ANALYSIS AND IMPLEMENTATION OF ERROR CORRECTING DOUBLE BUNDLE REED SOLOMON CODES FOR UK TELETEXT

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Abstract

The introduction of double bundle Reed Solomon (RS) error correcting codes in UK teletext is proposed. In order to implement this one or two rows of parity-check bytes must be added. Single bundle and double bundle codes are defined. The performance of UK teletext system is evaluated, when rows of parity-check bytes in the bundles are used. Then computer simulations are described. Next, both a software written coder and a decoder are described which have been tested in a laboratory environment. Results obtained from simulations and implementations are in good agreement with analytical results. The performance of the teletext system is found to be enhanced by the introduction of the RS codes.

I. INTRODUCTION

Recently, the performance results obtained by using error detection only, double error correction per page and an 8/4 Hamming code in UK teletext have been reported in [1], [2]. In [1] enhanced graphics is introduced in the Teletext system using a Differential Chain Coding (DCC) scheme. DCC appears to be more sensitive to bit-errors; therefore, it is proposed to introduce more powerful methods of error-correction by using single or double bundle RS coding schemes. This paper proposes the introduction of double bundle Reed Solomon (RS) error correcting codes in UK teletext. Its performance analysis is presented and compared with the performance of single and no bundle schemes. The performance is evaluated in terms of the probability of receiving a correct page for bit error probabilities of 10^{-3} and 10^{-4}. The use of single and double bundle RS codes in the North American Basic Teletext System (NABTS) is described in [3]-[7]. The teletext channel is a simplex channel and reception of teletext is sensitive to errors because there is no return channel. Therefore, the introduction of error correcting codes is very useful, especially if they are used to transmit software in the case of datacast. In particular, under such cases, it appears from the results presented in this paper that introducing double bundle RS codes may be a good solution. The Reed Solomon error correcting coding is based on the Galois field over 2^7 ($GF(2^7)$). The $GF(2^7)$ can be implemented by a 7-bit code, or a one-byte code (8 bits) in case one bit is used as a parity check. The basic data entity in UK teletext is 7 bits and one parity bit. This leads to a direct use of error correcting theory in practice. However, in implementation the byte level approach has advantages. The calculations can be carried out bytewise as is illustrated in [8]. A comparison between the performance of UK teletext and NABTS is presented by changing the parameters of NABTS into the parameters of UK Teletext because NABTS defines a row length of 32 bytes while UK Teletext uses 40 information bytes for each row. In [9] a study has been made to find the required amount of decoding capacity.

The paper is organized as follows. Section II describes the UK teletext. Coding and decoding properties are described in section III. Several types of probabilities appropriate to each of the three types of bundle schemes are explained in this section. The three bundle schemes are compared in section IV. Sections V and VI present the coding and decoding theory for error correcting teletext, respectively. The performance results obtained by computer simulations are discussed in section VII. An implementation scheme and experimental results are given in section VIII. Finally, conclusions are given in section IX.

II. TELETEXT

The teletext signal is transmitted during the Vertical Blanking Interval (VBI) of the television broadcasting signal. According to the Broadcast Teletext Specification (BTS) [10] a teletext dataline can be divided into synchronisation, addressing and information as shown in Fig. 1. First two Clock Run-In bytes appear for bit synchronisation. Then a framing code follows for byte synchronisation. The teletext data is then divided into magazines numbered from 1 to 8. These magazines consist of pages numbered from 0 to 99. Every page consists of a maximum of 32 rows. Usually the synchronisation is followed by the prefix or addressing, consisting of a magazine and line number. Then the characters follow, with 40 information bytes per row. Instead of the 40 information bytes a page header is provided with 8 bytes, Hamming coded page number, time and control data. The header is terminated with character bytes. These are displayed on the Teletext screen. It is always assumed that the synchronisation is established. Thus, the probability of a minimum of necessary data for a correct decoded page header is given by :

$$P_{header} = P_{prefix} (q^8 + 8pq^7)^8 \qquad (1)$$

Here p and q are, respectively, the probabilities of an incorrectly and correctly decoded bit. In Fig. 2 the major features of a teletext page are illustrated. After the page header a maximum of 23 information rows follows. Rows 24 to 29 are the so called ghost rows, which are used for linking purposes. In the present paper these consist of a designation code "DC" and page numbers. Rows 30 and 31 are page independent.
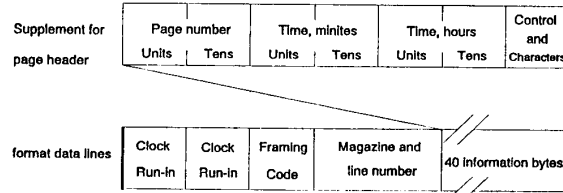
Supplement for page header	Page number		Time, minites		Time, hours		Control and Characters
	Units	Tens	Units	Tens	Units	Tens	

format data lines	Clock Run-In	Clock Run-In	Framing Code	Magazine and line number	40 information bytes

Figure 1   A teletext Data-Line

row
nr

0
1
2

synchronisation + address bytes

| page nr. | general information | time |

23 rows of 40 bytes

visible information

23

27

| D C | page number | page number | page number | page number | page number | ghost row
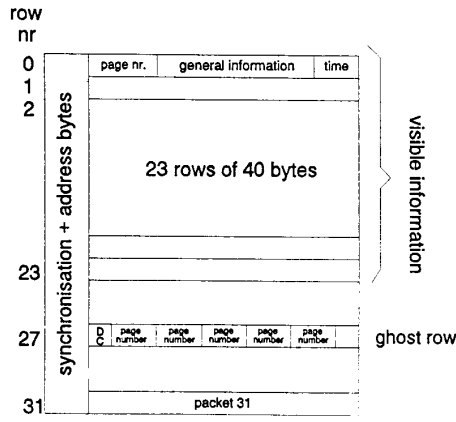
31

packet 31

Figure 2   Outline of a teletext page

III. CODING AND DECODING PROPERTIES

A. The decoder

The decoder can be seen as a black box. We have to add some extra parity bytes to the information of the teletext row, although the total number of bytes in a row must be 40. Then the decoding possibilities are as shown in table 1. According to the definition, the location of an erroneous byte cannot be found, while the location of an erasure byte is known, and is found by parity checking. The outcome of a decoding can be either correct, a failure or a false error correction. A correct decoding occurs if all bytes are correct or the decoder has to use its full capabilities to obtain the correct decoding. A failure occurs if the number of failure bytes is greater than the correction capacity of two erasures bytes or if one or both parity bytes are erasures. A false

Table 1 Decoding possibilities of the decoder

errors	erasures
1	0
0	1
0	2

error correction occurs if the parity bytes are erroneous, or with specific combinations of erasures and erroneous bytes. In this paper, we have not considered the undetectable and uncorrectable errors [9], [13], because the probability of false error correction conveys us enough information for implementing the codes for teletext. The probability of an erasure byte, defined as every possible combination of odd erroneous bits, p_{Bnf} equals:

$$P_{Bpf} \triangleq \sum_{a=0}^{8} \binom{8}{2a+1} p^{2a+1} q^{8-(2a+1)} \tag{2}$$

where a varies from 0 to 8. The probability of an erroneous byte p_{Bnpf}, which occurs if there is an even number of erroneous bits, can be defined as :

$$P_{Bnpf} \triangleq \sum_{a=1}^{4} \binom{8}{2a} p^{2a} q^{8-2a} \tag{3}$$

B. The row decoding

A teletext row consists of 40 information bytes. A row decoding succeeds, with probability p_{cr}, if both parity bytes are correct, the information bytes are received properly or the decoder has to use its full capabilities.

$$P_{cr} = P_{cpB} ((q^8)^{38} + P_{r1Be} + P_{r2Be} + P_{r1Br}) \tag{4}$$

Here p_{cpB} is the probability of correct parity bytes given by:

$$P_{cpB} = (q^8)^2 \tag{5}$$

The probability of a row decoding with one erasure and 37 correct bytes (p_{r1Be}) equals:

$$P_{r1Be} = 38 P_{Bpf} (q^8)^{37} \tag{6}$$

The probability of two horizontal erasures, combined with 36 correct bytes (p_{r2Be}) can be written as:

$$P_{r2Be} = \binom{38}{2} (P_{Bpf})^2 (q^8)^{36} \tag{7}$$

The probability of a row decoding, leading to a correct result, with one erroneous byte (p_{r1Br}) equals:

$$P_{r1Br} = 38 P_{Bnpf} (q^8)^{37} \tag{8}$$

The cause of a row false error correction can be twofold. The first reason is due to the possibility of erroneous parity bytes. One or two erroneous parity bytes do lead to a false error correction. The probability of such an occurrence equals:

$$\begin{aligned} P_{fcr,1} &= P_{fcpB} \\ &= 2 P_{Bnpf} q^8 + P_{Bnpf}^2 \\ &= P_{Bnpf} (2q^8 + P_{Bnpf}) \end{aligned} \tag{9}$$

The second reason is specific combinations of erasures and errors when the parity bytes are correct. This occurs when the decoder first counts the number of parity failures and the rest of the executions based on the numbers and locations of the parity failures. Within the area of bit error probability under research some combinations leading to a false error correction are:

- One erasures byte and an erroneous byte (A).
- Two erasures and an erroneous byte (B).

This is illustrated in Fig. 3(a). In these cases the erroneous bytes are not corrected while the erasures bytes are corrected wrongly. This leads to a total probability of a row false error correction of:

$$P_{fcr} = P_{fcpB} + P_{cpB} (2 \binom{38}{2} P_{Bpf} P_{Bnpf} (q^8)^{36} + 3 \binom{38}{3} P_{Bpf}^2 P_{Bnpf} (q^8)^{35}) \tag{10}$$

The case of more erroneous and erasures bytes leading to a false error correction is not considered. If there is not a correct row decoding or a false error correction, then there will be a decoding failure.

$$P_{fr} = 1 - P_{cr} - P_{fcr} \qquad (11)$$

Mortimer et al. [4,6] have given two definitions of a correct row. According to the first one it is assumed that any combination of one or two faulty bits do lead to a correctly decoded row. Using the results in [4], for UK teletext parameters:

$$P_{cr\_M\_1} = q^{320} + 320pq^{319} + 51040p^2q^{318} \qquad (12)$$

According to the second approach in [6] more combinations do lead to a correct decoding.

$$P_{cr\_M\_2} = q^{320}[1 + 40((1+\frac{p}{q})^8 - 1) + \binom{40}{8}\sum_{t=2}^{8}\sum_{s=sup(t-3,1)}^{inf(t,4)}\binom{8}{2s-1}\binom{8}{2t-2s+1}(\frac{p}{q})^{2t}] \qquad (13)$$

C. The packet properties

A data packet is a combination of a prefix and the row described in [11]. The probability of a correct prefix (p_{prefix}) is:

$$P_{prefix} = (q^8 + 8pq^7)^2 \qquad (14)$$

If the probability of an undetected and a falsely error-decoded prefix is neglected, then the probability of a lost prefix ($p_{lprefix}$) equals:

$$P_{lprefix} = 1 - P_{prefix} \qquad (15)$$

A packet is correctly decoded (p_{cp}) if both prefix and row decoding are correctly executed.

$$P_{cp} = P_{prefix} P_{cr} \qquad (16)$$

A false error correction occurs if the prefix is decoded correctly and the row decoding is made erroneously.

$$P_{fcp} = P_{prefix} P_{fcr} \qquad (17)$$

Now the calculation of the probability of a specific falsely corrected byte is described as follows. A specific byte, denoted with an arrow in Figs. 3(b) and 3(c), is falsely corrected, if the parity byte is erroneous and the byte has a parity failure (A), or it is erroneous (B), or if it has a byte with failures in combination with another erroneous (C) or erasure byte (D) or in combination with another erasure and erroneous byte (E).

If the parity is correct, then there is a combination of an erasure or an erroneous byte (F) and (G), or the specific byte is an erasure in combination with an erasure and an erroneous byte (H) or the specific byte is erroneous in combination with two erasures bytes (I).

$$P_{fcB} = P_{prefix}(P_{fcpB}(P_{Bpf}q^{296} + P_{Bnpf}q^{296} + \\ 37P_{Bpf}\ P_{Bnpf}q^{288} + 37P_{Bpf}\ P_{Bpf}q^{288} + \\ 666P_{Bpf}\ P_{Bpf}\ P_{Bnpf}q^{280}) + \\ P_{cpB}(37P_{Bpf}\ P_{Bnpf}\ q^{288} + \\ 37P_{Bnpf}\ P_{Bpf}\ q^{288} + \\ 666P_{Bpf}\ P_{Bpf}\ P_{Bnpf}\ q^{280} + \\ 666P_{Bnpf}\ P_{Bnpf}\ P_{Bpf}\ q^{280})) \qquad (18)$$
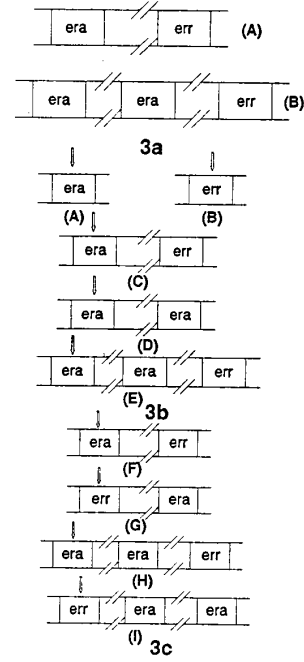

Figure 3   (a) False error corrected row, correct parity.
(b) Specific false error corrected byte, erroneous parity. (c) Specific false error corrected byte, correct parity.

After decoding of a row a byte is falsely corrected or correctly decoded:

$$P_{cB} = 1 - P_{fcB} \qquad (19)$$

A packet decoding failure occurs if the prefix was not found, or if the prefix was found, but a decoding failure occurred during the row decoding:

$$P_{fp} = P_{lprefix} + P_{prefix} P_{fr} \qquad (20)$$

A row without error correcting coding is correctly decoded if the prefix is correctly decoded and all 40 bytes are received correctly. This time 38 bytes are used to achieve a fair comparison.

$$P_{crwec} = (q^8)^{38} \qquad (21)$$

D. The no bundle

Assume we only receive λ rows for decoding as is illustrated in Fig. 4(a). This scheme is known as the no bundle. A no bundle page (P) is correctly decoded if the header and all λ packets (p) are correctly decoded.

$$P_{cPnb}(\lambda) = P_{header}(P_{cp})^\lambda \qquad (22)$$

A page using the no bundle coding scheme is falsely corrected if one or more rows are falsely corrected:

$$P_{fcPnb}(\lambda) = P_{header}(1 - (1 - P_{fcp})^\lambda) \qquad (23)$$

This is the probability of 1 or more false error corrections in an arbitrary pattern. A no bundle has a decoding failure if the header is lost or one or more packets in it do have a decoding failure.

$$P_{fPnb}(\lambda) = P_{lheader} + 1-(1-p_{fp})^{\lambda} \qquad (24)$$

The no bundle has only horizontal error detection and correction possibilities. To achieve vertical error capabilities the single- and double bundle coding scheme are introduced.

E. The probabilities of packet failures

The probability of all packets being decoded either correctly or falsely corrected equals:

$$P_{0fp}(\lambda) = (1-p_{fp})^{\lambda} \qquad (25)$$

This predicts the possibility of packet failures. If there is one failure of a packet in a bundle, then the single bundle or double bundle is able to restore the bundle, as will be explained later on. The probability of such an occurrence

equals:

$$P_{1fp}(\lambda) = \lambda p_{fp}(1 - p_{fp})^{\lambda-1} \qquad (26)$$

Only the double bundle is able to correct two failures in a packet. This probability equals:

$$P_{2fp}(\lambda) = \binom{\lambda}{2}(p_{fp})^2(1-p_{fp})^{\lambda-2} \qquad (27)$$

F. The single and double bundle schemes

i) Coding both bundles

The coding schemes for a page using single and double bundle error correcting are shown in Figs. 4(b) and 4(c). When using the single bundle, the data is vertically coded in an appropriate manner so that the bundle has error correcting properties in the columns. One extra row is added although each column uses two parity bytes. The trick is done by making the vertical information polynomial 2 λ long [8]. Then the information polynomial is divided in half and is separated in two different columns. One of the encoded parity bytes is placed in each of the columns. The parity bytes are all placed in the last row. The two columns, each one half of the information polynomial, are separated by 19 bytes. This is done for the improvement of burst error correcting properties. Coding the double bundle the first λ - 2 packets consist of the information and check bytes. The last two packets consist of only check bytes. Every column will be vertically coded. The check bytes in every column can be found in the last two rows of the column.

ii) Decoding the bundles

After all incoming rows are decoded sequentially we make the following observations :
- no erasures are left because one or two erasures are corrected, and more erasures or a non-decoded prefix lead to a vertical decoding failure.
- errors due to false error corrections remain.

iii) The bundles analysed

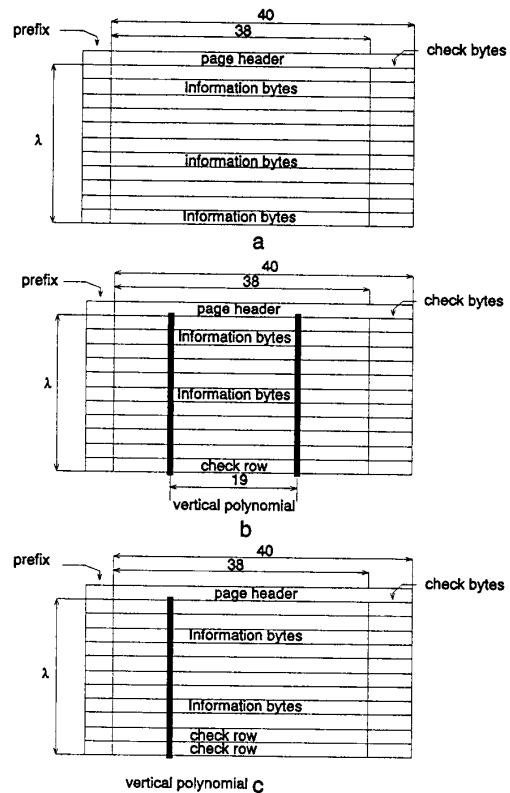Decoding analysis is discussed in three parts, the cases of no, one or two packet failures.

Figure 4   (a) No bundle coding scheme. (b) Single bundle coding scheme. (c) Double bundle coding scheme.

No packet failures signalled

First if no packet failure is reported, then a correct vertical decoding is achieved when all bytes in the vertical polynomial are correct or one of the information bytes is falsely corrected. This means for the single bundle, that the probability of correct decoding of a vertical codeword is:

$$P_{vert0sb}(\lambda) = (p_{cB})^{2\lambda} + \binom{2\lambda-1}{2\lambda-3}P_{fcB}(p_{cB})^{2\lambda-1} \qquad (28)$$

The single bundle is correctly decoded if all 19 vertical decodings are correct then

$$P_{csb0}(\lambda) = (P_{vert0sb}(\lambda))^{19} \qquad (29)$$

The probability of a double bundle correctly decoded column is:

$$P_{vert0db}(\lambda) = (p_{cB})^{\lambda} + \binom{\lambda-2}{\lambda-3}(p_{cB})^{\lambda-1}P_{fcB} \qquad (30)$$

There must be 38 correct decodings to form a correctly decoded bundle:

$$P_{cdb0}(\lambda) = (P_{vert0db}(\lambda))^{38} \qquad (31)$$

A bundle column is falsely error corrected if 2 or more information bytes are falsely corrected. This means that for the single bundle:

$$P_{fccsb0}(\lambda) = \sum_{x=2}^{2(\lambda-1)} \binom{2(\lambda-1)}{2(\lambda-1)-x} p_{cB}^{2(\lambda-1)-x} p_{fc1B}^{x} \quad (32)$$

For the double bundle:

$$P_{fccdb0}(\lambda) = \sum_{x=2}^{\lambda-2} \binom{\lambda-2}{\lambda-2-x} (p_{cB})^{\lambda-2-x} (p_{fcB})^{x} \quad (33)$$

The single bundle will be falsely corrected (p_{fcsb0}) if one or more columns or the check row is falsely corrected:

$$P_{fcsb0}(\lambda) = 19 P_{fccsb0}(\lambda) + P_{fcp}(\lambda) \quad (34)$$

The probability of a falsely corrected double bundle will be:

$$P_{fcdb0}(\lambda) = 38 P_{fccdb0}(\lambda) + 2 P_{fcp}(\lambda) P_{cp}(\lambda) + (P_{fcp}(\lambda))^{2} \quad (35)$$

The probability of a falsely corrected double bundle is higher than the falsely corrected single bundle. If there is no correct (p_{cb0}) or falsely corrected bundle (p_{fcb0}), then the bundle will have a probability of failures given by:

$$P_{fb0}(\lambda) = 1 - P_{cb0}(\lambda) - P_{fcb0}(\lambda) \quad (36)$$

One packet failure signalled
The outcome of the decoding depends upon where the failure is made. If the failure is made in the information packets, then the bundle should use its decoding capacity, and the rest of the information bytes in each decoding should be correct. If the failure is made in the check row(s), then the bundle is declared to be decoded with no bundle properties. In the case of the single bundle:

$$P_{csb1}(\lambda) = \frac{\lambda-1}{\lambda} ((p_{cB}^{2(\lambda-2)} p_{cB}^{2})^{19} + \frac{1}{\lambda} (p_{cp})^{(\lambda-1)}) \quad (37)$$

In the case of the double bundle:

$$P_{cdb1}(\lambda) = \frac{\lambda-2}{\lambda} ((p_{cB})^{\lambda-3} (p_{cB})^{2})^{38} + \frac{2}{\lambda} (p_{cp})^{\lambda-2} \quad (38)$$

False corrections can be introduced by the packets without decoding failures or by the check packets. Therefore, a falsely corrected single bundle can be expressed as:

$$P_{fcsb1}(\lambda) = \frac{\lambda-1}{\lambda} (p_{fcp} + 1 - (1-p_{fcp})^{\lambda-2})) + \frac{1}{\lambda} (1 - (1-p_{fcp})^{\lambda-1}) \quad (39)$$

Similarly for the double bundle:

$$P_{fcdb1}(\lambda) = \frac{\lambda-2}{\lambda} (2 p_{fcp} p_{cp} + (p_{fcp})^{2} + 1 - (1-p_{fcp})^{\lambda-3}) + \frac{2}{\lambda} (1 - (1-p_{fcp})^{\lambda-2}) \quad (40)$$

The failure probability of the single and double bundles can be expressed by adapting (36) to the case of one failure packet.

Two packet failures reported
In this case only the double bundle correction scheme is able to deliver a correct decoded bundle. If the failures occur in packets 1 to λ - 1, then rows λ - 1 and λ must be correctly decoded. If both packets λ - 1 and λ have a decoding failure, then the bundle is said to be decoded with the no bundle properties, and the probability of correct decoding is

$$P_{cdb2}(\lambda) = \frac{(\lambda-2)(\lambda-3)}{\lambda(\lambda-1)} ((p_{cB})^{\lambda-4} (p_{cB})^{2})^{38} + \frac{2}{\lambda(\lambda-1)} (p_{cp})^{\lambda-2} \quad (41)$$

A falsely corrected bundle occurs if packets λ - 1 or λ or both are falsely corrected or a false correction has been made on one or more of the rest of the packets.

$$P_{fcdb2}(\lambda) = \frac{2(\lambda-2)(\lambda-3)}{\lambda(\lambda-1)} (2 p_{fcp} p_{cp} + (p_{fcp})^{2} + 1 - (1-p_{fcp})^{\lambda-4}) + \frac{2}{\lambda(\lambda-1)} (1 - (1-p_{fcp})^{\lambda-2}) \quad (42)$$

If one failure is reported in packet λ - 1 or λ and the other in packets 1 to λ - 2, then a double bundle failure appears. It is not possible to repair such a failure. Therefore, the probability of a double bundle failure equals:

$$P_{fdb2}(\lambda) = 1 - P_{cdb2}(\lambda) - P_{fcdb2}(\lambda) \quad (43)$$

iv) The overall properties of single bundle decoded pages
A page using single bundle coding is correctly decoded if the header is correctly decoded and if no or one failure of a packet occurs. This can be expressed as:

$$P_{cPsb}(\lambda) = P_{header} (P_{sb0fp}(\lambda) P_{0fp}(\lambda) + P_{sb1fp}(\lambda) P_{1fp}(\lambda)) \quad (44)$$

The page is falsely corrected if the header is decoded and the bundle is falsely corrected. It is expressed by rewriting equation (44) for false error correction case. If the page is neither correct nor falsely corrected then it will lead to failures. This occurs when the header is lost, or when the header is detected, and there is no correct or falsely corrected bundle.

$$P_{fPsb}(\lambda) = P_{lheader} + P_{header} (1 - P_{csb}(\lambda) - P_{fcsb}(\lambda)) \quad (45)$$

v) The overall properties of a page using double bundle decoding
A page using double bundle error correction can be correctly decoded if the header is correctly decoded and if no, one or two failures of packets occur.

$$P_{cPdb}(\lambda) = P_{header} (P_{cdb0}(\lambda) P_{0fp}(\lambda) + P_{cdb1}(\lambda) P_{1fp}(\lambda) + P_{cdb2}(\lambda) P_{2fp}(\lambda)) \quad (46)$$

The probability of a false error correction of the double bundle can also be calculated using (46) adapted to the false error correction case.

$$P_{fcPdb}(\lambda) = P_{header}(P_{fcdb0}(\lambda)P_{0fp}(\lambda) +$$
$$P_{fcdb1}(\lambda)P_{1fp}(\lambda) + \qquad (47)$$
$$P_{fcdb2}(\lambda)P_{2fp}(\lambda))$$

The page using double bundle will lead to a failures decoding if the header is not found or if the bundle is not correct or falsely corrected:

$$P_{fPdb}(\lambda) = P_{lheader} +$$
$$P_{header}(1 - P_{cdb}(\lambda) - P_{fcdb}(\lambda)) \qquad (48)$$

IV. A COMPARISON OF THE THREE BUNDLES

The probability of packet failure is calculated using equations (25), (26) and (27). For a bit error rate of 10^{-4} the probability of no packet failure equals nearly one as is illustrated in Fig. 5. When the bit error probability is 10^{-3}, then the p_{0fp} is still nearly 1 but decreases with bundle length. The probability of one failure of a packet has some influence. The probability of two packet failures is negligible for both values of bit error rate (10^{-3} and 10^{-4}). The probability of correct bundles given the number of packet failures is shown in Fig. 6. Given the number of packet failures the correcting properties of a single or double bundle for one or two failures are the same, (a..f; g,h). Only when two failures occur (j and k) is the double bundle decoding scheme able to deliver a correct bundle. Fig. 7 illustrates that it can be very useful to introduce error correcting coding. For a bit error probability of 10^{-3} the pages will be blurred (k). Bundle decoding still delivers a correct page. Because the probability of two row failures is very low within the considered area, the probabilities of a correct single or double bundle are about equal (Compare b and c, and f and g). According to the approach of Mortimer et al. [6] the double bundle coding scheme will still deliver a correct bundle at p = 10^{-3} (e). At that point according to our approach there is a little chance the bundle will not be correctly decoded. A page with a decoding failure
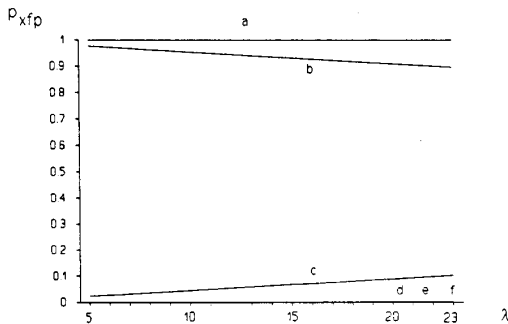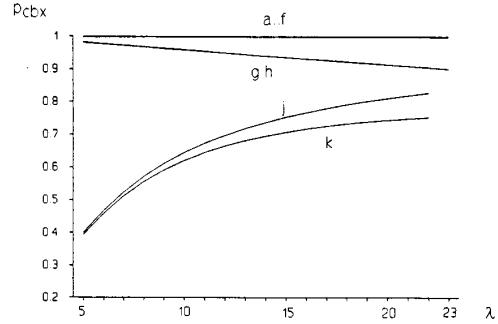

Figure 6   Probability of a correct single bundle p_{csbx} or double p_{cdbx} given the number of packet failures

(a) p = 10^{-4} no packet failures p_{csb0}, (b) p = 10^{-4} 1 packet failure p_{csb1}, (c) p = 10^{-4} no packet failures p_{cdb0}, (d) p = 10^{-4} 1 packet failure p_{cdb1}, (e) p = 10^{-3} no packet failures p_{csb0}, (f) p = 10^{-3} no packet failures p_{cdb0}, (g) p = 10^{-3} 1 packet failure p_{csb1},
(h) p = 10^{-3} 1 packet failure p_{cdb1}, (j) p = 10^{-4} 2 packet failures p_{cdb2}, (k) p = 10^{-3} 2 packet failures p_{cdb2}.
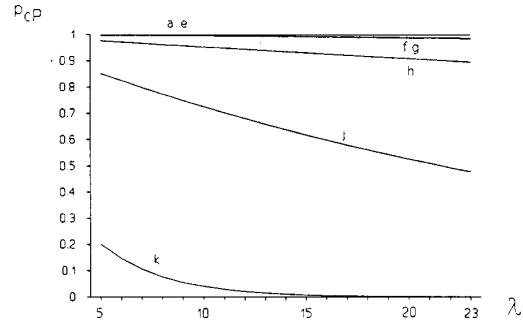

Figure 7   Probability of correct pages (p_{cp})

(a) p = 10^{-4} using no bundle p_{cPnb}, (b) p = 10^{-4} using single bundle p_{cPsb}, (c) p = 10^{-4} using double bundle p_{cPdb}, (d) p = 10^{-3} according to [6] p_{cPdb_M}, (e) p = 10^{-4} according to [6] p_{cPdb_M}, (f) p = 10^{-3} using single bundle p_{cPsb}, (g) p = 10^{-3} using double bundle p_{cPdb},
(h) p = 10^{-3} using no bundle p_{cPnb}, (j) p = 10^{-4} without error correction p_{cPwec}, (k) p = 10^{-3} without error correction p_{cPwec}.

Figure 5   Probability of packet failures

(a) p = 10^{-4} no packet failures p_{0fp}, (b) p = 10^{-3} no packet failures p_{0fp}, (c) p = 10^{-3} one packet failure p_{1fp}, (d) p = 10^{-3} two packet failures p_{2fp}, (e) p = 10^{-4} one packet failure p_{1fp}, (f) p = 10^{-4} two packet failures p_{2fp}.

will not be correctly decoded. A page with a decoding failure can be detected, and therefore it is not such a problem as a falsely corrected page. The detection can be signalled, for instance, to the user. The probability of a decoding failure increases with bundle length as is shown in Fig. 8. The case without error correcting coding does not yield a decoding failure, except when a prefix is not correctly decoded. However, because a decoding failure of the Hamming coded bytes is very low, the probability of a page decoding failure is also very small. In both cases, p = 10^{-3} and p = 10^{-4}, a decoding failure of the no bundle is assumed to be the most probable. This occurs because a no bundle decoding failure can be caused by a prefix failure or a horizontal decoding

failure while the others (single bundle and double bundle) can correct one or more horizontal decoding failures. A falsely corrected page is a decoded page which still contains erroneous bytes. This probability increases also with the bundle length. The calculated results are shown in Fig. 9. This probability is also known as the average number of errors per page (ANEP). The ANEP can be calculated from:

$$ANEP(\lambda) = 40 \ \lambda \ p_{fcP} \tag{49}$$

As with the page decoding failures, the vertical error correction gives an improvement in the quality of the page. At both bit error probabilities the probability of a falsely corrected page without error correction (a and b) is very
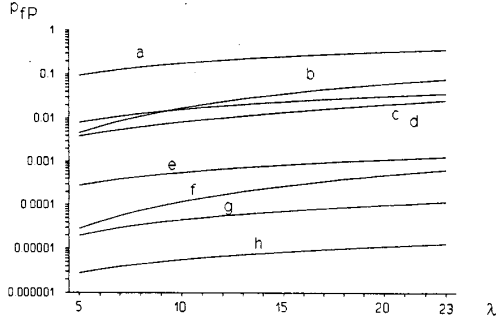
Figure 8   Probability of a page decoding failure (p_{fp})

(a) $p = 10^{-3}$ using no bundle p_{fPnb}, (b) $p = 10^{-3}$ using single bundle p_{fPsb}, (c) $p = 10^{-4}$ using no bundle p_{fPnb}, (d) $p = 10^{-3}$ using double bundle p_{fPdb}, (e) $p = 10^{-3}$ without error correction p_{fPwec}, (f) $p = 10^{-4}$ using single bundle p_{fPsb}), (g) $p = 10^{-4}$ using double bundle p_{fPdb}, (h) $p = 10^{-4}$ without error correction p_{fPwec}.
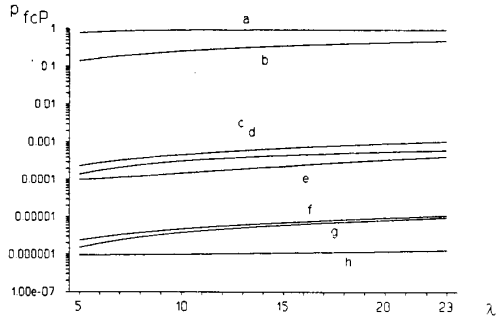
Figure 9   Probability of false error corrected pages (p_{fcp})

(a) $p = 10^{-3}$ without error correction p_{fcPwec}, (b) $p = 10^{-4}$ without error correction p_{fcPwec}, (c) $p = 10^{-3}$ using no bundle p_{fcPnb}, (d) $p = 10^{-3}$ using double bundle p_{fcPdb2}, (e) $p = 10^{-3}$ using single bundle p_{fcPsb}, (f) $p = 10^{-4}$ using no bundle p_{fcPnb}, (g) $p = 10^{-4}$ using double bundle p_{fcPdb}, (h) $p = 10^{-4}$ using single bundle p_{fcPsb}

high. The vertical decoding by bundle decoding d, e and g and h gives an improvement by using only horizontal decoding (c and f). Although still negligible, the probability of a falsely corrected page using double bundle decoding is somewhat higher compared to the single bundle.

V. ENCODER THEORIES FOR TELETEXT WITH ERROR CORRECTION

In the encoder there must be an extension of the coding techniques. In the encoder parity check data are added to the information bytes u(x). This leads to a transmitted codeword c(x). We used the encoder as it is described in [8], adapted to the UK Teletext environment. The encoders are implemented by using shift registers with feedback connections. To find the right connections and calculation constants the generator polynomial g(x) must be known.

$$g(z) = z^2 + \alpha^{32}z + \alpha^3 \tag{50}$$

where $\alpha^{32} = 06_H$
$\alpha^3 = 08_H$ α^{32} and α^3 both coefficients in GF(2^7)
z : one clock delay

An encoder for UK teletext is shown in Fig 10. During the first k periods the switches can be found in the A - position. The "k" should be 38, $2\lambda - 2$ and $\lambda - 2$ respectively for row, single bundle and double bundle decoding. Then the characters found in the input stream are passed on to the output and fed into the register. They form the input at I1 of the adder. The output O1 is multiplied at the byte level by, respectively, the coefficients in GF(2^7). Using the internal summation the input I2 is formed according to

$$I(z) = \alpha^3 z^2 + \alpha^{32} z \tag{51}$$

During the last 2 periods the switches are in position B. Then the check bytes are fed out of the shift register to yield the output codeword.

VI. DECODER THEORIES FOR ERROR-CORRECTING TELETEXT

In the encoder the check data are added to the information bytes. This leads to the transmitted codeword c(x). In the Teletext channel errors are introduced, as is illustrated in Fig. 11. The decoder extracts the check data from the received data stream. With this data the errors are estimated. The errors are added in GF(2^7) to the received data to estimate the transmitted data by using:
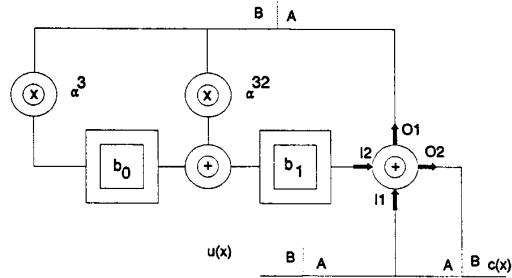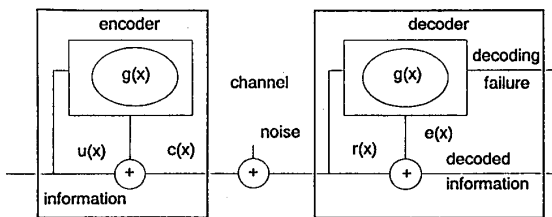
Figure 10   Coder for UK Teletext

Figure 11   Block diagram of a system with coder, channel and decoder

$$decoded\ information = e(x) + r(x) \quad (52)$$

If the decoder detects erasure check data in the rows or too many erasures in the information, then a decoder overflow or decoding failure is indicated. The decoding theories introduced by Mortimer et al. in [3] are, of course, usable in the case of UK Teletext, but they must be adapted. The reason for this is due to the changed coding and decoding strategy.

A. One erasure

If one parity failure in the information bytes is detected, then the erasures byte must to be corrected. The erasure polynomial e(x) is given by:

$$e(x) = e_a x^a \quad (53)$$

The error value is denoted by e_a, and the location is denoted by x^a where "a" indicates where the error can be found. According to error correcting theory the error can be found by calculating the syndrome $s(\alpha)$.

$$E(\alpha) = s(\alpha) \quad (54)$$

In the particular case of UK Teletext the syndrome can be represented as:

$$S(x) = r_0 + r_1 x + r_2 x^2 \ldots r_{39} x^{39} \quad (55)$$

Where:

r_0 : received parity byte number 0
r_1 : received parity byte number 1
r_2 : received information byte number 0
.
r_{39} : received information byte number 37

Because S(x) can be found and x^a is known, the error value can be calculated as follows:

$$e_a = \frac{E(\alpha)}{\alpha^a} = \frac{s(\alpha)}{\alpha^a} \quad (56)$$

B. Two erasures

In the case of two erasures information bytes, with the rest of the bytes correct, the erasure polynomial can be denoted as:

$$E(x) = e_a x^a + e_b x^b \quad (57)$$

The erasures values at location "a" and "b" are respectively e_a and e_b.
Now the syndrome of α ($s(\alpha)$) and α^2 ($s(\alpha^2)$) must be

calculated, while the resulting two independent equations make it possible to find the two unknowns e_a and e_b:

$$s(\alpha) = e_a \alpha^a + e_b \alpha^b$$
$$s(\alpha^2) = e_a \alpha^{2a} + e_b \alpha^{2b} \quad (58)$$

The e_b can be written as:

$$e_b = \frac{\alpha^{2a} s(\alpha) + \alpha^a s(\alpha^2)}{\alpha^{b+2a} + \alpha^{2b+a}} \quad (59)$$

The erasure value at point "a" can be found with:

$$e_a = \frac{s(\alpha) + e_b \alpha^b}{\alpha^a} \quad (60)$$

C. One error

If no parity failures are signalled, then the decoder will determine whether there are any errors. This is done by calculating the syndrome. If the syndrome is not equal to zero, then it is assumed that an error has been made. When an error occurs, then the location of the error "a" can be found from:

$$\frac{s(\alpha^2)}{s(\alpha)} = \frac{e_a \alpha^{2a}}{e_a \alpha^a} = \alpha^a \quad (61)$$

The logarithm of the result provides the location "a". Then the error value itself can be found as follows:

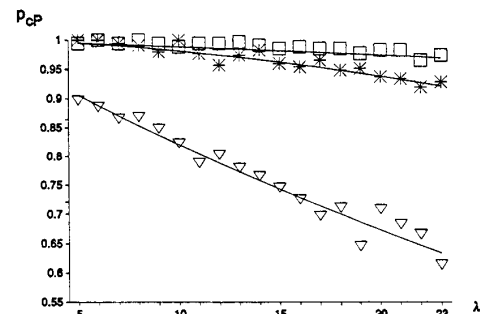$$\frac{s(\alpha)}{\alpha^a} = \frac{e_a \alpha^a}{\alpha^a} = e_a \quad (62)$$

Figure 12   Simulation results, probability of correct pages (p_{cp}) p = 10^{-3}

□ double bundle, * single bundle, ∇ no bundle

VII. SIMULATIONS

Simulations have been executed to check the performance analysis. The simulations have been executed on a HP Vectra AT-386 PC. All programs are written in Turbo Pascal 5.5. For every bundle length λ, 350 simulation runs were made. The channel is based on the Pascal defined random number generator. This generator is initialised with the chosen bit error rates. The actual measured bit error rate during simulations is most of the time, within an accuracy of 10% but deviations of 25% were indicated. We began by discussing the outcome of the simulation when p is taken to be 10^{-3}. The probability of correct pages is shown in Fig. 12. Fig. 13 shows the simulation results for the
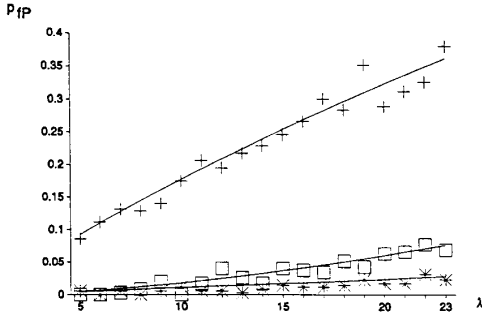
P_fp

Figure 13   Simulation results, probability of decoding failures (p_{fp}) $p = 10^{-3}$

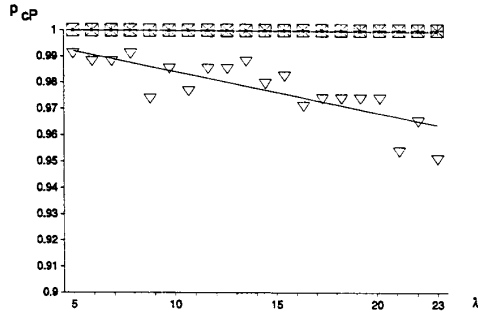+ no bundle, □ single bundle, * double bundle

P_cP

Figure 14   Simulation results, probability of correct pages (p_{cp}) $p = 10^{-4}$

□ double bundle, * single bundle, ∇ no bundle

probability of decoding failures. The solid lines are the analytical results. The probability of the correct page for decoded no bundle ("∇"), single bundle ("*") as well as the double bundle ("□") are as expected. The outcome of the decoding failures can be found in Fig. 13. These also agree with analytical results. The simulations were also executed with a bit error probability of 10^{-4}. These are illustrated in Fig. 14. The no bundle performance is somewhat lower, but it is still nearly one.

VIII. IMPLEMENTATION

To check the practical significance of the developed theories, implementations were realized. The transmitter as well as the receiver were formed by a PC. The experimental setup is illustrated in Fig. 15. The transmitter, a HP Vectra PC, was equipped with a special card. The card consists of memory consisting of encoded pages as well as a teletext transmitter. The channel noise was generated by a noise generator. The receiver is also a HP Vectra PC provided with a teletext receiver card. The received rows can be extracted from the card and after forming bundles they can be processed. The first question which arises is what noise level is to be applied to the channel. The configuration setup is presented later. Finally, the results are shown and discussed. During the experiments a bit error probability of 10^{-3} was chosen.
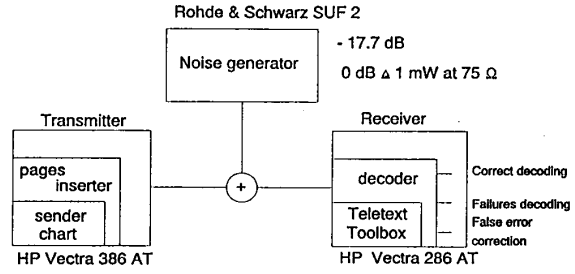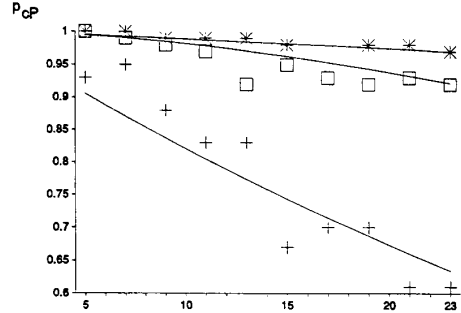

Figure 15   Experimental setup.

P_cP

Figure 16   Implementation results, probability of correct pages (p_{cp}) $p = 10^{-3}$

□ double bundle, * single bundle ∇ no bundle

In case of binomial distributions the normalised power can be calculated as follows:

$$\sigma^2 = 7360 \; \frac{1}{1000} \; (1 - \frac{1}{1000}) \; \approx 7 \qquad (63)$$

This is also explained in [12]. Now the noise level (N_l) can be calculated according to:

$$N_l = 20 \log \left(\frac{\frac{\sigma}{r}}{\sqrt{P \; r}} \right) \qquad (64)$$

Where :

  σ : Normalised voltage

  r   : Impedance of the generator and the rest of the circuit

  P   : reference power of the generator

For $\sigma^2 = 7$, $r = 75\Omega$ and $p = 10^{-3}$, using (64) $N_l = -17.7$ dB. When this level was indicated, the measured bit error rate at the receiver side was as expected. The results can be found in Fig. 16. Because of some practical problems, the difference between experimental and analytical results is higher compared to the simulations. One can conclude that the probabilities of correct decodings are as expected.

IX. CONCLUSIONS

The introduction of error correcting coding provides an improvement of the teletext system performance even with single reception. The computational results presented in this

paper are for bit error probabilities 10^{-3} and 10^{-4}. It is observed that the probability of a correct decoded page increases and the probability of obtaining incorrect information due to false error correction is almost negligible. The simulation and implementation results are very close to the analytical expectations. The solution can be implemented in the existing teletext transmission system without any problem. It can be made downward compatible by transmitting the check data on seperate pages. The single bundle performance is always found to be better than the no bundle case. The performance of the single bundle in terms of the probability of a falsely corrected page is better then the double bundle performance. For that reason one should use the single bundle coding scheme to transmit software or, for instance, DCC coded pictures. The double bundle performance is superior compared to the single bundle in terms of correction and failure decoding properties. It is a solution for more general problems. One of the disadvantages of the developed error correcting system is a decrease of useful visible information per page. This can be overcome by sending the check data on separate pages with the described linking method.

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