FWI sensitivity analysis in the presence of free-surface multiples

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SUMMARY

It is generally believed that full waveform inversion needs very low frequencies in the data to avoid convergence to a local minimum, which would lead to an incorrect velocity model. Often, low frequencies are not present in the data. Then, a kinematically accurate initial model is required that contains the long-wavelength structures that cannot be reconstructed from the data. Mora (1989), however, showed that reflector below the target area may help in recovering some of the long-wavelength information. Here, we extend his analysis to the case with a free surface, generating multiples. At first sight, multiples should improve the sensitivity and resolution. Indeed, our study shows that sensitivity becomes higher in some parts of the model spectrum when multiples are included. If we consider thin layers and only first-order multiples, then the sensitivity is improved. However, when the layer is thick enough to allow for normal modes of higher orders, areas of high sensitivity appear and this leads to a poorer conditioning of the inverse problem in the wavenumber domain.

INTRODUCTION

Full waveform inversion requires a kinematically correct initial velocity model. Otherwise, the iterations will converge to an incorrect velocity model that corresponds to the nearest local minimum of the least-squares cost function. The initial model should contain those slowly varying spatial structures that cannot be reconstructed from the data because of missing low frequencies. Mora (1989) showed, however, that a reflector below the target area will provide some long-wavelength information. He considered the spatial wavenumber spectrum of a perturbation in a homogeneous velocity model, located between the survey line at the surface and a deeper reflector for a given frequency of the data and found that there is non-zero sensitivity for small wavenumbers, hence long wavelengths, at some angles. This suggests that the presence of a reflector together with large offsets in the acquisition should help in avoiding local minima, even if the data lack reliable low-frequency information. Kazei et al. (2012) extended this analysis to head waves. Here, we further extend this analysis to the case with a free surface that generates surface multiples.

METHOD

The solution of the 2-D constant-density acoustic wave equation in a medium with a constant velocity c_0 can be represented as

$$\hat{p}(\mathbf{r},t) = \int_{-\infty}^{\infty} dk_x' e^{-ik_x'x} \int_{-\infty}^{\infty} d\omega' e^{i\omega' t} p(k_x', z, \omega'). \tag{1}$$

The pressure $\hat{p}(\mathbf{r},t)$ depends on position $\mathbf{r}$ and time t and the transformed pressure $p(k_x, z, \omega)$ on horizontal wavenumber k_x, depth z and angular frequency ω. The solution of the wave equation is a linear combination of upgoing $(+)$ and downgoing $(-)$ wavefields of the form $\exp\left(\pm iz\sqrt{k_0^2 - k_x^2}\right)$, with $k_0 = \omega/c_0$. Note that the chosen Fourier convention is the conjugate of the customary one. In the case of two halfspaces, where the upper one with sources and receivers and scatterers has a velocity c_0, the incident downgoing wavefield is $p_0 = \exp\left(-iz\sqrt{k_0^2 - k_x^2}\right)$ for a source at zero depth. The full wavefield is $p = p_0 + p_1$ with the reflected upgoing field

$$p_1 = C(k_x) e^{+iz\sqrt{k_0^2 - k_x^2}}, \quad C(k_x) = \frac{1-b}{1+b}, \quad b = \sqrt{\frac{k_1^2 - k_x^2}{k_0^2 - k_x^2}},$$

where $C(k_x)$ is the reflection coefficient and $k_1 = \omega/c_1$ contains the velocity c_1 of the deeper halfspace. Next, we include

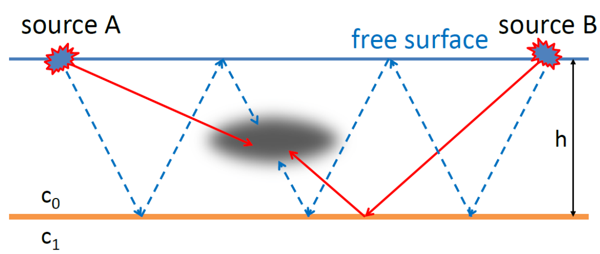

Figure 1: The wavepaths from sources to a perturbation in the layer. The paths marked by red lines were considered by Mora (1989). Blue dashed wavepaths appear only when we add a free surface into the model.

a free surface as sketched in Figure 1. If all multiples are included,

$$\begin{aligned} p' &= p\left(1 + \sum_{n=1}^{\infty} \left[-C(k_x) e^{-2ih\sqrt{k_0^2 - k_x^2}}\right]^n\right) \\ &\to p \Big/ \left(1 + C(k_x) e^{-2ih\sqrt{k_0^2 - k_x^2}}\right). \end{aligned} \tag{2}$$

For thick layers, with h large, the higher-order multiples can be neglected as they arrive late and have small amplitudes. For thin layers, they have to be included, but then the sum in equation 2 may not converge for reflections near the critical point and beyond, where $|C(k_x)|$ approaches 1. In that case, attenuation is needed to obtain a convergent sum. To avoid these problems, we will only consider the first-order multiples and drop all terms in equation 2 except the one for $n = 1$.

Given expression 2 for the wavefield, we can generalize Mora's (1989) technique of sensitivities. The scattered field is determined by the Born approximation. After Fourier transform in the horizontal source and receiver coordinates, the wavefield perturbation becomes

$$\delta u_{sr}(k_s, k_r) = \int k_0^2 \, \delta W(\mathbf{r}) G(k_s, \mathbf{r}) G(k_r, \mathbf{r}) \, d\mathbf{r}, \tag{3}$$

with $\delta W(\mathbf{r})$ a small perturbation of the background squared slowness. We earlier studied the contributions of head waves in case of two halfspaces without a free surface (Kazei et al., 2012; Mora, 1989). By replacing the Green functions with those presented here, we obtain a similar but a bit more complicated linear relation between the spatial perturbation spectrum and the transformed data in terms of sensitivities:

$$S(K_x,K_z) = \left| \frac{\delta u_{sr}\left(k_s(K_x,K_z),k_r(K_x,K_z)\right)}{\delta \widetilde{W}(K_x,K_z)} \right|. \qquad (4)$$

Here K_x, K_z are coordinates in the spatial perturbation spectrum $\delta\widetilde{W}(K_x,K_z)$ (Wu and Toksöz, 1987, e.g.).

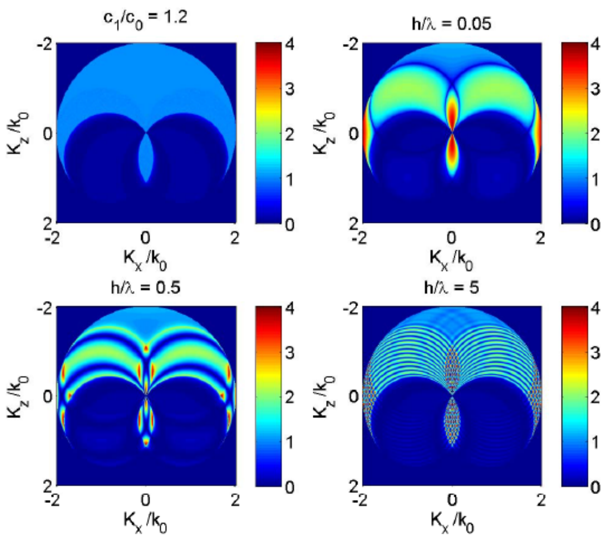

Figure 2: Sensitivities for a low contrast $c_1/c_0 = 1.2$ in case of two halfspaces without a free surface (left upper) and with a free surface and first-order multiples for the same contrast but different ratios between the layer thickness and wavelength, h/λ. The color scales are normalized relative to the maximum of the result without multiples.

SENSITIVITIES: RESULTS & DISCUSSION

We computed sensitivities for the case with first-order multiples and compared them to the case without. The last is shown in the left upper panel of Figure 2. The other panels are scaled relative to the maximum in this panel. Note that we flipped the direction of the K_z-axis compared to (Kazei et al., 2012), so that the lower part of the figures correspond to waves that were reflected at least once. The criterion for existence of the first normal mode in case of a free layer over a halfspace is (Brekhovskikh and Godin, 1998) $h/\lambda \geq 1/4\sqrt{1-(c_0/c_1)^2}$, where h is the layer thickness and $\lambda = 2\pi c_0/\omega$ is the wavelength in the upper layer. For $c_1/c_0 = 1.2$, the critical ratio $h/\lambda = 0.5$, meaning that the first mode is just appearing in the left lower panel of Figure 2, giving rise to peaks in several spots. Its interaction with the direct wave produces additional bright circular arcs. For a different contrast, the left lower panel in Figure 3 for the same $h/\lambda = 0.5$ provides much larger spots of high sensitivity. Therefore, the first-order multiples can be useful for inversion for some of the spatial wavenumbers of the perturbation. If large offsets are present in the data, high sensitivities at large K_x values can be observed in the panels for $h/\lambda = 0.05$ provided by the zeroth-order mode. In that case, there are no other normal modes and the pattern of the sensitivities is similar to the case without free surface, in the left upper panels, yet the overall sensitivity is improved.

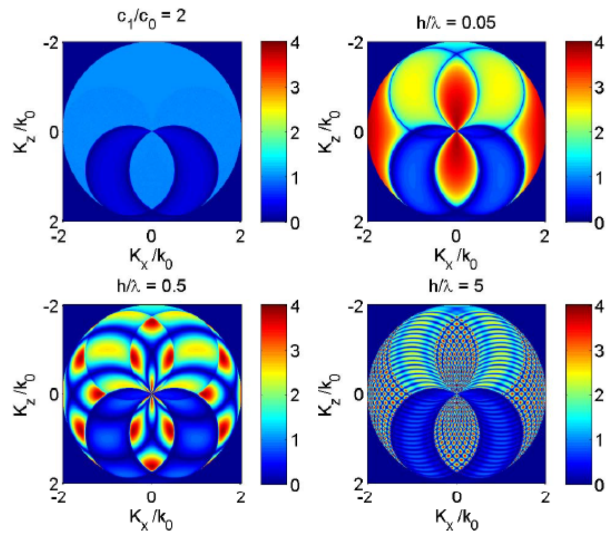

Figure 3: As Figure 2, but for a contrast $c_1/c_0 = 2$.

CONCLUSIONS

We have shown by using spectral sensitivities technique (Mora, 1989, e.g) that the presence of first-order multiples in the data can be helpful for full waveform inversion if the layer containing the perturbation is sufficiently thin to avoid the occurrence of normal modes. For thicker layers, the first-order multiples lead to an increased condition number of the matrix relating parameters to data, making inversion more difficult the thicker the layer. However, multiples may still be beneficial if the inversion is focused somehow on a discrete wavenumbers subset in the spatial spectrum of the perturbation.

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