shape of the cable.
Armor bedding	Often a polypropylene yarn, which provides structural integrity to the armor system and protection to the PE sheath.
Armor	Metal wires that give tensional strength to the cable during laying, while also protecting the cable from corrosion and impact damage while on the seabottom.
Outer serving	A protective layer for the armor wires. Can be made of a polymeric yarn or extrusion. Protects against scratches that can weaken the corrosion protection of the armor.
Sea water	The unoccupied space within a submarine cable is often filled with seawater, intentionally or not. In modern cables, the seawater provides a conductive medium between the metallic sheaths to dissipate circulating currents.

4.1.1. The Conductor of an SPC

In order to transmit electric power from one place to another, a conductive medium is required. This means a medium that allows for easy movement of electrons within itself. A medium that meets this need is said to have a high value of **conductance** (G) which is equal to the reciprocal of **resistance** (R). The level of conductance of the conductor dictates the losses and thereby the production of heat in the cable. High temperatures are damaging to cable components,

most importantly the insulation layer, and should therefore be avoided. Temperature is the limiting factor for the cable loadability (ampacity).

The formula for conductance is the following:

$$G = \frac{I}{V} = \frac{\sigma A}{l} \quad (4.1)$$

From this equation we see that there are three design parameters that influence G , of which there are two that are 'free to choose', given a distance over which we require to transmit power. The **conductivity** (σ) is a material property, and is a product of the amount of free electrons within a material and their freedom to move when subject to an electric field [34]:

$$\sigma = -\rho_e \mu_e \quad (4.2)$$

Where ρ_e is the free-electron charge density (which is negative as electrons are negatively charged) and μ_e the mobility of the electrons in the material. In submarine power cables, the most well-known conductor is copper (there is even the International Annealed Copper Standard (IACS), describing a conductivity relative to a certain standard [35]). Besides copper, the main materials are aluminum alloys [36].

The other parameter that can be chosen in the design process is the cross-sectional area. A larger cross section will always result in a better conductance, as can be seen in Equation 4.1. To achieve this, a solid conductor is often chosen as it is easy to manufacture and has good longitudinal water tightness [36]. However, in addition to having a cross sectional area, there is a phenomenon that has an impact on the effectiveness of the cross section: the **skin effect** (more on this in subsection 4.3.5). The skin effect comes from the EMF produced by the conductive current, which repels the current toward the peripherals of the conductor [36]. To counteract this, the cross section can be broken up into many individual and smaller strands of wire. Arrangements that implement this idea are the stranded conductors, segmental conductors (often the Milliken type) and the keystone conductor [36].

With regards to the emission of EMFs, conductors can have another important parameter. Because of mechanical strength improvements, the armor layer can be twisted (subsection 4.1.5), which creates torsion in the cable when loaded in tension. To counteract this torsion, the conductors (and consequently the whole phases) are twisted in opposite direction. The length over which the phases make a 360° rotation is called the lay-length. Lay-length can have a large impact on the EMF emissions, as it results in increased, destructive interference between the EMF emissions of different phases. Next to their effect on EMFs, the phase lay-length, together with the armor lay-length, dictate the required length of a model.

4.1.2. Electrical Insulation of SPCs

Electrical insulation is of paramount importance for a SPC to function, as well as for safety. Due to the high potential difference between a conductor and its environment, as well as a different phase in the case of three-phase AC, effective insulation is essential to prevent leakage [36].

To create an effective barrier, the resistance of the insulation layer should be high enough to limit the interaction with its environment to an acceptable level. Rewriting Equation 4.1 results in the resistance formula:

$$R = \frac{l}{\sigma A} \quad (4.3)$$

Where l is the length in the radial direction (that is, the thickness of the insulation layer), σ the conductivity, and A the average area of the insulation tube around the conductor. Given a conductor geometry, we can create a high resistance by choosing a large thickness l and a low conductivity σ (where A follows automatically from l).

The most common isolation material in SPCs is **cross-linked polyethylene (XLPE)**. An important property of the insulation layer is the relative permittivity of the material, which is relatively high for XLPE (≈ 2.3 [37]). Relative permittivity dictates the capacitance of the cable (subsection 4.3.7), which has a significant effect on the current distribution along the length of the conductor and, depending design, on the current in the metallic sheath (subsection 4.3.6).

4.1.3. Use of Metallic Sheaths in SPCs

Metallic sheaths (also called screens) are used to protect the insulation layer from water intrusion, provide a fault current return path, a path for charging currents (subsection 4.3.6) and a neutral to provide safety [38] [36]. Submarine cables, although they are buried in the sea floor, are almost always in contact with water. To truly isolate the cable from water is complicated; therefore, manufacturers often opt for a semi-wet design. In a semi-wet design, the infiltration of water within the cable is calculated, and only the insulation is shielded by use of a metallic layer, called the sheath/screen.

Another use of the metallic sheath is to provide electrical safety to the system. It is used as a ground connection, providing a path for fault currents and induced capacitive charge currents (see subsection 4.2.1). The ground connection is also a means of making the cable safe to touch, as well as restricting leakage of the electric field to the environment.

4.1.4. Modern Design of Polyethylene Outer Sheaths

The metallic sheath that protects the insulation from water intrusion is subject to galvanic corrosion when it comes into contact with seawater. For this reason, there has always been an insulating polyethylene layer around the metallic sheaths to protect them from corrosion. This layer was isolating to stop the conduction between the metal and the seawater. However, nowadays manufacturers have opted to use a semi-conductive PE layer instead. The reason for this is that there are large heat and safety advantages in connecting the sheaths together (discussed in subsection 4.3.2 and subsection 4.3.6), while the corrosive effect is deemed acceptable by experts.

The heat and safety advantages of using a semiconductive layer instead of insulation are based on the common equipotential that exists between the three-phase conductors and by extension their metallic sheaths. The circulating currents (from capacitive and inductive effects) that would historically flow through the metallic sheath (explained in more detail in subsection 4.3.6 and subsection 4.3.2), are said to dissipate because the voltage and current in each conductor is phase shifted by 120° . A phase shift of 120° means that the voltage and current vectors add up to zero, which in turn means that they can be considered grounded when connected electrically.

In their design process, cable manufacturers nowadays assume that this method creates a perfect ground connection along the length of the sheath, which completely cancels out the charging and circulating currents. This greatly decreases the losses within the cables and also decreases the stress on the metallic and polyethylene sheaths. However, it can be argued that this is not completely true, as we will see in chapter 5.

4.1.5. The Armor Layer and Short Twisted Periodicity

Submarine cables can experience heavy tensional stresses during the laying process, as well as impact stresses from the installation tools. And, depending on their burial depth (subsection 4.3.8) they can also be damaged by anchors or fishing gear [36]. For this reason, the cable's internals are protected by an armor layer.

The armor has a great impact on the mechanical properties of the cable. Armor wires are twisted so that they can convert tensional forces into torsional forces, increasing the strength of the cable. Depending on the desired properties, the lay length L_{arm} (length for full rotation) of the armor can be chosen. To create torsional balance, this will often also require a twist in the opposite direction within the conductors (with lay length L_{pha}).

A armor wires are often made of steel, which because it is a magnetic material introduces extra losses during operation. The amount of relative twist between the conductors and the armor affects the electromagnetic performance of the cable. The periodicity of this relative twist is called the cross-pitch (CP), which is the length after which both the armor and the phases first make an integer number of 360° turns. Due to electromagnetic interactions between the armor and phases (section 4.3) CP has been recognized as the minimum length required for accurate modeling, because it would capture every periodic effect [39],[40]. The value of CP can be calculated by the following:

$$CP_{\text{cab}} = \frac{360^\circ}{T_{\text{pha}} - T_{\text{arm}}} = \frac{1}{\frac{1}{L_{\text{pha}}} - \frac{1}{L_{\text{arm}}}} \quad (4.4)$$

Where $T_{\text{pha}} = \frac{360^\circ}{L_{\text{pha}}}$ and $T_{\text{arm}} = \frac{360^\circ}{L_{\text{arm}}}$. There are two main effects of the cross-pitch, which are based on the same principle. Due to the changing magnetic field within the three conductors, electric voltages will be induced along the length of the armor wires. The periodic relative twist results in periodically changing induced voltages along the length of the wires. To clarify, a visualization is made in COMSOL, which is shown in Figure 4.2.

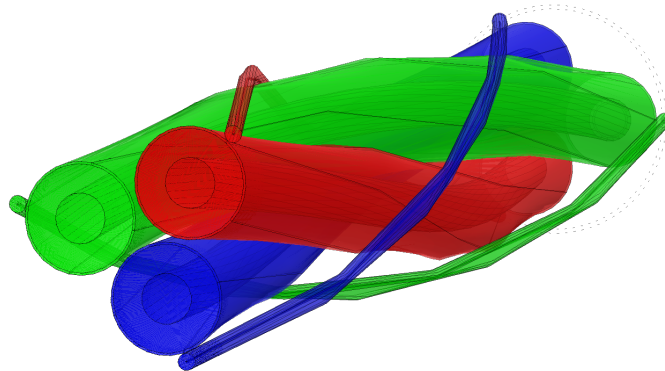


Figure 4.2: 3D drawing indicating the meaning of the short-twisted periodicity between armor wires and phases. In this cable section only three armor wires are shown to allow for visibility. The short-twisted periodicity is the length after which an armor wire-phase pair meet again[40].

To understand the effect of cross-pitch, it convenient to first see what happens without it. In the case that the wires

remain parallel with the phases along the length, this will create longitudinal currents, in the opposite direction to the conduction current, thereby reducing the magnetic field emitted. However, these currents also create losses in the system, increasing the temperature of the cable and thereby decreasing its ampacity.

This effect is almost completely neglected when the armor has the opposite twist to the phases. Instead of having a unidirectional induced voltage over the length of a wire, the armor wires now encounter an induced voltage from all three phases consecutively within a length equal to the cross-pitch. Each phase induces an electrically phase-shifted voltage in the wire $\pm 120^\circ$, which combines to a total of zero within the cross-pitch. Because of this periodicity, all the electromagnetic interactions that occur between the cable components along its entire length can be observed within a single cross-pitch. Therefore, CP is the cable's shortest complete period.

In addition to the positive effect on losses, this also makes analysis easier. Longitudinal currents along the length of armor wires do not have to be calculated before being used as input to a model. Armor wires therefore act as a passive element within the model that can reduce the emitted magnetic field because of their high magnetic permeability (for steel wires).

To increase computational efficiency, research has been done to see how much the length of the section could be reduced without compromising the accuracy of models [39], [41], [42], [43]. This resulted in the minimal periodicity of the cable, called the **short twisted periodicity** (L_{sp} : the distance after which a pair of armor and wire arrive at the same relative position. The short-twisted periodicity can be calculated by

$$L_{sp} = CP_{cab}/N_{arm} \quad (4.5)$$

This method of shortening the cable section modeled for computational efficiency is promising but should be tested, which is done in subsection 6.3.4.

4.2. Brief Overview of Power Transmission System Design

This section provides a detailed look at the concept of reactive power balancing in transmission systems and grounding mechanisms of submarine cables, including schematic representations and discussions on the implications of grounding techniques such as bonding at both ends and the interaction between cable sheaths and armor.

When considering the electromagnetic fields produced by a cable, it is important to also look at the system around it. The design of the transmission system has an impact on the operating conditions of the cable and thereby on the voltage and current distribution along the length of the cable. Transmission from offshore windparks to the main onshore grid has as a goal to deliver electric power with minimal losses. In addition to the obvious solution of creating a cable with a low impedance, the configuration on the offshore and onshore side can have a large impact on the quality of transmission. Additionally, the method of grounding the cable has an impact, not only on safety but also on circulating currents in the metallic sheath and armor, as will be discussed in later sections. subsection 4.2.1 will explain the main methods of reactive power balancing that are employed in windfarms today. subsection 4.2.2 will explain how a submarine cable is grounded

4.2.1. Active and Reactive Power

The primary objective of power transmission is to provide usable energy, known as **active power**, to all connected users. However, alternating-current transmission systems inherently exhibit capacitive and inductive properties, producing undesired **reactive power**. It should be noted that all components in AC transmission produce and/or consume reactive power, including the consumers. Therefore, producing extra reactive power in the transmission system can be beneficial. This subsection will discuss the concept and means of balancing the reactive power, which is required for the analysis of current distribution along the cable.

The Concept of Reactive Power

Capacitive effects generate **reactive power**, while inductive effects consume it. Reactive power is equal to the power that is lost due to an electrical phase shift ϕ between the transmission voltage and the current, which reduces the efficiency of active power delivery.

In an ideal case, the voltage and current waves are exactly aligned with each other, so that their product, which is the active power: $P = v \cdot i = V \sin(\omega t) \cdot I \sin(\omega t)$ is maximized. To visualize this, we can look at the upper circuit of Figure 4.3, where voltage (blue) and current (green) are perfectly aligned, to provide optimal and positive active power.

When we then introduce a forward electrical phase shift of the current, which occurs due to the capacitance of the cables, we lose active power because, over a part of the period, the product of v and i becomes negative. The extreme case, with a phase shift ϕ of $+90^\circ$ is depicted in the middle of Figure 4.3, where the net active over the whole period decreased to zero. Inductance has the opposite effect as capacitance because it results in a backward phase shift in the current relative to the voltage. However, with the same result: losses in active power (the extreme case of -90° is shown at the bottom of Figure 4.3). Also, note that there is still the same amount of current flowing through the line, but without

delivering power. In practice, the phase shift ϕ between voltage and current will be somewhere between $-90^\circ \leq \phi \leq 90^\circ$ and the **apparent** (instantaneous) power flowing through the line is the sum of active and reactive power. Mathematically, this is written as follows:

$$\vec{S} = P + jQ = VI \cos \phi + jVI \sin \phi \quad (4.6)$$

Where S denotes the **apparent power** in the line, P the active power and Q the reactive power on the imaginary axis j . The AC transmission voltage and current are represented by V and I respectively, which are the root mean square (rms) values of v and i [44].

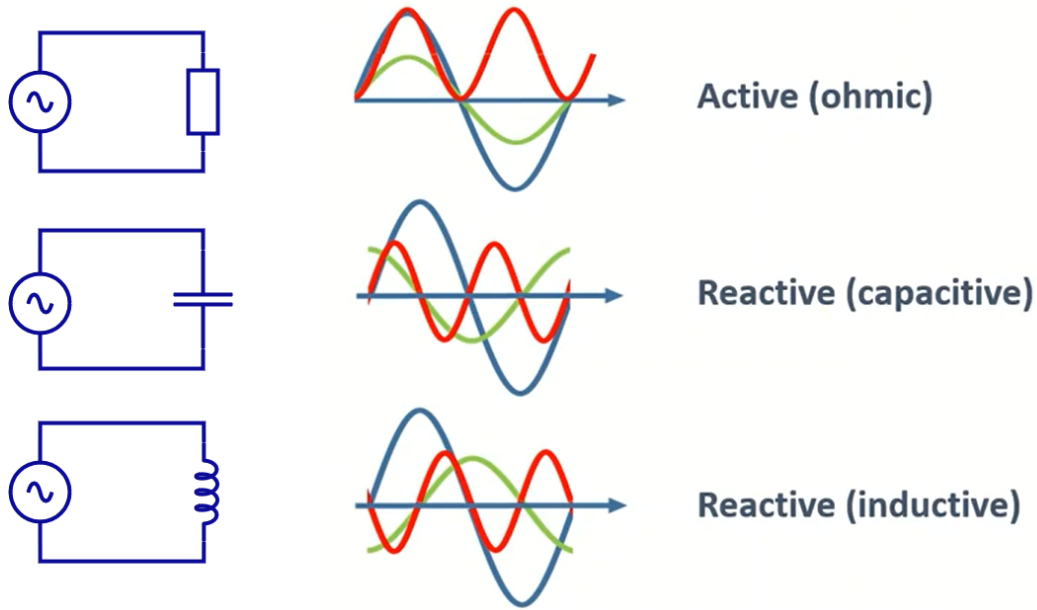


Figure 4.3: Visualization of the effect capacitance and inductance have on the current (blue), voltage (green), and active power (red). [45]

From this understanding, **reactive power** can be defined as the component of alternating current electricity that oscillates between the source and the load, resulting in no net transfer of energy over a complete cycle. It is measured in volt-amperes reactive (VAR) and is essential for maintaining the voltage levels necessary for active power to be delivered efficiently

Reactive Power Management in Submarine Power Cable Systems

Reactive power must be managed in order to achieve efficient transmission, which can generally be done in two ways:

- Compensation at the load side
- Compensation in the transmission system at both ends of a cable (Q_1 sending end and Q_2 receiving)

Since we are looking at a submarine power cable, compensation on the load side will not be analyzed here (although a description can be found in [44]). Reactive power compensation in the submarine power transmission system, however, has a large impact on the current and voltage distribution along its length, so its mechanism of operation will be described here.

Due to their large capacitance, submarine power cables (SPCs) introduce substantial reactive power challenges within transmission grids, which can be compensated for using an inductive component. SPCs already have inherent inductance, but there is still often a surplus of reactive power. To mitigate this, shunt reactors play a vital role. Connected in Y-formation between the three phases (Figure 4.4), shunt reactors absorb reactive power, manage voltage levels and influence the distribution of current flow through the cables. Their placement is the sending and / or receiving end of long submarine cable connections [46] or within substations. For applications demanding precision and flexibility, variable shunt reactors with tap changers offer a highly adaptable solution.

It is worth noting that while shunt reactors primarily handle reactive power compensation, transmission lines often incorporate series reactors that play a crucial role in overall system safety and durability. Series reactors, typically installed in line

with the power cables, are used to limit short-circuit currents and modify load flow patterns. By introducing impedance into the system, they help manage fault currents and improve system stability. These reactors are usually placed at the sending and receiving ends or if the design allows it at specific intervals along the cable length, to optimize their effectiveness.

This type of system is described in Figure 4.4, where inductive line reactors are placed at both ends of the transmission line to compensate for the capacitive transmission line interfaces. Line reactors are star-connected between phases and combined with a series neutral grounding reactor (NGR) to protect against overvoltages. This combination ensures both reactive power management and enhanced system protection, illustrating the dual roles of inductive components in submarine cable systems.

Submarine power cables, due to their significant capacitance, introduce substantial reactive power challenges within transmission grids, which must be compensated for using an inductive component. SPCs already have an inductance, but there is still a surplus of reactive power. To mitigate this, shunt reactors play a vital role. Connected between line voltage and ground, shunt reactors absorb reactive power, managing voltage levels, and influencing the distribution of current flow through the cables. Their strategic placement is often at the ends of long submarine cable connections or within substations. For applications demanding precision and flexibility, variable shunt reactors with tap changers offer a highly adaptable solution. Note that while series reactors are not directly involved in reactive power management, they are used in conjunction with submarine cable systems to limit short-circuit currents and modify load flow patterns.

This type of system is described in Figure 4.4, where both ends of the transmission line are inductive line reactors compensating for the capacitive transmission line interfaces. The line reactors are star-connected between the phases, but are also combined with a (series) neutral grounding reactor (NGR) to protect from overvoltages.

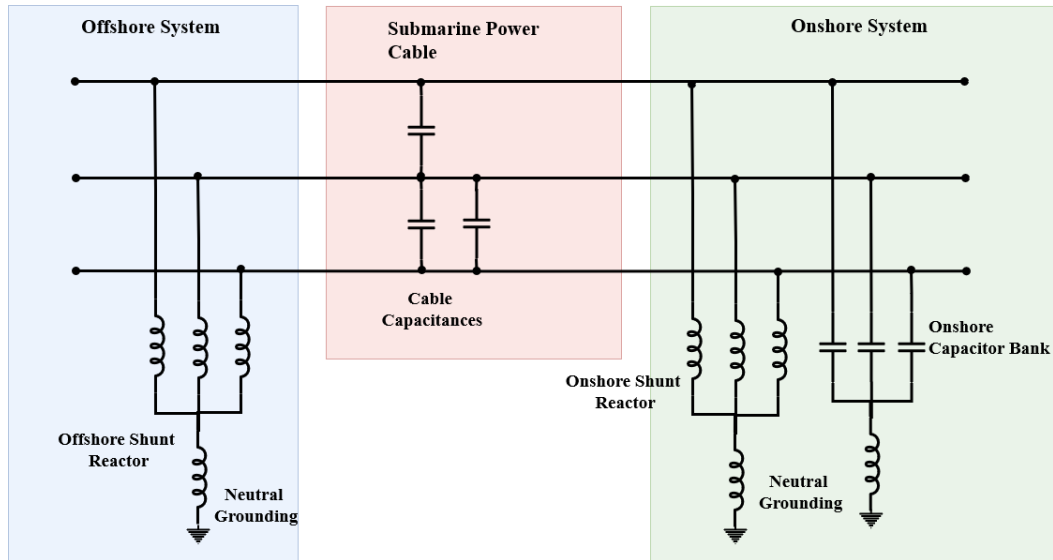


Figure 4.4: Schematic showing a reactive power compensation mechanism at both ends of a transmission line [47]

From the viewpoint of electromagnetic field emissions, it is essential to take into account these reactive power compensation mechanisms. It is common to simply look at the active power for predicting the magnetic fields produced by an SPC, but for long-distance transmission lines the reactive power can often be of comparable magnitude to the active power, and the total current at the sending end should be calculated according to Equation 4.7 [48]:

$$I_s = \sqrt{\left(\frac{P}{3V_s \cos \theta}\right)^2 + \left(\frac{Q_s}{3V_s}\right)^2} \quad (4.7)$$

Where P is the active power, V_s is the voltage at the sending end, Q_s is the reactive power compensated at the sending end and θ the angle between active- and apparent power.

Reactive Power Distribution Along the Cable

The distributed capacitance in a submarine power cable (SPC) affects its reactive power characteristics. Longer cables exhibit higher capacitance, leading to increased reactive power generation and higher compensation requirements. In long-distance HVAC submarine cables, reactive power becomes the limiting factor due to its impact on real power transmission, thermal losses, and cable heating.

Without compensation equipment, the charging current will be very high at the generation end and decreases linearly

until it reaches the pure conduction current at the receiving end. Shunt line reactors (Figure 4.4) change the shape of this current distribution so that the average and peak values are lower.

The Apparent Current Distribution in Transmission Lines

The combination of active and reactive power in a transmission line results in apparent power. The apparent power is the vector magnitude of the the active power on the real axis and reactive power on the imaginary axis. Due to the large capacitance of a submarine HVAC cable, creating large amounts of reactive power, and the measures taken to absorb reactive power, the apparent power can be significantly larger than the active power. Therefore, it is important to take all reactive components into account. This is a known phenomenon, as it involves losses and power quality in a transmission system, and TenneT has created some models for different operating conditions, for which the vector magnitudes of the current distributions are displayed in Figure 4.5.

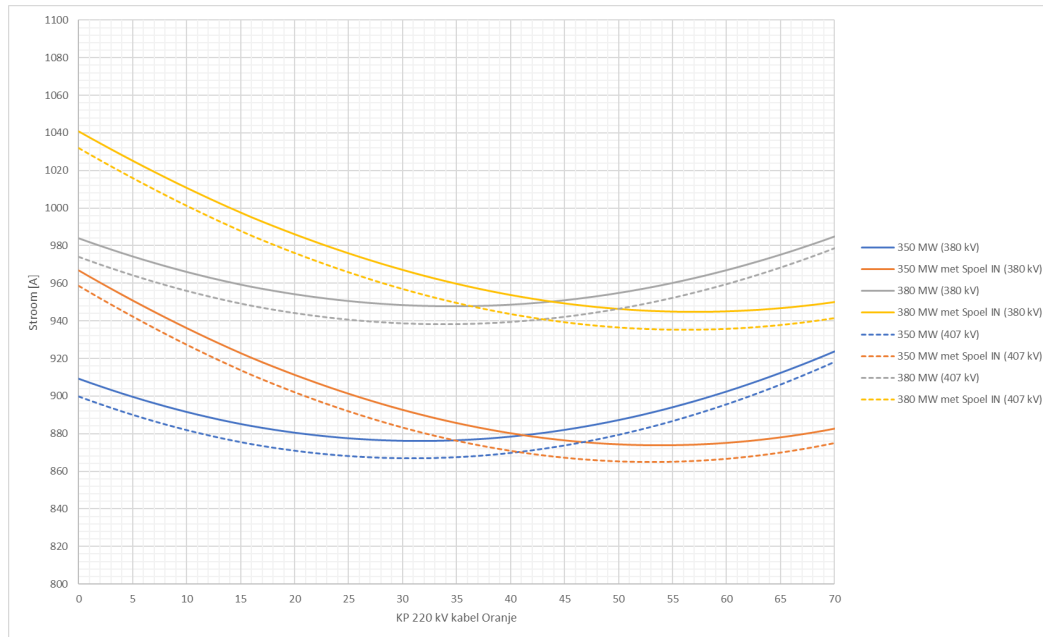


Figure 4.5: Current distribution from PowerFactory simulations by TenneT. For a range of reactive power settings.

It is clear that in realistic operating conditions, the (apparent) current can be up to 140 A higher than the active current, a difference of more than 13%.

4.2.2. Grounding Method

In addition to managing the power quality of a transmission system, a transmission system operator should ensure electrical safety. Electrical safety includes protecting itself from damage due to excessive buildup of voltages over the insulation layers (conductor and sheath insulation), resulting in partial discharge at imperfections in the layer. Partial discharge can create a permanent path of low resistance and short circuiting between the conductor and the metallic sheath. However, electrical safety also means that the cable should be safe to touch.

Both of these issues are addressed through grounding the neutral points of the cable that are supposed to be at zero voltage. Electrical grounding means creating a connection to a potential of zero, preferably with a large potential for absorbing current. This type of medium is often called an infinite sink in idealized form. In practice this is often done, as the name would suggest, by connecting something electrically to the earth. The Earth is considered to have an electric potential of zero, to which all other potentials are measured.

To use the Earth for electrical grounding, systems are physically connected to it, through burying large conductive rods in it and connecting them. For submarine power cables this is done at the sending and receiving ends, at connection sockets, but sometimes also along its length through copper wires extending from out of the buried cable into the environment. However, the Earth is not the only large sink of zero potential. When sources of equal but opposite potential are connected together, they form a potential of zero. This is the case for the three phases of a HVAC submarine power cable, which, due to their electrical phase shift of 120° , have a sum of zero potential. In modern submarine cables, this is used to dissipate the potential on the metallic sheath, by creating a conductive medium between them, thereby suppressing circulating currents. This method reduces losses and buildup of voltage. At the ends of a submarine cable line, the metallic sheaths, as well as the armor wires that can also experience buildup of voltage and current along their length,

are bound together with a highly conductive joints, to use their common equipotential (zero) for grounding. Due to this bonding, where the sheaths are connected in series, circulating currents can flow between the metallic sheath of different phases. Next to the use of the common equipotential for grounding, the joint itself is connected to the Earth for additional safety.

Grounding is a very important part of electrical engineering, as it ensures the durability of the cable and the safety of people interacting with the products. However, as will be seen in subsequent parts of this thesis, the connection to a zero potential, as well as the series connection of the metallic sheaths, also have negative consequences for transmission losses.

4.3. Electromagnetism in HVAC Submarine Power Cables

Now that the important cable components for this analysis and the transmission system surrounding the cable have been discussed and the important electromagnetic laws have been defined, it is time to explain the interactions that occur within a submarine power cable.

The first topic that has to be discussed is the interface between the conductors and the metallic sheaths and between the metallic sheaths, as it is the basis of a large part of this research. Interfaces between metallic sheaths can dissipate circulating currents, but the extent of their impact will be analyzed in this thesis. The concepts of induced sheath currents and capacitive charging currents are then discussed to understand the formation of circulating currents in the metallic sheath, emphasizing the importance of the capacitance and inductance parameters. Next, other possible factors influencing the internal currents and EMF emissions are discussed: the concept of Eddy currents, the effect of temperature on conductivity, and the skin effect.

4.3.1. Grounding and Interface of Conductors and Sheaths

As explained in subsection 4.2.2, in modern cable designs, which are the topic of this thesis, the metallic sheaths are grounded through a connection with their common equipotential. The main reason that grounding of the sheaths is necessary is the capacitive charging of the insulation layer (more on this in subsection 4.3.6), which results in a buildup of potential on the metallic sheath if no compensation measures are taken. visualization of this, for an infinitesimal slice of cable, is shown in Figure 4.6.

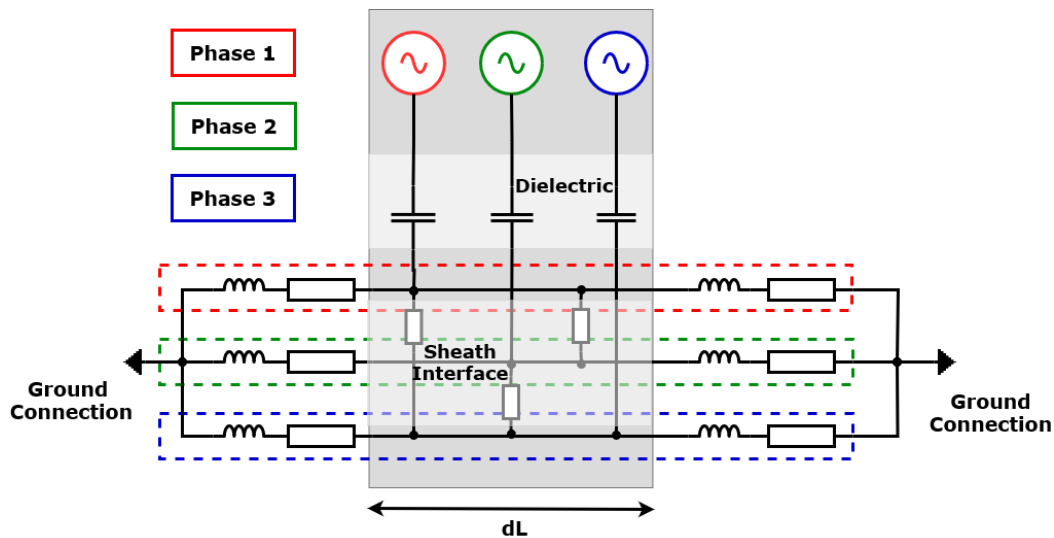


Figure 4.6: Circuit representation of an infinitesimally small length of cable, showing the interaction between the conductor phases and corresponding metallic sheaths, as well as between the different metallic sheaths. Metallic sheaths are grounded at each end.

This concept is important as it is the basis for understanding how current distributions in the conductor and metallic sheaths change when changing design, material, and operational parameters.

As this interface is present over the whole length of the cable, it could be expected that all sheath currents are dissipated over this medium and that no longitudinal currents remain. However, an absolute cancellation of these currents was deemed unlikely as there could never be a perfect connection between the sheaths. The question of the extent of this dissipation effect is answered in this thesis.

4.3.2. Induced Currents In The Metallic Sheath

Faraday's law (subsection 3.1.3) is one of the most important phenomena in physics, as it enables the operation of electricity transformers and many other electromagnetic tools, but in the design of submarine power cables, it has a large drawback. As Equation 3.10 states, a changing magnetic field creates an electric field that acts to create a current in the opposite direction to the current that created the magnetic field. In the context of the metallic sheath of an HVAC cable, which is connected to the ground and the other two ($\pm 120^\circ$ phase-shifted) metallic sheaths at its start and end point, this creates circulating currents. This results in power losses due to the sheath resistance, which is directly bad for efficiency, but also hurts the ampacity (max conduction current) of the cable due to heating. This effect is mostly neglected in modern designs due to the semiconductive layer connecting the three sheaths over the entire length. However, its effect should still be incorporated in the analysis of the net current flowing through the cable. Therefore, this section will show how to calculate the magnitude of the induced electric field $\mathbf{E}_s$ within the metallic sheath due to the conduction current I_0 .

Induced Voltage in the Metallic Sheath of AC Power Cables

A method has already been developed to estimate the induced voltage in the metallic sheath and consequently the induced current [49],[50]. For a single-phase cable, the induced electric field in the sheath follows from the sheath reactance per unit length X_s and the conduction current in positive longitudinal direction I_0 to be[50]:

$$\mathbf{E}_s = -jX_s I_0 \quad \text{V/m} \quad (4.8)$$

where X_s is the impedance per unit length, calculated by

$$X_s = 2\omega \cdot 10^{-7} \ln \left(\frac{2R}{D_s} \right) \quad \Omega/\text{m} \quad (4.9)$$

R the outer radius of the metallic sheath and D_s the inner diameter.

However, as we know, modern SPCs are often three-core cables that all $\pm 120^\circ$ phase shifted w.r.t. each other. Looking at the induced voltage in a single core requires taking the induction from other phase conductors into account. Because of the phase shift, the total induced voltage per sheath is reduced. The equations for the total induced electric field for each phase are indicated below:

$$\mathbf{E}_{sa} = -jI_1 X_1 + jI_3 X_a \quad \text{V/m} \quad (4.10)$$

$$\mathbf{E}_{sb} = jI_1 X_1 + jI_3 X_3 \quad \text{V/m} \quad (4.11)$$

$$\mathbf{E}_{sc} = -jI_3 X_3 + jI_1 X_b \quad \text{V/m} \quad (4.12)$$

for which the reactances X can be calculated by[50]:

$$X_1 = 2\omega 10^{-7} \ln \left(\frac{2S_1}{d_s} \right) \quad \Omega/\text{m} \quad (4.13)$$

$$X_3 = 2\omega 10^{-7} \ln \left(\frac{2S_2}{d_s} \right) \quad \Omega/\text{m} \quad (4.14)$$

$$X_a = 2\omega 10^{-7} \ln \left(\frac{S_3}{S_1} \right) \quad \Omega/\text{m} \quad (4.15)$$

$$X_b = 2\omega 10^{-7} \ln \left(\frac{S_3}{S_2} \right) \quad \Omega/\text{m} \quad (4.16)$$

Note that now S describes the distances between the central axes of the conductor, as depicted in Figure 4.7.

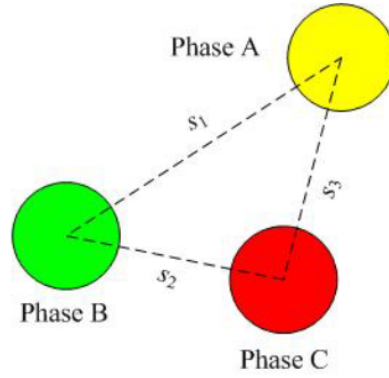


Figure 4.7: Depiction indicating the meaning of parameters S_1 , S_2 and S_3 [50]

Induced Voltage in Equilateral Three-Core HVAC Cables

For an **equilateral triangle** ($S_1 = S_2 = S_3 \equiv S$) this can be simplified [51]. Reactances become:

$$X_1 = X_2 = 2\omega 10^{-7} \ln \left(\frac{2S}{d_s} \right) \quad \Omega/\text{m} \quad (4.17)$$

$$X_a = X_b = 2\omega 10^{-7} \ln \left(\frac{S}{S} \right) = 0 \quad \Omega/\text{m} \quad (4.18)$$

which combined with the fact that $I_3 = I_1 * e^{-j\frac{2\pi}{3}}$ results in the induced electric fields being:

$$\mathbf{E}_{sa} = -jI_1 X_1 \quad \text{V/m} \quad (4.19)$$

$$\mathbf{E}_{sb} = jI_1 X_1 + jI_3 X_3 = \left(\frac{\sqrt{3}}{2} + \frac{j}{2} \right) X_1 I_1 \quad \text{V/m} \quad (4.20)$$

$$\mathbf{E}_{sc} = -jI_3 X_3 = \left(-\frac{\sqrt{3}}{2} - \frac{j}{2} \right) X_1 I_1 \quad \text{V/m} \quad (4.21)$$

Note that the term $X_1 I_1$, although phase shifted, has the same amplitude. This means that we can write the induced field in phasor form as:

$$\mathbf{E}_{sp} = 2\omega I_{cp} \ln \left(\frac{2S}{D_s} \right) \cdot 10^{-7} \quad \text{V/m} \quad (4.22)$$

Where ω is the radial frequency of transmission, I_{cp} is the conduction current phasor for a single phase, S is the spacing between conductor centerlines and D_s is the average diameter of the metallic sheath.

Induced Longitudinal Sheath Current

Knowing the induced electric field and consequently the voltage induced in the sheath, the current can be calculated by dividing the induced voltage by the sheath impedance.

$$I_s = \frac{E_s}{R_s + jX_s} \quad \text{A} \quad (4.23)$$

However, it must be noted that for the modern cable design, a more complex formulation is required due to the current dissipation created by the common equipotential of three metallic sheaths that are electrically connected through the semiconductive PE sheath and the conductive seawater in the cable (more on this in section 5.4). The resistance of the sheath per unit length can be expressed by the following relation[50]:

$$R_s = \frac{\rho_s}{A_s} [1 + \alpha_s(\theta \cdot \eta - T_{\text{ref}})] \quad \Omega/\text{m} \quad (4.24)$$

where ρ_s is the resistivity of the metal sheath material, A_s is the cross-sectional area of the metal sheath, α_s is the temperature coefficient of the material, θ is the temperature of the conductor in degrees Celsius and η is the ratio of the sheath temperature to the conductor temperature, which typically ranges from 0.7 to 0.8 depending on the insulation thermal resistance[50]. The reactance X_s for the phase of interest is calculated from Equation 4.9.

4.3.3. Induced Eddy Currents

Next to the longitudinally circulating currents due to electromagnetic induction, there are also locally circulating currents that arise from the same principle. These are called Eddy currents because of the nature of their direction. A depiction of the effect of Eddy currents is shown (in Figure 4.8) for two wires carrying a current in the same direction.

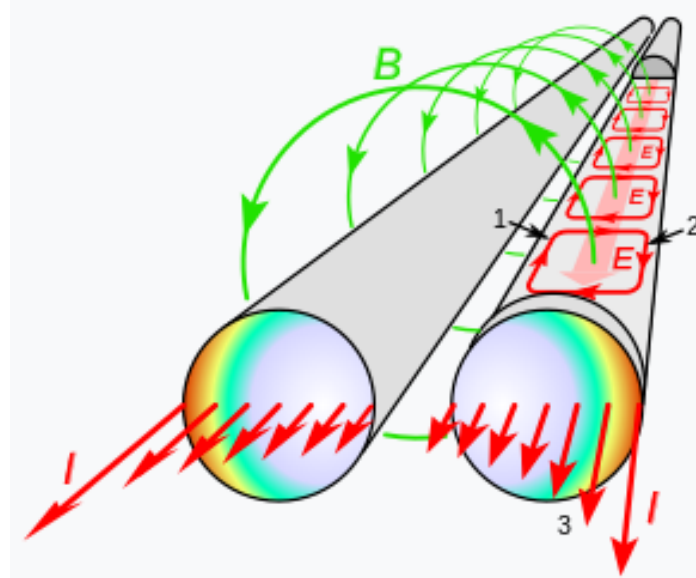


Figure 4.8: Schematic showing the effect of Eddy currents within two parallel cables carrying current in the same direction[52].

Eddy currents are practically whirlpools of current that arise from a gradient in changing magnetic field. At point 3 in the figure we see how the changing magnetic field produced by the left conductor has created a current gradient in the right conductor, based on the induced voltage in the direction opposite to the current, which decreases with distance to the center of the left conductor and vice versa. This gradient produces the Eddy's as indicated by the circles with notation 'E' as shown in the conductor on the right.

With regard to the analysis of electromagnetic field emissions, Eddy currents are important because they create a shielding effect. An estimation of their intensity can be made based on equations from the literature ([53]), but this is beyond the scope of this research.

Since the COMSOL Multiphysics Magnetic Fields module [17] solves Maxwell's equations in 3D, this will automatically incorporate the effect of Eddy currents.

4.3.4. Thermal Effect on Conductivity

In the design and operation of submarine power cables, it is crucial to consider the thermal effects on the materials' conductivity. Temperature significantly impacts material conductivity, which in turn affects the efficiency and performance of the cables. For most practical scenarios, particularly under high current conditions, the cable temperature can approach its maximum operational limit, typically around 80 degrees Celsius.

At elevated temperatures, the resistance of the conductor material generally increases, which can lead to a decrease in overall conductivity. Modeling the thermal dependency of conductivity is crucial in the context of electromagnetic field (EMF) simulations for submarine cables. Temperature variations, particularly near the upper operating limit of approximately 80 degrees Celsius under high current scenarios, significantly affect the electrical properties of cable materials and the seawater. These changes in conductivity directly influence the distribution and intensity of EMFs within the cable and thereby also the emissions. Accurately accounting for thermal effects is therefore essential for ensuring that EMF models reflect realistic operational conditions and provide reliable predictions.

4.3.5. The Skin Effect

In conductors, the alternating current (AC) typically flows near the surface rather than uniformly through the entire cross section, a phenomenon known as the skin effect. This effect is driven by induced magnetic fields that generate eddy currents, which oppose the flow of the main current. The result is an increase in resistance within the conductor, particularly noticeable at higher frequencies [34].

At a standard frequency of 50 Hz, commonly used in many power transmission systems, the skin effect is present, but not as pronounced as at higher frequencies. However, it still significantly impacts the behavior of the system, particularly in how currents are induced in the sheaths and armor wires of submarine cables. These induced currents can affect the

overall magnetic field distribution around the cable and need to be carefully considered in the design and operation of these systems.

The general formula for the skin depth δ is [34]

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad (4.25)$$

where

$$\begin{aligned} f &= \text{power transmission frequency} \\ \mu &= \text{magnetic permeability of the conductor} \\ \sigma &= \text{conductivity of the conductor} \end{aligned}$$

This formula is valid at frequencies away from strong atomic or molecular resonances [54], which is the case for standard conduction frequencies in transmission systems. For a transmission frequency of 50 Hz, the skin depth for standard copper is ≈ 9.39 mm and for aluminum ≈ 11.6 mm.

4.3.6. Capacitance and Charging Currents

Capacitance is an important parameter to consider in designing submarine power cables and analyzing current distribution and EMF emissions. Capacitance in electrical cables refers to their ability to hold a charge. In the context of HVAC systems, when voltage is initially applied to a submarine cable, the electrical field within the cable is not established instantaneously. Instead, the cable must first accumulate an electrical charge to polarize the insulation layer, much like a reservoir that needs to be filled before it can discharge water. This initial electrical buildup is termed the charging current.

The concept of charging currents is integral to operating HVAC (High Voltage Alternating Current) submarine cables, which are essential for transmitting electrical power over long underwater distances. These cables exhibit significant capacitive properties because of their insulated construction and the dielectric nature of their surrounding environment, primarily seawater.

As discussed in subsection 4.2.1, the charging current is a non-power-bearing component of the electrical system, meaning it does not contribute directly to the transmission of usable power but is essential for setting up the conditions under which power can be efficiently transmitted. In three-phase systems, each phase of the cable will draw a charging current influenced by the transmission frequency, voltage level, and capacitance of the cable.

For submarine cables, the length and environment significantly amplify the capacitive effects, thereby increasing the magnitude of charging currents. These currents must be carefully managed because they affect the thermal and electrical load on the cable. In addition, they influence the power supply system's design, including the transformers' specifications and the protective equipment's configuration.

Understanding the behavior and impact of charging currents is critical in optimizing the design and functionality of submarine HVAC cable systems, ensuring both efficiency and reliability in power transmission.

Mechanism Behind Charging Currents

Now that the concepts of capacitance and charging current are clarified, the mechanism behind charging currents can be discussed, starting with the standard case of an insulated metallic sheath. As the term suggests, charging currents are essential for priming the cable's insulation system, which functions as a capacitor, to conduct alternating current. This charging process recurs with each cycle of the alternating current, which, for transmission cables in Europe, occurs at a frequency of 50 Hz [55], but can also be 60 Hz in other parts of the world [56].

Each alternating current cycle requires the capacitor to be recharged. This charging requires a current that must be provided by the conductor. Consequently, an additional input current is necessary to maintain the desired output current at the line's termination. Additionally, across the dielectric barrier of the cable, a corresponding charge is needed to counterbalance the conductor's polarization. In traditional cable designs that feature an insulated metallic sheath and solid bonding, this compensatory charge was historically sourced from the ground, thus establishing a circuit through the metallic sheath.

The charging currents are directly proportional to the capacitance of the cable. Given the extended lengths common in modern applications, these currents can be substantial, leading to significant ohmic losses in the system and requiring compensation mechanisms to be built. Considering the cable as a simple capacitor, the charging current drawn as a function of voltage is shown in Equation 4.26.

$$I_c = C_{cab} \cdot \frac{dV}{dt} \quad (4.26)$$

To gain a better understanding of how this works, Figure 4.9 shows a highly simplified model of a single-side bonded single-phase AC cable. This allows us to understand the concept, which can be easily extended to three phases and solid-bonding.

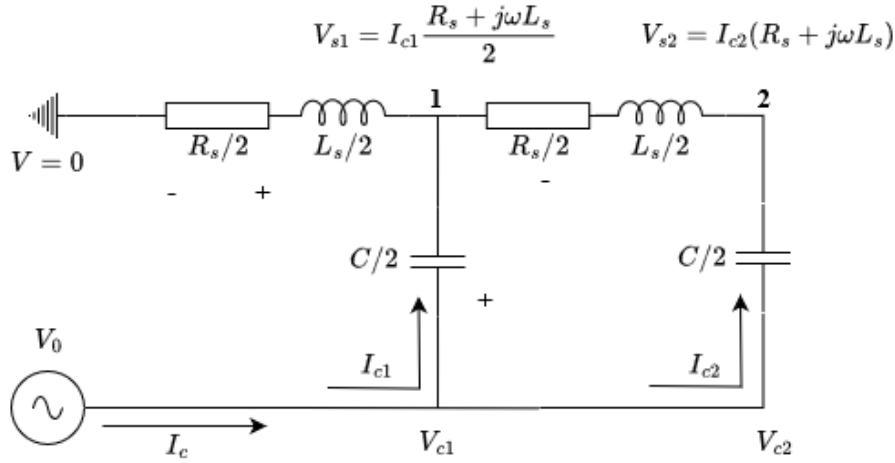


Figure 4.9: Schematic of a simplified single-phase, single-bonded HVAC cable with lumped parameters, each representing half the cable length. Illustration to support the understanding of charging currents in conventional, isolated metallic sheaths.

The electrical circuit under analysis cuts a longitudinal part of the cable into two sections. It consists of a voltage source V_0 representing the conductor, while the other side of the circuit is connected to neutral (ground). The insulation layer of the cable is represented by the two capacitors, both half of the total cable capacitance $C/2$. Each half of the cable also has a resistance $R_s/2$ and an inductance $L_s/2$. Each capacitor has a corresponding charging current, I_c , which is based on the conductor voltage alone, as in reality, it is so much higher than the sheath voltage that the sheath voltage can be neglected for this calculation.

Calculating the charging current through each capacitor is done by using the total impedance Z_i of its path to the ground. Impedance is calculated by:

$$Z_i = R_i + jX_i \quad (4.27)$$

The capacitive and inductive reactances, X_C and X_L , for each capacitor, are defined by the relation

$$X_C = -\frac{1}{\omega C} \quad X_L = j\omega L \quad (4.28)$$

where $\omega = 2\pi f$ is the angular frequency of the AC source C is the capacitance and L is the inductance. The total impedance Z for each series combination of a capacitor and the line impedance $R_s + j\omega L$ is given by:

$$Z_1 = \frac{R_s + j\omega L_s}{2} - \frac{1}{2\omega C} \quad \text{and} \quad Z_2 = R_s + j\omega L_s - \frac{1}{2\omega C} \quad (4.29)$$

from which the charging currents over each capacitor can be calculated by

$$I_{c1} = V_{c1}/Z_1 \quad \text{and} \quad I_{c2} = V_{c2}/Z_2 \quad (4.30)$$

The voltage at each junction on the sheath (1,2 as indicated in Figure 4.9) can be calculated by applying Kirchoff's voltage law on the path of their corresponding charging currents I_{c1} and I_{c2} . From that, it follows that the sheath voltage V_s at each junction is equal to:

$$V_{s1} = I_{c1} \frac{R_s + j\omega L_s}{2} \quad \text{and} \quad V_{s2} = I_{c2}(R_s + j\omega L_s) \quad (4.31)$$

Assuming that the conductor voltage is approximately equal over the length, which it often is, the charging current becomes constant over the length. This in turn means that while the sheath current increases from right to left, as more current joins the same path, the sheath voltage decreases in that direction as the impedance encountered by the current becomes smaller.

The same principle can be applied to the modern metallic sheath that has a conductive polyethylene layer around it.

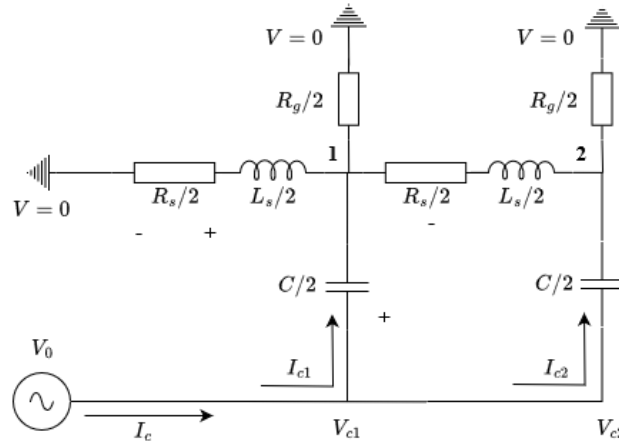


Figure 4.10: Schematic of a simplified single phase, single-bonded HVAC cable with lumped parameters, each representing half the cable length. Illustration to support the understanding of charging currents in modern cables with conductive outer sheaths.

However, in this case, there is not only a ground at the end of the cable but multiple distributed ground connections along its length. A simplified model of this case is shown in Figure 4.10.

It can be deduced from this that the charging current distribution will be highly dependent on the relation between sheath impedance Z_s and the ground resistance R_g . Extensions of this schematic will be used in later analysis of the charging currents.

4.3.7. Capacitance of Transmission Lines

To better understand what charging (capacitive) currents are, and to be able to do a detailed analysis, the concept and means of calculating its value will be discussed here. Capacitance is a measure of the capability for energy storage in electrical devices. Sometimes, this is done on purpose, but it is an unavoidable by-product in the case of HVAC power cables. A capacitor can be seen as a device that stores electric flux (whereas an inductor stores magnetic flux).

Under a potential difference, two conductors will become polarized relative to each other. This means that one side (with the negative charge) stores more electrons, whereas the other creates an equal magnitude but opposite-charge electron deficit (positive charge). The amount of charge stored on both depends on the voltage applied and on a combination of material and structural properties (capacitance). The capacitance is defined as the amount of charge per applied voltage:

$$C = \frac{Q}{V_0} \quad (4.32)$$

The capacitance is a function of only the physical dimensions of the system and the permittivity of the homogeneous dielectric. Capacitance is measured in farads, which are Coulombs per Volt.

The capacitance of cylindrical screens around circular conductors is given by [57]

$$C = \frac{2\pi\epsilon_0\epsilon}{\ln\left(\frac{D_i}{d_c}\right)} C_{LL} \quad (\text{F/m}) \quad (4.33)$$

where

- ϵ_0 is the permittivity of the vacuum $\approx 8.854 \cdot 10^{-12} \text{ F/m}$;
- ϵ is the relative permittivity of the insulation;
- D_i is the external diameter of the insulation (excluding screen) (mm);
- d_c is the diameter of the conductor, including the screen, if any (mm);
- C_{LL} is the length correction factor when considering laying up cores. The calculation is given in Annex A [57].

When analyzing a system for which the capacitance is unknown, this method can be used to calculate it. However, in many cases, the capacitance is a known value that comes from the cable manufacturer.

The focus here is on understanding the electromagnetic effects that are observable in the environment surrounding submarine power cables. This includes considerations of the geomagnetic field and its interactions with the cable system.

4.3.8. EMFs in the Environment

This thesis aims to help advancement in the analysis of EMF emissions in the environment for use in ecological research and should, therefore, consider the EMFs from the view of a marine animal. Taking the exact complications into account is left for the implementation of these methods by ecologists, but there are some concepts that should be noted from a physics perspective.

Modeling electromagnetic fields requires knowledge of the cable's internal components and interactions but the environment the cable is placed in can also have important consequences for the emissions experience by marine animals.

Most notable is the burial depth of the cable. As electromagnetic fields decrease with distance, the depth at which the cable is buried has a large effect on the EMFs that can be experienced by marine animals. Burial depth, the depth at which the cable is buried, is often subject to standards and is often equal to 1.5 m in the Netherlands. Increasing the burial depth is considered because it provides greater safety to damage by ship anchors and corrosion, which has the additional benefit of lowering EMFs experienced by animals. However, this has a major downside: it increases the cost of installation and hurts the temperature balance of the cable. Burial depth is also unpredictable. As there is often just a layer of sand on top of the cable, that layer is dependent on the sedimentation processes in the sea. Cables can be buried much deeper than originally intended or can be exposed. Dynamic cables are even exposed by design.

As is already explained in this thesis, cables always have a means of shielding the electric field arising from their conduction voltage within themselves. However, as explained in section 3.1, due to the duality in electromagnetic fields, electric fields can still be created through magnetic induction (also seen in subsection 4.3.2). These electric fields can be sensed by animals and should, therefore, be considered when interpreting simulations. When using FEM software such as COMSOL, which is used in this thesis, there are already modules available for this.

Another means of EMF interactions with animals is through the concept of movement-induced electric fields (MIVs). MIVs arise from the concept of the Lorentz force (section 3.2), which ecologists should consider when analyzing the field a certain animal will experience. This can be easily done through the application of Equation 3.15 on a known magnetic field.

4.4. Calculating the Conductance at Sheath Interfaces

As becomes clear from the previous sections in this chapter, the conductance between the sheath interfaces is dominant in the dissipation of circulating currents. The calculation of conductance can be done on the basis of two methods that both have different applications.

Cable sheaths can be compressed together, making their interface consist solely of semi-conductive polyethylene. section 4.4 describes how to calculate the conductance based on the compression deformation.

A different arrangement occurs when the polyethylene layers are not touching but apart, with conductive seawater in between. For this arrangement, it is easiest to perform a quick COMSOL analysis of the conductance, which will be discussed in subsection 4.4.1.

Conductance Calculation for Compressed Cables

The conductance between cores is influenced by the size of the contact surface between semiconducting polyethylene sheaths. This contact surface width is determined by the forces between the cores and the modulus of elasticity of the polyethylene material (as in Figure 4.11). The width of the contact surface between two polyethylene sheaths is dependent of the compression force between them. This force deforms the circular surface into a flat contact surface.

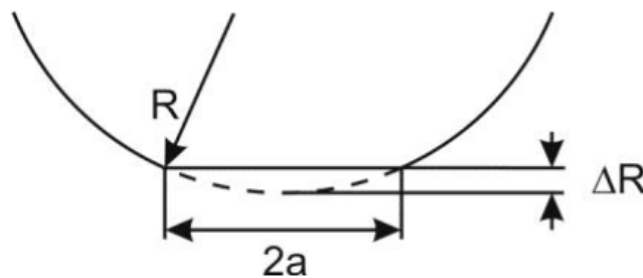


Figure 4.11: A sheath deformed by a force F .

The width $2a$ for a 1-meter cable can be estimated using the following equation that is based on mechanical deformation:

$$\epsilon E = \frac{\Delta R}{R} \cdot E = \frac{R - \sqrt{R^2 - a^2}}{R} \cdot E = \frac{F}{2a} \quad (4.34)$$

Note the mechanical engineering properties of strain (ϵ) and module of elasticity E that should not be confused with permittivity and electric field intensity. Using some algebraic operations, the assumption that $a \ll R$ and finally substituting $R = \frac{D}{2}$ results in.

$$2a = \sqrt[3]{\frac{2FD^2}{E}} \quad (4.35)$$

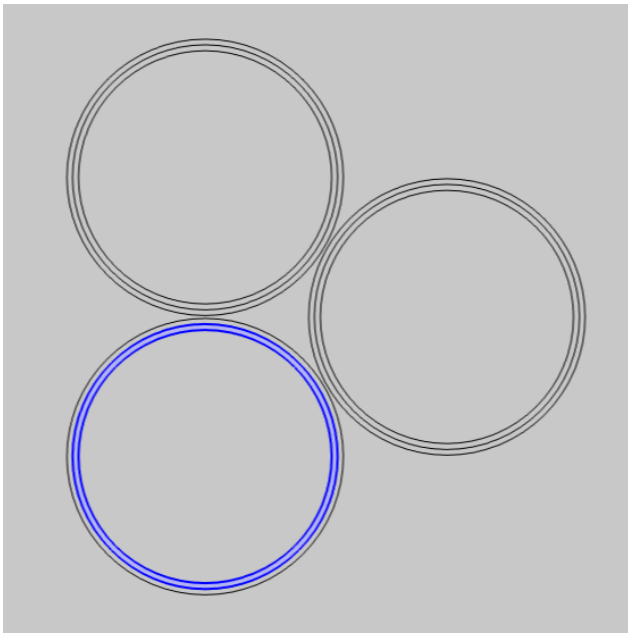
From this the conductance G between two metallic sheaths can be calculated

$$G = \sigma \cdot \frac{A}{\ell} = \sigma \frac{2a}{2t} \quad (4.36)$$

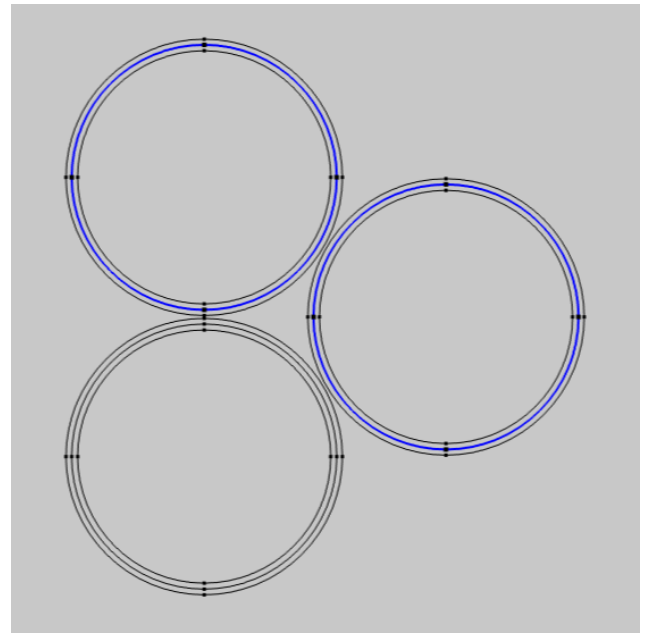
Where σ is the conductance of the semi-conductive sheath, $2a$ is the contact surface due to deformation, and t is the thickness of a single semiconductive layer.

4.4.1. Calculating the Conductance for Separated Sheaths Using COMSOL

For the case of sheaths separated by seawater, COMSOL Multiphysics can be used to quickly calculate the conductance. This is done by creating a simplified 2D model of the cross-section, which includes only the metallic and polyethylene sheath of all three phases surrounded by seawater. An electric potential is applied on the outer edge of the metallic sheath in one of the phases (Figure 4.12a), while grounding is applied along the same edges at the other two phases (Figure 4.12b).



(a) Application of voltage potential on the edge of the metallic sheath.



(b) Application of grounding on the edges of the metallic sheaths.

Figure 4.12: Schematics of the arrangement of sheaths used to calculate electric conductance between them.

In the modeling of submarine cables, it's essential to accurately represent the oceanic environment to understand its impact on electromagnetic fields. This section addresses the electrical properties of seawater, specifically conductivity and permittivity, which significantly influence electromagnetic interactions. The conductivity of seawater is calculated by:

$$\sigma = 0.18C^{0.93}[1 + 0.02(T - 20)] \quad \text{S/m} \quad (4.37)$$

This formula is relevant for frequencies below GHz, salinity (C) in (‰), and temperature (T) in (°C) [58]. The rated temperature for the conductors is 80°C, which we can assume the water in between the sheaths has as well. This leads to a conductivity of

$$\sigma_{water} = 0.18 \cdot 35^{0.93}[1 + 0.02(80 - 20)] = 10.8 \text{ S/m} \quad (4.38)$$

which will be used in the analysis of conductance between the metallic sheaths.

This system is then simulated using the Electric Currents module in the frequency domain. The result is a current flowing from the electric potential that is applied toward the grounded edges, displayed in Figure 4.13

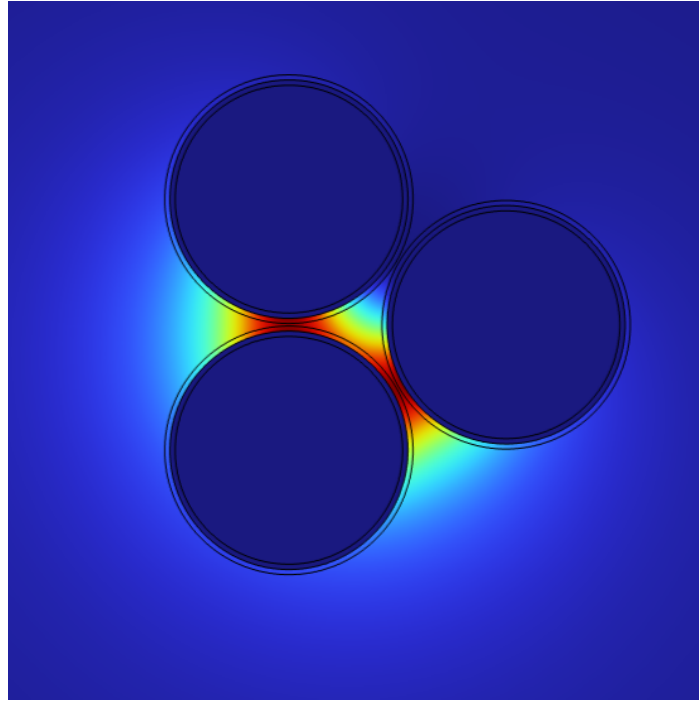


Figure 4.13: Depiction of the current density between the metallic sheath with an applied voltage and two grounded sheaths.

The value of current per unit length can be determined by calculating the line integral over the edge where the electric potential is applied (Figure 4.12a), or on both edges where the grounding is applied (Figure 4.12b). This should both lead to the same value from the current balance. From this current the conductance per unit length can be calculated, noting that the total conductance here is the combination of both conductances G :

$$2G = \frac{I}{V} \rightarrow G = \frac{I}{2V} \quad \text{S/m} \quad (4.39)$$

where I is the current calculated by the line integral, and V is the applied electric potential.

4.4.2. Calculating the Effective Conductance From Electric Phase-Shift

Now to calculate the effective conductance as seen from a single sheath, one wants to get the conductance towards the effective ground that the three combined sheaths create, which is more complicated than simply multiplying by two. Because of the alternating nature of the transmission and the phase shift, the path of the current is also alternating with time. An analysis of this effect is done here.

The conductance will be calculated from the reference of a single metallic sheath. There are two cases that will be compared to find the effect of the alternating voltage in the other two sheaths on the effective conductance:

- The other phase-sheaths are simply assumed have zero potential (ground). This is depicted in Figure 4.14, where we only have $V_{s1} = V_{sp}e^{j\omega t}$ that combines the current phasor V_{sp} with the time component $e^{j\omega t}$ and the other phases are $V_{s2} = V_{s3} = 0$.
- The other two phases do differ in voltage in the same manner as V_{s1} but are phases shifted by $\pm \frac{2\pi}{3}$ ($\pm 120^\circ$): $V_{s2} = V_{sp}e^{j(\omega t - \frac{2\pi}{3})}$ and $V_{s3} = V_{sp}e^{j(\omega t + \frac{2\pi}{3})}$. This is shown in Figure 4.15.

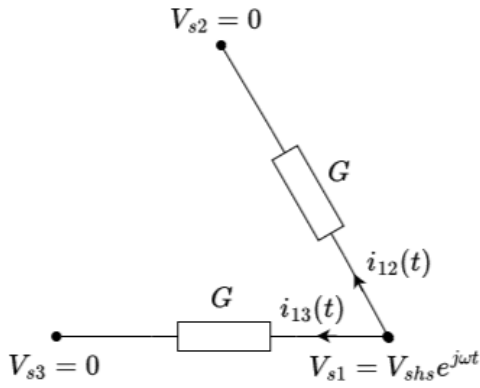


Figure 4.14: Simplified schematic of inter-sheath conductance with the other two sheaths at $V=0$.

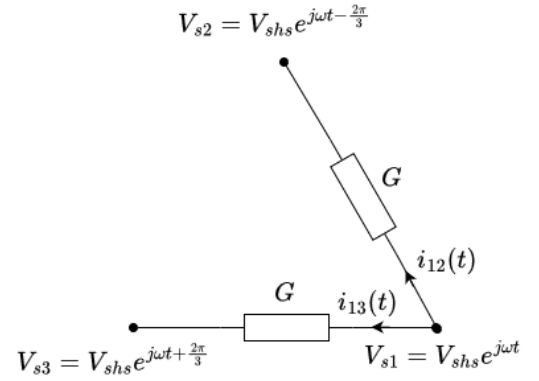


Figure 4.15: Simplified schematic of inter-sheath conductance with the other two sheaths at 120° phase shifted AC voltage.

In both figures, the exchange currents between the phases, as seen from the view of sheath 1, are represented as $i_{12}(t)$ and $i_{13}(t)$. Together they add up to the total current flowing out of sheath 1 at any instant in time.

For the simple case, where the other two sheaths are assumed to be zero, the real instantaneous current flowing out of sheath 1 is equal to

$$i_{s1}(t) = 2G \cos(\omega t) V_{sp} \quad (4.40)$$

On the other hand, applying AC voltage on sheaths 2 and 3 results in a more complicated interaction with a different current flowing out of sheath 1. The current flowing out of sheath 1 as a function of time can be written in terms of the voltage difference between 1,2 and between 1,3.

$$i_{s1}(t) = i_{12}(t) + i_{13}(t) = (V_{s1}(t) - V_{s2}(t))G + (V_{s1}(t) - V_{s3}(t))G = G(2V_{s1}(t) - V_{s2}(t) - V_{s3}(t)) \quad (4.41)$$

Simplifying this and substituting the exponential functions for voltage results in:

$$i_{s1}(t) = GV_{sp}(2e^{j\omega t} - e^{j\omega t - \frac{2\pi}{3}} - e^{j\omega t + \frac{2\pi}{3}}) \quad (4.42)$$

The real instantaneous current flowing out of sheath 1 can now be calculated to be:

$$i_{s1}(t) = GV_{sp}(2 \cos \omega t - \cos \omega t \cos \frac{2\pi}{3} - \cos \omega t \cos \frac{2\pi}{3}) = 3G \cos(\omega t) V_{sp} \quad (4.43)$$

Comparing Equation 4.40 and 4.43, it can be concluded that the current flowing out of sheath 1 is 3/2 times higher for the case where the alternating voltage is applied on the other two sheath. Therefore, when the value of conductance is calculated for the simple case, which is easiest to do (using one of the methods described in section 4.4 and subsection 4.4.1), the conductance should be multiplied by a factor 3/2, to get the effective conductance.

4.5. Conclusion on Relevant Modeling Parameters

This chapter aimed to understand the workings of submarine power cables and their surrounding transmission system and environment and identify which components and parameters should be considered when modeling electromagnetic fields.

This chapter discussed the different components of submarine power cables and the electromagnetic interactions that occur within and between them, thereby identifying the components that have an electromagnetic impact. The overall transmission system that dictates the distribution of voltage and current, and consequently the active, reactive, and apparent power through the length of the cable, was also discussed. Finally, the effect of burial depth and induction in the water was explained.

From this chapter multiple important parameters and concepts to consider when modeling electric currents in cables and the electromagnetic fields resulting from those currents can be identified.

It becomes clear that the conduction voltage and current distribution along the length of a transmission line are dictated by only a few parameters:

- Required voltage and current at the sending or receiving end.

- Phase resistance R_c .
- Phase capacitance C .
- Phase inductance L_c .
- Reactive power compensation at sending and/or receiving end $Q1, Q2$.

However, the conduction current is always paired with circulating currents in the metallic sheath, which are also affected by those parameters. Therefore, some more important design parameters can be identified that dictate the sheath's current distribution.

- Sheath resistance R_s .
- Sheath inductance L_s .
- Inter-sheath conductance G .

These overall parameters are influenced by the geometry and material characteristics and are, therefore, encapsulating the cable geometry and material parameters. The method of calculation of these parameters is already established, and will not be considered further in this thesis. However, due to a lack of data, a method for calculating inter-sheath conductance is displayed at the end of this chapter.

The knowledge gained in this chapter will be used for the analysis of the current distribution in the conductor and metallic sheath in the following chapter 5 and for the choice of parameters to include in the 3D modeling of cables in chapter 6.

5

Current Flow Prediction in SPCs

This chapter will outline the analysis of the different flows of current within the three-phase submarine power cable. In order to have a chance at creating an accurate model for electromagnetic fields, it is essential to know what the current is that flows through the cable the point of interest. Specifically, the apparent current should be determined, as this is what effectively creates the magnetic field. As described in chapter 4 this is not a trivial problem.

The current that is being delivered to the connection at the end of the transmission line is not the same as the current at the generation side. At any point in the cable, the apparent current is different, due to a combination of inductive (subsection 4.3.2) and capacitive (subsection 4.3.6) effects within the cable and level and method of reactive power compensation (subsection 4.2.1) at the start and end of the line.

Next to the difference in the current through the conductor, the capacitive and inductive effects also result in the flow of currents within the metallic sheath, as well as the armor wires. Due to the new method of submarine cable design (subsection 4.1.4), where a semiconductive PE sheath creates a connection between the metallic sheaths of different phases, which utilizes the common equipotential of the three phases to relieve the once high longitudinal leakage currents, there will be a distribution between longitudinal and inter-sheath current.

In industrial practice, the connection to this common equipotential has been assumed to be a perfect connection to the ground, resulting in zero longitudinal current in the metallic sheath. However, this is not the case, as the semiconductive layer is far from a perfect conductor, so it has to be seen if neglecting the longitudinal sheath-currents is a fair assumption. Because this has been neglected until now and because there is as good as no academic literature describing the analysis of three-phase cables with a semiconductive PE sheath, a new method of predicting the current in the metallic sheath has been developed.

This chapter is structured as follows. First, the question of the existence of sheath currents while using a conductive outer sheath is tested in section 5.1. Next, the voltage and current distributions along the length of an HVAC-SPC are described in section 5.2, as these are the source of every other electromagnetic effect within and around the cable. This model is then verified by comparing it to the outcome of simulations by TenneT in section 5.3. Section 5.4 then describes how the sheath currents can be modeled based on transmission line analysis, using the conductor voltage, current and cable design parameters. A simpler method to describe sheath current based on a junction-series representation of the metallic sheath was developed in section 5.5. This method is applied in section 5.6 to the Borssele Alpha cable for different operational modes to verify its use as well as to see the level of circulating current dissipation through the conductive outer sheath. This chapter is concluded with a discussion of the results in section 5.7.

5.1. Conductive Outer Sheath and Remaining Currents

This section will go into the modeling approach for the sheath currents. It will first discuss how the charging and inductive currents can be modeled. Then it will prove that both still exist by showing that charging currents still exist. This is the only way to do this for now, as the 'Magnetic Fields' module does not allow for conductive currents and, therefore, does not show any radial dissipation. However, proving that charging currents still exist, directly proves that this will also be the case for inductive currents as dissipation works with the same principle for both.

The current industry standard is to use a semiconductive PE sheath such that charging- and circulating currents are minimized (subsection 4.3.64.3.2) However, this connection is imperfect, as described in subsection 4.1.4, and the expectation is that this will still result in longitudinal currents in the sheath. The extent of this should be tested. In order to do this, a simplified 3D model was used to show that there are indeed still longitudinal currents. It has to be 3D because this is the only way to test the ratio of longitudinal over radial current distribution. A form of saturation can also be expected,

since, as a longer length increases the amount of current built up, it also increases the area over which the current can diffuse to the common equipotential that three metallic sheaths that are electrically phase shifted by 120° would create. This proof of existence for charging-current and a semiconductive PE sheath is tested here, and the findings can be extended to induced currents, as the principle of a nonzero ratio of longitudinal over inter-sheath currents will hold.

To verify the assumption that cable charging currents exist, even though there is a common equipotential between the phases, is done using a simplified model. Charging currents are a matter of current-flow analysis, for which the Electric Currents COMSOL-module was used [18]. The simplified model uses only a single phase, with grounding applied at the end of the sheath and everywhere around the semiconductive layer. This is a much more effective way of grounding the sheath than in the real three-phase model, but it is used to prove that this is an existing effect. The single phase model is shown in Figure 5.1b, where the grounding that is applied is indicated with blue shading.

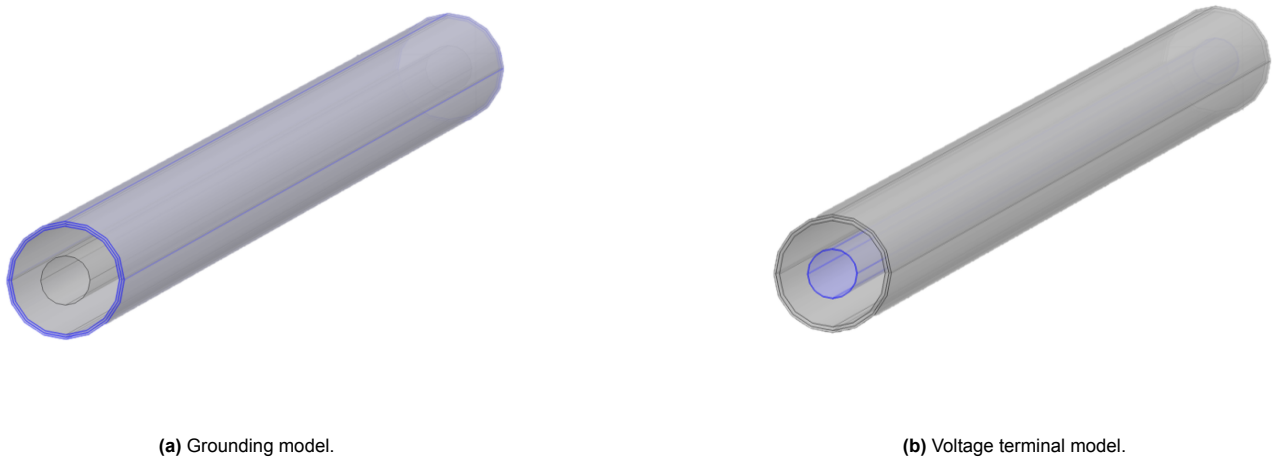


Figure 5.1: Model of the simplified single-phase system with indication of applied grounding (left) and voltage terminal (right).

The parameters for this model were taken close to the values of a realistic system to gain an indication of the real magnitude of current in metallic sheaths. The exact values used for these parameters can be found in Table 5.1.

Table 5.1: Parameters for the Electromagnetic Design of the single-phase test case

Parameter	Value
Radius of Conductor	20.8 mm
Radius over Insulation	45.78 mm
Radius over Metallic Sheath	47.78 mm
Radius over Semiconductive PE Sheath	49.78 mm
XLPE Conductivity	1×10^{-14} S/m
XLPE Relative Permittivity	2.3
Conductor Conductivity	1×10^8 S/m
Conductor Relative Permittivity	1
Water Conductivity	4.167 S/m
Water Relative Permittivity	80
Semiconductive PE Conductivity	1×10^6 S/m
Semiconductive PE Relative Permittivity	10 S/m
Terminal Voltage (Conductor)	180 kV

First, the model's validity should be verified. This is done by checking what happens without the circumferential grounding. Charging currents are capacitive currents that are phase shifted (explained in subsection 4.3.6) w.r.t. the input voltage. This means that the result can be found as the imaginary part of the current in the z-direction, using the expression $\text{imag}(ec.Jz)$ in our results plot. To show the distribution over the length, a 3D Cut-Line selection is made that goes through the middle of the metallic sheath through its length.

For both single (Figure 5.3a) as well as solid bonding (Figure 5.2b), the sheath current response is shown in Figure 5.2.

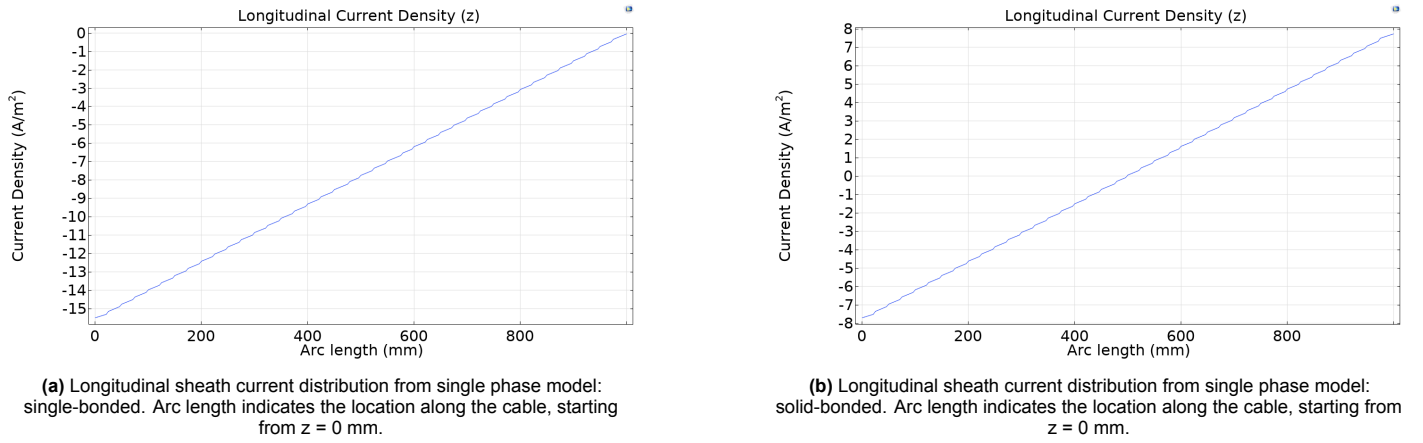


Figure 5.2: Comparison of the charging current in the metallic sheath of a single bonded vs. solid bonded cable section.

In both Figures 5.2a and 5.2b we see linear behaviour for the current density. This is exactly as expected, as the capacitance per unit length is constant as well as the conduction voltage. We see that with solid bonding there is a distribution towards both ends of the cable section, also as expected.

Inspecting the value of the charging current, also gives a correct result. The capacitance of this piece of cable is calculated by:

$$C = \frac{2\pi\epsilon_0\epsilon}{\ln\left(\frac{D_1}{d_c}\right)} = \frac{2 \cdot \pi \cdot 8.854 \cdot 10^{-12} \cdot 2.3}{\ln\left(\frac{2 \cdot 45.78}{2 \cdot 20.8}\right)} = 1.622 \cdot 10^{-10} \text{ F} \quad (5.1)$$

and

$$I_c = 2\pi f C V_c = 2 \cdot \pi \cdot 50 \cdot 1.622 \cdot 10^{-10} \cdot 180 \cdot 10^3 = 9.17 \cdot 10^{-3} \text{ A} \quad (5.2)$$

now to get the to the current density we need

$$A_{sh} = \pi \cdot (R_s^2 - R_i^2) = \pi \cdot (47.78^2 - 45.78^2) = 5.88 \cdot 10^{-4} \text{ m}^2 \quad (5.3)$$

to arrive at

$$i_z = I_c / A_{sh} = \frac{9.17 \cdot 10^{-3}}{5.88 \cdot 10^{-4}} = 15.6 \text{ A/m}^2 \quad (5.4)$$

which exactly fits the result found in Figure 5.2a. For the solid bonded variant (Figure 5.2b) we have to divide $i_z/2 = 8.3 \text{ A/m}^2$, which also perfectly the maximum and minimum values of the plot. From this we can conclude this model works correctly.

Now we can use this model to prove that there will also still be longitudinal currents when grounding is applied around the circumference of the semiconductive sheath. This type of grounding will be a large overapproximation of what would happen if three 120° phase-shifted sheaths are connected, but this proves even stronger that longitudinal currents are still produced. Once again, the simulation was run for both single (Figure 5.3a) and solid (Figure 5.3b) bonding.

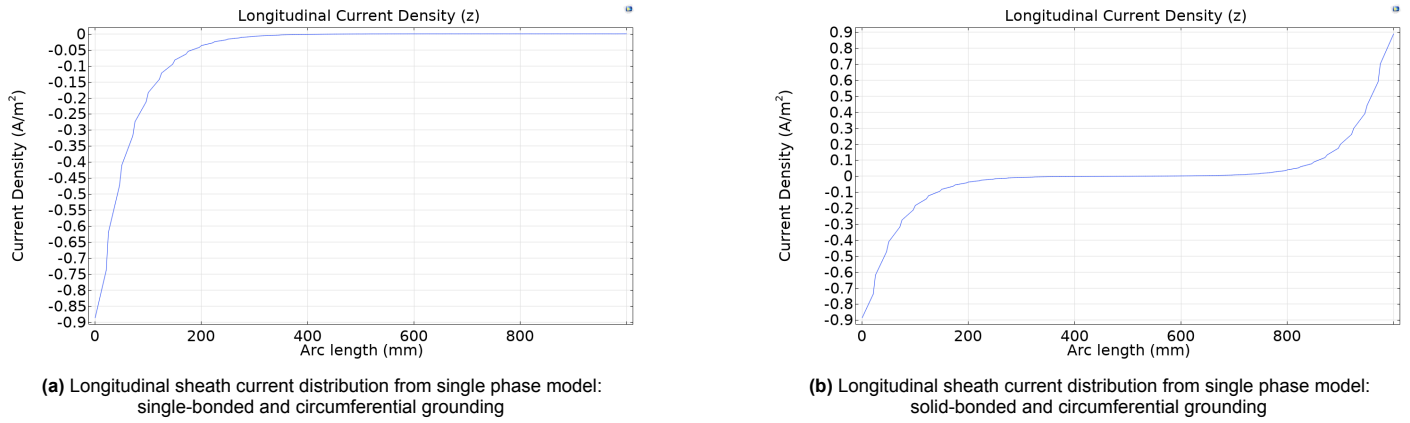


Figure 5.3: Single bonded vs. solid bonded model of capacitive current a small single phase cable in COMSOL.

From both graphs, we see that there is now a very different distribution, with much lower peak values and even lower average values of longitudinal current. To better see what the resulting effect is, we should zoom in to the area where there is more distance to the ground. Due to the complexity of an increase in total charging current and a decrease in total ground resistance when increasing the length of the cable section, it is not clear what the result would be for a real cable system. However, in this model, together with the argument that the ratio of longitudinal resistance to radial resistance can never really become infinite, it can be considered proven that longitudinal charging currents still exist when a semiconductive PE sheath is used. By extension of this principle, it is also proven that induced circulating currents remain to be produced.

On the basis of this knowledge, the need for a model describing these currents arises. Due to the computational limits of numerical modeling, this cannot be done in COMSOL; however, a mathematical model can be constructed, which can then be used as input to model a smaller cable section of interest. The following sections will be dedicated to constructing such a model based on transmission line analysis.

5.2. Conductor Voltage and Current Distribution Using General Wave Equations

This section will introduce an analytical methodology to calculate the distribution of voltage and current along the length of the conductors of a power cable. This method is based on the analysis of the transmission lines [34][59][60][48] and was originally used to predict signal transmission in telegraph systems, leading to the name of the 'Telegraphers Equations'. It is based on equations describing wave propagation within transmission lines as a function of a power cable's conductive, resistive, capacitive, and inductive properties. The method is described extensively in Appendix B, but the important steps will be discussed here such that the general approach can be applied to the metallic sheath in section 5.4.

First, a derivation of the general transmission line equations will be shown in subsection 5.2.1, after which subsection 5.2.2 discusses the simplifications and mathematical definitions used in the solution. The solution of the established system of equations is done in subsection A.0.6, which allows the prediction of a current and voltage distribution for any cable system for which the design parameters and operating conditions are known. This is verified in section 5.3 by applying it to the Borssele Alpha cable without reactive power compensation and by comparing the established method's predictions for realistic with modeling efforts by TenneT.

5.2.1. General Transmission Line Equations

The analysis begins with describing the mechanism behind transmission line analysis. It is based on a schematic that represents a small slice of the cable, from which difference equations are deduced. These difference equations are then mathematically adjusted to produce the **telegrapher's equations** as well as the **general wave equations** for current and voltage within a transmission line. To achieve this, we begin by constructing a circuit model for an incremental segment of the transmission line (Figure 5.4). We include in our model the electromagnetic parameters per unit length: inductance (L), capacitance (C), shunt conductance (G), and series resistance (R). These constants are essential, as they dictate the electrical properties of the transmission line:

- The series resistance (R) accounts for losses due to the finite conductivity σ_c of the conductors, commonly represented by ρ_c (Ω/m).
- The shunt conductance (G) models the leakage currents through the dielectric, commonly represented by σ_d (S/m).
- The series inductance L (H/m) models the phase delay due to the self inductance of the unit length.
- The shunt capacitance C (F/m) models the leakage currents through the dielectric to polarize it and allowing for transmission.

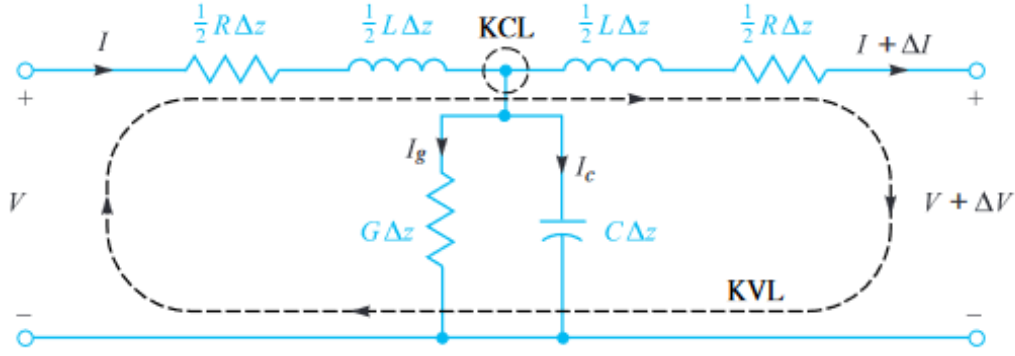


Figure 5.4: Schematic of a small slice of transmission line with electromagnetic parameters L, C, G and R , voltage and current V and I and the locations for applying Kirchoffs laws.[34]

Applying Kirchoffs voltage and current law on the locations indicated in Figure 5.4 and substituting that for a very small section of length Δz , $\Delta I = \frac{\partial I}{\partial z} \Delta z$ and $\Delta V = \frac{\partial V}{\partial z} \Delta z$, the Telegraphers equations can be derived. The Telegraphers equation for voltage is Equation 5.5 and for current is Equation 5.6

$$\frac{\partial V}{\partial z} = - \left(RI + L \frac{\partial I}{\partial t} \right) \quad (5.5)$$

$$\frac{\partial I}{\partial z} = - \left(GV + C \frac{\partial V}{\partial t} \right) \quad (5.6)$$

These equations describe broadly how the voltage and current change with z . Both equations make intuitive sense. Equation 5.5 describes how voltage drops due to the current flowing through a resistance and the change in current over an inductance. Equation 5.6 describes how the current drops due to a voltage seeing a conductance to the environment and the change in voltage over a capacitance. However, these equations cannot yet describe the complete distribution over the length. For that, some mathematical operations are required.

Differentiating the telegraphers equations w.r.t. z and t allows for making substitutions that lead to the general wave equations for the transmission line: 5.7 and 5.8.

$$\frac{\partial^2 V}{\partial z^2} = LC \frac{\partial^2 V}{\partial t^2} + (LG + RC) \frac{\partial V}{\partial t} + RGV \quad (5.7)$$

$$\frac{\partial^2 I}{\partial z^2} = LC \frac{\partial^2 I}{\partial t^2} + (LG + RC) \frac{\partial I}{\partial t} + RGI \quad (5.8)$$

The solutions to the general wave equations, combined with the correct boundary conditions can be used to describe the voltage and current along a transmission line.

5.2.2. Mathematical Definitions Used For Solving Wave Equations

Before continuing to solve the general wave equations, it is required to set some definitions that can be used to simplify the problem. As might be expected from the analysis of transmission of electromagnetic waves, the analysis can be simplified by assuming that the current and voltage are represented by sinusoidal functions. In particular, the use of the Euler identity is essential as it accounts for the real as well as complex part of the voltage and current waves (active and reactive power). The Euler identity allows for writing the instantaneous voltage (and current) as combinations exponential functions, which can be split up into a time component and phasor (time independent) component:

$$V_c(z, t) = V_0 e^{\pm \gamma z} e^{j\omega t} \quad (5.9)$$

with complex $\gamma = \alpha + \beta j$ and V_0 is dependent on boundary conditions. The phasor is the interesting term, since it is descriptive of the distribution along the length of the line. The analysis can be done without the time component, after which the time component can be reintroduced at any time. As the phasor has a forward and backward propagation term, it can be written as follows:

$$V_{cp} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (5.10)$$

where V_0^+ and V_0^- correspond to the forward and backward propagation. Implementation of the instantaneous voltage and current equation in the general wave equation for voltage (5.7) and simplifying by removal of the time component, leads to the differential equation for the transmission line voltage.

$$\frac{d^2 V_{cp}}{dz^2} = \underbrace{(R + j\omega L)}_Z \underbrace{(G + j\omega C)}_Y V_{cp} = \gamma^2 V_{cp} \quad (5.11)$$

From Equation 5.11, the complex exponent $\gamma = \sqrt{ZY}$ can be determined, such that the solution to the differential equation is of the same form as in 5.10.

Since Equation 5.8 is of the same form as Equation 5.7, the current phasor will also be of the form

$$I_{cp}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad (5.12)$$

and plugging this into the telegraphers equations leads to the relation between the current phasor and voltage.

$$I_{cp}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \quad (5.13)$$

The characteristic impedance $Z_0 = Z/\gamma$ can be identified. This design parameter describes the relation between voltage and current in along the cable and is a function of both the geometry as well as material characteristics [61].

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (5.14)$$

When the load impedance Z_L is matched to the line impedance Z_0 , the transmission line is said to be fully matched, meaning no reflections occur. No reflections occurring means that there is no backward propagating wave and $V_0^- = 0$. In this case, Z_0 is the ratio between voltage and current. However, in reality, this is not the case. This would mean that the line impedance is equally high to the load's impedance, so 50% of power is lost in transmission. Moreover, it would be very coincidental if, by accident, the cable and complex main power grid would exactly match. For this reason, it is best to just deal with the difference between the two, which can be solved by describing the boundary conditions properly.

5.2.3. Solution To Transmission Line Equations: Conductor

Solving the transmission line equations for the conductor is relatively easy. Using equations 5.10, 5.12 and 5.13 the forward and backward propagation constants V_0^+ and V_0^- can be written in terms of the load voltage V_L and load current I_L (known quantities from operational settings) and the characteristic impedance.

$$V_0^+ = \frac{V_L + Z_0 I_L}{2} e^{\gamma \ell} \quad \text{and} \quad V_0^- = \frac{V_L - Z_0 I_L}{2} e^{-\gamma \ell} \quad (5.15)$$

Here, ℓ is the length of the cable. From this follow the equations for conductor voltage and current phasors.

$$V_{cp}(z) = \frac{V_L + Z_0 I_L}{2} e^{\gamma \ell} e^{-\gamma z} + \frac{V_L - Z_0 I_L}{2} e^{-\gamma \ell} e^{\gamma z} \quad (5.16)$$

$$I_{cp}(z) = \frac{V_L + Z_0 I_L}{2 Z_0} e^{\gamma \ell} e^{-\gamma z} - \frac{V_L - Z_0 I_L}{2 Z_0} e^{-\gamma \ell} e^{\gamma z} \quad (5.17)$$

Equation 5.17 and 5.16 can be readily used to predict the voltage and current along the length of a transmission line, given that the characteristic impedance Z_0 , the propagation constant γ and the load-side voltage V_L and current I_L are known. These equations will be used in subsequent analysis of the metallic sheath.

Because Equation 5.17 and 5.16 are defined from the load-side, they can be simplified using a coordinate transformation $z' = \ell - z$.

$$V(z') = V_L \cosh(\gamma z') + I_L Z_0 \sinh(\gamma z') \quad \text{and} \quad I(z') = \frac{V_L}{Z_0} \sinh(\gamma z') + I_L \cosh(\gamma z') \quad (5.18)$$

Now that the method is established (again more details are available in Appendix A) it can also be applied to inspect the voltage and current along the metallic sheath.

For the conductor case specifically, there is a more complete formula from the literature [48],[60] that can be used to determine the impact of reactive power compensation:

$$\begin{bmatrix} \dot{V}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} \cosh(\gamma \ell) + Z_0 Y_2 \sinh(\gamma \ell) & Z_0 \sinh(\gamma \ell) \\ \left(\frac{1}{Z_0} + Z_0 Y_1 Y_2 \right) \sinh(\gamma \ell) + (Y_1 + Y_2) \cosh(\gamma \ell) & \cosh(\gamma \ell) + Z_0 Y_1 \sinh(\gamma \ell) \end{bmatrix} \begin{bmatrix} \dot{V}_2 \\ \dot{I}_2 \end{bmatrix} \quad (5.19)$$

which corresponds to the schematic shown in Figure 5.5.

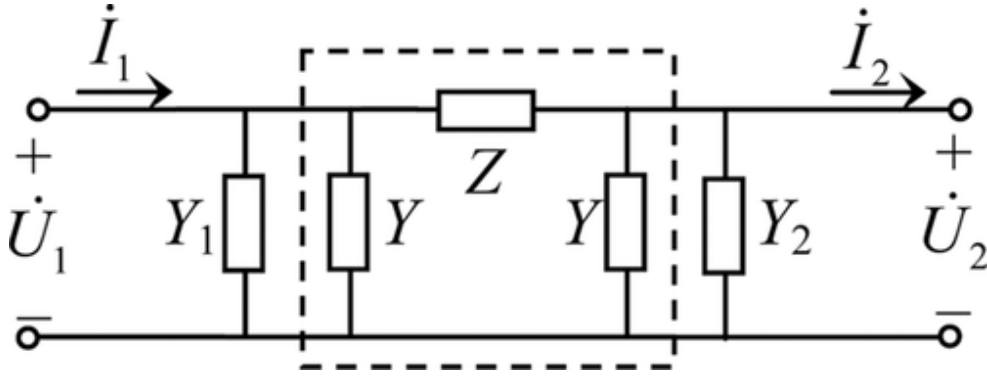


Figure 5.5: Schematic of a cable system with shunt compensation at both ends[60].

Here the additional components Y_1 and Y_2 can be identified, which represent the shunt reactors at sending and receiving end respectively. Again Z_0 represents the characteristic impedance ($Z_0 = \sqrt{Z/Y}$) of the line and $\gamma = \alpha + \beta i = \sqrt{ZY}$ is the same as before. Currents $\dot{I}_1$ and $\dot{I}_2$ and voltages $\dot{U}_1$ and $\dot{U}_2$ represent the sending and receiving end.

To specifically look at the impact of reactive power compensation at either end, this equation can be rewritten as follows. Note that $\angle \varphi_{u_2}$ indicates that the real part of the voltage phasor U_2 must be used.

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 + jQ_1/U_1 \end{bmatrix} = \begin{bmatrix} \cosh \gamma \ell & Z_0 \sinh \gamma \ell \\ \frac{\sinh \gamma \ell}{Z_0} & \cosh \gamma \ell \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ \dot{I}_2 - jQ_2/U_2 \angle \varphi_{u_2} \end{bmatrix} \quad (5.20)$$

Where Q_1 and Q_2 are the amounts of reactive power compensated at the sending and receiving end, respectively, note that voltage is now denoted by the terminal voltages $\dot{U}_1$ and $\dot{U}_2$. Equation 5.20 allows us to calculate the voltage and current distribution along the transmission line's length. This equation is indispensable in the analysis of electromagnetic field emissions since it describes how the majority of the current that flows through the cable is distributed along the length. Additionally, this equation functions as an input to the analysis of the metallic sheath voltage and current.

5.3. Verification of Transmission Line Solution: Borssele Alpha

Before using these equations in predictions of sheath voltage and current as well as electromagnetic field models, their effectiveness should be verified. This will be done by first checking if equations Equation 5.16 and Equation 5.17 give sensible results when applied using the Borssele Alpha cable parameters (section 5.3). Next, in Figure 5.3, Equation 5.20 will be tested against results from simulations by TenneT under different modes of operation of Borssele Alpha with reactive power compensation. subsection 5.3.1 will summarize the findings in this section and establish the mode of operation to be used in the subsequent analysis of the metallic sheath as a transmission line.

Verification of Transmission Line Solution Without Reactive Power Compensation

To test their validity, Equation 5.16 and 5.17 can be tested with a realistic case for which the values are known: Borssele Alpha 1. In this transmission line model, the properties of the cable will be considered constant along the length of the cable, which is not the case in reality as the onshore cable section has a different layout and use of materials. Due to the properties of submarine cables (section 4.1), that are designed to have minimal losses, it can be assumed that $G \approx 0$ and that $R \ll j\omega L$, such that the low loss conditions apply, simplifying the calculation of γ and Z_0 (Appendix A). Then γ can be calculated as follows:

Due to $G \approx 0$, the formula for α (A.45) simplifies to:

$$\alpha = \frac{1}{2} R \sqrt{\frac{C}{L}} = 1.7636 \cdot 10^{-7} \quad (5.21)$$

and the formula for β (A.46) simplifies to:

$$\beta = \omega \sqrt{LC} \left[1 + \frac{1}{8} \left(\frac{R}{\omega L} \right)^2 \right] = 2.6419 \cdot 10^{-6} \quad (5.22)$$

where $\gamma = \alpha + \beta i$. The characteristic impedance is calculated Z_0 using the following relation, which is a simplified version of the complete characteristic impedance equation (Equation A.47):

$$Z_0 \doteq \sqrt{\frac{L}{C}} \left\{ 1 + \frac{1}{2\omega^2} \frac{1}{4} \left(\frac{R}{L} \right)^2 - \frac{j}{2\omega} \left(\frac{R}{L} \right) \right\} = 42.91 - 2.87i \quad \Omega \quad (5.23)$$

However, because the rated current is the rms of the current vector, the real losses in voltage and current amplitude are not from $P_{loss} = I \cdot Z_0$ but from $P_{loss} = I \cdot \sqrt{2}Z_0$. Therefore the characteristic impedance used in subsequent calculations will be

$$Z_0 = 60.68 - 4.06i \quad \Omega \quad (5.24)$$

Combining that with the length of the cable $\ell = 70$ km and the fact that we want the parameters at the load side (here called $V_2 = 220$ kV and $I_2 = 919$ A) to be real numbers (to provide 100% apparent power), we can calculate the required input voltage and current using the following matrix equation:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh(\gamma\ell) & Z_0 \sinh(\gamma\ell) \\ \frac{1}{Z_0} \sinh(\gamma\ell) & \cosh(\gamma\ell) \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Where $z' = \ell$ means we are looking at the point $z = \ell - z' = 0$. However, because we want to know the voltage distribution, this is instead plotted for the full range $0 \leq z' \leq \ell$ and then converted back to the z -frame of reference (Matlab script in Appendix B). The results of the simulation are depicted in Figure 5.8.

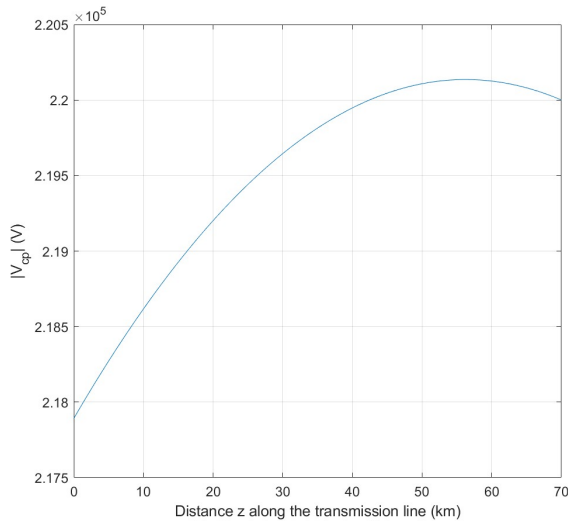


Figure 5.6: Simulated voltage distribution for a Borssele Alpha cable phase without reactive power compensation.

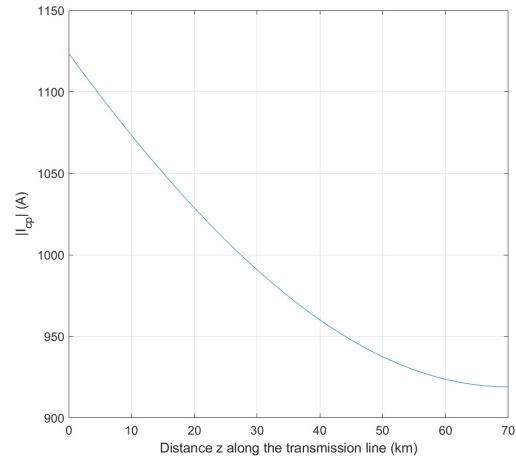


Figure 5.7: Simulated current distribution for a Borssele Alpha cable phase without reactive power compensation.

Figure 5.8: Results of simulating the voltage and current distribution for the uncompensated Borssele Alpha cable.

Figure 5.8 shows promising results. The voltage distribution, as shown in Figure 5.6, depicts an increasing voltage curve when looking from the sending end ($z=0$ to the receiving end ($z = \ell$)). This can be explained by the difference in capacitance that a charge moving along the length of the cable experiences. At $z = 0$ the capacitance from the viewpoint of a unit of charge is equal to that of the whole cable, which decreases the voltage (current flows easier with higher capacitance). When traveling towards the $z = \ell$, the capacitance, as seen from the moving charge, is smaller, as the capacitance it can experience drops. However, at some point, the reduced resistance towards the sending end starts to play a role, which explains the peak in voltage that occurs around $z = 56$ km, after which it decreases again. The current distribution is also sensible, as it decreases over the whole length, following a parabolic shape. This

Verification of Conductor Current and Voltage Distribution With Reactive Power Compensation

For testing the distribution of the conductor current and voltage, the best verification method is deemed to be a comparison with data and models from TenneT. TenneT has a large amount of expertise in-house and is constantly measuring the inputs and outputs of the cable system. Therefore their model can be seen as a valid verification method for the mathematical conductor model. It should be noted beforehand that, again, the assumption is used that the cable has constant properties along its length, which is not the real case. Therefore, some inherent errors can be expected.

This verification is done using the operational settings in PowerFactory schematics by TenneT (see Table 5.2). The settings include the load-side current and voltage (I_2 and V_2) and the sending-side current and voltage (I_1 and V_1), as well as reactive power drawn by the offshore and onshore high voltage shunt reactor (**Q1-HV** and **Q2-HV**) and onshore medium voltage (**Q2-MV**). The schematics did not include reactive power use by the onshore transformer that connects to the main grid, which must be kept in mind when interpreting results.

The offshore shunt reactor has a constant reactive power setting of 55 MVar, and the onshore shunt reactor is fixed 80 MVar (note that these differ slightly between modes due to different cable voltages and currents).

The onshore medium voltage setting combines a capacitor ($Q = -32.5$ MVar) and a shunt reactor ($Q = 65$ MVar). Both can be turned on (indicated as IN) or off (indicated as UIT). However, in nominal conditions, the capacitor is turned on and the reactor off, as the reactive power drawn by the whole system at full load already balances the capacitive reactive power production by the cable (SP UIT C IN).

Powerfactory simulations run by TenneT also consider some differentiated operational conditions. The cases modeled are: standard nominal (380 kV 350 MW), increased voltage nominal power (407 kV 350 MW), standard voltage max power (380 kV 380 MW) and increased voltage at max power (407 kV 380 MW) as is clearly indicated in the table.

Table 5.2: Operational data for different modes of operation. Retrieved from PowerFactory schematics by TenneT.

Mode of Operation	Q1-HV [MVar]	Q2-HV [MVar]	Q2-MV [MVar]	I_2 [A]	V_2 [V]	I_1 [A]	V_1 [V]
380 kV 350 MW SP IN	54.7	78.7	30.44	$883 + 391j$	229.476	$877 - 69j$	223.186
380 kV 350 MW	54.1	79.2	-33.99	$881 + 226j$	228.146	$882 - 230j$	223.841
407 kV 350 MW SP IN	55.9	80.4	31.09	$874 + 390j$	231.779	$869 - 77j$	225.583
407 kV 350 MW	55.2	80.9	-34.7	$872 + 223j$	230.434	$873 - 0.239j$	226.244
380 kV 380 MW SP IN	55.2	78.9	30.54	$956 + 411j$	230.429	$948 - 45j$	223.514
380 kV 380 MW	54.6	79.4	-34.11	$954 + 246j$	229.101	$954 - 206j$	224.171
407 kV 380 MW SP IN	56.3	80.6	31.19	$946 + 409j$	232.713	$939 - 53j$	225.901
407 kV 380 MW	55.7	81.1	-34.83	$944 + 242j$	231.369	$944 - 215j$	226.564

For each mode of operation Equation 5.20 is solved (first part of Matlab script in Appendix B. Here the values for **Q2-HV** and **Q2-MV** are combined into a single value Q2. The propagation constant $\gamma = 1.763 \cdot 10^{-7} + 2.636 \cdot 10^{-6}i$ and characteristic impedance $Z_0 = 60.68 - 4.06i$ for a single phase are the same as in section 5.3.

The resulting current distributions from this simulation are shown in Figure 5.9 while Figure 5.10 depicts the results from TenneT.

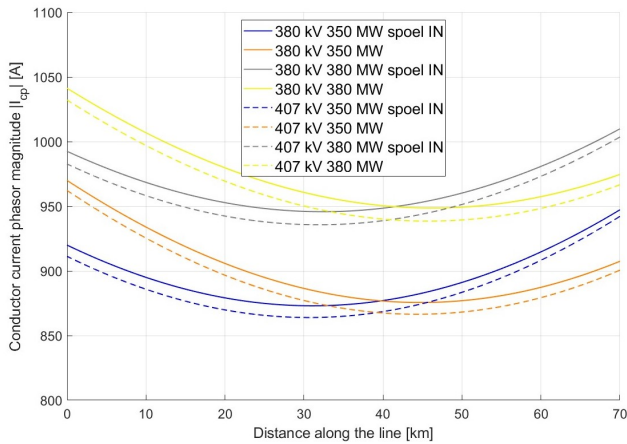


Figure 5.9: Modeling results of the current distribution along the Borssele Alpha cable by TenneT based on different reactive power compensation modes.

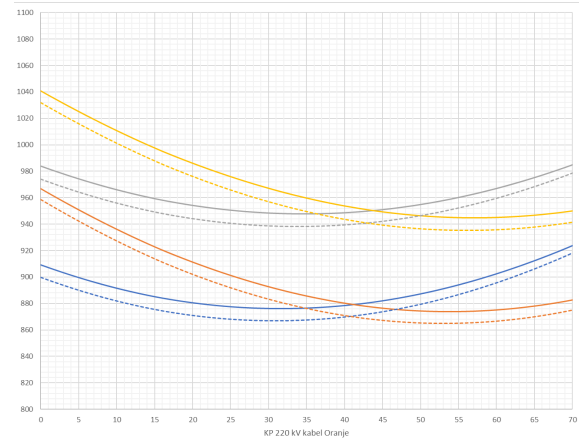


Figure 5.10: Modeling results of the current distribution along the Borssele Alpha cable by TenneT based on different reactive power compensation modes.

Figure 5.11: A comparison of modeling results from the transmission line method against the PowerFactory results from TenneT indicates an almost perfect match.

From Figure 5.11, it can be seen that there is a good match with the results from the modeling done by TenneT. The curves have almost the exact same shape, and the difference in values at the end-points of the cable is is. Therefore, it can be concluded that this method of calculating the conductor current can be used to accurately predict conductor current at a location along the length of the cable. This is an excellent result as the current conduction current is the most significant chunk of total current, meaning that predicting it accurately is already an enormous step in the direction

of an accurate electromagnetic emissions model.

However, there is still the question of the intensity of metallic sheath currents. Metallic sheath currents are coupled to the conduction current through magnetic coupling (as described in subsection 4.3.2) and to the conductor voltage through the capacitive charging of the cable (subsection 4.3.6). Therefore it should also be checked if this method produces correct voltage distributions. For verification of the voltage distribution, the output of the simulation (Figure 5.12) is again compared to the output of the simulations by TenneT (Figure 5.13).

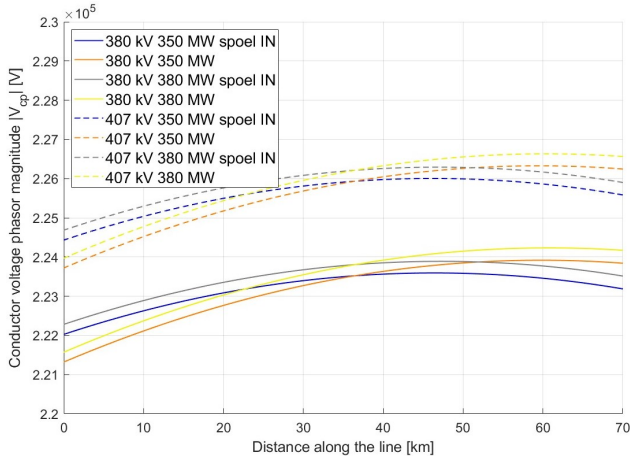


Figure 5.12: Modeling results of the voltage distribution along the Borssele Alpha cable by TenneT based on different reactive power compensation modes.

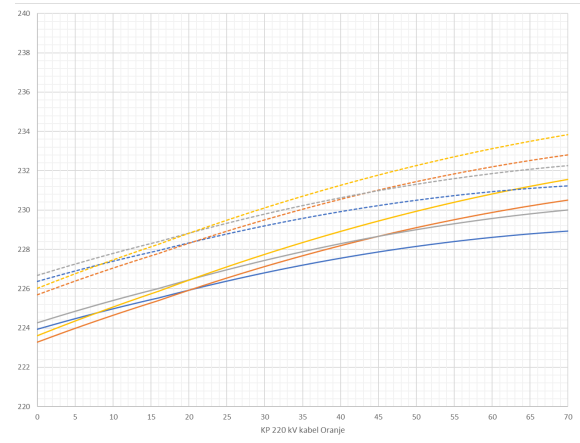


Figure 5.13: Modeling results of the voltage distribution along the Borssele Alpha cable by TenneT based on different reactive power compensation modes.

Figure 5.14: Comparison of modeling results from the transmission line method against the PowerFactory results from TenneT.

From Figure 5.14, it can be seen that the voltage along the cable length drops much quicker for the transmission line model than for the PowerFactory model. This could be due to different cable parameters being used in the PowerFactory model, especially close to the receiving (onshore) side. Next to that, the ambiguity in the reactive power consumption of the onshore transformer might have an influence on the voltage distribution as well. Since the difference between the models is still small, this is not considered an issue.

Operation Mode	Transmission Line	PowerFactory	Relative Error (I)	Relative Error (V)
380 kV 350 MW SP IN	Sending: $I = 919$ A, $V = 222.03$ kV Receiving: $I = 947$ A, $V = 223.19$ kV Middle: $I = 873$ A, $V = 223.50$ kV	Sending: $I = 910$ A, $V = 223.95$ kV Receiving: $I = 924$ A, $V = 229.48$ kV Middle: $I = 876$ A, $V = 227.20$ kV	0.99% 2.59% 0.34%	1.63% 2.74% 1.63%
380 kV 350 MW	Sending: $I = 968$ A, $V = 221.33$ kV Receiving: $I = 907$ A, $V = 223.84$ kV Middle: $I = 880$ A, $V = 223.47$ kV	Sending: $I = 967$ A, $V = 223.841$ kV Receiving: $I = 883$ A, $V = 228.15$ kV Middle: $I = 886$ A, $V = 227.65$ kV	0.1% 2.72% 0.68%	1.12% 1.89% 1.84%
407 kV 350 MW SP IN	Sending: $I = 911$ A, $V = 224.44$ kV Receiving: $I = 941$ A, $V = 225.55$ kV Middle: $I = 865$ A, $V = 225.91$ kV	Sending: $I = 900$ A, $V = 225.583$ kV Receiving: $I = 919$ A, $V = 231.78$ kV Middle: $I = 868$ A, $V = 229.60$ kV	1.22% 2.39% 0.35%	0.51% 2.69% 1.61%
407 kV 350 MW	Sending: $I = 963$ A, $V = 223.71$ kV Receiving: $I = 900$ A, $V = 226.25$ kV Middle: $I = 871$ A, $V = 225.88$ kV	Sending: $I = 958$ A, $V = 226.24$ kV Receiving: $I = 875$ A, $V = 230.43$ kV Middle: $I = 876$ A, $V = 230.50$ kV	0.52% 2.86% 0.57%	1.11% 1.81% 2.00%
380 kV 380 MW SP IN	Sending: $I = 992$ A, $V = 222.28$ kV Receiving: $I = 1010$ A, $V = 223.52$ kV Middle: $I = 946$ A, $V = 223.78$ kV	Sending: $I = 984$ A, $V = 223.51$ kV Receiving: $I = 984$ A, $V = 230.43$ kV Middle: $I = 948$ A, $V = 227.85$ kV	0.81% 2.64% 0.21%	0.55% 3.00% 1.79%
380 kV 380 MW	Sending: $I = 1040$ A, $V = 221.57$ kV Receiving: $I = 975$ A, $V = 224.17$ kV Middle: $I = 956$ A, $V = 223.75$ kV	Sending: $I = 1040$ A, $V = 224.17$ kV Receiving: $I = 950$ A, $V = 229.10$ kV Middle: $I = 960$ A, $V = 228.39$ kV	0.00% 2.63% 0.42%	1.16% 2.15% 2.03%
407 kV 380 MW SP IN	Sending: $I = 982$ A, $V = 224.79$ kV Receiving: $I = 1004$ A, $V = 225.90$ kV Middle: $I = 935$ A, $V = 226.19$ kV	Sending: $I = 974$ A, $V = 225.90$ kV Receiving: $I = 978$ A, $V = 232.71$ kV Middle: $I = 938$ A, $V = 230.22$ kV	0.92% 2.66% 0.32%	0.49% 2.93% 1.75%
407 kV 380 MW	Sending: $I = 1032$ A, $V = 223.97$ kV Receiving: $I = 966$ A, $V = 226.56$ kV Middle: $I = 944$ A, $V = 226.16$ kV	Sending: $I = 1032$ A, $V = 226.56$ kV Receiving: $I = 942$ A, $V = 231.37$ kV Middle: $I = 950$ A, $V = 230.75$ kV	0.00% 2.55% 0.63%	1.14% 2.08% 1.99%

Table 5.5: Comparison of Transmission Line and PowerFactory values with Relative Errors

The comparison between the results of the transmission line model against the model by TenneT, as displayed in Table 5.5, indicates that the model can be considered highly accurate. The maximum relative error for the current is 2.84%, and for the voltage, 3%, both very low values. The average relative error for the current is 1.21%, and for the voltage, 1.74%.

5.3.1. Conclusions on Verification of the Conductor Transmission Formulas

Based on the excellent performance shown in Figure 5.3, the method of using the telegrapher's equations can be deemed valid. This means that it can be used in further modeling efforts to determine the voltage and current at any desired point along the length of a conductor, given that the usage and design parameters are known. Because of its simplicity, this method is a valuable tool for anyone attempting to predict the intensity of electromagnetic field emissions by (submarine) power cables.

Due to the effectiveness of this model, it will also be deployed to calculate the sheath current along the length of the cable. In this way, if, and if so, to what extent the sheath current should be considered in further modeling can be verified. Lastly, it should be noted that, even though the percentage errors are low, the inaccuracy in the slope of the conductor voltage can paint a distorted picture of the sheath current distribution, as will be seen in section 5.4. Therefore, in cases where the sheath currents play a significant role, more care should be taken when setting up the model, such that the exact design and operational parameters are used.

5.4. Analytical Derivation of Charging Currents in the Sheath

This section will apply the existing methodology of analyzing transmission lines to the current and voltage within the metallic sheath of a cable. The results following from this section are new adaptations of the Telegraphers equations and General Wave Equations and the voltage and current distributions following from them. Describing the voltage and especially the current flowing through the metallic sheath can be invaluable for analyzing magnetic field emissions, as is the aim of this thesis, but can also provide insight into the thermal performance of a power cable.

5.4.1. Application of Wave Equations to Metallic Sheaths

To describe the voltage and current distribution within the metallic sheath, a new version of the wave equations (subsection 5.2.1) is developed for the metallic sheath. This is a more complicated scenario than for the conductor as it includes a distributed capacitive current instead of a simple supply from the generation end. In Figure 5.15, the electrical interactions between the metallic sheath, the conductor, and the common equipotential between the sheaths (subsection 4.3.1) are represented in the form of an electrical circuit.

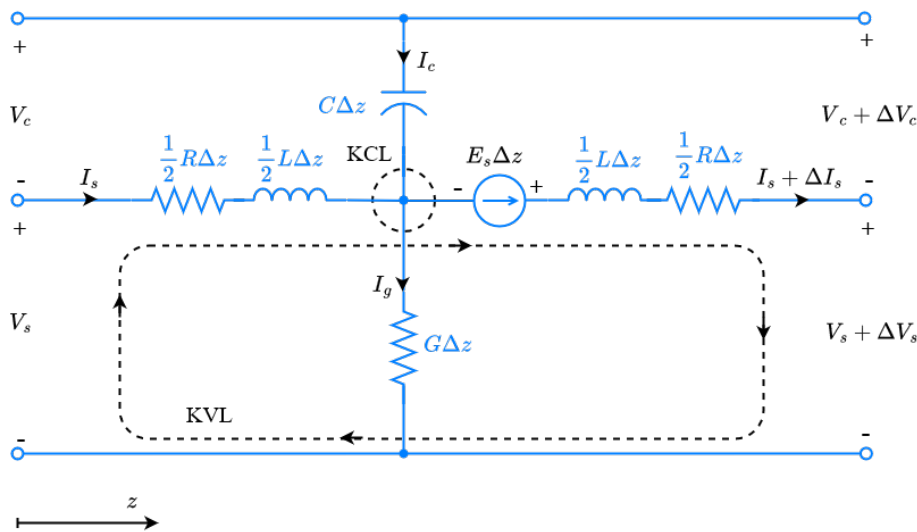


Figure 5.15: Schematic of the electrical interaction of the metallic sheath with the conductor (top) and ground (bottom). It displays the lumped elements that represent the electrical properties per unit length.

In the schematic (Figure 5.15), several voltages of interest can be observed. The sheath voltage V_s , which is to be calculated, and the conductor voltage V_c , which can independently be calculated (subsection A.0.6) and serves as an independent input to the metallic sheath. Finally, the induced voltage $E_s\Delta z$, which is also the result of the analysis of the conductor is also considered an independent input (subsection A.0.6, subsection 4.3.2).

Additionally, several currents can be identified:

- **Sheath current I_s :** The primary current to be calculated.
- **Capacitive current I_c :** The charging current polarizes the insulation along the length. This is also a direct result of the conductor analysis, as the sheath voltage is negligible compared to the conductor voltage.
- **Conductive current to ground I_g :** The current flowing through the ground conductance (i.e. to the other electrically phase-shifted metallic sheaths).

The design parameters are:

- R : Longitudinal sheath resistance per unit length.
- G : Radial conductance to ground per unit length.
- L : Sheath self-inductance per unit length.
- C : Capacitance of the dielectric insulation per unit length.

As is done for the conductor, Kirchoff's voltage and current law are applied to the circuit as indicated in Figure 5.15. Transforming from a difference equation by using that for small sections of length Δz , the differences in voltage and current become $\Delta V_s = \frac{\partial V_s}{\partial z} \Delta z$ and $\Delta I_s = \frac{\partial I_s}{\partial z} \Delta z$, the **new distributed Telegraphers equations** follow.

$$\frac{\partial V_s}{\partial z} = - \left(R I_s + L \frac{\partial I_s}{\partial t} \right) + E_s \quad (5.25)$$

$$\frac{\partial I_s}{\partial z} = -G V_s + C \frac{\partial V_c}{\partial t} \quad (5.26)$$

These equations describe how the sheath voltage and current change with position z along its length as a function of the voltage and current through itself (as is the case for a conductor) but with the additional effect of distributed charging current (represented by $C \frac{\partial V_c}{\partial t}$) and the voltage induced by the conduction current (E_s).

Differentiating these w.r.t. z and t and using substitution results in the **new form of the general wave equations** for transmission lines with distributed current and voltage. The wave equation for voltage along the metallic sheath is shown in Equation 5.27, for the current in Equation 5.28.

$$\frac{\partial^2 V_s}{\partial z^2} = R G V_s + L G \frac{\partial V_s}{\partial t} - R C \frac{\partial V_c}{\partial t} - L C \frac{\partial^2 V_c}{\partial z \partial t} + \frac{\partial E_s}{\partial z} \quad (5.27)$$

$$\frac{\partial^2 I_s}{\partial z^2} = R G I_s + (L G - R C) \frac{\partial I_s}{\partial t} - L C \frac{\partial^2 I_s}{\partial t^2} - G E_s + C \frac{\partial E_s}{\partial t} \quad (5.28)$$

These second order differential equations can be solved to find the distribution of voltage and current along the length of the metallic sheath, given that the conductor distributions and the design parameters of the cable are known. The solution to both the voltage and current distribution can be found by solving one of the two wave equations (5.27 or 5.28) and plugging the result into one of the distributed Telegraphers equations (Equation 5.25 or 5.26). Substitution of the instantaneous forms of sheath, conductor and induced voltage into Equation 5.27, and solving differentials, allows for large simplification. Division by the time component and grouping terms results in the differential equation for the voltage phasor (details Appendix A).

$$\frac{\partial^2 V_{sp}}{\partial z^2} = (R G + j \omega L G) V_{sp} + (L C \omega^2 - j \omega R C) V_{cp} + j \omega E_{sp} \quad (5.29)$$

Here V_{sp} , V_{cp} and E_{sp} are the phasor forms of V_s , V_c and E_s . Noting that the induced voltage phasor is a function of the conductor current phasor ($E_{sp} = -j X_{sf} I_{cp}$), and thereby the conductor voltage (Equation 5.13), it simplified so being related to V_{sp} and V_{cp} alone:

$$\begin{aligned} \frac{\partial^2 V_{sp}}{\partial z^2} &= (R G + j \omega L G) V_{sp} + (L C \omega^2 - j \omega R C) (V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}) \\ &\quad - \omega X_{sf} \left(\frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \right) \end{aligned} \quad (5.30)$$

Equation 5.30 can be written in its simplest form to be

$$\frac{\partial^2 V_{sp}}{\partial z^2} = k^2 V_{sp} + (\tau^2 + \zeta) V_0^+ e^{-\gamma z} + (\tau^2 - \zeta) V_0^- e^{\gamma z} \quad (5.31)$$

where the propagation constants $k = \sqrt{G(R + j \omega L)}$, $\tau = \sqrt{\omega C(L \omega - j R)}$ and $\zeta = -\omega \frac{X_{sf}}{Z_0}$ can be identified. It becomes apparent that the propagation constant for the sheath voltage itself (k) is analogous to the propagation constant of the conductor (γ) as seen in subsection 5.2.2, whereas τ and ζ are specific to the distributed voltage and current input.

This differential equation has to be solved differently than for the conductor because it does not have a specified input at one of the ends of the line. Instead, it has the distributed current input I_c and voltage E_s . The way this can be solved is by finding the set of solutions solution of Equation 5.31 and applying the boundary conditions of zero voltage at each end ($z = 0$ and $z = \ell$). The set of solutions to the differential equation for the sheath voltage phasor is:

$$V_{sp}(z) = C_1 e^{-kz} + C_2 e^{kz} + A e^{-\gamma z} + B e^{\gamma z} \quad (5.32)$$

where coefficients A and B follow directly from the particular solution and C_1 and C_2 have to be found by application of the boundary conditions (Appendix A). The coefficients for Equation 5.32 are as follows.

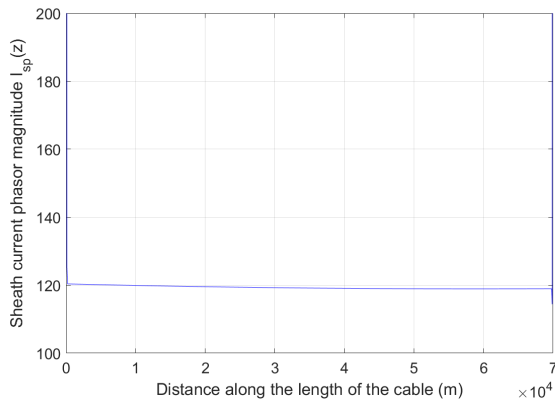
$$\begin{aligned} A &= \frac{(\tau^2 + \zeta)V_0^+}{\gamma^2 - k^2} & B &= \frac{(\tau^2 - \zeta)V_0^-}{\gamma^2 - k^2} \\ C_1 &= \frac{-(A + B)e^{k\ell} + Ae^{-\gamma\ell} + Be^{\gamma\ell}}{(e^{k\ell} - e^{-k\ell})} & C_2 &= \frac{(A + B)e^{-k\ell} - Ae^{-\gamma\ell} - Be^{\gamma\ell}}{(e^{k\ell} - e^{-k\ell})} \end{aligned} \quad (5.33)$$

Knowing these coefficients allows for calculating the sheath voltage phasor at any point along the length of the transmission line. From the sheath voltage phasor, it is possible to determine the distribution for the sheath current as well, using Equation 5.25. Working out the differentials to remove the time component, substituting $E_{sp} = -jX_{sf}I_{cp}$, and rearranging the equation to isolate the sheath current, results in the sheath voltage phasor I_{sp} in terms of the known conductor current phasor I_{cp} , the differential of the sheath voltage phasor V_{sp} and known design parameters.

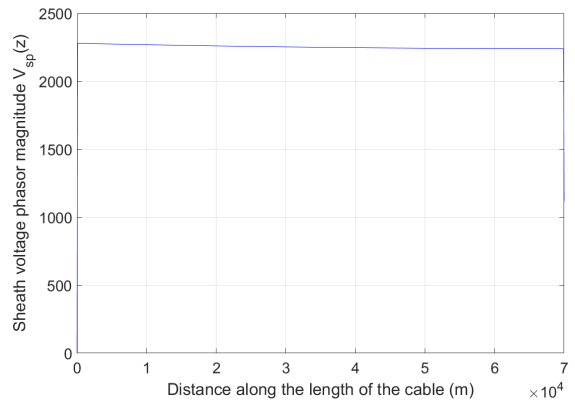
$$I_{sp}(z) = \frac{jX_{sf}I_{cp}(z) - \frac{dV_{sp}(z)}{dz}}{R + j\omega L} \quad (5.34)$$

This equation can be used to calculate the circulating current flowing through the metallic sheath, allowing for the verification of the assumption that the interconnection between the three phase-shifted sheaths creates a perfect ground. If that is the case, it can be concluded that the analysis of electromagnetic field emissions only requires knowledge of the conductor current distribution. However, as the implementation of this method and the method in section 5.5 indicate, this is not the case.

It must be noted that there was limited time to test this method, but there were interesting results that correspond to the results from the junction-series approximation in section 5.5. Simulation using this model for non reactive power compensated operation was done. Design parameters for this simulation are equal to those described in the Table 6.4. The resulting current and voltage distribution is depicted in Figure 5.16.



(a) Current distribution in the metallic sheath for uncompensated operation. Based on simulation using the new transmission line method.



(b) Voltage distribution in the metallic sheath for uncompensated operation. Based on simulation using the new transmission line method.

Figure 5.16: Results from simulating the metallic sheath using the transmission line method.

Figure 5.16 shows surprising results. Not only does it indicate that there is a persistent current in the metallic sheath, but it also indicates a high voltage. The voltage indicated is highly questionable as this would indicate a large current between the metallic sheaths. Therefore this method still requires refinement (code included in section B.2).

A note for further analysis of this method is that some care has to be taken when calculating the distributions on a computer. The coefficients C_1 and C_2 can quickly become higher than the computer can represent or can be rounded to zero and, therefore, should not be calculated separately. The main problem occurs when calculating the values $e^{-k\ell}$ and $e^{k\ell}$, but also for $e^{-\gamma\ell}$ and $e^{\gamma\ell}$ if γ becomes sufficiently high. Suppose the equations are instead written directly in terms of the components of the coefficients. In that case, these high/low coefficients are balanced by the corresponding low/high exponential terms, including location z .

There are two reasons why a second method should be developed. Since no data is available for the voltage and current distributions in the metallic sheath, this modeling method should be verified using a method known to work for

comparable cases. The next section will construct a model using a matrix formulation based on the same principles as for the transmission line.

5.5. Junction-Series Approximation of the Sheath Current and Voltage

The method of differential equations allows for the direct calculation of voltage and current at a particular value of z , which, if implemented correctly, is computationally efficient and accurate. However, there is a conceptually simpler way to do this, which we can use to verify that the method of transmission line equations accurately represents reality. This method is based on the representation of the sheath by breaking it up into multiple small junctions to which Kirchhoff's Circuital Law (KCL) can be applied. The dependency that consecutive junctions have on each other, combined with proper boundary conditions, allows the system to be solved by simple matrix operations.

This approach must be valid because it uses Kirchhoff's laws to model each small sheath segment's voltage and current behavior. The system's overall behavior can be simulated by breaking down the sheath into smaller parts. This method provides a straightforward way to verify the results obtained from the differential equation approach, ensuring consistency and reliability in our calculations.

5.5.1. Representation of the Metallic Sheath Using Junctions

The metallic sheath can be seen as a transmission line that is grounded at both ends instead of being connected to a source at one end and a load at the other. It has distributed current inflow due to the charging current (subsection 4.3.6), distributed induced voltage due to the conduction current (subsection 4.3.2), and distributed leakage current due to the semiconductive PE-layer being attached to 'ground' by touching the other ($\pm 120^\circ$ phase shifted) metallic sheaths (see subsection 4.1.3). Dividing this into many small sections, a lumped parameter representation can be made, where the voltage and current are calculated for each node simultaneously using matrix representation. Applying KCL to each junction determines the balance of current and the interdependency between sections. Knowledge about the electric parameters of the sheath, known conductor voltage and -current, and proper boundary conditions creates a linear system of equations that can be solved numerically. A schematic showing the junction-series representation is depicted in Figure 5.17.

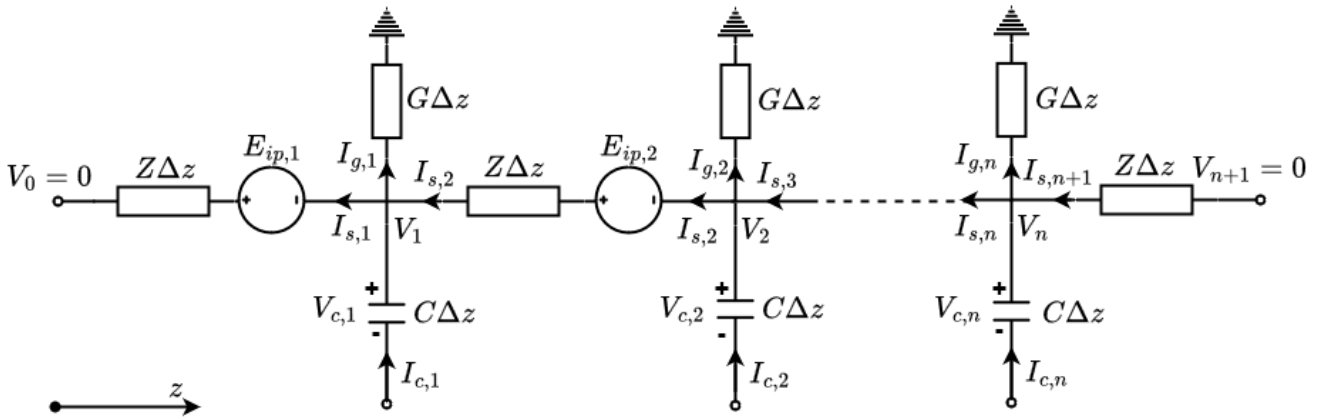


Figure 5.17: Junction-series representation of the metallic sheath

Analyzing the schematic (Figure 5.17), it can be seen that there are n junctions which have a corresponding voltage V_i and a charging current $I_{c,i}$ and sheath current $I_{s,i+1}$ flowing in. The sheath current $I_{s,i}$ and ground-current $I_{g,i}$ flow out of the junction. Every sheath current $I_{s,i}$ flows through a lumped impedance $Z\Delta z$, giving rise to the voltage difference $\Delta V_i = V_i + E_{s,i}\Delta z - V_{i-1}$. The ground currents are equal to $I_{g,i} = V_i \cdot G\Delta z$, and charging currents are equal to $I_{c,i} = C\Delta z \frac{dV_{c,i}}{dt}$. Solid bonding (subsection 4.2.2) is applied by setting $V_0 = V_{n+1} = 0$.

5.5.2. Matrix Representation Using KCL

Now that the circuit is defined, the junctions can be analyzed using KCL. KCL states that all current flowing into a junction must also flow out. This leads to the current balance for each junction, which is the basis of our solution. Note that the complete analysis is done in phasor form to simplify since the magnitude is the value of interest. The time component can be reintroduced whenever deemed necessary by multiplying the outcome by $e^{j\omega t}$. The current balance is defined in Equation 5.35 for each individual junction.

$$I_{c,i} + I_{s,i+1} = I_{s,i} + I_{g,i} \quad \text{for } 1 \leq i \leq n \quad (5.35)$$

where we know for conductor voltage $V_c = V_{cp}e^{j\omega t}$ that:

$$I_{c,i} = C\Delta z \frac{dV_{c,i}}{dt} = j\omega C\Delta z V_{cp,i} \quad (5.36)$$

where ω (rad/s) is the radial frequency of transmission $\Delta z = \frac{\ell}{n}$ (m) and conductor voltage $V_{cp,i}$ can be calculated using Equation A.58 combined with the relation $z = \frac{i\ell}{n}$ to be:

$$V_{cp,i} = V_{cp}\left(\frac{i\ell}{n}\right) = \frac{V_L + Z_0 I_L}{2} e^{\gamma \ell} e^{-\gamma \frac{i\ell}{n}} + \frac{V_L - Z_0 I_L}{2} e^{-\gamma \ell} e^{\gamma \frac{i\ell}{n}} \quad (5.37)$$

with known line parameters V_L (load side voltage), I_L (load side current), Z_0 (characteristic impedance for the conductor), and γ (propagation constant). Applying Ohm's Law on the sheath elements and the ground connections yields:

$$I_{s,i} = \frac{V_i + E_{s,i}\Delta z - V_{s,i-1}}{Z\Delta z} \quad \text{for } 1 \leq i \leq n \quad (5.38)$$

$$I_{s,n+1} = \frac{V_{n+1} - V_n}{Z\Delta z} \quad (5.39)$$

$$I_{g,i} = G\Delta z V_i \quad \text{for } 1 \leq i \leq n \quad (5.40)$$

with $E_{s,i}$ as the phasor of induced electric field (from Equation 4.22) :

$$E_{s,i} = 2\omega 10^{-7} \ln\left(\frac{2S}{D_s}\right) I_{cp,i} \quad (5.41)$$

Where μ_0 is the magnetic permeability of free space, ω the radial frequency of transmission frequency, S is the distance between centerlines of the phases, and D_s is the inner diameter of the metallic sheath. The sign is now positive as the field was already defined to be in the negative z -direction in the circuit (Figure 5.17).

Note that this formula for induced electric fields is based on the assumption of an infinite line, which is not completely valid at the start and end of the cable. However, this can still assumed to be valid due to the effect only occurring over a short length. The conductor current phasor as a function of junction index i can be taken from the predetermined conductor current distribution (Equation 5.20) that has an equal amount of nodes.

$$I_{cp,i} = I_{cp}\left(\frac{i\ell}{n}\right) \quad (5.42)$$

Now that all the currents are defined in terms of sheath voltage and conductor voltage, a matrix A can be constructed to solve for all sheath voltage simultaneously. To do this, Equation 5.35 is rearranged by filling in equations 5.40 and grouping everything known on the left side of the equation (setting it equal to solution χ_i) and everything unknown on the right (setting it equal to $A(\vec{i}) \cdot \vec{V}_s$):

$$\chi_i = I_{c,i} + \frac{(E_{s,i+1} - E_{s,i})}{Z} = V_i\left(\frac{2}{Z\Delta z} + G\Delta z\right) - \frac{V_{i-1}}{Z\Delta z} - \frac{V_{i+1}}{Z\Delta z} = A(\vec{i}) \cdot \vec{V}_s \quad \text{for } 2 \leq i \leq n-1 \quad (5.43)$$

For the edge cases where $i = 1$ and $i = n$ the equation includes V_0 and V_{n+1} , $E_{s,n+1}$ respectively which we take out of the solution vector to be able to solve an $n \times n$ system. Since $V_0 = V_{n+1} = 0$ it does not affect the outcome. This results in the following two equations:

$$\chi_1 = I_{c,1} + \frac{(E_{s,2} - E_{s,1})}{Z} = V_1\left(\frac{2}{Z\Delta z} + G\Delta z\right) - \frac{V_2}{Z\Delta z} = A(\vec{1}) \cdot \vec{V}_s \quad (5.44)$$

$$\chi_n = I_{c,n} - \frac{E_{s,n}}{Z} = V_n\left(\frac{2}{Z\Delta z} + G\Delta z\right) - \frac{V_{n-1}}{Z\Delta z} = A(\vec{n}) \cdot \vec{V}_s \quad (5.45)$$

The coefficients from the voltages on the right side of the Equation 5.43 and 5.45 are then used to build $n \times n$ matrix A . The voltage for each junction along the length of the cable can then be solved simultaneously by the matrix operation in Equation 5.46.

$$\vec{V}_s = A^{-1} \vec{\chi} \quad (5.46)$$

This concludes the method of calculating the voltage distribution along the length of the metallic sheath using a representation of many junctions. However, the value of interest is the sheath current, which can be determined by applying Ohm's law between two junctions, as it was used to set up Equation 5.40.

$$I_{s,i} = \frac{V_i + E_{s,i}\Delta z - V_{s,i-1}}{Z\Delta z} \quad \text{for } 1 \leq i \leq n \quad (5.47)$$

5.6. Validation of Zero Sheath Current Assumption

Modern industrial cable design uses a semiconductive polyethylene sheath to dissipate charging and induced currents between the electrically phase-shifted metallic sheaths. In the design process for these cables, the assumption is made that this completely removes longitudinal currents in the metallic sheath. However, the concern was raised that this is not realistic, as the connection between phases can never be perfect.

Methods for evaluating the sheath current have been developed in this thesis and will be used to verify the assumption of zero sheath currents in this section. Due to its simplicity, the junction representation method of the metallic sheath will be used (code in Appendix B). Initially, this will be done using the parameters from the Borssele Alpha cable, for which the voltage and current distributions have been verified in Figure 5.3.

From the application of the conductance method using COMSOL (subsection 4.4.1), a conductance of $G = 16.7 \text{ S/m}$ has been calculated a single metallic sheath and its two counterparts that are electrically phase shifted by $\pm 120^\circ$. For the cable parameters from TenneT and using the nominal operating condition 380 kV 350 MW as input to the induced voltage and charging current, the sheath voltage and current distribution are shown in Figure 5.11. Plotting this distribution for the Borssele Alpha cable design (see Appendix A) with standard (380 kV 350 MW) operational settings gave some surprising results. Including the induced voltage results in much higher sheath currents than would be expected from a cable that is grounded over its circumference. The sheath voltage and current distributions are calculated for a range of values of inter-sheath conductance G and the results are included in Figure 5.20.

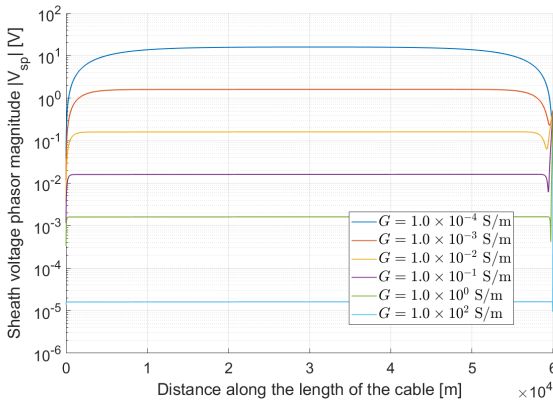


Figure 5.18: Sheath voltage distribution for multiple values of inter-sheath conductance G at standard operating conditions (380 kV 350 MW).

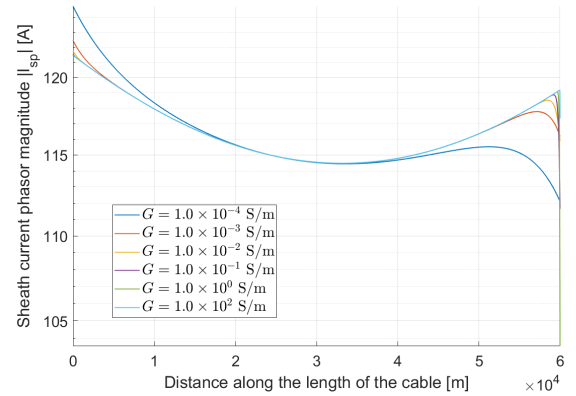


Figure 5.19: Sheath current distribution for multiple values of inter-sheath conductance G at standard operating conditions (380 kV 350 MW).

Figure 5.20: Comparison of sheath voltage and current distributions for multiple values of inter-sheath conductance G at standard operating conditions (380 kV 350 MW).

Looking at Figure 5.20, some interesting results can be seen. The most surprising is Figure 5.19 as it indicates that the sheath current above 100 A for all value of conductance in the range, which is completely contradictive to the assumption of complete current dissipation. Moreover, the range of conductances is even exaggerated w.r.t. reality, as the conductance of the Borssele cable is only $G = 16.7 \text{ S/m}$. The voltage distribution on the other hand, is something that was expected beforehand, as there cannot be a large buildup of potential when each sheath is electrically connected to ground by means of an electrical connection with 2 other $\pm 120^\circ$ phase shifted sheaths. However, what is surprising, is that this low voltage occurs while high currents flow through the sheath.

These results are strange, but there is a possible explanation for this that requires more research. Since the junction model assumes that it is not the absolute value of the sheath voltage that dictates the current, but also the induced electric field, it is logical that there would be a current that is proportional to the electric field over the impedance:

$$I_{s,i} \propto \frac{E_{s,i}\Delta z}{Z\Delta z} \quad (5.48)$$

When the values of sheath voltage are negligible, this becomes the complete equation for sheath current. In Equation 5.48 it can be seen that the length component z cancels out. This would also suggest that whatever happens over a length Δz (i.e. current dissipation to other phases) does not have an effect on the longitudinal current. Calculation of the sheath voltage using Equation 5.48 with $E_s = 0.046$ V/m and a sheath impedance of $Z = 3.8 \cdot 10^{-4} + 5.0 \cdot 10^{-5}i$ (both realistic values) results in a current $I_s = 119 - 15.7i$ A which is what can be seen in Figure 5.19.

If the method of junction representation is correct, which is very likely as it is not a new concept in electric circuit analysis, this would mean that the induced current in a metallic sheath is barely affected by introducing a conductive outer layer. This does not mean that the metallic sheath has no overall effect because this method can still reduce the charging current.

To gain an understanding of the distribution of current, the same model is used to see what happens when it is only inductive (Figure 5.21) or only capacitive (Figure 5.22) current in the sheath for different values of conductance.

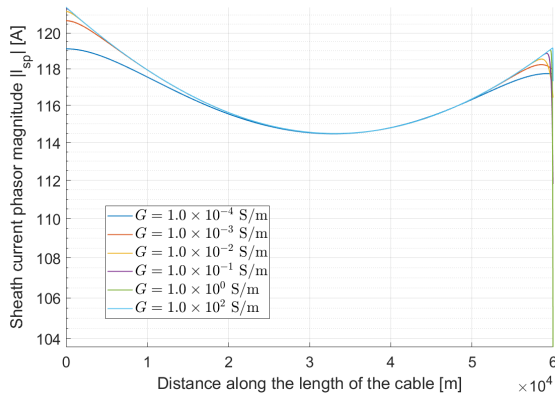


Figure 5.21: Sheath current distribution for multiple values of inter-sheath conductance G at standard operating conditions (380 kV 350 MW).

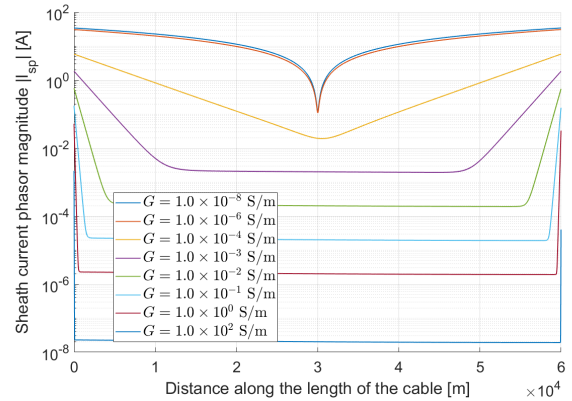


Figure 5.22: Sheath current distribution for multiple values of inter-sheath conductance G at standard operating conditions (380 kV 350 MW).

From Figure 5.21 and 5.22 it can be seen that for the conductance values used, the induced currents have a much larger effect than the capacitive currents. It can be seen that using this junction model, the capacitive current distribution behaves exactly as expected. The capacitive current is negligible for high conductances, but even then, it starts to increase close to the ends of the cable, where the ratio of conductance to endpoint-grounding over the inter-sheath conductance starts to rise. Additionally, when decreasing the inter-sheath conductance, the charging current starts to behave like it would for a cable with an insulated sheath: very low current in the middle of the cable and substantial near the ends.

Due to the fact that there is no data on this, it is hard to verify whether this model accurately predicts reality. For now, the only verification method is to compare it to the results of the transmission line model, which is not yet working. However, it is clear that the assumption that sheath currents are completely canceled out due to the conductive outer sheath is incorrect. The extent to which the current flows through the metallic sheath remains to be seen. To really know the intensity requires experimental testing. For the rest of this thesis, this new finding will be considered true and will be taken into account when parameter analysis of the cable design and use cases is performed.

5.7. Conclusion on the Current Distribution Analysis

The aim of this chapter was to create an understanding of the current distribution along the length of an HVAC power cable and a methodology for accurately predicting the net current. It can be concluded that this aim was more than met, as, next to establishing an understanding of how currents are distributed along the length, multiple valuable tools were created to aid in the analysis of current flow. These tools included two methods for analyzing current distribution in the metallic sheath that can easily be adapted by anyone intending to predict EMF emissions. It was also shown that, even though modern submarine power cables use a conductive outer sheath, there are still large circulating currents due to induction.

Using a simple COMSOL model, it was first proven that even though the polyethylene sheath is conductive, the currents in the metal sheath can never be zero as the connection is imperfect. The case was made that this principle can be extended to the induced currents, thereby highlighting the need for an in-depth analysis of the sheath current.

An existing method of predicting the current along a transmission line was found and verified by showing that it can reproduce results from simulations by TenneT. In further analysis, this method can be used to predict conductor current at a location of interest, serving as an input to metallic sheath current models and to EMF models in COMSOL.

A new method for predicting the circulating currents in the metallic sheath was created based on the transmission line method used for predicting the current in the metallic sheath. For the development of this method, the metallic sheath was assumed to be a transmission line. It used a distributed inflow of current due to capacitive charging and continuous induced electric fields due to the conduction current, combined with the assumption of a conductive connection to the ground along its length and the boundary conditions of zero voltage at each end. Promising results were achieved by predicting charging currents, but it has not yet been effectively implemented for induced currents due to a lack of time.

Because the transmission line method was not implemented correctly, and because a method of verification would always be required, a second means of predicting sheath current was developed. This method, based on a representation using a series of junctions, showed very interesting results. Rather than dissipating through the conductive connection between the different metallic sheaths, the model predicts very strong currents in the metallic sheaths. The effect of the conductive outer sheath appeared to affect only the charging current, whereas the induced current was unaffected. The invariability of induced current to the conductive outer sheath was shown by isolating the effect of capacitive and induced currents for different conductance values between the sheaths. This is a surprising conclusion, as it would be expected that short-circuiting conductors of opposite potential, such as three metallic sheaths of a submarine cable, result in a loss of longitudinal current. However, due to the nature of the induced electric field in the longitudinal direction of the metallic sheath, the voltage difference between the sheaths seems to be only an emergent phenomenon that arises from the current that flows. The simulations also show that the voltage on the metallic sheath is small, indicating the induced current's independence from the voltage differential over the length.

The charging currents, however, are a result of a voltage differential. It was clearly shown that the charging current within the sheath becomes negligible for high values of conductance, whereas it mimics the well-known linear distribution for low values of conductance.

6

Parameter Impact Analysis for EMF Modeling

This chapter aims to determine the relative importance of parameters included in electromagnetic field (EMF) emission models for submarine power cables. Accurately modeling these parameters is crucial for ecological research, though incorporating every detail can be challenging and often unnecessary. Therefore, an analysis of the influence of individual parameters is appropriate.

chapter 4 already selected the parameters that are likely to be important for current and EMF emissions. chapter 5 developed methods to describe the current along the cable length, helping to refine the importance of these parameters. This chapter will assess the relative importance of the selected parameters by comparing their impact on the outcomes of the current and EMF models.

The structure of this chapter is organized as follows: First, the general methodology used for modeling in this chapter is explained in section 6.1. Next, in section 6.2 the design and operational parameters affecting the current are analyzed. This is followed by an examination of the design and operational parameters impacting EMF emissions in section 6.3. Additionally, to provide extra insight specifically for ecological research, the impact of animal movement on the perceived field is explored in section 6.4. Finally, the chapter concludes with a summary of findings in section 6.5.

6.1. Methodology of the Parameter Impact Analysis

The parameters selected for this sensitivity analysis are considered significant based on a thorough review of the underlying physics as discussed in chapter 4 and consultations with field experts. It also considers the parameters that are represented in the current distribution of the conductors and metallic sheaths (chapter 5). Each parameter will be rigorously tested to evaluate its influence on the EMFs generated by the cable system, ensuring a comprehensive assessment of model sensitivities.

6.1.1. Simulation Tools and Software

Modeling of EMFs is done through the use of COMSOL Multiphysics. The choice for this software was made based on the availability, applicability to EMF modeling and its the ease of doing parametric analyses.

COMSOL is a multi-purpose modeling tool, and has useful modules for modeling EMFs, with clear tutorials for modeling submarine cables, used as a reference to create the models for this parameter analysis ([62], [63], [64], [40]).

The use of a geometry-tree structure in COMSOL complements this parametric approach, as it allows for automatic building of models for a predefined list of parameter values. Each layer in the tree represents a successive step in constructing the model, building on the previous steps. While this method may initially complicate the geometry setup, it significantly benefits the overall modeling process.

Modeling of currents along the length of the cable was done with the use of Matlab code. As will be indicated at the corresponding sections, important Matlab scripts for reproduction of the results are included in Appendix B.

6.1.2. Construction of a Standard COMSOL Model

A standard model was used to perform that analysis, such that the effect of individual parameters on the outcome of a model could be calculated. This model is based on a state-of-the-art three-phase high-voltage submarine power cable design based on the Borssele offshore transmission system by TenneT. The Borssele transmission system uses a typical

modern three-phase cable. However, there are a few specific factors to focus on. It uses semiconductive polyethylene layers around its metallic sheaths (as described in section 4.1). It is semi-wet, meaning that non-insulated cavities are assumed to be filled with seawater. Finally, the armor wire sequence is broken up with synthetic wires, which are assumed to be water here.

This model is made based on the method described in the COMSOL tutorial for modeling submarine power cables [64],[40]. The model was set up for the Magnetic Fields module, which only calculates for the effects of induction. Since the conductors, metallic sheaths, and armor wires are the only highly conductive and/or highly permeable components, all other components can be neglected as they will have no relevant effect on the outcome. Note that this is only true given a pre-calculated conduction current that is used as input to the COMSOL model because some components clearly have an effect on the currents flowing through the cable (as is concluded in chapter 5).

Geometry Sequence in COMSOL

The geometry of the cable section is created using a geometry sequence, where every dimension is based on a set of parameters that are defined at the start of the model tree. The geometric design parameters for the cross section are outlined in Table 6.1. Included are the parameters that describe the conductor, metallic sheath and the armor wires directly. Next to that are the three-phase diameter D_{pha3} and the polyethylene sheath thickness T_{pe} that are used for creating the model as it would be with all components included.

It should be noted that Table 6.1 gives the parameter descriptions for the general states. However, there are a few cases where these parameters are adjusted to create dependency on a parameter being analyzed.

Table 6.1: Geometric parameters for the simplified cross section used in parameter analysis.

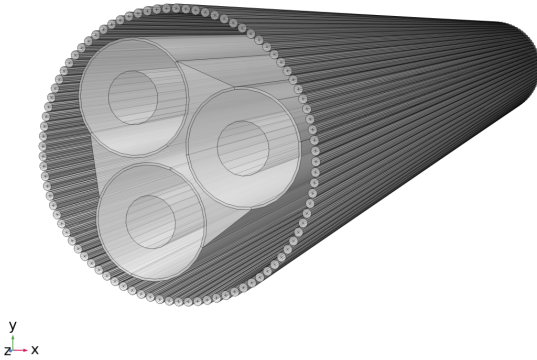
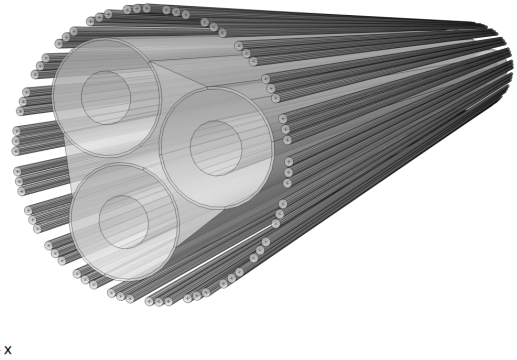
Parameter	Description	Value
Dcon	Conductor diameter	41.6 mm
Dins	Diameter over the insulation	89.6 mm
Tpbs	Diameter of the lead sheath	2.0 mm
Tpe	Diameter of the polyethylene sheath	2.0 mm
Tarm	Diameter/thickness of an armor wire	7.0 mm
Rsp	Spacing ratio (%) of circumference of armor layer covered	88
Dpha	Diameter of the phases ($D_{ins} + 2 \cdot T_{pbs} + 2 \cdot T_{pe}$)	$D_{ins} + 2 \cdot T_{pbs} + 2 \cdot T_{pe}$ mm
Dpha3	Diameter over the three phases ($D_{pha} \cdot (\frac{2}{\sqrt{3}} + 1)$)	$D_{pha} \cdot (\frac{2}{\sqrt{3}} + 1)$ mm
Darm	Diameter of the armor layer centerline	$0.22715 \cdot 1 + 2 \cdot (T_{arm} - 0.007)$ mm
Dcab	Diameter of the cable	$D_{arm} \cdot (\frac{0.244}{0.22715})$ mm
Narm	Number of armor wires	$\text{floor}(\frac{R_{sp} \cdot \pi \cdot D_{arm}}{T_{arm} \cdot 100})$ or 88

As discussed in subsection 4.1.5 describing the periodicity of a submarine power cable, the minimum length required for modeling is the cross pitch divided by the number of wires: $L_{sec} = CP_{cab} \cdot 1/N_{arm}$ [43]. However, there also exists a non-uniform type of armor layer (used in Borssele Alpha), where every fourth armor wire is made of glass fiber instead of steel. Because of the difference in electromagnetic properties, a new periodicity is proposed (which will be tested later). Instead of after each armor wire, the periodicity can now be identified as after every fourth wire such that the length becomes: $L_{sec} = CP_{cab} \cdot 4/N_{arm}$.

These main parameters together form the basis of the model. Only the parameters that are being inspected will differ between simulations. To account for the distortion of the cross-sectional surfaces in geometry, the surface correction factors SCF_{pha} and SCF_{arm} are implemented. To aid in meshing, the conductors and armor wires are simplified to icosagons and hexadecagons, which are then size-corrected by applying truncation correction factors TCF_{p20} and TCF_{p16} , respectively. The complete model with a continuous armor layer is shown in Figure 6.1 and for the pattern used in the Borssele Alpha cables in Figure 6.2.

Table 6.2: Geometric parameters for the twist and resulting length modeled (following the short-twisted periodicity [43])

Parameter	Description	Value
LLpha	Lay-length of the phases	$18 \cdot D_{pha3} \text{ m}$
LLarm	Lay-length of the armor wires	$-15 \cdot D_{arm} \text{ m}$
CPcab	Cross pitch of the cable	$\frac{1}{\left(\frac{1}{LLpha} - \frac{1}{LLarm}\right)} \text{ m}$
Nper	Number of periods (cross-pitches) included in the model	$\frac{1}{N_{arm}} \text{ or } \frac{4}{N_{arm}}$
Lsec	Length of the section that is modeled	$CPcab \cdot Nper \text{ m}$

**Figure 6.1:** Simplified cable model of electromagnetic components with complete armor coverage.**Figure 6.2:** Simplified cable model of electromagnetic components with armor wires as in the Borssele Alpha cables.**Figure 6.3:** Comparison of 3D models: continuous armor layer (left) vs. intermittent fibers (right)

Both models include only the electromagnetic (metallic) components. The environment and insides are modeled as a generic insulator that has a much lower conductivity than the metallic components. The metallic components have typical values for conductivity σ and relative permeability μ_r applied to them, whereas the generic insulator has a relatively high (but much lower than the metallic components) stabilizing conductivity to deal with numerical accuracy problems [40]. For dealing with temperature differences, the model uses the linearized resistivity to calculate the conductivity of components based on the values ρ_0 , α and T (and $T_{ref} = 20^\circ$): $\sigma = \frac{\sigma_0}{(1+\alpha(T-20))}$. Table 6.3 lists the standard electromagnetic material parameters used. Note that they can be changed when their effects are analyzed.

Table 6.3: Electromagnetic properties of the components and environment modeled in COMSOL.

Material	μ_r	σ_0 [S/m]	ϵ_r	α [1/K]	T [°C]	σ
Generic Insulator	1	50	1	-	-	50 S/m
Copper	1	5.96e7	1	3.9e-3	89.5	4.69e7
Lead	1	4.55e6	1	3.86e-3	82.7	3.66e6
Steel	100-50-j	4.03e6	1	5.3e-3	70.2	3.18e6

Meshing the Standard Model

The standardized model was meshed using the methodology from the COMSOL Cable modeling tutorial [64]. However, some deviations were made to increase the robustness of the parametric sweep. A short outline of the method will be given here.

- Mesh control entities are created on the cross-section of the conductors to aid COMSOL in creating an efficient and robust mesh. This is aimed at increasing the accuracy near the skin of the conductors by using a more dense and structured quadrilateral mesh.
- The cross-section first meshes using 2D mesh entities. The inside of the conductors, the insulation, and the free space between the phases and the armor layer use a free triangular mesh. A mapped mesh (which follows the predefined mesh entities) is applied to the conductor skin and the metallic sheaths.

- Two separate swept mesh distributions are used to translate the 2D mesh to its opposing surface at the other end of the cable section. The first is applied only to the phases, whereas the second is applied to the armor layer, twisting in opposite directions.
- A convert and copy face operation is used to mesh the inner spaces of the cable as well as the outer surface, after which a free tetrahedral 3D mesh is created.

The mesh that follows from this method is as shown in Figure 6.4.

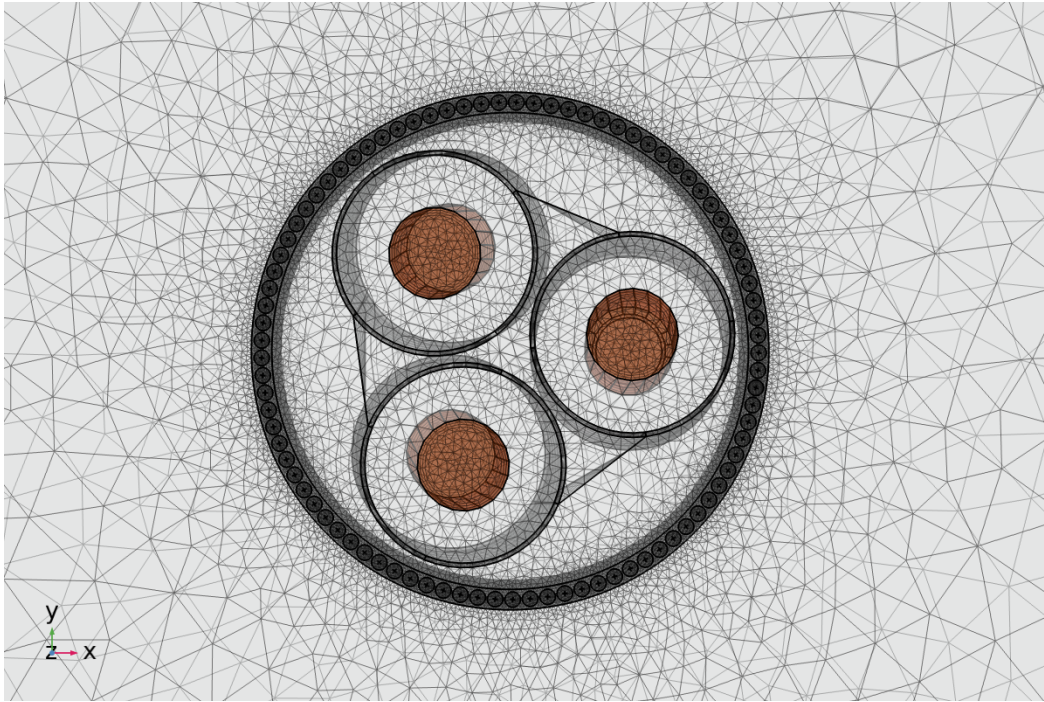


Figure 6.4: Cross sectional view of the standard mesh used in the parametric sweep.

It should be noted that mesh creation and stability depends on the geometry that it is based on. Some geometries can be harder to properly mesh and need additional mesh intities. Meshing requires iteration and sometimes making concessions. The mesh used in this chapter is slightly different from the mesh in the COMSOL tutorial, simply because the model would not converge with the original. Additionally, this mesh is much finer than in the tutorial, for the same reason, which increases computational time. However, due to the large decrease in length of the modeled section it still has much quicker results (between 5-10 mins for a parametric sweep instead of 30-60 mins for a single computation).

Setting Up The Magnetic Fields Analysis

The next step is setting up the Magnetic Fields module, which requires some additional operations to represent the system we want to analyze.

First, a periodic condition is applied to represent the continuity of the cable. This connects both cable cross-sections so that their inputs and outputs can interact. Then, the three phases are implemented using a coil domain feature. Each coil receives a current equal to $I_0 \cdot e^{\pm j\phi}$ where I_0 is the conduction current amplitude and ϕ the electrical phase shift of that current (which is zero for phase 1). The input and output planes are then defined to be the same for each phase, as the phase shift already dictates the correct direction. The metallic sheaths and the cable armor wires are created as extra Ampere's law entities to apply the linearized resistivity that components in the general Ampere's law domain do not have.

Finally, a stabilization compensation entity was added to compensate for the increased conductivity that was applied by the general insulator to all 'insulator' domains. This entity is an external current density that is exactly equal to the current density that follows from the increased conductivity. Thereby neglecting its effect, which keeps the system numerically solvable [40].

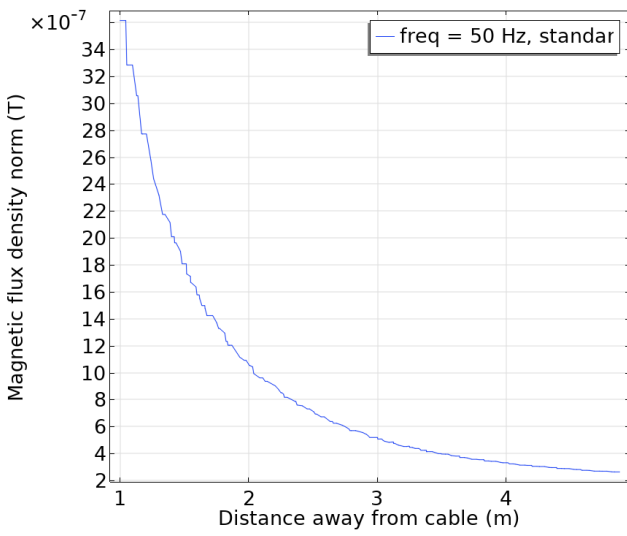
Frequency Domain Analysis in COMSOL

To limit simulation time, the frequency domain was chosen as the mode of analysis. This looks solely at the magnitude of the harmonic frequencies at the moment in time at which the input values occur. In reality everything would change with time, following sinusoidal relations with angular frequency $\omega = 2\pi f$ [rad/s].

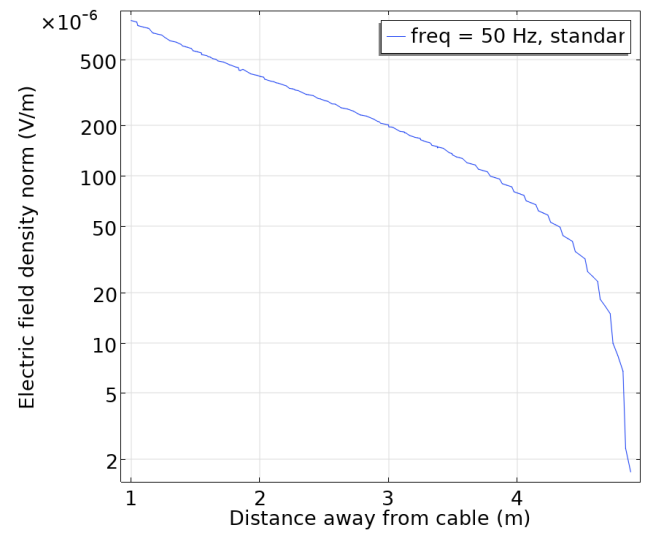
Ideally, one would simulate everything with a simulation frequency of 50 Hz as is the grid frequency in Europe. However, due to the requirement of building a model that can consistently converge to a solution for different parameters, the frequency used in this parameter analysis is 500 Hz. As described in subsection 4.3.2, this will result in induced electric fields that are ten times higher than for 50 Hz. Consequently, the electromagnetic shielding because of induced fields is increased largely. Because of this, the choice was made to adjust the conductivity of the metallic sheath and armor wires by a factor of 10, which should account for the increase in induced currents. This factor comes from equalizing the induced current w.r.t. the case for 50 Hz:

$$I_{ind} = \frac{E(\omega)}{Z(\omega)} = \frac{-j\omega LI_0}{\frac{1}{\sigma_{50}} + j\omega L} = \frac{-j10\omega LI_0}{\frac{1}{\sigma_{500}} + j10\omega L} \quad (6.1)$$

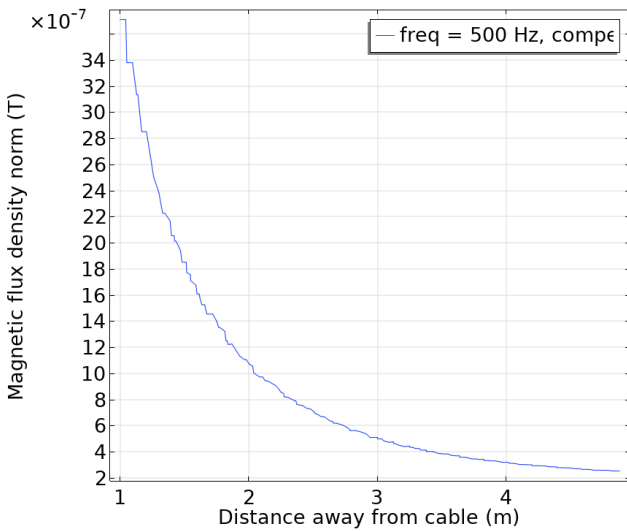
To account for the temperature dependency, this factor $\frac{1}{10}$ is applied to both the reference conductivity σ_0 and the temperature coefficient α for both lead and steel. It should be noted that this might not be 100% accurate due to the possible difference in skin and proximity effects, but as can be seen in Figure 6.5 it is enough for the purpose of analyzing the relative effects of parameters.



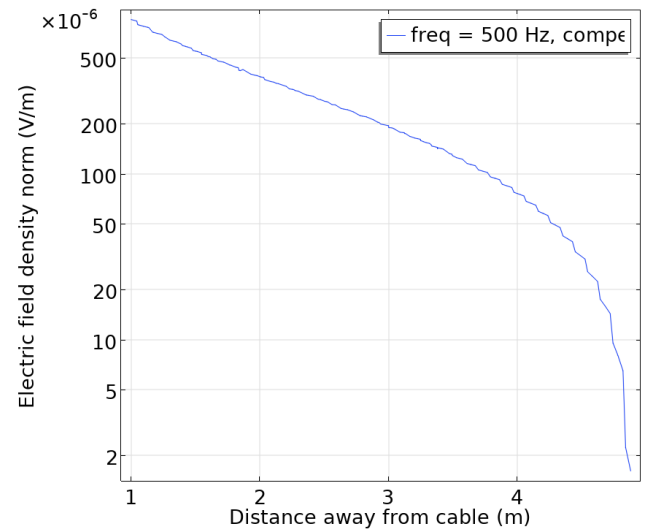
(a) Magnetic flux density using standard conductances at $f = 50$ Hz.



(b) Electric field intensity using standard conductances at $f = 50$ Hz.



(c) Magnetic flux density using compensated conductances at $f = 500$ Hz.



(d) Electric field intensity using compensated conductances at $f = 500$ Hz.

Figure 6.5: Comparison of the magnetic flux and electric field emission models in COMSOL by using the standard conduction frequency and material parameters vs. using frequency-compensated material parameters and another frequency compensation factor for the electric field.

Looking at both the magnetic flux density plot (6.5a, 6.5c) and the electric field intensity plots (6.5b, 6.5d), almost no difference between the two can be visually identified. Both method show exactly the same shape of distribution, the only

small difference can be seen at the left hand side of the magnetic flux curves, where the flux is $\approx 5 \mu\text{T}$ higher for the 500 Hz case. From the excellent performance it can be concluded that there is no relevant difference in outcome between the two methods. Therefore the rest of this parameter analysis will be done using the 500 Hz - frequency compensated settings.

Study Settings for Parametric Sweep

The final step in the configuration of the model is to set up the study. A coil geometry analysis step was added to make sure the phases were represented properly. The frequency domain analysis was implemented for 500 Hz (following the conclusion that frequency compensation is valid).

Lastly, the parametric sweep operation was set up. This has to be adjusted for each run to fit the parameters that one wants to sweep. The parametric sweep runs the model for each parameter in a list. For each parameter, the whole sequence of input parameters is recalculated according to their dependencies. Then, the geometry is created following the geometry sequence, and finally, the mesh is built to fit the geometry of that parameter. This is an great feature for testing the effect of different parameters as it allows for easy comparison of their relative impact. However, much care has to be taken that dependencies are setup correctly, such that the geometry and consequently the mesh can be built.

6.2. Parameter Impact Analysis on Total Phase Current

This section will give an outline of the effects that changing different parameters has on the net current that flows through each phase (meaning the conductor and the sheath combined). Thereby, it creates an initial list of parameters that are important to use in modeling. It will be used to provide input values to COMSOL models, to test the impact of these parameters on magnetic field emissions. This will be done based on the method developed in chapter 5.

The parameters that have an impact on the flow of current along the conductor and the metallic sheath are the design and material parameters of the cable, summarized by their resulting electromagnetic properties R_c, R_s, L, C, L_s and G . However, as has been discussed in chapter 4 and chapter 5, the inter-sheath conductance G for modern HVAC submarine cables is high, in order to reduced sheath current. Therefore this section will focus its analysis on the case for a conductive sheath interface.

This parametric analysis is done by using a standard set of design and use parameters, for which a single parameter is changed to understand its impact. A list of all standard parameters is shown in Table 6.4. For each parameter of interest a separate script is created that plots the conductor, sheath and combined current along the length of the cable.

Table 6.4: Standard parameters used in current distribution calculation script (Matlab)

Parameter	Value	Description
Q1	55 MVar	Sending end reactive power compensation
Q2	47.5 MVar	Receiving end reactive power compensation
I_L	881 + 226i A	Load current
V_L	223.841 V	Load voltage
L	70e3 m	Total length of the cable in meters
alpha	1.7636e-7	Real part of propagation constant (per meter)
beta	2.6419e-6	Imaginary part of propagation constant (per meter)
gamma	alpha + 1j*beta	
Zc	60.68 - 4.04i Ω	Characteristic impedance
Dout	94e-3 m	Diameter over the outside of the metallic sheath
Ds	90e-3 m	Diameter over the inside of the metallic sheath
tsc	2e-3 m	Thickness of semiconductive PE layer
tsw	1e-3 m	Distance between two semiconductive layers
S	Dout + 2 * tsc + tsc m	Distance between two conductor cores
Rc	0.0151e-3 Ω /m	Phase resistance per meter
Lc	0.3592e-6 H/m	Phase inductance per meter
C	0.196e-9 F/m	Phase capacitance per meter
omega	100 * pi rad/s	Radial frequency of transmission
sigma	0.455e7 S/m	Conductivity of lead sheath
As	$\pi / 4 * (Dout^2 - Ds^2) m^2$	Area of sheath cross section
Rs	1 / (As * sigma) Ω /m	Sheath resistance per meter
Ls	$2 * 10^{-7} * \log(2 * S / Ds)$ H/m	Sheath inductance per meter
G	16.7 S/m	Sheath conductance per meter
Xsf	$2 * 10^{-7} * \omega * \log(2 * S / Ds)$ Ω /m	Sheath reactance per meter

The relative impact of these is analyzed by applying an array of multiplication factors in the standard value (Table 6.4) and plotting the results using Matlab (Appendix B). The multiplication factors are chosen to be close to the standard value to give an indication of real-world situations.

6.2.1. Impact Analysis of Design Parameters on Phase Current

As has become clear in chapter 5, the design parameters with the largest impact on the current distribution are the electrical properties of the conductor and the metallic sheath. Although they arise from a combination of geometric and material properties, only the phase resistance R_c , phase capacitance C and phase inductance L_c are tested to keep analysis simple. For the metallic sheath, the sheath inductance L_s and sheath resistance R_s are tested.

The Impact of Conductor Parameters on Phase Current

The conductor is the largest current carrying medium in a power cable and is therefore analysed first in this parameter analysis. Included parameters are the resistance per meter (R_c), the inductance per meter (L_c) and the capacitance per meter (C). It should be noted that both capacitance and inductance depend on the design of the conductor as well as the metallic sheath, but are grouped with the conductor here. The transmission line parameters are updated to match the values for R_c , L_c and C by using Equation A.48 for characteristic impedance Z_0 and Equation A.45 and A.46 for propagation constant γ .

For the range of conductor resistances R_c the simulation results of the total phase current and the sheath current alone are shown in Figure 6.8.

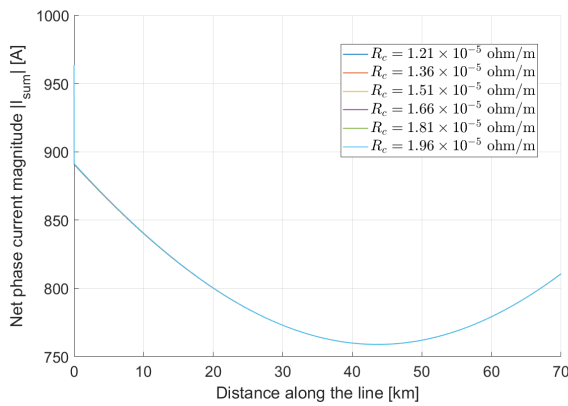


Figure 6.6: Combined current phasor magnitude along the length of the cable for different values of conductor resistance R_c , keeping all other parameters constant.

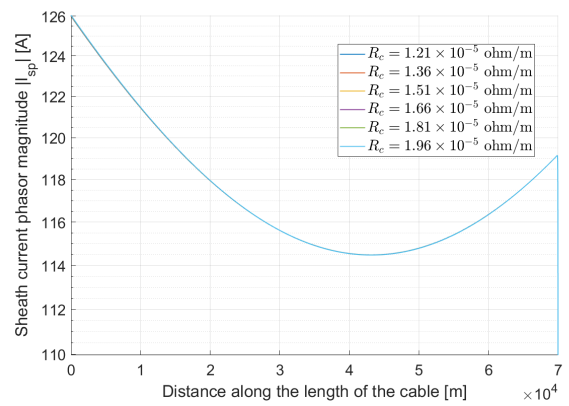


Figure 6.7: Sheath current phasor magnitude along the length of the cable for different values of conductor resistance R_c , keeping all other parameters constant.

Figure 6.8: Comparison of the impact of different values of conductor resistance on the current distribution along the length of the cable. Includes the combined (left) and metallic sheath value (right).

From Figure 6.8 it can be seen that there is no visible impact on the currents due to the conductor resistance. Both the sum of the currents and the sheath current are relatively unaffected. This is somewhat surprising as resistance will result in losses, however, the reason for this is the low dependence on cable voltage. Due to the semiconductive layer, the charging currents that result from voltage are dissipated. Therefore, even though the conductor does experience a voltage drop over its length, there is no impact on the conductor current and, consequently, no change in the sheath current.

Next, the phase inductance is tested. The simulation results for different values of phase inductance is shown in Figure 6.11.

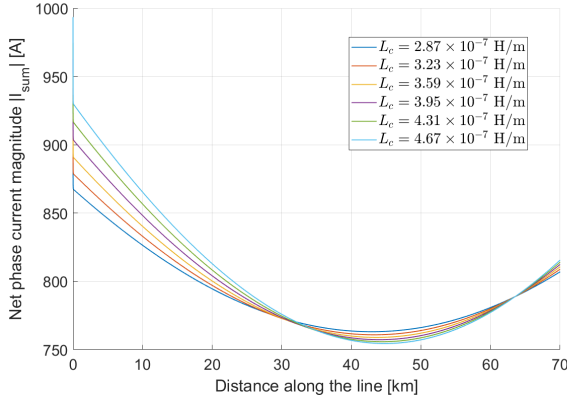


Figure 6.9: Combined current phasor magnitude along the length of the cable for different values of conductor inductance L_c , keeping all other parameters constant.

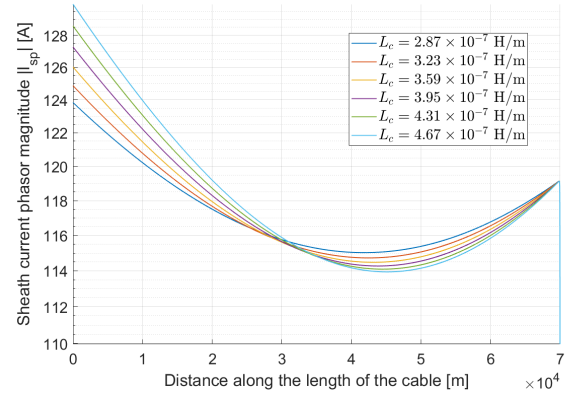


Figure 6.10: Sheath current phasor magnitude along the length of the cable for different values of conductor resistance L_c , keeping all other parameters constant.

Figure 6.11: Comparison of the impact of different values of phase inductance on the current distribution along the length of the cable. Includes the combined (left) and metallic sheath value (right).

From Figure 6.11 it can be seen that the phase inductance also has relatively little effect on the outcome of both the total and sheath current. This is somewhat surprising as in reality inductance has not only a direct effect on the conductor current, but on the induced current as well. However, it must be noted that this simulation does not couple the values of phase- and sheath inductance, while they would be coupled in reality. Therefore this solution should be interpreted with the consideration that the induced current changes more than indicated here.

A better effort could be achieved by coupling both L_c and L_s through describing them in terms of common geometric properties. Another option is to sweep both at the same time, by assuming they are linearly related to each other. This would require multiplying them by the same array of values in one simulation.

The simulation results for the impact analysis of the phase capacitance is shown in Figure 6.14, where much larger influence is seen than for the previous parameters.

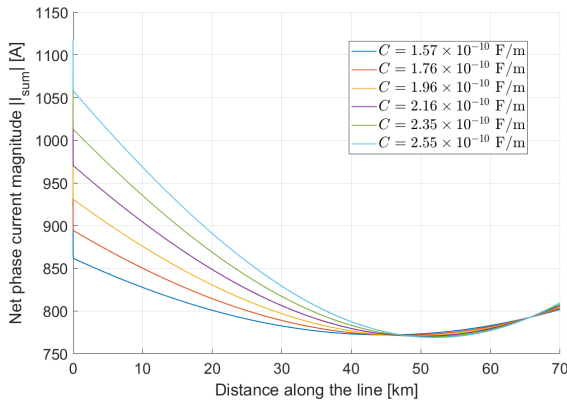


Figure 6.12: Combined current phasor magnitude along the length of the cable for different values of conductor capacitance C , keeping all other parameters constant.

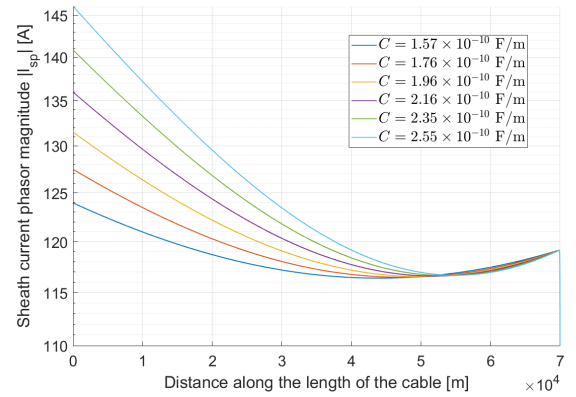


Figure 6.13: Sheath current phasor magnitude along the length of the cable for different values of conductor resistance C , keeping all other parameters constant.

Figure 6.14: Comparison of the impact of different values of phase capacitance on the current distribution along the length of the cable. Includes the combined (left) and metallic sheath value (right).

From Figure 6.14 it can be seen that both the combined phase current and the sheath current are strongly affected by a change in capacitance C . From the view of the conductor, this is logical, as the capacitance of submarine cables is known to have a large impact on the reactive power production of the cable and corresponding current distribution. However, it is usually compensated by using shunt reactors at one or both ends of the cable. Changing its value while keeping others constant therefore directly impacts the current. The linear relation between the sending end current and the capacitance is captured in the equation for charging current 4.26.

Interestingly, due to the use of a conductive connection to ground over the length of the cable, it could be expected that

the capacitance has no effect on the current in the metallic sheath due to dissipation. However, as the capacitance has such a strong effect on the conductor current, it has an indirect, but large, effect on the sheath current as well.

The Impact of Metallic Sheath Parameters on Phase Current

In section 5.6 it was shown that even though the metallic sheath can have a conductive interface with the ground (through potential cancellation between the other electrically phase shifted metallic sheaths), there are still circulating currents within it.

The parameters of the metallic sheath that can affect these currents are tested for their impact. Sheath inductance L_s dictates the induced electric field intensity, whereas the sheath resistance R_s impacts the impedance to induced currents. As can be expected from their linear impact on induced electric field and sheath impedance, both variables have a similar impact on the simulation results. However, there are some differences that must be discussed. Starting with the sheath inductance L_s , for which the results are depicted in Figure 6.16.

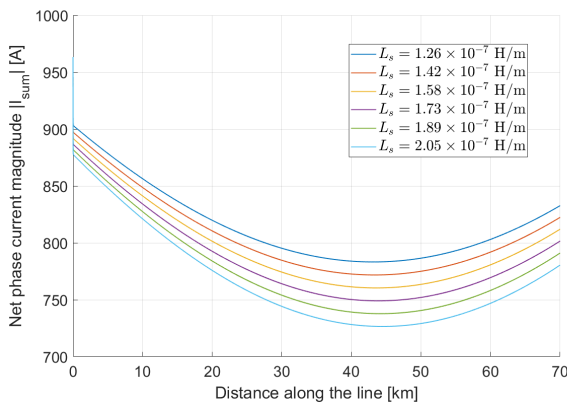


Figure 6.15: Combined current phasor magnitude along the length of the cable for different values of sheath inductance L_s , keeping all other parameters constant.

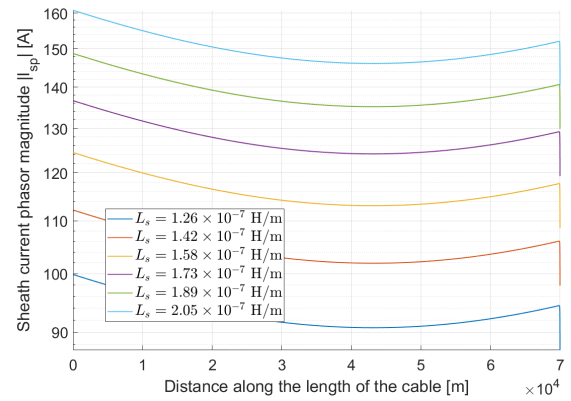


Figure 6.16: Sheath current phasor magnitude along the length of the cable for different values of sheath inductance L_s , keeping all other parameters constant.

Figure 6.17: Comparison of the impact of different values of phase capacitance on the current distribution along the length of the cable. Includes the combined (left) and metallic sheath value (right).

From Figure 6.17 it can be seen that the sheath inductance has a significant effect on the sheath current, which would be expected, as the induced voltage is the dominant driver of current in the sheath. However, as the figure indicates, there is a diminishing effect. This diminishing effect can be explained by the diminishing relative increase in induced voltage, when the steps are constant, as is the case here.

For the effect of sheath resistance R_s , the results are shown in Figure 6.20.

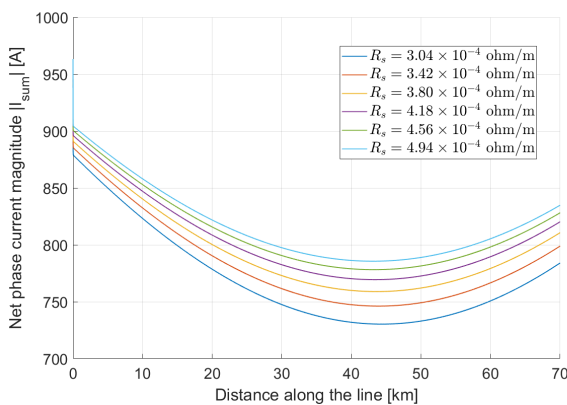


Figure 6.18: Combined current phasor magnitude along the length of the cable for different values of sheath resistance R_s , keeping all other parameters constant.

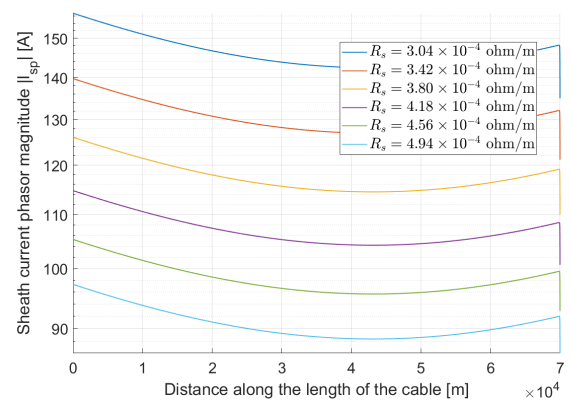


Figure 6.19: Sheath current phasor magnitude along the length of the cable for different values of sheath resistance R_s , keeping all other parameters constant.

Figure 6.20: Comparison of the impact of different values of phase capacitance on the current distribution along the cable length. Includes the combined (left) and metallic sheath value (right).

Figure 6.20 displays comparable results to the inductance simulation, but the impact is reversed. Here, an increase in the value of R_s increases the impedance, thereby decreasing the current. As for inductance, the relative value of the increase in R_s with each step diminishes as the steps are constant.

Conclusion on the Design Parameter Impact

It can be concluded that the cable design parameters have a significant impact on the current distribution along the length of the cable, with one exception.

Even though the phase resistance would result in power losses, heating and thereby reduced ampacity, it does not have an impact on the flow of current along the conductor or metallic sheath. Therefore it can be taken out of consideration when modeling EMFs.

Capacitance was shown to be the largest contributor to the current distribution along the length. Even though it does not directly alter the sheath current, the charging currents produced by capacitance are so large that they strongly contribute to the sheath current through the induced voltage.

Phase inductance, sheath inductance, and sheath resistance were shown to have a comparable magnitude of impact, although through different mechanisms. Phase inductance draws more current from the load side of the conductor, resulting in an increase in sheath current in the opposite direction at the load side, which has a compensatory effect on the net current. On the other hand, inductance and resistance were shown to have the effect of translating the sheath current curve in the vertical direction, thereby translating the magnitude of the combined current in the opposite direction vertically.

6.2.2. Impact Analysis of Operational Parameters on Phase Current

As we want to see the effect of usage parameters, the analysis is based on a realistic model of a modern HVAC power cable (see subsection 6.1.2). By systematically varying current loading, frequency, and imbalance within a detailed 3D model, we aim to assess their impacts on the system's electromagnetic fields. The table below details the parameters included in this analysis and their corresponding test ranges.

Table 6.5: Summary of Electromagnetic Operational Parameters and Their Test Ranges

Parameter	Description	Test Values
Load Current I_L	Output current dictates the current and voltage along the length	$I_L = 616.70, 704.80, 792.90, 881.00, 969.10, 1057.20, 1145.30$ A
Load Voltage V_L	Output voltage dictates the current and voltage along the length	$V_L = 156.69, 179.07, 201.46, 223.84, 246.23, 268.61, 290.99$ V
Reactive Power Compensation	Reactive power compensation has a large effect on the distribution of current along the length	$Q_1 = 0.00, 18.33, 36.67, 55.00, 73.33, 91.67, 110.00$ MVar $Q_2 = 0.00, 37.50, 75.00, 112.50, 150.00, 187.50, 225.00$ MVar

Due to the induced field being linearly related to the conductor current, the sheath current is also linearly related. This is because that is the only relevant component as charging current has no impact. Therefore it is only interesting to look at one of the distributions, noting that the sheath current will have a proportional magnitude in opposite direction, cancelling in the conduction current partially.

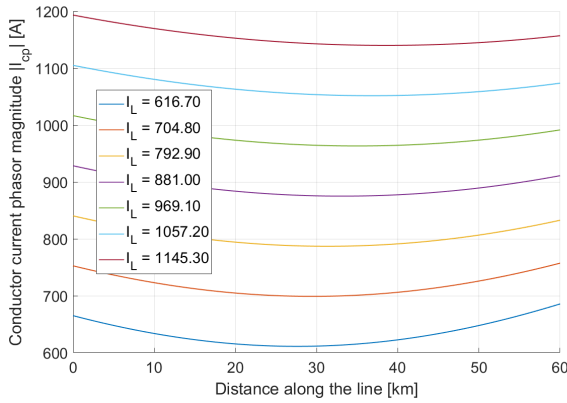


Figure 6.21: Conductor current distribution for different values of load current I_L .

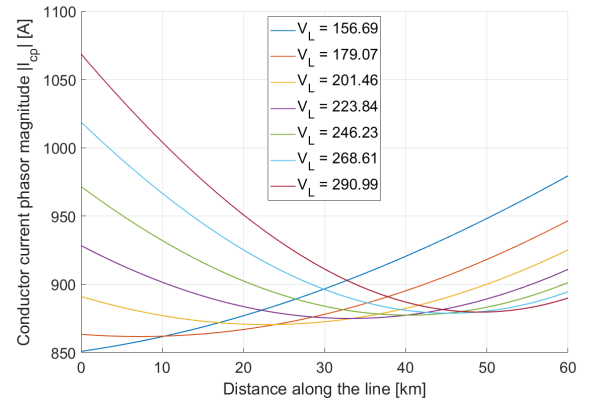


Figure 6.22: Conductor current distribution for different values of load voltage V_L .

Figure 6.23: Conductor current distribution comparison for different values of load voltage and current

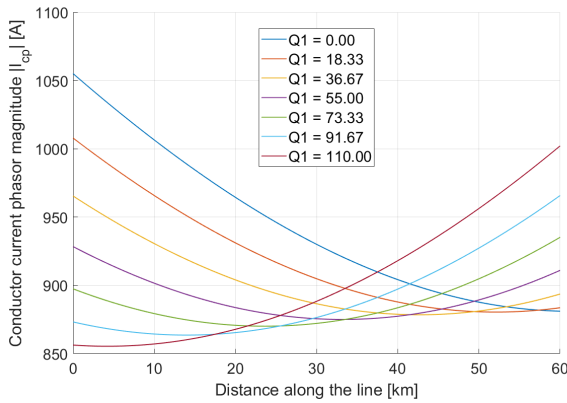


Figure 6.24: Conductor current distribution for different values of sending end reactive power compensation Q_1 .

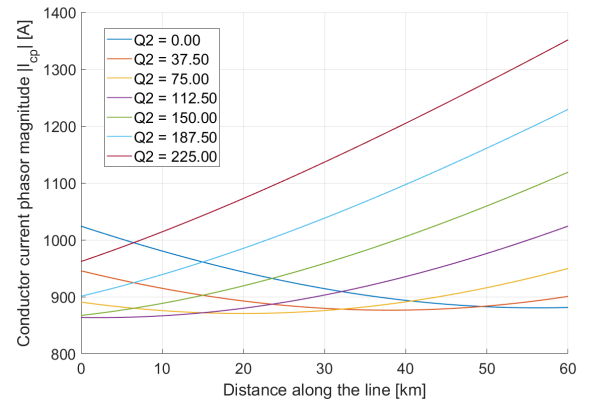


Figure 6.25: Conductor current distribution for different values of receiving end reactive power compensation Q_2 .

Figure 6.26: Conductor current distribution comparison for different values of reactive power compensation at sending and receiving end.

Most interesting here are the distributions for current and reactive power compensation, as those could actually be changed to values comparable in these ranges.

Think this is good to do for ranges of logical values, i.e. those that already exist in live cable systems or that can be expected to be used in the future.

6.3. The Impact of Design and Operational Parameters on EMF Emissions

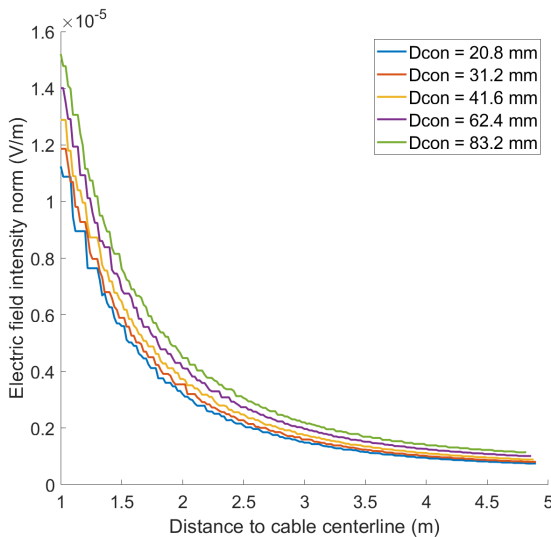
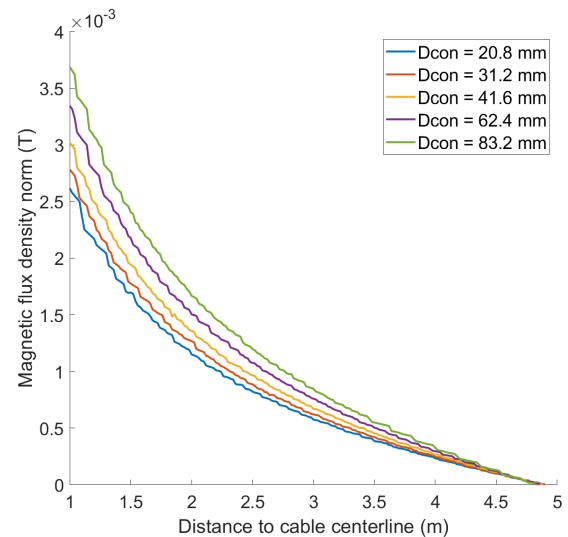
The design parameters are the things that are within the control of the industry and are, therefore, the most important to analyze. To keep it simple, the focus will be on the parameters of which an effect can be expected. Important parameters are those that directly impact the magnetic field (ferromagnetic) or indirectly (conductive materials). Ferromagnetic materials provide an easy path for the magnetic field, containing it within the cable, whereas conductive materials determine the flow of charge and, thereby, the resulting magnetic field. In Table 6.7, each parameter is described with its expected effects.

Table 6.7: Outline of the cable design parameters that will be modeled and the reasoning behind them.

Metallic Sheath	Reasoning
Thickness	Conductance, magnetic shielding, induction
Resistivity	Conductance, indirect magnetic shielding, induction
Conductor	Reasoning
Resistivity	Skin depth, current concentration
Diameter	Current concentration
Armor Wires	Reasoning
Number of wires	Magnetic shielding, induction
Resistivity	Induced currents
Permeability	Magnetic shielding
Diameter	Magnetic shielding, induced currents
Twist	Reasoning
Periods Modeled	Effect of periodic phenomena due to twist
Armor Lay Length	Induced currents, reluctance
Phase Lay Length	Interference with own EMF emissions

6.3.1. Impact of Conductor Design Parameters on EMF Emissions

The design parameters of the conductors in a submarine cable are essential for describing the voltage and current distribution along the length of the transmission line, but their impact on local emissions is not clear yet. Two conductor parameters that can be of interest. The conductor diameter will clearly have an effect on the data points used in this analysis, as it results in a larger total cable diameter and thereby increases the proximity of the 'measurement' to the conductors. However, it will also have the effect of conducting the current over a larger cross-section, which can affect the emissions' attenuation, as seen in Figure 6.29.

**Figure 6.27:** Magnetic flux density norm (magnitude) for different values of Dcon, keeping all other parameters constant.**Figure 6.28:** Electric field intensity norm (magnitude) for different values of Dcon, keeping all other parameters constant.**Figure 6.29:** Comparison of the impact of conductor diameter at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

From the magnetic field plot 6.27, it is most notable that the curves seem to converge after a certain distance. This is likely because, with distance, the assumption of an infinite conductor rod (section 3.3) becomes true.

The resistivity of the conductor will have an effect on the distribution of current throughout its cross-section. Therefore, is also likely to have an effect on the EMF emissions in the same manner as the conductor diameter. Simulation results for the range of R0cup values are shown in Figure 6.29.

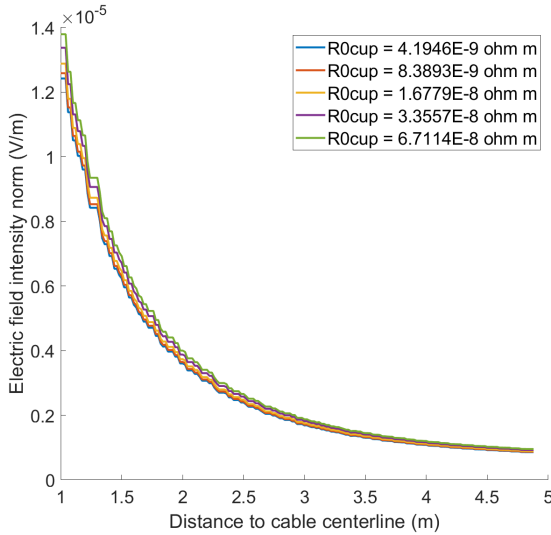


Figure 6.30: Magnetic flux density norm (magnitude) for different values of R_{0cup} , keeping all other parameters constant.

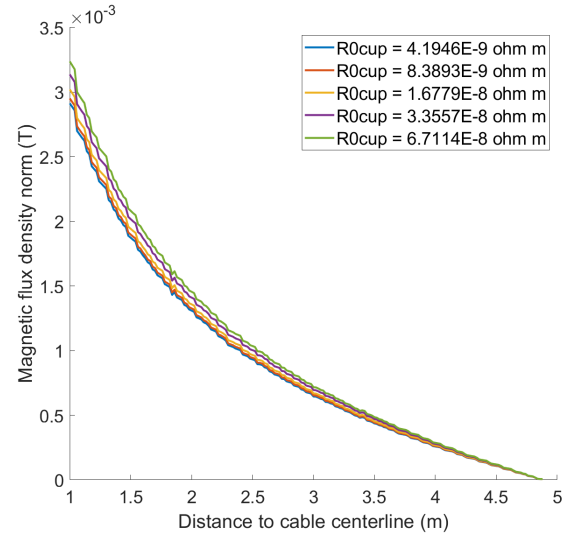


Figure 6.31: Electric field intensity norm (magnitude) for different values of R_{0cup} , keeping all other parameters constant.

Figure 6.32: Comparison of the impact of conductor resistivity at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

As for the conductor diameter, it can be seen from Figure 6.32 that there is an effect of changing the resistivity, but it is mainly in the area close to the cable. This is also explained by the effect of distance, making the three conductors seem more like infinitely thin rods.

6.3.2. Impact of Metallic Sheath Design Parameters

The metallic sheath is expected to have a significant shielding effect due to being relatively conductive and close to the conductor. Large induced currents will be created in the metallic sheath, which counteracts the conductors' EMF emissions. As lead is not a magnetic material, the shielding comes purely from the ability of the sheath to conduct current. The parameters that affect the conductance of the sheath are its cross-sectional area and its conductivity. The sheath (lead) thickness T_{pbs} determines the cross-sectional area and, keeping everything else constant, the insulation layer thickness T_{ins} . Because the standard model uses the linearized resistivity, the reference resistivity R_{0pbs} will be analyzed instead of the conductance.

First, the effect of the sheath thickness is tested using a parametric sweep, for which the results are shown in Figure 6.35.

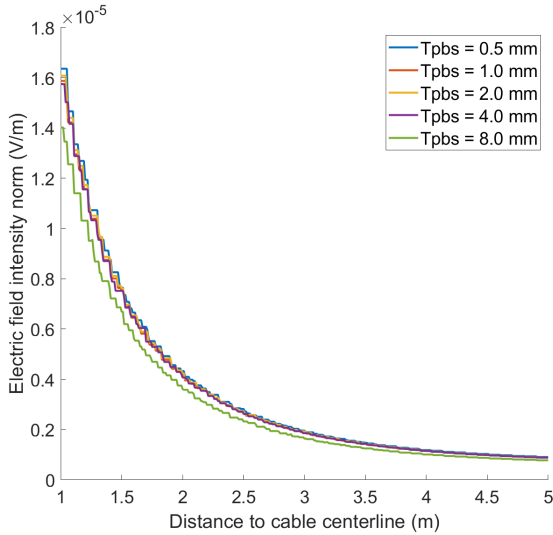


Figure 6.33: Magnetic flux density norm (magnitude) for different values of T_{pbs} , keeping all other parameters constant.

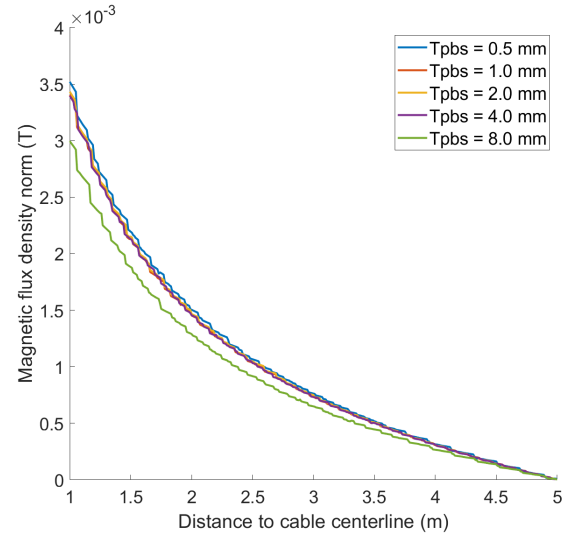


Figure 6.34: Electric field intensity norm (magnitude) for different values of T_{pbs} , keeping all other parameters constant.

Figure 6.35: Comparison of the impact of metallic sheath thickness at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

Figure 6.35 indicates a relatively small effect on the sheath thickness. For the relative amount of change compared to its standard value, the metallic sheath thickness has little effect on the outcome of the simulation. Even though the amount of material it constitutes is small, and the absolute effect per amount of material might be large, it is not so relevant for the total outcome. This should be held in mind when interpreting the effect of the insulation thickness.

Electric fields are most intense at small distances away from their source. Therefore, it can be expected that the shielding done by a metallic sheath would be strongest when the insulation thickness T_{ins} is small. However, the cross-sectional area does increase with T_{ins} . Figure 6.38 shows the electromagnetic field emissions from different values of insulation thickness.

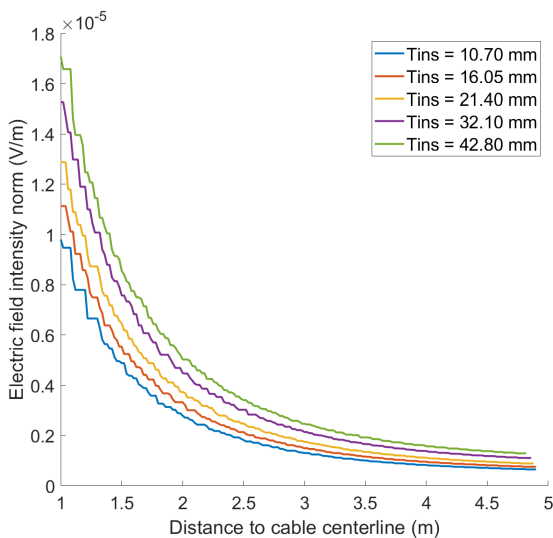


Figure 6.36: Magnetic flux density norm (magnitude) for different values of T_{ins} , keeping all other parameters constant.

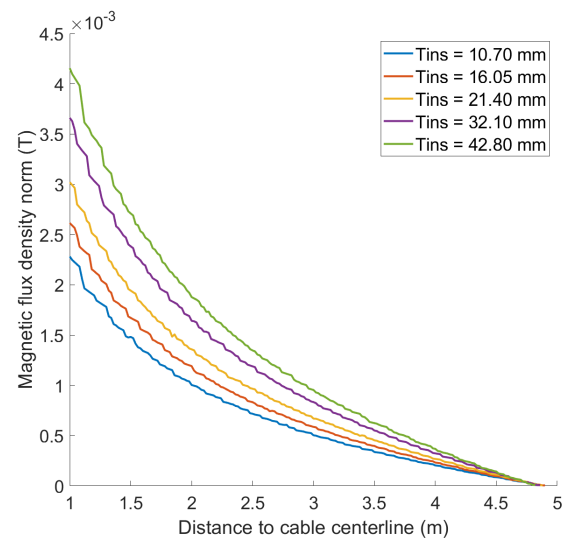


Figure 6.37: Electric field intensity norm (magnitude) for different values of T_{ins} , keeping all other parameters constant.

Figure 6.38: Comparison of the impact of insulation layer thickness at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

It becomes clear that insulation thickness also has a significant effect on the EMF emissions, which might, at first sight, be unexpected as the sheath thickness had little effect. However, this can be explained by the fact that increasing the

insulation thickness has an effect on the whole geometry sequence and results in the armor wires being further away from the conductors. Therefore, this effect is more likely to come from the armor than from the sheath.

Lastly, the resistivity of the sheath was tested, for which the results are displayed in Figure 6.41.

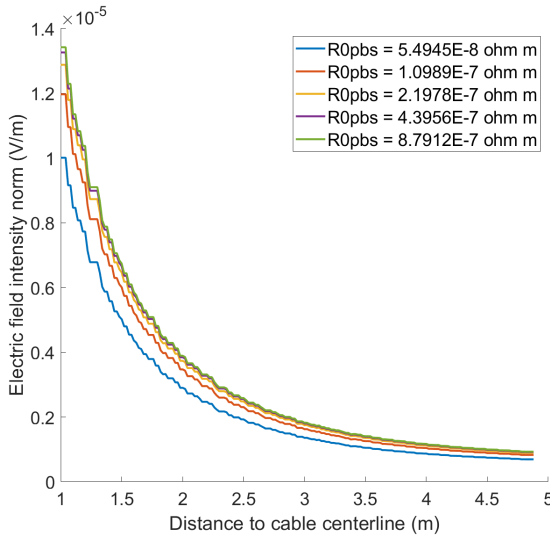


Figure 6.39: Magnetic flux density norm (magnitude) for different values of R_{0pbs} , keeping all other parameters constant.

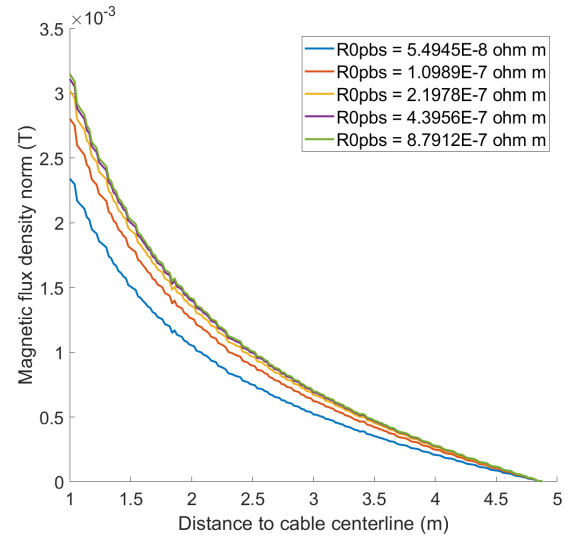


Figure 6.40: Electric field intensity norm (magnitude) for different values of R_{0pbs} , keeping all other parameters constant.

Figure 6.41: Comparison of the impact of metallic sheath resistivity at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

The results shown in Figure 6.41 are as expected, with no large changes in the emissions for most parameters. However, when the resistance becomes really low, the effect becomes more noticeable ($R_{0pbs} = 5.4945E-8$), which can be explained by the linear relationship between induced current and conductivity ($1/R_{0pbs}$).

For high levels of resistivity, there is barely any shielding. For the curves linked to $R_{0pbs} = 8.7912E-7$ and $R_{0pbs} = 4.3956E-7$, the electric field emissions are comparable and seem to approach a value resembling the absence of a sheath. However, lowering the resistivity by a certain factor (factor 2 in this case) shows an almost linear relation of the amount of shielding to that reduction factor of the resistivity. This is not very special on its own, but it strengthens the assumption that EMF emissions can be modeled accurately using the (linear) frequency compensation factor.

This parameter analysis of the metallic sheath makes it even more clear how critical it is in the analysis of electromagnetic fields. Parameters related to the metallic sheath dictate the conduction current (chapter 5) but on top of that, they counteract the EMF emissions associated with the conduction current.

6.3.3. Sensitivity to Armor Design Parameters

As discussed in subsection 4.1.5, the armor wires can have a significant effect on the electromagnetic field emissions of the cable. They are often magnetic (high μ_r), which creates a preferred path for magnetic field lines, and conductive, which can counteract electromagnetic field through induction.

To start with, two different armor wire designs must be compared. The simplest design is a constant armor pitch whereas the design used in the Borssele cable switches up the pattern by introducing fiber cables. Fiber cables break up the connection between multiple armor wires such that they do not form a magnetic core (subsection 4.1.5). This is beneficial in terms of losses, but it also means that more of the field can escape the cable. A comparison of the field emissions to the environment is shown in Figure 6.44.

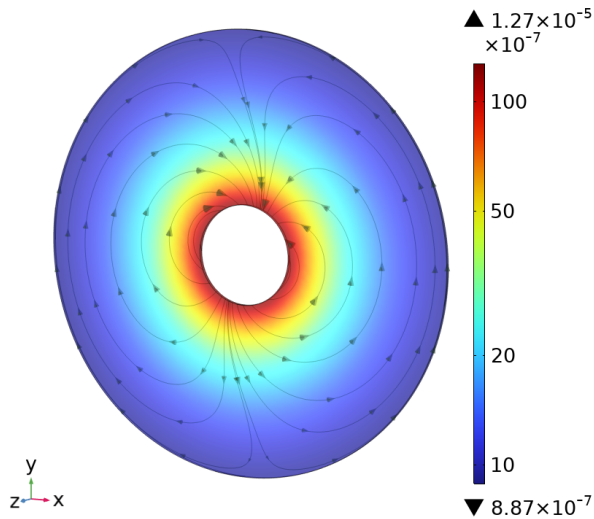


Figure 6.42: Magnetic field distribution with constant steel armor wires.

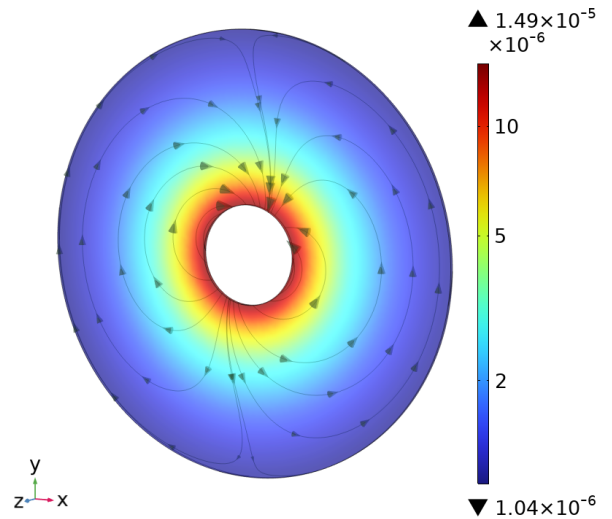


Figure 6.43: Magnetic field distribution with intermittent fiber armor wires.

Figure 6.44: Comparison between the magnetic fields emitted by constant steel armor wire pitch vs. intermittent fiber wires, showing the effect of the magnetic core created by the constant pitch.

It can be concluded that the use of intermittent fiber armor wires in the armor wires sequence increases the EMF emissions of the cable. This indicates the effect of a magnetic core being imitated by the armor wires in case they are closely connected. When connected, the armor forms a path of low reluctance, which concentrates the magnetic field within it, resulting in a shielding effect. This effect is also displayed in Figure 6.47, where the total amount of steel wires is changed while keeping everything else constant. This is done for the constant steel armor wire case.

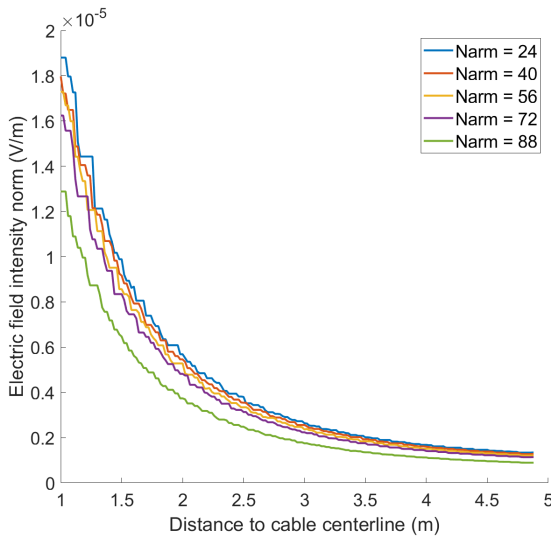


Figure 6.45: Magnetic flux density norm (magnitude) for different amounts of armor wires, keeping all other parameters constant.

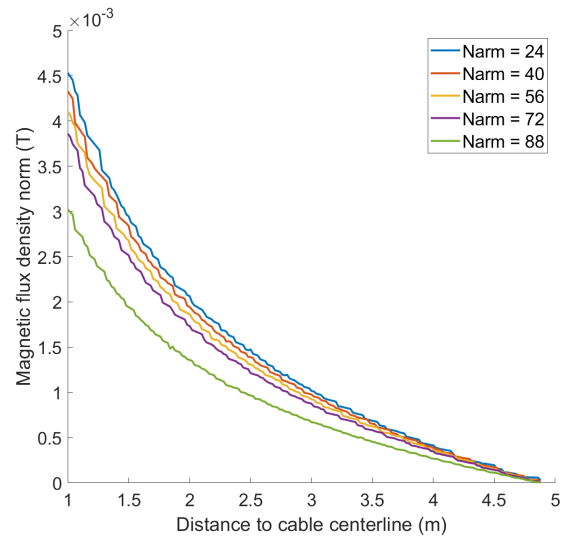


Figure 6.46: Electric field intensity norm (magnitude) for different amounts of armor wires, keeping all other parameters constant.

Figure 6.47: Sensitivity to the amount of armor wires at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

From Figure 6.47 it can be seen that there is a large impact of the amount of armor wires used. Particularly, the difference between $N_{arm} = 88$ and $N_{arm} = 72$ is interesting, as it is much larger than for the other steps. This is likely to be mainly due the change from an almost continuous path of steel along the armor, to the more discontinuous case. It can also be observed that even though there is no more magnetic core for lower values of N_{arm} , there is still a shielding effect. This is because the reluctance of the armor layer is still much lower than elsewhere, and because induction in the armor wires is still present, contributing to the shielding.

As indicated in Table 6.7 the thickness of individual armor wires is also relevant to the shielding. Thicker armor wires created a wider path of low reluctance. Thickness also reduces the resistance of the wires, such that induced currents are increased. The effect of the thickness T_{arm} is shown in Figure 6.50.

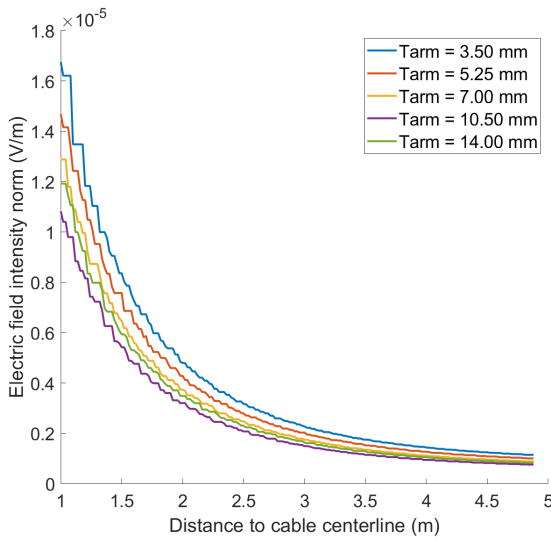


Figure 6.48: Magnetic flux density norm (magnitude) for different values of armor wire thickness, keeping all other parameters constant.

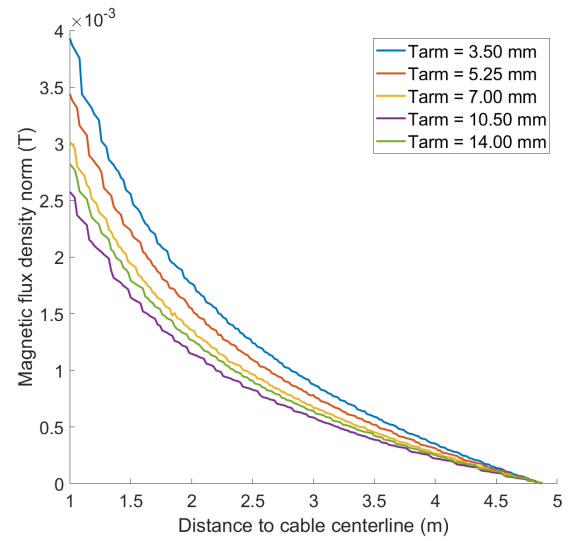


Figure 6.49: Electric field intensity norm (magnitude) for different values of armor wire thickness, keeping all other parameters constant.

Figure 6.50: Sensitivity to the armor wire thickness at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

From Figure 6.50, it can be seen that an increase in armor wire thickness can increase the amount of shielding, since for most of the curves, the field intensity is decreased with wire thickness. However, it seems there is a value of negative returns, as the field intensity is lowest for $T_{arm} = 10.5$ mm instead of for $T_{arm} = 14$ mm.

There are two possible reasons for this. First, it must be noted that the amount of armor wires is set to be dependent on the armor wires thickness (see Table 6.1). therefore, increasing the armor wire thickness, decreases the amount of wires and gaps between wires, and increases the size of the gaps. More, but smaller, gaps could be preferred over less, but larger, gaps. Another explanation is that the induced currents are lower, as the average distance of the steel in the armor wires to the conductors is larger, decreased the induced shielding effect.

The impact of the relative permeability μ_r of the armor wires is measured by simply applying a parametric sweep over a range of values. The range of values is chosen around the standard value for magnetic permeability of carbon-steel: $\mu_r = 100$.

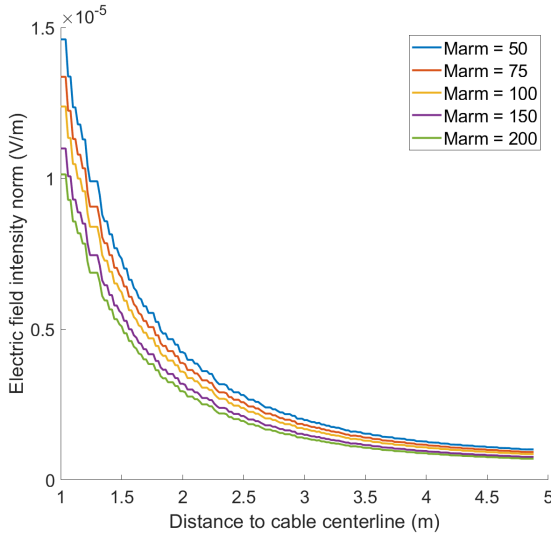


Figure 6.51: Magnetic flux density norm (magnitude) for different values of armor permeability, keeping all other parameters constant.

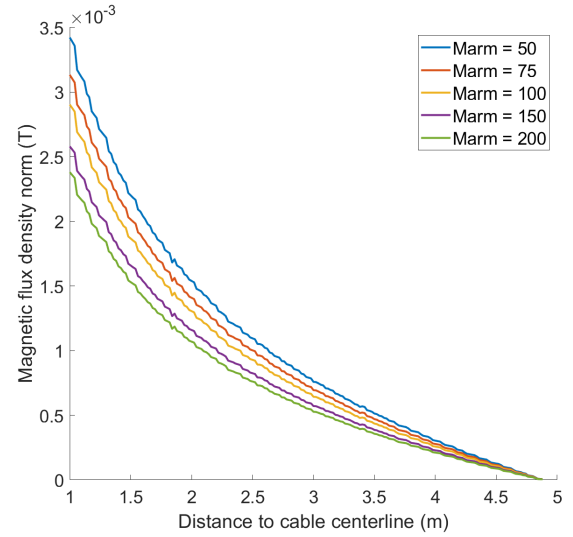


Figure 6.52: Electric field density norm (magnitude) for different values of armor permeability, keeping all other parameters constant.

Figure 6.53: Sensitivity to relative permeability of the armor at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

The results shown in Figure 6.53 are as can be expected from the magnetic permeability. An increase in permeability results in a decrease of the magnetic field emissions. What can be identified by looking at each set of equal steps of increased μ_r (Marm = 50, 75, 100 and Marm = 100, 150, 200), is that there seems to be a diminishing effect of shielding when increasing the permeability. This can be explained by the gaps between the armor wires, as the reluctance of the armor layer becomes limited by them. However, it is clear that permeability of the armor wires plays a very large role in the intensity of EMF emissions.

Next to providing a low reluctance path for magnetic field lines, the steel armor wires can have a shielding effect through induced currents. As the resistivity of the wires is limiting the induced currents, decreasing the resistivity will increase the induced currents and consequently the shielding of EMF emissions. The effect of changing armor wire reference resistivity R0arm is shown in Figure 6.56.

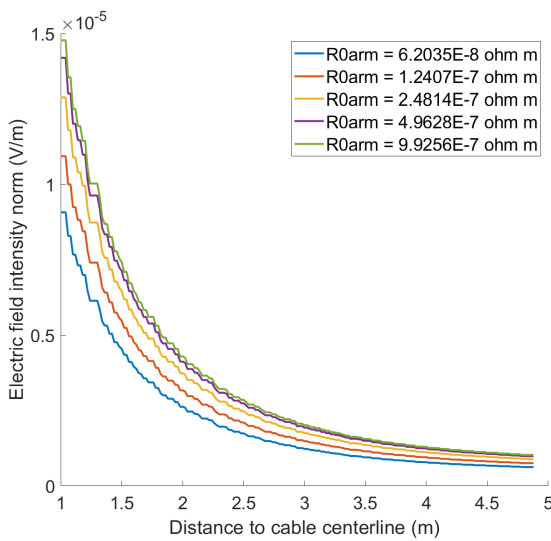


Figure 6.54: Magnetic flux density norm (magnitude) for different values of armor resistivity, keeping all other parameters constant.

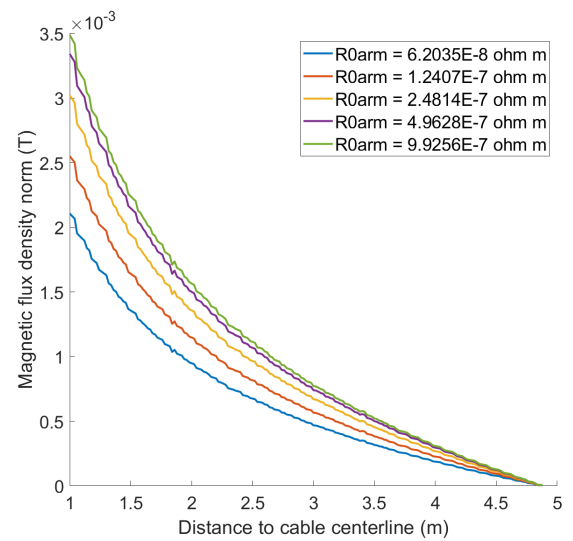


Figure 6.55: Electric field density norm (magnitude) for different values of armor resistivity, keeping all other parameters constant.

Figure 6.56: Sensitivity to armor resistivity at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

Figure 6.56 displays a large impact of the resistivity of the armor wires on the EMF emissions. The effect is most clearly depicted in Figure 6.54, where the magnetic flux density increases in equal steps when going from the lowest value $R_{0arm} = 6.20E-8 \Omega m$ to $R_{0arm} = 1.24E-7 \Omega m$ to $R_{0arm} = 2.48E-7 \Omega m$. This would indicate that for that range of values the magnetic shielding due to induction is logarithmically related to $\sigma = 1/R_{0arm}$, which is strengthened by the fact that the effect quickly drops for higher values of R_{0arm} .

It can be concluded that the design of the armor layer has a significant impact on the electromagnetic field emissions by a submarine power cable. The most eminent effect was seen due to the difference of having a densely populated armor layer, creating a path of low reluctance for the magnetic field, which results in high induced current in the wires and strong shielding of the field. Having a conductive (low resistivity) armor layer also creates a strong shielding effect as the induced currents are increased. The thickness and permeability of the armor wires are also clearly important, but have less extreme effects.

6.3.4. The Impact of Lay-Length

As discussed in subsection 4.1.1 and subsection 4.1.5, the lay-length of the phases LL_{pha} and armor LL_{arm} can have an impact on the electromagnetic field emissions. Because of the relation to these parameters, the impact of modeling different fractions of a full cross-pitch CP is analyzed here. The effect of these parameters is analysed using a parametric sweep of their values while keeping everything else constant.

The phase lay-length is expected to have the largest impact on the field emissions as it directly dictates the amount of destructive interference between the fields emitted by the different phases. The results are displayed in Figure 6.57.

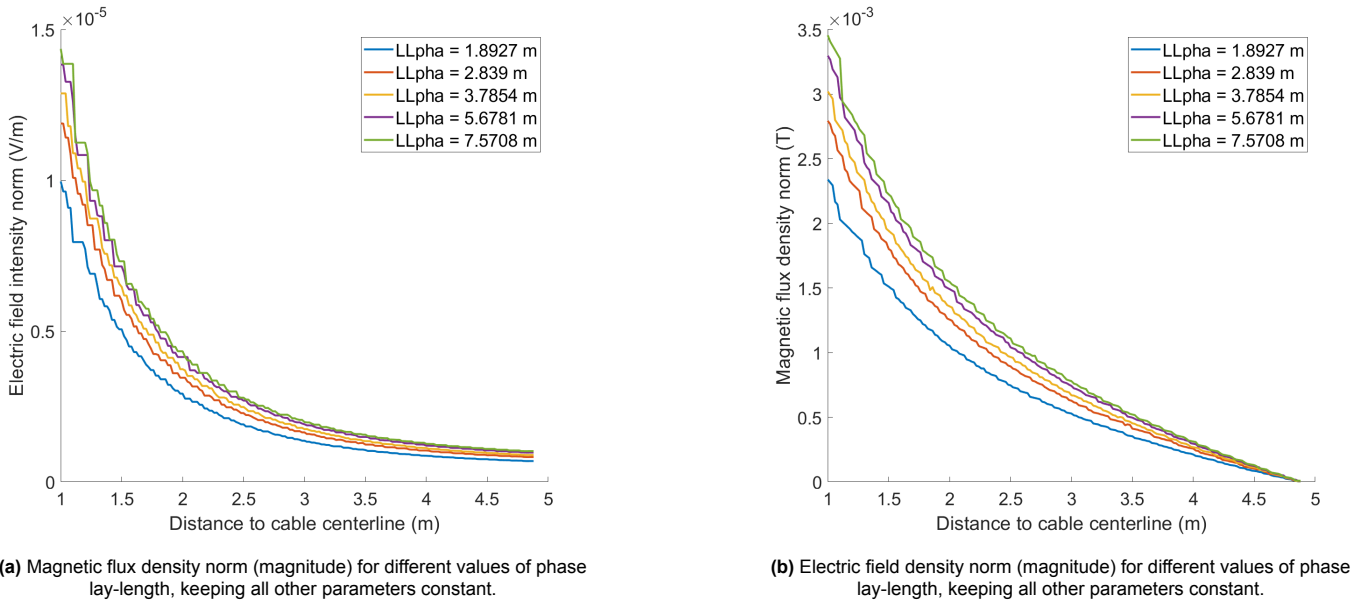


Figure 6.57: Sensitivity to phase lay-length at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

Figure 6.57 displays a clear impact of the phase lay-length. The shorter the lay-length of the phases, the stronger the lower the emitted field. This is likely due to the increase in destructive interference. Strengthening that argument is the fact that the simulation shows an increasing effect of lowering the lay length ($LL_{pha} = 2.839$ to $LL_{pha} = 1.8927$), which can be explained by the emissions of different phases overlapping increasingly.

The armor lay length impacts the amount of induced current in the wires and the reluctance of the armor layer due to the change in orientation of the wires and gaps between them. The results are shown in Figure 6.58.

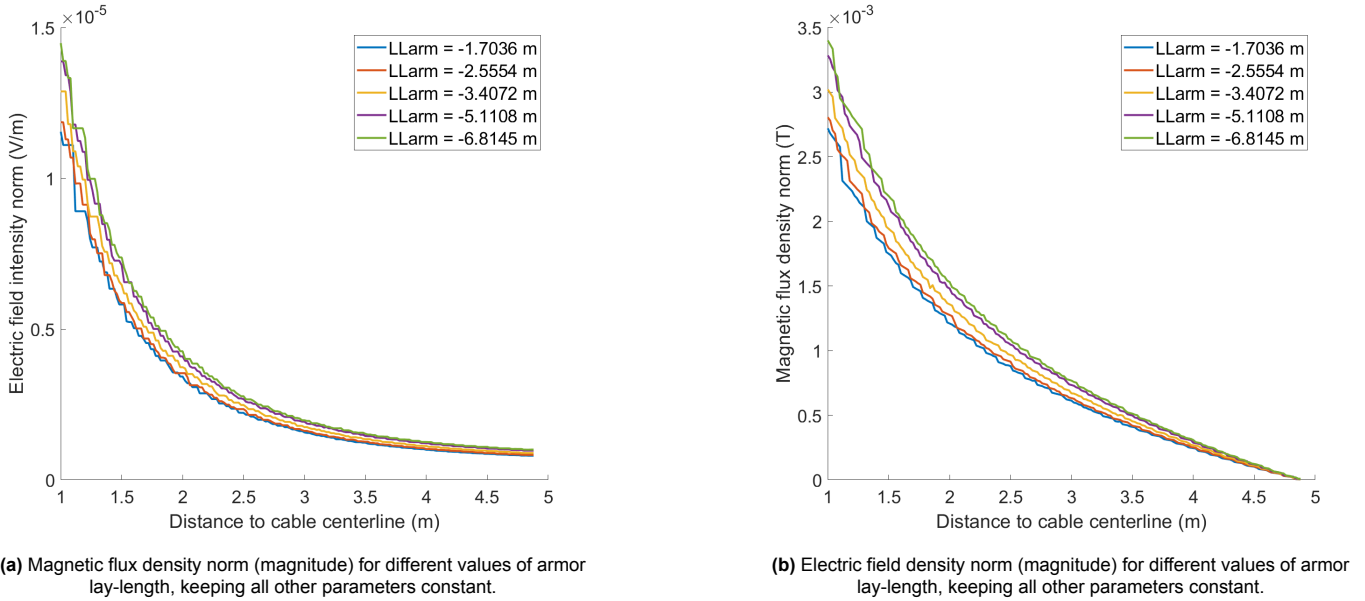


Figure 6.58: Sensitivity to armor lay-length at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

Although the effect of the lay length of the armor wires is much smaller than the effect of the phase lay length, its effect is still significant. The direction of this effect, the shorter lay length equals more shielding, is somewhat surprising. It could be expected from the view of induced current in the armor wires that the more the wires are aligned to the phases; the more shielding would occur (direction of induced electric fields subsection 4.3.2). On top of that, the length over which an armor wires meets the electromagnetic field of the phases is shorter, which should also create less currents. However, the simulation indicates the opposite. This could be the case due to the orientation that the armor wires have w.r.t. the magnetic field from the phases, when the lay length is shortened. A shorter lay length means the armor wires become more aligned with the magnetic field lines of the phases, which could create a slanted path of lower reluctance (indicated in COMSOL tutorial as well [40]).

As indicated in subsection 4.1.5 literature has stated that a model can be created using only the most minimal form of periodicity as the length of the section. For an armor layer of only steel wires, this shortest form of periodicity is $L_{sp} = CP_{cab}/N_{arm} = CP_{cab} \cdot N_{per}$, such that for the standard case in this model, $N_{per} = 1/88 = 0.011364$. In Figure 6.59 the effect of changing the number of short-periodicities is shown.

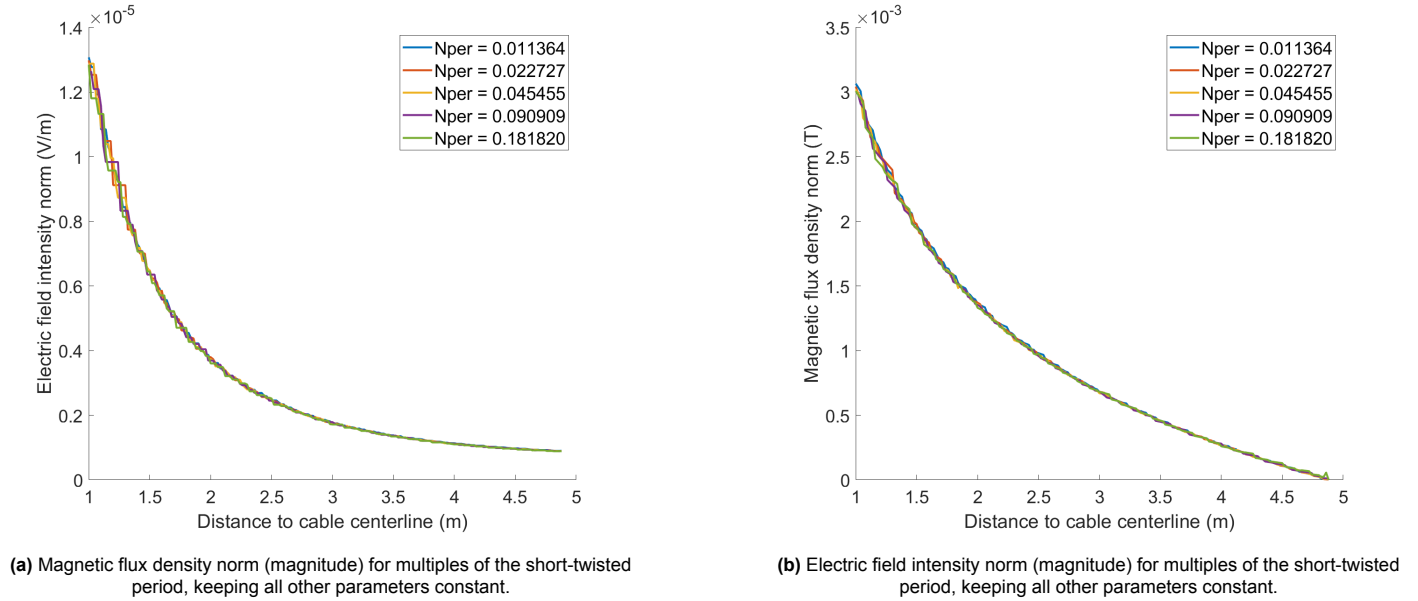


Figure 6.59: Sensitivity to the amount of short-twisted periods modeled at 500Hz. Frequency corrected using compensated conductance and a frequency factor in post-processing.

From this, it can be concluded that there is no visible difference for the different values of N_{per} , which concludes that this method (developed by Del-Pino-Lopez et al. [43]) can be extended to the external analysis of submarine power cables. This is a very convenient result, as a shorter length requires a smaller mesh, which greatly reduces that simulation time w.r.t. the analysis of a full cross pitch. As stated in Table 6.1.2 the parameter sweep operations used in this analysis took only 10 minutes, whereas earlier analyses to model the complete cross pitch took a full hour or more, with lower mesh density.

This part about the impact of lay length and section length yielded some useful results. It became clear that the lay length of the phases greatly impacted the EMF emissions, which has also been seen in independent simulations by TenneT. The lay length of armor also has a canceling impact, which is mainly due to the creation of a path of even lower reluctance for the magnetic field to concentrate in. Finally, it was verified that the short-twisted periodicity is valid by showing that an increase in length did not change the outcome of the simulations.

6.3.5. Results and Limitations of Design Parameter Impact Analysis

From the impact analysis of the electromagnetic field emissions to the selected design parameters, it became clear that each of them is relevant to consider when modeling. Therefore it can be concluded that much care has to be taken to use realistic values for each of these parameters. However, it should also be noted that the parameter ranges used in the analysis often spanned beyond their realistic values. This was done such that an insight into the extremes could be gained. To expand on the insight in this section, an analysis of the relative change in percentages was done, which is summarized in Table 6.8. The percentages in the table are normalized with respect to the corresponding percentage change in parameter value.

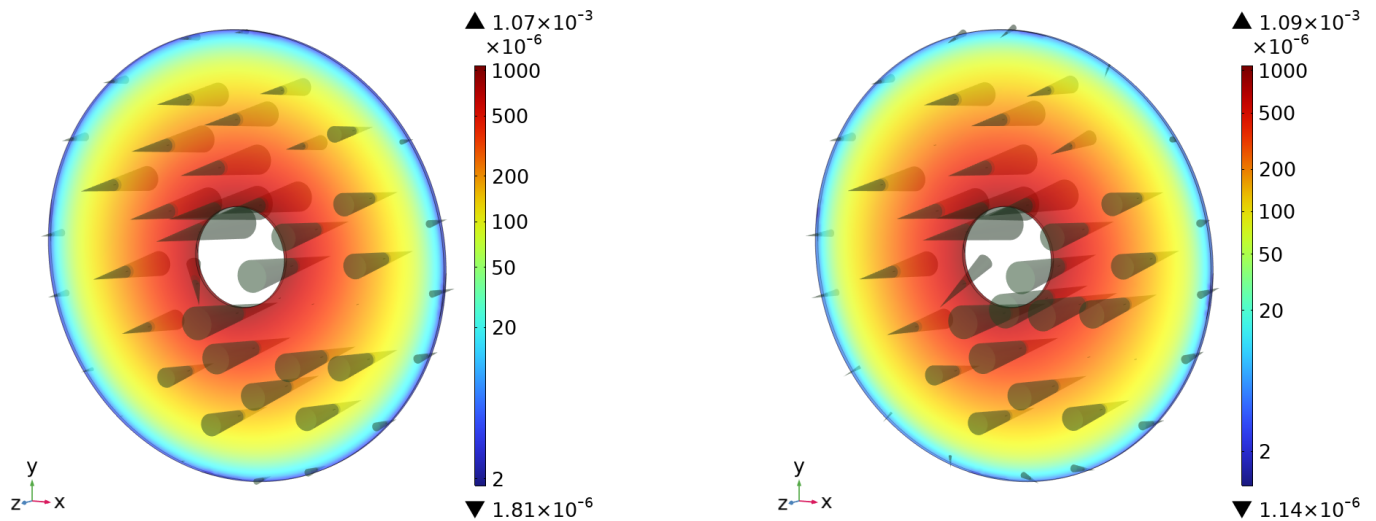
Table 6.8: Normalized Percentage Changes for M-Field and E-Field

Parameter Swept	Standard	Parameter Values	Normalized Percentage Change M-Field	Normalized Percentage Change E-Field
LLarm	-3.4072	-1.7036, -2.5554, -5.1108, -6.8145	0.214, 0.298, 0.185, 0.136	0.202, 0.292, 0.171, 0.124
Marm	100	50, 75, 150, 200	0.358, 0.317, 0.224, 0.181	0.359, 0.317, 0.224, 0.181
Narm	56	24, 40, 72, 88	0.857, 0.894, 1.040, 1.599	0.689, 0.789, 0.981, 1.510
Nper	0.045455	0.011364, 0.022727, 0.090909, 0.181820	0.036, 0.045, 0.036, 0.029	0.020, 0.026, 0.015, 0.006
R0arm	2.4814E-7	6.2035E-8, 1.2407E-7, 4.9628E-7, 9.9256E-7	0.402, 0.309, 0.105, 0.051	0.399, 0.307, 0.104, 0.050
R0cup	1.6779E-8	4.1946E-9, 8.3893E-9, 3.3557E-8, 6.7114E-8	0.048, 0.046, 0.039, 0.024	0.048, 0.046, 0.038, 0.024
R0pbs	2.1978E-7	5.4945E-8, 1.0989E-7, 4.3956E-7, 8.7912E-7	0.299, 0.142, 0.030, 0.014	0.298, 0.142, 0.030, 0.014
Tarm	7.00	3.50, 5.25, 10.50, 14.00	0.596, 0.563, 0.292, 0.072	0.587, 0.544, 0.294, 0.060
Tins	21.40	10.70, 16.05, 32.10, 42.80	0.489, 0.556, 0.415, 0.366	0.507, 0.557, 0.443, 0.400
l0	1286.9	321.73, 643.47, 1930.4, 2573.9	0.667, 0.500, 1.000, 1.000	0.667, 0.500, 1.000, 1.000
LLpha	3.7854	1.8927, 2.839, 5.6781, 7.5708	0.460, 0.316, 0.201, 0.148	0.454, 0.305, 0.178, 0.137
Dcon	41.6	20.8, 31.2, 62.4, 83.2	0.281, 0.350, 0.219, 0.223	0.288, 0.335, 0.237, 0.244
Tpbs	2.0	0.05, 1.0, 4.0, 8.0	0.045, 0.020, 0.014, 0.044	0.024, 0.023, 0.017, 0.042

From Table 6.8, it can be seen that almost all of the parameters analyzed in this section can be considered essential, meaning that their normalized percentage impact is $\geq 0.1\%$, except for the number of minimum periodic lengths modeled (Nper) and surprisingly enough for the sheath thickness (Tpbs). It must be noted that the per However, it should be noted that this analysis had a limitation of size. The length of the section that was analyzed is small compared to the length of a real cable, which can have an impact on the results of $\pm 5\%$ as described in subsection 4.1.5. The environment that could be modeled was also limited, because COMSOL is not able to handle the size differences in geometry that arise when the environment becomes larger.

6.4. The Impact of Animal Movement on Perceived E-Field

This thesis is essentially aimed at helping ecological researchers determine the electromagnetic fields from the perception of marine animals. section 3.2 introduced the concept of movement-induced electric fields, which is an aspect that has not been considered in the models used. Since there has been some concern raised by experts ([4]), the impact of this will be explained here. A comparison between the electric field experienced by a static object in the environment and the electric field experienced by a moving object in the environment is shown in Figure 6.60. Here Figure 6.60a shows the static case, whereas Figure 6.60b displays the result of applying a velocity in the z-direction: $V_z = 5$ m/s (a value deemed reasonable for marine animals). The combined field is calculated by adding the cross product of the velocity with the local magnetic flux: $E_{ind} = V_z \times B$.



(a) Volume plot of the electric field in the near environment for $V_z = 0$ m/s.

(b) Volume plot of the electric field in the near environment for $V_z = 5$ m/s.

Figure 6.60: Comparison of the electric field as experienced by a non-moving fish vs. a fish moving along the cable at $V_z = 5$ m/s.

This comparison yields interesting insights. Looking at the range of values that the electric field has in the environment shows that the maximum value is almost unaffected, whereas the minimum value is 1.5 times as large as the static case. The effect on the minimum field is most interesting because it could indicate that electric fields can be perceived at larger distances. The reason for this difference in impact between the near vicinity of the cable and near the edge of the simulated environment could be explained by the difference in attenuation between the magnetic flux density and electric field intensity.

6.5. Conclusion of The Parameter Analysis

The aim of this chapter was to establish an order of importance for the parameters that are included in electromagnetic field modeling. As was already understood from chapter 5, modeling of electromagnetic fields requires not only a perfect COMSOL model but also insight from the operation of the cable and the current distribution following from it. Depending on the operational factors, the current and EMF emissions can be vastly different. In addition, the EMF emissions greatly depend on the cable's electromagnetic properties and dimensions.

To gain an understanding of how to analyze these phenomena, an outline of a simple method for modeling was made. This method included the calculation of sheath current and voltage distributions, as well as instructions for modeling in COMSOL.

From the impact analysis of operational and transmission line parameters on the current distribution along the cable, it became clear that the location along the transmission line greatly impacts the net current in the cable. Phase capacitance and inductance impact the distribution of conductor current, as well as the sheath current. However, the effect of capacitive currents is only visible for cables with an insulating outer sheath. Induced currents are independent of the conductance of the outer sheath and are, therefore, only related to the sheath inductance, current loading, and sheath impedance. When considering the inputs for an EMF model in COMSOL, one should use the correct operational settings and transmission line parameters to predict the current at the location of interest accurately.

The impact analysis of the EMF emissions due to the different electromagnetic parameters verified the importance of the chosen parameters. Except for the metallic sheath thickness, all design parameters showed to have a great impact on the outcome of the EMF simulations. However, due to constraints of the mesh flexibility, the impact of the conductor diameter and insulation thickness are still questionable, as their effect could not be isolated from their change in the overall geometry. It was shown that many aspects of the armor layer significantly affect the EMF emissions due to the large conductance and low reluctance. Interestingly, the abrupt increase in shielding when the armor wires form a dense sequence indicates that they might act like a magnetic core.

Lastly, a means of using an ultra-shortened section length ($L_{sp} = CP/N_{arm}$) for modeling electromagnetic field emissions was verified. Multiples of this shortened length were tested, and no significant difference was seen between them. This is a great result as it decreases the simulation time needed from around an hour for a single simulation to only 10 minutes for a parametric sweep.

Conclusion

This thesis aimed to develop guidelines on how to model electromagnetic fields (EMF) for submarine power cable systems to aid in-depth research into their ecological effects. To address the main research question—*what are the most effective parameters to include in electromagnetic emission models to aid research into the ecological effects of submarine power transmission?*—the research was broken down into several subquestions, each tackled in different chapters.

7.1. Summary of Findings

7.1.1. Requirements for an EMF-emission model

What are the requirements for an EMF-emission model that can be utilized in ecological research? The necessary criteria for an EMF-emission model suitable for ecological research were established, focusing on induced electric fields, alternating magnetic fields, and movement-induced fields. These criteria were derived from understanding the potential impact of EMF on marine species, particularly Elasmobranchii. The thresholds identified are:

- Induced electric fields ≥ 500 nV/m (5 nV/cm).
- Alternating magnetic fields of ≥ 0.8 μ T (NOEL) for local dwelling behavior.
- Alternating magnetic fields of ≥ 8 μ T (LOEL) for local dwelling behavior.
- Alternating magnetic fields of ≥ 0.4 μ T (NOEL) for disturbance during embryogenic development.

These criteria led to the conclusion that a minimal magnetic field emission model should be able to predict magnetic fields as small as 0.4 μ T and an induced electric field emission as small as 0.5 μ V/m. As the rest of this thesis is focused on creating a best-effort methodology for EMF emissions, this is a standalone conclusion to take into account when attempting to simulate a specific situation for ecological research.

7.1.2. Prediction of Net Current

How can the current at a location along the length of a submarine power cable be predicted based on its design- and usage parameters? This was the most important question of this thesis, as the modern submarine cable design concept that uses a conductive layer around the metallic sheath was introduced, for which no calculation methods were available. Two methods for predicting the net current along the length of a submarine power cable based on design and usage parameters were constructed for the main conductors and for the metallic sheath. The first method, which is an implementation of transmission line theory, can be used to predict the conductor voltage and current distribution, which was verified by comparing its predictions to PowerFactory simulations by TenneT.

A new implementation of the transmission line method was created with the aim of predicting current in the metallic sheath. However, due to time limitations, the implementation of this method has not been verified yet. For this reason, a different method of approximation using multiple junctions was created. The effectiveness of this method cannot be tested due to the lack of data on this topic, but its simplicity allows for easy mathematical verification.

Using these methods, it was demonstrated that current distribution in a single phase is a complex superposition of conductor current and metallic sheath current but can be calculated using a set of basic design- and operational parameters:

- Required voltage (V) and current (I) at the sending or receiving end directly impact all electromagnetic interactions within the cable.
- Phase resistance (R), impacts the flow of current and the losses along the length.
- Phase capacitance (C), impacts the amount of current lost to polarization of the insulation layer along the length and the phase angle ϕ between conduction voltage and current.

- Phase inductance (L), impacts the flow of current and the power factor, and the phase angle ϕ between conduction voltage and current.
- Reactive power compensation at sending and/or receiving end (Q_1, Q_2), impacts the voltage and current distribution and compensates for ϕ .
- Sheath resistance (R_s), impacts the flow of induced currents along its length.
- Sheath inductance (L_s), impacts the production and flow of induced currents along its length.
- Inter-sheath conductance (G), impacts the dissipation of charging currents between metallic sheaths.

Most of these parameters can be determined using existing methods based on geometric and material characteristics from datasheets (R, C, L, R_s, L_s), or the operational conditions of the cable (V, I, Q_1, Q_2). However, simple calculation methods were developed as the conductance between metallic sheaths is a new phenomenon in literature. Each of these parameters is essential in describing the current along the cable, as was proven by using a parameter analysis and the EMF emissions resulting from it.

7.1.3. Important Parameters in COMSOL Multiphysics Models

What are the most important parameters to include in a submarine power cable model in COMSOL Multiphysics by means of their impact on electromagnetic field emissions? Critical parameters for inclusion in a COMSOL Multiphysics model of submarine power cables were identified based on their impact on the essential electromagnetic parameters. The impact of these parameters was tested by use of a parametric sweep simulation in COMSOL, which resulted in the following list of essential EMF modeling parameters.

- Conduction current, which is the main contributor to the EMF emissions.
- Conductor geometry and resistivity impact the current concentration.
- Metallic sheath geometry and resistivity, impacting the level of inductive shielding.
- Armor wire geometry, spacing, and permeability impact the reluctance of the armor layer, and resistivity impacts the level of inductive shielding.
- Phase lay-length, impacting the destructive interference between electrically phase-shifted EMFs.
- Armor lay-length, impacting the reluctance of the armor layer.

Due to the large impact of changing these parameters on the outcome of simulations, they are all essential parameters to consider when attempting to create an accurate model of EMF emissions.

7.1.4. EMF Prediction of the Borssele System

This is something left to do if there is time By modeling the Borssele system with essential parameters, the thesis verified the accuracy and significance of these parameters. The study confirmed that omitting any parameter that increases error beyond $\pm 1\%$ could lead to significant inaccuracies in EMF predictions. The findings underscore the importance of comprehensive modeling to accurately predict EMF emissions.

7.2. Comprehensive Conclusion

The research effectively answered the main research question by systematically addressing the subquestions through detailed analysis and modeling. The thesis established that a comprehensive EMF model for submarine power cables must include a range of parameters that affect both conductor and sheath currents and the electromagnetic interactions within and between the cable components.

Key parameters such as phase resistance, capacitance, inductance, sheath resistance, and inductance, along with operational settings, significantly influence the outcome of EMF models. The development and verification of methodologies for current distribution prediction and EMF modeling in COMSOL provided robust tools for future ecological impact assessments of submarine power cables.

The findings emphasize the need for considering both the electrical and geometric properties of cables, as well as operational factors, in modeling efforts. This research contributes valuable insights and methodologies that can be adapted for further studies, enhancing the understanding of EMF emissions and their ecological effects.

Recommendations for Further Research

Verification of Current Flow Prediction Method

Further research should focus on verifying the current flow prediction method developed in this thesis. Both current prediction methodologies indicate that induced circulating currents are unaffected, even when the metallic sheath is conducting. The transmission line method even suggests high voltages, which seems unlikely and warrants further investigation. This verification can be done through a simple test setup, involving the connection of multiple small conductor sections that are phase-shifted together through a conductive medium, and measuring the current at the end of the section. This will help in gaining a better understanding of the exact flow of the leakage current that crosses the metallic sheath.

Larger Scale Models

Further research should also employ larger scale models. The models used in this parameter analysis had a 5 m radius and could not fully represent the region that could affect marine animals. To address this, a means of dealing with the differences in scale must be found. In this thesis, creating a region of more than 6m radius around the cable resulted in large distortions in cable geometry. Finding a solution to this issue is crucial for accurate modeling of larger regions.

Cable Design and Temperature Management

There is ambiguity regarding cable manufacturers' design processes, particularly concerning the assumption of no circulating currents. If this assumption is indeed made, there is a need for additional work on temperature management calculations. It is also unclear if these calculations consider the fact that part of the submarine cable is placed on land, where water intrusion might not occur. For cables placed at relative heights (e.g., in dunes), there might be no water at all, leading to dry sections that might experience more circulating current and thereby increased heat production. This aspect requires further exploration to ensure accurate temperature management in varying environmental conditions.

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A

Appendix A: The Transmission Line Equations

A.0.1. General Transmission Line Equations

Note that subsection 5.2.1-subsection A.0.2 are a condensed quote from the book [34]. The analysis begins with describing the mechanism behind transmission line analysis. It is based on a schematic, from which difference equations are deduced. These difference equations are then mathematically adjusted to produce the **telegrapher's equations** as well as the **general wave equations** for current and voltage within a transmission line. To achieve this, we begin by constructing a circuit model for an incremental segment of the transmission line (Figure A.1). We include in our model the primary constants per unit length: inductance (L), capacitance (C), shunt conductance (G), and series resistance (R). These constants are essential, as they characterize the physical properties of the transmission line:

- The series resistance (R) accounts for losses due to the finite conductivity σ_c of the conductors, commonly represented by ρ_c (Ω/m).
- The shunt conductance (G) models the leakage currents through the dielectric, commonly represented by σ_d (S/m).
- The series inductance L (H/m) models the phase delay due to the self inductance of the unit length.
- The shunt capacitance C (F/m) models the leakage currents through the dielectric to polarize it and allowing for transmission.

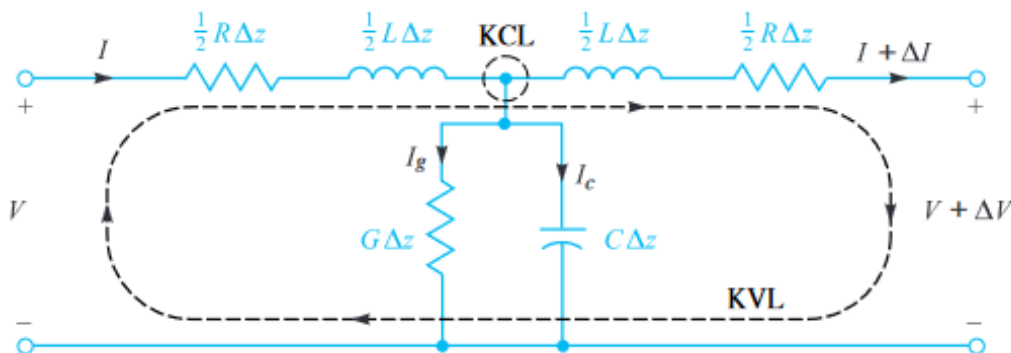


Figure A.1: Transmission General Schematic

In the model, we analyze a section of the length line Δz , consisting of resistance $R\Delta z$, inductance $L\Delta z$, conductance $G\Delta z$, and capacitance $C\Delta z$. To maintain symmetry in the network, these elements are divided equally throughout the segment. This configuration could alternatively be rearranged with half the conductance and capacitance positioned at each end of the segment.

Our goal is to understand how the output voltage and current differ from their input counterparts as Δz approaches a very small value. By applying Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL), we derive a pair of differential equations. These equations express the changes in voltage and current with respect to the longitudinal

coordinate z . As depicted in Figure A.1, the input and output voltages and currents differ by incremental amounts ΔV and ΔI , which are determined through successive applications of KVL and KCL. These differential equations ultimately govern the propagation of signals along the transmission line.

First, Kirchhoff's Voltage Law (KVL) is applied to the loop that encompasses the entire section length, as shown in Figure A.1:

$$V = \frac{1}{2}RI\Delta z + \frac{1}{2}L\frac{\partial I}{\partial t}\Delta z + \frac{1}{2}L\left(\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t}\right)\Delta z + \frac{1}{2}R(I + \Delta I)\Delta z + (V + \Delta V) \quad (\text{A.1})$$

We can solve this equation for the ratio, $\frac{\Delta V}{\Delta z}$, obtaining:

$$\frac{\Delta V}{\Delta z} = -\left(RI + L\frac{\partial I}{\partial t} + \frac{1}{2}L\frac{\partial \Delta I}{\partial t} + \frac{1}{2}R\Delta I\right) \quad (\text{A.2})$$

Next, we introduce the following relationships:

$$\Delta I = \frac{\partial I}{\partial z}\Delta z \quad \text{and} \quad \Delta V = \frac{\partial V}{\partial z}\Delta z \quad (\text{A.3})$$

which are then substituted into the previous equation to yield:

$$\frac{\partial V}{\partial z} = -\left(1 + \frac{\Delta z}{2}\frac{\partial}{\partial z}\right)\left(RI + L\frac{\partial I}{\partial t}\right) \quad (\text{A.4})$$

In the limit as Δz approaches zero (or a value small enough to be negligible), this equation simplifies to its final form:

$$\frac{\partial V}{\partial z} = -\left(RI + L\frac{\partial I}{\partial t}\right) \quad (\text{A.5})$$

Equation A.5 is the first of the two equations we are seeking. To find the second equation, we apply Kirchhoff's Current Law (KCL) to the upper central node in the circuit of Figure A.1, noting from the symmetry that the voltage at the node will be $V + \frac{\Delta V}{2}$:

$$I = I_g + I_c + (I + \Delta I) = G\Delta z\left(V + \frac{\Delta V}{2}\right) + C\Delta z\frac{\partial}{\partial t}\left(V + \frac{\Delta V}{2}\right) + (I + \Delta I) \quad (\text{A.6})$$

Then, using Equation A.3 and simplifying, we obtain

$$\frac{\partial I}{\partial z} = -\left(1 + \frac{\Delta z}{2}\frac{\partial}{\partial z}\right)\left(GV + C\frac{\partial V}{\partial t}\right) \quad (\text{A.7})$$

Again, we obtain the final form by allowing Δz to be reduced to a negligible magnitude. The result is as follows.

$$\frac{\partial I}{\partial z} = -\left(GV + C\frac{\partial V}{\partial t}\right) \quad (\text{A.8})$$

Coupled differential equations A.5 and A.8 describe the evolution of current and voltage in any transmission line. Historically, they have been referred to as the telegraphist's equations. Their solution leads to the wave equation for the transmission line, which we now undertake. We begin by differentiating Equation A.5 with respect to z and Equation A.8 with respect to t , obtaining the following:

$$\frac{\partial^2 V}{\partial z^2} = -R\frac{\partial I}{\partial z} - L\frac{\partial^2 I}{\partial t \partial z} \quad (\text{A.9})$$

and

$$\frac{\partial I}{\partial z \partial t} = -G\frac{\partial V}{\partial t} - C\frac{\partial^2 V}{\partial t^2} \quad (\text{A.10})$$

Next, Equation A.8 and A.10 are substituted into Equation A.9. After rearranging terms, the result is:

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} + (LG + RC)\frac{\partial V}{\partial t} + RGV \quad (\text{A.11})$$

An analogous procedure involves differentiating Equation A.5 with respect to t and Equation A.8 with respect to z . Then, Equation A.5 and its derivative are substituted into the derivative of Equation A.8 to obtain an equation for the current that is in identical form to that of Equation A.11:

$$\frac{\partial^2 I}{\partial z^2} = LC \frac{\partial^2 I}{\partial t^2} + (LG + RC) \frac{\partial I}{\partial t} + RGI \quad (\text{A.12})$$

Equations A.11 and A.12 are the general wave equations for the transmission line. Their solutions under various conditions form the major part of our study.

A.0.2. Lossless Propagation of Sinusoidal Waves

Understanding sinusoidal waves on transmission lines is crucial because any transmitted signal can be broken down into a series of sinusoids. This decomposition forms the basis for frequency domain analysis, allowing the study of how transmission lines affect different frequency components. By analyzing these effects, we can study how a signal changes as it moves along a transmission line by examining how the line's properties, which can vary with different frequencies, influence each part of the signal's frequency spectrum. This section aims to explore sinusoidal propagation and its impact on signals in the context of a lossless transmission line.

It is first necessary to state Equation A.13 from the Lossless Propagation section of the book [34], which is a solution of Equation A.11 where $R = G = 0$.

$$V(z, t) = f_1\left(t - \frac{z}{v}\right) + f_2\left(t + \frac{z}{v}\right) = V^+ + V^- \quad (\text{A.13})$$

We begin by assigning sinusoidal functions to the voltage functions in Equation A.13. Specifically, we consider a specific frequency, $f = \frac{\omega}{2\pi}$, and write $f_1 = f_2 = V_0 \cos(\omega t + \phi)$. By convention, the cosine function is chosen; the sine function is obtainable, as we know, by setting $\phi = -\frac{\pi}{2}$. We next replace t with $t \pm \frac{z}{v_p}$, obtaining

$$\mathcal{V}(z, t) = |V_0| \cos\left[\omega\left(t \pm \frac{z}{v_p}\right) + \phi\right] = |V_0| \cos[\omega t \pm \beta z + \phi] \quad (\text{A.14})$$

where we have assigned a new notation to the velocity, which is now called the phase velocity, v_p . This is applicable to a pure sinusoid (having a single frequency) and will be found to depend on frequency in some cases. Choosing, for the moment, $\phi = 0$, we obtain the two possibilities of forward or backward z travel by choosing the minus or plus sign in (A.14). The two cases are:

$$\mathcal{V}_f(z, t) = |V_0| \cos(\omega t - \beta z) \quad (\text{A.15})$$

(forward z propagation) and

$$\mathcal{V}_b(z, t) = |V_0| \cos(\omega t + \beta z) \quad (\text{A.16})$$

(backward z propagation) where the magnitude factor, $|V_0|$, is the value of $\mathcal{V}$ at $z = 0, t = 0$. We define the phase constant β , obtained from (A.14), as

$$\beta \equiv \frac{\omega}{v_p} \quad (\text{A.17})$$

We refer to the solutions expressed in Eqs. (A.15) and (A.16) as real instantaneous forms of the transmission line voltage. They are mathematical representations of what one would measure experimentally. The terms ωt and βz , appearing in these equations, have units of angle and are usually expressed in radians. We know that ω is the radian time frequency, which measures the phase shift per unit time, and has units of rad/s. Similarly, we see that β will be interpreted as a spatial frequency, which in the present case measures the phase shift per unit distance along the z direction. Its units are rad/m. If we fixed the time at $t = 0$, the equations would become

$$\mathcal{V}_f(z, 0) = \mathcal{V}_b(z, 0) = |V_0| \cos(\beta z) \quad (\text{A.18})$$

which we identify as a simple periodic function that repeats every incremental distance λ , known as the wavelength. The requirement is that $\beta\lambda = 2\pi$, and so

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f} \quad (\text{A.19})$$

A.0.3. Complex Analysis of Sinusoidal Waves

Expressing sinusoidal waves as complex functions is useful (and essentially indispensable) because it greatly eases the evaluation and visualization of the phase that will be found to accumulate by many mechanisms. In addition, we will find many cases in which two or more sinusoidal waves must be combined to form a resultant wave—a task made much easier if complex analysis is used. Expressing sinusoidal functions in complex form is based on the Euler identity: Expressing

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$$e^{\pm jx} = \cos(x) \pm j \sin(x)$$

from which we can write the cosine and sine, respectively, as the real and imaginary parts of the complex exponent:

$$\cos(x) = \text{Re} [e^{\pm jx}] = \frac{1}{2} (e^{jx} + e^{-jx}) = \frac{1}{2} e^{jx} + \text{c.c.} \quad (\text{A.20})$$

$$\sin(x) = \pm \text{Im} [e^{\pm jx}] = \frac{1}{2j} (e^{jx} - e^{-jx}) = \frac{1}{2j} e^{jx} - \text{c.c.} \quad (\text{A.21})$$

where $j \equiv \sqrt{-1}$, and where *c.c.* denotes the complex conjugate of the preceding term. The conjugate is formed by changing the sign of j wherever it appears in the complex expression. We may next apply Equation A.20 to our voltage wave function, Equation A.14:

$$\mathcal{V}(z, t) = |V_0| \cos[\omega t \pm \beta z + \phi] = \frac{1}{2} \underbrace{(|V_0| e^{j\phi})}_{V_0} e^{\pm j\beta z} e^{j\omega t} \quad (\text{A.22})$$

Note that we have arranged the phases in Equation A.22 so that we identify the complex amplitude of the wave as $V_0 = (|V_0| e^{j\phi})$. In future usage, a single symbol (V_0 in the present example) will usually be used for the voltage or current amplitudes, with the understanding that these will generally be complex (having magnitude and phase).

Two additional definitions follow from Equation A.22. First, we define the complex instantaneous voltage as:

$$V_c(z, t) = V_0 e^{\pm j\beta z} e^{j\omega t} \quad (\text{A.23})$$

The phasor voltage is then formed by dropping the $e^{j\omega t}$ factor from the complex instantaneous form:

$$V_s(z) = V_0 e^{\pm j\beta z} \quad (\text{A.24})$$

The phasor voltage can be defined provided we have sinusoidal steady-state conditions, which means that V_0 is independent of time. This has in fact been our assumption all along, because a time-varying amplitude would imply the existence of other frequency components in our signal. Again, we are treating only a single-frequency wave. The significance of the phasor voltage is that we are effectively letting time stand still and observing the stationary wave in space at $t = 0$. The processes of evaluating relative phases between various line positions and of combining multiple waves are made much simpler in phasor form. Again, this works only if all waves under consideration have the same frequency. With the definitions in Equation A.23 and Equation A.24, the real instantaneous voltage can be constructed using Equation A.22:

$$\mathcal{V}(z, t) = |V_0| \cos[\omega t \pm \beta z + \phi] = \text{Re} [V_c(z, t)] = \frac{1}{2} V_c + \text{c.c.} \quad (\text{A.25})$$

Or, in terms of the phasor voltage:

$$\mathcal{V}(z, t) = |V_0| \cos[\omega t \pm \beta z + \phi] = \text{Re} [V_s(z) e^{j\omega t}] = \frac{1}{2} V_s(z) e^{j\omega t} + \text{c.c.} \quad (\text{A.26})$$

In other words, we may obtain our real sinusoidal voltage wave by multiplying the phasor voltage by $e^{j\omega t}$ (reincorporating the time dependence) and then taking the real part of the resulting expression.

A.0.4. Transmission Line Equations and Their Solutions in Phasor Form

We now apply our results from the previous section to the transmission line equations, starting with the general wave equation, Equation A.11. This is rewritten as follows, for the real instantaneous voltage, $\mathcal{V}(z, t)$:

$$\frac{\partial^2 \mathcal{V}}{\partial z^2} = LC \frac{\partial^2 \mathcal{V}}{\partial t^2} + (LG + RC) \frac{\partial \mathcal{V}}{\partial t} + RGV \quad (\text{A.27})$$

We next substitute $\mathcal{V}(z, t)$ as given by the far right-hand side of Equation A.25, noting that the complex conjugate term (c.c.) will form a separate redundant equation. We also use the fact that the operator $\partial/\partial t$, when applied to the complex

form, is equivalent to multiplying by a factor of $j\omega$. After substitution and after taking all the time derivatives, the factor $e^{j\omega t}$ is divided. We are left with the wave equation in terms of the phasor voltage:

$$\frac{d^2 V_s}{dz^2} = -\omega^2 L C V_s + j\omega(LG + RC)V_s + RGV_s \quad (\text{A.28})$$

Rearranging terms leads to the simplified form:

$$\frac{d^2 V_s}{dz^2} = \underbrace{(R + j\omega L G)}_Z \underbrace{+ j\omega C}_Y V_s = \gamma^2 V_s \quad (\text{A.29})$$

where Z and Y , as indicated, are, respectively, the net series impedance and the net shunt admittance in the transmission line, both measured per unit distance. The propagation constant in the line is defined as

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY} = \alpha + j\beta \quad (\text{A.30})$$

The significance of the term will be explained in subsection A.0.6. For our immediate purposes, the solution of Equation A.29 will be

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad (\text{A.31})$$

The wave equation for the current will be identical in form to Equation A.29. We therefore expect the phasor current to be in the form:

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} \quad (\text{A.32})$$

The relation between the current and voltage waves is now found, as before, through the telegraphist's equations, Equation A.5 and Equation A.8. In a manner consistent with Equation A.25, we write the sinusoidal current as

$$\mathcal{I}(z, t) = |I_0| \cos(\omega t \pm \beta z + \xi) = \frac{1}{2} \underbrace{(|I_0| e^{j\xi})}_{I_0} e^{\pm j\beta z} e^{j\omega t} = \frac{1}{2} I_s(z) e^{j\omega t} \quad (\text{A.33})$$

Substituting the far right-hand sides of Equation A.25 and Equation A.33 into Equations A.5 and A.8 transforms the latter equations as follows:

$$\frac{\partial \mathcal{V}}{\partial z} = - \left(R\mathcal{I} + L \frac{\partial \mathcal{I}}{\partial t} \right) \Rightarrow \frac{dV_s}{dz} = -(R + j\omega L)I_s = -ZI_s \quad (\text{A.34})$$

and

$$\frac{\partial \mathcal{I}}{\partial z} = - \left(G\mathcal{V} + C \frac{\partial \mathcal{V}}{\partial t} \right) \Rightarrow \frac{dI_s}{dz} = -(G + j\omega C)V_s = -YV_s \quad (\text{A.35})$$

We can now substitute Equation A.31 and Equation A.32 into either Equation A.34 or A.35 [we will use Equation A.31] to find:

$$-\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{+\gamma z} = -Z(I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}) \quad (\text{A.36})$$

Next, equating coefficients of $e^{-\gamma z}$ and $e^{+\gamma z}$, we find the general expression for the line characteristic impedance:

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{Z}{\gamma} = \frac{Z}{\sqrt{ZY}} = \sqrt{\frac{Z}{Y}} \quad (\text{A.37})$$

Incorporating the expressions for Z and Y , we find the characteristic impedance in terms of our known line parameters:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = |Z_0| e^{j\theta} \quad (\text{A.38})$$

Note that with the voltage and current as given in Equation A.25 and Equation A.33, we would identify the phase of the characteristic impedance, $\theta = \phi - \xi$.

A.0.5. Low-Loss Propagation in the Conductor

The first step in the implementation of this method towards predicting the current in the metallic sheath is to determine the voltage in the conductor as a function of z and t (the reason for this will be explained in section 5.4). This step is relatively simple as we can assume the transmission occurs with losses so low that they are negligible ($R \ll j\omega L$ and $G \ll j\omega C$).

Having obtained the phasor forms of voltage and current in a general transmission line (Equation A.31 and A.32), we can now look more closely at their meaning. First we incorporate A.30 into A.31 to obtain

$$V_{cp}(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z} \quad (\text{A.39})$$

Next, multiplying A.39 by $e^{j\omega t}$ and taking the real part gives the **real instantaneous voltage**:

$$\mathcal{V}_c(z, t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{\alpha z} \cos(\omega t + \beta z) \quad (\text{A.40})$$

Here we assume V_0^+ and V_0^- to be real. Equation A.40 describes the forward and backward propagation of waves that diminish in amplitude with distance (following $e^{-\alpha z}$ for forward propagation and $e^{\alpha z}$ for backward propagation). The decrease in the wave amplitude is also called *attenuation*, and α is called the *attenuation constant*. The phase constant β is found by taking the imaginary part of A.30, which will be a complicated function, as will be seen later. However, it is also still more simply defined as ω/v_p and the wavelength is still defined as the distance over which a phase shift of 2π rad occurs: $\lambda = 2\pi/\beta$.

Equation A.30 can be rewritten to:

$$\begin{aligned} \gamma &= \alpha + j\beta = [(R + j\omega L)(G + j\omega C)]^{1/2} \\ &= j\omega\sqrt{LC} \left[\left(1 + \frac{R}{j\omega L}\right)^{1/2} \left(1 + \frac{G}{j\omega C}\right)^{1/2} \right] \end{aligned} \quad (\text{A.41})$$

For the assumption of low losses: $R \ll \omega L$ and $G \ll \omega C$, this can be simplified by using a power series approximation:

$$\sqrt{1+x} \approx 1 + \frac{x}{2} - \frac{x^2}{8} \quad (x \ll 1) \quad (\text{A.42})$$

We use Equation A.42 to expand the terms in large parentheses in Equation A.41, resulting in:

$$\gamma = j\omega\sqrt{LC} \left[\left(1 + \frac{R}{j\omega L} + \frac{R^2}{8\omega^2 L^2}\right) + \left(1 + \frac{G}{j\omega C} + \frac{G^2}{8\omega^2 C^2}\right) \right] \quad (\text{A.43})$$

Then, all the products of Equation A.43 are calculated, neglecting the terms involved RG^2 , R^2G , and R^2G^2 , as these, following the same reasoning as before, are negligible compared to all others. The result is:

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC} \left[1 + \frac{1}{2j\omega} \left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}} \right) + \frac{1}{8\omega^2} \left(\frac{R^2 L^2 - 2RG\sqrt{LC} + G^2 C^2}{LC} \right) \right] \quad (\text{A.44})$$

Separating real and imaginary parts of Equation A.44 gives α and β :

$$\alpha \doteq \frac{1}{2} \left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}} \right) \quad (\text{A.45})$$

$$\beta \doteq \omega\sqrt{LC} \left[1 + \frac{1}{8} \left(\frac{G}{\omega C} - \frac{R}{\omega L} \right)^2 \right] \quad (\text{A.46})$$

We note that α scales proportional to R and G , as expected. Observing that the terms in Equation A.46 involving R and G lead to a phase velocity, $v_p = \omega/\beta$, which is frequency-dependent. Moreover, the group velocity, $v_g = d\omega/d\beta$, will depend on frequency and will lead to signal distortion, as explored in Chapter 12. Note that with non-zero R and G , phase and group velocities that are constant with frequency can be obtained when $R/L = G/C$, known as Heaviside's condition. In this case, Equation A.46 becomes $\beta = \omega\sqrt{LC}$, and the line is distortionless. Further complications occur when accounting for possible frequency dependencies in R , G , L , and C . Consequently, conditions of low-loss or distortion-free propagation will usually occur over limited frequency ranges. Typically, loss increases with increasing frequency, mainly because of the increase in R with frequency. This nature of latter effect, known as skin effect loss, comes from the forced concentration of the current into the peripherals of the conductor due to self inductance (subsection 4.3.5).

Finally, the low-loss approximation to the characteristic impedance is created. Using Equation A.42 results in:

$$\begin{aligned} Z_0 &= \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{j\omega L \left(1 + \frac{R}{j\omega L}\right)}{j\omega C \left(1 + \frac{G}{j\omega C}\right)}} \\ &\doteq \sqrt{\frac{L}{C}} \left[\left(1 + \frac{R}{j\omega L} + \frac{R^2}{8\omega^2 L^2}\right) + \left(1 + \frac{G}{j\omega C} + \frac{G^2}{8\omega^2 C^2}\right) \right] \end{aligned} \quad (\text{A.47})$$

Next, multiplying Eq. A.37 by a factor of 1, in the form of the complex conjugate of the denominator of Eq. A.37 divided by itself. The resulting expression is simplified by neglecting all terms on the order of R^2G , G^2R , and higher. Additionally, the approximation $1/(1+x) \doteq 1-x$, where $x \ll 1$ is used. The result is:

$$Z_0 \doteq \sqrt{\frac{L}{C}} \left\{ 1 + \frac{1}{2\omega^2} \left[\frac{1}{4} \left(\frac{R}{L} + \frac{G}{C} \right)^2 - \frac{G^2}{C^2} \right] + \frac{j}{2\omega} \left(\frac{G}{C} - \frac{R}{L} \right) \right\} \quad (\text{A.48})$$

A.0.6. Solution of the Transmission Line Equations for an Uncompensated Power Line

To verify that the developed transmission line equations produce realistic distributions of voltage and current along a transmission line, it is necessary to apply them to a realistic case for which the outcome is known. For this, the equations developed in this chapter will be solved to arrive at a useful system of equations that describe input vs. output voltage as a function of length. The analysis starts with a review Equations A.49, A.50 and (a rearranged version of) A.36 :A.51:

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (\text{A.49})$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad (\text{A.50})$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} = \frac{1}{Z_0} (V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) \quad (\text{A.51})$$

We will start by writing A.49 and A.51 in terms of their value at the load side ($z = l$):

$$V_L = V_s(l) = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} \quad (\text{A.52})$$

$$I_L = I_s(l) = \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l} \quad (\text{A.53})$$

Now the aim is to write V_0^+ and V_0^- as a function of the other variables. For this, the equation for current (A.53) is multiplied by Z_0 to get

$$Z_0 I_L = I_s(l) = V_0^+ e^{-\gamma l} - V_0^- e^{\gamma l} \quad (\text{A.54})$$

We can then determine V_0^+ and V_0^- by respectively adding and subtracting this from the A.52. Adding equations (1) and (3) gives us

$$\begin{aligned} V_L + Z_0 I_L &= 2V_0^+ e^{-\gamma l} \\ V_0^+ &= \frac{V_L + Z_0 I_L}{2} e^{\gamma l} \end{aligned} \quad (\text{A.55})$$

Subtracting (3) from (1):

$$V_L - Z_0 I_L = 2V_0^- e^{\gamma l} \quad (\text{A.56})$$

$$V_0^- = \frac{V_L - Z_0 I_L}{2} e^{-\gamma l} \quad (\text{A.57})$$

Substituting V_L and I_L into the voltage equation:

$$V_s(z) = \frac{V_L + Z_0 I_L}{2} e^{\gamma l} e^{-\gamma z} + \frac{V_L - Z_0 I_L}{2} e^{-\gamma l} e^{\gamma z} \quad (\text{A.58})$$

Applying an exchange of variables:

$$\begin{aligned} z' &= l - z \rightarrow z = l - z' \\ V(z') &= \frac{V_L + Z_0 I_L}{2} e^{\gamma z'} + \frac{V_L - Z_0 I_L}{2} e^{-\gamma z'} \end{aligned} \quad (\text{A.59})$$

Which can be written as:

$$V(z') = V_L \cosh(\gamma z') + I_L Z_0 \sinh(\gamma z') \quad (\text{A.60})$$

The same method is used to arrive at the equation for current:

$$I(z') = \frac{V_L}{Z_0} \sinh(\gamma z') + I_L \cosh(\gamma z') \quad (\text{A.61})$$

A.0.7. Application of Wave Equations to Metallic Sheaths

This subsection will derive a new version of the wave equations described in subsection A.0.1 for the metallic sheath that is placed around the dielectric layer of a single phase HVAC cable. This is a more complicated scenario than for the conductor as it includes a distributed capacitive current instead of a simple supply from the generation end. In Figure A.2 this is represented in the form of an electrical circuit.

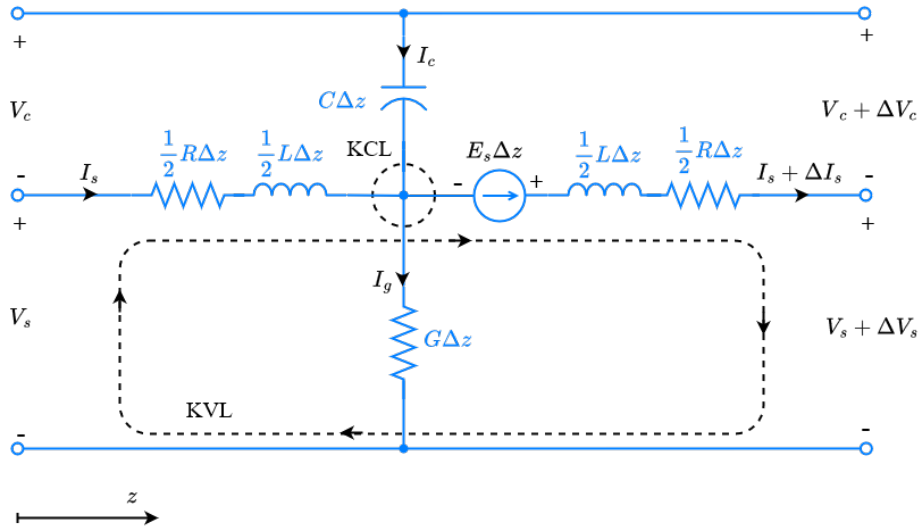


Figure A.2: Schematic of the electrical interaction of the metallic sheath with the conductor (top) and ground (bottom). It displays the lumped elements that represent the electrical properties per unit length.

In the schematic (Figure A.2), several voltages of interest can be observed: the sheath voltage V_s , and the conductor voltage V_c , which can independently be calculated (subsection A.0.6) and serves as an input to the system. Finally, the induced voltage $E_s \Delta z$, which is also resultant from the analysis of the conductor (subsection A.0.6, subsection 4.3.2). Additionally, several currents can be identified.

- **Sheath current I_s :** The primary current which this method aims to calculate.
- **Capacitive current I_c :** The current involved in charging the current along the length. This is also a direct result of the conductor analysis, as the sheath voltage is negligible compared to the conductor voltage.
- **Conductive current to ground I_g :** The current flowing through the ground conductance (i.e. to the other phase shifted metallic sheaths).

We also identify the design parameters:

- R : Longitudinal sheath resistance per unit length.
- G : Radial conductance to ground per unit length.
- L : Sheath self-inductance per unit length.
- C : Capacitance of the dielectric insulation per unit length.

These design parameters are assumed to be known and constant over the length.

Applying Kirchoff's Voltage Law along the dotted line indicated in Figure A.2 results in:

$$V_s = R \Delta z \left(I_s + \frac{\Delta I_s}{2} \right) + L \Delta z \left(\frac{\partial I_s}{\partial t} + \frac{1}{2} \frac{\partial \Delta I_s}{\partial t} \right) - E_s \Delta z + V_s + \Delta V_s \quad (\text{A.62})$$

Solving for $\frac{\Delta V_s}{\Delta z}$:

$$\frac{\Delta V_s}{\Delta z} = - \left(RI + L \frac{\partial I}{\partial t} + \frac{1}{2} L \frac{\partial \Delta I}{\partial t} + \frac{1}{2} R \Delta I \right) + E_s \quad (\text{A.63})$$

We can also write the following two equations based on their definition and assuming Δz is sufficiently small:

$$\Delta I_s = \frac{\partial I_s}{\partial z} \Delta z \quad \text{and} \quad \Delta V_s = \frac{\partial V_s}{\partial z} \Delta z \quad (\text{A.64})$$

Substituting this into Equation A.63 and then simplifying results in:

$$\frac{\partial V_s}{\partial z} = -R \left(I_s + \frac{1}{2} \frac{\partial I_s}{\partial z} \Delta z \right) - L \left(\frac{\partial I_s}{\partial t} + \frac{1}{2} \frac{\partial^2 I_s}{\partial z \partial t} \Delta z \right) + E_s \quad (\text{A.65})$$

In the limit as $\Delta z \rightarrow 0$ this simplifies even more, yielding the so called Telegraphers equations for voltage, which turns out to be the same as for the standard situation:

$$\frac{\partial V_s}{\partial z} = - \left(RI_s + L \frac{\partial I_s}{\partial t} \right) + E_s \quad (\text{A.66})$$

Now applying Kirchoff's Current Law to the central node indicated in Figure A.2 and substituting the expressions for each current:

$$\begin{aligned} I_s &= I_g + (I_s + \Delta I_s) - I_c \\ I_s &= G \Delta z \left(V_s + \frac{\Delta V_s}{2} \right) + (I_s + \Delta I_s) - C \Delta z \frac{\partial V_c}{\partial t} \end{aligned} \quad (\text{A.67})$$

Simplifying this equation and using the definitions of ΔV and ΔI from A.64 results in the differential of I w.r.t. z :

$$\frac{\partial I_s}{\partial z} = -G \left(V_s + \frac{1}{2} \frac{\partial V_s}{\partial z} \Delta z \right) + C \frac{\partial V_c}{\partial t} \quad (\text{A.68})$$

As for A.65 taking the limit as $\Delta z \rightarrow 0$ to find the **new version of the Telegraphers current equation**:

$$\frac{\partial I_s}{\partial z} = -GV_s + C \frac{\partial V_c}{\partial t} \quad (\text{A.69})$$

Solving this equation in conjunction with Equation A.66 will allow us to determine the voltage and current distribution within a metallic sheath that is subject to a distributed capacitive current. This can be used to predict current and voltage within a metallic sheath, with an insulating outer sheath, or with a semiconductive outer sheath as the values of C_g and G can be freely chosen.

We will now continue writing down the general transmission line equations for metallic sheaths. First, differentiating A.66 w.r.t. t to find

$$\frac{\partial^2 V_s}{\partial z \partial t} = -R \frac{\partial I_s}{\partial t} - L \frac{\partial^2 I_s}{\partial t^2} + \frac{\partial E_s}{\partial t} \quad (\text{A.70})$$

and A.69 w.r.t. z

$$\frac{\partial^2 I_s}{\partial z^2} = C \frac{\partial^2 V_c}{\partial z \partial t} - G \frac{\partial V_s}{\partial z} \quad (\text{A.71})$$

Combining A.69 and A.71 into A.70 and then grouping terms results in

$$\begin{aligned} \frac{\partial^2 I_s}{\partial z^2} &= C \left(-R \frac{\partial I_s}{\partial t} - L \frac{\partial^2 I_s}{\partial t^2} + \frac{\partial E_s}{\partial t} \right) - G \left(-RI_s - L \frac{\partial I_s}{\partial t} + E_s \right) \\ \frac{\partial^2 I_s}{\partial z^2} &= RGI_s + (LG - RC) \frac{\partial I_s}{\partial t} - LC \frac{\partial^2 I_s}{\partial t^2} - GE_s + C \frac{\partial E_s}{\partial t} \end{aligned} \quad (\text{A.72})$$

Which is the analogous to the general wave equation for current as described in subsection A.0.1. Doing the same for A.66 w.r.t. z and A.69 w.r.t. t :

$$\frac{\partial^2 V_s}{\partial z^2} = -R \frac{\partial I_s}{\partial z} - L \frac{\partial^2 I_s}{\partial z \partial t} + \frac{\partial E_s}{\partial z} \quad (\text{A.73})$$

$$\frac{\partial^2 I_s}{\partial z \partial t} = C \frac{\partial^2 V_c}{\partial z \partial t} - G \frac{\partial V_s}{\partial t} \quad (\text{A.74})$$

Substituting A.69 and A.74 into A.73 results in

$$\begin{aligned}\frac{\partial^2 V_s}{\partial z^2} &= -R \left(C \frac{\partial V_c}{\partial t} - G V_s \right) - L \left(C \frac{\partial^2 V_c}{\partial z \partial t} - G \frac{\partial V_s}{\partial t} \right) + \frac{\partial E_s}{\partial z} \\ \frac{\partial^2 V_s}{\partial z^2} &= R G V_s + L G \frac{\partial V_s}{\partial t} - R C \frac{\partial V_c}{\partial t} - L C \frac{\partial^2 V_c}{\partial z \partial t} + \frac{\partial E_s}{\partial z}\end{aligned}\quad (\text{A.75})$$

Which is the analogous to the general wave equation for voltage as described in subsection A.0.1. Using the same method, the specific case of the general wave equation for current can be found:

$$\frac{\partial^2 I_s}{\partial z^2} = R G I_s + (L G - R C) \frac{\partial I_s}{\partial t} - L C \frac{\partial^2 I_s}{\partial t^2} - G E_s + C \frac{\partial E_s}{\partial t} \quad (\text{A.76})$$

This part derived a specialized case of the **general wave equations** for transmission lines (also called the **transmission line equations**) described in [34], adjusted to the specialized Telegraphers Equations A.66 and A.69.

A.0.8. Application of the Specialized Wave Equations

The analysis of the voltage and current in the metallic sheath is continued, using the derivation for transmission line equations from the book [34]. First taking the definition of the real instantaneous voltage $\mathcal{V}_s(z, t)$ as a function of z (the **phasor voltage** $V_{sp}(z)$) and the time component $e^{j\omega t}$:

$$\mathcal{V}_s(z, t) = \frac{1}{2} V_{sp}(z) e^{j\omega t} \quad (\text{A.77})$$

The same can be written for the real instantaneous voltage of the conductor $\mathcal{V}_c(z, t)$, the relation can be found as the result from the earlier derivation of a standard transmission line (section 5.2). $\mathcal{V}_c$ is written in terms of the **conductor phasor voltage** $V_{cp}(z)$ as

$$\mathcal{V}_c(z, t) = \frac{1}{2} V_{cp}(z) e^{j\omega t} \quad (\text{A.78})$$

Finally, a similar equation for the induced electric field in the sheath can be written, since it is dependent on the conductor current, which is equal in form to the conductor voltage in Equation A.78.

$$E_s(z, t) = \frac{1}{2} E_{sp}(z) \cdot e^{j\omega t} \quad (\text{A.79})$$

Using only the first terms of A.77, A.78 and A.79 since the *c.c.* term will produce a redundant solution. Substituting them in the transmission line equation for voltage (A.75) results in the differential equation for the real instantaneous voltage $\mathcal{V}_s(z, t)$.

$$\frac{\partial^2 (V_s e^{j\omega t})}{\partial z^2} = R G V_s e^{j\omega t} + L G \frac{\partial (V_s e^{j\omega t})}{\partial t} - R C \frac{\partial (V_c e^{j\omega t})}{\partial t} - L C \frac{\partial^2 (V_c e^{j\omega t})}{\partial z \partial t} + \frac{\partial (E_s e^{j\omega t})}{\partial z} \quad (\text{A.80})$$

Calculating the differentials and dividing everything by the exponential term results in the second order differential equation for the sheath voltage phasor V_{sp} in terms of conductor voltage phasor V_{cp} and induced electric field phasor E_{sp} :

$$\frac{\partial^2 V_{sp}}{\partial z^2} = R G V_{sp} + j\omega L G V_{sp} - j\omega R C V_{cp} - (j\omega)^2 L C V_{cp} + j\omega E_{sp} \quad (\text{A.81})$$

Working out the j^2 terms and simplifying A.81 again leads to:

$$\frac{\partial^2 V_{sp}}{\partial z^2} = (R G + j\omega L G) V_{sp} + (L C \omega^2 - j\omega R C) V_{cp} + j\omega E_{sp} \quad (\text{A.82})$$

Where it is known from subsection A.0.4 that the conductor voltage phasor V_{cp} can be written as

$$V_{cp} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (\text{A.83})$$

with V_0^+ and V_0^- defined by Equation A.55 and Equation A.57 and γ is the conductor propagation constant. Next to that, it is known from subsection 4.3.2 that the induced electric field in the sheath is equal to:

$$E_{sp} = -j X_{sf} I_{cp} \quad (\text{A.84})$$

where X_{sf} is the effective reactance of the sheath, combining both the effect of its own conduction current, as well as the conduction currents of the other two phases. This can be done when using assumption of a perfect trefoil arrangement

of the three phases and perfect load balance between the phases. We also know from Equation A.51 that the conductor current phasor is equal to

$$I_{cp} = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \quad (\text{A.85})$$

where Z_0 is the characteristic impedance of the conductor. Substituting A.83, A.84 and A.83 into A.82 produces

$$\begin{aligned} \frac{\partial^2 V_{sp}}{\partial z^2} = & (RG + j\omega LG)V_{sp} + (LC\omega^2 - j\omega RC)(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}) \\ & - \omega X_{sf} \left(\frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \right) \end{aligned} \quad (\text{A.86})$$

Where we take $k = \sqrt{G(R + j\omega L)}$, $\tau = \sqrt{\omega C(L\omega - jR)}$ and $\zeta = -\omega \frac{X_{sf}}{Z_0}$ such that the equation can be simplified to

$$\frac{\partial^2 V_{sp}}{\partial z^2} = k^2 V_{sp} + \tau^2 (V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}) + \zeta (V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) \quad (\text{A.87})$$

Here, k can be calculated in the same manner as for γ described in subsection A.0.5. Equation A.87 can be solved, starting with its homogeneous equation:

$$\frac{\partial^2 V_{sp}}{\partial z^2} - k^2 V_{sp} = 0 \quad (\text{A.88})$$

This has a simple solution: $V_{sp,h} = c_1 e^{-kz} + c_2 e^{kz}$. For the **particular solution** we assume $V_{sp,p} = Ae^{-\gamma z} + Be^{\gamma z}$. Substituting in A.87:

$$\frac{\partial^2}{\partial z^2} (Ae^{-\gamma z} + Be^{\gamma z}) = k^2 (Ae^{-\gamma z} + Be^{\gamma z}) + (\tau^2 + \zeta)V_0^+ e^{-\gamma z} + (\tau^2 - \zeta)V_0^- e^{\gamma z} \quad (\text{A.89})$$

Calculating the differentials and grouping exponential terms we can find coefficients A and B to be

$$\begin{aligned} A\gamma^2 &= k^2 A + (\tau^2 + \zeta)V_0^+ & B\gamma^2 &= k^2 B + (\tau^2 - \zeta)V_0^- \\ A &= \frac{(\tau^2 + \zeta)V_0^+}{\gamma^2 - k^2} & B &= \frac{(\tau^2 - \zeta)V_0^-}{\gamma^2 - k^2} \end{aligned} \quad (\text{A.90})$$

Combining the homogenous and particular solutions, while keeping constants C_1, C_2, A, B yields the **complete solution for the sheath voltage phasor**:

$$V_{cp}(z) = C_1 e^{-kz} + C_2 e^{kz} + Ae^{-\gamma z} + Be^{\gamma z} \quad (\text{A.91})$$

Now solving for the boundary conditions to get coefficients C_1 and C_2 . We know for a solid-bonded cable that the voltages at both ends of the metallic sheath are zero. Applying this at the sending end ($z = 0$) gives

$$V_{cp}(0) = 0 = C_1 + C_2 + A + B \Rightarrow C_2 = -C_1 - A - B \quad (\text{A.92})$$

such that the differential equation becomes:

$$V_{sp}(z) = C_1 (e^{-kz} - e^{kz}) - (A + B)e^{kz} + Ae^{-\gamma z} + Be^{\gamma z} \quad (\text{A.93})$$

Applying boundary condition at the receiving end ($z = \ell$) condition:

$$V_{sp}(\ell) = 0 = C_1 (e^{-k\ell} - e^{k\ell}) - (A + B)e^{k\ell} + Ae^{-\gamma\ell} + Be^{\gamma\ell} \quad (\text{A.94})$$

Such that we can solve for C_1 :

$$C_1 = \frac{-(A + B)e^{k\ell} + Ae^{-\gamma\ell} + Be^{\gamma\ell}}{(e^{k\ell} - e^{-k\ell})} \quad (\text{A.95})$$

from which we can solve for C_2 :

$$C_2 = \frac{(A + B)e^{-k\ell} - Ae^{-\gamma\ell} - Be^{\gamma\ell}}{(e^{k\ell} - e^{-k\ell})} \quad (\text{A.96})$$

We now have the coefficients C_1, C_2, A, B in terms of known usage parameters. This gives us a function for the sheath voltage phasor V_{sp} that can be used to calculate the amplitude of the voltage at any point $0 \leq z \leq \ell$.

Filling in the identities $V_s = V_{sp}e^{j\omega t}$ and $E_s = E_{sp}e^{j\omega t}$ in the specialized Telegraphers equation for voltage A.66 and removing the term $e^{j\omega t}$, we can find that

$$\frac{dV_{sp}}{dz} = -(RI_{sp} + j\omega L \cdot I_{sp}) + E_{sp} = -(R + j\omega L)I_{sp} - jX_{sf}I_{cp} \quad (\text{A.97})$$

so that we now also have the sheath current phasor I_{sp} in terms of known material parameters and a previously determined sheath voltage phasor V_{sp} and induced electric field phasor.

$$I_{sp}(z) = \frac{jX_{sf}I_{cp}(z) - \frac{dV_{sp}(z)}{dz}}{R + j\omega L} \quad (\text{A.98})$$

B

Appendix B: Matlab Code

B.1. Calculation of Current and Voltage Distribution (MATLAB)

This script uses transmission line theory for the conductor current and the junction method for the sheath voltage and current.

Listing B.1: Example MATLAB Code

```
1
2
3 % Define the parameters for all modes of operation
4 modes_of_operation = { ...
5     '380 kV 350 MW SP IN C IN', '380 kV 350 MW SP UIT C IN', ...
6     '380 kV 380 MW SP IN C IN', '380 kV 380 MW SP UIT C IN', ...
7     '407 kV 350 MW SP IN C IN', '407 kV 350 MW SP UIT C IN', ...
8     '407 kV 380 MW SP IN C IN', '407 kV 380 MW SP UIT C IN' ...
9 };
10
11 Q1_HV = [54.7, 54.1, 55.2, 54.6, 55.9, 55.2, 56.3, 55.7];
12 Q2_HV = [78.7, 79.2, 78.9, 79.4, 80.4, 80.9, 80.6, 81.1];
13 Q2_MV = [30.44, -33.99, 30.54, -34.11, 31.09, -34.7, 31.19, -34.83];
14 I_L_values = [883 + 391j, 881 + 226j, 956 + 411j, 954 + 246j, ...
15               874 + 390j, 872 + 223j, 946 + 409j, 944 + 242j];
16 V_L_values = [223.186, 223.841, 223.514, 224.171, 225.583, 226.244, 225.901, 226.564];
17
18 % Define line styles and colors
19 line_styles = {'-', '-', '-', '-', '--', '--', '--', '--'};
20 line_colors = {[0, 0, 0.85], [1, 0.5, 0], [0.5, 0.5, 0.5], [0.95, 0.95, 0], ...
21               [0, 0, 0.85], [1, 0.5, 0], [0.5, 0.5, 0.5], [0.95, 0.95, 0]};
22
23 % Initialize arrays to store results
24 num_points = 1000; % number of points along the line
25 L = 60e3; % total length of the cable in meters
26 l_values = linspace(0, L, num_points); % length values from 0 to 60 km
27 all_V1_values = zeros(length(modes_of_operation), num_points);
28 all_I1_values = zeros(length(modes_of_operation), num_points);
29 all_Vs_values = zeros(length(modes_of_operation), num_points);
30 all_Is_values = zeros(length(modes_of_operation), num_points);
31
32 % Loop through each mode of operation
33 for k = 1:length(modes_of_operation)
34     mode = modes_of_operation{k};
35     Q1_HV_k = Q1_HV(k);
36     Q2_HV_k = Q2_HV(k);
37     Q2_MV_k = Q2_MV(k);
38     I_L_k = I_L_values(k); % Convert string to complex number
39     V_L_k = V_L_values(k);
40
41     % Combined Script
42
43     %% Transmission Line Distribution
44     % Given parameters
45     alpha = 1.7636e-7; % real part of propagation constant (per meter)
46     beta = 2.6419e-6; % imaginary part of propagation constant (per meter)
```



```

47 gamma = alpha + 1i * beta; % propagation constant
48 Zc = 42.91 - 2.86i; % characteristic impedance
49 V2 = V_L_k * 1e3; % receiving end voltage in volts
50 I2 = I_L_k; % receiving end current per phase in amperes
51 Q2 = (Q2_HV_k + Q2_MV_k)*10^6;
52 Q1 = Q1_HV_k * 10^6; % reactive power at the sending end in VARs (example value)
53
54 % Loop through the range of l values
55 for i = 1:num_points
56     l = l_values(i);
57     cosh_gamma_l = cosh(gamma * l);
58     sinh_gamma_l = sinh(gamma * l);
59
60     % Define the matrix A and vector B with reactive power compensation
61     A = [cosh_gamma_l, Zc * sinh_gamma_l; sinh_gamma_l / Zc, cosh_gamma_l];
62     B = [V2; I2 - 1j * Q2 / V2];
63
64     % Solve for U1 and i1 + jQ1 / U1
65     X = A \ B;
66     U1 = X(1); % Voltage at the sending end
67     I1_Q1 = X(2); % Current and reactive power term at the sending end
68
69     % Subtract the reactive power term jQ1/U1 from I1_Q1 to get the actual current I1
70     I1 = I1_Q1 - 1j * Q1 / U1;
71
72     all_V1_values(k, i) = abs(U1); % store the absolute value of U1
73     all_I1_values(k, i) = abs(I1); % store the absolute amplitude value of I1
74 end
75
76 %% Sheath Voltage Calculation
77 % Initialize parameters
78 n = num_points;
79 z = linspace(0, L, n); % Discretized spatial points
80 delta_z = L / n;
81
82 % Given parameters
83 mu0 = 4 * pi * 10^-7; % magnetic permeability of free space
84 Dout = 94e-3; % diameter over the outside of the metallic sheath
85 Ds = 90e-3; % diameter over the inside of the metallic sheath
86 tsc = 2e-3; % thickness of semiconductive pe layer
87 tsw = 1e-3; % distance between two semiconductive layers
88 S = Dout + 2 * tsc + tsw; % distance between two conductor cores
89 Rc = 0.0155e-3; % phase resistance per meter
90 Lc = 0.3592e-6; % phase inductance per meter
91 C = 0.196e-9; % phase capacitance per meter
92 omega = 100 * pi; % radial frequency of transmission
93 sigma = 0.455e7; % conductivity of lead sheath
94
95 % Calculated parameters
96 As = pi / 4 * (Dout^2 - Ds^2); % area of sheath cross section
97 Ls = 2e-7 * log(Dout / Ds); % sheath inductance per kmeter
98 G = 7.784; % Given G value
99 Xsf = 2 * omega * log(2 * S / Ds) * 10^-7; % effective sheath reactance
100 alpha = .5 * Rc * sqrt(C / Lc);
101 beta = omega * sqrt(Lc * C) * (1 + 0.125 * (Rc / omega * Lc)^2);
102 gamma = alpha + 1j * beta; % conductor propagation constant
103 Z0 = sqrt(Lc / C) * (1 + 1 / (8 * omega^2) * (Rc / Lc)^2 - 1j * Rc / (2 * omega * Lc)); % phase
    characteristic impedance
104 zeta = -omega * Xsf / Z0; % conductor current coefficient
105
106 % Initialize variables
107 Rs = 1 / (As * sigma); % sheath resistance per kmeter
108 Zs = Rs + 1j * omega * Ls; % sheath impedance per kmeter
109 tau = sqrt(omega * C * (Ls * omega - 1j * Rs)); % conductor voltage coefficient
110 Vck = zeros(1, n); % Placeholder for Vc(z)
111 Ic = zeros(n, 1); % Ensure Ic is a column vector
112 Es = zeros(n, 1);
113 % Initialize matrix
114 A_matrix = zeros(n, n);
115 Icon = zeros(n, 1);
116 sol = zeros(n, 1);
117 % Add line properties to matrix
118 for j = 2:n-1

```

```

119     A_matrix(j-1, j) = -1 / (Zs * delta_z); % Vi-1
120     A_matrix(j, j) = G * delta_z + 2 / (Zs * delta_z); % Vi
121     A_matrix(j+1, j) = -1 / (Zs * delta_z); % Vi+1
122 end
123
124 % Boundary conditions
125 A_matrix(1, 1) = G * delta_z + 2 / (Zs * delta_z);
126 A_matrix(2, 1) = -1 / (Zs * delta_z);
127 A_matrix(n-1, n) = -1 / (Zs * delta_z);
128 A_matrix(n, n) = G * delta_z + 2 / (Zs * delta_z);
129
130 for j = 1:n
131     Vck(j) = all_V1_values(k, n+1-j);
132     Ic(j) = all_I1_values(k, n+1-j);
133     Es(j) = 2 * omega * log(2 * S / Ds) * 10^(-7) * all_I1_values(k, n+1-j);
134 end
135
136 for j = 1:n-1
137     sol(j) = Ic(j) + (Es(j+1) - Es(j));
138 end
139
140 sol(n) = Ic(n) - Es(n);
141
142 % Solve for V using matrix inversion
143 V = A_matrix \ sol;
144
145 % Initialize Vcp matrix with V0 included to calculate Is
146 Vs_full = zeros(1, n+1);
147 Vs_full(2:n+1) = V;
148
149 % Calculate Is seeing backward as positive
150 Is = zeros(1, n);
151 for j = 2:n
152     Is(j-1) = (Vs_full(j) - Vs_full(j-1)) / (Zs * delta_z);
153 end
154
155 all_Vs_values(k, :) = abs(V); % store the absolute value of sheath voltage
156 all_Is_values(k, :) = abs(Is); % store the absolute value of sheath current
157 end
158
159 figure_size = [100, 100, 1200, 666]; % [left, bottom, width, height]
160 font_size = 14;
161 line_width = 1; % Thickness of the plot lines
162 tick_font_size = 12; % Font size for the axis ticks
163
164 %% Plot Transmission Line Voltage Distribution
165 figure('Position', figure_size);
166 hold on;
167 for k = 1:length(modes_of_operation)
168     plot((L - l_values) / 1e3, all_V1_values(k, :), 'LineWidth', line_width, ...
169         'LineStyle', line_styles{k}, 'Color', line_colors{k}); % convert meters to kilometers for plotting
170 end
171 xlabel('Distance along the line [km]', 'FontSize', font_size);
172 ylabel('Conductor voltage phasor magnitude |V_{cp}| [V]', 'FontSize', font_size);
173 % title('Voltage Distribution along the Transmission Line', 'FontSize', font_size);
174 legend(modes_of_operation, 'Location', 'northeastoutside', 'FontSize', font_size);
175 set(gca, 'FontSize', tick_font_size); % Set font size for axis ticks
176 grid on;
177
178 %% Plot Transmission Line Current Distribution
179 figure('Position', figure_size);
180 hold on;
181 for k = 1:length(modes_of_operation)
182     plot((L - l_values) / 1e3, all_I1_values(k, :), 'LineWidth', line_width, ...
183         'LineStyle', line_styles{k}, 'Color', line_colors{k}); % convert meters to kilometers for plotting
184 end
185 xlabel('Distance along the line [km]', 'FontSize', font_size);
186 ylabel('Conductor current phasor magnitude |I_{cp}| [A]', 'FontSize', font_size);
187 % title('Current Distribution along the Transmission Line with Reactive Power Compensation', 'FontSize',
188     font_size);
189 legend(modes_of_operation, 'Location', 'northeastoutside', 'FontSize', font_size);
190 set(gca, 'FontSize', tick_font_size); % Set font size for axis ticks
191 grid on;

```

```

191
192 %% Plot Sheath Voltage Distribution
193 figure('Position', figure_size);
194 hold on;
195 for k = 1:length(modes_of_operation)
196     plot(z, all_Vs_values(k, :), 'LineWidth', line_width, ...
197         'LineStyle', line_styles{k}, 'Color', line_colors{k});
198 end
199 xlabel('Distance along the length of the cable [m]', 'FontSize', font_size);
200 ylabel('Sheath voltage phasor magnitude |V_{sp}| [V]', 'FontSize', font_size);
201 % title('Sheath Voltage over the length of the power cable', 'FontSize', font_size);
202 legend(modes_of_operation, 'Location', 'northeastoutside', 'FontSize', font_size);
203 set(gca, 'FontSize', tick_font_size); % Set font size for axis ticks
204 grid on;
205
206 %% Plot Sheath Current Distribution
207 figure('Position', figure_size);
208 hold on;
209 for k = 1:length(modes_of_operation)
210     semilogy(z, all_Is_values(k, :), 'LineWidth', line_width, ...
211         'LineStyle', line_styles{k}, 'Color', line_colors{k});
212 end
213 set(gca, 'YScale', 'log'); % Manually set the Y-axis to logarithmic scale
214 xlabel('Distance along the length of the cable [m]', 'FontSize', font_size);
215 ylabel('Sheath current phasor magnitude |I_{sp}| [A]', 'FontSize', font_size);
216 % title('Sheath current over the length of the power cable', 'FontSize', font_size);
217 legend(modes_of_operation, 'Location', 'northeastoutside', 'FontSize', font_size);
218 set(gca, 'FontSize', tick_font_size); % Set font size for axis ticks
219 grid on;

```

B.2. Sheath Current and Voltage Distribution: Transmission Line (MATLAB)

```

1 % Given parameters
2 mu0 = 4 * pi * 10^-7; % magnetic permeability of free space
3 Dout = 94e-3; % diameter over the outside of the metallic sheath
4 Ds = 90e-3; % diameter over the inside of the metallic sheath
5 tsc = 2e-3; % thickness of semiconductive pe layer
6 tsw = 1e-3; % distance between two semiconductive layers
7 S = Dout + 2 * tsc + tsc; % distance between two conductor cores
8 omega = 100 * pi; % radial frequency of transmission
9 sigma = 0.455e7; % conductivity of lead sheath
10 V_L = 223.84i; % Example value for V_L
11 I_L = 881 + 226j; % Example value for I_L
12 R = 0.0151e-3; % phase resistance per meter
13 L = 0.3592e-6; % phase inductance per meter
14 C = 0.196e-9; % phase capacitance per meter
15
16 % Calculated parameters
17 As = pi / 4 * (Dout^2 - Ds^2); % area of sheath cross section
18 Ls = 2e-7 * log(2*S / Ds); % sheath inductance per
19 G = 22.26*3/4; % G value from comsol
20 Rs = 1 / (As * sigma); % sheath resistance per meter
21 Zs = Rs + 1j * omega * Ls; % sheath impedance per meter
22
23 Z0 = (42.91 - 2.86i) * sqrt(2); % Example value for Z0 (characteristic impedance)
24 tau = sqrt(omega*C*(Ls*omega-1j*Rs)); % Example value for tau
25 X_sf = 2*omega*1e-7*log(2*S/Ds); % Example value for X_sf
26 l = 70e3; % Example value for l
27
28 % Define z range
29 z = linspace(0, l, 1000); % Example range for z
30
31 % Calculate propagation constants
32 k = sqrt(G * (Rs + 1i * omega * Ls));
33 gamma = sqrt(omega * C * (L * omega - 1i * R));
34 zeta = -omega * X_sf / Z0;
35
36 % Calculate V0+ and V0-
37 V0_plus = (V_L + Z0 * I_L) / 2 * exp(gamma * l);
38 V0_minus = (V_L - Z0 * I_L) / 2 * exp(-gamma * l);
39
40 % Calculate Icp

```

```

41 I_cp = V0_plus/Z0 * exp(-gamma * z) - V0_minus/Z0 * exp(gamma * z);
42
43 % Calculate coefficients
44 A = ((tau^2 + zeta) * V0_plus) / (gamma^2 - k^2);
45 B = ((tau^2 - zeta) * V0_minus) / (gamma^2 - k^2);
46
47 C2 = ((A + B) * exp(-k * l) - A * exp(-gamma * l) - B * exp(gamma * l)) / (exp(k * l) - exp(-k * l));
48
49 % Calculate Vsp(z)
50 V_sp = -(A + B) * exp(-k * z) + A * exp((-gamma - k) * l - k * z) + B * exp(gamma * l - k * (l - z)) + ...
51     (A + B) * exp(-2 * k * l + k * z) - A * exp((-gamma - k) * l + k * z) - B * exp((gamma - k) * l + k * z)
52     + A * exp(-gamma * z) + B * exp(gamma * z);
53
54 % Calculate dVsp/dz
54 dV_sp_dz = k*(A + B) * exp(-k * z) - k * A * exp((-gamma - k) * l - k * z) - k * B * exp(gamma * l - k * (l +
55     z)) + ...
55     k * (A + B) * exp(-2 * k * l + k * z) - k * A * exp((-gamma - k) * l + k * z) - k * B * exp((gamma - k) *
56     l + k * z) - gamma * A * exp(-gamma * z) + gamma * B * exp(gamma * z);
57
58 Eind = 1i * X_sf * I_cp;
59
60 % Calculate Isp(z)
60 I_sp = (1i * X_sf * I_cp - dV_sp_dz) / (Rs + 1i * omega * Ls);
61
62 % Plot Vsp(z)
63 figure('Position', [100, 100, 1000, 666]);
64 plot(z, abs(V_sp), 'b', 'DisplayName', 'Real(V_{sp}(z))');
65 xlabel('Distance along the length of the cable (m)', 'FontSize', 20);
66 ylabel('Sheath voltage phasor magnitude V_{sp}(z)', 'FontSize', 20);
67 % title('Voltage V_{sp}(z)', 'FontSize', 20);
68 % legend;
69 set(gca, 'FontSize', 18); % Set tick font size
70 grid on;
71
72 % Plot Isp(z)
73 figure('Position', [100, 100, 1000, 666]);
74 plot(z, abs(I_sp), 'b', 'DisplayName', 'Real(I_{sp}(z))');
75 ylim([100, 200]);
76 xlabel('Distance along the length of the cable (m)', 'FontSize', 20);
77 ylabel('Sheath current phasor magnitude I_{sp}(z)', 'FontSize', 20);
78 % title('Current I_{sp}(z)', 'FontSize', 20);
79 % legend;
80 set(gca, 'FontSize', 18); % Set tick font size
81 grid on;

```