

High-Resolution Numerical Modelling and Sensitivity Analysis of Lunar Lava Tube Gravity Anomalies

by

Jasper Kuba Geijsberts

to obtain the degree of Master of Science

at the Delft University of Technology,

to be defended publicly on Monday, September 14, 2026, at 14:00.

Student number:	5331587	
Project duration:	December, 2025 – August, 2026	
Thesis committee:	Dr. Hom Nath Gharti,	Queen's University, daily supervisor
	Dr. ir. W. van der Wal,	TU Delft, Responsible supervisor
	Dr. ir. E. J. O. Schrama,	TU Delft, Chair
	Dr. ir. B. C. Root,	TU Delft, Independent Member

Preface

This thesis completes a now six-year journey of mine toward becoming an aerospace engineer. I intended to start this thesis, hoping to learn something new, as well as work on something related to lunar exploration, instead of just applying knowledge I've already learned. I think I can say, with certainty, that this desire was more than fulfilled, as I got to dive deeper into the geophysical and geological side of space exploration.

Writing this thesis at the Department of Geological Sciences and Geological Engineering at Queen's University in Kingston, Canada, has been one of the most rewarding and wonderful experiences in my time as a student. I met some of the most superb people, who've made my time in Canada extremely fulfilling. I learned from every person I met, while also enjoying free time to experience Canada's people, culture and nature.

I am deeply grateful to my supervisor, Hom Nath, who, through all our weekly meetings, both taught and guided me through this master's. His ambition and support inspired me to go above and beyond as much as I could. I also want to thank Wouter and Alex for your precise feedback and for making this experience a possibility. It wouldn't have been possible without you.

I would also like to thank my family and friends, who are closest to me. Firstly, my mom, my dad, and my brothers, who always supported me with whatever I chose to do, for which I am forever grateful. I want to thank Tori for always being there to support me and making me smile throughout my master's. I also want to thank everyone at the DESEL lab (and friends), including Jack, Andre, Daniel, and Emmanuel; you made my time in Miller and Bruce amazing and unforgettable. Ball time. I want to thank Ian, Niko, and Serban, with whom I spent most of my time during my master's in Delft: you guys always upped my spirits, so thank you for being there.

I am excited to see what the future brings, and I hope to keep creating and exploring for a long time to come.

*Jasper Kuba Geijsberts
Kingston, Canada, & Delft, The Netherlands, August 2026*

Abstract

Caves on the Moon provide safe shelter for humans against high temperatures, radiation and micrometeorites. They could also provide useful knowledge about the layers and composition of the Moon. Lava tubes, a type of cave, are expected to occur on the Moon, and tentative collapsed examples have already been found. Finding these lava tubes is a great priority in the exploration of the Moon. One method of searching for these lava tubes is by measuring small variations in gravitational attraction caused by the lack of mass due to these caves.

This thesis investigates whether lava tubes can be detected using current sensors and lunar DTMs. For this, a comparison of accuracy is investigated between the SPECFEM and GBOX software, which can calculate the gravity anomaly of 3D models using the spectral element method and direct integral method, respectively. 3D lunar lava tube and DTM models are created and meshed for simulation with SPECFEM. Different sensitivity analyses are performed on the mesh resolution, depth, shape, density, and scale of the lava tube, as well as on the DTM mesh resolution and DTM height. The models are from two different regions: A small lava tube that could potentially exist in the Marius Hills region, and a large lava tube that could potentially exist in the Gruithuisen K region.

SPECFEM was found to be more accurate than GBOX for the same mesh size. In terms of compute time, GBOX is found to have a quicker compute time per computation; however, for areas or volumes, many computations are necessary, and the compute time for GBOX explodes in these cases. SPECFEM is the only viable method of the two for high-resolution solutions across a volume. To understand detectability, a sensor error of 0.3 mGal, as well as a calculated total error of 5.9 mGal, which includes uncertainty in DTM height of ± 4 m RMS and density of $\pm 80 \text{ kg m}^{-3}$ RMS, were used as reference. The DTM height uncertainty contributed approximately 0.6 mGal error RMS to the total error, while the DTM density uncertainty contributed most of the error, at approximately 5.275 mGal.

From the results, it was found that a mesh size of 0.0125 km (8% of the minimum diameter of the lava tube) sufficed in giving accurate results to within 0.3 mGal for all seven shapes considered for the Marius Hills lava tube. The Marius Hills lava tube model was found to be detectable up to a depth of 3.4 km using the sensor error, and 0.28 km using the total error. The Gruithuisen K lava tube was detectable at least to a depth of 6.5 km using the sensor error, although no exact bound was found, while only detectable to 0.59 km using the total error for a narrow part of the lava tube. In a part with a wider cavity, the detectability increased to a depth of 2.95 km. The shape of the lava tube was found to be distinguishable at depths lower than 25 m using the sensor error, but a simulation at 200 m depth showed that this distinguishability stops somewhere between 25 and 200 m depth. Inverted keyhole and triangular-shaped cross-sections were found to have sharper V-shaped gravity signals across the lava tube, while rectangular, keyhole, and elongated shapes had U-shaped gravity signals. Using the total error as the detection threshold, the cross-sectional shapes become indistinguishable for the Marius Hills model. For the DTM, a resolution of 59 m was required at the surface to stay within the sensor error.

Overall, these results provide a quantitative basis for designing future lunar gravimetric surveys targeting the detection and characterisation of such lava tubes in their depth, shape, and scale. This directly supports survey designers in finding lava tubes that can help provide shelter for future lunar explorers.

Contents

Preface	iii
List of Figures	xi
List of Tables	xvii
Nomenclature	xix
1 Introduction	1
1.1 Motivation	1
1.2 Objective	2
1.3 Research Questions	2
1.4 Scope	3
2 Literature Study	5
2.1 Lava Tubes	5
2.1.1 Earth	5
2.1.2 Moon	10
2.1.3 Lunar Lava Tube Discovery	11
2.1.4 Artificial Lava Tubes and Terrestrial Analogues	19
2.1.5 Lunar Lava Tube Mission Proposals	21
2.2 Lunar Digital Terrain Models	22
2.3 Gravimetry	23
2.3.1 Lunar Gravimeters	23
2.3.2 Gravimetry Corrections	24
2.4 Gravity Anomaly Calculation Methods	28
2.4.1 Direct Integral Methods	28
2.4.2 Spectral Methods	32
2.4.3 Spectral Element Method	32
2.4.4 Gravity Anomaly Calculation Software	36
2.5 The Spectral Element Method	39
2.5.1 Governing Equations and Spectral Element Formulation	39
2.5.2 Spectral Element Method Example	41
2.6 Forward Method	46
2.6.1 Infinite Elements	47
2.7 Previous Studies on Gravity-Based Lunar Lava Tube Detection	50
3 Methods	53
3.1 Assumptions and Model Setup	53
3.1.1 Assumptions	53
3.1.2 Consistency in Performance Cluster	55
3.1.3 Lava Tube Model Basis	55
3.1.4 Cross-Section Selection	56
3.2 Acquisition of the DTM and Lava Tubes	56
3.2.1 Obtaining the DTM	57
3.2.2 Obtaining the Lava Tube	57
3.3 GBOX Simulation	58
3.4 SPECFEM Simulation	59
3.5 Sensitivity Analysis	59
4 Simulations	61
4.1 Parametric Study Overview	61

4.2	Models	62
4.2.1	Rectangular Prism and Spherical Salt Dome	62
4.2.2	Lava Tube Models	63
4.2.3	DTM	67
4.3	High-Resolution Considerations	68
4.4	DTM Bounds Considerations	70
4.4.1	Rationale	70
4.5	Test Plan Simulations	72
4.5.1	Parameter Comparisons	73
4.5.2	Performed Simulations	74
5	Results	79
5.1	Error Budget	79
5.1.1	Sensor Error	80
5.1.2	DTM Height Error	80
5.1.3	DTM Density Error	81
5.2	Accuracy and Computational Speed of SPEC-FEM and GBOX	81
5.2.1	Accuracy Analysis of SPEC-FEM and GBOX	82
5.2.2	Compute Time	88
5.3	Lava Tube Sensitivity Analysis	92
5.3.1	Resolution	93
5.3.2	Depth	98
5.3.3	Scale	108
5.3.4	Shape	112
5.3.5	Density	116
5.4	DTM Sensitivity Analysis	119
5.4.1	Resolution	119
5.4.2	Height	120
6	Conclusions and Recommendations	125
6.1	Conclusion	125
6.2	Answers to Research Questions	127
6.3	Recommendations	129
6.3.1	Data Visualization	129
6.3.2	Narrowing the Lunar Rock Density	129
6.3.3	Boundary Effects	129
6.3.4	Meshing Methods and SPEC-FEM Convergence	129
6.3.5	Inverse Method	129
	References	131
A	Code	137
A.1	generate_stl.py	137
A.2	testing_error_bounds.py	142
A.3	Analytical_Prism.py	142
A.4	constants.py	145
A.5	potential_library.py	145
B	DTM and Lava Tube Model Generation	147
B.1	Acquisition of the DTM	147
B.2	Acquisition of the Lava Tube	149
C	Analytical Solutions for Gravitational Anomaly Benchmarks	155
C.1	Rectangular Prism	155
C.2	Sphere	155
C.3	Multiple Spheres	156
C.4	Rectangular Prism with a Parabolic Density Distribution	156
D	Detailed Insight GBOX and SPEC-FEM software	159
D.1	GBOX Simulations	159

D.1.1	GBOX code	159
D.1.2	3D Model Preparation	160
D.1.3	Performing Simulations	161
D.1.4	Analysis	161
D.2	SPECFEM Simulations	162
D.2.1	Preparing the Simulation	162
D.2.2	Performing Simulations	164
D.2.3	Analysis	165

List of Figures

2.1	The inflation method for the creation of lava tubes. The dashed line parts indicate rapidly cooled and now solidified lava, while the clear parts indicate flowing lava. If flowing lava can drain (instead of solidifying), or when, during prolonged activity, a gas space forms above the flow (Kempe et al. 2018), a lava tube is formed (Kempe 2009).	6
2.2	The different ways lava tubes can form. (A-B) : Formation through overcrusting. (A) shows overcrusting, where the roof is made by broken pieces of rock being welded together. (B) shows overcrusting as the roof grows over time by lava slowing down and cooling, which solidifies to create a taller wall. This continues until the walls combine to make a roof. (C-D) : Formation through shallow inflation. (C) shows inflation combined with downward thermal/mechanical erosion, creating a rectangular shape. (D) shows inflation in aa flows. (E-F) : Formation through deep inflation. (E) shows pre-existing tubes and how erosion and breakdown can make bigger lava tubes. (F) shows lava tubes being created along already present fissures, which are weak links and therefore the inception horizons along which the bigger lava tube is formed. Taken from Sauro et al. (2020).	8
2.3	The Kula Kai Caverns. Taken from Losarcos and Dombard (2025).	9
2.4	Various possible lava tube cross-sections. (A) Keyhole cross-section, forms because of large amounts of down-cutting, while (B) the inverted keyhole is created because of a large amount of ledge growth. C-E Triangular, arched, and rounded cross-sections form after each other through ledge growth. (F) Rectangular cross-sections form with the accumulation of slabs of rock on top of the lava flow. (G) Elongated cross-sections form in unchanneled flows, which are spread out and on very gentle slopes. (H) Multiple levels of lava tubes form when an active flow inside a bigger cross-section cools and crusts over. Image taken from Losarcos and Dombard (2025).	10
2.5	The Marius Hills with various sinuous rilles present. Most important are the three rilles (A, B, and C) present in this area, of which C and part of B are speculated to be collapsed lava tubes (Greeley 1971). Image taken from Greeley (1971).	13
2.6	A comparison between a sinuous rille on the Moon, approximately 100 km south-west of the Gruithuisen domes (left), and a collapsed lava tube in California (right). Taken from Oberbeck, Quaide, and Greeley (1969).	14
2.7	Overview of the locations on the Moon that indicate evidence of lunar lava tubes. Exact locations can be readily found online. Image made with use from QuickMap (Arizona State University, NASA, and Applied Coherent Technology Corp. 2024).	15
2.8	The Marius Hills Hole (Robinson et al. 2012).	16
2.9	A schematic of the Traverse Gravimeter Experiment (TGE) instrument (Talwani, Thompson, et al. 1973).	24
2.10	The geometrical aspects used in calculating the free-air coefficient. A typical use case for the free-air coefficient is when measurements are taken from an aeroplane. The schematic is not to scale.	25
2.11	Visual representation of the Bouguer correction infinite slab model. The slab is given in white, while mountains or other features are given in brown. Sea level can also be replaced by a datum on bodies that do not have a predefined sea level. Adapted from Garland (1971).	26
2.12	The geometrical setup used in Talwani, Worzel, and Landisman (1959)	28

2.13	The gravity anomaly of a buried prism. This prism (mesh size of 0.5 km) encompasses a density anomaly of -400 kg m^{-3} , as well as a width, length, and depth of 5000 m, 10000 m, and 500 m, respectively. The prism is located a total of 2000 m from the observation point to the top face of the prism. The sphere (mesh size of 0.25 km) is a density anomaly of -200 kg m^{-3} . The radius is 4000 m, with the observation point being 6000 m above the centre of the sphere. The anomalies were calculated using subroutines given in Blakely (1995).	30
2.14	The model used to obtain the analytical solution for the gravitational attraction of an infinite cylinder. The red surface represents the imaginary Gaussian surface. Adapted from Connor (2015).	31
2.15	The meshed buried rectangular prism used in Gharti, Tromp, and Zampini (2018). The left figure shows the domain and the buried prism, while the right-hand figure also shows various infinite elements that were used in this implementation.	34
2.16	The gravity anomaly at four elevations for the multiple buried spheres example. The analytical solution, the SPECFEM solution, and the GBOX solution are compared. Taken from Gharti, Tromp, and Zampini (2018).	37
2.17	A schematic showing the boundaries and definitions used in the boundary conditions to which the governing equations are subject.	40
2.18	Lagrange polynomials for four GLL points with the range from -1 to 1. The red dots indicate the location of the GLL points.	43
2.19	The quadrature points used in the 3D case, already mapped and present in natural coordinates. Taken from Gharti, Tromp, and Zampini (2018).	47
2.20	The mapping for an infinite element to the natural coordinates (Zienkiewicz, Emson, and Bettess 1983).	48
2.21	The mapping for an infinite element to the natural coordinate in 3D (Gharti, Tromp, and Zampini 2018).	48
3.1	An overview of the process used to run GBOX simulations.	58
3.2	An overview of the process required for performing a gravity simulation using SPECFEM.	59
4.1	The model of the buried rectangular prism. No bounding box mesh is present in this figure, although it is present when simulating. In this case the mesh size is 2 km.	62
4.2	The model of the spherical salt dome. No bounding box mesh is present in this figure, although it is present when simulating. The mesh size is 1 km.	63
4.3	The cross-section used for the Marius Hills lava tubes.	64
4.4	The model of the Marius Hills keyhole lava tube, where $h = 0.1 \text{ km}$. No bounding box is present in this figure, although there will be one in the simulations.	65
4.5	The model of the Marius Hills keyhole lava tube, where $h = 0.05 \text{ km}$. No bounding box is present in this figure, although there will be one in the simulations.	65
4.6	The model of the Marius Hills keyhole lava tube, where $h = 0.025 \text{ km}$. No bounding box is present in this figure, although there will be one in the simulations.	66
4.7	The model of the Marius Hills keyhole lava tube, where $h = 0.0125 \text{ km}$. No bounding box is present in this figure, although there will be one in the simulations.	66
4.8	The Gruithuisen K lava tube model, with a mesh size of 0.2 km.	67
4.9	The model for the square DTM with a width of 40 km, meshed at 177 m mesh size, located in the Marius Hills region. The bounding box which would be present in the simulations is not present here.	68
4.10	The gap between a cube with a density of 3320 kg m^{-3} , giving 0.6 mGal gravitational attraction and the observation point plotted for depths from 1 m to 100 m.	69
4.11	The depth in km plotted against the radius in km of a cube with a density of 3320 kg m^{-3} such that the gravitational attraction is equal to 0.6 mGal for every depth. Note that the figure is in a log-log scale.	69
4.12	A schematic showcasing a lava tube inside a box, which represents the surrounding rock. In this case, no DEM is present on the surface.	71
4.13	A schematic of the cylinder and the variables and distances used in deriving the gravitational attraction formula for it.	71

4.14	The thickness of the DTM vs the minimum width required to include all information.	72
5.1	Comparison of varying element sizes on the gravity anomaly of a buried prism.	82
5.2	Comparison of varying element sizes on the gravity anomaly of a buried prism, zoomed in.	82
5.3	Comparison of varying element sizes on the gravity anomaly of a buried sphere vs the analytical solution, $h=1$ km.	83
5.4	Comparison of varying element sizes on the gravity anomaly of a buried sphere vs the analytical solution, $h=0.5$ km.	83
5.5	Comparison of varying element sizes on the gravity anomaly of a buried sphere vs the analytical solution, $h=0.25$ km.	84
5.6	Comparison of varying element sizes on the gravity anomaly of a buried sphere, using SPECFEM.	85
5.7	Comparison of varying element sizes on the gravity anomaly of a buried sphere, using GBOX.	85
5.8	Comparison between SPECFEM and GBOX on the accuracy of representing a buried rounded lava tube (25 m depth) with resolution equal to 0.1 km.	86
5.9	Comparison between SPECFEM and GBOX on the accuracy of representing a buried rounded lava tube (25 m depth) with resolution equal to 0.05 km.	86
5.10	Comparison between SPECFEM and GBOX on the accuracy of representing a buried rounded lava tube (25 m depth) with resolution equal to 0.025 km.	87
5.11	Comparison between SPECFEM and GBOX on the accuracy of representing a buried rounded lava tube (25 m depth) with resolution equal to 0.0125 km.	87
5.12	Comparison of varying element sizes on the gravity anomaly of a buried rounded lava tube (25 m depth) in the Marius Hills region, using SPECFEM.	88
5.13	Comparison of varying element sizes on the gravity anomaly of a buried rounded lava tube (25 m depth) in the Marius Hills region, using GBOX.	88
5.14	The compute time vs the number of observation points for three lava tube meshes, where $h = 0.023, 0.018$, and 0.015 km. Note that the plot is on a log-log scale.	89
5.15	The compute time vs the number of observation points for three rectangular prism meshes, where $h = 2, 1$, and 0.5 km	90
5.16	The compute time vs the number of observation points for three spherical salt dome meshes, where $h = 1, 0.5$, and 0.25 km	90
5.17	The compute time vs the number of computations for three lava tube meshes, where for SPECFEM $h = 0.1, 0.05$, and 0.025 km, and for GBOX the number of observation points has been varied (44, 64, 101) to approximately match the number of computations for the SPECFEM simulations. Note the logarithmic scale for the y-axis.	91
5.18	The compute time vs the mesh density $1/h$ (defined as the inverse of the mesh size h) for a rounded lava tube. For the GBOX simulation, 1001 observation points as well as 1 million observation points are present. For SPECFEM, three mesh sizes are presented. Note the logarithmic scale for the y-axis.	92
5.19	Comparison of varying element sizes on the gravity anomaly of a buried <i>arched</i> lava tube (25 m depth) in the Marius Hills region, using SPECFEM.	93
5.20	Comparison of varying element sizes on the gravity anomaly of a buried <i>elongated</i> lava tube (25 m depth) in the Marius Hills region, using SPECFEM.	94
5.21	Comparison of varying element sizes on the gravity anomaly of a buried <i>inverted keyhole</i> lava tube (25 m depth) in the Marius Hills region, using SPECFEM.	94
5.22	Comparison of varying element sizes on the gravity anomaly of a buried <i>keyhole</i> lava tube (25 m depth) in the Marius Hills region, using SPECFEM.	95
5.23	Comparison of varying element sizes on the gravity anomaly of a buried <i>rectangular</i> lava tube (25 m depth) in the Marius Hills region, using SPECFEM.	95
5.24	Comparison of varying element sizes on the gravity anomaly of a buried <i>triangular</i> lava tube (25 m depth) in the Marius Hills region, using SPECFEM.	96

5.25	The difference between the lower-resolution results of the Marius Hills lava tube simulations and the higher-resolution results, which in this case is assumed to be close enough to an ‘analytical’ solution. The vertical lines indicate where the graph crosses the 0.3 mGal boundary last, with each colour corresponding to the line of the same colour. z represents the depth of the lava tube. Some numerical artefacts can be seen.	97
5.26	The difference between the lower-resolution results of the Gruithuisen K lava tube simulations and the higher-resolution results, which in this case is assumed to be close enough to an ‘analytical’ solution. The vertical lines indicate where the graph crosses the 0.3 mGal boundary last, with each colour corresponding to the line of the same colour. z represents the depth of the lava tube. Some numerical artefacts can be seen.	98
5.27	The gravitational attraction of a lava tube vs the distance from the lava tube, Marius Hills region. The sensor and total error are calculated in section 5.1, and describe the error of just the sensor, and the sensor including errors in height and density of the DTM, respectively.	99
5.28	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region. No lava tube is included.	100
5.29	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a rounded lava tube at 25 m depth.	100
5.30	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a rounded lava tube at 200 m depth.	101
5.31	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a rounded lava tube at 1000 m depth.	101
5.32	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a rounded lava tube at 2500 m depth.	102
5.33	The 2D gravitational attraction (magnitude) field plot of just the lava tube, at 1000 m depth, plotted such that one can see the effect at the surface of the DTM. Note that only three bins are used to approximate the 0.6 mGal accuracy bin that the sensor used would provide. Additionally, note that the value in the graph is a magnitude value.	103
5.34	The gravitational attraction of a lava tube vs the distance from the lava tube, Gruithuisen K region ($x = 0$, $y = 0$ m). The sensor error value is not given, as the simulation did not extend far enough to reach this value.	103
5.35	The gravitational attraction of a lava tube vs the distance from the lava tube, Gruithuisen K region ($x = 7000$, $y = 3000$ m). The sensor error value is not given, as the simulation did not extend far enough to reach this value.	104
5.36	The 2D gravitational attraction field plots of just the Marius Hills DTM, 30 km by 30 km. The left plot uses the full range of values, while the right a smaller subset, which allows for a high contrast.	105
5.37	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM, Marius Hills region, including a Gruithuisen K lava tube at a depth of 35 m at minimum, and 295 m at the centre.	105
5.38	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 by 10 km is shown), Marius Hills region, including a Gruithuisen K lava tube at a depth of 35 m at minimum, and 295 m at the centre.	106
5.39	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a Gruithuisen K lava tube at a depth of 210 m at minimum, and 470 m at the centre.	106
5.40	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a Gruithuisen K lava tube at a depth of 1010 m at minimum, and 1270 m at the centre.	107
5.41	The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a Gruithuisen K lava tube at a depth of 2510 m at minimum, and 2770 m at the centre.	107
5.42	The 2D gravitational attraction field plot of the Gruithuisen K lava tube at 2510 m (2770 m) depth, plotted on a Marius Hills DTM of 10 by 10 km. Only the results of the lava tube are present, in magnitude. Note that the value in the graph is a magnitude value.	108

5.43	Comparison of the gravity anomaly of a <i>rounded</i> lava tube at a depth of 25 m when varying the scale of the lava tube, Marius Hills region. Mesh size is 0.025 km.	109
5.44	Comparison of the gravity anomaly of a <i>rounded</i> lava tube at a depth of 200 m when varying the scale of the lava tube, Marius Hills region. Mesh size is 0.025 km.	109
5.45	Ratio of gravity anomalies between the smaller- and larger-scale lava tubes vs the regular lava tube, at 900 m depth in the Marius Hills region. Numerical artefacts result in noise in the data.	110
5.46	The 2D field plots for a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown) at 25 m and 200 m depth for the 0.9× and 1.1× scale.	111
5.47	The gravity anomaly for a rounded lava tube at 25 m depth, with a scale of 0.9 times the nominal, as well as a nominal rounded lava tube with a depth of 62 m. Both lava tubes are Marius Hills lava tubes. Note the roughly equal values at the centre of the gravity signal.	112
5.48	Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube. The distance between the observation point and the centre of the lava tube is 900 m.	113
5.49	Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, zoomed in. The distance between the observation point and the centre of the lava tube is 900 m.	113
5.50	Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, at a closer distance of 150 m from the centre of the lava tube to the observation point, zoomed in.	114
5.51	Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, at a distance of 25 m from the top of the lava tube to the observation point, zoomed in.	114
5.52	Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, at a distance of 200 m from the top of the lava tube to the observation point, zoomed in.	115
5.53	Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, at a distance of 500 m from the top of the lava tube to the observation point, zoomed in.	115
5.54	The 2D field plots for a 10 km by 10 km piece of DTM at 25 m and 200 m depth for the various shapes, excluding the rounded shape, which has already been shown.	117
5.54	The 2D field plots for a 10 km by 10 km piece of DTM at 25 m and 200 m depth for the various shapes, excluding the rounded shape, which has already been shown. (cont.)	118
5.55	Comparison of the gravity anomaly of a lava tube when varying the density of the lava tube.	119
5.56	The differences between lower-resolution DTM simulations (0.118 km to 0.472 km) and the highest-resolution DTM (0.059 km). The highest-resolution DTM was taken as the ‘truth’ to be compared against.	120
5.57	The gravitational attraction of a square DTM in the Marius Hills region, with a width of 10 km: a nominal DTM, a DTM with all heights lowered by 4 m, as well as a DTM with all heights increased by 4 m. Zoomed in for convenience on a point that would be near the surface.	121
5.58	The gravitational attraction of a square DTM in the Marius Hills region, with a width of 20 km: a nominal DTM, a DTM with all heights lowered by 4 m, as well as a DTM with all heights increased by 4 m. Zoomed in for convenience on a point that would be near the surface.	122
5.59	The gravitational attraction of a square DTM in the Marius Hills region, with a width of 30 km: a nominal DTM, a DTM with all heights lowered by 4 m, as well as a DTM with all heights increased by 4 m. Zoomed in for convenience on a point that would be near the surface.	122
5.60	The difference between the −4 m DTM, the +4 m DTM and the nominal DTM over the distance or height to the DTM. DTM width is 10 km, Marius Hills region.	123
5.61	The difference between the −4 m DTM, the +4 m DTM and the nominal DTM over the distance or height to the DTM. DTM width is 20 km, Marius Hills region.	123
5.62	The difference between the −4 m DTM, the +4 m DTM and the nominal DTM over the distance or height to the DTM. DTM width is 30 km, Marius Hills region.	124

B.1	An overview of the steps used to obtain the DTM 3D model of any area on the lunar surface.	148
B.2	An overview of the steps used to obtain the lava tube 3D model.	149
B.3	The curves drawn in QuickMap (Arizona State University, NASA, and Applied Coherent Technology Corp. 2024), with the high-resolution mosaic used as a reference.	150
B.4	The Marius Hills DTM with and without the curves drawn for creation of the lava tube.	151
B.5	The Gruithuisen K area and the curves drawn to obtain the lava tube.	152
B.6	An overview of the steps taken inside Rhino 8 and Grasshopper to obtain the lunar lava tube models. The orange process block indicates a custom block that was made to automate the scaling of the cross-section widths.	153
B.7	The custom block implemented in Grasshopper ensures that, whatever cross-section is used, the cross-section is scaled to match the predefined curves of the lava tube. On the left, the inputs are the lava tube with scaling that has been manually fine-tuned once, as well as the area of the lava tube cross-sections.	154
D.1	A flow diagram describing the basic procedure for using the gpotential program in calculating the gravity anomaly of an arbitrary 3D shape.	160
D.2	The procedure for obtaining a VTI file containing the discretised 3D model that includes density information. The exported file is ready to be used by the GBOX gravity potential software.	161
D.3	A 3D view of an example lava tube model (Just shown for explanatory purposes) and its gravitational anomaly map, created using a 2D Delaunay filter in ParaView.	162

List of Tables

2.1	The different estimates on possible sizes for lunar lava tubes, as well as analyses on stability of various lunar lava tube sizes.	12
2.2	Papers from the 60s until now which have provided evidence for and against the existence of lunar lava tubes, and if they are optimistic or pessimistic about the existence.	18
2.3	Summarising table of lunar lava tube characteristics.	19
2.4	Terrestrial lava tube datasets. Taken from Romio, Lobosco, et al. (2026).	20
2.5	Details of the numerical comparisons used to assess the SPECSEM software for gravity anomalies. The "" marks indicate that this parameter did not change compared to the original (Gharti, Tromp, and Zampini 2018). Note that the agreement was summarised from the original remarks found in Gharti, Tromp, and Zampini (2018). These remarks were checked and verified by the author of this thesis and thus agreed with.	35
3.1	Assumptions used in the simulations.	53
3.2	The specifications of the clusters and supercomputers on which simulations were performed.	55
4.1	The specific characteristics of the buried rectangular prism as well as the spherical salt dome, obtained from Gharti, Tromp, and Zampini (2018).	62
4.2	The seven cross-sectional shapes for which a lava tube model was created, as well as their origin. All lava tube cross-sections are taken from the dataset provided in Romio, Lobosco, et al. (2025).	64
4.3	The characteristics of the Marius Hills lava tube model.	66
4.4	The characteristics of the Gruithuisen K lava tube model.	67
4.5	The different graphs that shall be produced from the simulations. All comparisons which are not between GBOX and SPECSEM use SPECSEM for the simulations.	73
4.6	Simulations to be performed for the comparison of compute speed and accuracy of SPECSEM and GBOX.	75
5.1	The grid search for the errors of using a higher vs. lower-resolution DTM (Marius Hills region), dependent on the height from the surface. Similar to the graph, the results are compared to the 0.059 km DTM. Green cells indicate values below 0.3 mGal, while red above 0.3 mGal.	120

Nomenclature

Abbreviations

Abbreviation	Definition
2D	Two-Dimensional
3D	Three-Dimensional
ABC	Artificial Boundary Condition
API	Application Programming Interface
AS	Assumption
CAD	Computer-Aided Design
CT	Compute Time
DAEDALUS	Descent And Exploration in Deep Autonomy of Lava Underground Structures
DEM	Digital Elevation Model
DFKI	Deutsches Forschungszentrum für Künstliche Intelligenz (German Research Center for Artificial Intelligence)
DTM	Digital Terrain Model
ESA	European Space Agency
GDAL	Geospatial Data Abstraction Library
GIS	Geographic Information System
GK	Gruithuisen K
GLL	Gauss-Legendre-Lobatto
GPS	Global Positioning System
GRAIL	Gravity Recovery and Interior Laboratory
IGMAS+	Interactive Gravity and Magnetic Application System+
LiDAR	Light Detection And Ranging
LOLA	Lunar Orbiter Laser Altimeter
LRO	Lunar Reconnaissance Orbiter
LT	Lava Tube
MH	Marius Hills
MHH	Marius Hills Hole
MIH	Marius Ingenii Hole
MTH	Marius Tranquilitatis Hole
NASA	National Aeronautics and Space Administration
RMS	Root Mean Square
RO	Research Objective
RP	Rectangular Prism
RQ	Research Question
SB	Small bodies
SD	Spherical Salt Dome
SELENE	Selenological and Engineering Explorer
SEM	Spectral Element Method
SIEM	Spectral Infinite Element Method
SM	Spectral Method
TGE	Traverse Gravimeter Experiment

Symbols

Symbol	Definition	Unit
g	Gravitational acceleration	$[\text{m s}^{-2}]$
g_0	Mean Gravitational Attraction	$[\text{m s}^{-2}]$
δg_{BG}	Bouguer correction	$[\text{m s}^{-2}]$
δg_{FA}	Free-air correction	$[\text{m s}^{-2}]$
δg_{TC}	Terrain correction	$[\text{m s}^{-2}]$
g	Gravitational acceleration	$[\text{m s}^{-2}]$
h	Height	$[\text{m}]$
M	Mass	$[\text{kg}]$
r	Radius/distance	$[\text{m}]$
S	Surface	$[\text{m}^2]$
V	Volume	$[\text{m}^3]$
W	Collapse Width	$[\text{m}]$
z	Depth	$[\text{m}]$
ϵ_h	Height Error	$[\text{m s}^{-2}]$
ϵ_{sensor}	Sensor Error	$[\text{m s}^{-2}]$
ϵ_{total}	Total Gravity Error	$[\text{m s}^{-2}]$
ϵ_ρ	Density Error	$[\text{m s}^{-2}]$
Φ	Gravitational potential	$[\text{m}^2 \text{s}^{-2}]$
ρ	Density	$[\text{kg/m}^3]$

Constants

Symbol	Definition	Value	Unit
G	Gravitational Constant	6.67430×10^{-11} (NIST 2022)	$[\text{m}^3 \text{kg}^{-1} \text{s}^{-2}]$
GM_{moon}	Lunar Gravitational Parameter	4902.800 (Konopliv et al. 2002)	$[\text{km}^3 \text{s}^{-2}]$
R_{moon}	Lunar Radius	1738.0 (Roncoli 2005)	$[\text{km}]$

1

Introduction

Lunar lava tubes are caves on the Moon, created by volcanic activity and left behind underground. Such caves provide incredible insight into the Moon's geological layers, allowing scientists to learn about the Moon's history, which in turn can help humanity learn about the Earth's history. In addition, such lunar lava tubes could provide shelter for future human explorers. To confirm the existence of such lava tubes, a gravity survey on the surface of the Moon can be performed. To plan a successful gravity survey, simulations using lunar lava tubes can prove beneficial in determining where to measure, as well as how to measure, and what accuracy is necessary. Therefore, in this thesis, a high-performance simulation of gravity anomalies is performed using the software SPECDEM.

In this chapter, the motivation for performing a simulation to calculate the gravity anomaly of a lunar lava tube is explained, after which the objective of this thesis is mentioned. Then, the research questions and scope of the thesis are presented.

1.1. Motivation

In the exploration of small bodies (SBs) in the solar system, accurate knowledge of the gravitational field is essential for mission planning, navigation, and scientific investigations. For example, when landing on small bodies, the exact local gravity experienced can be crucial in performing a successful landing manoeuvre. Additionally, understanding the gravitational field can provide insights into the internal structure and composition of these bodies, which is valuable for both scientific research and resource utilisation (Noeker et al. 2022).

Indeed, using gravimetry, one is able to constrain whether there are any abnormalities (density variations) beneath the surface, such as a deposit with a contrasting density to its surroundings (Blakely 1995, Ch. 7). This method has been used on Earth to find, for example, salt-rich deposits, as is done in Santolaria et al. (2024).

The same method can be used to find cavities beneath the surface (one can understand here that the density is zero). These cavities can be any kind of cavity, but on the Moon, lava tubes are of special importance.

Lunar lava tubes are, in essence, tunnels created when lava flows, but then cools and dries to create rock-solid structures, while leaving cavities behind (Kempe 2009). These lava tubes are of great importance in lunar exploration, as they can provide incredible geological and subsurface information about the Moon, which in turn can help humanity learn about the Earth (Wagner and Robinson 2022).

Additionally, lava tubes can also provide shelter to future lunar explorers (Coombs and Hawke 1992). Both radiation, micrometeorites, and the thermal environment of the Moon are hazards which humans can substantially mitigate by housing in lunar lava tubes (Losarcos and Dombard 2025; Horz 1985; Muñoz et al. 2025). For example, the radiation due to galactic cosmic rays dies down to negligible levels after 6 meters of lunar regolith (De Angelis et al. 2002). The thermal levels inside a lava tube

are estimated to remain fairly constant throughout the lunar diurnal temperature cycle, around -20°C according to Horz (1985), and 290K according to Horvath, Hayne, and Paige (2022).

Confirming lava tubes exist remains a high priority in lunar exploration. Since the 70s, the existence of lunar lava tubes has been speculated (Murase and McBirney 1970; Greeley 1971), with evidence starting from Apollo mission samples. Later, even further evidence from lunar missions was found that argues in favour of their existence (Chappaz, Sood, et al. 2017). The Marius Hills Hole, a lunar skylight, is currently seen as the prime subject as a possible entrance into an intact lunar lava tube (Robinson et al. 2012).

To confirm the existence of lava tubes, multiple mission options have been proposed. In fact, ESA requested ideas from anyone to suggest novel ways to confirm and explore lunar lava tubes with a mission on the surface of the Moon (ESA 2021). One of the groups that participated was the group including Canadensys and Queen's University. They came up with the idea of first sending an inexpensive rover mission to the Moon, which included a gravimeter (specifically the VEGA gravimeter) and ground-penetrating radar. The rover would encircle the Marius Hills Hole to first classify whether this skylight actually leads to a lunar lava tube entrance or not. For this, the concept of using the gravimeter to detect gravity anomalies was used.

The forward method, or simulations of lava tube models that calculate the gravity anomalies these models would give, can help to understand and interpret gravity data from real-life surveys such as the one this rover would be doing. Doing such simulations with a supercomputer also allows for highly accurate and precise models to be simulated, enhancing fidelity such that almost all useful details are captured. The sensitivity analyses performed with simulations will also help with planning of future missions. Inputs are varied in the 3D CAD model, which can help create sensitivity maps, which are maps that show which points contain the most information. These points are then of most interest in a gravity survey, as they help you characterise the subsurface quickest. Simulations could also provide insight into what characteristics of the lava tube can be constrained, once the measurements are actually made.

A software package that is able to perform such simulations with high accuracy and high speed is the SPECSEM software, which uses the spectral-infinite-element method, as given in Gharti, Tromp, and Zampini (2018). Because the software allows for hexahedral meshes that follow the shape of the original model, highly accurate results can be obtained. Next to that, due to the possibility of parallelisation, the time needed to compute solutions can be reduced. Lastly, a solution is always calculated for the whole model, resulting in the fact that once a model has been simulated once, the exact results from a specific part of that model can be obtained at any moment thereafter. Then, no additional simulations are needed as long as the desired results are within the model.

1.2. Objective

Based on the motivation and background of this research, one research objective can be established:

- RO1: To determine whether a typical lunar lava tube can be detected using typical lunar gravimeters, given uncertainties in the DTM, density, and lava tube geometry.

1.3. Research Questions

To fulfil the research objective, multiple research questions have to be answered. These research questions are outlined in this section and can be found in the following list:

- RQ1: How can a representative 3D model of a lunar lava tube be obtained?
- RQ2: How can a representative 3D model of the lunar surface be obtained?
- RQ3: How does SPECSEM compare to other methods in terms of accuracy and computational cost for computing gravity anomalies of lunar lava tubes?
- RQ4: How can the SPECSEM software be used to obtain the gravity anomaly of a 3D model that includes a lunar lava tube as well as a lunar digital terrain model?

- RQ5: How does the resolution of the model influence the accuracy and compute time of the simulation and analysis?
- RQ6: How sensitive is the computed gravity anomaly to incremental variations in the scale, shape, density, and depth of a 3D model which includes a lava tube and DTM?

The answers to these research questions will help contribute to research on lunar lava tubes by showing what considerations geologists must take when planning a geological survey of the Moon.

1.4. Scope

First, a lunar lava tube model is obtained, after which it is prepared and meshed for use in SPEC-FEM. Similarly, a 3D model of the DTM is obtained, which is prepared similarly. Using the meshed models, the simulation is performed. Next, a sensitivity analysis of the 3D CAD model is explored by varying the lava tube in shape, scale, and depth, while the DTM is varied in height, density, and mesh resolution. Then, it will be examined how such variations change the gravity anomaly. Lastly, noise that can be expected in a nominal lunar mission is applied to the simulation to see if lava tubes would still be discernible given noise in the instrument, environment, and model.

The work in this thesis is limited to using models that have either been created artificially or through a terrestrial analogue. This does not preclude the method from being used in a real lunar lava tube model if such a model becomes available. Only seven shapes, for the seven different categories of lava tube cross-sections, are made. One additional lava tube that contains multiple cross-sectional shapes is also made. The simulations will have a high number of nodes and degrees of freedom. Because of this, they will have to be performed on supercomputers available at Queen's University, Kingston.

Tools to be used for the general simulation are the SPEC-FEM software, Coreform Cubit (Coreform LLC 2024) for meshing (exact version to be determined), and Python, while GBOX (Blakely 1995, pp. 377) is to be used in the comparison with SPEC-FEM. The exact version of SPEC-FEM to be used is SPEC-FEMX.

This research excludes performing work on confirming whether the 3D CAD model of the lunar lava tube that will be obtained could structurally exist. Next to that, this research excludes any other work on confirming the existence of lava tubes — it only provides a means of obtaining a gravity anomaly of a potential lunar lava tube, as well as how sensitive the gravity anomaly is to such a model.

The simulations performed in this thesis do not take density variations throughout the region of interest, nor the presence of layers, into account. Instead, the density is always assumed to be homogeneous throughout the whole DTM.

Although the work in this thesis is focused on lunar lava tubes, the method can be applied to terrestrial lava tubes, as well as any other subsurface anomaly on practically any planetary body.

2

Literature Study

In this chapter, the background of the thesis is explained. This includes historical information about the existence of lava tubes, as well as the current evidence available to help prove their existence. Next to that, various basic notions used in gravimetry are explained. Lastly, the various methods that have been used and developed are explained, with the current state of the art showcased.

2.1. Lava Tubes

Lava tubes are geological structures created by volcanic activity through various formation methods. Although on Earth they can be explored in person, on other planetary bodies, like Mars and the Moon, they cannot be verified but are expected to exist (Sauro et al. 2020). On Earth, they are often found near volcanoes, such as on Hawaii (Kempe 2009) or near Mt. Etna (Sawłowicz 2020). On the Moon, they are speculated to exist (Zhu et al. 2024; Greeley 1971), but have yet to be confirmed. This section will talk about the importance of lava tubes for geological insights as well as habitats for future lunar explorers.

Lava tubes have been defined in various ways. The most used definition is the one by Halliday (2004), who acknowledged that lava tubes have many ‘contradictory meanings’, and defined them as ‘roofed conduits of flowing lava, either active, drained, or plugged’. Other researchers have different interpretations, such as Greeley (1971), who instead defines it as ‘A volcanic structure formed by the flow of basalt beneath a cooled surface crust which eventually drains and leaves behind a hollow void’. On top of that, the exact naming used for the structure known to be lava tubes is also uncertain. Kempe (2009) states that the term ‘lava tube’ should in fact not be used, but instead be replaced by the term ‘pyroduct’, which originated earlier (and therefore should have precedent), and was coined by Coan (1844). Using the term ‘lava tubes’ implies that the caves and tunnels that are formed have a circular cross-section and that the lava can flow up and down as if it is in pipes under pressure — This should be avoided. However, the term ‘lava tubes’ is used in most literature as the standard term for the tunnels through which lava flows. In this thesis, the terms ‘lava tube’ and ‘pyroduct’ will be used interchangeably.

All in all, a lava tube must be seen as a tunnel made through volcanic activity. It can be either plugged, drained, or active (meaning lava is still flowing in the tube).

2.1.1. Earth

Lava flows have been characterised in many ways. First, an Italian nine-class-based system was proposed by Mario Gemmellaro in 1858. However, since the 1800s, the system used by American researchers in Hawaii came into wide use (Harris et al. 2017). Terms used in this system are Hawaiian in nature and used in modern times. An explanation of this system was given by Macdonald (1953). In total, three types of flows can continuously transform into each other (Macdonald 1953):

- Pahoehoe: Smooth, glassy, ropy flow. There are spheroidal vesicles (Macdonald 1953).
- Aa: A fragmented flow with pieces of rock on top of the flow. It is rough and brecciated (Sawłowicz

2020).

- Block: Looks similar to aa flows, but has less irregular fragments, and is sometimes not mentioned. It is also more silicic (more silica content) than basalt flows (Harris et al. 2017).

An important distinction between pahoehoe and aa flows is also that lava tubes are abundant in pahoehoe flows, whereas they are less common in aa flows (Rowland and Walker 1990; Kauahikaua, Sherrod, et al. 2003).

On Earth, generally three main methods exist by which lava flows can create the original lava tube structures (Losarcos and Dombard 2025):

- Overcrusting
- Shallow inflation
- Deep inflation and thermal/mechanical erosion

All three methods of creation exist only in pahoehoe lava flows, contrary to aa flows. Note that these methods describe how the tunnels are created, but not how they are drained, which is explained afterwards.

Overcrusting

Overcrusting is a method by which flows of lava meet air, cool off, solidify, and form a crust around the flowing lava. This crust then insulates the flow from the air, allowing the lava to flow further. Once lava has stopped flowing, a cave made of solid volcanic rock remains (Losarcos and Dombard 2025). In overcrusting flows, which are gravity-driven, the tubes always flow in the direction of steepest descent (Sauro et al. 2020).

Shallow inflation

The shallow inflation method for creating lava tubes, which, on Earth, is more prevalent than overcrusting, is a method where the roof is formed by the repetition of a lava flow advancing and inflating (Kempe 2009). This method involves a lava flow solidifying quickly, but such that gas bubbles are able to contain themselves in the rock, lowering the rock's density. Then a next 'pulse' of lava will flow under this lower-density sheet, lift it up and advance further, as can be seen in Figure 2.1 (Losarcos and Dombard 2025).

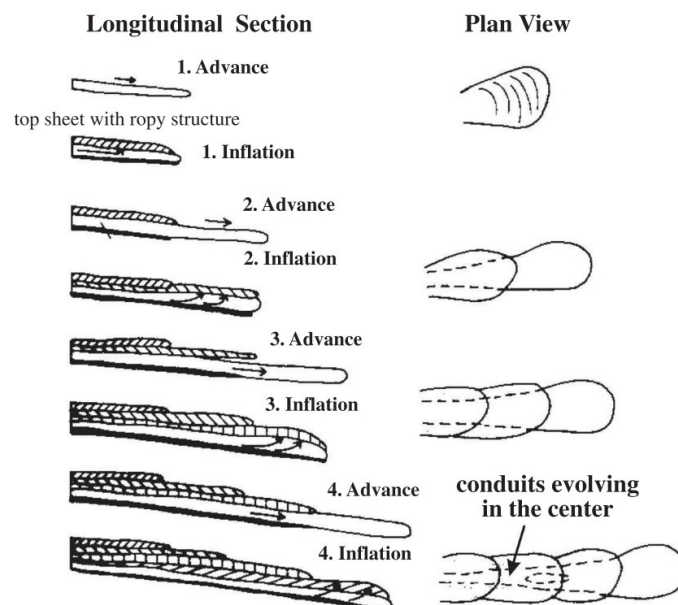


Figure 2.1: The inflation method for the creation of lava tubes. The dashed line parts indicate rapidly cooled and now solidified lava, while the clear parts indicate flowing lava. If flowing lava can drain (instead of solidifying), or when, during prolonged activity, a gas space forms above the flow (Kempe et al. 2018), a lava tube is formed (Kempe 2009).

Inflation-type lava flows are pressure-driven (Sakimoto, Crisp, and Baloga 1997), instead of being gravity-driven. This can cause them to sometimes not follow the line of steepest descent (Sauro et al. 2020), as the flow instead follows the weakest inception horizon. The weakest inception horizon is a sequence of rock that is more susceptible to cave development because of the rock it contains. It might, for example, be weaker in strength or erode more quickly.

Deep inflation

Deep inflation is another form of the creation of lava tubes where, when a large amount of lava is present, a main 'flow' is created, which applies downward thermal and mechanical erosion. This is usually applied to existing lava tubes. Deep inflation deepens the lava tube to deeper levels, where its direction is influenced by the weakest point of the rocks surrounding the flow (Losarcos and Dombard 2025).

Drainage of lava tubes

Although the methods of creation are accepted as understood, lava tubes still must be drained of lava after creation. The drainage of the lava is key to actually creating lava tubes that can be considered caves. One way the lava tubes are drained is when an empty lava tube forms by its flow being diverted (National Park Service n.d.). The diversion allows the lava to flow on, which then results in drainage of the original tubes. Kempe et al. (2018) states differently that lava tubes are not created when lava is drained at the end of an event, but that they are created when 'prolonged activity' allows a gas space to form above the 'down-cutting' lava flow. Down-cutting means that the flow thermally and mechanically erodes the bottom of the river and therefore changes the shape. This can be seen, in combination with inflation, in Figure 2.2. Grimes (2004) shows that flows can erode under the surface, creating a bigger cross-section (Kauahikaua, Cashman, et al. 1998), and if the amount of lava is limited, empty caves are left behind as the lava keeps eroding downward until it cools off and solidifies. Whether they get plugged or not depends on various factors such as the temperature, supply rate, viscosity, and velocity of the flow (Coombs and Hawke 1992). It is the authors' impression that lava tubes inhabit a combination of the previous methods of drainage. It is likely that some form of erosion and down-cutting is always present, as lava is a fluid. However, if flows are close to the surface, it seems likely that overcrusted flows simply flow downhill until the lava supply ends and the earlier tubes are drained while the lava solidifies further downstream.

If lava tubes do form, they can still collapse, especially when created close to the surface. They can exist as sinuous (meaning meandering) ridges, due to, possibly, the inflation process, where the surface is broken to sometimes elevate the roof above the lava tube (Oberbeck, Quaide, and Greeley 1969). Once lava tubes from old volcanic flows collapse, they become rilles and channels, which could be filled with sediments (Sawłowicz 2020).

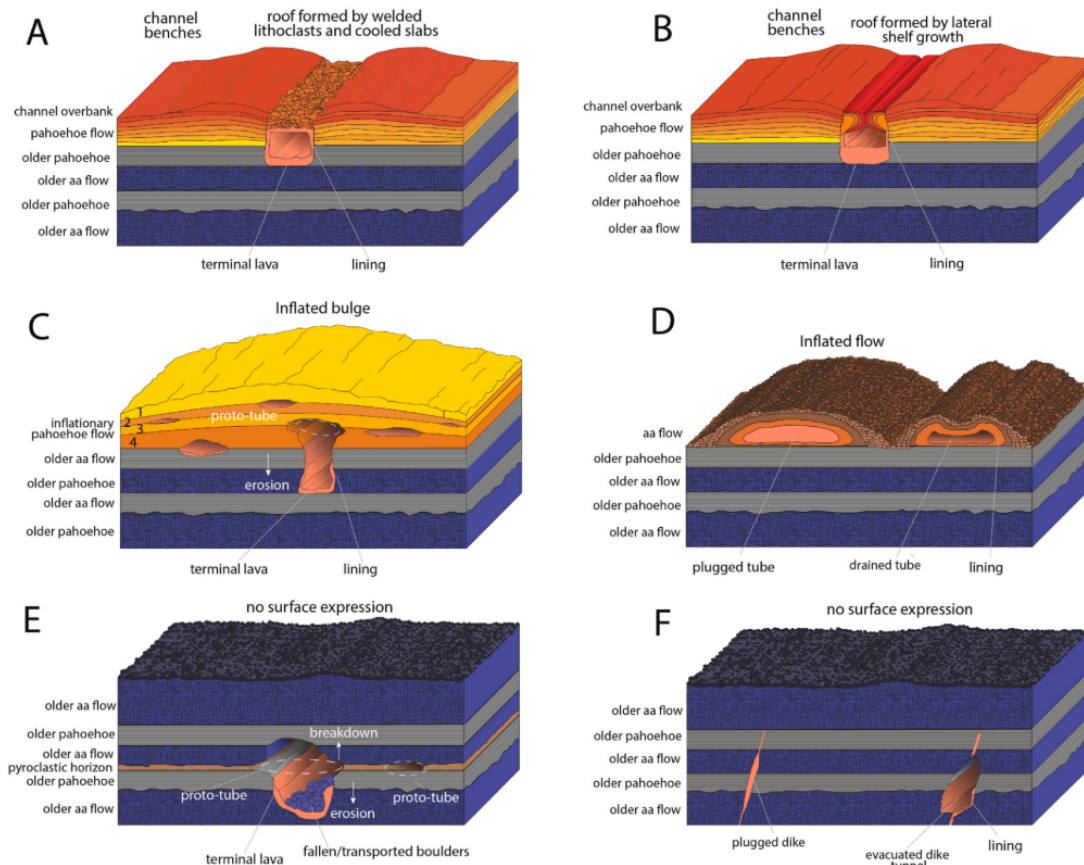


Figure 2.2: The different ways lava tubes can form. **(A-B):** Formation through overcrusting. **(A)** shows overcrusting, where the roof is made by broken pieces of rock being welded together. **(B)** shows overcrusting as the roof grows over time by lava slowing down and cooling, which solidifies to create a taller wall. This continues until the walls combine to make a roof. **(C-D):** Formation through shallow inflation. **(C)** shows inflation combined with downward thermal/mechanical erosion, creating a rectangular shape. **(D)** shows inflation in aa flows. **(E-F):** Formation through deep inflation. **(E)** shows pre-existing tubes and how erosion and breakdown can make bigger lava tubes. **(F)** shows lava tubes being created along already present fissures, which are weak links and therefore the inception horizons along which the bigger lava tube is formed. Taken from Sauro et al. (2020).

Multi-level lava tubes also exist, with the Kazamura cave serving as a notable example in Hawaii (K. Allred and C. Allred 1997). Such systems can form through down-cutting within an active lava tube. Then, if the roof of this pyroduct is too weak, it could collapse, creating a skylight on an active lava tube. This skylight, however, allows cool air to flow in, giving the flowing lava inside the tube the chance to potentially solidify. As such, a second, smaller lava tube could be created inside the larger one, which results in a multi-level system (Kempe 2009).

Another interesting concept is the concept of braided lava tubes, in which multiple tubes connect and diverge into branches. This occurs when existing lava tubes are insufficient to accommodate new eruptions, resulting in the formation of new branches. Such branching is commonly observed (Sawłowicz 2020) and can result in incredibly complex systems of lava tubes (Losarcos and Dombard 2025). An example of such a system is shown in Figure 2.3.

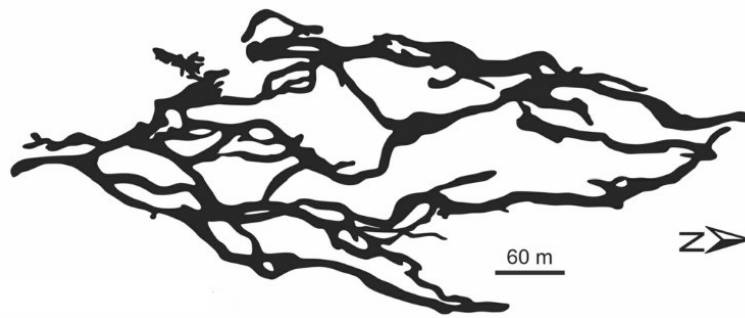


Figure 2.3: The Kula Kai Caverns. Taken from Losarcos and Dombard (2025).

Sauro et al. (2020) also points out that in braided systems, ‘collapse chains follow the development of the shallowest part of the tube while other segments can remain stable and thus without any expression at the surface’. Therefore, it is possible to see only a single collapse chain on the surface, while a whole system of lava tubes is actually present.

The width of collapses in lava tubes, namely ‘jameos’, compared to the width of the intact lava tube they are a part of, varies immensely. That is, the size of the collapse could be much smaller than the width of the lava tube, or much larger (Sauro et al. 2020).

Tumuli

Lava tubes can form through overcrusting, shallow inflation, or a combination of both. The resulting crust is initially thin, but under favourable circumstances, it may remain intact while lava continues to flow. Then, by way of conduction through the roof, sides, and floor of the lava tube, the outer lava flow cools, and the crust thickens. On the inside of the lava tube, the cross-sectional area decreases. As such, with a continued lava flow, the pressure inside the tube increases. Then, when the flow continuously inflates, the tube can crack or open on the roof, creating ‘tumuli’, or ‘pressure ridges’ (Orr et al. 2015).

Cross-sections

Lava tube cross-sections vary considerably in geometry. The cross-sectional geometry is governed by the formation method, the surrounding environment, and the rheological properties of the lava itself. For example, the size of the cross-section of the lava tube is positively correlated with slope. For slopes of 2 to 4 degrees, the cross-sections are more likely to be large, whereas they tend to be smaller or collapse entirely for slopes of less than 1 degree.

Five main types of cross-sections are identified for lava tubes, the first three of which are formed in channelised flows (Losarcos and Dombard 2025):

- **Keyhole:** This cross-section forms when mechanical and thermal erosion on the floor of the active conduit is high, and ledge growth is low. An inverted keyhole cross-section forms under opposite conditions: there is more ledge growth than down-cutting (Marraffa et al. 2025).
- **Arched:** This cross-section develops through thermal erosion and significant ledge growth, but starts as triangular, which then progresses to an arch. When the lava drains from the tube, the arched tube becomes circular or rounded.
- **Rectangular:** This cross-section forms when slabs of rock that float on top of the lava accumulate.
- **Elongated:** This cross-section forms in unchanneled flows with a gentle slope. Erosion is mostly present at the sides of the flow, instead of the top and bottom.
- **Multiple:** This cross-section is created when a new roof is formed on an active lava flow inside a bigger tube. This can happen in one event or multiple events. Kempe (2009) showed that an active puka can create this by allowing inflow of cool air into the active lava tube. Losarcos and Dombard (2025) states that another method of formation is that a ‘gradual lowering of the lava stream surface’ allows the flow to cool, enabling the formation of multi-level systems.

The different shapes for cross-sections can be seen in Figure 2.4.

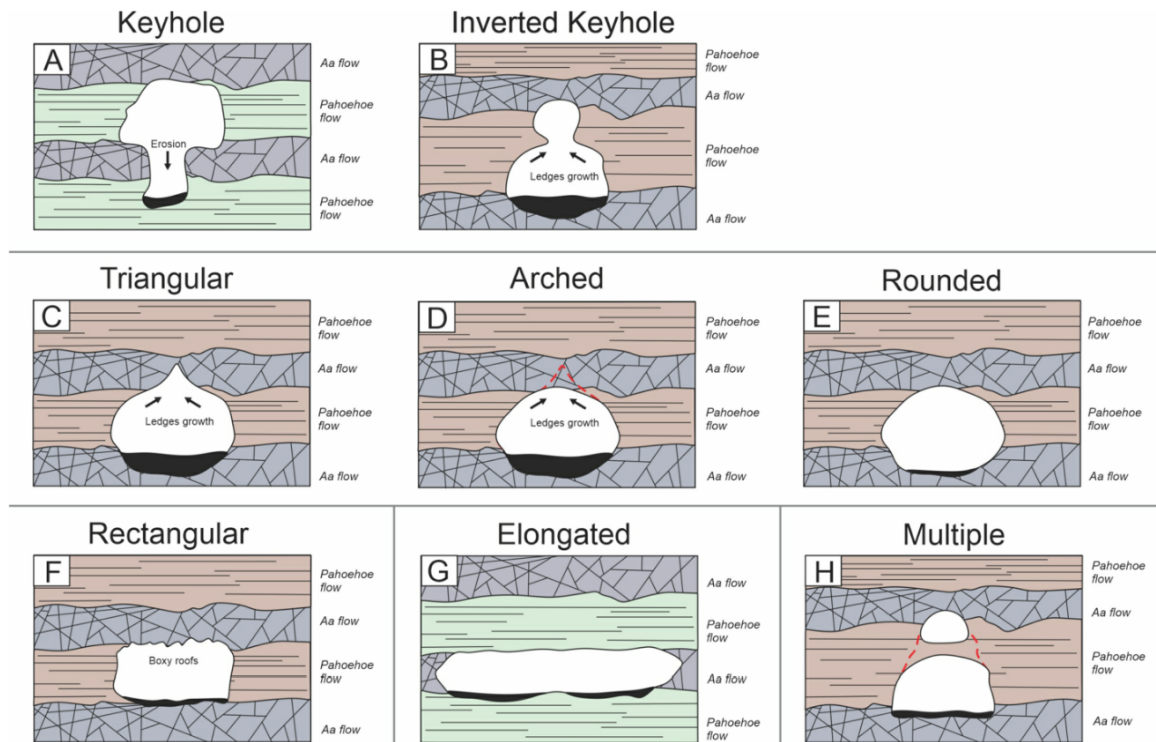


Figure 2.4: Various possible lava tube cross-sections. (A) Keyhole cross-section, forms because of large amounts of down-cutting, while (B) the inverted keyhole is created because of a large amount of ledge growth. C-E Triangular, arched, and rounded cross-sections form after each other through ledge growth. (F) Rectangular cross-sections form with the accumulation of slabs of rock on top of the lava flow. (G) Elongated cross-sections form in unchanneled flows, which are spread out and on very gentle slopes. (H) Multiple levels of lava tubes form when an active flow inside a bigger cross-section cools and crusts over. Image taken from Losarcos and Dombard (2025).

2.1.2. Moon

The Moon is an incredibly volcanic planetary body, with 16% of the Moon's surface covered in solidified lava flows, which accounts for about 1% of the total volume of the lunar crust (Zhao et al. 2023; Losarcos and Dombard 2025). The viscosity of lunar rock found on the Moon, which is lower than that of all previously studied volcanic rocks on Earth, contributes heavily to the potential existence of intact lunar lava tubes (Murase and McBirney 1970). In this subsection, the exact formation of these lava tubes and their potential morphometry is discussed.

Formation

On the Moon, the same methods present on Earth are in use, but to varying degrees. According to Losarcos and Dombard (2025), any of the methods present on Earth operate on the Moon, at least at small scales. However, on larger scales, by which kilometre-wide lava tubes are meant, only two methods are predominant: deep inflation and thermal/mechanical erosion. At such scales, formation methods such as overcrusting are insufficient to produce a structurally stable roof.

The lunar rilles that are visible on the surface of the Moon according to Zhao et al. (2023) could have been made by long-lasting eruptions (100–300 days) which had to have had fluxes between 10^4 and $10^5 \text{ m}^3 \text{ s}^{-1}$. It has been proposed that lunar sinuous rilles may have formed under such flux and duration conditions.

Morphometry

Due to the lower gravity present on the Moon, lava tubes are able to become much larger, as their structure does not need to support its mass with the same magnitude as on Earth. Oberbeck, Quaide, and Greeley (1969) show that a flat roof model 385 m wide, under a roof 65 m thick can be stable. On the other hand, Blair et al. (2017) shows an arched model could be stable with a width of 1600 m, under a roof of 200 m, both assuming no thermal loads. One of the smallest roof thicknesses still stable would

be 5 m; this would allow a maximum width of 990 m. Theinat et al. (2020) show that lunar lava tubes with a width of 1000 m, whose existence is predicted in Chappaz, Sood, et al. (2017), are possible. They show that a 100 m thick roof would allow for stability in such a lava tube.

On the Moon, inflation is considered the predominant formation mechanism for lava tubes, as overcrusting is unlikely to produce stable lava tubes at the expected widths, given their low tensile strength (Theinat et al. 2020). Inflation features such as inflation plateaus (the individual layers which are lifted in the inflation mechanism, illustrated in Figure 2.1) have not yet been definitively identified near the locations on the Moon most likely to house lava tubes (Losarcos and Dombard 2025; Robinson et al. 2012).

Most volcanic activity is found on the near side of the Moon. Here, the crust is 15 km thinner than on the far side, which means volcanic activity and the rising of magma are much more prevalent.

To summarise the different sizes of lava tubes that could supposedly exist, a table is made, which can be seen in Table 2.1.

As shown in Table 2.1, the range of lava tube size estimations varies quite a lot. Overall, however, the size estimations range between 65 m and 1000 m. The three lunar lava tube skylights, MHH, MTH, and MIH, are expected to house lava tubes on the lower end of the spectrum. When looking at collapse chains, the sizes of the lava tubes ranged from 500 to 900 m. The range is so large that in simulations, a decision has to be made about what types of sizes are useful to get results for.

Composition

Areas that have features that indicate lava tubes, such as the Marius Hills (in the Oceanus Procellarum), where sinuous, for example, winding or meandering, rilles are present, as well as many other features, are also seen in areas where mare basalts are prominent. Lava tubes are therefore generally expected to be found in basaltic areas, and are expected to be made of mare basalt (Greeley 1971).

The exact density of lunar rock is important in the context of gravity simulations. The density of the rock is in direct correlation with the calculation of gravity and, therefore, must be studied closely for an accurate representation. Kiefer et al. (2012) estimates the bulk density of multiple Apollo samples, including multiple mare basalts. The average of his findings, using all Apollo basalt samples, is found to be 3190 kg m^{-3} . The highest single bulk density for a mare basalt found in this paper was found to be 3270 kg m^{-3} , with an error of $\pm 50 \text{ kg m}^{-3}$, resulting in the highest possible density of 3320 kg m^{-3} . However, other measurements had various errors ranging from 30 to 100 kg m^{-3} RMS. Still, in gravitational studies, a single density for the mare basalt is usually assumed (Kiefer et al. 2012). Chappaz, Sood, et al. (2017), performing a gravitational anomaly study using GRAIL data, used a density of 2560 kg m^{-3} , which was derived from GRAIL data. It was, however, mentioned that this density was likely not accurate for the actual density of the lava, which should be between 3090 and 3170 kg m^{-3} . Zhu et al. (2024) assumes a density of 3000 kg m^{-3} for its forward model, which similarly represents mare basalt.

As can be seen, an exact value to be used for the density is not agreed upon, as variability is always present. Still, most values for the mare basalt density are found to be around 3000 kg m^{-3} .

2.1.3. Lunar Lava Tube Discovery

In this subsection, the discovery of lunar lava rilles, lunar skylights, and their connection to lunar lava tubes is explained. From the first finding of lunar 'rivers' on the Moon to potential entrances to underground lava tubes, the exact rationales for the existence of lunar lava tubes are discussed. First, the observation of lunar rilles is stated, after which the exact volcanic regions on the Moon are discussed. The discovery of lunar skylights such as the Marius Hills Hole is highlighted. Lastly, the author's opinion on the existence of lava tubes is explained.

Lunar Rilles and Volcanism

Christiaan Huygens was one of the first to identify valleys on the Moon (Cruikshank and Wood 1972). Another famous sighting of a valley on the Moon, which resulted in the name 'Schröter's valley', was made in the late 1700s by Schröter. The valley was described as a 'snake-like valley', indicating its sinuosity (Schröter 1788). These rilles, which have been photographed many times since by lunar orbiters, can be split into a minimum of three groups (Greeley 1971):

Table 2.1: The different estimates on possible sizes for lunar lava tubes, as well as analyses on stability of various lunar lava tube sizes.

Source	Size estimation	Method
Lunar Lava Tube Size Estimates		
Oberbeck, Quaide, and Greeley (1969)	385 m width at depth of 65 m, with no arch. Additional conservative stability estimate: 500 m width.	Used beam theory with lunar properties.
Chappaz, Melosh, and Howell (2014) and Chappaz, Howell, and Melosh (2014)	1 km	Correlation between 1 km wide lava tube and Rima Sharp region Bouguer anomaly.
Sauro et al. (2020)	500 m to 900 m	Looked at the widths of collapse chains.
Chwala, Komatsu, and Haruyama (2024)	Widths of tubes beneath Marius Hills Hole (MHH), Mare Tranquillitatis Hole (MTH), and Mare Ingenii Hole (MIH) (of which the significance is explained in subsection 2.1.3) range between 65 and 285 m. Roofs of <10 m are unlikely to be stable with tube widths >500 m.	Finite element limit analysis.
Lunar Lava Tube Stability Estimates		
Blair et al. (2017)	Arched roof: 1600 m wide and 200 m deep should be stable. 990 m width with 5 m depth is stable.	Numerical modelling to understand the stresses in the proposed lava tubes. Thermal stresses have to be low; otherwise, lava tubes would collapse. Smaller tensile strength compared to compressive strength of basaltic rock was not considered (Modiriasari, K., et al. 2019).
Modiriasari, Theinat, et al. (2018)	4 km width with 40 m roof stable (elliptical/circular profile).	ABAQUS simulations
Modiriasari, K., et al. (2019)	1000 m width with 100 m roof stable.	ABAQUS simulations
Theinat et al. (2020)	1000 m width with 100 m roof is stable. 4000 m width with 1000 m roof is stable.	ABAQUS simulations

- Linear rilles: Large, wide rilles. They most likely originate from events that start deep in the crust, and usually appear in the form of faults, fractures, or fissures.
- Sinuous rilles: These rilles tend to occur in high areas topographically. They have a higher sinuosity, which means they meander and turn. They are typically a couple of hundred meters wide and can be tens of km long.
- Tightly meandering rilles: These rilles meander incredibly tightly, giving high sinuosity.

Each of these types of rilles most likely has a different origin (Greeley 1971). Sinuous rilles have been the subject of much speculation and numerous hypotheses regarding their origins. An example of sinuous rilles can be seen in Figure 2.5.

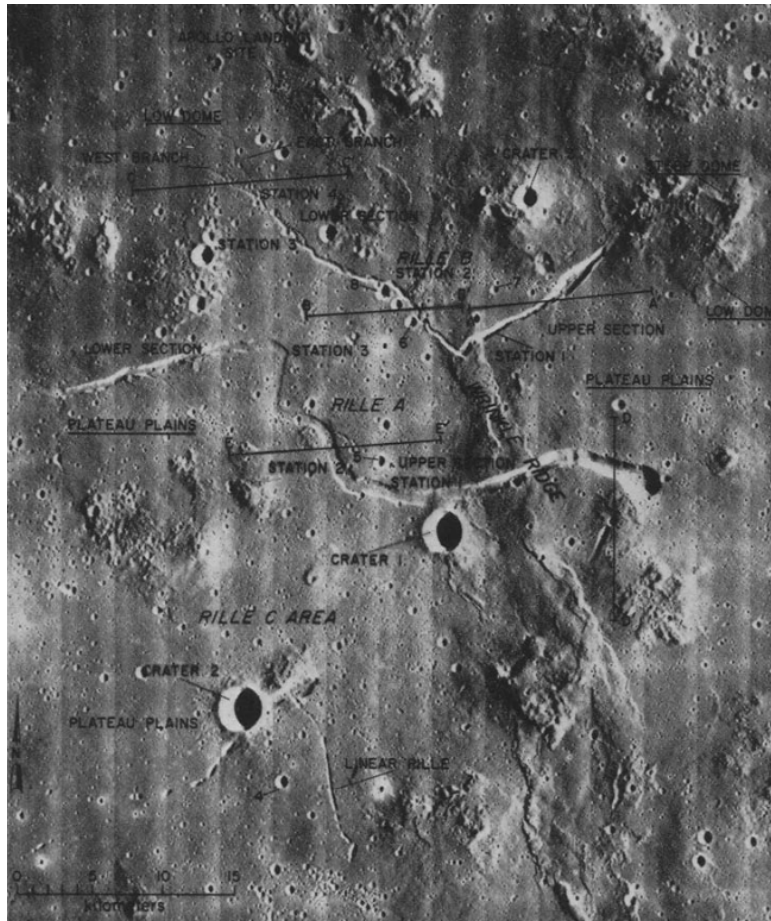


Figure 2.5: The Marius Hills with various sinuous rilles present. Most important are the three rilles (A, B, and C) present in this area, of which C and part of B are speculated to be collapsed lava tubes (Greeley 1971). Image taken from Greeley (1971).

In the 1960s, sinuous rilles were initially attributed to flowing water because of their similarity to rivers and channels on Earth (Urey 1967; Gilvarry 1960; Gilvarry 1968). However, this view has been contested, with alternate studies proposing lava could have created them. Murase and McBirney (1970) stated that the viscosity of a synthetic copy of a lunar rock (brought back by Apollo) was found to be low (compared to all volcanic rock on Earth), which indicates that lava flows should be thin, as well as indicating that lava tubes exist and that they should be large. Both Oberbeck, Quaide, and Greeley (1969) and Greeley (1971) propose that sinuous rilles are more likely to represent lava channels or collapsed lava tubes. For this, Oberbeck, Quaide, and Greeley (1969) gave evidence from Lunar Orbiter V that showed what they interpreted to be a lunar sinuous rille in the making. To prove their point, they also provided an image of an analogous terrestrial feature. These images can be seen in Figure 2.6.

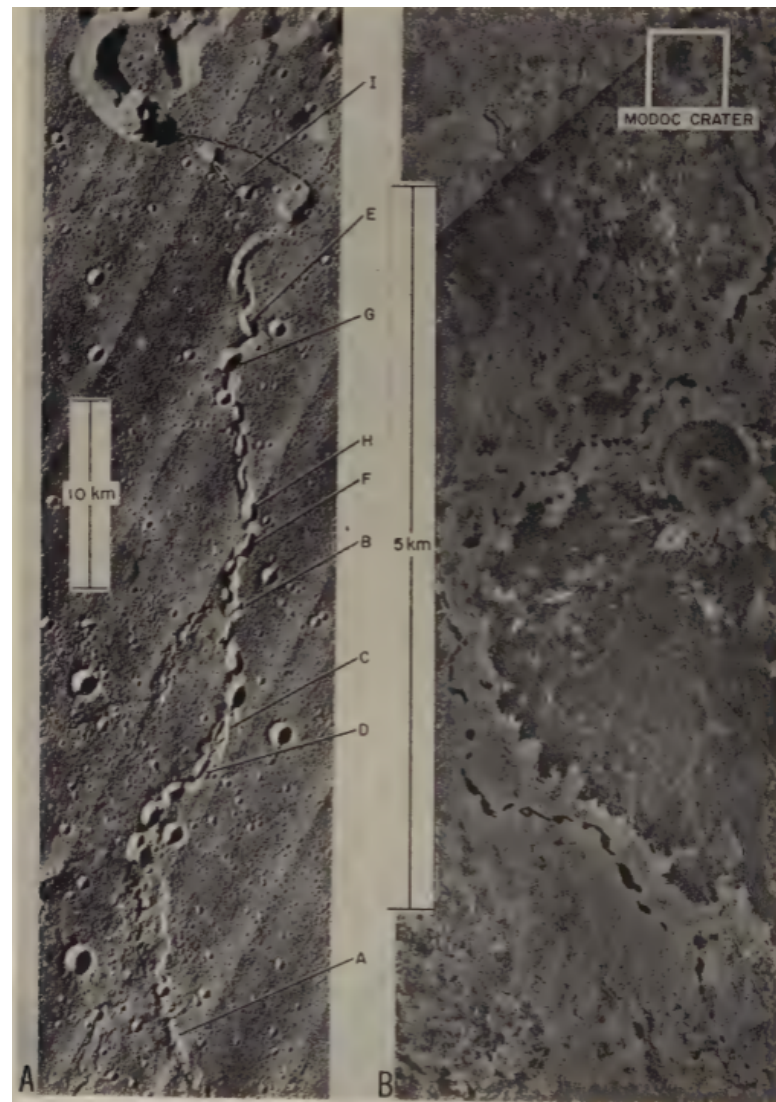


Figure 2.6: A comparison between a sinuous rille on the Moon, approximately 100 km south-west of the Gruithuisen domes (left), and a collapsed lava tube in California (right). Taken from Oberbeck, Quaide, and Greeley (1969).

As can be seen, this type of feature is hard to explain by water erosion. Erosion by water should leave a much smoother surface, instead of a cratered feature as seen in Figure 2.6. A more recent study by Zhao et al. (2023) states that such sinuous rilles could have been created by a high flux of magma (10^4 to $10^5 \text{ m}^3 \text{ s}^{-1}$), which stayed alive for durations between 100 and 300 days. Such events would cause extensive thermal erosion, which would in turn allow sinuous rilles to form.

Greeley (1971) also looked at the Marius Hills, a known volcanic region on the Moon, and found similarities with terrestrial lava tubes. He suggests that the Marius Hills is a place with a high likelihood of lava tubes. Similarly, Cruikshank and Wood (1972) states that there are a couple of areas on the Moon of specific interest for the existence of lava tubes, citing the Marius Hills, the area near Prinz, and the eastern Mare Serenitatis. The Marius Hills can be seen in Figure 2.5. Coombs and Hawke (1992) tried to locate any possible lava tubes by inspecting known lunar sinuous rilles. They found 67 tube segments, which they believed were likely to be 'open and intact'. The segments were found in four regions: the south of Gruithuisen K, the Marius Hills region, south-east Mare Serenitatis, and east Mare Serenitatis. These locations can be seen in Figure 2.7.



Figure 2.7: Overview of the locations on the Moon that indicate evidence of lunar lava tubes. Exact locations can be readily found online. Image made with use from QuickMap (Arizona State University, NASA, and Applied Coherent Technology Corp. 2024).

A chain found in the Gruithuisen area, which can also be seen in Figure 2.6, is a typical collapse chain created by inflation flows of lava (Sauro et al. 2020). In the top left, a putative vent can be seen, which Sauro et al. (2020) states is also a characteristic of a collapse chain coming from a lava tube, instead of another geological process, noting that these ‘criteria are solely based on the analogies with terrestrial lava tubes’. The Gruithuisen K lava tube could also have been a sinuous ridge, due to its elevated features in some parts of the collapse chain.

For the Marius Hills region, both Coombs and Hawke (1992) and Greeley (1971) agreed that rille C, which ‘flows’ south-east and can be seen in Figure 2.5, is most likely a collapsed lava tube.

Sauro et al. (2020) defined the morphotype of multiple collapse chains on the Moon and states the following:

- **Marius Hills:** There are two sinuous rilles, A (13.096° N, 57.056° W) and B (13.603° N, 58.047° W), of which A is sinuous. Both show collapses, with a large depression on one side, which is deemed to be a putative vent. B also contains a skylight. The width of the collapses in this area is on average $W = 574$ m for A and $W = 527$ m for B (Sauro et al. 2020).
- **Gruithuisen:** Is defined as a sinuous collapse chain, containing a big depression on one side, which is a putative vent. Next to that, a wrinkle ridge is present. The width of the collapses was found to be on average $W = 858$ m (Sauro et al. 2020).
- **Hyginus Rill Tectonic:** This feature is defined as a ‘Graben-bounded’ tectonic pit chain, and is therefore not created through lava.

As can be seen, there is substantial evidence that some sinuous rilles can possibly trace their origin to the collapse of lunar lava tubes, and that many of these are present in the Marius Hills area. Although not every sinuous rille is assumed to be a collapsed lava tube, some exhibit characteristics very similar to partly collapsed lava tubes.

More than a decade later, newer technology enabled a repeat survey of the lunar surface, in hopes of finding stronger evidence of lava tubes. Haruyama, Hioki, et al. (2009) used the SELENE Terrain Camera and Multi-band Imager to do exactly this. They found a single hole in the Marius Hills region, previously described by Coombs and Hawke (1992) and Greeley (1971) as most likely to contain lava

tubes. The hole does not have the features of a crater, lacking an elevated rim and deposits around it that should have been ejected there by the impact. As such, this hole is deemed to be of a different origin. The location of the hole is inside rille A as seen in Figure 2.5, and could therefore directly be connected to a lava tube. The hole is about 65 m wide, with a depth estimated at 80 to 88 m. Additionally, two other holes were found through SELENE in Haruyama, Hara, et al. (2010), located in the Mare Tranquillitatis and the Mare Ingenii. These holes are speculated to be skylights that house entrances into lava tubes.

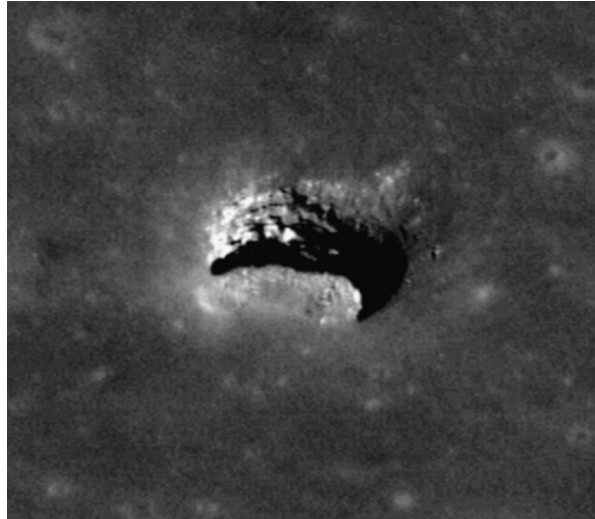


Figure 2.8: The Marius Hills Hole (Robinson et al. 2012).

Skylights

Skylights, also named ‘pukas’ in Hawaii, or ‘jameos’ in the Canary Islands (Sawłowicz 2020), are places on the surface of a body, where the roof of a lava tube has collapsed (Kempe 2009). They can also be formed on Earth due to overpressure from a lava flow, such that initially a tumulus is created, which then later collapses (Sauro et al. 2020).

Skylights are of great interest in the field of lunar exploration, as they might provide an easy entry point into a lava tube. Various skylights have been found, but it is still unclear what size lava tube they lead to, if any. Some pits, like the ‘Seven Sisters’ on Mars near Arsia Mons, look like skylights, but have then been linked to sinkhole formation of another kind, similar to ones in Arabia (Kempe 2017). It is therefore important to understand the context of sinkholes before drawing a conclusion about whether a skylight is present. On Earth, skylights are often found alongside collapse chains (Sauro et al. 2020), but it is unclear whether this is true on the Moon as well. Robinson et al. (2012) raised the question of whether the three different pits could potentially be part of a lava tube system:

- Mare Ingenii Hole (MIH)
- Mare Tranquillitatis Hole (MTH)
- Marius Hills Hole (MHH)

These pits show behaviour inconsistent with impact events, but could be part of a lava tube system or other collapsed geological features.

Recent efforts

In 2017, NASA’s GRail mission, which studied the Moon’s gravity field (Zuber et al. 2013), was used by Chappaz, Sood, et al. (2017) to investigate the presence of lava tubes. To do this, gradiometry and cross-correlation (Chappaz, Melosh, and Howell 2014) were used to detect subsurface anomalies. The analysis they performed had a west-east bias because the individual spacecraft tracks were visible in the spherical harmonic expansion of the data. Furthermore, the method is most likely to find subsurface features which are at least a few km wide, hundreds of metres high, and tens of kilometres long (Chappaz, Sood, et al. 2017). Zhu et al. (2024) used the same data, but a different method, to detect

potential lava tubes without bias in a specific direction. They examined the Marius Hills pit, also known as the Marius Hills Hole (MHH), and found that this hole could potentially be a skylight and an entrance to a lava tube.

Another method to identify lava tubes was used in Kaku et al. (2017). They used the SELENE (Kaguya) Radar Sounder to identify an echo pattern on the Moon consistent with subsurface cavities, such as lava tubes. Using this method, they managed to find data that could serve as evidence of a lava tube. Notably, most of the locations found were in areas where the GRAIL gravity data showed mass deficits. However, the claims from Kaku et al. (2017) were refuted by Kobayashi et al. (2021), which used three ambiguity tests to show that it is much more likely that the lava tube supposedly detected was actually just surface clutter.

Another paper that tried to make detecting lunar lava tubes easier was Donini, Bovolo, et al. (2018), where a fuzzy rule-based system was used on radargrams to automatically detect lava tubes, which they also tested on SELENE data, and developed further in Donini, Carrer, et al. (2022).

Lastly, the evidence for lunar lava tubes collapsing to form sinuous rilles (also called 'collapse chains') is summarised in Sauro et al. (2020), which presents an extensive comparison between terrestrial and lunar collapse chains.

To summarise the evidence for and against lava tubes, Table 2.2 is presented below.

As can be seen, eight papers are optimistic about the existence of lava tubes or state there is good evidence, while one refutes the evidence provided and is pessimistic. Additionally, two others are pessimistic and argue for water as the origin of sinuous rilles. Next to that, two sources do provide data which they state could be evidence for lunar lava tubes, but never themselves state that what they were seeing were lava tubes.

The author of this thesis found that most papers published were in favour of the existence of lava tubes, or were at least providing evidence that could be used to prove the existence of lava tubes. Many state that there is a high probability they exist, and some provide arguments supported by data.

It is the author's opinion that although sinuous rilles specifically could possibly originate from collapsed lava tubes, and that therefore lava tubes do exist, many rilles do not indicate that lava tubes created them. For example, the Rima Hyginus, or Hyginus Rille, is a linear rille, which, although it looks similar to sinuous rilles in some ways (collapse pits are present along the rille), is created by deep volcanic activity (Greeley 1971; Wilson et al. 2011), instead of the collapse of lava tubes.

Still, some collapse chains, especially those in the Marius Hills and Gruithuisen area, exhibit characteristics remarkably similar to terrestrial collapse chains. The chains are sinuous, have a deep depression at the beginning, and show the presence of skylights (in the case of Marius Hills). As such, if terrestrial characteristics for lava tubes are representative on the Moon as well, which does not have to be the case, it seems that these chains originate from lava tubes.

Also, it's worth noting that papers pessimistic about the existence of lava tubes are hard to find. Most authors seem to be optimistic about the *existence* of lava tubes, however, to confirm that a specific chain will house an *intact* lava tube is much less common. Robinson et al. (2012) does explicitly state and show that the Marius Hills Hole that was photographed, although confirming the existence of a sublunar void, could be just that, a void, without an entry into a lava tube. The rebuttal of the data provided by Kaku et al. (2017) by Kobayashi et al. (2021) also shows that when analysing lunar data, one must be careful about making statements that are too optimistic. Overall, the author of this thesis believes it is likely that lunar lava tubes exist, but also that confirming them can only be done using better measurements. This could be done by a lunar mission that can come into close contact with a lunar skylight such as the MHH. Then a direct survey can be conducted by either using instruments around the MHH or by entering the skylight to confirm through optical methods.

Summary of lunar lava tube characteristics

To summarise all the information gained on lunar lava tubes, Table 2.3 has been created. Here, the approximate size, shape, and other characteristics of lunar lava tubes are summarised.

Table 2.2: Papers from the 60s until now which have provided evidence for and against the existence of lunar lava tubes, and if they are optimistic or pessimistic about the existence.

Paper	Evidence provided	Optimistic or pessimistic?
Gilvarry (1960) and Gilvarry (1968)	States that the meandrous nature of sinuous rilles indicates an origin of rivers and water, and argues against the origin from lava hypothesis.	Pessimistic
Urey (1967)	States that the meandrous nature of sinuous rilles indicates an origin of rivers and water, and argues against the origin from lava hypothesis.	Pessimistic
Oberbeck, Quaide, and Greeley (1969)	Compares rille structures from Lunar Orbiter V photos to terrestrial collapsed lava tubes to argue for the existence of lava tubes on the Moon.	Optimistic
Murase and McBirney (1970)	Synthetic silicate liquid with composition of lunar rock has low viscosity. This means lava tubes should be more common, as lower viscosity increases the potential for lava tube formation.	Optimistic
Greeley (1971)	Uses characteristics of rille structures to argue for the existence of lava tubes.	Optimistic
Cruikshank and Wood (1972)	Compares lunar rille structures to terrestrial lava tubes on Hawaii to argue for the existence of lava tubes on the Moon.	Optimistic
Coombs and Hawke (1992)	Found 67 tube segments which they think are likely 'intact and open'.	Optimistic
Haruyama, Hara, et al. (2010) and Haruyama, Hioki, et al. (2009)	Found the Marius Hills Hole, suspects it's an entrance into a lava tube.	Optimistic
Robinson et al. (2012)	Using data from the New Lunar Reconnaissance Orbiter Camera, it could be seen that the Marius Hills Hole for sure houses a sublunar void, although unsure if it is a lava tube. They do state it's possible, but do not make any statement themselves.	Neither
Chappaz, Melosh, and Howell (2014) and Chappaz, Sood, et al. (2017)	Used GRAIL data (which is gravity data) to try to find subsurface voids. Some evidence of extensions of lunar rilles and other structures was found, which could be used to support the presence of lunar lava tubes.	Neither
Kaku et al. (2017)	Used the SELENE Lunar Radar Sounder data to figure out if there are any characteristics that could be used as evidence for lunar lava tubes near the MHH.	Optimistic
Kobayashi et al. (2021)	Rejects the research done by Kaku et al. (2017) and states whatever was seen is likely surface clutter.	Pessimistic
Zhu et al. (2024)	Extended on Chappaz, Sood, et al. (2017) to find more evidence for potential lunar lava tubes in the Marius Hills region.	Optimistic

Table 2.3: Summarising table of lunar lava tube characteristics.

Characteristic	Value
Formation Method	Deep inflation and thermal erosion (Losarcos and Dombard 2025).
Length	Tens of kilometres (Chappaz, Sood, et al. 2017).
Cross-sections	Rounded or arched in stability estimates (Blair et al. 2017; Modiriasari, Theinat, et al. 2018).
Good candidates for collapsed lava tubes	Gruithuisen K (Oberbeck, Quaide, and Greeley 1969; Coombs and Hawke 1992), Marius Hills (Greeley 1971; Coombs and Hawke 1992; Haruyama, Hioki, et al. 2009), south-east Mare Serenitatis, and east Mare Serenitatis (Coombs and Hawke 1992).
Locations with collapse chains	Gruithuisen K (Oberbeck, Quaide, and Greeley 1969; Coombs and Hawke 1992) and Marius Hills (Greeley 1971; Coombs and Hawke 1992).

2.1.4. Artificial Lava Tubes and Terrestrial Analogues

To simulate lunar lava tubes, an accurate 3D CAD model of a lunar lava tube must be obtained. This model must either be taken from real examples or created. As the existence of lunar lava tubes is yet to be confirmed, there are only two available alternatives:

- Create an artificial lunar lava tube.
- Use a lava tube on Earth as an analogue for a lunar lava tube.

The first option is usually done in a very simplified way. Most often, either a cylindrical or semicircle/semi-ellipse cross-section is used (Chwała, Komatsu, and Haruyama 2024), as in Blair et al. (2017), Modiriasari, Theinat, et al. (2018), Modiriasari, K., et al. (2019), and Theinat et al. (2020). These models are obviously quite simple but useful for comparing gravity-calculation software or for getting an initial idea of what gravity anomalies of lunar lava tubes could look like. Next to that, a simple model might meet the measurement requirements of current state-of-the-art gravimeters.

The second option is to use a terrestrial analogue to simulate a lunar lava tube. This is, for example, done by Muñoz et al. (2025), which uses the Ape Cave in Washington State, USA. They made a 3D LiDAR model of the cave and conducted a gravity survey on the surface. Then, they created a forward gravity model of the 3D model they obtained and assessed whether the gravity survey could accurately identify the cave.

Lava tube datasets

An accurate model of a lava tube includes an accurate cross-section. Therefore, if one were to use a terrestrial analogue to model a lunar lava tube, having access to terrestrial lava tube models or cross-sections would be extremely beneficial for modelling lunar lava tubes. There are many volcanic regions on Earth that have lava tubes, many of which have been modelled. Most are present in Hawaii, Italy, or the Canary Islands, as well as some other places. Romio, Lobosco, et al. (2026) summarised several lunar lava tube models made with LiDAR and where they come from. The table produced there has been adapted here and is shown in Table 2.4.

As can be seen, there are several LiDAR datasets that could be used to model terrestrial analogues of lunar lava tubes.

Romio, Lobosco, et al. (2026) also created software to help use these LiDAR lava tube datasets to model any type of lunar lava tube. In the software, the user can input the curve the lava tube follows, as well as the exact cross-sections used and their sizes. This software, 'pyroduct', an extension for Grasshopper in Rhino 8, provides an extensive and quick method for creating an example of a lava tube on Earth, the Moon, or Mars. For example, one could use the lava tube cross-sections of the 'Grotta Corsaro', a lava tube in Sicily, Italy. These cross-sections can follow a curve input by the user, as well as a DTM input by

Table 2.4: Terrestrial lava tube datasets. Taken from Romio, Lobosco, et al. (2026).

Reference	Lava tube(s)	Location	Survey Instruments	LiDAR data availability
(Tomasi et al. 2022)	Corona Lava Tube	Lanzarote Island, ES	Leica P40, Leica Pegasus	Data will be made available upon request.
(Bell et al. 2022)	Skull Cave, Ship Cave, Incline Cave	Lava Beds National Monument (California), US	Riegl VZ-400	doi.org/10.13016/ovqx-j09w
(Grechi et al. 2024)	Tabernacle Hill Lava Tube	Tabernacle Hill Lava field (Utah), US	Apple iPhone 13 LiDAR	Data will be made available on request.
(Lemaire et al. 2024)	Grotta lavica negli efflussi vesuviani del 1858	Somma-Vesuvius (Campania), IT	Riegl VZ-400	Data will be made available on request.
(Hidding et al. 2025)	Grotta di Monte Intra-leo	Mt. Etna (Sicily), IT	Unitree L1	doi.org/10.4121/778...
(Romio, Hidding, et al. 2025)	Grotta di Monte Intra-leo	Mt. Etna (Sicily), IT	Unitree L1	doi.org/10.4121/0cc...
(Yang et al. 2024)	Shenyang Cave	Jingpo Lake Geopark (Heilongjiang Province), CN	NA	Data will be made available on request.
(Muñoz et al. 2025)	Ape Cave	Mount St. Helens volcano (Washington), US	Geoslam Zeb Horizon	doi.org/10.5281/zen...

the user, to obtain a lava tube that can move up and down exactly as desired. Lastly, the user can also use two existing curves for the outside of a lava tube to create the exact sizes the cross-sections should take at each point of the curve. As such, there is immense freedom in creating custom lava tube models.

2.1.5. Lunar Lava Tube Mission Proposals

Since the renewed interest in both human presence on the Moon and the existence of lunar lava tubes to help accommodate that presence, various proposals for lunar missions have been made.

ESA, in fact, invited various institutions from member states to propose ideas for a lunar mission to explore these caves (ESA 2021). This was done through the SysNova initiative (ESA n.d.).

One of the proposals, from Canadensys and Queen's University, involved the idea that it is still not yet confirmed whether the MHH provides actual entrances to a lunar lava tube, as their existence has not been confirmed. Therefore, this proposal suggests first sending an *inexpensive* mission to roam around the lunar skylights and perform geophysical surveys. These surveys would include both gravimetry and ground-penetrating radar to determine whether subsurface features such as lava tubes are present. In this first, inexpensive mission, a rover would be used to determine whether the hole is actually an entrance to a lava tube, rather than descending only to find that the hole is much smaller and not an entrance. Then, once it has been confirmed that a hole is an entrance, a more sophisticated, expensive cave-diver mission can be sent to explore from the inside. The gravity survey's success depends on the exact locations where gravity is measured. For example, a square grid might perform worse in constraining the subsurface anomalies than a spiral. Therefore, a Flask Applet named GravSim was made, by which a gravity survey can be planned on-the-fly. It allows a user to input the locations of survey points, and the gravity anomaly field produced by those points is shown. The gravimeter, which is developed by Canadensys, would be the VEGA space gravimeter. More information on the functioning of the gravimeter is shown in subsection 2.3.1.

Queen's University, where this thesis was written, partnered with Canadensys on their proposal and provided all the gravity simulation software for the planning of the gravity survey. Additionally, more advanced software has been developed at Queen's to calculate gravity anomalies with high precision, using the Spectral Infinite Element Method (SIEM). This method is explained more in depth in subsection 2.4.3. Such high-quality simulations would enable high-precision analyses of lunar lava tubes and can help interpret what lunar lava tubes would look like from the inside, once gravity data have been obtained.

It must also be noted that gravimetry will not only help determine whether a lunar lava tube is present near the Marius Hills Hole; it can also help find other lava tubes and predict their size and depth inexpensively. Therefore, conducting extensive research in this field is incredibly useful for future lunar exploration missions and even for a future human presence on the Moon.

2.2. Lunar Digital Terrain Models

To simulate the gravity anomaly of a lunar lava tube, an accurate digital terrain model is needed. This terrain model is then used not only to obtain the terrain correction but also to model the lunar lava tube, as it can follow the lunar surface elevation for a more realistic lava tube.

Digital Terrain Models (DTMs) or Digital Elevation Models (DEMs) are 3D models used to calculate terrain corrections. Such corrections are then used in further calculations to obtain the Bouguer anomaly.

Noeker et al. (2022) applied a DTM of the Apollo 17 landing site to calculate the terrain correction in that context, using a new wedge-pentahedra method. For the DTM, JMARS (Christensen et al. 2009) was used to obtain the *Blended LRO/LOLA and SELENE/Kaguya DEM* dataset. This dataset combines LOLA data and SELENE data to obtain a DTM of the Moon. The dataset covers latitudes of $\pm 60^\circ\text{C}$, 512 pixels per degree. It has a resolution of $\approx 59\text{ m}$ at the equator, with an error in height of $\pm 3\text{--}4\text{ m}$ (U.S. Geological Survey 2015).

The Lunar Reconnaissance Orbiter dataset and the SELENE dataset can also be obtained individually.

2.3. Gravimetry

In this section, gravimetry is explored through the subjects of hardware used in gravimetry, as well as corrections needed to analyse data measured by gravimeters. In subsection 2.3.1, multiple gravimeters that are operable or have operated on the Moon are discussed, while in subsection 2.3.2 the corrections used in calculating gravity anomalies are shown, including various formulas needed.

2.3.1. Lunar Gravimeters

In this subsection, various lunar gravimeters, both operating and operable, are stated and explored. This will include gravimeters that operate from orbit, as well as surface gravimeters. Their accuracy and applications are mentioned.

Orbital Gravimeters

One mission which measured the gravity field of the Moon from orbit was the GRAIL mission (Zuber et al. 2013). The mission consists of two spacecraft, 'Ebb' and 'Flow', both of which include the 'Lunar Gravity Ranging System'. This instrument measures the change in distance between the two spacecraft. Then, as the spacecraft fly over the Moon, their orbits get perturbed by the Moon's local variations in gravity. These variations are caused by subsurface features, and as such, once the orbits are corrected for various disturbances, the gravity field can be calculated and analysed for subsurface features. Once obtained, the data can be used to create a spherical harmonic representation of the gravity field. The accuracy of this mission was found to be such that features needed multiple kilometres of width and length, and hundreds of metres in height to be distinguishable (Chappaz, Sood, et al. 2017). Therefore, in this case, the resolution of GRAIL was insufficient to confirm the existence of lava tubes. If the resolution of an orbital gravimeter such as GRAIL could be increased, for example through a new mission, it could become possible to detect the very large lava tubes. Still, for smaller ones, with a maximum width of a couple of hundred meters, the orbital gravimeters are currently too inaccurate.

Surface Gravimeters

On the Apollo 17 mission, a surface gravimeter was used. This instrument was part of the Traverse Gravimeter Experiment (TGE). The experiment involved the astronauts measuring the lunar gravity at preplanned locations. Using the measurements, attempts were made to uncover anomalies beneath the lunar surface, as was done in Talwani, Thompson, et al. (1973).

The instrument used in the mission was completely self-contained, with its own battery and thermal insulation. The main sensor was a Bosch Arma D4E vibrating string accelerometer, which was extremely sensitive to temperature. This was taken into account by ensuring that a deviation of more than 0.01 K in the instrument temperature was never achieved. The instrument also had a self-levelling mechanism, which makes sure the instrument stays within $0^{\circ}3'$ of arc (Talwani, Thompson, et al. 1973).

A schematic of the whole instrument can be seen in Figure 2.9.

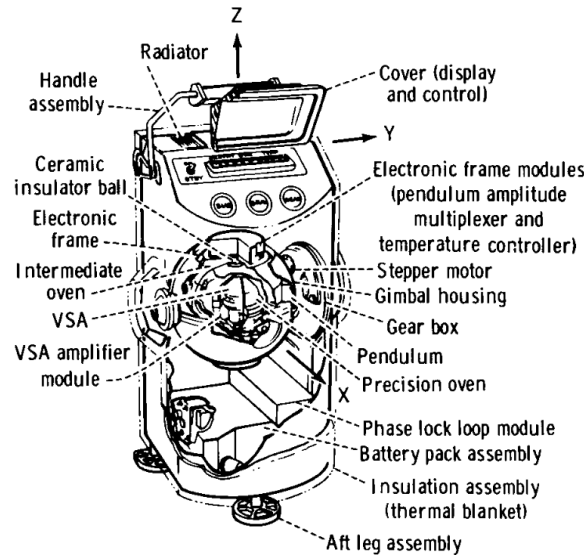


Figure 2.9: A schematic of the Traverse Gravimeter Experiment (TGE) instrument (Talwani, Thompson, et al. 1973).

Unfortunately, because the instrument is prone to drift, additional measurements were needed to better understand the extent of this effect. This extra measurement was obtained by taking a reading with the sensor in an inverted position. All observations during the mission were either made with the instrument on the lunar roving vehicle or with the instrument on the surface, and all took approximately 3 min or more.

The gravimeter had an accuracy of 0.035 mGal on the Moon, but including all error sources, the instrument never exceeded an accuracy of 2 mGal (Talwani, Thompson, et al. 1973).

VEGA Gravimeter

One gravimeter, which is used in the proposal for a lunar mission to find lunar lava tubes from Canadensys, on which more information can be found in subsection 2.1.5, is the VEGA gravimeter. The VEGA Space Gravimeter is an absolute vector gravimeter developed by Canadensys that enables measurement of gravity on the Moon (normal terrestrial gravimeters would saturate on the Moon). The gravimeter has a size of about $10 \times 10 \times 20$ cm and a mass of 2.1 kg. It is able to obtain a measurement accurate to $0.3 \mu\text{g}$ root-mean-squared for a 10-minute measurement on the Moon (ESA 2021). This is equal to an accuracy of about 0.3 mGal root-mean-squared.

It must also be noted that more accurate gravimeters are available. For example, in Muñoz et al. (2025), two gravimeters with an accuracy of $1 \mu\text{Gal}$ were used, which in practice had uncertainties of about $20 \mu\text{Gal}$. Still, such gravimeters are designed to function on Earth, and due to the extreme environment present on the Moon, lunar gravimeters must be designed with this environment in mind.

2.3.2. Gravimetry Corrections

In this section, common gravimetry corrections are mentioned, including the free-air, Bouguer, and terrain corrections.

Free-air correction

The free-air correction is a correction made to a gravity measurement to account for the fact that a measurement was made at an elevation other than sea level. This correction is needed, as when a correction is made farther from a body's centre of gravity, the measured gravity is lower. To allow for correct analyses of measurements, all data must be processed and equalised to a single reference datum, usually sea level. For the Moon, this would instead be a predefined datum. The general gravity equation is shown in Equation 2.1,

$$g = \frac{GM}{r^2}. \quad (2.1)$$

When this equation is differentiated with respect to the radius, the gradient can be found,

$$\frac{\partial g}{\partial r} = \frac{-2GM}{r^3}. \quad (2.2)$$

Using Equation 2.1, and plugging it back into Equation 2.2, one gets,

$$\frac{\partial g}{\partial r} = \frac{-2g}{r}. \quad (2.3)$$

Then, to actually find the change in gravity anomaly, one multiplies this factor by the height h :

$$\delta g_{\text{FA}} = \frac{-2g}{r}h. \quad (2.4)$$

Here, r is the distance from the centre of the Earth to sea level. As is mentioned in Blakely (1995, pp. 140–142), the free air correction is not a correction where one can think of the measurement point being moved down to sea level. One must remember that in subsequent calculations, the measurement point should still be assumed to be at the original measurement location.

A schematic where a measurement is taken from a plane can be seen in Figure 2.10. In the case on the Moon, there would not be an aeroplane, but instead there would be an astronaut or rover taking measurements. Note that it is *not* necessary to have air or free space between you and the surface to apply the free-air correction. Instead, one only needs the distance between the reference datum and the measurement.

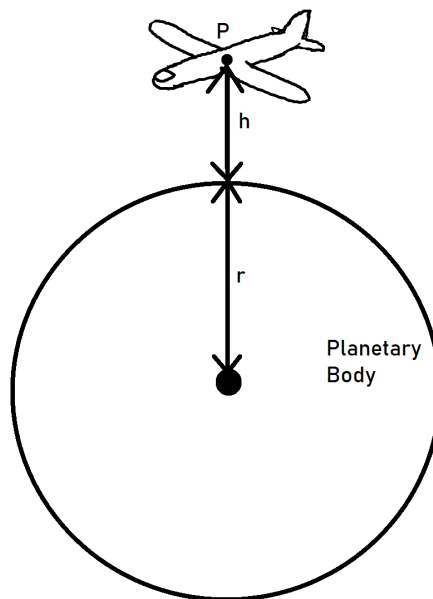


Figure 2.10: The geometrical aspects used in calculating the free-air coefficient. A typical use case for the free-air coefficient is when measurements are taken from an aeroplane. The schematic is not to scale.

Bouguer correction

Most measurements, especially on Earth, take place above sea-level. While the free-air correction accounts for a higher elevation (compared to sea level or a datum), the Bouguer correction accounts for the mass that is usually present beneath the point of measurement but above the reference datum. This mass would increase gravity and should not be ignored. To calculate this correction, Bouguer suggests using an infinite slab with an average density ρ and a height/thickness h (typically taken from the datum to the point of measurement, although this does not have to be the case (i.e. aeroplane measurements)).

The Bouguer correction then is (Blakely 1995, pp. 143),

$$\delta g_{BG} = 2\pi\rho hG \quad . \quad (2.5)$$

A visual representation of the Bouguer correction infinite slab model is shown in Figure 2.11.

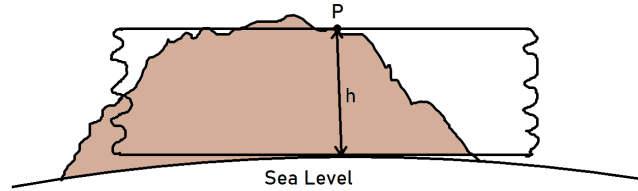


Figure 2.11: Visual representation of the Bouguer correction infinite slab model. The slab is given in white, while mountains or other features are given in brown. Sea level can also be replaced by a datum on bodies that do not have a predefined sea level. Adapted from Garland (1971).

Terrain correction

Most readers will realise that the Bouguer correction seems simplistic, as large variations in elevation can be present in mountainous areas. Assuming one can correct for this with an infinite slab is often inadequate.

Therefore, a third correction is typically present, especially in mountainous areas or on SB's in the solar system. This terrain correction is then used to account for the variations in topography with respect to the Bouguer plate. To compute the terrain correction, various methods are available. As an example, some methods include the discretisation of the topography into rectangles or prisms on a grid. Then, for each smaller prism volume, the gravity it exerts on the measurement point can be calculated and summed to get the total terrain correction.

Bouguer anomaly

Applying all three different corrections, the *reduced* or *refined* Bouguer anomaly can be calculated using Equation 2.6 (Blakely 1995, pp. 143),

$$g_B = g_{\text{measured}} - \delta g_{\text{free-air}} - \delta g_{\text{Bouguer}} - \delta g_{\text{terrain}} - g_0 \quad (2.6)$$

This equation combines all corrections needed to be able to compare two different measurements 'fairly'. To explain the combination of all corrections, assume a measurement is taken in the Swiss Alps, with mountains surrounding the measurement point. Then, because the measurement point is above sea level, one has to account for the gravity loss in measuring higher than sea level. This is done by the *free-air correction*. Next to that, because of the amount of rock beneath the measurement point but above the reference datum (which is sea level), gravity will be perceived as higher in this case compared to a different measurement at sea level. This is the *Bouguer correction*. Lastly, the topography of the mountains in the Swiss Alps varies immensely over short horizontal distances. This means some points of the terrain result in less gravity being perceived, while other points result in more gravity being perceived. To account for this final problem, the *terrain correction* is used. Combining all three corrections allows one to calculate the *reduced* Bouguer anomaly, while leaving the terrain correction out can be done to calculate the *simple* Bouguer anomaly.

The Bouguer anomaly is then used to analyse the subsurface density variations. In fact, geological subsurface structures can be inferred, as well as possible density variations. If the measured gravity (after corrections) is higher than the calculated gravity, a positive Bouguer anomaly is present, indicating

a higher density below the measurement point. Conversely, the opposite is true for a negative Bouguer anomaly. It must be noted that for simulations, calculating the anomaly is useful when trying to simulate measurements that would occur in real life. Then, one can try to understand what the Bouguer anomaly would look like if it were calculated for these measurements.

2.4. Gravity Anomaly Calculation Methods

In this section, various methods of calculating the gravitational attraction of 2D and 3D models are explored and explained. Such calculations will support the construction of forward models of any geometry and the calculation of their gravitational attraction. Additionally, such calculations can be used to obtain the gravity anomaly of these models when they are assumed to be buried beneath the ground. These could then be compared with gravity anomalies obtained by applying corrections to measurements, thereby improving gravity surveys.

2.4.1. Direct Integral Methods

In this subsection, direct integral methods for calculating gravity are explored. This will include both the historical perspective and the current state of the art. Direct integral methods rely on integrating the gravitational acceleration of an infinitesimal point over any volume to obtain the gravitational attraction. The method can also involve using sums of various smaller, easier shapes, such as bricks or polygons.

2D Anomalies

The simplest calculation method for gravity anomalies is to use a 2D coordinate system and assume uniform density anomalies. For a ridge, for example, one can (up to a certain extent) assume that along its length, the shape of the density anomaly does not change. In this case, using a 2D cross-section could be sufficient to model the gravity anomaly and, as such, to figure out the density distribution. Talwani, Worzel, and Landisman (1959) describes a method where an arbitrary 2D shape can be described by a polygon with an arbitrary number of vertices, allowing for arbitrary accuracy. Then, for this polygon, the gravitational attraction can be calculated by summing the contributions of each edge of the polygon around the boundary:

$$g = 2G\rho \oint z d\theta$$

Where z is the positive downward/vertical coordinate, and θ is the angle between the field point (where the gravity is being calculated) and the calculation point (a point on the edge). These geometrical elements can be seen in Figure 2.12.

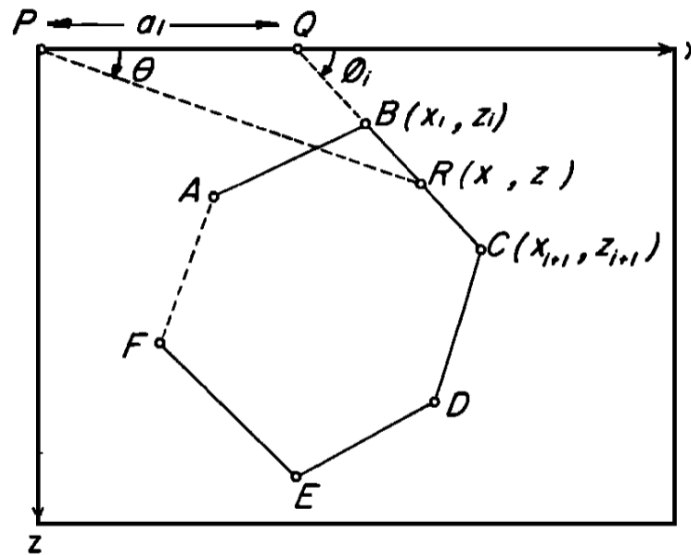


Figure 2.12: The geometrical setup used in Talwani, Worzel, and Landisman (1959)

The gravity anomaly of 2D structures with non-uniform density has also been studied. For example, Pohánka (1998) and Hansen (1999) provided methods for including linear variation in calculations of the gravity anomaly of 2D polyhedral bodies.

Even more advanced are methods that allow for variations using polynomial functions, as D'Urso (2015) provided. Such closed-form expressions allow for more complex calculations in the 2D domain.

3D Uniform Anomalies

Obviously, in real-world applications, the gravity anomalies will not always be represented accurately by a 2D uniform anomaly. Therefore, being able to model a gravity anomaly in 3D is necessary.

Talwani and Ewing (1960) expanded on his 2D uniform gravity anomaly theory (Talwani, Worzel, and Landisman 1959) into 3D through the use of summing multiple horizontal lamina. The 3D anomaly is split into lamina, or thin sheets, each an arbitrary 2D shape defined on a Cartesian coordinate system. This 2D shape can then be approximated by a polygon with n sides. The gravity anomaly of this polygon is then calculated as Vdz , where V is the anomaly per unit thickness and dz is the infinitesimal thickness.

To get the gravity anomaly of the entire 3D anomaly, an integration is performed:

$$\Delta g_{\text{total}} = \int_{z_{\text{bot}}}^{z_{\text{top}}} V dz. \quad (2.7)$$

To perform this calculation on a computer, the coordinates of the vertices that describe the contour for each lamina are needed, as well as the height of the lamina. The integration in Equation 2.7 is then performed numerically.

Another solution to the 3D problem is to find a method of calculating the gravity anomaly of any 3D prism. Nagy (1966) provided a method of calculating the gravitational attraction of an arbitrary right rectangular prism, which could then be used successively for the gravity anomaly of any arbitrary shape. A subroutine would use these individual ‘bricks’ as building blocks of the whole gravity anomaly. Banerjee and Das Gupta (1977) improved this method by using a different derivation, allowing the user not to worry about the prism crossing the x and y axes, as is necessary in the method of Nagy (1966). This derivation also showed that, if the right rectangular prism/parallelepiped is extended to infinity in the x and y directions, the Bouguer anomaly equation, described in Equation 2.5, is obtained.

Plouff (1976) also provided additional extensions of the method developed by Talwani and Ewing (1960). In fact, the method from Talwani and Ewing (1960) had the disadvantage that measurements close to a lamina (which, in the case of real-world measurements on a surface, would be the case) could not be estimated correctly. Plouff (1976) shows a way to integrate the formulas for the lamina in the depth direction to obtain an exact analytical solution for a 3D polygonal prism. Plouff (1976) also derived the analytical solution for a rectangular prism. This formula can be seen in Equation 2.8, in subsection 2.4.4.

Plouff (1976)’s analytical solution of a rectangular prism is used in Blakely (1995) to provide a Fortran subroutine, called ‘gbox’ or ‘GBOX’. This subroutine has since been used, for example, in Gharti, Tromp, and Zampini (2018) and Cai and Wang (2005) to provide a comparison of accuracy between numerical methods and semi-analytical methods. In these cases, a model is discretised into boxes or rectangular prisms, each of which can have its anomaly calculated using the GBOX subroutine.

The GBOX subroutine, although displayed in code in Blakely (1995, pp. 377), has its formula given in the same book in Blakely (1995, pp. 186–187), which is originally obtained by Plouff (1976) and can be found in Equation 2.8.

$$g = \gamma \rho \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \mu_{ijk} \left[z_k \arctan \frac{x_i y_j}{z_k R_{ijk}} - x_i \log(R_{ijk} + y_j) - y_j \log(R_{ijk} + x_i) \right] \quad (2.8)$$

Where,

$$R_{ijk} = \sqrt{x_i^2 + y_j^2 + z_k^2},$$

$$\mu_{ijk} = (-1)^i (-1)^j (-1)^k.$$

The x_i , y_j , and z_k , describe the endpoints of the brick in each dimension. γ and ρ describe the gravitational constant and the density of the prism, respectively.

Because a calculation is performed for every cell, the number of cells used scales linearly with compute time. Similarly, because every cell must be calculated for every observation point, the compute time

also scales linearly with the number of observation points. This results in the fact that a large model with many observation points could quickly take too long to simulate for practical purposes.

After all the attractions are calculated, they are summed to give the total gravitational attraction. This calculation is time-consuming, depending on the number of observation points and elements/cells, and not perfectly accurate, and is therefore taken as a benchmark to compare to other numerical methods.

Additionally, because the formula nominally calculates the gravitational attraction in one direction only, it must be performed three times in total to obtain a full 3D vector. In practice, the vertical component in gravity anomalies is always the biggest, and vector values are usually unnecessary.

Examples of gravity anomalies for a buried sphere (for which a subroutine was also given in Blakely (1995)), as well as a buried box, can be seen in Figure 2.13.

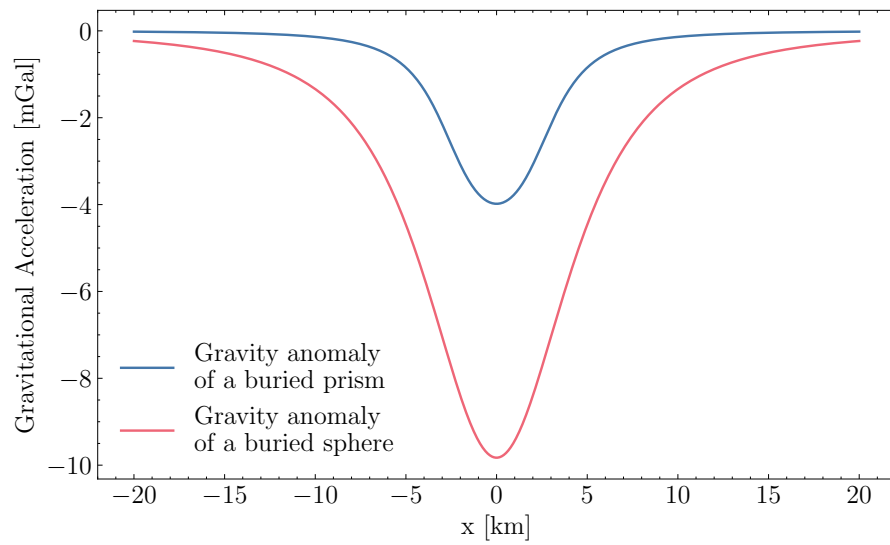


Figure 2.13: The gravity anomaly of a buried prism. This prism (mesh size of 0.5 km) encompasses a density anomaly of -400 kg m^{-3} , as well as a width, length, and depth of 5000 m, 10000 m, and 500 m, respectively. The prism is located a total of 2000 m from the observation point to the top face of the prism. The sphere (mesh size of 0.25 km) is a density anomaly of -200 kg m^{-3} . The radius is 4000 m, with the observation point being 6000 m above the centre of the sphere. The anomalies were calculated using subroutines given in Blakely (1995).

Another expansion of Talwani and Ewing (1960) was developed by Johnson and Litehiser (1972), in which the coordinate system was changed from Cartesian to spherical and first-order effects of the ellipticity of the Earth were included. This is relevant in applications where the gravity anomaly is present over a large area, such that excluding the ellipticity of the Earth would provide errors that are too significant to ignore. Similar to the original method, laminas are used, over which numerical integration is performed. Now, however, the laminas are portions of concentric ellipsoids of revolution, where the sides of the polygons are arcs of great circles.

Homogeneous Polyhedrons

Another method of modelling a 3D shape is by using 3D polyhedra. The gravitational acceleration (from which the gravitational attraction can be found using Newton's formula $F = ma$) of a 3D polyhedron is described by Werner (1994). This is done by using both Gauss' divergence theorem and Green's theorem, which allow the 3D polyhedron to be described by just its edges and vertex angles. Using this method, local variations that require high accuracy can be modelled more accurately by using many small polyhedron faces, while less varying areas can still be modelled accurately using larger faces. The solution is in closed form and uses only natural logarithms and arc tangents to calculate the final potential and acceleration.

Other methods of finding the gravity of a 3D polyhedron have been suggested by Paul (1974), Okabe (1979), Götze and Lahmeyer (1988) (which was used in the IGMAS+ software, on which more is said in subsection 2.4.4), and various others (D'Urso 2015).

Analytical Solution Buried Infinite Cylinder

To understand the results of simulations of lunar lava tubes, an analytical solution for a buried cylinder could be useful. Although such a formula was not easily found for a finite cylinder, the solution is relatively easy for an infinite one.

Connor (2015) provides the method and solution through the use of Gauss' Law. First, imagine an *infinite* cylinder. For this cylinder, two points can already be established. At any external point, outside the cylinder, the gravitational attraction will point towards the axis of the cylinder and be perpendicular to the axis. This results in the fact that no 'arrows' in the gravity field have a component that is either along the axis of the cylinder.

From this fact, a useful hypothetical Gaussian surface can be established, as can be seen in Figure 2.14.

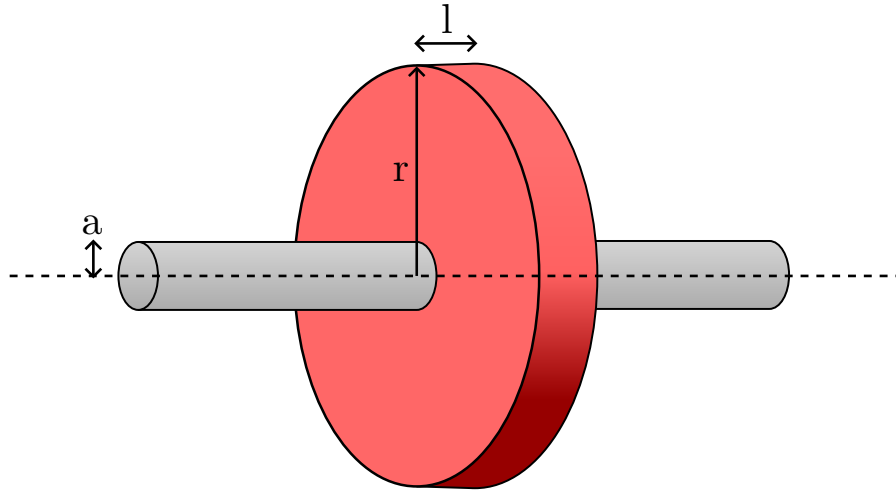


Figure 2.14: The model used to obtain the analytical solution for the gravitational attraction of an infinite cylinder. The red surface represents the imaginary Gaussian surface. Adapted from Connor (2015).

a denotes the radius of the cylinder of interest, r the radius of the imaginary Gaussian surface, which also denotes the distance between the observation point and the centre of the cylinder. Lastly, l denotes the thickness of the Gaussian cylinder.

This Gaussian surface captures all the gravitational flux on the curved surface, allowing for a simple calculation.

Using the integral form:

$$\oint_S \vec{g} \cdot \hat{n} dA = -4\pi GM. \quad (2.9)$$

Here, S denotes the Gaussian surface, whereas $\vec{g}$ denotes the gravity field vector, $\hat{n}$ the outward normal, perpendicular to the Gaussian surface, and dA the infinitesimal area element. G denotes the gravitational constant, while M denotes the mass encompassed by the Gaussian surface.

As such, the following simplifications can be made to Equation 2.9.

$$\begin{aligned} \text{Using } M &= M_l l = \pi a^2 \delta \rho l, \\ -2\pi r l g &= \pi a^2 \delta \rho l \cdot -4\pi G, \\ -2\pi r g &= -4\pi^2 a^2 \delta \rho G, \end{aligned}$$

Where M_l is the mass per unit length of the cylinder. Then, simplifying, one obtains,

$$g = \frac{2G\pi a^2 \delta \rho}{r} \quad (2.10)$$

As can be seen, a quadratic relationship is obtained between the gravitational attraction and the cylinder's radius a . Next to that, an inverse relationship is obtained for the distance to the centre of the cylinder. It is therefore expected to also see such relationships once simulations of lava tubes are performed. Still, it must be remembered that the analytical solution obtained in Equation 2.10 is for an *infinite* cylinder, which is not the case for lava tubes. In some situations, such infiniteness could be assumed. For example, when measurements are performed close to the cylinder, and the cylinder is long and thin, such assumptions could potentially be made. Overall, the analytical solution provided here is to be used as a guideline or for comparison with the simulations performed later in this thesis.

2.4.2. Spectral Methods

In this subsection, the use of spectral methods in geophysical applications is explored. Spectral methods rely on a sum of basis functions, which are able to approximate the solution of the underlying differential equation. Spherical harmonics are then the basis functions for the spectral method on a sphere. Such methods are useful as they are highly accurate. In addition, they provide the coefficients of the equations that describe the solution, resulting in the fact that, in the area of interest used, the solution can be obtained at any location. This contrasts with direct integral methods, where one chooses a specific location and only then calculates the gravitational attraction. One of the downsides of the spectral method is that one uses the whole domain, which can result in large computations that take too long or require too much memory. This can be alleviated by using the spectral element method, which splits the domain up into elements.

Unboundedness

In spectral methods, a problem that occurs is that when solving the Poisson equation, one has to solve an unbounded problem (the boundary condition is applied at infinity). To solve this numerically, one would need an infinite-sized domain, which is, of course, impossible. Therefore, a workaround is needed. This is often done by using artificial boundary conditions (ABCs). ABCs restrict the domain and allow numerical methods to be used to solve this problem. In essence, there are two options (Tsynkov 1998):

- Global ABCs: Are highly accurate and robust, but require expensive computation.
- Local ABCs: Are simple and computationally cheap, but lack accuracy.

Therefore, there is a trade-off between accuracy and computational speed.

Spherical Harmonics

To solve the equation for the gravity potential, which is a form of the Poisson equation, given in Equation 2.11, spherical harmonics can also be used. This has been readily employed in geophysical applications. Spherical harmonics are useful in these applications precisely because Earth is close to a sphere, and as such spherical harmonics provide a natural basis on which solutions can be calculated. However, for purposes such as calculating lava tube gravitational attractions, spherical harmonics are not a good choice, since they are used for global solutions and are much smoother than is useful for lava tubes.

$$\nabla^2 \delta\Phi = 4\pi G \delta\rho(x) \quad (2.11)$$

2.4.3. Spectral Element Method

In this subsection, the usage of the spectral element method (SEM) is discussed. Here, again, the Poisson equation is the equation to be solved, resulting in an unbounded problem that can be solved using artificial boundary conditions. Furthermore, in this subsection various historical uses of the spectral element method are mentioned, as well as its current use in calculating gravity anomalies.

Spectral Element Method Origin

The field where the spectral element method gained popularity early is in the field of fluid dynamics. In fact, one of the first papers on SEM (Patera 1984) used this method precisely for solving the incompressible Navier–Stokes equations. In Patera (1984), the main benefit of using SEM was stated to be that, while the spectral method is accurate, it is difficult to use in complex and large problems.

Whereas the finite element method is very flexible and has low computing cost, it is less accurate. Therefore, SEM combines both methods to be flexible and accurate. Various ways of using the spectral element method have also been described in various books, such as Canuto et al. (1988) and Deville, Fischer, and Mund (2002).

Spectral element method for gravity anomalies

In terms of uses of the spectral element method for gravity anomalies, Martin et al. (2017) uses Gauss-Lobatto-Legendre (GLL) quadrature to discretise the model using the spectral element method (SEM) and then performs the numerical integration using these points. SEM showed faster calculation times compared to GBOX by a factor of 40. This is due to GBOX using arctangent and natural logarithm functions, which are generally computationally expensive. Its accuracy was also better than GBOX when modelling a subsurface sphere.

Approximating boundary conditions

One way to solve the unbounded problem is to allow for approximations of the boundary, as was done in Cai and Wang (2005). Here, the assumption was made that the gravitational potential at a distance r from the density anomaly was of the order of r^{-4} . Applying this enables the creation of a fast finite element method (FFEM), which has been shown to be faster than the standard integration method (GBOX) by a factor of 10–100.

Wave propagation

A research field where SEM is found particularly useful is in geophysics, specifically wave propagation Faccioli et al. (1997), which performed elastic wave propagation using a pseudo-spectral method, aiming to combine the positives of both the finite element method and spectral methods. For this, spectral approximations and domain decomposition techniques were used. Peter et al. (2011) also performed SEM simulations for seismic wave propagation (coupled acoustic and (an)elastic). For this, both forward and adjoint simulations were performed. In Seriani and Oliveira (2008), the accuracy of such SEM simulations was analysed, specifically for the elastic wave equation. Their analysis was one of the first theoretical accuracy analyses for elastic conditions, which they performed using a dispersion analysis. Lastly, a review of the most commonly used SEM techniques in wave propagation was shown in Tromp, Komatitsch, and Liu (2008), where both techniques for the forward and adjoint/inverse problem were discussed. The inverse problem is a problem where, instead of generating how the wave would influence the object that is being studied, the object's movements are measured, and synthetic simulations are made. Then, one minimises the differences between the simulations and the measurements to make the simulations resemble real life as closely as possible. This is an interesting concept that could potentially be used for trying to understand how lunar lava tubes look when looking at their gravity signal. Through iteration, a first estimate of the lava tube size, depth, and shape could be made in advance.

Infinite Element

Another method introduced to alleviate the unbounded problem is to use what is called an infinite element. This is essentially an element which is not finite, but instead has one boundary that is fixed in the domain of interest, and then the opposite side of this element extends out to infinity. In general, there are two ways of using infinite elements:

- Displacement Descent.
- Coordinate Ascent.

Displacement Descent

The displacement descent approach involves using shape functions, which essentially also include decay. Bettess (1977) showed that the shape functions then look analogous to Lagrange polynomials, with an added factor, such that the shape functions N_i , for $i = 1$ to $n - 1$ would look as follows.

$$N_i = e^{(x_i - x)/L} \prod_{\substack{j=1 \\ j \neq i}}^{n-1} \frac{x_j - x}{x_j - x_i}.$$

Medina and Taylor (1983) also showed a version of the displacement descent approach, where some conditions of using infinite elements were outlined:

- Finiteness
- Radiation (Sommerfeld radiation condition)

Finiteness means that all derivatives of the shape functions must vanish at infinity, whereas radiation means the shape functions do not create inward waves in the far field. Medina and Taylor (1983) also used Gauss-Laguerre quadrature for the numerical integration, as normal Gauss-Legendre quadrature could not be used in the infinite element without modifications. El-Esnawy, Akl, and Bazaraa (1995) does provide a method where Gauss-Legendre quadrature can be used, after modifications.

Coordinate Ascent

Unlike displacement descent, coordinate ascent can automatically use Gauss-Legendre quadrature. This is because coordinate ascent uses a mapping of the infinite element, by which the mapped version is finite; as such, general quadrature can be used. Beer and Meek (1981), Zienkiewicz, Emson, and Bettess (1983), and Kumar (1985) all provide variations of this method. It is shown that this method can have the advantage of not needing additional nodes, and thus additional degrees of freedom, which means less computation.

In Gharti, Tromp, and Zampini (2018), the coordinate ascent approach is combined with the spectral element method to create the spectral infinite element method (SIEM). The method starts with the derivation of the weak form, given in subsection 2.5.1.

The method was tested against both analytical and semi-analytical methods, specifically GBOX, given by Blakely (1995, pp. 377). In fact, the following test cases were used:

- Rectangular prism
- Spherical salt dome
- Multiple spheres with varying size and density
- Rectangular prism with a parabolic density distribution

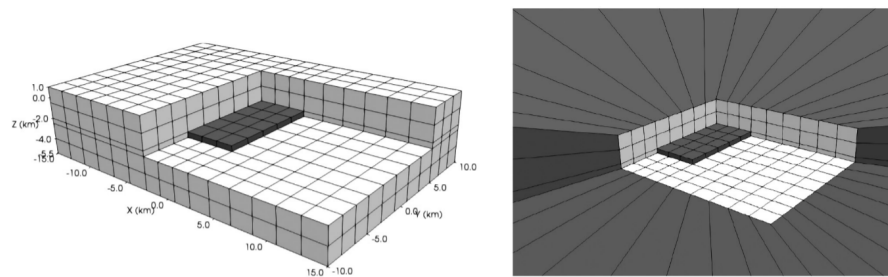


Figure 2.15: The meshed buried rectangular prism used in Gharti, Tromp, and Zampini (2018). The left figure shows the domain and the buried prism, while the right-hand figure also shows various infinite elements that were used in this implementation.

For each of these test cases, both an analytical solution and SPECSEM were implemented, except for the case involving multiple spheres with varying sizes and densities, where GBOX was also used as a benchmark.

How these analytical solutions were calculated is shown in Appendix C.

In the numerical examples that were tested, the solution very closely aligned with the analytical solution, sometimes requiring a smaller mesh size, as in the spherical salt dome. The exact details of each numerical comparison can be seen in Table 2.5. In this table, the agreement column was summarised using remarks from the original authors of the paper. The statements were verified by the author of this thesis and agreed with.

Table 2.5: Details of the numerical comparisons used to assess the SPECFEM software for gravity anomalies. The "" marks indicate that this parameter did not change compared to the original (Gharti, Tromp, and Zampini 2018). Note that the agreement was summarised from the original remarks found in Gharti, Tromp, and Zampini (2018). These remarks were checked and verified by the author of this thesis and thus agreed with.

Numerical Comparison	Domain	Density, size, and location object	Elements	Agreement
Rectangular Prism				
Rectangular Prism	$30 \times 20 \times 6.5$ km.	Density anomaly: prism of $5 \times 10 \times 0.5$ km, at 2 km depth, with density of 400 kg m^{-3} .	Element size of 2 km, total No. of elements: 825.	Good.
Rectangular Prism (refined)	""	""	Element size of 1 km, total No. of elements: 6600.	Very good.
Spherical Salt Dome				
Spherical salt dome	$40 \times 30 \times 30$ km	Density anomaly Radius: 4 km, depth to centre sphere: 6 km, with density of -200 kg m^{-3} .	Element size of 1 km inside the dome, 2 km outside the dome. Total No. of elements: 6702.	Decent, discrepancy in the middle of the dome.
Spherical salt dome (refined)	""	""	Element size of 0.5 km inside the dome, 1 km outside the dome. Total No. of elements: 53615.	Good, still discrepancy at centre.
Spherical salt dome (refined)	""	""	Element size of 0.25 km inside the dome, 0.5 km outside the dome. Total No. of elements: 156538.	Nearly Perfect.
Multiple Spheres with Varying Size and Density				
Multiple spheres with varying size and density	$206 \times 80 \times 170$ km	100 spheres, varying radii and density. Ten equidistant lines in the vertical plane with a vertical spacing of 10 km. In the horizontal direction, the spheres are also equidistant from each other at a spacing of 14 km. The exact radius and density vary linearly, and their formulas can be found in the paper (Gharti, Tromp, and Zampini 2018). The location of the first sphere is at an (x,y,z) coordinate of (40,0,-5) km.	Element size is $0.2r_s$ inside a sphere and $0.4r_s$ outside, where r_s is the sphere's radius. As the spheres increase in size towards the bottom, the bottom spheres have a coarser mesh. Total element size was 0.4-0.76 km inside the spheres, and 0.8-1.52 km outside the spheres. Total No. of elements: 2886000.	Time needed: 8 min. Excellent agreement with analytical solution.

Continued on next page

Continued from previous page

Numerical Comparison	Domain	Density, size, and location object	Elements	Agreement
Multiple spheres with varying size and density (GBOX)	""	The same model as used in SPECSEM was used, but discretised into cubic cells.	Cell size used: 0.2 km. Total No. of cells: 1611488.	Time needed: 28 min. Worse than SPECSEM. Unable to represent spherical shapes accurately.
Multiple Spheres with Varying Size and Density				
Rectangular prism with a parabolic density distribution	$14 \times 4 \times 4$ km	Density anomaly: prism of $2 \times 1 \times 1$ km, at a depth of 1 km. Anomaly distribution is given by $\delta\rho(z) = \frac{\delta\rho_0^3}{(\delta\rho_0 - \alpha z)^2}$. $\delta\rho_0$ is the anomaly observed at ground surface, while α is a constant and equal to 0.001. Four density anomalies were used: $\delta\rho = 5, 10, 100$ and 100 kg m^{-3} .	Element size of 0.4 km, total No. of elements: 4235.	Good.
Rectangular prism with a parabolic density distribution (refined)	""	""	Element size of 0.2 km, total No. of elements: 33880.	Excellent.

A close look must be taken at the example containing multiple buried spheres, as it compares the analytical solution, GBOX, and SPECSEM. Firstly, the cell size used in GBOX was 0.2 km, which was half of the smallest cell size in the SPECSEM model (0.4 km). Still, the GBOX solution performs worse than the SPECSEM solution. This is most likely caused by the difficulty of modelling a sphere using cubes, as well as the use of variable densities in the model. The exact discrepancy in accuracy can be seen in Figure 2.16.

Another interesting aspect of this example is that to obtain the gravity anomaly, a total of 157 observation points were used at four elevations (0, 5, 15, and 25 km), for a total of 628 observation points. This is important because the computational cost of SPECSEM is independent of the number of observation points, whereas for GBOX the calculation must be performed again for each observation point, increasing computation time. This highlights the superiority of SPECSEM over simpler methods such as GBOX in certain situations.

Overall, it can be seen that SIEM is a highly accurate method of forward modelling, allowing for calculations in large models with a high number of nodes. One method that has not been tested against SPECSEM is a polyhedral method.

2.4.4. Gravity Anomaly Calculation Software

To calculate the gravity anomaly of a given model, it is useful to use software to perform the calculations, especially for complicated models. Software for this has been developed since the late 1950s, as in Talwani, Worzel, and Landisman (1959).

Since then, many improvements have been made. Using polygons and polyhedra to calculate the gravity anomaly of a model is widely used because it enables many simple calculations, which computers

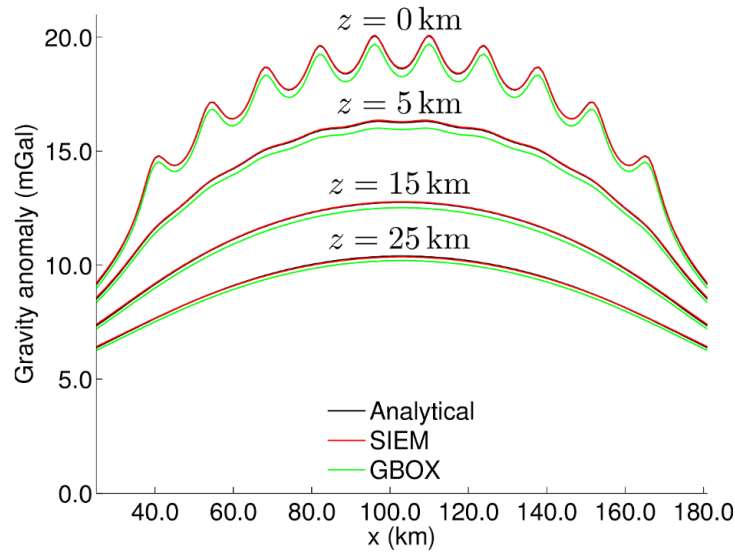


Figure 2.16: The gravity anomaly at four elevations for the multiple buried spheres example. The analytical solution, the SPECIFEM solution, and the GBOX solution are compared. Taken from Gharti, Tromp, and Zampini (2018).

excel at. Methods such as those described in Werner (1994) are among the most recently used for this purpose. Other proposed methods use spheres or bricks, combined in various sizes to estimate the gravity anomaly of a more complex body.

Three important software packages that are used for comparisons and actual analysis are:

- GBOX
- IGMAS+
- SPECIFEM

GBOX

One specific example used for comparisons with other methods, as is done in Gharti, Tromp, and Zampini (2018), is GBOX. GBOX is a single subroutine given in Blakely (1995, pp. 377) that calculates the gravity of a rectangular parallelepiped, also sometimes called a 'brick'. This subroutine can be used in any model that consists solely of bricks. Usually, a model is discretised into bricks, cubic cells, or a variation, after which the subroutine can be applied to each cell. The cell's gravities can then be summed, yielding the full gravity anomaly. Obviously, using only cubic cells or bricks results in spherical objects being modelled relatively inaccurately compared to some other methods.

The formula for this method was given earlier in Equation 2.8.

IGMAS+

IGMAS+ (Götze and Lahmeyer 1988; Schmidt et al. 2020) is software currently maintained and developed by the GFZ Helmholtz Centre for Geosciences. It has existed for more than four decades. The current program runs in Java and is named 'IGMAS+'. For its method of calculating the gravity anomaly, it uses the polyhedral method developed by Götze (1978) and the GFZ Helmholtz Centre for Geosciences (2025). Other than for calculating gravity anomalies, the software can be used for magnetic field modelling, as both the gravity and magnetic fields are potential fields.

SPECIFEM

SPECIFEM is software with multiple versions, all used in geophysical applications, with current use cases being in seismic wave propagation (Peter et al. 2011), where SPECIFEM3D was used, or in 3D slope stability analysis (Gharti, Komatitsch, et al. 2012), where SPECIFEM3D_GEOTECH was used. Another version, named SPECIFEMX, which has not been publicly released yet, can be used for calculating

the gravity anomaly of any model. This software is to be used in this thesis for calculating gravity anomalies. The spectral finite element method is explained in more depth in chapter 3.

Overall, SPECFEM is expected to be the preferred software in high-resolution simulations, in cases where shapes are complex and organic and must be followed as closely as possible. Similarly, it is preferred when the solution must be known over a large area or volume. It is expected to perform better in accuracy, as GBOX discretises models into cubes, which compromises accuracy, especially in spherical geometries. IGMAS+, which uses polyhedra, is expected to also perform worse for the same model, as the polyhedral method discretises a model into flat-faced polyhedra, for which a direct integral method is then used. SPECFEM, which uses SEM, can approximate curves with a high degree of accuracy using a spectral element method, allowing it to obtain a great fit with the same model. Still, no direct comparison between IGMAS+ and SPECFEM has been performed, which means that such a comparison would be of high interest for quantifying these predictions. In terms of computational speed, GBOX is assumed to be the quickest, since it only needs a single calculation per box: if one discretises a model into a million boxes, only a million computations are necessary. One advantage that SPECFEM has over the other codes is that it calculates independently of the number of observation points—this is especially useful in applications where the gravity anomaly must be known at multiple locations. This is obviously very much the case in gravity surveys. It is therefore expected that overall, SPECFEM will perform better, especially in high-resolution simulations for complex models.

2.5. The Spectral Element Method

In this section, the mathematical principles underlying SPEC-FEM are explained. The basic governing equations of the SPEC-FEM software used in the thesis and the solutions are stated, after which the spectral element method is explained. Because SPEC-FEM also uses infinite elements, a brief clarification on this aspect is provided. First, the governing equations are explained in subsection 2.5.1, after which an example of a spectral element method is given for the 1D case, in subsection 2.5.2. This section provides an entry point into understanding the spectral element method. Lastly, the equations and formulas used to create the SPEC-FEM version of the spectral element method are explained in section 2.6. For readers familiar with the spectral element method, this section can be skipped, as it only provides a good entry point into understanding how SPEC-FEM works.

2.5.1. Governing Equations and Spectral Element Formulation

To understand the problem to be solved, one first has to start by defining the starting principles. Because SPEC-FEM is the tool to be used, the derivations will be in line with how that tool is implemented, namely following the spectral element method.

The spectral element method is a numerical method for solving differential equations. It combines the flexibility of the finite element method while keeping accuracy high, as spectral methods do. Normal spectral methods have to calculate their coefficients considering the entire domain, while spectral element methods do not have this limitation. Instead, they can be parallelised, speeding up computation drastically.

The problem statement for SPEC-FEM, considering a body in space, is as follows. The governing equations can be defined as (Gharti, Tromp, and Zampini 2018),

$$\begin{aligned}\nabla^2 \delta\Phi &= 0, \text{ in subdomain outside the density anomaly} \\ \nabla^2 \delta\Phi &= 4\pi G \delta\rho(x, y, z), \text{ in subdomain inside the density anomaly.}\end{aligned}$$

Which are subject to the boundary conditions given by,

$$\begin{aligned}[\delta\Phi]_{-}^{+} &= 0 \text{ on } \Gamma \\ [\hat{\mathbf{n}} \cdot \nabla \delta\Phi]_{-}^{+} &= 0 \text{ on } \Gamma\end{aligned}\tag{2.12}$$

$$\delta\Phi = 0 \text{ on } \Gamma_{\infty}\tag{2.13}$$

Note that $[\cdot]_{-}^{+}$ indicates a jump from the $-$ side to the $+$ side of the boundary Γ . This can be seen in Figure 2.17:

Then, to derive the weak form, which is necessary for SEM, both sides of the equation are multiplied by a test function w and integrated over the entire domain $\mathcal{O}$:

$$\int_{\mathcal{O}} w \nabla^2 \delta\Phi \, dV = 4\pi G \int_{\mathcal{O}} w \delta\rho \, dV.\tag{2.14}$$

First, integration by parts is applied to the left side. This is done by using Green's first identity.

$$\nabla \cdot (w \nabla \delta\Phi) = w \nabla^2 \delta\Phi + \nabla w \cdot \nabla \delta\Phi,\tag{2.15}$$

which can be rearranged to give:

$$w \nabla^2 \delta\Phi = \nabla \cdot (w \nabla \delta\Phi) - \nabla w \cdot \nabla \delta\Phi.\tag{2.16}$$

Plugging this in, the left-hand side of Equation 2.14 becomes:

$$\int_{\mathcal{O}} w \nabla^2 \delta\Phi \, dV = \int_{\mathcal{O}} \nabla \cdot (w \nabla \delta\Phi) \, dV - \int_{\mathcal{O}} (\nabla w) \cdot (\nabla \delta\Phi) \, dV.\tag{2.17}$$

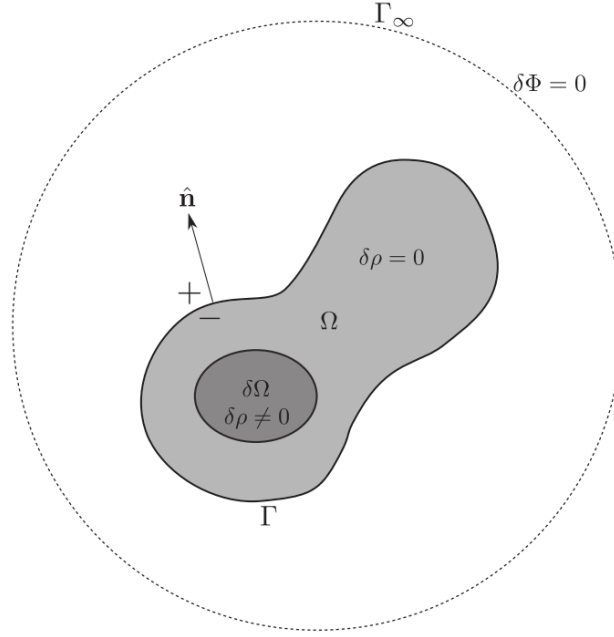


Figure 2.17: A schematic showing the boundaries and definitions used in the boundary conditions to which the governing equations are subject.

Now the divergence theorem can transform the volume integral into a surface integral. However, before this happens, the volume integral is split up into the two domains which are present in the problem.

$$\int_{\Omega} \nabla \cdot (w \nabla \delta \Phi) dV = \int_{\delta \Omega} \nabla \cdot (w \nabla \delta \Phi) dV + \int_{\delta \Omega^c} \nabla \cdot (w \nabla \delta \Phi) dV, \quad (2.18)$$

Now, the divergence theorem,

$$\int_V (\nabla \cdot \mathbf{F}) dV = \oint_S (\mathbf{F} \cdot \hat{\mathbf{n}}) dS, \quad (2.19)$$

Is applied to the first term on the right-hand side,

$$\int_{\delta \Omega} \nabla \cdot (w \nabla \delta \Phi) dV = \int_{\Gamma} w (\hat{\mathbf{n}} \cdot \nabla \delta \Phi_-) dS, \quad (2.20)$$

Where $\hat{\mathbf{n}}$ is the outward-pointing normal vector on the boundary Γ , and the subscript $-$ indicates the value just inside the boundary. The same is applied to the second term, where it can be seen that two terms arise:

$$\int_{\delta \Omega^c} \nabla \cdot (w \nabla \delta \Phi) dV = \int_{\Gamma} w (-\hat{\mathbf{n}} \cdot \nabla \delta \Phi_+) dS + \int_{\Gamma_{\infty}} w (\hat{\mathbf{n}} \cdot \nabla \delta \Phi) dS, \quad (2.21)$$

Where the subscript $+$ indicates the value just outside the boundary, and Γ_{∞} indicates the surface at infinity. Combining these two results, the last term of Equation 2.18 becomes:

$$\int_{\delta \Omega^c} \nabla \cdot (w \nabla \delta \Phi) dV = \int_{\Gamma} w (\hat{\mathbf{n}} \cdot \nabla \delta \Phi_- - \hat{\mathbf{n}} \cdot \nabla \delta \Phi_+) dS + \int_{\Gamma_{\infty}} w (\hat{\mathbf{n}} \cdot \nabla \delta \Phi) dS. \quad (2.22)$$

This can be written as follows,

$$\int_{\delta \Omega^c} \nabla \cdot (w \nabla \delta \Phi) dV = - \int_{\Gamma} w [\hat{\mathbf{n}} \cdot \nabla \delta \Phi]_+^- dS + \int_{\Gamma_{\infty}} w (\hat{\mathbf{n}} \cdot \nabla \delta \Phi) dS, \quad (2.23)$$

Now, using boundary condition Equation 2.12, the left term on the right-hand side reduces to zero.

All in all, Equation 2.14 can now be written as:

$$\int_{\Omega} (\nabla w) \cdot (\nabla \delta \Phi) dV = -4\pi G \int_{\delta \Omega} w \delta \rho dV + \int_{\Gamma_{\infty}} w (\hat{\mathbf{n}} \cdot \nabla \delta \Phi) dS, \quad (2.24)$$

Which is the final weak form. This weak form is the basis of the spectral element method. Any future derivation uses some form of this weak form. For example, in subsection 2.5.2, a 1D version of this is used as the basis; see Equation 2.28.

Using the general weak form, any fitting numerical method can be applied to solve the differential equation for the gravitational potential. Once the gravitational potential has been calculated, the gravitational attraction can be calculated using a numerical derivative method. For SIEM, this is shown in section 2.6.

Here, one does have the problem of the unbounded domain. Solving an unbounded problem is also not easy, and therefore usually the domain is restricted to be bounded. For this new domain, new artificial boundary conditions (ABCs) must be defined. Normally, there are two different options, as mentioned in subsection 2.4.2: Global ABCs and Local ABCs.

2.5.2. Spectral Element Method Example

To fully understand the spectral element method, which is used in SPEC-FEM, an example will be used. The weak form provided in subsection 2.5.1 will be used as a starting point. First, the spectral method, on which the spectral element method relies, is explained. Afterwards, the link with the spectral element method is clarified.

Spectral Method: An example using GLL and Lagrange Polynomials

The spectral element method relies on the spectral method, which is used over multiple coupled elements. Therefore, the spectral method must be understood first. To see the spectral method in action, an example will be shown here. This example should help illustrate the advantages of using the spectral method, as well as the use of GLL points and Lagrange polynomials.

The Poisson equation in 1D will be considered, with $\delta \Phi$, which is to be solved for (since using a numerical derivative δg , the gravitational attraction, can be obtained), and $f = 4\pi G \delta \rho(x)$, which is a forcing function. In this case, the boundary term,

$$\frac{d^2 \delta \Phi}{dx^2} = 4\pi G \delta \rho(x), \quad (2.25)$$

which will be written more simply in the following way:

$$\delta \Phi_{xx} = 4\pi G \delta \rho(x). \quad (2.26)$$

This shall be written in the weak form, using the range -1 to 1 for simplicity. This can later be changed to any range through mapping. The weak form will then be:

$$\int_{-1}^1 w (\delta \Phi_{xx} - (4\pi G \delta \rho(x))) dx = 0, \quad (2.27)$$

Where w is the weighting function. This weak form is a 1D case of the weak form given in Equation 2.24. Using integration by parts, this becomes:

$$[w \delta \Phi_x]_{-1}^1 - \int_{-1}^1 w_x \delta \Phi_x dx = 4\pi G \int_{-1}^1 w \delta \rho(x) dx. \quad (2.28)$$

Now, it will be assumed that the solution to $\delta \Phi$ can be approximated by basis functions multiplied by coefficients. The coefficients are unknowns that have to be found, whereas the basis functions can be chosen by the user.

In this case, Lagrange polynomials are used, such that each $\delta\Phi$ can be written as:

$$\delta\Phi(x) = \sum_{i=1}^N a_i \ell_i(x),$$

where a_i are the coefficients to be found, and ℓ_i are the Lagrange polynomials. N is the number of points and therefore also the number of basis functions.

The Lagrange polynomials are defined as follows:

$$\ell_j = \prod_{\substack{0 \leq m \leq k \\ m \neq j}} \frac{x - x_m}{x_j - x_m},$$

Where j indicates the index of the current Lagrange polynomial, the interpolation is defined on $k + 1$ quadrature points, while m acts as an index that goes over all the other quadrature points, except point j (this would result in a value of zero in the denominator).

Imagine now that the number of points and basis functions to be used is chosen to be four. Therefore, $N = 4$.

In this case, Equation 2.28 can be written four times, for the four points, where each point (that corresponds to a row) will look as follows:

$$[w_i \delta\Phi_x]_{-1}^1 - \int_{-1}^1 w_{i_x} \delta\Phi_x dx = 4\pi G \int_{-1}^1 w_i \delta\rho dx, \quad (2.29)$$

where i is the index of the point, which can be 1, 2, 3, or 4.

In the current example, the weighting function w has not yet been defined. In this case, the Galerkin approach is used to select the functions. This means that the weighting functions are chosen to be identical to the basis functions, which are Lagrange polynomials. Therefore, $w_i = \ell_i$.

$\delta\Phi_x$ will now also be replaced with the approximate solution:

$$\left[\ell_i \sum_{j=1}^n a_j \ell_{j_x} \right]_{-1}^1 - \int_{-1}^1 \ell_{i_x} \sum_{j=1}^n a_j \ell_{j_x} dx = 4\pi G \int_{-1}^1 \ell_i \delta\rho dx, \quad (2.30)$$

Equation 2.30 is now one row of a 4-row system of equations (one for each quadrature point).

To facilitate numerical calculations, the integrals will be approximated using numerical quadrature. The numerical quadrature has the following form:

$$\int_{-1}^1 g(x) dx \approx \sum_{k=1}^n w_k g(x_k), \quad (2.31)$$

Where w_k are weights, and x_k are the quadrature points for Gauss-Legendre-Lobatto (GLL).

The GLL points are defined as the roots of the following equation:

$$(x^2 - 1)L'_n(x) = 0, \quad (2.32)$$

from -1 to 1. L_n is the Legendre polynomials of degree n . There are many ways of finding the Legendre polynomials, one of which is by using Rodrigues' formula:

$$L_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n. \quad (2.33)$$

There is a specific advantage to using GLL points in combination with Lagrange polynomials.

The property that these polynomials have is that the basis function defined at a certain quadrature point is equal to one at that point, while zero at all the other points. This is called the Kronecker delta property, and it is useful as calculations are simplified. The Kronecker Delta is defined as:

$$\delta_{ik} = \begin{cases} 1 & \text{if } i = k, \\ 0 & \text{if } i \neq k. \end{cases} \quad (2.34)$$

Of course, instead of GLL points, any points could have been chosen, but GLL points have the advantage of providing an exact result for polynomials of degree $2n - 1$ or less, where n is the number of quadrature points.

An example of Lagrange polynomials for four GLL points is shown in Figure 2.18.

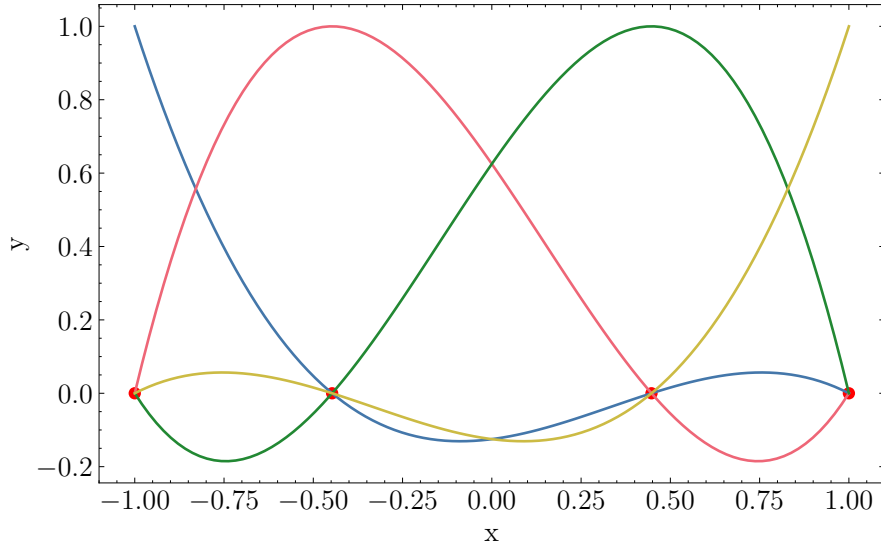


Figure 2.18: Lagrange polynomials for four GLL points with the range from -1 to 1. The red dots indicate the location of the GLL points.

As can be seen in Figure 2.18, each of the polynomials defined from -1 to 1 is equal to 1 at one of the quadrature points, while being equal to zero at all the others.

The quadrature can now be applied to both the right-hand side and the left-hand side. The right-hand side, per row, becomes:

$$4\pi G \int_{-1}^1 \ell_i \delta \rho \, dx \approx 4\pi G \sum_{i=1}^n w_k \ell_i(x_k) \delta \rho(x_k). \quad (2.35)$$

Now, the Kronecker delta property is used, which states that $\ell_i(x_k) = \delta_{ik}$. This gives:

$$4\pi G \int_{-1}^1 w_k \delta \rho \, dx \approx 4\pi G w_i \delta \rho(x_i). \quad (2.36)$$

The right-hand side vector can then be written as:

$$\mathbf{f} = 4\pi G \begin{bmatrix} w_1 \delta \rho(x_1) \\ w_2 \delta \rho(x_2) \\ w_3 \delta \rho(x_3) \\ w_4 \delta \rho(x_4) \end{bmatrix} \quad (2.37)$$

For the left-hand side, the second term can be extended to:

$$\int_{-1}^1 \ell_{i_x} \sum_{j=1}^n a_j \ell_{j_x} \, dx \approx \int_{-1}^1 \ell_{i_x} (a_1 \ell_{1_x} + a_2 \ell_{2_x} + a_3 \ell_{3_x} + a_4 \ell_{4_x}) \, dx. \quad (2.38)$$

It is easy to see that this integral can be split into four:

$$\int_{-1}^1 \ell_{1_x}(a_1 \ell_{1_x}) dx + \int_{-1}^1 \ell_{1_x}(a_2 \ell_{2_x}) dx + \int_{-1}^1 \ell_{1_x}(a_3 \ell_{3_x}) dx + \int_{-1}^1 \ell_{1_x}(a_4 \ell_{4_x}) dx \quad (2.39)$$

Considering all rows simultaneously, this system can be expressed in matrix form. The a_i coefficients can be separated into a vector $\vec{a}$, and the integrals can be put into a matrix K . This gives, in our example, for all four rows:

$$\begin{bmatrix} \left[\ell_1 \sum_{j=1}^n a_j \ell_{j_x} \right]_{-1}^1 \\ \left[\ell_2 \sum_{j=1}^n a_j \ell_{j_x} \right]_{-1}^1 \\ \vdots \\ \vdots \end{bmatrix} - \begin{bmatrix} \int_{-1}^1 \ell_{1_x} \ell_{1_x} dx & \int_{-1}^1 \ell_{1_x} \ell_{2_x} dx & \int_{-1}^1 \ell_{1_x} \ell_{3_x} dx & \int_{-1}^1 \ell_{1_x} \ell_{4_x} dx \\ \int_{-1}^1 \ell_{2_x} \ell_{1_x} dx & \int_{-1}^1 \ell_{2_x} \ell_{2_x} dx & \int_{-1}^1 \ell_{2_x} \ell_{3_x} dx & \int_{-1}^1 \ell_{2_x} \ell_{4_x} dx \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \mathbf{f} \quad (2.40)$$

Note that the dots indicate that the pattern continues for the other rows.

Next, Equation 2.40 can be written as:

$$\mathbf{M}_b - \mathbf{K}\mathbf{a} = \mathbf{f} \quad (2.41)$$

Also, the quadrature Equation 2.31 can now be applied to get, for the middle matrix:

$$\begin{bmatrix} \sum_{k=1}^n w_k \ell_{1_x}(x_k) \ell_{1_x}(x_k) & \sum_{k=1}^n w_k \ell_{1_x}(x_k) \ell_{2_x}(x_k) & \sum_{k=1}^n w_k \ell_{1_x}(x_k) \ell_{3_x}(x_k) & \sum_{k=1}^n w_k \ell_{1_x}(x_k) \ell_{4_x}(x_k) \\ \sum_{k=1}^n w_k \ell_{2_x}(x_k) \ell_{1_x}(x_k) & \sum_{k=1}^n w_k \ell_{2_x}(x_k) \ell_{2_x}(x_k) & \sum_{k=1}^n w_k \ell_{2_x}(x_k) \ell_{3_x}(x_k) & \sum_{k=1}^n w_k \ell_{2_x}(x_k) \ell_{4_x}(x_k) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \quad (2.42)$$

Now this matrix can also be written in a much more convenient form, using a differentiation matrix D and a weighting matrix W . They are defined as follows:

$$\begin{aligned} D_{kj} &= \ell_{j_x}(x_k), \\ W_{kj} &= w_k \delta_{kj}, \end{aligned} \quad (2.43)$$

Where W is a diagonal matrix. For clarity, in the example, they would look as such:

$$\begin{aligned} \mathbf{D} &= \begin{bmatrix} \ell_{1_x}(x_1) & \ell_{1_x}(x_2) & \ell_{1_x}(x_3) & \ell_{1_x}(x_4) \\ \ell_{2_x}(x_1) & \ell_{2_x}(x_2) & \ell_{2_x}(x_3) & \ell_{2_x}(x_4) \\ \ell_{3_x}(x_1) & \ell_{3_x}(x_2) & \ell_{3_x}(x_3) & \ell_{3_x}(x_4) \\ \ell_{4_x}(x_1) & \ell_{4_x}(x_2) & \ell_{4_x}(x_3) & \ell_{4_x}(x_4) \end{bmatrix} \\ \mathbf{W} &= \begin{bmatrix} w_1 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 \\ 0 & 0 & w_3 & 0 \\ 0 & 0 & 0 & w_4 \end{bmatrix} \end{aligned} \quad (2.44)$$

The middle matrix K then would have to be written as:

$$\mathbf{K} = \mathbf{D}\mathbf{W}\mathbf{D}^T \quad (2.45)$$

Therefore, Equation 2.41 now becomes:

$$\mathbf{M}_b - \mathbf{D}\mathbf{W}\mathbf{D}^T \mathbf{a} = \mathbf{f} \quad (2.46)$$

Evaluating the left-hand boundary term using the Kronecker delta property, it can be seen that the internal rows become zero, as those Lagrange polynomials are evaluated at the first and last GLL points and therefore equal zero.

Currently, applying all this to Equation 2.40 gives:

$$\begin{bmatrix} \left[\ell_1 \sum_{j=1}^n a_j \ell_{j_x} \right]_{-1}^1 \\ 0 \\ 0 \\ \left[\ell_4 \sum_{j=1}^n a_j \ell_{j_x} \right]_{-1}^1 \end{bmatrix} - \mathbf{DWD}^T \mathbf{a} = \mathbf{f} \quad (2.47)$$

For the first and last rows, Dirichlet boundary conditions are applied, and they are therefore replaced in the final matrix S , which is defined as:

$$\mathbf{S} = \mathbf{M}_b - \mathbf{DWD}^T \quad (2.48)$$

These final rows ensure that the coefficients a_1 and a_4 match the boundary conditions at -1 and 1, respectively. This can be explained by the Kronecker delta property: The Lagrange polynomials defined at that point are equal to one, so the actual value the user gives through the boundary condition is equal to the coefficient at that point. $\mathbf{f}$ will hold the value of the boundary condition.

As such, the final matrix equation to be solved is:

$$\mathbf{Sa} = \mathbf{f}, \quad (2.49)$$

Or written out:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & S_{22} & S_{23} & 0 \\ 0 & S_{32} & S_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \delta\Phi(-1) \\ 4\phi Gw_2 \delta\rho(x_2) \\ 4\phi Gw_3 \delta\rho(x_3) \\ \delta\Phi(1) \end{bmatrix} \quad (2.50)$$

This equation can be solved for the coefficients a_i . Then these coefficients, applied to the Lagrange polynomials, will yield an approximation of $\delta\Phi$, which can then be used to estimate the gravitational attraction.

Mapping

In subsection 2.5.2, the differential equation that was solved approximately was defined from -1 to 1. Obviously, in practical applications, this is not the case. Therefore, a mapping usually occurs to map the actual range to -1 to 1 to allow for the calculations. Consider the following integral:

$$\int_a^b g(x) dx. \quad (2.51)$$

To map this to -1 to 1, a linear relationship between x and ξ (the variable from -1 to 1) is defined:

$$x = A\xi + B. \quad (2.52)$$

Knowing that $\xi = -1$ when $x = a$, and $\xi = 1$ when $x = b$, the constants A and B can be found:

$$\begin{aligned} a &= A(-1) + B, \\ b &= A(1) + B, \end{aligned}$$

This gives:

$$\begin{aligned} A &= \frac{b-a}{2}, \\ B &= \frac{a+b}{2}. \end{aligned} \quad (2.53)$$

As such, the mapping is:

$$x = \frac{b-a}{2}\xi + \frac{a+b}{2}.$$

Applying this to Equation 2.51, the following is obtained:

$$\int_a^b g(x) dx = \int_{-1}^1 g\left(\frac{b-a}{2}\xi + \frac{a+b}{2}\right) \frac{dx}{d\xi} d\xi \quad (2.54)$$

Where $dx/d\xi = (b-a)/2$.

From Spectral Method to Spectral Element Method

Now that the spectral method has been explained, it can be used to create the spectral element method (SEM). The one change used when creating the SEM is that multiple elements now have to be coupled through their boundary conditions. Imagine two elements side by side. The right-hand boundary of the first element will have to be equal to the left-hand boundary of the right element (at least in the case of Dirichlet boundary conditions). This means that a_N of the left element is equal to a_1 of the right element. This, in fact, means that both of these coefficients do not need to be present in the $\vec{a}$ vector. As such, a two-element system with a polynomial degree $N = 3$ (four points per element), $\vec{a}$ will have seven entries. The middle entry will then have both the row of the S matrix from the left element and the right element merged. That row would look as follows:

$$\left[(\ell_{ij,1}), (\ell_{ij,1}), (\ell_{ij,1}), (\ell_{ij,1}) + (\ell_{ij,2}), (\ell_{ij,2}), (\ell_{ij,2}), (\ell_{ij,2}) \right] \quad (2.55)$$

Where $\ell_{ijk} = \int_{-1}^1 \ell_{ix} \ell_{jx} dx$ for element k , with i and j being the row and column indices respectively, and columns are separated by commas.

Using this, the global matrix can be set up and solved similarly using linear algebra.

2.6. Forward Method

The forward method shall be performed on a computer running SPECFEM, which uses SIEM. This method is applied in 3D, but essentially uses the spectral element method, explained in subsection 2.5.2, combined with an infinite element. This is formulated as follows, and is taken from Gharti, Tromp, and Zampini (2018).

We start with the weak form:

$$\int_{\Omega} (\nabla w) \cdot (\nabla \delta \Phi) dV = -4\pi G \int_{\delta \Omega} w \delta \rho dV + \int_{\Gamma_{\infty}} w (\hat{\mathbf{n}} \cdot \nabla \delta \Phi) dS, \quad (2.56)$$

Then, this equation is discretised into any number of hexahedral elements that one desires:

$$\delta \Phi(\xi) = \sum_{\alpha=1}^n \delta \Phi_{\alpha} N_{\alpha}(\xi), \quad (2.57)$$

Where ξ_{α} is a quadrature point and $\delta \Phi_{\alpha}$ indicates the field value at the quadrature point. The quadrature points used can be seen in Figure 2.19. α indicates the index of the point, while N_{α} is a basis function, which in this case denotes the Lagrange polynomials:

$$N_{\alpha}(\xi) = \prod_{j=1}^3 N_{\alpha j}^j(\xi^j). \quad (2.58)$$

Again, a Galerkin approach is taken, resulting in the weighting function being equal to the basis function.

Then, the whole equation can be rewritten with a stiffness matrix,

$$\mathbf{K}_e = \int_{\Omega_e} \mathbf{B}^T \mathbf{B} dV, \quad (2.59)$$

A force vector:

$$\mathbf{F}_e = -4\pi G \int_{\delta \Omega_e} \delta \rho \mathbf{N}_e dV + \int_{\Gamma_{\infty}} (\hat{\mathbf{n}} \cdot \nabla \delta \Phi)_e \mathbf{N}_e dS. \quad (2.60)$$

Where,

$$\mathbf{B} = \mathbf{D} \mathbf{N}_e^T, \quad (2.61)$$

Where,

$$\mathbf{D} = \left\{ \frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \right\}^T, \quad (2.62)$$

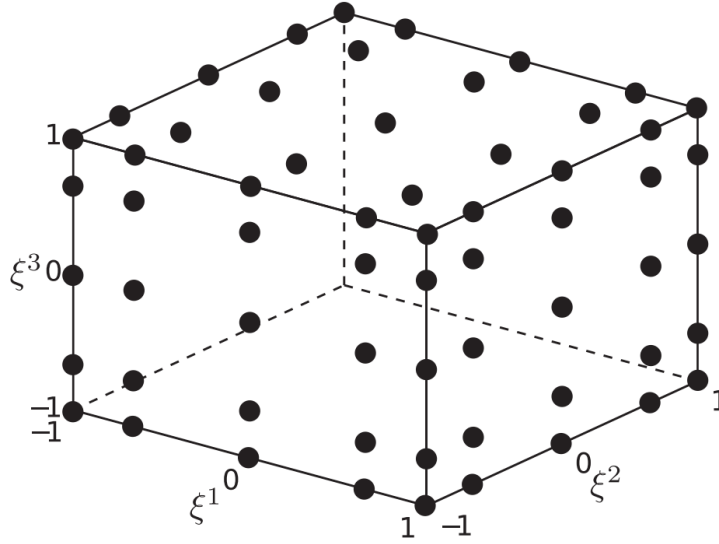


Figure 2.19: The quadrature points used in the 3D case, already mapped and present in natural coordinates. Taken from Gharti, Tromp, and Zampini (2018).

$$\mathbf{N}_e = \{N_1 N_2 N_3 \dots N_n\}^T, \quad (2.63)$$

And,

$$\delta\Phi_e = \{\delta\Phi_1 \delta\Phi_2 \delta\Phi_3 \dots \delta\Phi_n\}^T \quad (2.64)$$

The element system can be written in matrix form using $\mathbf{K}_e$ and $\mathbf{F}_e$:

$$\mathbf{K}_e \delta\Phi_e = \mathbf{F}_e \quad (2.65)$$

Then, combining elements, one gets the whole system:

$$\mathbf{K} \delta\Phi = \mathbf{F}. \quad (2.66)$$

In Equation 2.24, we can calculate the first term on the right-hand side, but the second term, which is present on the boundary Γ_∞ , is assumed to be unknown by assuming the gradient of the gravity potential to be unknown on this boundary. As such, one can split Equation 2.66 into a matrix as follows, where one has a known component and an unknown component:

$$\begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \begin{bmatrix} \delta\hat{\Phi}_1 \\ \delta\Phi_2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1 \\ \hat{\mathbf{F}}_2 \end{bmatrix}, \quad (2.67)$$

Where the hat denotes the known component. The matrix given in Equation 2.67 can be solved with simple linear algebra and knowing that $\delta\hat{\Phi}_1 = 0$.

Once $\delta\Phi$ is known, the gravitational attraction can be calculated using the following equation:

$$\delta\mathbf{g} = -\nabla\delta\Phi = -\sum_{\alpha=1}^n \delta\Phi_\alpha \mathbf{J}_\alpha^{-1} \cdot \nabla N_\alpha, \quad (2.68)$$

Where J indicates the Jacobian at node α .

2.6.1. Infinite Elements

Outside the domain, on the + side of the boundary, a single infinite element is used. This infinite element maps the infinite domain onto the natural domain and was initially derived in Zienkiewicz, Emson, and Bettess (1983).

We start with a one-dimensional element, which starts at x_1 , goes through x_2 , until x_3 , which lies at infinity. x_0 will be used to define a 'pole'. To map this element to the natural finite domain, the following method is used. We use the following transformations:

$$x = \tilde{N}_0(\xi) \cdot x_0 + \tilde{N}_2(\xi) \cdot x_2 \quad (2.69)$$

Where,

$$\tilde{N}_0(\xi) = \frac{-\gamma'}{1-\gamma'}, \quad (2.70)$$

$$\tilde{N}_2(\xi) = 1 + \frac{\gamma'}{1-\gamma'}, \quad (2.71)$$

$$(2.72)$$

Then, one obtains:

$$\xi = +1, \quad x = \frac{\xi}{1-\xi}(x_2 - x_0) + x_2 = \infty = x_3 \quad (2.73)$$

$$\xi = 0, \quad x = x_2 \quad (2.74)$$

$$\xi = -1, \quad x = \frac{x_0}{2} + \frac{x_2}{2} = x_1. \quad (2.75)$$

The mapping can be seen in Figure 2.20.

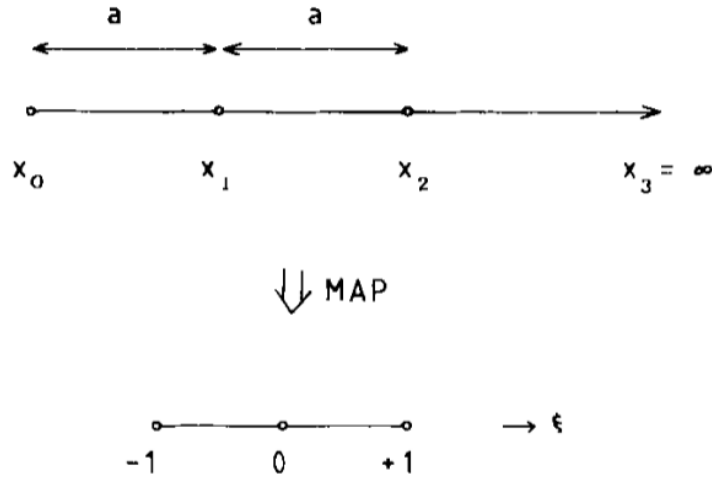


Figure 2.20: The mapping for an infinite element to the natural coordinates (Zienkiewicz, Emson, and Bettess 1983).

For a 3D element, which is actually what is used in SPECFEM, the mapping can be seen in Figure 2.21.

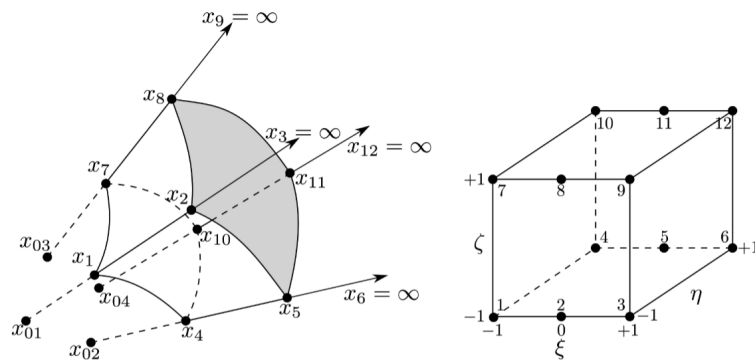


Figure 2.21: The mapping for an infinite element to the natural coordinate in 3D (Gharti, Tromp, and Zampini 2018).

The pole of the mapping, x_0 , is defined in this case to be the centre of mass of the density anomaly:

$$\bar{x} = \frac{\int_V \mathbf{x} \delta \rho(\mathbf{x}) dV}{\int_V \delta \rho(\mathbf{x}) dV} \quad (2.76)$$

Using these relations, the spectral element method can now be used efficiently to solve gravity anomaly problems.

For inputs in the software, the following are important:

- The model. This defines the coordinates of the elements and their quadrature points. These are basically the inputs to the element system provided in matrix form in Equation 2.66. Included in the model is also the surface to be used for infinite elements. SPECSEM uses this surface to define what the infinite elements look like, which can then later be mapped onto a natural domain.
- The pole for the infinite elements. This defines where into infinity the infinite elements 'point', and is critical as its location needs to be such that infinite elements do not overlap (in the case the outside surface is not convex). If it is chosen as the centre of mass of the density anomaly, then Equation 2.76 is used.
- The number of GLL points.
- The definition of the density of the model, which is included in the force matrix given in Equation 2.66. This is inserted into SPECSEM using a dedicated file.

This mathematics, which has already been implemented in the software SPECSEM, will then be used on a high-performance supercomputer to perform a gravity anomaly calculation of a lunar lava tube model. For this, the following steps are taken:

- Obtain a high-resolution lunar lava tube model. Since lunar lava tubes have yet to be detected, an artificial lava tube or a terrestrial analogue can be used.
- Mesh the lunar lava tube model. Using Coreform Cubit (Coreform LLC 2024), the lava tube model can be discretised into a hex mesh to be used in SPECSEM.
- The model is prepared in terms of assigning materials and, therefore, densities to each element.
- The simulation is performed. The gravity anomaly at multiple locations is obtained.
- A sensitivity analysis is performed by slightly changing the input and analysing what happens to the output.

2.7. Previous Studies on Gravity-Based Lunar Lava Tube Detection

Several studies have investigated the use of gravimeters to detect and characterise lunar lava tubes. The studies use terrestrial analogue lava tubes to experimentally test their hypotheses and scaled 3D models of such lava tubes to compare the experimental results with computer simulations.

One such study was conducted by Muñoz et al. (2025), who used measurements from the Washington state Ape Cave and compared them with computer simulations of the same lava tubes to try to understand where they are alike and where they differ. They use the actual dimensions of the Ape Cave and produce a model (Model 1) in which the environment is lunar, but the geometry is that of Ape Cave. The cave is located at a depth beneath the ground of 26 m, which is chosen because the Marius Hills Hole skylight is that deep. Model 2 is made by using the same lava tube, increasing the size fivefold, and placing it at the depth where the same roof thickness is achieved. In the model, three layers depicting the layers that would be present on the Moon are included in the model. These are two layers of regolith, at 1 and 4 m, as well as 1719 and 2221 kg m⁻³. For the third layer, which was basalt, a density of 3270 kg m⁻³ was used. The lava tube was meshed at a 5 cm resolution and simulated using the IGMAS+ software GFZ Helmholtz Centre for Geosciences (2025).

For the real-life measurements of Ape Cave, gravimeters with an accuracy of 1 µGal were used (two LaCoste and Romberg (LCR) G-Model relative gravimeters (G-127 and G-801)), which are found to be accurate to 20 µGal in field settings. A total of five measurements, which were semi-perpendicular, were measured along Ape Cave, although only four were used due to bad weather conditions on the last day of the survey. The measurements were processed to account for outliers, the Earth tide effect, and instrumental drift. Next to that, the free-air and terrain corrections were applied.

In the simulations, it was found that for shallower and more elongated lava tubes, the gravity signal was more irregular and variable, whereas for deeper lava tubes, the curve became smoother. When the lava tubes are shallow, the geometry strongly influences the shape of the gravity signal. In the field measurements, such intricacies were not available, as the signal varied too much to distinguish such variations.

Next to that, it was found that resolutions of 15 and 25 µGal would be able to measure more than 90% and 63% of the forward model, respectively. Therefore, these resolutions would be needed to provide all the information necessary for detecting lava tubes as they were present in the 3D model.

When the simulations for the gravity anomaly and the measurements are compared, a clear distinction is seen in that for three of the four measurements or transects, the field gravity has a higher amplitude. This was attributed to the assumption of homogeneous rock surrounding Ape Cave. In conclusion, the simulations were stated to be a good predictor for estimating the order of magnitude to be expected in field measurements. Additionally, for shallow depths, lunar lava tubes should be at a minimum detectable level.

As can be seen, Muñoz et al. (2025) provides a good initial overview of the challenges and insights in both the simulations of lunar lava tubes and detection of such caves using gravimeters. However, some limitations and future research needs become quite apparent. As is also stated in the paper, the lava tube models used for the simulations are *small* for typical lava tube sizes expected on the Moon. Therefore, a limitation of this research is that gravity simulations and the possibility of detecting a true *lunar-sized* lava tube are not considered in depth. Instead, the possibility of detecting a terrestrial-sized lava tube (and a fivefold-larger one, which still does not represent lunar scales) on the Moon is explored.

Another area for future work is to place greater focus on a sensitivity analysis of the lava tubes, where the influence of scale, shape, depth, and density is explored in a more quantifiable way. Muñoz et al. (2025) does state that for shallow lava tubes the gravity signal is much more influenced by the shape of the cross-section, but this does not go into detail on the different types of groups seen in lava tube cross-sections.

Carroll et al. (2015) also includes lunar lava tube simulations and gravimetry. However, in this conference paper, an extremely simple cylinder with a radius of 150 m and a depth to its centre of 200 m is considered. The rock surrounding this lava tube is assumed to be 1.800 kg m⁻³. Simulating a line plot at $z=0$ for this model gives a symmetrical bell-shaped curve with an absolute peak at about 9 mGal. From this simulation and other sizes and depths, it is concluded that a gravimeter with an accuracy of

1 mGal should be able to sense the typical range of values (2 mGal to 10 mGal) for lava tubes present on the Moon. Obviously, many more things can be investigated, as stated in comments on the previous paper discussed. The main thing missing from previous work is sensitivity analyses discussing the behaviour of the gravity signal with respect to input parameters of the lava tube. Such analyses could provide immense value for understanding the lava tube when measuring the gravity signal, as they are closely related.

The limitations found in the previous work discussed here are focused on as part of this thesis.

3

Methods

In this chapter, the methods used to answer the research questions, as well as to fulfil the research objective, are proposed.

3.1. Assumptions and Model Setup

In this section, the initial conditions and assumptions used in the thesis are explained. This includes the specs of the computers on which the simulations take place, as well as rationales for the location, shape, and basis for the lava tube.

3.1.1. Assumptions

For the simulation of the lunar lava tube, various assumptions have to be made. These assumptions are the basis of the ‘playing field’ in which the thesis will give results, and are therefore incredibly important to be stated clearly. In this subsection, all the assumptions used in this thesis are stated simply and clearly in Table 3.1.

Table 3.1: Assumptions used in the simulations.

ID	Assumption	Rationale
Geometry Assumptions		
AS-1	The collapsed structure of the Marius Hills and Gruithuisen K regions are assumed to be collapsed lava tubes.	As is expanded on a lot in chapter 2, literature states that these structures are likely to be collapsed lava tubes.

Continued on next page

Continued from previous page

ID	Assumption	Rationale
AS-2	Cross-sections of terrestrial lava tubes can be used in analogy for lunar lava tubes, and these analogous lava tubes can exist structurally.	Because no cross-sections of actual lunar lava tubes are available, a proxy must be created. This can be done artificially, by using, for example, a cylinder, or by using existing cross-sections from terrestrial lava tubes. In this thesis, terrestrial cross-sections are used, which are assumed to be good proxies for lunar lava tubes. The process of lava tube formation on the Moon is similar to that on Earth, although much larger in scale. This could result in different shapes, especially regarding which types of shapes are structurally stable. Certain terrestrial cross-sections could be stable, but would become unstable if they were to be 'created' on the Moon (they would not be created but would collapse instantly). It is assumed these analogous 'copies' can exist on the Moon.
AS-3	The width of the MH and GK collapse chains can be used as a proxy for the width of the lava tube itself.	Generally, the width of the collapse chain does not have to indicate the width of the lava tube. However, because there is no other measurement to base the width of the lava tube on, this is assumed to be a good proxy for the lava tube. Because, for the thesis, a real-life situation is not necessary, this assumption is deemed valid.
Physical Parameter Assumptions		
AS-4	The density of the lunar rock is assumed to be 3320 kg/m ³ .	Although this choice is not necessarily relevant for the sensitivity analysis, it is important when defining what mesh size is necessary at what distance from the observation point, as is outlined in section 4.3. This is also the only place where this assumption is used.
Sensitivity Analysis Assumptions		
AS-5	The lunar regolith is deemed homogeneous and of constant density.	Although this is known not to be completely true, as multiple layers do exist in the Moon, this assumption is deemed valid as it should not affect the essence of the sensitivity analyses performed in this thesis. It would be necessary for analyses in which the absolute gravitational attraction must be calculated. In the sensitivity analysis, only relative values are of interest. Then, when a conclusion needs to be made on the detectability of the lava tube, the use of a homogeneous density can be quantified as an error.
AS-6	The curvature of the Moon is assumed not to influence the gravitational attraction.	The Moon is obviously known to be a sphere; however, for this thesis, the usage of a flat or curved surface does not influence the essence of the thesis. Therefore, this assumption is deemed valid.
AS-7	The cluster/processors on which the simulations are performed are assumed to be consistent throughout their usage.	Since the simulations are performed using identical nodes on Trillium and the same computer for the case of using Savant, and the simulations are mostly performed in the same time frame, the computer is assumed to be consistent in processing power throughout all these simulations.

Continued on next page

Continued from previous page

ID	Assumption	Rationale
AS-8	Human errors in establishing the exterior bounds of the lava tube are negligible for results.	When creating the lava tube, the outline of the collapse chain was performed by hand. This inevitably includes errors. In this thesis, because the exactness of the lava tube is not of importance for the results, these errors are assumed to have a negligible effect.

3.1.2. Consistency in Performance Cluster

The simulations performed in this thesis are performed either on the Trillium cluster or on the local supercomputer ‘Savant’. The Trillium cluster is a ‘large parallel cluster built by Lenovo Canada and hosted by SciNet at the University of Toronto’ Digital Research Alliance of Canada (n.d.). For a fair comparison, when simulations must be compared in compute time, all relevant simulations are run on the same cluster or computer, which allows for a fairer comparison. It is assumed that the Trillium cluster has a similar, consistent performance during all simulations and does not influence the comparison in computational speed. The same is assumed for Savant.

The Trillium cluster and Savant have the following specifications:

Table 3.2: The specifications of the clusters and supercomputers on which simulations were performed.

Computer	Nodes	Cores	Memory	CPU	GPU
Trillium	1224	192	749G		2 x AMD EPYC 9655 (Zen 5) 2.6 GHz, 384 MB cache L3
Savant	-	64 (2x 32 core)	251G	Intel Xeon Gold 6438Y+	-

3.1.3. Lava Tube Model Basis

As there are no direct measurements of lunar lava tubes, it is impossible to obtain an accurate model or representation of such structures. However, it is still possible to obtain a decent lava tube using analogous lava tubes on Earth. In this thesis, it is assumed that the cross-sections of terrestrial lava tubes can be used as analogues to create models of lunar lava tubes.

In addition to using terrestrial cross-sections, the exact shape, size, length, and trajectory of the lava tube must also be assumed. However, in this instance, a lunar structure can be used to derive these. There are multiple collapse chains that the literature has identified as the most likely candidates for collapsed lava tubes. These collapse chains are the Marius Hills collapse chain, as well as the Gruithuisen K collapse chain. As these collapse chains showcase, on the surface, the approximate length of the lava tubes, as well as a proxy for the width of the lava tube at various points along the length, a good estimate for the shape and size can be made. It is assumed that using such a collapse chain to obtain a lava tube model is a representation of a lunar lava tube accurate enough for the purposes outlined in this thesis. For example, for the sensitivity analyses, the lava tube needs to be in the range of lava tubes that could exist on the Moon, in length, width, size, shape, and depth. Using the collapse chains definitely qualifies for these requirements.

To create a model of the lunar lava tube, a software called Rhinoceros 8 (Developed by Robert McNeel & Associates), including its Visual Programming Language ‘Grasshopper’, is used. Romio, Lobosco, et al. (2026) provides an add-on for this program, named ‘pyroduct’, which allows users to use existing terrestrial lava tube cross-sections to obtain a lava tube model. The user can insert a path the lava tube has to follow using a curve, and then can manually select the exact locations along this path where a certain cross-section should occur. As such, a high level of customizability is allowed. Next to that, another option that is available in ‘pyroduct’ is for the curve or path of the lava tube to be placed ‘on

top' of a mesh, allowing it to follow, for example, a downward slope. This helps with creating vertical variations in the lava tube model.

In this thesis, the decision is made for the lava tube to follow the collapse chain present in the Marius Hills and Gruithuisen K regions. Therefore, there will be two lava tube models, one for the Marius Hills collapse chain, and one for the Gruithuisen K collapse chain. Next, the width of the lava tube shall match the width of these collapse chains as closely as possible. It is assumed that the width of the collapse chain can be used as the width of the lava tube model. Although it is true that the width of the collapses can vary significantly, while the lava tube below does not, as is mentioned in subsection 2.1.1, it is assumed in this thesis that the widths of the collapses match the widths of the intact lava tube.

The Marius Hills collapse chain is measured to be approximately 300 m wide, while the Gruithuisen K collapse chain is measured to be approximately 800 m wide. Although the 300 m estimate is a little on the small side of the general 500 to 900 m range that is usually taken for lunar lava tubes, it is still assumed that the lava tube model is the same width as the collapse chain width. The Gruithuisen K collapse chain width is perfectly within the range, so it is assumed that the lava tube model derived from this collapse chain has the same width as the collapse chain.

3.1.4. Cross-Section Selection

Another problem that must be faced when modelling lunar lava tubes is the decision on what cross-sections to use for the lava tube. As there are many variations on Earth, as can be seen in Figure 2.4, a good assumption has to be made about what cross-sections should be used. For this, the literature provides a decent precedent: Usually, an arched, rounded, or elliptical cross-section is assumed in lunar lava tube simulations. This is the most common shape for lava tubes, although they can have immense variability (Chwała, Komatsu, and Haruyama 2024).

Additionally, the large size as well as the ease of simulation could play a role as well in choosing such cross-sections for simulations. For the current simulations, all relevant shapes are first simulated to understand their effects on the gravity anomaly. Then, once it is understood that using an elliptical, arched, or rounded shape is fine as a representation for the other lava tubes, only one of these cross-sections is used for further simulations.

Another factor in selecting these types of cross-sections for future simulations (if the shape sensitivity analysis finds that the shape is not particularly relevant for the other simulations) is that such shapes are much easier to mesh. As such, they will allow the other sensitivity analyses to be performed on time. In addition, a bias is also given to specifically rounded cross-sections that are simple and do not have small, quick variations over the cross-section. This is again due to simpler cross-sections being easier to mesh, and therefore easier to simulate in SPECFEM.

It is noted that an interesting simulation to perform would consist of a lava tube with some smaller characteristics present in its cross-section. This could be useful in determining if space gravimeters are able to detect such differences in the lava tube cross-section. As such, this is deemed future work.

As for the location of the lava tube, it is deemed sufficient to place the lava tube in the same general area where the collapse chain was found. This is the most realistic option, as the collapse chains were found where intact lava tubes are most likely to occur. The exact position of the lava tube is not of greatest interest, as this is not necessarily relevant for the sensitivity analyses, which only care about relative differences between simulations.

3.2. Acquisition of the DTM and Lava Tubes

To be able to perform a high-resolution simulation with SPECFEM, a high-resolution model must first be obtained. Such a model is necessary for the sensitivity analyses, as the error caused by the variation in height for the DTM, as well as variation in DTM resolution, must be established. In addition, for the practical case, it is useful to combine the lava tube and DTM simulations to get an idea of the measurements that would be obtained if one were to measure on the surface of the Moon.

Lunar DTMs have been created from satellite observations and are readily available at a decent resolution (highest resolution of ~60 m at the lunar equator (Barker et al. 2016)), and are therefore relatively easy to

use. Obtaining DTMs is explained briefly in subsection 3.2.1, while a more detailed explanation is given in section B.1.

As stated in chapter 2, lunar lava tubes have yet to be confirmed. This results in the fact that a ‘real’ lunar lava tube model is impossible to obtain. Instead, a different approach must be taken. The two options that are most viable are:

- To use a model of a lava tube on Earth, which is more readily available, and change the geometry such that it aligns with sizes that would be found on the Moon. Then, any other modifications needed to more accurately model lunar lava tubes as they are expected to appear would have to be applied. This model can then be used (after meshing) in SPECFEM.
- To create an artificial lava tube with the dimensions expected in scientific literature. Obviously, such a lava tube would be much less complicated and would require some form of alteration to be characterised as high-fidelity.

In this thesis, the first option was chosen, as this would allow some type of realistic lava tube, instead of performing a relatively simplistic one. This choice was made after the discovery of the pyroduct software (Romio, Lobosco, et al. 2026), which allows for easy generation of such lava tubes.

The process used to obtain the lunar lava tube models is explained briefly in subsection 3.2.2, while a more detailed explanation is found in section B.2.

3.2.1. Obtaining the DTM

Some simulations include a DTM of the lunar surface. This DTM is used for more realistic gravitational attraction simulations. However, to perform such simulations, the DTMs must first be obtained. In this subsection, the procedure for how the DTMs in this thesis were obtained is briefly explained. This procedure will then also provide the answer to RQ4. A more detailed procedure is present in section B.1.

To obtain the DTM, first a DTM dataset containing the region of interest must be obtained. In this thesis, the DTM was obtained from U.S. Geological Survey (2015). This DTM, currently in TIF format, needs to be transformed into an STL file format. For this, a Python script was created that performed multiple tasks to end up with a ready-to-use STL. The TIF is first cropped using ogr2ogr or a relevant Python module (such as rasterio). Then, using either GIS software or more Python code, outliers in the dataset that either do not make sense or result in problems in meshing later on are fixed by using the values of the pixels surrounding the outliers to ‘flatten’ them to reasonable values. Once this is done, the STL can be generated. This is done by manually setting each pixel square to be composed of two STL mesh triangles, which are connected to each other at specific heights. The Python code that does this uses the ‘rasterio’ and ‘trimesh’ Python modules. This STL is then exported and ready to be used. Note that this STL, in its current state, only includes the topography surface. By this, it is meant that it is not a 3D object yet, and that more modifications have to be made to make it into a 3D object. These modifications are done inside the meshing software, Coreform Cubit (Coreform LLC 2024), and more info on this can be found in section B.1.

3.2.2. Obtaining the Lava Tube

A crucial part of this thesis is the process used for obtaining the model of the lava tube. The process starts with obtaining the 2D map curves from QuickMap (or JMARS or any other similar program) of the potential lava tube of interest. This is done by having a satellite image of the region of interest and then using drawing tools to outline the lava tube to obtain the two outer curves. Afterwards, the GDAL program ‘ogr2ogr’ is used to convert the shapefile of these curves to a ‘dxf’ file, which can be imported more easily into Rhino 8. Once in Rhino 8, the lava tube is generated using Pyroduct, which must be installed inside Grasshopper (A graphical programming tool inside Rhino 8). Pyroduct uses the two curves, creates a middle curve between these two curves, and uses this as the basis for the line that the lava tube should follow. Next to that, a DTM mesh can be imported for the lava tube to follow, which introduces movement in height. Using user input, the exact cross-sections of interest can be chosen for each part of the lava tube, as desired, and the exact scale of the cross-section can be changed to the desired state as well. The scaling part was automated in this thesis, and a more detailed explanation of this is found in section B.2. Then, using this lava tube as a basis, it can be ‘capped’, ‘lofted’, and ‘baked’, which results in a model that can be exported as a STEP file. This constitutes the basic process

of obtaining the lava tube.

3.3. GBOX Simulation

In this section, an explanation is provided for how GBOX is used to obtain the gravity anomaly for an arbitrary 3D model, and the process of simulating using GBOX is stated.

An overview of the process used to run GBOX simulations, once the lava tube model has been obtained, can be seen in Figure 3.1.

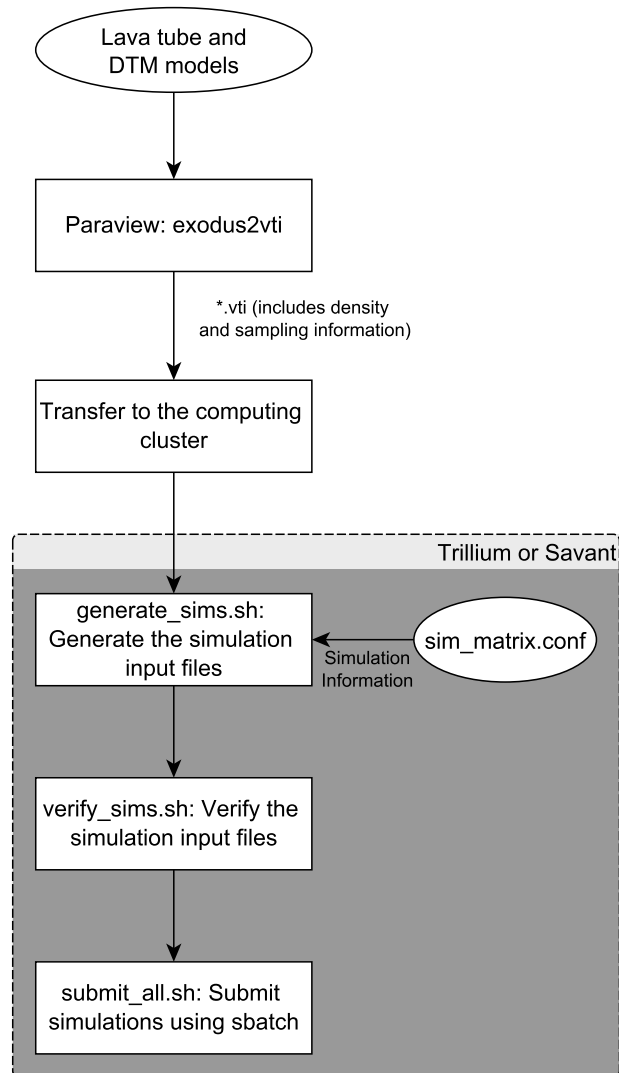


Figure 3.1: An overview of the process used to run GBOX simulations.

As shown in Figure 3.1, the two important steps in running GBOX simulations are converting Exodus files into VTI files, which should include density, and generating the simulation files that define how the simulation will run.

A more detailed explanation of this process can be found in section D.1, which clarifies the in-depth process of using GBOX.

3.4. SPECFEM Simulation

In this section, the procedure to simulate lava tubes using the SPECFEM software is clarified. For this, an explanation is provided of the exact process used to obtain the final results.

An overview of the process for the SPECFEM simulation can be found in Figure 3.2.

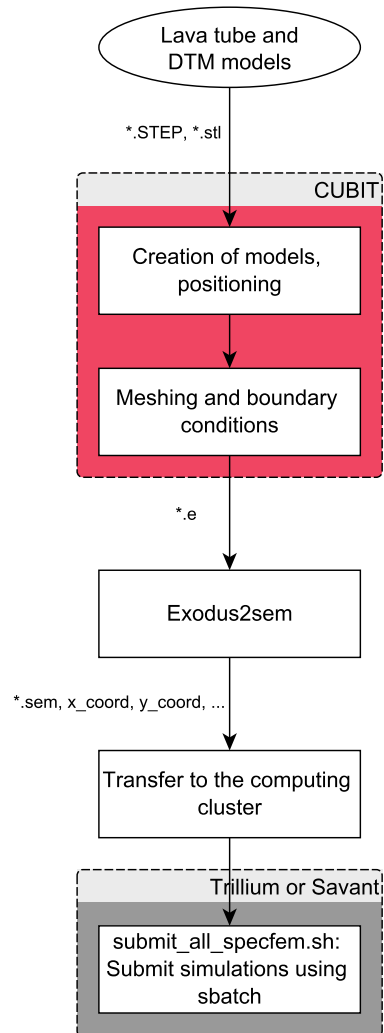


Figure 3.2: An overview of the process required for performing a gravity simulation using SPECFEM.

As shown in Figure 3.2, the process involves converting Exodus files into SPECFEM-readable files, while also preparing 'submit' files to run the simulations easily. This process is cumbersome, but necessary to provide the correct inputs for the simulations. More information on performing SPECFEM simulations can be found in section D.2.

3.5. Sensitivity Analysis

The sensitivity analysis can be done in many ways. In this thesis, it is done by varying the 3D lunar lava tube and DTM model in two ways:

- Varying the materials used, and therefore the density.
- Varying the geometry used.

Making changes to these variables can help clarify what a gravity survey can detect, as well as the

accuracy with which it can do so. Next to that, it helps in understanding what is necessary to obtain a satisfactory inversion of gravity data in a real survey. If one knows that 5% changes in the material only vary the gravity anomaly by a negligible amount, one can be more certain of the material being used. The same applies to the geometry. For the sensitivity analyses, a minimum of three simulations will always be performed, such that, if necessary, at least a parabolic function can be fit through the lines. This was chosen as a compromise, as too many simulations would be hard to finish in a timely manner, while using only two simulations could be extrapolated only linearly and would therefore not yield sufficient results.

4

Simulations

In this chapter, the simulations performed in this thesis are summarised. Their exact parameters are explored, the basis of the models is explained, and the exact models used in the simulations are shown. Lastly, a theoretical basis for the bounds of the simulations is established and discussed.

4.1. Parametric Study Overview

In this section, the exact parameters to be varied in the sensitivity analysis are briefly discussed and explained. It is already known that the effect of density and geometry on the gravity anomaly is to be assessed. However, many various aspects could be varied. Therefore, a quick explanation is given here to show the exact parameters that will be varied.

In the lunar mare, where the Marius Hills and Gruithuisen K regions are located, basalt is the most abundant material. Therefore, a value for the density of this basalt is taken from the literature, after which a sensitivity analysis is performed through variations in the density. It is expected that the density will have a 1-to-1 relationship with the gravity anomaly, as the GBOX equation, Equation 2.8, also predicts a 1-to-1 relationship. If the density is doubled, the gravity anomaly should be doubled as well.

For the geometry, multiple things can be varied for a sensitivity analysis. In this thesis, for the lava tube, the following is varied.

- Scale: The diameter of the lava tube is varied.
- Cross-sectional Shape: Since almost all terrestrial cross-sections can be categorised by seven shapes, these seven shapes can be tested against each other in terms of gravity anomaly.
- Depth: The depth of the lava tube, one of the most important characteristics, is varied to help understand when the lava tube can no longer be detected by the lunar gravimeter used as a reference in this thesis.
- Resolution: The mesh size of the lava tube is varied to help understand what the lowest mesh size is that could still be used to obtain accurate enough results.

For the DTM, the height and density are varied to understand the exact impact of uncertainty in DTM height. Note that the density variation simulations for the lava tube and DTM are identical, as the lava tube consists of empty space, so the only density that could be changed is that of the rock surrounding the lava tube.

To ensure that all the sensitivity analyses give accurate results, first, the accuracy of the potential programs to be used must be assessed. Therefore, simulations are also performed to check the accuracy and computation time of both GBOX and SPECFEM. For this, models from Gharti, Tromp, and Zampini (2018) are used as extra verification.

4.2. Models

In this section, the 3D models used in the simulations are explained and shown. First, the simple shapes such as the rectangular prism and the spherical salt dome (which are used for the accuracy comparison between GBOX and SPECFEM) are shown, followed by the Marius Hills and Gruithuisen K lava tube. Lastly, an example DTM model is shown.

4.2.1. Rectangular Prism and Spherical Salt Dome

In this subsection, the rectangular prism and the spherical salt dome models are shown. The rectangular prism does not necessarily represent any real-life deposit, but showcases the SIEM being tested on a simple case. The spherical salt dome is similar to the salt dome located 125 miles (ca. 201 km) southeast of Galveston, Texas, USA. These examples are taken from Gharti, Tromp, and Zampini (2018) and are modelled as a rectangular prism and sphere. The exact info about each example can be found in Table 4.1.

Table 4.1: The specific characteristics of the buried rectangular prism as well as the spherical salt dome, obtained from Gharti, Tromp, and Zampini (2018).

Model	Rectangular prism	Spherical Salt Dome
Geometry Object	5×10×0.5 km	Radius=4km
Depth	2 km	6 km
Bounds Simulation	30×20×6.5 km	40×30×30 km
$\delta\rho$	400 kg m ⁻³	-200 kg m ⁻³
Element sizes simulated	0.5, 1 km, and 2 km	Dome: 1 km, 0.5 km, and 0.25 km. Outside is double the size of inside the dome

These models are created inside Coreform Cubit and, after meshing, appear as shown in Figure 4.1.

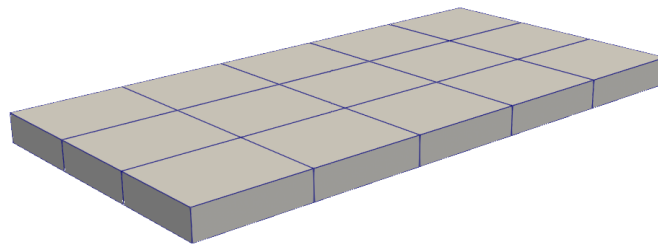


Figure 4.1: The model of the buried rectangular prism. No bounding box mesh is present in this figure, although it is present when simulating. In this case the mesh size is 2 km.

As can be seen in Figure 4.1, the buried prism is quite simple and does not include complex meshing. The spherical salt dome model can be seen in Figure 4.2.

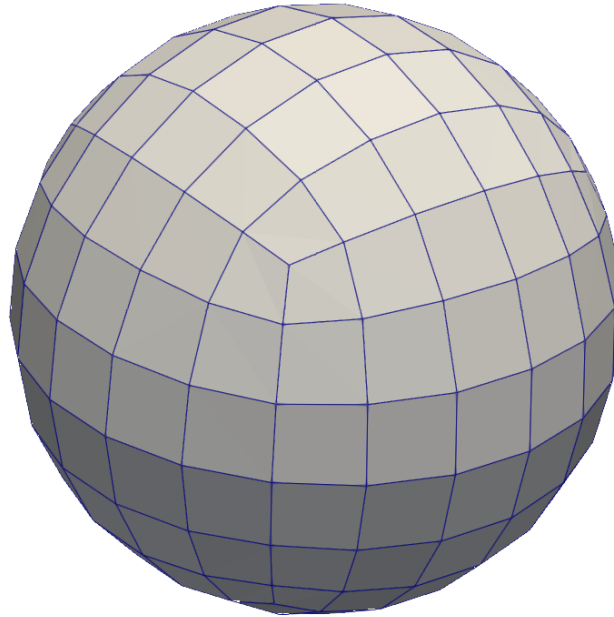


Figure 4.2: The model of the spherical salt dome. No bounding box mesh is present in this figure, although it is present when simulating. The mesh size is 1 km.

As can be seen in Figure 4.2, the model represents a sphere. In SPECSEM, the model is not limited to rectangular prisms, but any type of hex elements, which means that the shape of the sphere can be followed much more accurately.

4.2.2. Lava Tube Models

In this subsection, the exact lava tube models created using Rhino 8 are shown. In total, seven different lava tube models are used for the Marius Hills region, each with their own type of cross-section. Additionally, for the Gruithuisen K region, a single lava tube is created which uses many different shapes. First, the seven Marius Hills lava tubes created are explained, after which the Gruithuisen K lava tube is shown.

Marius Hills Lava Tube

As explained in subsection 2.1.1, there are a total of eight distinct shapes for the cross-section of a lava tube. One of these is the 'multiple' shape, which encompasses cases where lava tubes can be found on top of each other. This shape was excluded, as it essentially represents two stacked lava tubes, rather than a fundamentally different cross-section.

For the seven shapes that were left, a selection was made of the real-life cross-sections that would represent these shapes. They are shown in Table 4.2.

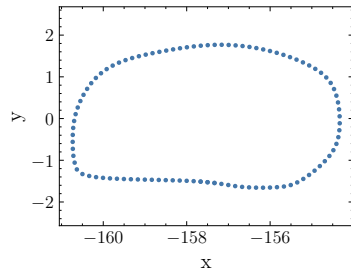
A visual of each cross-section can be seen in Figure 4.3.

As can be seen in Figure 4.3, although some shapes look similar because they are part of the same group, such as the rounded, arched, and triangular shapes, all have distinct characteristics and are interesting to study separately. When creating the lava tubes, it was ensured that each had approximately the same cross-sectional area, such that comparisons between shapes would only reflect the actual geometry and not be influenced by the volume of the lava tube.

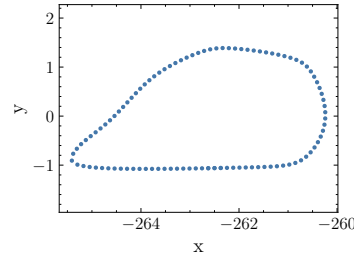
To understand exactly what the lava tube model looked like, the keyhole lava tube is shown for various mesh sizes used in Figures 4.4–4.7.

Table 4.2: The seven cross-sectional shapes for which a lava tube model was created, as well as their origin. All lava tube cross-sections are taken from the dataset provided in Romio, Lobosco, et al. (2025).

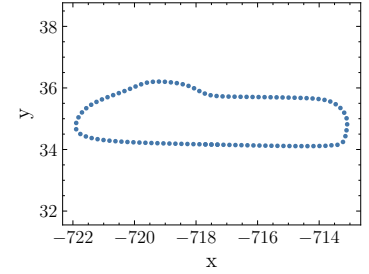
Shape	Cave	Cross-section number
Rounded	USA.HAW.18 Kazumura I	59
Arched	USA.HAW.18 Kazumura I	56
Elongated	USA.HAW.18 Kazumura I	1
Keyhole	USA.HAW.18 Kazumura I	15
Rectangular	USA.HAW.18 Kazumura I	67
Inverted Keyhole	IT.SIC.06 Grotta Marasca	1
Triangular	IT.SIC.06 Grotta Marasca	36



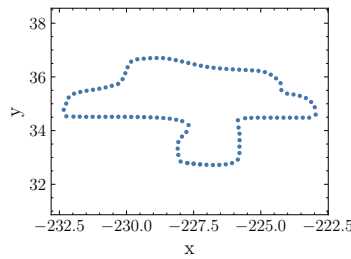
(a) The cross-section used for the 'rounded' shape.



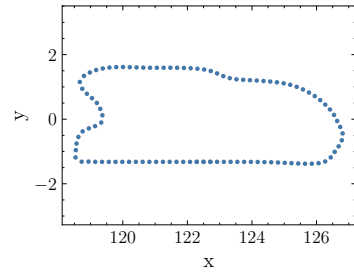
(b) The cross-section used for the 'arched' shape.



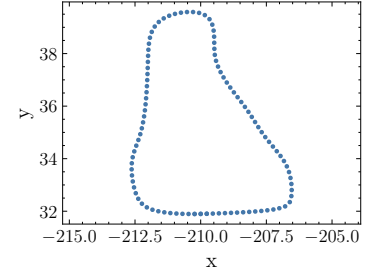
(c) The cross-section used for the 'elongated' shape.



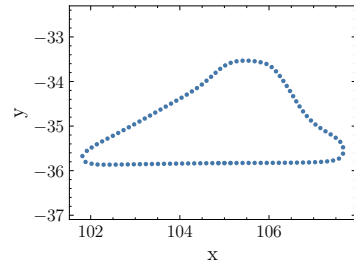
(d) The cross-section used for the 'keyhole' shape.



(e) The cross-section used for the 'rectangular' shape.



(f) The cross-section used for the 'inverted keyhole' shape.



(g) The cross-section used for the 'triangular' shape.

Figure 4.3: The cross-section used for the Marius Hills lava tubes.

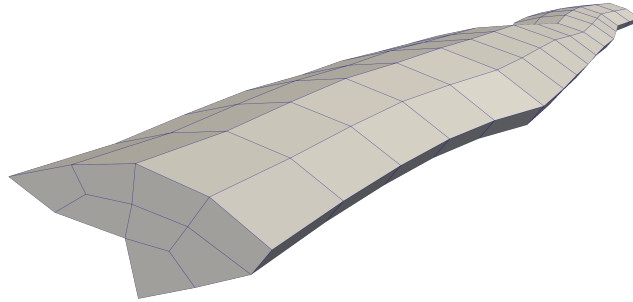


Figure 4.4: The model of the Marius Hills keyhole lava tube, where $h = 0.1$ km. No bounding box is present in this figure, although there will be one in the simulations.

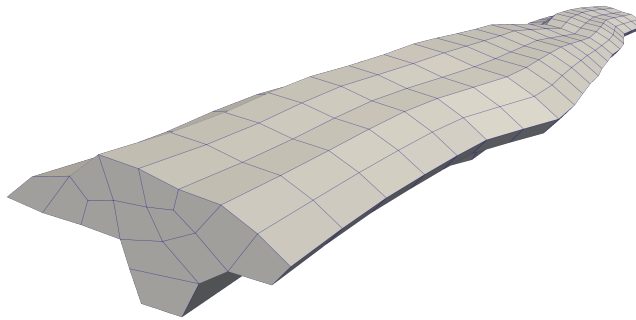


Figure 4.5: The model of the Marius Hills keyhole lava tube, where $h = 0.05$ km. No bounding box is present in this figure, although there will be one in the simulations.

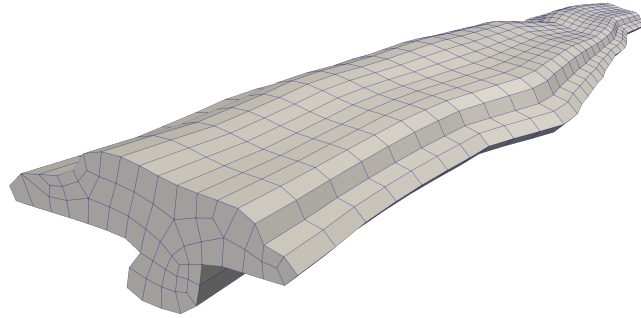


Figure 4.6: The model of the Marius Hills keyhole lava tube, where $h = 0.025$ km. No bounding box is present in this figure, although there will be one in the simulations.

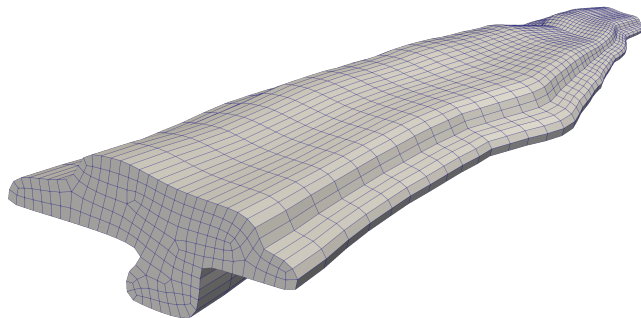


Figure 4.7: The model of the Marius Hills keyhole lava tube, where $h = 0.0125$ km. No bounding box is present in this figure, although there will be one in the simulations.

As can be seen in Figures 4.4–4.7, the lava tube originally resolves poorly due to the low resolution, and with refinement, the exact shape is increasingly captured.

More information on the characteristics of the Marius Hills lava tubes can be found in Table 4.3.

Table 4.3: The characteristics of the Marius Hills lava tube model.

Characteristic	Value
Length	≈ 4 km
Minimum Diameter	≈ 140 m
Maximum Diameter	≈ 400 m

As can be seen, the Marius Hills lava tube is quite a small variant of lunar lava tubes, with a maximum

diameter of 400 m. This is useful for the depth comparison, as the size creates a more conservative case for the detectability of the lava tube.

Gruithuisen K Lava Tube

The Gruithuisen K lava tube is much larger and more complicated than the Marius Hills lava tube. It consists of 15 sections, defined by 16 different cross-sections.

The Gruithuisen K lava tube model can be seen in Figure 4.8.

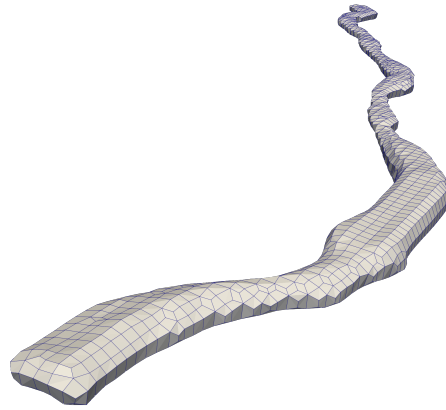


Figure 4.8: The Gruithuisen K lava tube model, with a mesh size of 0.2 km.

The exact characteristics of this lava tube can be seen in Table 4.4.

Table 4.4: The characteristics of the Gruithuisen K lava tube model.

Characteristic	Explanation/Value
Lava tube cross-sections	USA.HAW.11 Terraced Cave, sections 0–16
Length	≈40 km
Minimum Diameter	≈200 m
Maximum Diameter	≈1.2 km

Table 4.4 shows that the Gruithuisen K lava tube is much larger and representative of a conventionally sized lunar lava tube. Nevertheless, both the Marius Hills and Gruithuisen K lava tubes are derived from real collapsed lava tubes, and so both should be representative of actual lunar conditions.

4.2.3. DTM

In this subsection, the DTM models are presented and explained. The DTMs are from the locations where the collapsed lava tubes have been found. The DTMs themselves have no specific size constraints, except that they should be large enough to fully enclose the lava tube. As a general rule, for the Marius Hills simulations, the DTM always had the length of the Marius Hills lava tube as padding on all sides. Therefore, at a minimum, each side had an additional 4 km of DTM. Aside from width, the DTMs vary only in resolution. Because the original DTM has a resolution of approximately 59 m (Barker et al. 2016), this is the highest resolution at which the DTM could be meshed. A model of the DTM with a mesh size of 177 m is shown in Figure 4.9.

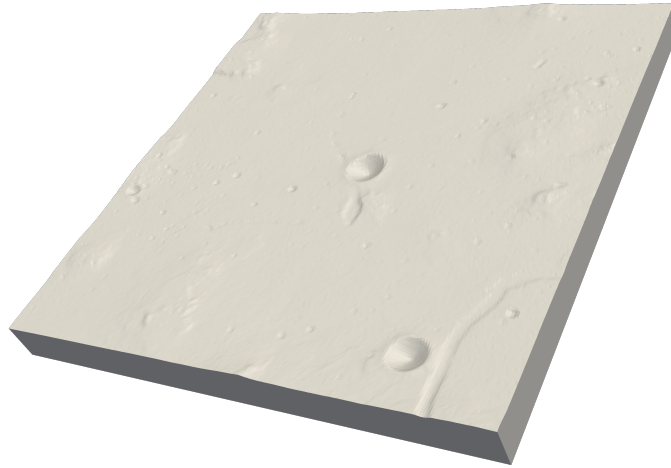


Figure 4.9: The model for the square DTM with a width of 40 km, meshed at 177 m mesh size, located in the Marius Hills region. The bounding box which would be present in the simulations is not present here.

Figure 4.9 shows that the DTM is represented as a box with topography imprinted on top of the box.

In this thesis, a lack of time resulted in only the Marius Hills DTM being meshed. This poses no hindrance to the sensitivity analyses performed on the lava tubes, however, and is only relevant when trying to obtain truly accurate, *absolute* (not relative) values for the gravitational attraction. In this thesis, this is not necessary.

4.3. High-Resolution Considerations

This section explains the rationale for performing a high-resolution simulation (compared to a low-resolution simulation). This is done using a Python script to show the smallest cube size the VEGA space gravimeter can detect; its specifications are discussed in subsection 2.3.1.

For the main simulations, the finest mesh size found to be necessary from the sensitivity analysis is adopted throughout. This resolution is determined by the highest accuracy that measurements on the Moon allow for. Using an even higher resolution would result in wasted compute, since gravimeters would not be able to sense the extra information that such simulations give.

The VEGA space gravimeter, as mentioned in subsection 2.3.1, has an accuracy of ± 0.3 mGal RMS for a 10-minute measurement. This means that any anomalies with a deviation of 0.3 mGal or more are true anomalies in 68% of the cases. As such, working back from this accuracy, it is possible to find the smallest possible entity that the gravimeter is still able to detect.

To understand the smallest object that the gravimeter would be able to detect for each depth, a Python script is written. This Python script calculates the size of a cube that would give a 0.6 mGal gravity anomaly, and does this for each depth, in a given range. 0.6 mGal is chosen here, as this would allow for a 2σ or 95% confidence in anomaly detection. Similarly, the choice of a cube allows for the usage of GBOX for implementing this code, while also representing the approximate mesh size of hexahedral elements.

For this simulation, the density chosen for the rock is assumed to be 3320 kg m^{-3} , as this is the highest density found for lunar rock, given errors, as explained in subsection 2.1.2. The reason for using a higher density is that this results in a smaller minimum size, and therefore a higher resolution required for the simulations. This is the conservative choice.

Using this density, it is then possible to figure out what size cube would be resolvable/detectable at any specified distance. For example, a cube at a distance of 10 m from the observation point will have a specific width that results in a 0.6 mGal gravitational attraction. This can be done for any distance to obtain a graph showcasing the minimum resolution for each distance from the observation point. It

was quickly found that for small depths, the width of the cubes would extend further upward than the observation point, resulting in a nonsensical situation. This happens until a depth of approximately 5.2 m. This can be seen quite visibly when the depth is plotted against the gap between the observation point and the edge of the cube, as can be seen in Figure 4.10.

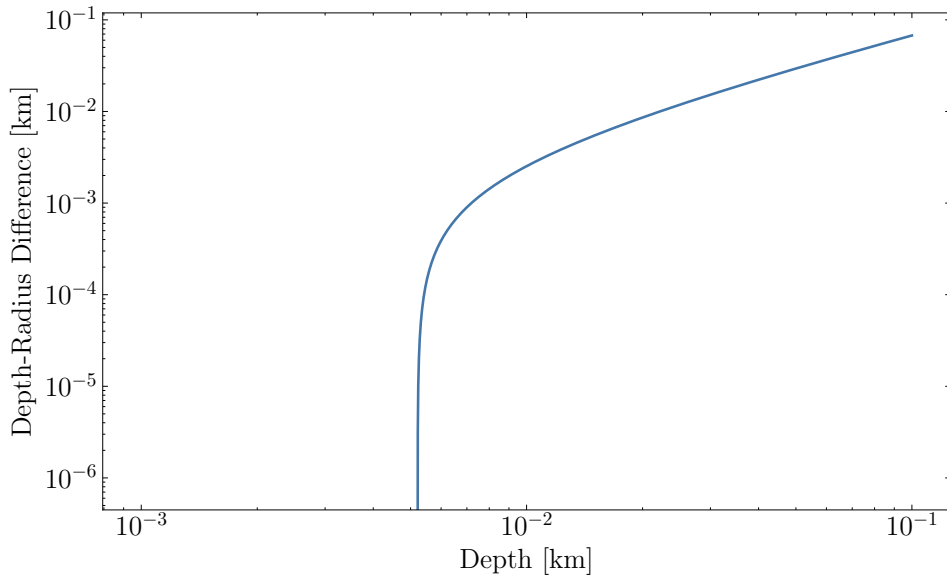


Figure 4.10: The gap between a cube with a density of 3320 kg m^{-3} , giving 0.6 mGal gravitational attraction and the observation point plotted for depths from 1 m to 100 m.

Figure 4.10 shows that the depth at which the cube radius equals the depth (where their difference tends to zero) is between 5 and 6 m, with a more precise estimate of 5.213 m.

While calculating the radii of cubes at a variety of depths, it can now be kept in mind that depths smaller than 5.213 m are to be discarded. A graph showcasing the radii for depths from 5.213 m to 100 km are shown in Figure 4.11. The figure is in a log-log scale.

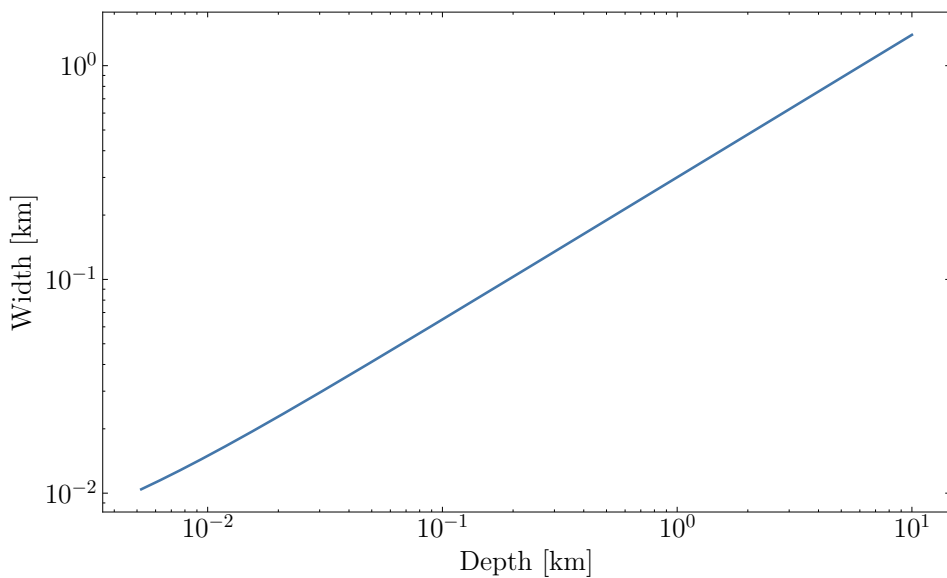


Figure 4.11: The depth in km plotted against the radius in km of a cube with a density of 3320 kg m^{-3} such that the gravitational attraction is equal to 0.6 mGal for every depth. Note that the figure is in a log-log scale.

As can be seen in Figure 4.11, the graph is quite linear, and as such it can be established that a power law is likely in effect. As such, the width scales predictably with the depth.

Although this graph is calculated for depth, it can be used as a graph that uses *distance* instead of depth, as the cubes do not necessarily have to be below the observation. Therefore, the size of the cubes can give some insight into the mesh sizes necessary. This is due to the elements in meshes being hexahedrons, which one tries to get as close to cubes as possible when meshing. Using this graph can then help speed up calculations by using more coarse grids far away from observation points, while using highly refined grids close to observation points.

As such, when observation points are, for example, only held on the surface, the resolution of the mesh can be pre-defined for each depth, but also for each horizontal distance in the model. Using Figure 4.11, one can, for example, state that a crater 1 km away should use a maximum mesh size of approximately 300 m.

However, assigning a different mesh size to each point in a DTM is impractical, and therefore a single mesh size is used throughout this thesis. To obtain the appropriate mesh size, a sensitivity analysis is done to compare various (single) mesh sizes for the DTM, which helps quantify the effect they have on the gravitational acceleration. After this effect is understood, a single mesh size can be chosen for any future simulations.

4.4. DTM Bounds Considerations

This section establishes the minimum bounds required for a simulation. A mathematical analysis is presented to understand the exact characteristics of widening the DTM.

4.4.1. Rationale

A key consideration in the simulations is the effect of truncating the DTM to a finite area. Limiting the DTM introduces an error that must be quantified to determine whether the chosen DTM bounds are sufficient.

Increasing the bounds in a simulation makes it more accurate, as it takes more information about the environment into account. However, this comes at a computational cost. Therefore, one would like to use the smallest possible simulation bounds while remaining accurate to the simulation's requirements. To understand the exact required dimensions for each simulation, it is useful to know how accurate the simulation has to be.

Usually, in the lava tube simulations, a box is created with a DTM surface on top. In this box, the lava tube is placed. A schematic of such a situation, without DTM, can be seen in Figure 4.12.

From the literature study, it is known that gravimeters have an accuracy of approximately 0.3 mGal on the Moon, as can be seen in subsection 2.3.1.

As such, it is preferred to perform the simulation using a DTM with a width that, at most, gives a deviation of 0.3 mGal from a simulation that includes a practically infinitely wide DTM. To do this, the gravitational attraction of a finite DTM must be compared to the gravitational attraction of an infinite DTM.

In this case, the DTM is represented as a box. However, the accuracy of a finite upright cylinder relative to an infinite one can be used as a proxy. If the box is then considered to be encapsulated by the cylinder, this becomes a conservative estimate.

Consider a cylinder as seen in Figure 4.13.

The gravitational attraction of this cylinder when extended to infinity in the horizontal direction can be calculated using the Bouguer plate anomaly,

$$g_z = 2\pi G\rho h \quad (4.1)$$

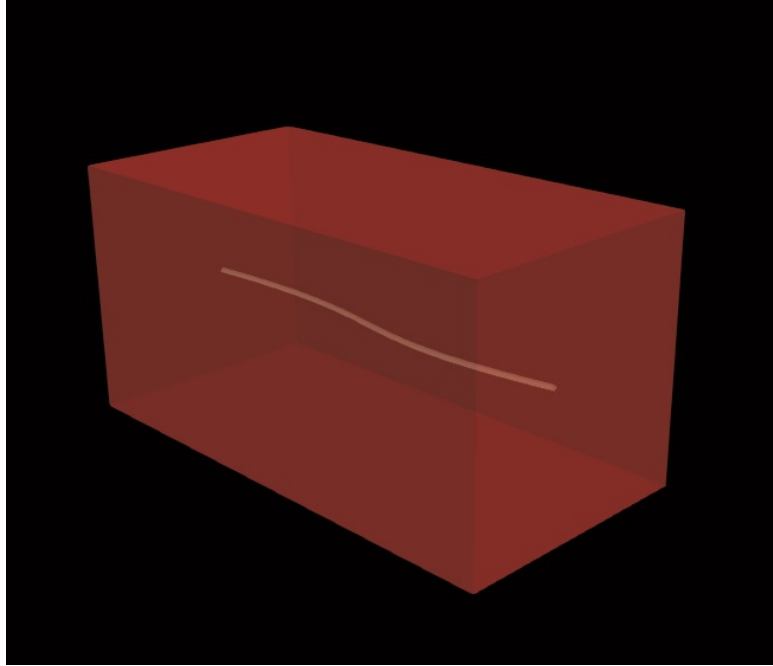


Figure 4.12: A schematic showcasing a lava tube inside a box, which represents the surrounding rock. In this case, no DEM is present on the surface.

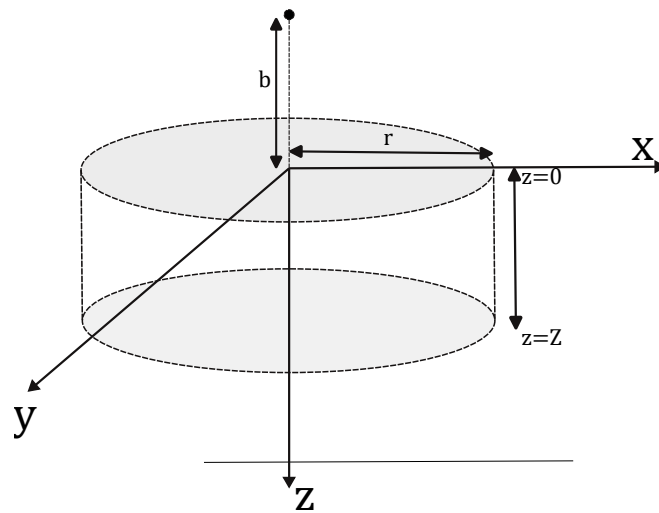


Figure 4.13: A schematic of the cylinder and the variables and distances used in deriving the gravitational attraction formula for it.

This formula can then be compared to the circular cylinder formula (Ramsey 1961; MacMillan 1958),

$$g_z = 2\pi G \int_0^h \rho(z) \left[1 - \frac{z+b}{[R^2 + (z+b)^2]^{1/2}} \right] dz, \quad (4.2)$$

Equation 4.2 can be derived from the gravitational attraction of a thin ring. Using the difference between this circular cylinder formula and the Bouguer plate anomaly, the exact radius or distance at which a circular cylinder differs by less than 0.3 mGal from the infinite cylinder can be found. This distance is then the minimum DTM width to include all relevant information.

A code was written for this purpose, which can be seen in section A.2. Using this code, Figure 4.14 can be obtained.

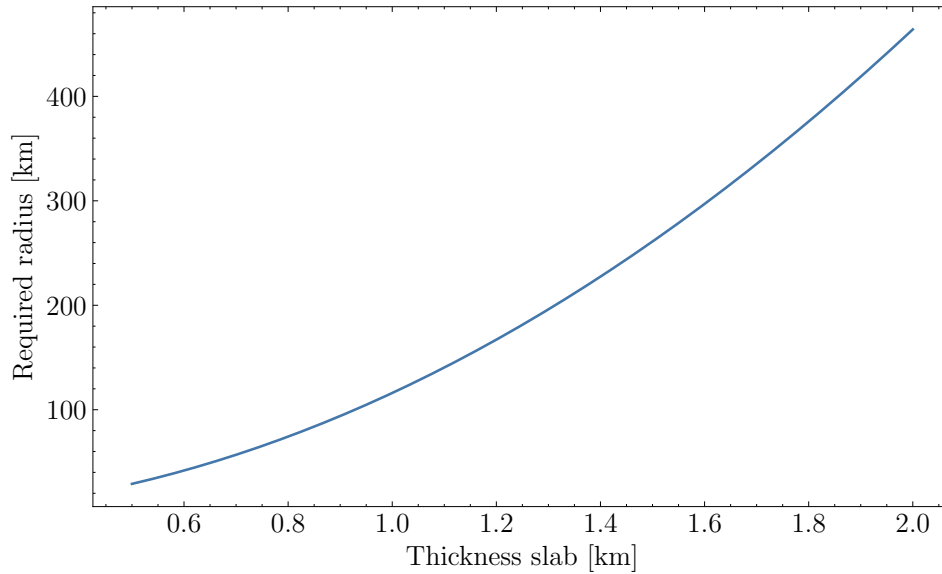


Figure 4.14: The thickness of the DTM vs the minimum width required to include all information.

As can be seen in Figure 4.14, the minimum width for a DTM depends on the thickness of the slab used and increases rapidly with slab thickness. Therefore, using a slimmer DTM reduces the width necessary.

Although this is an approximate approach, it supplies a conservative estimate for a minimum DTM width. In fact, using other methods, such as numerical ones, the minimum DTM width can only be made smaller. The method also assumes that the thickness of the DTM is much smaller than the width of the DTM.

Still, the width defined by this theoretical method can be seen to be quite large. To try to mesh DTMs with widths of more than a 100 km is impractical and cannot be seen as a reasonable way to proceed. Therefore, in this thesis, a smaller DTM size was used for the DTM sensitivity analyses and other results. This results in an extra error that must be included in the error budget, especially when absolute solutions that have to be compared to field measurements are to be obtained. In this thesis, the DTM was created such that its width and length would span three lava tubes long for the relevant lava tube. This was deemed enough for the sensitivity analyses performed in this thesis. As such, for Marius Hills, since the lava tube is about 4 km, which is explained in more depth in subsection 4.2.2, the minimum width to be used is 12 km. As such, one length is used as ‘padding’ on each side.

It must also be noted that in such DTMs, the actual region of interest for results must be limited such that boundary effects are minimised. Therefore, results were limited to a 4 km width square (-2 to 2 km) as much as possible.

4.5. Test Plan Simulations

In this section, the test plan for the simulations is explained. This includes showing what graphs are planned to be made and what simulations are necessary to produce these graphs. Simulations will

include those for accuracy, compute time, and the sensitivity analyses.

4.5.1. Parameter Comparisons

This subsection outlines the comparisons to be performed between SPECfEM and GBOX, along with the sensitivity analyses that will be performed. The objects for which the compute time, accuracy, and other comparisons will be established are:

- The Rectangular Prism,
- The Spherical Salt Dome,
- The Lunar Lava Tubes,
- The Lunar DTM.

For the first three geometries, multiple simulations are performed that will help understand the differences in accuracy between SPECfEM and GBOX, as well as differences in their compute time. The sensitivity analyses are performed on the lunar lava tubes and DTMs, which help gain a quantitative assessment of the effect of each varied parameter on the gravitational attraction.

An overview of the figures to be produced is shown in Table 4.5. The graph abbreviations follow the convention [geometry]_[comparison]_[additional] (except for compute time), where *geometry* specifies the model, *comparison* the comparison, and *additional* provides additional information, such as the mesh size, which programs are compared, or the depth of the simulation.

Table 4.5: The different graphs that shall be produced from the simulations. All comparisons which are not between GBOX and SPECfEM use SPECfEM for the simulations.

Abbreviation of Graph	Comparison	Note
Rectangular Prism		
RP_AC_SA	SPECfEM vs Analytical (equal to GBOX)	The resolution of SPECfEM is varied from 2 to 0.5 km.
Spherical Salt Dome		
SD_AC_H1_SGA	SPECfEM vs GBOX vs Analytical	$h = 1$ km
SD_AC_H05_SGA	SPECfEM vs GBOX vs Analytical	$h = 0.5$ km
SD_AC_H025_SGA	SPECfEM vs GBOX vs Analytical	$h = 0.25$ km
Lava Tube, Marius Hills, Rounded		
LTMR_AC_H01_SGA	Accuracy, SPECfEM vs GBOX	$h = 0.1$ km, 25 m depth.
LTMR_AC_H005_SGA	Accuracy, SPECfEM vs GBOX	$h = 0.05$ km, 25 m depth.
LTMR_AC_H0025_SGA	Accuracy, SPECfEM vs GBOX	$h = 0.025$ km, 25 m depth.
LTMR_AC_H00125_SGA	Accuracy, SPECfEM vs GBOX	$h = 0.0125$ km, 25 m depth.
LTMR_CR_SM	Resolution, SPECfEM	25 m depth, Rounded LT.
LTMA_CR_SM	Resolution, SPECfEM	25 m depth, Arched LT.
LTME_CR_SM	Resolution, SPECfEM	25 m depth, Elongated LT.
LTMik_CR_SM	Resolution, SPECfEM	25 m depth, Inverted-keyhole LT.
LTMK_CR_SM	Resolution, SPECfEM	25 m depth, Keyhole LT.
LTMR _e _CR_SM	Resolution, SPECfEM	25 m depth, Rectangular LT.
LTMT_CR_SM	Resolution, SPECfEM	25 m depth, Triangular LT.
LTMR_CR_GB	Resolution, GBOX	25 m depth, Rounded LT.
LTMR_CRD	Resolution, over depth	Comparing $h = 0.1, 0.05$ with $h = 0.0125$ km.
LTMR_CD	Depth	From 0 to 4.3 km, Using $h = 0.0125$ km.
LTMR_CSC_25m	Scale	$\pm 10\%$ of the diameter of the lava tube. 25 m depth.

Continued on next page

Table 4.5 – *continued from previous page*

Abbreviation of Graph	Comparison	Note
LTMR_CSC_200m	Scale	$\pm 10\%$ of the diameter of the lava tube. 25 m depth.
LTMR_CSC_sens	Scale	$\pm 10\%$ of the diameter of the lava tube divided by the original size. 25 m depth.
LTMR_CSCD	Scale and Depth	A 25 m deep lava tube with scale 0.9 is compared with a 62 m deep but nominal scale lava tube.
LTMR_CDY	Density of lava tube	This is an indirect way of measuring the variation in the density of the surrounding rock.
Lava Tube, Marius Hills: 7$\times$ Shape		
LTM_CSH_25m	Seven shapes	25 m roof thickness.
LTM_CSH_200m	Seven shapes	200 m roof thickness.
LTM_CSH_500m	Seven shapes	500 m roof thickness.
DTM, Marius Hills		
DTM_CR	Resolution	The mesh size of the DTM is varied.
DTM_HV	Height Variation	± 4 m simulations are performed to understand the difference from the nominal height simulation.
Lava Tube, Gruithuisen K		
LTG_CRD	Resolution, over depth	Comparing $h = 0.2, 0.1$ with $h = 0.075$ km.
LTG_CD	Depth	From 0 to 6.5 km, Using $h = 0.075$ km.
LTG_CD_wide	Depth	From 0 to 6.5 km, Using $h = 0.075$ km and values at $x = 7000, y = 3000$ m.
Compute time		
CT_LT_G_CR	GBOX, Resolution	$h = 0.023, 0.018, 0.015$ km.
CT_RP_G_CR	GBOX, Resolution	$h = 2, 1, 0.5$ km.
CT_SD_G_CR	GBOX, Resolution	$h = 1, 0.5, 0.25$ km.
CT_SG_COMP	Number of computations	Simulations for GBOX were performed to match the number of computations.
CT_SG_MS	Mesh density	Two lines are present for GBOX for a varying number of observation points.

4.5.2. Performed Simulations

This subsection includes information on the simulations that were performed to obtain the results and graphs listed in Table 4.5. The simulations are summarised and listed in Table 4.6.

Note that the simulation ID follows the convention [S/G]_[geometry]_[h]_[n], where S and G denote SPECFEM and GBOX respectively, [geometry] identifies the simulated object, [h] denotes the mesh size in kilometres (except for the DTM, where it is in metres), and [n] denotes the number of observation points (GBOX only). The geometry abbreviations are: RP – rectangular prism, SD – spherical salt dome, LTMR – Marius Hills rounded lava tube, and LTG – Gruithuisen lava tube. DY denotes a density variation

simulation, while p4 and m4 denote the +4 and -4 m simulations for the DTM.

Table 4.6: Simulations to be performed for the comparison of compute speed and accuracy of SPECFEM and GBOX.

ID	Abbreviation	No. of Com- puted Cells	No. of Observation Points	Performed on
SPECFEM				
1	S_RP_2	1415	[-]	Trillium
2	S_RP_1	8960	[-]	Trillium
3	S_RP_05	62240	[-]	Trillium
4	S_SD_1	7064	[-]	Trillium
5	S_SD_05	49760	[-]	Trillium
6	S_SD_025	371072	[-]	Trillium
7	S_LTMR_01	31885	[-]	Savant
8	S_LTMR_005	45547	[-]	Savant
9	S_LTMR_0025	75400	[-]	Savant
10	S_LTMR09_0025	69880	[-]	Savant
11	S_LTMR11_0025	83140	[-]	Savant
12	S_LTMR_0025_DY09	75400	[-]	Savant
13	S_LTMR_0025_DY11	75400	[-]	Savant
14	S_LTMA_01	33910	[-]	Savant
15	S_LTMA_005	45751	[-]	Savant
16	S_LTMA_0025	78460	[-]	Savant
17	S_LTMA_00125	194632	[-]	Trillium
18	S_LTME_005	49423	[-]	Savant
19	S_LTME_0025	94660	[-]	Savant
20	S_LTME_00125	241504	[-]	Savant
21	S_LTME_000625	886240	[-]	Trillium
22	S_LTMK_01	34360	[-]	Savant
23	S_LTMK_05	46108	[-]	Savant
24	S_LTMK_0025	77140	[-]	Savant
25	S_LTMK_00125	190384	[-]	Trillium
26	S_LTMK_01	34900	[-]	Savant
27	S_LTMK_05	49270	[-]	Savant
28	S_LTMK_0025	93940	[-]	Savant
29	S_LTMK_00125	262240	[-]	Trillium
30	S_LTMRe_01	33865	[-]	Savant
31	S_LTMRe_05	45139	[-]	Savant
32	S_LTMRe_0025	78580	[-]	Savant
33	S_LTMRe_00125	191176	[-]	Trillium
34	S_LTMT_01	34180	[-]	Savant
35	S_LTMT_05	46720	[-]	Savant
36	S_LTMT_0025	85660	[-]	Savant
37	S_LTMT_00125	226960	[-]	Trillium
38	S_LTG_02	252308	[-]	Savant
39	S_LTG_01	393824	[-]	Savant
40	S_LTG_0075	777764	[-]	Savant
41	S_LTG_0075_DY09	777764	[-]	Savant
42	S_LTG_0075_DY11	777764	[-]	Savant
43	S_LTG09_0075	762784	[-]	Savant
44	S_LTG11_0075	797508	[-]	Savant
45	S_10kmDTM_59m	1056508	[-]	Trillium
46	S_10kmDTM_118m	1059496	[-]	Trillium
47	S_10kmDTM_177m	1045600	[-]	Trillium

Continued on next page

Table 4.6 – continued from previous page

ID	Abbreviation	No. of Com- puted Cells	No. of Observation Points	Performed on
48	S_10kmDTM_236m	1056292	[-]	Trillium
49	S_10kmDTM_295m	1056760	[-]	Trillium
50	S_10kmDTM_354m	1056796	[-]	Trillium
51	S_10kmDTM_413m	1055968	[-]	Trillium
52	S_10kmDTM_472m	1060072	[-]	Trillium
53	S_10kmDTM_177m_nom	147392	[-]	Trillium
54	S_10kmDTM_177m_p4	147392	[-]	Trillium
55	S_10kmDTM_177m_m4	144032	[-]	Trillium
56	S_20kmDTM_177m	514715	[-]	Trillium
57	S_20kmDTM_177m_p4	514715	[-]	Trillium
58	S_20kmDTM_177m_m4	514715	[-]	Trillium
59	S_30kmDTM_177m	1086740	[-]	Trillium
60	S_30kmDTM_177m_p4	1086740	[-]	Trillium
61	S_30kmDTM_177m_m4	1086740	[-]	Trillium
GBOX				
62	G_RP_2_401	15	401	Trillium
63	G_RP_2_4001	15	4001	Trillium
64	G_RP_2_40001	15	40001	Trillium
65	G_RP_1_401	120	401	Trillium
66	G_RP_1_4001	120	4001	Trillium
67	G_RP_1_40001	120	40001	Trillium
68	G_RP_05_401	960	401	Trillium
69	G_RP_05_4001	960	4001	Trillium
70	G_RP_05_40001	960	40001	Trillium
71	G_SD_1_401	720	401	Trillium
72	G_SD_1_4001	720	4001	Trillium
73	G_SD_1_40001	720	40001	Trillium
74	G_SD_05_401	3768	401	Trillium
75	G_SD_05_4001	3768	4001	Trillium
76	G_SD_05_40001	3768	40001	Trillium
77	G_SD_025_401	23144	401	Trillium
78	G_SD_025_4001	23144	4001	Trillium
79	G_SD_025_40001	23144	40001	Trillium
80	G_LTMR_01_1001	310	1001	Savant
81	G_LTMR_005_1001	2050	1001	Savant
82	G_LTMR_0025_1001	13292	1001	Savant
83	G_LTMR_00125_1001	102660	1001	Savant
84	G_LTMR_0023_1001	18014	1001	Savant
85	G_LTMR_0018_1001	37035	1001	Savant
86	G_LTMR_0015_1001	61310	1001	Savant
87	G_LTMR_0023_1002001	18014	1002001	Savant
88	G_LTMR_0018_1002001	37035	1002001	Savant
89	G_LTMR_0015_1002001	61310	1002001	Savant
9044	G_LTMR_0023_44	18014	44	Savant
91	G_LTMR_0023_64	18014	64	Savant
92	G_LTMR_0023_101	18014	101	Savant

As such, the simulations in Table 4.6 shall be performed to obtain the results given in chapter 5. Note that the DTM simulations have similar numbers of elements in the model. This is due to the fact that the initial STL was meshed at the appropriate size (i.e. 59 to 472 m), which was then imported into the

meshing software, but meshed at a mesh size of 59 m. Therefore, the number of elements is roughly the same, but the actual geometry is still indicative of the mesh size given in the simulation name.

5

Results

In this chapter, the results of the simulations, along with a discussion of those results, are presented. First, the error budget is presented, on which all future conclusions are based. This includes an explanation of the sensor error and the total error, which accounts for uncertainty in height and density. Next, an accuracy analysis of SPECFEM is performed, comparing it with analytical solutions and GBOX. Then, the sensitivity analyses for shape, scale, depth, and density are showcased. Afterwards, the effect of resolution on the gravitational attraction for lava tubes and DTMs is detailed.

5.1. Error Budget

In this section, the detectability of the lava tube is discussed and calculated. A case study is conducted for the Marius Hills lava tube. Finally, a value for the total error, including uncertainty in DTM height and density, is obtained.

For a lava tube to be detectable, the gravity signal must exceed the total measurement error. Therefore, to assess if a typical lunar lava tube is detectable, the total error must first be calculated.

In total, there are three important errors to be considered:

- The sensor error
- DTM height error
- DTM lunar rock density error

These can all be characterised using the Bouguer anomaly formula, given in Equation 2.6, which was also given in subsection 2.3.2,

$$g^B = g_{\text{measured}} - \delta g_{\text{FA}} - \delta g_{\text{BG}} - \delta g_{\text{TC}} - g_0. \quad (2.6)$$

In this formula, the sensor error, the height error, and the density error are individually introduced and propagated through. Note that Blakely (1995), from which this equation was taken, states that the sign of the terrain correction is always negative, such that it always brings a positive contribution. Then, afterwards, the contributions can be combined to obtain the total error, using the formula given in Equation 5.1,

$$\epsilon_{\text{total}} = \sqrt{\epsilon_{\text{sensor}}^2 + \epsilon_h^2} + \epsilon_\rho, \quad (5.1)$$

Where ϵ_{sensor} is the sensor error, ϵ_h the height error, and ϵ_ρ the density error.

The sensor and height errors are treated as random, while the density error is treated as systematic. For the sensor and height errors, these values are given, while for the density error, a base value of 3170 kg m^{-3} is assumed, with an RMS error of $\pm 80 \text{ kg m}^{-3}$. This value is taken as an average of values presented in Kiefer et al. (2012).

5.1.1. Sensor Error

The sensor error is the error with which a gravity sensor can measure gravitational attraction. As such, it defines the resolution of measurements. Having a lower sensor error allows for more detail in measurements and for higher confidence in detecting anomalies in measurements as true anomalies, not noise.

In this thesis, the sensor error was defined to be 0.3 mGal RMS. This was chosen based on the fact that, out of existing gravimeters, the most suitable recent one, which was specifically designed to work on the Moon, achieved this accuracy. The sensor of interest is the VEGA space gravimeter, which was discussed more extensively in subsection 2.3.1. This sensor error is specifically valid for the Moon; on Earth, due to its larger mass, the error would be different. Since the error is a 1σ RMS error, a measurement lies within ± 0.3 mGal of the true value with a probability of 68%.

The sensor error is relevant to the detectability of the lava tube, as its accuracy defines the depth of lava tubes that can be measured. If the lava tube is too deep, it can fall below the resolution threshold, resulting in it being indistinguishable from noise.

The sensor error can be directly established in the Bouguer anomaly formula. First, it can be seen that the sensor error only influences g_{measured} , and does so directly. The error is therefore 0.3 mGal RMS.

It must be noted that errors in positioning on the Moon could influence the measurement error, especially when performing a survey. For example, the GPS has an uncertainty that also influences how accurately one thinks is located. This effect is not included in the error budget, as it is outside the scope of this thesis. However, such errors must be considered in surveys.

5.1.2. DTM Height Error

Since a DTM is used for terrain corrections, which in turn are used to more easily find underground anomalies, the effect that the DTM height resolution has on the error of this terrain correction is important to quantify.

To obtain the error the height introduces, the effect on each component in Equation 2.6 must be discussed. The height does not have a direct influence on the measurement error. It does affect the free-air correction, the Bouguer correction, as well as the terrain correction. To understand how they affect the error, partial derivatives with respect to the height can be taken,

$$\begin{aligned}\frac{\partial \delta g_{\text{FA}}}{\partial h} &= \frac{\partial \left(\frac{-2GM_{\text{moon}}}{r^3} h \right)}{\partial h} = \frac{-2GM_{\text{moon}}}{r^3}, \\ \frac{\partial \delta g_{\text{BG}}}{\partial h} &= \frac{\partial (2\pi h \rho G)}{\partial h} = 2\pi \rho G, \\ \frac{\partial \delta g_{\text{T}}}{\partial h} &\approx \frac{\Delta \delta g_{\text{TC}}}{\Delta h} = \frac{g_{\text{TC}_2} - g_{\text{TC}_1}}{h_2 - h_1},\end{aligned}$$

Where the free-air and Bouguer errors can be calculated algebraically, while the terrain correction error must be obtained from numerical simulations. In this case, the terrain correction was calculated for the nominal 30 by 30 km simulation, as well as for the simulations with 4 m of extra mass on top. These provide g_{TC_2} and g_{TC_1} , with $h_2 - h_1 = 4$ m.

Filling in the numbers, one obtains,

$$\begin{aligned}\frac{\partial \delta g_{\text{FA}}}{\partial h} &\approx -0.1869 & \left[\frac{\text{mGal}}{\text{m}} \right], \\ \frac{\partial \delta g_{\text{BG}}}{\partial h} &\approx 0.139 & \left[\frac{\text{mGal}}{\text{m}} \right], \\ \frac{\partial \delta g_{\text{TC}}}{\partial h} &\approx 0.0985 & \left[\frac{\text{mGal}}{\text{m}} \right].\end{aligned}$$

Summing these using the Bouguer formula, the total *height* error can be calculated using a RMS height error of 4 m,

$$\epsilon_h = \left(-\frac{\partial \delta g_{FA}}{\partial h} - \frac{\partial \delta g_{BG}}{\partial h} - \frac{\partial \delta g_{TC}}{\partial h} \right) h \approx 0.585 \text{ mGal}.$$

5.1.3. DTM Density Error

To calculate the DTM density error, a similar strategy is used as for the height variation.

First, the partial derivatives are obtained. The density is relevant only for the Bouguer correction and the terrain correction. For the terrain correction, a simplification can be performed. Instead of performing two simulations with varying densities, the linear relationship between the density and gravitational acceleration can be exploited. For the Bouguer correction, h is the distance in height between the mean radius of the Moon and the measurement height, which in this case is -1582.750 m, found using Quickmap (Arizona State University, NASA, and Applied Coherent Technology Corp. 2024).

$$\begin{aligned} \frac{\partial \delta g_{BG}}{\partial \rho} &= \frac{\partial (2\pi h \rho G)}{\partial \rho} = 2\pi h G, \\ \frac{\partial \delta g_T}{\partial \rho} &= \frac{\delta g_{T1}}{\rho}. \end{aligned}$$

Filling in the numbers, one obtains,

$$\begin{aligned} \frac{\partial \delta g_{BG}}{\partial \rho} &\approx -0.066 \quad \left[\frac{\text{mGal}}{\text{kg m}^{-3}} \right], \\ \frac{\partial \delta g_T}{\partial \rho} &\approx -0.00043 \quad \left[\frac{\text{mGal}}{\text{kg m}^{-3}} \right]. \end{aligned}$$

Then, using a density uncertainty of $\pm 80 \text{ kg m}^{-3}$, the final error can be obtained,

$$\epsilon_\rho = \left(-\frac{\partial \delta g_{BG}}{\partial \rho} - \frac{\partial \delta g_{TC}}{\partial \rho} \right) \Delta \rho \approx 5.275 \text{ mGal}$$

Using these errors, the total error can be calculated using Equation 5.1, giving a total error of,

$$\boxed{\epsilon_{\text{total}} \approx 5.9 \text{ mGal}} \quad (5.2)$$

In the case a double RMS value is chosen, the error increases to,

$$\epsilon_{\text{total}} \approx 11.2 \text{ mGal} \quad (5.3)$$

As can be seen throughout the calculation, the density error dominates the total error. In this case, the density was assumed to vary systematically. Still, this error assumes that density is homogeneous throughout the DTM, which is not necessarily the case. Region-to-region variability of density is possible and expected, but outside the scope of this thesis, and therefore not included.

5.2. Accuracy and Computational Speed of SPECFEM and GBOX

In this section, the accuracy and computational speed of the programs SPECFEMX and GBOX are demonstrated. First, simulations are performed at the same mesh size, at which point accuracy can be compared. Afterwards, the effects of mesh size and the number of observation points on the computation time are investigated for both programs.

5.2.1. Accuracy Analysis of SPECFEM and GBOX

In this subsection, the accuracy of both SPECFEM and GBOX is compared with each other and with analytical solutions. In total, three situations are used as models:

- Rectangular Prism
- Spherical Salt Dome
- Lava tube (Marius Hills version)

The first two examples are taken from Gharti, Tromp, and Zampini (2018).

For these examples, the accuracy of SPECFEM and GBOX is compared, as well as the increase in accuracy with a smaller element size. In Figure 5.1 and Figure 5.2, the comparison is shown for the rectangular prism, while in Figures 5.3–5.7 the spherical salt dome is shown. In Figures 5.8–5.10, the same comparison is also shown for a lava tube.

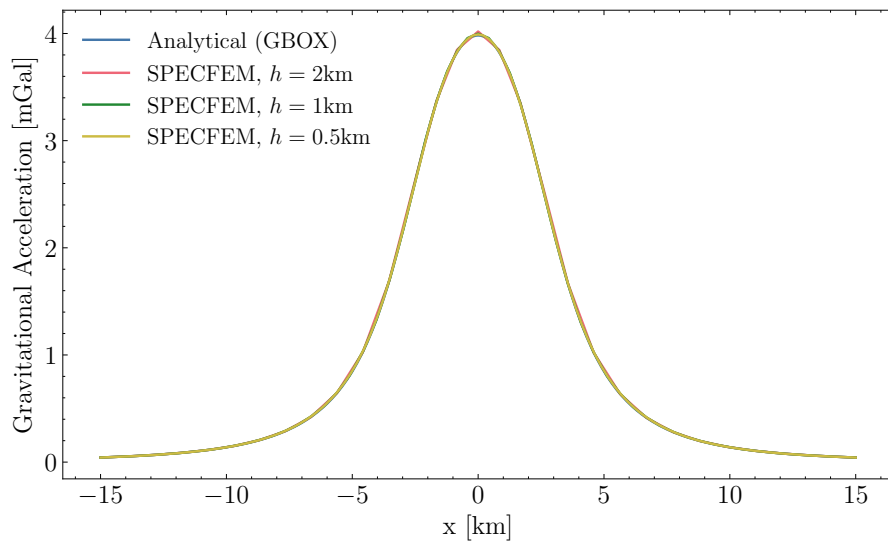


Figure 5.1: Comparison of varying element sizes on the gravity anomaly of a buried prism.

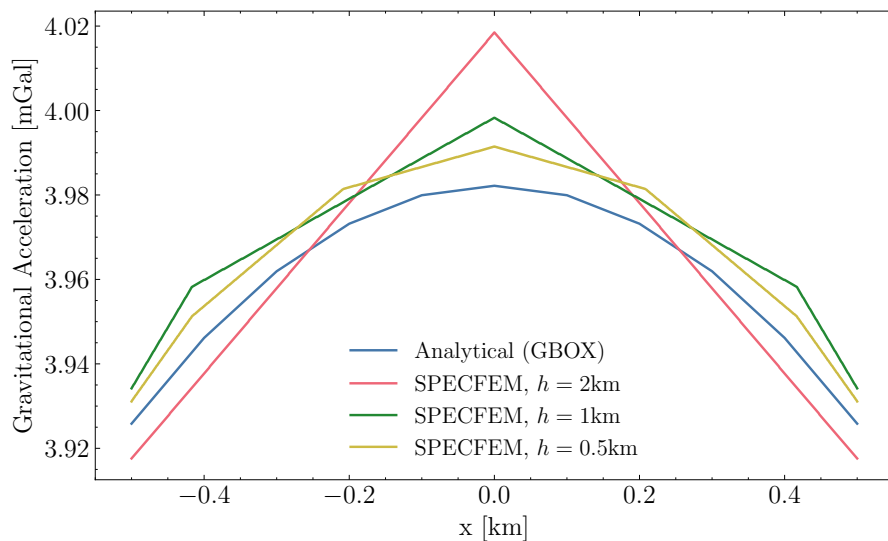


Figure 5.2: Comparison of varying element sizes on the gravity anomaly of a buried prism, zoomed in.

As shown in Figure 5.1 and Figure 5.2, the analytical solution and SPECFEM lie quite close to each

other. The progression toward the analytical solution can also be clearly seen as the mesh size is refined. Overall, the SPECFEM simulations yield quite accurate results even with coarse meshes. Even with a mesh size of 2 km, the results differ by at most 0.02 mGal, which is far below the measuring accuracy used in this thesis. Still, this is for a prism at a depth of 2 km. For shallower depths, a larger difference is likely to be measured. In this specific case, a mesh size of 2 km suffices, but since the extra cost of using a 0.5 km mesh size is so insignificant, a 0.5 km mesh size might as well be used. Also of note is the fact that this model is a *rectangular* prism. This means the model can be perfectly meshed with hexahedral elements, which also contributes to high accuracy.

For the spherical salt dome, results can be found in Figures 5.3–5.7.

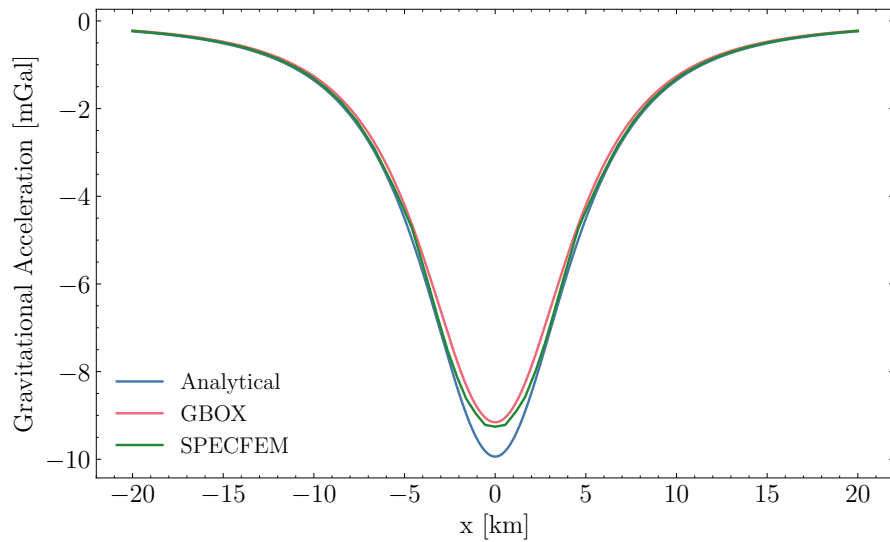


Figure 5.3: Comparison of varying element sizes on the gravity anomaly of a buried sphere vs the analytical solution, $h=1$ km.

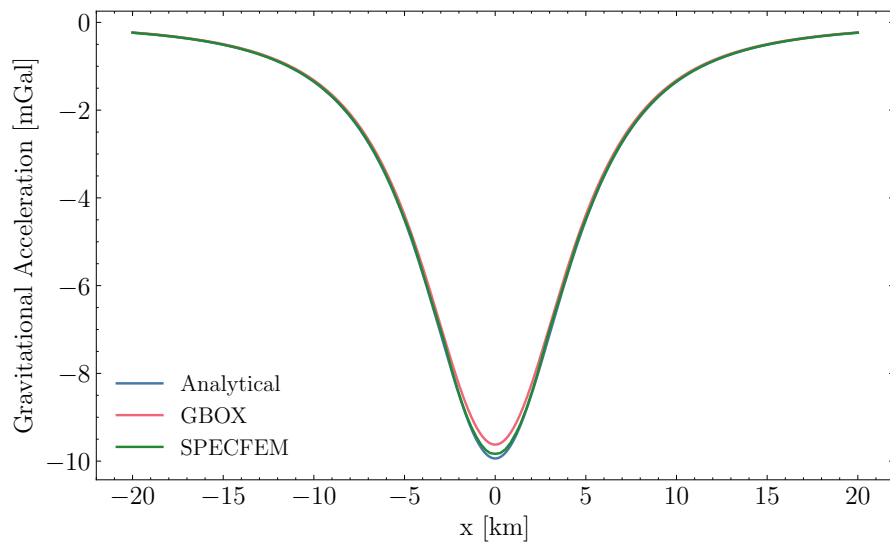


Figure 5.4: Comparison of varying element sizes on the gravity anomaly of a buried sphere vs the analytical solution, $h=0.5$ km.

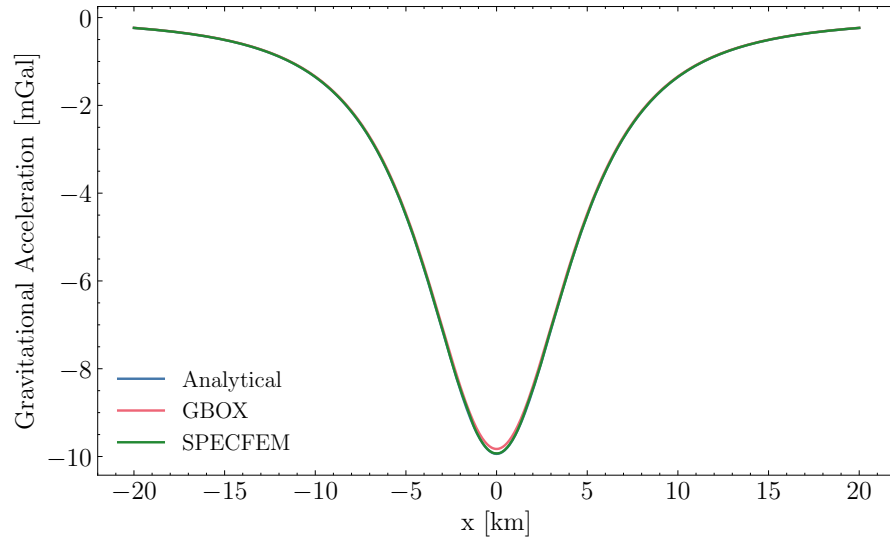


Figure 5.5: Comparison of varying element sizes on the gravity anomaly of a buried sphere vs the analytical solution, $h=0.25$ km.

As can be seen in Figures 5.3–5.5, when the mesh size is kept rough, at $h=1$ km, both the SPECFEM and GBOX simulations approximate the analytical solution for the spherical salt dome well at X-distances further away from the origin. At the centre, both SPECFEM and GBOX have significant errors compared to the analytical solution. This is substantially alleviated when the resolution is doubled to $h=0.5$ km. When $h=0.25$ km, the SPECFEM solution matches perfectly with the analytical solution, while the GBOX solution just lags the minimum of the gravity anomaly. This shows that SPECFEM, at least in this case, is more accurate than GBOX for the same mesh size. This is likely because SPECFEM can more accurately represent shapes, especially spherical ones, than GBOX.

Overall, it is found that at a mesh size of 0.5 km, the maximum difference between the analytical and SPECFEM solution is 0.1 mGal, while for a mesh size of 0.25 km, this drops to 0.04 mGal. A general rule of thumb can be derived using this mesh size and the radius of the spherical object. For spherical objects in lunar gravimetry, with a sensor accuracy of 0.3 mGal RMS, it is recommended to use a mesh size no larger than $\frac{0.25}{4} = \frac{1}{16}$ of the object's radius.

One noticeable fact when comparing the graphs of Gharti, Tromp, and Zampini (2018) and the ones present here is that for the spherical salt dome, at a mesh size of 1 km, the graphs are not identical. In fact, a large difference is present. This difference is most likely due to the meshes simulated being slightly different. The number of elements in the mesh was found to be different, and the difference in results is likely to stem from this discrepancy.

The progression of how much more accurate the results are with a higher resolution can be seen in Figure 5.6 and Figure 5.7, for SPECFEM and GBOX, respectively.

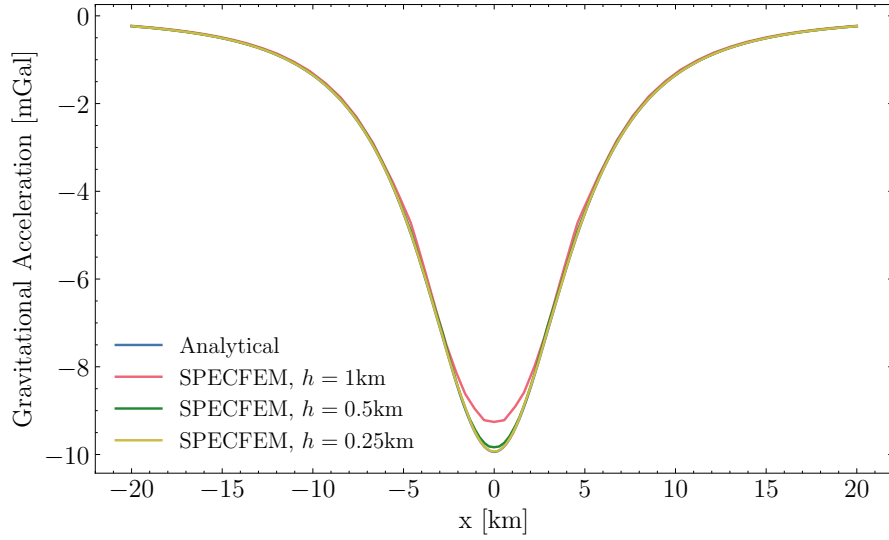


Figure 5.6: Comparison of varying element sizes on the gravity anomaly of a buried sphere, using SPECFEM.

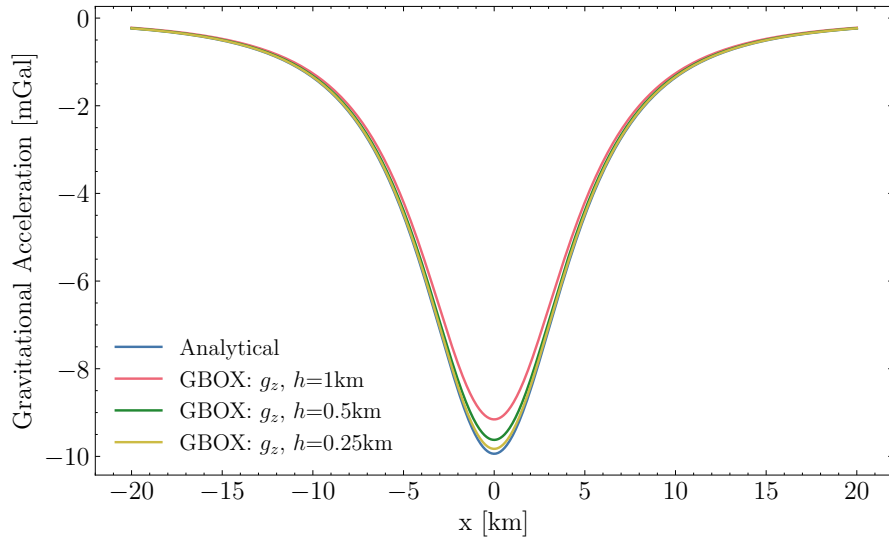


Figure 5.7: Comparison of varying element sizes on the gravity anomaly of a buried sphere, using GBOX.

As can be seen in Figure 5.6 and Figure 5.7, the convergence of SPECFEM happens more quickly.

However, such a better resolution must be compared in terms of other parameters, such as the time it takes to compute the solution. If SPECFEM takes much longer to compute, it might not be worth using SPECFEM, and vice versa. Such a comparison is performed in subsection 5.2.2.

The same simulations were performed on a buried lava tube. The results of these can be seen in Figures 5.8–5.10.

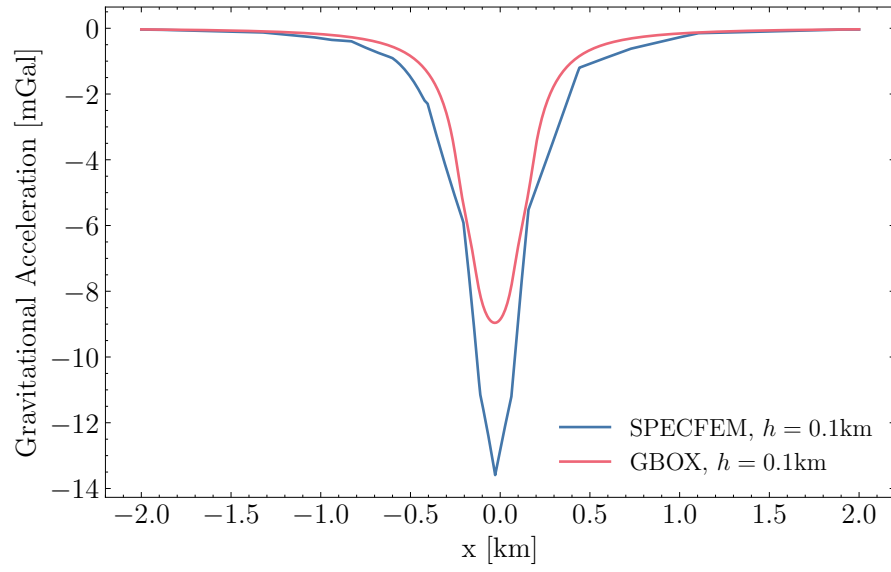


Figure 5.8: Comparison between SPECfEM and GBOX on the accuracy of representing a buried rounded lava tube (25 m depth) with resolution equal to 0.1 km.

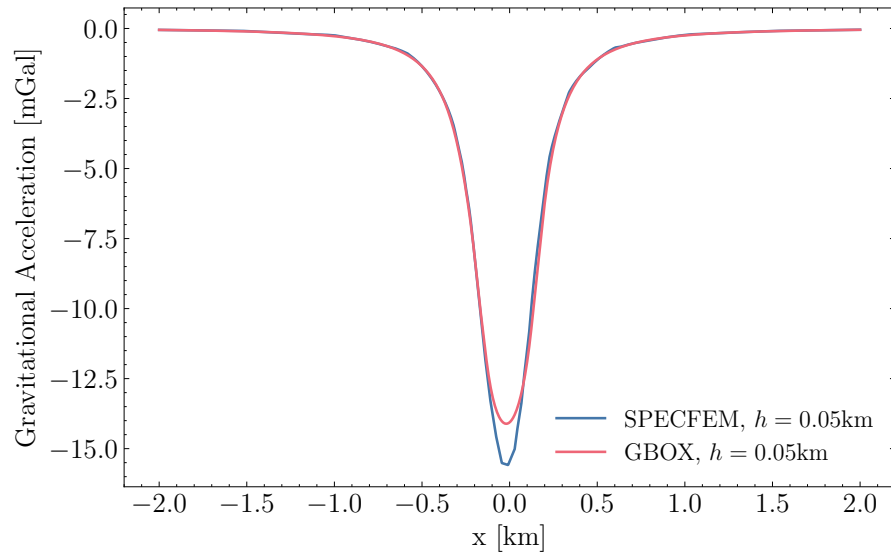


Figure 5.9: Comparison between SPECfEM and GBOX on the accuracy of representing a buried rounded lava tube (25 m depth) with resolution equal to 0.05 km.

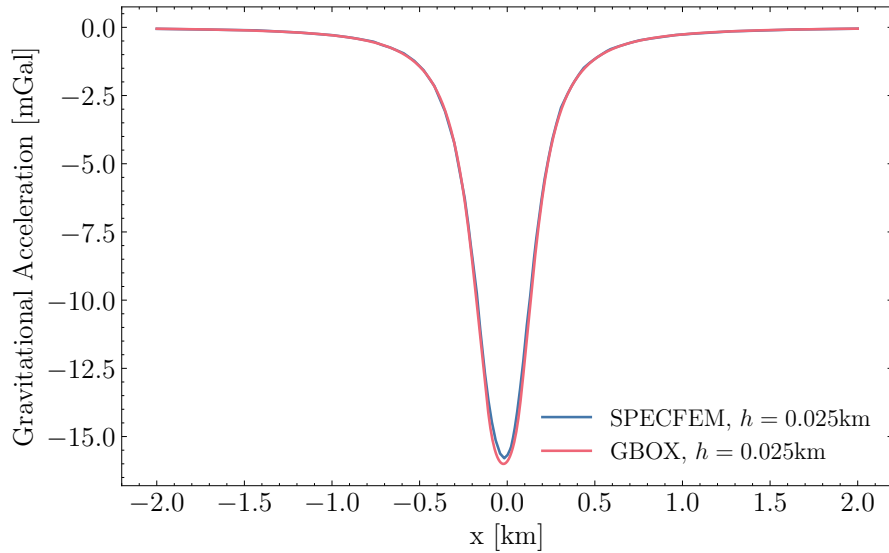


Figure 5.10: Comparison between SPECfEM and GBOX on the accuracy of representing a buried rounded lava tube (25 m depth) with resolution equal to 0.025 km.

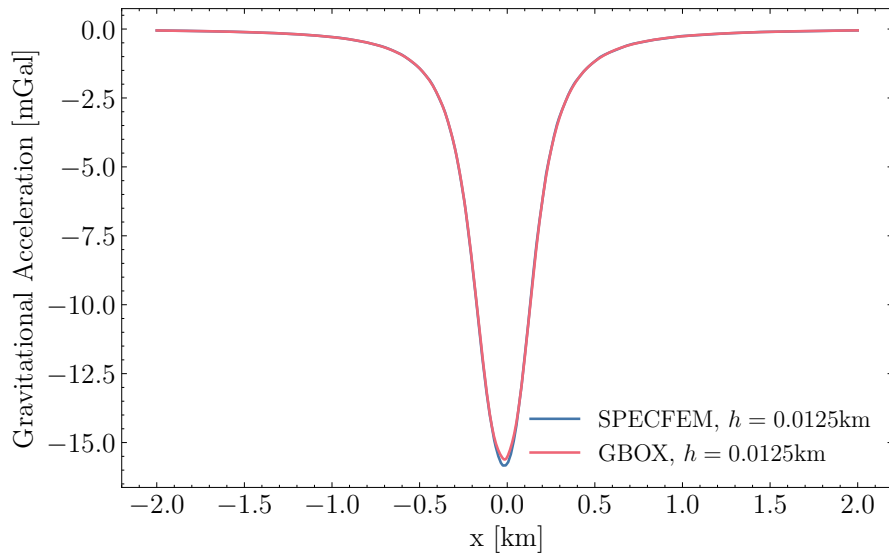


Figure 5.11: Comparison between SPECfEM and GBOX on the accuracy of representing a buried rounded lava tube (25 m depth) with resolution equal to 0.0125 km.

As can be seen in Figures 5.8–5.11, both SPECfEM and GBOX seem fairly comparable. One very noticeable difference is that the SPECfEM solutions are much more accurate at low resolutions compared to GBOX. GBOX, unlike SPECfEM, tends to significantly underestimate or overestimate gravitational attraction, especially at these low resolutions. This could be because GBOX uses rectangular prisms to compute gravitational attraction, which could leave a lot of space in models with many curves.

This, however, is only relevant for large mesh sizes, as when one looks at the highest-resolution results, it is clear that the GBOX and SPECfEM solutions both converge. The progressions of these simulations can be seen in Figure 5.12 and Figure 5.13.

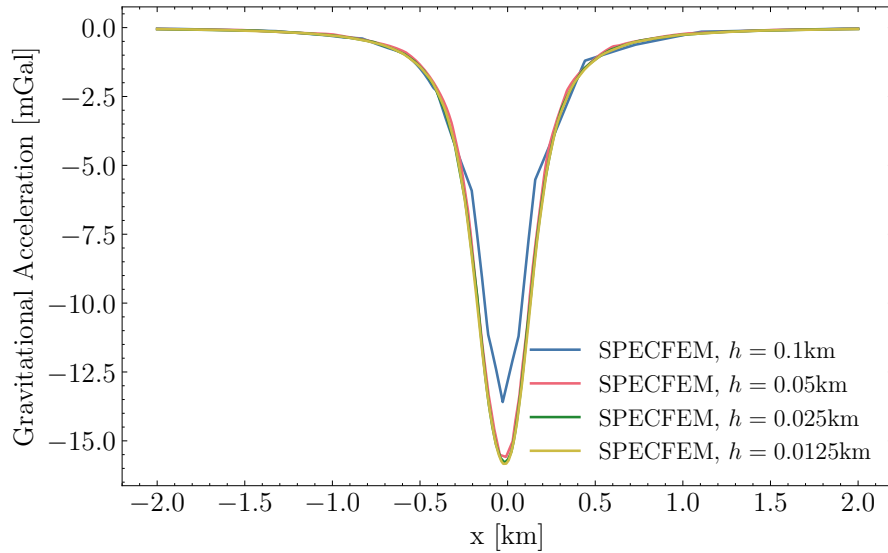


Figure 5.12: Comparison of varying element sizes on the gravity anomaly of a buried rounded lava tube (25 m depth) in the Marius Hills region, using SPECfEM.

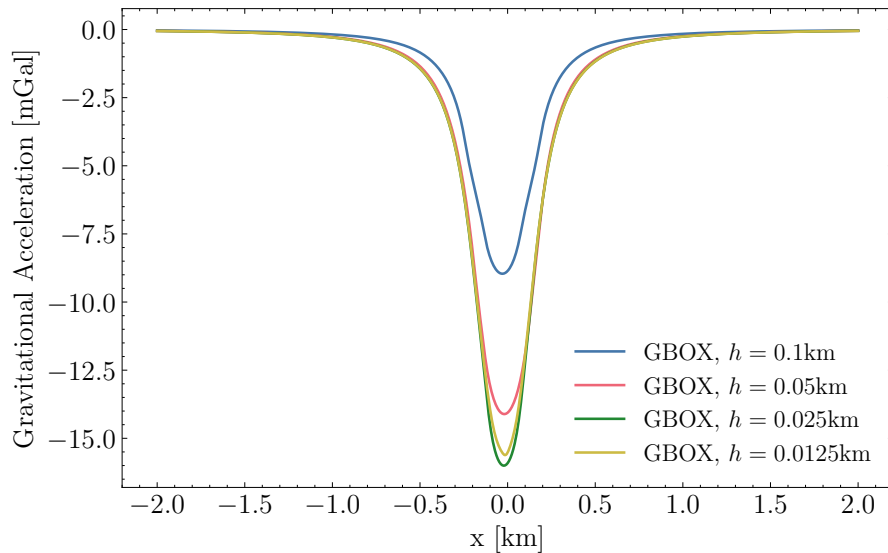


Figure 5.13: Comparison of varying element sizes on the gravity anomaly of a buried rounded lava tube (25 m depth) in the Marius Hills region, using GBOX.

As can be seen, SPECfEM, as in the rectangular prism and spherical salt dome simulations, outperforms GBOX in accuracy. Overall, it can be established that SPECfEM is more accurate for the same mesh size, although at high resolutions, this effect is not significant. Still, looking at a mesh size of 0.05 km and comparing it to the 0.0125 km mesh size, it can be seen that SPECfEM only differs by approximately 0.3 mGal, while for GBOX this difference is closer to approximately 1.8 mGal. For a mesh size of 0.025 km, a similar effect was found. SPECfEM differs by approximately 0.1 mGal, while GBOX differs by approximately 0.5 mGal.

Therefore, for accuracy, especially at lower mesh sizes, SPECfEM is preferred.

5.2.2. Compute Time

To determine whether GBOX or SPECfEM is more useful for obtaining the gravity anomaly, a close look must be taken at the time required for the simulations. If, for example, SPECfEM takes ten times

longer to simulate, but the difference in accuracy is only marginal, it becomes quite easy to argue for using GBOX instead. To understand the exact compute times, more simulations were performed, where, for GBOX, the number of observation points and the number of computed cells are varied, both independently, whereas for SPECFEM, just the number of elements is varied.

Because the impact is more interesting for the lava tube, this was done for the Marius Hills lava tube. First, the effect of changing the number of observation points, which can only be done for GBOX, is shown. Next, a comparison is made between GBOX and SPECFEM for the same number of computations, as well as one for the same number of ‘elements’, although these ‘elements’ are defined differently for each software.

For the observation point variation, a total of three different simulations were performed, all with different mesh sizes, where for each the number of observation points is increased. The results can be seen in Figure 5.14.

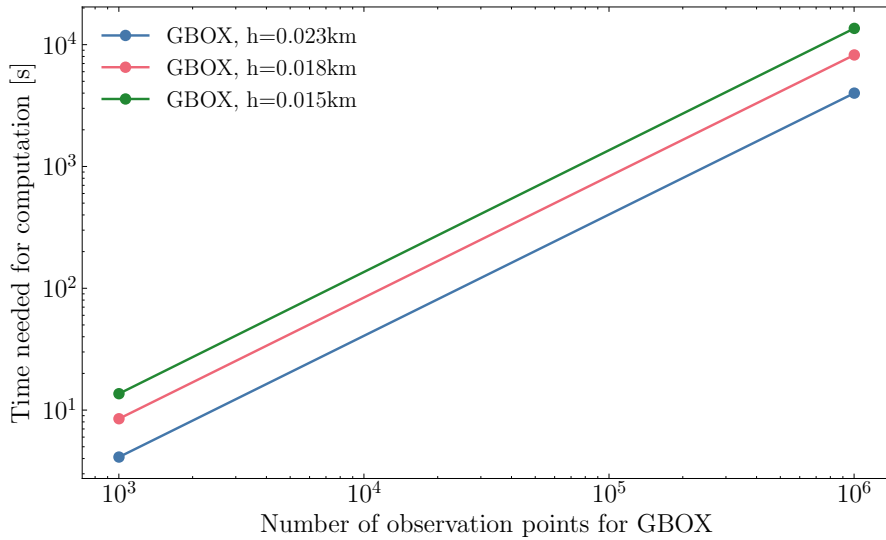


Figure 5.14: The compute time vs the number of observation points for three lava tube meshes, where $h = 0.023, 0.018$, and 0.015 km. Note that the plot is on a log-log scale.

As can be seen in Figure 5.14, although only two data points were taken for each model, the conclusion is quite clear: The compute time of the simulation depends linearly on the number of observation points. The more observation points, the longer the compute time, and if you have twice the number of observation points, you should get twice the time needed. To confirm this using more data, the same simulation was performed for the rectangular prism and the spherical salt dome, as seen in Figure 5.15 and Figure 5.16.

As can be seen in Figure 5.15 and Figure 5.16, although a slight bump can be seen for the second column of data points, mostly the same conclusion can be made.

Next, the effect of an increase in model size is compared for both GBOX and SPECFEM. Important for understanding this variation is that GBOX and SPECFEM, due to their different methods of calculating the gravitational attraction, have different characteristics that would define the time needed for computing solutions. For GBOX, computations are always done on several cells of interest. At every observation point, for each of these cells, a single formula is used to calculate the contribution of this specific cell for this specific observation point. As such, for GBOX, the time required T for a simulation can be calculated as given in Equation 5.4.

$$T = N_{\text{comp}} \cdot t_c, \quad N_{\text{comp}} = N_{\text{obs}} \cdot N_c, \quad (5.4)$$

Where N_{comp} is the number of computations, N_{obs} is the number of observation points, N_c is the number of cells, and t_c is the time required per computation. t_c depends on the specific processor or computer

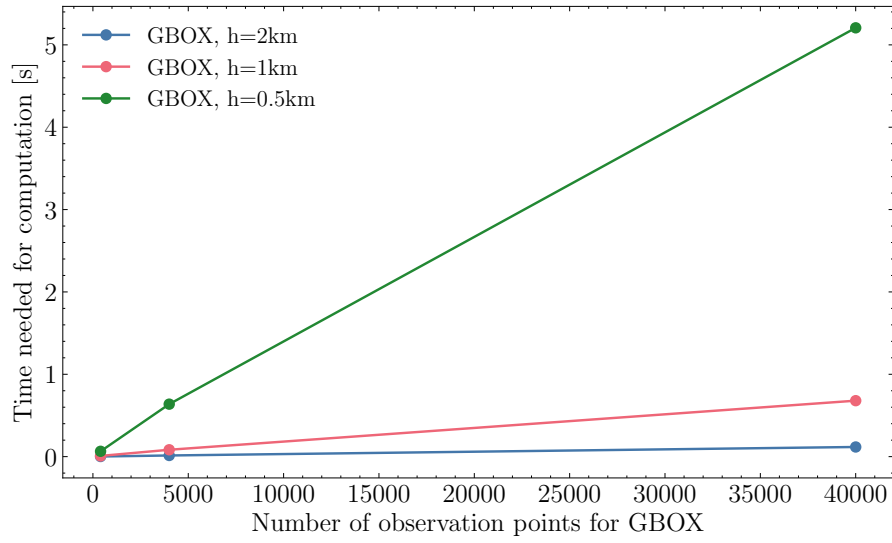


Figure 5.15: The compute time vs the number of observation points for three rectangular prism meshes, where $h = 2, 1$, and 0.5 km

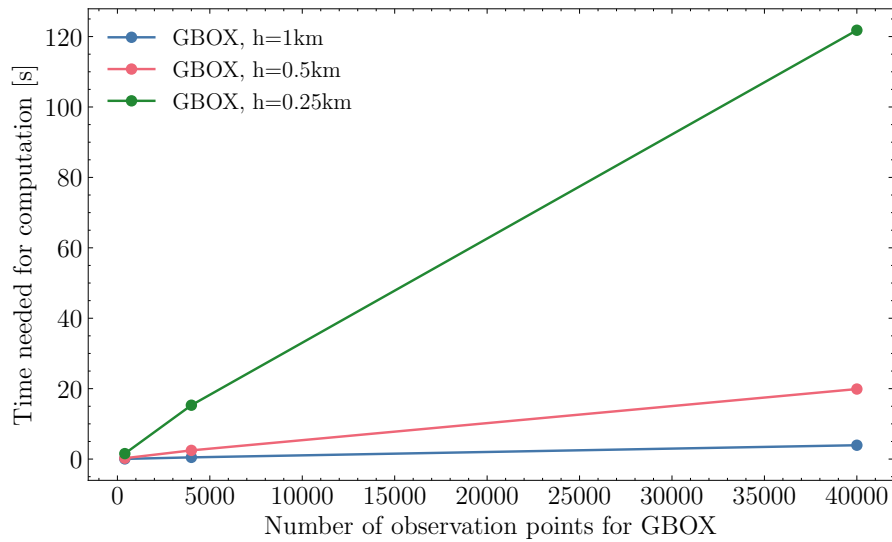


Figure 5.16: The compute time vs the number of observation points for three spherical salt dome meshes, where $h = 1, 0.5$, and 0.25 km

used in the simulation. If an area or volume is involved in the simulation, an increase in the density of observation points can prove rather costly for GBOX. If, for example, one wants to have twice as many points on both sides of a rectangle, for example, from 100 by 100 observation points to 200 by 200, it is quickly obvious that the total number of observation points increases by $2^2 = 4$. Similarly, if instead of an area a volume was used, which changed from 100 by 100 by 100 to 200 by 200 by 200, this value would have been $2^3 = 8$. This square law and cubic relationship create an immense increase in compute time when the density of observation points is increased.

Similarly, if one has been investigating lines, which could only use 1000 observation points, and now wants to investigate a volume (for example, a cube), the increase in compute time would be,

$$\frac{1000^3}{1000} = 1 \times 10^6.$$

Since lines can take anywhere between 0.1 ms ($N_c = 15$) and tens of seconds ($N_c = 4 \times 10^6$), the volume case would take minutes for small models, but months for large ones.

For SPECfEM, understanding the time necessary for calculating the solution required more attention. Firstly, unlike for GBOX, elements are present everywhere a solution is wanted. Therefore, extra elements are almost always created outside the area of interest. For these, cell calculations are made as well. Next to that, each cell has multiple GLL points, for which computations are made. Therefore, it is not just one computation per cell. For the case where there are three GLL points in each direction, the number of computations can be roughly calculated as,

$$N_{\text{comp}} = N_c \cdot N_{\text{GLL}}^3 = N_c \cdot 3^3. \quad (5.5)$$

One comparison that can be made is therefore one in which an equal number of computations are performed. This will show which software performs computations more quickly. Another comparison is how choosing a certain mesh size affects the compute time for both GBOX and SPECfEM.

For the computations, the results are shown in Figure 5.17.

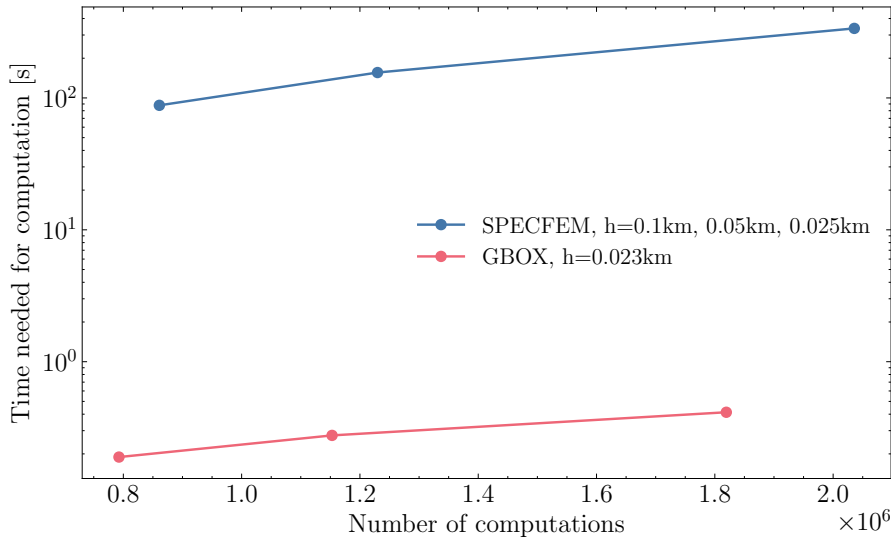


Figure 5.17: The compute time vs the number of computations for three lava tube meshes, where for SPECfEM $h = 0.1, 0.05$, and 0.025 km, and for GBOX the number of observation points has been varied (44, 64, 101) to approximately match the number of computations for the SPECfEM simulations. Note the logarithmic scale for the y-axis.

As shown in Figure 5.17, for an equal number of computations, GBOX is much quicker, staying more than two orders of magnitude quicker than SPECfEM per computation. From this, it can be concluded that the time needed per computation is much less for GBOX. Therefore, if a simulation requires a similar number of computations, GBOX is preferred.

In practice, the number of computations necessary for a simulation could be much higher in the GBOX case. The next figure, Figure 5.18, shows exactly this. The mesh density (number of mesh points per kilometre) is varied, but it is shown for two different numbers of observation points, which increase the necessary number of computations.

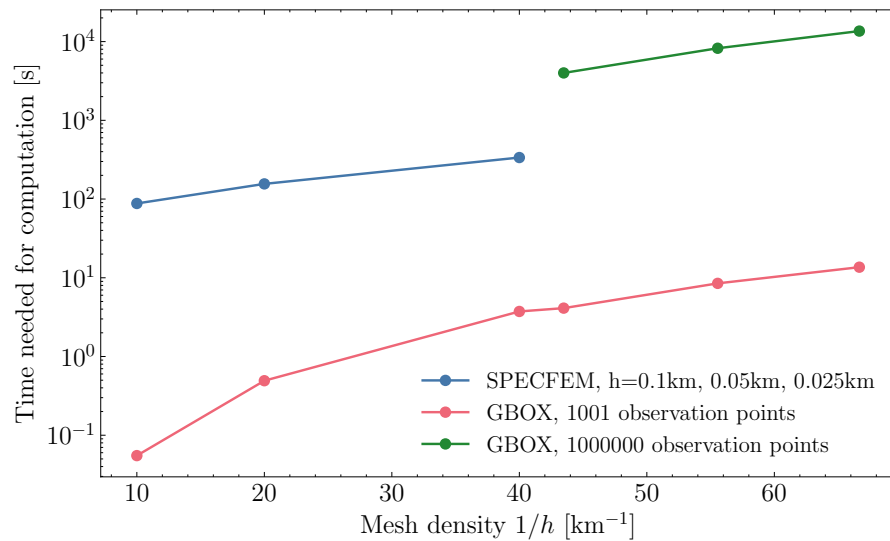


Figure 5.18: The compute time vs the mesh density $1/h$ (defined as the inverse of the mesh size h) for a rounded lava tube. For the GBOX simulation, 1001 observation points as well as 1 million observation points are present. For SPECFEM, three mesh sizes are presented. Note the logarithmic scale for the y-axis.

As can be seen in Figure 5.18, all three lines increase when the mesh density is increased. However, more interesting is the fact that the number of observation points influences the compute time for GBOX immensely. In this case, there are three distinct simulations. The two GBOX lines consider two different cases: 1001 observations are used to obtain a line plot, whereas the 1000000 observations can be seen as performing a 2D surface plot, where both the x and y axes are split up into 1000 different rectangles. For SPECFEM, the observation points can be seen to be present throughout the whole model. At any point, once the model has been simulated, and as long as the point of interest is within the model, then, when of interest to the user, the observation point only has to be *chosen*. As such, in the case where a whole region, such as an area or volume, is to be investigated, it becomes evident that SPECFEM is the tool to use.

In conclusion, GBOX is much faster per computation. However, as the number of observation points and the number of computed cells increase, the number of computations can increase substantially for GBOX, making SPECFEM a more viable option. As such, it is recommended to use GBOX for quick, small investigations, where only a line or a small area is required for the results. When a full volume or large area is of interest, the use of SPECFEM is recommended.

Based on these comparisons between GBOX and SPECFEM in both accuracy and compute time, the SPECFEM software is chosen for future simulation and sensitivity analyses. SPECFEM is found to be more accurate than GBOX for the same mesh size, and quicker to simulate when observation points are needed over a large volume. In this case, because the 2D characteristics of lava tube gravity anomalies are to be investigated as well, and therefore 2D field plots must be obtained, SPECFEM provides a much more useful entry point. A single simulation allows the author to obtain the 2D field plot at any specified height or shape (such as following the topography) at any point in time (after completion of the simulation). As such, in comparisons for height, continuous plots are much more easily obtained.

5.3. Lava Tube Sensitivity Analysis

In this section, a sensitivity analysis of the lava tube models is conducted with respect to their gravity anomalies. First, the resolution is varied. Afterwards, the depth is varied, after which the scale of

the cross-section is varied. Then, the shape is varied, after which the density is varied. The effects of varying these parameters are investigated and then discussed.

5.3.1. Resolution

To perform a sensitivity analysis of resolution, lava tube simulations can be run at a variety of resolutions. This will help understand the effect of increasing resolution and the point at which the increase in accuracy plateaus. This comparison has in fact already been done earlier in the thesis in subsection 5.2.1. However, the discussion has not yet been performed and is presented below.

Looking back at the plots given in subsection 5.2.1, specifically Figure 5.12, which refers to the *rounded* lava tube, conclusions can already be made about the variation of resolution. It can be seen that there is not a large increase in accuracy from 0.05 km to 0.025 km, as well as from 0.025 km to 0.0125 km. For the current case presented here, this is less than the sensor error of 0.3 mGal. From this, it can be concluded that a 0.025 km lava tube is a good mesh size to use for the other sensitivity analyses, as it provides a high enough accuracy while still being quick and simple to mesh and simulate.

However, it must be noted that this comparison is done for a rounded lava tube. It could be argued that, because the shape of a rounded lava tube is quite simple, lower-resolution meshes could suffice, while for more complex shapes, higher-resolution meshes are needed. Therefore, similar simulations were performed for the other 6 shapes. These can be seen in Figure 5.19–Figure 5.24.

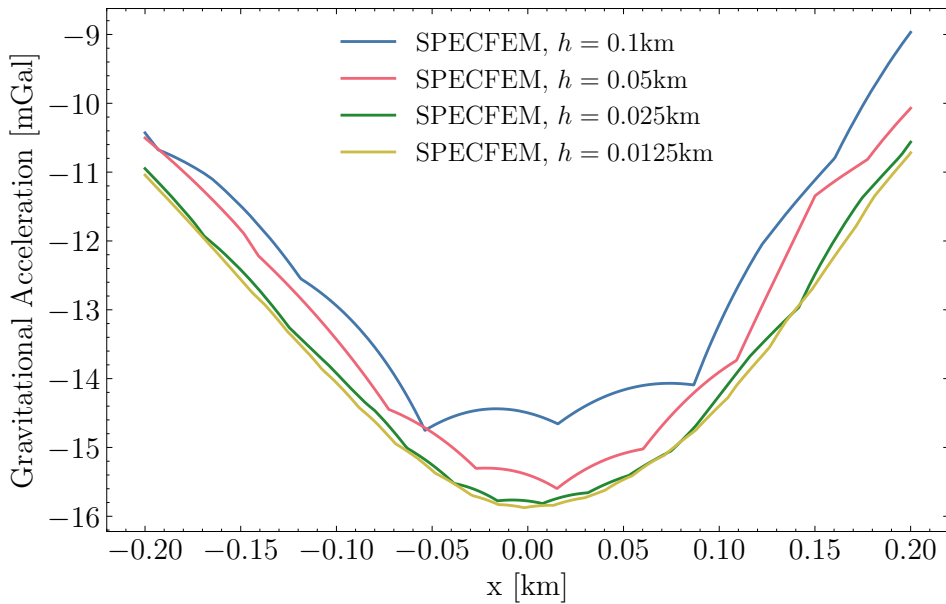


Figure 5.19: Comparison of varying element sizes on the gravity anomaly of a buried *arched* lava tube (25 m depth) in the Marius Hills region, using SPECfem.

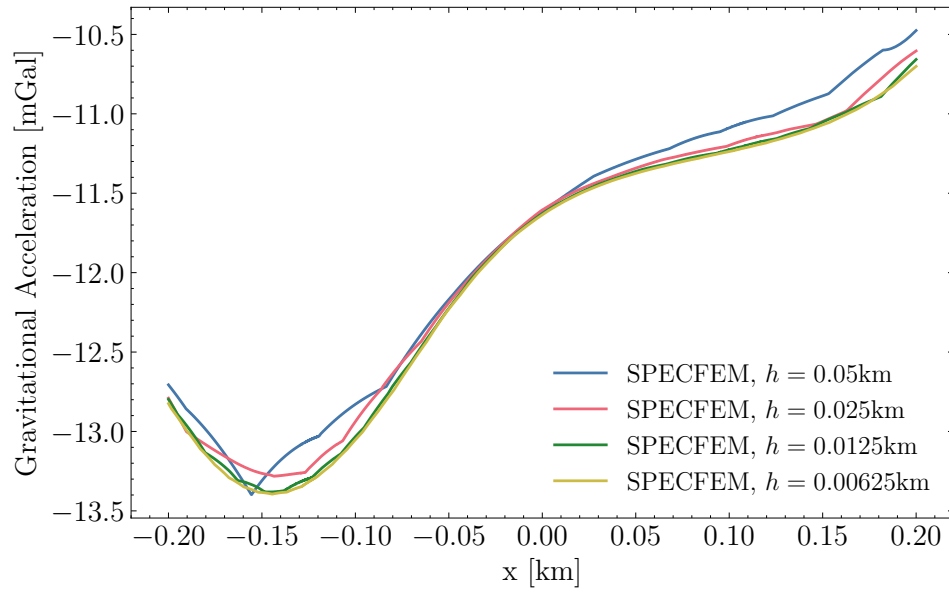


Figure 5.20: Comparison of varying element sizes on the gravity anomaly of a buried *elongated* lava tube (25 m depth) in the Marius Hills region, using SPECfEM.

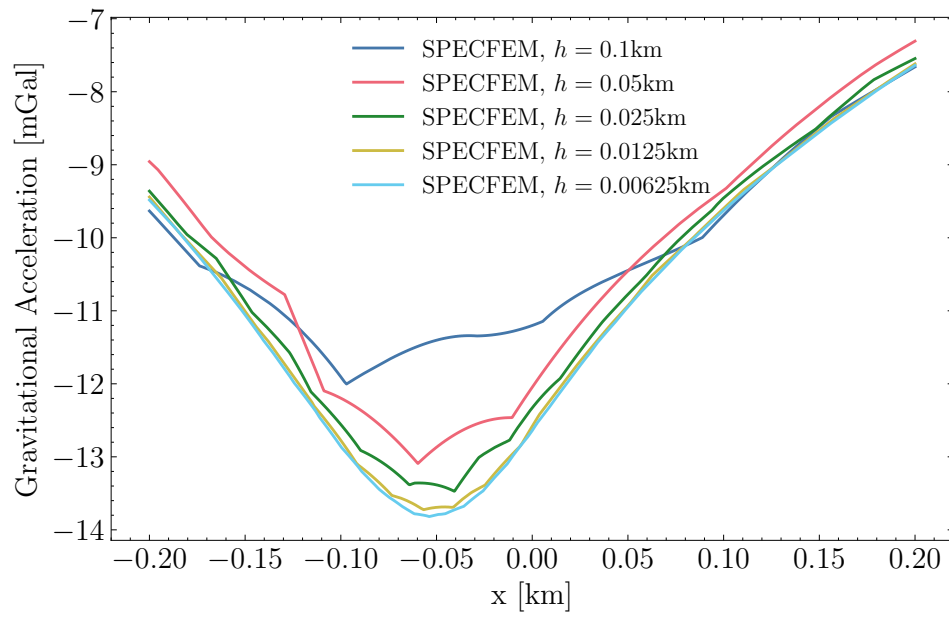


Figure 5.21: Comparison of varying element sizes on the gravity anomaly of a buried *inverted keyhole* lava tube (25 m depth) in the Marius Hills region, using SPECfEM.

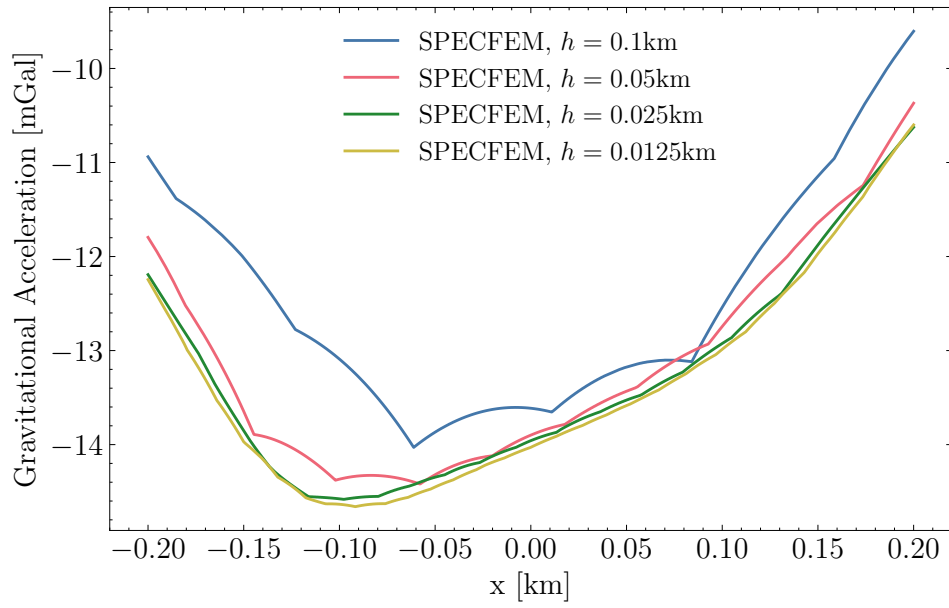


Figure 5.22: Comparison of varying element sizes on the gravity anomaly of a buried *keyhole* lava tube (25 m depth) in the Marius Hills region, using SPEC-FEM.

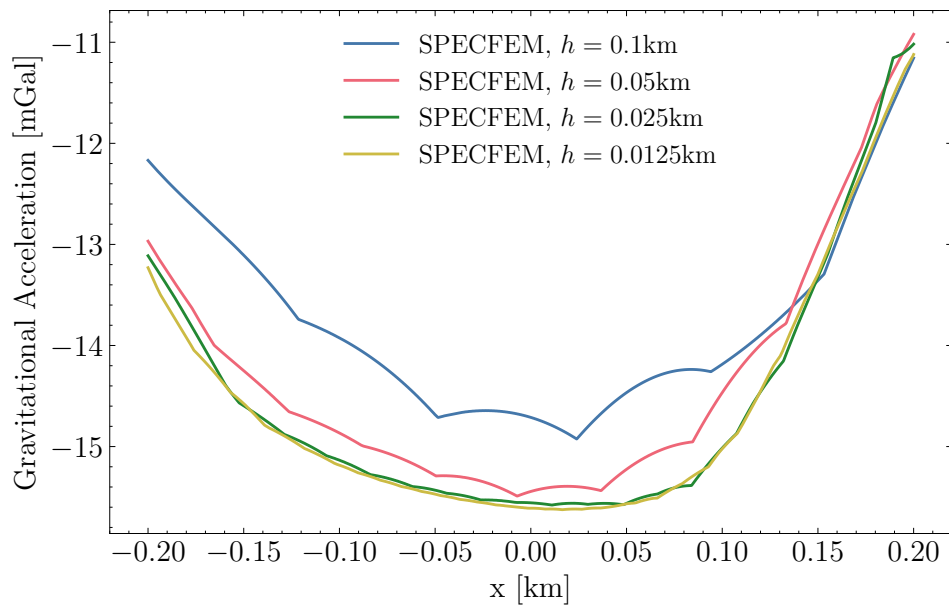


Figure 5.23: Comparison of varying element sizes on the gravity anomaly of a buried *rectangular* lava tube (25 m depth) in the Marius Hills region, using SPEC-FEM.

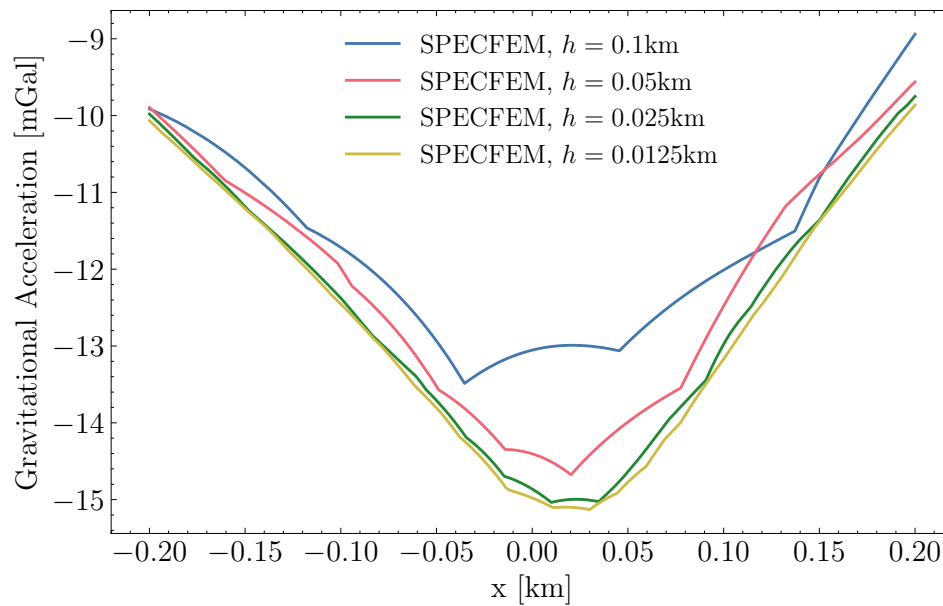


Figure 5.24: Comparison of varying element sizes on the gravity anomaly of a buried *triangular* lava tube (25 m depth) in the Marius Hills region, using SPECfEM.

As can be seen in Figure 5.19–Figure 5.24, for most shapes, a similar conclusion can be made. In fact, after more careful inspection, it is found that all shapes, except the inverted keyhole shape, can be sufficiently represented by a mesh size of 0.025 km. The rounded and rectangular meshes were found to both contain a single location in the region from -0.2 to 0.2 km where 0.3 mGal was crossed, but these are thought to originate from numerical artefacts, as they are single outliers. Due to the fact that for the inverted keyhole, one needs to be one mesh size higher, it is recommended to use a mesh size of 0.0125 km. Still, this recommendation should come with a note that it is derived from a relatively small sample size of seven lava tube cross-sections. Additional analyses should be done on many more lava tube shapes to make a more solid rule of thumb.

In the case of the Marius Hill lava tubes, which are used here, one can still, taking into account the considerations, deduce a mesh size per diameter or radius. However, since the Marius Hills lava tube diameter varies along the lava tube, an exact number is hard to pinpoint. If one errs on the conservative side, then it can be said that the mesh size should be at least $\frac{h}{D_{\min}} = \frac{12.5}{140} \approx 8\%$ of the *minimum* diameter of the lava tube. However, if the lava tube has many small features, which the seven shapes mostly do not have, higher resolutions might be necessary.

It must be noted that the resolution comparison was done for Gruithuisen K as well. However, it was found that, at a depth of 35 m, the $h = 0.075$ km mesh size differed too much from the mesh size of $h = 0.1$ km that it could not generally be considered close enough to the analytical solution. A higher resolution would be necessary. This is also generally found if the 8% rule is applied: $8\% \cdot 200 \text{ meter} = 16 \text{ m}$, which is smaller than the $h = 0.075$ km used before. However, problems in meshing such a large model at such a high resolution meant that the resolution check could not be completely verified. Therefore, the 0.075 km mesh size will be used in future graphs and comparisons as if it were the analytical solution. This is generally valid, as the scale and size of the lava tube decide the most important comparison: at what depth is the lava tube still detectable. Therefore, such results are not influenced. What is influenced is the detection of shapes, as well as the comparison in resolution over depth, as these comparisons rely on high-resolution simulations.

Effect of Depth

To understand what effect having a higher-resolution lava tube has on simulations while varying the depth, a sensitivity analysis can be performed. To do this, the SPECfEM simulations are performed for the Marius Hills and Gruithuisen K lava tubes with varying resolutions. The highest-resolution lava tube for the Marius Hills lava tube was already found to be close enough to the analytical solution

that it can be used for comparisons with lower-resolution lava tubes. For Gruithuisen K, as mentioned before, the highest resolution lava tube was not close enough to the analytical solution. Still, it will be taken as one, and the exact error will be discussed in the text below.

As such, the difference between the highest-resolution solution and the lower-resolution solutions is established for each distance from the centre of the lava tube. Then, if this difference is less than 0.3 mGal, which is the sensor error, the lower-resolution result is indistinguishable from the higher-resolution result when it comes to sensing such an object. In other words, one can obtain the distance of measurement at which using a lower resolution lava tube is good enough. The sensor error, being an extremely optimistic case, can be expanded to the total error due to errors in the correction performed in surveys. This error, calculated in section 5.1, was found to be 5.9 mGal. This accuracy provides an estimate of the actual sensitivity, including errors in model topography and density, but excluding survey errors. If the difference between a lower-resolution lava tube and a higher-resolution lava tube is less than this sensitivity, it is practically identical for purposes of calculations in the Bouguer anomaly.

The graphs for the Marius Hills and Gruithuisen K region can be seen in Figure 5.25 and Figure 5.26, respectively.

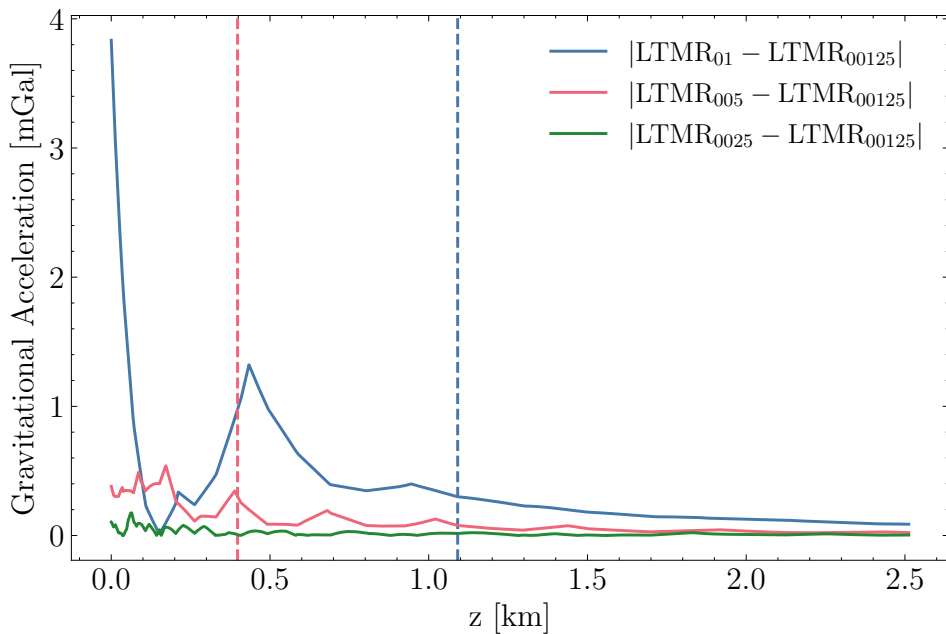


Figure 5.25: The difference between the lower-resolution results of the Marius Hills lava tube simulations and the higher-resolution results, which in this case is assumed to be close enough to an ‘analytical’ solution. The vertical lines indicate where the graph crosses the 0.3 mGal boundary last, with each colour corresponding to the line of the same colour. z represents the depth of the lava tube. Some numerical artefacts can be seen.

As can be seen in Figure 5.25 and Figure 5.26, the higher-resolution simulation ‘hits’ the 0.3 mGal boundary much earlier, while none of the lines reaches a difference of 5.9 mGal.

This means, in a practical sense, that using the total error, a mesh size of 0.1 km is fine for a Marius Hills lava tube at any depth, while for Gruithuisen K, a 0.2 km mesh size can be used for the lava tube at any depth.

Looking at the sensor error only, other conclusions can be made. For both the Marius Hills lava tube and the Gruithuisen K lava tube, it can be seen that the earlier boundary crossing happens around 400 m from the lava tube boundary, while the second one happens around 900 m from the lava tube boundary (conservatively speaking). This means that, in the instance the depth of the lava tube is known to be 400 m or more, one could simulate the MH lava tube with a mesh size of 0.05 km, while if the MH lava tube is at a depth of 900 m, one could use a mesh size of 0.1 km. Still, in practice, the total error should be used.

It must be noted, as previously done, that for the Gruithuisen K lava tube, the highest-resolution mesh

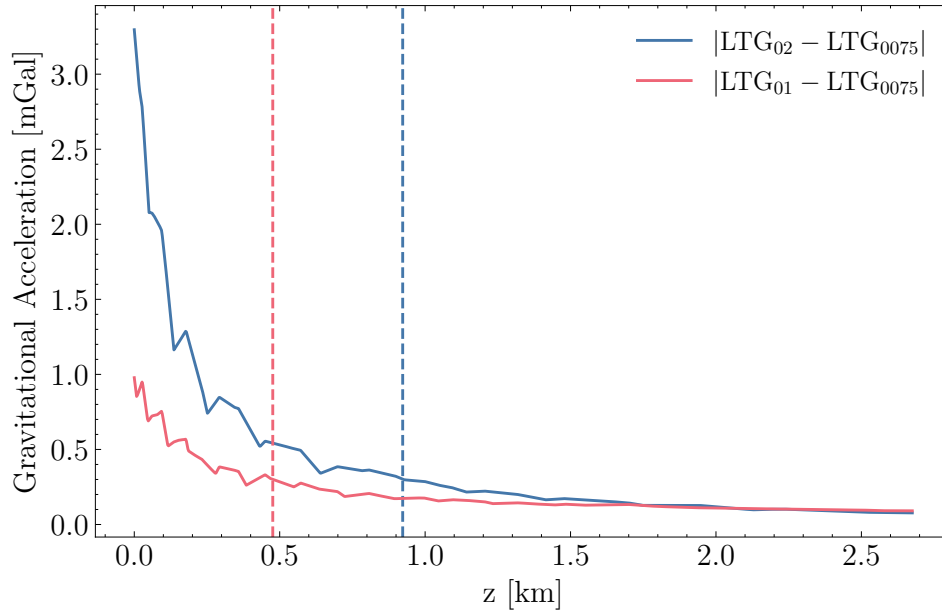


Figure 5.26: The difference between the lower-resolution results of the Gruithuisen K lava tube simulations and the higher-resolution results, which in this case is assumed to be close enough to an ‘analytical’ solution. The vertical lines indicate where the graph crosses the 0.3 mGal boundary last, with each colour corresponding to the line of the same colour. z represents the depth of the lava tube. Some numerical artefacts can be seen.

of $h = 0.075$ km is not enough to completely accurately model the lava tube. As such, Figure 5.26 must be looked at as the comparison for a Gruithuisen K model where variations in the model smaller than 75 m are ignored. If one were able to mesh at a higher resolution, then values close to the surface, and so on to the left in Figure 5.26, would be more accurate. Therefore, the depth values at which the difference of 0.3 mGal is hit are actually further to the right or left than expected. When looking into the resolution graphs at the depths of 500 and 900 m, it is found that the highest resolution mesh sizes differ by 0.2 mGal and 0.1 mGal. As such, using this value as a maximum difference between the analytical solution and the $h = 0.075$ km solution, the lines of hitting 0.3 mGal could potentially be as far as to the right as 1 and 1.5 km, instead of 0.5 and 0.8 km in Figure 5.26.

This resolution problem is however not relevant for the depth variation, given in subsection 5.3.2, as at a depth of approximately 590 m, the difference in gravitational attraction is estimated to differ by less than 0.2 mGal, which would keep the detectability depth at approximately 0.6 km.

Lastly, some numerical artefacts are also visible. They are the random bumps up and down in the graphs. These originate from using coarse mesh sizes and are expected in the SPEC-FEM software. The exact extent of these artefacts is outside the scope of this thesis; however, they are presented as recommendations for future work.

5.3.2. Depth

In this subsection, the depth of the lava tube is varied to quantify the gravity anomaly as a function of distance. In this case, by depth refers to *roof thickness*, instead of the distance from the surface to the centre of the lava tube, unless specified elsewhere.

It is also important to note that the 25 m depth is calculated using the top of the lava tube roof directly below, not the top of the whole model, as the exact depth varies along the lava tube. For Gruithuisen K, because the diameter varies much more, the depth is taken as the distance to the top of the model. Directly below the centre of the Gruithuisen K model, at $x = 0$, $y = 0$, the thickness is approximately 0.26 km larger.

First, the depth variation is performed for the lava tube in the Marius Hills region, while afterwards, it is performed for the Gruithuisen K region lava tube. For Marius Hills, these results can be seen in Figure 5.27.

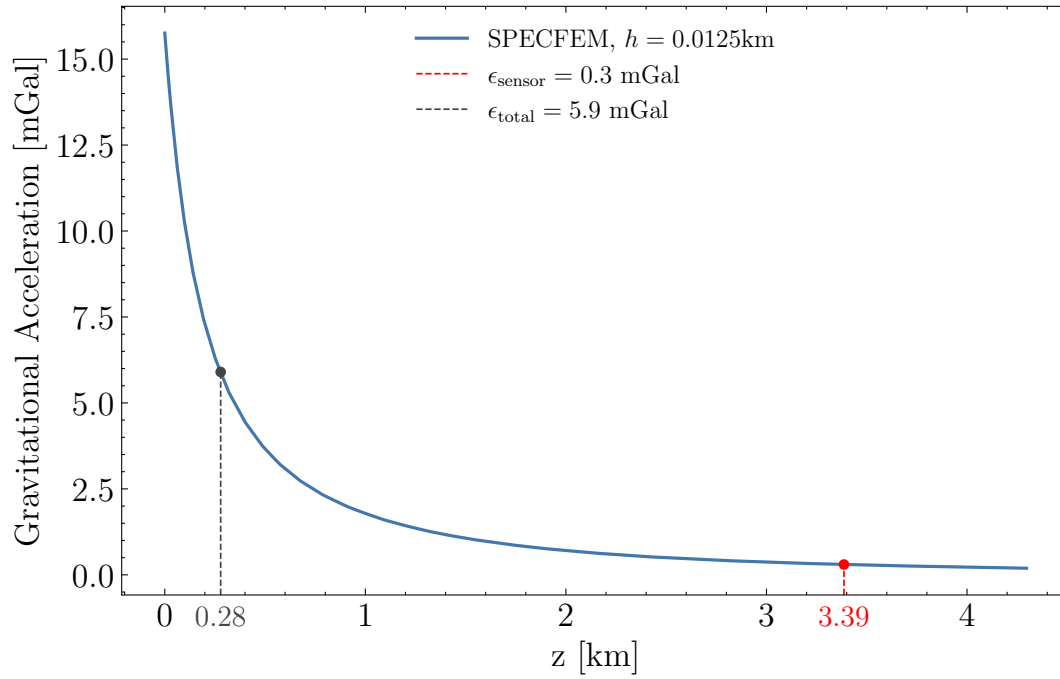


Figure 5.27: The gravitational attraction of a lava tube vs the distance from the lava tube, Marius Hills region. The sensor and total error are calculated in section 5.1, and describe the error of just the sensor, and the sensor including errors in height and density of the DTM, respectively.

In this graph, it is quite clear that the gravitational attraction rapidly decreases, after which a long tail is present. Looking at the analytical equation of an infinite buried cylinder, given in Equation 2.10, in subsection 2.4.1, one expects this relationship to be an inverse one. This was checked using the SciPy function 'curve_fit' to fit a curve to the data and to obtain the coefficient of determination R^2 . It was found that for a general equation,

$$y = \frac{a}{(x - c)} + b,$$

Where a, b and c are constants, that $R^2 = 0.9987$. This is quite a high coefficient of determination, and therefore it is quite certain that the relationship to depth is an inverse one, aligning with the analytical version of a buried cylinder.

At a distance of approximately 3.39 km, the gravity anomaly crosses the sensitivity point of 0.3 mGal. Therefore, considering only the sensor, the maximum detectable depth of a lava tube of such size is at a depth of approximately 3.39 km.

This depth exceeds the expected roof thickness of lunar lava tubes. However, the sensor error does not include various other errors relevant to performing a survey and calculating the corrections to obtain the Bouguer anomaly. As was seen in section 5.1, including errors in the corrections increases the detection threshold to 5.9 mGal, which would decrease the depth to approximately 280 m. Still, the error does not include errors in the performance of a potential survey, which could increase the error further. As such, it can be concluded that lava tubes with geometry similar to the MH lava tube can be detected up to roof thicknesses of approximately 0.28 km.

A further understanding of the effect of depth can be gained by plotting 2D field plots of measurements on the *surface* at relevant distances from lava tubes. For this, plots of the round lava tube at 0.025 km resolution were plotted at a 25 m depth, a 200 m depth, which are both very possible depths according to the literature (see subsection 2.1.2), a 1 km depth, and a 2.5 km depth. The 1 km depth is realistic, while the 2.5 km one is not. The results for these depths are, in practice, however, not as useful, because their depth would make them much harder to reach for both humans and robots. They do provide some insight into what measurements can be expected if a lava tube were to exist at such a depth. Firstly, a

DTM only 2D field is shown, with the results in a square of 10×10 km. Note that the actual results were for a 30×30 km DTM, as this provided the least error. For this, two graphs are presented: One with the full range of values present in the area and one with a more local range of values, which provides higher contrast and accentuates the lava tube more. Afterwards, the 2D field plots, which include lava tubes, are shown. In all graphs, the colours are discretised in bins of 0.6 mGal wide, which is the RMS error of the sensor (± 0.3). The 2D field plots can be seen in Figures 5.28–5.33.

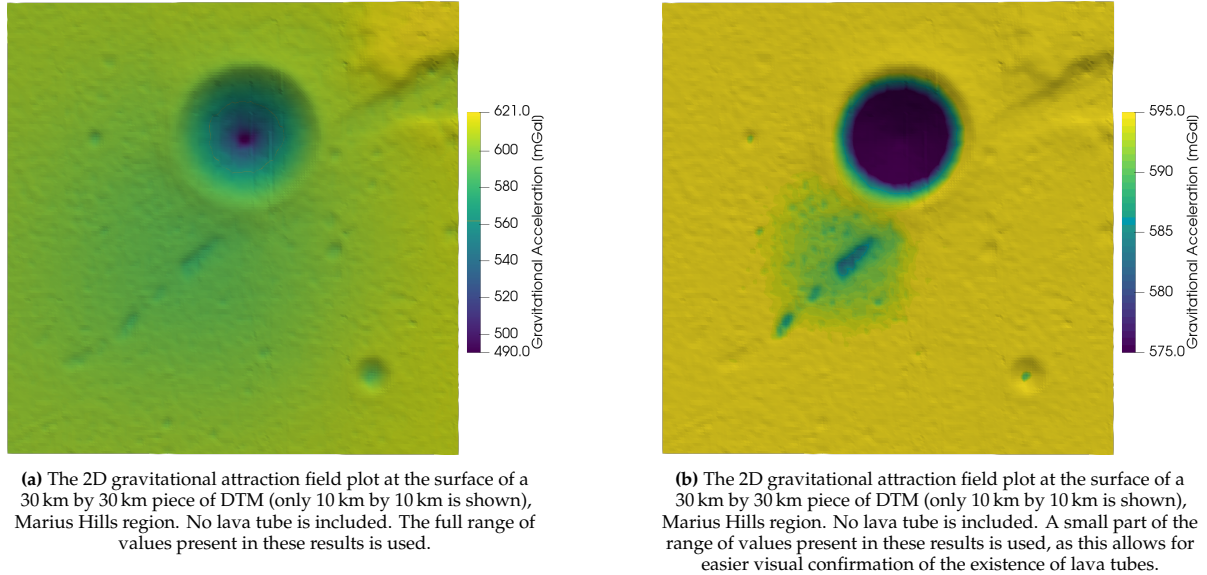


Figure 5.28: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region. No lava tube is included.

Figure 5.28 shows the gravitational attraction of the DTM without a lava tube. The left figure displays the full range of values, while the right figure uses a narrower colour range, which provides a higher contrast and will aid in the identification of the lunar lava tube gravity anomaly.

When a lava tube is included at different heights, the effect can be readily seen, as is visible in Figures 5.29–5.32.

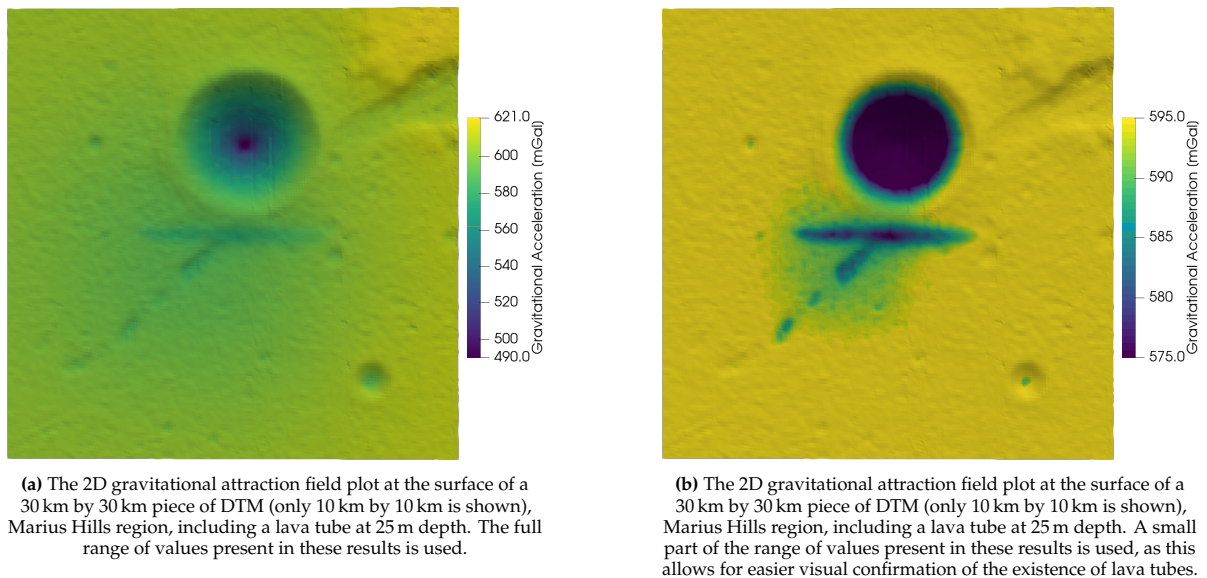


Figure 5.29: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a rounded lava tube at 25 m depth.

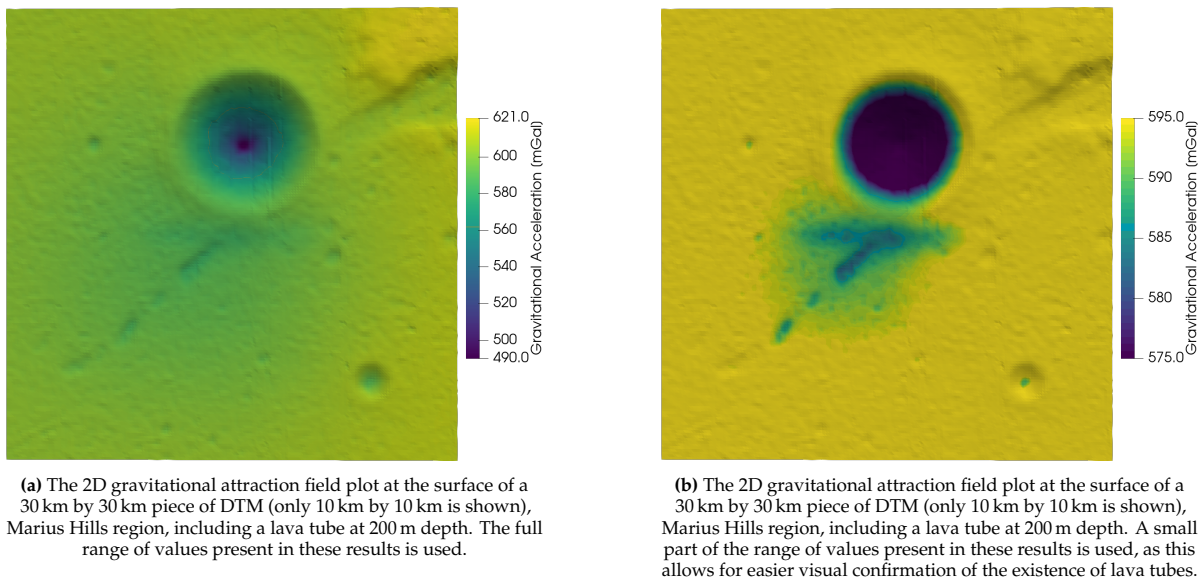


Figure 5.30: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a rounded lava tube at 200 m depth.

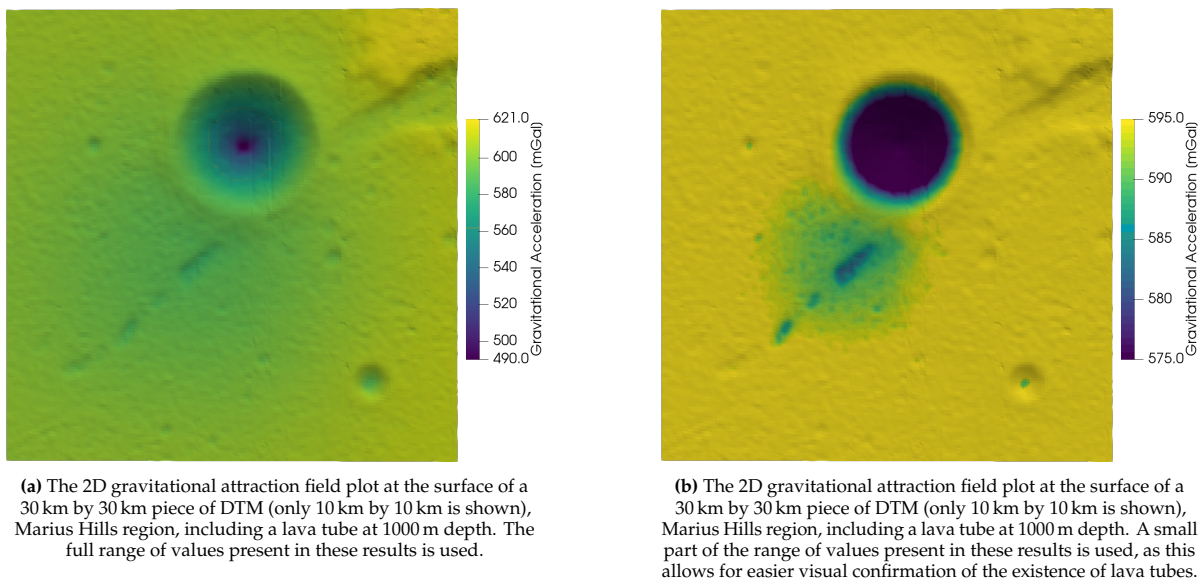


Figure 5.31: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a rounded lava tube at 1000 m depth.

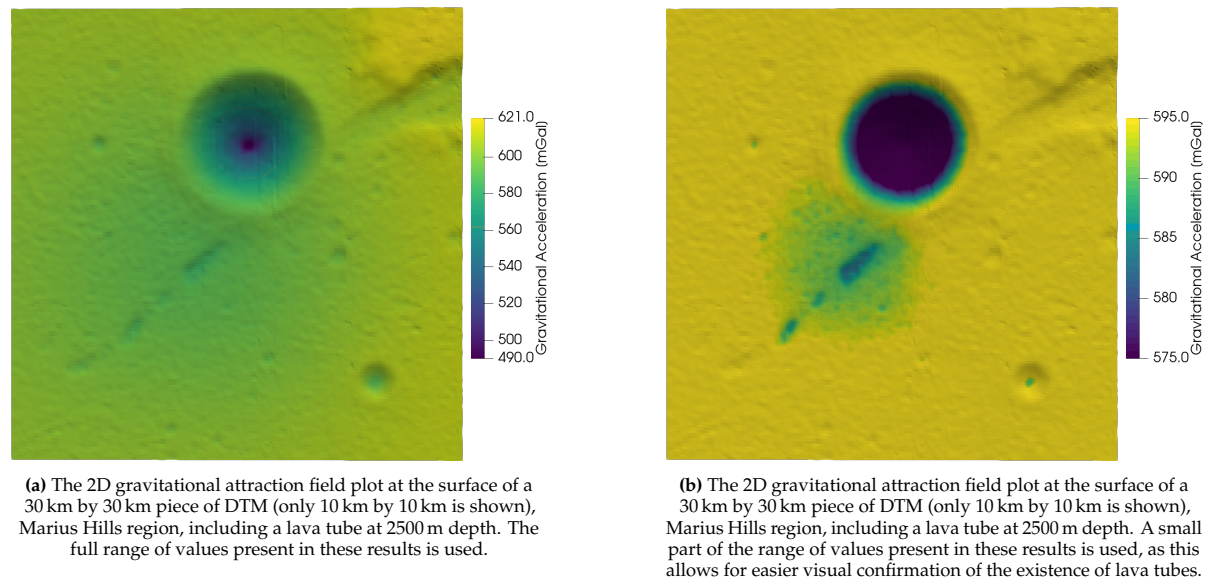


Figure 5.32: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a rounded lava tube at 2500 m depth.

As can be seen in Figures 5.29–5.32, the lava tube is clearly visible in the 25 m and 200 m cases, which are the expected depths of lava tubes, while in the 1000 m case it is unclear if the gravity anomaly is still distinguishable from background noise.

Currently, it was stated that the maximum depth at which a lava tube could be detected was 3.39 km. Checking this for the 2.5 km case, this is found to be true: A closer look at Figure 5.28b and Figure 5.32b reveals that the overall area near the lava tube takes a shade darker than in the DTM only case (it is quite difficult but possible). In technical terms, although it is correct to say the lava tube is *detectable*, it is quite dubious to state that it is *recognisable*. In practice, such a small change would indicate any subsurface structure to a survey team, such as a variation in density, not necessarily a lava tube.

For the 1000 m depth case, a similar conclusion can be made. Although the strength of the signal of the lava tube is much higher (relatively) than in the 2500 m case, the exact shape of the 2D field map shows that in practice it would be hard to distinguish the subsurface anomaly from any random variation in density, as can be seen in Figure 5.33.

Figure 5.33 illustrates that although there is definitely enough to be sensed by the sensor, the shape of what is sensed is impossible to recognise as a lunar lava tube anomaly. Any change in density, whether it be due to a cavity, a different material, or a change in porosity (which could result in a change of density), could result in such a gravity anomaly, if it were big enough. Next to that, including other errors, recognising lava tubes at such a depth would be hard and possibly not realistic anymore.

For Gruithuisen K, because the lava tube varies noticeably in roof thickness along its length, two graphs were made. One shows the depth variation at $x = 0, y = 0$ m, where the Gruithuisen K lava tube is narrower (width and height of approximately 520 and 200 m). The other shows the depth variation graph at a wider part of the lava tube (width and height of approximately 1000 and 550 m), at $x = 7000, y = 3000$ m. The depth variation graphs are shown in Figure 5.34 and Figure 5.35.

As can be seen in Figure 5.34 and Figure 5.35, a similar pattern is visible for the Gruithuisen K lava tube as for the Marius Hills one, although the distance at which the lava tube is still discernible is much larger. This is because the lava tube is much wider. The simulation performed for the Gruithuisen K lava tube did not include distances large enough for the lava tube to induce a variation as small as 0.3 mGal. However, the total error, again, calculated in section 5.1, does provide an estimate of the depth at which the Gruithuisen K lava tube is realistically detectable. It is found that roof thicknesses up to approximately 590 m allow for lava tubes to be discernible from other errors for the narrow part of the lava tube. As such, lava tubes with the geometry of the Gruithuisen K lava tube can be detected up to roof thicknesses of about 0.59 km at the thin part. For the wider part, this roof thickness is much

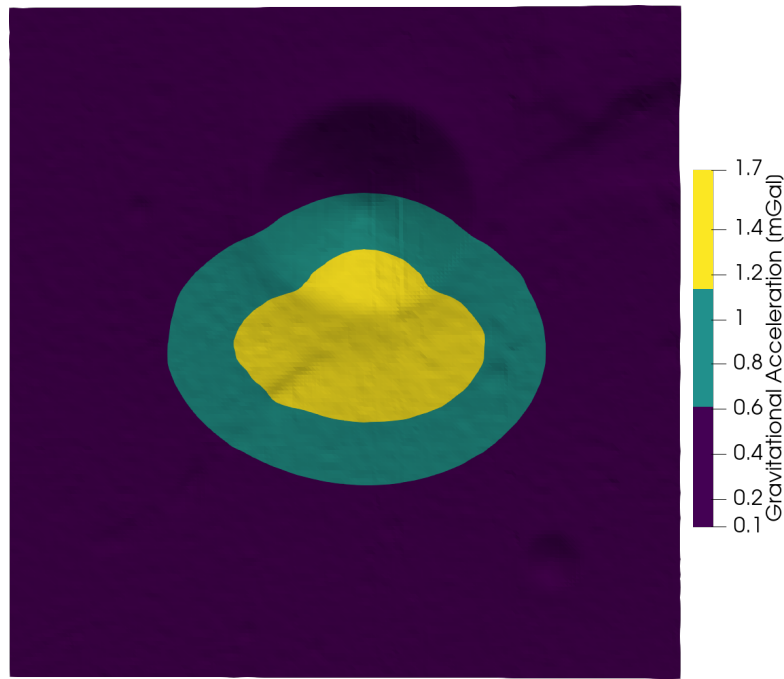


Figure 5.33: The 2D gravitational attraction (magnitude) field plot of just the lava tube, at 1000 m depth, plotted such that one can see the effect at the surface of the DTM. Note that only three bins are used to approximate the 0.6 mGal accuracy bin that the sensor used would provide. Additionally, note that the value in the graph is a magnitude value.

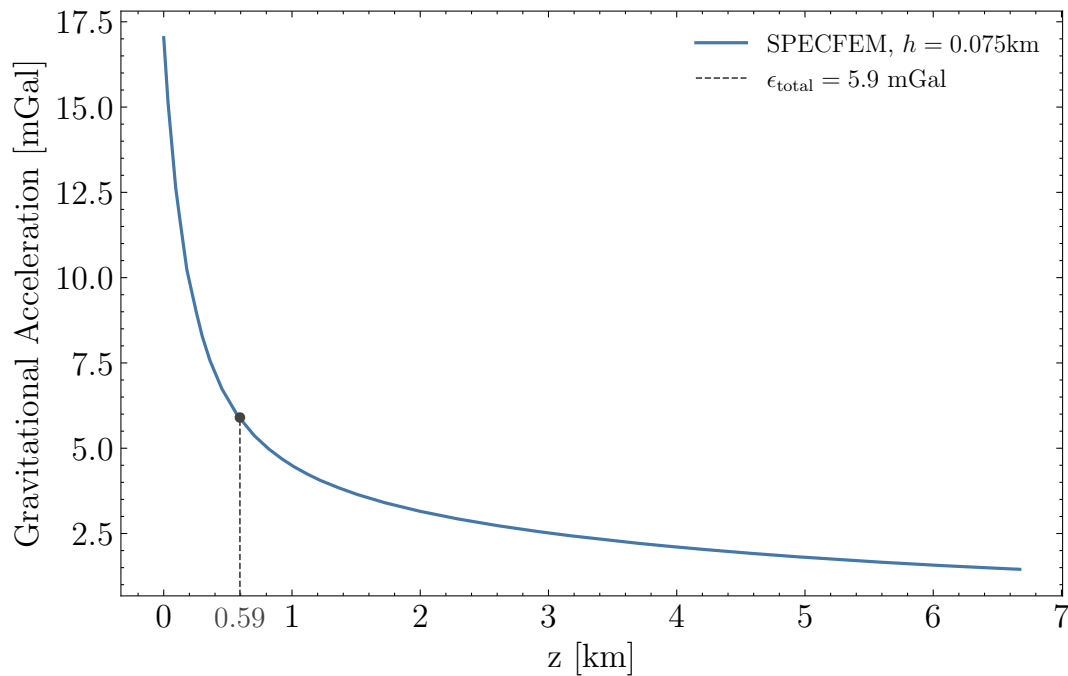


Figure 5.34: The gravitational attraction of a lava tube vs the distance from the lava tube, Gruithuisen K region ($x = 0$, $y = 0$ m). The sensor error value is not given, as the simulation did not extend far enough to reach this value.

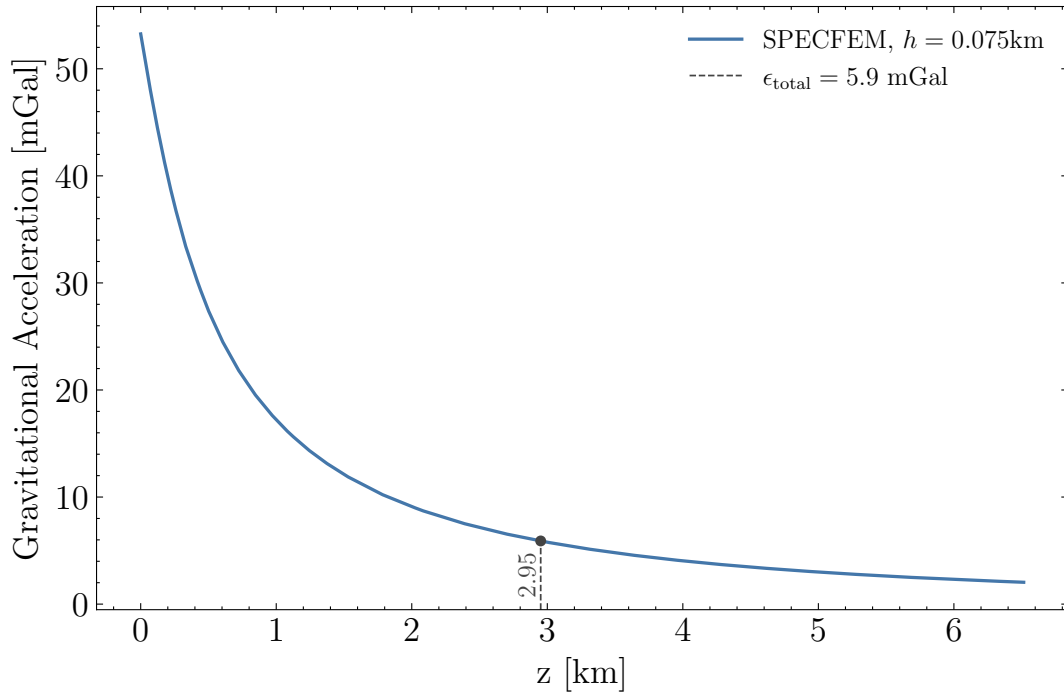


Figure 5.35: The gravitational attraction of a lava tube vs the distance from the lava tube, Gruithuisen K region ($x = 7000$, $y = 3000$ m). The sensor error value is not given, as the simulation did not extend far enough to reach this value.

larger, at about 2.95 km. As can be seen, a large variation is present, but a conservative estimate would place the detectability at the value of 0.59 km.

This is also more visible in the 2D field plots, which clearly show the size and signal of the lava tube. Note that in these plots, the Marius Hills DTM had to be used due to lack of time in simulating the Gruithuisen K DTM. Still, since the areas are both in volcanic areas in the Mare, and since the same density is used for both, the sensitivity analysis is not impacted.

First, the whole 30 km Marius Hills DTM for which a simulation was performed can be seen in Figure 5.36.

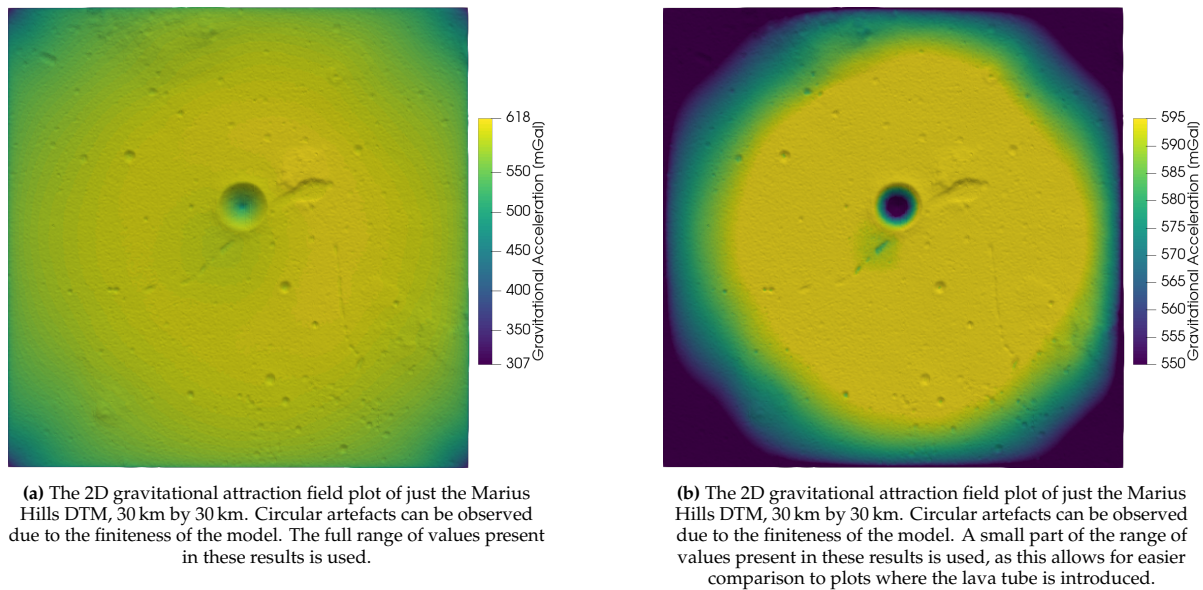


Figure 5.36: The 2D gravitational attraction field plots of just the Marius Hills DTM, 30 km by 30 km. The left plot uses the full range of values, while the right a smaller subset, which allows for a high contrast.

When the entire lava tube model is included, as is done in Figure 5.37, the outline of the lava tube can be readily seen.

Note that the exact depth of the lava tube is hard to determine. In the following cases, a depth is mentioned, but this is the depth to the highest z-point in the lava tube model. Because the lava tube model varies in size and rises and falls, the roof thickness at $x = 0, y = 0$ is actually just under 260 m. Therefore, exact values connected to the depth of the lava tube should be treated with care. Instead, the approximate relationship between the depth and the gravitational acceleration is to be discussed.

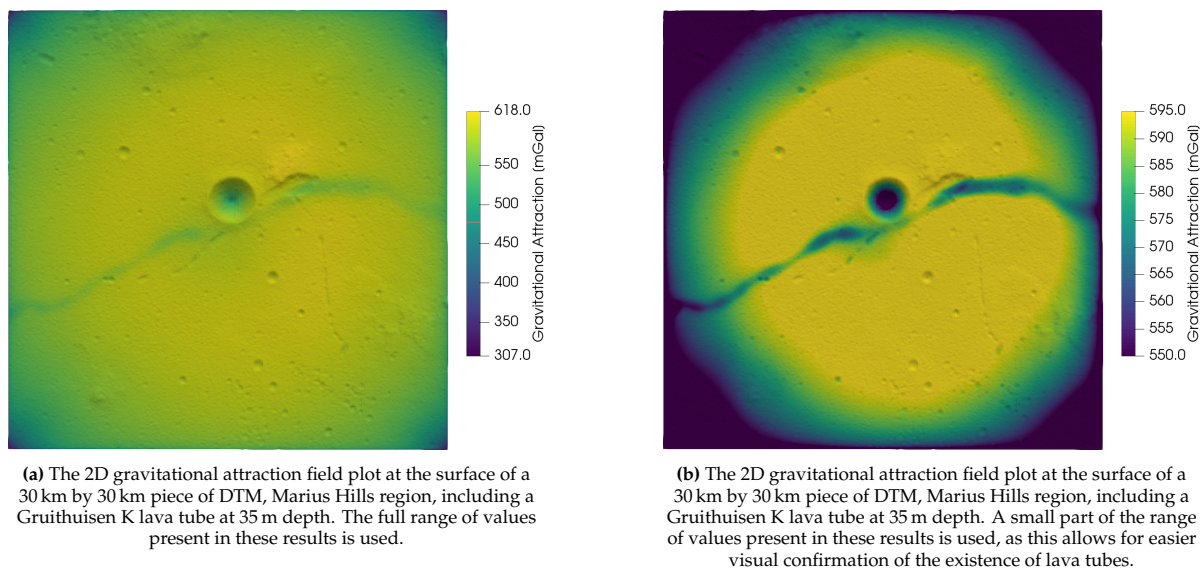


Figure 5.37: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM, Marius Hills region, including a Gruithuisen K lava tube at a depth of 35 m at minimum, and 295 m at the centre.

As can be seen, the lava tube is quite visible at shallow depths. There is no mistaking that the object is a lava tube, and although multiple measurements will always be necessary to confirm such facts in practice, with enough measurements, the outcome of a lava tube should prove relatively easy.

To further understand the relationship with depth, other depths are also plotted. However, to provide a fair comparison, only a 10 by 10 km square is shown, as this will limit the data to a region where there will be another 10 km on each side. This is necessary because the DTM should be at least one lava tube wider on each side. Therefore, although the lava tube extends further than the 10 by 10 km, only the results in this 10 by 10 km square are of interest.

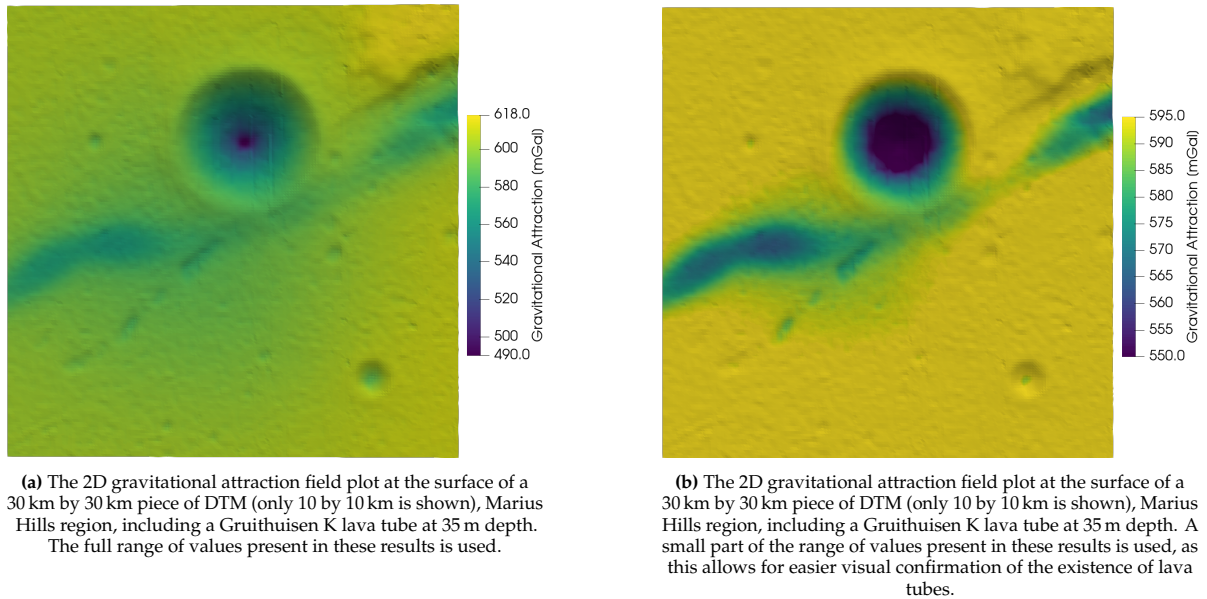


Figure 5.38: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 by 10 km is shown), Marius Hills region, including a Gruithuisen K lava tube at a depth of 35 m at minimum, and 295 m at the centre.

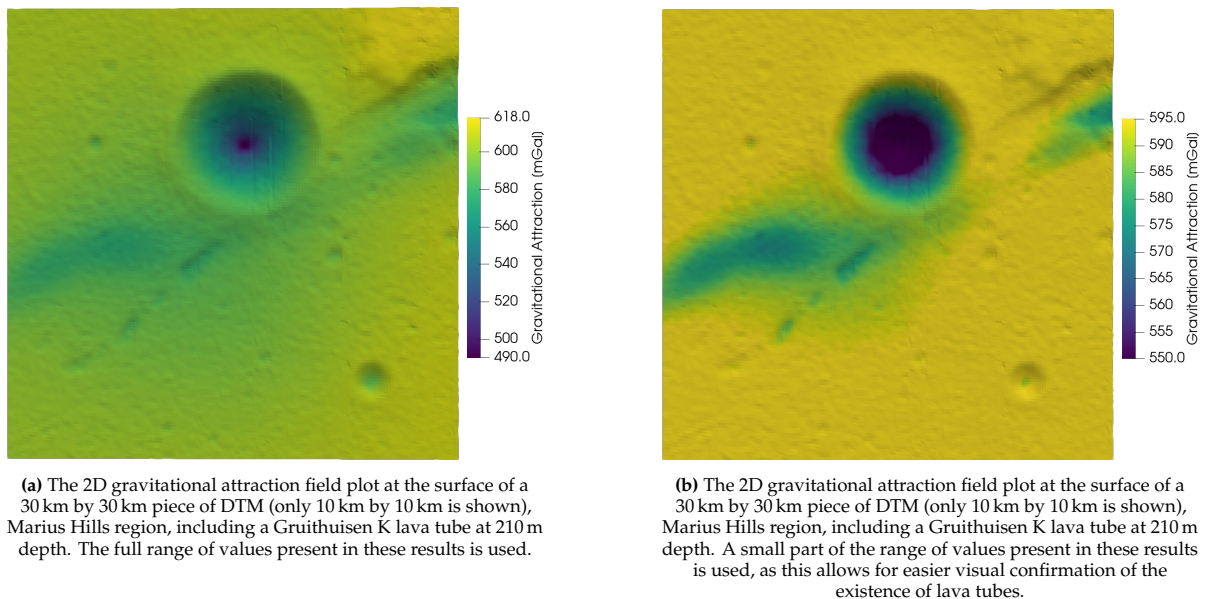


Figure 5.39: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a Gruithuisen K lava tube at a depth of 210 m at minimum, and 470 m at the centre.

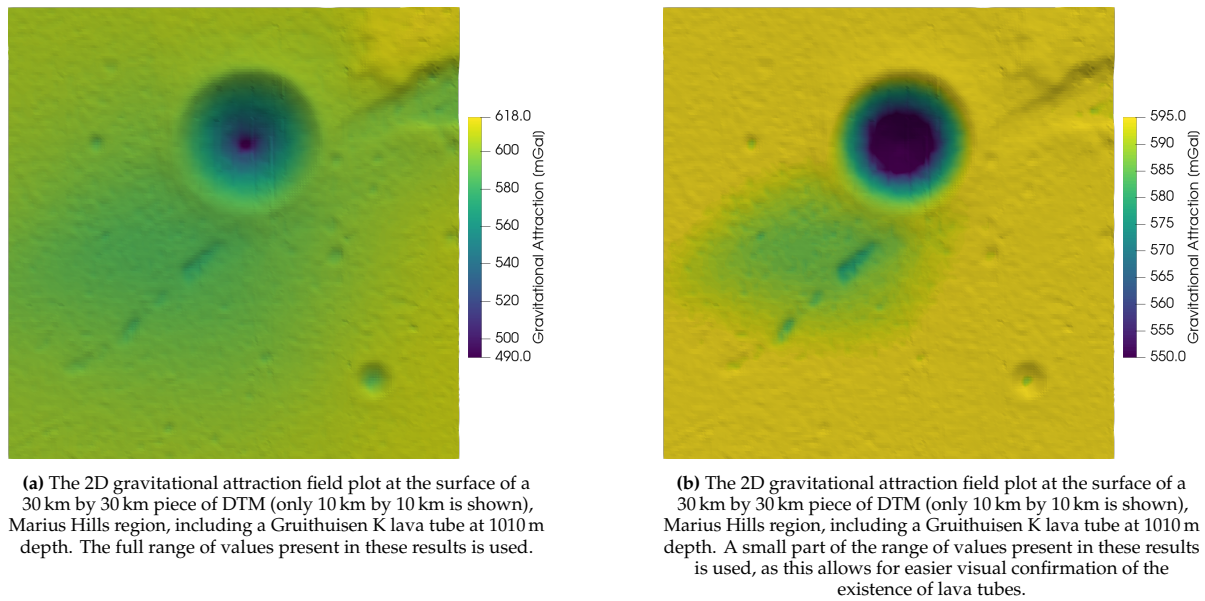


Figure 5.40: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a Gruithuisen K lava tube at a depth of 1010 m at minimum, and 1270 m at the centre.

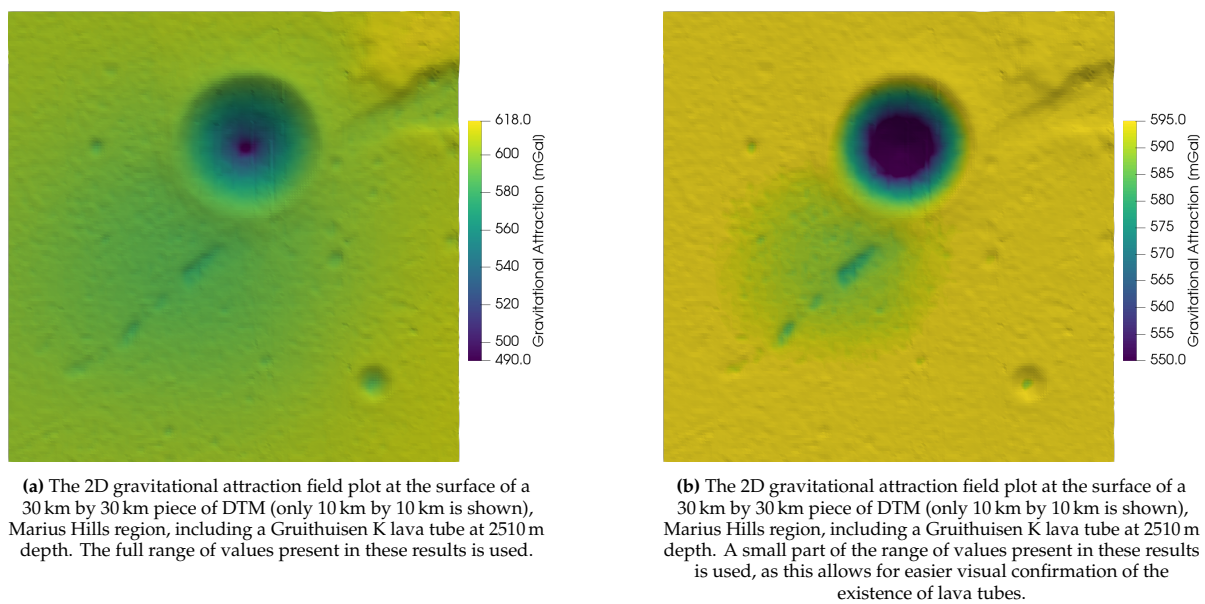


Figure 5.41: The 2D gravitational attraction field plots of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a Gruithuisen K lava tube at a depth of 2510 m at minimum, and 2770 m at the centre.

As can be seen in Figures 5.39–5.41, the lava tube is easily visible at depths below 200 meters. When the lava tube is more than 1000 m away, a patch of darkness can still be seen, but the existence of a lava tube specifically is hard to understand. Therefore, this lava tube is still detectable, but not recognisable anymore.

To make this clearer, a lava tube-only graph depicting a depth of 2510 m is shown in Figure 5.42.

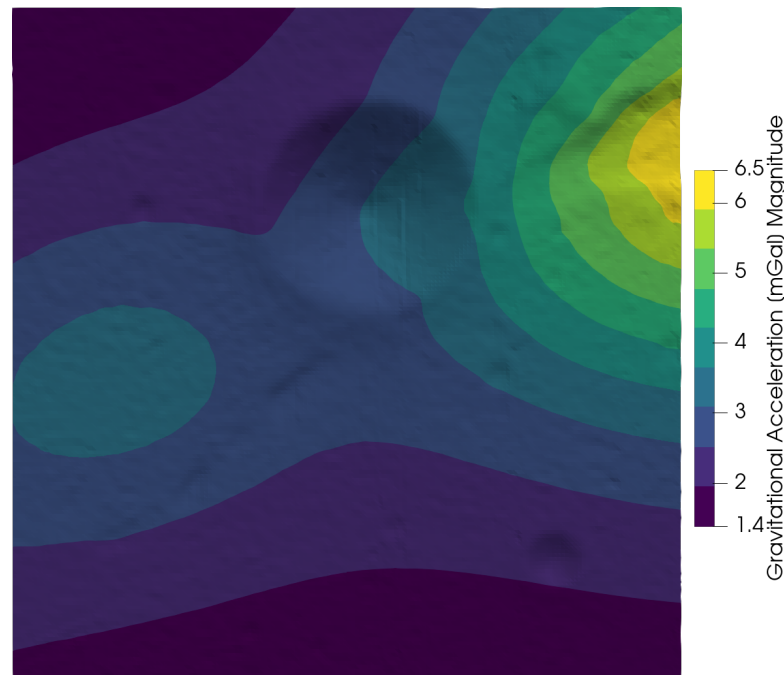


Figure 5.42: The 2D gravitational attraction field plot of the Gruithuisen K lava tube at 2510 m (2770 m) depth, plotted on a Marius Hills DTM of 10 by 10 km. Only the results of the lava tube are present, in magnitude. Note that the value in the graph is a magnitude value.

As can be seen in Figure 5.42, the lava tube still gives a signal that could be detected by a lava tube, especially in the top right area of the graph. As such, it can be concluded that lava tubes with sizes similar to Gruithuisen K should be detectable up to a depth of even more than 2500 m. Since lava tubes are expected to be closer to the surface, it can be concluded that any lava tube of such size is detectable. Still, the anomaly's recognizability as a lava tube is not guaranteed. Because the anomaly looks like a patch, and because it is so wide, it would be hard to recognise this anomaly as a lava tube. It would be very easy to argue that such anomalies could be density anomalies. Therefore, although it is possible to detect the Gruithuisen K lava tube at depths of even 2500 m, it is useful in practice because the shape of the lava tube is not immediately recognisable.

5.3.3. Scale

In this subsection, the *scale* (size) of the lava tubes is varied to understand how this affects the gravity anomaly. First, the effect is shown for the lava tube in the Marius Hills region, while afterwards, the Gruithuisen K region lava tube is used.

As was specified in chapter 4, the size is increased and decreased by 10%, after which the gravity anomaly is calculated for both, at a depth of 25 m and 200 m. The result can be seen in Figure 5.43 and Figure 5.44.

As can be seen, and as expected, the larger the lava tube is, the bigger the anomaly it creates. It can be seen that the increase seems to be regular, occurring at each point along the curve. To understand exactly how this varies, the ratio between lines can be plotted. This can be seen in Figure 5.45.

As can be seen in Figure 5.45, the ratio is constant for both lines, although some noise is present. As can be seen, the factor by which the gravity anomaly increases is not represented by a linear relationship. This is expected, as when the diameter is increased by 1.1, the area of the lava tube is increased by 1.1^2 . As the length of the lava tube does not increase, we expect the total increase in gravitational attraction to be $1.1^2 = 1.21$. This is approximately the value the line in Figure 5.45 shows. Therefore, a quadratic relationship can be established between the scale of the lava tube and the gravity anomaly. This is also confirmed by the analytic solution for an infinite buried lava tube, given in Equation 2.10, where the gravitational acceleration and the radius of the cylinder have a quadratic relationship.

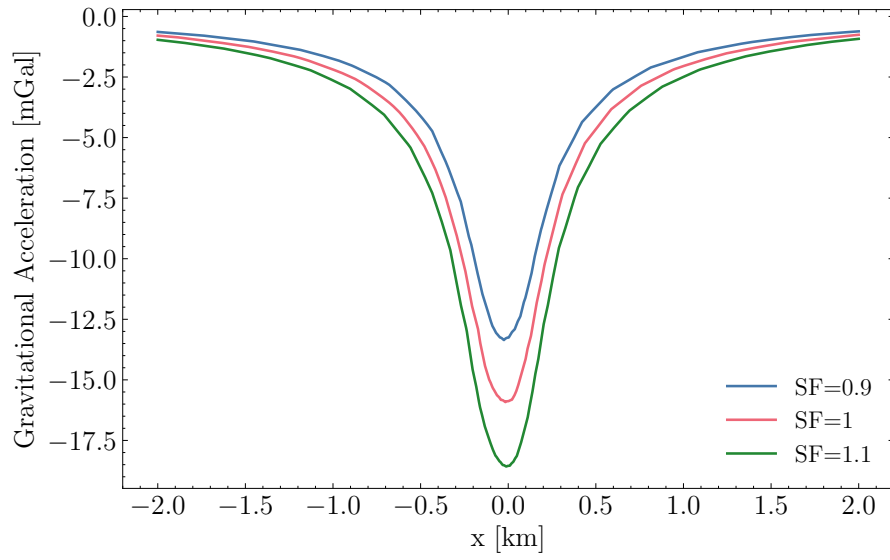


Figure 5.43: Comparison of the gravity anomaly of a *rounded* lava tube at a depth of 25 m when varying the scale of the lava tube, Marius Hills region. Mesh size is 0.025 km.

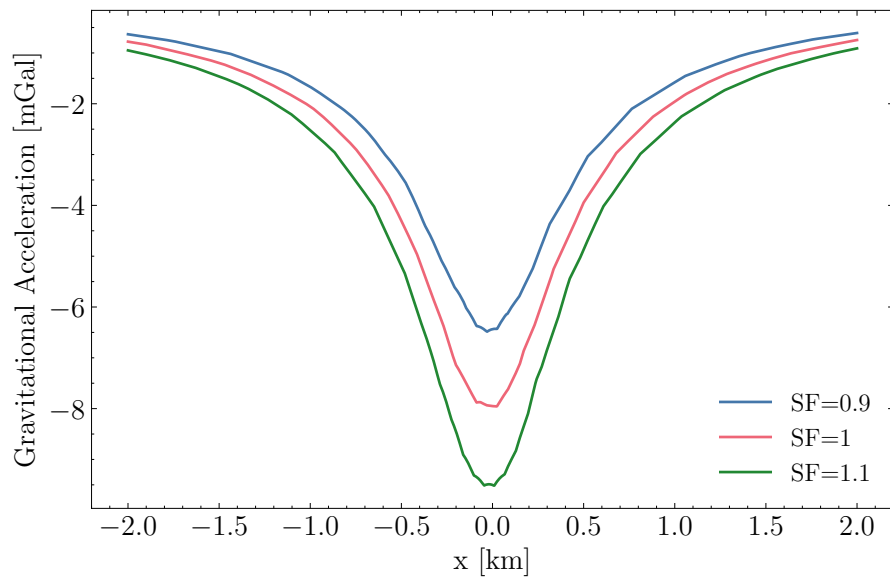


Figure 5.44: Comparison of the gravity anomaly of a *rounded* lava tube at a depth of 200 m when varying the scale of the lava tube, Marius Hills region. Mesh size is 0.025 km.

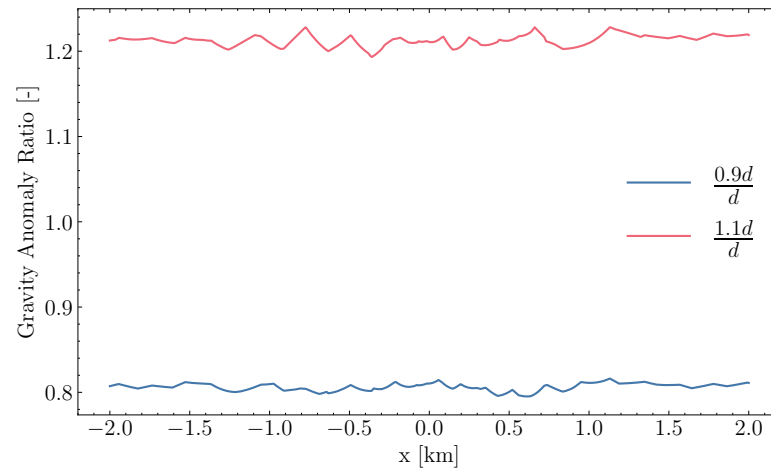


Figure 5.45: Ratio of gravity anomalies between the smaller- and larger-scale lava tubes vs the regular lava tube, at 900 m depth in the Marius Hills region. Numerical artefacts result in noise in the data.

For Gruithuisen K, a similar relation can be found.

To understand the effect of the scale in a broader sense, the 2D field plots can also be made. These are shown in Figure 5.46.

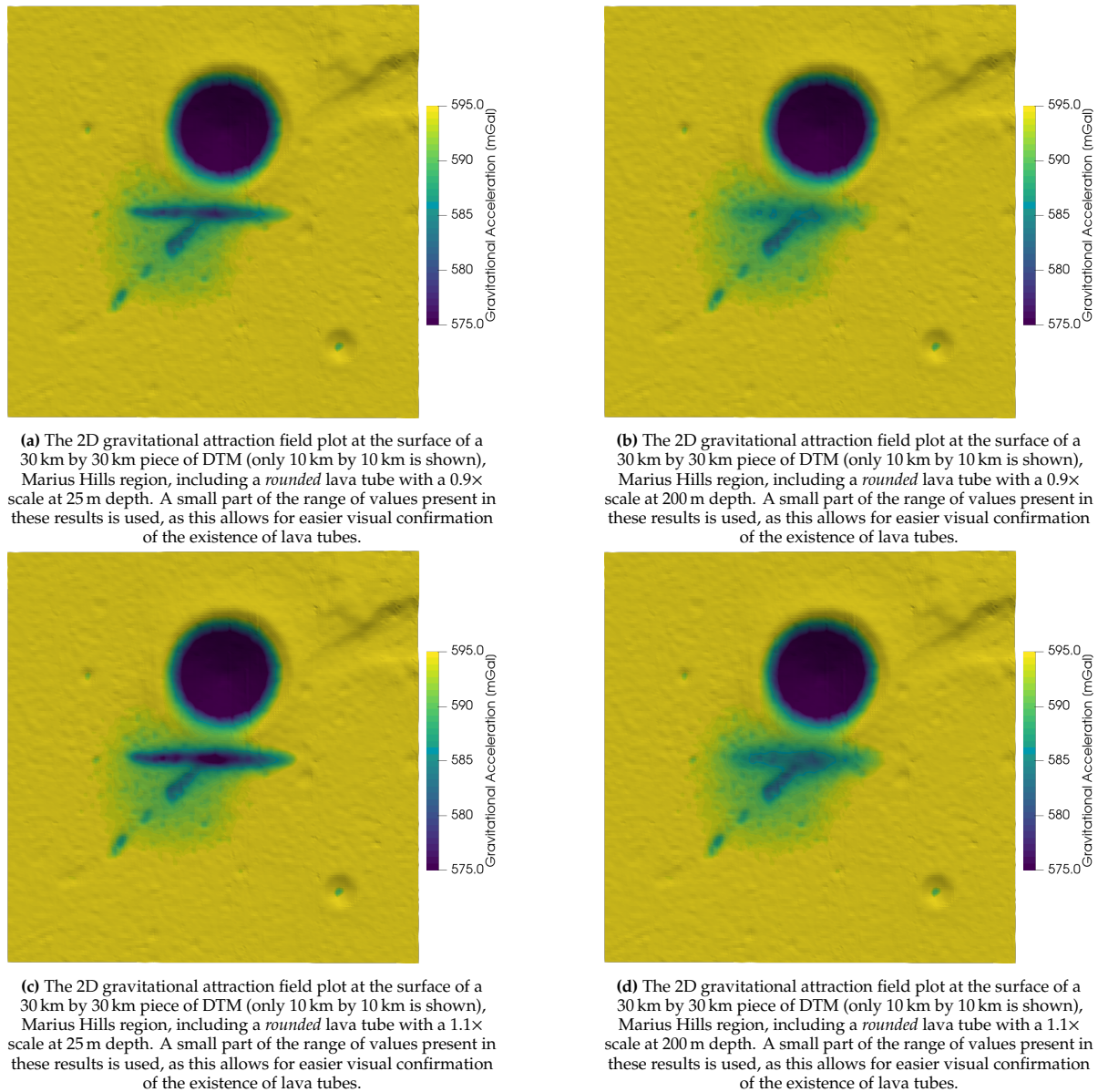


Figure 5.46: The 2D field plots for a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown) at 25 m and 200 m depth for the 0.9 $\times$ and 1.1 $\times$ scale.

As can be seen, a clear difference is present between the 0.9 and 1.1 cases. A stronger anomaly is present for the 1.1 case, and a smaller one for the 0.9 case.

One important note to mention is that in the case of a survey, one might be able to sense a certain value for the gravity anomaly, but not immediately be able to discern either the height or the scale. For example, a lava tube with a scale of 0.9 times the nominal could, at its maximum, easily be mistaken for a lava tube that has the nominal size but is deeper, at 62 m. This situation can be seen in Figure 5.47.

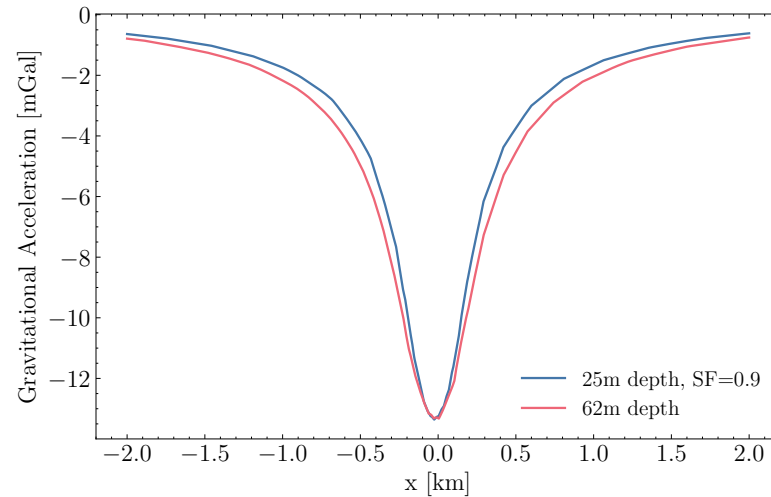


Figure 5.47: The gravity anomaly for a rounded lava tube at 25 m depth, with a scale of 0.9 times the nominal, as well as a nominal rounded lava tube with a depth of 62 m. Both lava tubes are Marius Hills lava tubes. Note the roughly equal values at the centre of the gravity signal.

As can be seen in Figure 5.47, when measuring right above the lava tubes, it is unclear if the lava tube is small in scale or present at a larger depth. More measurements are then necessary to constrain the anomaly to one of the situations (or another). As can be seen, a difference can be seen in measurements further away from the lava tube in the horizontal direction. Such measurements would then allow one to distinguish smaller lava tubes from deeper ones, if using the sensor error as the reference. Using the total error as the reference would result in these two lava tubes being possibly indistinguishable with measurements only from the surface. The most optimal way to then verify which lava tube is which is to measure at various heights, since the scale and depth vary quadratically and with the inverse, respectively. However, depending on the exact circumstances, this might not be possible either.

Because the results for the scale sensitivity analysis for the Gruithuisen K lava tube are similar, the plots have been omitted.

5.3.4. Shape

In this subsection, the *shape* of the lava tubes is varied to understand how this affects the lava tube gravity anomaly. First, the effect is shown for the lava tube in the Marius Hills region, while afterwards, the Gruithuisen K region lava tube is used. Note that for Figures 5.48–5.50, the distance to the lava tube is to the centre of the lava tube, not the roof of the lava tube.

As can be seen in Figures 5.48–5.50, the shape of the lava tube introduces some interesting curves. Figure 5.48 and Figure 5.49 show the curves at a distance of 900 m, where it is quite obvious that most lava tubes are very close to each other, with all of them being within 0.1 mGal at the minimum. This results in the fact that differences in shape are not possible to be distinguished at such distances.

However, at shallower depths near the lava tube, a different gravity signal profile is observed. As can be seen in Figure 5.50, placing the centre of each lava tube 150 m away from the observation point, we find that certain shapes incur much more different gravity anomalies. Especially interesting is the inverted keyhole, which has a much larger absolute gravitational attraction. Next to that, the elongated lava tube exhibits the opposite behaviour, appearing much flatter.

Many of these differences seen in the figures can be explained by looking at where the observation point is in relation to the lava tube. For the inverted keyhole, it can be seen that, because it is so tall, it can create a much larger gravitational attraction. Its depth (or roof thickness) to the top of the lava tube is much less, and therefore it can be seen as an *unfair* comparison. Therefore, a second graph must be visualised where the depth of each lava tube to its roof is equal.

These results, where all the ‘roofs’ are located at the same height, can be seen in Figure 5.51.

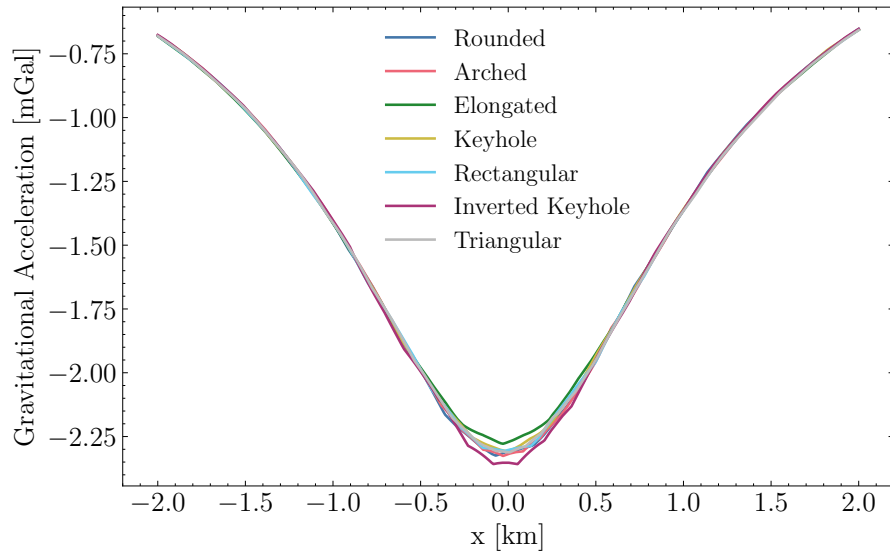


Figure 5.48: Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube. The distance between the observation point and the centre of the lava tube is 900 m.

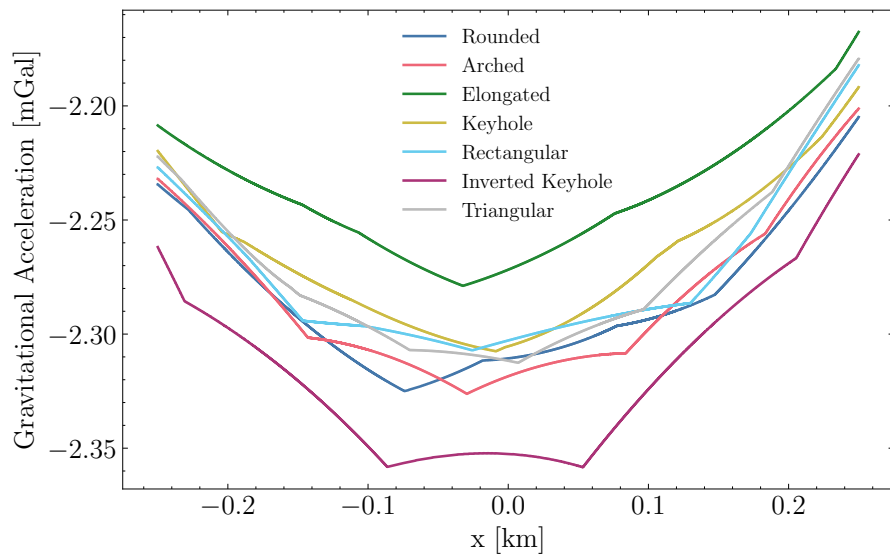


Figure 5.49: Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, zoomed in. The distance between the observation point and the centre of the lava tube is 900 m.

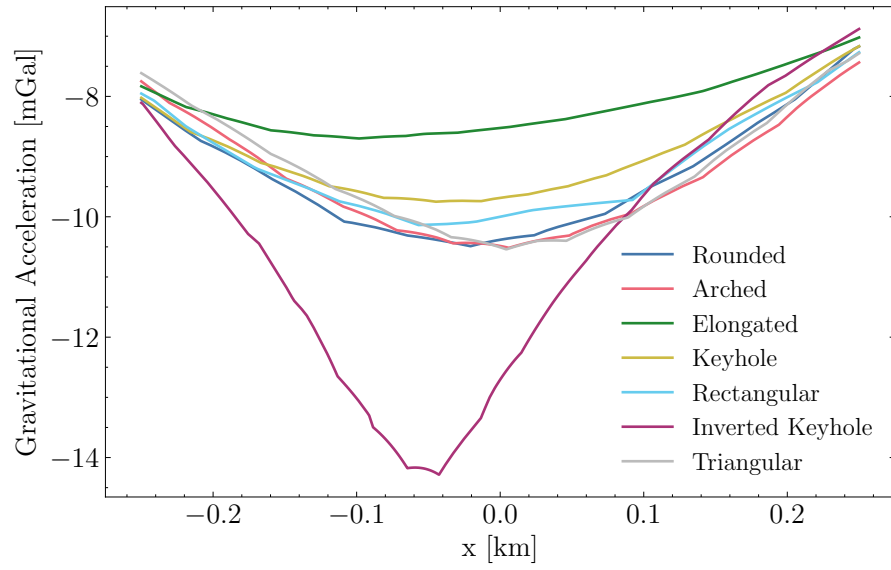


Figure 5.50: Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, at a closer distance of 150 m from the centre of the lava tube to the observation point, zoomed in.

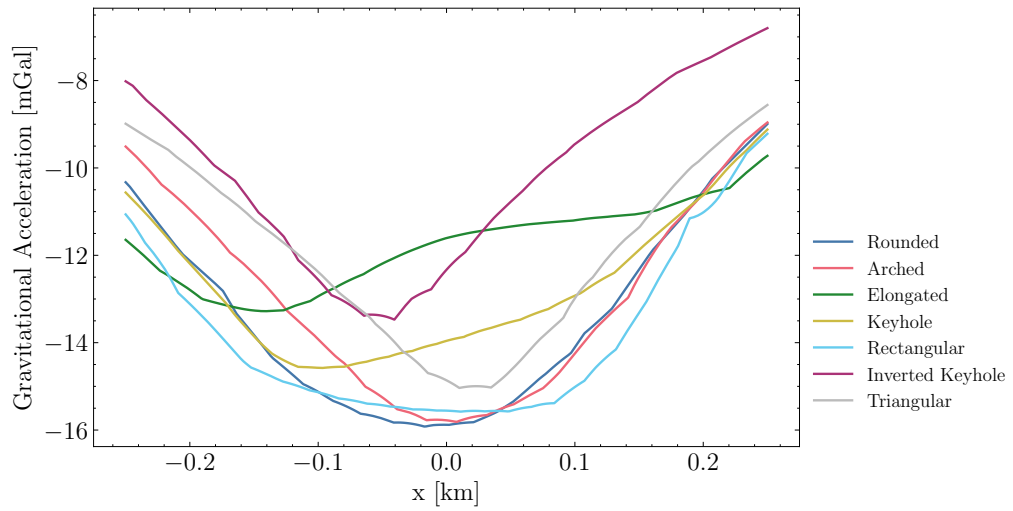


Figure 5.51: Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, at a distance of 25 m from the top of the lava tube to the observation point, zoomed in.

As can be seen, a clear difference is present, and the curves lie closer to each other. A clearer comparison can be made in terms of the shape of the graphs that are seen in Figure 5.51.

A number of generalities can be noticed. First, the ‘arched’ group (which consists of the arched, rounded, and triangular shapes) are seen to exhibit similar graphs. They do differ in magnitude; however, this is to be expected due to the observation points only being 25 m away in vertical height.

To verify this, two graphs with observation points 200 m and 500 m away are also shown. These can be seen in Figure 5.52 and Figure 5.53.

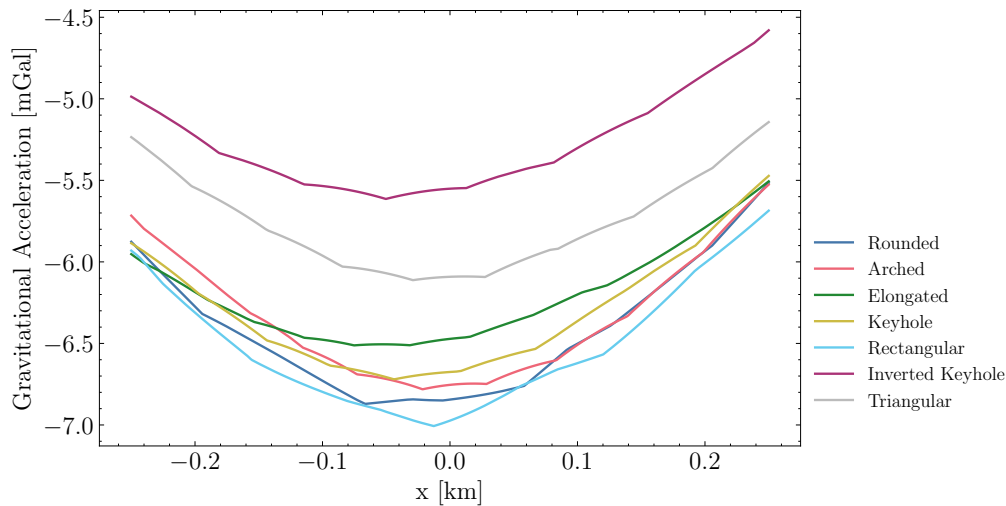


Figure 5.52: Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, at a distance of 200 m from the top of the lava tube to the observation point, zoomed in.

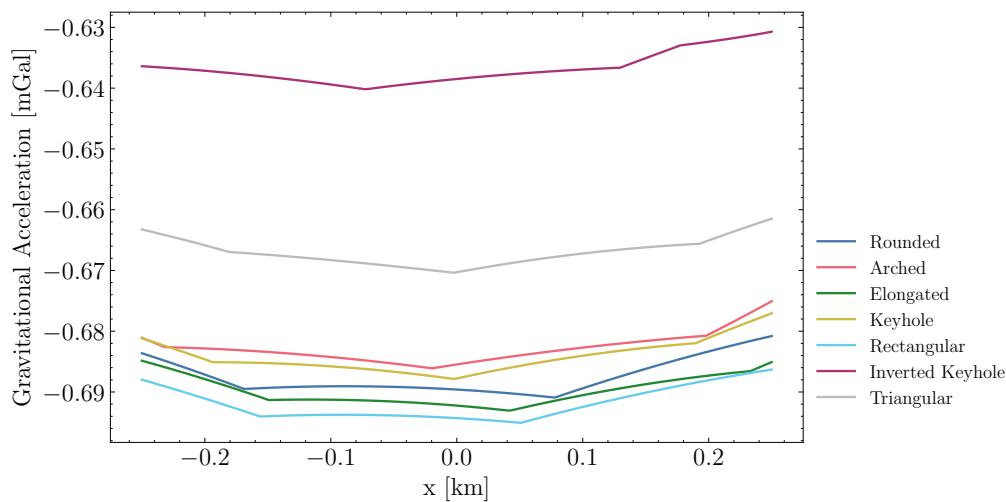


Figure 5.53: Comparison of the gravity anomaly of a lava tube when varying the shape of the lava tube, at a distance of 500 m from the top of the lava tube to the observation point, zoomed in.

As can be seen, except for the triangular and inverted keyhole shapes, all other shapes are similar to each other, especially at a distance of 500 m. The ‘arched’ group all lie close to each other, also in magnitude. Still interesting is the fact that in the 500 m case, the triangular and inverted keyhole still differ by a large amount, about 0.3 mGal each. This can be explained by the fact that both cross-sections have more ‘area’ further away from the observation point, resulting in a smaller gravitational anomaly. In this case, the inverted keyhole exhibits this effect more significantly.

Another generality that can be noticed in Figure 5.51 is that shapes that tend to have more flat parts horizontally, such as the rectangular, keyhole, and elongated shapes, tend to have a flat bottom near their

minimum, similar to a U-shape. This shape precisely allows such distinctions to be possible; however, as will be shown in the figures that follow, this becomes much harder as the observation point is placed farther away. Shapes with less flatness on top, such as the triangular and inverted keyhole shapes, produce graphs that have a sharp minimum, in an almost discontinuous fashion, similar to a V-shape. From this, it can be concluded that the geometry of the roof of the lava tube greatly influences the shape of the gravity signal. The rectangular, keyhole, and elongated shapes all have flat and wide roofs, while the triangular and inverted keyhole have much sharper roofs, similar to an inverted V-shape. This geometry, due to being closer to the observation point, has a greater influence on the gravity signal shape.

Still, although being able to distinguish shapes is interesting, a reference error of 0.3 mGal has been assumed in this case. Using a total error of 5.9 mGal, such intricacies become impossible to see and therefore, inference on the shape becomes impractical. Still, with an imagined error of 1 or 2 mGal, it can be seen from Figure 5.51 that distinguishing the shape would be possible.

A noticeable deduction that can also be made, when comparing the sensitivity analysis of the shape to that of scale, is that, at a distance of 200 m, the inverted and triangular shapes seem to give a lower gravitational attraction than the other shapes. For the case of comparing a lava tube with an inverted keyhole shape, this difference is about 1 mGal. From this, it can be concluded that a lava tube with an inverted keyhole shape could be indistinguishable from a lava tube with a different shape, but which lies deeper. As such, for deeper lava tubes (more than 200 m), constraining the shape becomes practically impossible.

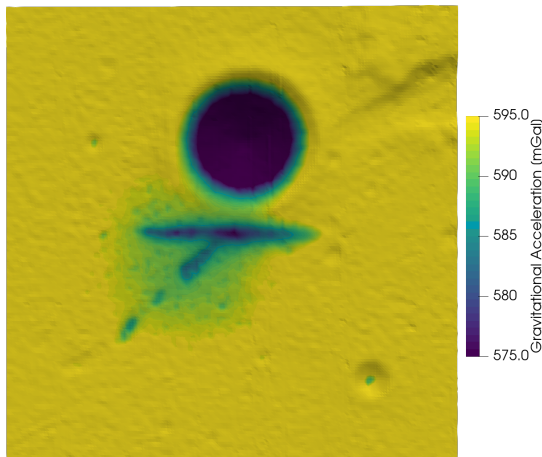
To understand the effect of the shape even more, 2D field plots can be generated. These can be seen in Figure 5.54.

As can be seen in Figure 5.54, for both a 25 m and 200 m depth, the lava tube 2D field plots look quite similar to each other. However, there are still quite some differences visible. Especially in the 25 m depth case, the inverted keyhole shape can be seen to be *sharper* than the other shapes, while the elongated shape can definitely be seen to have a wider shape. This was also evident in the plot given earlier in Figure 5.51. Such intricacies fade away in the 200 m case, although differences between gravitational anomalies can still be observed. Still, it is hard to link these graphs to different shapes, and the author of this thesis does not believe this is viable.

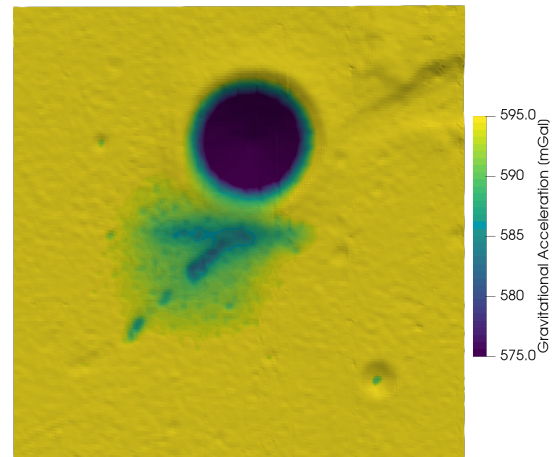
All in all, it seems that the geometry of the lava tube could certainly be at the minimum constrained, as long as you are quite close to the lava tube, and the roof is thin (25 m). Similarly, a small error is necessary; in this case, a 1–2 mGal suffices. This is no longer possible when measurements are taken from further distances.

5.3.5. Density

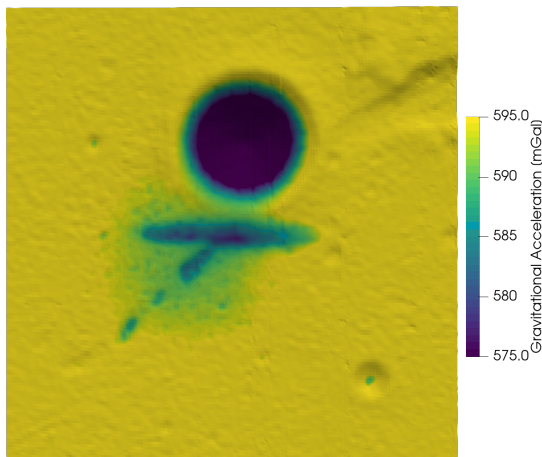
In this subsection, the *density* of the lava tubes is varied to understand how this affects the lava tube gravity anomaly. First, the effect is shown for the lava tube in the Marius Hills region, while afterwards, the Gruithuisen K region lava tube is used. Note that changing the density of the lava tube is actually an indirect way to see the effect of changing the density of the surrounding rock. Obviously, the lava tube itself is not changing density, since it should always have a density of zero. Therefore, this comparison should be seen as a comparison of the contrast between the lava tube and surrounding rock.



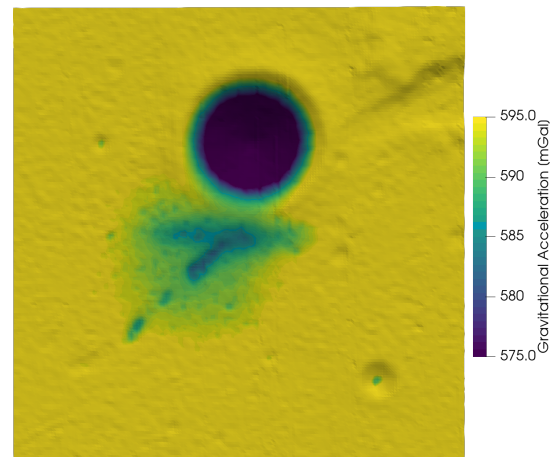
(a) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including an *arched* lava tube at 25 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.



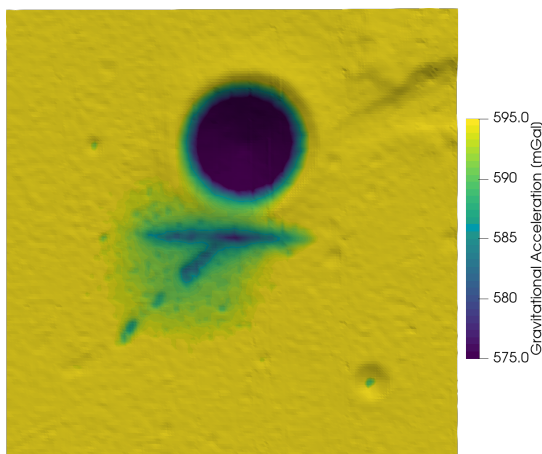
(b) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including an *arched* lava tube at 200 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.



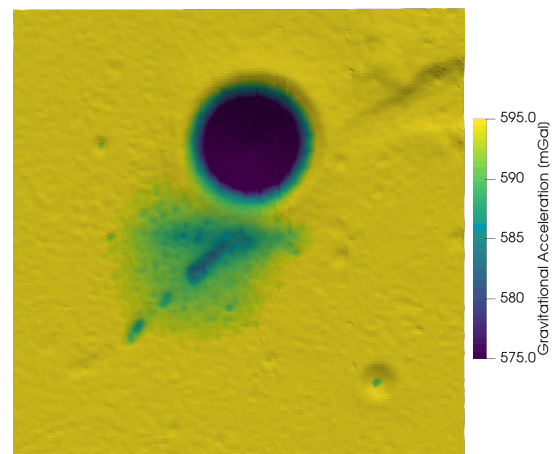
(c) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including an *elongated* lava tube at 25 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.



(d) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including an *elongated* lava tube at 200 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.

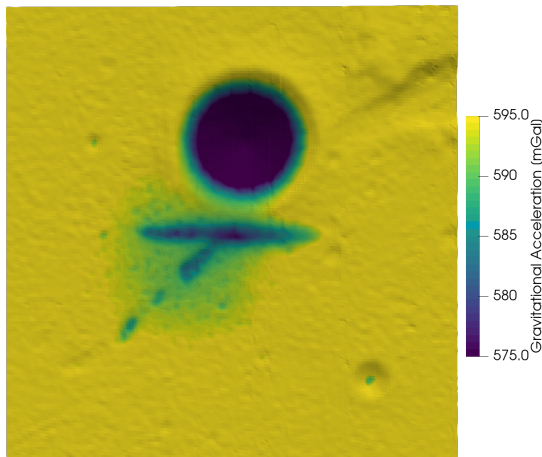


(e) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including an *inverted keyhole* lava tube at 25 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.

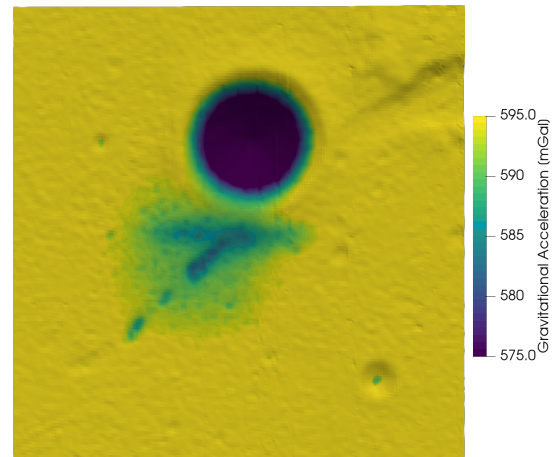


(f) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including an *inverted keyhole* lava tube at 200 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.

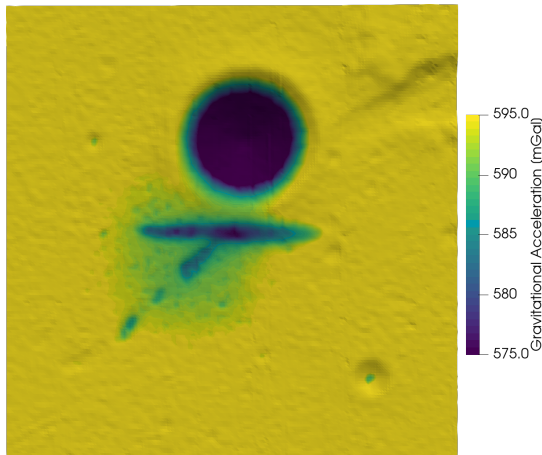
Figure 5.54: The 2D field plots for a 10 km by 10 km piece of DTM at 25 m and 200 m depth for the various shapes, excluding the rounded shape, which has already been shown.



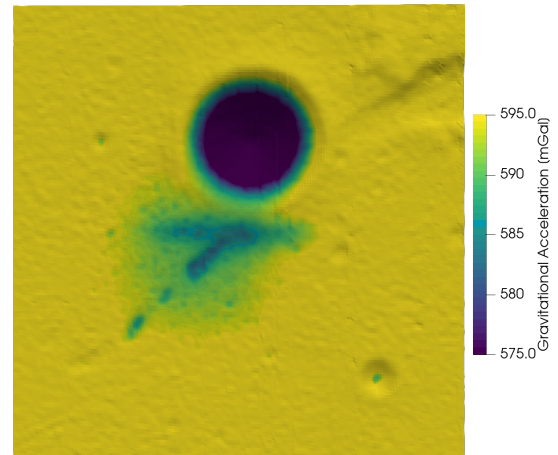
(g) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a *keyhole* lava tube at 25 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.



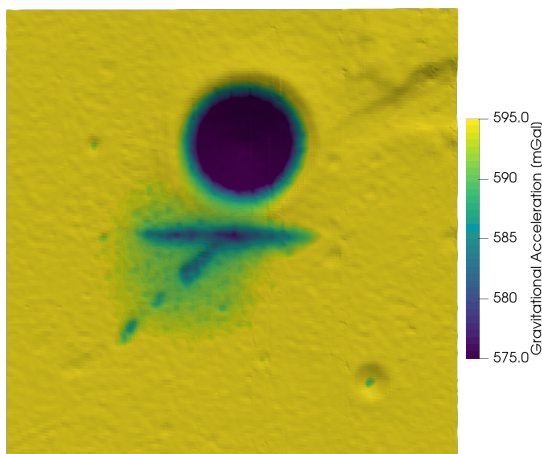
(h) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a *keyhole* lava tube at 200 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.



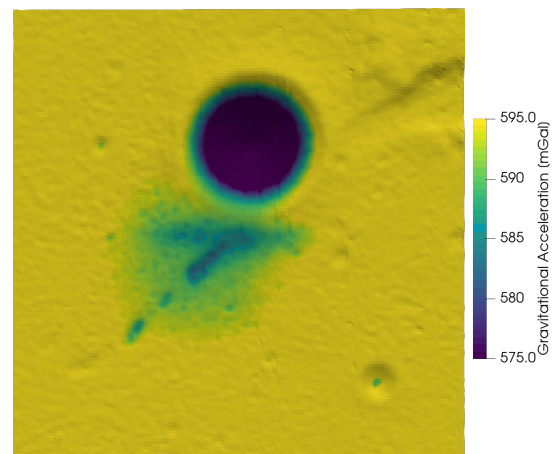
(i) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a *rectangular* lava tube at 25 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.



(j) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a *rectangular* lava tube at 200 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.



(k) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a *triangular* lava tube at 25 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.



(l) The 2D gravitational attraction field plot at the surface of a 30 km by 30 km piece of DTM (only 10 km by 10 km is shown), Marius Hills region, including a *triangular* lava tube at 200 m depth. A small part of the range of values present in these results is used, as this allows for easier visual confirmation of the existence of lava tubes.

Figure 5.54: The 2D field plots for a 10 km by 10 km piece of DTM at 25 m and 200 m depth for the various shapes, excluding the rounded shape, which has already been shown. (cont.)

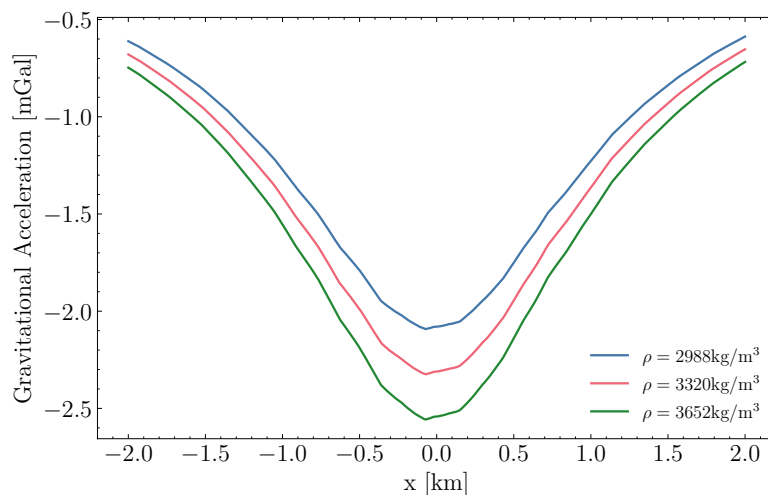


Figure 5.55: Comparison of the gravity anomaly of a lava tube when varying the density of the lava tube.

As can be seen, and as expected, a larger density induces a larger gravity anomaly, while a smaller density induces a smaller gravity anomaly. As the GBOX equation, Equation 2.8, has density in a linear relationship, it is expected that this relationship is linear. A 10% increase in density should result in a 10% increase in gravity anomaly. It is readily and easily calculated that this is indeed the case.

For Gruithuisen K, the same effect is found.

5.4. DTM Sensitivity Analysis

In this section, a sensitivity analysis of the DTM models is performed in relation to their gravity anomalies. First, the resolution is varied. Afterwards, a variation in the height of the DTMs is explored.

5.4.1. Resolution

To understand the effect of changing the resolution of the DTM on the gravitational attraction, a 10 km DTM was meshed at 7 different mesh sizes. Then, a vertical observation line was taken from the DTM surface to a height of 1750 m. By subtracting the results of the lower-resolution DTMs from the highest-resolution DTM (which was the original resolution, 0.059 km), it can be shown at what height the lower-resolution DTMs are within 0.3 mGal of the highest-resolution DTM.

The figure can be seen in Figure 5.56.

As can be seen in Figure 5.56, although the errors are great around the surface, as soon as the observation point is put 500 m away, all the DTMs fall within the 0.3 mGal boundary. This could be useful information, as measurements from orbit might allow for the use of lower-resolution DTMs. However, since most measurements will not have this benefit, the errors on the surface are of greatest interest.

Also interesting in Figure 5.56, is that boundary effects can be seen on the right side of the graph, where suddenly the difference increases in a zigzag pattern. This is something to be studied further in future work.

What the reader must still remember is that these results are obtained for a line right through the centre of the DTM ($x = 0$, $y = 0$). The errors could be greater in different parts of the DTM, due to the fact that the DTM is irregular. To understand if these errors are significantly different, a grid search is performed to see how the errors compare throughout the grid. Then, using such a grid search, the maximum error can be used as the worst-case measurement.

This was done for a square of 4 by 4 km with points every 1000 m in the y-direction, and approximately every 30 m in the x-direction. The results can be seen in Table 5.1.

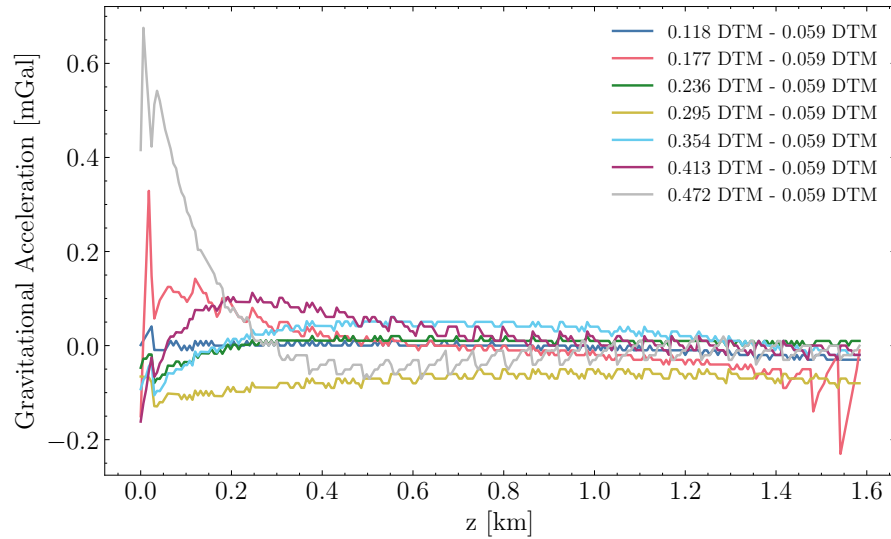


Figure 5.56: The differences between lower-resolution DTM simulations (0.118 km to 0.472 km) and the highest-resolution DTM (0.059 km). The highest-resolution DTM was taken as the ‘truth’ to be compared against.

Table 5.1: The grid search for the errors of using a higher vs. lower-resolution DTM (Marius Hills region), dependent on the height from the surface. Similar to the graph, the results are compared to the 0.059 km DTM. Green cells indicate values below 0.3 mGal, while red above 0.3 mGal.

Height [m]	0.118 km	0.177 km	0.236 km	0.295 km	0.354 km	0.413 km	0.472 km
0	0.516	0.490	1.623	1.875	3.115	4.035	4.523
100	0.120	0.134	0.222	0.431	0.780	1.267	2.444
200	0.071	0.103	0.141	0.292	0.363	0.654	1.472
300	0.051	0.096	0.090	0.222	0.221	0.376	0.981
400	0.041	0.079	0.070	0.159	0.220	0.214	0.681
500	0.040	0.070	0.071	0.141	0.171	0.132	0.511

As can be seen in Table 5.1, once the observation point is placed 100 m above the surface, a mesh size of 0.236 km will provide the same accuracy as a mesh size of 0.059 km. This is quite remarkable and useful because it shows that using a very high-resolution mesh of 0.059 km is unnecessary in most cases. In the worst case, it would only be necessary to use a mesh size of 0.059 km for two cells in any direction, because afterwards one would already be 100 m away from the observation point, resulting in the fact that a larger mesh size would suffice again. Using the total error as a reference, it can be seen that any resolution is indistinguishable from the 59 m resolution.

When simulating a DTM, the author recommends using a 59 m resolution mesh, especially when the simulations are for surface measurements. However, if possible, it is also recommended to adaptively change this mesh size to a coarser mesh; in this case, 236 m would suffice for parts of the DTM further away than 100 m.

Additionally, it was found to be quite difficult to mesh and simulate large meshes using a resolution of 59 m. In several cases, meshing could not be completed within a reasonable time frame, or meshes became non-manifold and unusable. Therefore, in some cases, a coarse resolution or some type of adaptive resolution *must* be used.

5.4.2. Height

The error of height was already computed in subsection 5.1.2, and found to be approximately 0.585 mGal. However, to understand this error more in depth, numerical simulations can be performed.

In the case of the DTMs used in this thesis, the error in height is around 3–4 m (U.S. Geological

Survey 2015). These errors are RMS errors, and therefore random in nature. To effectively simulate this numerically, one could use a normal distribution on each DTM surface point to randomly change the height. This can be done for every point per simulation. Then, by performing a sufficiently large number of such random simulations and averaging them to perform a Monte Carlo simulation, the actual expected error can be obtained. However, this was too computationally prohibitive for this thesis. Therefore, an upper bound is calculated by treating the error as systematic.

As such, DTMs were simulated using the actual DTM, as well as versions with every point 4 m higher and every point 4 m lower. Then, the effect on the gravity anomaly was simulated. This was done for multiple DTM widths, namely 10 km, 20 km, and 30 km. Doing this can estimate upper and lower bounds on the error that the height introduces. It must be noted that the error will be overestimated, since height is a random error, but treated as systematic here.

A plot was made where the x-axis displays the distance from the DTM or the height, while the y-axis displays the gravitational attraction. In the full-scale plots, the three simulations cannot be distinguished, so plots zoomed in on distances close to the lava tube are made. The plots can be seen for the three DTM widths in Figures 5.57–5.59.

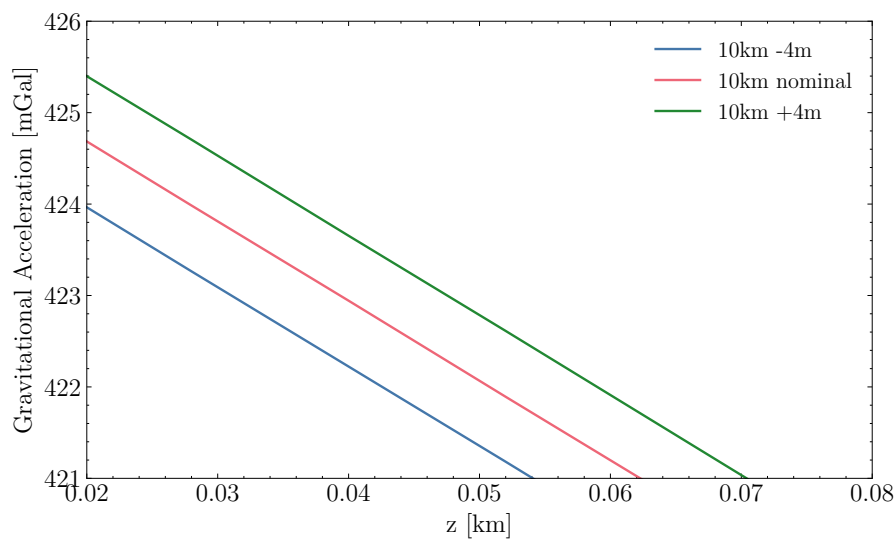


Figure 5.57: The gravitational attraction of a square DTM in the Marius Hills region, with a width of 10 km: a nominal DTM, a DTM with all heights lowered by 4 m, as well as a DTM with all heights increased by 4 m. Zoomed in for convenience on a point that would be near the surface.

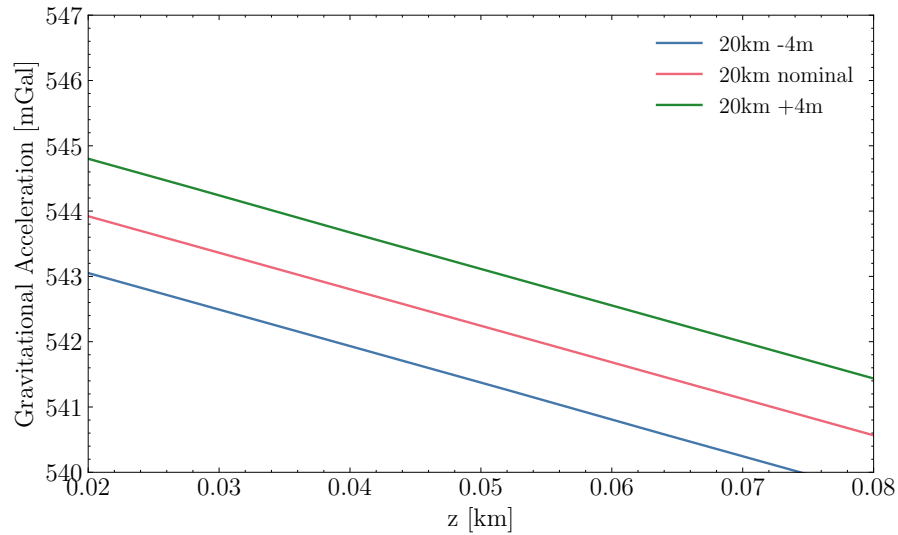


Figure 5.58: The gravitational attraction of a square DTM in the Marius Hills region, with a width of 20 km: a nominal DTM, a DTM with all heights lowered by 4 m, as well as a DTM with all heights increased by 4 m. Zoomed in for convenience on a point that would be near the surface.

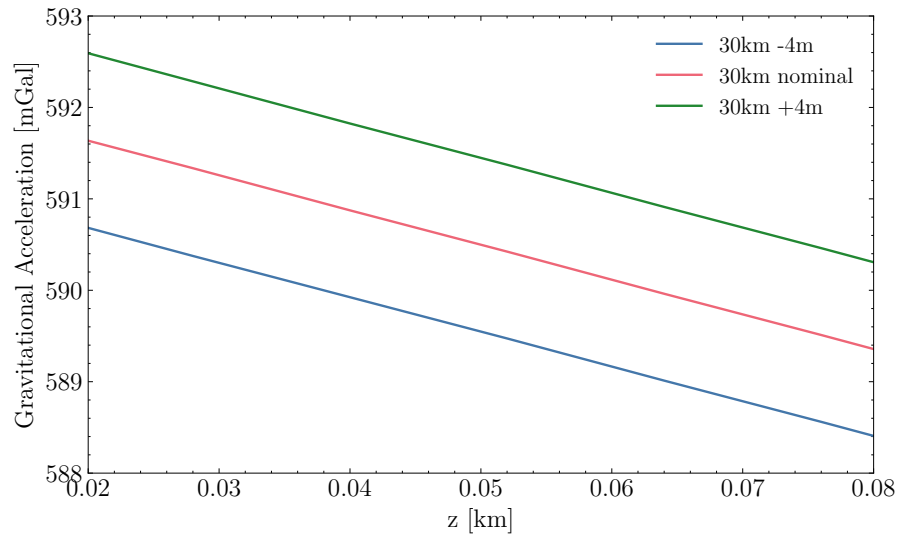


Figure 5.59: The gravitational attraction of a square DTM in the Marius Hills region, with a width of 30 km: a nominal DTM, a DTM with all heights lowered by 4 m, as well as a DTM with all heights increased by 4 m. Zoomed in for convenience on a point that would be near the surface.

As can be seen, the gravitational attraction does differ significantly, with the 30 km case the difference being the most significant at almost a whole mGal. It is quite quickly obvious that the exact amount by which the gravitational anomaly differs depends on the width of the DTM, as well as the height of the observation point. The wider the DTM, the larger the error, and the farther away the observation point, the smaller the error.

To further quantify the effect of height variation, the difference in gravitational acceleration between the ± 4 m and nominal simulations is presented in Figures 5.60–5.62 as a function of height in the DTM.

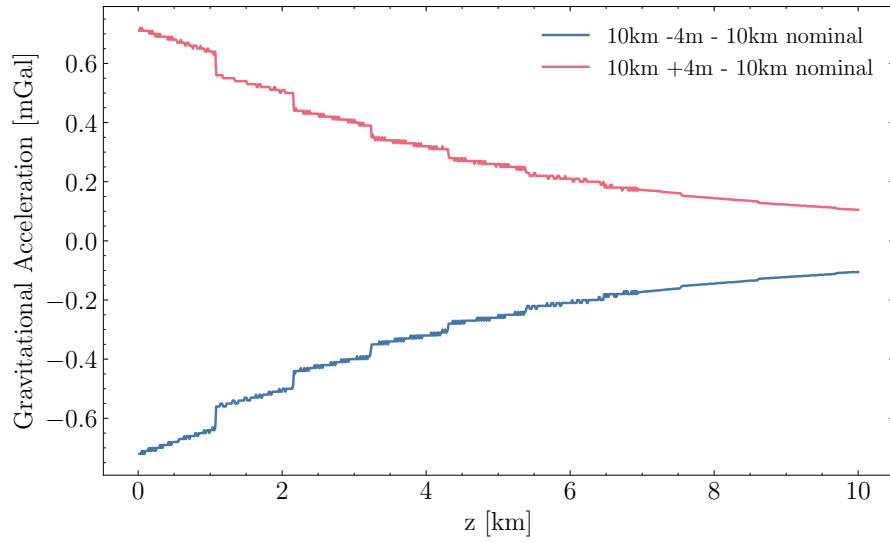


Figure 5.60: The difference between the -4 m DTM, the $+4$ m DTM and the nominal DTM over the distance or height to the DTM. DTM width is 10 km, Marius Hills region.

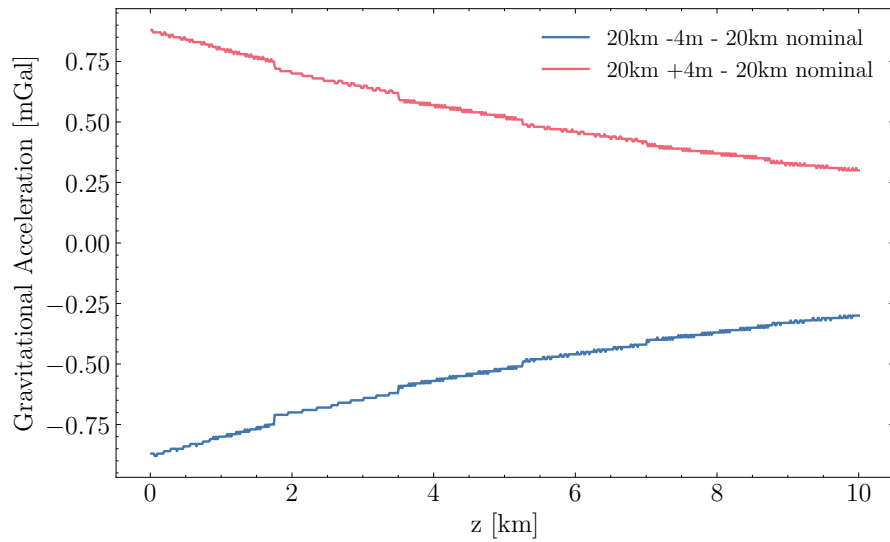


Figure 5.61: The difference between the -4 m DTM, the $+4$ m DTM and the nominal DTM over the distance or height to the DTM. DTM width is 20 km, Marius Hills region.

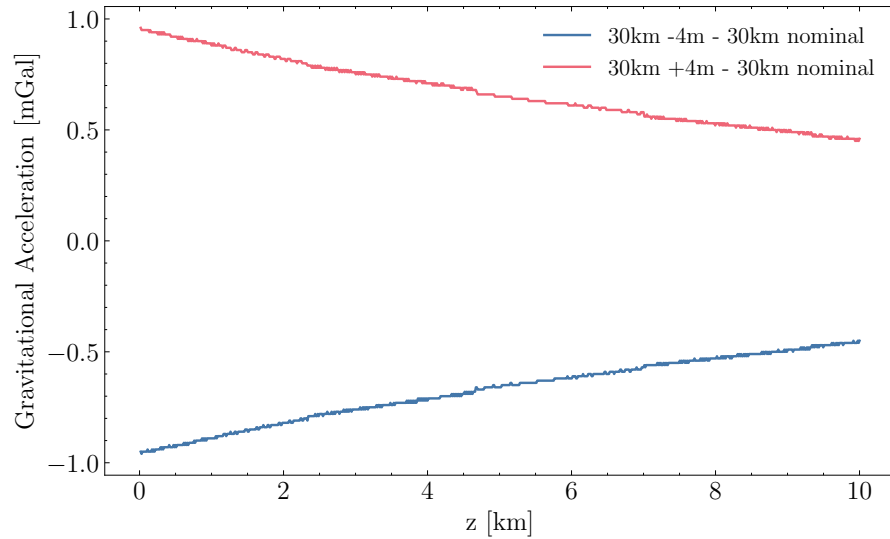


Figure 5.62: The difference between the -4 m DTM, the $+4$ m DTM and the nominal DTM over the distance or height to the DTM. DTM width is 30 km, Marius Hills region.

Firstly, as can be seen clearly in Figure 5.60, some numerical artefacts are clearly present in these results, which most likely come from the fact that two simulations which are quite close to each other are subtracted from each other. These numerical artefacts are to be expected and should be ignored.

Additionally, the results show that some error is present, which diminishes over the distance, and this error is greater when the DTM is wider. Therefore, the exact error present depends on the width of the DTM and the height of the observation point of interest. Still, some general information can be obtained. For example, surveys that are taken on the ground will always be interested in simulations and errors at the surface. These errors can more easily be stated in a table format and used. An interesting fact is that it seems that the error of the height variation is greater the larger the DTM you use. This would indicate the fact that a smaller DTM brings less error. However, this is not the case. A wide enough DTM must be used such that a more precise error is taken into account. Using a smaller DTM introduces other errors which must then be taken into account, and this height error will depend on that error. As such, all errors can still be taken into account. The exact minimum width for the DTM is discussed more in-depth in section 4.4.

One more sensitivity analysis that could be performed is the density sensitivity analysis. However, this analysis was skipped as it was deemed unnecessary. This is due to the fact that changing the density was confirmed to have a 1 to 1 relationship with the gravitational attraction, as is found through simulations in subsection 5.3.5.

Conclusions and Recommendations

In this chapter, the thesis is concluded by restating the research questions, summarising the findings, showing how the research questions were answered, and recommending future work.

6.1. Conclusion

The purpose of this thesis was to perform high-resolution simulations of a lunar lava tube embedded in a DTM, and to conduct a sensitivity analysis of the gravity signal. The motivation was to assess whether lunar lava tubes are detectable using current gravimeters, while accounting for errors in DTMs. These findings contribute to the planning of future lunar missions by providing a foundation for more reliable detection and characterisation of lunar lava tubes, which could offer shelter for humans.

To obtain the models and perform the simulations and sensitivity analyses, the DTM and the lava tube were split, which resulted in easier meshing and analysis. First, the lava tube and DTM models were obtained. This was done using the pyproduct software in Rhino 8 and Grasshopper, as well as using lunar DTMs and Python STL generation code, respectively. Two models were obtained. The first model, which is small, simple, and could have been present in the Marius Hills region, has a length of 4 km and minimum and maximum diameters of 140 and 400 m. The second model, which is larger, more complicated, and could have been present in the Gruithuisen K region, has a length of 40 km and a minimum and maximum diameter of 200 m and 1.2 km. Using the models, various situations were meshed using Coreform Cubit, after which simulations were performed.

Simulations were performed using the SPECFEM and GBOX software, which employ the spectral element and direct integral methods, respectively. Their accuracy and computation time were investigated using simple models, such as a rectangular prism, a sphere, and a lava tube. A sensitivity analysis was performed on both lava tubes, with more extensive parameter variation conducted for the Marius Hills lava tube. The resolution, scale, density, depth, and shape of the cross-section were varied. For the shape variation, seven different cross-sectional shapes were used, one for each common shape found in terrestrial lava tubes. For the DTM, a sensitivity analysis was performed on the density, DTM height, and DTM resolution.

It was found that, for the same mesh size, SPECFEM gained higher accuracy than GBOX. For the lava tube simulation, a 0.05 km mesh size in SPECFEM would differ from the most accurate simulation (mesh size 0.0125 km) by approximately 0.3 mGal, while GBOX would differ by 1.8 mGal. For a 0.025 km mesh size, SPECFEM differed by approximately 0.1 mGal, while GBOX differed by 0.5 mGal. Therefore, SPECFEM is preferred for the same mesh size in terms of accuracy, especially at lower resolutions.

In terms of compute time, GBOX scales linearly with both the number of elements and the number of observation points. It was found that for the same number of computations, GBOX is more than two orders of magnitude quicker. However, when the solution must be obtained for large areas or volumes, the number of computations increases quadratically (for areas) or cubically (for volumes) with the number of observation points in one direction. For example, a large model, with a number of

cells of $N_c = 4 \times 10^6$, could take months to compute if the solution must be provided for a volume with 1000 observation points on each side (1000^3 observation points total). For an area, SPECSEM can be up to an order of magnitude quicker. Still, in such a case, SPECSEM would then also provide solutions anywhere else in the initially simulated volume. As such, for cases where results are needed over a volume, SPECSEM becomes the only viable option.

Using a sensor error of 0.3 mGal RMS, a DTM height error of 4 m RMS, and a density uncertainty of 80 kg m^{-3} RMS, the total error was estimated to be 5.9 mGal. Using the sensor error and total error values, conclusions were drawn about the detectability of lava tubes from the sensitivity analyses.

A mesh resolution study for a Marius Hills lava tube at a roof thickness of 25 m revealed that a mesh size of 0.0125 km produces results within the sensor error for all seven shapes investigated. A mesh size of 0.025 km is sufficient for all shapes as well, with the exception of the inverted keyhole. As a rule of thumb, a mesh size equal to 8% of the *minimum* diameter of the lava tube is recommended to achieve results within the sensor error of 0.3 mGal. Using this error, for both the Marius Hills and Gruithuisen K lava tubes, the mesh size could be increased to 0.05 km and 0.1 km if the lava tube is deeper than 400 m, respectively, and to 0.1 km and 0.2 km if the depth is more than 900 m (conservatively speaking).

For the Marius Hills lava tube, using the sensor error indicates that depths up to approximately 3.4 km can be detected, whereas using the total error decreases this depth to approximately 0.28 km. For Gruithuisen K, using the sensor error, any lava tube up to a depth of 6.5 km could be detected, although larger simulations are necessary to confirm the exact point at which detectability stops. Using a total error, a narrow part of the lava tube was detectable up to 0.59 km, while a part with a wider cavity was detectable up to 2.95 km.

Two-dimensional field plots reveal that for both the Marius Hills and Gruithuisen K lava tubes, the anomaly remains identifiable at a depth of 200 m. At depths of 1000 m and beyond, the gravity anomaly is unrecognisable as a lava tube, since the shape of the lava tube is too attenuated to with certainty be attributed to a lava tube.

Varying the scale or diameter of the lava tube was found to have a quadratic relationship with the gravitational attraction, while the density was found to have a linear relationship. Because a larger lava tube increases the gravity anomaly, it was found that in some instances, a smaller lava tube becomes indistinguishable from a deeper one. One such case was a Marius Hills lava tube at a depth of 25 m, with a diameter 0.9 times the nominal diameter, compared against a nominal-diameter lava tube at a depth of 62 m. With the total error as a reference, the two lava tubes would be indistinguishable using measurements on the surface.

Varying the shape shows that the inverted keyhole and triangular shapes provide sharper, V-shaped gravity signals, while shapes with flat parts near the surface, such as the elongated, keyhole, and rectangular shapes, provide more U-shaped gravity signals. Using the sensor error, such intricacies are recognisable at a shallow depth of 25 m. At a depth of 200 m, it is impossible to distinguish the shape. Using the total error, the shape cannot be distinguished even at a depth of 25 m.

Varying the DTM resolution, it is found that at the surface, a resolution of 59 m, the highest available, had to be used for measurements to be within the sensor error. Adaptive meshing could allow users to use a 59 m mesh size close to the observation points, while allowing a 236 m mesh size to be used for any geometry further than 100 m from observation points. Using the total error, any resolution tested (118 to 472 m) is indistinguishable from the 59 m resolution.

It was also found that, by varying the height of the DTM experimentally for three different DTM widths (10, 20, and 30 km width squares), by ± 4 metres, an experimental upper bound for the height error of 1 mGal was found for the 30 km DTM.

There are several limitations of the work in this thesis. Firstly, the number of shapes used in the sensitivity analysis, at only seven, is too small a sample to infer general conclusions that apply to all lava tube morphologies. Similarly, the two lava tube models made can be seen as two case studies. They therefore provide insight, but the values of gravity signals and detectability thresholds should not be seen as universally applicable to other situations. Another limitation is the consideration of the stability of the lava tube models produced. Because determining stability was outside the scope of this thesis, the stability of the Marius Hills and Gruithuisen K lava tubes was not treated. It is possible

that the cross-sectional shapes considered in this thesis would collapse if present on the Moon. The assumption of a homogeneous density was also a limitation present in this work. It is expected for the density on the Moon to vary from region to region, and such density variations were not accounted for. Lastly, the curvature of the Moon was not included in simulations and is therefore not accounted for.

These limitations do not invalidate the conclusions drawn, but should be understood and considered when interpreting the results of this thesis.

6.2. Answers to Research Questions

This section provides the answers to the research questions asked at the beginning of the thesis, as well as stating if the research objective was achieved.

RQ1: How does SPECSEM compare to GBOX in terms of accuracy and computational cost for computing gravity anomalies of lunar lava tubes?

SPECSEM is more accurate than GBOX for the same mesh size. Comparing the gravitational attraction at a mesh size of 0.05 km for the Marius Hills lava tube, SPECSEM differs from the most accurate resolution by approximately 0.3 mGal, whereas GBOX differs by 1.8 mGal. At high-resolution mesh sizes, accuracy is comparable, although SPECSEM always provides a more accurate gravity signal.

In terms of computational cost, GBOX is quicker than SPECSEM per computation by more than two orders of magnitude. In situations where only a low number of computations are necessary, such as line plots, GBOX is preferred. On the other hand, in situations where the solution must be calculated for many observation points, such as an area or a volume, the number of computations for GBOX explodes, resulting in the fact that SPECSEM becomes the only viable solution. For example, when simulating a lava tube, SPECSEM is one order of magnitude quicker than GBOX if the region of solutions is an area (assuming 1 million observation points for GBOX).

RQ2: How can the SPECSEM software be used to obtain the gravity anomaly of a 3D model that includes a lunar lava tube as well as a lunar digital terrain model?

A gravity anomaly of a lunar lava tube, as well as a lunar DTM, can be obtained through SPECSEM by first obtaining models of the lava tube and DTM in question. Then, using meshing software, the models have to be meshed using hexahedral elements. In this thesis, the models were meshed separately. The region of space for which answers are calculated has to be provided in this mesh as well. Then, the SPECSEM software takes the mesh, in addition to other inputs for the spectral element method, such as the location of the pole for the infinite elements, as well as the number of GLL points per element. A simulation can then be performed, after which a post-processing tool, in this thesis, ParaView, can be used to visualise the results.

RQ3: How can a representative 3D model of a lunar lava tube be obtained?

The representative model needed for the sensitivity analysis required realistic cross-sectional shapes, and a realistic radius, depth, and length. For accurate simulations, the mesh size should be at least 0.0125 km. If more realism is needed (i.e., to obtain the actual values expected on the Moon), the exact twisting and turning of the lava tube and height variation must be included.

A representative 3D model of a lunar lava tube can be obtained by finding collapse chains on the Moon and drawing the 2D curves on the edges of this collapse chain. Using these 2D curves together with the pyroduct software inside Rhino 8 and Grasshopper, a lava tube can be created artificially. The lava tube model can also follow a DTM topography if desired, and uses terrestrial cross-sections to representative lunar lava tube characteristics.

RQ4: How can a representative 3D model of the lunar surface be obtained?

A representative 3D model of the lunar surface can be obtained by downloading a lunar DTM, which can then be transformed and cropped to the desired location. Then an STL can be generated from the DTM using Python, which can be made into a 3D model using various methods. In this thesis, the method chosen was to create a spline surface from the mesh, which was extruded using Coreform Cubit. This operation creates a solid 3D model of the lunar DTM, which can be used later in simulations as a STEP file. In this thesis, the representative model needed for the sensitivity analysis required the highest amount of accuracy possible from the DTM. This meant using a mesh size of 59 m. For other simulations, a mesh size of 236 m can suffice, as long as a higher resolution (i.e. 59 m) is used within 100 m of the observation point.

RQ5: How does the use of a high-resolution model influence the accuracy and compute time of the simulation and analysis?

In SPECFEM, for the lava tube, using a resolution of 0.025 km covers all accuracy demands for the sensor error of 0.3 mGal for the lava tube shapes in this thesis, except the inverted keyhole one. For the inverted keyhole, a resolution of 0.0125 km is necessary. For GBOX, a resolution of 0.0125 km is the only resolution within 0.3 mGal of the most accurate simulation available (SPECFEM simulation of 0.0125 km). Therefore, a mesh size of 0.0125 km is recommended for geometries similar to the Marius Hills lava tube to ensure accuracy across all shapes.

For the compute time, the same conclusion is found as in RQ1.

RQ6: How sensitive is the computed gravity anomaly to incremental variations in the 3D model of the lava tube and DTM in materials?

Varying the depth of the lava tube was found to have an inverse linear relationship with the gravity anomaly. Using the sensor error, Marius Hills lava tubes up to approximately 3.39 km could be detected, while for Gruithuisen K, the minimum depth reached beyond the simulation bounds, further than 6.5 km. Looking at the total error, the depth of detectability was 0.28 km for Marius Hills lava tubes, while 0.59 km for a narrower part of the Gruithuisen K lava tube, and 2.95 km for a wider part.

Varying the scale or diameter of the lava tube was found to have a square-law relationship with the gravity anomaly, which aligns with the analytical formula for an infinite cylinder.

Varying the shape had the profound impact that, as long as the lava tube depth was shallow (25 m), the shape of the cross-section of the lava tube could be constrained to either a flatter shape near the top of the lava tube (elongated, rectangular keyhole) or a flatter shape near the bottom of the lava tube with a sharper one near the top (inverted keyhole, triangular). This was possible due to the U-shaped and V-shaped gravity anomalies seen when plotted across the lava tube. Still, such intricacies can only be distinguished with high-accuracy measurements and are not discernible when the total error is considered. Therefore, the errors would have to be constrained more to allow the lava tube shapes to be distinguished.

Varying the density was found to have a linear relationship with the gravity anomaly.

For the DTM, the height of the DTM was varied through simulations, which was found to change the gravity anomaly by approximately 1 mGal for a 30 by 30 km DTM. In such a case, the error is treated systematically, which is conservative. The density was also varied, with the same conclusion reached as for the lava tube.

RO1: To determine whether a typical lunar lava tube can be detected using typical lunar gravimeters, given uncertainties in the DTM, density, and lava tube geometry.

Typical lava tubes are detectable, given uncertainties in the sensor, as well as in the DTM height and density. Detectability depends on the size of the lava tube, as well as the error. In this case, a total error of 5.9 mGal was used. This meant that the Marius Hills model was detectable up to a roof thickness of 0.28 km, while for the Gruithuisen K model, this depth increases to 0.59 km for a narrower part of the lava tube, and 2.95 km for a wider part. Shapes were distinguishable using the sensor error as a reference at a depth of 25 m. At a depth of 200 m, shapes cannot be distinguished.

6.3. Recommendations

The author has a couple of recommendations for future work and research. They are summarised in this section.

6.3.1. Data Visualization

The data obtained from simulations has been visualised mostly through line plots, as well as 2D field plots. Still, one way the author would have liked to visualise the data is through histograms, as they could also possibly provide more information on the sensitivity analyses. Patterns might come out in such a visualisation method, or possibly even other methods. The author recommends such different visualisation techniques to be performed on simulations similar to the ones in this thesis to potentially find new ways of finding and distinguishing lunar lava tubes.

6.3.2. Narrowing the Lunar Rock Density

Because the biggest source of error (resulting in most of the 5.9 mGal total error) in understanding if a lava tube can be detected is the error in density, lowering this error can drastically increase the depth at which lava tubes could be sensed. In the literature, many different densities are used, and more research on the composition of lunar rock present in a general lunar DTM is highly recommended for future work.

If either the homogeneity of a region or the exact variation in density could be estimated, this could go a long way to reduce the total error. Similarly, more effective methods for estimating the true density of a lunar rock sample could help with this problem.

Additionally, performing a sensitivity analysis of density variations in simulations could provide a useful first approximation of the types and magnitudes of errors to be addressed. For example, such simulations can constrain the location and size of density variations.

6.3.3. Boundary Effects

Some boundary effects were found in the graphs presented in this thesis, such as in Figure 5.56. Such boundary effects, present in SPECFEM, should receive more attention, as they can bring unwanted effects, and their extent is not precisely known. Conducting an extensive sensitivity analysis to determine at what point such errors render results unreliable will help future engineers run accurate simulations with minimal edge effects.

6.3.4. Meshing Methods and SPECFEM Convergence

One of the hardships the author experienced during this thesis was meshing models. This was done in Coreform Cubit, and because many simulations had to be performed, it was intuitive to script these simulations. However, on numerous occasions, the meshing of the models resulted in meshes with negative scaled Jacobians or other non-manifoldness. This was quite a bottleneck, and more research into the reliability of meshing programs is a useful endeavour.

6.3.5. Inverse Method

In the simulations performed in this thesis, only the forward method is used: a model is created, and then the gravitational attraction is calculated. The inverse method tries to do the opposite. Using the gravitational anomaly measured by a sensor, a 3D model is created and iterated upon until the gravitational attraction of this generated model differs insignificantly from the original gravity anomaly. Thus, the model of the lava tube is generated from the gravity anomaly. Future surveys would follow this exact procedure to obtain the most accurate representation possible of the lava tube. Therefore, a sensitivity analysis of the inverse method for lava tubes such as the models presented in this thesis

could provide useful information for constraining a lava tube's geometry once real measurements are performed on the Moon.

The work presented in this thesis gives an initial understanding of the detectability of lunar lava tubes. Using sensitivity analyses and error estimation, a basis is provided for future work on gravimetry as a method for finding subsurface structures such as lava tubes. As such, this work contributes to lunar gravimetry, the scientific goals of researching the Moon, as well as to future human exploration.

References

- Allred, K. and C. Allred (Aug. 1997). "Development and Morphology of Kazumura Cave, Hawaii". In: *Journal of Cave and Karst Studies* 59 (2), pp. 67–80. URL: <https://caves.org/journal-of-cave-and-karst-studies/jcks-articles/development-and-morphology-of-kazumura-cave-hawaii/>.
- Arizona State University, NASA, and Applied Coherent Technology Corp. (2024). *Lunar QuickMap*. We acknowledge the use of imagery from Lunar QuickMap, a collaboration between NASA, Arizona State University, and Applied Coherent Technology Corp.
- Banerjee, B. and S. P. Das Gupta (Aug. 1977). "Gravitational attraction of a rectangular parallelepiped". In: *Geophysics* 42 (5), pp. 1053–1055. ISSN: 0016-8033. DOI: 10.1190/1.1440766.
- Barker, M. K. et al. (July 2016). "A new lunar digital elevation model from the Lunar Orbiter Laser Altimeter and SELENE Terrain Camera". In: *Icarus* 273, pp. 346–355. ISSN: 0019-1035. DOI: 10.1016/J.ICARUS.2015.07.039.
- Beer, G. and J. L. Meek (Jan. 1981). "'Infinite domain' elements". In: *International Journal for Numerical Methods in Engineering* 17 (1), pp. 43–52. ISSN: 1097-0207. DOI: 10.1002/NME.1620170104.
- Bell, E. et al. (June 2022). "Field Mapping and Modeling of Terrestrial Lava Tube Magnetic Anomalies as an Analog for Lunar Lava Tube Exploration and Prospecting". In: *Journal of Geophysical Research: Planets* 127 (6). ISSN: 2169-9100. DOI: 10.1029/2021JE007140.
- Bettess, P. (Jan. 1977). "Infinite elements". In: *International Journal for Numerical Methods in Engineering* 11 (1), pp. 53–64. ISSN: 1097-0207. DOI: 10.1002/NME.1620110107.
- Blair, D. M. et al. (Jan. 2017). "The structural stability of lunar lava tubes". In: *Icarus* 282, pp. 47–55. ISSN: 0019-1035. DOI: 10.1016/J.ICARUS.2016.10.008.
- Blakely, R. J. (Jan. 1995). "Potential Theory in Gravity and Magnetic Applications". In: *Potential Theory in Gravity and Magnetic Applications*. DOI: 10.1017/CB09780511549816.
- Cai, Y. and C. Y. Wang (Sept. 2005). "Fast finite-element calculation of gravity anomaly in complex geological regions". In: *Geophysical Journal International* 162 (3), pp. 696–708. ISSN: 1365-246X. DOI: 10.1111/J.1365-246X.2005.02711.X.
- Canuto, C. et al. (1988). *Spectral Methods in Fluid Dynamics*. Springer Berlin Heidelberg. ISBN: 978-3-540-52205-8. DOI: 10.1007/978-3-642-84108-8.
- Carroll, K. et al. (2015). "Exploring Subsurface Lunar Voids Using Surface Gravimetry". In: *46th Lunar and Planetary Science Conference*.
- Chakravarthi, V., H. M. Raghuram, and S. B. Singh (Feb. 2002). "3-D forward gravity modeling of basement interfaces above which the density contrast varies continuously with depth". In: *Computers & Geosciences* 28 (1), pp. 53–57. ISSN: 0098-3004. DOI: 10.1016/S0098-3004(01)00080-2.
- Chappaz, L., K. C. Howell, and H. J. Melosh (Aug. 2014). "Strategies for Detection of Buried Empty Lava Tubes with GRAIL Data". In: *American Institute of Aeronautics and Astronautics*. DOI: 10.2514/6.2014-4371. URL: <https://arc.aiaa.org/doi/10.2514/6.2014-4371>.
- Chappaz, L., H. J. Melosh, and K. C. Howell (2014). "Surface and Buried Lava Tube Detection with GRAIL Data". In: *LPI (1777)*, p. 1746. DOI: 10.1126/science.1231753.
- Chappaz, L., R. Sood, et al. (Jan. 2017). "Evidence of large empty lava tubes on the Moon using GRAIL gravity". In: *Geophysical Research Letters* 44 (1), pp. 105–112. ISSN: 1944-8007. DOI: 10.1002/2016GL071588.
- Christensen, P. R. et al. (2009). "JMARS - A Planetary GIS". In: *AGUFM 2009, IN22A-06*. URL: <https://ui.adsabs.harvard.edu/abs/2009AGUFMIN22A..06C/abstract>.
- Chwała, M., G. Komatsu, and J. Haruyama (Mar. 2024). "Structural stability of lunar lava tubes with consideration of variable cross-section geometry". In: *Icarus* 411, p. 115928. ISSN: 0019-1035. DOI: 10.1016/J.ICARUS.2023.115928.
- Coan, T. (1844). "Letter of March 15, 1843, describing the Mauna Loa eruption of 1843". In: *The Missionary Herald*.

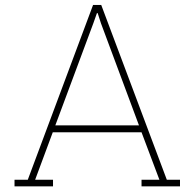
- Connor, C. B. (2015). *Plotting the gravity anomaly due to a horizontal cylinder based on Gauss's Law using gnuplot*. URL: https://gscommunitycodes.usf.edu/geoscicommunitycodes/public/geophysics/Gravity/horizontal_cylinder.php#top.
- Coombs, C. R. and B. R. Hawke (1992). "A search for intact lava tubes on the Moon: Possible lunar base habitats". In: *NASA. Johnson Space Center, The Second Conference on Lunar Bases and Space Activities of the 21st Century, Volume 1*.
- Coreform LLC (2024). *Coreform Cubit*. Computer software. Orem, UT. URL: <https://coreform.com>.
- Cruikshank, D. P. and C. A. Wood (Mar. 1972). "Lunar rilles and Hawaiian volcanic features: Possible analogues". In: *The Moon* 3 (4), pp. 412–447. ISSN: 00270903. DOI: 10.1007/BF00562463/METRICS.
- D'Urso, M. G. (May 2015). "The Gravity Anomaly of a 2D Polygonal Body Having Density Contrast Given by Polynomial Functions". In: *Surveys in Geophysics* 36 (3), pp. 391–425. ISSN: 15730956. DOI: 10.1007/S10712-015-9317-3/FIGURES/12.
- De Angelis, G. et al. (2002). "Lunar lava tube radiation safety analysis." In: *Journal of radiation research* 43 Suppl. ISSN: 04493060. DOI: 10.1269/JRR.43.S41.
- Deville, M. O., P. F. Fischer, and E. H. Mund (Aug. 2002). "High-Order Methods for Incompressible Fluid Flow". In: *High-Order Methods for Incompressible Fluid Flow*. DOI: 10.1017/CB09780511546792.
- Digital Research Alliance of Canada (n.d.). *Trillium - Alliance Doc*. URL: <https://docs.alliancecan.ca/wiki/Trillium>.
- Donini, E., F. Bovolo, et al. (Oct. 2018). "An approach to lava tube detection in radar sounder data of the moon". In: *International Geoscience and Remote Sensing Symposium (IGARSS) 2018-July*, pp. 8424–8427. DOI: 10.1109/IGARSS.2018.8519146.
- Donini, E., L. Carrer, et al. (2022). "An Unsupervised Fuzzy System for the Automatic Detection of Candidate Lava Tubes in Radar Sounder Data". In: *IEEE Transactions on Geoscience and Remote Sensing* 60. ISSN: 15580644. DOI: 10.1109/TGRS.2021.3062753.
- ESA (2021). *ESA plans mission to explore lunar caves*. URL: https://www.esa.int/Enabling_Support/Preparing_for_the_Future/Discovery_and_Preparation/ESA_plans_mission_to_explore_lunar_caves.
- (n.d.). *ESA - SysNova*. URL: https://www.esa.int/Enabling_Support/Preparing_for_the_Future/Discovery_and_Preparation/SysNova2.
- El-Esnawy, N. A., A. Y. Akl, and A. S. Bazaraa (Feb. 1995). "A new parametric infinite domain element". In: *Finite Elements in Analysis and Design* 19 (1-2), pp. 103–114. ISSN: 0168-874X. DOI: 10.1016/0168-874X(94)00060-S.
- Faccioli, E. et al. (1997). "2D and 3D elastic wave propagation by a pseudo-spectral domain decomposition method". In: *Journal of Seismology* 1 (3), pp. 237–251. ISSN: 13834649. DOI: 10.1023/A:1009758820546/METRICS.
- Garland, G. (1971). *Introduction to Geophysics (Mantle, Core and Crust)*. W. B. Saunders Company.
- GDAL/OGR contributors (2026). *GDAL/OGR Geospatial Data Abstraction software Library*. Open Source Geospatial Foundation. DOI: 10.5281/zenodo.5884351. URL: <https://gdal.org>.
- GFZ Helmholtz Centre for Geosciences (2025). *About | IGMAS+*. URL: <https://igmas.git-pages.gfz-potsdam.de/igmas-pages/about>.
- Gharti, H.N., D. Komatitsch, et al. (July 2012). "Application of an elastoplastic spectral-element method to 3D slope stability analysis". In: *International Journal for Numerical Methods in Engineering* 91 (1), pp. 1–26. ISSN: 0029-5981. DOI: 10.1002/nme.3374.
- Gharti, H.N., J. Tromp, and S. Zampini (Nov. 2018). "Spectral-infinite-element simulations of gravity anomalies". In: *Geophysical Journal International* 215 (2), pp. 1098–1117. ISSN: 0956-540X. DOI: 10.1093/GJI/GGY324.
- Gilvarry, J. J. (1960). "Origin and Nature of Lunar Surface Features". In: *Nature* 1960 188:4754 188 (4754), pp. 886–891. ISSN: 1476-4687. DOI: 10.1038/188886a0. URL: <https://www.nature.com/articles/188886a0>.
- (1968). "Observational Evidence for Sedimentary Rocks on the Moon". In: *Nature* 1968 218:5139 218 (5139), pp. 336–341. ISSN: 1476-4687. DOI: 10.1038/218336a0. URL: <https://www.nature.com/articles/218336a0>.
- Götze, H. J. (June 1978). "Ein numerisches Verfahren zur Berechnung der gravimetrischen Feldgrößen dreidimensionaler Modellkörper". In: *Archiv für Meteorologie, Geophysik und Bioklimatologie Serie A* 27 (2), pp. 195–215. ISSN: 00666416. DOI: 10.1007/BF02246695/METRICS.

- Götze, H. J. and B. Lahmeyer (Aug. 1988). "Application of three-dimensional interactive modeling in gravity and magnetics". In: *Geophysics* 53 (8). issn: 0016-8033. doi: 10.1190/1.1442546.
- Grechi, G. et al. (Sept. 2024). "Modal Analysis of a Lava Tube Roof Complex: Tabernacle Hill, Utah, USA". In: *Rock Mechanics and Rock Engineering* 57 (9). issn: 1434453X. doi: 10.1007/S00603-024-03868-9/FIGURES/4.
- Greeley, R. (Dec. 1971). "Lava tubes and channels in the lunar Marius Hills". In: *The Moon* 3, pp. 289–314. issn: 00270903. doi: 10.1007/BF00561842/METRICS.
- Grimes, K. (2004). "VOLCANIC CAVES AND RELATED FEATURES". In: *Field Guide: Limestone Coast*. URL: <https://www.hamilton-field-naturalists-club-victoria.org.au/images/pdf/Publications/Geology/volcanic-caves-kg2004.pdf>.
- Halliday, W. R. (Aug. 2004). *Encyclopedia of Caves and Karst Science*. Ed. by John Gunn. Routledge, pp. 1624–1632. isbn: 9780203483855. doi: 10.4324/9780203483855.
- Hansen, R. O. (Feb. 1999). "An analytical expression for the gravity field of a polyhedral body with linearly varying density". In: *Geophysics* 64 (1), pp. 75–77. issn: 0016-8033. doi: 10.1190/1.1444532.
- Harris, A. J. L. et al. (Jan. 2017). "Pāhoehoe, 'a'ā, and block lava: an illustrated history of the nomenclature". In: *Bulletin of Volcanology* 79 (1). issn: 14320819. doi: 10.1007/S00445-016-1075-7/FIGURES/26.
- Haruyama, J., S. Hara, et al. (2010). "New Discoveries of Lunar Holes in Mare Tranquillitatis and Mare Ingenii". In: *LPI* (1533), p. 1285. doi: 10.1029/2009GL040635.
- Haruyama, J., K. Hioki, et al. (2009). "Possible lunar lava tube skylight observed by SELENE cameras". In: *Geophysical Research Letters* 36. doi: 10.1029/2009GL040635.
- Hidding, A. et al. (2025). "Robotic Lava Tube Mapping and Multimodal Data Collection Using Quadruped and Lidar". In: *Frontiers in Robotics and AI*. doi: 10.2139/SSRN.5238812. URL: <https://papers.ssrn.com/abstract=5238812>.
- Horvath, T., P. O. Hayne, and D. A. Paige (July 2022). "Thermal and Illumination Environments of Lunar Pits and Caves: Models and Observations From the Diviner Lunar Radiometer Experiment". In: *Geophysical Research Letters* 49 (14). issn: 1944-8007. doi: 10.1029/2022GL099710.
- Horz, F. (1985). "Lava tubes - Potential shelters for habitats". In: *Lunar Bases and Space Activities of the 21st Century*, pp. 405–412.
- Johnson, L. R. and J. J. Litehiser (Dec. 1972). "A method for computing the gravitational attraction of three-dimensional bodies in a spherical or ellipsoidal Earth". In: *Journal of Geophysical Research* 77 (35), pp. 6999–7009. issn: 2156-2202. doi: 10.1029/JB077I035P06999.
- Kaku, T. et al. (Oct. 2017). "Detection of Intact Lava Tubes at Marius Hills on the Moon by SELENE (Kaguya) Lunar Radar Sounder". In: *Geophysical Research Letters* 44 (20), pp. 10, 155–10, 161. issn: 1944-8007. doi: 10.1002/2017GL074998.
- Kauahikaua, J., K. V. Cashman, et al. (Nov. 1998). "Observations on basaltic lava streams in tubes from Kilauea Volcano, island of Hawai'i". In: *Journal of Geophysical Research: Solid Earth* 103 (B11), pp. 27303–27323. issn: 2156-2202. doi: 10.1029/97JB03576.
- Kauahikaua, J., D. Sherrod, et al. (2003). "Hawaiian lava-flow dynamics during the Pu'u 'Ō'Ō-Kūpaianaha eruption: A tale of two decades". In: *U.S. Geological Survey*.
- Kempe, S. (2009). "Principles of pyroduct (lava tunnel) formation". In: *Newsletter Hawai'i Speleological Survey of the National Speleological Society*.
- (July 2017). "What if the "Seven Sisters" are not volcanic? A look at deep hypogene sinkholes in Arabia". In: *International Congress Of Speleology* 1, pp. 371–374.
- Kempe, S. et al. (Jan. 2018). *Genesis of Pyroducts of the Galápagos Islands, Ecuador*. URL: https://www.academia.edu/64464205/Genesis%5C_of%5C_Pyroducts%5C_of%5C_the%5C_Gala_pagos%5C_Islands%5C_Ecuador.
- Kiefer, W. S. et al. (Apr. 2012). "The density and porosity of lunar rocks". In: *Geophysical Research Letters* 39 (7). issn: 00948276. doi: 10.1029/2012GL051319.
- Kobayashi, T. et al. (Sept. 2021). "Nadir Detection of Lunar Lava Tube by Kaguya Lunar Radar Sounder". In: *IEEE Transactions on Geoscience and Remote Sensing* 50 (9), pp. 7395–7418. issn: 15580644. doi: 10.1109/TGRS.2020.3033033.
- Konopliv, A. S. et al. (Dec. 2002). "A Global Solution for the Gravity Field, Rotation, Landmarks, and Ephemeris of Eros". In: *Icarus* 160 (2). issn: 0019-1035. doi: 10.1006/ICAR.2002.6975.
- Kumar, P. (Nov. 1985). "Static Infinite Element Formulation". In: *Journal of Structural Engineering* 111 (11), pp. 2355–2372. issn: 0733-9445. doi: 10.1061/(ASCE)0733-9445(1985)111:11(2355).

- Lemaire, T. et al. (Nov. 2024). "Lava flow field development and lava tube formation during the 1858-1861 eruption of Vesuvius (Italy), unravelled by historical documentation, lidar data and 3D mapping". In: *Journal of Volcanology and Geothermal Research* 455. ISSN: 0377-0273. DOI: 10.1016/J.JVOLGEORES.2024.108197.
- Losarcos, J. M. and A. J. Dombard (Dec. 2025). "Lava Tubes on Earth, the Moon, and Mars: Detection, Evolution, and Exploration Potential". In: *Space Science Reviews* 2025 221:8 221 (8). ISSN: 1572-9672. DOI: 10.1007/S11214-025-01260-9.
- Macdonald, G. A. (Mar. 1953). "Pahoehoe, aa, and block lava". In: *American Journal of Science* 251 (3), pp. 169–191. ISSN: 0002-9599. DOI: 10.2475/AJS.251.3.169.
- MacMillan, W. D. (1958). *The Theory of the Potential*. Dover Publications.
- Marraffa, A. et al. (Mar. 2025). "Morphometric analysis of lava tubes". MA thesis. Università degli Studi di Padova. DOI: 10.5194/EGUSPHERE-EGU25-18326.
- Martin, R. et al. (Apr. 2017). "A high-order 3-D spectral-element method for the forward modelling and inversion of gravimetric data-Application to the western Pyrenees". In: *Geophysical Journal International* 209 (1), pp. 406–424. ISSN: 1365246X. DOI: 10.1093/gji/ggx010.
- Medina, F. and R. L. Taylor (Aug. 1983). "Finite element techniques for problems of unbounded domains". In: *International Journal for Numerical Methods in Engineering* 19 (8), pp. 1209–1226. ISSN: 1097-0207. DOI: 10.1002/NME.1620190808.
- Modiriasari, A., Theinat, A. K., et al. (June 2019). "Stability Analysis of Lunar Lava Tubes for Permanent Extraterrestrial Habitation". In: *Proceedings of the 53rd US Rock Mechanics/Geomechanics Symposium*. ARMA 19–387.
- Modiriasari, A., A. K. Theinat, et al. (2018). "Size and Structural Stability Assessment of Lunar Lava Tubes". In: *49th Lunar and Planetary Science Conference*, p. 2803.
- Muñoz, S. M. et al. (Nov. 2025). "Gravity Modeling of Lunar Lava Tubes: Insights From a Terrestrial Analog". In: *Journal of Geophysical Research: Planets* 130 (11). ISSN: 2169-9100. DOI: 10.1029/2025JE009130.
- Murase, T. and A. R. McBirney (1970). "Viscosity of lunar lavas". In: *Science* 167 (3924), pp. 1491–1493. ISSN: 00368075. DOI: 10.1126/SCIENCE.167.3924.1491.
- Nagy, D. (Apr. 1966). "The gravitational attraction of a right rectangular prism". In: *Geophysics* 31 (2), pp. 362–371. ISSN: 0016-8033. DOI: 10.1190/1.1439779.
- National Park Service (n.d.). *Lava Tubes - Hawai'i Volcanoes National Park (U.S. National Park Service)*. URL: <https://www.nps.gov/havo/learn/nature/lava-tubes.htm>.
- NIST (2022). *CODATA Value: Newtonian constant of gravitation*. URL: <https://physics.nist.gov/cgi-bin/cuu/Value?bg>.
- Noeker, M. et al. (2022). "Application of the Wedge-Pentahedra Method (WPM) to the Apollo 17 Landing Site and Comparison to the Traverse Gravimeter Experiment Measurements". In: *Modern Geology* 1, pp. 75–80. URL: <https://ui.adsabs.harvard.edu/abs/1969ModGe...1...750/abstract>.
- Okabe, M. (Apr. 1979). "Analytical expressions for gravity anomalies due to homogeneous polyhedral bodies and translations into magnetic anomalies". In: *Geophysics* 44 (4), pp. 730–741. ISSN: 0016-8033. DOI: 10.1190/1.1440973.
- Orr, T. R. et al. (Jan. 2015). "A sinuous tumulus over an active lava tube at Kilauea Volcano: Evolution, analogs, and hazard forecasts". In: *Journal of Volcanology and Geothermal Research* 291, pp. 35–48. ISSN: 0377-0273. DOI: 10.1016/J.JVOLGEORES.2014.12.002.
- Patera, A. T. (June 1984). "A spectral element method for fluid dynamics: Laminar flow in a channel expansion". In: *Journal of Computational Physics* 54 (3), pp. 468–488. ISSN: 0021-9991. DOI: 10.1016/0021-9991(84)90128-1.
- Paul, M. K. (May 1974). "The gravity effect of a homogeneous polyhedron for three-dimensional interpretation". In: *Pure and Applied Geophysics PAGEOPH* 112 (3), pp. 553–561. ISSN: 00334553. DOI: 10.1007/BF00877292/METRICS.
- Peter, D. et al. (Aug. 2011). "Forward and adjoint simulations of seismic wave propagation on fully unstructured hexahedral meshes". In: *Geophysical Journal International* 186 (2), pp. 721–739. ISSN: 1365-246X. DOI: 10.1111/J.1365-246X.2011.05044.X.
- Plouff, D. (Aug. 1976). "Gravity and magnetic fields of polygonal prisms and application to magnetic terrain corrections". In: *Geophysics* 41 (4), pp. 727–741. ISSN: 0016-8033. DOI: 10.1190/1.1440645.

- Pohánka, V. (July 1998). "Optimum expression for computation of the gravity field of a polyhedral body with linearly increasing density". In: *Geophysical Prospecting* 46 (4), pp. 391–404. ISSN: 1365-2478. DOI: 10.1046/J.1365-2478.1998.960335.X.
- Ramsey, A.S. (1961). *An introduction to the theory of Newtonian attraction*. Cambridge University Press.
- Robinson, M. S. et al. (Aug. 2012). "Confirmation of sublunarean voids and thin layering in mare deposits". In: *Planetary and Space Science* 69 (1), pp. 18–27. ISSN: 0032-0633. DOI: 10.1016/J.PSS.2012.05.008.
- Romio, F. A. P., A. Hidding, et al. (Nov. 2025). "LiDAR and Photogrammetry Data Evaluation from a Lava Tube Mission in Sicily". Preprint. Not peer reviewed. DOI: 10.22541/AU.176278707.79300810/V1.
- Romio, F. A. P., G. Lobosco, et al. (Nov. 2025). *Pyroduct: The First Realistic and Parametric Lava Tube Generator for Earth, Moon, and Mars*. DOI: 10.5281/ZENODO.17740882.
- (Mar. 2026). "Pyroduct: A novel parametric software for the simulation of terrestrial, lunar, and Martian lava tubes". In: *Icarus* 447. ISSN: 00191035. DOI: 10.1016/j.icarus.2025.116904.
- Roncoli, R. B. (2005). *Lunar Constants and Models Document - JPL Technical Document D-32296* (2005). Tech. rep. JPL.
- Rowland, S. K. and G. P.L. Walker (Nov. 1990). "Pahoehoe and aa in Hawaii: volumetric flow rate controls the lava structure". In: *Bulletin of Volcanology* 52 (8), pp. 615–628. ISSN: 02588900. DOI: 10.1007/BF00301212/METRCS.
- Sakimoto, S. E. H., J. Crisp, and S. M. Baloga (Mar. 1997). "Eruption constraints on tube-fed planetary lava flows". In: *Journal of Geophysical Research: Planets* 102 (E3), pp. 6597–6613. ISSN: 2156-2202. DOI: 10.1029/97JE00069.
- Santolaria, P. et al. (May 2024). "Salt Distribution in the South Pyrenean Central Salient: Insights From Gravity Anomalies". In: *Tectonics* 43 (5). ISSN: 1944-9194. DOI: 10.1029/2024TC008274.
- Sauro, F. et al. (Oct. 2020). "Lava tubes on Earth, Moon and Mars: A review on their size and morphology revealed by comparative planetology". In: *Earth-Science Reviews* 209, p. 103288. ISSN: 0012-8252. DOI: 10.1016/J.EARSCIREV.2020.103288.
- Sawłowicz, Z. (2020). "A short review of pyroducts (lava tubes)". In: *Annales Societatis Geologorum Poloniae* 90 (4), 513–534—doi: 10.14241/asgp.2020.34. ISSN: 02089068. DOI: 10.14241/asgp.2020.34.
- Schmidt, S. et al. (2020). "IGMAS+ - a tool for interdisciplinary 3D potential field modelling of complex geological structures". In: *EGU General Assembly 2020*. DOI: 10.5194/egusphere-egu2020-8383.
- Schröter, J. H. (1788). "Ueber den Mondfleck Aristarchus". In: *In Astronomisches Jahrbuch für das Jahr 1791*, pp. 201–203.
- Seriani, G. and S. P. Oliveira (June 2008). "Dispersion analysis of spectral element methods for elastic wave propagation". In: *Wave Motion* 45 (6), pp. 729–744. ISSN: 0165-2125. DOI: 10.1016/J.WAVEMOTI.2007.11.007.
- Talwani, M. and W. M. Ewing (Feb. 1960). "Rapid computation of gravitational attraction of three-dimensional bodies of arbitrary shape". In: *Geophysics* 25 (1), pp. 203–225. ISSN: 0016-8033. DOI: 10.1190/1.1438687.
- Talwani, M., G. Thompson, et al. (1973). *Traverse Gravimeter Experiment*. Tech. rep.
- Talwani, M., J. L. Worzel, and M. Landisman (Jan. 1959). "Rapid gravity computations for two-dimensional bodies with application to the Mendocino submarine fracture zone". In: *Journal of Geophysical Research* 64 (1), pp. 49–59. DOI: 10.1029/JZ064I001P00049.
- Theinat, A. K. et al. (Mar. 2020). "Lunar lava tubes: Morphology to structural stability". In: *Icarus* 338. ISSN: 0019-1035. DOI: 10.1016/J.ICARUS.2019.113442.
- Tomasi, I. et al. (June 2022). "Inception and Evolution of La Corona Lava Tube System (Lanzarote, Canary Islands, Spain)". In: *Journal of Geophysical Research: Solid Earth* 127 (6). ISSN: 2169-9356. DOI: 10.1029/2022JB024056.
- Tromp, J., D. Komatitsch, and Q. Liu (2008). "Spectral-element and adjoint methods in seismology". In: *Communications in Computational Physics* 3 (1), pp. 1–32. ISSN: 1815-2406.
- Tsynkov, S. V. (Aug. 1998). "Numerical solution of problems on unbounded domains. A review". In: *Applied Numerical Mathematics* 27 (4), pp. 465–532. ISSN: 0168-9274. DOI: 10.1016/S0168-9274(98)00025-7.
- Turcotte, D. and G. Schubert (Apr. 2014). *Geodynamics*. Cambridge University Press. ISBN: 9780511843877. DOI: 10.1017/CB09780511843877.
- U.S. Geological Survey (2015). *Moon LRO LOLA - SELENE Kaguya TC DEM Merge 60N60S 59m*. URL: https://astrogeology.usgs.gov/search/map/moon_lro_lola_selene_kaguya_tc_dem_merge_60n60s_59m (visited on 10/22/2025).

- Urey, H. C. (Dec. 1967). "Water on the Moon". In: *Nature* 1967 216:5120 216 (5120), pp. 1094–1095. issn: 1476-4687. doi: 10.1038/2161094a0.
- Wagner, R. V. and M. S. Robinson (Aug. 2022). "Lunar Pit Morphology: Implications for Exploration". In: *Journal of Geophysical Research: Planets* 127 (8). issn: 2169-9100. doi: 10.1029/2022JE007328.
- Werner, R. A. (July 1994). "The gravitational potential of a homogeneous polyhedron or don't cut corners". In: *Celestial Mechanics & Dynamical Astronomy* 59 (3), pp. 253–278. issn: 09232958. doi: 10.1007/BF00692875.
- Wilson, L. et al. (Oct. 2011). "An igneous origin for Rima Hyginus and Hyginus crater on the Moon". In: *Icarus* 215 (2), pp. 584–595. issn: 0019-1035. doi: 10.1016/J.ICARUS.2011.07.003.
- Yang, Q. et al. (Aug. 2024). "Cross-section extraction and model reconstruction of lava tube based on L1-medial skeleton". In: *International Journal of Applied Earth Observation and Geoinformation* 132, p. 104062. issn: 1569-8432. doi: 10.1016/J.JAG.2024.104062.
- Zhao, J. et al. (Oct. 2023). "Volcanism and Deep Structures of the Moon". In: *Space: Science and Technology (United States)* 3. issn: 26927659. url: <https://spj.science.org/doi/10.34133/space.0076>.
- Zhu, K. et al. (Jan. 2024). "GRAIL gravity gradients evidence for a potential lava tube at Marius Hills on the moon". In: *Icarus* 408. issn: 0019-1035. doi: 10.1016/J.ICARUS.2023.115814.
- Zienkiewicz, O. C., C. Emson, and P. Bettess (Mar. 1983). "A novel boundary infinite element". In: *International Journal for Numerical Methods in Engineering* 19 (3), pp. 393–404. issn: 1097-0207. doi: 10.1002/NME.1620190307.
- Zuber, M. T. et al. (Feb. 2013). "Gravity field of the Moon from the Gravity Recovery and Interior Laboratory (GRAIL) mission". In: *Science (New York, N.Y.)* 339 (6120). issn: 1095-9203. doi: 10.1126/SCIENCE.1231507.



Code

In this appendix, various codes used to obtain the results in this thesis are presented.

A.1. generate_stl.py

```
1 import pymeshlab
2 from pathlib import Path
3 import numpy as np
4 import rasterio
5 from rasterio.windows import Window
6 import trimesh
7
8 def crop_square(input_tif, output_tif, xc, yc, r_meters):
9     """Crops a tif to a square format
10
11     Args:
12         input_tif (Path): The path to the input tif file.
13         output_tif (Path): The path to the output tif file.
14         xc (float): The center in x-coordinate of the cropped square in meters.
15         yc (float): The center in y-coordinate of the cropped square in meters.
16         r_meters (float): The radius of the cropped square in meters.
17     """
18     with rasterio.open(input_tif) as src:
19
20         pixel_size = src.res[0]
21
22         row_c, col_c = src.index(xc, yc)
23
24         half = int(round(r_meters / pixel_size))
25
26         row_off = row_c - half
27         col_off = col_c - half
28         height = 2 * half + 1 # plus one to ensure odd
29         width = 2 * half + 1 # plus one to ensure odd
30
31         print(f"Center pixel: row={row_c}, col={col_c}")
32         print(f"Half-width (pixels): {half}")
33         print(f"Window size: {height} x {width}")
34
35         window = Window(col_off, row_off, width, height)
36
37         data = src.read(window=window)
38
39         print(f"Center value: {data[0, half, half]}")
40
41         out_meta = src.meta.copy()
42         out_meta.update({
43             "height": height,
44             "width": width,
45             "transform": rasterio.windows.transform(window, src.transform)
```

```

46     })
47
48     with rasterio.open(output_tif, "w", **out_meta) as dst:
49         dst.write(data)
50
51 def fix_pixel_xy(in_file, out_file, points_xy: list[tuple[int, int]]):
52     """Average a pixel value to its eight neighbour cells.
53
54     Args:
55         in_file (Path): input tif
56         out_file (Path): output tif
57         points_xy (list[tuple[int, int]]): the x and y points in meters which define the
58             pixels to be averaged.
59
60     """
61     with rasterio.open(in_file) as src:
62         Z = src.read(1)
63         p = src.profile
64         for x, y in points_xy:
65             row, col = src.index(x, y)
66
67             if not (0 <= row < Z.shape[0] and 0 <= col < Z.shape[1]):
68                 continue
69
70             nbrs = Z[row-1:row+2, col-1:col+2].copy()
71             nbrs[1, 1] = np.nan
72             Z[row, col] = np.nanmean(nbrs)
73
74     with rasterio.open(out_file, 'w', **p) as dst:
75         dst.write(Z, 1)
76
77 def read_dem(filepath):
78     """Read the DEM from tif and return x, y, and z (elevation) numpy array
79
80     Args:
81         filepath (Path): The input file tif.
82
83     Returns:
84         tuple[NDArray[np.float64], NDArray[np.float64], NDArray[np.float64]]:
85             x: 1D array of x coordinates
86             y: 1D array of y coordinates
87             elevation: 2D array of elevation values
88
89     """
90     with rasterio.open(filepath) as src:
91         elevation = src.read(1)
92
93         transform = src.transform
94         nrows, ncols = elevation.shape
95         res_x = transform.a
96         res_y = transform.e
97         x = np.arange(ncols, dtype=np.float64) * res_x
98         y = np.arange(nrows, dtype=np.float64) * res_y
99
100         return x, y, elevation.astype(np.float64)
101
102 def build_surface(_x, _y, _Z, stride=1):
103     """Builds a 3D triangulated mesh using x-y coordinates and the elevation. First centers x
104         and y to zero, then downsamples through the stride. A mesh is then created using a
105         meshgrid and column stack to create the vertices after which the faces are made.
106         Lastly, the surface is created from these using the trimesh module.
107
108     Args:
109         _x (NDArray[np.float64]): 1D array of x coordinates.
110         _y (NDArray[np.float64]): 1D array of y coordinates.
111         _Z (NDArray[np.float64]): 2D array of elevation values with shape (nrows, ncols).
112         stride (int, optional): Downsampling factor - every nth point is kept.
113             Higher values = coarser mesh, faster processing.
114             Defaults to 1 (no downsampling).
115
116     Returns:
117         trimesh.Trimesh: A 3D triangulated surface mesh ready for export or further
118             processing.

```

```

112 """
113 _x = _x - np.max(_x)/2
114 _y = _y - np.min(_y)/2
115
116 stride = int(stride)
117 x_ds = np.asarray(_x)[::stride]
118 y_ds = np.asarray(_y)[::stride]
119 Z_ds = np.asarray(_Z)[::stride, ::stride]
120
121 nrows, ncols = Z_ds.shape
122
123 X, Y = np.meshgrid(x_ds, y_ds)
124
125 vertices = np.column_stack([X.ravel(), Y.ravel(), Z_ds.ravel()])
126
127 ii, jj = np.meshgrid(np.arange(nrows - 1),
128                      np.arange(ncols - 1), indexing='ij')
129 ii = ii.ravel()
130 jj = jj.ravel()
131
132 v00 = ii * ncols + jj          # top-left
133
134 v01 = ii * ncols + jj + 1      # top-right
135 v10 = (ii + 1) * ncols + jj    # bottom-left
136 v11 = (ii + 1) * ncols + jj + 1 # bottom-right
137
138 faces = np.vstack([
139     np.column_stack([v00, v01, v11]),
140     np.column_stack([v00, v11, v10]),
141 ])
142
143 surface = trimesh.Trimesh(vertices=vertices, faces=faces,
144                           process=False)
145
146 return surface
147
148 def center_stl(infile, outfile):
149     """Center STL using MeshLab centering on Scene Bounding Box. Immediately exports output
150     file.
151
152     Args:
153         infile (Path): Path to STL input file
154         outfile (Path): Path to STL output file
155     """
156     ms = pymeshlab.MeshSet()
157     ms.load_new_mesh(str(infile))
158     ms.compute_matrix_from_translation(
159         traslmethod='Center on Scene BBox'
160     )
161     ms.save_current_mesh(str(outfile))
162
163 def center_stl_layer(infile, outfile):
164     """Center STL using MeshLab centering on Layer Bounding Box. Immediately exports output
165     file.
166
167     Args:
168         infile (Path): Path to STL input file
169         outfile (Path): Path to STL output file
170     """
171     ms = pymeshlab.MeshSet()
172     ms.load_new_mesh(str(infile))
173     ms.compute_matrix_from_translation(
174         traslmethod='Center on Layer BBox'
175     )
176     ms.save_current_mesh(str(outfile))
177
178 def center_stl_trimesh(infile, outfile):
179     """Center STL using trimesh. Immediately exports output file.
180
181     Args:
182         infile (Path): Path to STL input file

```

```

181         outfile (Path): Path to STL output file
182         """
183         mesh = trimesh.load_mesh(infile)
184
185         cx, cy, cz = mesh.bounds.mean(axis=0)
186
187         T = np.eye(4)
188         T[:3, 3] = [-cx, -cy, 0.0]
189         mesh.apply_transform(T)
190
191         mesh.export(outfile)
192
193 def center_stl_for_varying_bounds(infile, outfile, ray_origin: np.array, ray_dir: np.array):
194     """Center STL in x and y and vertically. Ensures alignment of multiple STLs with varying
195         widths but the same center.
196
197     Args:
198         infile (Path): Path to input STL file
199         outfile (Path): Path to output STL file
200         ray_origin (np.array): The origin of the ray that will be used to determine the
201             height of the STL.
202         ray_dir (np.array): The direction the ray will be shot in. Usually [0,0, -1] or
203             [0,0,1].
204
205     Exports output file immediately.
206     """
207     mesh = trimesh.load_mesh(infile)
208     locs, idx_ray, idx_tri = mesh.ray.intersects_location(
209         ray_origins=ray_origin,
210         ray_directions=ray_dir,
211         multiple_hits = False
212     )
213
214     if len(locs) == 0:
215         print(f"[WARN] No intersection at x=0,y=0 for {infile}")
216         return 0
217     z_hit = float(np.atleast_2d(locs)[0, 2])
218     T = np.eye(4)
219     T[:3, 3] = [0.0, 0.0, -z_hit]
220     mesh.apply_transform(T)
221     mesh.export(outfile)
222
223 if __name__ == "__main__":
224     #####INPUTS#####
225     NAME = "mh_DTM" # marius hills
226     # NAME = "gk_DTM" # gruithuisen K
227
228     CROP = True
229     FIX_PIXEL = True
230     CENTER = True
231     CENTER_VB = True
232     RAY_ORIGIN = np.array([[0.0,0.0,1e6]])
233     RAY_DIR = np.array([[0.0,0.0,-1]])
234
235     #####PIXELS TO BE FIXED#####
236     # # Marius Hills DTM 1
237     XC = -1141
238     YC = 400494
239
240     # # Marius Hills DTM 2
241     # XC = -1332.5
242     # YC = 361837.9
243
244     # # Gruithuisen K DTM 1
245     # XC = 2280.44
246     # YC = 1046895.09
247
248     POINT1 = [-11577.53, 397787.04] #Marius Hills
249     POINT2 = [-11517.4, 397785.4] #Marius Hills
250     POINT3 = [5774.7, 397076.1] #Marius Hills

```

```

249 POINT4 = [-4587.8, 1073842.6] #Gruithuisen K
250
251
252 POINTS = [POINT1, POINT2, POINT3, POINT4]
253
254 #####DEFINITION WIDTH OF SQUARE#####
255 WIDTH_SQUARE = np.arange(10, 50, 10) #Marius Hills
256 WIDTH_SQUARE = np.arange(10, 55, 5) #Marius Hills
257
258 # WIDTH_SQUARE = np.arange(50, 80, 10) #Gruithuisen K
259 # WIDTH_SQUARE = [50] #Gruithuisen K
260
261 #####FOR LOOP WITH EVERY WIDTH#####
262 for width_square in WIDTH_SQUARE:
263     STRIDE=[1]
264
265     SCRIPT_DIR = Path(__file__).resolve().parent
266     PROJECT_DIR = SCRIPT_DIR.parent
267
268     in_file = Path("Marius_Hills/mh_DTM_40km_f.tif")
269     # in_file = Path("Gruithuisen_K/LunarLROLOLAKaguya_MAP2_34.727N316.473_EQUI.tif")
270
271     if not in_file.is_absolute():
272         in_file = (PROJECT_DIR / in_file).resolve()
273
274     input_dir = in_file.parent / "mh_DTM_sens_newr"
275     filename = f"{NAME}_{width_square}km"
276
277     output_folder = input_dir / f"{NAME}_{width_square}km"
278     output_folder.mkdir(parents=True, exist_ok=True)
279
280     output_filename_crop = Path(filename)
281     output_filename_fix_pixel = Path(output_filename_crop.stem + "_f")
282
283     output_file_crop = output_folder / output_filename_crop.with_suffix('.tif')
284     output_file_fix_pixel = output_folder / output_filename_fix_pixel.with_suffix(".tif")
285
286     with rasterio.open(in_file) as src:
287         row, col = src.index(XC, YC)
288         XC, YC = src.xy(row, col)
289         print(XC, YC)
290
291     if CROP:
292         crop_square(in_file, output_file_crop, XC, YC, 1000*width_square/2)
293
294     if FIX_PIXEL:
295         fix_pixel_xy(output_file_crop, output_file_fix_pixel, POINTS)
296
297     x,y,Z = read_dem(output_file_fix_pixel)
298     for s in STRIDE:
299         output_filename_stride = Path(output_filename_fix_pixel.stem + f"_str{s}")
300         output_file_stride = output_folder / output_filename_stride.with_suffix('.stl')
301         surf = build_surface(x,y,Z, stride=s)
302
303         surf.export(output_file_stride, file_type='stl')
304
305     if CENTER:
306         output_filename_center = Path(output_filename_stride.stem + f'_c')
307         output_file_center = output_folder / output_filename_center.with_suffix('.stl')
308         try:
309             # center_stl(output_file_stride, output_file_center) #for Scene centering
310             # center_stl_layer(output_file_stride, output_file_center) # for layer
311             # centering
312             center_stl_trimesh(output_file_stride, output_file_center) # for layer
313             # centering
314
315         except Exception as e:
316             print(f"Centering failed for {output_file_stride.name}: {e}")
317
318     if CENTER_VB:

```

```

317         if CENTER == True:
318             output_filename_center_vb = Path(output_filename_center.stem + f'_{cvb}')
319             output_file_center_vb = output_folder / output_filename_center_vb.
320                 with_suffix('.stl')
321             center_stl_for_varying_bounds(output_file_center, output_file_center_vb,
322                 RAY_ORIGIN, RAY_DIR)
323
324         if CENTER == False:
325             output_filename_center_vb = Path(output_filename_stride.stem + f'_{cvb}')
326             output_file_center_vb = output_folder / output_filename_center_vb.
327                 with_suffix('.stl')
328             center_stl_for_varying_bounds(output_file_stride, output_file_center_vb,
329                 RAY_ORIGIN, RAY_DIR)

```

A.2. testing_error_bounds.py

```

1  import numpy as np
2  import matplotlib.pyplot as plt
3  import os
4
5  RHO = 3320 #kg/m^3
6  G = 6.67430e-11 # m3 kg-1 s-2
7
8  script_dir = os.path.dirname(os.path.abspath(__file__))
9  print(script_dir)
10
11 def error(_R, _Z, _b):
12     """calculates the error for the gravitational attraction formula.
13
14     Args:
15         R (double): the radius of the disk
16         Z (double): the height or thickness of the disk
17         b (double): the height of the measurement above the disk
18     """
19     return np.sqrt(_R**2+_b**2) - np.sqrt(_R**2+(_b+_Z)**2)
20
21 def R_required(_Z, _e, _b):
22     """calculate the radius of the slab required to obtain a certain accuracy.
23
24     Args:
25         _Z (float): thickness or height of slab
26         _e (float): error [m/s^2]
27         _b (float): height of observation point
28     """
29     epsilon = _e / (2*np.pi * G * RHO)
30     num = 2 * _b * _Z + _Z**2 - epsilon**2
31     den = 2 * epsilon
32     return np.sqrt( (num/den)**2 - _b**2)
33
34 Z_range = np.linspace(500, 2000, 1000)
35 # Z_range = np.linspace(4, 5, 1000)
36
37 R_required_list = R_required(Z_range, 0.000003, 0)
38
39 plt.figure(dpi=300)
40 plt.plot(Z_range/1000, R_required_list/1000)
41 plt.xlabel("Thickness slab [km]")
42 plt.ylabel("Required radius [km]")
43 plt.title("The required radius of the bounding box for every thickness of the slab.")
44 plt.show()

```

A.3. Analytical_Prism.py

```

1  from potential_library import gbox
2  import numpy as np
3  from tqdm import tqdm
4  from constants import *
5  import matplotlib.pyplot as plt
6  from plotting import plot_main
7  from pathlib import Path

```

```

8
9 PROJECT_ROOT = Path(__file__).resolve().parents[0]
10 FIGURES_DIR = PROJECT_ROOT / "figures"
11
12 def prism(depth, radius):
13     '''Creates the x, y, and z coordinates for the two corners of a prism/cube
14
15     :param depth: distance to the prism center
16     :param radius: radius of the prism/cube
17
18     Note: Assume that x1, y1, and z1 are lower than x2, y2, and z2, respectively. All values
19     should be positive
20     '''
21     if depth < 0:
22         raise ValueError('Depth should be positive')
23
24     if radius < 0:
25         raise ValueError('Radius should be positive')
26
27     z1 = depth - radius
28     z2 = depth + radius
29     x1 = -radius
30     y1 = -radius
31     x2 = radius
32     y2 = radius
33
34     return x1, y1, z1, x2, y2, z2
35
36
37
38 def gz_minus_target(_depth, _radius, _rho, target=0.3):
39     """Calculates the residual value between the current gravitational acceleration gz and
40     the target (default=0.3)
41
42     Args:
43         _depth (float): distance to the prism center
44         _radius (float): radius of the prism/cube
45         _rho (_type_): density of the prism
46         target (float, optional): The target gz. Defaults to 0.3.
47
48     Returns:
49         float: difference between current gz and the target.
50     """
51     x1, y1, z1, x2, y2, z2 = prism(_depth, _radius)
52     _gz = gbox(0.0, 0.0, 0.0, x1, y1, z1, x2, y2, z2, _rho)
53     return _gz - target
54
55 def find_radius(depth, rho, target=0.3, tol=1e-6, rmax=100.0):
56     """_summary_
57
58     Args:
59         depth (_type_): _description_
60         rho (_type_): _description_
61         target (float, optional): _description_. Defaults to 0.3.
62         tol (_type_, optional): _description_. Defaults to 1e-6.
63         rmax (float, optional): _description_. Defaults to 100.0.
64
65     Raises:
66         ValueError: _description_
67
68     Returns:
69         _type_: _description_
70     """
71     r_lo = 1e-10      # avoid zero-radius singularity
72     r_hi = rmax
73
74     # Ensure target is reachable
75     if gz_minus_target(depth, r_hi, rho, target) < 0:
76         raise ValueError("Target gravity not reached - increase rmax")

```

```

77
78     while (r_hi - r_lo) > tol:
79         r_mid = 0.5 * (r_lo + r_hi)
80         if gz_minus_target(depth, r_mid, rho, target) > 0:
81             r_hi = r_mid
82         else:
83             r_lo = r_mid
84
85     return 0.5 * (r_lo + r_hi)
86
87 # OBSERVATION POINTS
88 X0 = 0.0
89 Y0 = 0.0
90 Z0 = 0.0
91 RHO = 3320 # [kg/m^3]
92
93 if __name__ == "__main__":
94     range_distances = np.logspace(-3,-1, 10000)
95     # range_distances = np.logspace(np.log10(0.00261),2, 10000)
96
97     gravity = []
98     depth = []
99     width = []
100
101     for zc in tqdm(range_distances):
102
103         r = find_radius(zc, RHO, tol=1e-8, rmax = zc+0.1)
104         x1, y1, z1, x2, y2, z2 = prism(zc, r)
105
106         gz = gbox(
107             0.0, 0.0, 0.0,
108             x1, y1, z1,
109             x2, y2, z2,
110             RHO
111         )
112
113         gravity.append(gz)
114         depth.append(zc)
115         width.append(2 * r)
116
117     radius = np.array(width)/2
118     depth = np.array(depth)
119     difference = depth - radius
120
121     idx = np.argmin(np.abs(depth-radius))
122     print(idx)
123     print(f'At depth = radius: {depth[idx]=}, {width[idx]=}, {radius[idx]=} {gravity[idx]=}')
124
125     plot_main(depth, difference, 'Depth [km]', 'Depth-Radius Difference [km]', 'Prism Depth
        vs Depth-Radius Difference for Target Gravity of 0.3 mGal,\n assuming a density of
        3320 kgm$^{-3}$', plot_type='loglog', save_path=FIGURES_DIR / "
        prism_depth_radius_difference.png")
126
127
128
129     range_distances = np.logspace(np.log10(depth[idx]),1, 1000)
130     # range_distances = np.logspace(np.log10(0.00261),2, 10000)
131
132     gravity = []
133     depth = []
134     width = []
135
136     for zc in tqdm(range_distances):
137
138         r = find_radius(zc, RHO, tol=1e-8, rmax = zc+0.1)
139         x1, y1, z1, x2, y2, z2 = prism(zc, r)
140
141         gz = gbox(
142             0.0, 0.0, 0.0,
143             x1, y1, z1,
144             x2, y2, z2,

```



```

145         RHO
146     )
147
148     gravity.append(gz)
149     depth.append(zc)
150     width.append(2 * r)
151
152     plot_main(depth, width, 'Depth [km]', 'Width [km]', 'Prism Depth vs Width for Target
        Gravity of 0.3 mGal,\n assuming a density of 3320 kgm$^{-3}$', save_path=FIGURES_DIR
        / "prism_depth_width_regular.png")
153
154     plot_main(depth, width, 'Depth [km]', 'Width [km]', 'Prism Depth vs Width for Target
        Gravity of 0.3 mGal,\n assuming a density of 3320 kgm$^{-3}$', plot_type='loglog',
        save_path=FIGURES_DIR / "prism_depth_width_loglog.png")

```

A.4. constants.py

```

1 from numpy import pi
2
3 GRAV_CONS = 6.67408e-11 # [m^3 kg^-1 s^-2]
4
5 KM2M = 1e+3 # km to m
6 G2mGAL = 1e5 # m/s^2 to milliGal
7
8 # MATH
9 ZERO = 0.0
10 HALF = 0.5
11 ONE = 1.0
12 TWO = 2.0
13 THREE = 3.0
14 FOUR = 4.0
15 ONE_THIRD = ONE / THREE
16 TWO_THIRD = TWO / THREE
17 FOUR_THIRD = FOUR / THREE
18 PI = pi
19 HALFPI = HALF * PI
20 TWOPI = TWO * PI
21
22 # Tolerance values
23 INFTOL = 1.0e32
24 ZEROTOL = 1.0e-12

```

A.5. potential_library.py

```

1
2 import numpy as np
3 import math
4
5 from constants import *
6
7 def gbox(x0,y0,z0,x1,y1,z1,x2,y2,z2,rho):
8     '''
9     Docstring for gbox
10
11
12     :param x0: observation point x coordinate in [km]
13     :param y0: observation point y coordinate in [km]
14     :param z0: observation point z coordinate in [km]
15     :param x1: x coordinate one side prism in [km]
16     :param y1: y coordinate one side prism in [km]
17     :param z1: z coordinate one side prism in [km]
18     :param x2: x coordinate other side prism in [km]
19     :param y2: y coordinate other side prism in [km]
20     :param z2: z coordinate other side prism in [km]
21     :param rho: density of prism in [kg/m^3]
22
23     Output: gravity g in [mGal]
24
25     '''
26     ISIGN = np.array([-1,1])

```

```

27
28
29
30 x = np.array([x0-x1,x0-x2])
31 y = np.array([y0-y1,y0-y2])
32 z = np.array([z0-z1,z0-z2])
33
34 rsum = 0
35
36 for i in range(len(x)):
37     for j in range(len(y)):
38         for k in range(len(z)):
39             rijk = np.sqrt(x[i]**2+y[j]**2+z[k]**2)
40             ijk = ISIGN[i]*ISIGN[j]*ISIGN[k]
41             arg1 = np.arctan2((x[i]*y[j]), (z[k]*rijk))
42             if arg1 < 0:
43                 arg1 = arg1 + TWOPI
44             arg2 = rijk + y[j]
45             arg3 = rijk + x[i]
46             if arg2<=0:
47                 print('GB0X: bad field point (arg2)')
48             if arg3<=0:
49                 print('GB0X: bad field point (arg3)')
50             arg2 = np.log(arg2)
51             arg3 = np.log(arg3)
52             rsum = rsum+ijk*(z[k]*arg1-x[i]*arg2-y[j]*arg3)
53 g = rho*GRAV_CONS*rsum*G2mGAL*KM2M
54 return g

```

B

DTM and Lava Tube Model Generation

In this appendix, a more detailed explanation is given of the processes for obtaining both the DTMs and the lava tubes in this thesis. They can be found in section B.1 and section B.2, respectively.

B.1. Acquisition of the DTM

In this section, the way that DTMs were obtained in this thesis is explained. The general procedure is outlined in Figure B.1.

As can be seen in Figure B.1, to obtain the DTM, a couple of different steps are needed. First, using GDAL (Geospatial Data Abstraction Library, GDAL/OGR contributors (2026)), the downloaded JP2 or TIF is translated and warped into a system that falls in line with the Moon. Using Python code or GIS software, bad pixels can be filtered out of the DTMs. This was necessary a handful of times, as otherwise meshing in CUBIT would be hampered.

Afterwards, a Python script is used to transform the TIF into an STL file. This STL file can then be read into Coreform Cubit, where the surface becomes a volume by first synthesising a net surface, which is then swept.

It is important that GDAL is first used to set the coordinate reference system (CRS), such that the correct measurements are taken, as well as the right projections.

```
1 gdal_translate.exe SLDEM2015_128_60S_60N_000_360.jp2 output.tif
2 gdal_translate.exe -projwin_srs "+proj=longlat +R=1737400 +no_defs" -projwin -57.465
   13.738 -56.375 12.707 output.tif mariushills_1L.tif
```

Because a raster with a higher resolution was found online directly in the TIF format (U.S. Geological Survey 2015), this conversion was not actually performed in reality. Instead, just the projection conversion was performed:

```
1 gdal_translate.exe -projwin_srs "+proj=longlat +R=1737400 +no_defs" -projwin -57.465
   13.738 -56.375 12.707 .\LunarLR0L0LAKaguya_MAP2_13.6N303.1-EQUI.tif mariushills_1L.
   tif
```

This process, which was originally done using GDAL, ended up being automated in this thesis using Python code. Existing Python packages wrap GDAL, which allows for easy use in automating obtaining the right TIF file.

After obtaining the TIF in the correct projections, the lava tube is ready to be turned into an STL. For this, a Python code was written and can be seen in section A.1.

As such, the STL is created. This STL can then be used in the obtainment of the lava tube, which is specified in subsection 3.2.2. However, for the model to be easily usable in Boolean operations in CUBIT, it must first be turned into a STEP file.

This is done inside CUBIT and happens as follows:

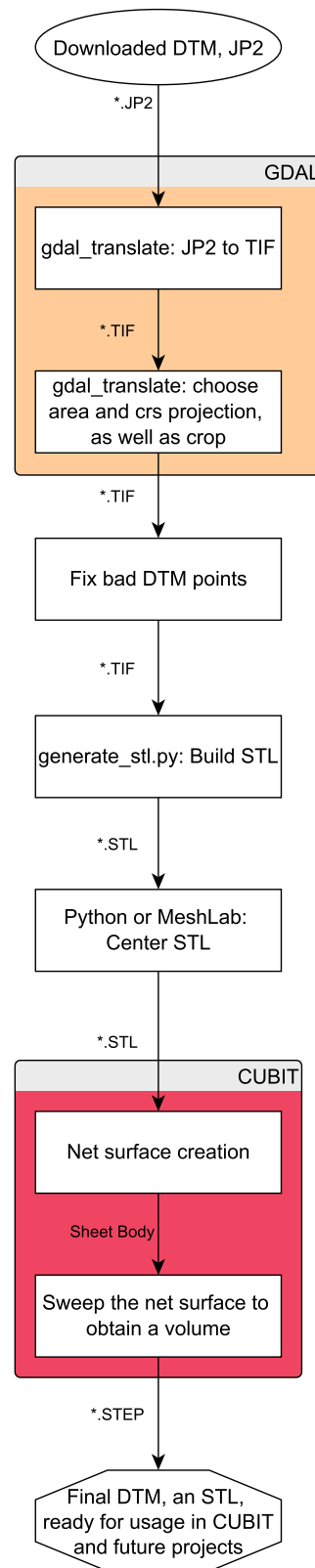


Figure B.1: An overview of the steps used to obtain the DTM 3D model of any area on the lunar surface.

- The STL is imported into Coreform Cubit.
- A net surface is created from the STL surface.
- The net surface is swept downward to obtain a volume.
- The volume is centred at (0,0,0).
- The volume is cut on all of its sides *except* the DTM side with all the heights. This is to ensure that each of those sides is flat and planar.

This file can then be exported to be used in future meshing and, afterwards, simulations. During the thesis, this last step was performed directly in the script of each simulation, as it would allow for more flexibility in creating meshes.

B.2. Acquisition of the Lava Tube

The process for obtaining the lava tube is a more complex one than for obtaining the DTM. This is due to the fact that multiple steps have to be taken to obtain the exact shape of the lava tube desired. An overview of the process needed to obtain the lava tube can be seen in Figure B.2.

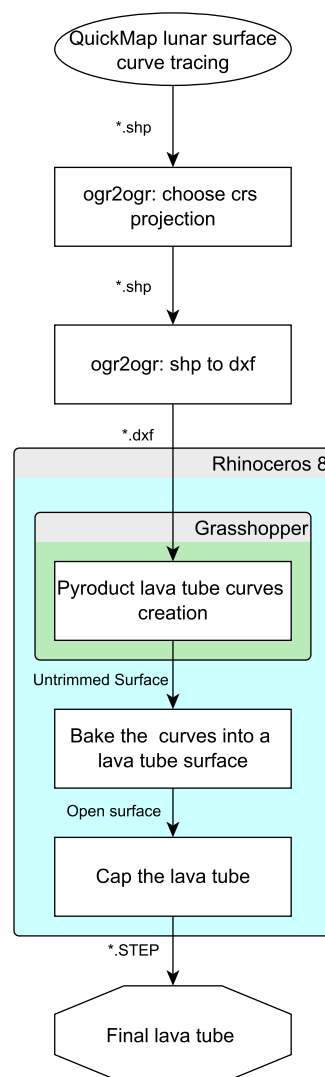


Figure B.2: An overview of the steps used to obtain the lava tube 3D model.

Figure B.2 illustrates that the first step is obtaining curves using QuickMap, after which Rhino 8 and

Grasshopper are used to generate the lava tube from these curves and other inputs.

Obtaining the lava tube 2D curves from QuickMap

The first step in the process of obtaining a model of a lunar lava tube is to obtain the 2D curves of the potential lava tube of interest. In this thesis, this is done by using the QuickMap tool, developed by NASA, Arizona State University & Applied Coherent Technology Corp. Here, the area of interest was first located, after which the curves around the lava tube were drawn by hand.

As is quite obvious, the curves are drawn by hand on the computer and are therefore not extremely accurate. This results in an error in the accuracy of the lunar lava tube model. However, for this thesis, the exact lava tube model is not of high interest, while the variations in shape and density are. Therefore, such human inconsistencies do not affect the outcome of this thesis.

The exact curve used for the Marius Hills lava tube can be seen in Figure B.3.

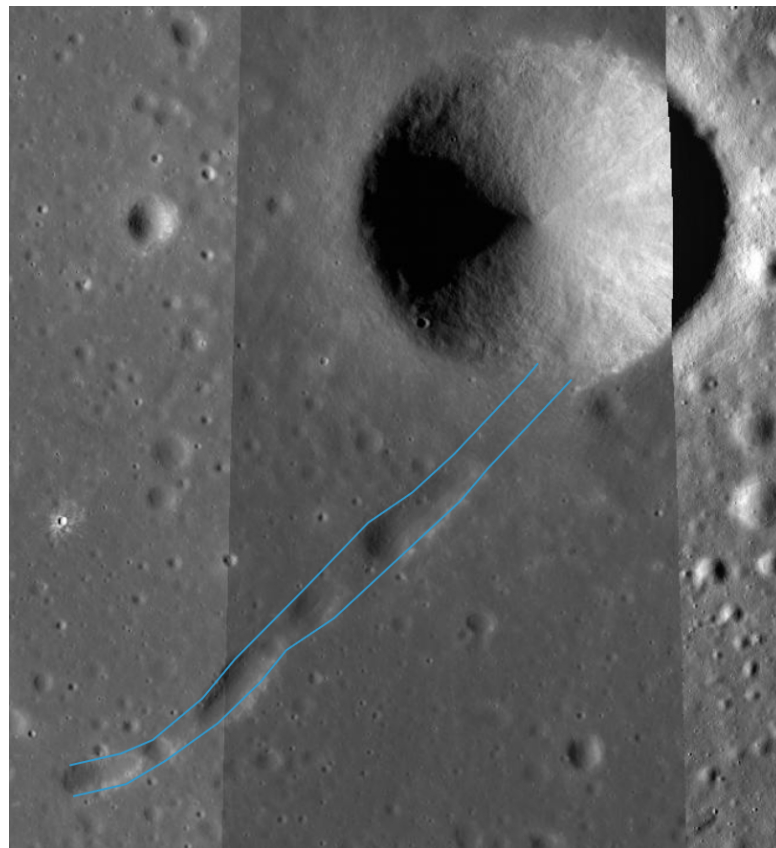
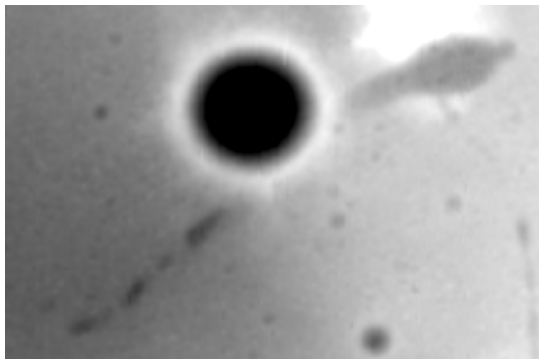
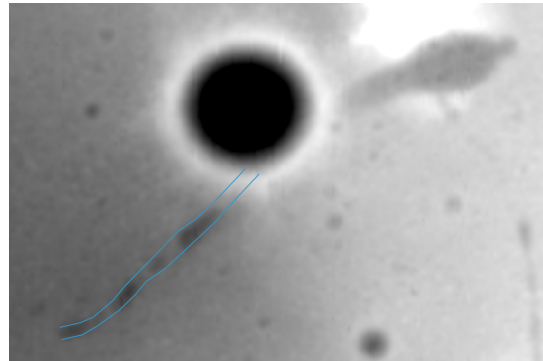


Figure B.3: The curves drawn in QuickMap (Arizona State University, NASA, and Applied Coherent Technology Corp. 2024), with the high-resolution mosaic used as a reference.



(a) A DTM map of the Marius Hills area, with the allegedly partially collapsed lava tube in sight.



(b) A DTM map of the Marius Hills area, with the allegedly partially collapsed lava tube in sight, as well as the curves drawn by the author, which were used to obtain the lava tube model.

Figure B.4: The Marius Hills DTM with and without the curves drawn for creation of the lava tube.

As can be seen in Figure B.4a and Figure B.4b, the lava tube was drawn until it reached the crater. This was a choice made by the author, as there is no real way to understand the size or shape of the partially collapsed lava tube at the location of this crater. Next to that, as stated in Sauro et al. (2020), a putative vent is present in the top-right part of the images in Figure B.4a and Figure B.4b. This vent is much larger than the actual lava tube part of this structure, so that part was also left out of the model. In addition, the accurate modelling of this feature is not part of the scope of this thesis and therefore not important.

Once the curves have been drawn, they can be downloaded from QuickMap in the shapefile (.shp) format. This file contains the information on the shape and location of the curves and will be input for the next steps.

For the Gruithuisen K lava tube, the same procedure was performed, and the curves drawn can be seen in Figure B.5.

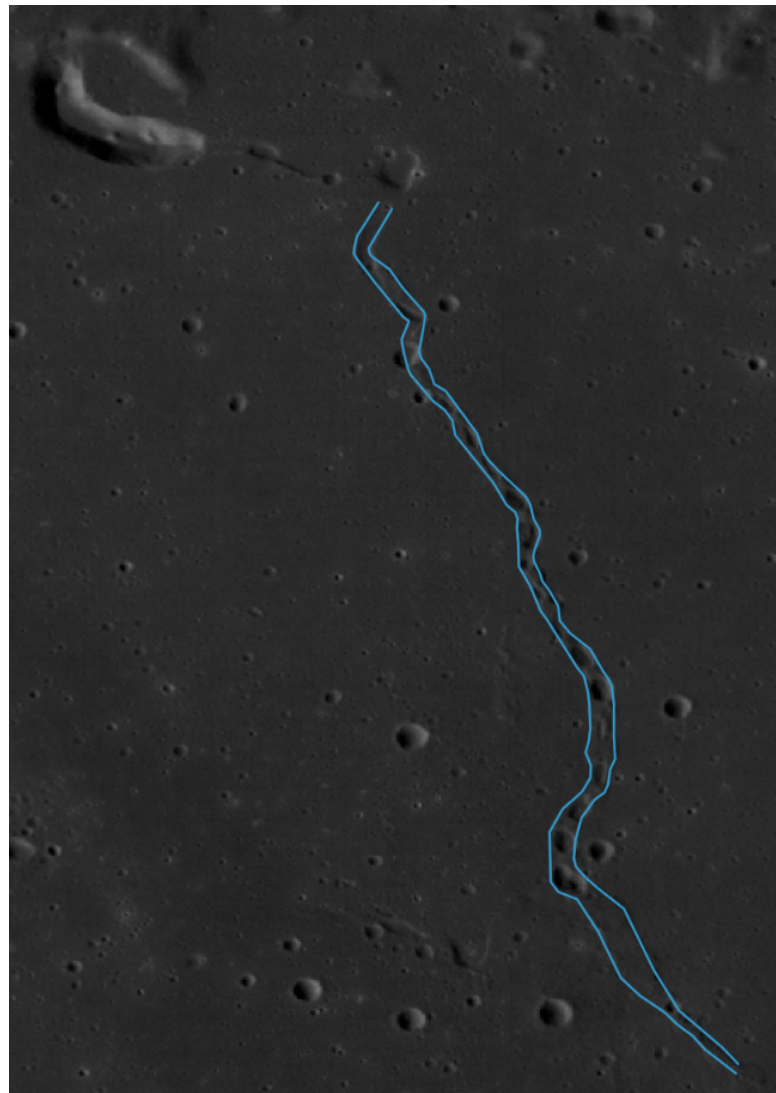


Figure B.5: The Gruithuisen K area and the curves drawn to obtain the lava tube.

As can be seen in Figure B.5, most of the lava tube was included, except the top part near the putative vent. This was not included as it was unclear to the author what parts still belonged to the collapse chain and what parts did not.

Using ogr2ogr

Once the lava tube curves have been obtained, the curves have to be converted to a format that can be used inside the Grasshopper and Rhino 8 environment. For this, the ogr2ogr tool was used. This tool, which originates from GDAL, allows users to perform operations on raster and vector geospatial data formats.

The following commands were used for the conversion from shapefile to DXF:

```
1 ogr2ogr -f "ESRI Shapefile" bottom_aligned.shp .\user-features.shp -s_srs "+proj=longlat
   +a=1737400 +b=1737400 +no_defs" -t_srs "+proj=eqc +lat_ts=13.6 +lon_0=-56.9 +a
   =1737400 +b=1737400 +units=m +no_defs"
2 ogr2ogr -f DXF bottom_aligned.dxf bottom_aligned.shp
```

As can be seen, a custom coordinate reference system (CRS) is specified by stating the radius of the body as well as the exact longitude, latitude and projection used. This helps convert it into a format more easily readable by Rhino 8. As such, the resulting DXF format is importable into Rhino 8.

Rhino 8 lava tube synthesis

Using the imported curves in Rhino 8, the lava tube is ready to be constructed. This is done using the Grasshopper tool of Rhino 8, as well as using the pyroduct software (Romio, Lobosco, et al. 2026), which was developed for the sole purpose of creating lava tube models.

The exact functioning of the pyroduct software is explained in more depth in Romio, Lobosco, et al. (2026), but a quick overview of the steps used in this thesis will be presented. Next to that, the exact Grasshopper files, including the curve files, are available on the GitHub page for download.

The quick overview can be seen in Figure B.6.

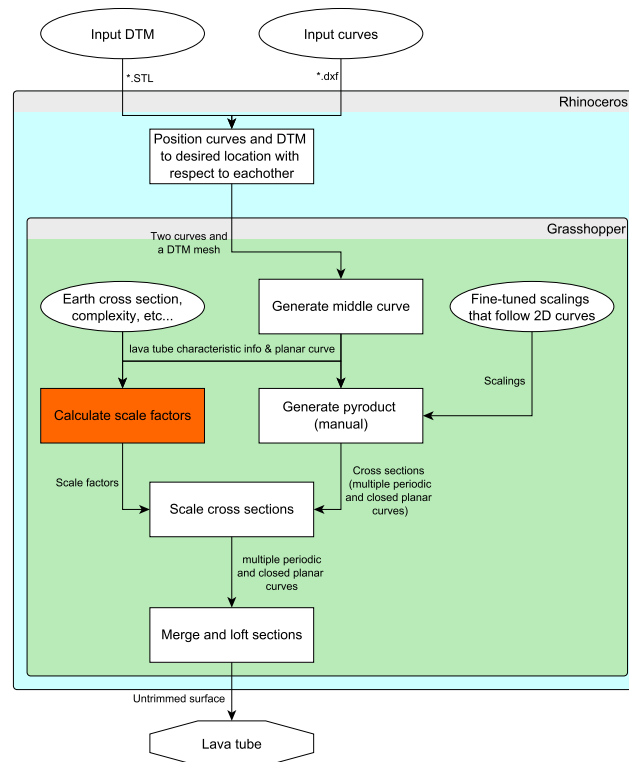
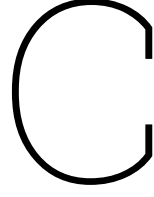


Figure B.6: An overview of the steps taken inside Rhino 8 and Grasshopper to obtain the lunar lava tube models. The orange process block indicates a custom block that was made to automate the scaling of the cross-section widths.

As can be seen in Figure B.6, the process inside Rhino 8 is not too complex. The two imported curves allow the creation of a middle curve, which is projected onto a DTM mesh. A choice is to be made on the lava tube cross-sections: many terrestrial examples are available, and all provide slightly different cross-sections. These cross-sections are then placed along the middle curve, interpolated and then lofted to obtain the final lava tube.

An interesting part, however, is that when the cross-sections are placed along the middle curve, they have to be manually scaled to the width of the initial curves. This is due to the fact that every cross-section has a different absolute size, and therefore scale. When a user decides to include many sections in their lava tube, the process becomes quite manual. Therefore, the author of this thesis implemented a block that can take the original cross-section widths, as well as the distance between the two initial curves at the beginning and end of each desired section, to automatically scale the cross-sections such that they are as wide as the distance between the two initial curves.

This custom block can be seen in detail in Figure B.7.



Analytical Solutions for Gravitational Anomaly Benchmarks

In this appendix, the analytical solutions for a number of situations are given, including their references. These analytical solutions were used in Gharti, Tromp, and Zampini (2018), where a comparison was made between GBOX and SPECFEM. Part of this comparison was redone and verified in this thesis, using the same analytical solutions. Therefore, the analytical solutions used as reference in Gharti, Tromp, and Zampini (2018) are presented here. Firstly, the analytical solution for a rectangular prism is given. Afterwards, the solution for a sphere is given, as well as one for multiple spheres. Lastly, the solution for a prism with a parabolic density distribution is given.

C.1. Rectangular Prism

The analytical solution for a rectangular prism, in this case, is defined by the GBOX formula. This was already given in Equation 2.8, but will be repeated here.

$$g = \gamma \rho \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \mu_{ijk} \left[z_k \arctan \frac{x_i y_j}{z_k R_{ijk}} - x_i \log(R_{ijk} + y_j) - y_j \log(R_{ijk} + x_i) \right] \quad (C.1)$$

Where,

$$R_{ijk} = \sqrt{x_i^2 + y_j^2 + z_k^2},$$
$$\mu_{ijk} = (-1)^i (-1)^j (-1)^k.$$

The x_i , y_j , and z_k describe the endpoints of the brick in each dimension. γ and ρ describe the gravitational constant and the density of the prism, respectively.

C.2. Sphere

The analytical solution to calculate the gravitational attraction due to a sphere is given in Equation C.2, taken from Turcotte and Schubert (2014).

$$\delta g_z(x, 0, z) = \frac{4\pi G r^3 \delta \rho}{3} \frac{z_s}{(x^2 + z^2)^{\frac{3}{2}}}, \quad (C.2)$$

Where r is the radius of the sphere, $\delta \rho$ the density anomaly, z_s the depth of the sphere, and G the gravitational constant.

C.3. Multiple Spheres

The analytical solution for multiple spheres is given quite easily and derived by summing the formula for an individual sphere over multiple spheres. It is given in Equation C.3 and taken from Gharti, Tromp, and Zampini (2018).

$$\delta g_z(x, 0, z) = \frac{4\pi G}{3} \sum_{s=1}^{n_s} r_s^3 \delta \rho_s \frac{z_s}{[(x - x_s)^2 + (z - z_s)^2]^{\frac{3}{2}}} \quad (\text{C.3})$$

C.4. Rectangular Prism with a Parabolic Density Distribution

The analytical formula for a rectangular prism with a parabolic density distribution is given by Chakravarthi, Raghuram, and Singh (2002). Gharti, Tromp, and Zampini (2018) states that the form given in the original has typos and is therefore incorrect; the correct form is then given in Equation C.4 and was taken from Gharti, Tromp, and Zampini (2018).

$$\begin{aligned} \delta g_z(x, 0, 0) = & 2G\delta\rho_0^3 \left\{ \frac{\alpha Y(x+T)[2\delta\rho_0^2 + \alpha^2(Y^2 + (x+T)^2)]}{L_1(Y^2\alpha^2 + \delta\rho_0^2)[\delta\rho_0^2 + \alpha^2(x+T)^2]} \ln \frac{L_3(\alpha r_4 L_1 + L_1^2 - \delta\rho_0 L_4)}{L_4(\alpha r_1 L_1 + L_1^2 - \delta\rho_0 L_3)} \right. \\ & - \frac{\alpha Y(x-T)[2\delta\rho_0^2 + \alpha^2(Y^2 + (x-T)^2)]}{L_2(Y^2\alpha^2 + \delta\rho_0^2)[\delta\rho_0^2 + \alpha^2(x-T)^2]} \ln \frac{L_3(\alpha r_3 L_2 + L_2^2 - \delta\rho_0 L_4)}{L_4(\alpha r_2 L_2 + L_2^2 - \delta\rho_0 L_3)} \\ & - \frac{\delta\rho_0}{\alpha(\delta\rho_0^2 + Y^2\alpha^2)} \left[\tan^{-1} \frac{Y r_4}{z_2(x+T)} - \tan^{-1} \frac{Y r_1}{z_1(x+T)} \right] \\ & + \frac{\delta\rho_0}{\alpha(\delta\rho_0^2 + Y^2\alpha^2)} \left[\tan^{-1} \frac{Y r_3}{z_2(x-T)} - \tan^{-1} \frac{Y r_2}{z_1(x-T)} \right] \\ & - \frac{\delta\rho_0}{\alpha[(\delta\rho_0^2 + (x+T)^2\alpha^2)]} \left[\tan^{-1} \frac{(x+T)r_4}{z_2 Y} - \tan^{-1} \frac{(x+T)r_1}{z_1 Y} \right] \\ & + \frac{\delta\rho_0}{\alpha[\delta\rho_0^2 + (x-T)^2\alpha^2]} \left[\tan^{-1} \frac{(x-T)r_3}{z_2 Y} - \tan^{-1} \frac{(x-T)r_2}{z_1 Y} \right] \\ & + \frac{Y}{2(\delta\rho_0^2 + Y^2\alpha^2)} \ln \frac{[(x+T) - r_4][(x+T) + r_1]}{[(x+T) + r_4][(x+T) - r_1]} \\ & - \frac{Y}{2(\delta\rho_0^2 + Y^2\alpha^2)} \ln \frac{[(x-T) - r_3][(x-T) + r_2]}{[(x-T) + r_3][(x-T) - r_2]} \\ & - \frac{x-T}{2[\delta\rho_0^2 + (x-T)^2\alpha^2]} \ln \frac{(Y-r_3)(Y+r_2)}{(Y+r_3)(Y-r_2)} \\ & + \frac{x+T}{2[\delta\rho_0^2 + (x+T)^2\alpha^2]} \ln \frac{(Y-r_4)(Y+r_1)}{(Y+r_4)(Y-r_1)} \\ & + \frac{1}{\alpha(\delta\rho_0 - \alpha z_2)} \left[\tan^{-1} \frac{Y(x+T)}{z_2 r_4} - \tan^{-1} \frac{Y(x-T)}{z_2 r_3} \right] \\ & - \frac{1}{\alpha(\delta\rho_0 - \alpha z_1)} \left[\tan^{-1} \frac{Y(x+T)}{z_1 r_1} - \tan^{-1} \frac{Y(x-T)}{z_1 r_2} \right] \Big\} , \end{aligned} \quad (\text{C.4})$$

where,

$$\begin{aligned} L_1 &= \left[((x+T)^2 + Y^2)\alpha^2 + \delta\rho_0^2 \right]^{\frac{1}{2}} , \\ L_2 &= \left[((x-T)^2 + Y^2)\alpha^2 + \delta\rho_0^2 \right]^{\frac{1}{2}} , \\ L_3 &= \delta\rho_0 - \alpha z_1 , \\ L_4 &= \delta\rho_0 - \alpha z_2 , \end{aligned}$$

and

$$r_1 = \left[(x+T)^2 + Y^2 + z_1^2 \right]^{\frac{1}{2}} ,$$

$$r_2 = \left[(x-T)^2 + Y^2 + z_1^2 \right]^{\frac{1}{2}} ,$$

$$r_3 = \left[(x-T)^2 + Y^2 + z_2^2 \right]^{\frac{1}{2}} ,$$

$$r_4 = \left[(x+T)^2 + Y^2 + z_2^2 \right]^{\frac{1}{2}} .$$

D

Detailed Insight GBOX and SPECFEM software

In this appendix, the exact way GBOX and SPECFEM were used to obtain the results in chapter 5 is shown. First, the procedure for performing a GBOX simulation is explained, after which the procedure for SPECFEM is discussed.

D.1. GBOX Simulations

In this section, the code used in GBOX is explained, after which the method of preparing the 3D model is shown. Then, a description of how to perform the simulation as well as the analysis of the results are displayed.

D.1.1. GBOX code

Firstly, the GBOX code itself must be explained. As stated in subsection 2.4.1, the code originates from Plouff (1976) and has been repeated in Blakely (1995) with an implementation in Fortran. For this thesis, the GBOX code implementation in Fortran, which was implemented by Gharti, Tromp, and Zampini (2018), is used. In addition, to obtain the code for the graph showcasing the minimum resolution that can be resolved for each depth, Figure 4.11, the GBOX code was transferred to Python.

The original code for GBOX used is provided by Blakely (1995):

```
1  subroutine gbox(x0,y0,z0,x1,y1,z1,x2,y2,z2,rho,g)
2  c
3  c Subroutine GBOX computes the vertical attraction of a
4  c rectangular prism. Sides of the prism are parallel to the x,y,z axes,
5  c and the z axis is vertical down,
6  c Input parameters:
7  c  Observation point is (x0, y0, z0). The prism extends from x1
8  c  to x2, from y1 to y2, and from z1 to z2 in the x, y, and z
9  c  directions, respectively. Density of prism is rho. All
10 c  distance parameters in units of km; rho in units of
11 c  kg/(m**3).
12 c
13 c Output parameters:
14 c  Vertical attraction of gravity g, in mGal.
15 c  real km2m
16 c  dimension x(2),y(2),z(2),isign(2)
17 c  data isign/-1,1/,gamma/6.670e-11/,twopi/6.2831853/,
18 c  si2mg/1.e5/,km2m/1.e3/
19 c  x(1)=x0-x1
20 c  y(1)=y0-y1
21 c  z(1)=z0-z1
22 c  X(2)=x0-x2
23 c  y(2)=y0-y2
24 c  z(2)=z0-z2
```

```

25      sum=0.
26      do 1 i=1,2
27          do 1 j=1,2
28              do 1 k=1,2
29                  rijk=sqrt(x(i)**2+y(j)**2+z(k)**2)
30                  ijk=isign(i)*isign(j)*isign(k)
31                  arg1=atan2((x(i)*y(j)), (z(k)*rij k))
32                  if(arg1.lt.0.) arg1=arg1+twopi
33                  arg2=rij k+y(j)
34                  arg3=rij k+x(i)
35                  if(arg2.le.0.) pause 'GBOX: Bad field point'
36                  if(arg3.le.0.) pause 'GBOX: Bad field point'
37                  arg2=aalog(arg2)
38                  arg3=aalog(arg3)
39                  sum=sum+ijk*(z(k)*arg1-x(i)*arg2-y(j)*arg3)
40              continue
41      g=rho*gamma*sum*si2mg*km2m
42      return
43      end

```

This code can then be used by repeatedly calling it to obtain the gravity anomaly of any 3D shape to any accuracy, as long as it has been discretised finely enough. The code for this, which was used in Gharti, Tromp, and Zampini (2018), is written in Fortran and used in this thesis for any GBOX calculations. This code will be called 'gpotential' and its basic workflow is as shown in Figure D.1. It must be noted that the name 'GBOX' mainly refers to the subroutine that can calculate the gravitational attraction in the z-direction, while 'gpotential' refers to the program that uses the GBOX subroutine to automatically calculate the gravity anomaly for any 3D shape that has been discretised into cubic/rectangular cells (all pointing in the same direction). However, in this thesis, the term 'GBOX' will also be used to refer to the gpotential program as a whole, as GBOX is a more well-known term.

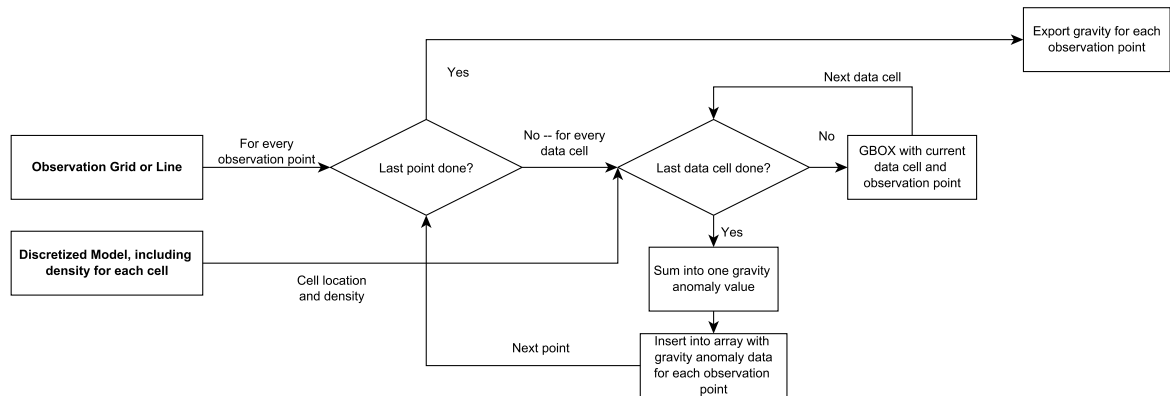


Figure D.1: A flow diagram describing the basic procedure for using the gpotential program in calculating the gravity anomaly of an arbitrary 3D shape.

It must be noted that as GBOX calculates the gravitational attraction only in one direction (namely the z-direction), gpotential will also be given for this direction only and is therefore smaller than if the absolute gravity anomaly were to be calculated.

D.1.2. 3D Model Preparation

In this subsection, the procedure for obtaining a 3D model suitable to be used in the implemented GBOX software to calculate the gravity anomaly for any 3D object is outlined. The exact steps and software necessary are explained such that anyone with the code and software should be able to perform the exact same steps and obtain the same results.

To obtain a 3D model, which includes density specifications, such that it can be imported into GBOX, the steps to be followed can be seen in Figure D.2.

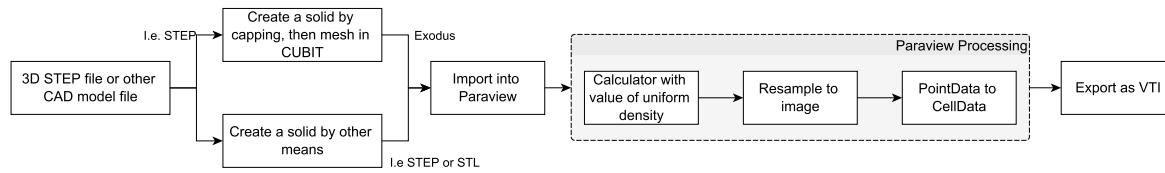


Figure D.2: The procedure for obtaining a VTI file containing the discretised 3D model that includes density information. The exported file is ready to be used by the GBOX gravity potential software.

As can be seen in Figure D.2, multiple software programs have to be used to obtain the right format for use in GBOX. In the current thesis, the initial 3D CAD model is made with Rhino 8. Here, the exported file is a STEP file.

D.1.3. Performing Simulations

In this subsection, the steps for performing the actual simulation once the prerequisites have been obtained are explained. This includes what an input file should look like, and what to pay attention to.

When using GBOX, one is actually using a Fortran-compiled executable called ‘gpotential’, which implements GBOX to allow calculation and summation over multiple cells and multiple observation points. The input file for using this executable is custom and consists of a template as given in Listing D.1.

Listing D.1: The GBOX gravity potential input file template

```

1 # Example:
2 GVTI
3 # Mass density file (Grid interval: )
4 input/
5 # Distance unit conversion factor
6 0.001
7 # Number of observation lines
8 1
9 # Observation line (x1,y1,z1/x2,y2,z2/dx,dy,dz)
10 0.0 0.0 0.0
11 100.0 0.0 0.0
12 1.0 1.0 1.0

```

As can be seen in Listing D.1, five main inputs need to be defined:

- Type of input file: In this case, GVTI indicates that a VTI file format is expected as input to GBOX. In the current code, this input file type is custom-made. To allow for inputting other file types, more code has to be written.
- Location of the input file: This describes the location of the ‘.vti’ file, which includes all the data of the 3D model.
- Unit conversion factor: This conversion factor can be used to convert a model described in kilometres to metres or vice-versa.
- The number of observation lines: This indicates the number of observation lines to be used.
- The observation line: This indicates the grid or line which holds the observation points to be used for the simulation. In Listing D.1, the observation points will be running from 0 to 100 km, at 1 km intervals (i.e. [0, 1.0, 2.0, ..., 99.0, 100.0])

Then, using the input file and the executable, the program can be run using the command `./bin/gpotential input/[input_filename].in`, where ‘[input_filename]’ indicates the name of the input file. This will create an output file with the same name as the input file, with a file type ‘.grav’. This file contains the gravity attraction information and can be analysed using any desired method.

D.1.4. Analysis

After performing the simulation, the data can be analysed. In the following list, various ways are shown that can help do this:

- ParaView. To import, ensure that the file type is changed to a readable format such as '.txt'.
- Gnuplot
- Python matplotlib module

ParaView allows for a good analysis by using its 2D Delaunay filter. This is especially useful when an observation *grid* has been used, instead of just a line. A smooth solution can then be obtained, such as the one shown in Figure D.3.

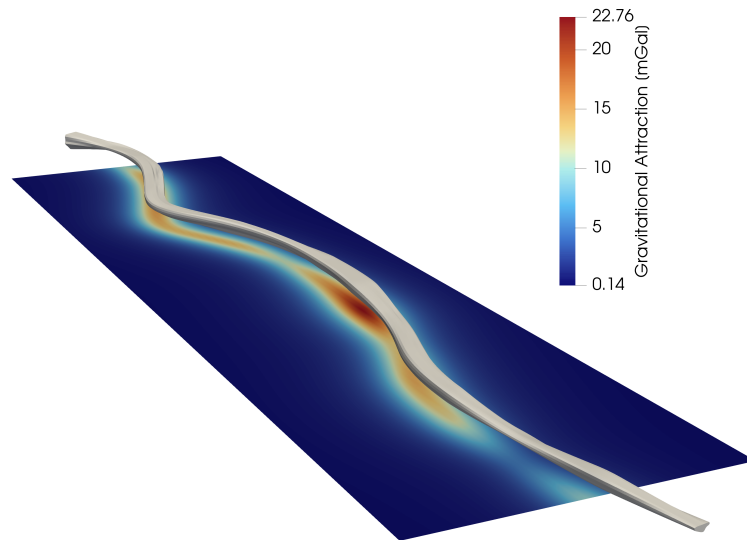


Figure D.3: A 3D view of an example lava tube model (Just shown for explanatory purposes) and its gravitational anomaly map, created using a 2D Delaunay filter in ParaView.

The exact graph as shown in Figure D.3 is obtained by importing the output to GBOX, with its file type changed to .txt, into ParaView. Afterwards, the following filters are applied:

- TableToPoints: This converts the .txt file into a table in ParaView that can then be manipulated further. The x, y, and z coordinate columns can be chosen.
- Delauney2D: This filter performs 2D Delaunay triangulation on the points in the field. The mesh that is created is then used for calculating gradients of planes between points, which allows for a 'smoothing' of field points.

To further analyse the data, custom code can be used. This is especially useful in the sensitivity analysis where multiple runs have to be compared against each other.

D.2. SPECFEM Simulations

In this section, the approach for preparing the SPECFEM simulation is described, after which the procedure for running the simulations and analysis afterwards is shown.

D.2.1. Preparing the Simulation

After the model has been meshed, and thus an exodus file has been obtained, the other necessary files for running the simulation can be created.

Firstly, the exodus format cannot be directly read by SPECFEM and first needs to be transformed. This is done using the exodus2sem tool that is provided together with specfem. Doing this creates the files listed as follows:

```
1 - SM_RP_05.sem
2 - SM_RP_05.txt
3 - SM_RP_05_connectivity
```

```

4 - SM_RP_05_coord_x
5 - SM_RP_05_coord_y
6 - SM_RP_05_coord_z
7 - SM_RP_05_infinite_recreated
8 - SM_RP_05_infinite_surface
9 - SM_RP_05_material_id
10 - SM_RP_05_nsbucx
11 - SM_RP_05_nsbucy
12 - SM_RP_05_nsbucz
13 - SM_RP_05_ssbucx
14 - SM_RP_05_ssbucy
15 - SM_RP_05_ssbucz

```

The sem file (also sometimes called psem to indicate a parallel run) looks as follows:

```

1 #pre information
2 preinfo: nproc=1, method='sem', ngllx=3, ngly=3, ngllz=3, nenode=8, ngnode=8, &
3 inp_path='./input', out_path='./output/', disp_dof=0, pot_dof=1, pot_type='gravity'
4
5 #mesh information
6 mesh: xfile='SM_RP_05_coord_x', &
7       yfile='SM_RP_05_coord_y', &
8       zfile='SM_RP_05_coord_z', &
9       confile='SM_RP_05_connectivity', &
10      fsfile='empty_free_surface', &
11      idfile='SM_RP_05_material_id'
12
13 #boundary conditions
14 bc: ubc=0, &
15     uxfile='SM_RP_05_ssbucx', &
16     uyfile='SM_RP_05_ssbucy', &
17     uzfile='SM_RP_05_ssbucz', &
18     phifile='empty_free_surface', &
19     infbc=1, add_infmesh=1, &
20     infrfile='SM_RP_05_infinite_surface', &
21     mat_type='define', &
22     imat_inf=3, &
23     pole0='user', &
24     coord0=0.0000 0.0000 -2250.0000, &
25     pole_type='point', &
26     rinf=50000.0, &
27     valinf=0
28 #material list
29 material: matfile='SM_RP_05_material_list', density=1
30
31 #control parameters
32 control: ksp_tol=1e-8, ksp_maxiter=20000, nl_tol=1e-4, nl_maxiter=1
33
34 #save data
35 save: gpot=1, agrav=1
36
37 #devel option
38 devel: nondim=0

```

The pre-information section defines what type of simulation it is, with what type of elements. In this case, a gravity simulation is performed. The mesh information indicates which files specify the coordinates. The boundary conditions section defines an important part of the simulation, namely the infinite elements. In this case, an infinite mesh is created, and a pole is defined at the origin. The material list section connects the material list file to SPECFEM. The control parameters section defines the parameters for convergence for the simulation, including the maximum number of iterations and the threshold for which to stop iterating. Lastly, the save data section defines what outputs to save, and the devel option section defines developer options: In this case, `nondim=0` specifies that results should already be in SI units, instead of being nondimensionalised.

An empty file is made so that it can be connected to the `fsfile` (free surface file), as this is not necessary for gravitational simulations. In the author's case, this was called `empty_free_surface`.

The `material_list` file defines what densities are given to what blocks in the mesh. As such, this

file defines one of the most important characteristics of each simulation. Here, also the density of the infinite elements is defined, which, in the case of all simulations in this thesis, is zero.

Then, once these files have been created according to the specifications, the input folder is complete.

Still, for the simulation to be run, especially for a parallel run, a dedicated run or submit file is useful in ensuring that each simulation is run properly. In this case, a `submit1` file is created, with contents similar to what can be seen as follows:

```

1 #!/bin/bash
2 #SBATCH --account=rrg-hngharti
3 #SBATCH -N 1 # node count
4 #SBATCH --ntasks-per-node=80
5 #SBATCH -t 03:50:00
6 #SBATCH --job-name=SM_RP_05
7 # clear old files
8 rm -r partition output tmp
9 mkdir -p partition output tmp
10 # load appropriate compiler
11 source ~/.login
12
13 cd $SLURM_SUBMIT_DIR
14
15 # problems with mpiexec/mpirun so use srun
16 ./bin/partmesh ./input/SM_RP_05.sem
17 mpiexec -n 1 ./bin/pspecfemx ./input/SM_RP_05.sem -ksp_monitor -ksp_type fgmres -pc_type asm
   -ksp_gmres_restart 300 -start_in_debugger

```

As can be seen, using SLURM (which includes sbatch as one of its commands), the parameters for how many nodes, the job name and how long the simulation can take maximally are defined. Then, the rest of the script runs like a normal bash script. In this case, the contextual programs are loaded through a `.login` file, after which the model is separated into parts by `partmesh`, followed by running the mesh through `mpiexec` and specifying the number of processors used.

Lastly, a symlink is made for the bin folder, which contains the `partmesh` and `pspecfemx` executables, such that these are easily accessed.

D.2.2. Performing Simulations

To run the SPECfEM simulations on Trillium, the cluster on which the simulations are performed, as well as on savant, the smaller computer that was available at the lab at Queen's University locally, a small bash script was written to automate submitting simulation runs. This script uses 'sbatch' and can be seen on GitHub.

This code can be used once all the folders with their individual files have been set up.

After running simulations, each folder will look as follows:

```

1 |-- SM_RP_05.log
2 |-- bin -> /home/hngharti/build/SPECfEMX/bin/
3 |   |-- partmesh
4 |   |-- pspecfemx
5 |   '-- specfemx
6 |-- input
7 |   |-- SM_RP_05.sem
8 |   |-- SM_RP_05.txt
9 |   |-- SM_RP_05_connectivity
10 |   |-- SM_RP_05_coord_x
11 |   |-- SM_RP_05_coord_y
12 |   |-- SM_RP_05_coord_z
13 |   |-- SM_RP_05_infinite_recreated
14 |   |-- SM_RP_05_infinite_surface
15 |   |-- SM_RP_05_material_id
16 |   |-- SM_RP_05_material_list
17 |   |-- SM_RP_05_nsbucx
18 |   |-- SM_RP_05_nsbucy
19 |   |-- SM_RP_05_nsbucz
20 |   |-- SM_RP_05_ssbucx
21 |   |-- SM_RP_05_ssbucy

```

```

22 | |-- SM_RP_05_ssbucz
23 | '-- empty_free_surface
24 |-- output
25 | |-- SM_RP_05_original_inf_proc0.case
26 | |-- SM_RP_05_original_inf_proc0.geo
27 | |-- SM_RP_05_original_proc0.case
28 | |-- SM_RP_05_original_proc0.geo
29 | |-- SM_RP_05_original_trinf_proc0.case
30 | |-- SM_RP_05_original_trinf_proc0.geo
31 | |-- SM_RP_05_proc0.case
32 | |-- SM_RP_05_proc0.geo
33 | |-- SM_RP_05_step0_proc0.gpot
34 | |-- SM_RP_05_step0_proc0.grav
35 | |-- SM_RP_05_step1_proc0.gpot
36 | '-- SM_RP_05_step1_proc0.grav
37 |-- partition
38 | |-- SM_RP_05_connectivity_proc0
39 | |-- SM_RP_05_coord_x_proc0
40 | |-- SM_RP_05_coord_y_proc0
41 | |-- SM_RP_05_coord_z_proc0
42 | |-- SM_RP_05_ghost_proc0
43 | |-- SM_RP_05_infinite_recreated_proc0
44 | |-- SM_RP_05_material_id_proc0
45 | '-- empty_free_surface_proc0
46 |-- slurm-1459089.out
47 |-- submit1
48 '-- tmp
49 |-- SM_RP_05_gdof
50 '-- SM_RP_05_gnum

```

Here, the outputs are obviously available in the output folder.

D.2.3. Analysis

Once the run has been performed, it must be analysed. This is done using a variety of methods. For SPECFEM, the main program used for visualising SPECFEM output data is ParaView. ParaView allows for visualising the output data format. In this case, the output files are '.case' and '.geo' files. The '.case' is imported into ParaView, after which the time is changed from 0 to 1, and the exact output data of interest, which in our case is 'gravity_acceleration'. Then, the colour spectrum can be set such that our results are clearly visible. To obtain further data, various additional filters can be added to show specific parts of the data.

In the case of this thesis, most visualisation was automated using the ParaView Python API.