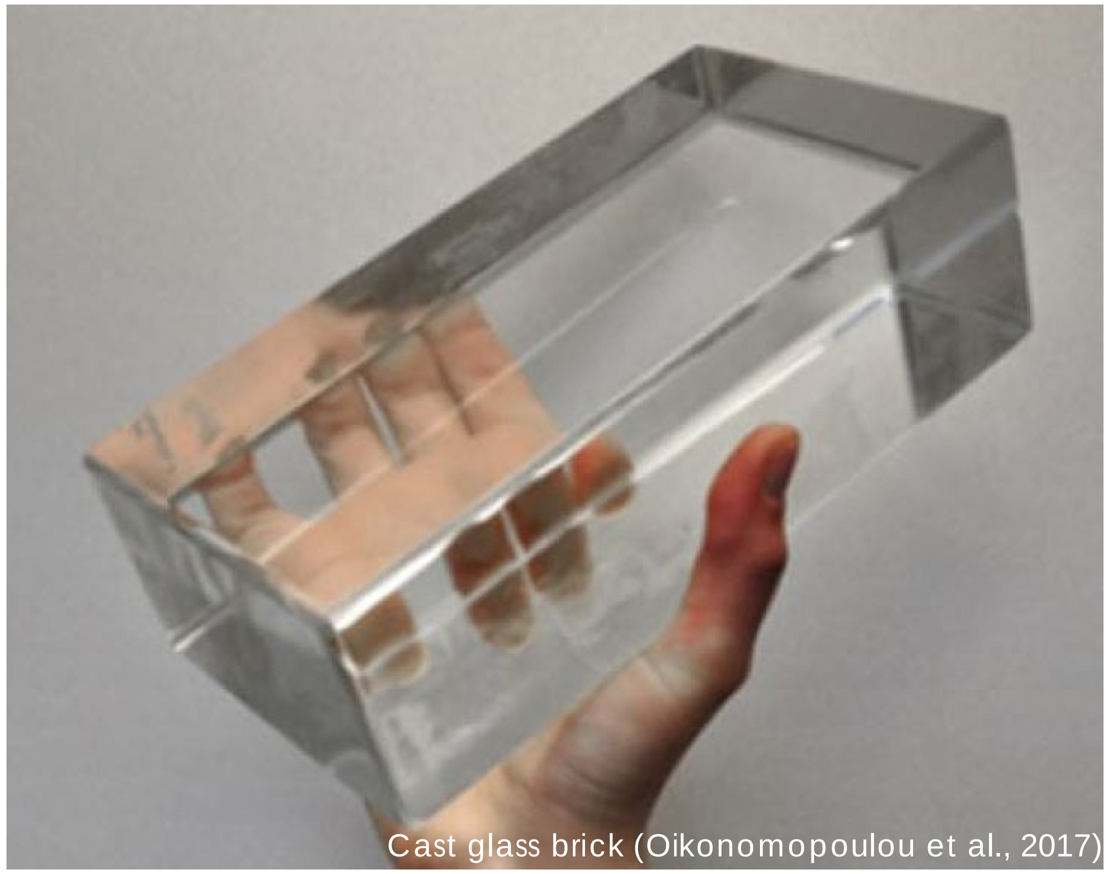
Glass Vaults

Introducing an adjustable mould for casting glass voussoirs for transparent shell structures

Felix van der Weijst

Cast glass


Cast glass brick (Oikonomopoulou et al., 2017)

High compressive strength
Low tensile strength
Resistance to buckling

Cast glass is a suitable material for shell structures

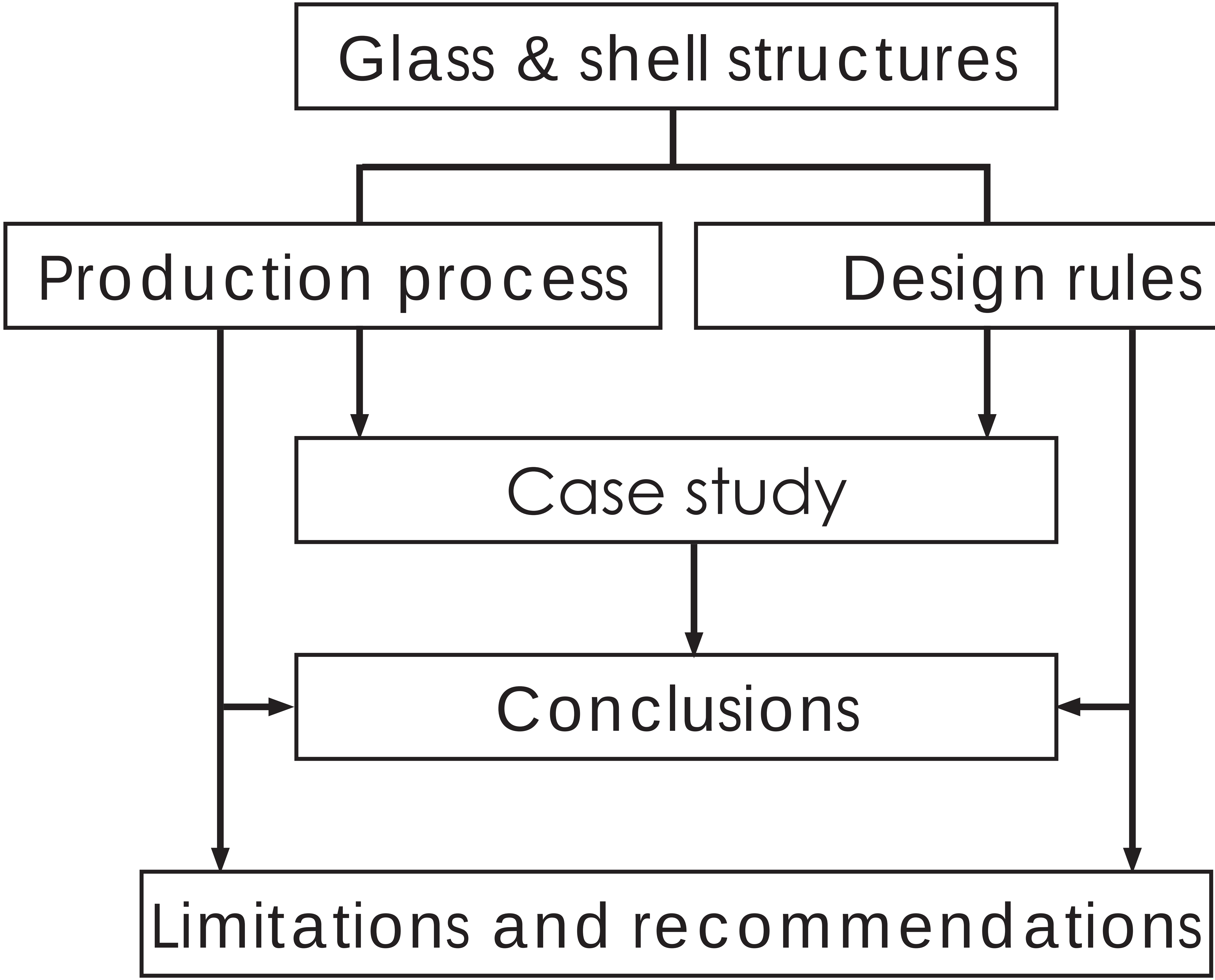
Tennishalle Grenchen (Source: Oliver Menge)

Traditional shells are dark and form enclosed spaces

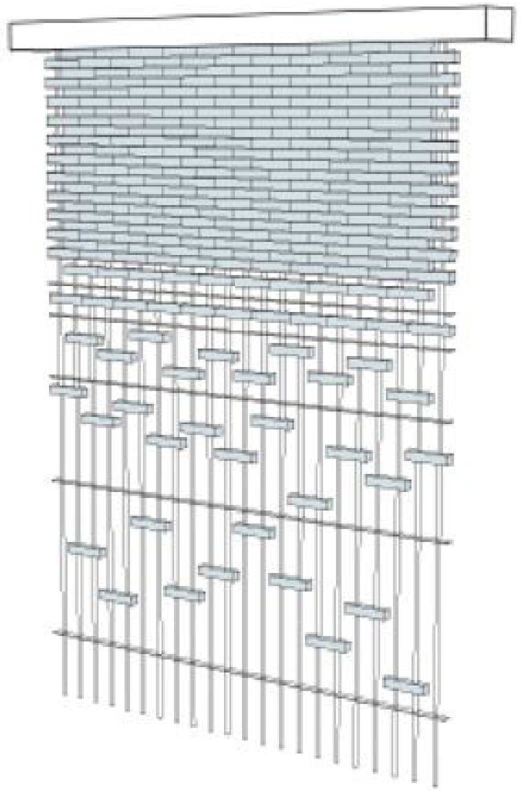
(Tang, 2015)

Little research is done on glass shell structures

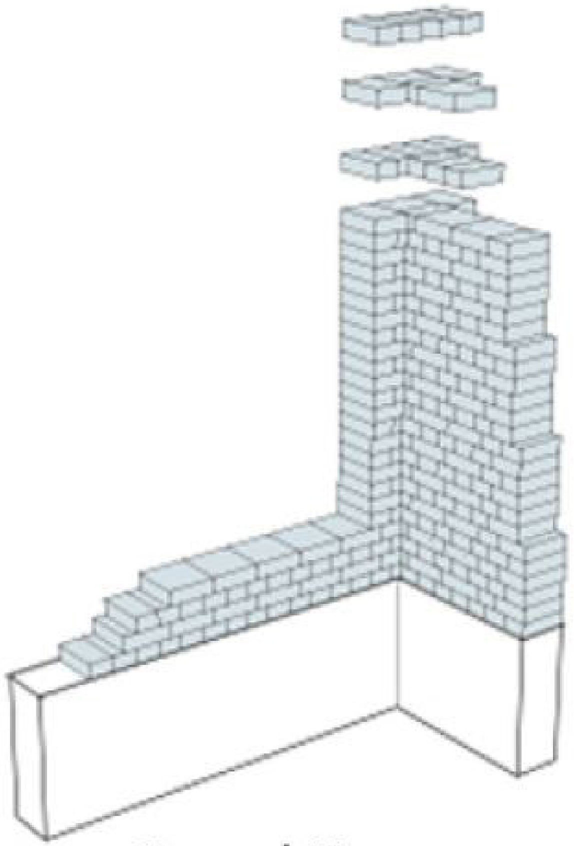
How can a transparent shell structure out of cast glass components be engineered to be fabricated in an efficient way?


Glass & shell structures
Production process
Design rules
Case study
Conclusions
Limitations and recommendations

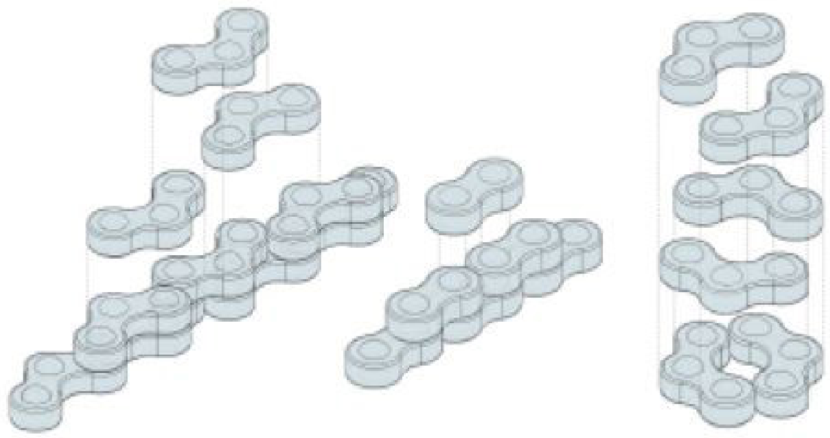
Connection methods



Mechanical



Adhesive



Dry-assembly

(Oikonomopoulou et al., 2018)

Mechanical connection


Optical House construction (Oshima, 2012)

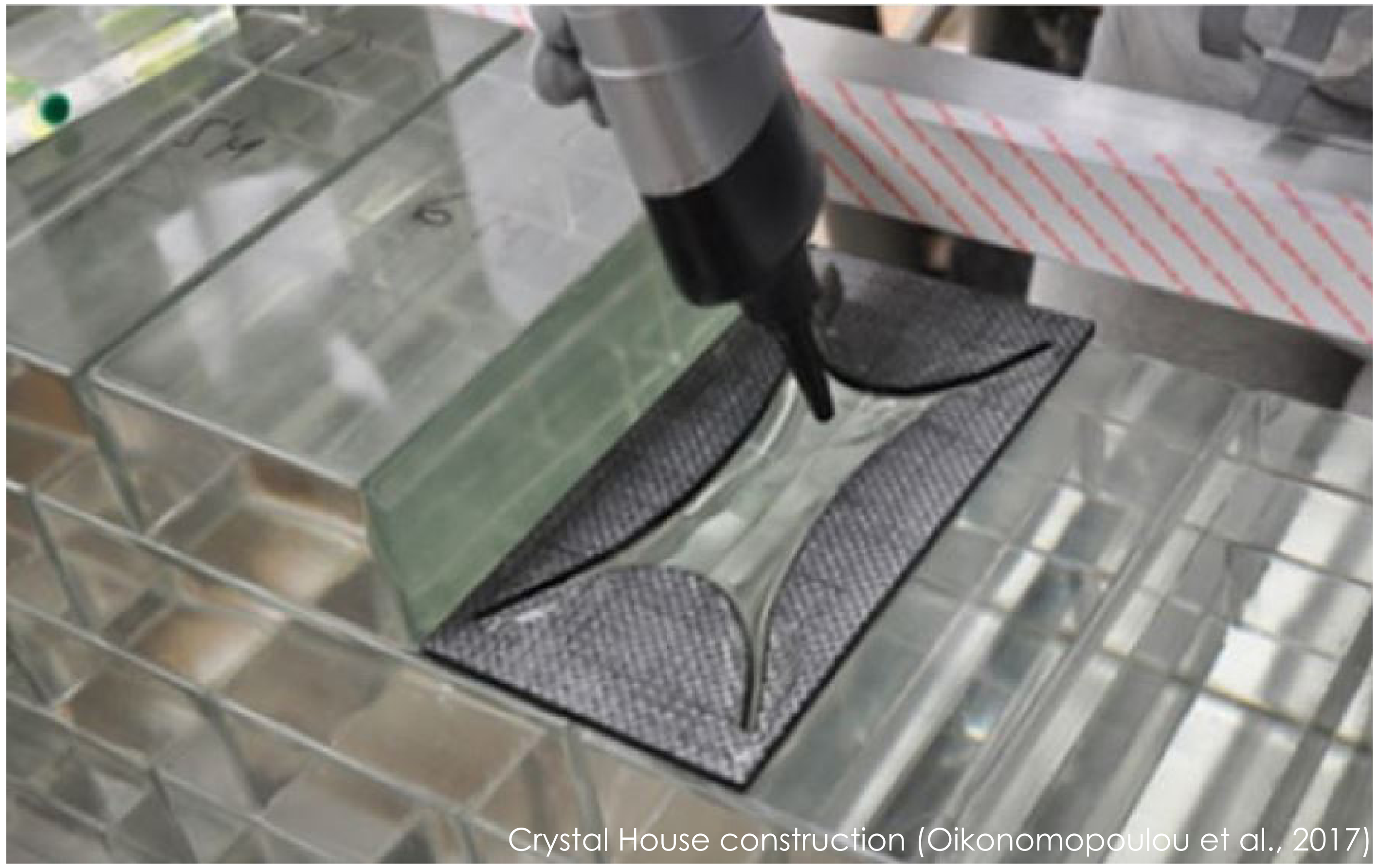
- Substructure resists lateral loads
- Interlayer accomodates tolerances
- Easily assembled

Mechanical connection


Optical House (Oshima, 2012)

- Compromised transparency
- Reversable

Adhesive connection



Crystal House construction (Oikonomopoulou et al., 2017)

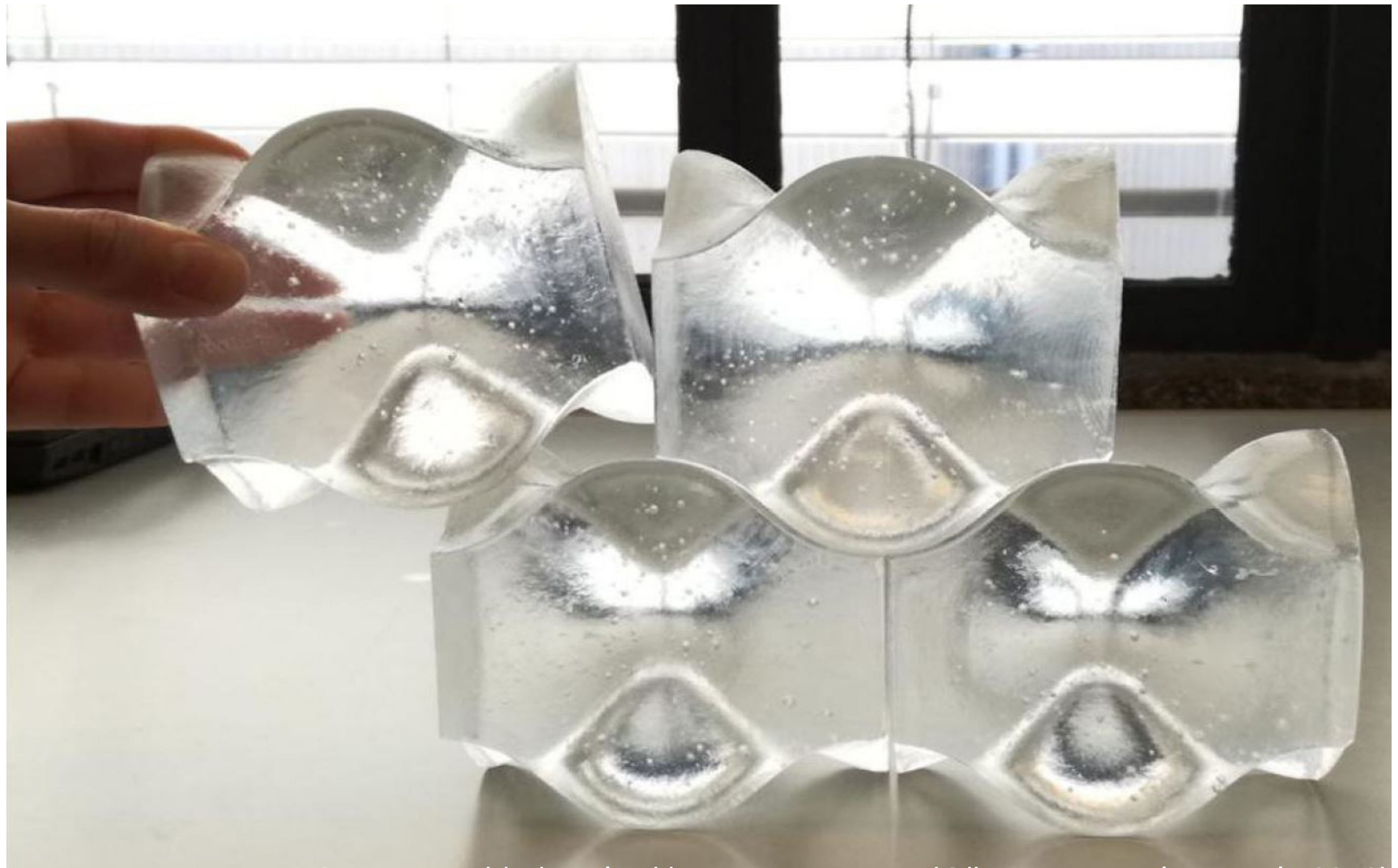
- Requires low tolerances (± 0.25 mm)
- Meticulous construction
- Time-consuming

Adhesive connection

Crystal House (Source: Bart van Vlijmen)

- High transparency
- Non-reversible

Dry-assembly


Osteomorphic interlocking components (Oikonomopoulou et al., 2018)

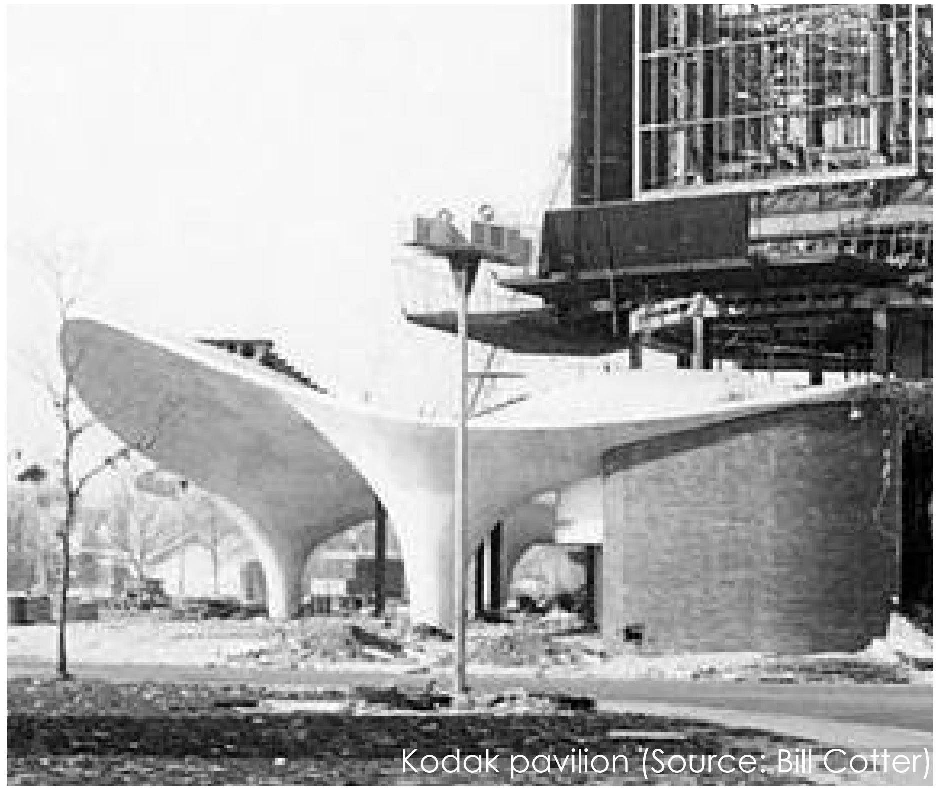
- Interlocking geometry
- Interlayer accomodates tolerances
- Easy construction

Dry-assembly


LEGO-inspired interlocking components (Oikonomopoulou et al., 2018)

- High transparency
- Reversable
- Safe structure

Shell structures



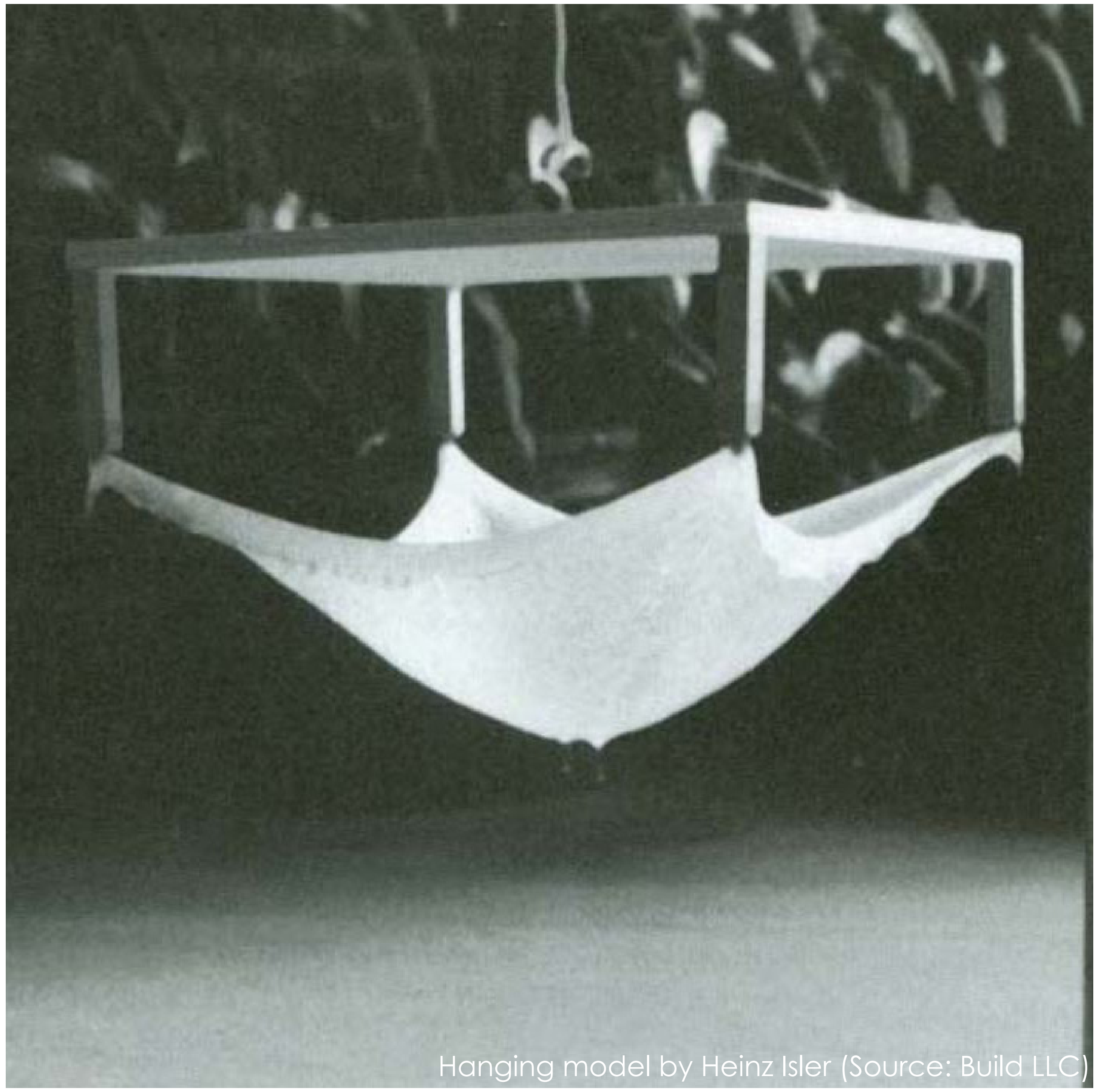
Freeform

Mathematical

Funicular

(Adriaenssens et al., 2014)

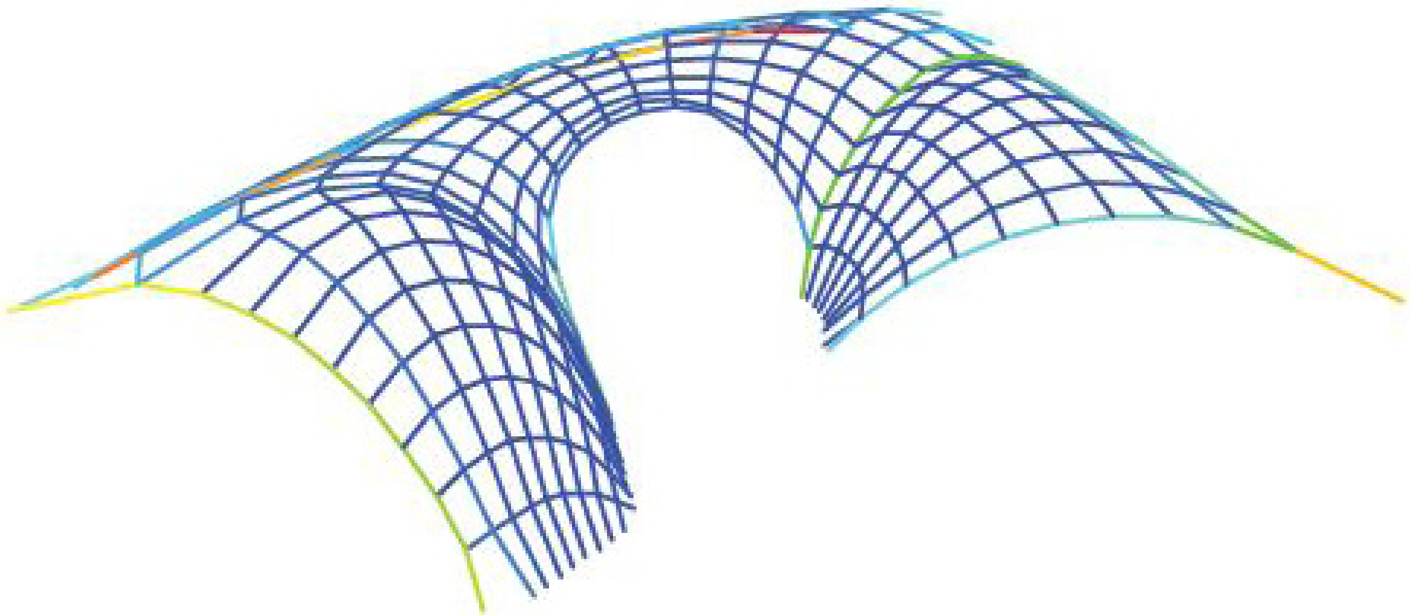
Physical form finding



Hanging model by Heinz Isler (Source: Build LLC)

Sicli Pavilion (Source: Patrimoine Suisse)

Computational form finding



Thrust network analysis (Source: Block Research Group)

Free-form tile vault (Source: Block Research Group)

Construction methods

Sicli Pavilion (Source: Patrimoine Suisse)

Monolithic

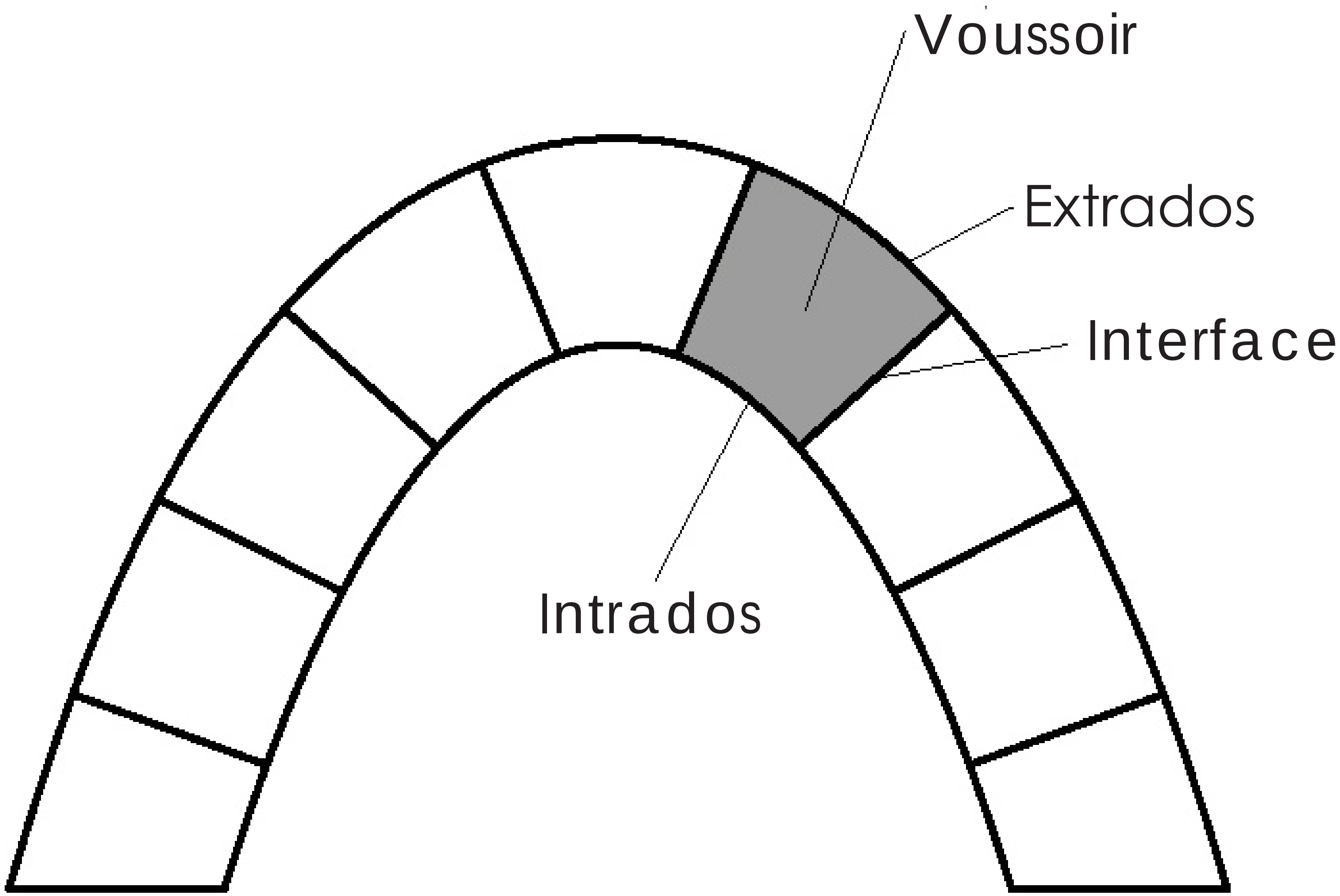

Torre Bujaco (Source: Ignacio Rodriguez Ponsoda)

Discrete with mortar

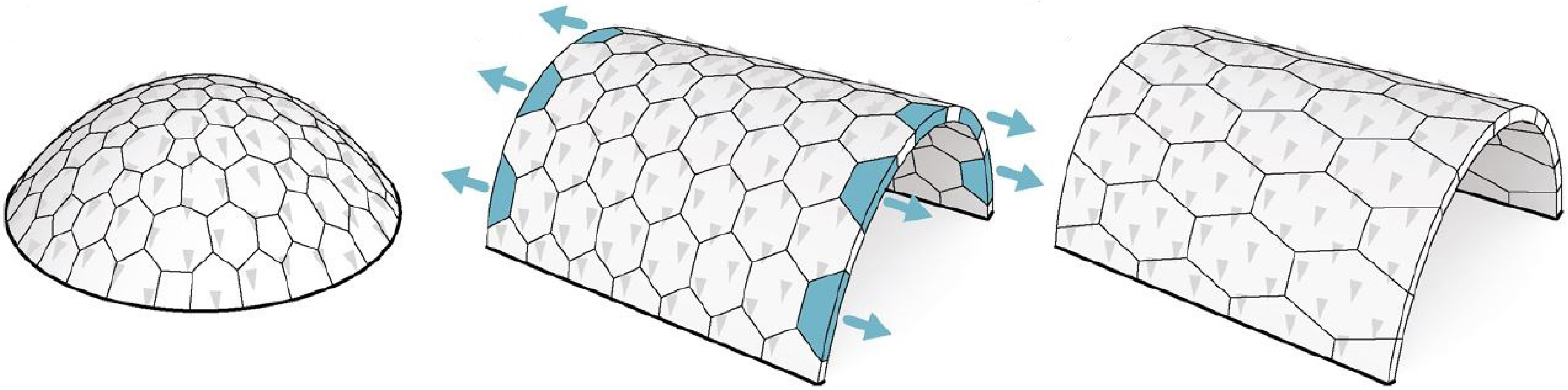

Hôtel de Ville (Source: Destination811375)

Dry-assembled discrete

Voussoir terminology

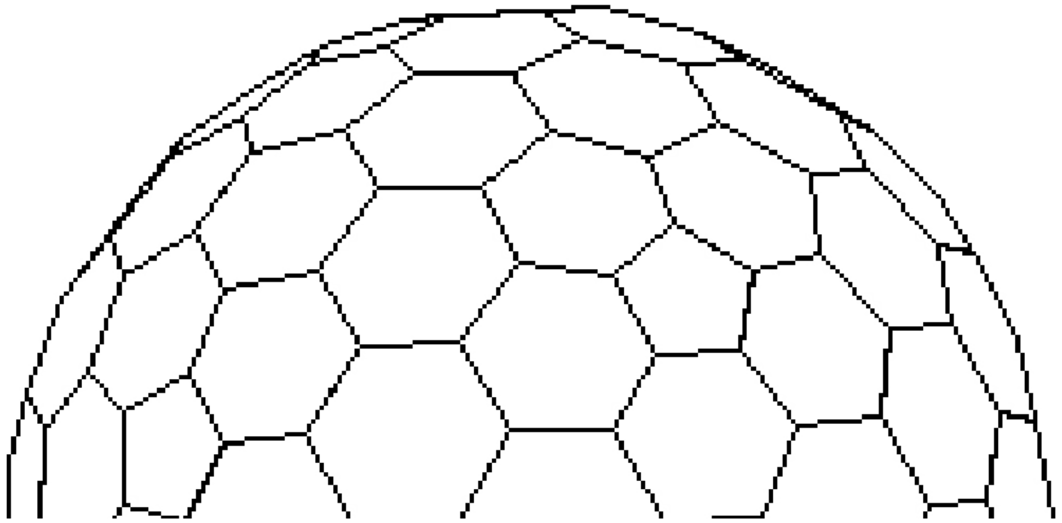


Interfaces perpendicular to the force flow



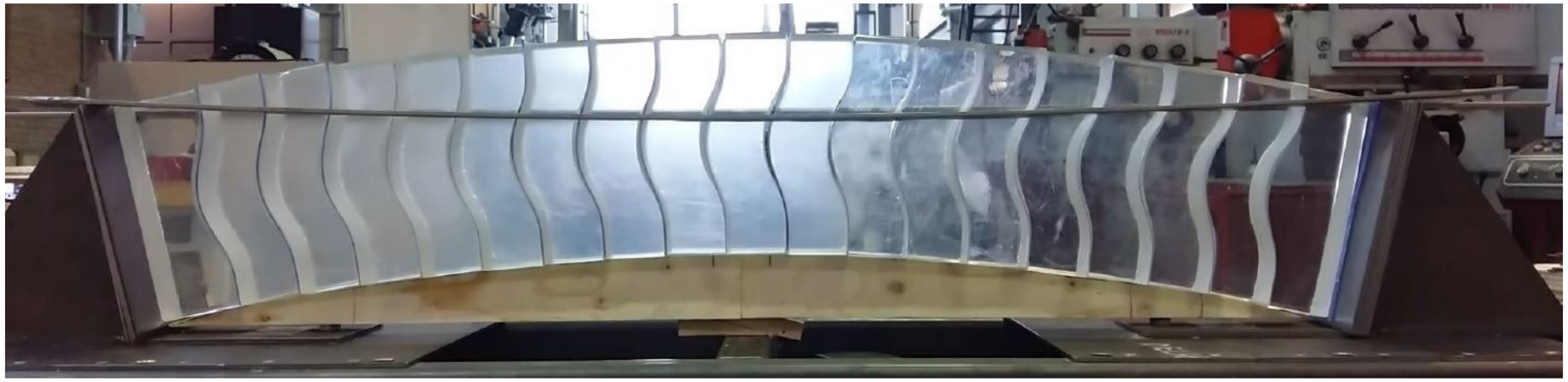
(Rippmann & Block, 2018)

Cast glass shell structures



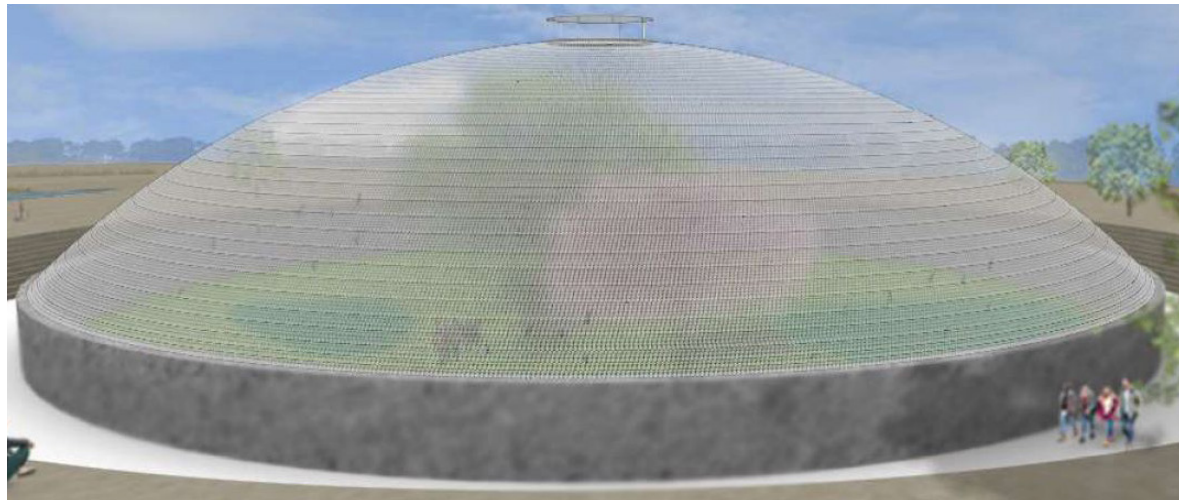
Dome

(Bristogianni et al., 2016)



Arch

(Aurik, 2017)



Dome

(Janssens, 2018)

One voussoir geometry

Spherical dome or circular arch

Funicular shells have varying curvature

Voussoirs of different geometry

An adjustable mould could be used to cast all these different voussoirs

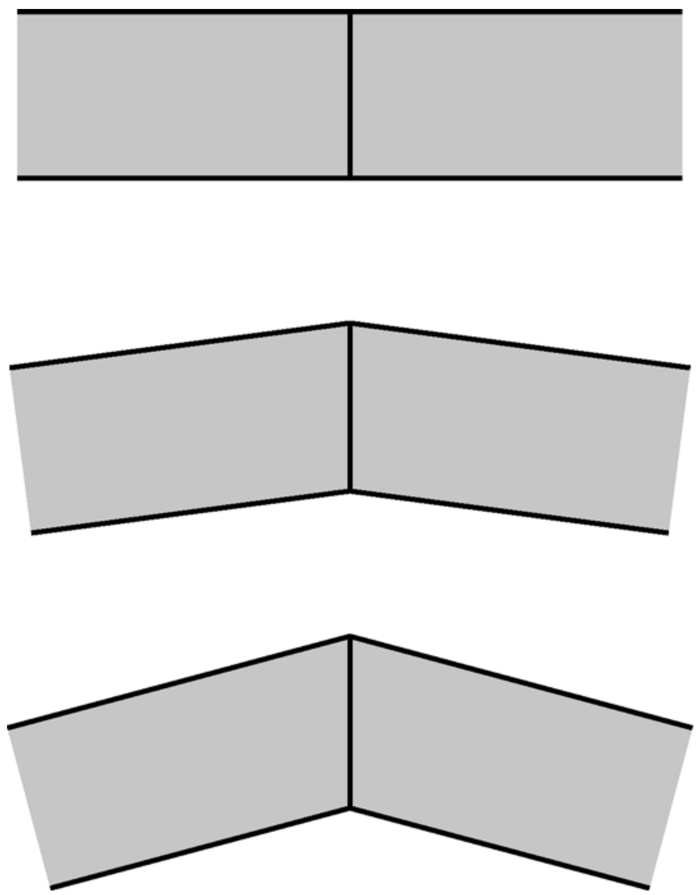
Adjustable mould variables

- Intrados/extrados
- Interfaces

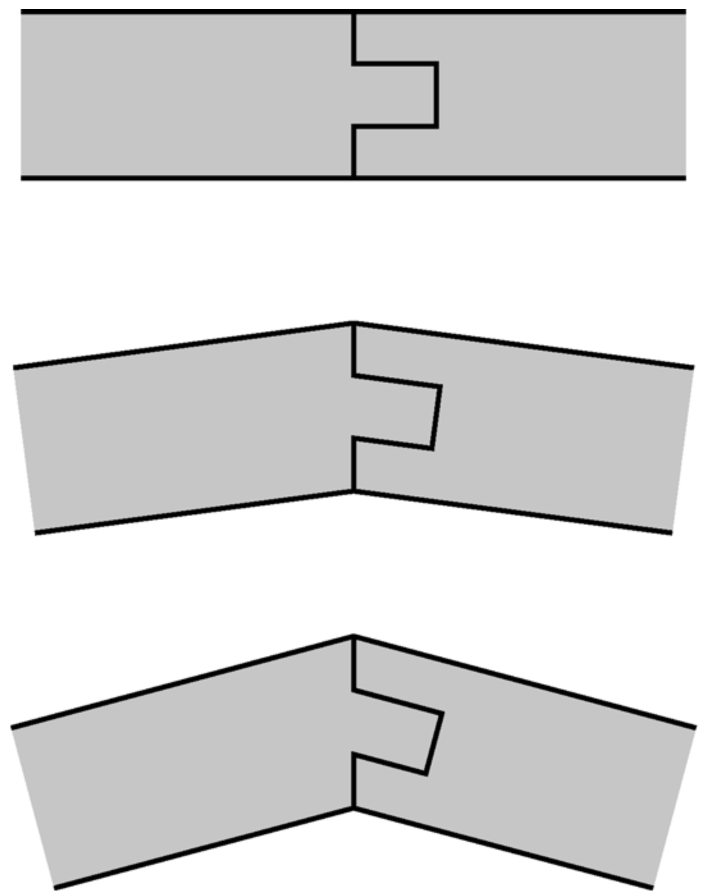
Interface variables

- Voussoir joining angle

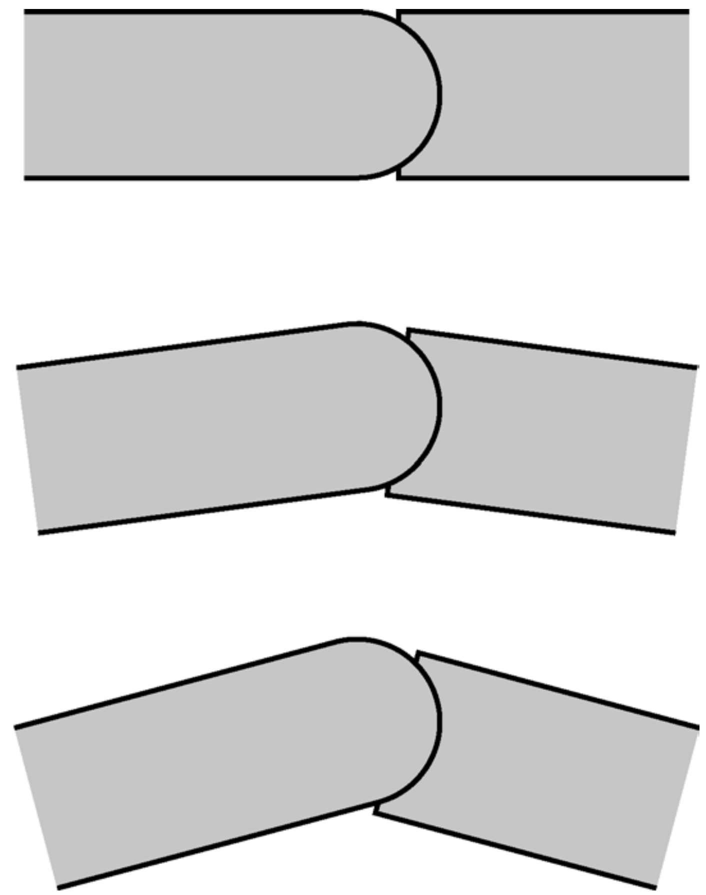
Interfaces



Planar



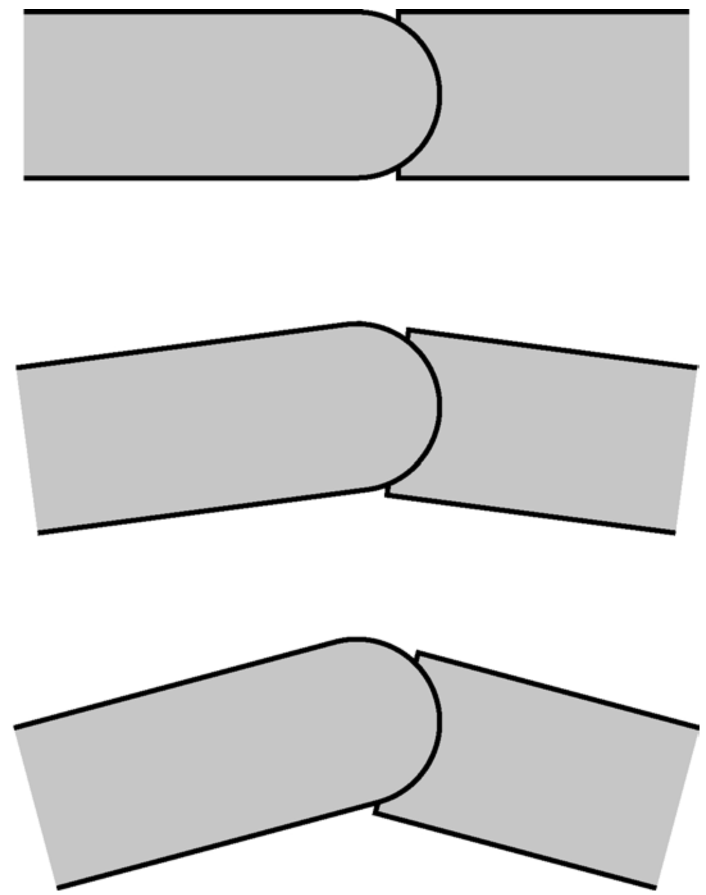
Tongue and groove



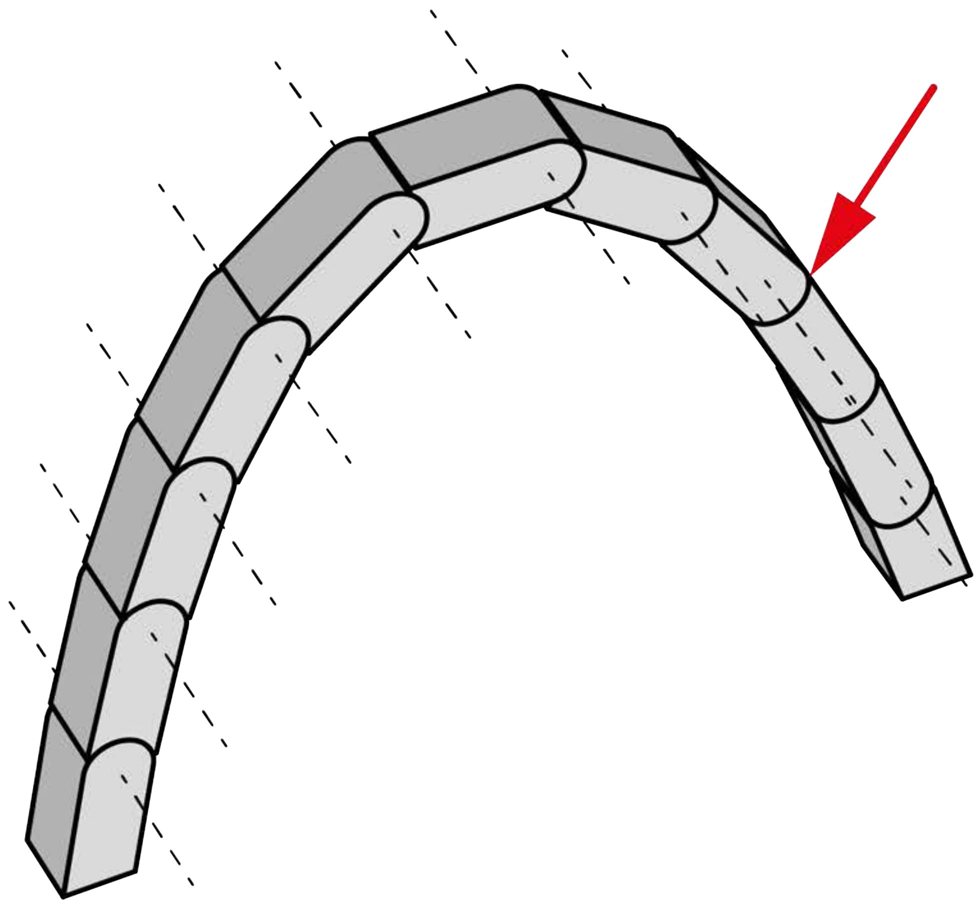
Convex-concave

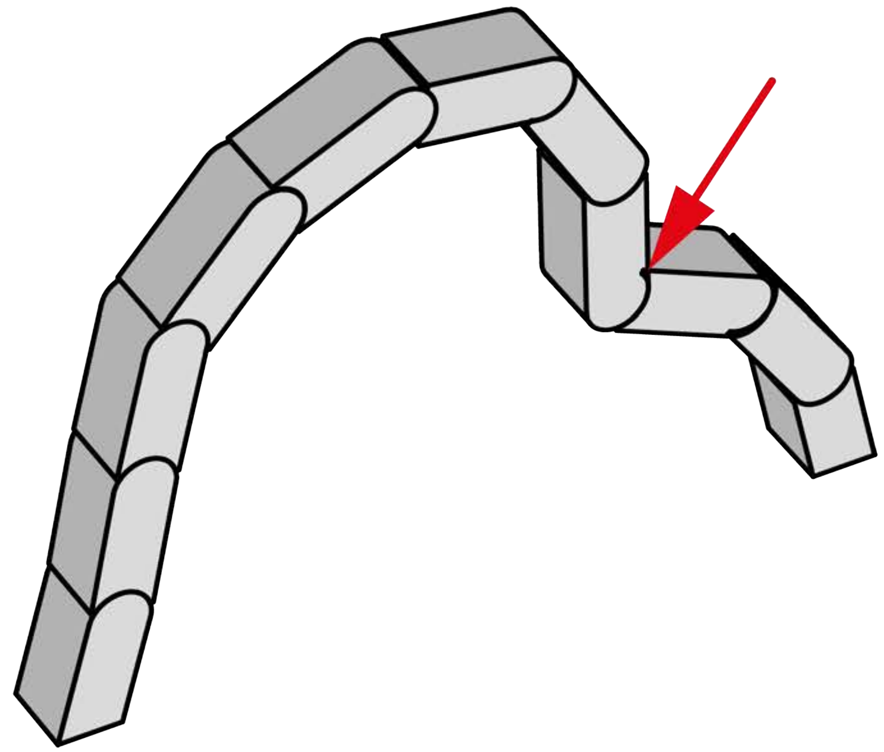
Interfaces

- Interlocking geometry
- Large contact area
- One geometry for all voussoir joining angles
- Act as a hinge

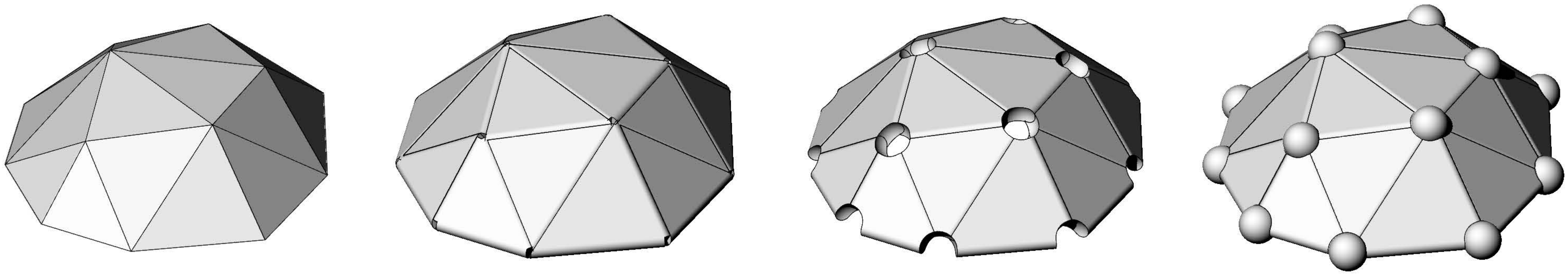


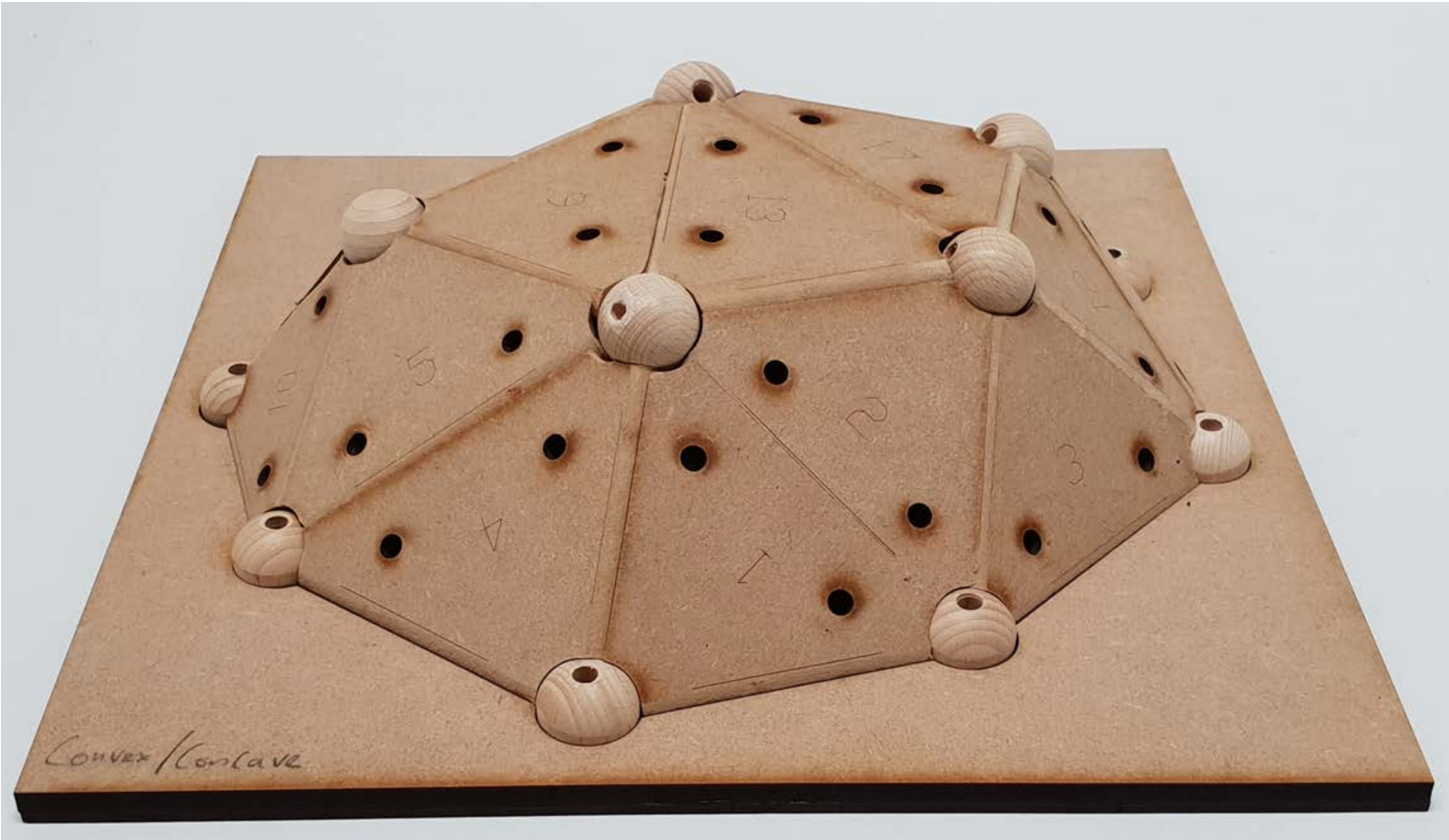
Convex-concave



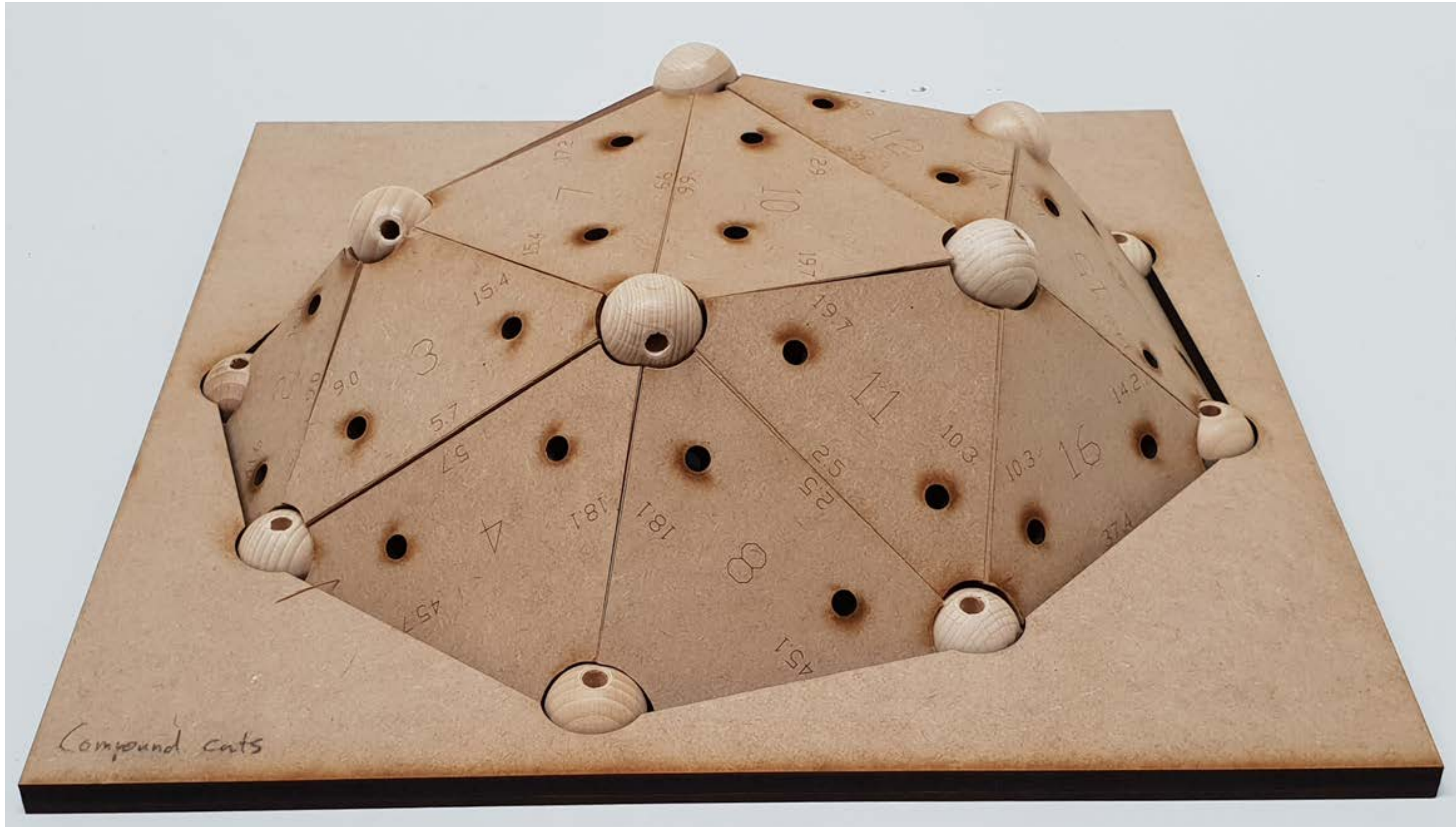


- All hinge lines are parallel
- Unstable equilibrium



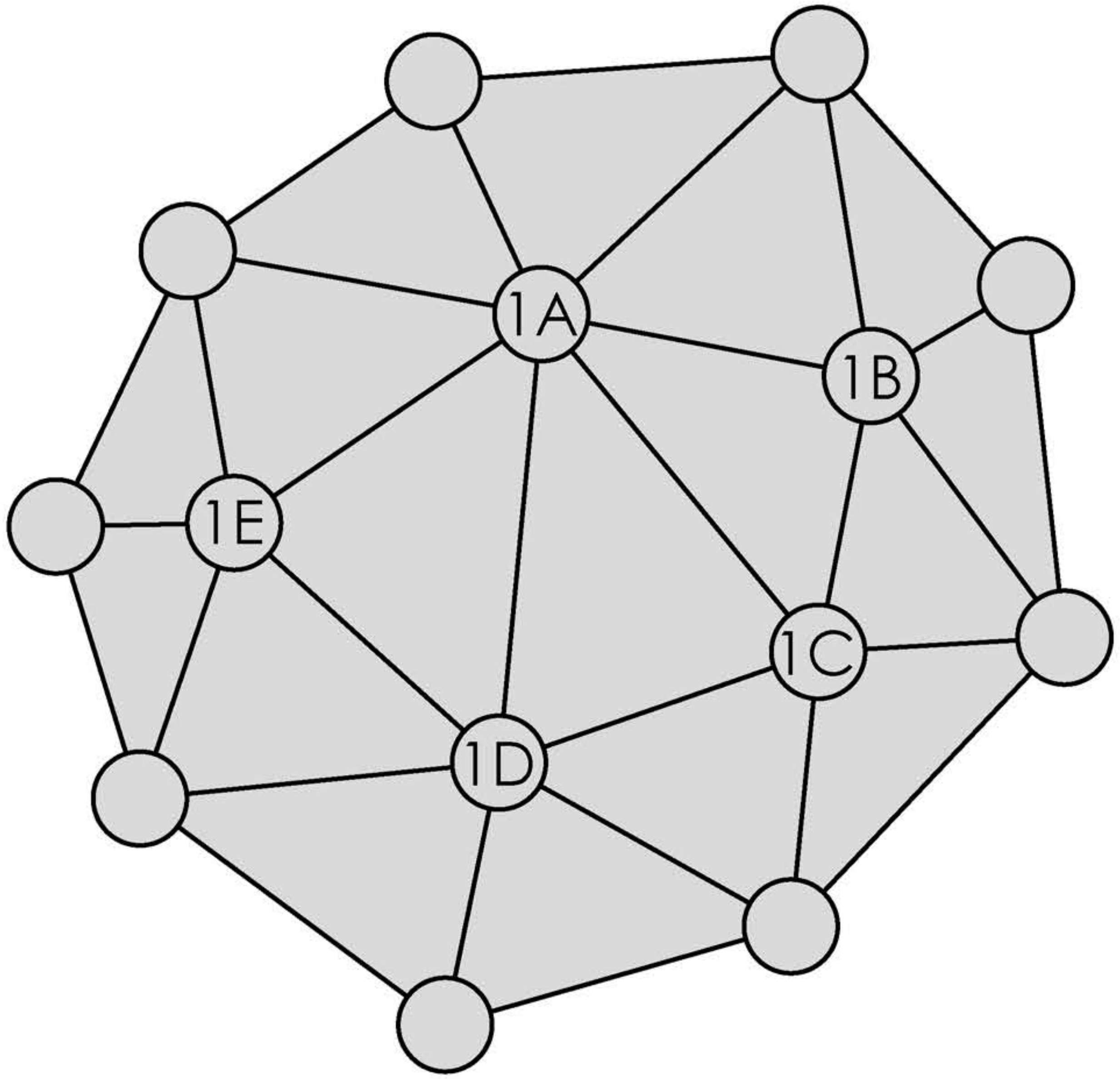


Convex-concave

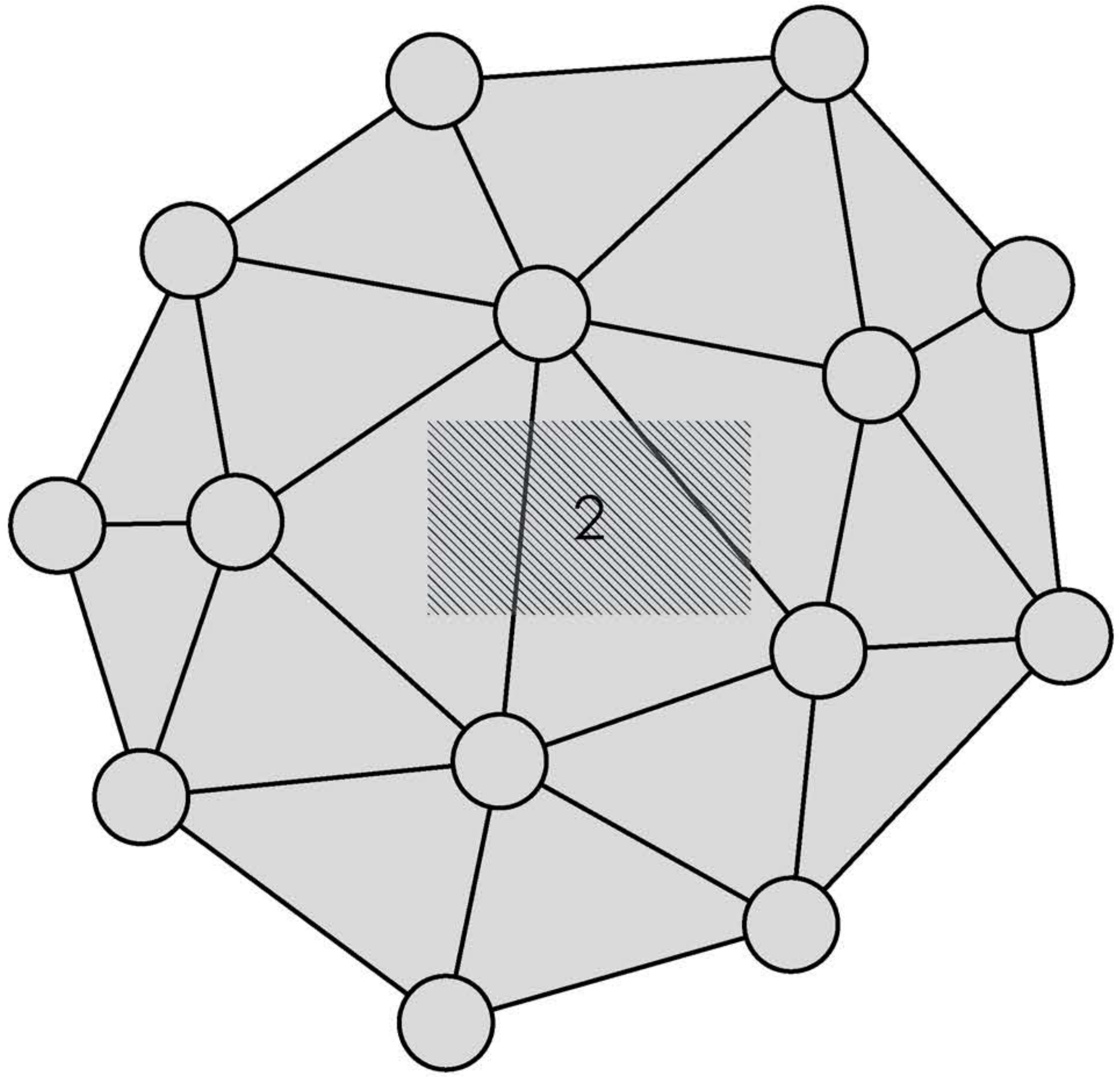


Planar

Load locations

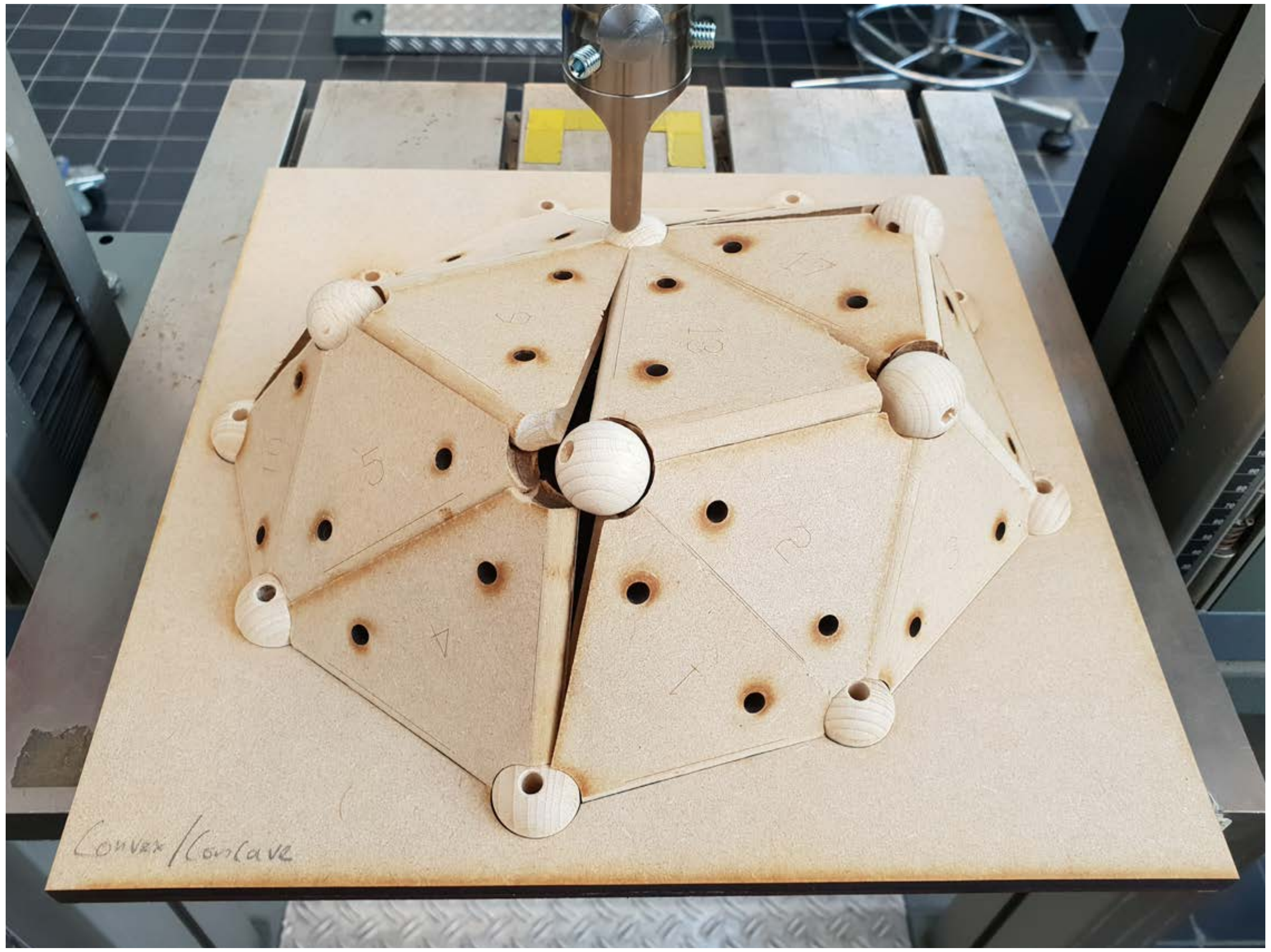


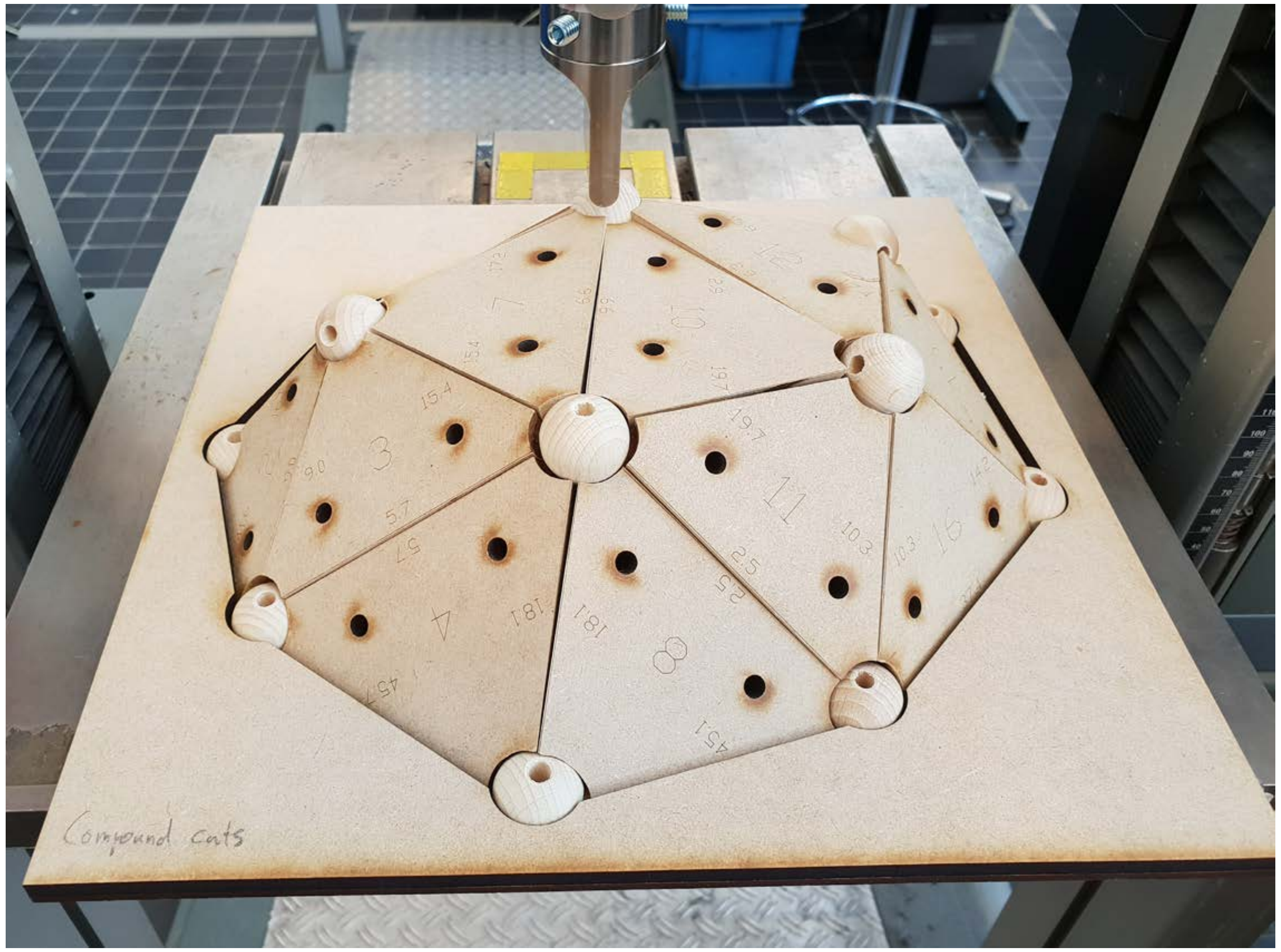
Point loads



Distributed load

Test 1 results



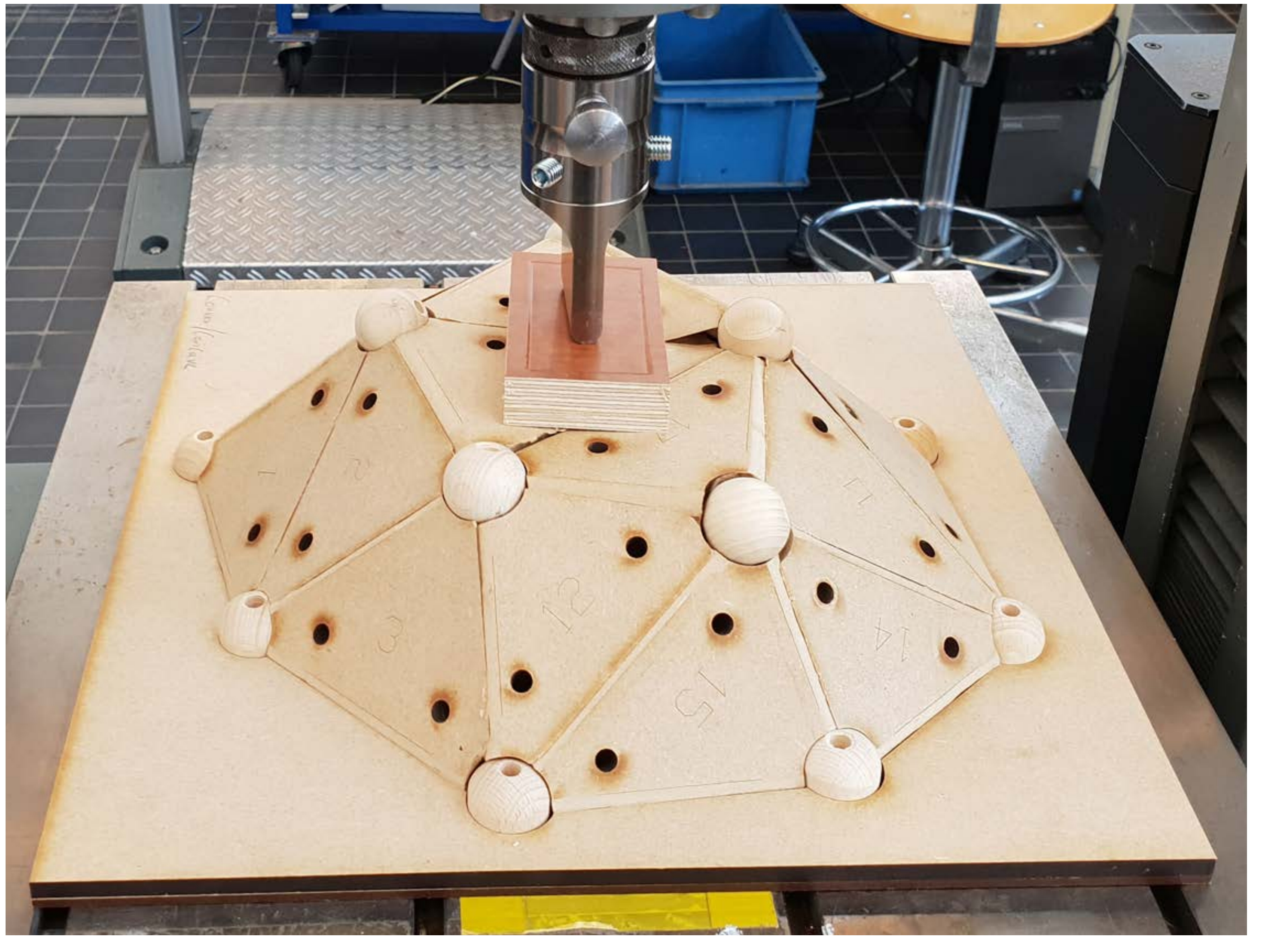


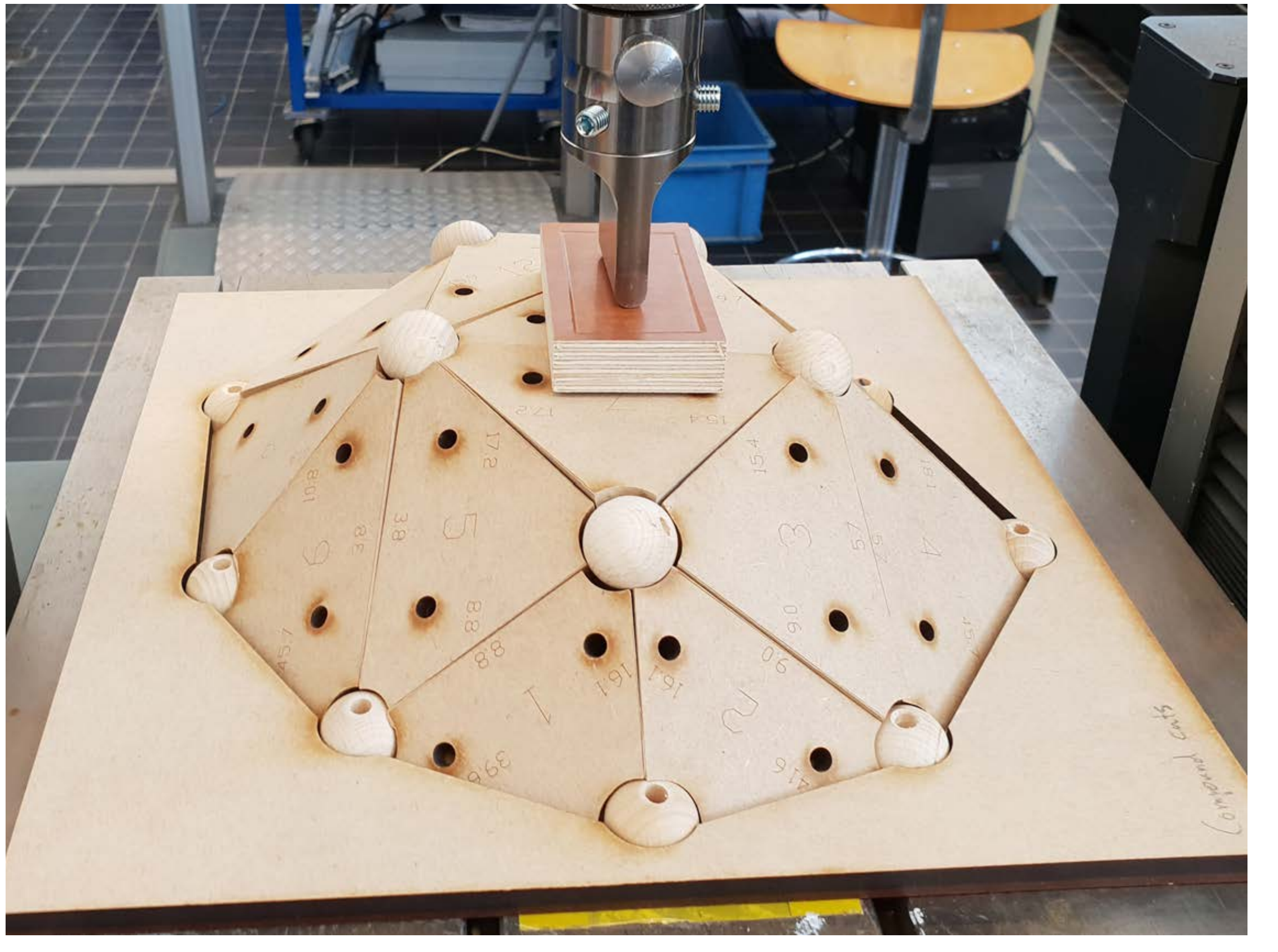
Failure due to destabilizing the equilibrium

Test 1 results

	Convex-concave	Planar
Maximum load	57 N	200 N

Test 2 results





Test 2 results

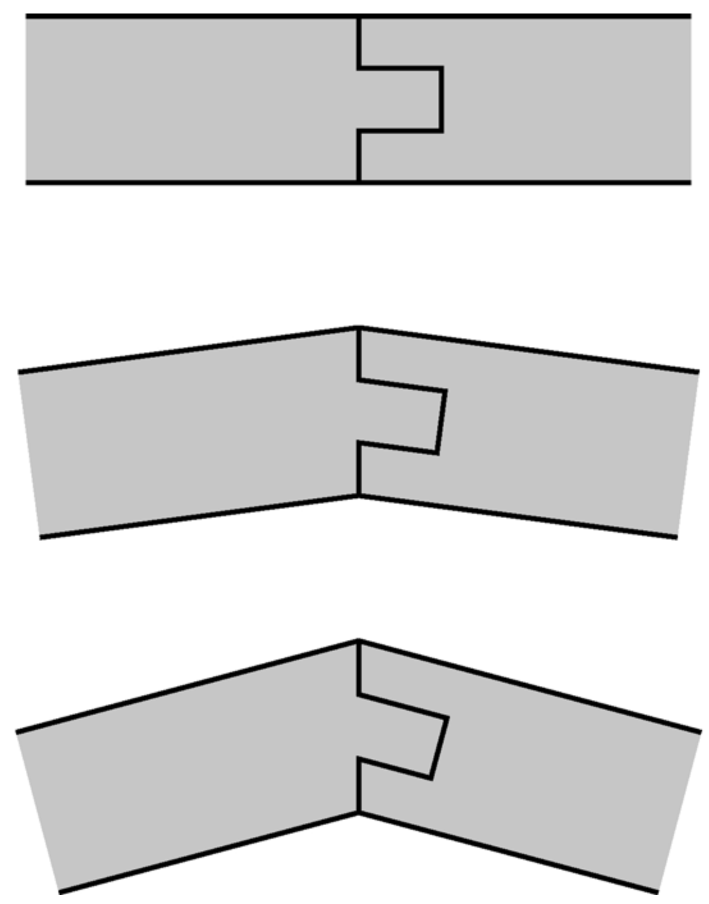
	Convex-concave	Planar
Maximum load	1011 N	88 N

Test 2 results

Failure due to stresses exeeding the yield strength

Ideal interface

- Interlocking geometry
- Non-hinging

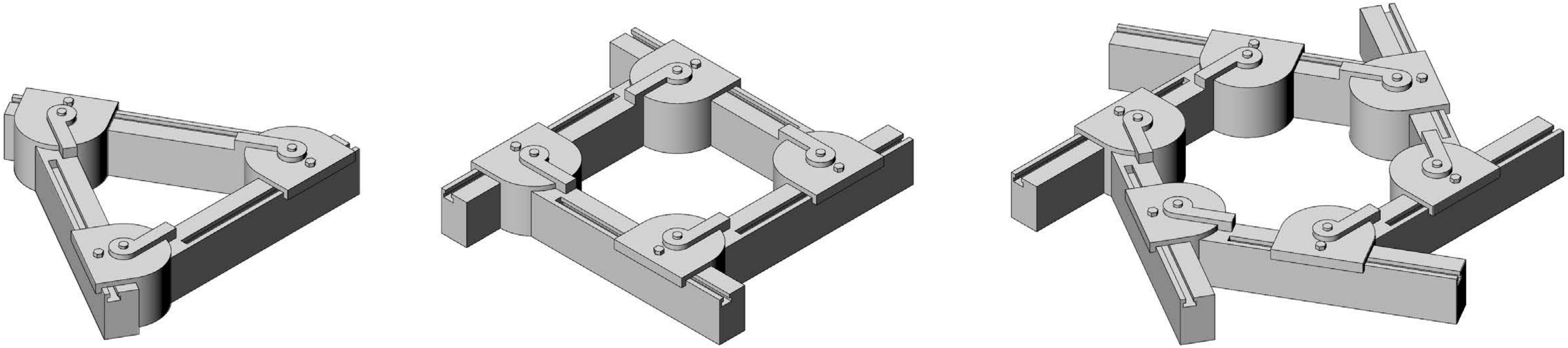


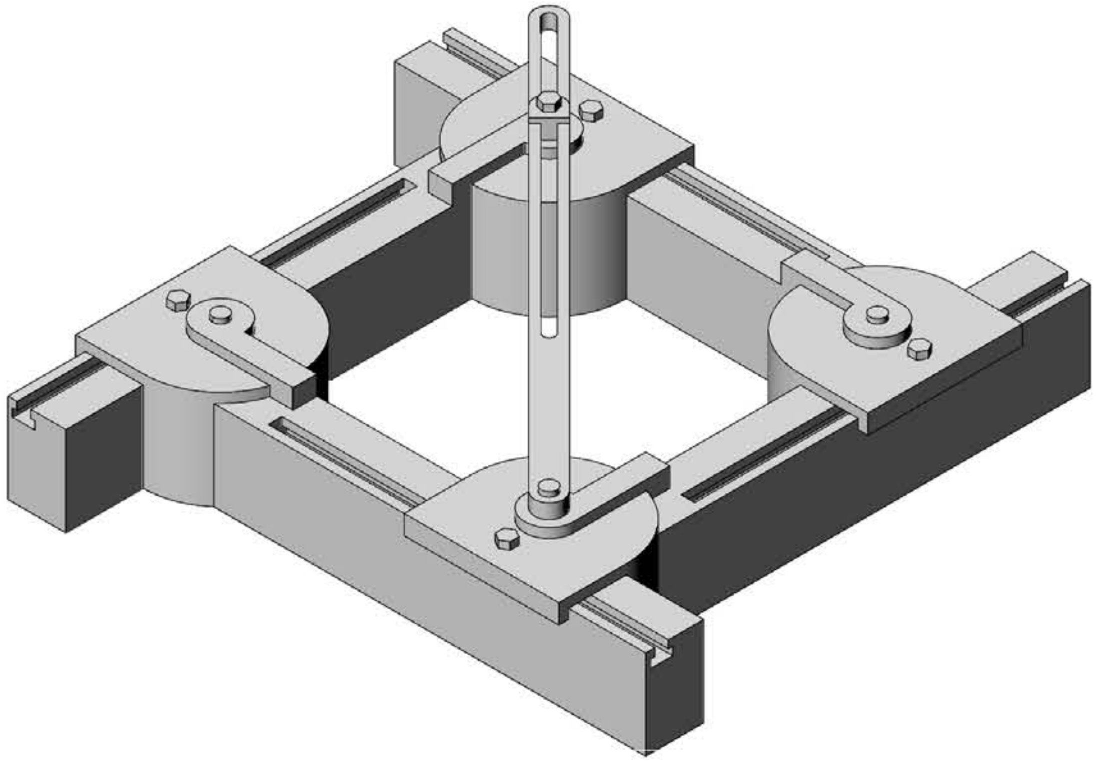
Interface variables

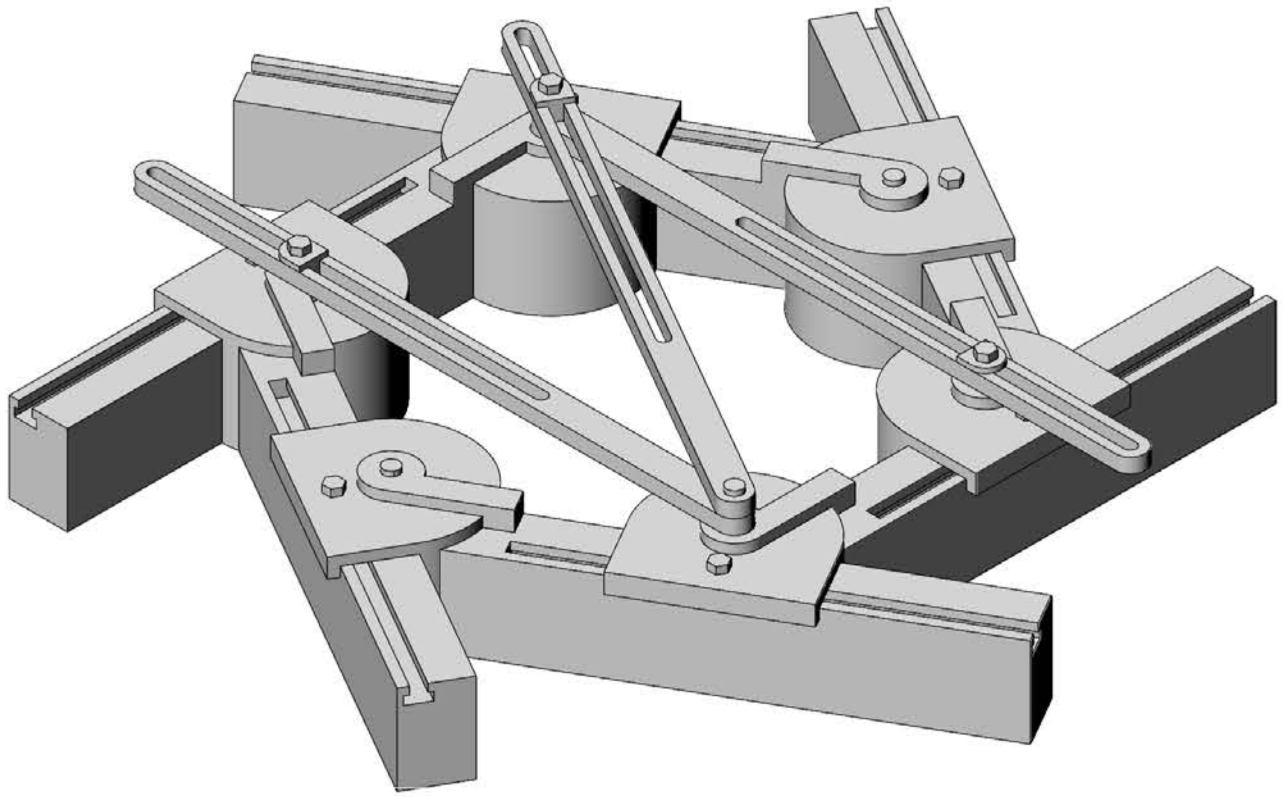
- Voussoir joining angle
- Tongue or groove

Intrados/extrados variables

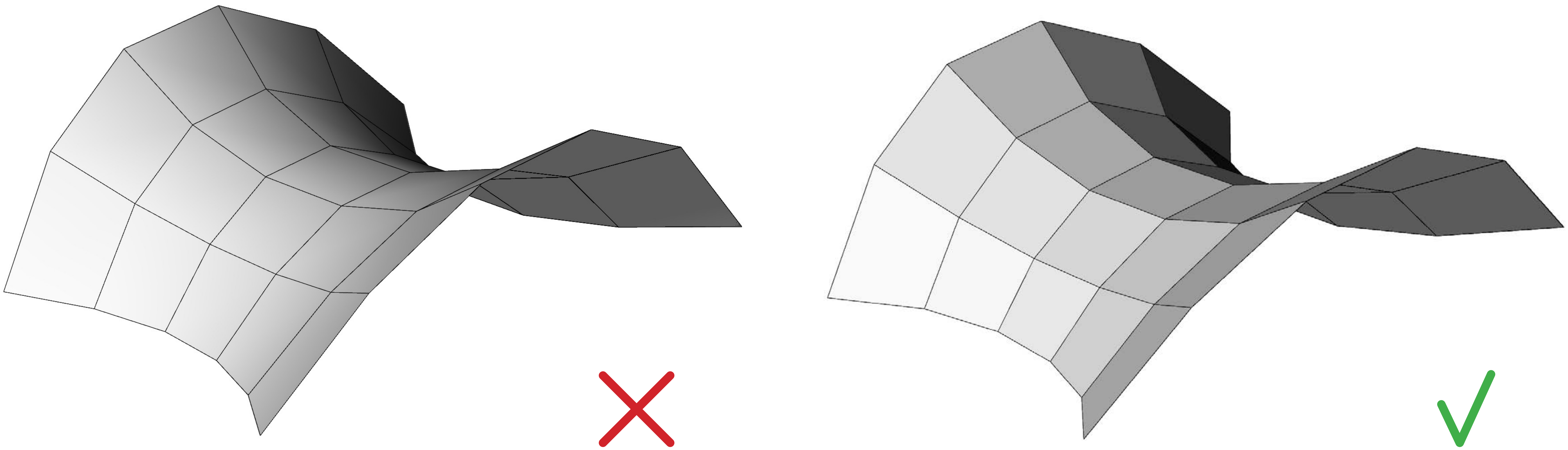
- Edge count (triangular, quadrangular etc.)
- Edge length
- Interior angle



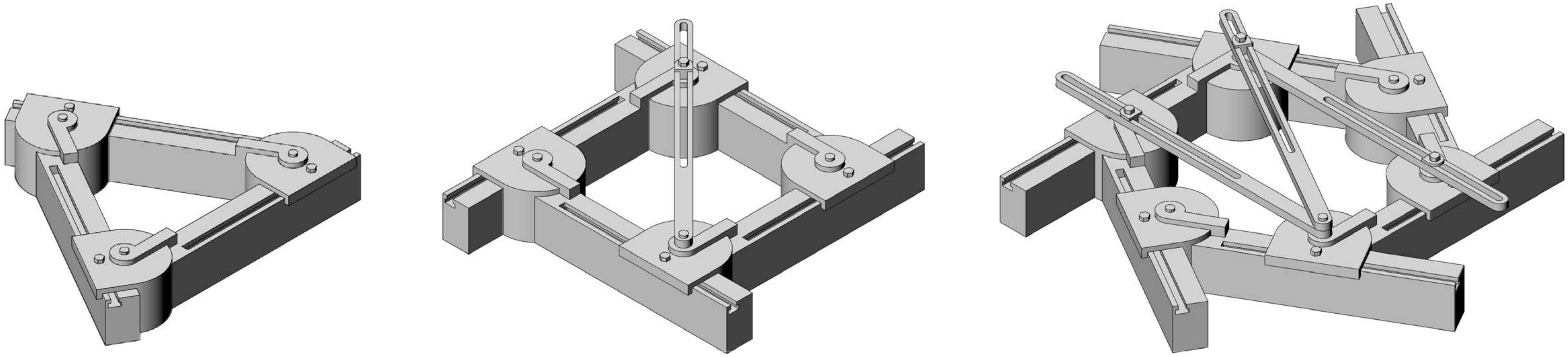




Planarity

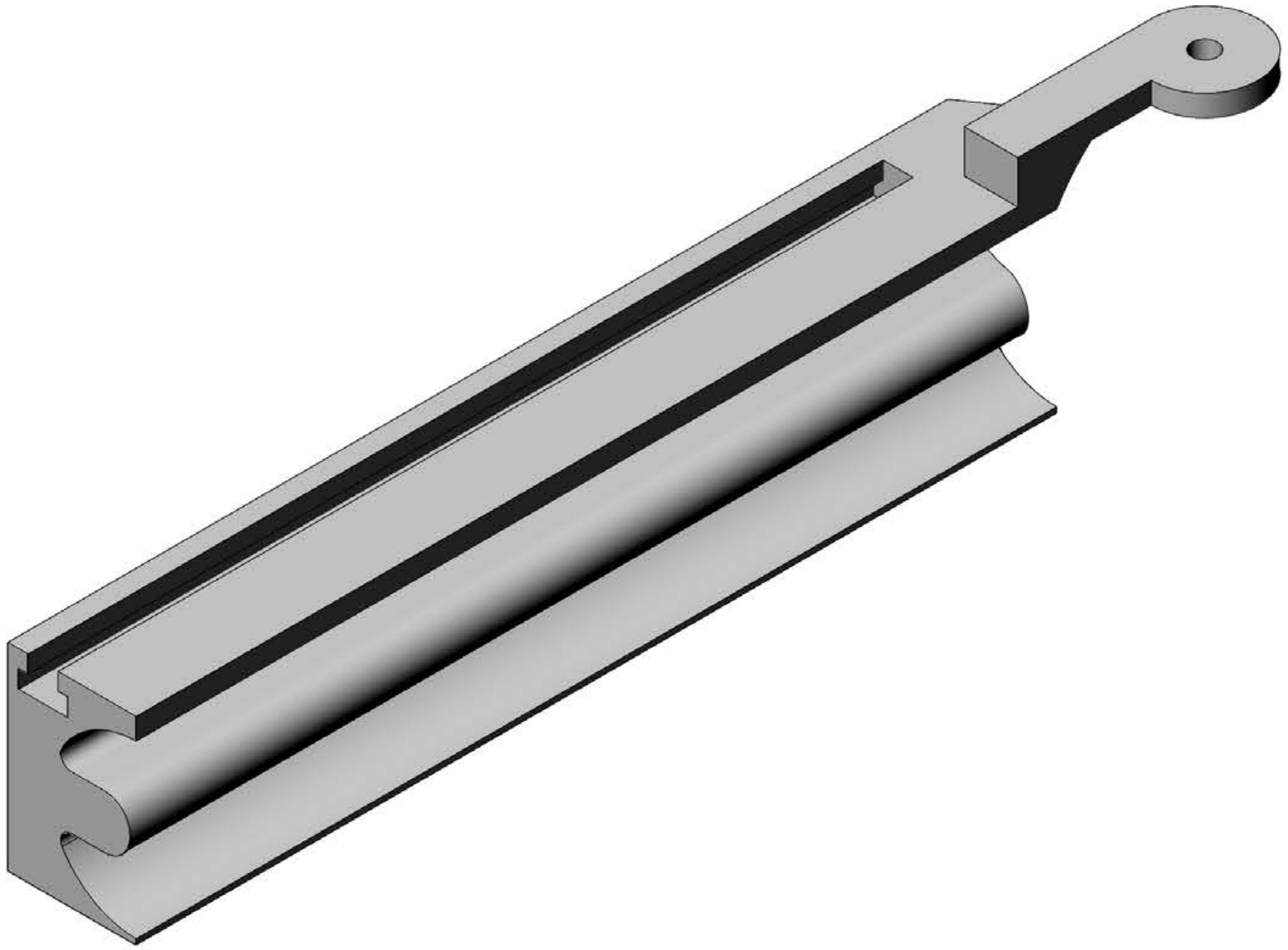


These moulds are only suitable for voussoirs with planar intrados and extrados

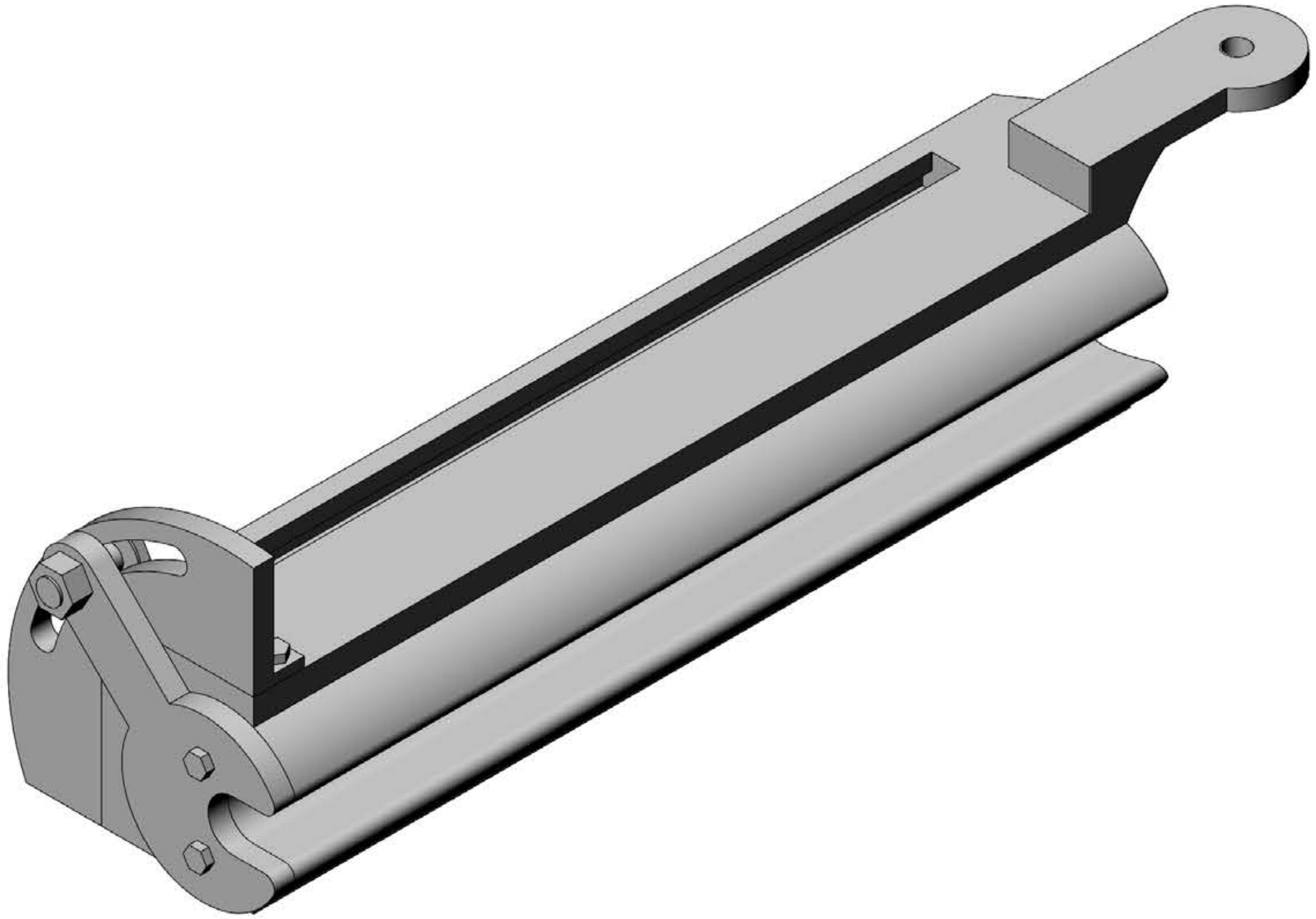


These moulds do not have ajustable interface features

Edge modules

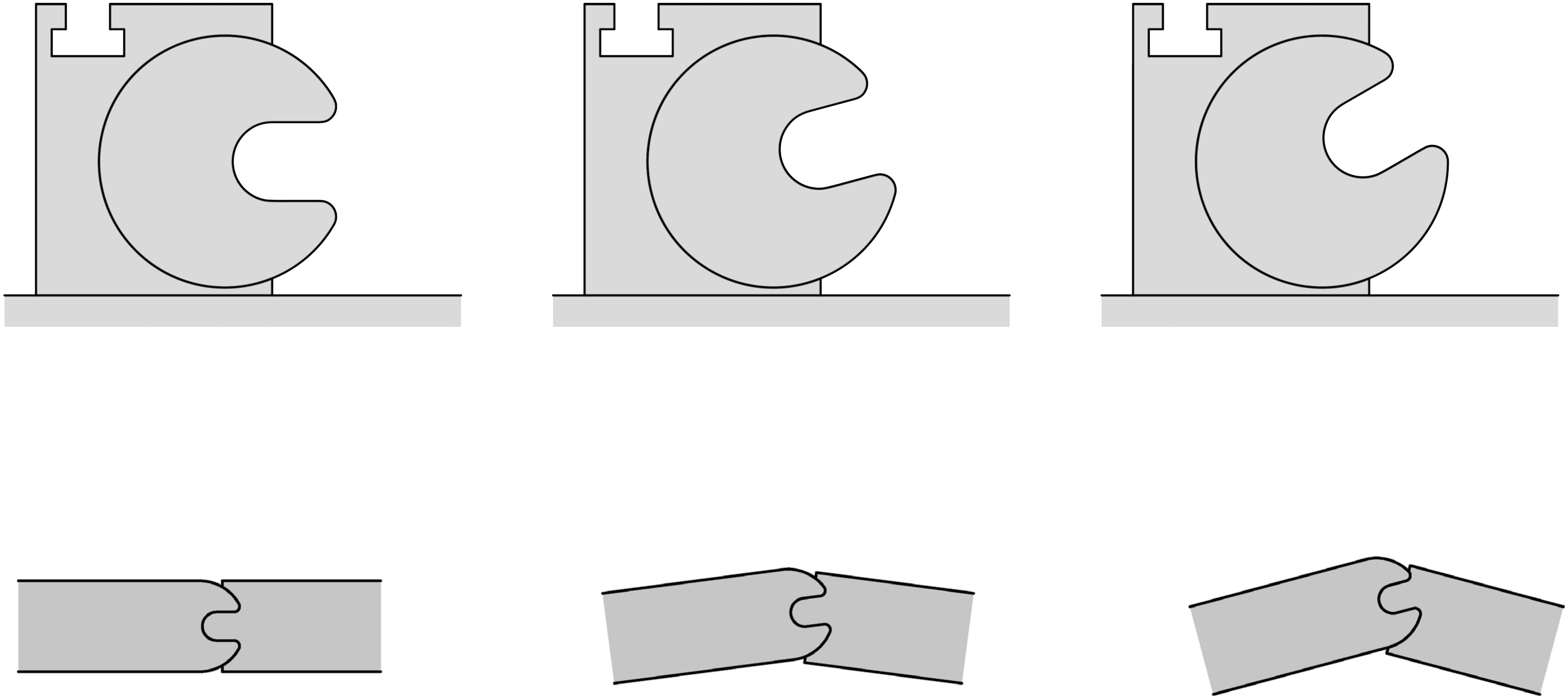


Groove edge module

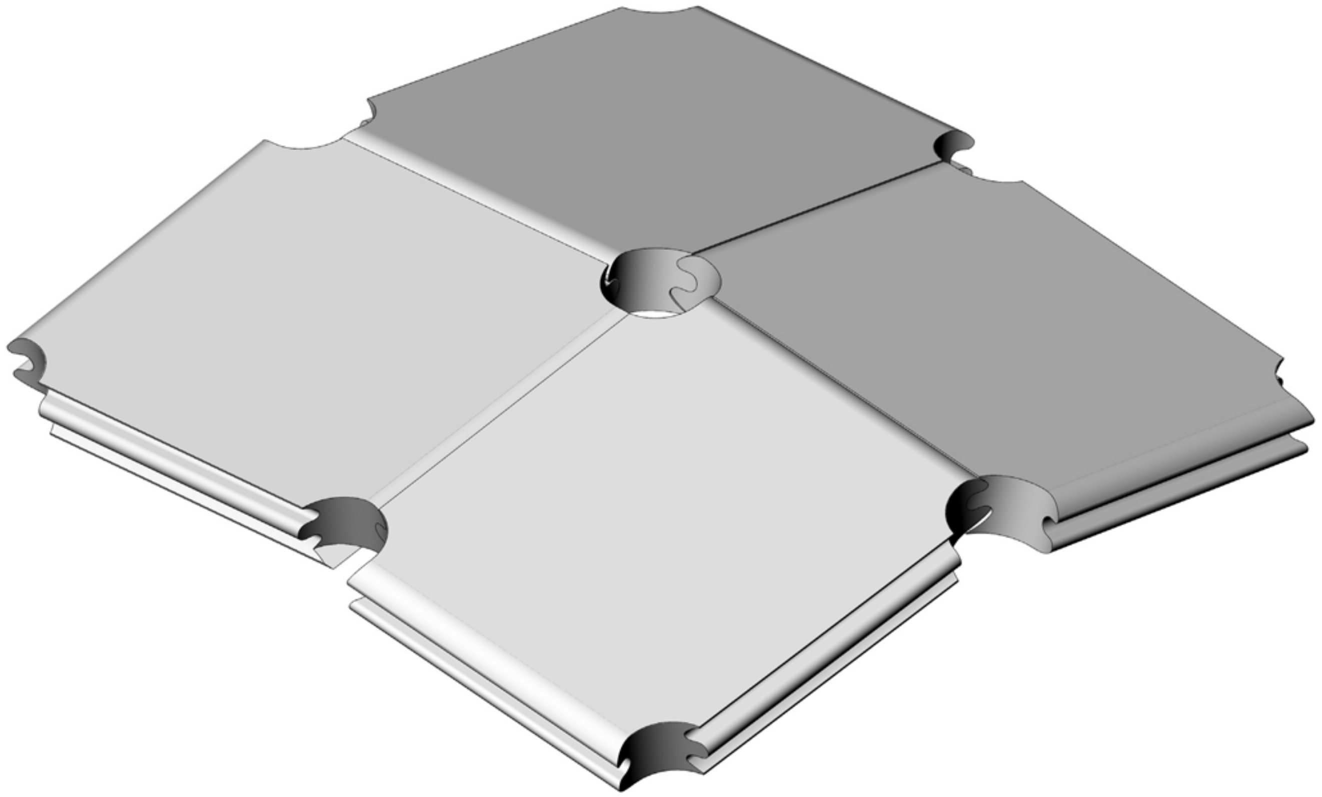


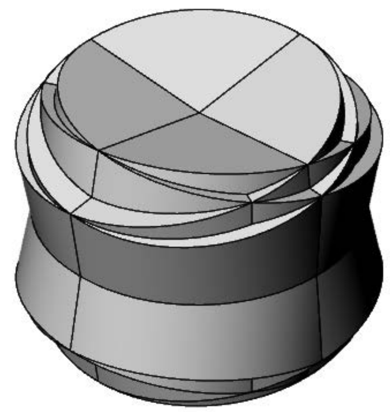
Tongue edge module

Tongue edge module

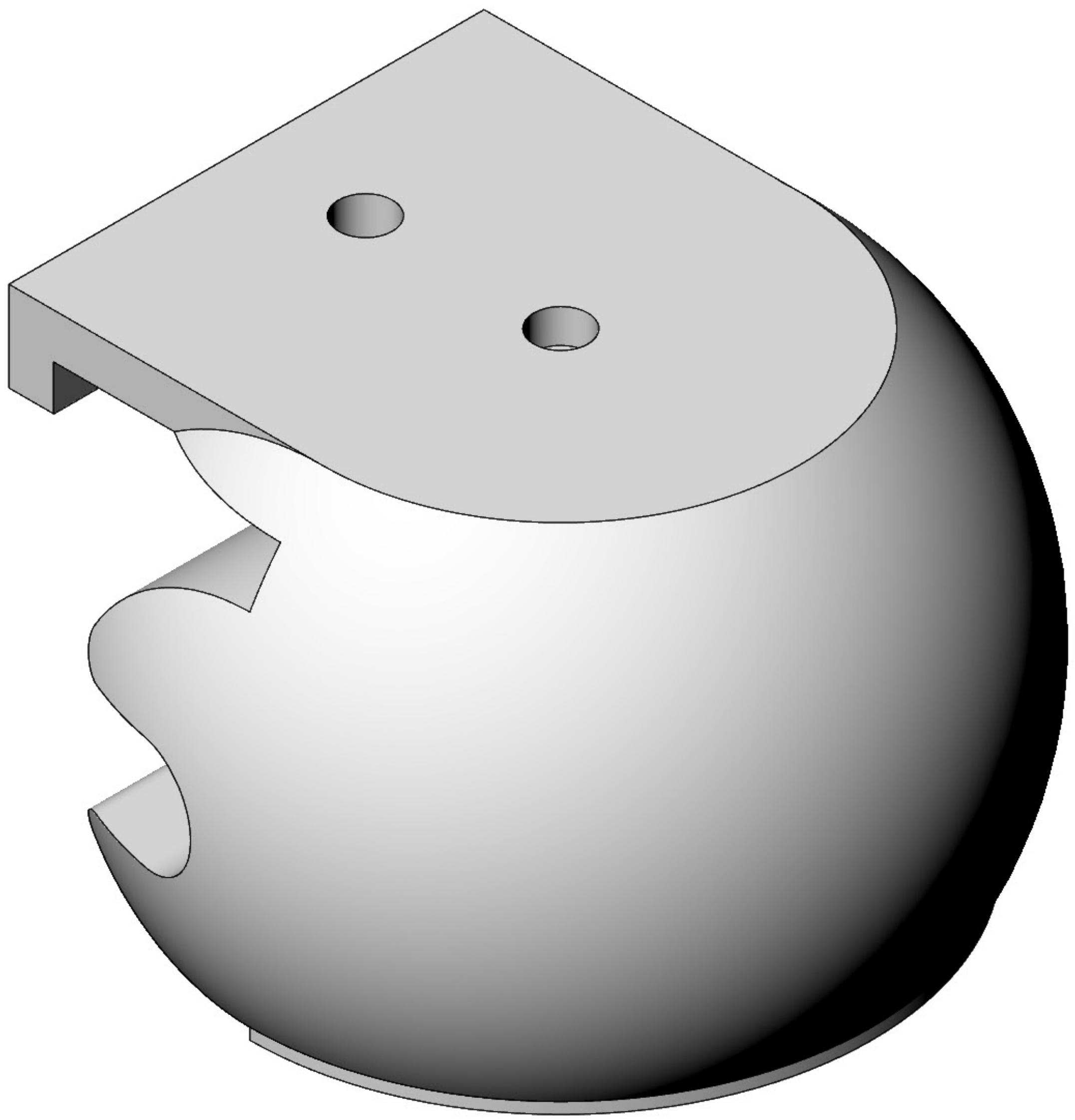


Holes at the nodes

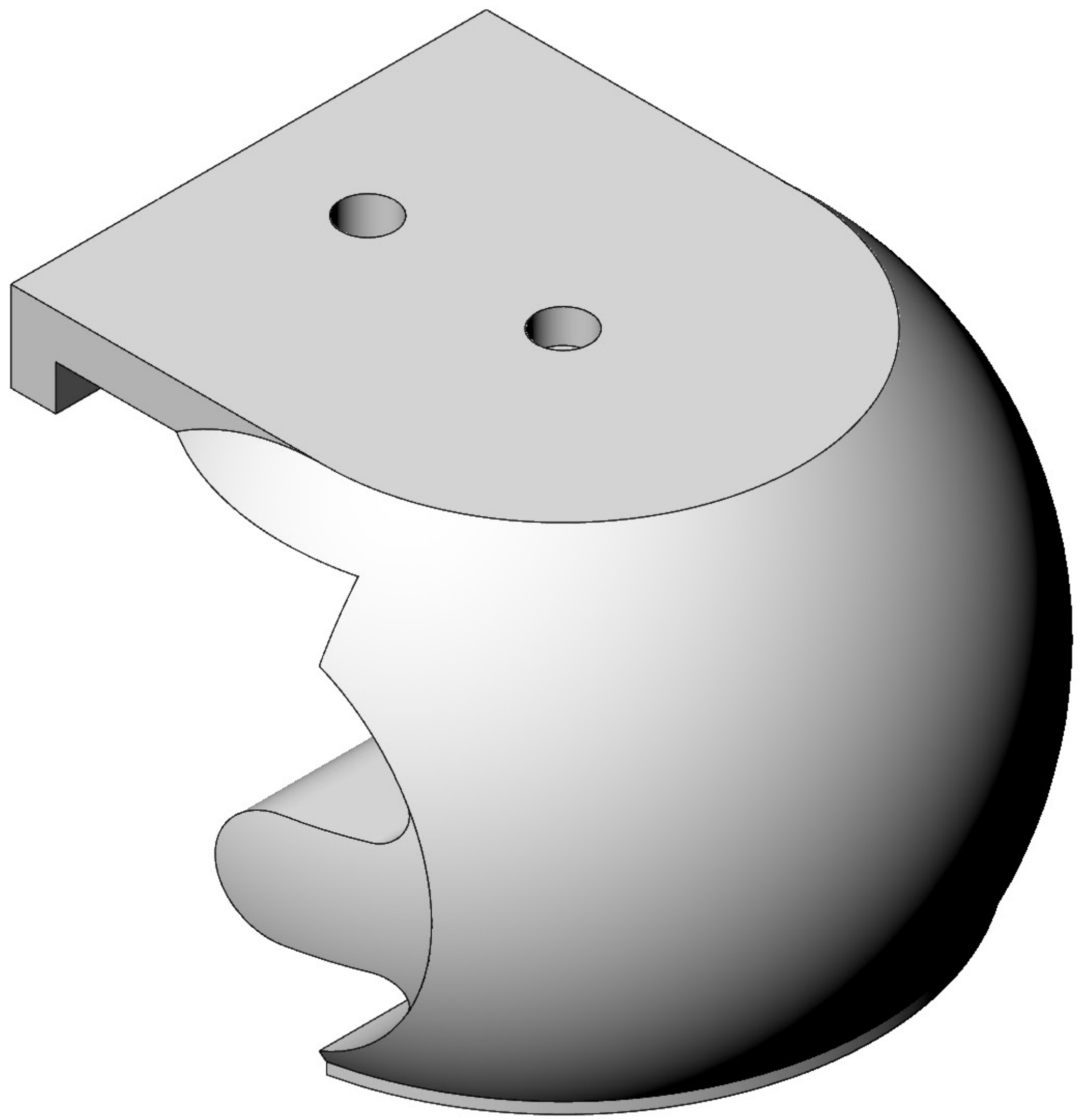




Spherical vertex modules

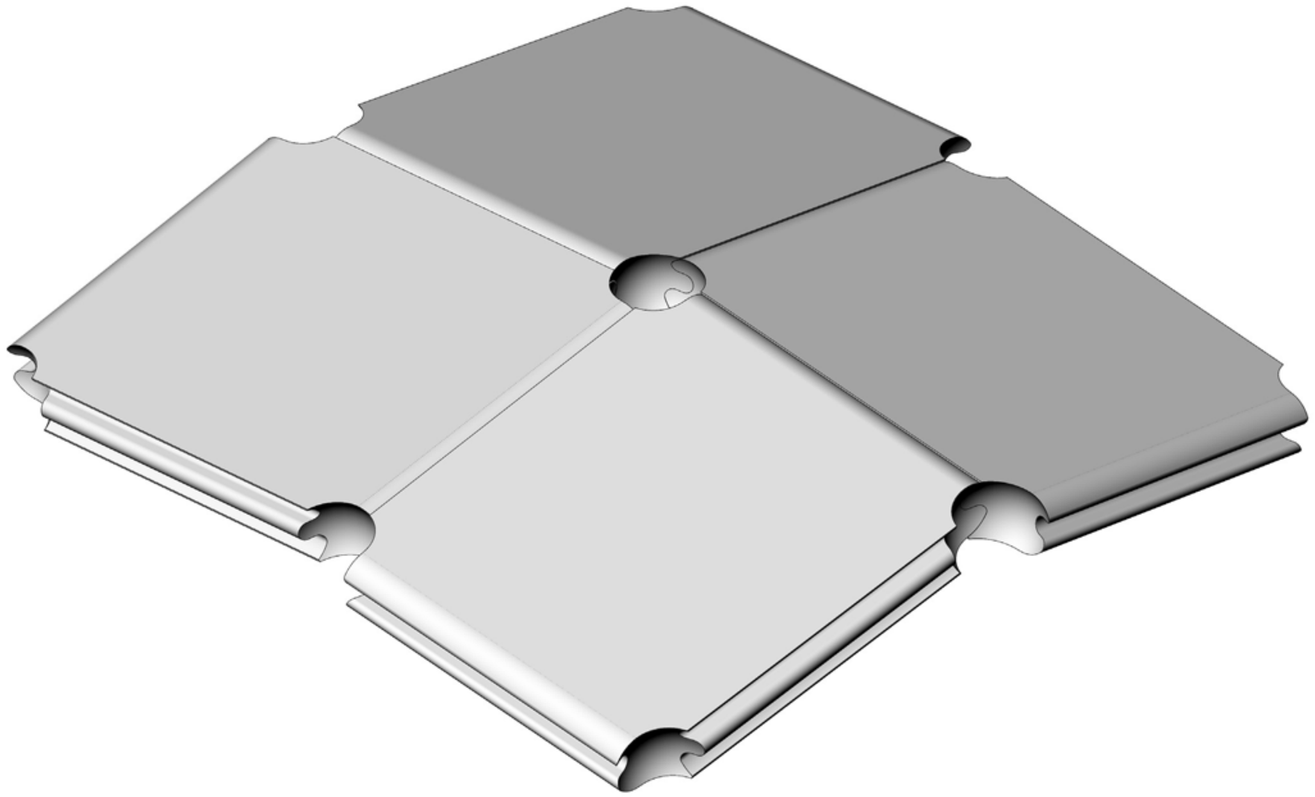


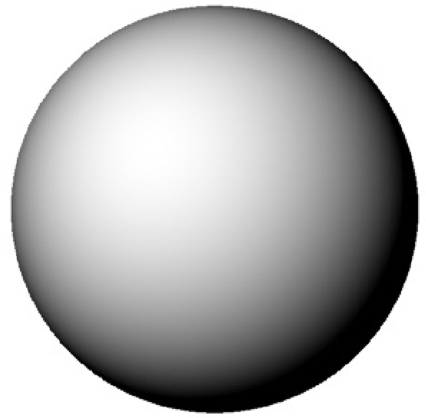
Groove vertex modules



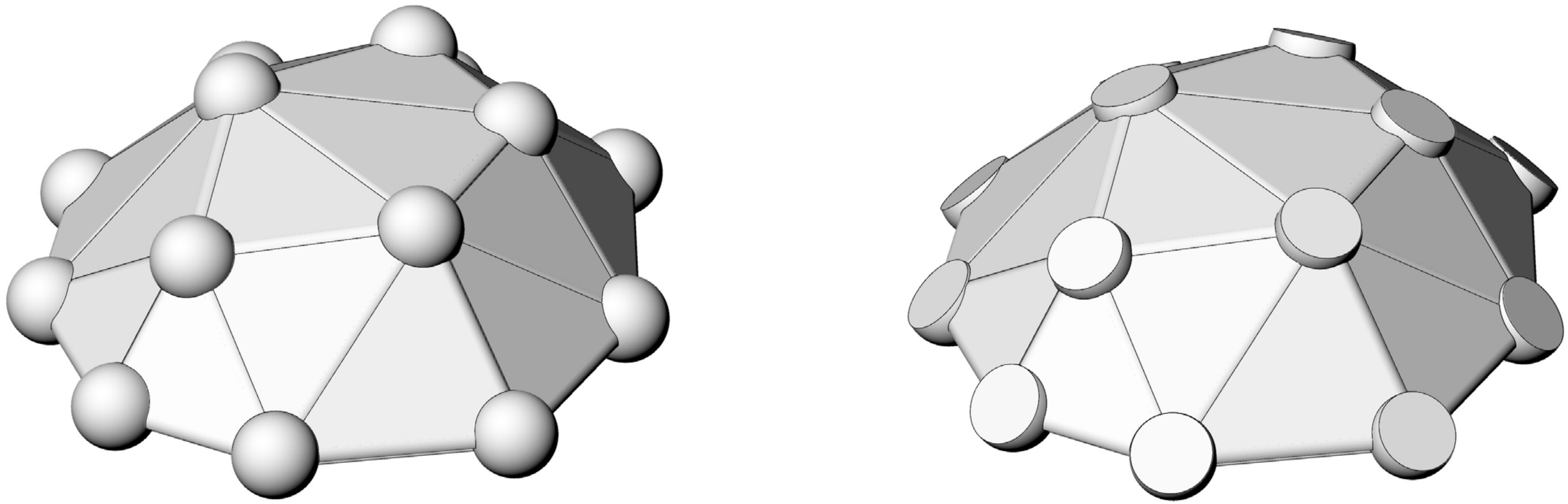
Tongue vertex modules

Spherical holes at the nodes

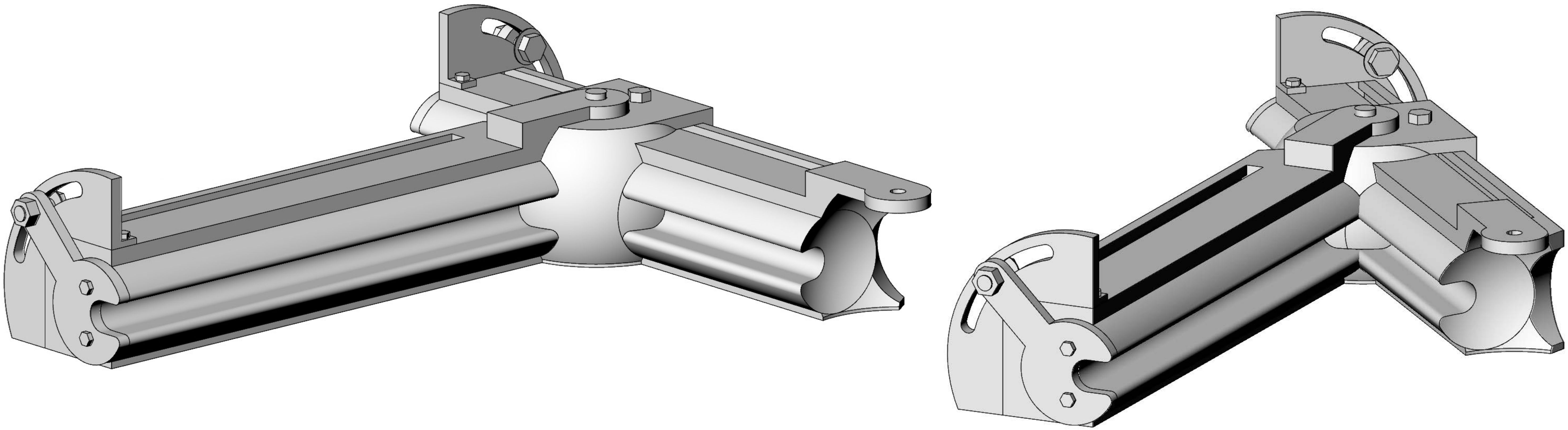




Node components



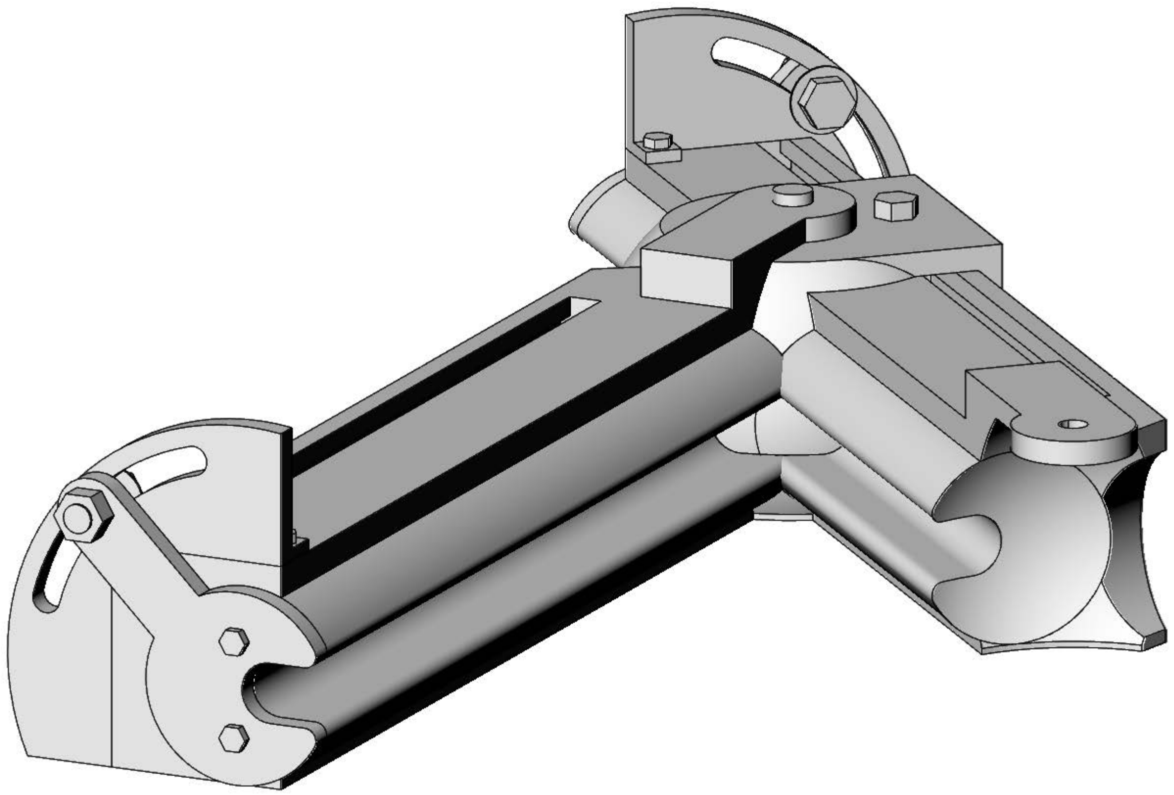
Small angles



Problem only occurs with tongue to tongue edge module connections

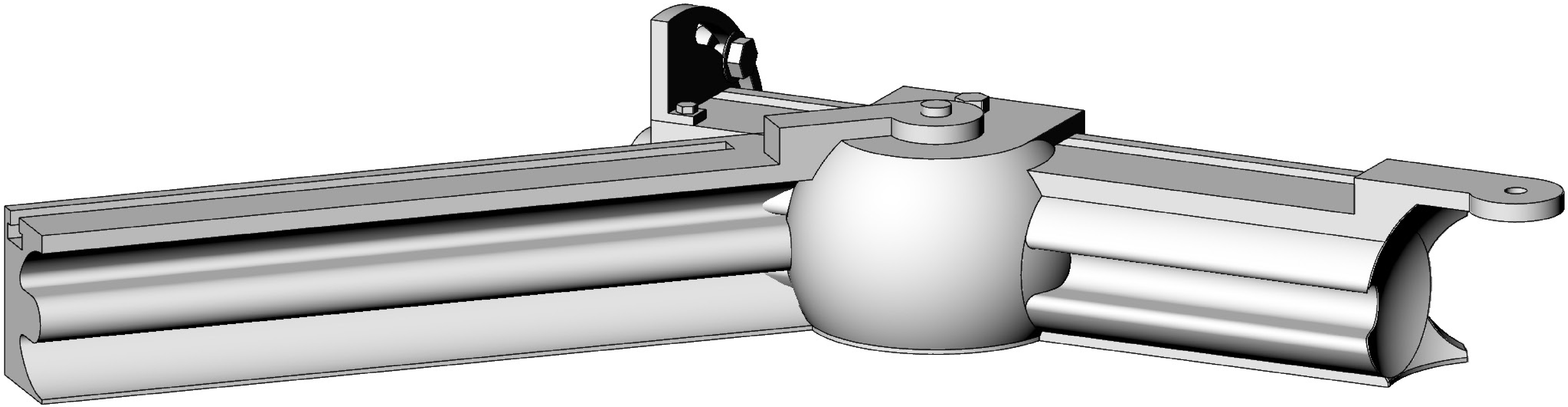
Small angles

$$r_v > \frac{t_v}{2\sin\left(\frac{1}{2}\alpha_{i,min}\right)}$$



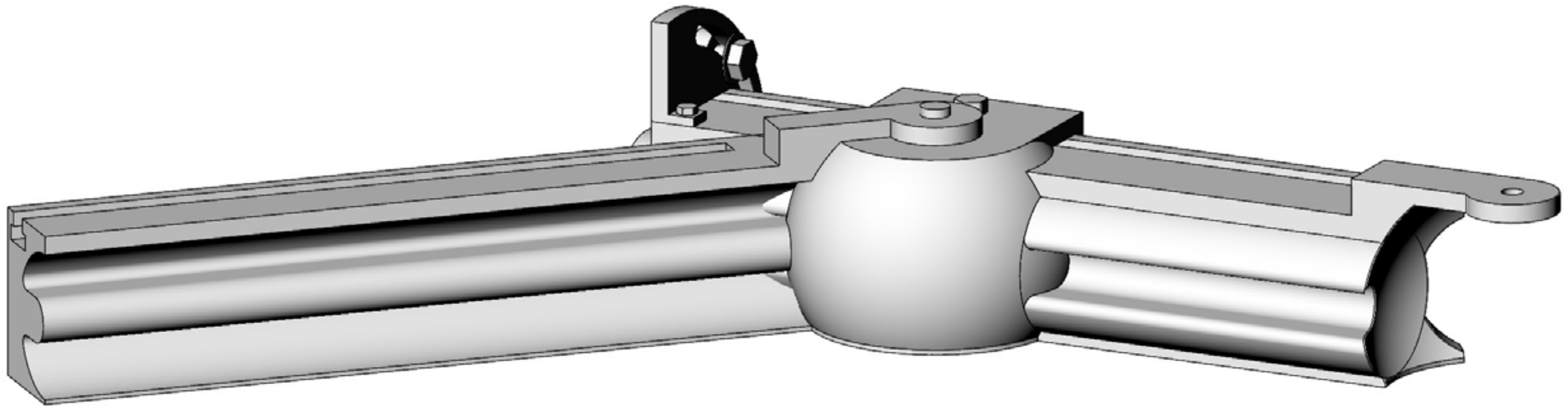
The smaller the minimum interior angle, the larger the radius
The larger the voussoir thickness, the larger the radius

Large angles



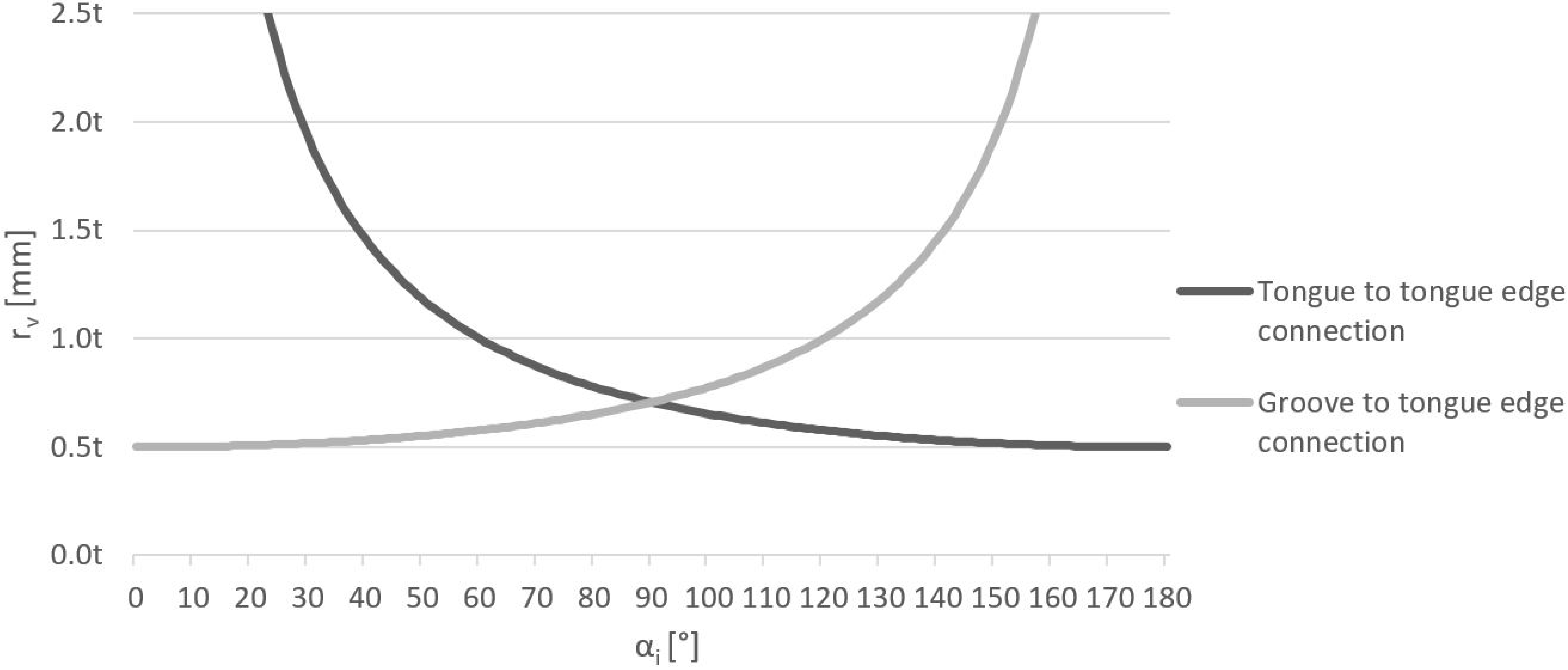
Problem only occurs with groove to tongue edge module connections

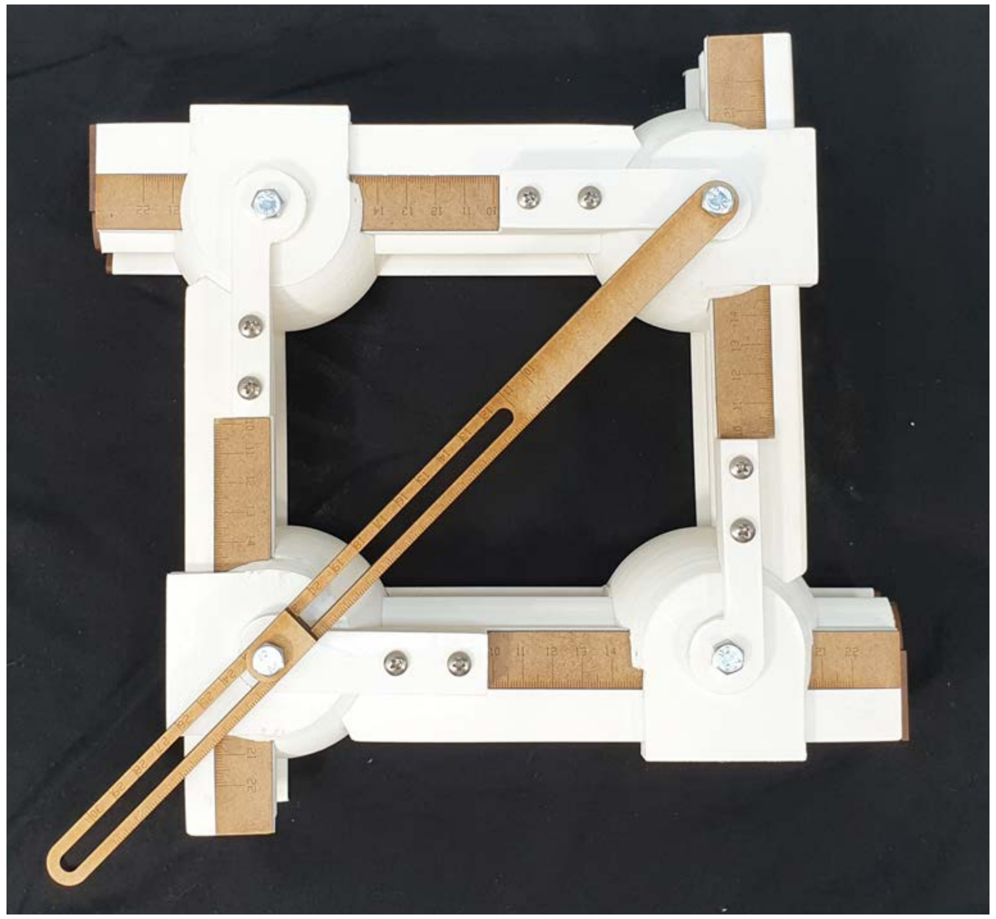
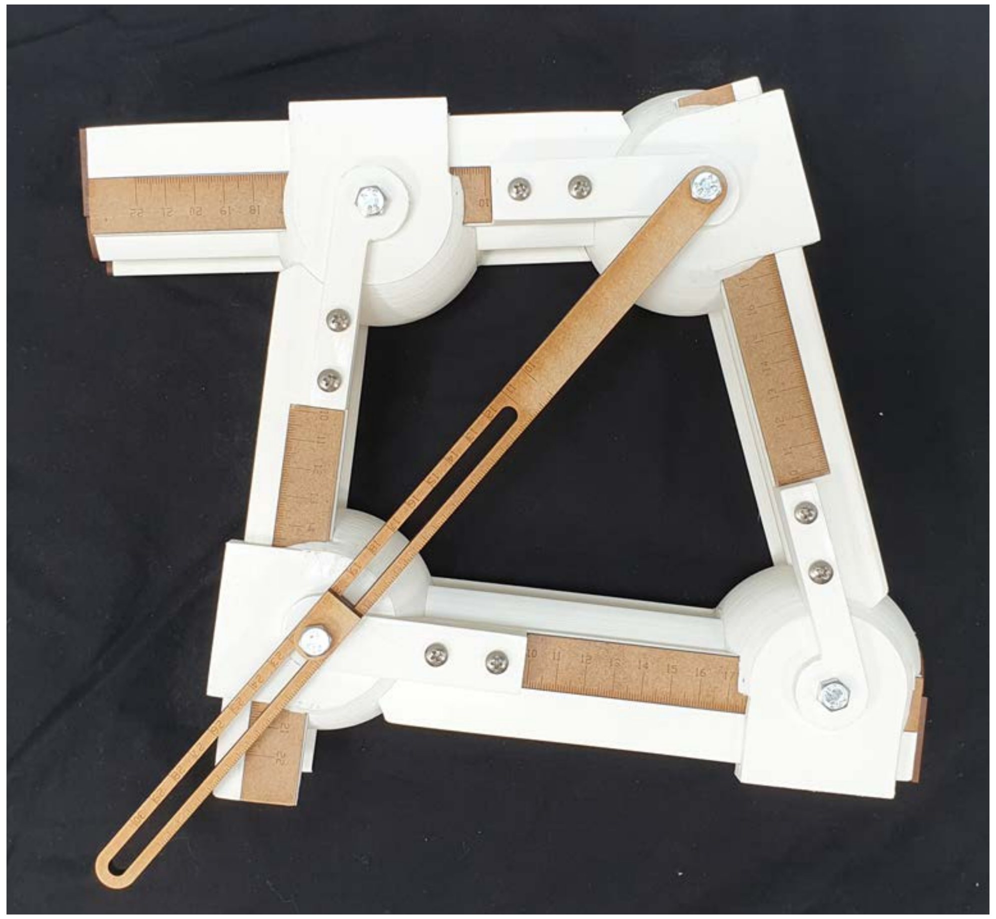
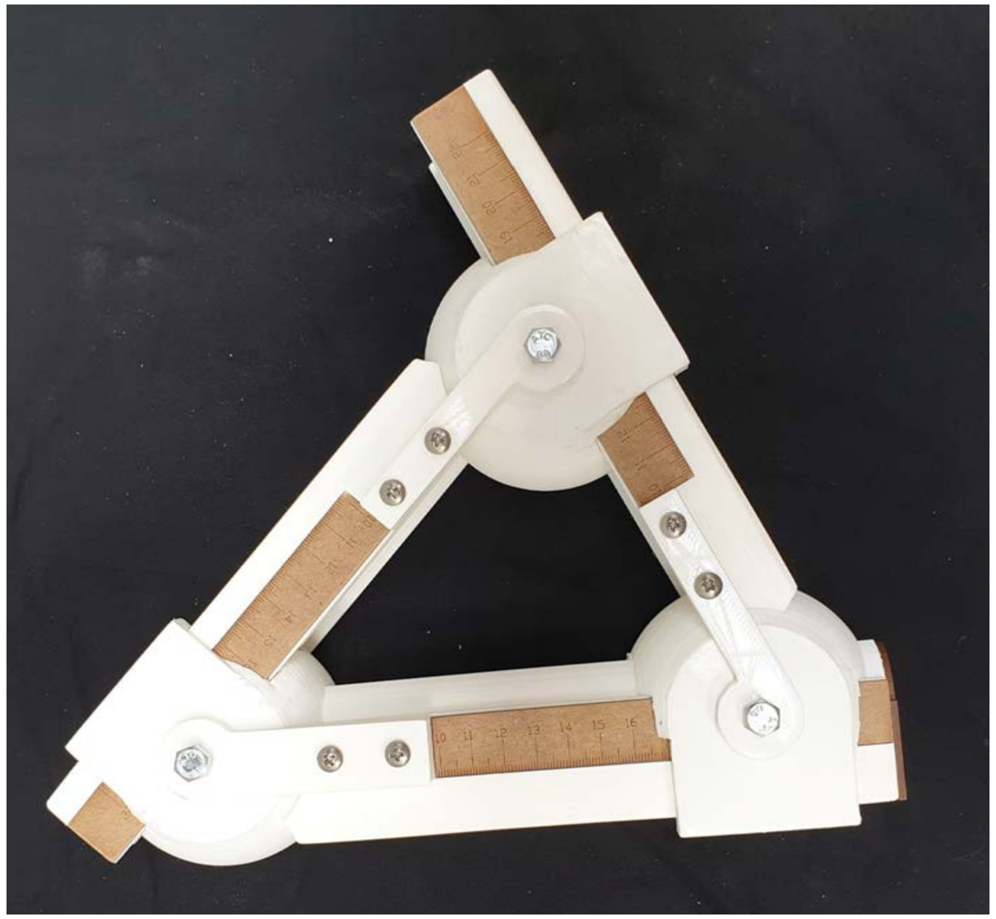
$$r_v > \frac{t_v}{2\cos\left(\frac{1}{2}\alpha_{i,max}\right)}$$

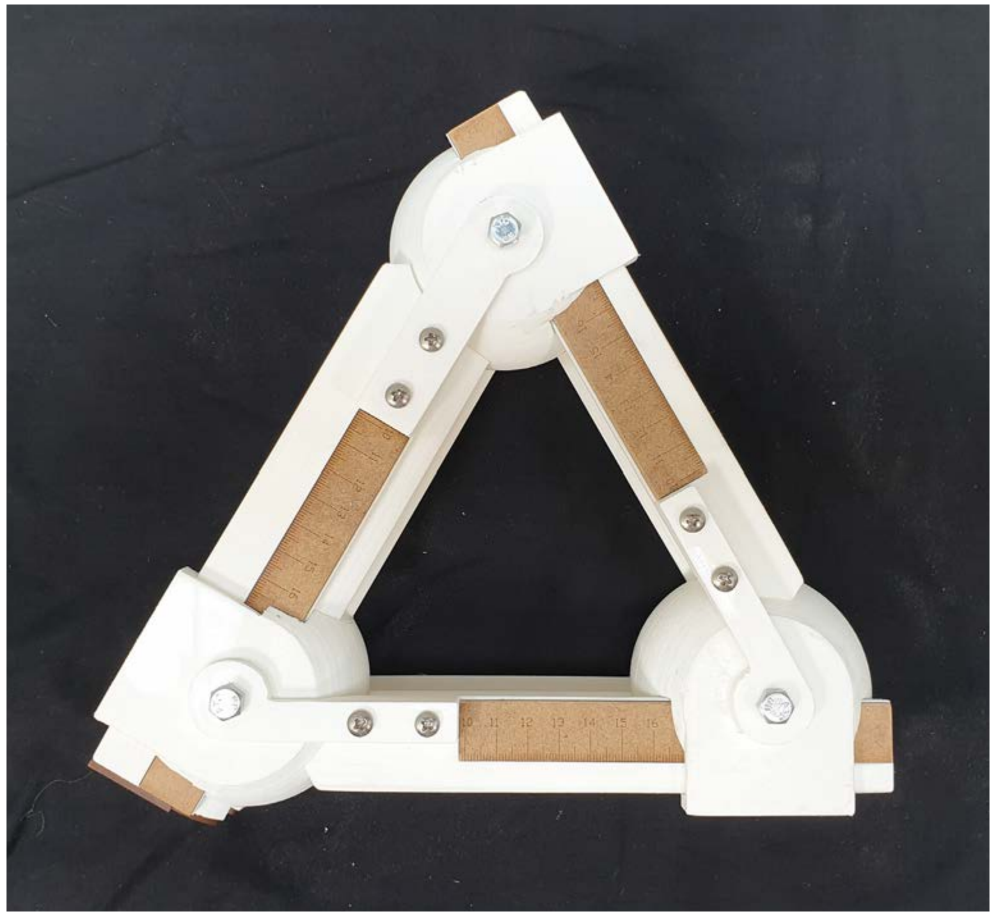


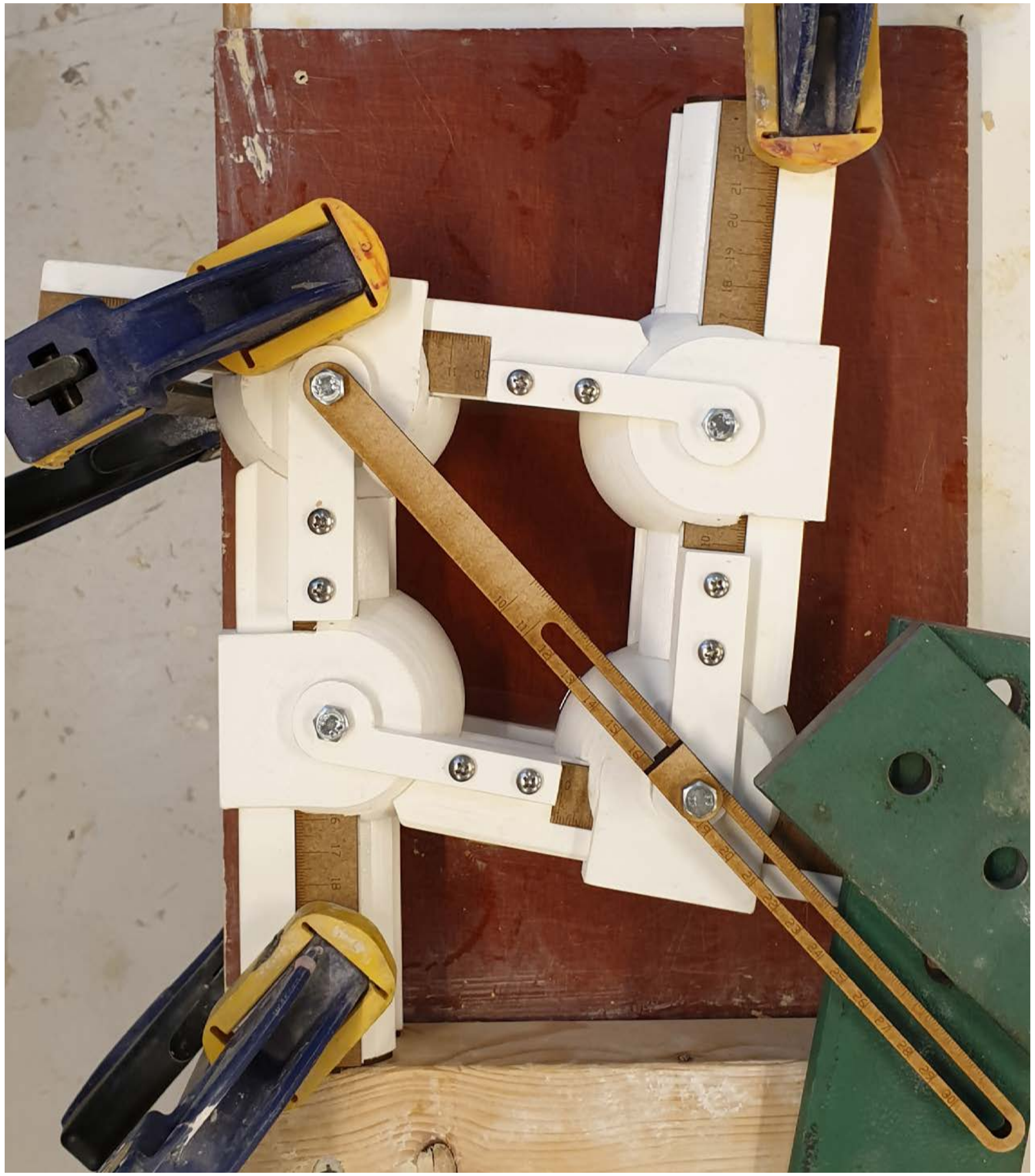
The larger the maximum interior angle, the larger the radius
The larger the voussoir thickness, the larger the radius

Radius of the vertex module



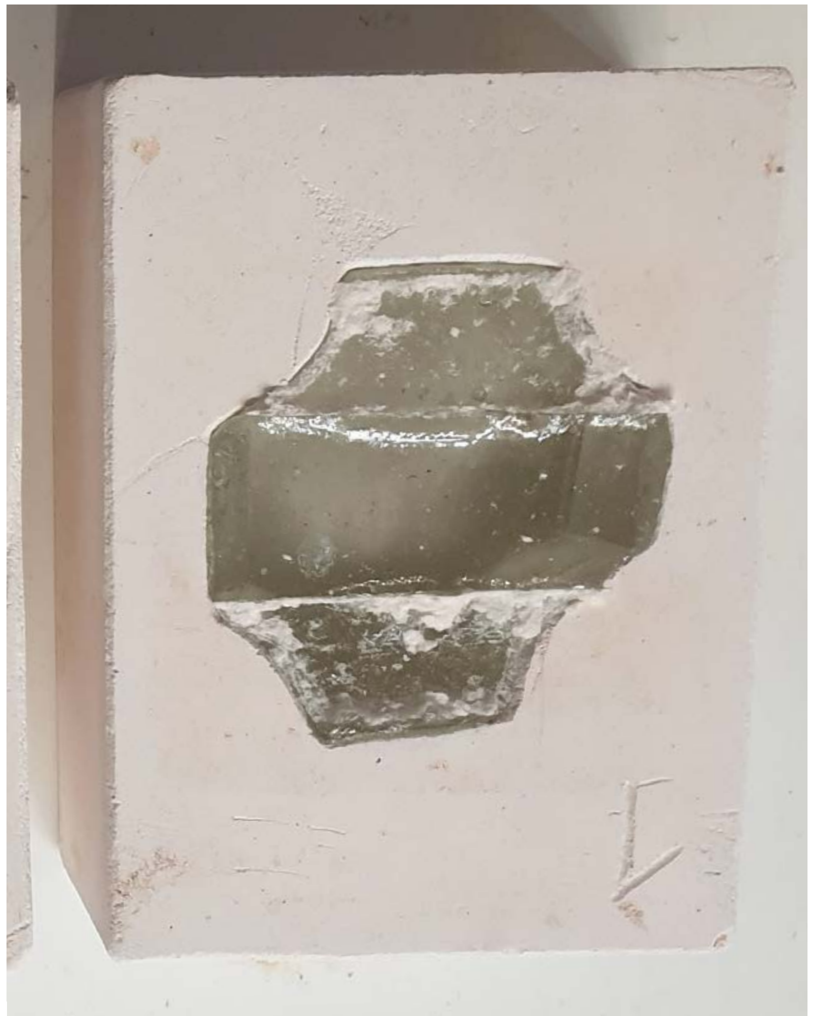









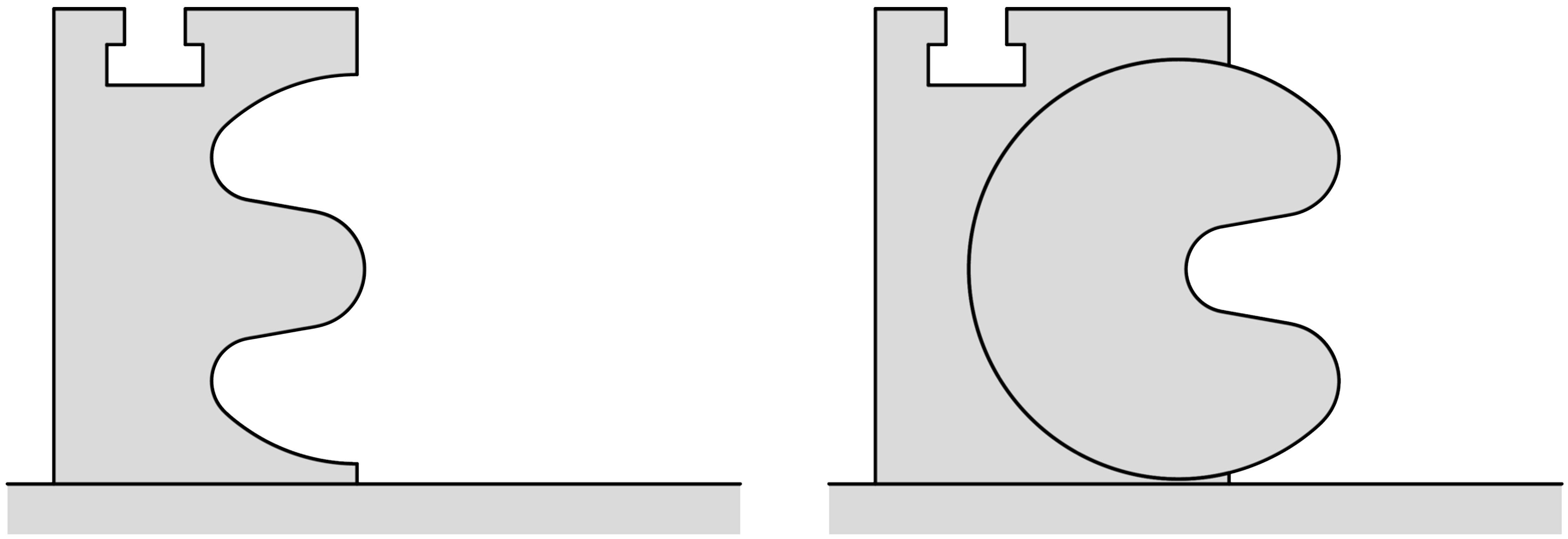
Wax casts

Lost-wax casting



Cast glass voussoirs

Improved design



Draft angle for easier demoulding
1.5 mm offset to accommodate for a 3 mm thick interlayer

Summary

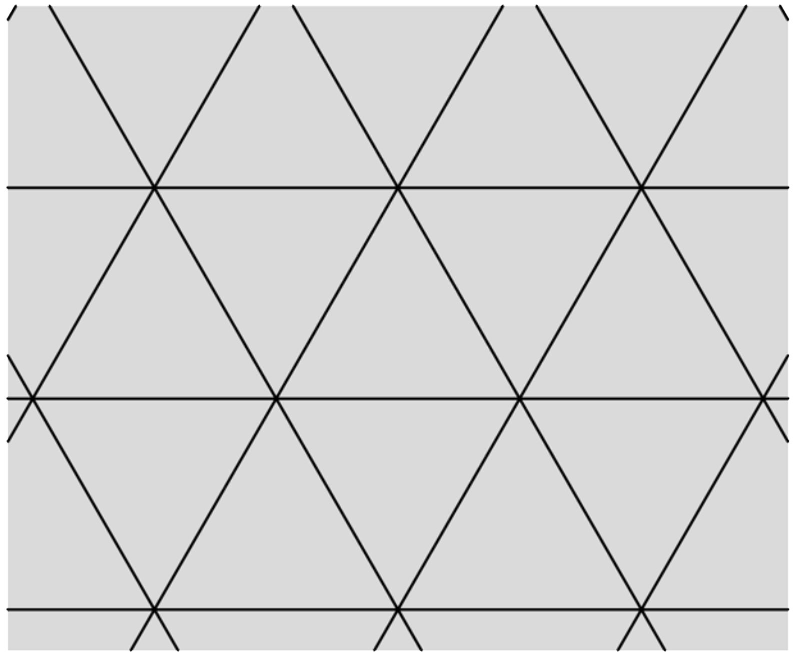
- Adjustable mould
    - Planar convex intrados/extrados
    - Tongue and groove interfaces

- Spherical compoment at the nodes

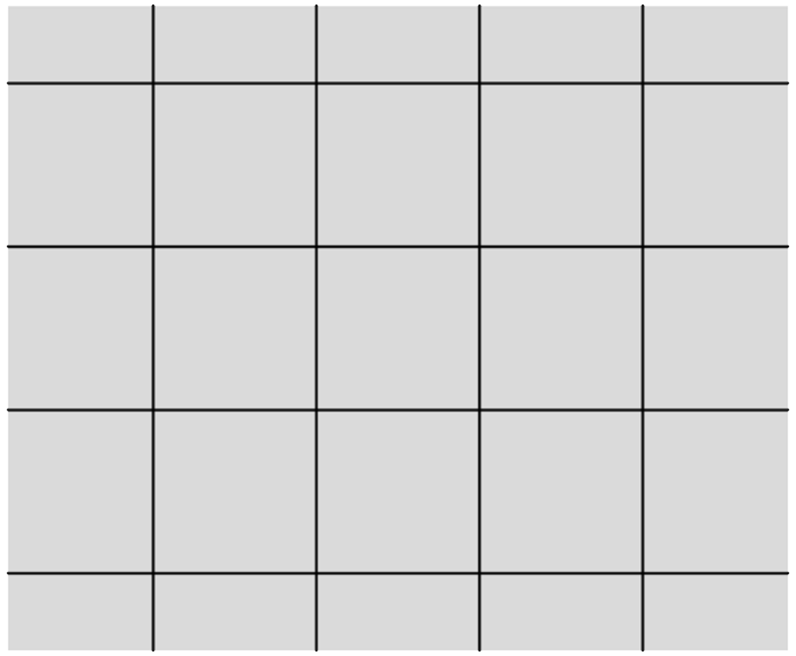
- Dimensions determined by the tessellation pattern

Tessellation pattern

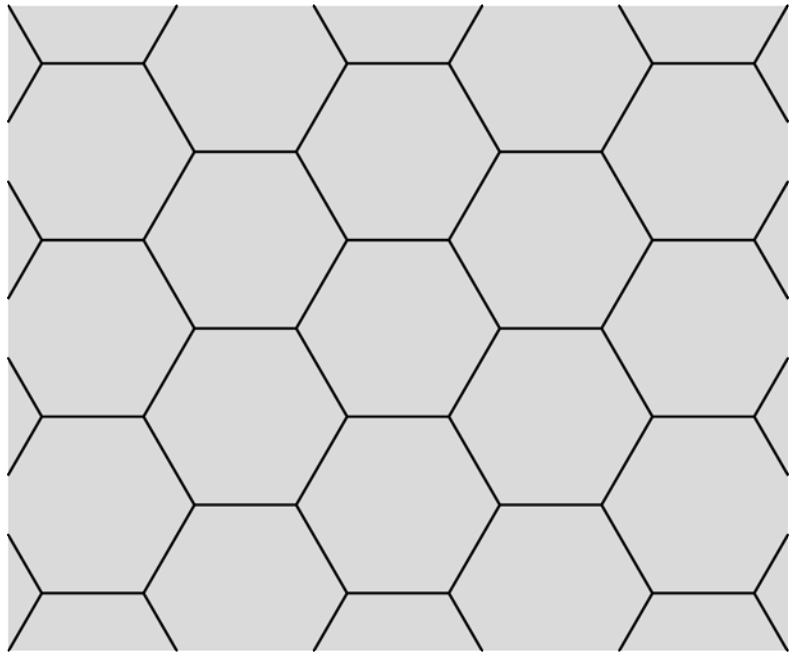
Tessellation pattern



Triangular

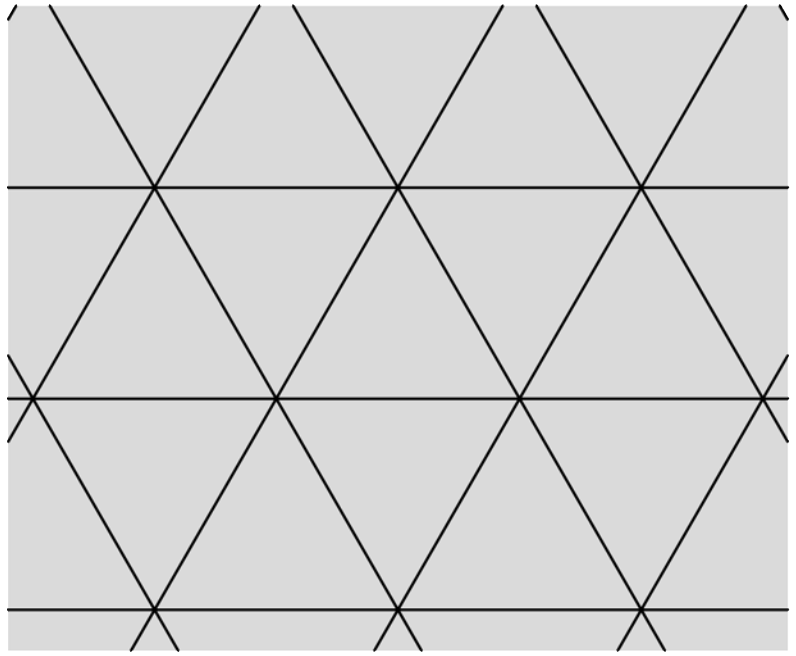


Quadrangular

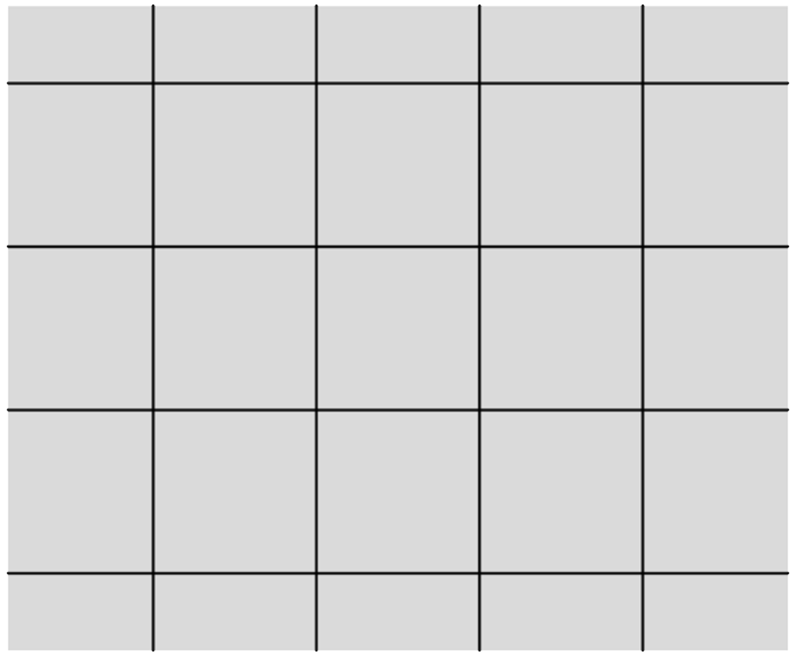


Hexagonal

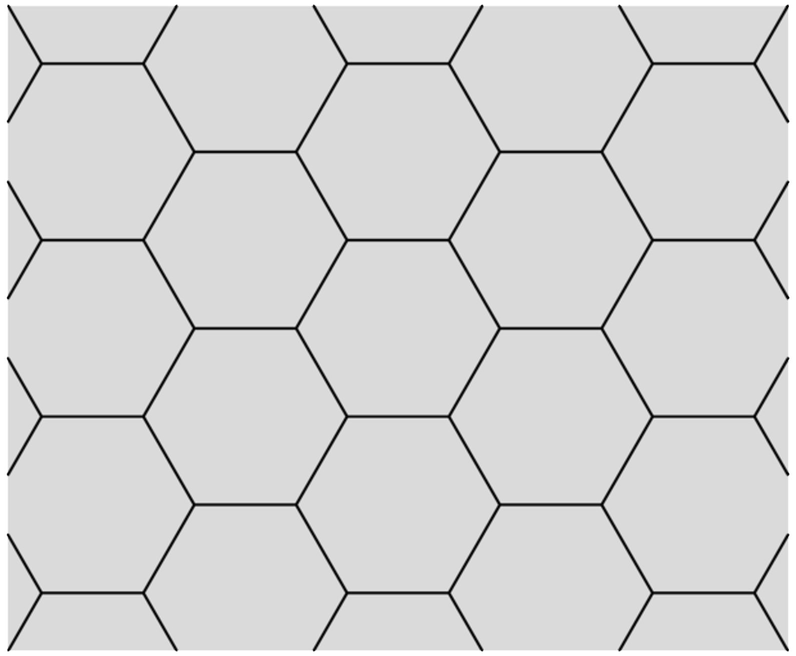
Planarity



Yes

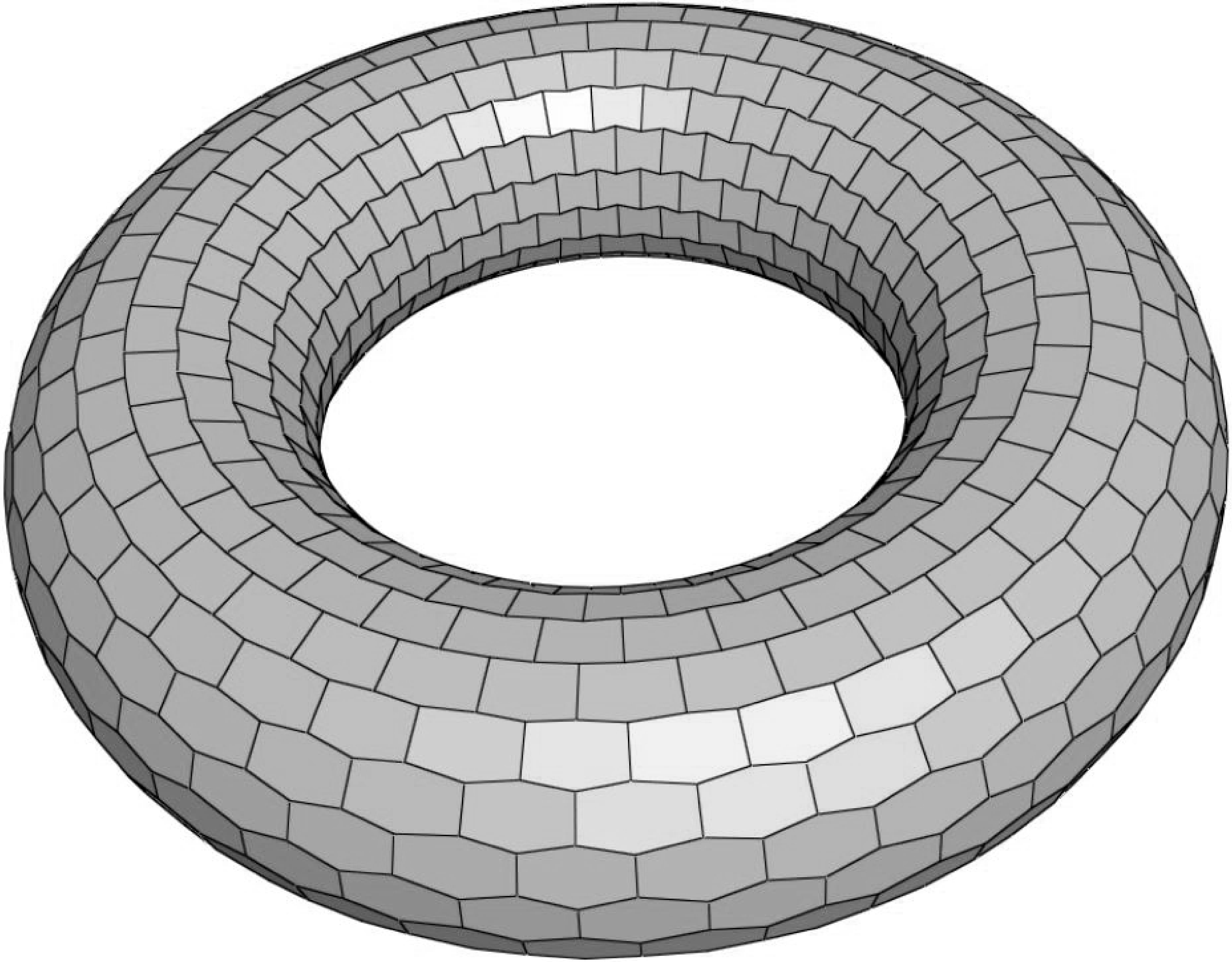


Not guaranteed



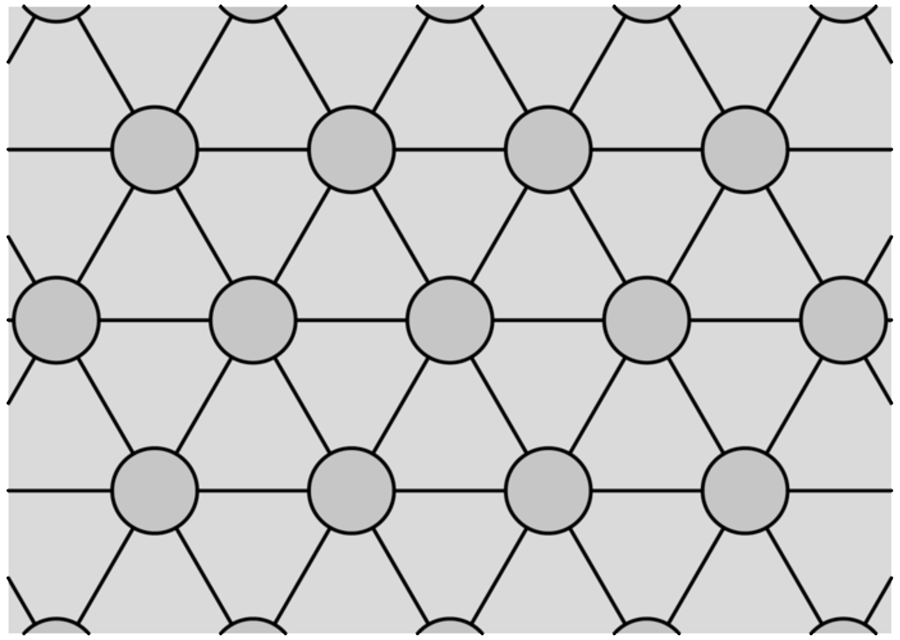
Not guaranteed

Planarity

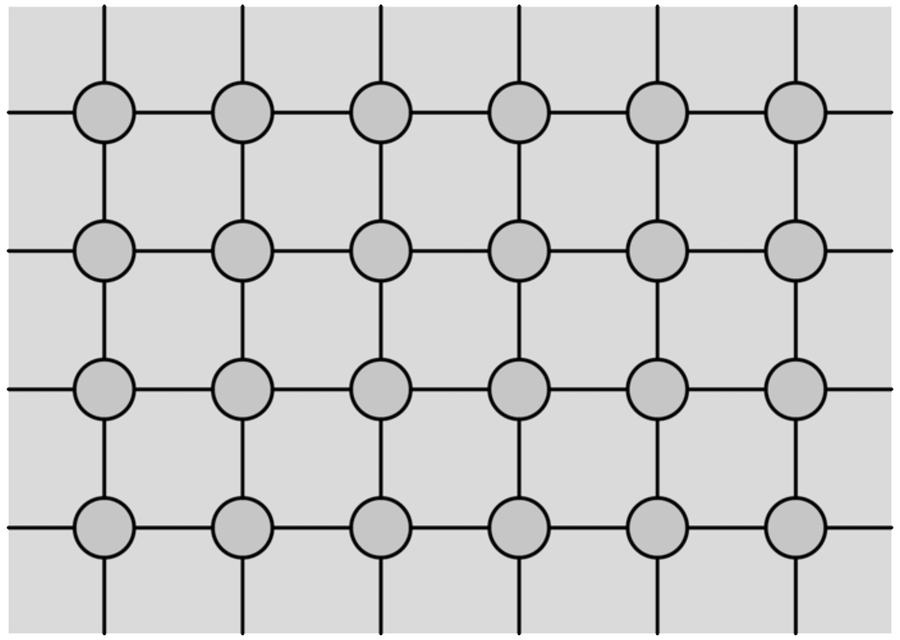


(Wang & Liu, 2009)

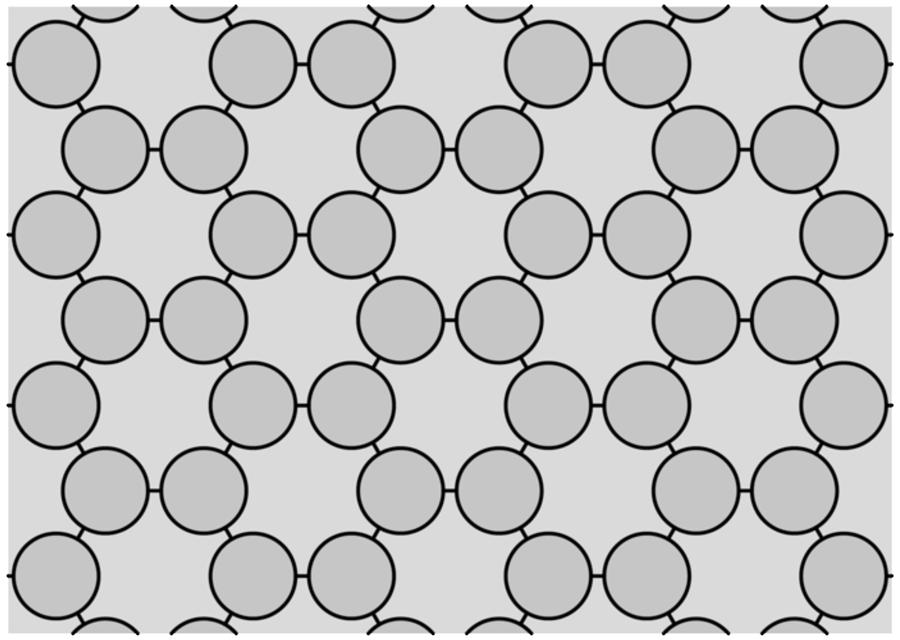
Node components



Large

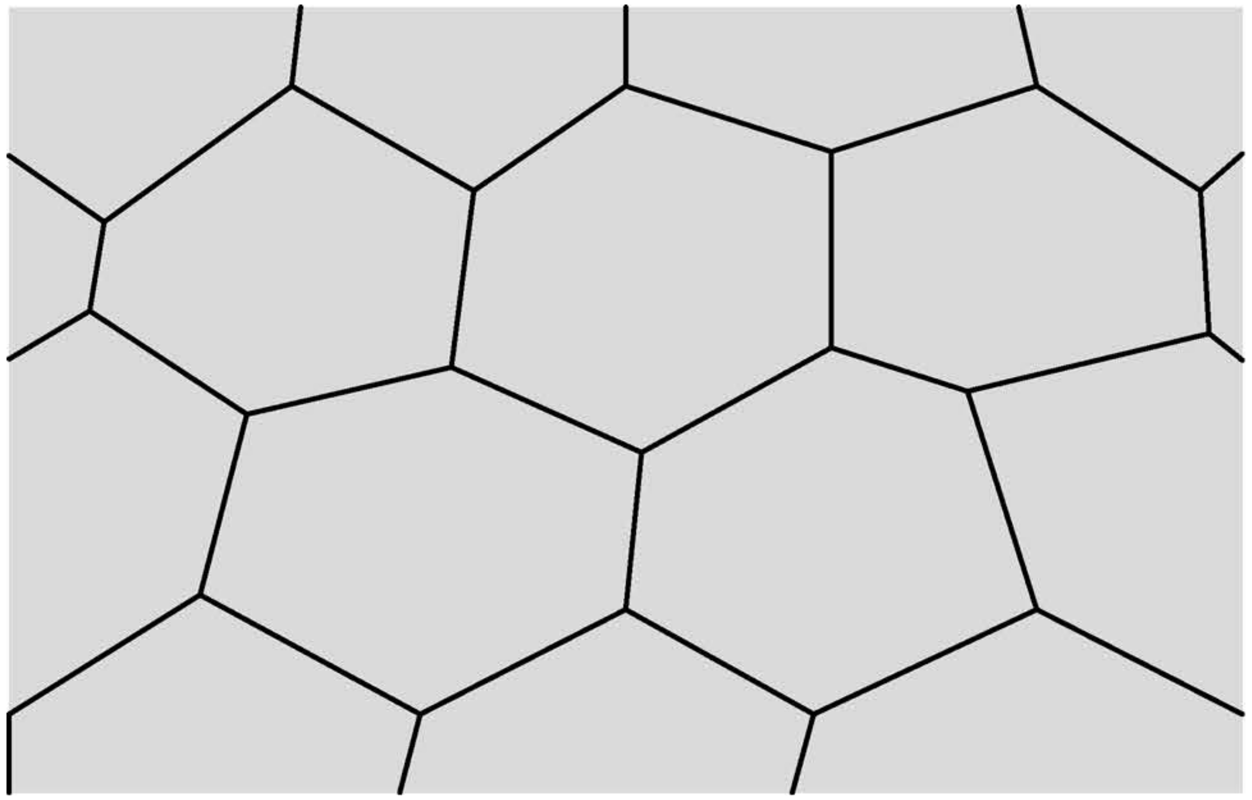


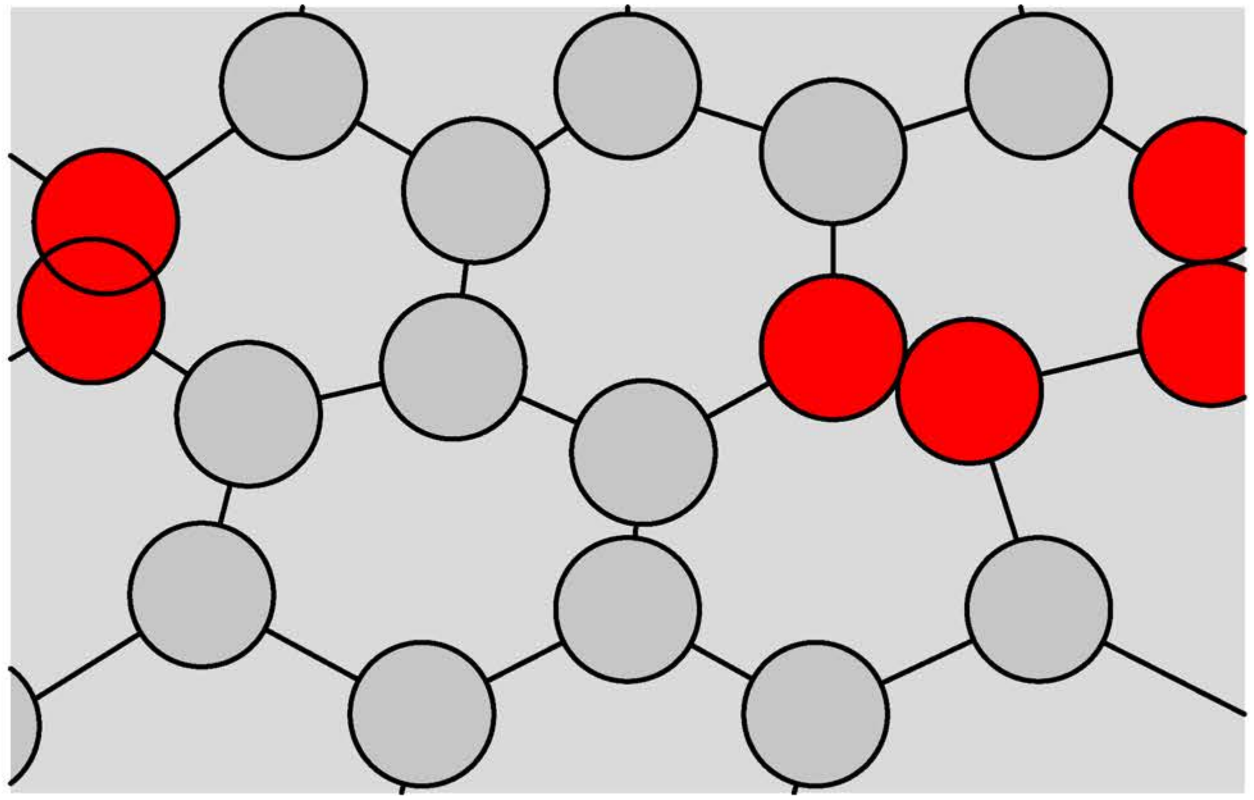
Small



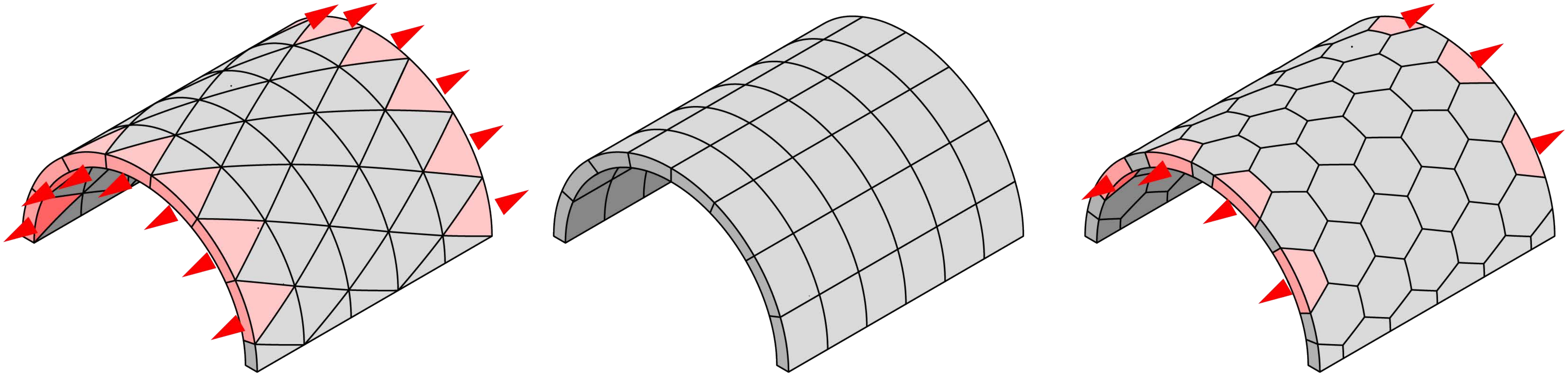
Large

Node components

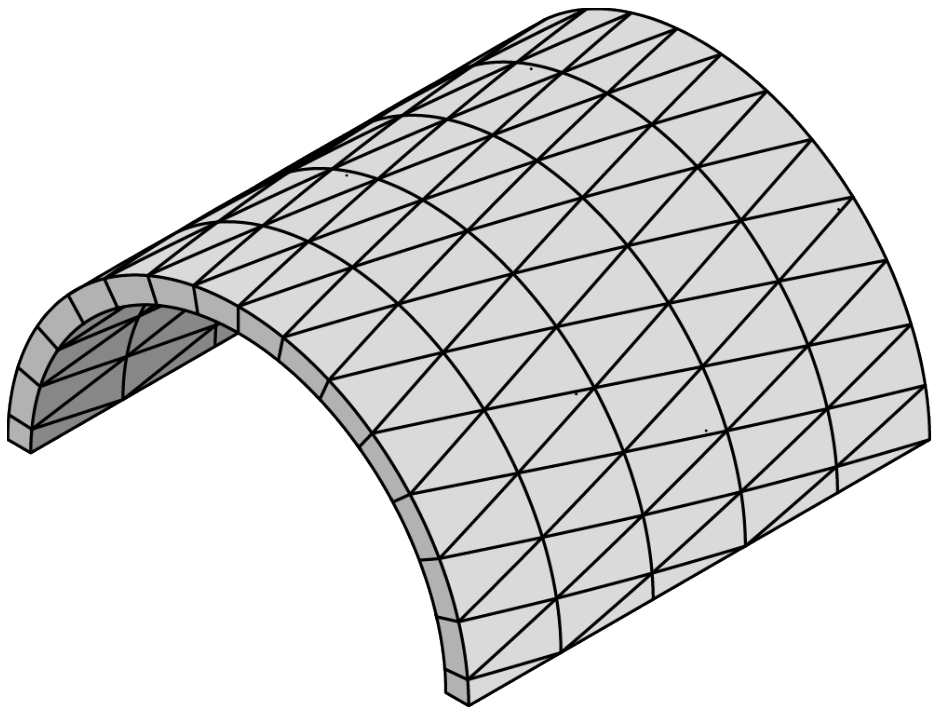


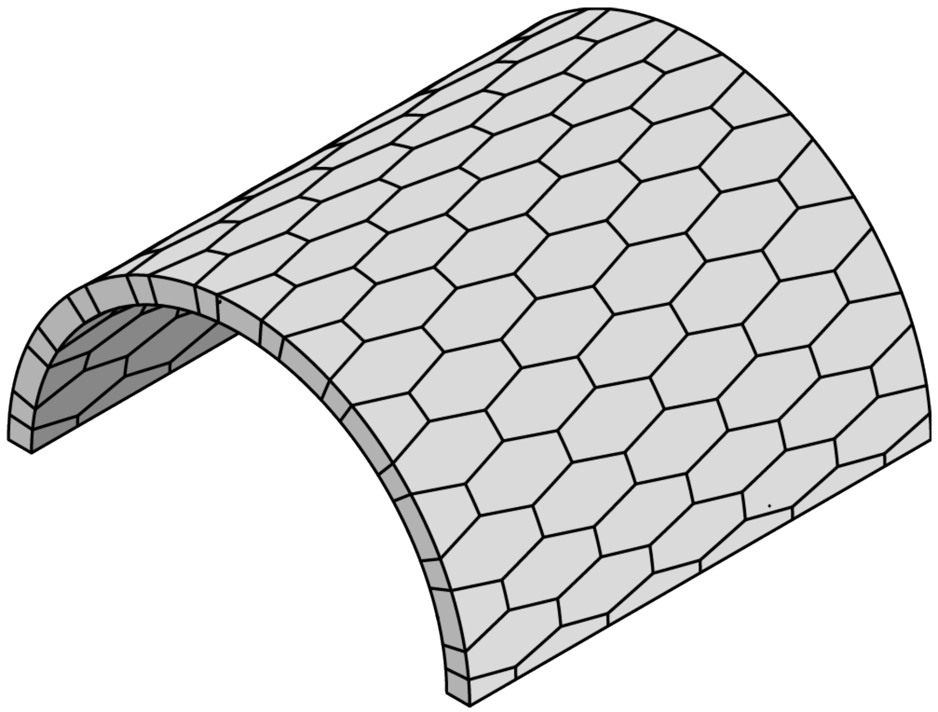


Risk of sliding at edges



Risk of sliding at edges





Summary

- Polygons have to be planar and convex

- Edge length should be larger than twice the sphere radius

- Tessellation should be alligned with the force flow

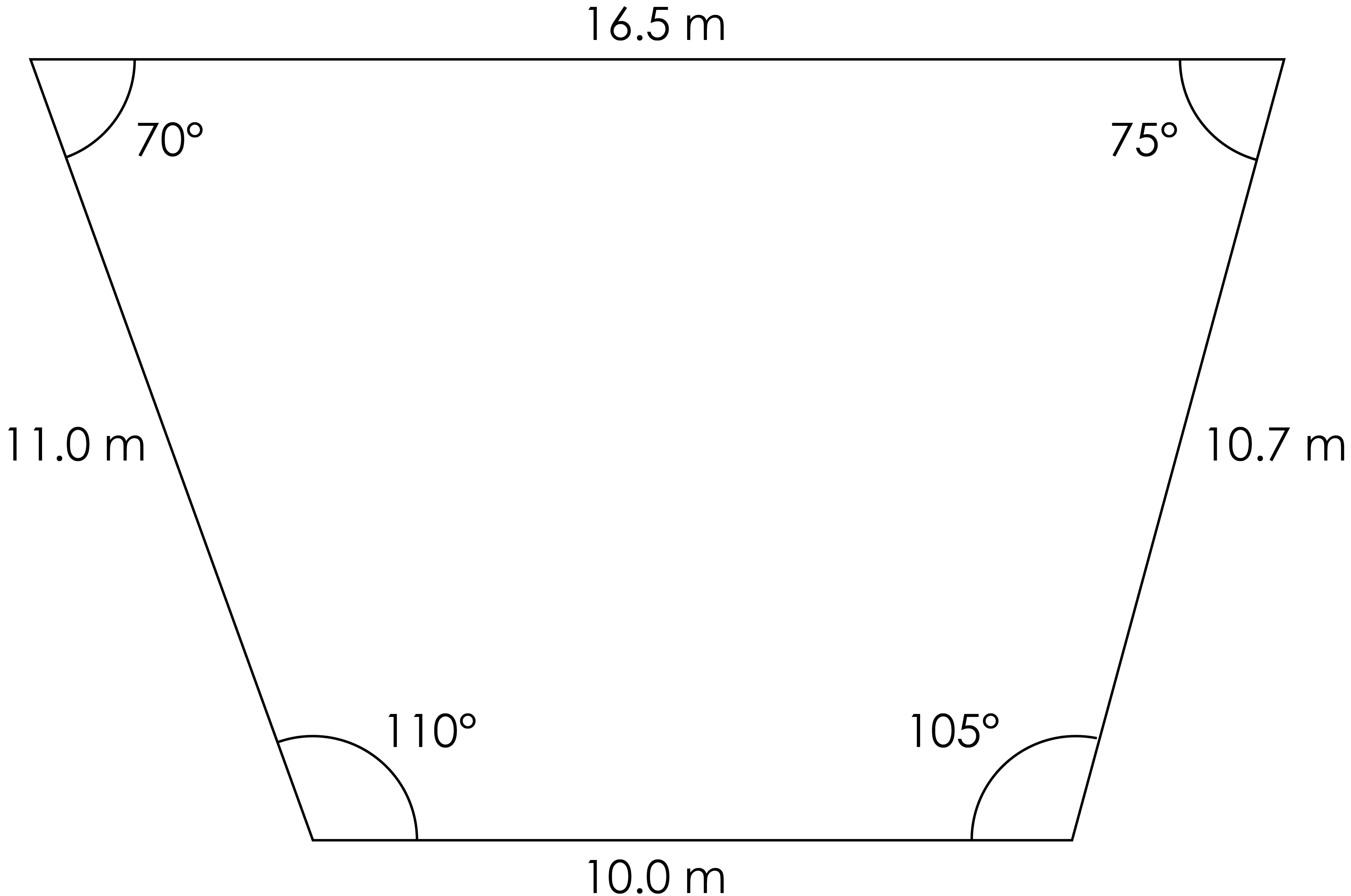
Case study

Great Court at the British Museum (Foster + Partners)

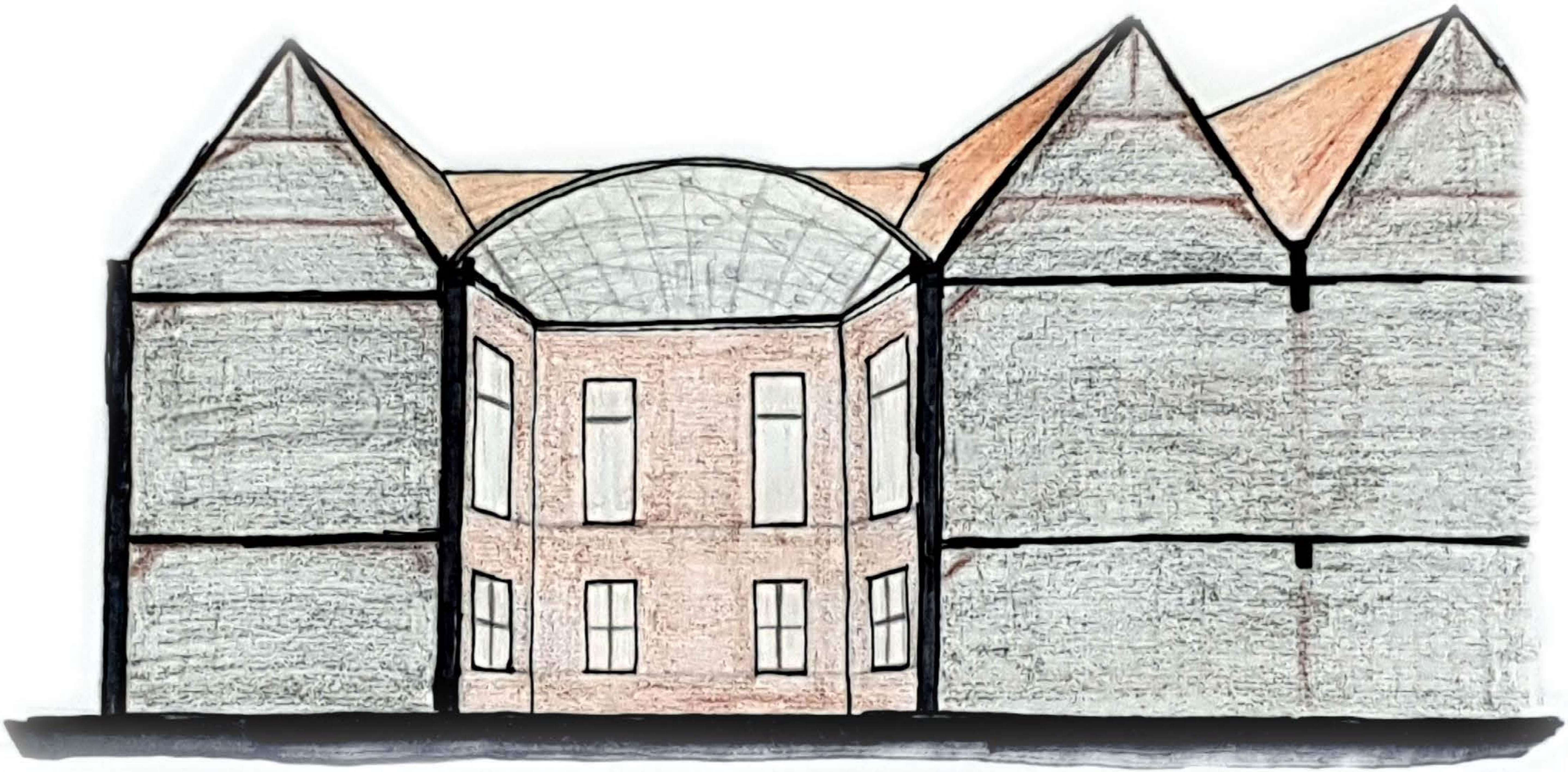
Dutch Maritime Museum (Ney & Partners)

Armamentarium Delft (cepezed projects)

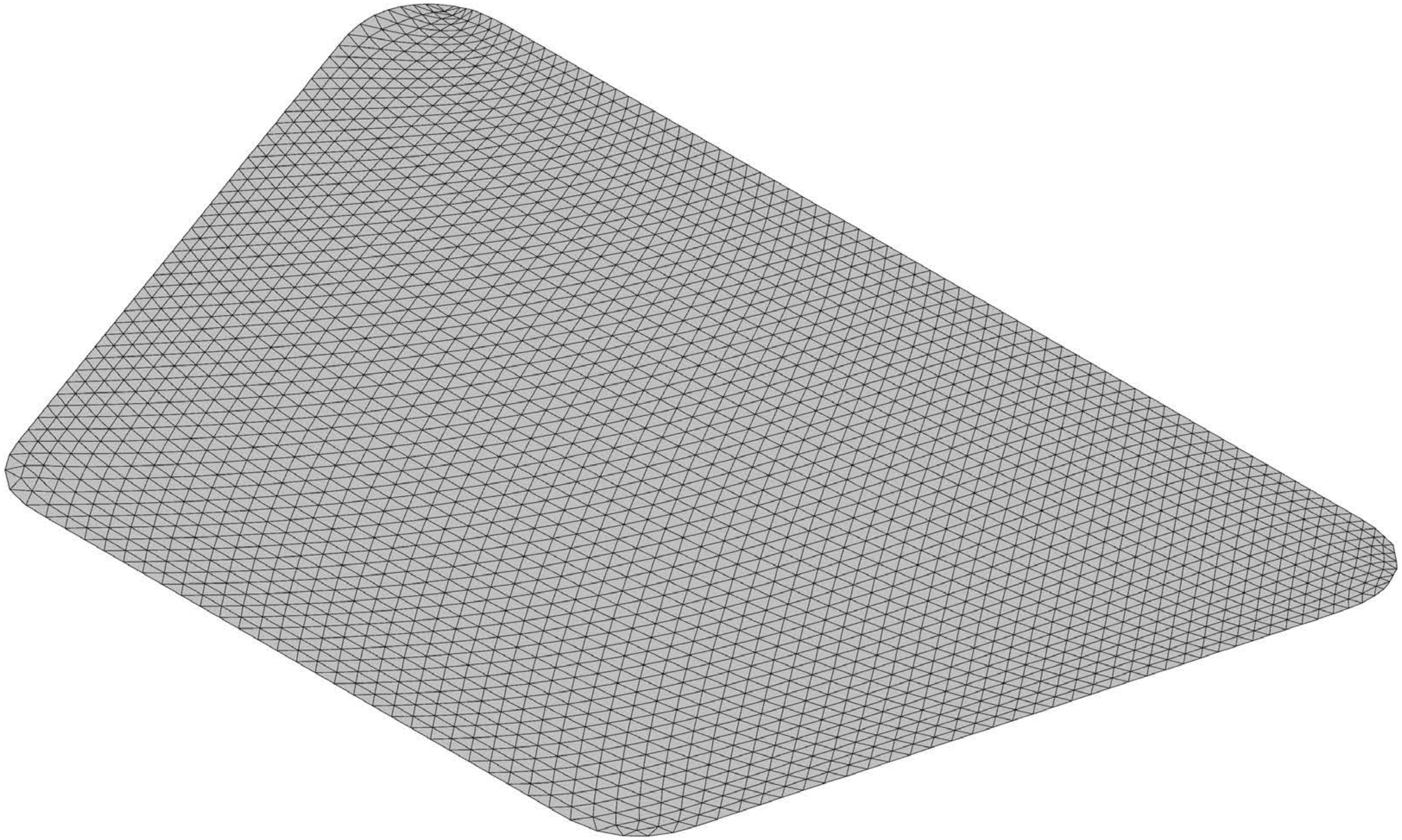
Plan



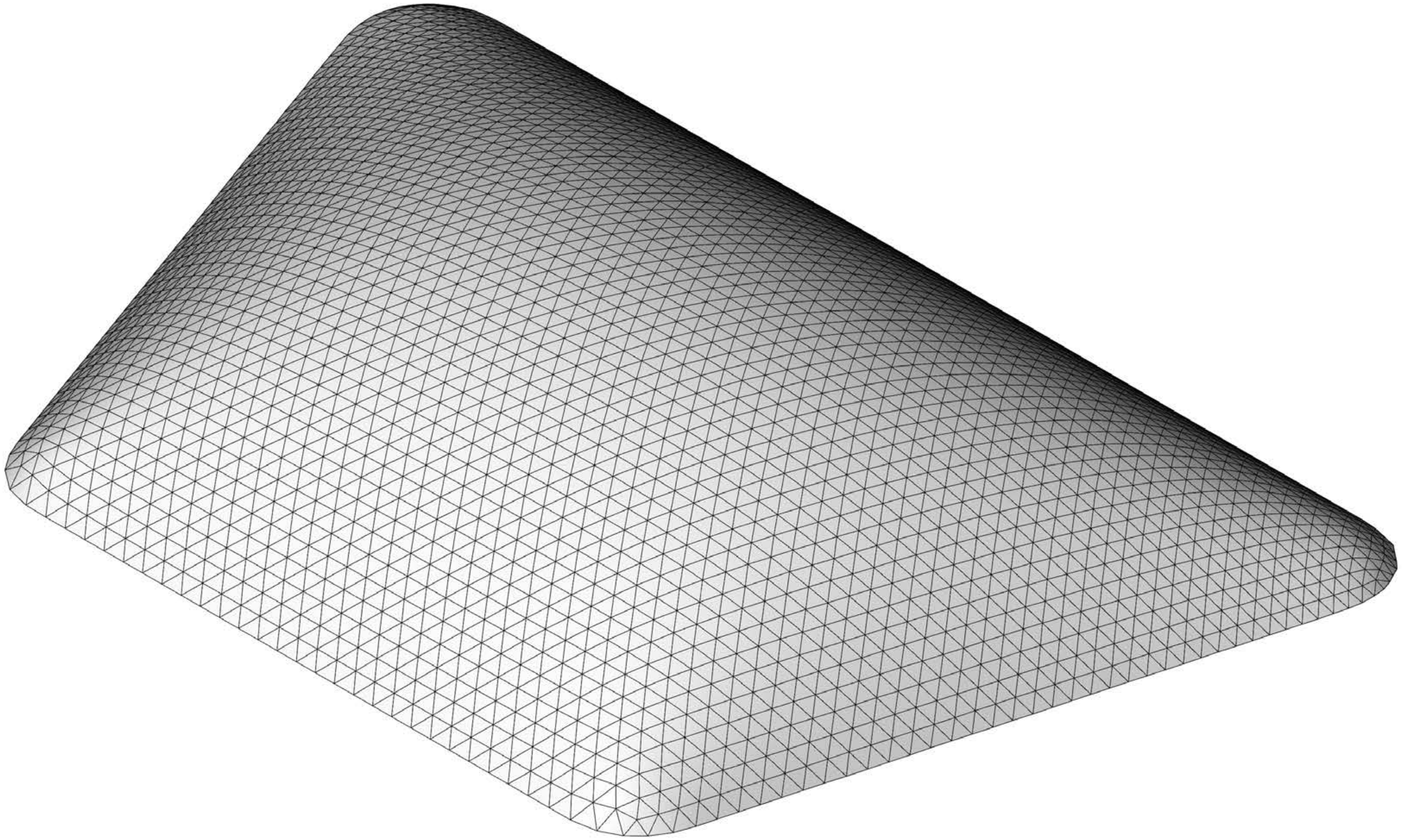
Concept



Form finding



Form finding



Structural validation

- Finite element method (FEM) analysis
  - Displacements
  - Stresses

- Discrete element method (DEM) analysis
  - Stability

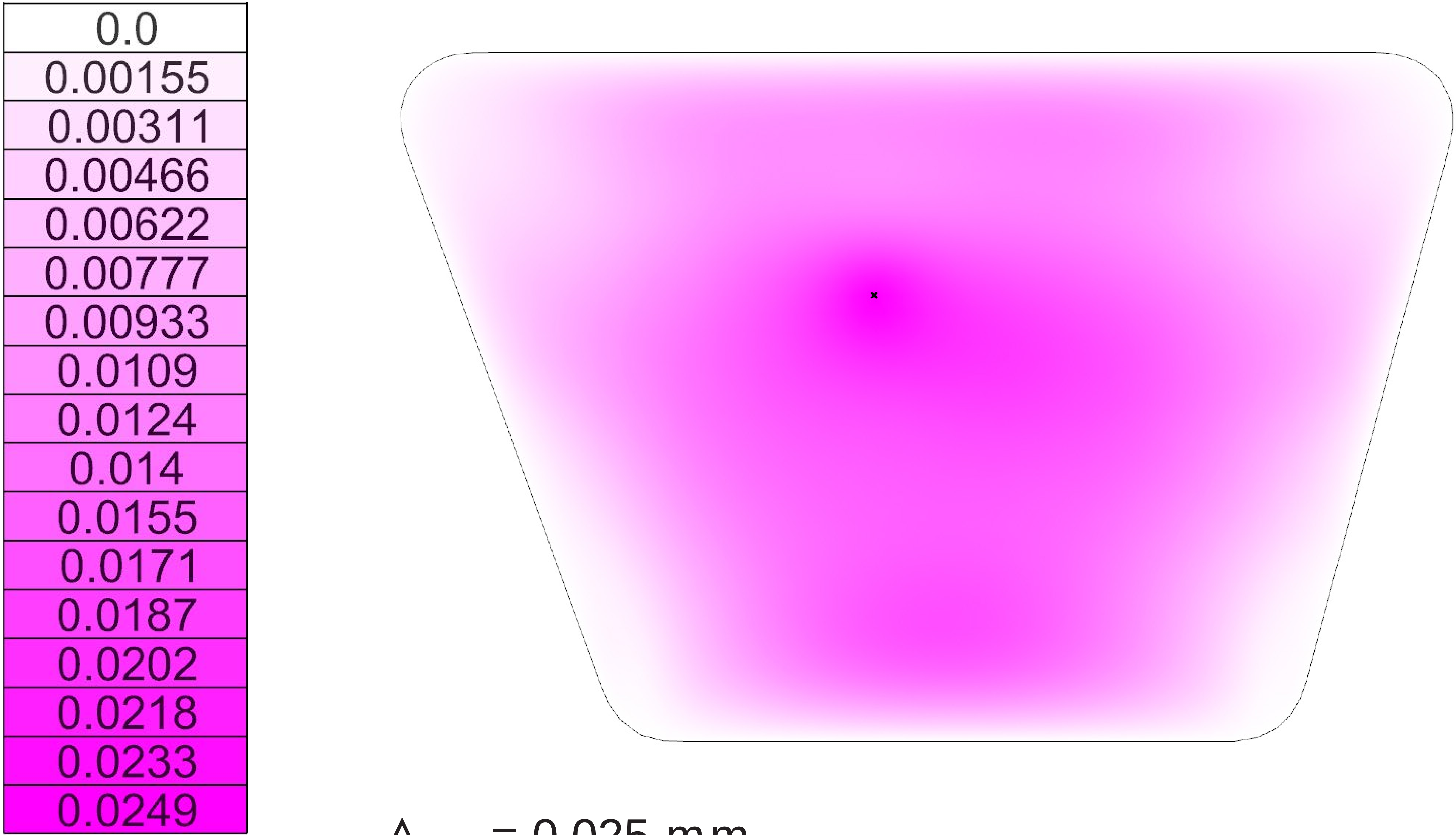
FEM analysis

Thickness	100 mm
Density	2500 kg/m^3
Young's modulus	70 GPa
Shear modulus	28 GPa
Yield strength	30 MPa

FEM analysis

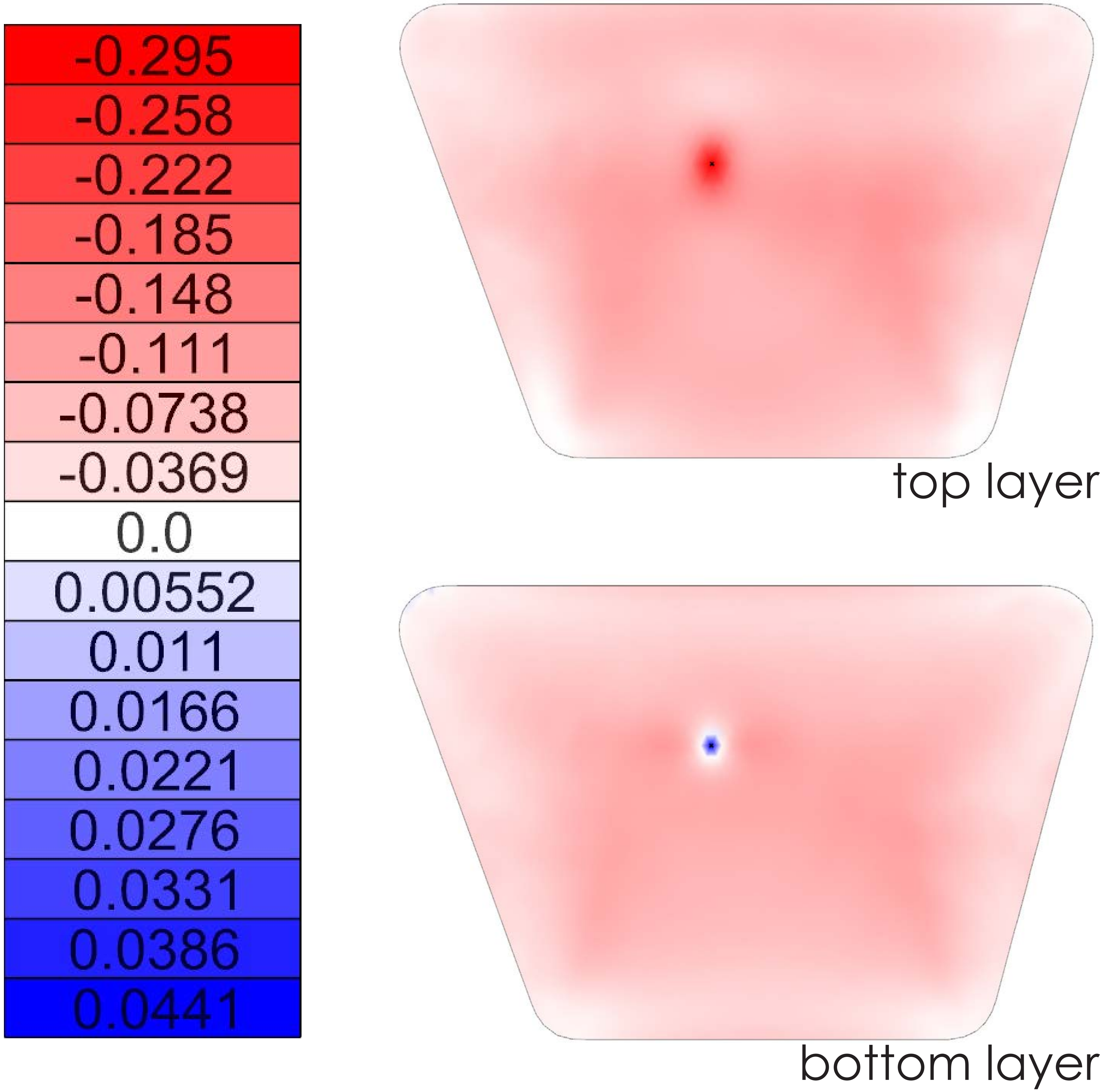
	Load	Safety factor
Dead load	2.5 kN/m²	1.2
Live load (concentrated)	1.5 kN	1.5
Wind load (asymmetric)	0 - 0.48 kN/m²	1.5
Snow load	1.1 kN/m²	1.5

Displacement [mm]



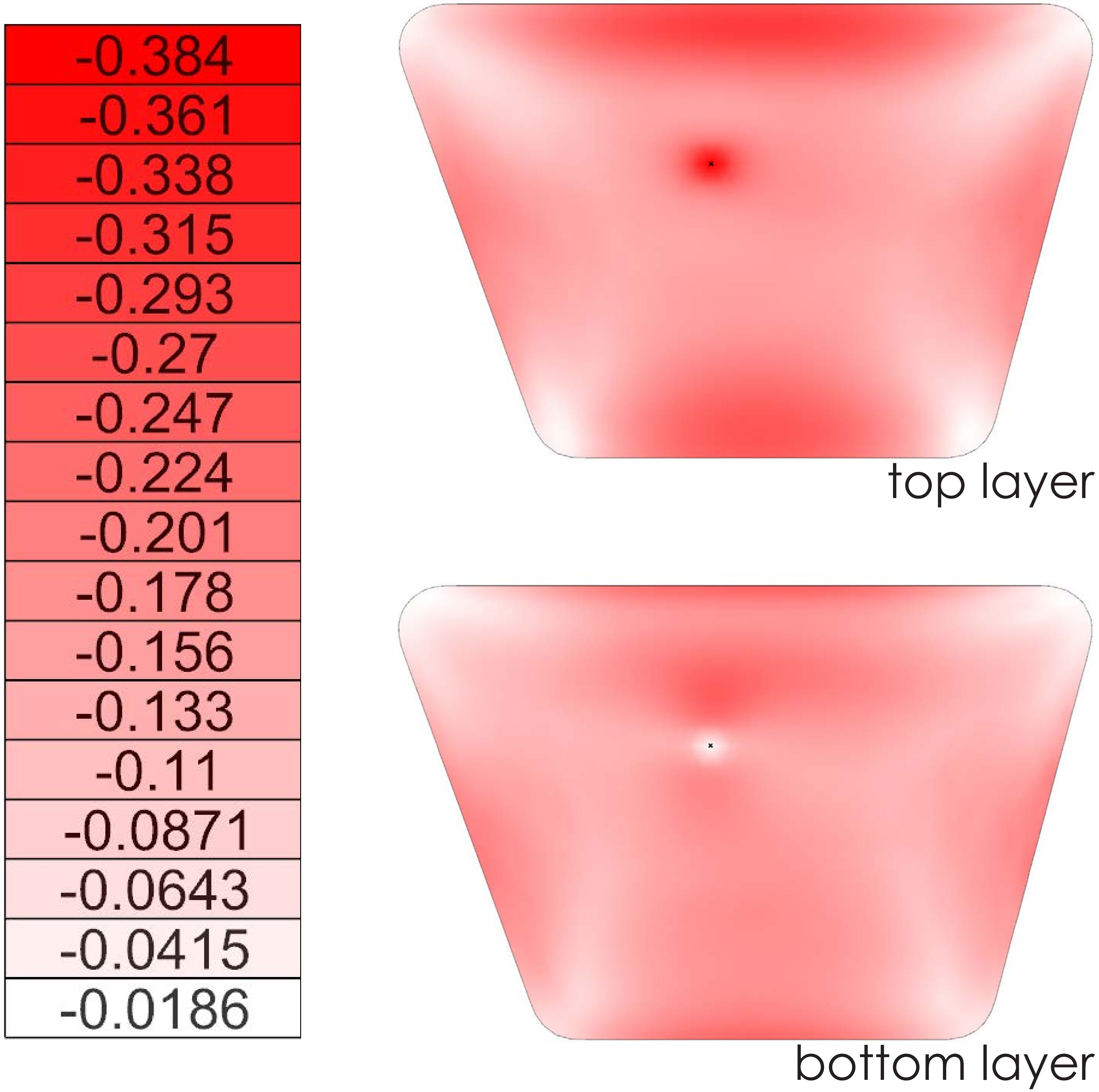
$\Delta_{max} = 0.025$ mm

Principal stress 1 [MPa]



σ_{min} = -0.30 MPa
σ_{max} = 0.04 MPa

Principal stress 2 [MPa]



$\sigma_{min} = -0.38$ MPa
$\sigma_{max} = -0.02$ MPa

Tessellation

- Fully supported along its boundary
- No risk of sliding between the voussoirs
- No need for allignment to the force flow

Tessellation constraints

- Maximize the smallest occuring edge length

- Maximize the smallest occuring interior angles

- Minimize area of each mesh face

Tessellation constraints

- Maximize the smallest occuring edge length

- Maximize the smallest occuring interior angles

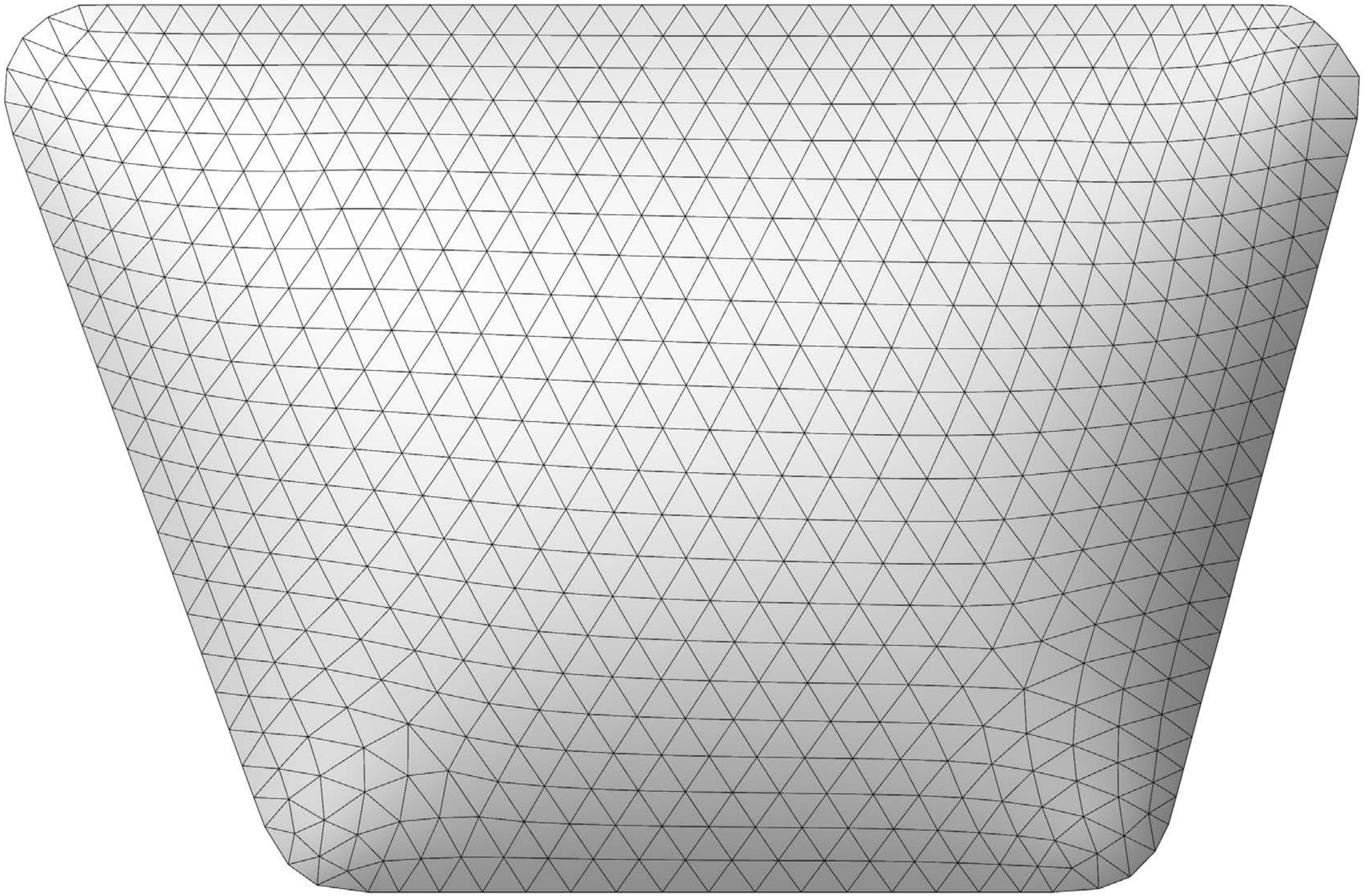
- Minimize area of each mesh face

Tessellation

- Tessellation generated using T.MAP

- Manual post-processing

Tessellation



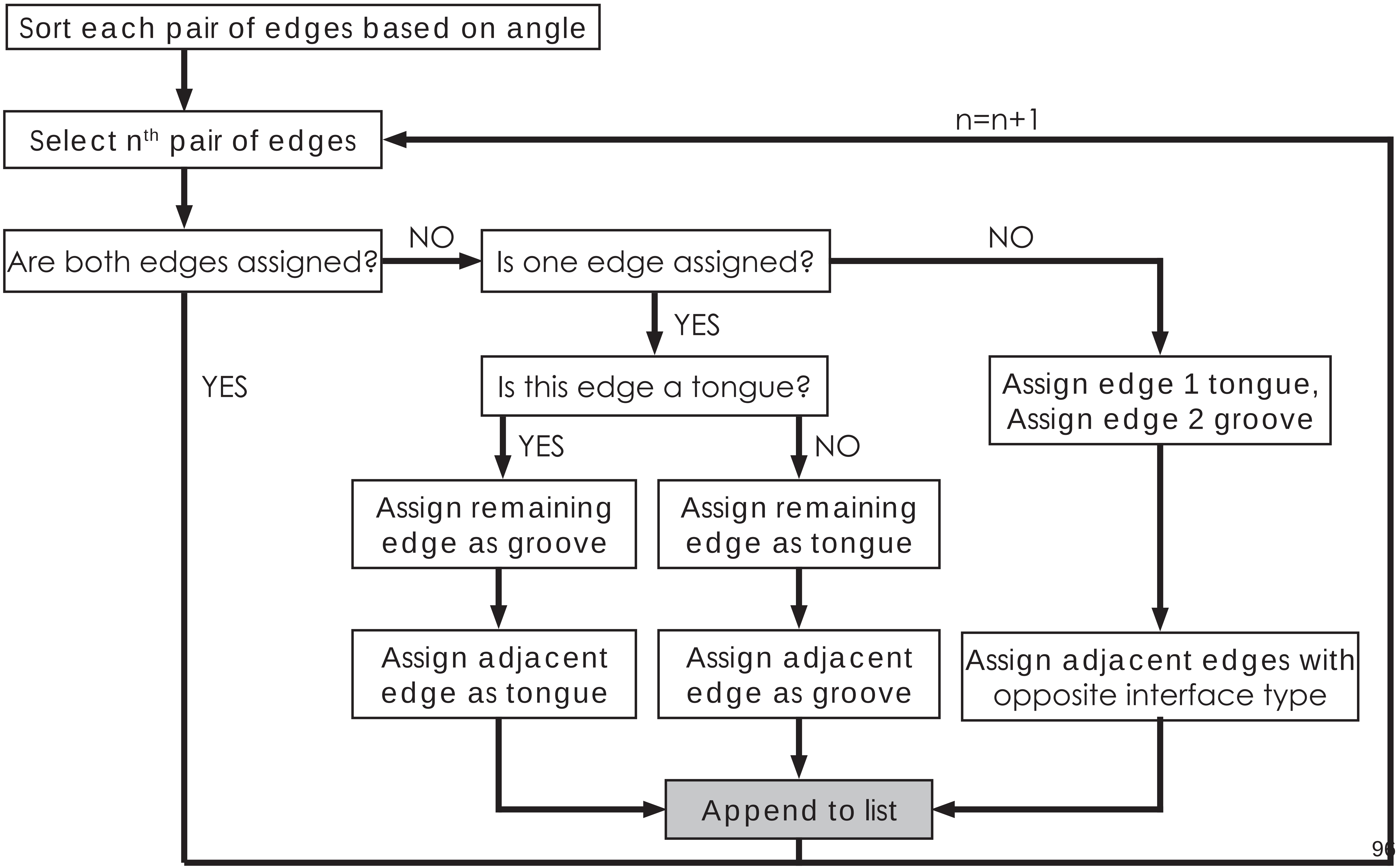
Tessellation

	Minimium	Maximum	Average
Interior angle	**34.2°**	97.3°	60.0°
Edge length	**330 mm**	782 mm	538 mm
Area	0.051 m^2	0.186 m^2	0.125 m^2
Voussoir joining angle	-6.9°	20.2°	2.1°

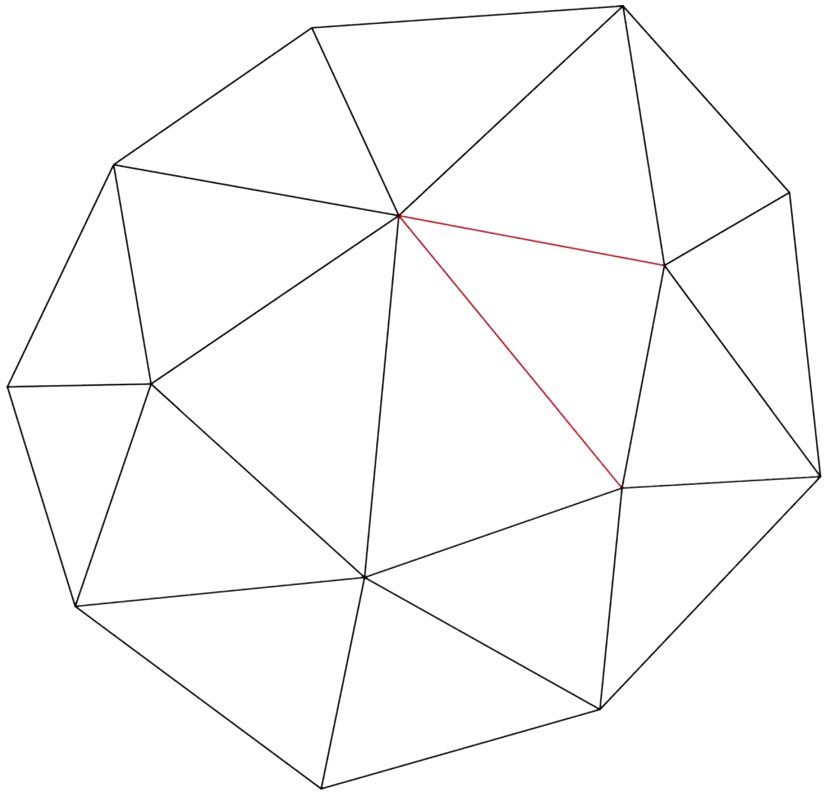
Voussoir generation

Avoid the assignment of two tongue edges on pairs of edges that form small interior angles

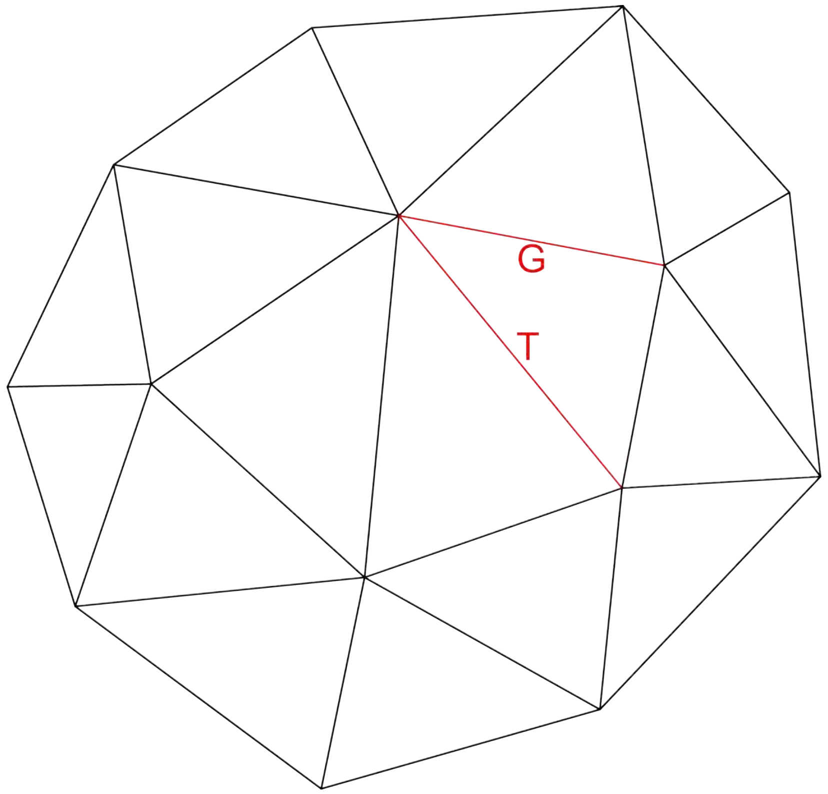
Interface assigning algorithm



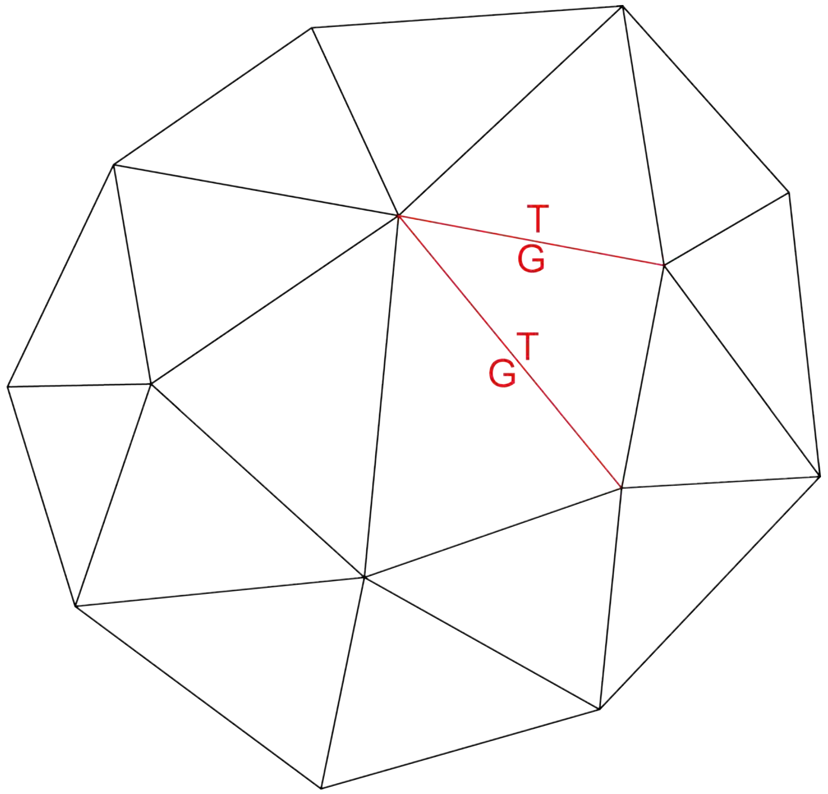
Interface assigning algorithm



Check edges

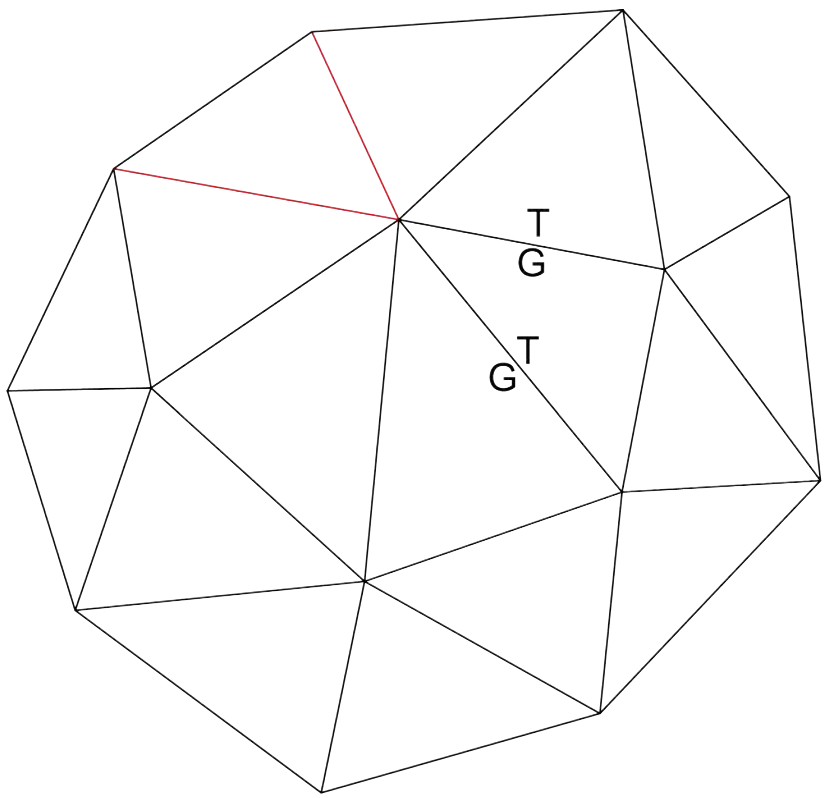


Assign edge 1 groove
Assign edge 2 tongue

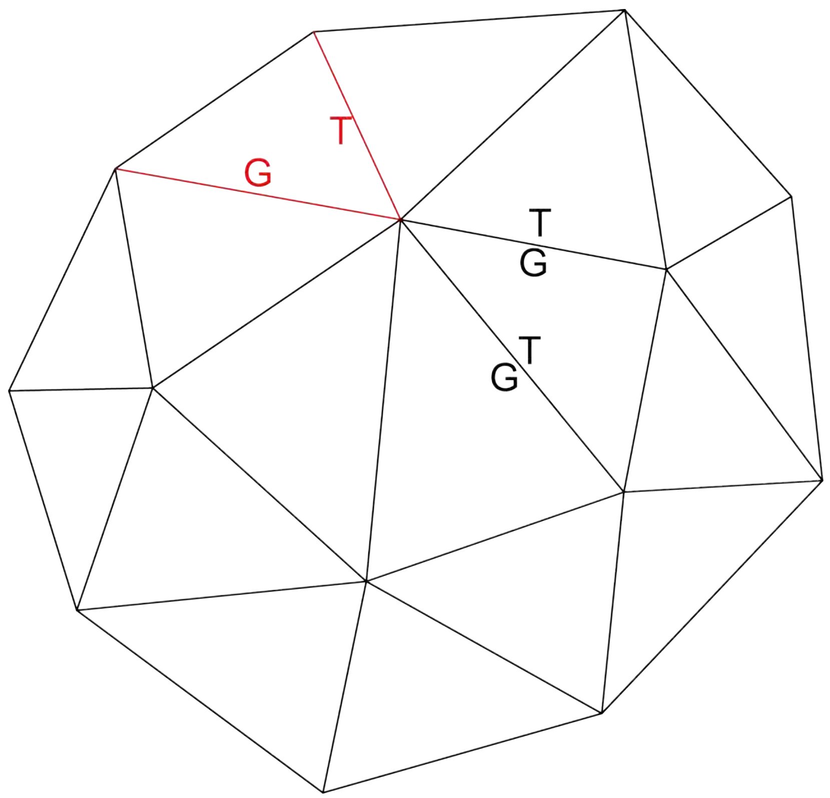


Assign adjacent edges
opposite interface type

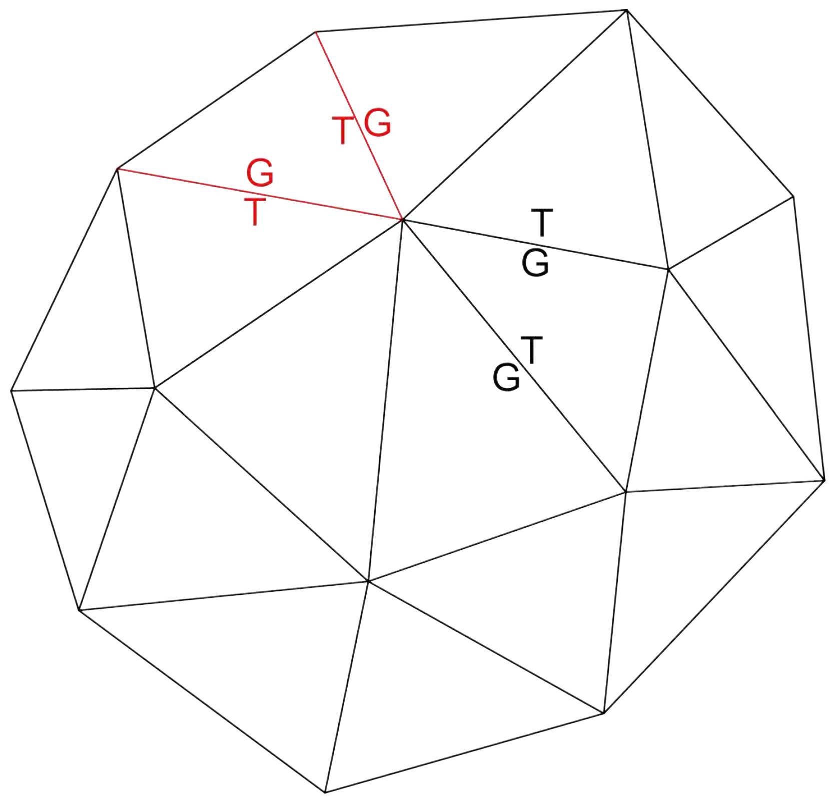
Interface assigning algorithm



Check edges

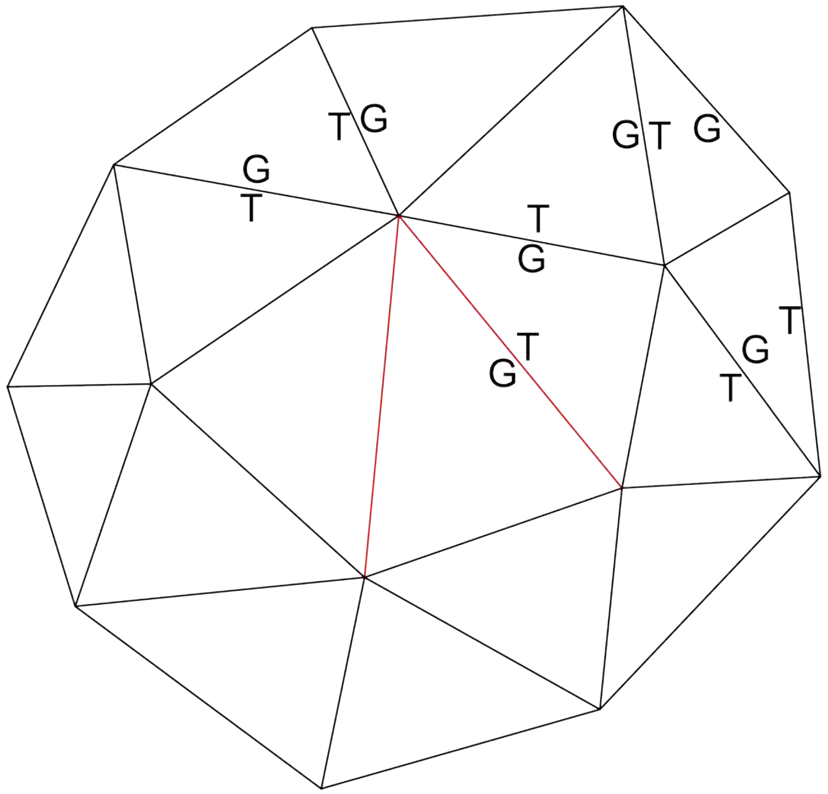


Assign edge 1 groove
Assign edge 2 tongue

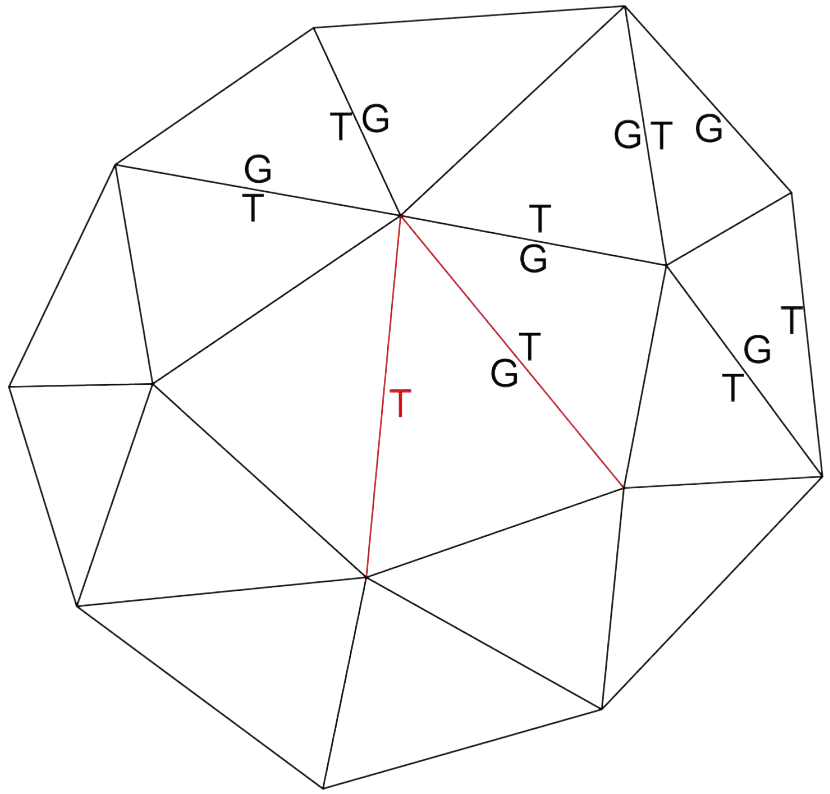


Assign adjacent edges
opposite interface type

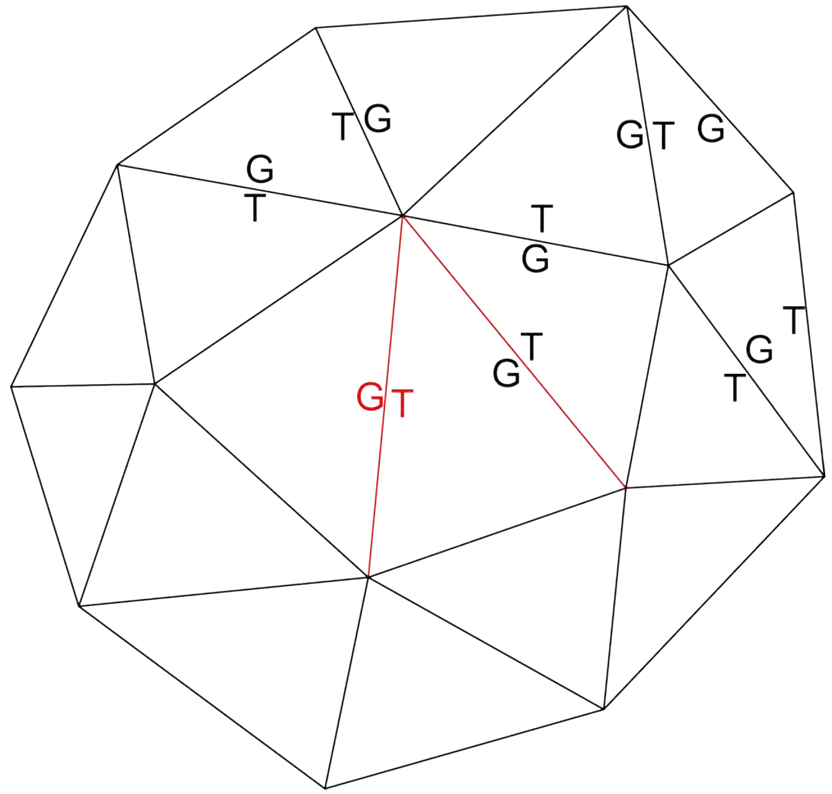
Interface assigning algorithm



Check edges

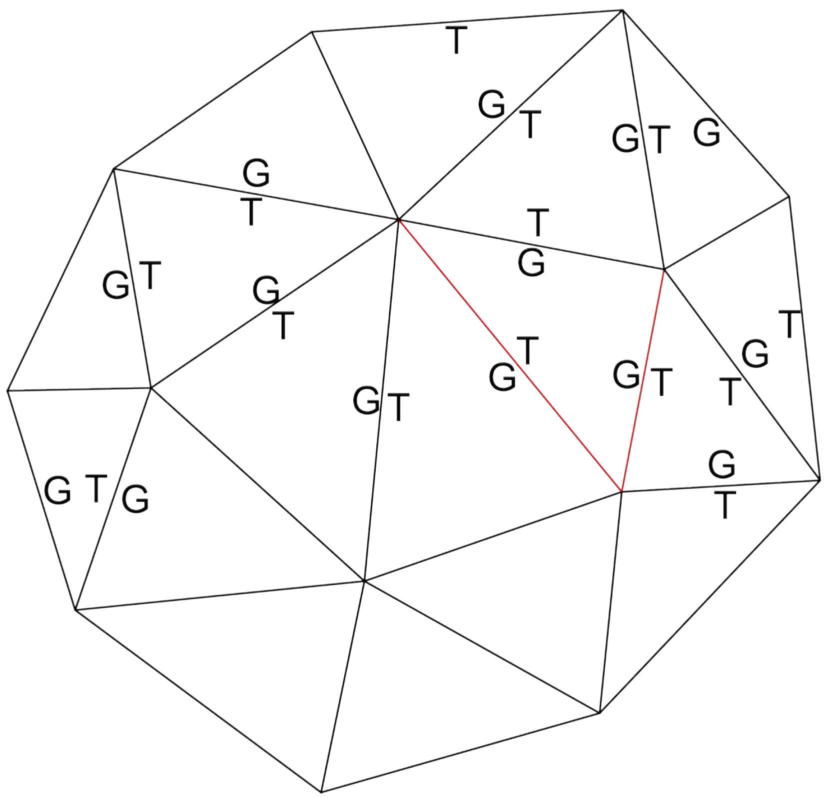


Assign remaining edge opposite interface



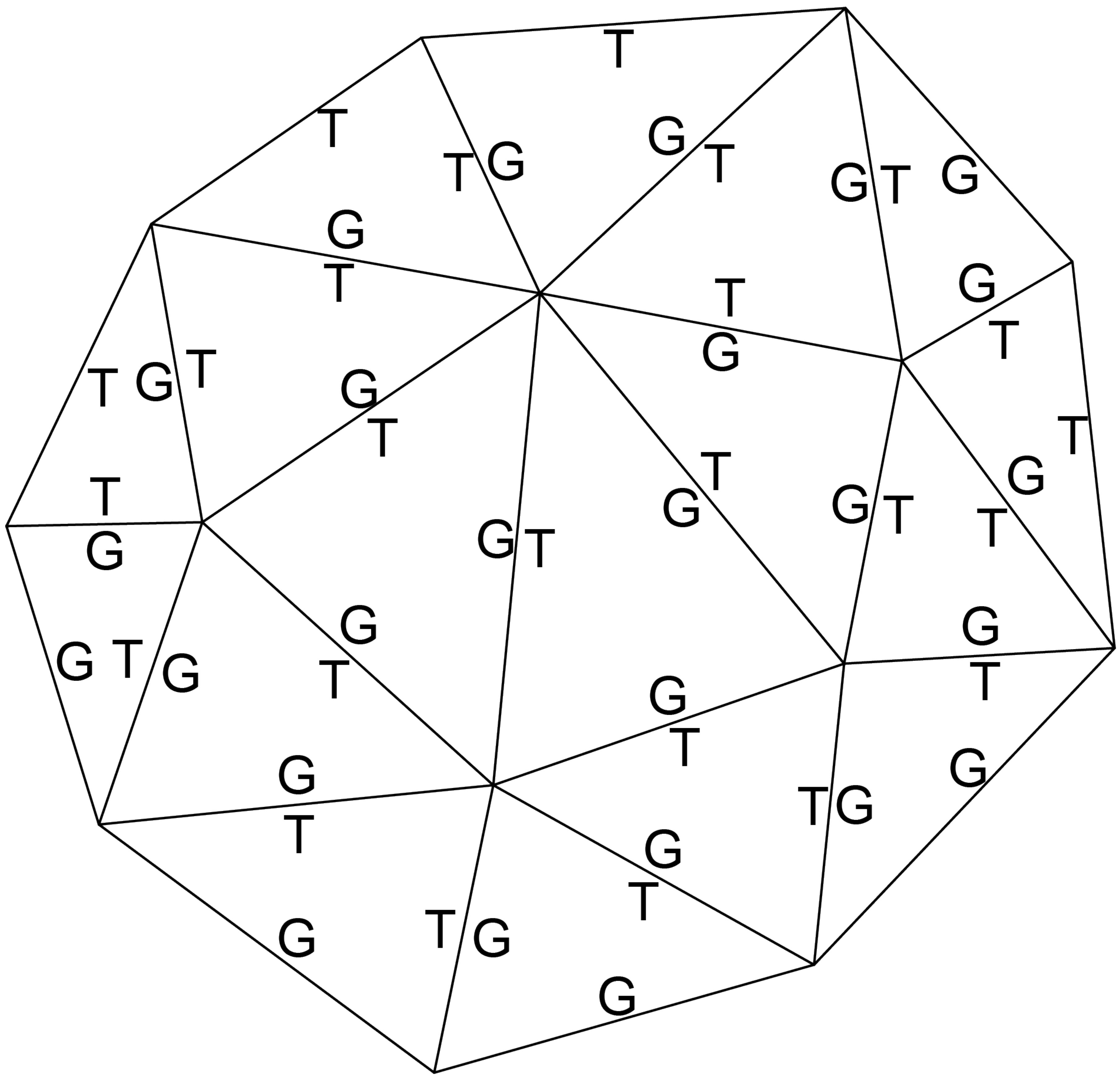
Assign adjacent edges opposite interface type

Interface assigning algorithm



Check edges → Skip

Interface assigning algorithm



Voussoir generation

Smallest angle at which two tongues are assigned	44.8°
Voussoir thickness	100 mm

$$r_v > \frac{t_v}{2\sin\left(\frac{1}{2}\alpha_{i,min}\right)}$$

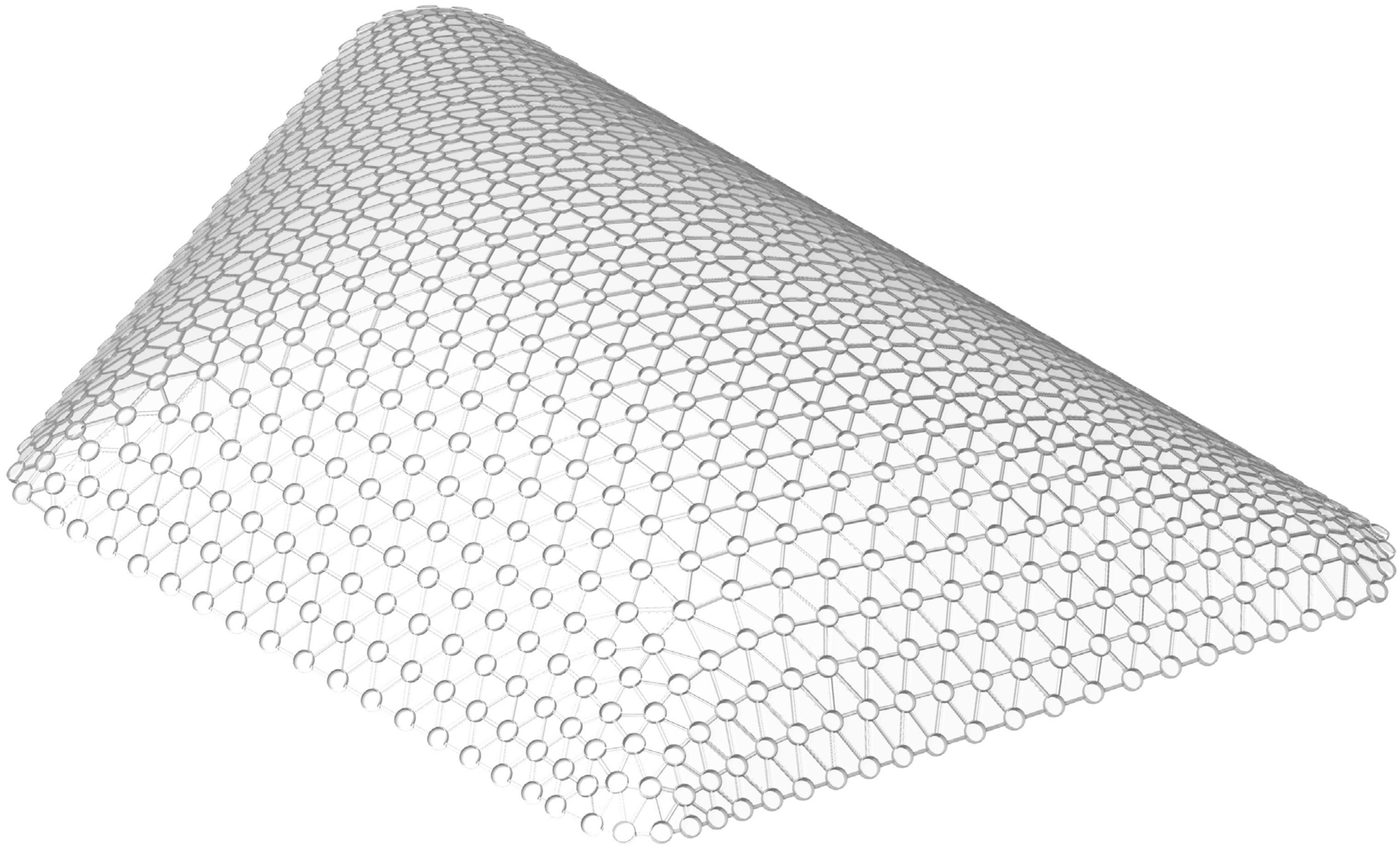
Minimum sphere radius of the vertex modules	132 mm

Voussoir generation

Adjustable mould input data list

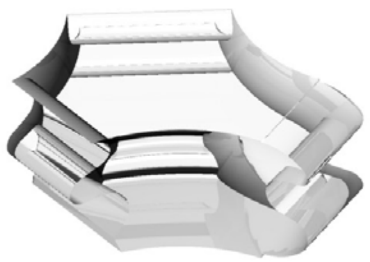
- Voussoir index number
- Types of interfaces (tongue or groove)
- Edge lengths
- Voussoir joining angles
- Adjacent voussoir index numbers

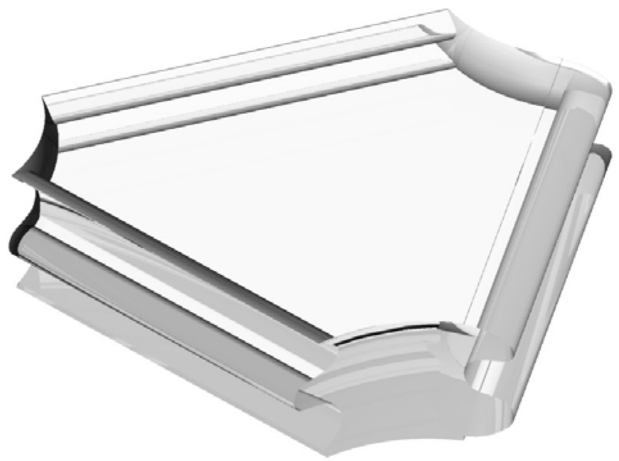
Cast glass shell structure



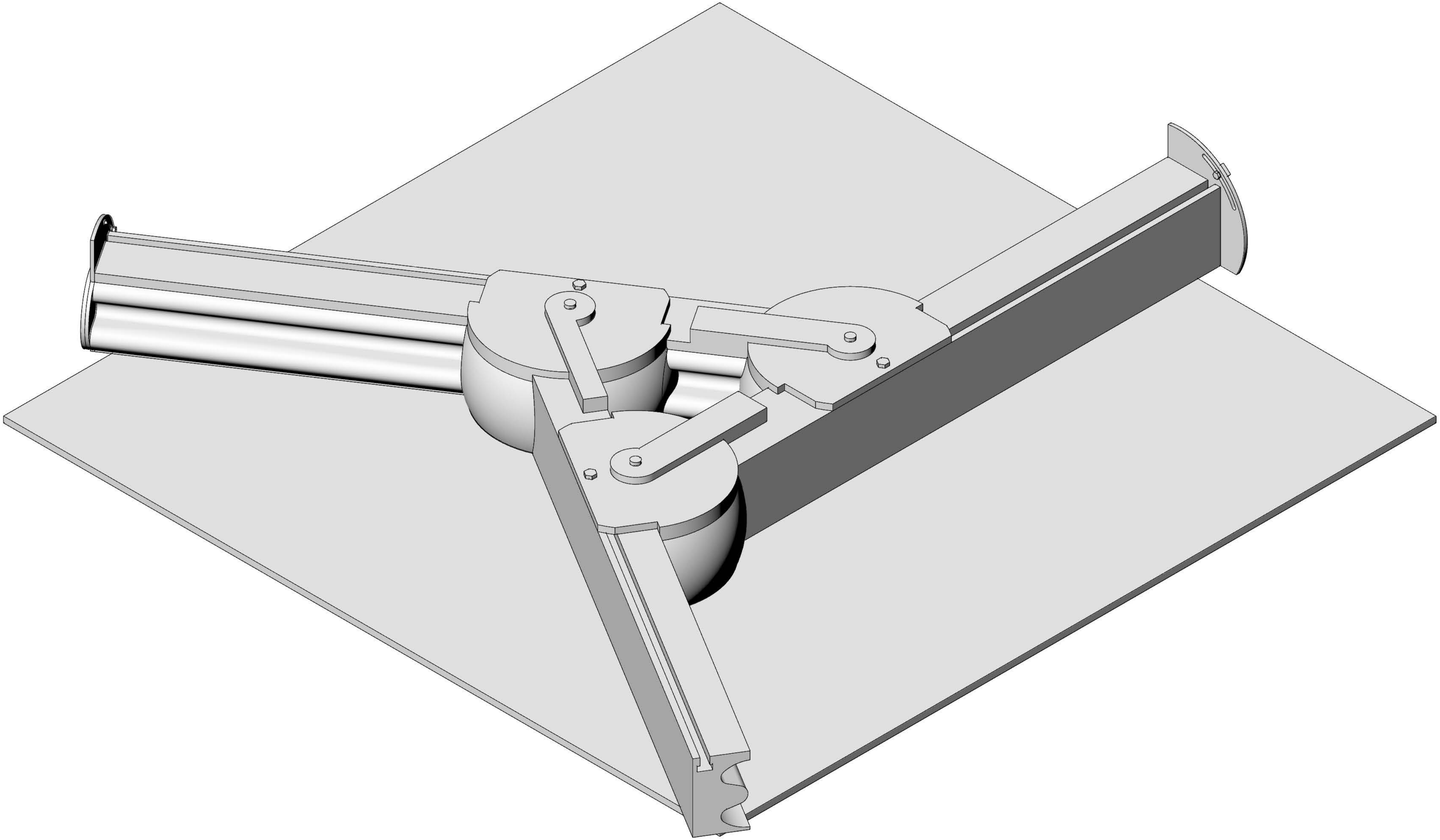
Voussoirs

	Minimium	**Average**	**Maximum**
Mass	5.41 kg	24.6 kg	38.5 kg

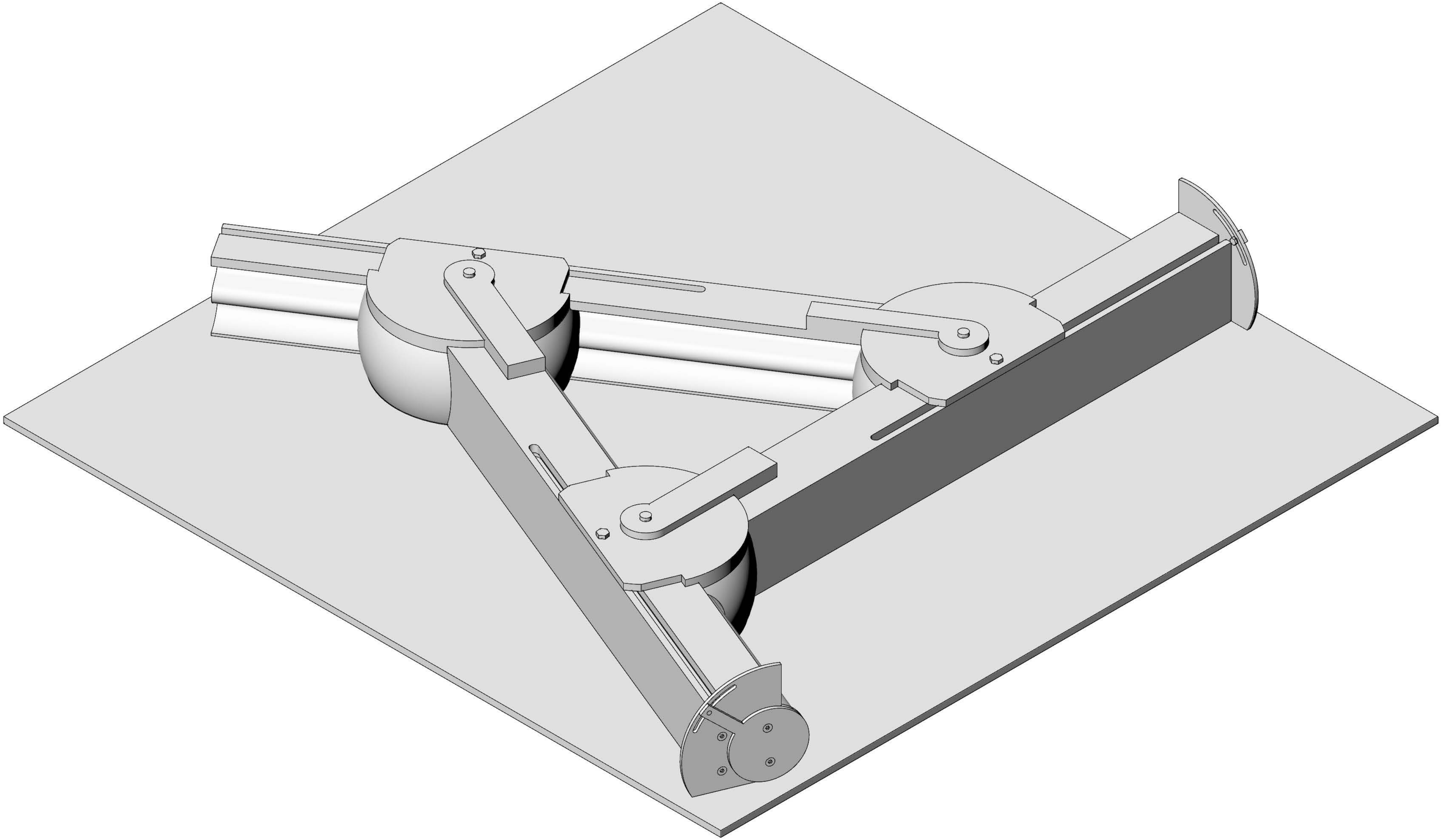




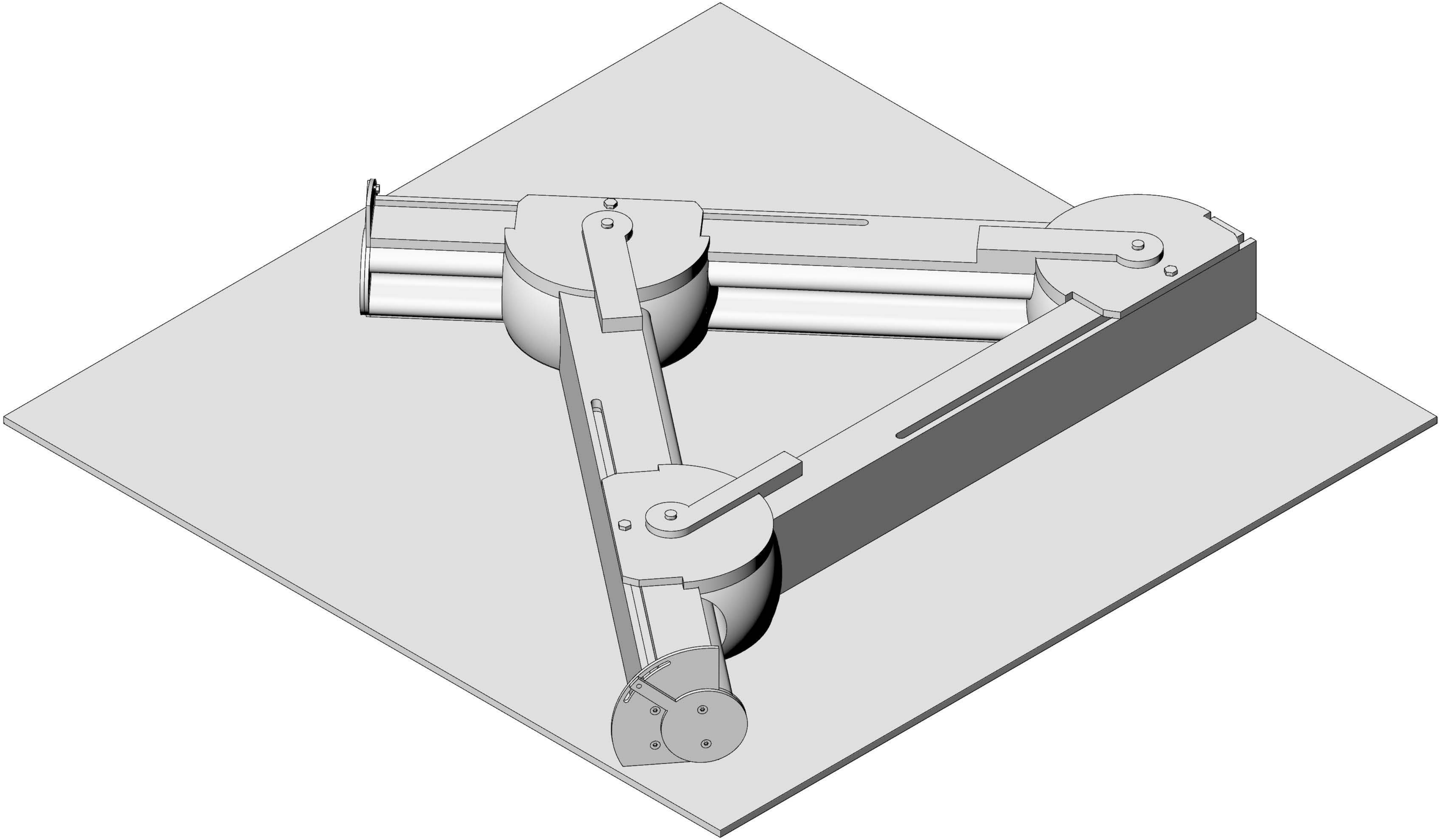
Adjustable mould



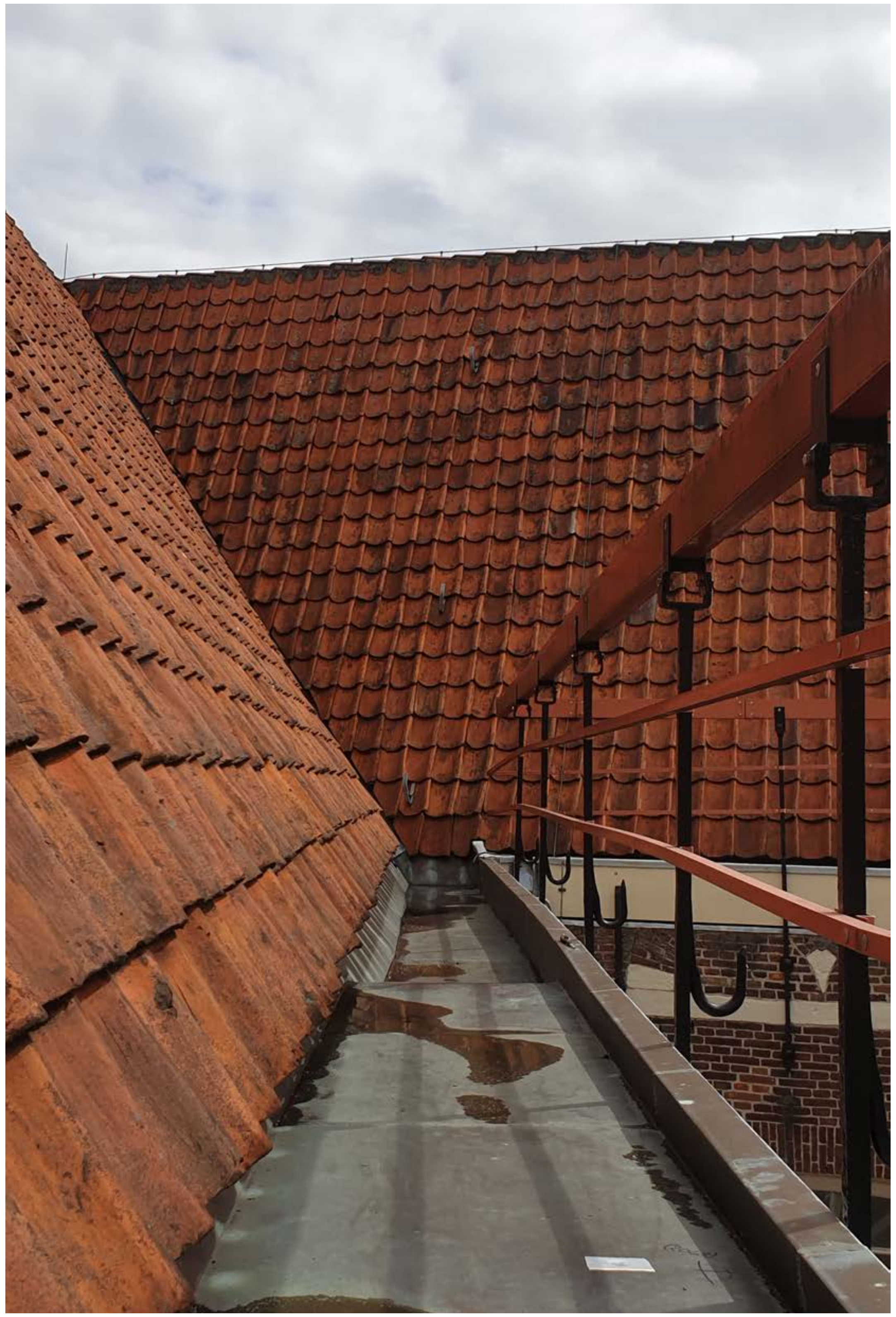
Adjustable mould



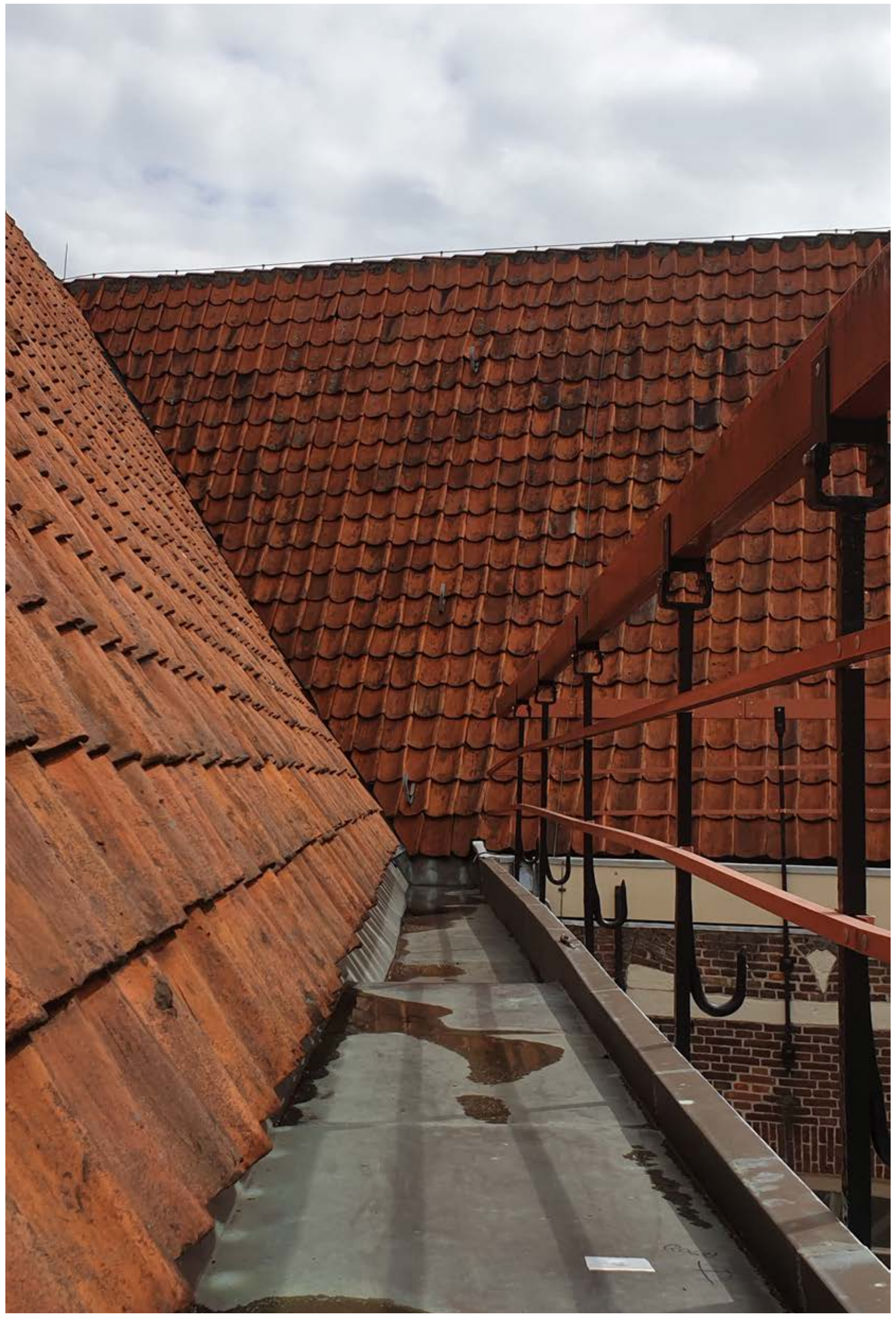
Adjustable mould

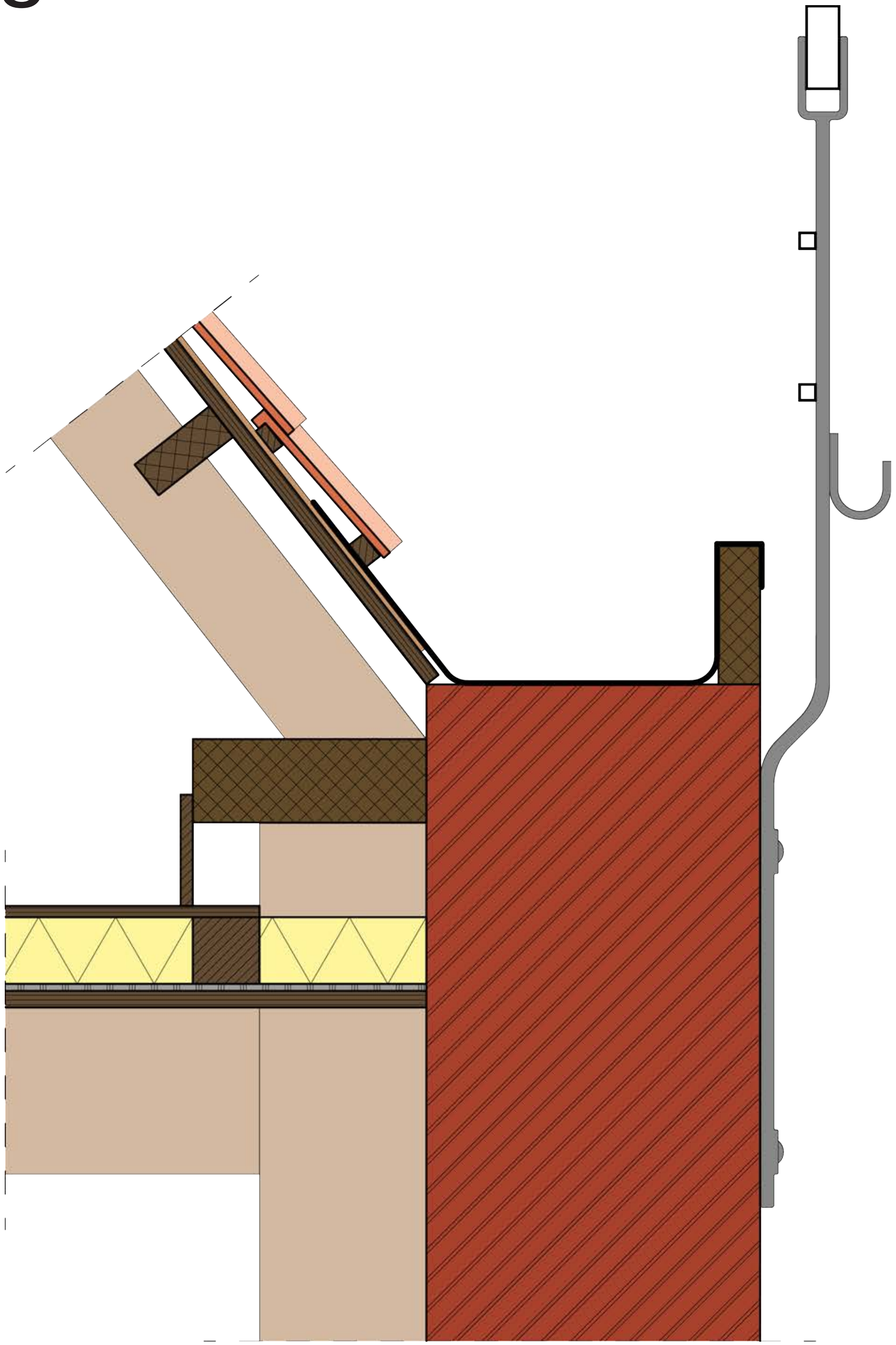


Connection with existing structure

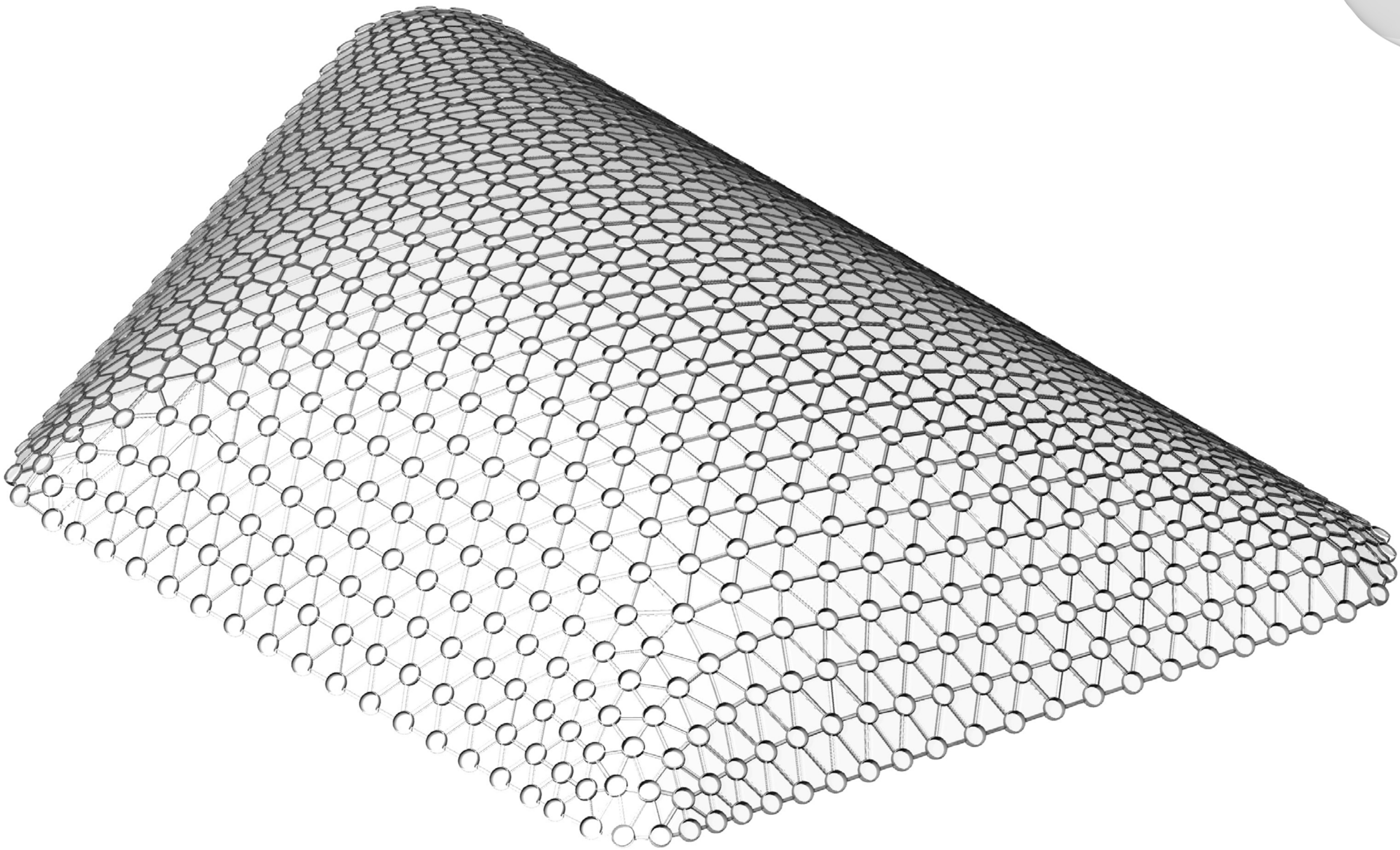


Connection with existing structure

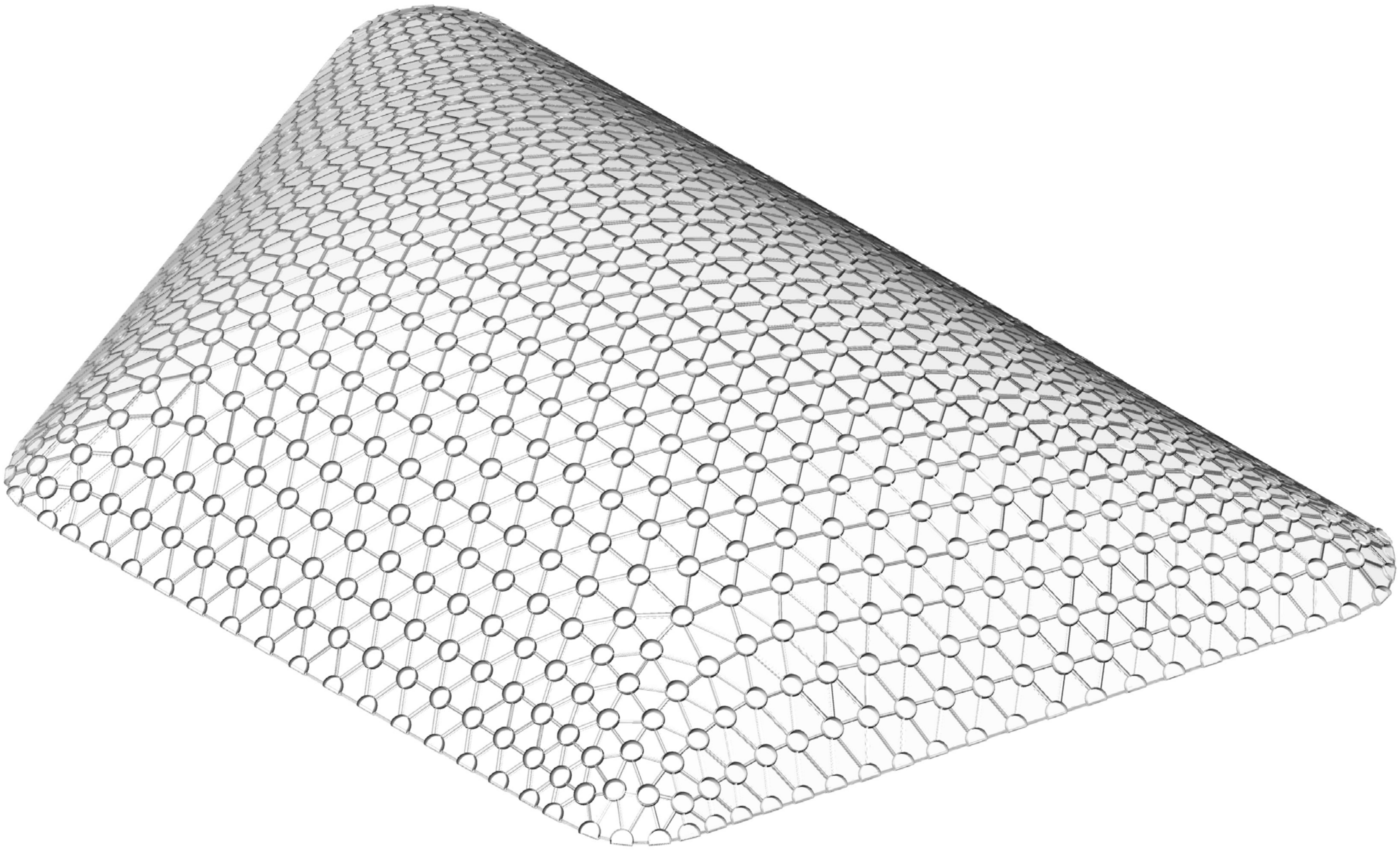




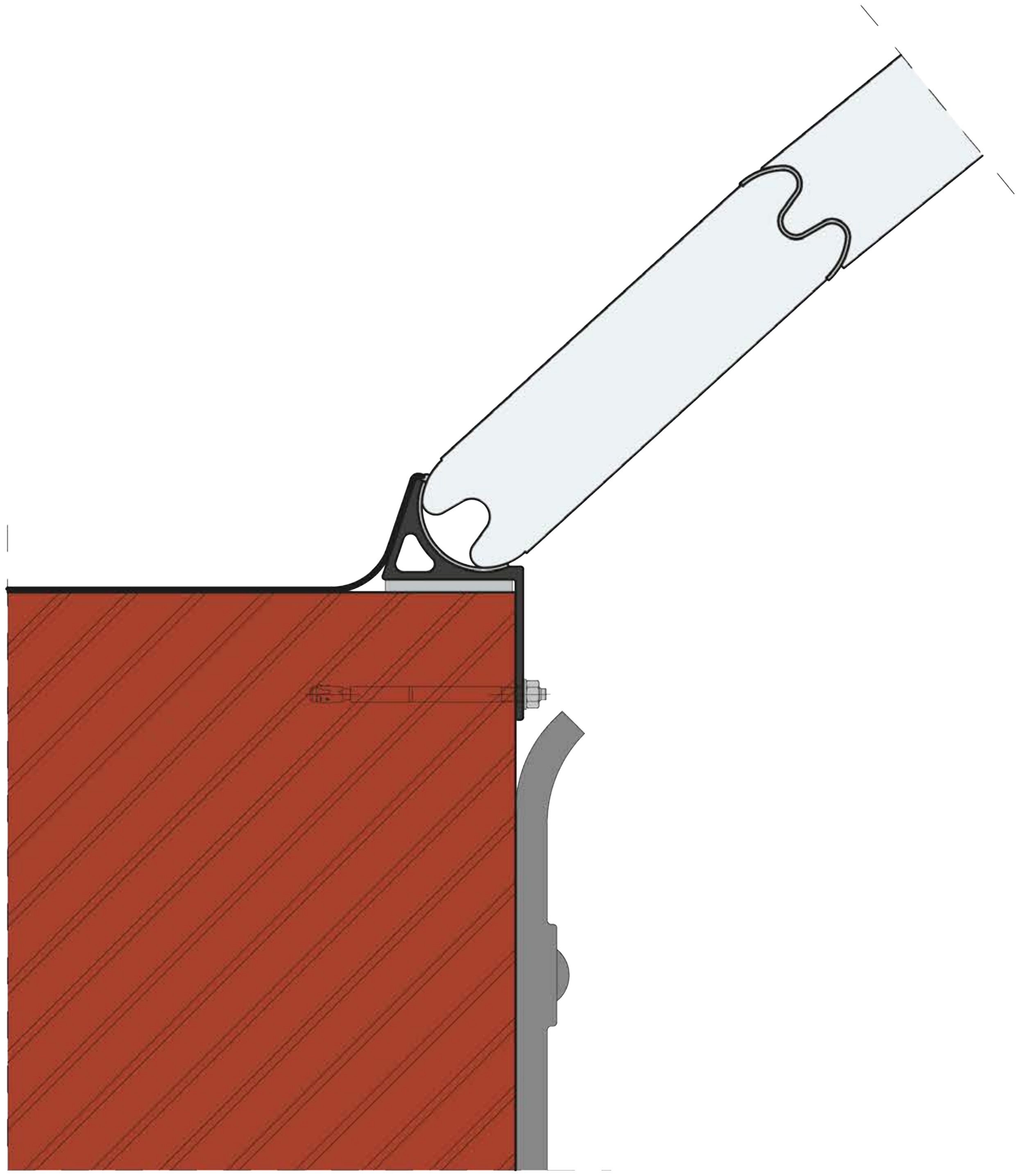
Interrupted boundary



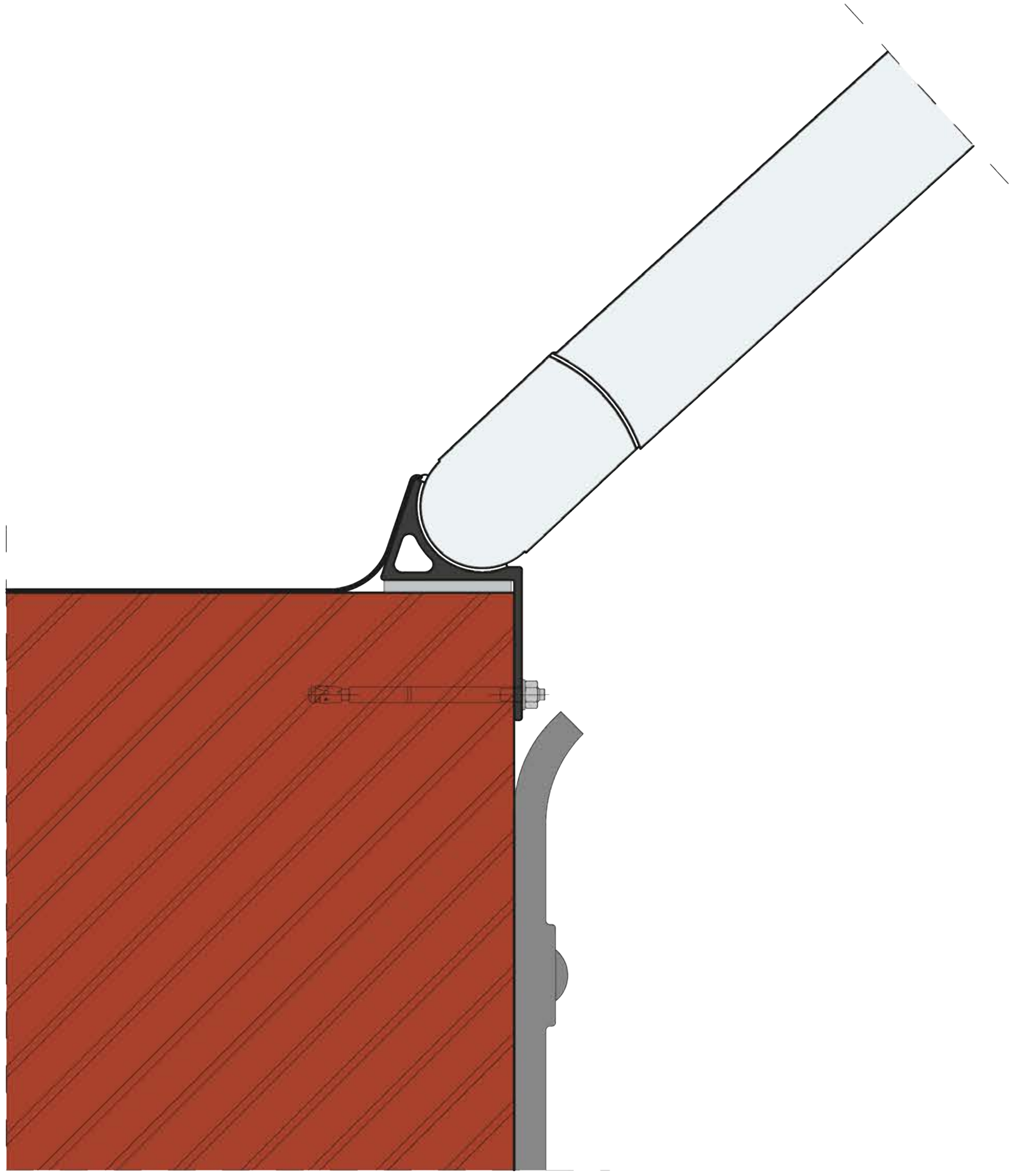
Continuous boundary



Support

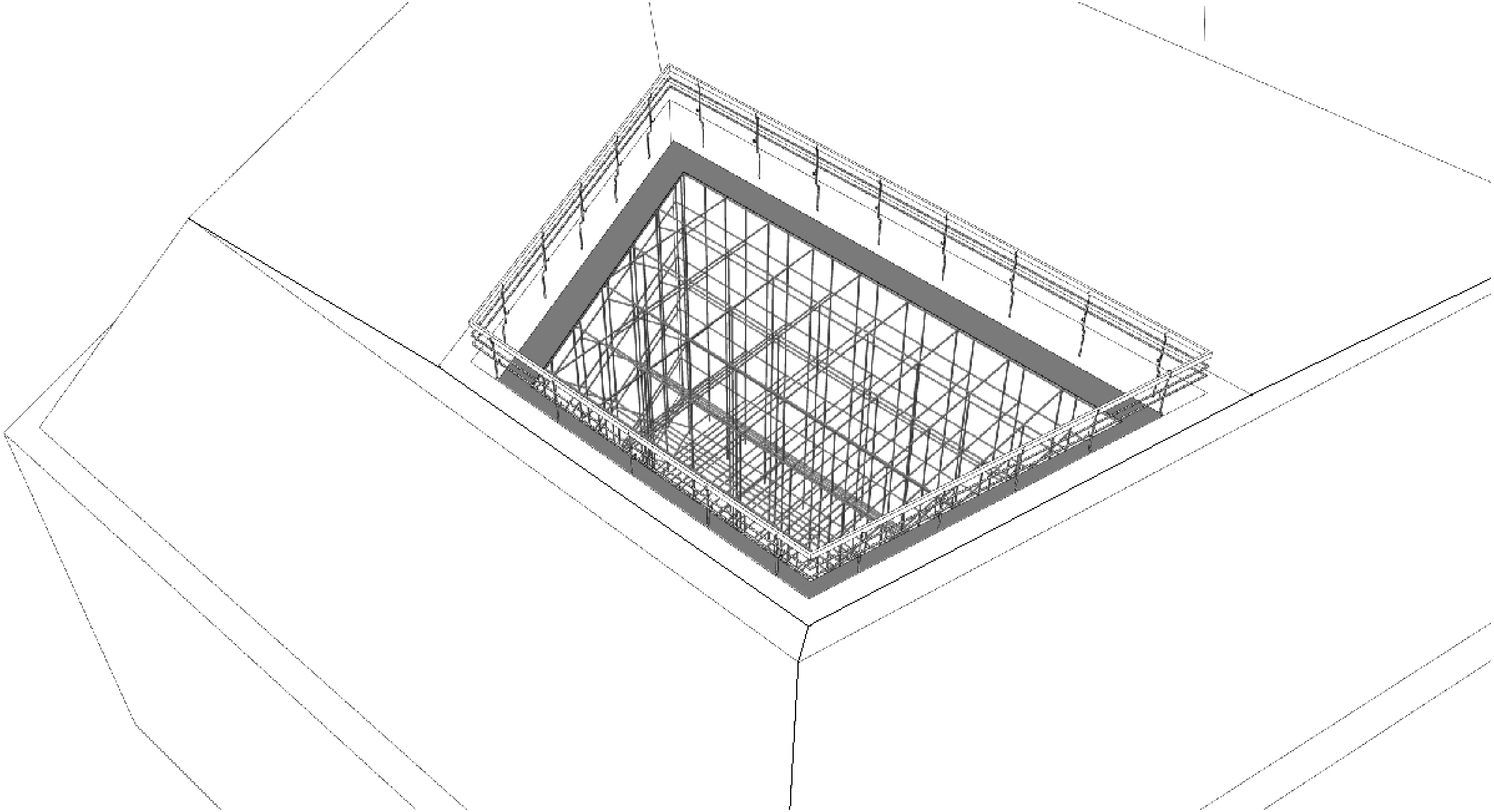


Voussoir (groove interface)



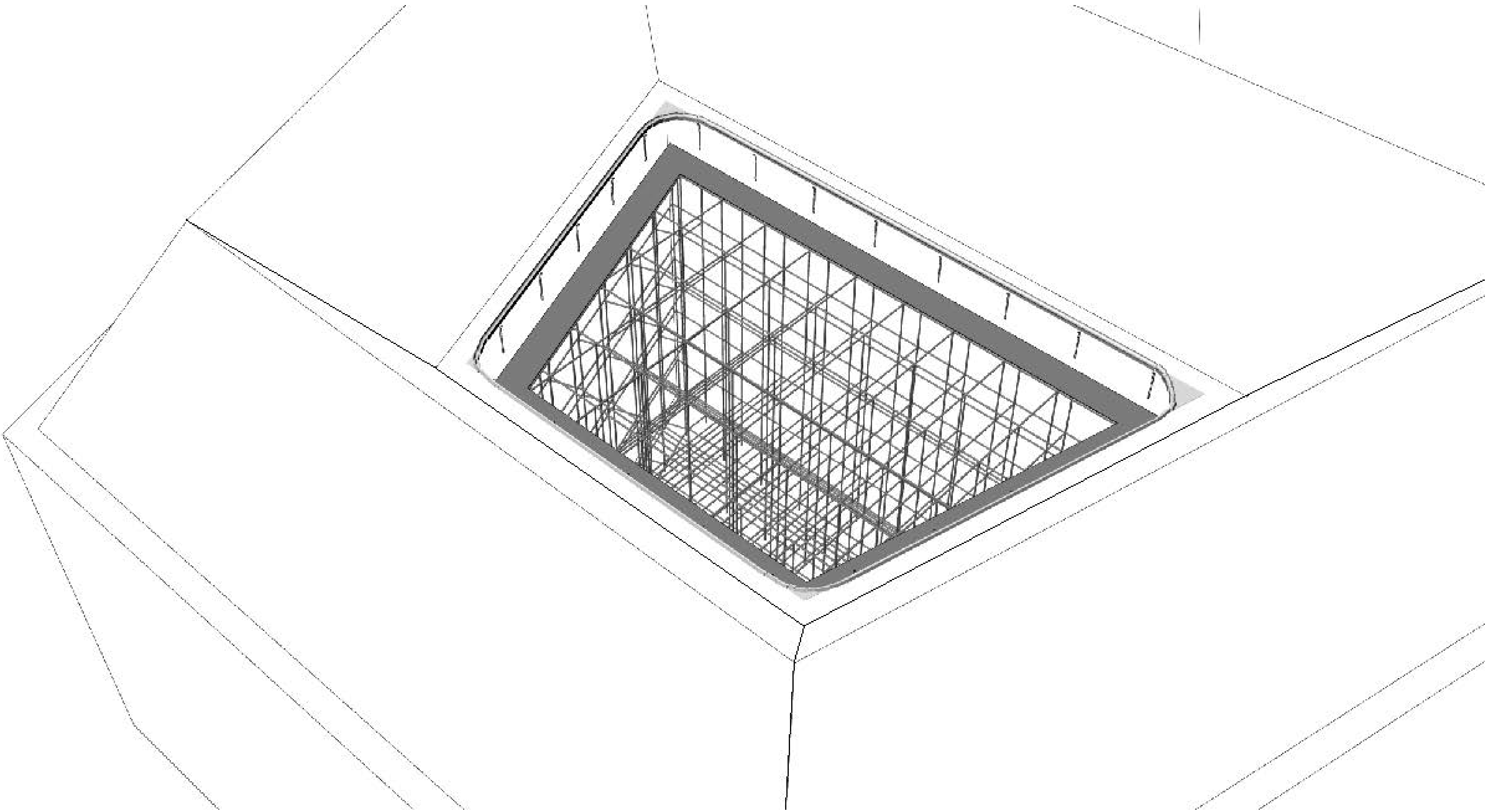
Boundary node component

Assembly



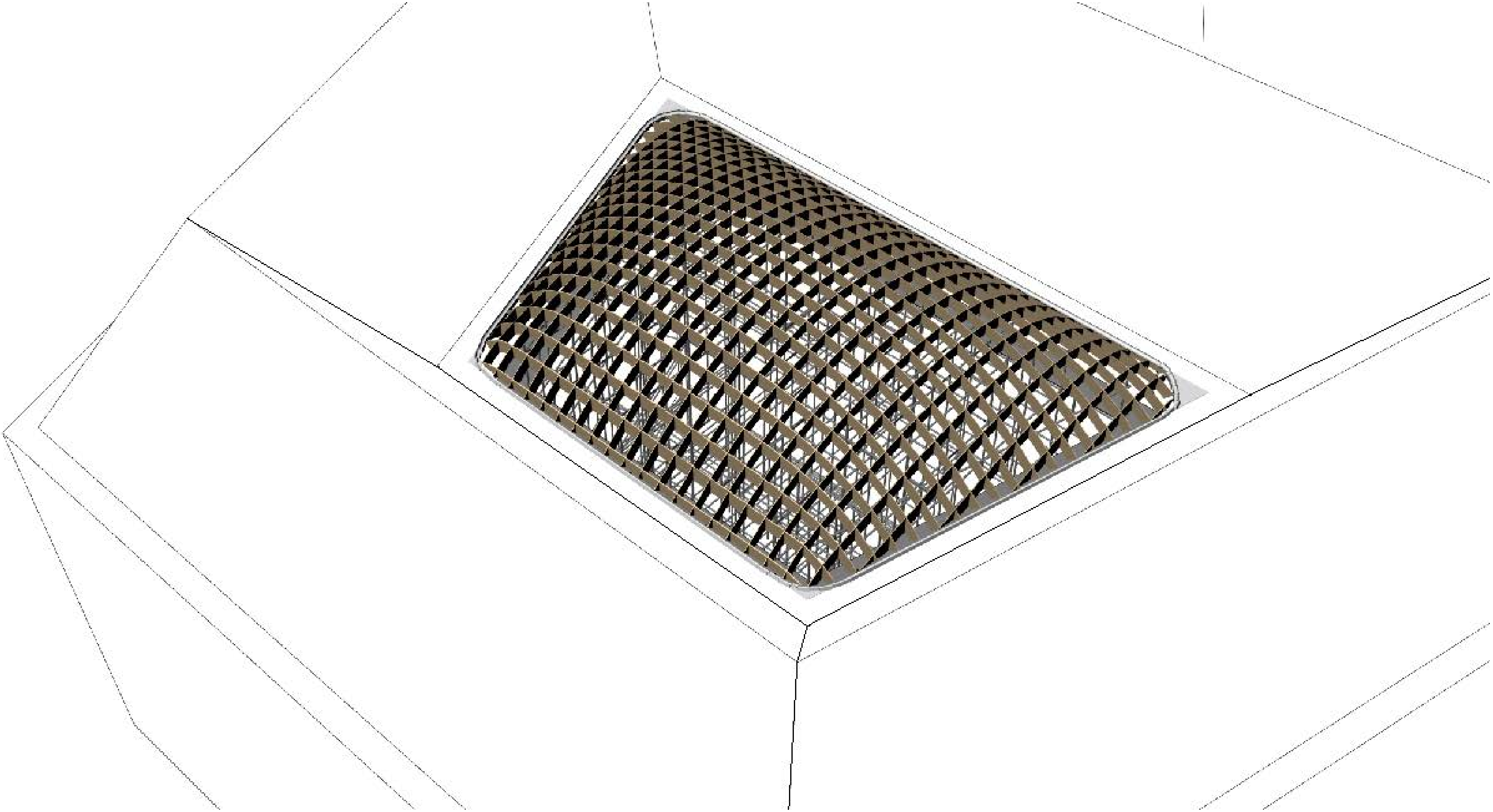
1. Construct scaffolding

Assembly



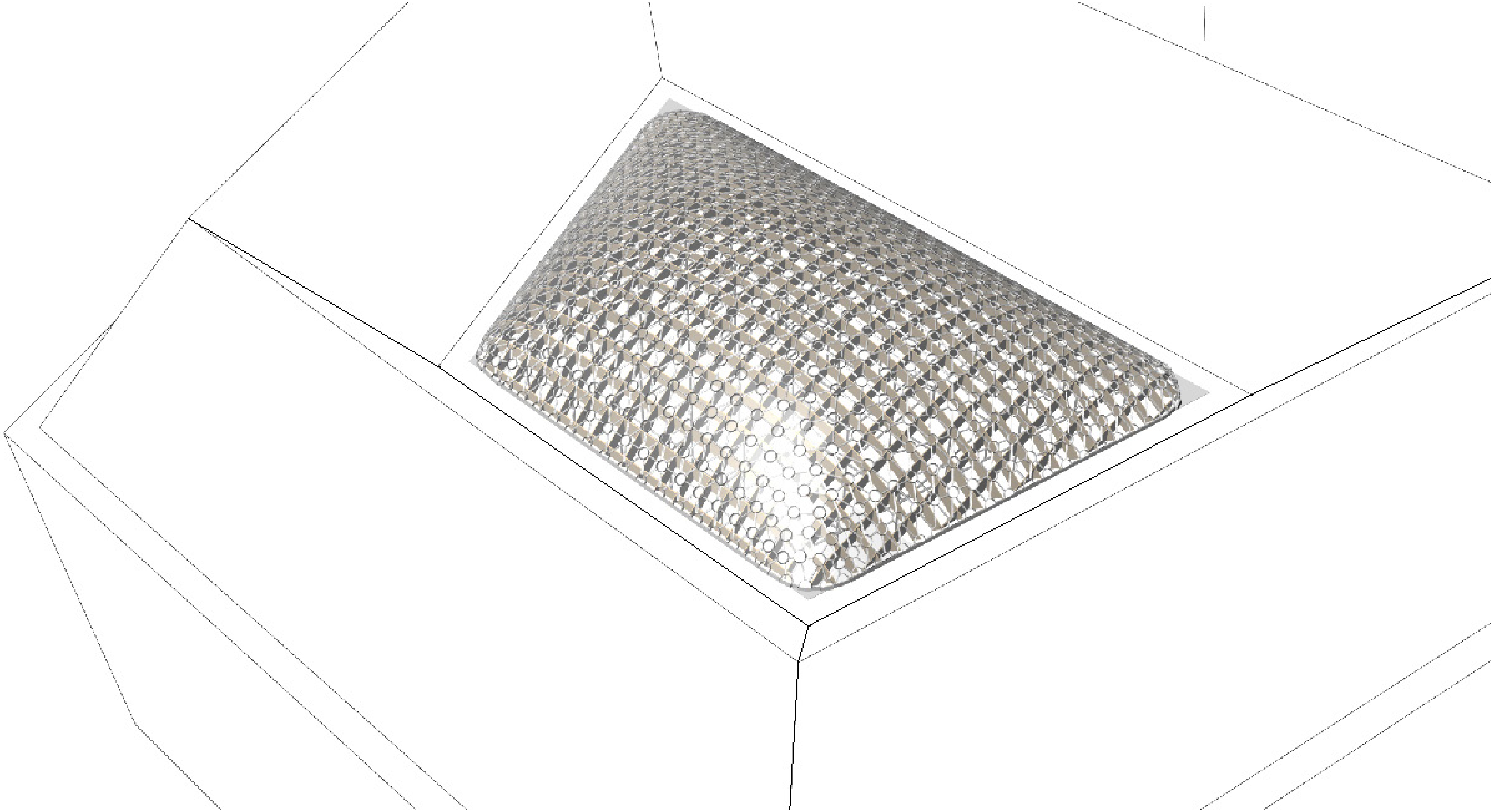
2. Install supports

Assembly



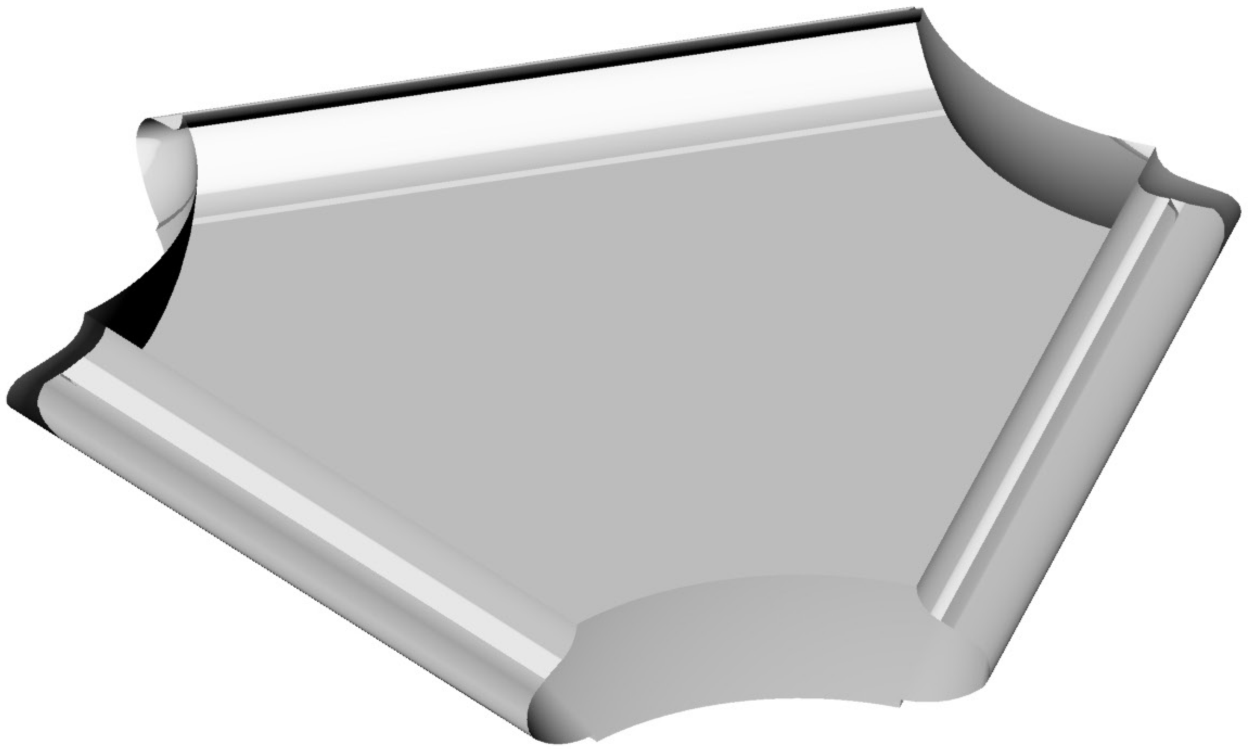
3. Build formwork

Assembly

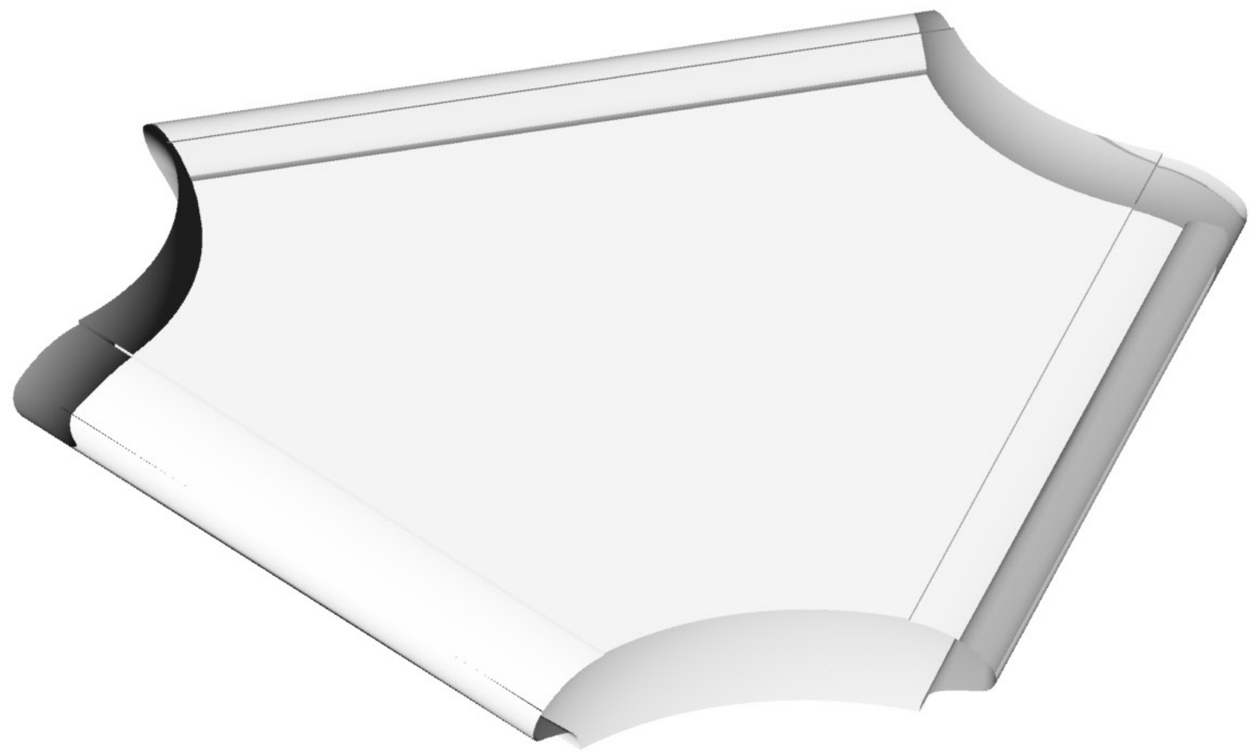


4. Assemble shell structure

Keystone

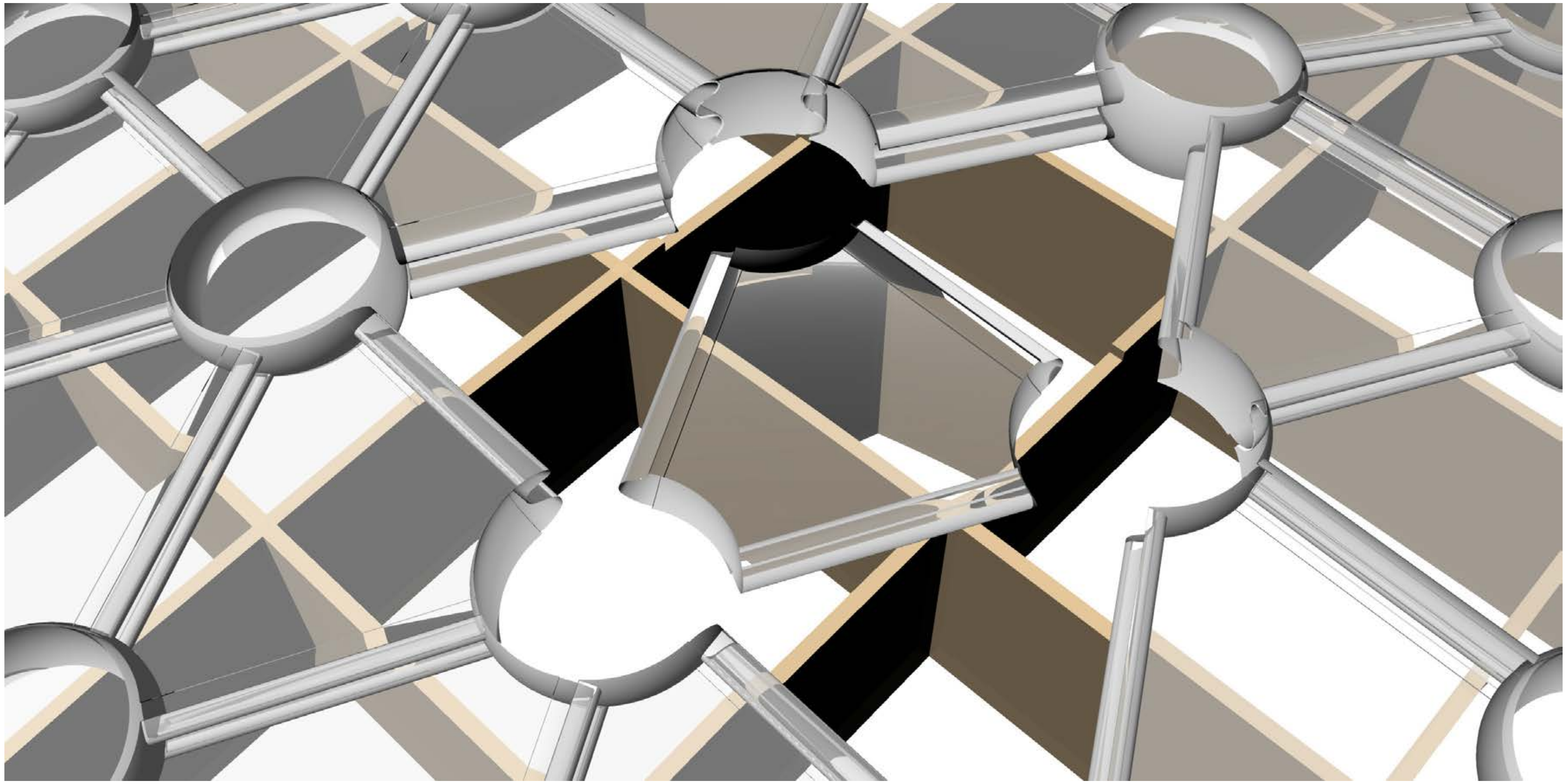


Bottom keystone



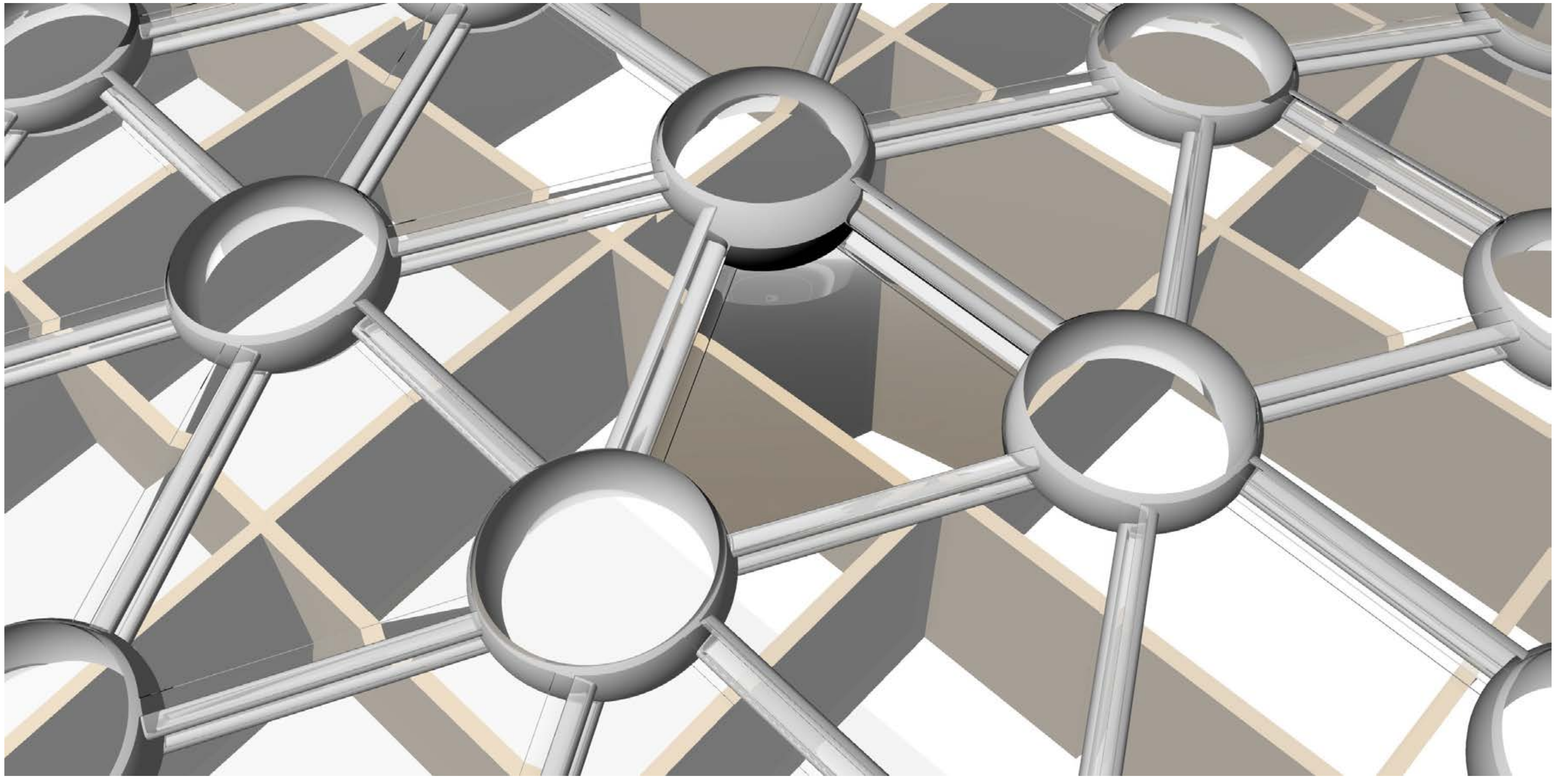
Top keystone

Keystone



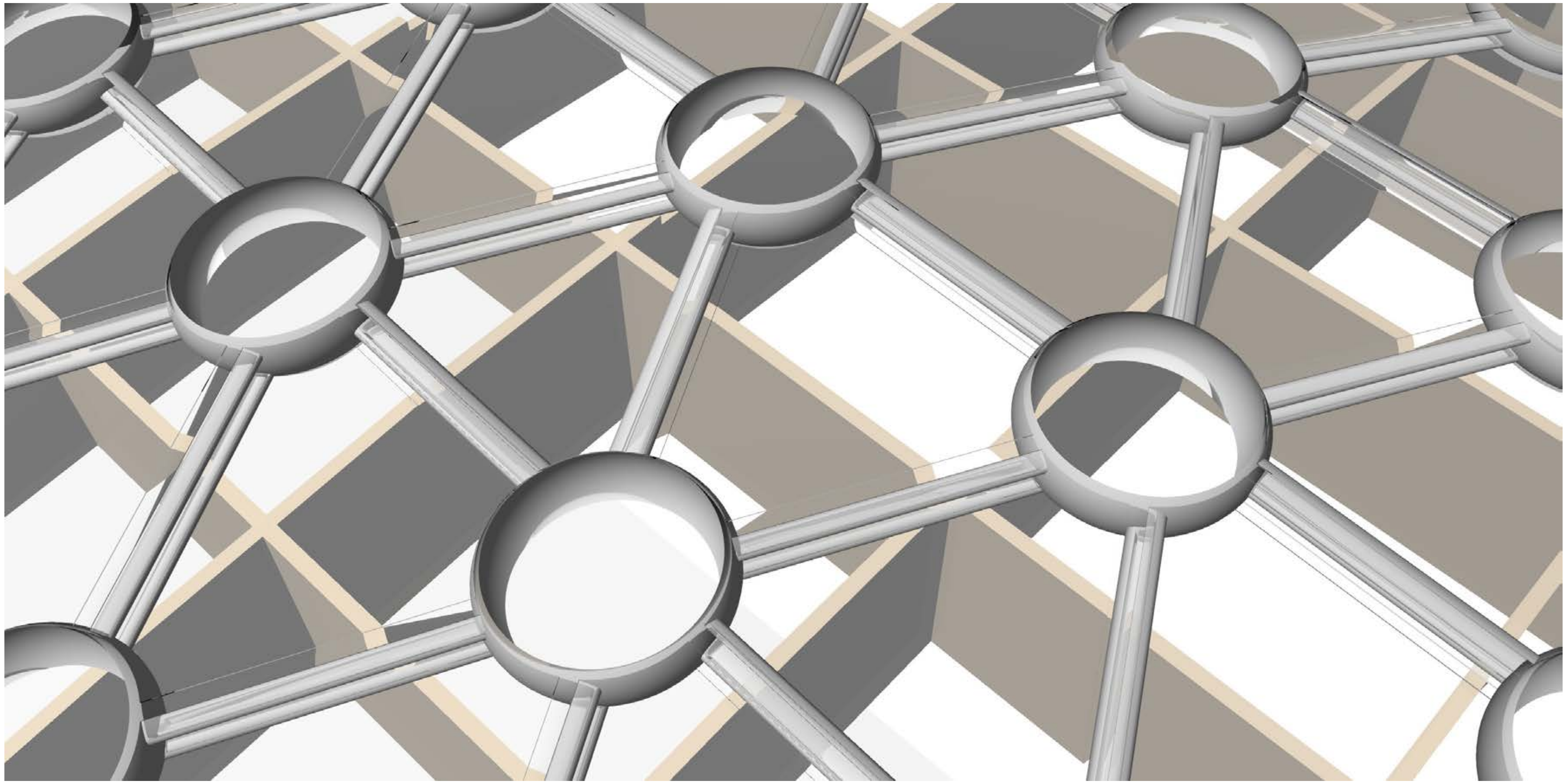
1. Place bottom keystone

Keystone



2. Place adjacent voussoirs

Keystone



3. Place top keystone

Conclusions

This research resulted in a set of **design rules** and a **production process** for the construction of fully transparent shell structures

Conclusions

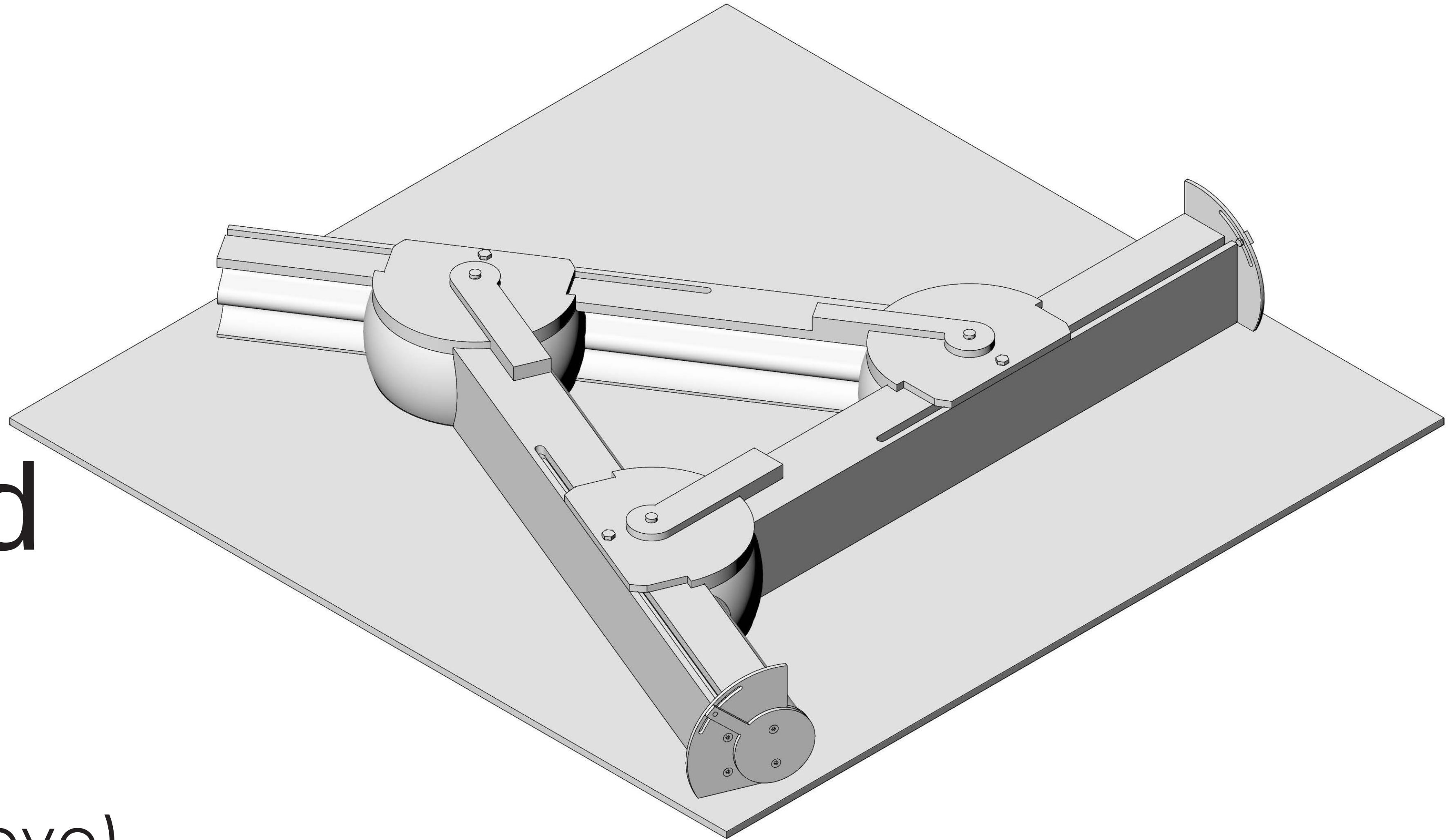
Design rules

- The shell needs to have a funicular shape
- Tessellating according to certain constraints

Tessellating constraints

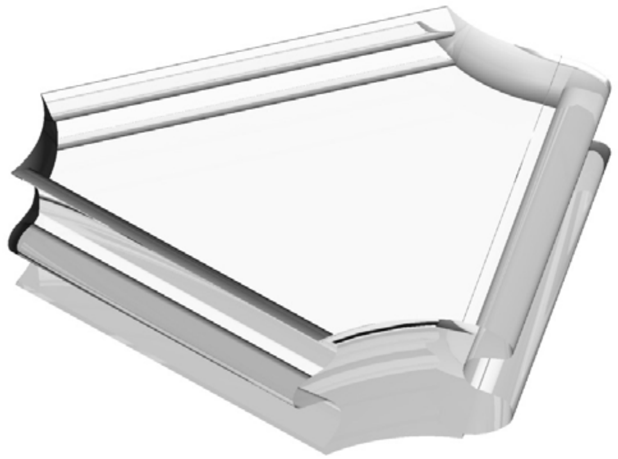
- Convex polygons
- Planar polygons
- Very large or small angles should be avoided
- Edge lenghts larger than twice the sphere radius
- Tesellation pattern should be alligned to the force flow
- The face area should be kept to a minimum

Conclusions



Adjustable mould

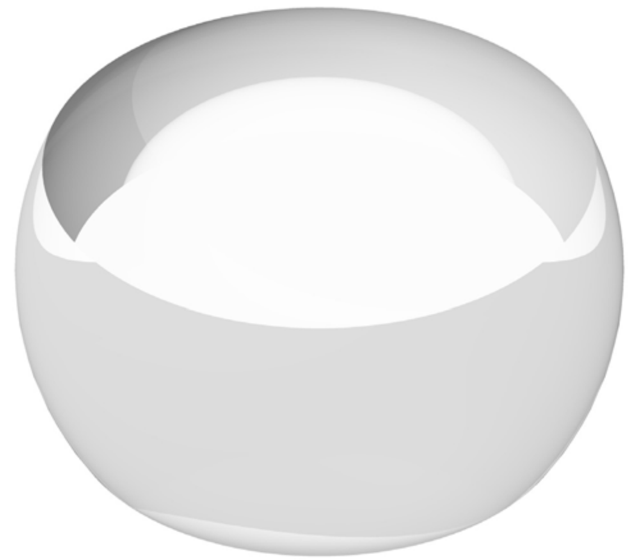
- Edge count
- Edge length
- Interior angles
- Interface type (tongue or groove)
- Voussoir joining angle



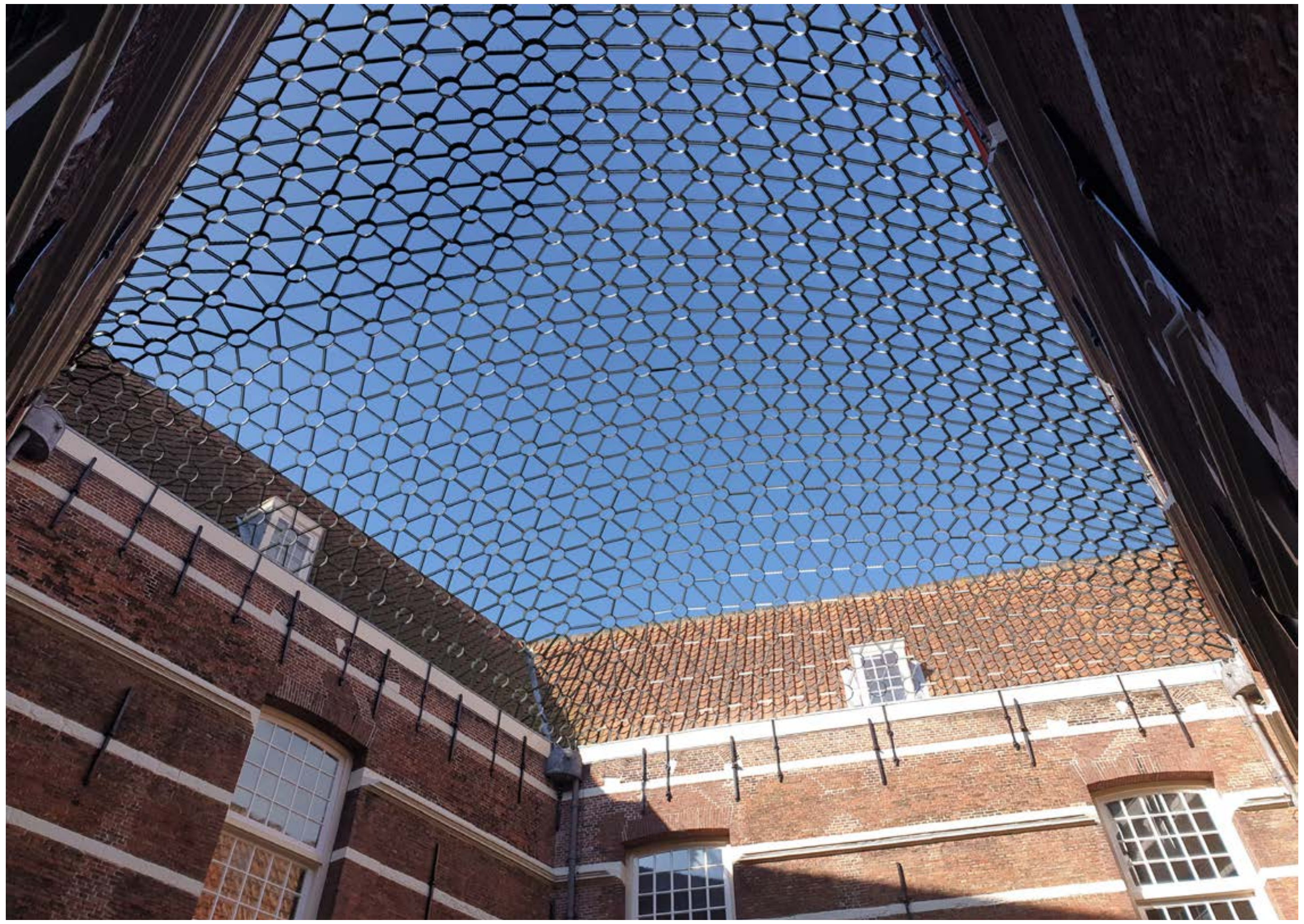
Conclusions

Node components

- Watertightness
- Identical component for every node
- Boundary node component



Comparison



Great Court at the British Museum (Foster + Partners)

Fully transparent
€€€
Formwork needed

Limited transparency
€
No formwork required

Restoration

Cast glass can show a historic building in its original and in its ruined state

(Barou, 2016)

Restoration

Notre Dame on fire (Source: ANP)

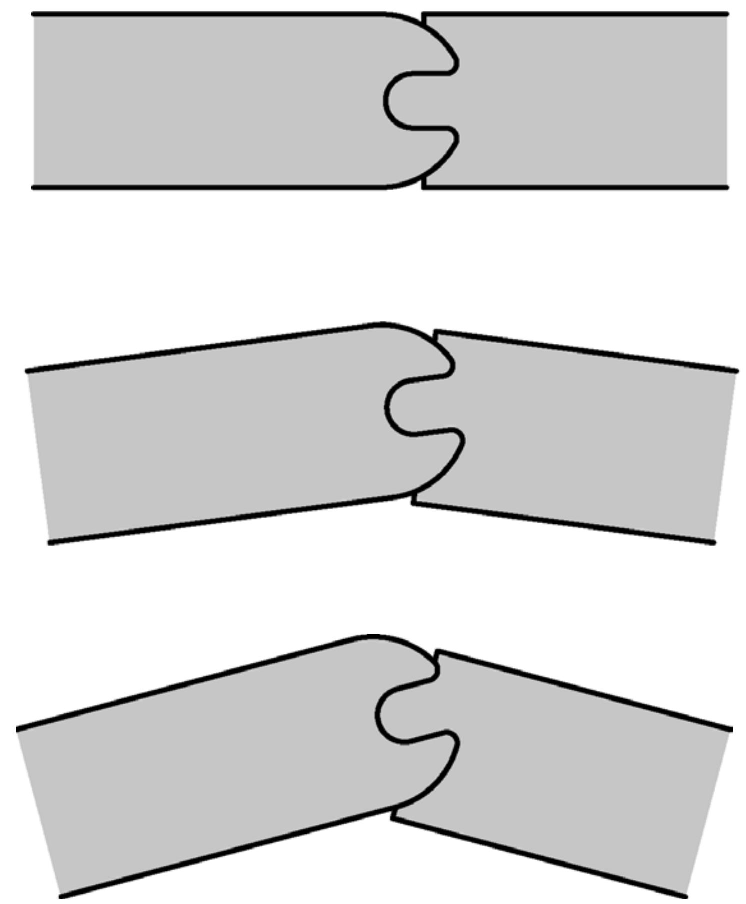
Damaged vault of the Notre Dame (Source: ANP)

Limitations and recommendations

Limitations and recommendations

Tongue and groove joint

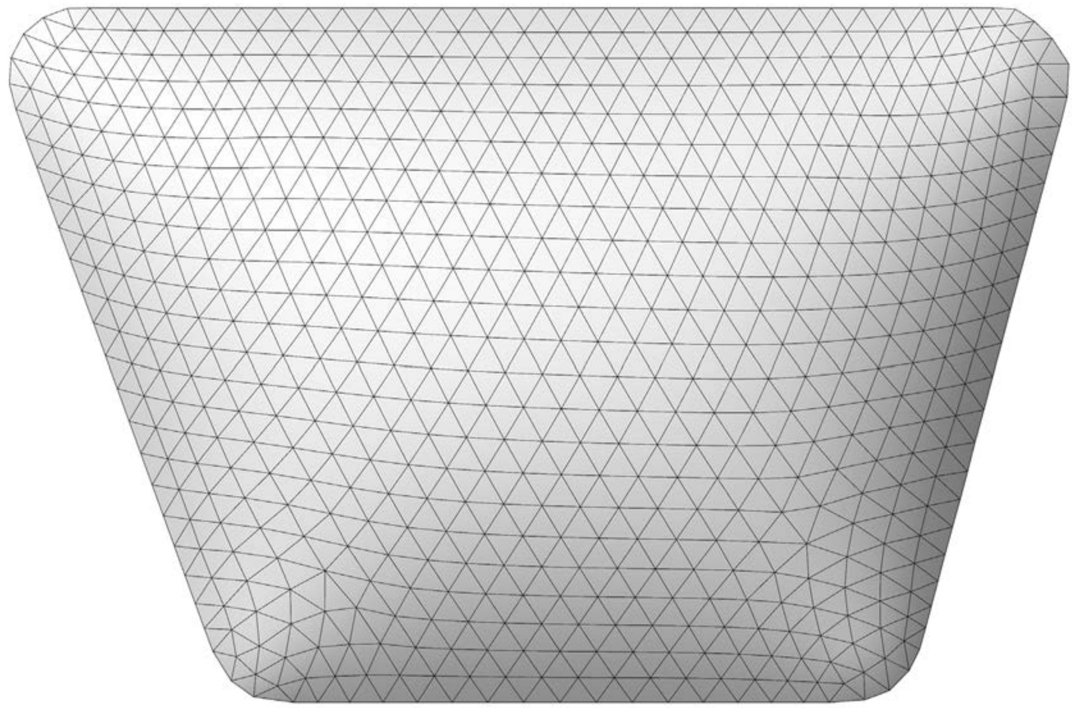
- Can withstand a bending moment
- How much?
- FEM analysis and physical testing



Limitations and recommendations

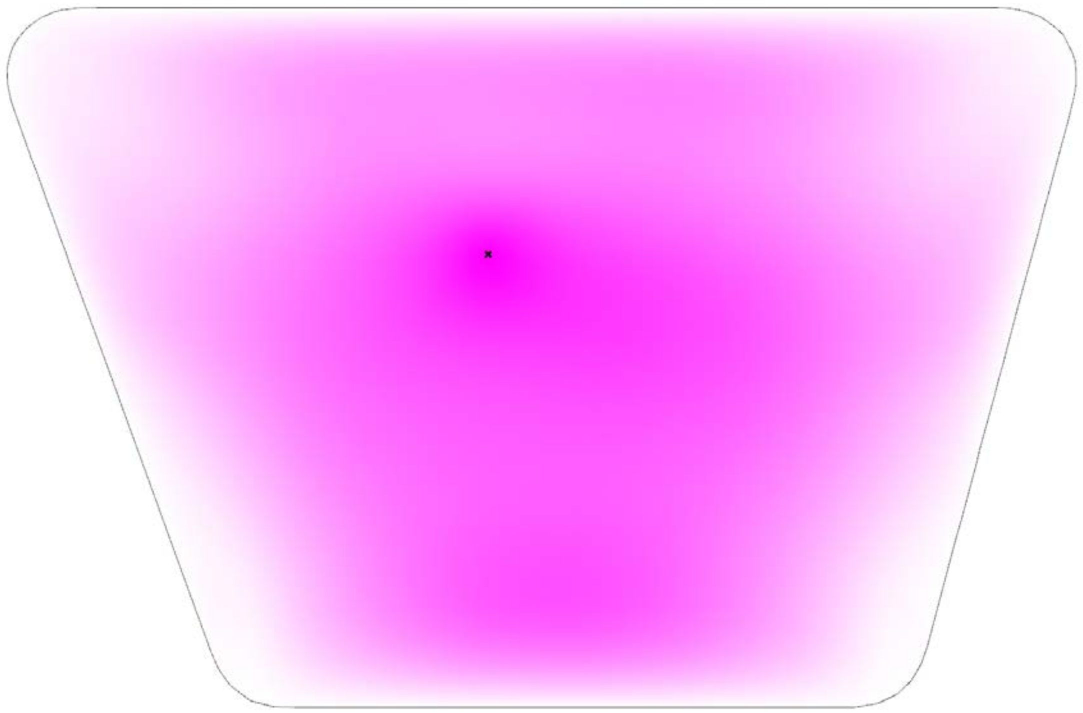
Tessellation pattern

- Not optimized
- Algorithm



Limitations and recommendations

Structural analysis
- Interlayer
- DEM analysis



Limitations and recommendations

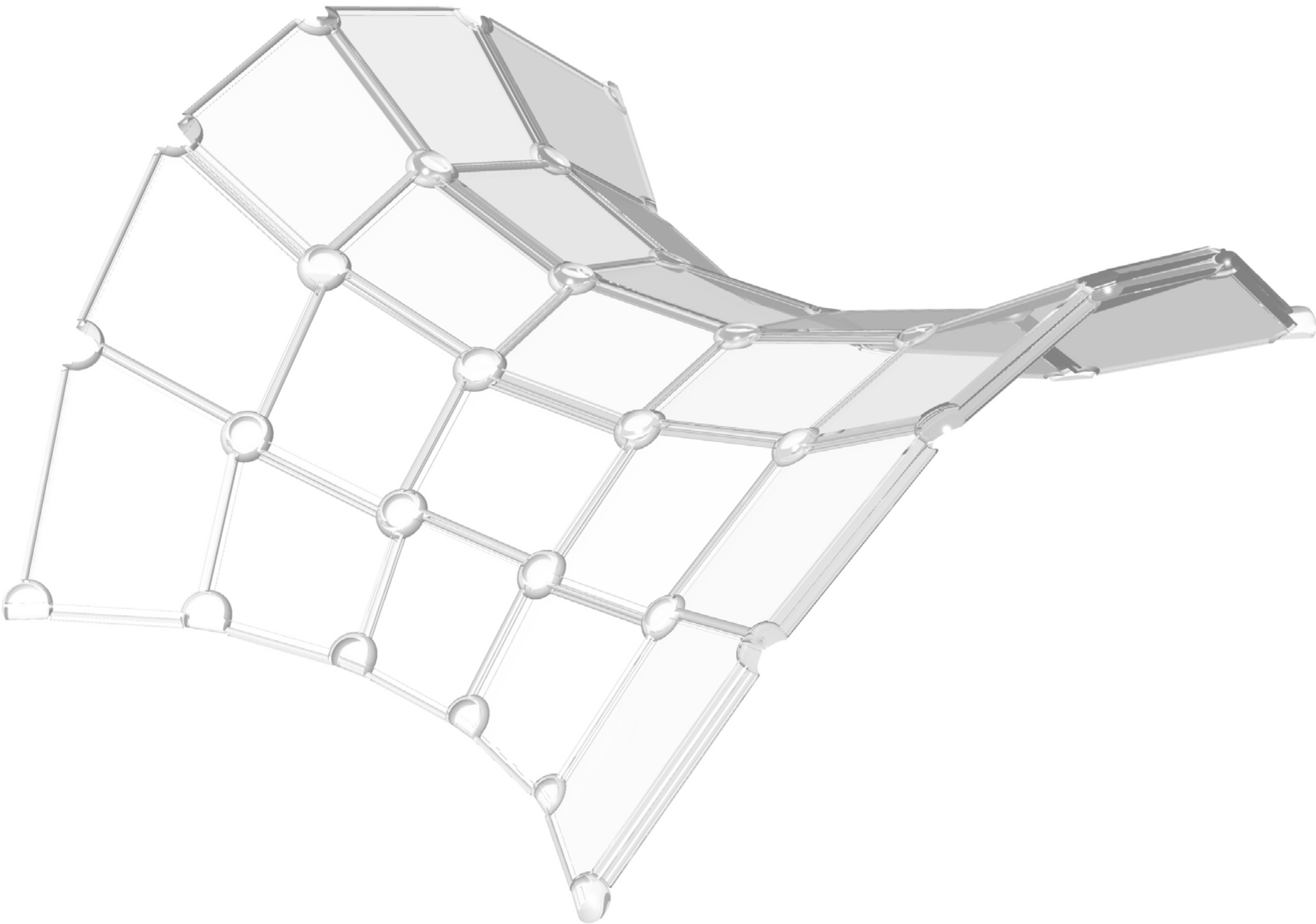
Voussoir mass

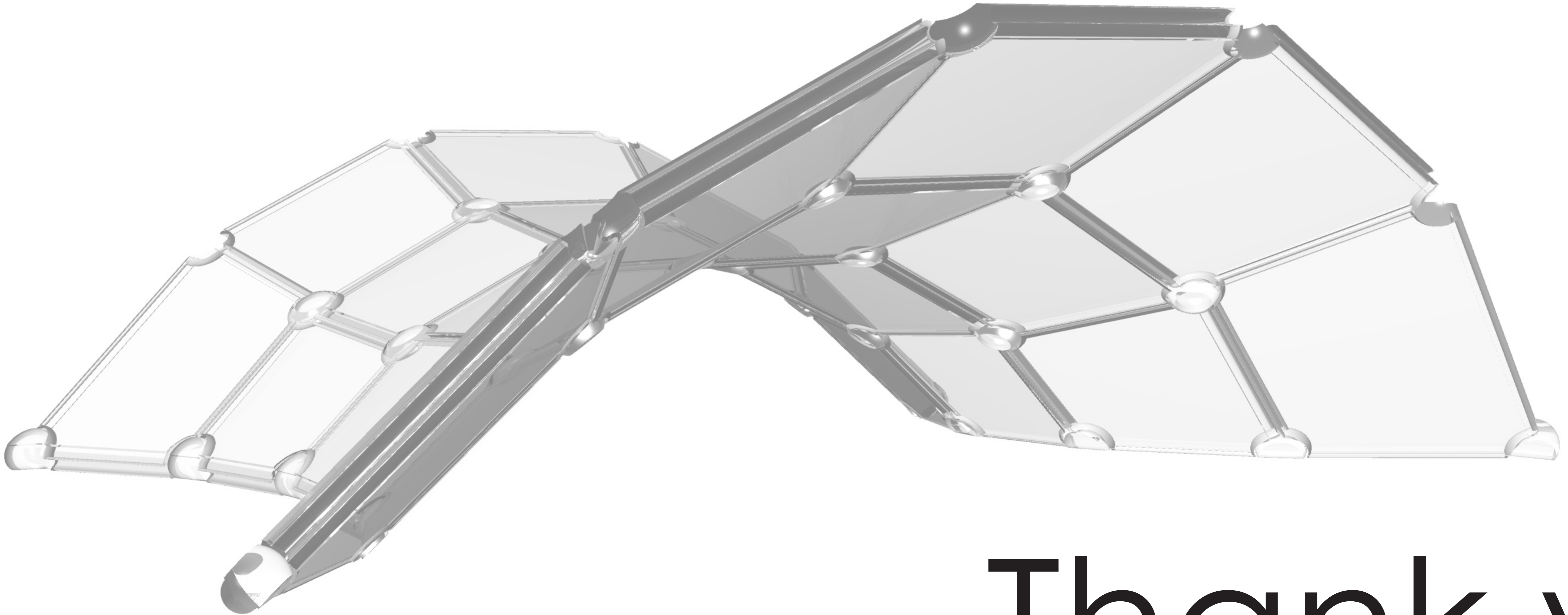
- Renders the shell economically infeasible
- Reduce weight by improving the tessellation pattern

What's next?

GlassTec 2020

- Span ~1 m
- 25 voussoirs
- 30 mm thick
- 2.3 - 5.6 kg





Thank you!