A Normalized Approach to the Design of Low-Loss Optical Waveguide Bends

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Abstract—This article presents a normalized approach for optimal design of abrupt junctions between straight and curved waveguides operating in the Whispering Gallery Mode regime. The optimalization includes the widths of both the straight and the curved waveguide, the lateral offset between them, and the bending radius of the curved waveguide. With this approach optimum bend design is possible from a simple set of formulas or normalized graphs. Predicted transmission losses for optimally designed junctions are well below 0.1 dB.

I. INTRODUCTION

THE first theoretical paper on bends in optical dielectric guides was published in 1969 by Marcatili [1]. Since then a large number of methods have been developed to analyze propagation through waveguide bends. A powerful technique is the conformal transformation method as described by Heiblum and Harris [2] in which the curved waveguide is translated into an equivalent straight one with a transformed index profile. A suitable method for solving the transformed problem is the Transfer-Matrix Method [3]. This method, which is well known in optics, was applied to the transformed index profile of curved waveguides by Thyagarajan *et al.* [4] and by Pennings [5].

The analysis shows that in curved waveguides the mode profile shifts to the outer edge of the bend, which causes a field mismatch at the junction between a straight and a curved waveguide. Neumann [6] proposed to apply an offset between both waveguides in order to correct for the field mismatch. Pennings [5] showed that an even better match is possible if not only the position of the straight waveguide, but also its width, is optimized. He provided normalized graphs for the computation of bending loss and optimal offset between straight and curved waveguides. In this paper the normalized analysis is extended so as to include all relevant parameters for optimal bend design, i.e., bending and coupling (field mismatch) loss, optimal offset, and optimal waveguide widths. Our method applies to low-contrast slab waveguides. Three-dimensional waveguides can be analyzed by combining our method and the well-known effective-index method.

II. ANALYSIS AND DESIGN OF WAVEGUIDE BENDS

Modes in circularly curved waveguides, as depicted in Fig. 1, can be described as $U_\gamma(r)e^{\pm \gamma_\phi \phi}$. They resemble the modes of straight waveguides, the main difference being that the phase fronts coincide with planes of constant ϕ instead of constant z. The constant γ_ϕ can be looked upon as a complex angular propagation constant

$$\gamma_\phi = \alpha_\phi + j\beta_\phi, \tag{1}$$

where α_ϕ is the angular attenuation coefficient and β_ϕ is the real angular propagation constant with dimension rad^{-1}. The admitted values of γ_ϕ and the corresponding mode profiles $U_\gamma(r)$ follow by solving the well-known Helmholtz equation in a cylindrical coordinate system in combination with the appropriate boundary conditions (being the finiteness of the field at the origin and the outward radiation condition). With the following transformation [2]:

$$u = R_t \ln(r/R_t), \tag{2}$$

in which R_t is an arbitrary reference radius, the equation for $U(r)$ is brought into the form

$$\left[\frac{\partial^2}{\partial u^2} + \{k_0^2 n_t^2(u) - \gamma_t^2\}\right]U_t(u) = 0, \tag{3}$$

in which

$$n_t(u) = n\{r(u)\}e^{u/R_t}, \tag{4}$$

$$\gamma_t = \gamma_\phi/R_t, \tag{5}$$

$$r(u) = R_t e^{u/R_t}, \tag{6}$$

and k_0 is the wave number in the free space. From (3) it is seen that the mode profile of a mode in a curved waveguide with index profile $n(r)$ can be computed as the mode profile of the corresponding mode in an equivalent straight waveguide with a transformed index profile $n_t(u)$ as shown in Fig. 1. The transformed index profile and the corresponding amplitude distribution of the fundamental mode are illustrated in Fig. 2 for different values of the bending radius.

Once the wave equation has been solved in the transformed domain the angular propagation and attenuation constants β_ϕ and α_ϕ follow from (5) as

$$\beta_\phi = \beta_t R_t, \tag{7a}$$

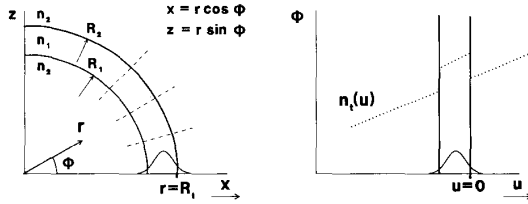$$\alpha_\phi = \alpha_t R_t. \tag{7b}$$

Fig. 1.   Curved waveguide geometry in a cylindrical coordinate system (left) and in the corresponding transformed coordinate system (right).
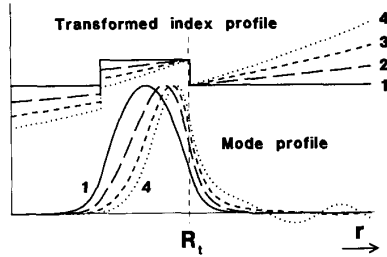

Fig. 2.   Transformed index profile and the corresponding mode profile for a straight waveguide (1) and curved waveguides with decreasing bending radius (2–4).

The radiation loss A_ϕ in db/90° follows from α_ϕ as

$$A_\phi = -20 \log_{10}\left\{\exp\left(-\alpha_\phi \pi/2\right)\right\} = 10\pi\alpha_\phi \log_{10} e. \quad (7c)$$

The mode profile $U(r)$ follows from the transformed mode profile $U_t(u)$ as

$$U(r) = U_t\{u(r)\}, \quad (8)$$

in which $u(r)$ is described by (2).

If R_t is chosen equal to the outer edge of the waveguide, as illustrated in Fig. 1, and $u/R_t \ll 1$ in the vicinity of the waveguide (i.e., R_t is much greater than the waveguide width), then it follows from (2) that r approximately equals $R_t + u$ and the transformed index profile reduces to

$$n_t(u) \simeq n(R_t + u)(1 + u/R_t). \quad (9)$$

In this case the mode profile $U(r)$ is found from $U_t(u)$ by simply shifting it over a distance R_t.

Fig. 2 illustrates the mode profile in a curve waveguide for different values of the bending radius. From the figure it can be seen that the mode profile will shift to the outer edge of the waveguide if the bending radius is decreased. If a sufficiently small bending radius is chosen the field strength at the inner edge vanishes and the mode will be fully guided by the outer edge (curves 3 and 4), so that the location of the inner edge becomes irrelevant. Such a mode is called a Whispering Gallery Mode (WG mode) after Lord Rayleigh [7], who explained this phenomenon in relation to the propagation of sound waves along a curved gallery.

Because the profile of a whispering gallery differs from that of a straight waveguide, coupling loss will occur at the junctions between curved and straight waveguides. It can be minimized by matching the two mode profiles as closely
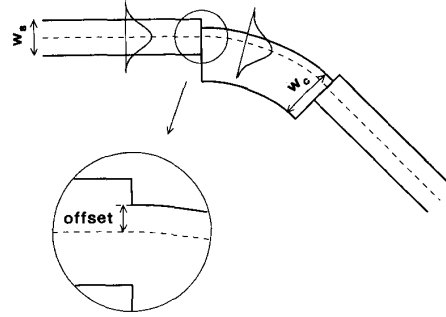

Fig. 3.   A curved and a straight waveguide section, which are optimally dimensioned and aligned for small transition loss.

as possible. This can be achieved through a proper choice of the width and the location of the straight waveguide relative to the curved one, as illustrated in Fig. 3, such that the overlap between the straight and the curved waveguide mode is optimal. The application of an offset between the straight and the curved waveguide in order to reduce transition loss was first proposed by Neumann [6]. Sheem and Whinnery [8] were the first to apply Whispering Gallery Modes to integrated optical circuits. Pennings [5] showed that the lowest total bending loss is obtained by employing curved waveguides which operate in the Whispering Gallery Mode regime.

The optimalization of the bending loss, as described above, is straightforward, but too complicated to be performed without dedicated software. The analysis can be simplified, however, by a proper normalization of the problem.

III. Normalized Approach to Optimal Bend Design

A normalized approach to the analysis of curved waveguides has been applied by Marcatili [1] and Pennings [5]. In this section that approach will be extended to provide normalized solutions to all relevant parameters for optimal design of waveguide bends, operating in the Whispering Gallery Mode regime, including the junctions with straight waveguides.

We normalize all spatial dimensions with respect to the wavelength $\lambda_2 = \lambda_0/n_2$ in the background medium ($\lambda_0 = 2\pi/k_0$):

$$\nu = \frac{u}{\lambda_2}. \quad (10)$$

In terms of the normalized coordinate ν and the relative index contrast profiles:

$$\Delta(\nu) = \frac{n(\nu) - n_2}{n_2} \quad \text{and} \quad \Delta_t(\nu) = \frac{n_t(\nu) - n_2}{n_2}, \quad (11)$$

the wave equation in the transformed domain (Eq. (3)) is transformed into

$$\frac{\partial^2}{\partial \nu^2} V_t(\nu) + 4\pi^2\left[\{1 + \Delta_t(\nu)\}^2 - \{1 + \Delta_{et}\}^2\right]V_t(\nu) = 0, \quad (12)$$

where Δ_{et} is the effective relative index contrast, which is

related to the effective index N_t ($= \gamma_t/k_0$) through

$$\Delta_{et} = \frac{N_t - n_2}{n_2}. \tag{13}$$

Using (9)–(11) the transformed relative index contrast $\Delta_t(\nu)$ can be approximated for $\Delta(\nu) \ll 1$ (i.e., a low index contrast) and $u/R_t \ll 1$ (i.e., in the vicinity of the waveguide) as

$$\Delta_t(\nu) = \Delta(\nu) + \frac{n(\nu + \rho_t)}{n_2}(\nu/\rho_t) \simeq \Delta(\nu) + \nu/\rho_t. \tag{14}$$

With these approximations (12) reduces to

$$\frac{\partial^2}{\partial \nu^2} V_t(\nu) + 8\pi^2 \{\Delta(\nu + \rho_t) + \nu/\rho_t - \Delta_{et}\} V_t(\nu) \simeq 0, \tag{15}$$

in which $\rho_t = R_t/\lambda_2$, and the transformed index profile has been substituted according to (9). If we introduce a new variable $\bar{\nu}$:

$$\bar{\nu} = \nu/a, \tag{16}$$

Eq. (15) appears to keep exactly the same form if the following substitutions are made (the transformed quantities are indicated with a bar):

$$\bar{\Delta}(\bar{\nu}) = a^2 \Delta(a\nu), \tag{17}$$

$$\bar{\Delta}_{et} = a^2 \Delta_{et}, \tag{18}$$

$$\bar{V}(\bar{\nu}) = V(a\nu), \tag{19}$$

$$\bar{\rho}_t = \rho_t/a^3. \tag{20}$$

From this result it follows that if $\{V(\nu), \Delta_{et}\}$ is a solution of (15) for the index profile $\Delta(\nu)$ and radius ρ_t, then $\{V(a\nu), a^2\Delta_{et}\}$ is a solution for the index profile $a^2\Delta(a\nu)$ and radius ρ_t/a^3, i.e., if the relative contrast profile is compressed by a factor a, its height is multiplied by a factor a^2, and its radius is divided by a factor a^3, then the mode profile is compressed by a factor a, but otherwise retains the same shape.

The propagation constant β_t and the attenuation coefficient α_t are related to the effective-index contrast Δ_{et} of the mode as

$$\beta_t = k_0 n_2 \{1 + \text{Re}\,(\Delta_{et})\}, \tag{21a}$$

$$\alpha_t = -k_0 n_2 \{\text{Im}\,(\Delta_{et})\}. \tag{21b}$$

From (21b), in combination with (18), it follows that the attenuation coefficient α_t transforms according to

$$\bar{\alpha}_t = a^2 \alpha_t. \tag{22}$$

The angular attenuation coefficient α_ϕ follows from the transformed constant α_t through multiplication by $\bar{\rho}_t$ (Eq. (5)) so that we find, for α_ϕ,

$$\bar{\alpha}_\phi = \bar{\alpha}_t \bar{\rho}_t = a^2 \alpha_t \rho_t/a^3 = \alpha_t \rho_t/a = \alpha_\phi/a. \tag{23}$$

Obviously, both the mode profile in curved (and straight) waveguides, and the radiation loss in curved waveguides transform in a very simple manner by introducing the variable $\bar{\nu}$ (Eq. (16)). This is an important result. It means

that, if we compute the radiation loss or the coupling loss at the junction between a straight and a curved waveguide as a function of the bending radius R for a given contrast Δ_0, then the properties for other contrasts Δ can be directly inferred (as long as both Δ and Δ_0 are small).

It should be noted that the normalization introduced above is restricted to small index contrasts (up to 10%, as will be shown in the sequel to this paper), and that the curved waveguides should operate in the Whispering Gallery Mode regime.

IV. NUMERICAL RESULTS AND EMPIRICAL CORRECTIONS

To design a waveguide bend with low loss and optimal junctions to the straight waveguides, the following five quantities have to be determined:

- The angular radiation loss.
- The minimal width of the curved waveguide.
- The optimal width of the straight waveguide.
- The optimal offset between the curved and the straight waveguide.
- The corresponding coupling loss.

On the basis of the normalization described in the previous section (Eqs. (16)–(20)), the analysis of a waveguide with arbitrary contrast Δ and bending radius R can be reduced to the analysis of a waveguide with a normalized index contrast Δ_0. The choice of the normalized contrast Δ_0 fixes the value of the transformation constant a through (17): $a = (\Delta/\Delta_0)^{1/2}$. The transformed waveguide has an (outer) bending radius $a^3(R_t/\lambda_2)$ according to (20).

For our analysis we chose $\Delta_0 = 0.01$. Fig. 4(a)–4(d) show the results of the analysis as a function of the (normalized) radius. The radiation loss (Fig. 4(b)) is computed for a Whispering Gallery Mode, i.e., the width was chosen so large that it no longer affects the propagation properties of the mode. The optimal offset and width of the straight waveguide (Fig. 4(c) and 4(d)) were found by optimizing the overlap between the straight-waveguide mode and the WG mode. The ultimate coupling loss (Fig. 4(a)) follows as the logarithm of the optimum overlap. It was empirically determined that for $w_c > 1.5w_s$ (w_s being the optimal straight-waveguide width as determined from the graphs), the results do not significantly depend on w_c. From this it follows that $w_c = 1.5w_s$ is a good choice for the curved waveguide to operate virtually in the WG mode regime.

For small contrasts the normalized solutions apply to TE-polarized as well as TM-polarized modes. For the maximal relative contrast analyzed in the present chapter ($\Delta = 0.16$), the TM-radiation loss was found to be greater by 30% than the TE-polarized loss. The normalized optimal offset for TM polarization was found to be smaller than the TE-polarized value by approximately 0.2 μm. The difference in optimal waveguide width is within 1%. Differences between the normalized solutions for both
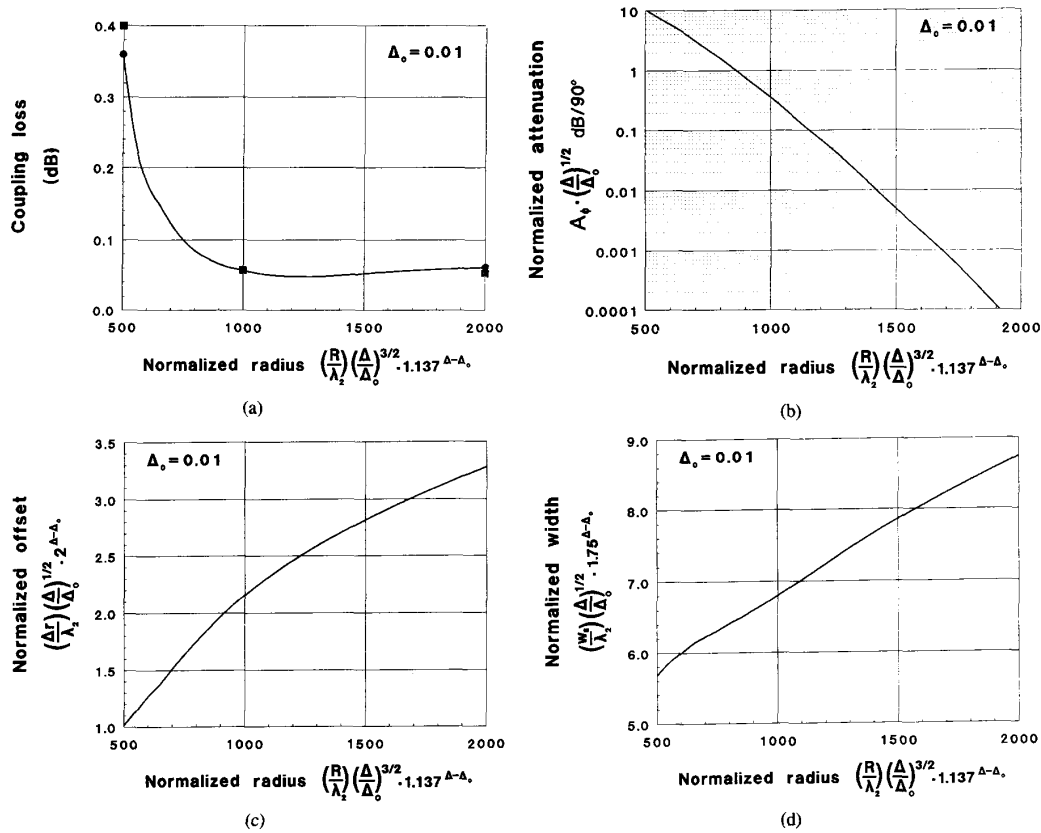
Fig. 4. (a) Normalized transition loss for a single junction. (b) Normalized radiation loss. (c) Normalized optimal offset between the outer edge of the curved waveguide and the center of the straight one. (d) Normalized optimal width of the straight waveguide.

polarizations are thus negligible for most practical purposes. It is stressed that in three-dimensional waveguides the polarization dependence may be greater because the effective indices of the transverse slab modes which form the starting point for the lateral computations may differ considerably. This effect can be analyzed, however, using the normalized approach.

Fig. 4(a)–4(d) are employed as follows. The normalized radius R_n is computed according to

$$R_n = a^3 R/\lambda_2 * 1.137^{\Delta - \Delta_0}, \qquad a = (\Delta/\Delta_0)^{1/2}, \qquad (24)$$

in which R and Δ are the actual radius and refractive index contrast of the waveguide, and $\Delta_0 = 0.01$ is the value of Δ for which the graphs were computed. The origin of the correction factor $1.137^{\Delta - \Delta_0}$ will be discussed in the sequel to this paper. The required properties can then be read from the relevant graph. The coupling loss is independent of the normalization and can be read directly. The other properties are determined on the basis of the normalized values, as read from the figure, through division by the product of a ($= \Delta/\Delta_0)^{1/2}$ and the correction factor as listed along the vertical axis. A polynomial description of the curves is provided in the Appendix.

To analyze the accuracy of the normalization, we have computed the radiation loss, the normalized width, and

the normalized offset for a series of contrasts, ranging from 0.0025 to 0.16, which cover a practical range from very low to rather high index contrasts. The relative error has been determined by dividing these results by those computed using the normalized solutions. Fig. 5(a), 5(c), and 5(e) show the results. From Fig. 5(a) we see that the relative error in A_ϕ is linear in both R and Δ. Because the dependence of the logarithm of A_ϕ on R is approximately linear, the error can be compensated with a correction term of the form $c^{\Delta - \Delta_0}$. Calculation yields $c = 1.137$ as a good fit. Fig. 5(b) shows the resulting error after correction. Its magnitude appears to be linear with Δ, from which we conclude that the normalization error will be within 20% for contrasts up to 0.2.

The errors in the normalized offset and width (of the straight waveguide) appear to be independent of R and linear in Δ. This again suggests a correction factor of the form $c^{\Delta - \Delta_0}$. For the offset a good fit is found with $c = 2$, for the optimal width with $c = 1.75$. Fig. 5(d) and 5(f) show the relative error after correction, which appears to be within 6% for the offset (within 2% for $R_n > 1000$) and within 2% for the width. The markers in Fig. 4(a) show the effects of the residual errors for the least and the greatest contrast (0.0025 and 0.16, respectively), at the extreme ends of the computation range. From these data
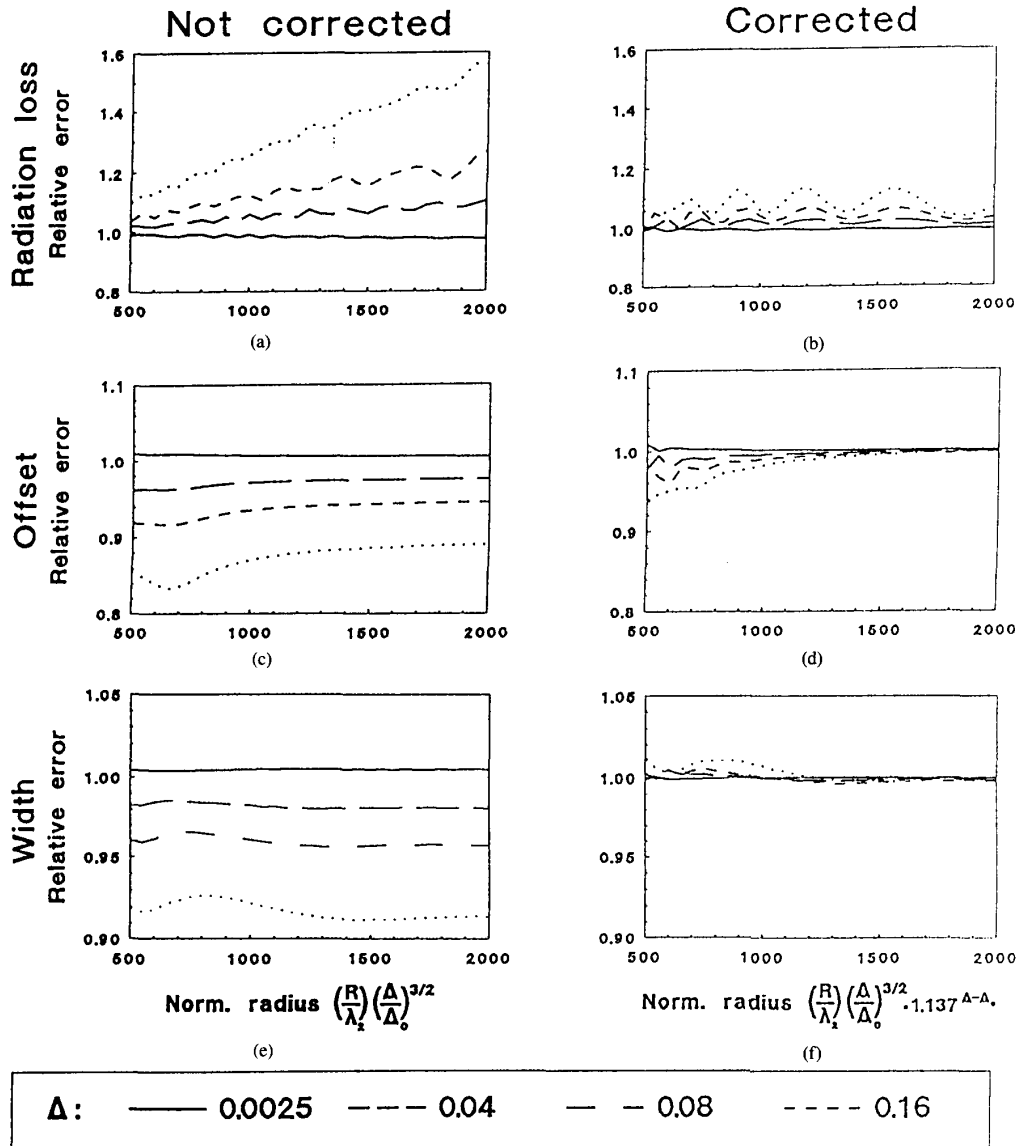
Fig. 5.   Relative errors in the normalized solutions for (a) the radiation loss, (c) the optimal offset, and (e) the optimal width, for different index contrast, without (a), (c), (e) and with empirical corrections as described in the text (b), (d), (f).

it is evident that the errors will be negligible for almost all practical purposes.

V. DISCUSSION AND CONCLUSIONS

Employing the normalized graphs of Figs. 4 or the regression formulas of the Appendix, optimal bend design can be performed with a pocket calculator for a broad variety of planar optical waveguides with low or medium optical contrast. Two different design strategies will be briefly discussed.

If the lowest possible loss is required, a normalized radius should be selected for which the sum of the radiation loss (over the relevant sector angle) plus twice the coupling loss is minimal. Except for very low contrasts, the

total loss will be dominated by the coupling loss and a normalized radius between 1000 and 1500 will be optimal, corresponding to a normalized radiation loss between 0.4 and 0.005 dB/90°. The corresponding optimal widths and the offset between the straight and the curved waveguides then follow from Fig. 4(c) and 4(d).

If the choice of the straight-waveguide width is not free, the radius of the bend has to be chosen such that the mode width matches that of the straight waveguide. This is done by reading the normalized radius corresponding to the prescribed (normalized) width of the straight waveguides from Fig. 4(d). The other parameters are fixed by this choice, and follow from the graphs.

The normalized approach as outlined above is particu-

larly suited for computer-aided design and simulation of planar optical circuits. We confined ourselves to the excitation of the fundamental modes which cover most of the practical applications. Radiation loss and coupling efficiencies for higher-order modes can be normalized equally well. For the method to be applicable, index contrasts should be low and the curved waveguide should be sufficiently wide (Whispering Gallery Mode regime).

APPENDIX

The normalized curves of Figs. 4(a)–4(d) are easily quantified with polynomial regression. The results are given below in terms of the real (i.e., not normalized) entities, for $\Delta_0 = 0.01$:

$$A_\phi = (100\Delta)^{-1/2} 10^{2.29 - 2.17R'_n - 0.58(R'_n)^2} \text{ (dB/90°)}$$
$$= \text{radiation loss per 90° in dB.} \tag{A1}$$

$$w_s = \frac{\lambda_0}{n_2} (100\Delta)^{-1/2} \{4.56 + 2.45R'_n - 0.18(R'_n)^2\}$$
$$\cdot 1.75^{0.01 - \Delta} \tag{A2}$$
$$= \text{optimal width of the straight waveguide.}$$

$$w_c = 1.5w_s \tag{A3}$$
$$= \text{minimal width of the curved waveguide.}$$

$$\Delta r = \frac{\lambda_0}{n_2}(100\Delta)^{-1/2}\{-0.9 + 4.7R'_n - 2.0(R'_n)^2$$
$$+ 0.35(R'_n)^3\}2^{0.01 - \Delta} \tag{A4}$$
$$= \text{optimal offset between the outer edge of the curved}$$
waveguide and the center of the straight one.

$$\eta = 10^{1.63 - 5.97R'_n + 3.92(R'_n)^2 - 0.82(R'_n)^3} \text{ (dB)}$$
$$= \text{coupling loss in dB at a (single) junction between a}$$
straight and a curved waveguide, optimized according to the above parameters. $\quad$ (A5)

$$R'_n = \frac{n_2 R}{\lambda_0}(100\Delta)^{3/2} 1.137^{\Delta - 0.01}/1000$$
$$= \text{normalized bending radius (at the outer edge). Note}$$
the factor 1000 in the denominator, which is included to avoid repetition of factors 0.001 in the regression formulas. $\quad$ (A6)

These formulas apply in the range $0.5 < R'_n < 2$, i.e., $500 < R_n < 2000$.

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Hans Blok was born in Rotterdam, The Netherlands, on April 14, 1935. He received a degree in electrical engineering from the Polytechnical School of Rotterdam in 1956. He then received the B.Sc. and M.Sc. degrees in electrical engineering and the Ph.D. degree in technical sciences, all from Delft University of Technology, in 1961, 1963, and 1970, respectively. Since 1968, he has been a member of the Scientific Staff of the Laboratory of Electromagnetic Research at Delft University of Technology. During these years, he has carried out research and lectured in the areas of signal processing, wave propagation, and scattering problems. During the academic year 1970–1971, he was Royal Society Research Fellow in the Department of Electronics of the University of Southampton, U.K., where he was involved in experimental and theoretical research on lasers and nonlinear optics. In 1972 he was appointed Associate Professor at Delft University of Technology, and in 1980 he was named Professor. From 1980 to 1982 he was Dean of the Faculty of Electrical Engineering. During the academic year 1983–1984 he was visiting scientist at Schlumberger-Doll Research, Ridgefield, CT. At present, his main interest is in guided wave optics and inverse scattering problems.