Multi-Period Line Planning and Timetabling for Varying Railway Passenger Demand

Renate J. H. van der Knaap

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Dissertation

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Renate Johanna Helena VAN DER KNAAP

This dissertation has been approved by the promotors.

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For every girl who doubted herself – may you know your worth.

Preface

What a journey it has been from the departure of my PhD research in 2021 to arriving at my defence in 2026. It has had both peaks and valleys, but above all it was made unforgettable by the many wonderful people I met along the way. Before diving into the content, I would like to take this opportunity to thank the people who have supported me on this journey.

First and foremost, I would like to express my sincere gratitude to my two promotors, Rob and Niels. Rob, I have greatly valued your guidance over the years, where our regular, engaging discussions have been truly enjoyable. Your fast and thorough feedback was invaluable in keeping me on track and enhancing the quality of my work. Niels, I am deeply grateful for your mentorship throughout this journey. I appreciate how you have encouraged me to embrace new challenges and gently pushed me beyond my comfort zone, while always taking the time to support me in feeling confident. Your talent for storytelling and conveying complex ideas to a non-scientific audience has been truly inspiring.

Furthermore, I would like to thank the remaining members of my PhD committee for the time and effort you have devoted to serving on it and evaluating my dissertation. Oded, I greatly appreciate how you co-lead the Smart Public Transport Lab and now also head the Department of Transport & Planning (T&P), always showing genuine interest in the well-being of those working with you. You motivate people to excel and perform high-standard research, while also encouraging them to make time to enjoy life. Mathijs, although our faculties are relatively close to each other, it took 3 years and a trip to Sardinia before we first met at the 2024 ODS conference. I have valued your insightful questions and the fresh perspective you brought to my work. Pieter, thank you for your sharp questions and comments, both when meeting throughout the years as on my draft thesis. It has been a pleasure to read your work and that of your colleagues. Christian, I want to thank you for the enthusiasm you have shown for my work and for your sharp eye and insightful comments, which helped me to improve my dissertation during the final stages. Ties, after meeting you during your farewell party when you left TU Delft, I am glad that now you will also be part of mine as a representative of NS.

I would like to extend my gratitude to the Dutch railway undertaking NS, for financially supporting this PhD project. In particular, my thanks go to Dennis and Menno for taking the time throughout the years to supervise me from the NS side: brainstorming ideas, providing insights from practice, and reading and providing feedback on my draft papers. Furthermore, I greatly appreciate the opportunities you gave me to share my work within NS. Many thanks to the colleagues from the π/DigOps department for welcoming me into your midst, for the interesting conversations, and the very fun team outings, both during my MSc thesis internship and the PhD project. I am also grateful to Pedro, Danny and Khuê at Erasmus University, for the interesting discussions, the fun, and the pep talks throughout the years: a problem shared is truly a problem halved. A special thanks also goes to Gábor, for the numerous enjoyable conversations about our shared passion for music, and for never missing the opportunity to add confusion to my brain and clarity to my figures.

I am grateful to all my colleagues at the T&P department, particularly those in the Digital Rail Traffic Lab and the Smart Public Transport Lab. Thank you for creating a warm, welcoming atmosphere that encourages discussion, collaboration, and mutual support. It was a pleasure to watch you grow and tackle challenges throughout the years, as we were learning and researching together, and your words of encouragement made a real difference. I was also fortunate to travel with many of you to conferences, which were often the highlight of the year. Above all, my thanks go to my office-mates from room 4.17. Together we have created a true home away from home, with continuous support, celebrating each other's successes, enriching cultural and language exchanges, and a never-ending list of inside jokes. Because of you, coming to the office was always a pleasure.

My deepest gratitude goes to my two paranymphs, Nina and Ziyulong. Nina, throughout my PhD project you have been a superb sounding board. Whether I was stuck on a bug, wrestling with a formulation, or simply needing to talk something through, you were always ready to listen, offer advice, or provide moral support, which made my PhD journey much more enjoyable. Thank you for initiating this mutual support system. I also treasure the memories of our foreign adventures travelling to conferences and the game nights with you and Colin. Ziyulong, thank you for always checking in, giving pep talks when needed, celebrating milestones, and just being a wonderful friend. I am amazed by your dedication both to your research and to your family and friends. Peter and I greatly enjoyed the meals we shared with you, Xinyan and your parents and to learn more about your culture. I deeply value the friendship you have both given me. Having been a paranymph at both your defences, it gives me the greatest pleasure that you will now stand beside me on this special day.

I want to thank family and friends for their continuous support along this journey. Thank you for listening to my struggles and complaints, for providing much-needed distractions, and for helping me recharge my batteries. I feel so fortunate to have such wonderful people in my life who readily arranged time off to attend my defence on a random Monday in June. Especially, I want to thank my parents for their unconditional love and support throughout my academic journey. I hope the 'daughter that travelled the world to talk about her work' has made you proud.

Most of all, I am grateful to my husband, Peter, for his unwavering love and support through the highs and lows of these past years. Thank you for listening to all my complicated stories about trains and optimisation models, and for encouraging me to balance all the hard work with relaxation, whether through hiking, making music together, playing board games, or simply drinking coffee together. This journey has been infinitely better with you by my side.

Renate van der Knaap
Voorburg, May 2026

Contents

Chapter 1

Introduction

1.1 Background

Society currently faces significant challenges in transitioning towards more environmentally-friendly mobility. In 2021, the European Parliament passed the European Climate Law, stating that the member states are committed to a 55% reduction in CO_2 emissions by 2030 and to achieve climate neutrality by 2050 (Regulation 2021/1119, 2021). In the Netherlands, greenhouse gas emissions from transport made up 14% of the total emissions in 2023 (Centraal Bureau voor de Statistiek, 2024). Public transport in general, and trains in particular, can play an important role in reducing the emissions from transport. After walking, cycling and electric scooters, electric trains are the most sustainable mode of transport (Milieu Centraal, 2023). Therefore, it is important to make railway services as attractive as possible in order to attract and retain passengers.

Railway undertakings (RUs) face several challenges while striving to deliver optimal service to their customers, which include limited resources and financial difficulties. First, RUs do not have unlimited resources to serve the passenger demand. The railway network capacity is constrained, particularly in densely used networks across Europe, which limits the opportunities to run additional trains. Moreover, RUs only have a finite train fleet (i.e., rolling stock) and personnel available to operate services. Although these can be expanded over time, new rolling stock is very expensive and tight labour markets make recruiting new staff difficult. Second, RUs could face financial challenges on both the cost and the revenue side. In the context of the Netherlands, where the largest passenger RU is Netherlands Railways (Nederlandse Spoorwegen, or NS), this is particularly relevant. Ticket revenues are lower than long-term forecasts, partly due to external factors such as the Covid-19 pandemic. Passenger demand declined significantly during the pandemic, and recovery has been slow. Ton et al. (2022) and Kroesen et al. (2023) show that in the Netherlands, especially frequent train commuters intend to travel less by train than before the pandemic, mostly due to working from home more often. Reflecting this trend, NS anticipates that passenger demand will only return to pre-Covid levels by 2030 (Nederlandse Spoorwegen, 2025a). Meanwhile, operational costs continue to rise, particularly for personnel. For example, the collective labour agreement for 2024 between the unions and NS includes an average salary increase of 6.6% (Nederlandse Spoorwegen, 2024b) and in the 2025 negotiations resulted in agreements for two additional salary increases of 4% in 2025 and 3% in 2026 (Nederlandse Spoorwegen, 2025d). Given the limits on resources and financial challenges, it is vital for railway undertakings to use their resources as effectively as possible.

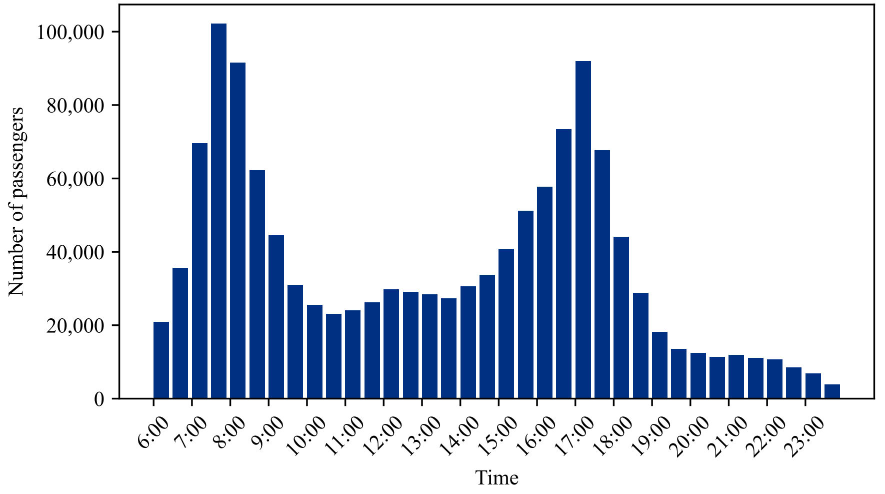


Figure 1.1: Number of NS passengers departing per half hour time slot on a regular Tuesday in 2019.

The optimal use of limited resources by RUs depends on the goals they have (e.g., maximising profits or minimising passenger journey times) and on the passenger demand. However, the demand for railway services is not constant, but instead changes throughout the day, week and year. Here, we illustrate how these fluctuations in passenger demand take shape in the Netherlands. Figure 1.1 shows the number of departing NS passengers on a regular Tuesday in 2019, aggregated per half hour time slot. This figure shows that there is a lot of variation in demand during the day, with higher demand at the start and end of the workday and less demand during the middle of the day and evening. Although we see this overall pattern at all stations, some stations have a greater variation between peak and off-peak demand than others. For example, the station at the international airport Schiphol has passengers arriving and departing quite consistently throughout the day, with 74% of the departing trips in 2023 starting outside the peak hours (Nederlandse Spoorwegen, 2025c). On the other hand, there are stations where more people depart during the peak hours. For instance, 55% of the passengers departing from the small commuter station Rilland-Bath do so during the peak hours (Nederlandse Spoorwegen, 2025b). The different days of the week also have different demand. For example, the peak hours are more crowded on Monday, Tuesday, and Thursday than on Wednesday and Friday (Ton et al., 2022). This is because Wednesday and Friday are popular days to work remotely from home, and many part-time workers often do not work on these days. Wednesdays are popular as most primary schools in the Netherlands have Wednesday afternoons off, so working parents want to be home to take care of their children. Lastly, there are also fluctuations during the year. The demand for railway services is lower during the school holidays, because many people are on holiday and hence do not travel to work or school. Furthermore, there is always a peak in demand in September, when the new academic year starts. During this month, there is a large group of students that travel to school or university and most people are back from their holidays and return to work. The demand from the student population gradually reduces during September and October. Some students drop out, some find housing closer to their educational institute, and others realise that not all lectures are worth travelling during the busy peak hours. Given all the variations listed above, we can conclude that there is not one type of railway demand.

Despite these numerous variations in demand, the railway line plan and timetable operated in many European countries, including the Netherlands, are (almost) fixed throughout the day. There are several reasons why adjusting the line plans and timetables to the demand has been limited in the past, including (1) the attractiveness of a fixed and cyclic schedule for passengers (2) the complexity of railway scheduling, and (3) the previously limited availability of demand data.

Reason (1) for having one fixed plan throughout the day lies in the multiple benefits it offers to passengers. The concept of cyclic railway timetables with well-coordinated connections was introduced by NS in 1970, aiming to compete with the growing popularity of cars (Wikipedia, 2025). Since then, the cyclic timetable has been adopted by several other European countries (Peeters, 2003). In the cyclic timetable, the schedule repeats every cycle, typically every hour. The primary benefit of a cyclic timetable is its memorability for passengers, which makes it easy to use. Cyclic timetables are often clock faced (i.e., trains always leave a certain number of minutes after the hour). Furthermore, they usually aim to have regular departure intervals between trains on the same line, so if a line has a frequency of 4, the time between two departures is 15 minutes. According to Wardman et al. (2004), these two aspects both improve the memorability of a timetable. Additionally, a cyclic timetable facilitates good connections to other public transport modes for the first and last mile, improving overall accessibility and convenience for passengers. A cyclic timetable is also advantageous for people travelling outside the peak hours as there are still multiple travel options available during less busy times. Previous studies have shown that having a cyclic timetable with well-planned connections improves the passenger satisfaction and as a result the demand and revenues for the railway undertaking (Wardman et al., 2004; Johnson et al., 2006). However, the drawback of a cyclic timetable is its inflexibility to the passenger demand. Cyclic timetables are usually optimised for the peak-hour demand and hence the resulting schedules fail to address the distinct travel patterns of off-peak travellers, resulting in a suboptimal service provision during these times. On the other hand, acyclic timetables that do not repeat every hour provide many opportunities to serve varying demand, but lack ease of use for the passengers. An overview of the benefits and drawbacks of cyclic and acyclic timetables, based on the work of Peeters (2003), is given in Table 1.1.

Table 1.1: Overview of benefits and drawbacks of cyclic and acyclic timetables (based on Peeters (2003)).

	Benefits	**Drawbacks**
Cyclic timetables	Easy to remember for passengers No gaps in train service when demand is low Consistent transfers throughout the day	Expensive, especially for varying demand Allocation of unused hourly time slots for non-hourly trains (e.g., freight or international trains)
Acyclic timetables	Many trains in periods with high demand Flexible to demand variations Less expensive for varying demand	More difficult to remember Less travel opportunities and/or gaps in service for periods with low demand Entire day needs to be considered during planning

Reason (2) why adjusting the line plan and timetable to time-varying demand has been limited in the past, is due to the complexity of railway scheduling. Before passengers can board a train according to a timetable, several planning steps have to be completed. These planning steps are often done sequentially, and consist of network design, line planning, timetabling, rolling stock scheduling, and crew scheduling (see Figure 1.2).

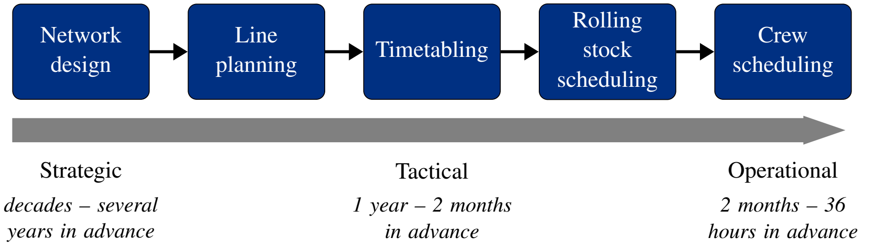


Figure 1.2: Steps in railway scheduling

Network design considers making changes to the railway infrastructure (tracks, stations, and switches), which are strategic choices that are made several years or even decades in advance (Abbink, 2014). Given the railway infrastructure, the next step is to determine the line plan. This is the set of lines that will be operated, where each line has a route through the network, a set of stations where the train will stop, and a frequency, i.e., how many trains per hour will be operated on the line. The line planning problem is also a strategic problem that is revisited every few years (Huisman et al., 2005).

Next, a timetable is created for this line plan which is categorised as a tactical planning problem. In the timetable, we determine for each train in the line plan the arrival and departure times at all the stations on its route. During this stage, we must ensure that trains do not use the same infrastructure at the same time. Furthermore, constraints can be included to make the timetable more attractive to passengers, for example by facilitating easy transfers between lines at large stations. Timetables can either be cyclic or acyclic. In the Netherlands, and many other European countries, a cyclic railway schedule is used. In a cyclic railway timetable, a timetable is created for the duration of one cycle (e.g., 1 hour) and then repeated to create a timetable for the entire day. At NS, a new basic timetable is created once per year and every 2 months a more detailed version is developed that also includes adjustments for specific events and detailed platform assignment (Abbink, 2014; Huisman et al., 2005). On the other hand, acyclic timetables do not have constraints on regularly running the same line. Instead, every hour of the day can have a different timetable, based on the demand.

The last two steps in railway scheduling, namely rolling stock and crew scheduling, are operational planning problems. These steps ensure that rolling stock, a driver, and a conductor are assigned to each train in the timetable, so that the train can actually be operated. Such problems are solved between 2 months and 36 hours in advance (Abbink, 2014; Huisman et al., 2005).

Each of these railway scheduling steps are already difficult to solve on their own. For example, different versions of the line planning problem are shown to be NP hard (see e.g., Bussieck (1998), Claessens et al. (1998), Schöbel & Scholl (2005)) as well as the cyclic timetabling problems (see e.g., Serafini & Ukovich (1989)). Due to this complexity, computers struggle to find feasible and attractive line plans and timetables for large instances (such as for the entire

Netherlands). Therefore, creating these plans still requires significant manual effort by railway undertaking employees. Hence, one can imagine that creating several plans for a single day to serve different types of demand is not an easy task for any railway undertaking.

Reason (3) relates to the limited availability of demand data. Since the introduction of the smart card fare collection systems, much more data about public transport demand has become available. Before the introduction of the smart card, a public transport operator had to estimate the demand using e.g., on-board count systems or surveys (Pelletier et al., 2011). However, while on-board count systems provide information about how many people use a vehicle, they are not able to give information about the origin and destination of passengers. This detailed origin-destination (OD) information can be gathered using surveys, but these are expensive to conduct and require the cooperation of passengers, which might lead to an incomplete view of demand. In contrast, smart cards provide both more and more detailed information about passenger demand. For example, the Dutch smart card systems records the check-in and check-out times and locations (Van Oort et al., 2015). By analysing this data, public transport operators can not only gain a better understanding from where to where passengers want to travel, but also at what times. Insight into this time-dependent OD data is vital for adapting the transport service to the changing demand.

Despite all the benefits of having a fixed, cyclic timetable, RUs face the need to use their constrained resources more effectively to serve the varying demand. This is already taken into account during the rolling stock planning phase, where train lengths are varied throughout the day to reduce operational costs. Long trains are used in the peak hours when demand is higher, and short trains during the off-peak periods. However, in the Netherlands, the line plan and timetable generally remain fixed and do not adjust to fluctuating demand, which is why this dissertation focusses specifically on these parts. Ideally, a schedule (i.e., line plan and timetable) would combine the benefits of both cyclic and acyclic railway schedules. Therefore, this dissertation introduces the multi-period railway timetable. To create a multi-period timetable, the day is divided into periods with similar demand, and for each period, a line plan and cyclic timetable are developed to match the demand. This scheduling design aims to combine the memorability of a cyclic schedule with the freedom to serve varying demand of an acyclic schedule.

This dissertation is not the first attempt of creating a public transport schedule that mixes cyclic and acyclic properties. For example, the multi-period line planning problem has been considered before by Şahin et al. (2020), Zhao et al. (2022), and Schiewe et al. (2023). However, these works mainly focus on line planning for urban transit (i.e., buses and metros), which have different characteristics from the conventional rail we consider in this thesis. For example, in urban transit it is common that a line stops at every station it passes, while in conventional rail it is common to have lines with different stopping patterns using the same infrastructure. Furthermore, we add to this literature by considering a larger number of line planning choices in our model, including route selection, stop planning, frequency setting, and allowing for varying stopping patterns in both directions of the same line. There is also literature that aims to create a timetable with both cyclic and acyclic characteristics (Robenek et al., 2017; Yin et al., 2019). However, a multi-period timetable that has multiple periods during the day with a separate line plan and cyclic timetable, has not been considered before in the literature. For a more detailed literature review of the different topics discussed in this thesis, we refer to the relevant sections in the corresponding chapters (Chapters 2-5).

While the previous paragraph and the literature reviews in the various chapters show that some literature exists on creating public transport schedules with cyclic and acyclic properties, several challenges remain unaddressed. As we aim to use the same schedule during periods with similar demand, a method is needed to determine appropriate time periods. Furthermore, having multiple plans during the day complicates the planning process. Instead of creating one line plan and one timetable for a single cycle that can be repeated throughout the day, several plans should be created: one for each period with similar demand. Moreover, we need to ensure a good transition between these different schedules. Hence, methods are needed that can create suitable multi-period line plans and timetables. Lastly, as multi-period railway scheduling is a new concept, we need to assess whether the multi-period schedule is better in serving the time-dependent demand than the cyclic timetable that is currently used in practice. In the next section, we provide the research questions used in this dissertation to tackle these challenges.

1.2 Research questions

Based on the challenges identified in the previous section, the main question that this thesis aims to answer is:

How can railway line plans and timetables be designed to effectively serve the varying demand?

To answer the main research question, the following subquestions are addressed. Each of the following chapters will focus on one of these questions.

- RQ1 What patterns and homogeneous periods can be identified in (Dutch) railway passenger demand? *(Chapter 2)*
- RQ2 Which line plan variations could be considered when creating a multi-period line plan to better serve the varying demand? *(Chapter 3)*
- RQ3 How can a multi-period timetable be created for a multi-period line plan? *(Chapter 4)*
- RQ4 How will the introduction of a multi-period timetable affect the railway demand? *(Chapter 5)*

1.3 Thesis contributions

This thesis provides contributions for both the scientific community and society, which are described in Sections 1.3.1 and 1.3.2, respectively.

1.3.1 Scientific contributions

In this thesis, several contributions are made to the scientific literature. Key contributions from each chapter are outlined below.

A hierarchical clustering method for determining periods with homogeneous railway demand.

In order to identify patterns and homogeneous periods in railway demand (RQ1), we provide a hierarchical clustering method in Chapter 2. First, we assess the transferability of an existing bus-demand clustering method to railway data. To better capture the railway demand patterns, we enhance this technique by incorporating both volume and structure of demand. The clustering technique is used in two ways: to find contiguous periods during the day, and to find periods with similar demand throughout the week. The method is tested on realised origin-destination (OD) data from NS.

A multi-period line planning model that includes an extensive set of line planning choices, transfers and asymmetric lines.

Chapter 3 investigates which line plan variations could be included in the multi-period line planning problem (RQ2). The chapter provides a mixed-integer linear programming model for determining a multi-period line plan. This model adds to the existing literature about multi-period line planning, by incorporating a large set of line planning decisions within the model. The proposed model considers selecting routes through the network, determining the stopping patterns and frequencies on these routes, and by considering asymmetric lines. With asymmetric lines, we mean that the stopping pattern and frequency on a route does not have to be the same in both directions. Furthermore, we aim to provide a schedule that minimises the Generalised Journey Time (GJT) of the passengers, including frequency related waiting and transfer times.

A novel model for computing a multi-period timetable for a multi-period line plan.

Chapter 4 answers RQ3 by presenting a novel model for multi-period timetabling. The multi-period timetable takes as input multiple periods during the day with a line plan that best serves the period-specific demand. The timetabling model creates a timetable for the multi-period line plan, when this is possible on the existing infrastructure. The multi-period timetable has a cyclic schedule during each period and ensures a conflict-free transition between schedules. The model's aim is to create a timetable that is attractive and memorable for passengers. It is attractive in the sense that the time waiting for trains and the time spend in vehicles is minimised. Furthermore, the memorability of the schedule is improved by penalising deviations from the new cyclic schedules during the transitions.

An evaluation of the multi-period timetable's impact on passenger demand, and a methodological framework for its design and iterative improvement using evaluation insights.

As the multi-period timetable is a new concept, it is uncertain if it can meet its goal of enhancing passenger services. To answer RQ4, we estimate the effect of the introduction of a multi-period timetable on the railway demand and average GJT in Chapter 5. The considered GJT consists of four components: adaptation time for boarding the first train, in-vehicle time, transfer time required for changing trains, and a fixed penalty for transfers. We compare the GJT of passengers under a completely cyclic schedule with their GJT under the multi-period schedule. Based on the changes in GJT we estimate how demand changes if the multi-period schedule is implemented. A case study based on a segment of the Dutch railway network is used for this analysis. Additionally, we introduce a methodological framework for designing a multi-period timetable, featuring an integrated feedback loop to iteratively enhance both the line plan and timetable using evaluation insights.

1.3.2 Societal relevance

The research presented in this dissertation offers significant contributions to both societal welfare and the operational efficiency of railway undertakings, including:

Benefits for passengers: faster journeys and increased attractiveness of rail travel.

The developed methodologies enable the design of line plans and timetables that reduce the average GJT without incurring additional costs for railway operators. By prioritising service improvements for high-demand origin-destination (OD) pairs, this approach reduces the GJT for the majority of passengers, though at the cost of reducing service for less-used OD pairs. Furthermore, the shorter generalised journey times improve the attractiveness of public transport, potentially encouraging more people to shift from private car use to rail. This modal shift benefits society by lowering the environmental impact of transport, as railways are among the most sustainable transport modes.

Benefits for railway undertakings: improved demand insights and more efficient resource utilisation.

Railway undertakings can use the methodological framework presented in this thesis to design and evaluate multi-period line plans and timetables. This framework includes a novel, data-driven technique to analyse railway demand patterns by dividing the operational day and week into periods with homogeneous demand patterns. These insights help railway undertakings determine during which periods a fixed schedule can be used and at which times it as advantageous to switch to a different schedule. Furthermore, the mathematical models developed support the creation of multi-period line plans and timetables that better align the railway services with fluctuating demand. Case studies demonstrate that such schedules can enhance resource utilisation and improve average generalised journey times while maintaining existing operational costs, resulting in more efficient railway operations.

1.4 Thesis outline

This section provides an overview of the dissertation's outline. A diagram illustrating the structure is presented in Figure 1.3. Chapter 2 focuses on analysing and identifying patterns and homogeneous periods in Dutch railway passenger demand. This analysis provides essential input for the multi-period line planning and timetabling models developed in subsequent chapters, as it provides information about when the same railway schedules can be used. In Chapter 3, the focus shifts to developing a multi-period line plan. The aim here is to create a line plan that effectively accommodates the varying passenger demand patterns identified in Chapter 2. Chapter 4 builds on this, by developing a multi-period timetable for the multi-period line plan devised in Chapter 3. Chapter 5 then examines the potential impacts of implementing this multi-period timetable on railway demand. This chapter provides critical insight into how the adjusted line plan and timetable influence the passengers' GJT and overall demand, highlighting the dynamic relationship between scheduling and demand. Furthermore, this chapter explores how these insights can be used to improve the line plan and/or timetable. The dissertation concludes with Chapter 6, which provides answers to the research questions outlined in Section 1.2 and also discusses the broader implications of the research findings.

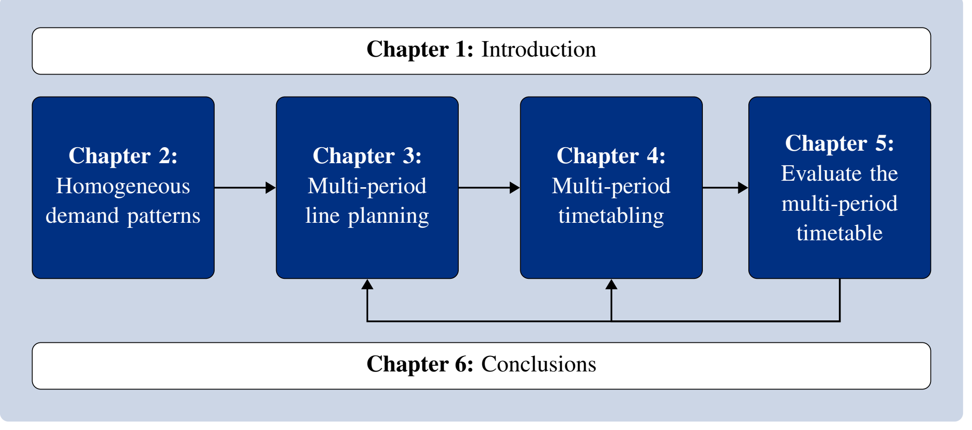


Figure 1.3: Overview of thesis structure

Chapter 2

Clustering railway passenger demand patterns from large-scale origin-destination data

In Chapter 1, the fluctuations in passenger demand for railway services throughout the day, week, and year were illustrated. In order to let train services, such as the line plan and timetable, match this fluctuating demand, it is essential to understand how the demand changes and identify periods during which the demand is relatively stable. This chapter uses hierarchical clustering on origin-destination (OD) data to determine continuous time-of-day periods on workdays during which the demand is homogeneous. The periods found for each workday are subsequently used as input in a clustering algorithm to look for similarities and differences between workdays. The methods for finding homogeneous periods during the day and week are applied to a case study covering a large part of the railway network in the Netherlands. This chapter will therefore provide insights that can be used in the creation of multi-period line plans (Chapter 3) and timetables (Chapter 4).

Apart from minor changes, this chapter is published as a journal article:

van der Knaap, R. J. H., de Bruyn, M., van Oort, N., Huisman, D. & Goverde, R. M. P. (2024). Clustering railway passenger demand patterns from large-scale origin-destination data. *Journal of Rail Transport Planning & Management*, 31, 100452.

2.1 Introduction

It is well known that the passenger demand for railway services fluctuates with the time of day. During the peak hours there is typically a high demand for transport, while in the middle of the day or in the evening the demand is much lower. The demand also fluctuates over the week: not all (week)days have the same demand. To illustrate these fluctuations, Figure 2.1 shows the number of arriving (in red) and departing (in blue) passengers and the total number of passengers (in black) per time of day for a station in the Dutch railway network. The figure displays the arrival and departure data of two days: an average Tuesday in 2019 (denoted by solid lines) and an average Friday in 2019 (denoted by dashed lines). Figure 2.1 shows the pattern of a typical commuter station with a lot of jobs in the vicinity: many people arriving during the morning peak period and leaving during the evening peak period and not so many people arriving and/or departing outside the peak hours. We can also observe in Figure 2.1 that the demand is not the same for each day of the week. The demand at the peak periods on Friday is much lower than the demand on Tuesday, while the demand is similar to Tuesday outside the peak hours. Due to the Covid-19 pandemic, these differences between days are expected to increase. Several studies in the Netherlands show that more people will partly work from home after the pandemic, with a preference for Friday and Wednesday (see e.g. Kennisinstituut voor Mobiliteitsbeleid et al. (2021), Van Hagen et al. (2021), Ton et al. (2022)). Although this may cause the difference between peak and off-peak periods to decrease, it will increase the difference between days.

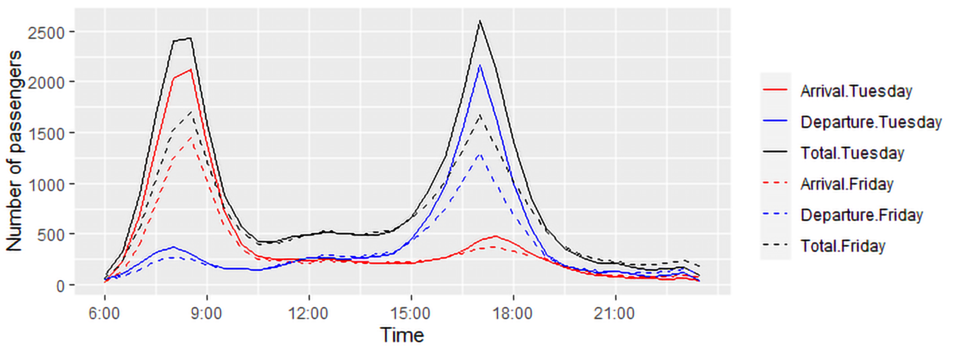


Figure 2.1: The figure displays the number of arriving (red), departing (blue), and total (black) passengers at a station in the Dutch railway network. The data of two days are given: Tuesday (solid lines) and Friday (dashed lines).

Nevertheless, many European countries including the Netherlands have fixed line plans and cyclic railway timetables. These timetables provide regular interval services throughout the day, including fixed departure times (e.g., 10 minutes past every hour) and sufficient time to transfer at stations where different services meet. Therefore, these timetables are easy to remember for passengers. Another benefit is that it is easier for people to travel outside the peak hours, since the service frequency is the same throughout the day. Wardman et al. (2004) and Johnson et al. (2006) confirm these aspects are beneficial for the customers, by showing that regularity of a timetable increases the passenger demand. One of the disadvantages however is the fact that the train lines and cyclic timetables are usually based on the peak hour demand, and are therefore not tailored for other periods outside the peak hours. This is both in terms of volume, and in terms of the structure of the demand (i.e., where passengers are coming from and/or going to). People who travel outside the peak hours are likely to have different travel purposes and hence destinations. This leads to heterogeneous travel demand patterns, which does not align well with the fixed cyclic service supply.

Better matching the train services (line plan and timetable) to the travel demand can have multiple benefits for both the passengers and the railway undertaking (RU). Passengers will have a better travel experience with faster trips and fewer transfers. Furthermore, the RU can save money on energy, rolling stock and personnel by only operating those trains for which there is demand. Lastly, when train services better match the demand, the train becomes a more attractive mode of transport. This would result in more people taking the train and higher ticket sale revenues for the RU.

Since both regular and irregular schedules have their benefits, it might be useful to have a schedule with both regular and irregular characteristics. One example of this could be to have multiple periods during a day, where there is a regular schedule within each period, and changes are made to the schedule when switching between periods to better match the demand in the next period. However, having multiple periods during a day also has some costs. For example, the RU who is used to having a regular schedule during the entire day would have to make several new schedules: one for each period. This is a challenging task since both line planning and timetabling problems are difficult to solve. Schöbel (2012) shows that even some special cases of line planning are NP hard. Furthermore, the Periodic Event Scheduling Problem, which is widely used for modelling cyclic railway timetabling is shown to be NP complete by Serafini & Ukovich (1989). Moreover, the literature review paper by Durán-Micco & Vansteenwegen (2022) about the line planning problem concludes that there exists a large gap between the problems addressed in the literature and the extremely complex problems that have to be solved in practice. In practice, the process of making a line plan and timetable is not fully automated yet, but still requires a large amount of manual labour. Therefore, there is a limit to the number of different schedules that the RU can create and hence operate in a day. Besides the cost for the RU, there is also a cost for the passenger, because they have to adjust to new schedules during the day. Therefore, the RU should only make changes to the schedule if significant improvements for the passenger and/or the RU can be realised.

Due to the reasons mentioned above, first a good insight is needed in how the demand patterns change throughout the day and week, before changes are made to the schedule. To aid the RU in finding periods for which a different schedule might be interesting, this chapter proposes a data-driven method to derive periods during the day and week in which the railway demand is homogeneous. We define periods that are homogeneous in demand as periods in which the travel patterns in terms of origin-destination flows are more or less the same. The idea behind this is that in periods of homogeneous demand and operational conditions, the public transport schedules can also be the same in terms of e.g., routes operated, stops, and frequencies. By determining the periods with homogeneous demand during the day and week, we can therefore put a maximum on the amount of periods an RU needs to consider for providing a different schedule to.

Since the emergence of Automatic Fare Collection (AFC) systems, collecting data about passenger trips has become much easier for the public transport agencies and researchers. Several papers analyse such data to describe periods of homogeneous demand or passenger groups with similar demand characteristics. Data mining techniques (like *k*-means, DBSCAN, and hierarchical clustering) are often used to find homogeneity in AFC data. The first ones to do this are Agard et al. (2006), who analyse user behaviours and determine different market segments among bus users. Other papers that use data mining techniques to get a better understanding of public transport passengers include Ma et al. (2013), Kieu et al. (2015), El Mahrsi et al. (2017), and Deschaintres et al. (2019). They suggest that these insights can be used by public transport operators to provide better pricing options and/or information to passengers. Furthermore, data mining techniques are also used on AFC data to investigate the change in passenger behaviour over time (Briand et al., 2017), the effect of long-term service disruptions (Eltved et al., 2021), and the effect of Covid-19 on public transport usage (Mützel & Scheiner, 2022; Henrion et al., 2023).

Besides the aforementioned goals, there are multiple papers that want to determine periods with homogeneous operational performance and/or demand, with the aim to improve the bus schedule. For example, Muller & Furth (2001) use the data from Trip Time Analysers on operational performance of buses, to determine periods in which the trip time is homogeneous. The trip time is defined as the time spent from the first to the last stop on the bus route. These homogeneous periods with corresponding trip times can then be used to improve the bus schedule, in order to minimise early and late arrivals. Lu & Reddy (2012) use hierarchical cluster analysis on the total hourly bus trip volumes to determine whether different days should have different bus schedules. Mahmoudzadeh & Wang (2020) also investigate the effects of days on travel patterns for a university campus bus shuttle service. Clustering based on the aggregate demand and aggregate delay is used to group days with similar travel patterns together and to find homogeneous periods of demand during the day. The knowledge gained is used for improving the shuttle schedule. Ji et al. (2011) take a different approach. They use hierarchical clustering on probability flow matrices to find contiguous periods during a normal workday in which the travel patterns are homogeneous. Probability flow matrices are OD matrices in which every cell is divided by the total number of trips in the matrix. Hence, the cells in a probability flow matrix sum to 1 and the value of a cell denotes the probability that a passenger travels from the origin to the destination corresponding to that cell. Ji et al. (2011) conclude that clustering probability flow matrices provides a better result than clustering that only looks at the total trip volume. However, De Bruyn & Mestrum (2021), who look at OD matrices of train demand, argue that both the volume and the structure of the demand are important when determining the similarity of demand in different periods. They introduce two measures that denote the similarity of two OD matrices in structure and volume, respectively.

Although there are several papers on finding homogeneous demand patterns in bus services, the literature on determining periods with homogeneous train demand is limited. To the best of our knowledge, the paper by De Bruyn & Mestrum (2021) is the only paper about finding periods with homogeneous demand patterns in the rail sector. Therefore, testing the performance of the current methods on train demand data is an important addition to the academic literature. Furthermore, we believe that both the structure and the volume of the demand are relevant when constructing the line plan and the timetable of a railway service. Although the demand structure plays an important role in constructing the train lines, the volume is also a crucial aspect in determining the frequencies of the lines. Therefore, it seems reasonable to take both the structure and the volume of demand into account when determining periods of homogeneous demand, as De Bruyn & Mestrum (2021) propose. However, the metrics that they propose are not directly suitable for constructing periods with homogeneous demand and the literature on bus demand only considers either the trip volumes or the structure of the demand (in the form of probability flow matrices). Therefore, we propose a new method using hierarchical clustering that takes into account both changes in structure and volume.

To summarise, the main contributions of this chapter are:

1. Providing a new method for finding periods of homogeneous demand, where both the structure and the volume of the demand are taken into account;
2. Applying the methods for finding periods of homogeneous demand based on the structure of demand and based on the structure and the volume of demand on a case study with railway passenger demand.

The remainder of this chapter is organised as follows. In Section 2.2 the methodology of the analysis is described. Next, Section 2.3 describes the case study data and Section 2.4 provides the results of applying the methods on this case. We conclude the chapter with some conclusions and recommendations for future research in Section 2.5.

2.2 Methodology

This section provides the methods used to derive homogeneous periods in railway travel demand during the day and week. First we describe the method employed to find homogeneous periods in demand during the day in Section 2.2.1. Next, in Section 2.2.2 we describe how this method is adapted to look for homogeneity during the week.

2.2.1 Homogeneous periods during the day

We propose to use clustering of OD matrices to determine periods of homogeneous demand. Clustering methods can be partitioned in two main groups: hierarchical approaches and partitional approaches (Jain et al., 1999). Partitional approaches (like *k*-means) produce only one partition of the data, while hierarchical methods produce a nested series of partitions. Since beforehand the optimal number of periods to divide a day in is not known, a hierarchical clustering approach seems to be the most fitting. Hierarchical clustering approaches can be agglomerative or divisive. In agglomerative approaches, all objects that need to be clustered start in their own cluster. In each iteration of the clustering algorithm, the two clusters which are closest to each other are merged, until only a single cluster is left. With divisive clustering all objects start in a single cluster and in each step of the algorithm a cluster is split into two parts. We refer the interested reader to Han et al. (2011) for an overview of data mining techniques including clustering.

Another important decision in hierarchical clustering algorithms is how the distance between clusters is defined, which is also known as the linkage measure. There are three widely used linkage measures: single linkage, complete linkage and average linkage. Each cluster contains one or more objects (which in this case are OD matrices). To determine the distance between two clusters, each of the linkage measures looks at the distance between each pair of objects such that the two objects are not from the same cluster. The linkage types differ in the way they use these distances to determine the distance between two clusters. In single linkage, the distance between two clusters is defined as the smallest distance between any pair of objects. When single linkage is used, this is sometimes referred to as nearest-neighbour clustering. Contrarily, in complete linkage, the distance between two clusters is defined as the largest distance between any pair of objects. Farthest-neighbour clustering is sometimes used to refer to clustering using this measure. Lastly, average linkage defines the distance between two clusters as the average distance between all object pairs.

In this chapter, we will use an agglomerative hierarchical clustering approach using complete linkage. This clustering method is already successfully applied in the work of Ji et al. (2011) to find passenger demand patterns in bus OD data. Complete linkage is chosen as it produces more compact clusters and since it has been found to be more effective across many applications (Jain et al., 1999). Furthermore, just like Ji et al. (2011), we will search for contiguous periods. The aim of this chapter is to find periods of homogeneous demand, for which a service plan (which includes a line plan and timetable) can be designed. Since switching between service plans is likely to be difficult, we add a constraint that periods must be contiguous, to reduce the potential number of switches during the day. Figure 2.2 provides a flowchart that describes the process that is used to find homogeneous periods during the day. All the terms in this figure will be explained in the remainder of this section.

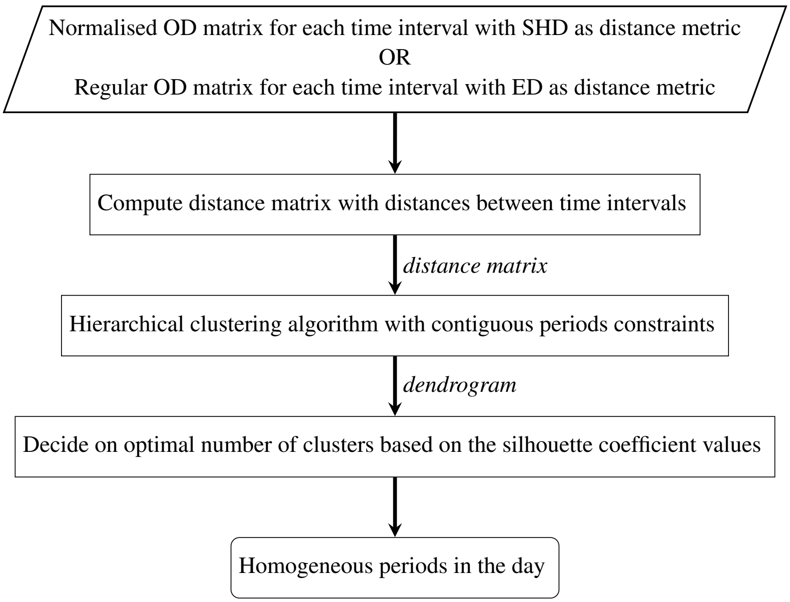


Figure 2.2: Flowchart describing the proposed process for finding homogeneous periods during the day.

In short, the hierarchical contiguous clustering algorithm works as follows. Suppose we want to cluster N objects, so in our case N OD matrices. The first step is to calculate an $N \times N$ distance matrix containing the distances between each pair of OD matrices. The distance between two OD matrices can also be seen as the degree of dissimilarity: if the distance is small, then the matrices are similar, and if the distance is large, the matrices are dissimilar. Once the distance matrix is calculated, the algorithm starts with N number of clusters (each containing one object). Then in each iteration of the algorithm, we determine which pairs of clusters are contiguous and find which of these pairs have the smallest distance to each other. This pair is then merged into a new cluster and the distance matrix is updated. This process is repeated until all clusters are combined into a single cluster. The result of the clustering algorithm can be visualised in a dendrogram. For a full description of the clustering algorithm, we refer to Ji et al. (2011).

In this chapter, we use two different methods to determine the distance matrix required as input for the clustering algorithm: one method using normalised OD matrices and another method using regular OD matrices. These methods are described in the next two paragraphs, respectively.

The first method using normalised OD matrices is based on the work of Ji et al. (2011) and McCord et al. (2012). Normalised OD matrices are also known as probability flow matrices. A regular OD matrix can be converted to a normalised OD matrix by dividing each cell of the OD matrix by the total sum of the matrix's cells. Let A_t^d be an OD matrix corresponding to day d and time t. Then the normalised OD matrix $\bar{A}_t^d$ can be calculated as:

$$\bar{A}_t^d(i,j) = \frac{A_t^d(i,j)}{\sum_{i=1}^n \sum_{j=1}^n A_t^d(i,j)}. \tag{2.1}$$

Here $A_t^d(i,j)$ ($\bar{A}_t^d(i,j)$) denotes the value in the cell of matrix A_t^d ($\bar{A}_t^d$) corresponding to the i^{th} row and the j^{th} column. Since each cell in an OD matrix denotes the number of passengers travelling from an origin to a destination, the value in each cell must be greater or equal to zero. The transformation to normalised OD matrices by Equation (2.1) preserves this non-negativity property. Furthermore, the total sum of all cells in a normalised OD matrix is equal to 1. Therefore, a normalised OD matrix can be seen as a discrete probability distribution. Hence, a similarity measure for probability distributions can be used to determine the distance between two normalised OD matrices. In this chapter the squared Hellinger distance measure (SHD) is used. SHD is chosen as distance measure, since it is a suitable measure for comparing probability distributions (Le Cam & Yang, 2000), and it has been used before to compare normalised OD matrices (Ji et al., 2011; McCord et al., 2012). The squared Hellinger distance between two $n \times n$ normalised OD matrices $\bar{A}_t^d$ and $\bar{A}_{t'}^d$, denoted by $\mathrm{H}(\bar{A}_t^d, \bar{A}_{t'}^d)$, is defined as:

$$\mathrm{H}(\bar{A}_t^d, \bar{A}_{t'}^d) = \sum_{i=1}^n \sum_{j=1}^n \left(\sqrt{\bar{A}_t^d(i,j)} - \sqrt{\bar{A}_{t'}^d(i,j)} \right)^2. \tag{2.2}$$

When the distance matrix is based on normalised OD matrices, only the structure of the demand is taken into account. The volume is cancelled out completely, since the OD matrices are divided by their total volume to create the normalised OD matrices. However, as mentioned before the passenger volumes also play an important role in determining what service is required. Therefore, we propose a new method to determine the distance matrix based on the original OD matrices, since these matrices include both the structure and the volume of the demand. As regular OD matrices usually do not meet the requirements of a probability density function, the squared Hellinger distance cannot be used in this case. Instead, we propose to use the well-known Euclidean distance (ED) as the distance metric. Given two non-normalised $n \times n$ OD matrices A_t^d and $A_{t'}^d$, the ED between the two matrices, denoted by $\mathrm{E}(A_t^d, A_{t'}^d)$ is calculated as:

$$\mathrm{E}(A_t^d, A_{t'}^d) = \sqrt{\sum_{i=1}^n \sum_{j=1}^n \left(A_t^d(i,j) - A_{t'}^d(i,j) \right)^2}. \tag{2.3}$$

The hierarchical clustering algorithm produces in each iteration a clustering result. Hence, if N objects are being clustered, the algorithm provides N different ways to cluster these objects (including all objects form a single cluster and each object forms its own cluster). To determine which of these results is the best, the silhouette coefficient is used. The silhouette coefficient was introduced by Rousseeuw (1987) as a measure for the quality of a clustering result. The result with the highest measured quality can then be selected as the final result.

We use the following example to explain the calculation of the silhouette coefficient. Let o be an object (e.g., an OD matrix) for which we calculate the silhouette coefficient and let O be the set of all objects. In the clustering result we want to assess, the objects from O are divided over k clusters $C_1,\ldots,C_k$. Furthermore, let object o belong to cluster C_i $(o \in C_i)$ with $1 \leq i \leq k$ and let $|C_i|$ denote the number of objects in cluster C_i. The equation given by Rousseeuw (1987) to calculate the silhouette coefficient of object o (denoted by $s(o)$) is

$$s(o) = \frac{b(o) - a(o)}{\max\{a(o), b(o)\}}. \tag{2.4}$$

The $a(o)$ in Equation (2.4) denotes the average distance from o to other objects within cluster C_i and hence is a measure of the cluster's compactness. Let $o, o' \in O$ and let $\text{dist}(o, o')$ denote the distance between objects o and o'. Then $a(o)$ is calculated as:

$$a(o) = \frac{\sum_{o' \in C_i, o \neq o'} \text{dist}(o, o')}{|C_i| - 1}. \tag{2.5}$$

The distance is measured using either the SHD or the ED, depending on which input is used: normalised OD matrices or regular OD matrices. On the other hand $b(o)$ in Equation (2.4) denotes the average distance from o to the objects in the nearest other cluster. This serves as a measure of closeness from object $o \in C_i$ to the k-1 other clusters and is calculated as:

$$b(o) = \min_{C_j : 1 \leq j \leq k, j \neq i} \left\{ \frac{\sum_{o' \in C_j} \text{dist}(o, o')}{|C_j|} \right\}. \tag{2.6}$$

The silhouette coefficient can take any value between -1 and 1. High values are desirable, since this denotes that an object is very close to other objects in its cluster and very far from objects in other clusters. On the other hand are negative values undesirable, since this denotes that an object is closer to the objects in another cluster than to the objects in its own cluster. The silhouette coefficient for an entire clustering result can be determined by computing the silhouette coefficient for each object and taking the average of these values. Let $|O|$ denote the number of objects that are clustered, then the silhouette coefficient for an entire clustering result (denoted by SC) is defined as Equation (2.7):

$$\text{SC} = \frac{1}{|O|} \sum_{o \in O} s(o). \tag{2.7}$$

As higher values of the silhouette coefficient are more desirable, one way to choose the appropriate number of clusters is by selecting the result which has the highest average silhouette coefficient over all objects (Rousseeuw, 1987).

2.2.2 Homogeneous periods during the week

With the methods described in Section 2.2.1, we can determine for each day separately a certain number of contiguous homogeneous periods (based on the values of the silhouette coefficient).

However, (parts of) certain days of the week might be very similar in terms of demand patterns. For example, the morning peak period on Tuesday might be similar to the morning peak on Thursday, because many people work at the office on those days. Therefore, in this section we describe the method used to compare the clusters found with the method described in Section 2.2.1. In Figure 2.3, a flowchart is given that displays the proposed process for finding homogeneous periods during the week. The remainder of this section will describe this process as well.

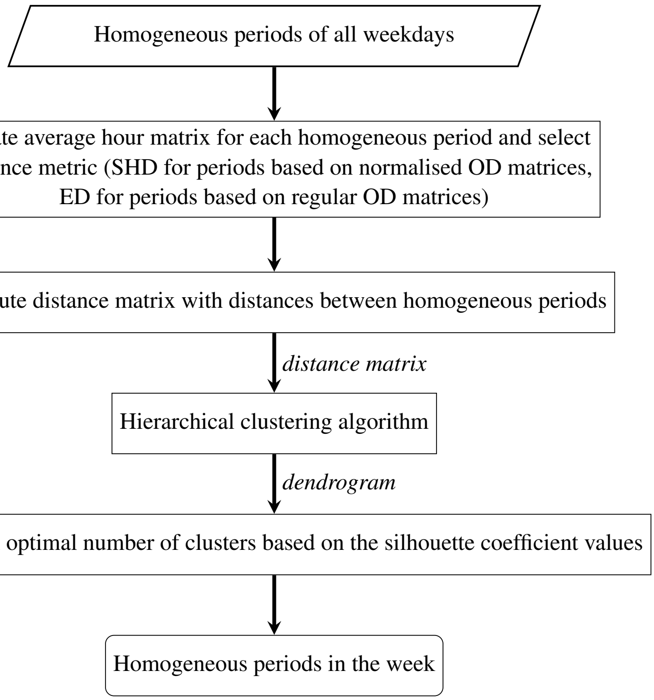


Figure 2.3: Flowchart describing the proposed process for finding homogeneous periods during the week.

The clustering algorithm used for determining homogeneous periods during the week is the same as the one described in Section 2.2.1, with one exception: the constraint to find only contiguous periods is dropped. Hence, in each iteration of the clustering algorithm any pair of clusters is eligible to be merged into a new cluster, not only the clusters that are contiguous in time. At this stage, we are looking for homogeneity during the week to determine if for example the service plan for a morning period on Tuesday can also be used for a morning period on Thursday. This cannot be found if the contiguity constraint would be maintained, therefore it is dropped.

The input for the algorithm is also slightly different compared to determining periods of homogeneous demand during the day. To keep the size of the clustering instance to an acceptable level, we construct an OD matrix for each of the determined periods. Each period consists of one or more subperiods which have their own OD matrices. The OD matrices of these subperiods are used as input in Section 2.2.1. An OD matrix for a larger period is established by adding up the OD matrices of all its subperiods and then dividing each cell by the total number of hours encompassed by those subperiods. This division is necessary to be able to compare periods consisting of a different number of subperiods more fairly.

The distance metrics used to create the distance matrix are the same as before. For the cluster results based on regular OD matrices, a distance matrix is created using the Euclidean distance. For the cluster results based on normalised OD matrices, the periods' OD matrices are transformed to normalised OD matrices using Equation (2.1) and a distance matrix is created using the squared Hellinger distance.

2.3 Case study data

The hierarchical clustering methods described in Section 2.2 require multiple time-dependent OD matrices as input. The time-span that is covered in an OD matrix can vary, but should be sufficiently small to get the desired level of detail, e.g., per hour or per half hour. Public transport operators that use smart card automatic fare collection systems, usually possess the data to create such time-dependent OD matrices (Pelletier et al., 2011), since smart cards often register time and location of check-in and/or check-out. This is also the case at the Dutch railway undertaking NS, which provides the data for the case study in this chapter. NS is the principal passenger railway undertaking in the Netherlands and serves 253 Dutch stations. Every working day in 2019, NS facilitated over 1.3 million passenger journeys (Nederlandse Spoorwegen, 2020). In 2019, NS operated essentially a cyclic timetable with a cycle time of 30 minutes. Adjusting the train services to the changing demand is mainly done by varying the length of the trains. Besides that, extra peak train lines are operated at a small part of the network and in the evening hours (after 22:00) the frequency of the service is gradually reduced. Given the great regularity of the schedule and the limited amount, size and frequency of the peak lines, the schedule is not expected to hinder the interpretation of the clustering result.

At NS, the main way in which the travel fare is calculated is via a check-in and check-out system using a smart card. Data recorded by this smart card includes the origin and destination station of the trip, and the check-in and check-out times. For a full description of the data recorded by the smart card, we refer to Van Oort et al. (2015). Since the check-in and check-out times and locations are recorded, the data can be easily translated into a time-dependent OD matrix. Besides using a (disposable) smart card, people can also order a ticket online. With these tickets, the origin, destination and travel date of the trip are known, but the exact travel time is not. The e-tickets are allocated over the day based on the distribution of trips made with similar smart-card products. In principle, ticket prices at NS are solely based on the distance travelled and not on time of day or the location in the network. However, to encourage people to travel outside the peak hours, NS has subscriptions that offer a discount on the trip price in the off-peak hours for a monthly fee. To get this discounted price, passengers need to check in outside of the defined peak hours: 6:30–9:00 and 16:00–18:30.

The data that is available to this research is the OD data in 2019, aggregated to half hour periods. A fictitious sample of the provided OD data is given in Table 2.1. The dataset contains for each half hour of each of the 365 days in 2019 how many trips were made between every possible OD pair. A possible OD pair is any combination of the 253 stations served by NS, which gives $253 * 252 = 63{,}756$ possible OD pairs. The check-in time determines in which half hour a trip is recorded. Hence, from the second row in Table 2.1 we conclude that on April 18, 2019, there are 53 trips from station 34 to station 196 that started between 15:30 and 15:59. The data from Table 2.1 can be used to fill OD matrices. As we have OD data for every day of the 365 days in 2019 and for every of the 48 half hours of those days, we can create $365 * 48 = 17{,}520$ OD matrices. For example, the data in the 2nd row of Table 2.1 belongs to OD matrix $A^{18.04.2019}_{15:30}$, and $A^{18.04.2019}_{15:30}(34,196) = 53$.

Table 2.1: Sample of fictitious OD data

Date (d)	**Start time of 30 minute period (t)**	**Check-In Station (i)**	**Check-Out Station (j)**	**Number of trips ($A^d_t(i,j)$)**
01/01/2019	00:00	1	2	1
18/04/2019	15:30	34	196	53
31/12/2019	23:30	253	252	0

The data of 2019 is used to exclude any effects of the Covid-19 pandemic. Furthermore, as we are interested in the passenger travel patterns during a normal workday, several days are excluded from the dataset, such as weekend days, school holidays, public holidays, and several (un)scheduled withdrawals from service as reported in NS' Annual Report of 2019 (Nederlandse Spoorwegen, 2020). Table 2.2 provides an overview of the number of days removed and the number of remaining days after each type of non-normal day is removed from the data.

Table 2.2: Data selection process

	Number of days removed in step	**Number of remaining days**
		365
No weekend days	104	261
No school holidays	86	175
No public holidays	4	171
No (un)scheduled major service withdrawals	17	**154**

After removing all these outlier dates, there are 154 days left. Let D denote the set of all dates left in the dataset. Since we also want to compare the demand patterns of different workdays, we split up set D into five sets based on the day of the week. Let these subsets of D be denoted by D^{Mo}, D^{Tu}, D^{We}, D^{Th}, D^{Fr}, for Monday until Friday, respectively. As we have the data of 154 days, distributed over five workdays, we have data of approximately 31 days per workday. Furthermore, we only focus on the period between 6:00 and 23:59, since this is the period in which most of the journeys occur. Let T denote the set of all half hour periods between 6:00 and 23:59. As we are interested in the travel patterns at ordinary workdays, we create a median OD matrix, by taking for each workday and for each half hour the median number of trips between each OD pair. So for every half hour period $t \in T$ and for every workday $x \in \{\text{Mo},\text{Tu},\text{We},\text{Th},\text{Fr}\}$ we create an OD matrix. The entry of the OD matrix for day x for period t for the trip between origin station i and destination station j is calculated as:

$$A^x_t(i,j) = \text{med}_{d \in D^x}\{A^d_t(i,j)\}, \tag{2.8}$$

where $\text{med}\{\cdot\}$ denotes the calculation of the median value. The median is chosen instead of the average, to reduce the effect of outliers. With 253 stations in the network, there are many OD pairs that are used only sporadically. By creating median OD matrices, instead of average OD matrices, the number of trips between these origins and destinations will be equal to zero instead of a very small positive number. As usually these OD trips are not made, the (median) value of zero seems the best choice. After an OD matrix is created for each day $x \in \{\text{Mo, Tu, We, Th, Fr}\}$ and each time period $t \in T$, the normalised OD matrices $\bar{A}_t^x$ can be created using Equation (2.1).

Note that although median OD matrices are used in this case study, the method presented works for any type of OD matrix. For example, if an operator feels that the data of one particular week is representative enough for the demand, then just the realised OD matrices of that week can be used as input. Alternatively, if an operator wants to know what demand patterns can be found in the forecasted demand, then an OD matrix with forecasted demand can be used as input. In this case study, we want to find demand patterns during regular workdays in 2019. Therefore, a median OD is selected as the best way to represent this regular demand.

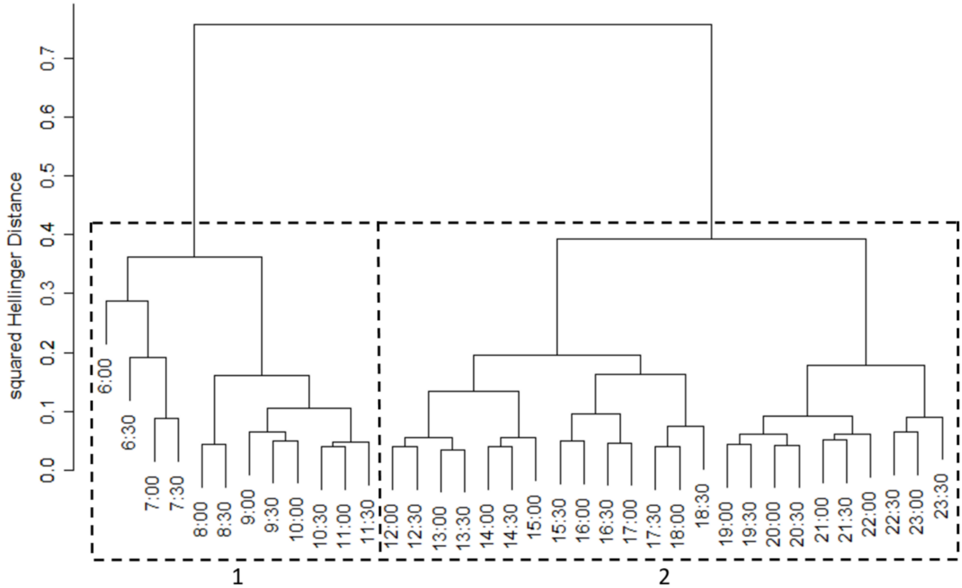
2.4 Results

This section presents the results of applying the methods described in Section 2.2 to the data described in Section 2.3 in order to find periods that are homogeneous in demand. The homogeneous demand periods within the different workdays are presented in Section 2.4.1. Next, Section 2.4.2 presents how homogeneous the demand is during the week. Lastly, in Section 2.4.3 the differences between the identified periods are illustrated by looking at some demand characteristics in the different periods.

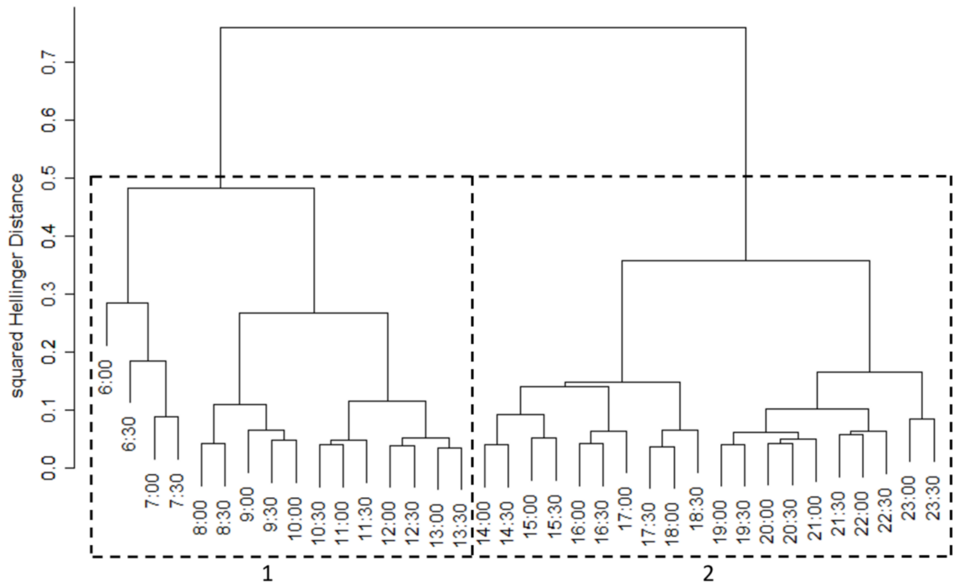
2.4.1 Homogeneous periods during the day

The results of the hierarchical clustering method can be visualised in a dendrogram plot, which shows the grouping of the (normalised) OD matrices. The dendrograms of the normalised and regular OD matrices of the five median workdays in 2019 are given in Figures 2.4 and 2.5, respectively. The dendrograms should be read as follows. The horizontal axis displays the time, where for example 6:00 denotes the OD matrix containing all trips that start between 6:00 and 6:29. All matrices start in separate clusters (as can be seen at the bottom of the dendrogram) and in every step two matrices are clustered together until all matrices are in a single cluster (as can be seen at the top of the dendrogram). The merging of two clusters is denoted in the dendrogram by a horizontal line connecting the two clusters. Note that when looking at homogeneous periods during the day, only two clusters that are adjacent in time are allowed to be combined into a new cluster. The vertical axis denotes the distance between two clusters at the moment they are combined. For each day, the cluster result with the highest silhouette coefficient is chosen as the result with the optimal number of clusters. This result is visualised in the dendrogram using dashed lines and each of these clusters is numbered for easier reference in the text. For example, for the Monday (see Figure 2.4a) the result with two clusters received the highest silhouette coefficient (namely 0.54), which resulted in the clusters 6:00 until 11:59 and 12:00 until 23:59.

When looking at the clustering results of the normalised OD matrices (see Figure 2.4), the results with the highest silhouette coefficient are the results that divide each workday into two parts: the morning hours on the one side (cluster 1) and the afternoon and evening hours on the other side (cluster 2). The transition from morning to afternoon is usually at 12:00, but is later on Tuesday (at 14:00) and earlier on Wednesday (at 10:30). A possible explanation for this division between morning and afternoon/evening is that people usually go somewhere in the morning

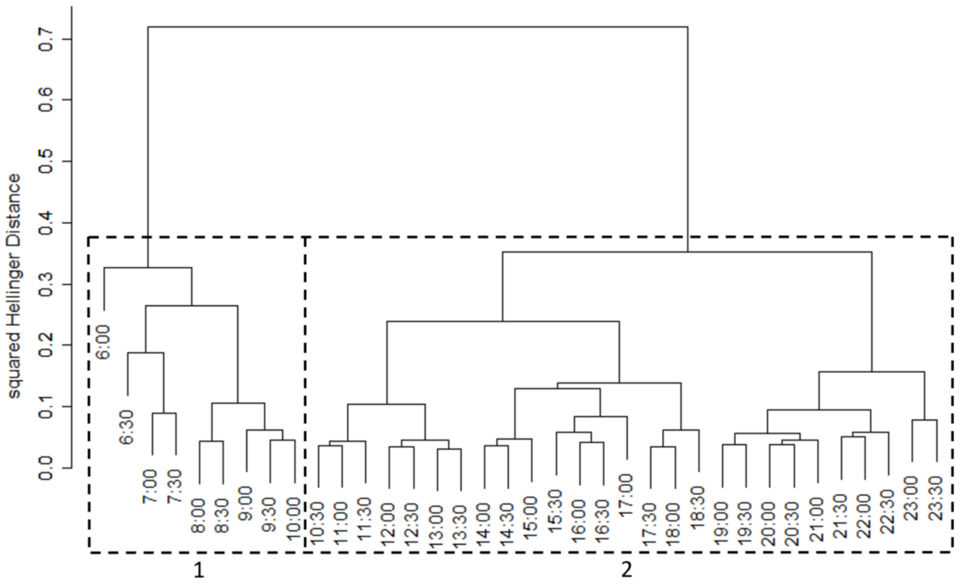


(a) Clustering results Monday - SC = 0.54.

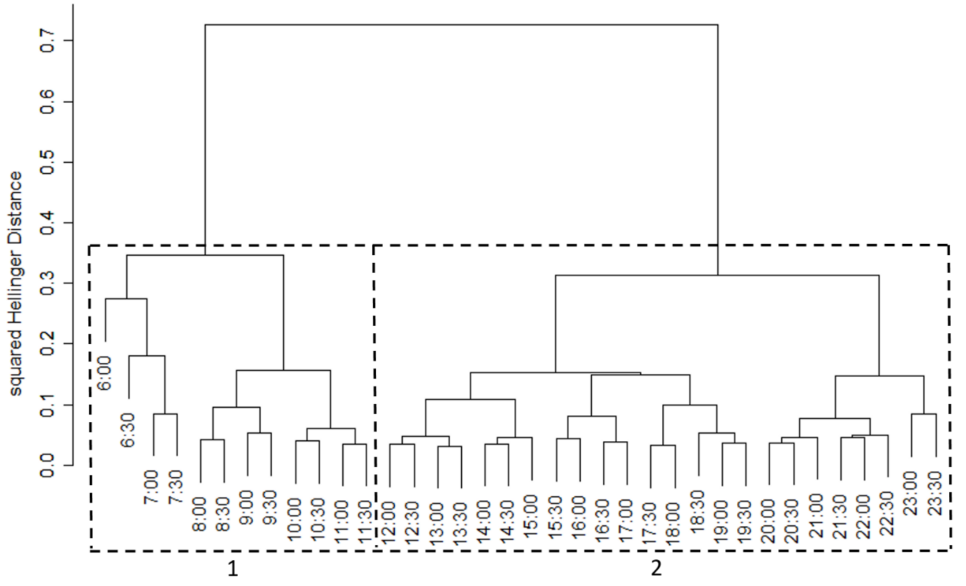


(b) Clustering results Tuesday - SC = 0.46.

Figure 2.4: Clustering results using squared Hellinger distance on normalised OD matrices. Dashed lines denote the optimal number of clusters, according to the silhouette coefficient (SC).

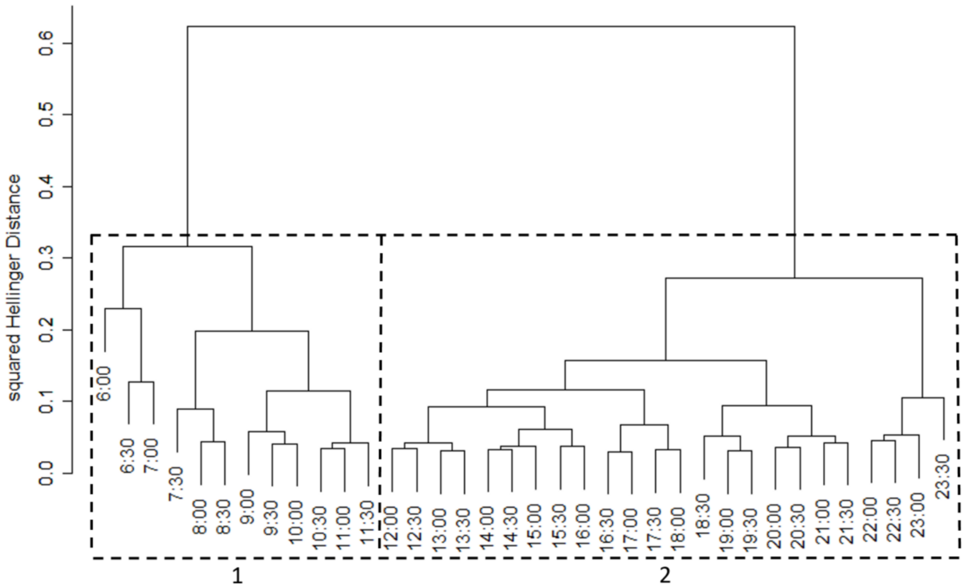


(c) Clustering results Wednesday - SC = 0.55.



(d) Clustering results Thursday - SC = 0.56.

Figure 2.4: (continued)



(e) Clustering results Friday - SC = 0.53.

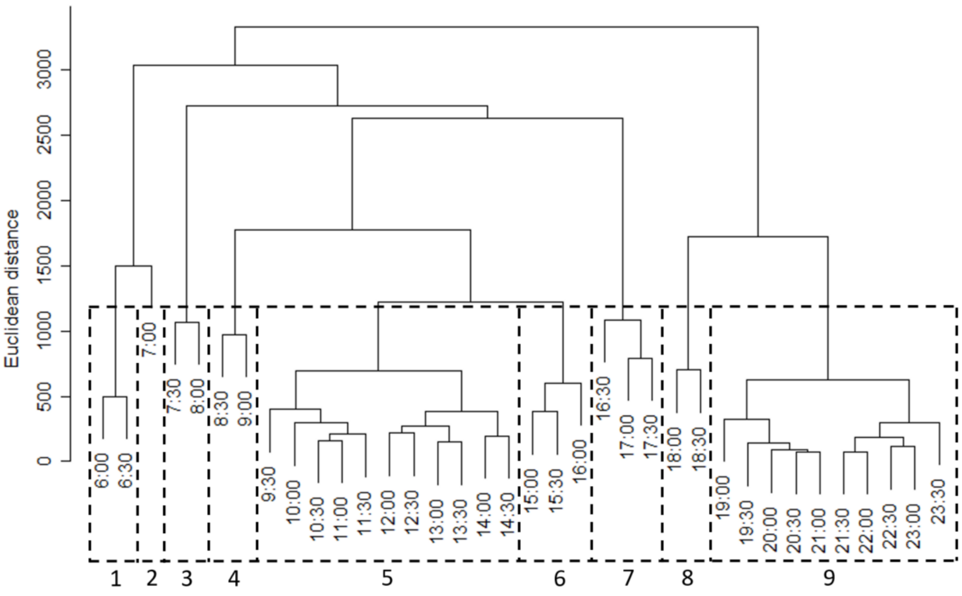
Figure 2.4: (continued)

and return to where they came from in the afternoon or evening of the same day, which is also clearly visible in Figure 2.1. To check the effect of this phenomenon on the clustering result, we compare the distance between the morning OD matrix and the afternoon/evening OD matrix to the distance between the morning OD matrix and the transpose of the afternoon/evening OD matrix. The results for this analysis are given in Table 2.3. The columns from this table show from left to right: the day, the squared Hellinger distance (SHD) between the morning and the afternoon/evening matrix, the SHD between the morning and the transposed afternoon/evening matrix, and the shortest distance from one of the day's matrices to one of the other days' matrices. This last column also denotes between brackets which matrices are the closest together. Here the first two letters denote the day (e.g., Mo stands for Monday) and the last letter denotes whether it is the morning matrix (.M) or the afternoon/evening matrix (.A). So for example for Monday, the distance between the morning and the afternoon/evening matrix is 0.2517, while the distance from the morning to the transposed afternoon/evening matrix is 0.0238. Furthermore, the shortest distance from one of the Monday matrices to another day's matrix is 0.0067, which is the distance between the Monday afternoon/evening matrix (Mo.A) and the Thursday afternoon/evening matrix (Th.A). The results show that the morning matrix is between 5 and 15 times closer to the transposed matrix than to the normal OD matrix of the afternoon/evening. Hence, it seems reasonable that the cluster result at least partly reflects this travel phenomenon. However, it is also clear that the "back and fort"-effect is not the only effect happening during the day. For each day the distance to another day's morning or afternoon/evening matrix is still a lot smaller than to its own transposed afternoon/evening matrix.

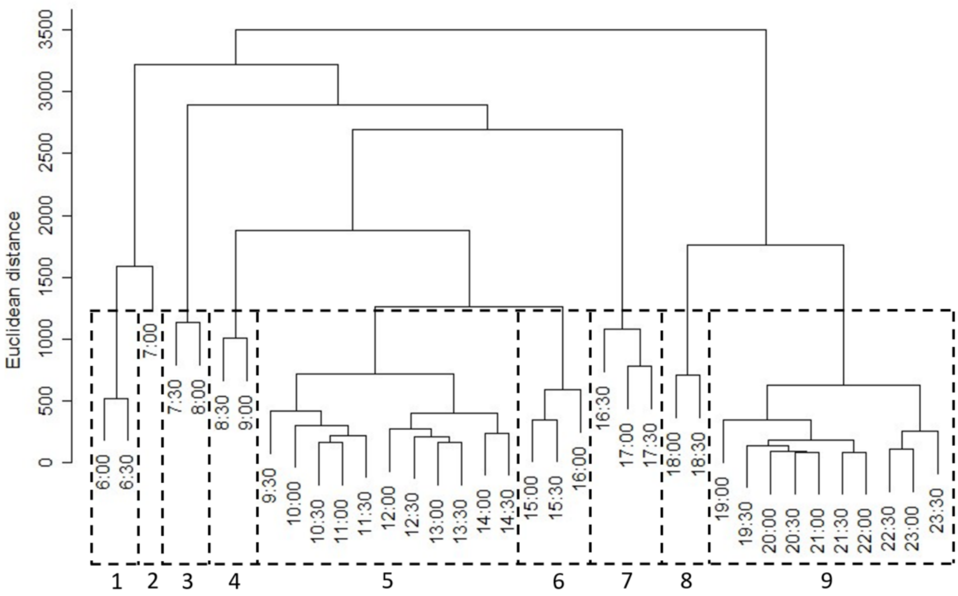
Table 2.3: Distances of the morning matrix to the regular afternoon/evening matrix (column 2), the transposed afternoon/evening matrix (column 3), and matrices of other days (column 4).

	Distance (SHD) of morning matrix to		
Day	**Afternoon/evening matrix**	**Transposed afternoon/evening matrix**	**Closest other matrix (closest matrices)**
Monday	0.2517	0.0238	0.0067 (Mo.A-Th.A)
Tuesday	0.2190	0.0144	0.0079 (Tu.A-Mo.A)
Wednesday	0.2319	0.0399	0.0060 (We.A-Th.A)
Thursday	0.2316	0.0237	0.0060 (Th.A-We.A)
Friday	0.1903	0.0358	0.0123 (Fr.M-Th.M)

When we take the volume into account by looking at the regular OD matrices (see Figure 2.5), the morning versus afternoon/evening pattern does not emerge. The recommended number of clusters (based on the silhouette coefficient) is also much higher: 10 clusters for the Wednesday, and 9 clusters for the other workdays. In the results, we see larger clusters during the day (from 9:30 until approximately 15:00) and in the evening (from 19:00/19:30 until 00:00). These clusters can be characterised as the midday off-peak (cluster 5 in Figure 2.5) and the evening period (cluster 10 on Wednesday and cluster 9 on other days), respectively. Around and during the peak hours we see many small clusters with between one and three matrices per cluster. On each day, the peak hours are divided into three periods, clusters 2, 3, and 4 for the morning peak and clusters 6, 7, and 8 for the afternoon peak. It is likely that the middle of these periods is the hyper peak (clusters 3 & 7), while the other periods can be seen as the shoulders of the peak. If we compare the start times of the different clusters with Figure 2.1, we see that the transition between clusters coincides with large changes in the passenger volume at the station. When comparing the clustering results of the different workdays, many similarities can be seen. For example, the first four periods of each workday have the same start and end time. Moreover, on Monday, Tuesday, and Friday, all the periods have all the same start and end times. Another noteworthy characteristic about the times, is that the peak hours according to the clustering result do not coincide completely with the peak hours that NS uses for pricing. For example, the morning peak hours end at 9:00 according to NS, while the midday off-peak demand only starts at 9:30 according to the clustering result. A similar result is seen at the afternoon peak. The peak hours used for pricing are 16:00–18:30, while the clustering result indicate that the peak hours start already at 15:00 (15:30 on Thursday) and end on most days at 19:00 (18:30 on Wednesday, 19:30 on Thursday). This result is not surprising: the pricing strategy is used to encourage people to travel outside the busiest periods. So, this result shows that the pricing strategy is working to some extent. In the next section, we check whether these periods are not only similar in start and end time, but also in terms of demand during those periods.

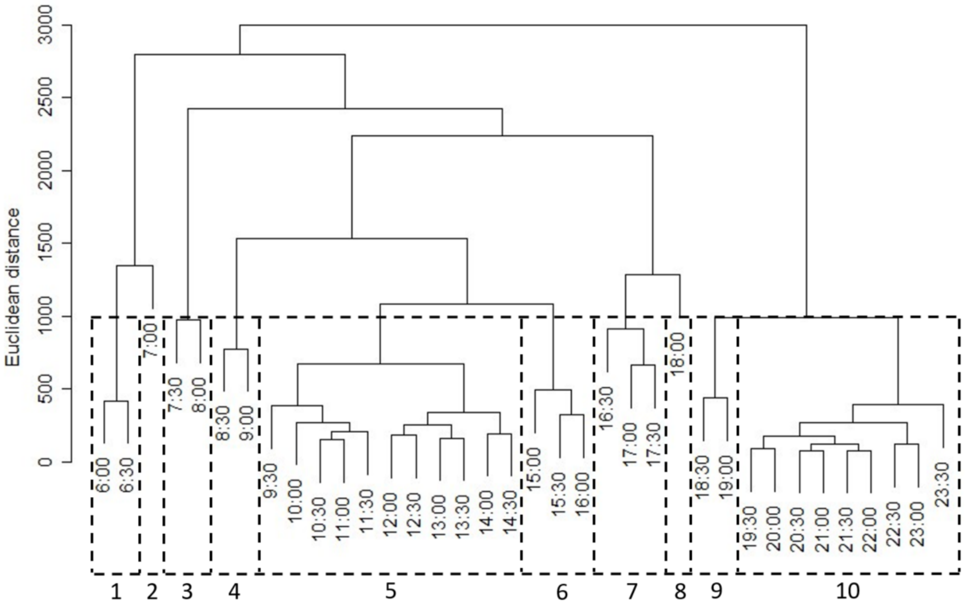


(a) Clustering results Monday - SC = 0.41.

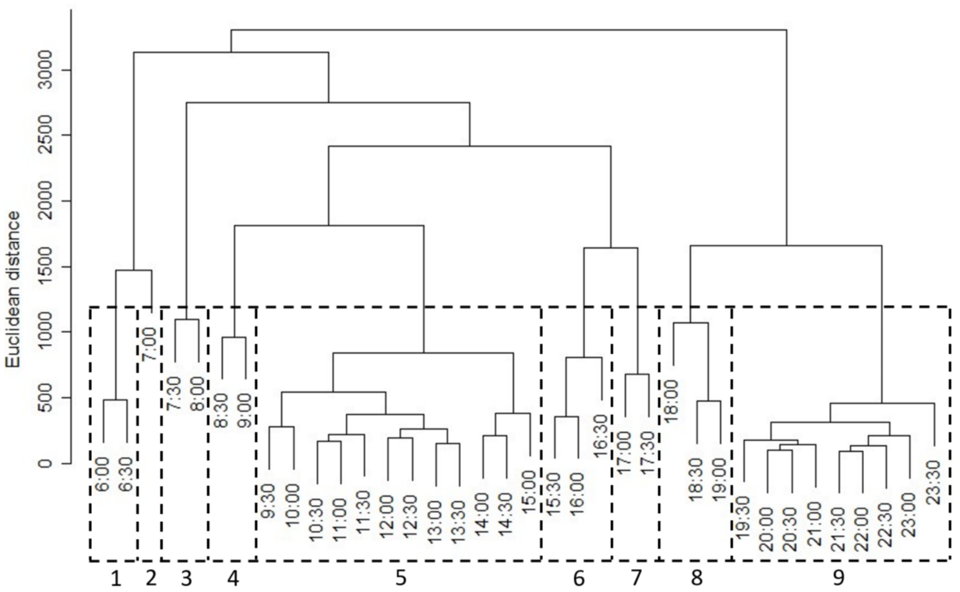


(b) Clustering results Tuesday - SC = 0.41.

Figure 2.5: Clustering results using Euclidean distance on regular OD matrices. Dashed lines denote the optimal number of clusters, according to the silhouette coefficient (SC).

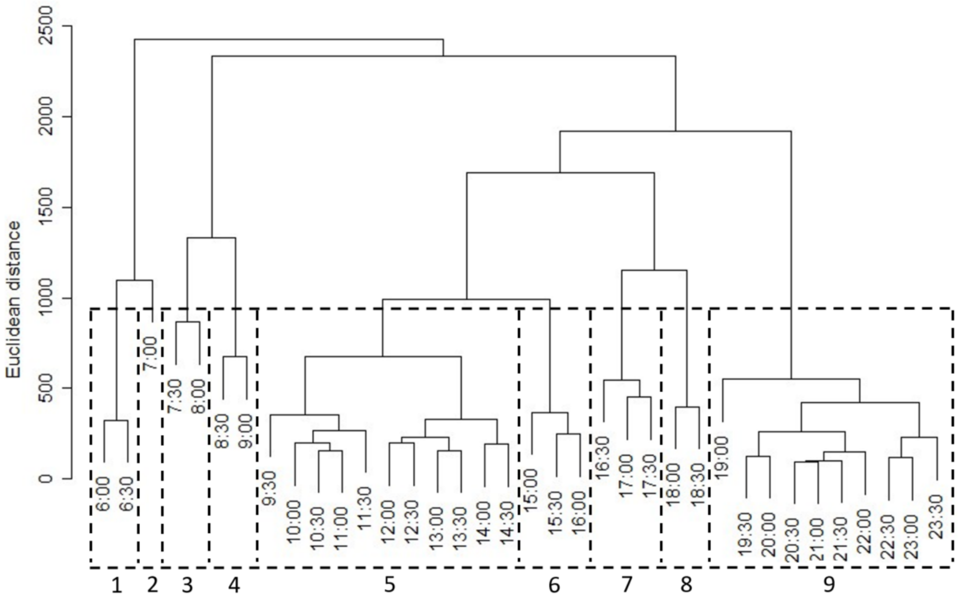


(c) Clustering results Wednesday - SC = 0.38.



(d) Clustering results Thursday - SC = 0.39.

Figure 2.5: (continued)



(e) Clustering results Friday - SC = 0.39.

Figure 2.5: (continued)

2.4.2 Homogeneous periods during the week

In Section 2.4.1, for each workday a number of homogeneous periods was determined in two different ways: based on the structure of the demand (using normalised OD matrices) and based on the structure and volume of the demand (using regular OD matrices). In this section, the demands in these periods are compared, to see if similarities exist across days of the work week.

First the periods based on the structure of demand are compared. As described in Section 2.3, this is done by creating a new OD matrix for each of the periods, and using these as input for the clustering algorithm. The constraint that contiguous periods must be formed is not taken into account at this stage. The dendrogram produced by the clustering algorithm is shown in Figure 2.6. In this figure, on the horizontal axis the different periods are depicted. In Section 2.4.1 we have found for each day two periods, which can be roughly viewed as a morning period and an afternoon/evening period. This is also reflected in the labelling on the horizontal axis: the first two letters denote the day (e.g., Mo stands for Monday) and the last letter denotes whether it is the morning matrix (.M) or the afternoon/evening matrix (.A). As before, the vertical axis displays the distance between two clusters at the moment when they are combined in a single clusters. This distance is measured by the squared Hellinger distance.

Looking at Figure 2.6, there is a clear division between morning and afternoon/evening matrices. When looking at the silhouette coefficient, the optimal number of clusters is two, where the morning matrices are in cluster 1 and the afternoon/evening matrices are in cluster 2. The silhouette coefficient corresponding to this clustering result is 0.94, which is very high since the silhouette coefficient can take a value between -1 and 1. The division in morning and

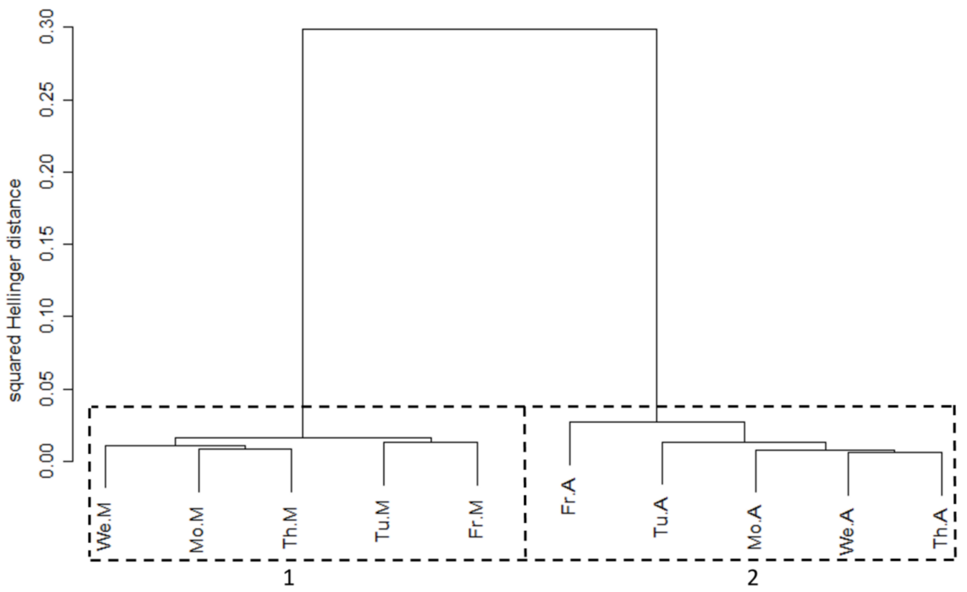


Figure 2.6: Clustering results for homogeneity during the week using squared Hellinger distance on normalised OD matrices. Dashed lines denote the optimal number of clusters, according to the silhouette coefficient (SC = 0.94).

afternoon/evening periods is not surprising given the analysis of the "back and forth"-effect in Section 2.4.1. This analysis shows that morning matrices and afternoon/evening matrices of the same day are relatively far apart, especially when compared to the distances to morning or afternoon/evening matrices of other days. Furthermore, this division is also illustrated in Figure 2.1: both on Tuesday and on Friday people travel towards the station mainly during the morning peak period, and travel back mainly during the afternoon peak period. Hence, it makes sense that a morning matrix is more similar to another morning's matrix than to an afternoon/evening matrix.

Next, the periods based on the structure and volume of the demand are compared. The dendrogram produced by the clustering algorithm is shown in Figure 2.7. These periods are labelled based on their occurrence during the day, so Mo.3 denotes the third cluster on Monday. In Figure 2.5 we can find that this is the cluster from 7:30 until 8:29.

Based on the silhouette coefficient values, we find that the optimal number of clusters is twelve. These clusters are denoted by dashed lines in Figure 2.7. Note that the majority of these clusters only contains matrices of the same time of the day. For example, the third cluster in Figure 2.7 contains the first period (.1) of each weekday. This homogeneity within the determined clusters implies that the demand develops similarly during the different weekdays.

Besides providing the optimal clusters, the shape of the dendrogram also gives information about the similarity of different clusters. When starting at the top of the dendrogram in Figure 2.7 and going down, the first division that is made is between low volume periods and the high volume (peak) periods. On the left side of the dendrogram, we see the three periods with a relatively low volume. From left to right, these periods can be characterised as the midday off-peak period (.5; +/- 9:30–15:00), the evening off-peak period (.9 or .10; +/- 19:00–00:00), and

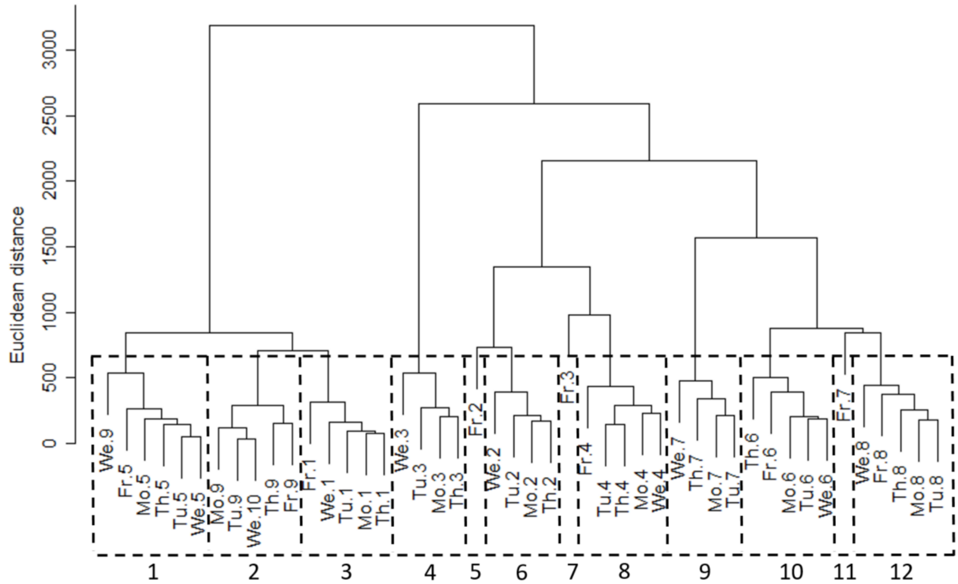


Figure 2.7: Clustering results for homogeneity during the week using Euclidean distance on regular OD matrices. Dashed lines denote the optimal number of clusters, according to the silhouette coefficient (SC = 0.54).

the early morning (.1; +/- 6:00–7:00). Note that although these periods are in separate clusters, we can see from the height of the horizontal lines joining them that they are quite close to each other and quite far apart from the rest of the periods. On the right side of this first division in the dendrogram, we see the peak periods. When zooming in on the peak periods, we see a division into three larger groups: hyper morning peak (.3; 7:30–8:30), the rest of the morning peak (.2 and .4; 7:00–7:30 and 8:30–9:30) and the evening peak (.6, .7 and .8; +/- 15:00–19:00). Note that the shoulders of the peak periods (e.g., .2 & .4 or .6 & .8) are more similar to each other than to the main peak period (.3 or .7), but are not so similar that they end up in the same cluster.

Although we see many similarities in the demand across the different days, the peak demand on Friday does not follow the same pattern as the peak demand on other workdays. Several of Friday's peak periods ended up in its own cluster (Fr.2, Fr.3 and Fr.7 are in clusters 5, 7, and 11). Furthermore, Friday's evening hyper peak period (Fr.7) is not clustered with the rest of the .7 periods, but instead is more similar to the evening peak shoulders (.6 and .8). The same holds for Friday's morning hyper peak period (Fr.3) which is closer to the morning peak shoulders (.4 and .2) than to the morning hyper peak of the other workdays (.3). Hence, Friday does not really show a hyper peak demand pattern. This result is not surprising, since people who work part-time often do not work on Fridays and Friday is a popular day to work from home (even before the Covid-19 pandemic).

2.4.3 Demand characterisation

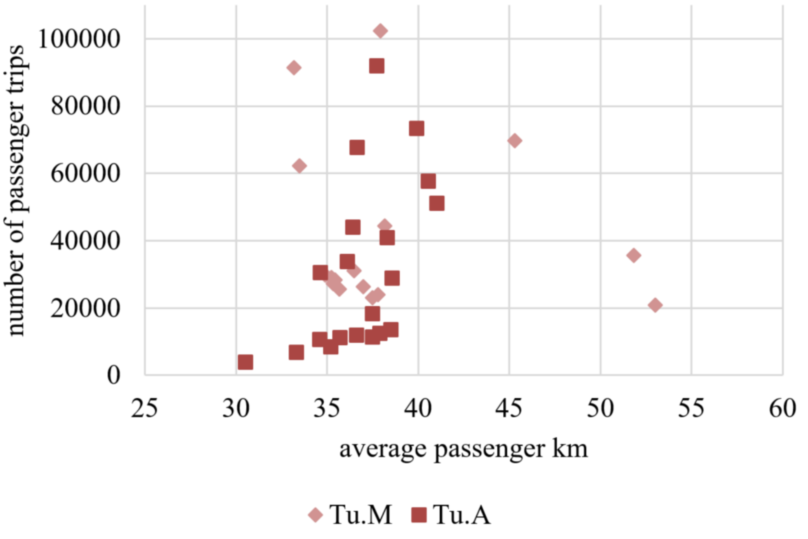
In Sections 2.4.1 and 2.4.2, we showed which periods are homogeneous in demand. However, the cluster results on its own do not give a good understanding of how the demand is changing throughout the day. Therefore, in this section we will illustrate this by providing some demand characteristics for the demand on Tuesday. The Tuesday is chosen because it is one of the most popular days of the week to travel by train, together with Thursday. We chose Tuesday instead of Thursday, since when we look at the clustering results, Tuesday has more similarities with the other days than Thursday and hence is more representative.

Besides giving a better understanding of changes in the demand, looking at demand characteristics of the found periods can also validate the clustering results. Investigating the demand characteristics can validate the clustering results, since when the demand is the same in two different adjacent periods, then there is no valid reason to define multiple periods. Furthermore, the changes in demand characteristics can provide input to the service plan. If it is known how demand differs throughout the day, then this also provides information about how the provided service should change throughout the day to match this demand.

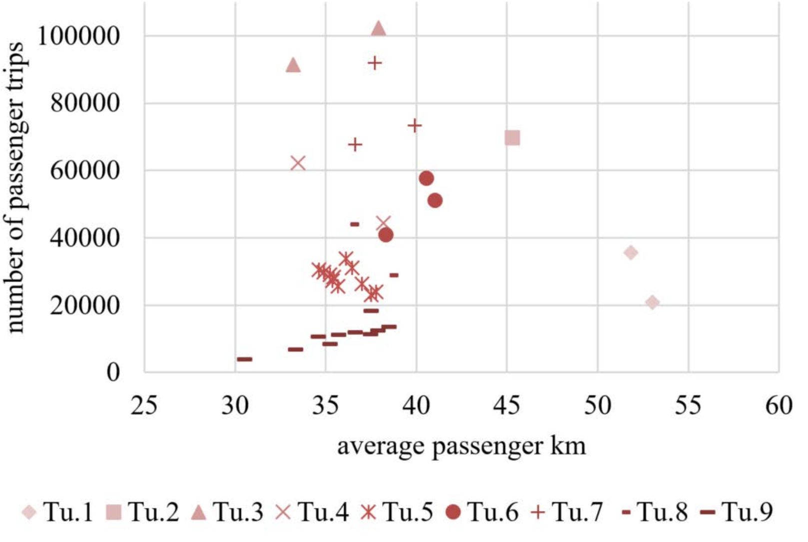
According to Balcombe et al. (2004), demand can be described by the total trip volume and the distance travelled, where the distance travelled is usually measured in passenger kilometres. Therefore, to show how the demand is changing throughout the day, for each half hour OD matrix of the Tuesday the total number of trips and the average number of passenger kilometres travelled in those trips is calculated. These results are then plotted in a scatter plot, which is given in Figure 2.8. In this figure, each of the 36 half hour OD matrices is a data point, where the average passenger kilometres travelled is displayed on the horizontal axis, and the total number of passenger trips is displayed on the vertical axis. To also visualise how the demand changes between periods, a different icon and colour is used for each period. In the plot, the lightest colour is used for the first period of the day, and the darkest colour is used for the last period of the day. As we have two types of cluster results (namely based on the structure of the demand and based on the structure and volume of the demand), we also have two scatter plots (Figure 2.8a and Figure 2.8b, respectively).

The first thing that stands out from Figure 2.8a is that periods Tu.M and Tu.A both show great variation in the demand characteristics. That periods Tu.M and Tu.A have a large variation in volume is not surprising. The clustering that found Tu.M and Tu.A is solely based on the structure of the demand, so the volume of demand is not taken into account at all. Therefore, it would have been more surprising if this clustering would provide a result in which the volumes are also neatly divided into different clusters. However, especially period Tu.M also displays a large variation in average passenger km, which could be seen as a (rough) measure of the structure of the demand. Thus, it would most likely be quite difficult to find a service plan that satisfies the demand in each half hour of Tu.M.

Figure 2.8b gives a much more detailed view of how the demand is changing throughout the day than Figure 2.8a. For example, we see that the early morning period Tu.1 (6:00–6:59), the midday off-peak period Tu.5 (9:30–14:59), and the evening period Tu.9 (19:00–23:59) are the periods with the lowest average volume. Furthermore, we see that during the morning peak (periods Tu.2, Tu.3, and Tu.4) and the evening peak (periods Tu.6, Tu.7, and Tu.8) the volume rises to a peak in the hyper peak periods Tu.3 and Tu.7, before declining again. Two data points that really stand out, are the data points from the early morning period Tu.1. Within this period there is not only a relatively low volume, but also a high average passenger km. Most likely, these are people who live far away from their job, and hence have to leave earlier to arrive on time at their job. A similar trend cannot be seen at the end of the workday, likely because at that time there are also more short-distance travellers in the train. However, note that the average passenger km during the evening peak periods (Tu.6, Tu.7, Tu.8), are higher than the average



(a) Clustering based on normalised OD matrices.



(b) Clustering based on regular OD matrices.

Figure 2.8: Average passenger km and volume for each half hour between 6:00 and 23:59 on the Tuesday. The symbols indicate in which period a half hour is placed. The two subfigures correspond to the two types of clustering results: based on normalised OD matrices (a) and based on regular OD matrices (b). The colour of the symbol relates to the time of day, where the lightest colour corresponds to the first and the darkest colour to the last period of the day.

passenger km during the majority of the morning peak periods (Tu.3 and Tu.4), which might be due to the passengers from periods Tu.1 and Tu.2 returning home.

When comparing Figures 2.8a and 2.8b, the clustering based on the regular OD matrices is more compact compared to the clustering based on the normalised OD matrices. When looking at the different volumes within clusters, in Figure 2.8a the cluster with the largest range in volume is Tu.A. The difference in volumes between the half hour period with the highest volume and the period with the lowest volume is 88,056. When we look at Figure 2.8b, the difference in volume within the clusters is much smaller. The cluster with the highest difference is cluster Tu.7, which has a difference in volume within the cluster of 24,272. So the difference in volume within the clusters is much lower for the result based on regular OD matrices compared to the result based on normalised OD matrices. Similarly, if we look at the difference in average passenger kilometres within the different clusters, the difference is much lower in the results based on regular OD matrices. The highest range in average passenger km is 7.96, which is observed in cluster Tu.9. However, if we look at the result based on normalised OD matrices, we find a highest range of 19.82 km within cluster Tu.M. Hence, the cluster results based on regular OD matrices are more compact in terms of volume and structure compared to the cluster results based on normalised OD matrices. Furthermore, if data points are close to each other in Figure 2.8b, we see that either these points are in the same period, or that these points are in periods that are not adjacent to each other and hence cannot be combined due to the contiguity constraint. In Figure 2.8a, this is not completely the case. For example, if we look at the points that are contained in Tu.5, we see that these points are split between Tu.M and Tu.A in Figure 2.8a. As the points in Tu.5 are very close together in terms of volume and average passenger km, it is questionable whether different service plans are needed to serve this demand. Therefore, the periods based on clustering regular OD matrices seem more suitable as input for creating demand-responsive railway schedules than the periods based on normalised OD matrices.

To strengthen this conclusion, we visualise the OD flows in the different clusters. These visualisations are provided in Figure 2.9 and are made using the open-source software Flowmap-Blue (Boyandin, 2019). To create these visualisations, an average one-hour OD matrix was created per cluster. Furthermore, to improve the readability of the figures, only a part of the network that includes the 4 largest cities of the Netherlands is displayed and nearby stations are grouped together. In this figure, the arcs display OD flows, where the colour and width of the arcs denote the volume of the OD flows. Furthermore, each group of stations is denoted by a circle on the map, where the size of the circle depends on the total volumes going to or departing from those stations. As shown by the legend in the top left corner, the colour of this circle also denotes whether the outgoing and incoming flows are balanced or if either the outgoing or incoming flow is higher.

Figures 2.9a and 2.9b display the OD flows of the cluster result based on the normalised OD matrices, where a morning and afternoon/evening cluster were found. These figures give further confirmation of the “back-and-forth” effect discussed in Section 2.4.1. In the morning (Figure 2.9a), the cities Amsterdam, Utrecht and The Hague all have more incoming flow than outgoing flow, while in the afternoon (Figure 2.9b) this is reversed. Furthermore, the direction of the flows reverses when we compare the morning to the afternoon. For example, in Figure 2.9a many people want to go from Haarlem to Amsterdam in the morning, while the other direction is not so popular. However, in the afternoon, the widest arrow points from Amsterdam towards Haarlem instead. We see similar a similar pattern in the flows between Alkmaar, Hoorn and Utrecht on the one side and Amsterdam on the other side.

(a) OD flows in Tu.M

(b) OD flows in Tu.A

(c) OD flows in Tu.3

(d) OD flows in Tu.5

(e) OD flows in Tu.7

Figure 2.9: Visualisation of OD flows in different clusters. Figures 2.9a and 2.9b show the OD flows from the clusters based on normalised OD matrices and Figures 2.9c, 2.9d, and 2.9e show the OD flows from the morning hyper peak, midday off-peak, and afternoon hyper peak clusters based on regular OD matrices.

From the clustering result based on regular OD matrices, we visualise three of the nine clusters: the morning hyper peak (Figure 2.9c), the midday off-peak (Figure 2.9d), and the afternoon hyper peak (Figure 2.9e). When we compare the three figures, we see a lot of difference in volumes between the three clusters, where the hyper peaks have a much higher demand than the midday off-peak period. Furthermore, similar to Figures 2.9a and 2.9b, we see a "back-and-forth" effect in the two peak hours. However, in the midday off-peak period this effect is not visible. In Figure 2.9d, most station groups have an incoming flow that is approximately equal to the outflow, as displayed by the single-coloured circles. Furthermore, for each OD pair in the figure, the arcs have approximately the same width and colour for both directions. From this we conclude that the demand in the midday off-peak is not only lower in volume, but also much more spatially balanced than the demand in the peak hours. Moreover, the midday off-peak is quite a long period (from 9:30 to 14:59). Hence, when we want to determine input for creating demand-responsive railway schedules, it seems logical that the midday off-peak gets its own cluster instead of being divided between a morning and afternoon cluster, as in the result based on normalised OD matrices. Since these visualisations show that there is more variation in demand during the day than just the "back and forth"-effect, we conclude that in this case the clusters based on regular OD matrices are better suited to be used as input for creating demand-responsive railway schedules.

2.5 Conclusions and recommendations

In this chapter, we investigated how homogeneity in railway demand can be discovered. This method could be useful for RUs that would like to create service plans (line plans and timetables) that better match the changing demand throughout the day and week, but still need a lot of manual labour to create their service plans. By determining the periods with homogeneous demand, the RU can reduce the amount of periods during the day that need to be considered for a different service plan. There have been several studies about homogeneous demand in bus services, but so far the academic literature for railway demand has been lacking. This chapter contributes to the academic literature by evaluating if the method for determining periods of homogeneous demand in bus services also work for railway demand. Furthermore, this chapter extends those methods to create homogeneous periods not only based on the demand structure, but also the demand volume, since volume is an important aspect when creating railway service plans.

The methods in this chapter are demonstrated on a case study from the Dutch passenger railway operator NS. When applying the clustering method on the normalised OD matrices (containing the structure of demand), each workday was split into two periods with the switch between periods around 12:00. An explanation for this is that people usually travel somewhere in the morning, and travel back in the afternoon or evening. The clustering result therefore provides some evidence that a symmetric service plan, in which each train line is operated with the same frequency and stops in both directions, is not the optimal choice when strictly looking at the demand structure. Applying the clustering method on the regular OD matrices (containing both structure & volume of demand) results in these periods: morning peak, midday off-peak, afternoon peak, and evening. The peak periods are more diverse, and hence divided into smaller time intervals, than the off-peak periods. The results of this analysis help to determine the time at which switching between service plans makes sense. The switching times between periods are quite consistent throughout the week. Hence, switching between service plans can happen at the same times every day, which is favourable for the customers. When looking at demand characteristics such as volume and average passenger kilometres per period, the introduced clustering method based on the regular OD matrices provides periods that are more compact and hence more suitable for creating railway service plans than the clustering method from the bus literature, which was based on the normalised OD matrices.

The analysis for homogeneity in demand during the week shows that the days of the week are quite similar. However, when looking at the structure & volume of the demand, the peak demand on Friday forms an exception to this. These results suggest that the service plan for Friday's morning and evening peak hours could be different from the service plan for the other weekdays' peak hours. The differences between days might increase further due to the Covid-19 pandemic (e.g, because people will work from home more on Wednesdays and Fridays). If this is the case then potentially the demand on other days will also change enough to warrant a separate service plan.

The methods discussed in this chapter provide valuable practical insights for public transport authorities and operators seeking a better understanding of their demand and developing more responsive schedules. This approach is particularly beneficial for those who wish to enhance demand-responsiveness without moving to a fully flexible timetable. The following key considerations can help guide the application of these methods in practice. Firstly, the clustering results provide information about the maximum number of periods that need to be considered during the day/week when creating demand-responsive schedules. In time intervals in which the demand is homogeneous, the demand can be served effectively by the same service schedule. Since there is a cost to creating and switching between service plans, the number of periods with homogeneous demand gives us the maximum number of schedules that we need to consider. However, this does not mean that it is also optimal to use this number of schedules in practice. For example, if some of the identified periods cover a very short time period (e.g., 30 minutes), it might not be beneficial to create a separate schedule for this. Secondly, the clustering result provides information on when the switch should be made between different schedules. For example, in the presented case study the midday off-peak period starts at 9:30, while the pricing peak hour at NS ends at 9:00. Therefore, if the RU wants to create a separate schedule for the morning peak hours, this result shows that it is better not to let the end of this peak period coincide with the end of the pricing peak period, but instead postpone it with 30 minutes. As a final note, we want to point out that the train passenger demand is sensitive to the train schedule and the pricing. Therefore, when a RU makes changes to its schedule or pricing strategy, this will influence the demand, which might change the clustering result as well. One way to deal with this is by using price and travel time elasticities to forecast the new demand and redo the clustering with this forecasted demand. If these clusters are in line with the new schedule, or with the old schedule if the pricing strategy is changed, then we can be more confident that the schedule can serve the new demand well. If instead the new clusters are not in line with the new schedule, further adjustments might be necessary.

In terms of future research, it would be very interesting to repeat this study with post-Covid data to see if there are (significant) changes in the results, given the expected changes in demand patterns. Furthermore, it would be interesting to test these methods on other datasets, to check the methods' validity. Our case study shows that creating homogeneous periods based on the regular OD matrices provides periods that are more compact and hence more suitable for creating service plans than the clustering based on normalised OD matrices. It would be interesting to see if this result also holds for different application contexts, like bus and metro. Another promising research direction would be to investigate whether adapting the service plan based on the results of such analysis can be beneficial for railway undertakings and its customers. Although the adapted service plan should fit the changing demand better, adapting the service plan also comes with some costs. For example, switching between service plans during the day can be difficult to plan or execute for railway undertakings. Furthermore, customers might find it difficult to get used to schedules that change throughout the day. Therefore, it should be investigated whether the benefits will outweigh these costs. Part of this investigation should deal with what indicators should be considered to go from the clustering result to the optimal number of schedules. The method presented in this chapter only determines the maximum number of periods that need to be considered when adapting the line plans and timetables to the changing demand. However, there could be several reasons to further merge periods together, including that the length of certain periods is too short to create a different schedule for, or that the realised reduction in travel time with a different schedule is too small. Hence, further research is needed to determine how we can go from the maximum to the optimal number of periods.

Chapter 3

Multi-period line planning for varying railway demand with asymmetric lines

The previous chapter analysed passenger demand data from NS and identified periods with homogeneous demand during the workday. While there are many variations in demand throughout the day, the different workdays of the week are quite similar to each other. Therefore, in the remainder of this dissertation, the focus lies on creating schedules that take into account demand fluctuations throughout the day.

Building on these insights, this chapter presents a mathematical model for creating a multi-period line plan that aligns with varying demand throughout the day. The model optimises route selection, stopping patterns, and service frequencies for each demand period to create a line plan that minimises the passengers' Generalised Journey Time (GJT). Additionally, the model incorporates asymmetric lines to address spatially unbalanced demand. A case study based on part of the Dutch railway network shows that the total GJT can be improved significantly when adapting the line plans throughout the day.

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3.1 Introduction

A railway line plan, consisting of routes, stops, and frequencies, determines a large part of the quality of the railway service for passengers. For example, the line plan determines if passengers can have a direct trip or if they need one or multiple transfers. Furthermore, a line's stopping pattern influences the passengers' in-vehicle times and hence their total travel time. Due to its impact on passengers and the high cost of public transport networks, the line planning problem gets a lot of attention in the literature (see e.g., Durán-Micco & Vansteenwegen (2022)). Moreover, it is well-known that the passenger demand is not fixed, but instead changes throughout the day and throughout the week (see e.g., Van der Knaap et al. (2024)). Hence, operating a single line plan throughout the day means that the plan's attractiveness may fluctuate with these demand changes. Schöbel (2012) points out that it is worthy to investigate whether demand in peak and off-peak hours are best served by different line plans or by the same line plan. Moreover, Zhang et al. (2020) show that when demand is diverse, flexible stopping rules are better able to find good solutions for the passengers compared to fixed stopping rules. While it is common in urban public transport to adjust the line plan during the day, such as changing frequencies or running special lines during peak hours (Van Oort, 2011), the line plans used by railway undertakings on conventional railways in the Netherlands and other parts of Europe are still more or less fixed throughout the day. Therefore, this chapter investigates how to create a train line plan that better matches the varying demand throughout the day.

Demand for public transport varies throughout the year (e.g., with less demand during the summer holiday as many people are on vacation), throughout the week (with different demand on different days of the week), and throughout the day (Van Oort, 2011). In this chapter we focus on demand variations during the day. There are several ways in which the demand can vary throughout the day. First, the demand volumes change: demand is higher in the peak periods at the beginning and the end of the workday. For instance, in the Netherlands on a regular working day in 2019, around 50% of all trips were made inside the peak hours (Van der Knaap et al., 2024), which are between 6:30–9:00 and 16:00–18:30. Hence, the off-peak hours in a working day have on average far fewer passengers. Second, the passenger demand distribution also varies throughout the day. For instance, during peak hours many people are commuting from home to their workplace or vice versa. Therefore, railway stations close to a business park might be highly visited during the peak hours, but not so much during working hours or in the evening. On the other hand, during the day there are more people travelling for leisure purposes, with for example a city centre as destination. Passengers travelling off-peak on a line designed around peak hour demand, with stops at stations near business parks, will experience longer travel times due to unnecessary stops at those stations. Furthermore, jobs might not be equally spread over the country. Compared to the average, some areas have more houses and other areas have more job locations. This results in two distinct transport flows during the day: in the morning more people travel from residential areas to the areas with jobs, while in the late afternoon the direction reverses as people return home (Van der Knaap et al., 2024).

Recent years have seen growing interest in multi-period line planning to better accommodate temporal demand fluctuations. For example, Şahin et al. (2020) are the first ones to introduce a line planning model that has multiple periods during the day to accommodate varying demand, while Nie et al. (2023) extend this to a weekly planning horizon. Despite such advances, several challenges remain. Most models have been developed and tested on corridor-like networks, whereas many European railway networks are highly connected. Since it is more difficult to find a line plan for a network than for a corridor, more research is needed into how a multi-period line plan can be created for a railway network. Furthermore, existing models often consider only a limited range of service adjustments such as frequencies or stopping patterns, but typically not a large range of aspects simultaneously.

Given the temporal and spatial differences in demand that are described above, possible service adjustments that can be used include changing the stopping patterns, the frequencies, the route choice, and including asymmetric lines. We define asymmetric lines as lines that are not operated with the same stopping pattern or frequency in both directions. Asymmetric lines can better accommodate demand that is not balanced in both directions. Moreover, in complex railway networks, providing direct connections for every origin-destination (OD) pair is often impractical. To achieve meaningful flexibility in route and stop pattern design, it is therefore essential to incorporate passenger transfers into multi-period line planning models. Combining these features into a comprehensive multi-period model remains an area with room for further development.

Multi-period line planning offers railway undertakings the opportunity to better match passenger demand throughout the day without increasing costs, thus improving both service quality and operational efficiency. Building on the existing research, the main contributions of this chapter are:

- A novel mixed-integer linear programming model for multi-period line planning in a railway network that simultaneously incorporates the selection of routes, stopping pattern, and frequency, the possibility for passengers to transfer between lines, and asymmetric lines.
- An analysis of the impact of asymmetric lines on minimising passengers' total Generalised Journey Time.

The proposed model is applied to a case study based on part of the Dutch railway network. The ε-constraint method (Mavrotas & Florios, 2013) is used to create several line plans with varying amounts of line plan adjustments throughout the day.

The remainder of the chapter is organised as follows. In Section 3.2, a literature review about (multi-period) line planning is given in order to briefly summarise previous work in the area. Next, Section 3.3 introduces a mathematical model for line planning that includes choice of route, frequency and stop pattern. Section 3.4 describes the case study and Section 3.5 provides the results of applying the proposed model to the case study. Lastly, the conclusions are provided in Section 3.6.

3.2 Literature review of line planning for varying demand

The scheduling of public transport services is often a sequential process, consisting of infrastructure planning, line planning, timetabling, vehicle scheduling, and crew scheduling (Ceder & Wilson, 1986). The focus in this chapter is on the Line Planning Problem (LPP). The output of the LPP is a line plan, which is a set of lines and their corresponding frequencies. Here, we define a line as a path through the public transport network and a set of served stations.

The Line Planning Problem, also called the Transit Network Design Problem, is a well-researched problem in literature. As a result, a rich variety of literature review papers exist, such as Schöbel (2012) and Durán-Micco & Vansteenwegen (2022). Schöbel (2012) provides an overview of the different types of line planning models found in the literature, including commonly used objectives and constraints. One direction for future research that is given is whether peak and off-peak demand should be served by the same or different line plans. The literature review of Durán-Micco & Vansteenwegen (2022) focuses on the different versions of the Transit Network Design Problem that have been discussed in the literature. They conclude that technological improvements, such as improved computer capacity and data collection techniques, have facilitated and increased research interest into the topic. However, although this is a well-researched problem, there still exists a significant gap between theory and practice due to the simplifying assumptions used in the literature.

3.2.1 Line planning considering constant demand

Most papers on the LPP have only considered constant demand. In the most simple cases, the demand for transport is given per edge in the public transport network, which connects two subsequent stations (Schöbel, 2012). Another option is that the number of passengers per origin-destination (OD) pair is given (see e.g., Borndörfer et al. (2008), Goossens et al. (2004)). Furthermore, the demand considered is typically for one period only. In some long-distance railways, like the high-speed railway network in China, trains do not run every hour. Therefore, the demand during the period of one day is used to determine which lines are suitable, like in Fu et al. (2015). On the other hand, when a cyclic railway timetable is the goal, the demand is usually given for one cycle of the timetable, for example one (peak) hour. Papers employing this strategy include Nachtigall & Jerosch (2008) and Bull et al. (2019). However, using a line plan during an entire day that is only based on one (peak) period might not be optimal for the passengers if the demand volumes and distribution varies throughout the day. Furthermore, Zhang et al. (2020) show that when passenger demand is diverse, having flexible stopping rules, which are not solely based on the type of station, is better for passengers. Under diverse demand, the flexible stopping rules generated a line plan with a lower average number of stops per line, which results in faster travel times for the passengers. Therefore, our study aims to address the limitations of constant demand assumptions and fixed stopping rules by considering dynamic demand patterns throughout the day and exploring flexible stopping strategies to improve passenger service and travel times.

3.2.2 Varying frequencies to address time-dependent demand

Some papers recognise that since the demand is fluctuating throughout the day or year, a line plan should be evaluated on more than one type of demand (e.g., Amiripour et al. (2014), Cyril et al. (2020), Durán-Micco et al. (2022)). However, to create a schedule that is convenient and clear for the passengers, the same lines with a fixed stopping pattern should be operated under each type of demand. Meanwhile, the frequencies of the selected lines can be adjusted to better match the varying demand. Amiripour et al. (2014) consider seasonal demand variations and aim to create a bus schedule that is robust to these changes. On the other hand, Cyril et al. (2020) and Durán-Micco et al. (2022) consider demand variations during the day. Cyril et al. (2020) determine for each line in the bus network a suitable frequency for both the peak and off-peak hours, while Durán-Micco et al. (2022) also consider the difference between morning and afternoon peak demand in their evaluation. Kaspi & Raviv (2013) propose an optimisation model that creates a line plan and cyclic timetable with the aim of minimising the operational cost and the total passenger travel time. Time-dependent demand that changes throughout the day is used to evaluate the timetable, but as the timetable is cyclic no service-adjustments are made throughout the day. While adjusting frequencies effectively addresses variations in demand volumes, this measure alone is insufficient to deal with varying demand distributions, which may also require changes to routes or stopping patterns.

3.2.3 Varying lines and frequencies to address time-dependent demand

The papers discussed in this section make changes to both the lines and the frequencies, instead of only changing the frequencies to match the demand as in the previous section. In these papers, the considered time interval is divided into different periods with different demand and the aim is to find a suitable set of lines and corresponding frequencies for each of these periods. The first paper on multi-period line planning is written by Şahin et al. (2020), and seeks to find a minimal cost line plan which considers the transfer of vehicles from period to period. Continuing on this work, Zhao et al. (2022) propose a model that considers time-dependent demand, determining line frequencies, transport capacity, stop-pattern and the transfers of rolling stock in a railway corridor. A round heuristic algorithm is used to solve a case study based on intercity railway in China. Nie et al. (2023) introduce the Weekly Line Planning (WLP) problem. The WLP problem determines a line plan for each time block of each weekday in order to maximise the total matching utility between supply and demand and to minimise the total train operational cost. A custom genetic algorithm is provided in order to solve two large-size case studies on the high-speed railway in China. Schiewe et al. (2023) highlight that unrelated line plans across different periods can lead to passenger confusion and operational challenges. To address this, an upper bound on the dissimilarity between line plans operated within a single day in the multi-period line planning problem is incorporated. Furthermore, the study introduces three distinct measures for assessing the (dis)similarity of these line plans. Avila-Ordóñez et al. (2022) also consider keeping changes to the line plan as small as possible, when creating a bus line plan for a major event. Their objective is to improve the total travel time, taking into account increased demand at the event location and congestion in that area, while changing the lines as little as possible to keep it attractive for the regular passengers. A heuristic based on Genetic Algorithms is presented to solve this flexible bus line planning problem.

The existing literature demonstrates that the selection of routes, frequencies, and stopping patterns are critical components when addressing varying demand. However, the majority of research assumes symmetric lines. Notably, Zhao et al. (2022) include asymmetric lines, but this consideration is confined to a single railway corridor. As a result, there remains a significant gap in the study of asymmetric lines within complex railway networks. Additionally, Schiewe et al. (2023) insightfully emphasise that substantial alterations to line plans within a single day can lead to passenger confusion and operational challenges. Therefore, our model will also incorporate mechanisms to address such considerations, aiming to balance the benefits of flexibility with the need to limit operational complexity.

3.2.4 Combining line planning and timetabling decisions for time-dependent demand

There are also several papers that combine line planning and timetabling decisions under time-dependent demand.

One group of papers only considers decisions about frequencies or departure times at the origin. While these can differ throughout the considered time period, the selected or given lines are kept the same. To serve asymmetric time-dependent demand on a metro line, Mo et al. (2021) consider having different frequencies, departure times, and train speed profiles for both directions on the line, while jointly optimising the rolling stock circulation. The objective of the proposed mixed-integer non-linear programming model is to optimise both the passenger waiting and travel times and the operating costs, including rolling stock utilisation, maintenance and energy. Li et al. (2019b) and Yuan et al. (2022) both want to improve passenger waiting time on a metro line by including the selection of a short-turning route in the model. By also including the rolling stock circulation, Yuan et al. (2022) show that using short-turning can significantly reduce the passenger waiting time compared to the timetable used in practice, while keeping the number of rolling stock the same.

Another group of papers combines timetabling with stop planning decisions. Qi et al. (2021) combine stop planning and timetabling to minimise the total travel time of the trains, while serving all the demand in their preferred time slot with a direct connection. The resulting model is tested on the Wuhan-Guangzhou high-speed railway line in China and the model is solved using CPLEX. Zhou et al. (2023b) developed a non-linear optimisation model for finding a time-dependent line plan, timetable, and differentiated ticket prices. The objective of the model is to both maximise the railway undertaking's revenue and minimise the total cost of the passengers. The model is solved using a simulated annealing algorithm. Moreover, in the context of congested systems, altering the stop patterns can help with reducing the travel time for passengers and the costs for the railway undertaking (Dong et al., 2020; Qu et al., 2023), improving the fairness among waiting passengers (Zhao et al., 2021), and improving the safety at stations (Shi et al., 2023).

Contrary to the papers previously mentioned in this section, which all create an acyclic timetable, Zhou et al. (2023a) aim to serve time-dependent demand by using a mixture of periodic and aperiodic lines. Periodic lines are operated during every hour and have the same route and stopping pattern throughout the day, while aperiodic lines are added to the cyclic schedule to serve specific time-dependent demand. The mixed-integer non-linear programming model is solved using a simulated annealing algorithm.

3.2.5 Research gap and contributions

Table 3.1 provides an overview of the key literature that incorporates line planning decisions under time-dependent demand and shows how this chapter relates to the literature. The first three columns denote the paper, the type of network analysed, distinguishing between a corridor and a full network, and the mode considered. The next four columns denote which line planning decisions are considered in the papers. "Line Selection" (LS) refers to the selection of a set of lines with pre-defined stopping pattern from a line pool. "Route Selection" (RS) involves selecting a path through the network without predefined stopping patterns. "Stop Pattern" (SP) indicates that the model sets the stopping patterns on predefined or selected routes. Lastly, "Frequency" (F) refers to determining the line frequencies. The last three columns indicate: whether passenger transfers are considered, if asymmetric lines with different stopping patterns or frequencies in opposite directions are allowed, and whether the aim is to determine a multi-period schedule.

Although there has been a lot of research already into the LPP in general and some research into multi-period line planning problem specifically, there still exist several important research gaps.

Table 3.1: Overview of the relevant literature considering line planning decisions under time-dependent demand.

Paper	N/C	Mode	Line planning choices included LS	RS	SP	F	T	Asym lines	Multi-period schedule
Li et al. (2019b)	C	M	-	✓	-	-	-	✓*	-
Dong et al. (2020)	C	R	-	-	✓	-	-	✓*	-
Mo et al. (2021)	C	M	-	-	-	✓	-	✓	-
Qi et al. (2021)	C	R	-	-	✓	-	-	✓	-
Zhao et al. (2021)	C	URT	-	-	✓	-	-	✓	✓
Nie et al. (2023)	C	HSR	✓	-	-	✓	✓	-	✓
Yuan et al. (2022)	C	M	-	✓	-	-	-	✓	-
Zhao et al. (2022)	C	R	-	-	✓	✓	-	✓	✓
Qu et al. (2023)	C	M	-	-	✓	-	✓	✓*	-
Shi et al. (2023)	C	M	-	-	✓	-	-	✓*	-
Zhou et al. (2023a)	C	HSR	-	✓	✓	-	✓	✓*	-
Kaspi & Raviv (2013)	N	R	✓	-	-	✓	✓	-	-
Amiripour et al. (2014)	N	B	✓	-	-	✓	✓	-	✓
Cyril et al. (2020)	N	B	-	-	-	✓	-	-	✓
Şahin et al. (2020)	N	B, M	-	-	-	✓	-	-	✓
Avila-Ordóñez et al. (2022)	N	B	-	✓	-	-	✓	-	-
Durán-Micco et al. (2022)	N	B	-	✓	-	✓	✓	-	✓
Schiewe et al. (2023)	N	B, M	✓	-	-	✓	-	-	✓
Zhou et al. (2023b)	N	HSR	-	-	✓	-	✓	✓*	-
This chapter	N	R	-	✓	✓	✓	✓	✓	✓

N/C: network (N) or corridor (C), Mode: bus (B), high-speed railway (HSR), metro (M), railway (R), urban rail transit (URT), Line plan choices: Line Selection (LS), Route Selection (RS), Stop Pattern (SP), Frequency (F). T: transfers considered, Asym lines: asymmetric lines considered, * only one direction considered.

First, most studies that consider time-dependent demand focus on bus lines or train lines in railway corridors. While adjusting frequencies and departure times at the origin station to fluctuations in demand throughout the day is common practice in bus services (see e.g., Van Oort et al. (2012)), such adjustments are not common in railway services. Railway operations, in contrast to buses, face strict infrastructure constraints that complicate frequency and departure time adjustments. Changing the departure time at the origin affects the departure time at all stations throughout the line. As trains could interact with other trains at each station on their route, even a small change in their departure time can create multiple conflicts due to trains claiming the same infrastructure at the same time. Additionally, while the railway network in China can be well represented by corridors, many European networks are highly connected networks where transferring between lines is common. Therefore, more research is needed into creating multi-period line plans for railway networks, where the possibility of transferring between lines is included.

Second, existing network-related research often only considers symmetric lines, which are either fixed or chosen from a predefined line pool. However, these requirements reduce the flexibility to deal with asymmetric demand. Line pools are typically based on numerous assumptions; for instance, the Dutch railway undertaking NS categorises lines into two types: Sprinter lines that stop at all stations, and Intercity lines that stop only at the large stations. Additionally, some medium-large stations are served by a subset of the Intercity lines. However, such categorisation restricts adjustments for stopping frequency at different times, limiting the ability to accommodate dynamic demand variations effectively. To provide more options to deal with time-varying and asymmetric demand, more flexibility in determining the stopping pattern and allowing for asymmetric lines should be considered.

In response to these gaps, this chapter proposes a model for the multi-period line planning problem in railway networks. The model's input includes a fixed number of periods that each have their own length and demand. To deal with the temporal and spatial differences between the demands of different periods, the model considers selecting routes in the network, setting the stopping patterns and the frequencies, and having asymmetric lines. Since the model addresses railway networks, the possibility of transferring between lines will also be taken into account. By integrating this comprehensive set of line planning decisions, the proposed model addresses several key limitations identified in the literature. Moreover, this study represents the first to analyse the impact of asymmetric lines on minimising passengers' total Generalised Journey Time in a (railway) network.

3.3 Mathematical model for multi-period line planning

In this section, the mathematical model for multi-period line planning is introduced. The aim of the model is to provide a line plan which minimises the passengers' total Generalised Journey Time (GJT) throughout the day, while not exceeding a fixed budget for the line plan. In this chapter, the GJT consists of the in-vehicle time, and the waiting time before boarding a train (for both the first train and when transferring). In Section 3.3.1 we introduce the notation and variables related to train lines. The variables and parameters used in the model are listed in Tables 3.2 and 3.3, respectively. Next, Section 3.3.2 introduces the change-and-go network that is used to model the passenger paths and calculate the GJT. In Section 3.3.3 the mathematical formulation of the multi-period line planning model is given.

Table 3.2: Variables used in the multi-period mathematical model.

Variables	
$x_s^{l,p}$	Binary variable which is equal to 1 if line l stops at station s in period p, 0 otherwise. This variable is defined for each candidate line $l \in \mathcal{L}$, for each station $s \in \mathcal{S}^l \setminus \{s_1^l, s_{n_l}^l\}$, and for each period $p \in \mathcal{P}$.
$f_i^{l,p}$	Binary variable which is equal to 1 if line l is operated with frequency i in period p, 0 otherwise. This variable is defined for each candidate line $l \in \mathcal{L}$, each frequency $i \in \mathcal{F}^l$, and each period $p \in \mathcal{P}$.
$y_s^{a,p}$	Real variable denoting the number of passengers that originate from station s and use arc a to travel to their destination during one hour in period p. This variable is defined for each origin station $s \in \mathcal{S}$, each arc $a \in \mathcal{A}$ in the change-and-go network, and each period $p \in \mathcal{P}$.
$\widetilde{f}^{l,p}$	Real variable denoting whether the frequency of line l has changed in period p compared to the previous period. This variable is defined for each candidate line $l \in \mathcal{L}$, and each period except the first one: $p \in \mathcal{P} \setminus \{p^1\}$.
$\widetilde{x}_s^{l,p}$	Real variable denoting whether the stop plan of line l at station s has changed in period p compared to the previous period. This variable is defined for each candidate line $l \in \mathcal{L}$, for each station $s \in \mathcal{S}^l \setminus \{s_1^l, s_{n_l}^l\}$, and each period except the first one: $p \in \mathcal{P} \setminus \{p^1\}$.

Table 3.3: Parameters used in the multi-period mathematical model.

Parameters	
$\mathcal{S}$	Set of stations.
$\mathcal{S}^T$	Set of stations at which a transfer is allowed, $\mathcal{S}^T \subseteq \mathcal{S}$.
$\mathcal{S}^E$	Set of terminal stations, $\mathcal{S}^E \subseteq \mathcal{S}$.
$\mathcal{L}$	Set of candidate lines.
$\mathcal{L}_s^-$	Set of candidate lines that have station $s \in \mathcal{S}^E$ as first station, $\mathcal{L}_s^- \subseteq \mathcal{L}$.
$\mathcal{L}_s^+$	Set of candidate lines that have station $s \in \mathcal{S}^E$ as last station, $\mathcal{L}_s^+ \subseteq \mathcal{L}$.
s_i^l	The i^{th} station on candidate line $l \in \mathcal{L}$.
s_1^l	The first station on candidate line $l \in \mathcal{L}$.
$s_{n_l}^l$	The last station on candidate line $l \in \mathcal{L}$. The number of stations on a candidate line n_l can be different for each line.
$\mathcal{S}^l$	Sequence of adjacent stations $(s_1^l, \ldots, s_{n_l}^l)$ defining the route of candidate line $l \in \mathcal{L}$.
$\mathcal{F}^l$	Set of allowed frequencies for candidate line $l \in \mathcal{L}$. The frequency is the number of trains per hour operated on the line, so all frequencies are positive integers.
f_{max}^l	The largest frequency in $\mathcal{F}^l$.
q^l	The capacity of a train on candidate line l.
k^l	Length of candidate line l in kilometres.
$\mathcal{P}$	Set of periods for which we want to determine a line plan.
p^j	The j^{th} period in $\mathcal{P}$. When the specific order is not relevant this is simplified to p.
b^p	The maximum budget for a line plan in period p, given as the total number of kilometres that trains can drive in one hour.
h^p	Length of period $p \in \mathcal{P}$ in hours.
$d_{s,u}^p$	Passenger demand from station s to station u $(s, u \in \mathcal{S})$ during an average hour in period $p \in \mathcal{P}$.
$\delta_s^{\nu,p}$	Demand at vertex $\nu \in V$ for passengers who originate from station $s \in \mathcal{S}$ during an average hour in period $p \in \mathcal{P}$.
t^a	Time needed to travel on arc $a \in \mathcal{A}$ in the change-and-go network. Details on how these times are calculated can be found in Section 3.4.3.
σ	Symmetry parameter, takes value 2 if symmetric lines are considered, 1 otherwise.

3.3.1 Modelling train lines

To minimise the passengers' total GJT when the demand varies throughout the day, the model should be able to create different lines for different periods. A train line is defined by its route through the network, its stopping pattern, and its frequency (i.e., how many times per hour a line is operated). Contrary to most literature, our proposed model does not select lines with fixed route and stopping pattern from a line pool. Instead, similarly to Zhou et al. (2023b) each candidate line has a fixed route, but its stopping pattern and frequency are determined by the model using decision variables. As part of the input, we need to provide a set of routes and per route how many candidate lines should be included. A route is a sequence of adjacent stations in the railway network, that determines the path through the railway network. Two stations are considered adjacent if they are directly connected by a track with no intermediate station between them. The first and last station of the sequence are the two terminal stations. Usually only a limited number of stations can function as a terminal station. To qualify as a terminal station, a station should for example have infrastructure for turning the trains or capacity for parking rolling stock that is temporarily not used in service. Hence, the selection of which stations are terminal stations greatly influences which routes can be included in the route set. Furthermore, as it could be desirable to have multiple lines with a different stopping pattern on the same route, it is possible to allow for more than one candidate line per route. The set of candidate lines is denoted by $\mathcal{L}$.

The stopping pattern of a candidate line is set by the stopping variables. Each station s on candidate line l gets a binary stopping variable $x_s^{l,p}$, which takes value one if the line stops at station s in period p and zero otherwise. We assume that a line has to stop at the terminal stations, so no stopping variable is created for these stations.

Besides stopping variables, candidate lines also have frequency variables. Each candidate line l has a set of frequencies $\mathcal{F}^l$, denoting how many trains per hour are allowed to be operated on the line. For each of these frequencies, a binary decision variable $f_i^{l,p}$ is created that is equal to one if the line is operated with frequency $i \in \mathcal{F}^l$ in period p and zero otherwise. If a candidate line is not selected in the final line plan, all its frequency variables will take value zero.

Rolling stock allocation is not taken into account in the model. Instead, for each route a fixed train capacity is given in the input. The capacity could for example be the capacity of the largest train that can be operated on that route.

3.3.2 Modelling passengers and travel time

The objective of the mathematical model is to minimise the total Generalised Journey Time of the passengers while staying within a fixed given budget for the line plan's costs. The GJT was first introduced by Tyler & Hassard (1973) to evaluate the timetable related attractiveness of different train services. To calculate the in-vehicle time and waiting times for the GJT, we assign the passengers to arcs in the change-and-go (C&G) network. The C&G network was first introduced by Schöbel & Scholl (2005) to model passenger transfers in a public transport network. Two versions of the C&G network are used: one with asymmetric lines and one with symmetric lines.

Modelling passengers with asymmetric lines

In this section, we describe the C&G network that is used when asymmetric lines are considered. In Figure 3.1, a small example of the C&G network is given. The example contains three

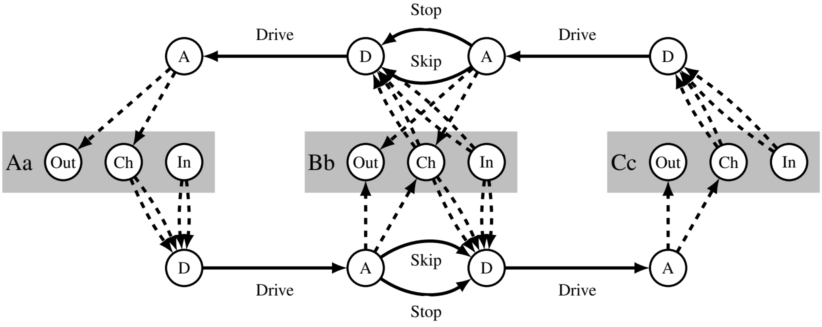

Figure 3.1: Example of a change-and-go network with two lines that operate in opposite directions.

stations (Aa, Bb, and Cc) and two (one-directional) train lines: one from station Aa to station Cc, and one from Cc to Aa. Both lines can be operated at two different frequencies. The different types of nodes and arcs are introduced in the next two paragraphs

In the C&G network, each train line gets a Departure (D) node at each station on the line except the destination station and an Arrival (A) node at each station except the first station on the line. Each station in the C&G network is represented by three nodes: the In, Change, and Out node. Passengers can access the public transport network at the In node and leave the network at the Out node. The Change node can be used to change lines at that station. Instead of creating a Change node for every station, it is possible to define Change nodes only for a certain set of transfer stations $\mathcal{S}^T$. Typically, larger stations are more suitable for transfers, due to their superior platform infrastructure and enhanced passenger facilities, such as the availability of amenities and shelter from the elements. Limiting transfer opportunities to these key stations not only reflects practical considerations in passenger experience, but also helps to reduce the model complexity, thereby improving computational tractability.

The C&G network also contains seven types of arcs connecting the different nodes. In Figure 3.1, the arcs related to train lines are denoted by solid lines, while the arcs used by passengers to board and alight trains are denoted by dashed lines. There are three types of arcs related to train lines: Drive arcs, Skip arcs and Stop arcs. Drive arcs are added from Departure nodes to the Arrival nodes at the next station and represent the train driving to the next station on the line. The Skip and Stop arcs are added at each intermediate station on the line. If the train line stops at the intermediate station, the Stop arc will be used, and otherwise the Skip arc will be used. Next, there are four types of arcs that are related to passengers: In, Out, In-change, and Out-change arcs. From each Arrival node, an Out arc connects to the Out node of the station for passengers that arrived at their destination and leave the network. The Out-change arc is created from the Arrival node to the Change node, for people who would like to change lines at this station. The In and In-change arcs are arcs from the In and Change nodes of the station, respectively, to the Departure node of the train line. Following the work of Bull et al. (2019), instead of one arc from the In and Change node to the train line, we introduce one arc per possible frequency of the train line. In the example of Figure 3.1, the train lines can be operated at two different frequencies and therefore there are two arcs going from each In and Change node to the Departure nodes of the train line. By introducing an arc for boarding the train for each frequency, we can set the cost of those arcs to a frequency dependent waiting time for boarding a line without introducing quadratic terms in our mathematical model.

The notation of the C&G network is given in Table 3.4. All arcs in the C&G network are linked to a certain candidate line: either arcs are used to describe a movement of a train on the line (Drive, Skip or Stop), or arcs are used to describe movements of passengers boarding or alighting trains on the line (In(-change) and Out(-change)). Furthermore, we can link all arcs, except the Drive arcs, to a specific station, because a line stops at or skips a station and passengers board or alight at a station. It is useful to have for each type of arc (e.g., Stop arcs), except Drive arcs, a separate set of arcs connected to a line l (denoted by $\mathcal{A}_l^{\text{Stop}}$) and a set of arcs connected to a station s (denoted by $\mathcal{A}_s^{\text{Stop}}$). Then, if we want to create a constraint for a Stop arc of candidate line l at station s, we can find the correct arc by taking the intersection of the sets: $\mathcal{A}_l^{\text{Stop}} \cap \mathcal{A}_s^{\text{Stop}}$.

The GJT can be calculated using the C&G network, by assigning a travel time related cost to each arc and determining the number of passengers that use that arc. The cost of using arc a is denoted by t^a, which is assumed to be given. For each type of arc, a different cost is set. When a

Table 3.4: Notation used for the change-and-go network.

V	Set of all nodes in the change-and-go network.
$\nu^{\text{Arr}}_{l,s}$	Denotes the arrival of line l at station $s \in \mathcal{S}^l$. This node is generated for all $l \in \mathcal{L}$ and for all stations $s \in \mathcal{S}^l$, $s \neq s^l_1$.
$\nu^{\text{Dep}}_{l,s}$	Denotes the departure of line l at station $s \in \mathcal{S}^l$. This node is generated for all $l \in \mathcal{L}$ and for all stations $s \in \mathcal{S}^l, s \neq s^l_{n_l}$.
ν^{In}_{s}	The source node of station s, which is generated for each $s \in \mathcal{S}$.
ν^{Out}_{s}	The sink node of station s, which is generated for each $s \in \mathcal{S}$.
ν^{Change}_{s}	The change node of station s, which is generated for each $s \in \mathcal{S}^T$.
$\mathcal{A}$	Set of all arcs in the change-and-go network.
$\mathcal{A}^+(\nu)$	Set of all arcs in the change-and-go network that enter node $\nu \in$ V.
$\mathcal{A}^-(\nu)$	Set of all arcs in the change-and-go network that leave node $\nu \in$ V.
$\mathcal{A}^{\text{Drive}}_l$	Set of arcs which represent driving from one station to the next on line $l \in \mathcal{L}$. A drive arc $(\nu^{\text{Dep}}_{l,s^l_j}, \nu^{\text{Arr}}_{l,s^l_{j+1}})$ is created for each candidate line $l \in \mathcal{L}$ and for each station $s^l_j \in \mathcal{S}^l \setminus s^l_{n_l}$.
$\mathcal{A}^{\text{Skip}}_l$	Set of arcs which represent skipping a stop on line $l \in \mathcal{L}$. A skip arc $(\nu^{\text{Arr}}_{l,s}, \nu^{\text{Dep}}_{l,s})$ is created for each candidate line $l \in \mathcal{L}$ and each station $s \in \mathcal{S}^l \setminus \{s^l_1, s^l_{n_l}\}$.
$\mathcal{A}^{\text{Stop}}_l$	Set of arcs representing a stop at a station on line $l \in \mathcal{L}$. A stop arc $(\nu^{\text{Arr}}_{l,s}, \nu^{\text{Dep}}_{l,s})$ is created for each candidate line $l \in \mathcal{L}$ and each station $s \in \mathcal{S}^l \setminus \{s^l_1, s^l_{n_l}\}$.
$\mathcal{A}^{\text{In}}_l$	Set of arcs representing passengers boarding line $l \in \mathcal{L}$ from their origin station. For each candidate line $l \in \mathcal{L}$, for each $s \in \mathcal{S}^l \setminus s^l_{n_l}$, and for each frequency $i \in \mathcal{F}^l$ an entrance arc $(\nu^{\text{In}}_{s}, \nu^{\text{Dep}}_{l,s})$ is created.
$\mathcal{A}^{\text{Out}}_l$	Set of arcs which represent passengers leaving line $l \in \mathcal{L}$ because they arrived at their destination station. For each candidate line $l \in \mathcal{L}$ and for each $s \in \mathcal{S}^l \setminus s^l_1$ an exit arc $(\nu^{\text{Arr}}_{l,s}, \nu^{\text{Out}}_{s})$ is created.
$\mathcal{A}^{\text{In-ch}}_l$	Set of arcs which represent passengers entering line $l \in \mathcal{L}$ after transferring at a station. For each candidate line $l \in \mathcal{L}$, for each station $s \in (\mathcal{S}^l \cap \mathcal{S}^T) \setminus s^l_{n_l}$, and for each frequency $i \in \mathcal{F}^l$ an incoming change arc $(\nu^{\text{Change}}_{s}, \nu^{\text{Dep}}_{l,s}; i)$ is created.
$\mathcal{A}^{\text{Out-ch}}_l$	Set of arcs which represent passengers leaving line $l \in \mathcal{L}$ to transfer at a station. For each candidate line $l \in \mathcal{L}$ and for each station $s \in (\mathcal{S}^l \cap \mathcal{S}^T) \setminus s^l_1$ an outgoing change arc $(\nu^{\text{Arr}}_{l,s}, \nu^{\text{Change}}_{s})$ is created.
$\mathcal{A}^X$	Set of all arcs of type X, where $X \in$ {Drive, Skip, Stop, In, Out, Ch-in, Ch-out}. $\mathcal{A}^X := \cup_{l \in \mathcal{L}} \mathcal{A}^X_l$.
$\mathcal{A}^X_s$	Set of arcs of type X linked to station $s \in \mathcal{S}$, where $X \in$ {Skip, Stop, In, Out, Ch-in, Ch-out}. Note that $\mathcal{A}^X_s \subset \mathcal{A}^X$.

is a Drive arc $(a \in \mathcal{A}^{\text{Drive}})$, t^a is the driving time between stations. For Stop arcs $(a \in \mathcal{A}^{\text{Stop}})$, t^a represents the dwell time at a station and a correction for the time loss due to braking before the stop and accelerating after the stop. If $a \in \mathcal{A}^{\text{Skip}}$, t^a is set to 0 to represent the absence of time loss by skipping the stop. For Out and Out-change arcs $(a \in \mathcal{A}^{\text{Out}} \cup \mathcal{A}^{\text{Out-ch}})$, t^a is set to a fixed penalty that represents the time-loss due to braking. Lastly, if $a \in \mathcal{A}^{\text{In}} \cup \mathcal{A}^{\text{In-ch}}$, the arc cost will be a frequency-dependent penalty for the waiting time plus a fixed correction for acceleration of the train. More details on how these times are calculated can be found in Section 3.4.3.

To track the routing of passengers through the C&G network, passenger flow variables are used. As the intention is to create a line plan that can be repeated every hour within a period, flow variable $y^{a,p}_s$ denotes the number of passengers that originate from station s and use arc a

during one hour in period p. Grouping passenger flows by origin station, introduced by Bull et al. (2019), significantly reduces the number of variables needed to represent the passenger flows compared to having a flow variable for each origin-destination pair. To ensure that all passengers are assigned a route through the network from their origin to their destination, the passenger demand is transformed to node demand. Following the work of Bull et al. (2019), let the demand at vertex ν for passengers who originate from station s during an average hour in period p be denoted by $\delta_s^{\nu,p}$. This demand represents the number of passengers entering the node minus the number of passengers leaving the node, and is defined as follows:

$$\delta_s^{\nu,p} = \begin{cases} d_{s,t}^p & \text{if vertex } \nu \text{ is the Out node for station } t, \\ -\sum_{t \in S} d_{s,t}^p & \text{if vertex } \nu \text{ is the In node for station } s, \\ 0 & \text{otherwise,} \end{cases}$$

where $d_{s,t}^p$ denotes the average hourly demand from station s to station t in period p. If the demand is satisfied at each node in the C&G network, all passengers travel from their origin to their destination and the passenger flow is conserved when a node is not the origin or the destination. With the flow variables, we can determine how many passengers use each arc in the C&G network. The GJT for these passengers can be calculated by multiplying the cost of each arc with the number of passengers using that arc and summing over all arcs. As the flow variables only represent the passengers during one hour per period and each period can have a different length, the flow variables are multiplied by the length of the period (h^p) to calculate the total GJT in the considered time frame. Hence, the total GJT can be computed with the following formula:

$$\sum_{p \in \mathcal{P}} \sum_{a \in \mathcal{A}} \sum_{s \in S} t^a h^p y_s^{a,p}. \tag{3.1}$$

Adaption for modelling symmetric lines

The previous section assumes that the candidate lines are defined in only one direction. Since each candidate line has its own route and variables for frequency and stop choice, the stopping patterns and frequencies of two lines traversing the same stations in opposite directions are determined independently. This independence gives the model more opportunities to reduce the total GJT, as it can adjust stopping patterns and frequencies for opposite directions when the demand on a route is asymmetric. However, currently in the Netherlands the line plan only contains symmetric lines. A symmetric line is operated in both directions with the same frequency and stopping pattern, which makes a line plan with only symmetric lines easier to understand and remember for passengers. Therefore, we also develop a version of the model featuring only symmetric lines, enabling us to evaluate the advantages and disadvantages associated with employing solely symmetric lines.

One way to ensure the creation of symmetric lines is by adding constraints stating that the value of the stop variables and frequency variables should be equal for two lines that run in opposite directions. However, by changing the C&G network instead, the number of nodes and arcs in the C&G network can be reduced as well as the number of variables and constraints needed. Since such a modification positively affects the scalability and solvability of the model, it has been selected as the preferred method. In the remainder of this section, we describe which changes should be made to the C&G network when we assume that lines are operated in two directions instead of one.

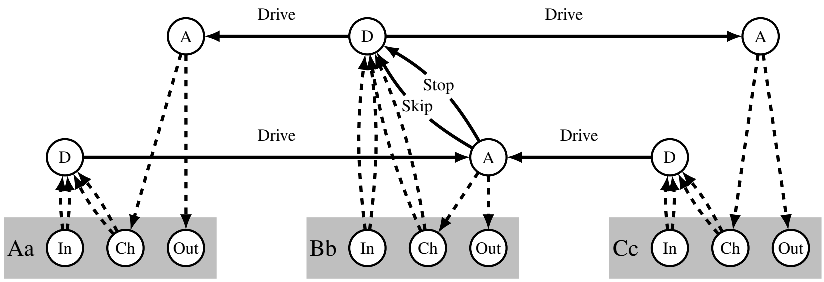

Figure 3.2: Example of a change-and-go network with one symmetric line that is operated in both directions.

In Figure 3.2, an example of a C&G network with a symmetric line is shown. The difference between this network and the C&G network with asymmetric lines, as displayed in Figure 3.1, consists of the following changes:

- An Arrival and Departure node is created for every station on the line. In the asymmetric case, no arrival node is created for the first station on the line and no departure node is created for the last station on the line.
- Drive arcs are added in both directions, instead of in only one direction.
- The In and In-change arcs are created for all stations. In the asymmetric case, these arcs are not created for the last station on the line.
- The Out and Out-change arcs are created for all stations. In the asymmetric case, these arcs are not created for the first station on the line.

By making these changes to the creation of the C&G network, we can use all arcs except the Drive arcs for travelling in both directions on the same line. Hence, we get a C&G with fewer arcs, which also means that fewer flow variables are needed as these variables are introduced for each arc. Besides fewer flow variables, the number of frequency and stop variables needed can also be halved since these now determine the frequency and stops in two directions. In order to account for these differences in the C&G network in the mathematical model, we introduce symmetry parameter σ. Parameter σ takes value 2 if symmetric lines are considered and 1 otherwise. In the next section, we describe how the symmetry parameter is used in the mathematical model.

3.3.3 Mathematical formulation

In this section, the variables, parameters, and notation described in Tables 3.2, 3.3, and 3.4 are used to define the constraints and objective of the multi-period line planning model.

Constraints

The constraints of the multi-period line planning model are given in (3.2)-(3.21).

$$\sum_{a\in\mathcal{A}^+(\nu)} y_s^{a,p} - \sum_{a\in\mathcal{A}^-(\nu)} y_s^{a,p} = \delta_s^{\nu,p} \qquad \forall s\in\mathcal{S},\ \nu\in\mathrm{V},\ p\in\mathcal{P} \tag{3.2}$$

$$\sum_{i\in\mathcal{F}^l} f_l^{i,p} \leq 1 \qquad \forall l\in\mathcal{L},\ p\in\mathcal{P} \tag{3.3}$$

$$\sum_{s\in\mathcal{S}} y_s^{a,p} \leq q^l \cdot \Big(\sum_{i\in\mathcal{F}^l} i\cdot f_l^{i,p}\Big) \qquad \forall l\in\mathcal{L},\ a\in\mathcal{A}_l^{\text{Drive}},\ p\in\mathcal{P} \tag{3.4}$$

$$\sum_{s\in\mathcal{S}} y_s^{a,p} \leq f_{max}^l \cdot x_u^{l,p}\cdot q^l\cdot\sigma \qquad \forall l\in\mathcal{L},\ u\in\mathcal{S}^l\setminus\{s_1^l,s_{n_l}^l\}, a\in\mathcal{A}_l^{\text{Stop}}\cap\mathcal{A}_u^{\text{Stop}},\ p\in\mathcal{P} \tag{3.5}$$

$$\sum_{s\in\mathcal{S}} y_s^{a,p} \leq f_{max}^l \cdot (1-x_u^{l,p})\cdot q^l\cdot\sigma \qquad \forall l\in\mathcal{L},\ u\in\mathcal{S}^l\setminus\{s_1^l,s_{n_l}^l\},\ a\in\mathcal{A}_l^{\text{Skip}}\cap\mathcal{A}_u^{\text{Skip}},\ p\in\mathcal{P} \tag{3.6}$$

$$\sum_{s\in\mathcal{S}} y_s^{a,p} \leq f_{max}^l \cdot x_u^{l,p}\cdot q^l\cdot\sigma \qquad \forall l\in\mathcal{L},\ u\in\mathcal{S}^l\setminus\{s_1^l,s_{n_l}^l\},\ p\in\mathcal{P},\ a\in\Big\{(\mathcal{A}_l^{\text{Out}}\cap\mathcal{A}_u^{\text{Out}})\cup(\mathcal{A}_l^{\text{In}}\cap\mathcal{A}_u^{\text{In}})\cup(\mathcal{A}_l^{\text{In-ch}}\cap\mathcal{A}_u^{\text{In-ch}})\cup(\mathcal{A}_l^{\text{Out-ch}}\cap\mathcal{A}_u^{\text{Out-ch}})\Big\} \tag{3.7}$$

$$y_s^{a,p} \leq f_i^{l,p}\cdot i\cdot q^l\cdot\sigma \qquad \forall l\in\mathcal{L},\ s\in\mathcal{S}^l\setminus\{s_{n_l}^l\},\ a\in(\mathcal{A}_l^{\text{In}}\cap\mathcal{A}_s^{\text{In}}),\ i\in\mathcal{F}^l,\ p\in\mathcal{P} \tag{3.8}$$

$$\sum_{s\in\mathcal{S}} y_s^{a,p} \leq f_i^{l,p}\cdot i\cdot q^l\cdot\sigma \qquad \forall l\in\mathcal{L},\ a\in\mathcal{A}_l^{\text{In-ch}},\ i\in\mathcal{F}^l,\ p\in\mathcal{P} \tag{3.9}$$

$$\sigma\cdot\Big(\sum_{l\in\mathcal{L}}\sum_{i\in\mathcal{F}^l} k^l\cdot i\cdot f_i^{l,p}\Big)\leq b^p \qquad \forall p\in\mathcal{P} \tag{3.10}$$

$$\sum_{l\in\mathcal{L}_s^-}\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{F}^l} h^p\cdot i\cdot f_i^{l,p} = \sum_{l\in\mathcal{L}_s^+}\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{F}^l} h^p\cdot i\cdot f_i^{l,p} \qquad \forall s\in\mathcal{S}^E \tag{3.11}$$

$$\sigma\cdot\Big(x_s^{l,p^j}-x_s^{l,p^{(j-1)}}\Big)\leq \widetilde{x}_s^{l,p^j} \qquad \forall l\in\mathcal{L},\ j\in\{2,\dots,|\mathcal{P}|\},\ s\in\mathcal{S}^l\setminus\{s_1^l,s_{n_l}^l\} \tag{3.12}$$

$$\sigma\cdot\Big(x_s^{l,p^{(j-1)}}-x_s^{l,p^j}\Big)\leq \widetilde{x}_s^{l,p^j} \qquad \forall l\in\mathcal{L},\ j\in\{2,\dots,|\mathcal{P}|\},\ s\in\mathcal{S}^l\setminus\{s_1^l,s_{n_l}^l\} \tag{3.13}$$

$$\sigma\cdot\Big(f_i^{l,p^j}-f_i^{l,p^{(j-1)}}\Big)\leq \widetilde{f}^{l,p^j} \qquad \forall l\in\mathcal{L},\ j\in\{2,\dots,|\mathcal{P}|\},\ i\in\mathcal{F}^l \tag{3.14}$$

$$\sigma\cdot\Big(f_i^{l,p^{(j-1)}}-f_i^{l,p^j}\Big)\leq \widetilde{f}^{l,p^j} \qquad \forall l\in\mathcal{L},\ j\in\{2,\dots,|\mathcal{P}|\},\ i\in\mathcal{F}^l \tag{3.15}$$

$$\sum_{l\in\mathcal{L}}\sum_{j=2}^{|\mathcal{P}|}\Bigg(\sum_{s\in\mathcal{S}^l\setminus\{s_1^l,s_{n_l}^l\}} \widetilde{x}_s^{l,p^j}+\widetilde{f}^{l,p^j}\Bigg)+e=\varepsilon \tag{3.16}$$

$$x_s^{l,p}\in\{0,1\} \qquad \forall l\in\mathcal{L},\ s\in\mathcal{S}^l\setminus\{s_1^l,s_{n_l}^l\},\ p\in\mathcal{P} \tag{3.17}$$

$$f_i^{l,p}\in\{0,1\} \qquad \forall l\in\mathcal{L},\ i\in\mathcal{F}^l,\ p\in\mathcal{P} \tag{3.18}$$

$$y_s^{a,p}\geq 0 \qquad \forall a\in\mathcal{A},\ s\in\mathcal{S},\ p\in\mathcal{P} \tag{3.19}$$

$$\widetilde{x}_s^{l,p^j}\geq 0 \qquad \forall l\in\mathcal{L},\ j\in\{2,\dots,|\mathcal{P}|\},\ s\in\mathcal{S}^l\setminus\{s_1^l,s_{n_l}^l\} \tag{3.20}$$

$$\widetilde{f}^{l,p^j}\geq 0 \qquad \forall l\in\mathcal{L},\ j\in\{2,\dots,|\mathcal{P}|\} \tag{3.21}$$

Constraint (3.2) ensures the conservation of passenger flow at all nodes. It guarantees that the flow entering node ν is equal to the flow leaving ν, except at stations' In and Out nodes, where the difference corresponds to the respective inflow and outflow of passengers. Next, (3.3) arranges that a maximum of one frequency is chosen for each line. Constraint (3.4) limits the passenger flow on a Drive arc to the capacity corresponding to the number of trains operated on the line. Therefore, if no frequency is selected, the line cannot be used to transport any passengers. Constraint (3.5) ensures that when line l will not stop at station s, the Stop arc cannot be used, while (3.6) ensures that the Skip arc cannot be used when l does stop at station s. Furthermore, constraint (3.7) assures that passengers can only enter or leave a train line when the train line stops at that station. To avoid creating quadratic constraints we use parameter f^l_{max} in constraints (3.5)-(3.7) instead of $\sum_{i \in \mathcal{F}^l} i \cdot f_l^{i,p}$ as was done in (3.4). Constraints (3.8) and (3.9) make sure that an arc from a station's In or Change node to a train line can only be used if the corresponding frequency is selected for the train line.

Constraints (3.5) through (3.9) collectively regulate if passengers may use the Stop, Skip, In, In-change, Out, and Out-change arcs. When only symmetric lines are considered, these arcs serve passengers travelling in both directions, effectively doubling their usage. Therefore, the capacities on these arcs are scaled accordingly by the symmetry multiplier σ. However, the capacity restrictions on Drive arcs, imposed by (3.4), ensure that the number of passengers travelling in each direction cannot exceed the frequency-dependent capacity. Together with the flow conservation constraints (3.2), these constraints guarantee that passenger flows on the Skip arcs, Stop arcs, and arcs for entering or leaving a line do not surpass vehicle capacities.

Constraint (3.10) ensures that the cost of the selected line plan does not exceed the given budget. The cost of a line plan depends on both the total number of kilometres driven and the total time it takes to execute (Pätzold et al., 2018). As we could not find appropriate weights to balance these two components, the cost of a period's line plan is defined as the total number of kilometres driven by the trains in one hour. These train kilometres are calculated by multiplying the length of each train line in one direction (k^l) by the chosen frequency for that line, and then summing over all lines. If we consider symmetric lines, each line is driven in both directions. Therefore, in that case we multiply the one-way cost of each line by two via the symmetry parameter σ.

Constraint (3.11) enforces that for each terminal station $s \in \mathcal{S}^E$, the total frequency of lines leaving the station should be equal to the total frequency of the lines entering it. By balancing the number of trains leaving and entering the terminal station, it is more likely that in a later planning stage a rolling stock plan can be found. When multiple train types are considered, the constraint can also be extended to ensure a balance per train type. Note that the constraint assumes that there are infinite parking capacities at the terminal stations: temporary imbalances in arriving and departing trains are permitted as long as the total number of arriving trains equals the total number of departing trains over the entire day. In the case of symmetric lines, the constraint can be removed as it will always be satisfied. This is because symmetric lines are operated in both directions, and hence the same number of trains will depart and arrive at each terminal station on the line.

Constraints (3.12)-(3.16) calculate and limit the number of adjustments made to the line plan throughout the day. We limit the number of adjustments, as making adjustments to the line plan during the day has a cost for both the passengers and the railway undertaking. Passengers value regularity in the timetable, because it makes it easy to use and remember the schedule. Wardman et al. (2004) and Johnson et al. (2006) show that a regular timetable increases the passenger demand. Furthermore, every adjustment to the line plan affects the timetable and possibly the rolling stock schedule. Since the public transport planning process still involves a great deal of manual labour, allowing many adjustments would result in a significant increase in cost for the railway undertaking.

In this work, we measure the dissimilarity between two successive line plans by counting the number of adjustments made to the stopping patterns and frequencies of the lines. Importantly, the comparison for each period (p^j) uses the line plan of the immediately preceding period (p^{j-1}) as a reference. For example, changes in P2 are measured relative to P1, and changes in P3 relative to period P2. This approach captures the cumulative number of changes made as the line plan evolves over the day. Previous work by Durán-Micco et al. (2022) and Schiewe et al. (2023) on line planning for bus and metro also provide metrics for measuring the (dis)similarity between different line plans. However, these metrics are mainly based on the common edges in both line plans, which makes them less suitable for appropriately evaluating the adjustments to the stopping pattern for railways. When a train would skip a stop, its route through the network would likely not change. Hence, we count the number of adjustments to the stopping pattern and frequencies instead. Note that the dissimilarity metric assumes that each adjustment made to the line plan carries equal weight. However, in practice, some adjustments, such as stopping at or skipping one extra station, are easier to implement than other adjustments. For example, increasing the frequency of a line from four to six trains per hour, while maintaining evenly spaced departures, is more complex. These differences could be reflected in constraint (3.16) by assigning different weights to adjustments. Nonetheless, as no established method currently exists to determine such weights, we have pragmatically chosen to assign equal weights.

Constraints (3.12) and (3.13) increase the lower bound of the change stop variable $\widetilde{x}_s^{l,p^j}$ to σ if the value of the corresponding stop variables is not the same in two consecutive periods. Similarly, constraints (3.14) and (3.15) set the lower bound of the change frequency variable $\widetilde{f}^{l,p^j}$ to σ if a different frequency is chosen in two consecutive periods. Note that each adjustment of the stopping pattern or frequency is counted once when asymmetric lines are considered. For symmetric lines each adjustment counts twice, as in that case the changes are made in both directions. Constraint (3.16) sums all change variables and, together with a slack variable e, requires the sum should be equal to a maximum number of changes ε. Lastly, constraints (3.17) and (3.18) denote that the stop variables and frequency variables are binary, and constraints (3.19)-(3.21) indicate that the flow, change stop, and change frequency variables should be non-negative.

Objective and ε-constraint method

The model's primary objective is to minimise the total GJT of passengers as defined in (3.1), while satisfying the constraints given in the previous section. In consultation with the railway undertaking NS, we have chosen not to include the operating cost of the line plan in the objective function. Instead, a maximum budget constraint is imposed, based on the available rolling stock. The purchase price (and consequently the depreciation) of a train represents the largest expense, far exceeding the operational costs. Therefore, once a train has been acquired, the aim is to utilise it as much as possible to transport passengers efficiently.

Besides the operating cost, there is also a cost associated with making adjustments to the line plan during the day for both passengers and the railway undertaking. Therefore, the number of allowed adjustments to the line plan are bounded by constraint (3.16). By solving the model for varying bounds on the number of allowed changes (ε), a set of Pareto optimal line plans can be generated. This provides a decision maker at the railway undertaking the opportunity to investigate multiple plans and select the one that is most to their liking. This so-called ε-constraint method is a popular method for solving multi-objective integer linear programming models (Mavrotas & Florios, 2013) and has been applied several times in the railway planning domain (see e.g., Binder et al. (2017), Yan et al. (2019), Stoilova (2020), Ning et al. (2022),Yang et al. (2024)).

In this work, the AUGMECON2 algorithm by Mavrotas & Florios (2013) is used to approximate the Pareto optimal solutions. In the original ε-constraint method, one objective is optimised while the other objective(s) are bounded by a changing right-hand side value to obtain different solutions. However, as pointed out by Mavrotas (2009), the original ε-constraint method does not guarantee that a solution is found that is not dominated by other solutions. In our case, there might be several line plans with the same total GJT but varying amounts of adjustments to the line plan over the day. In those cases, we would like to find the line plan with the least number of adjustments. Therefore, to find solutions that do not have more adjustments to the line plan than necessary, we add a small cost term to the model's objective to maximise the value of the slack variable e from constraint (3.16). This slack variable is multiplied by a small coefficient γ (usually between 10^{-3} and 10^{-6}), to ensure that the model first minimises the total GJT before minimising the number of adjustments. In the case study, $\gamma = 10^{-3}$ is used. Incorporating the cost term into the objective leads to the following optimisation model:

$$\begin{aligned} \text{Minimise} \quad & \sum_{p \in \mathcal{P}} \sum_{a \in \mathcal{A}} \sum_{s \in \mathcal{S}} h^p t^a y_s^{a,p} - \gamma \cdot e \\ \text{s.t.} \quad & \text{Constraints (3.2)} - \text{(3.21)} \end{aligned} \tag{MP-LPP}$$

A flow chart of the algorithm used to approximate the Pareto optimal solutions is given in Figure 3.3.

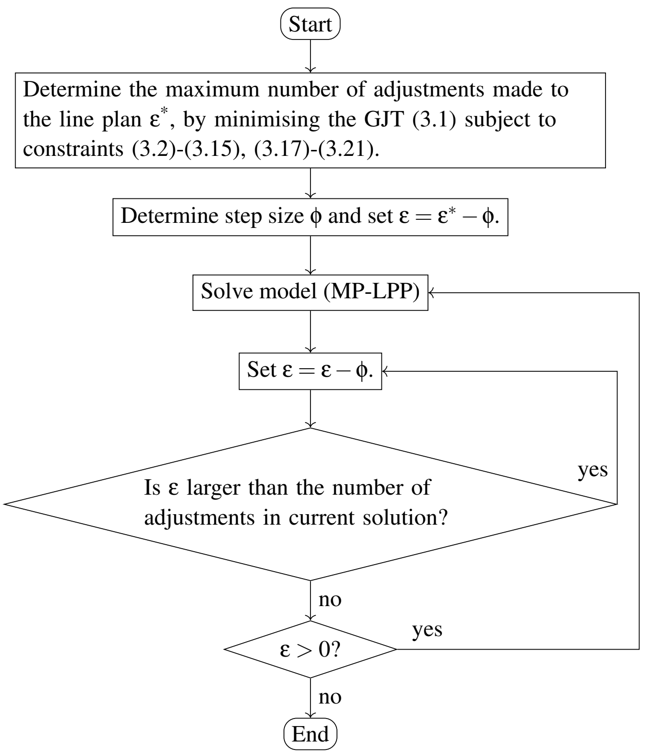

Figure 3.3: Algorithm used to approximate the Pareto optimal solutions.

3.4 Case study description

We test the model described in Section 3.3 on a case study based on part of the railway network in the Netherlands. The following subsections describe the network and candidate lines (Section 3.4.1), the time periods used (Section 3.4.2), and how the weights for the objective are computed (Section 3.4.3).

3.4.1 Network and candidate lines

The case study considers the network between the cities Leiden, The Hague, Rotterdam, and Utrecht. Rotterdam, The Hague, and Utrecht are the second, third, and fourth-largest municipalities of the Netherlands with populations between 673,000 and 377,000 (Centraal Bureau voor de Statistiek, 2025). Leiden is a smaller city with around 131,000 inhabitants, which makes it the third-largest city in the province South-Holland, after Rotterdam and The Hague. This network was selected as it has a lot of (directional) peak demand and different types of stations, including main stations of large cities and stations that are mostly interesting for commuters. Therefore, it is likely that different line plans are optimal in different periods, and hence that adjustments will be made during the day. A graphical representation of the network is depicted in Figure 3.4.

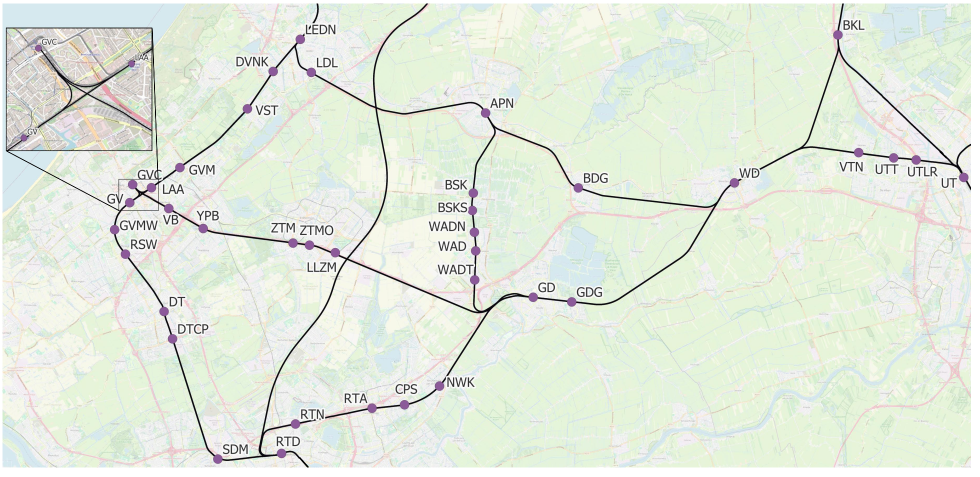

Figure 3.4: Part of the Dutch railway network between the cities Leiden (Ledn), The Hague (Gvc), Rotterdam (Rtd), and Utrecht (Ut).

In Table 3.5 an overview is given of the abbreviations used in Figure 3.4 and the corresponding station names. The table also denotes if a station is a terminal station, if transferring between trains is possible, and how many platforms and tracks are available at the station (Sporenplan, 2026). We have tested different sets of transfer stations and these tests showed that when transfers are possible at all stations, the same line plan was found as when transfers are only allowed at the larger stations. However, the solver needed either more time to solve the instance or had a larger optimality gap when transfers were possible at all stations. Therefore, we choose to only allow for transfers at the large stations in the network.

Table 3.5: Overview of stations used in the case study.

Abr.	**Station name**	**Terminal station**	**Transfer station**	**Num. platforms**	**Num. tracks**
Apn	Alphen a/d Rijn	✓	✓	3	4
Bdg	Bodegraven	-	-	2	2
Bkl	Breukelen	✓	✓	2	4
Bsk	Boskoop	-	-	2	2
Bsks	Boskoop Snijdelwijk	-	-	1	1
Cps	Capelle Schollevaar	-	-	2	2
Dt	Delft	-	✓	4	4
Dtcp	Delft Campus	-	-	4	4
Dvnk	De Vink	-	-	4	4
Gd	Gouda	✓	✓	5	8
Gdg	Gouda Goverwelle	✓	-	4	4
Gv	Den Haag HS	-	✓	5	10
Gvc	Den Haag Centraal	✓	✓	12	12
Gvm	Den Haag Mariahoeve	-	-	4	4
Gvmw	Den Haag Moerwijk	-	-	4	4
Gvy	Den Haag Ypenburg	-	-	2	4
Laa	Den Haag Laan van NOI	-	✓	4	4
Ldl	Leiden Lammenschans	-	-	1	1
Ledn	Leiden Centraal	✓	✓	6	10
Llzm	Lansingerland-Zoetermeer	-	-	2	2
Nwk	Nieuwerkerk a/d IJssel	-	-	2	2
Rsw	Rijswijk	-	-	4	4
Rta	Rotterdam Alexander	-	-	2	2
Rtd	Rotterdam Centraal	✓	✓	13	17
Rtn	Rotterdam Noord	-	-	2	2
Sdm	Schiedam Centrum	-	✓	4	4
Ut	Utrecht Centraal	✓	✓	16	16
Utlr	Utrecht Leidsche Rijn	-	-	2	4
Utt	Utrecht Terwijde	-	-	2	4
Vb	Voorburg	-	-	2	2
Vst	Voorschoten	-	-	4	4
Vtn	Vleuten	-	-	2	4
Wad	Waddinxveen	-	-	1	1
Wadn	Waddinxveen Noord	-	-	2	2
Wadt	Waddinxveen Triangel	-	-	1	1
Wd	Woerden	-	✓	4	6
Ztm	Zoetermeer	-	-	2	2
Ztmo	Zoetermeer Oost	-	-	2	2

The set of candidate lines used in the case study is given in Table 3.6. The first column displays the names of the candidate lines, which consist of the first station and last station. A subroute with an earlier terminal is considered a different route, since terminals cannot be skipped in the stop pattern. As we only consider one candidate line per route in the case study, the names are unique. Tests with multiple candidate lines per route indicated that the model

Table 3.6: Set of candidate lines for the case study with symmetric lines.

Name (l)	**Stations on route** ($\mathcal{S}^l$)	$\mathcal{F}^l$ **[trains/hr/dir]**	k^l **[km]**	q^l **[seats/train]**
Gd-Apn	Gd, Wadt, Wad, Wadn, Bsks, Bsk, Apn	{1, 2, 4}	17.5	314
Gvc-Gdg	Gvc, Vb, Gvy, Ztm, Ztmo, Llzm, Gd, Gdg	{1, 2, 4, 6}	30.5	816
Gvc-Ledn	Gvc, Laa, Gvm, Vst, Dvnk, Ledn	{1, 2, 4, 6}	15.6	816
Gvc-Ut	Gvc, Vb, Gvy, Ztm, Ztmo, Llzm, Gd, Gdg, Wd, Vtn, Utt, Utlr, Ut	{1, 2, 4, 6}	60.1	816
Ledn-Ut	Ledn, Ldl, Apn, Bdg, Wd, Vtn, Utt, Utlr, Ut	{1, 2, 4}	49.1	816
Rtd-Bkl	Rtd, Rtn, Rta, Cps, Nwk, Gd, Gdg, Wd, Bkl	{1, 2, 4, 6}	52.8	628
Rtd-Gdg	Rtd, Rtn, Rta, Cps, Nwk, Gd, Gdg	{1, 2, 4, 6}	26.0	628
Rtd-Gvc	Rtd, Sdm, Dtcp, Dt, Rsw, Gvmw, Gv, Gvc	{1, 2, 4, 6}	23.2	816
Rtd-Ledn	Rtd, Sdm, Dtcp, Dt, Rsw, Gvmw, Gv, Laa, Gvm, Vst, Dvnk, Ledn	{1, 2, 4, 6}	37.6	816
Rtd-Ut	Rtd, Rtn, Rta, Cps, Nwk, Gd, Gdg, Wd, Vtn, Utt, Utlr, Ut	{1, 2, 4, 6}	55.6	628

prefers one line per route with a high frequency over two lines with different stopping patterns and lower frequencies. Since we have a smaller solution space with one candidate line per route, and therefore can find better solutions in the same running time, we have chosen to include only one candidate line per route. If more lines per route are considered, an index can be added to create unique line names. The second column of Table 3.6 denotes the sequence of stations that a train passes in the network. This sequence determines the routing through the public transport network. Next, the third column displays the set of frequencies that a line is allowed to take when selected. The routes of lines between Gouda (Gd) and Alphen a/d Rijn (Apn) and between Leiden Centraal (Ledn) and Utrecht Centraal (Ut) partially go over single-track sections in the network. Therefore, the maximum frequency of these lines is set to 4 per hour instead of 6. Note that if the model does not select any frequency from $\mathcal{F}^l$ for line l, the line is excluded from the line plan. The second to last column displays the (one-way) length of the line in kilometres, which is taken as proxy for the cost of the line and the last column shows the seat capacity of trains on the line. The capacity is based on the largest train that can stop at all stations on a line's route. Note that Table 3.6 only shows the symmetric lines. For the case with asymmetric lines the reverse of these lines are also added to the set of candidate lines with the same frequency set, cost and capacity.

3.4.2 Periods and demand data

Three periods are considered in the case study: the morning peak, the midday off-peak, and the afternoon peak. Including these periods results in a large variation in demand. Passenger volumes are much larger in the peak periods than in the off-peak period and by looking at both the morning and afternoon peak, we also see a change in the direction of the passenger flows. For each period, a 1-hour OD matrix that represents the period's demand is needed as input for the model. In previous work of the authors (Van der Knaap et al., 2024), a method was developed to determine periods in which the railway demand is homogeneous. To apply the results of Van der Knaap et al. (2024), we use the same demand data set based on the realised demand of NS on Tuesdays in 2019. NS obtains this demand data from a smart card system with check-in and check-out gates. More details about the data set can be found in Van der Knaap et al. (2024) and details about the smart card system are provided in Van Oort et al. (2015).

According to Van der Knaap et al. (2024), the morning and afternoon peaks both have three different types of demand: start of the peak, hyper peak, and end of the peak. We choose to use the demand during the hyper peaks (in morning from 7:30–8:30 and in the afternoon from 16:30–18:00) to represent the peak demand. By using the demand from the busiest time during the peak, it is more likely that we find a line plan that can serve all the demand. Furthermore, Van der Knaap et al. (2024) found the demand characterising the midday off-peak to be the demand between 9:30 and 15:00. As the selected demand periods are not connected, we have to decide what schedule should be used in the periods 8:30–9:30 and 15:00–16:30. According to Van der Knaap et al. (2024), the demand during 8:30–9:30 is closer to the morning hyper peak than to the midday off-peak demand. Therefore, we propose to use the schedule found for the morning peak until the off-peak starts at 9:30. Similarly, the demand between 15:00 and 16:30 is closer to the demand in the afternoon hyper peak than the demand of the off-peak, so the schedule for the afternoon peak can be used from 15:00 onwards.

To create the 1-hour OD matrix for each period, the half hour OD matrices within that period are summed and then divided by the number of hours in the period. In the remainder of the chapter, the morning hyper peak period is referred to as P1, the midday off-peak period as P2, and the afternoon hyper peak period as P3. The period lengths are $h^{\text{P1}} = 1.5$, $h^{\text{P2}} = 5.5$, and $h^{\text{P3}} = 1.5$. Note that the length of P1 was increased by 0.5, such that P1 and P3 have the same length. This allows the model to create line plans that have imbalanced frequencies within periods but balanced frequencies over the whole day. While it is commonly observed that afternoon peak hours tend to last longer than morning peak hours, NS uses equal-length time windows for both in their peak pricing scheme, supporting the assumption of equal period lengths for P1 and P3. As the passenger volumes in the peak hours are much higher than the volumes during the off-peak, the budgets for the line plans reflect this difference. Budgets are set at $b^{\text{P1}} = b^{\text{P3}} = 3108$, based on the cost of the line plan operated in May 2023, with the off-peak budget halved to ($b^{\text{P2}} = 1554$) due to the significantly lower demand.

3.4.3 Arc costs used for GJT calculation

To calculate the GJT in the objective of (MP-LPP), we need to give a cost for each arc in the C&G network. For Drive arcs, the arc cost is the driving time between two stations, which is calculated using the length of each line section and the average speed on that section. For all other arcs, the costs are given in Table 3.7. The cost of the In and In-change arcs are dependent

Table 3.7: Used costs of arcs in the C&G-network. Costs are given in GJT minutes.

Arc type	**Cost of arc [min]**	**Arc type**	**Cost of arc [min]**
In-F1	55.85	In-change-F1	50.25
In-F2	31.85	In-change-F2	28.85
In-F4	17.35	In-change-F4	19.95
In-F6	10.85	In-change-F6	15.55
Out	0.7	Skip	0
Out-change	0.7	Stop	3.55

on the frequency of the line. For example, in Table 3.7 In-F2 denotes the cost of the In arc if the frequency of the line is two. The values in Table 3.7 of the In and In-change arcs include an acceleration penalty of 0.85 minutes to account for the increased travel time due to acceleration after the stop. Similarly, the costs of the Out and Out-change arcs include a deceleration time penalty of 0.7 minutes, to account for the extra travel time needed due to braking. The value of the Stop arc includes a dwell time of 2 minutes and both the acceleration and deceleration penalties.

The cost of the In and In-change arcs are based on the work of Guis et al. (2023) and de Bruyn et al. (2023), respectively. Both papers used a stated preference experiment to respectively estimate the entry and transfer resistance in terms of GJT for train passengers in the Netherlands. Guis et al. (2023) provide an entry resistance curve, which is shown in Figure 3.5. For each adaptation that a passenger makes from their preferred departure time, the entry resistance curve provides the GJT that the passenger experiences due to the change. Since the timetable is not yet known in this stage, and hence neither the adaptation time for the passengers, we assume that the passengers want to depart according to a uniform distribution. Furthermore, we assume that the trains on each line are equally spaced over the hour and that the network is not oversaturated, so passengers can generally fit in the train closest to their desired departure time. Given these assumptions, if a train has a frequency of two times per hour, the maximal adaptation time would be 15 minutes (earlier or later than the preferred departure time). If the passengers' preferred departure times are uniformly distributed, then the average adaptation time would be 7.5 minutes for a frequency of two. Using the entry resistance curve provided by Guis et al. (2023), we find that the cost of the corresponding in arc is equal to 31 GJT minutes, to which the previously discussed acceleration penalty of 0.85 is added. Similarly, the values of the other In arcs in Table 3.7 are derived from the same curve.

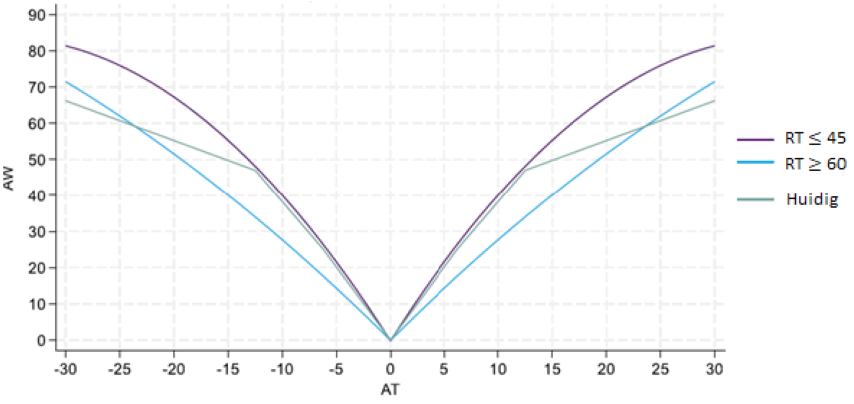

Figure 3.5: Entry resistance curve from Guis et al. (2023). Vertical axis denotes the entry resistance (in minutes), while the horizontal axis denotes the passenger's required adaptation from the desired departure time (in minutes). Values of the purple curve for travel times (denoted by RT in the legend) up to 45 minutes is used.

To calculate the GJT of a transfer, de Bruyn et al. (2023) derive a function that besides a fixed transfer penalty considers the type of transfer (e.g., cross-platform), the waiting time for the connecting train, and the time a passenger must wait for the next train if it misses the transfer. The function is used to calculate the values of the In-change arcs displayed in Table 3.7, under the assumption that the transfer will be cross-platform, the waiting time is half the headway, and the extra waiting time in case of a missed transfer is 60 divided by the frequency (assuming regular services). Ideally, a timetable offers relatively short transfer times for transfers with high passenger volumes, so assuming a waiting time equal to half the headway is somewhat pessimistic. However, since the timetable and the locations of high-volume transfers are not yet known at this stage, we have chosen to apply the half-headway waiting time after consultation with NS employees. The results of the case study are provided in the next section.

3.5 Results

In this section we demonstrate the model described in Section 3.3 by applying it to a case study using real data of part of the Dutch railway network as described in Section 3.4. The model was coded in Python 3.10.11 and solved with Gurobi Optimizer 10.0.1. All experiments were carried out on a laptop with Intel® Core™ i7-1185G7 @ 3.00GHz and 16 GB RAM. After Gurobi optimises the model with its presolve function, the scenarios with asymmetric lines had 33,709 constraints, 654 binary and 76,176 continuous variables, and 263,677 non-zeros, while the scenarios with symmetric lines had 23,368 constraints, 452 binary, 1 integer and 55,046 continuous variables, and 183,786 non-zeros.

3.5.1 General insights

Using the ε-constraint method, 10 different multi-period line plans with symmetric lines and 13 plans with asymmetric lines are generated by solving (MP-LPP) with different bounds ε on the number of line plan adjustments in constraint (3.16). Table 3.8 shows for each plan the type of lines used (symmetric or asymmetric), the total number of adjustments made to the line plan and how many of these are adjustments of the frequencies and adjustments of the stopping pattern. We count an adjustment to the line plan if the stopping pattern at a station or the frequency of a line is different relative to the previous period. The last three columns of Table 3.8 show the total GJT, the %-change in total GJT compared to the GJT of our reference scenario (denoted in bold), and the optimality gap of the solution. For calculating the optimality gap, we have used two options. The first option is the optimality gap reported by Gurobi after reaching the time limit. These gaps are denoted with a * in Table 3.8. As the gaps reported by Gurobi were quite large, we have also calculated a different lower bound by optimising the line plan for each period separately. Since the constraints concerning multiple periods (i.e., (3.11)-(3.16)) are not taken into account during these optimisation problems, they provide a lower bound for each period.

As reference scenario, we have selected the scenario with symmetric lines and 20 adjustments. In 2023, only symmetric lines were operated on the network, which is why we take the symmetric lines as reference. Furthermore, we need to allow for some adjustments in the reference scenario as in the midday off-peak (P2) only half the amount of train kilometres are allowed compared to the morning peak (P1) and afternoon peak (P3). Therefore, several adjustments of the frequency are needed to have line plans in P1 and P3 that (approximately) use the allowed budget. The scenario with 20 adjustments is the first scenario that can use at least 98.5% of the budget during each period, which is therefore used as the reference scenario. The need for changing the frequency can also be seen in the third and fourth column of Table 3.8. The first six scenarios with symmetric lines only adjust the frequency and the first five scenarios

*Table 3.8: Results of different line plans created by the ε-constraint method with a time limit of 2 hours (7200 seconds). The base scenario is the scenario with symmetric lines and 20 adjustments total (denoted in bold). Optimality gaps with a * were reported by Gurobi.*

Type of lines	Number of adjustments: Total	Freq.	Stopping pattern	Total GJT [min]	% difference in GJT w.r.t. base scenario	Optimality gap
Symmetric	0	0	0	6,744,095	19.74%	4.8%*
Symmetric	4	4	0	6,150,780	9.20%	16.5%
Symmetric	10	10	0	5,857,551	4.00%	10.9%
Symmetric	14	14	0	5,730,032	1.73%	8.5%
Symmetric	**20**	**20**	**0**	**5,632,353**	**-**	**6.7%**
Symmetric	24	24	0	5,576,424	-0.99%	5.6%
Symmetric	30	24	6	5,529,375	-1.83%	4.7%
Symmetric	32	24	8	5,527,530	-1.86%	4.7%
Symmetric	36	24	12	5,527,296	-1.87%	4.7%
Symmetric	40	24	16	5,523,326	-1.94%	4.6%
Asymmetric	0	0	0	6,574,592	16.73%	26.1%*
Asymmetric	5	4	1	6,119,410	8.65%	26.5%*
Asymmetric	10	10	0	5,804,499	3.06%	22.6%
Asymmetric	15	14	1	5,602,865	-0.52%	18.0%
Asymmetric	20	20	0	5,532,751	-1.77%	16.6%
Asymmetric	30	24	6	5,519,459	-2.00%	16.3%
Asymmetric	35	24	11	5,501,986	-2.31%	15.9%
Asymmetric	40	24	16	5,440,503	-3.41%	16.3%
Asymmetric	44	22	22	5,440,403	-3.41%	16.1%
Asymmetric	55	22	33	5,419,928	-3.77%	15.7%
Asymmetric	60	22	38	5,419,109	-3.79%	15.5%
Asymmetric	65	22	43	5,403,432	-4.06%	15.3%
Asymmetric	70	26	44	5,392,625	-4.26%	15.2%

with asymmetric lines also mainly focus on changing the frequency. After that, the focus shifts to making adjustments to the stopping pattern.

In Table 3.8, the reported line plans with the highest number of adjustments are the line plans that were found when no restrictions were imposed on the number of adjustments. For the case with symmetric lines, it is optimal to make 40 adjustments to the line plan, while for the case with asymmetric lines 70 adjustments are optimal. When comparing the GJT of these line plans with the GJT of the reference line plan, a maximal reduction of 1.94% in the GJT is observed for symmetric lines, while a maximal reduction of 4.26% occurs for asymmetric lines. Furthermore, comparing the total GJT of the scenarios with asymmetric lines for 70 and 20 adjustments reveals an improvement of 2.53%. Therefore, it can be concluded that the model can create a line plan with adjusted stops that better aligns with the demand, compared to the line plan that only makes minimal frequency adjustments to reduce service during the off-peak. Note that a 4.26% reduction in GJT can have a significant impact on a railway undertaking's revenue. According to a meta-analysis of time elasticities of travel demand in the United Kingdom, the GJT elasticity of trains is around -0.81 (Wardman, 2012), implying that a 4.26% reduction in

GJT would increase the revenue by about 3.45%. As the NS' revenue in 2023 in the Netherlands was €2877 million (Nederlandse Spoorwegen, 2024a), a 3.45% increase would be an increase of €99.3 million. The following two subsections examine more closely the effect of several design choices.

3.5.2 Impact of asymmetric lines

The first design choice examined is the selection of either asymmetric or symmetric lines. Table 3.9 compares the total GJT of the line plans with symmetric lines to the GJT of the line plans with asymmetric lines with the same number of adjustments. The first column denotes the number of adjustments that the line plans have, the second (third) column denotes the total GJT of the line plans with the symmetric (asymmetric) lines, and the last column denotes the percentage difference between the symmetric and asymmetric results. The last column shows that the line plans with asymmetric lines have a GJT that is between 0.2 and 2.5% lower than the GJT of line plans with symmetric lines and the same amount of adjustments. So if we allow for asymmetric lines, we can create line plans with lower total GJT given the same budget and allowed number of adjustments.

Table 3.9: Total GJT for line plans with symmetric and asymmetric lines with the same number of adjustments.

Number of	**Total GJT [min]**		
adjustments	**Symmetric lines**	**Asymmetric lines**	**%-difference**
0	6,744,095	6,574,592	-2.5%
10	5,857,551	5,804,499	-0.9%
20	5,632,353	5,532,751	-1.8%
30	5,529,375	5,519,459	-0.2%
40	5,523,326	5,440,503	-1.5%

In addition to achieving a lower GJT with the same budget and number of adjustments, considering asymmetric lines also allows for more opportunities to lower the GJT through changing the line plan. For the scenarios with asymmetric lines 70 adjustments were found, while for the symmetric lines a line plan with only 40 adjustments was created when no restriction was given for the number of adjustments. One reason is that, in the asymmetric case, adjustments can also be made in only one direction without the need to mirror these changes in the opposite direction. For example, if a station is skipped in the peak direction, it can still be served in the opposite direction to provide some service to these passengers. Such an approach saves GJT for passengers moving in the peak direction while still delivering service to the skipped station's passengers.

Since all solutions with symmetric lines are also allowed in the asymmetric case, the GJT is expected to be at least as good as in the symmetric case. However, the larger solution space associated with asymmetric lines results in a larger optimality gap compared to cases with symmetric lines. As can be seen in Table 3.8, the optimality gaps for the line plans with symmetric lines fall between 4.6 and 16.5%, while those for the line plans with asymmetric lines fall between 15.2 and 26.5%. So although we can find line plans that have lower GJT and are better adjustable to peak demand when we allow for asymmetric lines, it also comes with the cost of larger optimality gaps and potentially higher required running times. This could be problematic when good solutions need to be found in a relatively short time window.

3.5.3 Impact of frequency and stopping pattern adjustments

An analysis of the adjustments made to the line plan reveals that not all adjustments yield equal GJT savings. Furthermore, the adjustments selected in scenarios with a low number of allowed adjustments are often retained in scenarios with a higher number of allowed adjustments. For instance, the line plan with symmetric lines and 10 adjustments includes the 4 adjustments present in the line plan with only 4 adjustments. Figure 3.6 illustrates the decreasing GJT savings resulting from successive adjustments, showing the two approximated Pareto frontiers for both asymmetric and symmetric cases. In this figure, the horizontal axis represents the total GJT (in minutes) and the vertical axis shows the number of adjustments made to the line plan. Furthermore, the results for symmetric lines are marked by red squares, while asymmetric results are indicated by blue circles. Figure 3.6 clearly shows small changes in the GJT when many adjustments are used and large changes in GJT when adjustments are few. For example, the case with asymmetric lines going from 0 to 5 adjustments, reduces the GJT by 6.9%, while going from 40 to 44 adjustments only reduces the GJT by 0.002%. Given these diminishing GJT savings and the previously mentioned costs of adjustments for passengers and the railway undertaking, practitioners should carefully consider what the optimal number of adjustments is for their line plan.

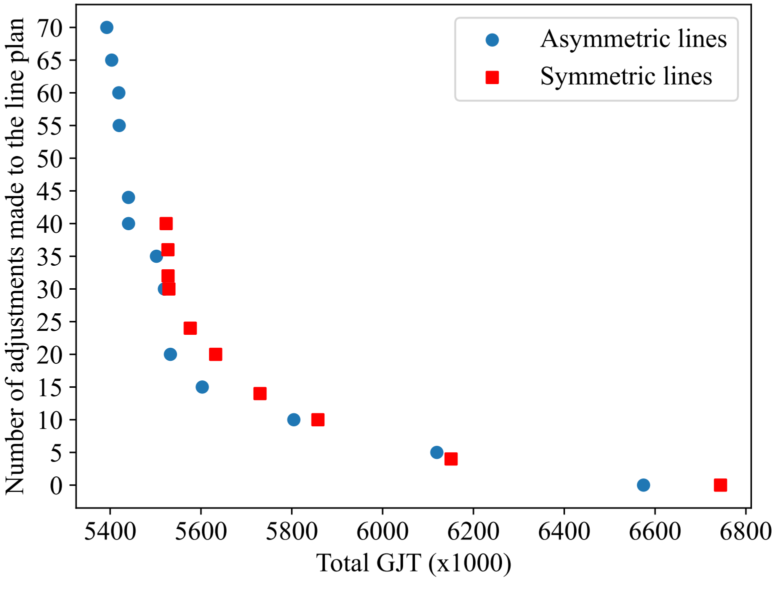

Figure 3.6: The approximated Pareto frontiers of the scenarios with symmetric lines and those with asymmetric lines

Looking at the third and fourth column in Table 3.8, the first 24 adjustments made to the line plan are mainly focused on the frequencies, while the additional adjustments after that primarily target the stopping pattern. Hence, the frequency adjustments apparently have a larger impact on the GJT than stopping pattern adjustments. This large impact can be explained, as the waiting time for boarding the first train and the transfer time are both frequency based. Since all passengers incur the cost of waiting to board the first train, all passengers that want to use a certain line will benefit if the frequency is increased. On the other hand, if an adjustment is made to the stopping pattern, then some passengers benefit (e.g., because they have a lower in-vehicle time due to a skipped stop) while few others incur higher costs (e.g., because they have to take a detour as their station is no longer served in their desired direction).

Line plan adjustments with symmetric lines

Figure 3.7 displays the reference line plan with symmetric lines and 20 adjustments. In the figure, a circle denotes that a line stops at a station. The frequencies of the lines are denoted at their terminal stations. Furthermore, this reference line plan has been compared to all line plans with symmetric lines and more than 20 adjustments. In Figure 3.7 we have also marked where the line plans are changed compared to the reference line plan. The colour denotes how often the line plan is changed at that location, where white denotes that no changes were made, and dark blue indicates that the change was made in many scenarios. The colour behind the frequency indicates how often it was changed in other scenarios. Similarly, for the stopping pattern: a coloured background behind a dot indicates a stop changed to a skip, while a coloured background behind a skip means the line stops at that station in other scenarios.

When comparing the frequencies over the periods of the reference line plan, it becomes evident that the frequencies of most lines are reduced by one step (e.g., from 6 to 4 or from 4 to 2) in the off-peak, and increased again for the afternoon peak. However, there are also two lines (between Rotterdam Centraal (Rtd) and Den Haag Centraal (Gvc) and between Gvc and Ut) that completely disappear in the off-peak. Note that the services that disappear in the off-peak are the fast 'Intercity' services that connect the larger stations. Since all stations in the network must be served in each period, the 'Sprinter' lines that stop at all stations must continue to operate. Not many changes are made to the frequencies in scenarios where more adjustments are allowed, indicating that adjustments chosen in the reference scenario are generally maintained in scenarios where more adjustments are allowed. However, the frequencies of the lines Rtd-Ledn, Rtd-Gvc and Gd-Apn are changed in most scenarios. In the scenarios with more adjustments, the frequency of Rtd-Gvc remains 6 throughout the day, while the frequency of Rtd-Ledn is reduced to 2 in the off-peak scenario. These adjustments keep the fast connection between the large stations between The Hague and Rotterdam, while still providing service to the smaller stations. The line between Gd and Apn usually takes frequency 2 in the peak periods and 1 in the off-peak, to provide better service in the peak periods.

When considering the changes made to the stopping pattern, considerable variation exists the colours. This indicates that once certain changes to the stopping pattern are made, they are usually repeated in the scenarios where more adjustments are allowed. The adjustments to the stopping pattern are mainly driven by large passenger flows during the peak hour. For example, the extra stop at Den Haag Laan van NOI (Laa) on the Gvc-Ledn line is often skipped in the peak hours, as there are many passengers who want to travel between Ledn and Gvc during the peak periods. By skipping this stop, the in-vehicle time of these passengers is reduced. However, the passengers travelling between Laa and Ledn are negatively affected by this adjustment, as their

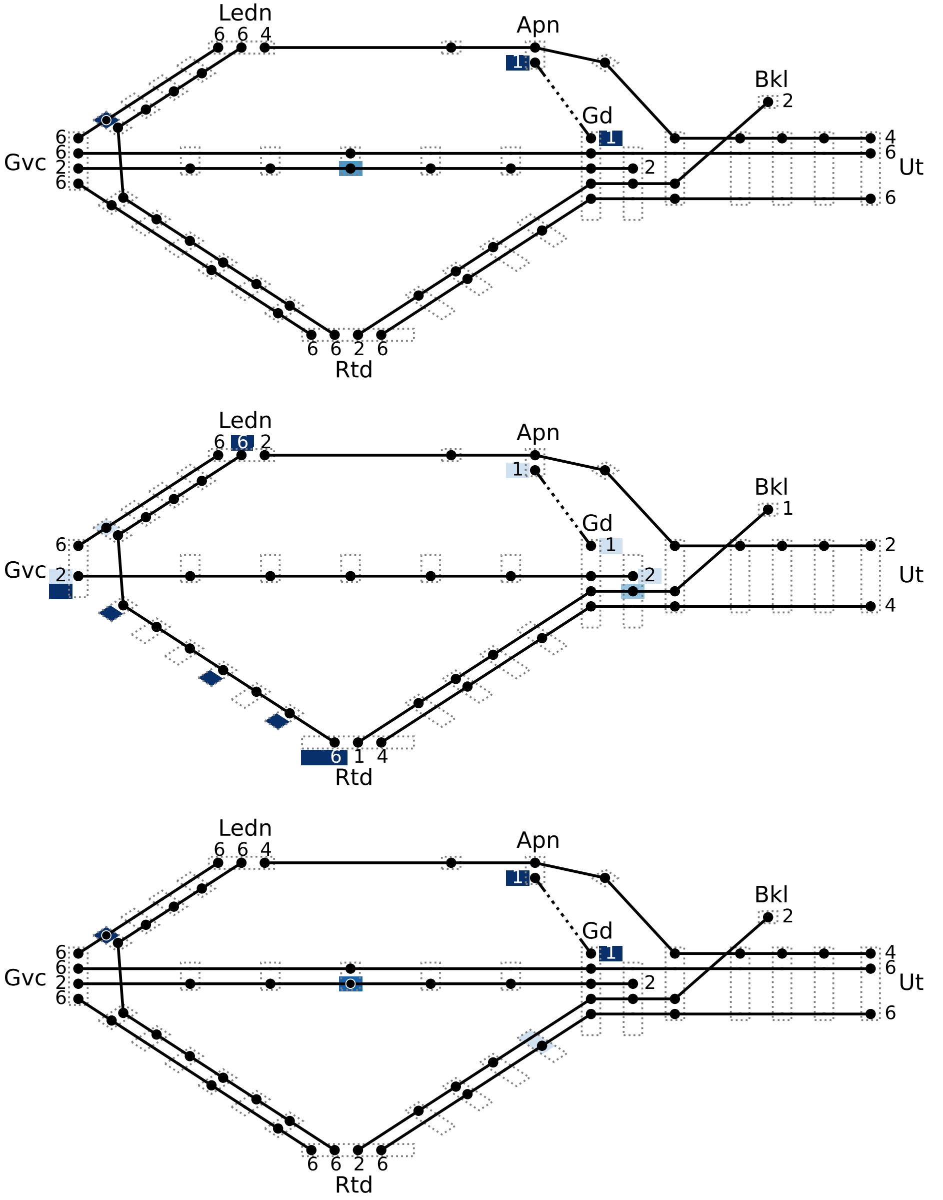

Figure 3.7: Line plan in P1, P2, and P3 (from top to bottom) with symmetric lines and 20 adjustments. Darker colours denote that the stopping pattern or frequencies were changed more often in scenarios with more adjustments. The line between Apn and Gd stops at all intermediate stations in all scenarios.

in-vehicle time is increased by the time of three additional stops. The increased GJT of these passengers turns out to be lower than the reduction in GJT for the passengers travelling between Gvc and Ledn, which is why the adjustment is made. However, during the off-peak there are fewer adjustments to the stopping pattern, since the small improvement in GJT for some OD pairs does not outweigh the inconvenience for other OD pairs.

Line plan adjustments with asymmetric lines

Similarly to Figure 3.7, Figure 3.8 displays the reference line plan with asymmetric lines and 20 adjustments. The figure shows lines operated in one direction, with arrows denoting the travel direction.

From the frequencies of the reference line plan it is apparent that on most routes the frequencies are symmetric and the same as in the reference line plan that uses symmetric lines. However, the lines between Rtd and Breukelen (Bkl) and between Apn and Gd do not have the same frequencies in the peak periods, with higher frequencies in the peak direction. The frequencies on these routes become symmetric during the off-peak. Interestingly, these asymmetric frequencies are changed very often and in most scenarios with asymmetric lines the frequencies are the same in both directions. For example, in almost all scenarios the lines between Rtd and Bkl have frequency 2 in both directions during the peaks and 1 during the off-peak. The lines between Apn and Gd have more variation in the frequency, but it is always symmetric in scenarios with more than 20 adjustments. For example, in 5 of the 8 additional scenarios both lines take frequency 4 in the morning peak and frequency 2 in the off-peak and afternoon peak. Another difference compared to frequencies in the symmetric line plans is observed on the lines between Gvc and Gouda Goverwelle (Gdg). In most scenarios the frequencies on this part of the network are 4 in P1 and P2, and 2 in P3, compared to a consistent frequency of 2 in the symmetric variants. The higher frequency in P1 and P2 is possible because the two lines between Rtd and Ut take frequency 4 instead of 6 in P1, and the frequency of the lines between Ledn and Rtd is reduced to 0 instead of 2 in P2.

Although most frequencies are symmetric, asymmetries in the stopping pattern occur in some parts of the network, most notably between Gvc and Ledn and between Gvc and Rtd. In the reference line plan, the stations Den Haag Mariahoeve (Gvm), Voorschoten (Vst), and De Vink (Dvnk) between Gvc and Ledn are all served by only one line (and hence in only one direction). Between Gvc and Rtd, most stations are served in both directions. However, the Intercity services that only serve the large stations and the Sprinter services that serve most or all stations are distributed over different routes. In the reference line plan, the lines Gvc-Rtd and Rtd-Ledn serve the large stations, while the lines Rtd-Gvc and Ledn-Rtd stop at most stations between Den Haag HS (Gv) and Rtd. During the off-peak, both lines between Gvc and Rtd have frequency 0, which results in the line Ledn-Rtd providing the Sprinter service and Rtd-Ledn the Intercity service between Gv and Rtd. In several other scenarios, the frequency of both lines between Ledn and Rtd are reduced to 0 in the off-peak, while the frequency of the lines between Gvc and Rtd are kept at 6. In those scenarios, the lines between Ledn and Rtd take an Intercity pattern during the peak periods, while the lines between Gvc and Rtd have a Sprinter pattern in both directions. During the off-peak, the line Rtd-Gvc keeps the Sprinter pattern, while Gvc-Rtd only stops at the large stations. This setup ensures that all stations receive service while facilitating faster connections in one direction.

When comparing the adjustments made in the scenarios with symmetric lines with the adjustments made in the scenarios with asymmetric lines there are some similarities. For example,

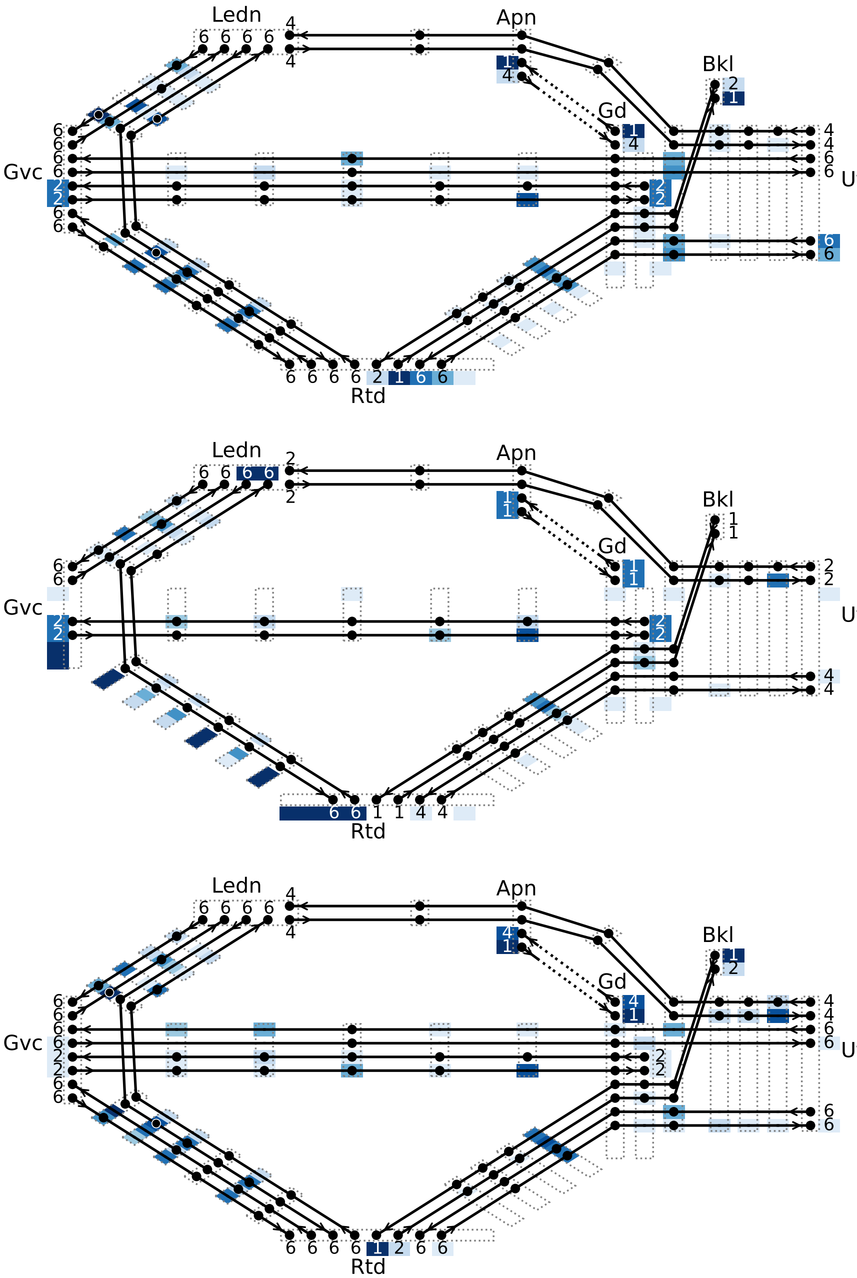

Figure 3.8: Line plan in P1, P2, and P3 (from top to bottom) with asymmetric lines and 20 adjustments. Darker colours denote that the stopping pattern or frequencies were changed more often in scenarios with more adjustments. The lines between Apn and Gd stop at all intermediate stations in all scenarios.

on most routes the frequencies are reduced by one step in the off-peak, and increased again during the afternoon peak. Furthermore, some lines that mostly provide Intercity services (e.g., between Gvc and Ut) are completely removed during the off-peak. However, by strategically serving some smaller stations with low demand in only one direction, the scenarios with asymmetric lines can achieve a lower GJT. By skipping these stations in one direction, faster connections can be provided to OD pairs with larger passenger demand, while keeping some level of service at the stations with lower demand. Furthermore, as both Sprinter and Intercity services can be combined on lines between Gvc and Rtd by giving each direction a different stopping pattern, the lines between Rtd and Ledn can be completely removed during the off-peak. This leaves budget to increase the frequency on the lines between Gvc and Gdg to 4 instead of 2, which benefits all passengers using this line.

3.6 Conclusion

The quality of the railway service offered to passengers is for a large part determined by the line plan, which influences factors such as direct trips, transfers, and in-vehicle times. The demand for railway services fluctuates throughout the day, with higher volumes during peak periods and varying demand structures. As the merits of a line plan depend on the demand it aims to serve, the quality offered by the line plan also varies throughout the day. Therefore, this chapter proposes a model that determines a train line plan with multiple periods that minimises the total GJT. Although in recent years there has been more interest in multi-period line planning, most models are proposed for and tested on corridors instead of networks and only incorporate a limited number of service adjustments and symmetric lines. To address these gaps, the mixed-integer linear programming model presented here incorporates the possibility for passengers to transfer, setting stopping patterns and frequencies, and having asymmetric lines. The proposed model was tested on a case study based on part of the Dutch railway network.

Multiple line plans were generated for the case study by varying the allowed number of adjustments to the line plan throughout the day. Comparing these plans to the reference line plan with symmetric lines and 20 adjustments reveals that allowing for more adjustments can significantly lower the total GJT. With symmetric lines, additional adjustments can reduce the GJT by up to 1.94%. Using asymmetric lines results in even greater reductions, lowering the GJT by 4.26% compared to the reference scenario. In general, line plans with asymmetric lines have a lower total GJT than line plans with symmetric lines under the same costs and number of adjustments. Furthermore, the additional GJT savings decrease when more adjustments are made. Early adjustments mainly target frequencies, which impact all passengers of the affected line by altering the waiting and transfer times. When the frequency of a line is increased, all passengers using that line benefit from it. Once frequency adjustments are implemented, the focus shifts to modifying stopping patterns. These adjustments offer less GJT reduction, since they benefit some passengers while inconveniencing others. In the case with symmetric lines, the adjustments to the stopping pattern are mainly skipping stops at small to medium large stations that are previously served by two lines. The majority of the passengers that are using the line do not want to (dis)embark at these stations and hence will have a shorter travel time. In the scenarios with asymmetric lines, stopping pattern adjustments allow for skipping certain stops in only one direction. This makes it possible to provide a fast service to passengers on large OD pairs, while still providing service to all stations by stopping in one direction.

While the model clearly demonstrates that asymmetric lines can reduce the total GJT, there are several factors to keep in mind for practical implementation. Asymmetric stopping patterns have a big impact on passengers travelling to or from a station in the opposite direction than served. These passengers will first need to travel farther away from their destination to be able to board a train in the right direction, potentially causing long detours. Furthermore, practitioners should consider how to clearly communicate asymmetric schedules to passengers. Traditional railway line maps typically show symmetric routes that remain fixed throughout the day. Visualising lines with differing routes and asymmetric stopping patterns that vary across the day is more challenging. Trip planner applications could assist by providing personalised journey information tailored to the trip and time of day. However, using these tools requires a certain level of computer literacy that some passengers may not possess. In addition to passenger impacts, operational considerations also apply. Line plans with a high degree of asymmetry can complicate rolling stock and crew scheduling, as less symmetric plans increase the likelihood of deadheading trips by crew members or rolling stock, raising operational costs. Given all these factors, practitioners should carefully weigh both passenger and operational trade-offs to determine whether, and to what extent, asymmetry is advantageous for their network.

In this chapter, we assume that the periods are fixed and part of the input. In the case study, demand for the morning hyper peak, midday off-peak and afternoon hyper peak are used, which are all very different. One question that would be interesting to tackle in a future study is how many periods can and should be provided to the model. Allowing more time periods may enable better alignment with demand patterns. However, transitioning between periods cannot be done instantly, as trains are driving a certain line somewhere on the network. Hence, another direction for future research is how the transition between two line plans can be done efficiently in the corresponding timetable planning. Scalability of the model is also an important aspect for determining the number of periods to include. Increasing the network size or the number of periods significantly increases the size of the model. Therefore, specialised heuristics to solve the resulting model could also be an interesting research direction. Lastly, while constraint (3.11) ensures an overall balance of incoming and outgoing trains at terminal stations across the entire day, it does not prevent short-term imbalances that could result in rolling stock shortages or insufficient parking capacities. To improve the operational feasibility of the determined line plans, future research could focus on integrating constraints that explicitly limit rolling stock imbalance within each time period and for each train type. Such enhancements would increase the likelihood that a feasible rolling stock circulation plan can be developed in conjunction with the line plan.

Chapter 4

Multi-period railway timetabling to serve time-dependent demand

The previous chapter introduced an optimisation model for creating multi-period line plans. This model takes as input several periods throughout the day, each with its own demand profile, and a set of allowed routes, and determines a line plan for each period that minimises the passengers' total Generalised Journey Time. The resulting multi-period line plan can be used as input for the timetabling problem, which is the focus of this chapter.

In this chapter, the novel concept of multi-period cyclic timetables is presented, together with a mixed-integer linear programming model to generate them. In the multi-period cyclic timetable, a cyclic timetable is maintained within each period, while feasible transitions are ensured between consecutive periods. The model aims to produce a timetable that is both attractive and memorable for passengers. Attractiveness is achieved by minimising in-vehicle times and ensuring regular departure intervals, while memorability is enhanced by penalising deviations from the cyclic schedules during transition periods. The model is validated on a case study based on part of the Dutch railway network, for which an optimal timetable is created.

Apart from minor changes, this chapter is published as a journal article:

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4.1 Introduction

The demand for passenger railway services exhibits significant variation during the day. At the beginning and end of the working day, there is a peak period in which a substantial number of commuters travel to or from their workplaces or educational institutions. Outside these peak periods, the passenger volumes are considerably lower and the travel patterns are different as a larger portion of the travellers are travelling for reasons other than commuting, like leisure activities or social visits. On a regular workday in the Netherlands, there can even be up to 10 distinct homogeneous demand patterns, according to Van der Knaap et al. (2024). Despite this variability in demand, in many European countries a fixed line plan and cyclic railway timetable is used, which is the same for every hour of the day. The primary benefit of a cyclic timetable is its memorability for passengers, which makes it easy to use. Cyclic timetables are often clock faced (i.e., trains always leave a certain number of minutes after the hour) and usually aim to have regular departure intervals between trains on the same line. According to Wardman et al. (2004), these two aspects both improve the memorability of a timetable. Furthermore, a cyclic timetable is also advantageous for people travelling outside the peak hours as there are still multiple travel options available during less busy times. Previous studies have shown that having a cyclic timetable with well-planned connections improves the passenger satisfaction and as a result the demand and revenues for the railway undertaking (Wardman et al., 2004; Johnson et al., 2006). However, the drawback of a cyclic timetable is its inflexibility to the passenger demand. Cyclic timetables are usually optimised for the peak-hour demand and hence the resulting schedules fail to address the distinct travel patterns of off-peak travellers, resulting in a suboptimal service provision during these times. On the other hand, acyclic timetables provide many opportunities to serve varying demand, but lack ease of use for the passengers. An overview of the benefits and drawbacks of cyclic and acyclic timetables, based on the work of Peeters (2003), is given in Table 4.1.

Table 4.1: Overview of benefits and drawbacks of cyclic and acyclic timetables

	Benefits	**Drawbacks**
Cyclic timetables	Easy to remember for passengers No gaps in train service when demand is low Consistent transfers throughout the day	Expensive, especially for varying demand Allocation of unused hourly time slots for non-hourly trains (e.g., freight or international trains)
Acyclic timetables	Many trains in periods with high demand Flexible to demand variations Less expensive for varying demand	More difficult to remember Less travel opportunities and/or gaps in service for periods with low demand Entire day needs to be considered during planning

In recognition of the benefits of both the cyclic and acyclic timetables, several studies have attempted to create a timetable that has both cyclic and acyclic characteristics. For example, Robenek et al. (2017) investigate three types of cyclicity in railway timetables. They show that out of the three options, a 'hybrid cyclic timetable' in which non-cyclic trains can only be scheduled in hours when also a cyclic train is scheduled, is best able to provide both regularity and flexibility. Other examples are the works of Yin et al. (2019) and Li et al. (2023), where a train is not necessarily scheduled in every hour, but whenever it is scheduled it always departs at the same number of minutes past the hour. However, one potential pitfall of creating a hybrid timetable that has both cyclic and acyclic characteristics, is that passengers do not recognise the regularity in the timetable and hence will not experience the benefits of the cyclicity.

In order to create a timetable that has as much cyclicity as possible while also following the varying demand, this chapter proposes a mathematical model for the multi-period timetabling problem. The mixed-integer linear programming (MILP) model takes as input several periods in a day in which the demand is homogeneous and for each of these periods a line plan that can serve this demand. The aim of the multi-period timetabling problem is then to create cyclic timetables for each of the defined periods and a good transition between the different cyclic timetables. Furthermore, regularity should be kept throughout the day wherever possible. Specifically, if lines are present in multiple period-specific line plans, the departures and arrivals of this line should deviate minimally to preserve passenger convenience. By having several periods with cyclic timetables and by keeping the regularity throughout the day wherever possible, we aim to provide both regularity to the passengers while simultaneously serving the demand better through the adapted lines. The proposed MILP model is tested on a case study based on part of the Dutch railway network. We show that the model can produce a multi-period timetable for this case study where the line plan has fixed frequencies but varying stopping patterns throughout the day. Changes in the stopping pattern, including both adding and removing stops, modify the minimal journey times of the trains. This in turn can create timetable conflicts that need to be resolved at some cost, e.g., by driving slower or having irregular intervals between departure. The proposed model aims to minimise these costs when creating the multi-period timetable.

The main contributions of this chapter are:

- A new concept of multi-period cyclic timetables to accommodate changing line plans corresponding to time-dependent demand.
- An optimisation model to compute multi-period timetables with smooth transitions between periods.
- A case study analysis that demonstrates the model's capability to generate a conflict-free timetable for a line plan with varying stopping patterns throughout the day.
- A sensitivity analysis of different objectives to illustrate the trade-off between minimising journey time and departure interval irregularity.

The remainder of this chapter is organised as follows. An overview of the relevant literature for the multi-period timetabling problem is given in Section 4.2. Next, in Section 4.3 the mathematical model is provided. The model is tested on a small case study of which a description and the results can be found in Section 4.4. A conclusion and suggestions for future research are provided in Section 4.5.

4.2 Literature review of timetabling for varying demand

The aim of this chapter is to provide a method to create a timetable with multiple cyclic periods that better matches the demand than a strictly cyclic timetable. As a multi-period timetable has both cyclic and acyclic characteristics, we briefly discuss these two types of timetables with a special focus on literature that aims to serve the time-dependent demand in Section 4.2.1. Next, Section 4.2.2 provides the literature that aims to add more flexibility to the timetable by allowing lines to have different cycle times. In Section 4.2.3, the literature that combines cyclic and acyclic timetables is discussed. Section 4.2.4 provides the literature that combines line planning and timetabling decisions, and the literature gaps are discussed in Section 4.2.5.

4.2.1 Cyclic and acyclic timetables

In many European countries, cyclic timetables are prevalent due to the ease-of-use for the passengers. Studies by Wardman et al. (2004) and Johnson et al. (2006) have demonstrated that cyclic timetables with well-coordinated connections enhance passenger satisfaction, thereby increasing demand and revenues for railway companies. The widespread adoption of cyclic timetables can be attributed to these benefits. In a cyclic timetable, the schedule recurs every cycle, usually every hour; for instance, a train departure at 8:05 will also occur at 9:05, 10:05, and so forth. Typically, such timetables are formulated using the Periodic Event Scheduling Problem (PESP) framework, initially developed by Serafini & Ukovich (1989). Given its frequent use in the literature, several overviews on timetabling with PESP exist, including Odijk (1996), Peeters (2003), and Caimi et al. (2017). However, PESP also has some challenges when the aim is to provide a passenger-friendly timetable. For example, a cyclic timetable is usually based on the demand of one cycle, for which often a peak hour is used. Therefore, variations in the demand during the day are not taken into account. Liebchen & Mohring (2007) show that modelling a balanced reduction of service during the off-peak hours is difficult within the PESP framework. Furthermore, they note that routing passenger flows is beyond the scope of PESP, which means that either assumptions are needed to approximate the passenger travel time or the PESP model needs to be extended.

Acyclic timetables, on the other hand, offer a lot of flexibility to serve demand that varies over time. Niu & Zhou (2013) minimise the number of waiting and denied passengers on an urban rail transit line with time-dependent demand and oversaturated conditions. They use a genetic algorithm to solve the proposed nonlinear mixed-integer program (MIP). Barrena et al. (2014a) create different MILP formulations for the timetabling problem with the objective of minimising the total waiting time and test them on a case study of the Madrid Metropolitan Railway. They show that when demand is fluctuating, significant improvements of the average waiting time can be realised when compared to a regular timetable. In a follow-up study, the same authors propose an Adaptive Large Neighbourhood Search (ALNS) algorithm to solve more realistic instances (Barrena et al., 2014b). Niu et al. (2015) aim to create a timetable on a corridor that minimises the passenger waiting time under time-dependent demand and predetermined skip-stop patterns. They propose a MIP model with a quadratic objective function and use the General Algebraic Modelling System to implement and solve it. Li et al. (2019a) investigate how equity can be incorporated in passenger-oriented railway timetabling with time-dependent demand. They try adding two fairness criteria, min-max fairness and α-fairness, to the objective of a non-linear integer programming model and show that in efficient timetables the waiting time is not always distributed fairly over the passengers. Yin et al. (2021) aim to reduce the crowdedness in (transfer)stations by coordinating the timetable of different lines to better serve the time-dependent demand. They propose two algorithms to solve this MILP model, namely an ALNS algorithm and a Decomposition-based ALNS.

Acyclic timetables are more suitable for considering time-dependent demand than strictly cyclic timetables. Since the goal of cyclic timetabling is to create a timetable that is repeated throughout the day, no changes to the timetable can be made to serve time-dependent demand. Instead, variations in demand volumes can be served by varying the rolling stock (e.g., using longer trains in the peak hours than during the off-peak). On the other hand, there have been several studies that consider time-dependent demand in acyclic timetabling in order to minimise the passenger waiting time or reduce crowdedness.

4.2.2 Timetabling with multiple cycle lengths

Besides the strictly cyclic and strictly acyclic timetable, there are also several studies that have attempted to make timetables that mix cyclic and acyclic characteristics in order to improve the service offered to customers. One direction of research to achieve this is by allowing lines to have different cycle times, e.g., some lines are operated every 2 or 3 hours instead of every hour. In that way, certain OD pairs can still have a direct connection even if there is not enough demand to provide this connection every hour. Caimi et al. (2011) present a framework with which a timetable can be created for lines with different cycle times. They create an augmented version of the periodic timetabling problem by projecting all lines on a single period, which can be modelled using e.g., PESP. Solving this augmented version results in slightly longer computation time, but eliminates the need for post-processing in the form of removing lines from certain periods and improves the solution quality of the final timetable. Zhou et al. (2017) propose a model for finding a periodic railway timetable that minimises the total train travel time, in which each train series can have its own period length. They use an algorithm based on Lagrangian relaxation decomposition to solve the proposed model for several case studies based on a small railway network and the high-speed railway between Beijing and Shanghai.

Although considering larger and varying cycle times can help to offer more direct connections, the aim of the aforementioned research is still to create a cyclic timetable that can be repeated throughout the day. However, when the demand is significantly different in different periods (for example in the peak and off-peak hours), simply lengthening the cycle time will not be enough to serve this period-specific demand as different lines could be needed.

4.2.3 Hybrid timetabling

Other studies have looked beyond varying the cycle times of different lines by including more acyclic characteristics. Robenek et al. (2017) investigate three types of cyclicity in railway timetables. In the 'θ shifted cyclic timetable', each line has a cyclic departure time, but trains are allowed to be scheduled with a tolerance of at most θ minutes from this departure time. Alternatively, in the 'ξ partially cyclic timetable', ξ% of the trains in the timetable should be cyclic, while the remainder of the trains have no departure time requirements. Lastly, the 'hybrid cyclic timetable' is investigated, in which non-cyclic trains can only be scheduled in hours in which also a cyclic train is scheduled. Tests on the Israeli Railways network showed that the 'hybrid cyclic timetable' is best able to keep a good level of regularity, while also achieving the flexibility of a non-cyclic timetable. Yin et al. (2019) propose a framework in which the arrival and departure times of trains are generally regular throughout the day, but the distribution of trains over the planning horizon depends on the passenger demand. The proposed MIP model minimises the weighted sum of the total train travel time and the total passenger waiting time and is solved using a 3-step heuristic. Li et al. (2023) propose a multi-objective integer programming model to solve an integration of the railway timetabling problem and the rolling stock circulation problem under time-dependent demand. The model's objective is to minimise the total passenger waiting time at their origin station and the total operating cost of train services. The model is solved exactly using Gurobi and the ε-constraint method is used to create a Pareto frontier.

These studies show that hybrid timetabling can create timetables for time-dependent demand that consider both passenger attractiveness and costs for the railway undertaking. However, a key challenge remains in maintaining the recognisability of the timetable's cyclicity, as most benefits of cyclic timetables originate from this recognisability.

4.2.4 Combining line planning and timetabling

Lastly, there are also several papers that combine line planning and timetabling decisions in order to better serve the time-dependent demand. Kaspi & Raviv (2013) combine line planning, timetabling and passenger routing decisions into one optimisation model. The model's objective is creating a line plan and a cyclic timetable that minimises the operational cost and the total passenger travel time. They use a cross-entropy metaheuristic to solve the model for a case study of Israel Railways. Yan & Goverde (2019) propose an approach to find both an optimal line plan and timetable, such that all OD pairs have at least one direct connection per time period. The suggested approach creates a timetable with both periodic and aperiodic characteristics: the lines' frequencies can take any positive integer value and the departure times of lines are not required to be strictly periodic but can deviate according to some tolerance parameter. The models for line planning and timetabling are solved consecutively, where in case of infeasibility the algorithm returns to the previous stage. Yang et al. (2021) propose a MILP model that uses a short-turning strategy to deal with asymmetric demand. Lagrangian relaxation is used to solve the model for a case study of the Yizhuang Line of Beijing Subway Network, and this shows that the short-turning strategy can effectively reduce the waiting time by almost 10%. Hao et al. (2022) aim to improve both the railway system's efficiency and the level of service to customers by considering the joint optimisation of line planning and timetabling under time-dependent demand. They use simulated annealing to solve the proposed MIP model. Zhang et al. (2021) propose a MILP model to solve the integrated line planning and timetabling problem with time-dependent demand. The model is solved for an instance on the Wuhan-Guangzhou high-speed railway corridor using Gurobi for varying amount of discrete time intervals. These tests show that when smaller time intervals are used, the trains are more equally distributed over the entire period. Qi et al. (2021) integrate train stop planning and timetabling to serve time-dependent demand. Their objective is to minimise the total travel time of the trains, while serving all the demand in their preferred time slot with a direct connection. The resulting model is tested on the Wuhan–Guangzhou high-speed railway line in China, where the model is solved using CPLEX. Xu et al. (2021) combine timetabling for the high-speed railway with stop-skipping, passenger assignment and platform decisions, to improve passenger convenience and better operating efficiency. They formulate the problem as a minimum-cost multicommodity network flow problem and the resulting MILP model is solved using a Lagrangian relaxation-based heuristic and CPLEX. Wu et al. (2021) try to find an equitable stopping pattern in an oversaturated urban rail transit network. The proposed MILP model which minimises the maximum waiting time of passengers in the network, is not only able to find solutions that are more equitable, but also improve other efficiency evaluation indicators such as the average waiting time. Real-life instances can be solved using the proposed Variable Neighbourhood Search algorithm. So when line planning and timetabling are combined to address time-dependent demand, often choices about stopping pattern and less frequently route selection, short-turning and frequency decisions are considered.

4.2.5 Literature gap

As discussed above, there are various studies on timetabling methods that aim to serve time-dependent demand, focusing primarily on acyclic or hybrid models. While strictly cyclic timetables benefit from their memorability, they fall short when the demand varies a lot throughout the day. Conversely, acyclic timetables offer a lot of flexibility, but lack clarity and ease-of-use for the passengers. While timetables with multiple cycle lengths can offer more direct connections, they still yield a cyclic pattern which assumes consistent demand throughout the day. This method becomes ineffective when the fluctuations in demand throughout the day are large. The works of Yan & Goverde (2019) and Li et al. (2023), which also aim for a cyclic timetable, suffer from similar limitations. Furthermore, minor deviations in scheduled departure times make them harder for passengers to remember, a challenge also present in the 'θ shifted cyclic timetable' by Robenek et al. (2017). Yin et al. (2019) and the 'hybrid cyclic timetable' of Robenek et al. (2017) propose scheduling based on travel demand, which can still result in service gaps in periods with low demand that would reduce the passenger demand during those periods even further. Not investigated before in the literature, are models that generate multiple cyclic timetables throughout the day in order to serve varying demand. Furthermore, we have not found any literature on transitioning from one cyclic timetable to another cyclic timetable. A timetable with multiple cyclic periods is better memorable for passengers than a completely acyclic one, as during several parts of the day, there would be a clock faced timetable. The memorability can be further improved by keeping the departure times on lines that occur in multiple periods the same throughout these periods. Moreover, by also providing a cyclic timetable in periods with less demand, passengers will not experience service gaps, which would reduce demand even further. In the remainder of this chapter, we will address these gaps in the literature, by proposing a model that creates a cyclic timetable for each period with homogeneous demand while maintaining regularity for lines that span multiple demand periods and creating smooth transitions between periods.

4.3 Mathematical model

In this section, we introduce the mathematical model for the multi-period timetabling problem. The notation and variables are introduced in Section 4.3.1, the objective is given in Section 4.3.2, and lastly the constraints of the model are provided in Section 4.3.3.

4.3.1 Notation

In this section, the notation and variables of the mathematical model are introduced. A full overview of the notation is provided in Table 4.2. Similarly to other timetabling problems, the multi-period timetabling problem takes as input a set of lines $\mathcal{L}$, where each line l has a certain route through the network, a stopping pattern, and a frequency f^l denoting how many times per cycle a line should be operated. The cycle time is denoted by C and could for example be 60 minutes. As lines are not necessarily operated during the entire planning horizon, each line also has a period in which the line should be operated, denoted by the start τ_l^{start} and end τ_l^{end} times (in

Table 4.2: Notation used in the mathematical model

Sets	
$\mathcal{L}$	Set of lines.
$\mathcal{L}^{\text{Trans}}$	Set of lines that is operated in the transition periods ($\mathcal{L}^{\text{Trans}} \subset \mathcal{L}$).
$\mathcal{P}$	Set of pairs of related lines.
$\mathcal{S}$	Set of stations in the network.
$\mathcal{S}^l$	Stations that line l passes on its route through the network.
$\mathcal{S}^l_{\text{Stop}}$	The set of stations where line l makes a stop.
$\mathcal{E}$	Set of all departure and arrival events. Each event $e \in \mathcal{E}$ corresponds to a line l_e, a specific train on that line t_e, a type u_e which can be departure or arrival,and a station s_e where the event takes place. Hence, event e can be defined by the tuple (l_e, t_e, u_e, s_e).
$\mathcal{A}$	Set of all activities. The types of activities are: drive, dwell, and headway.
$\mathcal{A}^{\text{headway}}$	Set of all headway activities ($\mathcal{A}^{\text{headway}} \subset \mathcal{A}$).
Parameters	
s^l_i	i^{th} station on line l $\left(s^l_i \in \mathcal{S}^l\right)$.
$s^l_{n^l}$	Last station online l.
τ^{start}_l	Start time (in minutes) of the period during which line l is operated.
τ^{end}_l	End time (in minutes) of the period during which line l is operated.
p^l	The number of cycles during which line l is operated.
f^l	Frequency of line f^l, i.e., how many times per cycle the line is operated.
t^l_i	The i^{th} train scheduled on line l.
b^{lower}_a	The lower bound on an activity time of activity $a \in \mathcal{A}$.
b^{upper}_a	The upper bound on an activity time of activity $a \in \mathcal{A}$.
h	Minimum headway between two events $e, e' \in \mathcal{E}$.
C	Duration of one cycle (in minutes), e.g., 60 minutes.
Variables	
π_e	Real variable denoting the time at which event $e \in \mathcal{E}$ takes place.
$o_{e,e'}$	Binary variable that takes value 1 if event e occurs before event e', 0 otherwise.
d^l_i	Binary variable which takes value 1 if the i^{th} train of line l is not scheduled, 0 otherwise.
$z^l_{i,s}$	Real variable denoting the absolute deviation from the cyclic departure time of (transition period) train t^l_i at station $s \in \mathcal{S}^l \setminus s^l_{n^l}$.
k^l_s	Real variable denoting the maximum time between two departures of trains of line $l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}$ at station $s \in \mathcal{S}^l_{\text{Stop}} \cup s^l_1$.

minutes) of this period. All the trains of this line should depart from its first station within this period. Lastly, lines also have an indicator denoting whether the line is operated in a transition period or not. To explain this transition period, consider the following example. Suppose we consider two different line plans, for which we want to create a multi-period timetable with a cycle time of 60 minutes. The first line plan should be operated in the period from 7:00 until 9:00 and the second line plan between 9:00 and 15:00. Hence, trains that depart between 7:00 and 9:00 use the first line plan, and trains that depart from 9:00 onwards use the second line plan. However, when we switch between the first and second line plan, there is some time period in which there are trains from both line plans running in the network. We will call this time period the transition period. In this chapter we assume that the trains departing during the previous cyclic schedule will keep their cyclic departure and arrival times. This way, we know that only passengers that depart during the transition period will have to deal with an irregular timetable. Trains that depart after 9:00 will use the second line plan. However, to facilitate the transition, we will not enforce the new cyclic schedule for the trains departing in the first cycle(s) of the new schedule. Instead, we try to schedule the trains as close as possible (in time) to the next cyclic schedule, but let the times deviate from the cyclic schedule or cancel the trains if this is necessary to create a feasible timetable. Note that the number of trains for which we should allow extra freedom depends on the case considered. In a case that has long train lines, which take more than C minutes to complete, we need to give freedom to trains in multiple cycles. The number of cycles depends on the length of the longest line in the previous period. Furthermore, it is unlikely that the transition period is an exact multiple of C minutes. However, this is not a problem because trains that depart after the transition period has finished shall be planned according to the new cyclic schedule, as deviations from the cyclic schedule are penalised in the objective. To easily distinguish between trains that start during the transition period (and hence get extra flexibilities), and trains that should be operated in a cyclic manner, two variants of lines are included in the line set. These lines have the same route, stopping pattern, and frequency, but a different period in which they are operated. The transition line is always operated directly before the period of the cyclic line. So returning to the example, we would have the first line plan operated cyclically between 7:00 and 9:00, and the second line plan operated cyclically between 10:00 and 15:00. From 9:00 until 10:00 we have trains departing according to the second line plan, but these trains can have departure and arrival times that deviate from the cyclic schedule. The transition line $l \in \mathcal{L}^{\text{Trans}}$ and the cyclic line $l' \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}$ are included as a pair in the related line set $(l, l') \in \mathcal{P}$. This set is used in the formulation of the constraints to compare the departure times of the transition trains with the departure times of the cyclic trains.

To create a timetable, we need to determine the arrival and departure times for all trains at each of the stations on their routes. We do this by creating departure and arrival events for each train, and for each of these events define an event time variable π_e. A departure event is created for each station $s \in \mathcal{S}^l \setminus s^l_{n^l}$, so for each station on the route of a train of line $l \in \mathcal{L}$, except for the last station. However, arrival events are only created for stops at large stations in order to reduce the number of variables needed. For example, at stations where the train does not stop, the arrival and departure times are the same. Hence, if we only have a departure event, and hence departure time at this station, we have all the information we need. Similarly, no arrival events are created for the small stations in the network. At these short stops, the dwell times are usually short as there is relatively low demand and usually no transfer possibilities. By assuming a fixed dwell time for these stations, which will be included in the minimal driving time towards the station, no arrival activity needs to be determined and therefore fewer variables are needed. Hence, arrival events are only created for stops at large stations, as at these stations you might want to dwell longer to facilitate transfers and to be able to fit trains on the infrastructure. The set of all departure and arrival events is denoted by $\mathcal{E}$.

Activities between two events ensure that there is a minimum and/or maximum time between two events. Three types of activities are considered. Drive activities represent a train driving from one station to the next. They are created between the departure event at one station and arrival event at the subsequent station. If the subsequent station is a small station or if the train does not make a stop at that station, then no arrival event is created. In that case, the drive activity is created between the departure events at the two stations. Dwell activities occur when a train stops at a large station. They are created between the arrival and departure events at such stations, indicating the train is dwelling at the station. Note that for stops at smaller stations, no arrival events are created, and thus no dwell activities are generated. Each drive and dwell activity $a \in \mathcal{A}$ has a lower (b_a^{lower}) and upper (b_a^{upper}) bound on the activity time, indicating the minimum and maximum time that the activity can take. Lastly, headway activities are created between the events of two trains utilising the same infrastructure. For trains of the same line, headway activities are only created between successive trains, as the order between trains is known in advance. In contrast, when two lines are involved, the order of events is not known in advance. In that case, headway events are created if trains could be operated around the same time. To determine this, an operating window is assigned to each train, factoring in both the earliest start time and the latest finish time. The earliest start time is set at the beginning of the cycle in which a train is scheduled to depart, under the assumption that, during each cycle, f^l trains of line l depart. The latest arrival time is defined as the end time of the cycle in which a train should depart plus the sum of upper bounds on the drive and dwell activities of that train. When the operating windows of two trains overlap and the trains use the same infrastructure, a headway activity is created. Headway activities $a \in \mathcal{A}^{\text{headway}}$ only have a lower bound (b_a^{lower}).

Besides the variables denoting the event times, there are four other types of variables. To ensure sufficient time between two events associated with a headway activity, an order variable $o_{e,e'}$ is created for each pair $(e,e') \in \mathcal{A}^{\text{headway}}$. This binary variable $o_{e,e'}$ takes value 1 if event e takes place before event e', and has value 0 otherwise. During the transition periods, we want to schedule the new trains to match the next cyclic schedule as closely as possible. However, to facilitate the transition between two cyclic railway schedules, the departure and arrival times from the transition trains can be shifted from the regular interval times. Variable $z_{i,s}^l$ indicates the absolute deviation from the cyclic departure time at station s by the i^{th} train of line $l \in \mathcal{L}^{\text{Trans}}$. Furthermore, trains of the transition lines can also be cancelled if necessary. Binary variable d_i^l takes value 1 if the i^{th} train of line l is not scheduled, and takes value 0 otherwise. Lastly, we like trains of the same line to be equally spread over the cycle. For example, if we have a line with frequency 4 and a cycle time of 1 hour, then ideally the difference between the departures of two subsequent trains is 15 minutes. Hence, we would like the maximum time between two departures to be as small as possible. Variable k_s^l denotes the maximum time between two departures of trains of line $l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}$ at station s. Note that this variable is not created for transition lines, as we already try to schedule these lines to match the cyclic schedule as closely as possible. Hence, if the cyclic lines are scheduled as equally spread as possible, the transition lines will automatically follow.

4.3.2 Objective

The aim is to create a timetable that is attractive for the passengers, so one that minimises the travel time. However, to calculate the exact travel times, passengers have to be assigned to trains, and this assignment makes the problem significantly more complicated. Therefore, in this work we try to approximate the travel time and the timetable attractiveness with the following components instead. The first component aims to minimise the in-vehicle time by minimising the (avoidable) total journey time:

$$\zeta_1 = \sum_{l \in \mathcal{L}} \sum_{i=1}^{f^l \cdot p^l} \left(\pi_{(l,i,arr,s_{n^l}^l)} - \pi_{(l,i,dep,s_1^l)} \right) - \sum_{a \in \mathcal{A} \setminus \mathcal{A}^{\text{headway}}} b_a^{\text{lower}}. \tag{4.1}$$

The journey time is defined as the difference between the arrival time at the final station and the departure time at the first station. The total journey time is calculated by summing over all trains. If the total journey time is minimised, then a train does not take more time than necessary to travel from A to B and does not dwell at a station longer than necessary. As a result, the in-vehicle time of passengers is minimised, as well as the operating costs since drivers and guards spend less time on the train and less rolling stock compositions are needed. A lower bound for the journey time is the sum of all lower bounds of the drive and dwell activities. By subtracting this lower bound, we obtain the 'avoidable' journey time. This is convenient, as this number is much closer in value to the values of the other components of the objective. Note that the lower bound of the journey time is based on the line plan in each period, so when the stopping pattern is changed the minimum journey time also changes accordingly. Hence, the avoidable journey time denotes the extra running and dwell time used on top of the minimum journey time given the line plan and lower bounds on drive and dwell activities.

The second objective aims to make the timetable more attractive by penalising irregular interval times between trains of the same line. This makes the timetable more attractive, as this provides regular travel opportunities throughout the hour. Regular headways also minimise the deviations from the passengers' desired departure times, when we assume that the desired departure times of the passengers are uniformly distributed over the hour. For example, if a train line has a frequency of 4 per hour, and the departure times of the trains are exactly 15 minutes apart, then the passengers should depart at most 7.5 minutes earlier or later than desired (3.75 minutes on average). Note that this considers a deviation from the desired departure time and not waiting time, since most passengers will check the timetable and arrive accordingly at the station when headways between trains are more than 10 minutes (Van Oort, 2011; Ingvardson et al., 2018). Furthermore, having equal intervals between trains of the same line increases the memorability of the timetable (Wardman et al., 2004). Hence, in the second part of the objective we penalise unequal spreading of the trains:

$$\zeta_2 = \sum_{l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}} \sum_{s \in \mathcal{S}^l \setminus s^l_{n^l}} \left(k^l_s - \frac{C}{f^l} \right) \cdot p^l. \tag{4.2}$$

Variable k^l_s denotes the largest difference between the departure times of two trains of line l at station s. If the trains are equally spread over the cycle, then the time between each two departures is the cycle time divided by the frequency. Only if the maximum time between two departures is larger than this optimum, this is penalised in the objective. Not all lines are operated for the same number of cycles. As we want to avoid these deviations as much as possible, we also need to take into account for how many cycles the line is operated. Namely, if an unequal spread is necessary to satisfy the headway constraints, then it is better if this can be restricted to a low number of cycles. Hence, we also multiply the deviation by the number of cycles in which a line l runs (p^l).

To make the schedule more memorable for passengers, we want to stick to the new cyclic pattern as much as possible in the transition periods. Hence, the third part of the objective is the total deviation from the new cyclic schedules, and is given in Equation (4.3):

$$\zeta_3 = \sum_{l \in \mathcal{L}^{\text{Trans}}} \sum_{s \in \mathcal{S}^l \setminus s^l_{n^l}} \sum_{i=1}^{f^l \cdot p^l} z^l_{i,s}. \tag{4.3}$$

For each transition train, we calculate how much the departure time deviates from the departure time in the cyclic periods at all its stations. By penalising this absolute deviation, we aim to schedule the trains in the transition period such that their departure times match the new cyclic schedule as closely as possible. If the new cyclic schedule fits immediately, we want to switch instantly to the new cyclic schedule, but this also gives some freedom in case the instant switch is not possible.

Lastly, to provide as much service as possible it would be ideal if all transition trains can be scheduled. Hence, a penalty is added to the objective for not scheduling transition trains, based on the number of trains that are cancelled, which is shown in Equation (4.4):

$$\zeta_4 = \sum_{l \in \mathcal{L}^{\text{Trans}}} \sum_{i=1}^{f^l \cdot p^l} d_i^l. \tag{4.4}$$

The multiple objectives are combined as a weighted sum. However, as all the objectives have different ranges, we first find the optimal (ζ^{ideal}) and nadir (ζ^{nadir}) values for each objective. These values are then used to normalise the objective and the weights are used to set the relative importance of each objective:

$$\text{minimise} \quad w_1 \frac{\zeta_1 - \zeta_1^{\text{ideal}}}{\zeta_1^{\text{nadir}} - \zeta_1^{\text{ideal}}} + w_2 \frac{\zeta_2 - \zeta_2^{\text{ideal}}}{\zeta_2^{\text{nadir}} - \zeta_2^{\text{ideal}}} + w_3 \frac{\zeta_3 - \zeta_3^{\text{ideal}}}{\zeta_3^{\text{nadir}} - \zeta_3^{\text{ideal}}} + w_4 \frac{\zeta_4 - \zeta_4^{\text{ideal}}}{\zeta_4^{\text{nadir}} - \zeta_4^{\text{ideal}}}. \tag{4.5}$$

Different timetables can be constructed by varying the weights of the different components. For example, by setting large weights for ζ_3 and ζ_4, we can ensure that trains are operated according to the new cyclic schedule as soon as possible, as deviations from the cyclic schedule are minimised. A sensitivity analysis for the weights is provided in Section 4.4.2.

4.3.3 Constraints

The following constraints are used in the mathematical model.

$$b_{e,e'}^{\text{lower}} \leq \pi_{e'} - \pi_e \leq b_{e,e'}^{\text{upper}} \qquad \forall (e,e') \in \mathcal{A} \setminus \mathcal{A}^{\text{headway}}, e,e' \in \mathcal{E} \tag{4.6}$$

$$\pi_{e'} - \pi_e \geq h - \left(1 - o_{e,e'}\right) M - (d_{t_{e'}}^{l_{e'}} + d_{t_e}^{l_e}) M \qquad \forall (e,e') \in \mathcal{A}^{\text{headway}}, e,e' \in \mathcal{E} \tag{4.7}$$

$$\pi_e - \pi_{e'} \geq h - o_{e,e'} M - (d_{t_{e'}}^{l_{e'}} + d_{t_e}^{l_e}) M \qquad \forall (e,e') \in \mathcal{A}^{\text{headway}}, e,e' \in \mathcal{E} \tag{4.8}$$

$$\tau_{l_e}^{\text{start}} \leq \pi_e \leq \tau_{l_e}^{\text{end}} \qquad \forall e \in \mathcal{E}, s_e = s_1^{l_e}, u_e = \text{dep} \tag{4.9}$$

$$\pi_e = \pi_{e'} + jC \qquad \forall e,e' \in \mathcal{E}, l_e = l_{e'} \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}, s_e = s_{e'} \in \mathcal{S}^{l_e}, \quad t_{e'} = 1,\ldots,f^{l_e}, j = 1,\ldots,(p^{l_e} - 1), t_e = t_{e'} + jf^{l_e} \tag{4.10}$$

$$k_{s_e}^{l_e} \geq \pi_e - \pi_{e'} \qquad \forall e,e' \in \mathcal{E}, l_e = l_{e'} \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}, u_e = u_{e'} = \text{dep}, \quad s_e = s_{e'} \in \mathcal{S}_{\text{Stop}}^{l_e} \cup s_1^{l_e}, t_e = 2,\ldots,f^{l_e}, t_{e'} = t_e - 1 \tag{4.11}$$

$$k_{s_e}^{l_e} \geq \pi_e + C - \pi_{e'} \qquad \forall e,e' \in \mathcal{E}, l_e = l_{e'} \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}, u_e = u_{e'} = \text{dep}, \quad s_e = s_{e'} \in \mathcal{S}_{\text{Stop}}^{l_e} \cup s_1^{l_e}, t_e = 1, t_{e'} = f^{l_e}, \tag{4.12}$$

$$k_s^l \geq \frac{C}{f^l} \qquad \forall l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}, s \in \mathcal{S}_{\text{Stop}}^l \cup s_1^l \tag{4.13}$$

$$z_{t_e,s_e}^{l_e} \geq \pi_{e'} - \pi_e - (p^{l_e} - j)C - M d_{t_e}^{l_e} \qquad \forall e,e' \in \mathcal{E}, l_e \in \mathcal{L}^{\text{Trans}}, (l_e, l_{e'}) \in \mathcal{P}, \quad u_e = u_{e'} = \text{dep}, t_{e'} = 1,\ldots,f^{l_{e'}}, \quad j = 0,\ldots,(p^{l_{e'}} - 1), t_e = t_{e'} + jf^{l_e} \tag{4.14}$$

$$z^{l_e}_{t_e,s_e} \geq -\pi_{e'} + \pi_e + (p^{l_e} - j)C - Md^{l_e}_{t_e} \quad \forall e,e' \in \mathcal{E}, l_e \in \mathcal{L}^{\text{Trans}}, (l_e,l_{e'}) \in \mathcal{P}, \tag{4.15}$$
$$u_e = u_{e'} = \text{dep}, t_{e'} = 1,\dots,f^{l_{e'}},$$
$$j = 0,\dots,(p^{l_{e'}} - 1), t_e = t_{e'} + jf^{l_e}$$
$$d^l_i = 0 \quad \forall l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}, i = 1,\dots,f^l \cdot p^l \tag{4.16}$$
$$\pi_e \in \mathbb{R}_{\geq 0} \quad \forall e \in \mathcal{E} \tag{4.17}$$
$$o_{e,e'} \in \{0,1\} \quad \forall e,e' \in \mathcal{E}, (e,e') \in \mathcal{A}^{\text{headway}} \tag{4.18}$$
$$d^l_i \in \{0,1\} \quad \forall l \in \mathcal{L}, i = 1,\dots,f^l \cdot p^l \tag{4.19}$$
$$z^l_{i,s} \in \mathbb{R}_{\geq 0} \quad \forall l \in \mathcal{L}^{\text{Trans}}, i = 1,\dots,f^l \cdot p^l, s \in \mathcal{S}^l \setminus s^l_{n^l} \tag{4.20}$$
$$k^l_s \in \mathbb{R}_{\geq 0} \quad \forall l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}, s \in \mathcal{S}^l_{\text{Stop}} \cup s^l_1 \tag{4.21}$$

Constraint (4.6) ensures that the difference between two event times is between a lower and upper bound if there exists a (non-headway) activity between these two. Constraints (4.7) and (4.8) ensure that there is an appropriate minimum headway between two trains that use the same infrastructure. Two constraints are needed to model this, as the order of the trains is not known in advance. By multiplying the order variable with a large value M, we ensure that only the relevant constraint is binding. Furthermore, the last term of both constraints ensures that if at least one of the trains is cancelled, then the minimum headway constraint is not binding. Note that although we use a fixed minimum headway between two events, this model can easily be extended such that the length of the headway time depends on the order in which the events take place or the types and/or station of the events. Constraint (4.9) ensures that the trains of each train line depart from the line's first station within the line's time interval. Next, constraint (4.10) states that the departures and arrivals of non-transition-period trains are exactly the same in each hour in which the line is operated, which ensures the cyclic departure times during the cyclic periods. Constraints (4.11) and (4.12) calculate what the maximum interval is between two departures of a line at a station. Constraint (4.11) does this for trains that depart during the same cycle, and constraint (4.12) checks the time difference between the last train of the first cycle and the first train of the next cycle. As we know that the maximum difference between two departures is minimised when the trains are equally spread over the hour, we set the cycle length divided by the frequency as the lower bound on the variable k^l_s in (4.13). Next, constraints (4.14) and (4.15) calculate the absolute deviation from the cyclic departure times over all cycles in the transition period. The variable indicates the absolute difference between the departure of a train in the transition period and the cyclic departure time of a train of the corresponding cyclic line. We compare the departure times of the i^{th} train of every cycle in the transition period with the departure times of the i^{th} train in the cyclic period. When the train is cancelled, this constraint should not be binding, so in that case a large value M is subtracted. Constraint (4.16) states that a train cannot be cancelled if it is not from a transition line. Lastly, constraints (4.17)-(4.21) define the signs of the variables in the model.

4.4 Case study & results

The model presented in the previous section is applied to a small case study. A short description of this case study is provided in Section 4.4.1. Gurobi is used to solve the model for this case study. Next, in Section 4.4.2 the results of the case study are presented.

4.4.1 Description of case study

The case study is based on a small part of the railway network in the Netherlands between the stations Leiden Centraal (Ledn), Den Haag Centraal (Gvc), and Rotterdam Centraal (Rtd). Figure 4.1 displays the (simplified) version of the track layout between stations Rtd, Gvc, and Ledn, which is used in this case study. In this figure, the tracks are denoted by lines and the platforms at stations are represented by grey rectangles.

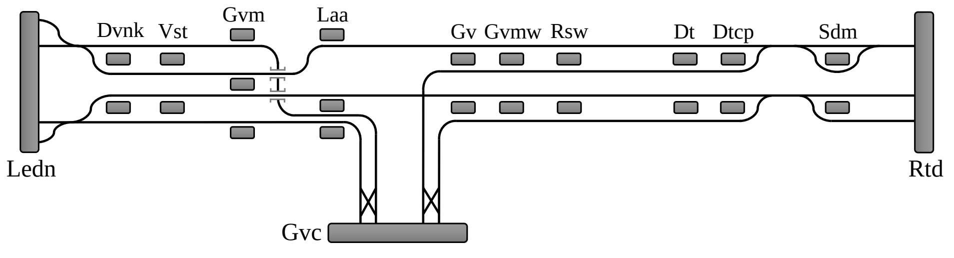

Figure 4.1: Simplified representation of the track layout of case study between stations Rotterdam Centraal (Rtd), Den Haag Centraal (Gvc), and Leiden Centraal (Ledn).

We consider a scenario in which the routes and frequencies of the lines stay the same throughout the day, but the stopping pattern of certain lines changes throughout the day to accommodate the varying demand over various periods. Furthermore, we consider asymmetric lines, where e.g., the stopping pattern of a line from Gvc to Ledn does not have to be the same as the stopping pattern from Ledn to Gvc. The line plan is given in Table 4.3. From left to right, the columns of the table denote the route through the network, the frequency of the lines, the transition time period in which we start operating this line and the extra flexibilities (shifting and cancelling) are allowed, the period during which the line is operated in a cyclic manner, the stopping pattern during these periods, and the minimum journey time of trains operating the lines under different stopping patterns. As can be concluded from the table, a planning time horizon between 7:00 and 17:00 is considered. Furthermore, in our case a route can have up to three different stopping patterns, as can be seen in e.g., the line between Ledn and Gvc (last group in Table 4.3). Between 7:00 and 8:59, this line only stops at station De Vink (Dvnk), between 9:00 and 14:59 it stops at Voorschoten (Vst), and between 15:00 and 16:59 the line stops at Vst, Den Haag Mariahoeve (Gvm) and Den Haag Laan van NOI (Laa). Note that the first hour in which the new stopping pattern is used is indicated in the column called transition period (9:00–9:59 for the second stopping pattern, and 15:00–15:59 for the third stopping pattern). Within these hours, the trains that depart on this line can be scheduled differently from the new cyclic schedule and can also be cancelled if necessary. In the remainder of the periods (10:00–14:59 and 16:00–16:59, respectively) the arrival and departures should be strictly cyclic. Hence, these are denoted in Table 4.3 under ‘Cyclic period’.

The following parameter values are used in the case study. The duration of a cycle is $C = 60$ minutes, so during the cyclic periods the timetable should be the same in each hour. The lower and upper bounds for the different activities are provided in Table 4.4. For the drive activities, the minimal driving time is calculated based on the length of the segments between two stations and the maximum speed between these two stations. Furthermore, if the train makes a stop at the station at the beginning and/or end of the track segment, an acceleration correction of 0.85

Table 4.3: Line plan of the case study.

Route	Freq.	Transition period	Cyclic period	Stop stations	Min. journey time
Rtd-Sdm-Dtcp-Dt-Rsw-Gvmw-Gv-Laa-Gvm-Dvnk-Ledn	6		7:00–16:59	Sdm, Dt, Gv, Laa	41
Ledn-Dvnk-Vst-Gvm-Laa-Gv-Gvmw-Rsw-Dt-Dtcp-Sdm-Rtd	6		7:00–8:59	Laa, Gv, Rsw, Dt, Sdm	43
	6	9:00–9:59	10:00–16:59	Laa, Gv, Dt, Sdm	41
Rtd-Sdm-Dtcp-Dt-Rsw-Gvmw-Gv-Gvc	4		7:00–8:59	Sdm, Dtcp, Dt	27
	4	9:00–9:59	10:00–14:59	Sdm, Dtcp, Dt, Rsw, Gv	31
	4	15:00–15:59	16:00–16:59	Sdm, Dtcp, Dt, Rsw, Gvmw	31
Gvc-Gv-Gvmw-Rsw-Dt-Dtcp-Sdm-Rtd	4		7:00–8:59	Gvmw, Dt, Dtcp, Sdm	29
	4	9:00–9:59	10:00–14:59	Gv, Gvmw, Rsw, Dt, Dtcp, Sdm	34
	4	15:00–15:59	16:00–16:59	Rsw, Dt, Dtcp, Sdm	30
Gvc-Laa-Gvm-Vst-Dvnk-Ledn	6		7:00–8:59	Laa, Gvm, Vst	21
	6	9:00–9:59	10:00–14:59	Gvm, Dvnk	17
	6	15:00–15:59	16:00–16:59	Dvnk	16
Ledn-Dvnk-Vst-Gvm-Laa-Gvc	6		7:00–8:59	Dvnk	16
	6	9:00–9:59	10:00–14:59	Vst	15
	6	15:00–15:59	16:00–16:59	Vst, Gvm, Laa	20

Table 4.4: Lower and upper bounds for different types of activities (in minutes)

Activity type	Lower bound $\left(b_a^{\text{lower}}\right)$	Upper bound $\left(b_a^{\text{upper}}\right)$
Drive	$\lceil(\text{Minimal driving time}) * 1.05\rceil$	$\lceil(\text{Minimal driving time}) * 3\rceil$
Dwell (cyclic train)	1	3
Dwell (transition train)	1	5
Headway	3	-

minutes and/or a deceleration correction of 0.7 minutes is added to the calculated value. To derive the lower bound for the drive activities, a 5% running time supplement is added, and then the value is rounded up to the nearest minute. For the upper bound of a drive activity, a 200% running time supplement is added before rounding up. A dwell time of a cyclic train at a large station should be between 1 and 3 minutes, while a transition train can dwell at a station longer: between 1 and 5 minutes. A dwell at a small station is set to be 0.5 minutes and this time is added to the calculated lower and upper bounds for the drive activities that arrive at the small station for a stop. The large stations in the network are Ledn, Laa, Gvc, Den Haag HS (Gv), Delft (Dt), Schiedam Centrum (Sdm), and Rtd, the other stations are considered small. By summing up the lower bound on all drive and dwell activities, we can calculate the minimum journey time for each line-stopping pattern combination. The last column of Table 4.3 reports these minimum journey times. This column shows that the absolute difference in journey time with changing stopping patterns lies between 1 and 5 minutes. The largest percentage change in journey time occurs at line Ledn-Gvc, which increases from 15 minutes during the off-peak to 20 minutes in the afternoon peak, an increase of 33.3%. Lastly, the headway between two trains that use the same infrastructure (h) is set to be 3 minutes.

4.4.2 Case study results

The objective of the proposed mathematical model, as shown in (4.5), is a weighted sum of four distinct sub-objectives. These sub-objectives are avoidable journey time, the deviation from the regular departure distribution of cyclic trains, departure time deviations of transition trains from the cyclic plan, and the number of cancelled transition trains. Given the differing units of these objectives, the first step is to determine their best (ideal) and worst case (nadir) values. These values serve as the basis for normalising the objectives, allowing us to set weights according to their relative importance. To derive the ideal (ζ^{ideal}) and nadir (ζ^{nadir}) values, we solve the model for each single objective lexicographically. Specifically, we first solve the model with one objective, then impose a constraint to fix its optimal value, and subsequently optimise the model with another sub-objective. This iterative process continues until the optimal value of each sub-objective is found, taking into account the previously optimised objectives. Given that we have four sub-objectives, there are 24 possible sequences in which the objectives can be optimised. Hence, the complete process needs to be repeated 24 times. Once the ideal and nadir values are obtained, each objective is normalised as shown in equation (4.5). We apply equal weights $w_1 = w_2 = w_3 = w_4 = 0.25$ to find a solution that equally optimises each objective. The model was solved to optimality in 27 minutes and 35 seconds on a laptop with Intel® Core™ i7-1185G7 @ 3.00GHz and 16 GB RAM, using Gurobi Optimizer version 11.0.3.

Although there are 24 sequences in which the objectives can be optimised, only four distinct timetables are generated, as each timetable is optimal for multiple sequences.

- When ζ_1 is optimised before ζ_2, the trains in the timetable have the highest deviation from being equally spread over the hour.
- When ζ_1 is optimised last, the timetable with the highest avoidable journey time is created.
- The plan with the highest number of cancelled trains is created when ζ_2 is optimised before ζ_1 and ζ_4 is optimised last.
- Lastly, the plan we obtain when ζ_2 is optimised before ζ_1 and ζ_3 is optimised last contains the highest shift from cyclic departure times in the transition periods.

Table 4.5 displays the optimal values of the different objectives for these optimisation orders. Furthermore, the last row of Table 4.5 shows the objective values for the result obtained using the equal weights in the objective function (4.5). In this table, the first column denotes the order of optimisation, and the subsequent columns show the four different objective values for each timetable.

Due to the definitions of the four objectives, we know that each objective has to be non-negative. From Table 4.5 it is clear that this minimum value of 0 can be achieved for all objectives except for ζ_2, which measures the train distribution irregularity. The minimum value for ζ_2 is 10, which is achieved when ζ_2 is optimised before ζ_1. For the different timetables that were obtained, Table 4.6 shows for which lines and periods the unequal distribution takes place, the follow-up times between consecutive trains, and at which station this unequal departure distribution is planned. Only the follow-up times of the first half hour are shown, as they

Table 4.5: Objective values of the four different objectives under different orders of optimisation.

	Objective values			
Order of optimisation	**ζ_1: Avoidable journey time**	**ζ_2: Unequal spread**	**ζ_3: Shift transition**	**ζ_4: Cancel transition**
ζ_1 before ζ_2	0	79.5	0	0
ζ_1 last	226	10	0	0
ζ_2 before ζ_1, ζ_4 last	198	10	0	10
ζ_2 before ζ_1, ζ_3 last	198	10	80	0
Weighted sum	100	34.5	0	0

Table 4.6: Unequal distribution of trains over the half hour

Order of optimisation	**Line**	**Period**	**Follow-up times**	**Stations**
ζ_2 before ζ_1, ζ_4 last; ζ_2 before ζ_1, ζ_3 last; ζ_1 last	Rtd-Ledn	7:00–16:59	9 – 10.5 – 10.5	Rtd, Sdm
ζ_1 before ζ_2	Rtd-Ledn	7:00–16:59	8 – 11 – 11	Rtd, Sdm, Dt, Gv, Laa
	Ledn-Rtd	7:00–8:59	10.5 – 9 – 10.5	Ledn, Laa, Gv, Rsw, Dt, Sdm
		10:00–16:59	10.5 – 9 – 10.5	Ledn, Laa, Gv, Dt, Sdm
	Rtd-Gvc	16:00–16:59	16 – 14	Rtd, Sdm, Dtcp, Dt, Rsw, Gvmw
Weighted sum	Rtd-Ledn	7:00–16:59	9 – 10.5 – 10.5	Rtd, Sdm
	Ledn-Rtd	7:00–8:59	10.5 – 9 – 10.5	Sdm
		10:00–16:59	10.5 – 9 – 10.5	Ledn, Laa, Gv, Dt, Sdm
	Rtd-Gvc	16:00–16:59	16 – 14	Rtd, Sdm, Dtcp, Dt, Rsw, Gvmw

remain consistent throughout the hour. When ζ_2 is prioritised over ζ_1, train departures for the line between Rtd and Ledn are spaced 9 – 10.5 – 10.5 minutes at stations Rtd and Sdm. Given there is only one track between Rtd & Sdm and Sdm & Delft Campus (Dtcp) that can be used for travelling towards Dtcp (see Figure 4.1), lines from Rtd to Ledn and Rtd to Gvc must share tracks. As Rtd-Ledn has a frequency of 6, Rtd-Gvc has a frequency of 4, and there is a headway requirement of 3 minutes, equal distribution across both lines is unattainable. By spacing departures of trains on Rtd-Led 9 – 10.5 – 10.5 at stations Rtd and Sdm, the unequal distribution is minimised. As the maximum follow-up time is 10.5 at two stations (Rtd and Sdm) during 10 hours, this gives the minimum objective value for ζ_2 of $(10.5-10)\cdot 2\cdot 10 = 10$.

Conversely, if ζ_1 is prioritised before ζ_2, the timetable has the most unequal departure distribution and no avoidable journey time. As shown in Table 4.6, in this timetable there are 3 lines in 4 periods for which the departures are unequally distributed. The line from Rtd to Ledn has the largest deviation from the equal distribution, where the first two departures of the half hour are only 8 minutes apart, while 10 minutes is ideal. The variations for lines from Ledn to Rtd and Rtd to Gvc are only 1 minute from the ideal separation. Note that for the affected lines, the unequal departures occur at all the stops on the line. When the avoidable journey time (ζ_1) is minimised, trains drive as fast as allowed and do not stop at stations longer than necessary.

However, this also means that if somewhere in the network an unequal distribution is needed due to headway constraints, trains should use this unequal distribution during their entire route. By maintaining the unequal distribution on the entire route, the value of ζ_2 is increased to its maximum of 79.5. Instead, if we allow for some longer drive times and/or dwell times, the unequal distribution can be remedied after the headway constraints are met. For example, if the line from Rtd to Ledn has the following departure times from Sdm, following the 9 – 10.5 – 10.5 distribution: 3, 12, 22.5, 33, 42, and 52.5. Suppose the minimum drive time to Dt is 5 minutes and a minimum dwell time is 1 minute, then the trains could be ready to depart at 9, 18, 28.5, 39, 48, and 58.5. However, if we would let the second and fifth train dwell an extra minute, and the third and sixth train dwell an extra 0.5 minutes, then all the trains would depart from Dt exactly 10 minutes apart at 9, 19, 29, 39, 49, and 59. Hence, there is a clear trade-off between minimising the avoidable journey time and minimising the unequal distribution of trains.

When we optimise ζ_2 before ζ_1 and either ζ_4 or ζ_3 last, we obtain a timetable in which deviations are made in the transition periods from the cyclic schedule. If ζ_4 is optimised last, 10 trains face cancellations during the transitions: all six trains between Ledn and Rtd from 9:00–9:59 and the four between Rtd and Gvc from 15:00–15:59. While this approach reduces the avoidable journey time by 28 minutes, this reduction mainly stems from the fact that the cancelled trains no longer have avoidable journey time (as they are not running). Meanwhile, the cancellations would make the timetable very unattractive for passengers of the affected lines during the transition periods.

In the timetable that is obtained when ζ_2 is optimised before ζ_1 and ζ_3 is optimised last, the avoidable journey time is reduced by shifting part of the transition trains. The unequal spread between departures is only calculated and penalised for cyclic trains. Since we want to schedule the transition trains as close as possible to the cyclic schedule, the equal spread is automatically also a goal for the transition trains. However, as the transition trains are not penalised for having an unequal distribution, we can reduce the avoidable journey time (ζ_1) for those trains without affecting the unequal spread penalty (ζ_2). Instead, we deviate from the cyclic departure times, which means an increase in ζ_3.

The timetable obtained by the weighted sum appears to blend elements from the timetables obtained when optimising ζ_1 before ζ_2 and when ζ_1 is optimised last. Table 4.6 shows that in this timetable the same lines have an unequal distribution as in the case where ζ_1 is prioritised over ζ_2. However, there are two notable improvements in the distribution by adding journey time. Firstly, the unequal spread is now confined to fewer stations. The lines from Rtd to Ledn (7:00–16:59) and from Ledn to Rtd (7:00–8:59) only have an unequal departure distribution at one or two stations, as opposed to all stations. Secondly, the departure distribution for the line from Rtd to Ledn has shifted from 8 – 11 – 11 to a more balanced 9 – 10.5 – 10.5, benefiting many passengers given its all-day operation. Table 4.5 shows that during the transition periods, the trains immediately depart according to the new cyclic schedule, as both objectives ζ_3 and ζ_4 have value 0. The time-distance diagram in Figure 4.2 shows the three cyclic schedules and the transitions between them. The horizontal axis depicts station distances and track layout, while the vertical axis represents time. The figure is divided into four parts: the top section displays the timetable from 8:00 and 11:00 and the bottom section from 14:00 and 17:00. To improve the readability of the figure, we exclude periods 7:00–8:00 and 11:00–14:00 from the figure. Due to the cyclicity between 7:00 and 9:00 and 10:00 and 15:00, the timetables from the missing periods can still be viewed by looking at periods 8:00–9:00 and 14:00–15:00. Furthermore, the railway corridor between Ledn and Rtd is shown in the left subfigures, while

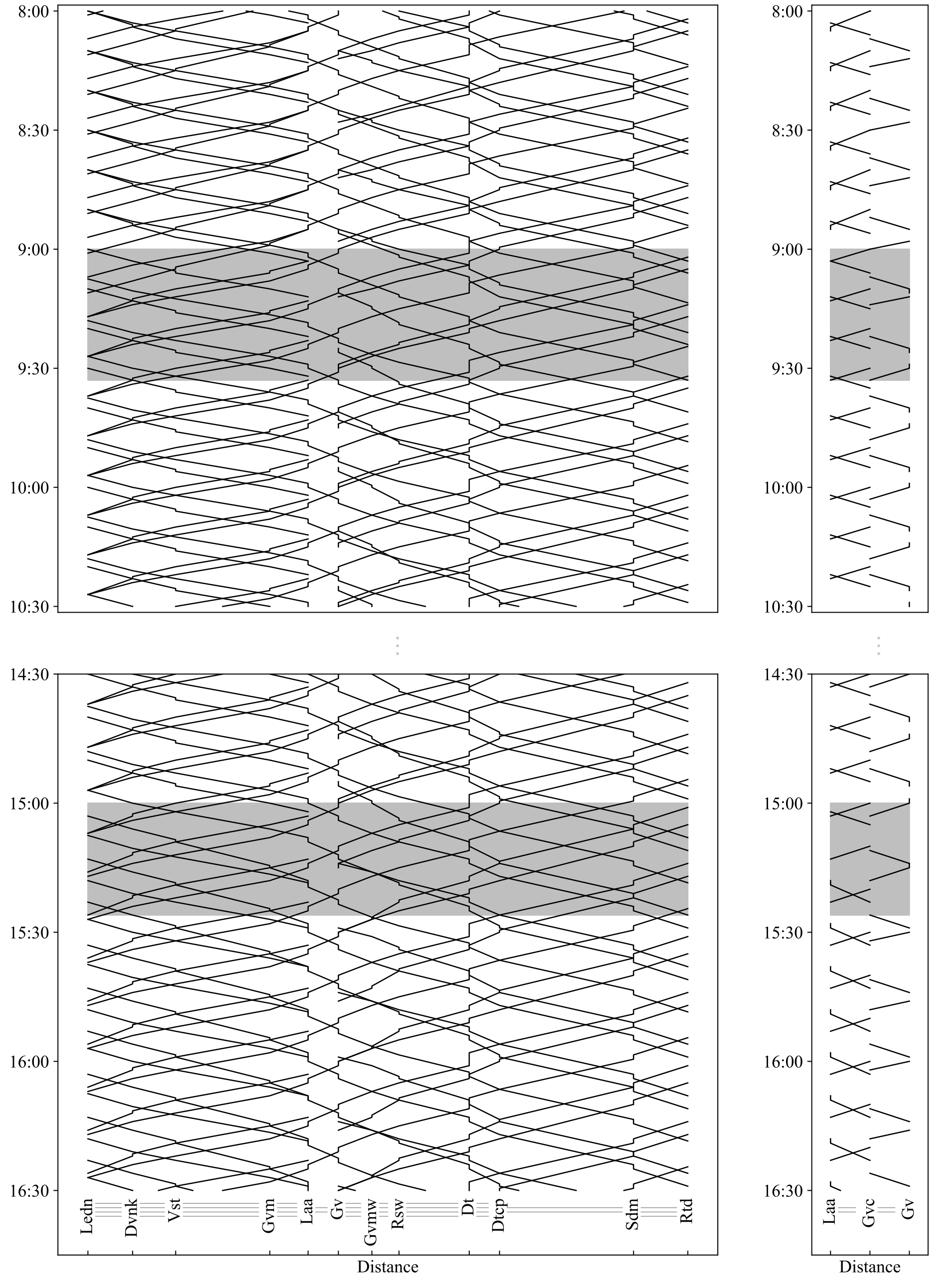

Figure 4.2: Time-distance diagram of timetable obtained with the equally weighted sum method. Transition periods are denoted in grey. The corridor between Ledn and Rtd is shown on the left, while the two corridors between Laa and Gvc and Gv and Gvc are shown on the right.

the tracks between Gvc and Laa and between Gvc and Gv are shown in the right subfigures. The transition periods, during which lines from both the old and the new schedule are operated, are shaded in grey. Figure 4.2 confirms that there are three distinct timetables throughout the day: if we compare the time-distance diagrams in the three unshaded parts, we see three different patterns. The transitions between the cyclic schedules take around 30 minutes, where the second transition is faster than the first transition. This is because during the second transition, the longest lines (between Ledn and Rtd and vice versa) do not change their stopping patterns and hence keep their cyclic times. The diagram also verifies that the timetable is conflict-free, with no overtaking and adequate headways on double-track sections.

As we have several weights in the objective function, we perform a sensitivity analysis on these weights. We do this by varying the weights between 0.1 and 0.7 with increments of 0.1, and under the condition that the sum of the 4 weights is equal to 1. The results confirm that there is mainly a trade-off between ζ_1 and ζ_2, as in most of the scenarios, ζ_3 and ζ_4 attain their minimum values. However, in some combinations where $w_3 = 0.1$ and/or $w_4 = 0.1$, either some trains are shifted from the cyclic departure times (up to 7 minutes) or one of the transition trains is cancelled. To further investigate the trade-off between ζ_1 and ζ_2, Figure 4.3 shows the objective values of ζ_1 (in blue circles) and ζ_2 (in red squares) under the different results that were found (with $\zeta_3 = \zeta_4 = 0$). The horizontal axis displays the relative weight of ζ_1 compared to the weight of ζ_2, so the value of $w_1/(w_1 + w_2)$. The plot shows that the trade-off between the two objectives occurs if the relative weight of ζ_1 is greater than 0.286 and smaller than 0.625. By varying the weights w_1 and w_2 such that the relative weight falls within this range, several timetables could be created based on the preferences of the railway undertaking. Note that in the case of equal weights, the relative weight falls within this range at 0.5.

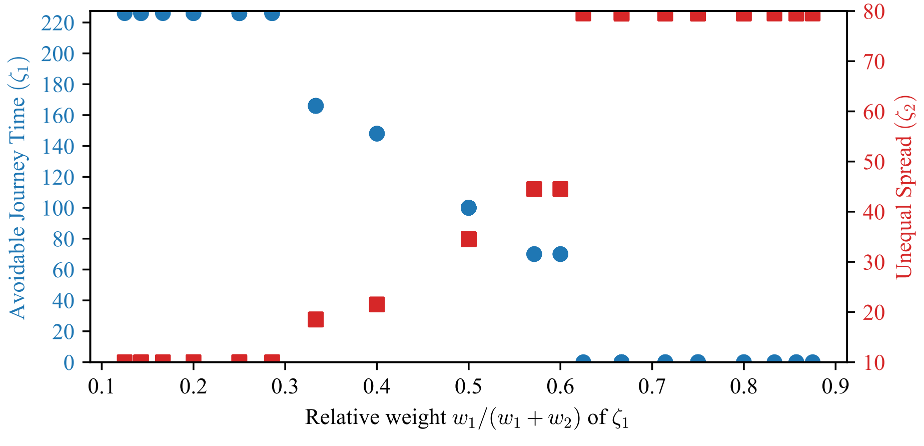

Figure 4.3: Sensitivity analysis of w_1 and w_2 when $\zeta_3 = \zeta_4 = 0$

4.5 Conclusion

The demand for passenger railway services varies significantly throughout the day in terms of volume and travel purposes, which influences travel destinations. However, in many European countries, a cyclic timetable is used that remains constant throughout the day. While such timetables offer ease-of-use to passengers, they are often optimised for peak-hour demand, leading to inefficiencies during off-peak times. On the other hand, acyclic timetables can more easily be adjusted to the fluctuating demand, but are more difficult to remember.

This study aimed to create a timetable that integrates both cyclic and acyclic elements to better serve the fluctuating demand. The proposed mixed-integer linear programming model creates multiple cyclic timetables within the day and transitions between them. To facilitate the transition between two cyclic timetables, departure times can be shifted and trains can be cancelled during the transition periods. To create a timetable that is optimal for the passengers, the model minimises the avoidable journey time, deviations from the ideal departure time distributions, and the shift from cyclic departure times and train cancellations during the transition periods.

The proposed model was successfully tested on a case study based on a small part of the Dutch railway network. In this case study, we assumed that there are three periods in which the demand is homogeneous: the morning peak, midday off-peak, and afternoon peak. For each of these periods, a line plan was provided that best serves the period-specific demand. For the case study, five different timetables were created. Four timetables were created by solving the model for each single objective lexicographically and the fifth timetable was created by using a weighted sum of the four objectives. Interestingly, in three of the five created timetables, no extra flexibilities were used for the trains that depart during the transition periods. The avoidable journey time and departure distribution are not largely impacted by directly using the cyclic event times and not cancelling trains in the transition periods. Furthermore, when these flexibilities are not used, the resulting timetable is more recognisable for the passengers during the transition periods. Hence, it seems to be favourable to immediately start using the new cyclic event times during the transition periods and not cancel any trains. Moreover, putting insufficient emphasis on avoiding train cancellations can lead to very poor quality timetables during the transition periods, while only minimal reductions of the avoidable journey time can be realised. Furthermore, we see a trade-off between minimising the avoidable journey time (which minimises the passenger in-vehicle time) and achieving an equal departure distribution (which minimises the waiting time at the station). When two trains with incompatible frequencies want to use the same infrastructure, some unequal departure distribution is needed. However, if a lot of weight is put on reducing the avoidable journey time, this unequal distribution is used at all the stations on the train's route. On the other hand, if more weight is put on having equal departure distributions, extra dwell and drive time is used to create equal departure distributions at the stations where this is possible.

We see several avenues for future research. Although the current case study provides some interesting insights, the differences between the line plans is relatively small as only the stopping pattern is adjusted. As a common challenge in practice is reducing or increasing the frequencies of a line or inserting new lines, future work should incorporate these variabilities as well. Furthermore, it will be interesting to apply the proposed model to larger case studies to test its scalability. If needed, one way to potentially improve the scalability of the model is by looking more closely between which trains a headway constraint is needed. For each pair of trains that need a headway constraint, also a binary variable is needed, which makes the problem harder to solve. If we can limit the number of headway activities, and therefore the number of binary variables needed, we could possibly speed up the solving process. Two potentially interesting extensions of the model include adding rolling stock turnaround times and adding transfer times. Considering the rolling stock turnaround times can reduce the cost of the timetable for the railway undertaking. If a timetable is designed such that there are short turnaround times, less rolling stock is needed to operate the timetable. As rolling stock is very costly, considering the turnaround times could greatly reduce the costs for the railway undertaking. Moreover, although the model aims to provide an optimal timetable for passengers, transfers between lines are not considered. In the considered case study, around 4000 transfers are required to transport all passengers from their origin to their destination. The service for these passengers could be improved by also considering the transfer times within the optimisation. Furthermore, while the two objectives of minimising the avoidable train journey time and the unequal spread of trains are used to create an attractive timetable, these do not take into account how many passengers are affected by extra journey time or an unequal distribution. Hence, in order to create a timetable that is favourable for the majority of the passengers, future research could investigate how the amount of affected passengers could be incorporated in the objectives. Another interesting research direction would be to investigate the robustness of the multi-period timetable. As both the timetable and demand are changing throughout the day, it is likely that the robustness of the schedule also varies. For example, when also changes in the frequency are considered, there are periods with more trains per hour, which could have a higher risk of delay propagation. Additional model extensions could be needed to create a schedule that is robust to disturbances in all periods. Lastly, additional information about the benefits and drawbacks would be needed before we can make a recommendation to a railway undertaking about implementing the multi-period timetable. Of course the demand for railway services always depends on the service offered. Therefore, it would be wise to estimate the effect of the new service on the demand and ticket revenues. For example, this could be done by comparing the travel times in the (current) fixed timetable with the travel times in the multi-period one and use travel time elasticities to estimate the effect on demand. When the new demand is determined, the effect on ticket revenues can also be calculated. Furthermore, having a multi-period timetable also adds complexity to creating the timetable, rolling stock plan, crew schedule, and to rescheduling the service after a disruption. Only after all these costs and expected benefits are approximated can we make a definitive recommendation to the railway undertaking.

Chapter 5

Evaluating the impact of multi-period railway timetabling on passenger demand

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The previous chapter introduced the novel concept of the multi-period cyclic timetable along with an optimisation model capable of generating such timetables for a multi-period line plan. Throughout this thesis, the focus has been on developing line plans and timetables that can better serve the varying demand throughout the day. Having established methodologies to create line plans and timetables that adapt to these demand variations, the current chapter shifts to evaluating their effectiveness. Specifically, this study assesses whether implementing a multi-period line plan actually improves service compared to a traditional, fixed cyclic timetable. For a case study on a section of the Dutch railway network, the passengers' Generalised Journey Time (GJT) under the multi-period timetable is compared with that under the reference cyclic timetable. Changes in GJT are then used, through incremental elasticity analysis, to estimate corresponding adjustments in passenger demand. Additionally, a methodological framework for designing multi-period timetables is presented, which includes an integrated feedback loop, enabling iterative refinement of both the line plan and timetable based on evaluation insights.

Apart from minor changes, this chapter has been submitted as:

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5.1 Introduction

The demand for passenger railway services exhibits significant variation during the day, not only in volume but also in travel purposes, which in turn influences the choice of destinations. On a regular workday in the Netherlands, there can even be up to 10 distinct homogeneous demand patterns (Van der Knaap et al., 2024). Despite this variability in demand, in many European countries a fixed line plan and cyclic railway timetable is used. A cyclic timetable is created for a base period (e.g., one hour) and then repeated to generate a full-day schedule. Benefits of cyclic timetables include its simplicity and predictability for passengers, which makes it easy to remember and use (Peeters, 2003). Previous studies have shown that having a cyclic timetable with well-planned connections improves the passenger satisfaction and as a result the demand and revenues for the railway undertaking (e.g., Wardman et al. (2004), Johnson et al. (2006)). However, the drawback of a strictly cyclic timetable is its inflexibility to the passenger demand. In practice, railway undertakings accommodate the varying demand by altering rolling stock compositions (e.g., longer trains during the peak), inserting extra train lines during the peak periods and/or reducing frequencies in the late evening when demand is low. Nonetheless, cyclic timetables and corresponding line plans are usually optimised for the peak-hour demand. Hence, the resulting schedules fail to address the distinct travel patterns of off-peak travellers, potentially resulting in a suboptimal service during these times. Acyclic timetables on the other hand are better suitable to meet time-varying demand variations, but can be harder to use for the passengers. For example, it is more difficult to remember an acyclic schedule due to the lack of regularity. Furthermore, there can be gaps in the service during periods with lower demand (e.g., during the midday off-peak or evening).

There have been several attempts to create railway timetables that combine cyclic and acyclic characteristics. The idea for these timetables is that we want to keep (some) of the positive characteristics of a cyclic timetable, while being able to make changes to better serve the time-dependent demand. Examples are the ‘hybrid cyclic timetable’ by Robenek et al. (2017) and the ‘hybrid timetable’ by Yin et al. (2019). These two timetable concepts have in common that a train on a specific line has a fixed departure time within each hour, but whether a train is actually scheduled within an hour depends on the demand. A downside of these concepts is that the cyclical pattern of the schedule is less clear to passengers, as not every hour is the same. To address this, Van der Knaap et al. (2025b) introduced the concept of the ‘multi-period timetable’. In multi-period timetabling, several periods during the day are considered in which the demand is homogeneous. For each of these periods, a line plan is given to serve the demand in that specific period. The multi-period timetable has a cyclic timetable for each period and a transition between the cyclic timetables of subsequent periods. Furthermore, if lines are operated in multiple subsequent periods, the departure times are kept the same throughout the periods to improve the memorability and ease of use for the passengers.

While the goal of the multi-period line plan (see e.g., Şahin et al. (2020), Schiewe et al. (2023), Van der Knaap et al. (2025a)) and timetable is to enhance passenger services, its effectiveness remains uncertain. Therefore, this chapter seeks to address this question through a case study focused on a segment of the Dutch railway network. For this network, two types of timetables have been created: a multi-period timetable and a fixed timetable resembling the schedule that was operated on this network in 2023. By calculating the passengers’ Generalised Journey Time (GJT) under both timetables and using incremental elasticity analysis (Ortúzar & Willumsen, 2011), we approximate the impact of a multi-period timetable on demand.

The main contributions of this chapter are:

- An evaluation of the multi-period timetable concept focussing on its impact on passenger demand and average Generalised Journey Time.
- A methodological framework for designing a multi-period timetable, featuring an integrated feedback loop to iteratively enhance both the line plan and timetable using evaluation insights.
- A sensitivity analysis showing that the case study results are insensitive to the value of the elasticity parameter.

The remainder of this chapter is organised as follows. Section 5.2 discusses the relevant literature on evaluating (public) transport systems using elasticities. Next, we present the methodology in Section 5.3 and the case study in Section 5.4. Section 5.5 provides the evaluation of two multi-period timetables and a sensitivity analysis on the elasticity parameter. Finally, the main conclusions, limitations and suggestions for future research are discussed in Section 5.6.

5.2 Evaluating transport systems using elasticities

Understanding service elasticities for (public) transport demand is fundamental for designing effective (public) transport policies and systems. Service elasticities indicate how much the demand will respond to changes in the service (Ortúzar & Willumsen, 2011):

$$\text{elasticity} = \frac{\text{\% change in passenger demand}}{\text{\% change in service metric (e.g., price)}}. \tag{5.1}$$

The intuition behind it is that if the elasticity is high, then a small change in the service will cause a large change in demand. Similarly, if the elasticity is low, then a change in service will only have a small impact on the demand. The elasticity values can be different for different modes, locations, and aspect of the journey. For example, for public transport there exist elasticities for fare (Kholodov et al., 2021; Wardman, 2022b), in-vehicle time, headway, and journey time (Wardman, 2022a). The different aspects of a trip can be combined in the Generalised Journey Time (GJT), where each element (e.g., waiting time, transfer) is weighed compared to the in-vehicle time based on the passenger valuation. When different aspects of the passenger journey change, the GJT elasticity can be used as a comprehensive metric to estimate the demand changes (Balcombe et al., 2004).

Many papers exist that estimate and/or use service elasticities in the transport domain. The large number of papers can be explained by the fact that elasticities are context dependent. Besides different elasticity values for different modes, elasticities can also change over time. For example, Redelmeier & El-Geneidy (2024) show how demand elasticities have changed after the Covid-19 pandemic for the bus and metro network in Montréal, Canada. They find that the demand has become more elastic after the pandemic, potentially due the rise of telework. Therefore, meta-analyses for existing contexts (e.g., Wardman, 2012, 2022b) or analyses for new contexts (e.g., Vasudevan et al. (2021), Mäkinen et al. (2025)) are regularly performed and published. Furthermore, the data collected from smartcards provide new opportunities to determine elasticities. For example, Wong & Yap (2023) use smartcard data from the Greater London area to find new GJT elasticities for public transport in urban and metropolitan public transport networks.

Besides papers that determine the service elasticities, there are also papers that use these elasticities to evaluate potential service changes. Van Oort et al. (2015) introduce an easy-tobuild elastic demand model that public transport operators can use to perform what-if analyses. In this model, the generalised cost elasticity is used to estimate the demand in new network scenarios. The case study considers changes in frequencies, fares, and line routes. Stoll et al. (2024) use an elasticity-based forecast method for passenger demand to estimate how the long-distance passenger demand will change when introducing an integrated periodic timetable in Germany. Besides elasticity on travel time, they also consider changes in ticket prices and train frequencies. Pandey & Lehe (2024) investigate how stop spacing and line lengths in a bus network affect different metrics such as number of passenger-kilometres travelled and total ridership under elastic demand.

Other papers focus on how elasticities can be useful to understand demand during disruptions. Eltved et al. (2021) analyse smart-card data to determine the impact of a long-term disruption of a rail line on passenger demand in the Greater Copenhagen area in Denmark. Yap & Cats (2022) determine demand elasticities for the GJT and Generalised Journey Cost during planned disruptions in urban public networks. They find a higher demand effect during the weekend than for regular users and during the peak periods on weekdays. Based on these elasticities they develop a neural network regression model that can predict passenger demand during closures with a high level of accuracy. Safitri & Chikaraishi (2023) use travel demand elasticity to determine when the disaster management phase changes (going from normal to emergency response, adaptation, and recovery) such that policymakers can take appropriate action. They show that travel demand becomes more elastic immediately after a disaster, and once the urgent disaster is over, the demand becomes less elastic again.

Service elasticities are also a useful tool when people can choose between different modes to make their trip and you would like to implement some policies to steer them to more sustainable ones. Vasudevan et al. (2021) determine the mode shift elasticity from private to public transport modes in an urban area in India based on household income and travel costs. Escañuela Romana et al. (2023) estimate the price elasticities of rail, road and air transportation, in order to evaluate the impact of public policies for sustainability. Mäkinen et al. (2025) create a tool that regions and municipalities in Finland can use to predict the effect of mobility measures on passenger transport emissions. The tool uses demand elasticities and diversion factors to estimate the measure's effect on the modal shift and corresponding emissions.

The previously mentioned papers mainly estimate the demand effects of introducing specific policies or networks after these scenarios have been created. However, since demand and supply always influence each other, there are some papers that already consider elastic demand during the design phase to find optimal services or pricing strategies. Xu et al. (2023) propose a model to find an optimal train stopping plan and seat allocation scheme under elastic demand. In the model, the demand per origin-destination (OD) pair is determined based on the pair's potential demand, the elasticity considering potential other modes, and travel cost. Given this elastic demand, the model's objective is to create a line plan that maximises the system's value, taking into account ticket revenues, consumer surplus and costs of stopping. Yao et al. (2025) aim to create a pricing strategy in order to ease the peak congestion in the Beijing metro. They propose an optimisation model that finds the optimal discount period and price such that the difference between actual and desired train load rates are minimised. A fare elasticity is used within the model to estimate the demand per time-window based on the pricing strategy. A downside of incorporating elasticities into optimisation frameworks is that it requires including demand and decisions on detailed passenger routing in the model. This further complicates the already computationally challenging line planning and timetabling problems. Many of these problems are proven to be NP-hard, including finding a feasible line plan (Bussieck, 1998) and creating cyclic timetables (Serafini & Ukovich, 1989).

To conclude, elasticities are a widely used and valuable tool in transport research for estimating demand responses for different service scenarios, as demonstrated in e.g., Van Oort et al. (2015), Stoll et al. (2024), and Mäkinen et al. (2025). One limitation of this method is that the quality of the evaluation is greatly dependent on the quality of the elasticity parameter. As many factors influence the elasticity (e.g., mode, travel purpose, travel distance (Wardman, 2022a,b)), selecting a representative value is not straightforward. On the other hand, key benefits of incremental elasticity analysis are that it is easy to implement and requires relatively little data. When the initial demand, the elasticity parameter, and change in service are known, the new demand can be approximated. Therefore, we will use incremental elasticity analysis in this chapter to evaluate the novel multi-period timetabling concept. Given the innovative nature of this schedule, it remains unclear whether passengers and railway undertakings can benefit from implementing such a schedule. By employing demand elasticities to estimate the potential changes in ridership resulting from introducing a multi-period timetable, this study provides new insights into its viability and effects on the railway system.

5.3 Methodological approach

The process of creating and evaluating a multi-period timetable is visualised in Figure 5.1. As shown in the figure, the process consists of four main steps: determine periods with homogeneous passenger demand, create a multi-period line plan, create a multi-period timetable for this line plan, and evaluate the created timetable compared to a reference timetable. In Section 5.3.1 we briefly discuss how one can determine periods in the day that have homogeneous passenger demand. In Sections 5.3.2 and 5.3.3 we describe the models used to create the multi-period line plan and timetable for this chapter's case study. Next, Section 5.3.4 describes the method used to evaluate the multi-period timetable, which includes calculating the GJT for each OD pair throughout the day, and using elasticities to estimate the new demand (Section 5.3.4). After analysing the changes in GJT and demand, we can either decide that the timetable is good enough and terminate the process or return to the line planning and/or timetabling stage to try to improve the solution. For example, we could decide that the timetable is good enough if we see a significant improvement in demand in all periods. Instead, if one or more periods show only a small increase or decrease in demand, we can decide to return to the line planning or timetabling stage. To determine which stage to return to, we can check which OD pairs lost the most demand in the new timetable and how the service for these OD pairs has changed. If the service for an OD pair is similar in both line plans (e.g., has the same number of required transfers), then we should check in the timetable if the transfer time between the relevant lines can be improved. However, if we see that these OD pairs have gained extra transfers due to changes in the line plan, we should return to the line planning phase to reconsider these decisions.

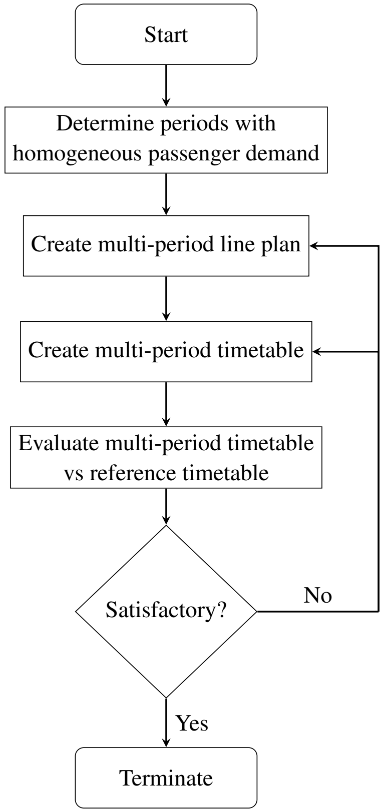

Figure 5.1: Process of creating and evaluating a multi-period timetable.

5.3.1 Determine periods with homogeneous passenger demand

The first step in the process of creating a multi-period timetable is determining for which periods in the day a different line plan and timetable is desirable. Within these periods, we want the demand to be homogeneous such that it can be served well with the same service plan. One way to determine these periods with homogeneous demand is by using the hierarchical clustering algorithm provided by Van der Knaap et al. (2024). This algorithm takes as input OD demand matrices for each (e.g., 30-minute) time window throughout the day and combines these windows to create contiguous periods with homogeneous demand. More background and details on finding periods with homogeneous demand can be found in Van der Knaap et al. (2024).

5.3.2 Multi-period line planning model

The multi-period line plan that is used in the case study of this chapter was created using the mixed-integer linear programming (MILP) model developed by Van der Knaap et al. (2025a). This model takes as input the time periods for which a line plan should be found (as determined in the previous step), the passenger demand in those periods, and a set of candidate lines. Each candidate line has a route through the railway network and a set of allowed frequencies, but not a fixed stopping pattern as this is set by the model. For each period, the model selects which candidate lines are operated, what their frequency is, and at which stations the line will stop, such that the total GJT of the passengers is minimised.

The GJT in the MILP model consists of the in-vehicle time and the waiting time before boarding a train (either during a transfer or before the first train). To calculate the GJT, the passengers are routed on a change-and-go (C&G) network (Schöbel & Scholl, 2005; Bull et al., 2019). This network contains arcs for alighting and boarding trains and arcs denoting drive and dwell activities of the trains. By assigning a weighted travel time cost to each of these arcs, the GJT can be calculated by finding for all passengers the shortest path from their origin to their destination in this network. As in the line planning phase there is no timetable available yet, the model makes some assumptions to estimate these travel time cost. First, the model assumes that trains on the same line are equally spread over the hour, and that the waiting times are half the headway between two subsequent trains. These values are weighed to reflect the entrance and transfer resistance in the Dutch setting based on the work of de Bruyn et al. (2023) and Guis et al. (2023). Table 5.1 shows for each of the used frequencies the corresponding cost in GJT minutes for boarding the first train and the cost for transferring between trains. Furthermore, the model assumes that the time needed to drive between two stations is the same for each train, where the driving time is calculated based on the distance between two stations and the maximum operational speed on that stretch. A correction for the braking time loss is added to the dwell and alighting arcs and a similar correction for the acceleration time to the dwell and boarding arcs. Lastly, the dwell time at a station is the same for each train.

Table 5.1: Used costs for boarding the first train and transferring in the C&G network. Costs are given in GJT minutes.

Frequency [trains/hour]	**Cost of entering first train [min]**	**Cost of transfer [min]**
1	55.85	50.25
2	31.85	28.85
4	17.35	19.95
6	10.85	15.55

There are several constraints that a line plan created by the model should satisfy. An important constraint is that the cost for the railway operator should stay below a given budget. The operator's budget is measured in terms of train kilometres per hour, and for every period a separate hourly budget is set. Furthermore, all demand should have a route through the C&G network. This means that every station in the network, that is the origin or destination of at least one passenger, should be served by at least one line. As the lines are assumed to be only operated in one direction, a station could be served in only one direction by a given line. Full specifications of the model can be found in Van der Knaap et al. (2025a).

The output of the model is a multi-period line plan, where each period has its own set of lines with corresponding stopping pattern and frequencies per hour. This line plan is used as input for the multi-period timetabling model, which is described in the Section 5.3.3. Note that it is not guaranteed that a feasible timetable can be constructed for the line plan created in this step. To make finding a feasible timetable more likely, the set of allowed frequencies of a candidate line can be adjusted. For example, if the route of the candidate line contains part of the network that only has a single track we can choose the set of allowed frequencies in such a way that it should be possible to create a timetable for this single track part.

5.3.3 Multi-period timetabling model

When the multi-period line plan has been created, the next step is to turn this into a multi-period timetable. In a multi-period timetable, each period has its own 1-hour cyclic timetable, which is repeated throughout the period. Furthermore, at the start of a new period, we need a transition from the previous cyclic timetable to the next cyclic timetable. A graphical representation of the structure of a multi-period timetable is given in Figure 5.2. In this figure, each colour denotes a different period with its own line plan. Moreover, the cyclic arrows denote hours in which trains should depart according to a cyclic schedule, while the 'T' denotes the transition between cyclic schedules.

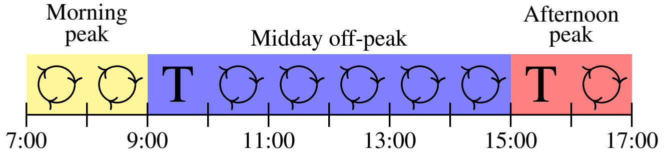

Figure 5.2: Structure of a multi-period timetable.

Van der Knaap et al. (2025b) provide a MILP model which can compute multi-period timetables. This MILP model takes as input a line plan, where for each line is specified during which times the line is operated, and parameters such as which lines use the same infrastructure and the minimum headway between two departure and/or arrival events. Each cyclic timetable and transition should be feasible, in the sense that two trains that use the same infrastructure should be scheduled such that there is a minimum headway (e.g., three minutes) between them.

To facilitate feasible transitions, the multi-period timetabling model offers extra flexibilities to trains departing in the first cycle of the new schedule (denoted by a 'T' in Figure 5.2). These flexibilities are that trains can be shifted from their cyclic departure times or that they can be cancelled. The transition time ends, when all trains in the network are running according to the new cyclic schedule. Therefore, the earliest time when the transition time ends is when all trains from the outgoing schedule have finished their route. When the extra flexibilities are used during the transition, the transition time would be extended until the trains with a changed timetable have finished their route. Hence, the transition time could be extended up to the time of one cycle (here one hour) plus the longest (deviating) train journey time in the considered network.

To create a timetable that is attractive for the passengers, the model's objective is a weighted sum consisting of four normalised parts. The first objective is to minimise the total train journey time, as lower train journey times also result in lower in-vehicle times for the passengers. The second objective ensures that departures of trains of the same line are scheduled evenly over the hour whenever possible. This is achieved by penalising the maximum headway between two consecutive trains in the objective. When trains are equally spaced over the hour, passengers have regular travel opportunities throughout the hour, which makes the timetable more attractive. The third and fourth objectives penalise the use of the extra flexibilities available during the transition, so the shift from the cyclic departure times and the cancellation of transition trains. This is done in order to create schedules that are easier to remember for the passengers. For this same reason, the model requires that when a line is operated in multiple consecutive periods, then this line should keep the same cyclic departure times throughout these periods. Note that the passengers' GJT is not explicitly minimised in the model, as this requires including the passenger routing in the model and this would make the problem significantly more complicated.

5.3.4 Evaluating the multi-period timetable

Once we have created a multi-period timetable, we can evaluate it compared to a reference timetable (e.g., the one currently operated). To do this, we first compute the GJT for each origin-destination pair at different times of the day. We then determine the percentage change in GJT when comparing the multi-period timetable with the reference timetable. Using the GJT variations and GJT elasticities, we calculate the resultant change in demand. Detailed explanations of both the calculation of the GJT and the estimation of the new demand are provided below. Finally, these changes in GJT and demand are thoroughly analysed, with results presented in Section 5.5.

Calculating the Generalised Journey Time

The concept of Generalised Journey Time (GJT) was introduced by Tyler & Hassard (1973) to assess the attractiveness of a timetable for passengers, and has been used in many studies since. In this evaluating stage, we consider a GJT that consist of four parts: adaptation time for taking the first train, the in-vehicle time (IVT), the transfer time needed to interchange between trains, and a fixed penalty for transferring. Note that since at this stage a timetable is available, the assumptions for adaptation and transfer times used in the line planning stage are not needed here. Instead, we assume that the desired departure time of passengers is equally distributed over the hour. As the service headways in the considered timetable are between 10 and 15 minutes, we assume that people will check which train(s) they want to take in advance and arrive accordingly at the station (Vuchic, 2005). The adaptation time is then calculated as the absolute difference between their desired departure time and the actual train departure time. Given the limited flexibility passengers have to adjust their departure times, we consider trains that depart within a 30-minute window before and after these desired times when calculating the GJT.

Recognising that passengers generally prefer the comfort of being on a train over waiting on a platform, we apply weighting factors to adaptation and transfer times to equate them to IVT minutes. For the adaptation and transfer time we use a weight of 2 (Wardman, 2004), and the fixed transfer penalty is set to 5 minutes (Balcombe et al., 2004). These weights have also been used before in case studies regarding the Dutch railway system (e.g., Ghaemi et al. (2018)).

To evaluate the attractiveness of the proposed timetables, we calculate the GJT for each OD pair throughout the day. This involves converting the timetable into a time-space graph, similarly as in Zhu & Goverde (2019). An example of the time-space graph is given in Figure 5.3. In this graph, nodes represent arrivals and departures of all trains. Moreover, for each station there is a demand node for every minute in the considered time period and an artificial 'exit' node. Besides these exit nodes, each node is time-stamped to indicate when the event occurs. In this graph, nodes related to train movements are linked by drive and dwell arcs, with drive arcs connecting departure nodes to the corresponding arrival nodes at the next station along the train's route. Dwell arcs connect the arrival and departure nodes at stations where the train halts. Additionally, transfer arcs are established from arrival nodes to all departure nodes at the same station within a 2 to 62-minute window post-arrival. This assumes a minimum transfer time of 2 minutes and as the considered timetables are cyclic with a 60-minute cycle,

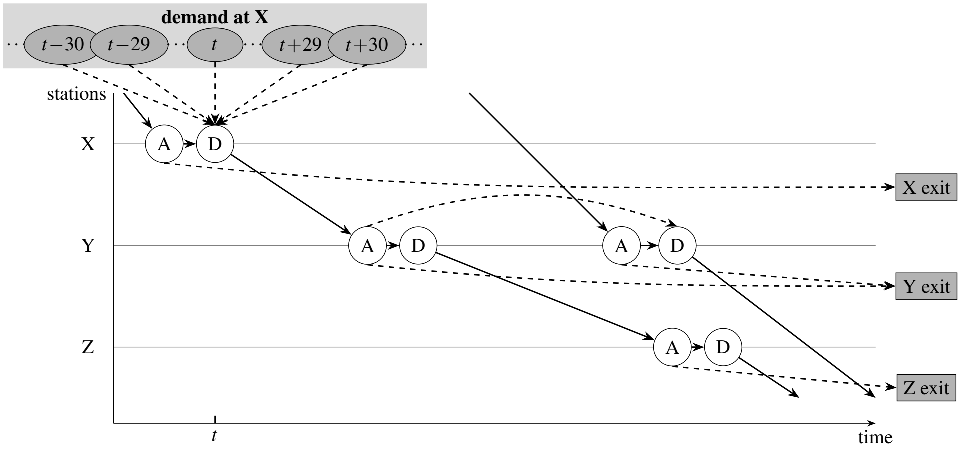

Figure 5.3: Example of time-space graph used to calculate the GJT

passengers will not have to wait longer than 62 minutes for a connecting train. Moreover, arcs are added from demand nodes to all departures occurring between 30 minutes before and after the desired departure time, as well as exit arcs leading from all arrival nodes to the station's exit node. The weight of each arc reflects the time between events multiplied by the event type's weight. For instance, a transfer from a train arriving at 7:35 to one departing at 7:45 would get a weight of 25 minutes, calculated as 2 (transfer weight) times the 10-minute interval plus the 5-minute fixed penalty.

The GJT for each demand node to each station's exit node, except for the station of origin, is determined by identifying the shortest path within this time-space graph. The analysed period in this chapter is restricted to a specific timeframe from 7:00 to 17:00. Since train operations extend beyond this timeframe, specific assumptions are required to accurately calculate the GJT for passengers intending to depart between 7:00–7:29 and 16:30–16:59. We assume that the timetable from 6:00 to 7:00 mirrors that from 7:00 to 8:00, and the timetable from 17:00 to 18:00 mirrors that from 16:00 to 17:00. This approach allows for a consistent determination of GJT for travellers at the edges of the specified timeframe.

Estimating the change in demand using elasticities

After calculating the GJT for each minute within the considered time period, the results are aggregated by determining the average GJT for each 30-minute interval. Given that two distinct timetables are being compared, analysing the GJT for each departure minute is less insightful, as trains may depart at different times in the respective timetables. Instead, an average GJT is calculated for each origin-destination (OD) pair across every half-hour segment i within the planning period. We denote the average GJT of a trip from o to d departing during time interval i by $\overline{T}_i[o,d]$ for the reference timetable, and by $\widetilde{T}_i[o,d]$ for the multi-period timetable. To estimate the demand under the multi-period timetable, the well-known incremental elasticity analysis method is used (Ortúzar & Willumsen, 2011).

Equation (5.2) shows how the new demand $\widetilde{D}_i[o,d]$ is calculated for each OD pair (o,d) and each time interval i:

$$\widetilde{D}_i[o,d] = \max\left\{ \left(\left(\frac{\widetilde{T}_i[o,d]}{\overline{T}_i[o,d]} - 1 \right) \varepsilon + 1 \right) \overline{D}_i[o,d], 0 \right\}. \tag{5.2}$$

In this equation, the percentage change in GJT $(\widetilde{T}_i[o,d]/\,\overline{T}_i[o,d] - 1)$ is multiplied with the GJT elasticity ε to determine the percentage change in demand. Next, we calculate the new demand by multiplying the percentage change plus 1 by the initial demand. In case the calculated reduction in demand is larger than the initial demand for a certain OD pair, the new demand for that OD pair is set to 0.

5.4 Case study

The case study presented in this chapter examines a segment of the Dutch railway network connecting the cities of Leiden, The Hague, and Rotterdam, as shown in Figure 5.4. This section of the network encompasses 13 stations and is analysed over the time span from 7:00 to 17:00. Two distinct timetables have been developed for this study: a reference timetable that is strictly cyclic from 7:00 to 17:00 and a multi-period timetable with three different periods.

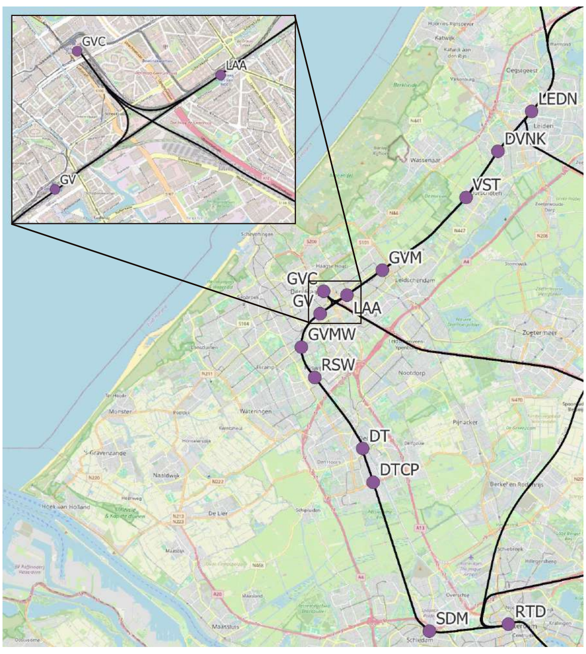

Figure 5.4: Network of case study

The reference timetable mirrors the schedule operated in this area in 2023, incorporating a line plan featuring ‘Intercity’ trains that serve routes between Den Haag Centraal (Gvc) and Rotterdam Centraal (Rtd), Gvc and Leiden Centraal (Ledn), and Rtd and Ledn, stopping only at major stations. Additionally, ‘Sprinter’ trains service the Gvc-Rtd and Gvc-Ledn routes, making stops at all stations. From this reference line plan, depicted in Figure 5.5a, a cyclic timetable with a 60-minute cycle time is created, which is repeated every hour. Furthermore, the trains of most lines are equally distributed over the hour (e.g., 15 minutes apart when the frequency is four). The only exception is the line between Ledn and Rtd, which has departure intervals alternating between 10 and 20 minutes, as was the case in the 2023 timetable.

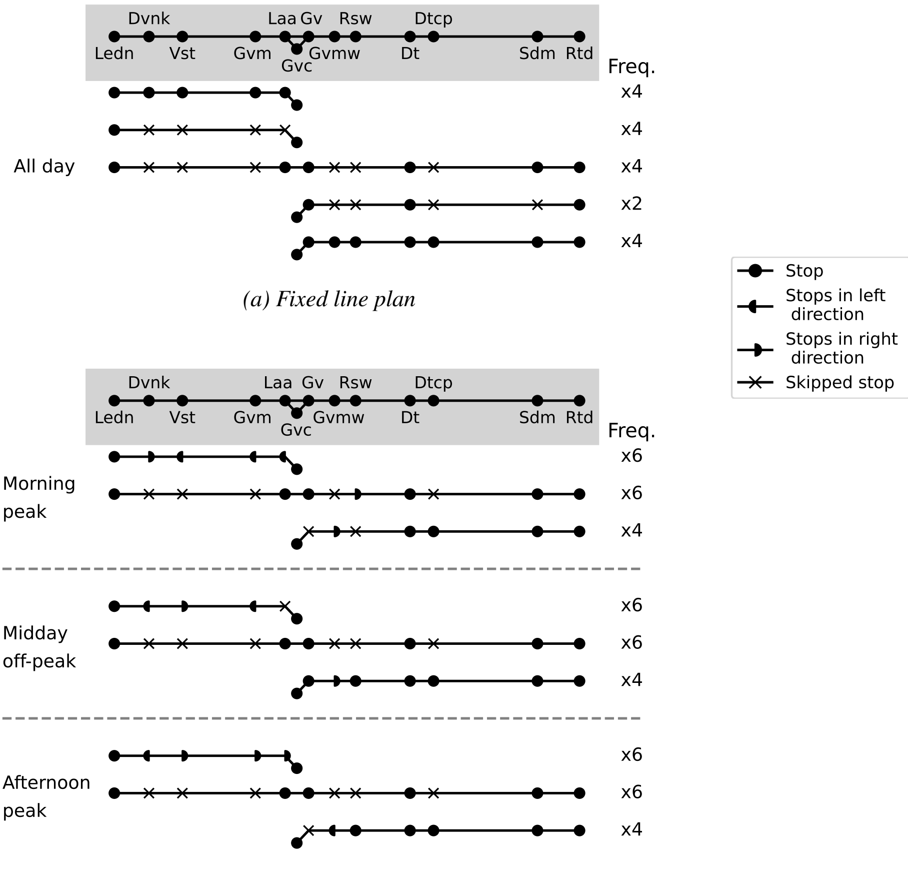

Figure 5.5: Fixed line plan (a) and multi-period line plan (b) of the case study.

For the multi-period timetable, the day has to be divided into different periods based on demand characteristics. According to the work of Van der Knaap et al. (2024), the time period between 7:00 and 17:00 can be divided into six periods with different demands. These periods vary in length, with shorter periods (between 30 and 90 minutes) during the peaks, and a long period during the middle of the day. Here we have chosen to combine the shorter periods during the morning and afternoon peak to create two larger periods, leaving us with three periods in total. The reason for combining the shorter periods is that transitioning between line plans and corresponding timetables cannot be done instantly. In our network it can already take up to 45 minutes before all trains from the previous timetable have completed their route, and hence only the new cyclic schedule is operated. Therefore, we have taken into account a minimum period length of two hours, which gives us three periods: the morning peak (7:00–8:59), the midday off-peak (9:00–14:59), and the afternoon peak (15:00–16:59).

For each of the three periods, a customised line plan is designed to minimise the GJT for passengers using the multi-period line planning model explained in Section 5.3.2. To fairly compare the multi-period schedule with the reference schedule, the hourly budget for the multi-period line plan is the same in each period, and equal to the hourly number of train kilometres in the reference line plan. The resulting line plans are illustrated in Figure 5.5b. The multi-period and reference line plan have comparable costs in terms of train kilometres: the reference line plan uses 8288 kilometres, while the multi-period line plan uses 8240 kilometres. Thus, the cost of the multi-period line plan is only 0.6% below the cost of the reference line plan. For this multi-period line plan, a multi-period timetable is created using the multi-period timetabling model given in Section 5.3.3. Transitions to new cyclic schedules occur at 9:00 and 15:00, introducing an overlap where trains from both the outgoing and incoming schedules coexist temporarily in the network. The extra flexibilities (i.e., shifting or cancelling transition trains) are not needed to create feasible transitions, so the transition time ends when all the trains from the outgoing schedule have finished their route.

Figure 5.6 displays the timetables, where the reference timetable is shown in red and is the same in every hour. The black lines denote the multi-period timetable and the shaded grey areas denote the time during which a transition takes place between two subsequent cyclic schedules in the multi-period timetable. As both the multi-period timetable and the reference timetable are cyclic between 10:00 and 15:00, we omit the schedule between 10:30 and 14:30 to save space. The left part in Figure 5.6 displays the corridor between Ledn and Rtd, while the two corridors between Laa and Gvc and Gv and Gvc are shown on the right. To assess the impact of implementing the multi-period line plan and timetable compared to the fixed reference timetable, we utilise the realised demand data provided by the Dutch railway operator, NS, from a regular Tuesday in 2019. For each OD pair, this demand data indicates the number of passengers departing from their origin within consecutive half-hour periods. These periods are defined as starting from the full hour up to 29 minutes past the hour (e.g., 9:00–9:29), and from 30 minutes past the hour up to 59 minutes past (e.g., 9:30–9:59), continuing in this pattern throughout the day. For the impact analysis, a GJT elasticity of -0.9 is applied, derived from a meta-analysis by Wardman (2022a). This elasticity reflects that a 1% increase (or decrease) in GJT results in a 0.9% decrease (or increase) in demand for the OD pair.

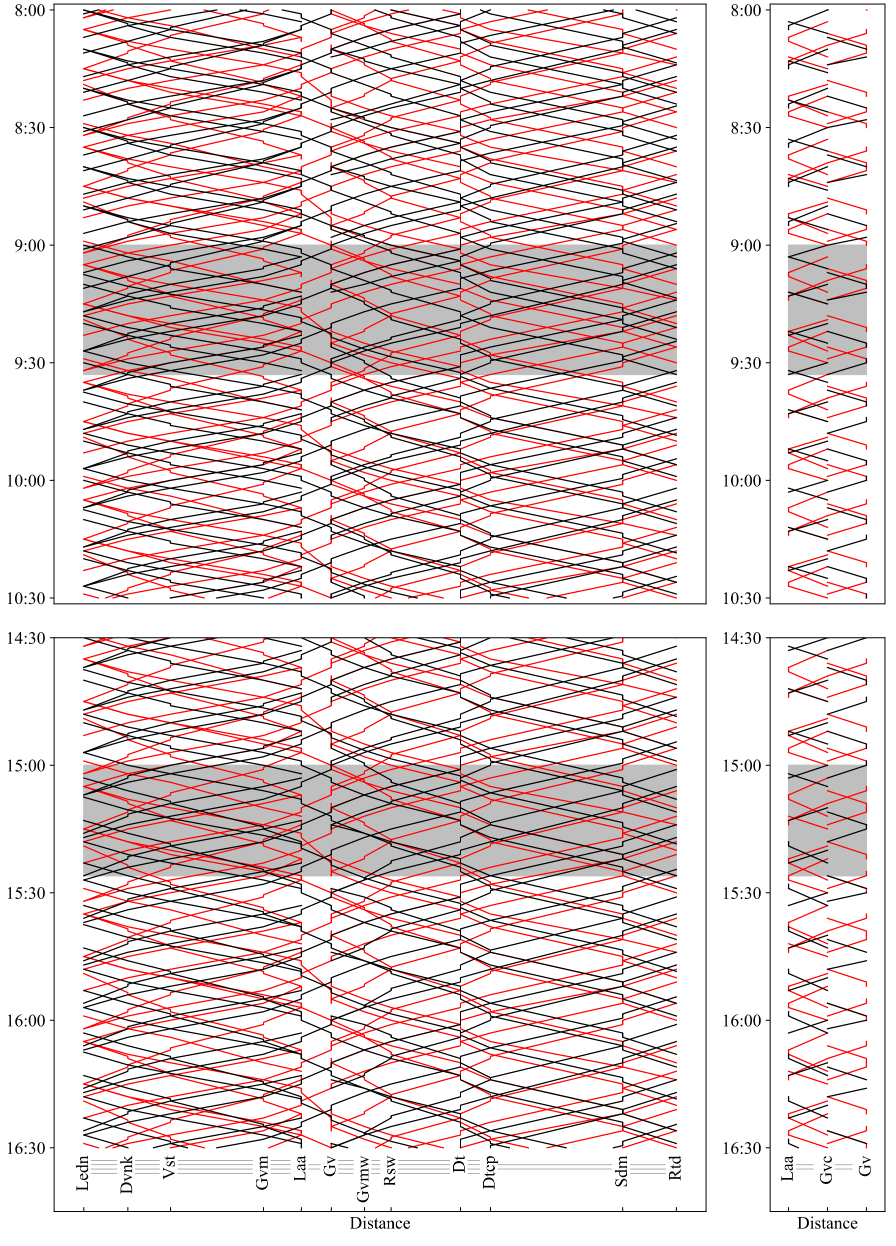

Figure 5.6: Time-distance diagram of the considered timetables, where time is displayed on the vertical axis and the distance on the horizontal axis. The schedule between 10:30 and 14:30 is omitted, as it remains constant between 10:00 and 15:00. The red lines denote the strictly cyclic reference timetable from 2023, the black ones the multi-period timetable. The grey horizontal lines on the bottom of the figure denote the track layout.

5.5 Results

Section 5.5.1 evaluates the multi-period timetable created for the case study described in Section 5.4. As this evaluation shows that the timetable does not improve the passenger demand in all periods, we use the iterative method described in Section 5.3 to improve the multi-period line plan and timetable. This adapted timetable is evaluated in Section 5.5.2. Lastly, Section 5.5.3 provides a sensitivity analysis on the elasticity parameter.

5.5.1 Evaluation of original multi-period timetable

The overall results over the entire period (7:00–17:00) are presented in Table 5.2. In this table, the first column denotes which timetable is under consideration, and the next three columns provide the number of passengers for the timetable, the total GJT, and the average GJT per passenger. For the reference timetable, the number of passengers is equal to the total number found in the input demand data. For the multi-period timetable, the number of passengers is estimated by factoring in the change in demand approximated with the change in GJT and the GJT elasticity. The last two rows of the table display the absolute and percentage change between the multi-period and reference timetables.

Table 5.2: Number of passengers and (average) GJT in both timetables from 7:00–17:00.

Case	Number of passengers	Total GJT [min]	Average GJT [min]
Reference timetable	63,750	1,611,108	25.27
Multi-period timetable	63,900	1,540,171	24.10
Absolute change	150	-70,936	-1.17
Percentage change	0.2%	-4.4%	-4.6%

From the table, we observe that the multi-period timetable has a subtle positive effect on the number of passengers: there are more passengers under the multi-period timetable compared to the reference timetable, reflecting an increase of 0.2%. Additionally, the multi-period timetable positively influences both the total GJT and the average GJT per passenger, with the average GJT improving by more than one minute, which translates to a reduction of 4.6%. This reduction in GJT is partly due to attracting more passengers on routes with lower GJT and losing passengers on routes with higher GJT.

When we break down these results per period (morning peak, midday off-peak, and afternoon peak), it is evident that the positive outcomes are not uniformly distributed across all time windows. Table 5.3 shows for each period with a different timetable and the two transitions the total change in demand (summed across all OD pairs) and the average change in demand per half hour. The last three columns of the table show the minimum, median and maximum percentage change in GJT over all OD pairs.

During the midday off-peak and the afternoon peak we see an overall increase in passenger numbers, while there is a decline of 372 passengers (1.6% of the demand) during the morning peak. This decline is surprising, since the reference timetable has similar costs to the multi-period timetable in terms of train kilometres, and the multi-period line planning model aims to minimise the passengers' GJT. However, the MILP model for multi-period line planning could not be solved to optimality within the given time window. The decrease in passengers indicates that the line plan and timetable for the morning peak period are suboptimal if the aim

Table 5.3: Change in demand (number of passengers) and percentage change in GJT, broken down per period with a different timetable.

	Change in demand [# pass]		% change in GJT over all OD pairs		
Period	**Total**	**Avg. per half hour**	**Min.**	**Med.**	**Max.**
Morning peak (7:00–8:59)	-372	-93	-62%	5%	476%
Midday off-peak (9:30–14:59)	500	45	-37%	-3%	406%
Afternoon peak (15:30–16:59)	26	9	-37%	-3%	533%
First transition (9:00–9:29)	1	1	-35%	2%	395%
Second transition (15:00–15:29)	-5	-5	-37%	-2%	382%

is to improve the passenger service. When comparing the multi-period line plan in the morning peak with the reference line plan, we see it has a lot less direct connections, which means that more transfers are necessary. An extra transfer adds at least 9 minutes to the GJT, as the minimum transfer time is 2 minutes (multiplied by transfer weight of 2) plus the 5-minute fixed transfer penalty. As the average GJT in the reference timetable is only 25.27 minutes, having an extra transfer likely means a large percentage increase in GJT and hence a large reduction in demand. The midday off-peak records the most significant rise in demand, averaging 84 additional passengers per hour, which is an increase of 1.9%. The afternoon peak shows a modest increase with an average of 10 passengers per hour, 0.2% more than in the reference situation.

Interestingly, during the transitions from morning peak to midday off-peak (approximately 9:00–9:29) and from midday off-peak to afternoon peak (approximately 15:00–15:29), the demand is much lower than in the rest of the corresponding period. This suggests that service quality during these transitions might be suboptimal. While the multi-period timetable design ensures feasible schedules, it currently overlooks aspects like minimizing waiting times during transitions. This potentially leads to extended waiting time between the last train of the previous schedule and the first trains of the new schedule. Addressing this would improve the quality of the multi-period timetable.

The percentage change in GJT in Table 5.3 displays a considerable variation; the maximum reduction is 62%, whereas the maximum increase peaks at 533%. Such substantial increases are attributed to the design of the multi-period line plan where not all stations are serviced bidirectionally. For instance, the route with a 533% increase in GJT, from Den Haag Moerwijk (Gvmw) to Den Haag HS (Gv) in the afternoon peak, exemplifies this issue. Previously, passengers could travel directly four times per hour, but in the updated schedule they need multiple transfers, extending the average GJT from 15.38 minutes to 71.73 minutes. Nevertheless, this does not have a large impact on the overall demand, as the initial demand for this OD pair is low.

In Figure 5.7 we can see for which OD pairs we do have a large change in the demand. This figure shows three heatmaps, corresponding to the morning peak, midday off-peak, and afternoon peak of the multi-period timetable. Each heatmap details the average hourly change in demand per OD pair, rounded to the nearest integer. The colour indicates whether the demand increases (blue) or decreases (red) and the magnitude of change.

The morning peak heatmap (7:00–8:59) in Figure 5.7 reveals a pronounced decrease in demand across many OD pairs, particularly towards stations Gvc, Laa, and Rsw and also departing

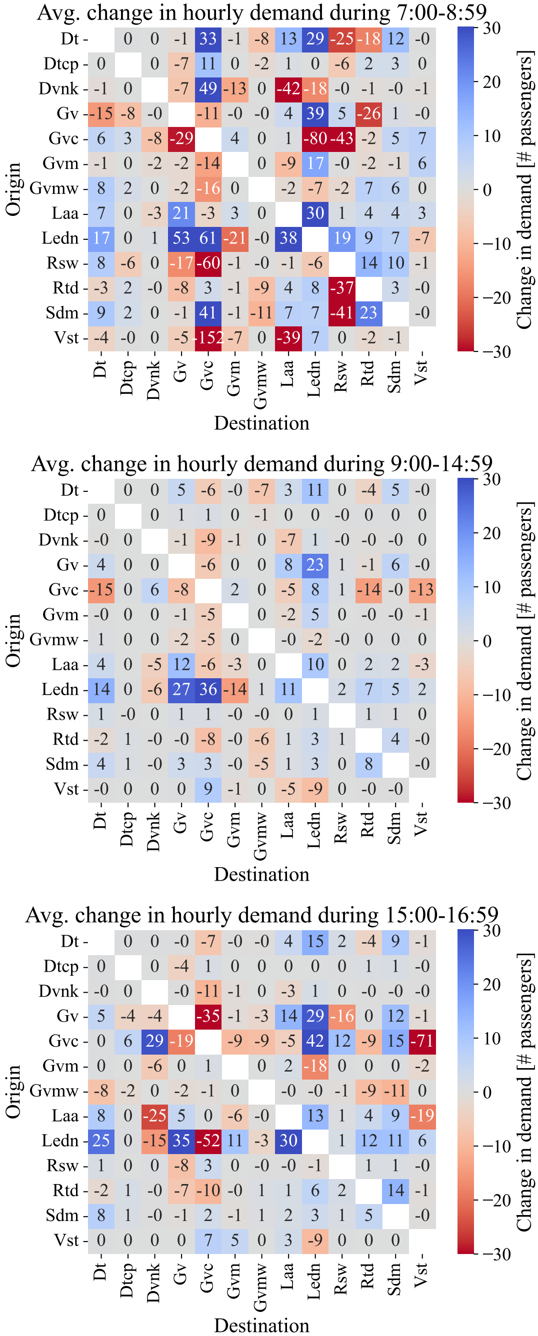

Figure 5.7: Heatmaps of average change in hourly demand under different timetables of the multi-period timetable

from Gvc. These stations share a common characteristic: a significant reduction in direct connections, coupled with high demand during peak hours. Due to the reduced direct connections, more passengers need to make transfers, raising the GJT, which in turn lowers demand. During the midday off-peak, the fluctuations in demand are less pronounced. In this period, stations like Gvc and Rsw regain some direct connections not available in the morning peak, which improves the GJT for several OD pairs to or from those stations. Furthermore, the demand is lower during off-peak hours, which results in smaller changes in demand. In the afternoon peak, again we see larger declines, particularly in the demand to and from station Gvc. The station that benefits the most from the new schedule is station Ledn: in all periods most passengers travelling to or from Ledn have a better GJT.

5.5.2 Adapted multi-period timetable

While the timetable evaluated in Section 5.5.1 has more passengers in the midday off-peak and afternoon peak, it has less demand in the morning peak. Based on this result, we could say that the current multi-period timetable is not satisfactory if we want to improve the service for passengers throughout the entire day. Hence, according to the flow chart in Figure 5.1 we should return to either the line planning or timetabling phase to make some improvements.

The morning peak is the only period in which the reference timetable is performing better than the multi-period timetable. Hence, we would like to make some changes in that period. As discussed in the previous section, the decrease in demand could be explained by the fact that the multi-period line plan has a lot less direct connections in the morning peak than the reference line plan. Hence, a possible improvement could be to use the reference line plan in the morning peak and the rest of multi-period line plan in the off-peak and afternoon peak. The multi-period timetabling model is used to create a timetable for this adapted line plan and next the adapted multi-period timetable is evaluated using the process described in Section 5.3.4.

Similarly to Table 5.2, Table 5.4 displays the overall results over the entire period (7:00–17:00), comparing the adapted multi-period timetable to the reference timetable. The adapted multi-period timetable has 941 more passengers than the reference timetable, which is an increase of 1.5%. Note that this increase is much higher than the increase of 150 passengers as achieved by the original multi-period timetable. Interestingly, while the average GJT in the adapted multi-period timetable (24.28) is almost one minute lower than the average GJT in the reference timetable (25.27 minutes), it is higher than the average GJT in the original multi-period timetable (24.10). This is due to the fact that the original multi-period timetable has more short trips (below 20 minutes) than the adapted timetable, which brings down the average GJT.

Table 5.4: Number of passengers and (average) GJT in reference and adapted multi-period timetables from 7:00–17:00.

Case	**Num. passengers**	**Total GJT [min]**	**Avg. GJT [min]**
Reference timetable	63,750	1,611,108	25.27
Adapted multi-period timetable	64,691	1,571,026	24.28
Absolute change	941	-40,082	-0.99
Percentage change	1.5%	-2.5%	-3.9%

The breakdown per period is given in Table 5.5, which shows the total and average change in demand and the minimum, median and maximum percentage change in GJT. The results show that there is an increase in demand in all periods. Even though the adapted multi-period timetable and the reference timetable now have the same line plan during the morning peak, the multi-period timetable manages to increase the demand with 116 trips per half hour. The reduction in GJT needed for this increase in demand comes from the redistribution of the trains over the hour. In the reference timetable, the lines between Ledn and Rtd and vice versa have departure intervals that alternate between 10 and 20 minutes. However, in the adapted timetable these lines have almost equal departure intervals. Therefore, most passengers on this line have lower waiting times. The last two rows of Table 5.5 show that the service during the transitions is worse than the service in the reference timetable, leading to a small decrease in demand. This strengthens the hypothesis that the transitions can be further optimised.

Table 5.5: Change in demand (number of passengers) and percentage change in GJT for the adapted multi-period timetable compared to the reference timetable. Results are broken down per period with a different timetable.

	Change in demand [# pass]		**% change in GJT over all OD pairs**		
Period	**Total**	**Avg. per half hour**	**Min.**	**Med.**	**Max.**
Morning peak (7:00–8:59)	463	116	-18%	-4%	24%
Midday off-peak (9:30–14:59)	479	44	-37%	-4%	406%
Afternoon peak (15:30–16:59)	18	6	-37%	-5%	512%
First transition (9:00–9:29)	-10	-10	-35%	-3%	296%
Second transition (15:00–15:29)	-8	-8	-37%	-1%	382%

When we compare these results with the results of the original multi-period timetable in Table 5.3, we see that there is mainly a significant improvement during the morning peak from a decrease of 372 passengers to an increase of 463 passengers. During the other periods, the original multi-period timetable has more passengers than the adapted one, but the difference is relatively small. Hence, we can conclude that overall the adapted multi-period timetable better serves the passengers than both the reference timetable and the original multi-period timetable.

5.5.3 Sensitivity analysis on elasticity parameter

The results presented in Sections 5.5.1 and 5.5.2 are influenced by the value of the elasticity parameter. Therefore, we perform an analysis to see how sensitive the results are to the value of this parameter. This is done by calculating the total number of passengers in both the original and adapted multi-period timetable, while the elasticity parameter varies between -1.2 and -0.6 (Wong & Yap, 2023). The value used in the case study is -0.9. Figure 5.8 shows the outcome of this sensitivity analysis, where the horizontal axis displays the value of the elasticity parameter and the vertical axis the total number of passengers. In this figure, the dash-dot line denotes the number of passengers for the original multi-period timetable and the dashed line denotes the number of passengers of the adapted timetable. The demand of the reference timetable is denoted by a horizontal line, as this is not dependent on the value of elasticity parameter.

Based on Figure 5.8 we can conclude that the result is not very sensitive to changes of the elasticity parameter. On average, the number of passengers increases (decreases) with about 18

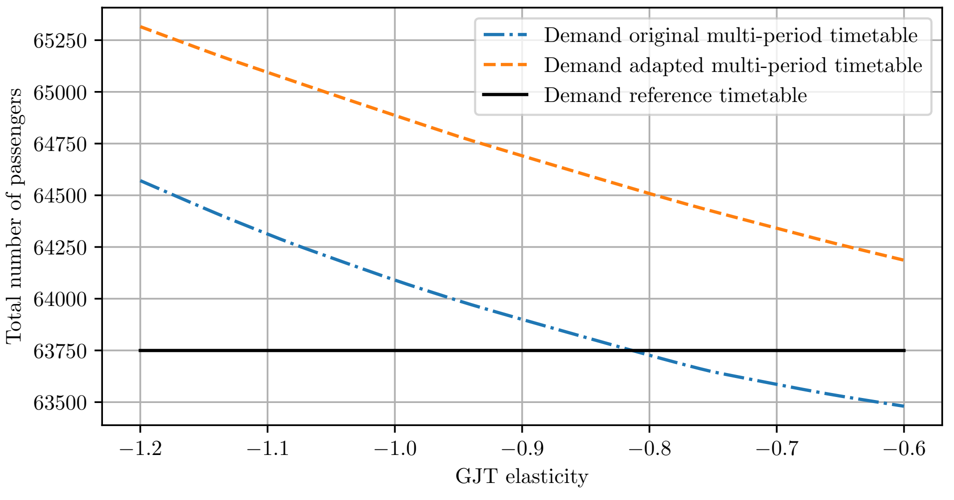

Figure 5.8: Total number of passengers in the original and adapted multi-period timetable, and in the reference timetable, under different values of the GJT elasticity.

when the elasticity is decreased (increased) by 0.01. However, if the elasticity is higher than -0.82, the demand of the original multi-period timetable drops below the demand of the reference timetable. Hence, we cannot definitively conclude that the original multi-period timetable can serve the demand better than the reference timetable. On the other hand, the demand in the adapted multi-period timetable is above the demand of the reference timetable for all the tested values of the elasticity parameter. Therefore, we can conclude that the adapted multi-period timetable can indeed serve the demand better than the reference timetable.

5.6 Conclusion

In this chapter, we have evaluated the multi-period timetabling concept by estimating its effect on passenger demand and Generalised Journey Time (GJT). Additionally, we have developed a methodological framework for designing and assessing multi-period timetables. Our case study, based on part of the Dutch railway network, demonstrates that adopting a multi-period timetable can substantially increase passenger demand and reduce the average GJT when considering fluctuating demand. These results were observed to be insensitive to the value of the elasticity parameter. Furthermore, the case study highlights the important role of the feedback loop within our framework. By iteratively refining the line plan and timetable based on the evaluation results, we were able to obtain a significantly improved timetable. This underscores the importance of evaluation and adjustment in the timetabling process.

There are a few limitations to the methodology used in this chapter. For example, we have only considered how the difference in GJT will affect the demand, while demand is also effected by other factors like punctuality and crowding in the vehicles. Furthermore, we have assumed that the value of the GJT elasticity is independent of the change in GJT, while elasticities usually only hold for small changes. In our case study, the maximum change in GJT observed was 533%, which cannot be considered as a small change. One upside is that the largest deviations in GJT are increases of the GJT, the largest decrease is -62%. Given an elasticity parameter of -0.9, any increase in GJT above 111% would result in a demand of 0, so the resulting demand is the same if the increase in GJT is 112% or 533%. However, further investigation is needed to determine how well the GJT elasticity of -0.9 represents changes in GJT between -62% and 112%.

There are several topics that future research could address. Firstly, several improvements could be made to the MILP models for multi-period line planning and timetabling. The original multi-period timetable that was fully created with these models did not provide a solution that improves the demand in all periods. Furthermore, the multi-period timetabling model currently only focusses on creating feasible transitions that resemble the next cyclic timetable as close as possible. However, the evaluation shows that the service during these transitions is worse than the service immediately before and after the transition. Including metrics on the waiting time during transitions in the timetabling model could potentially improve the service during these transitions. Lastly, operators must weigh the improved service and potential increase in passengers against the complexity and costs associated with more intricate (re)scheduling and train operations in the multi-period setting.

Chapter 6

Conclusions

This thesis explored how to design railway line plans and timetables that effectively accommodate passenger demand fluctuations throughout the day. To achieve this, several research questions were formulated in Chapter 1 and addressed in Chapters 2 to 5. In this chapter, we summarise the key findings and conclusions corresponding to each research question in Section 6.1. Additionally, we provide implications for practical application and directions for future research in Sections 6.2 and 6.3, respectively.

6.1 Main findings

The main question addressed in this thesis is *How can railway line plans and timetables be designed to effectively serve the varying demand?* Before providing the answer to this question, we will first answer the four subquestions below.

RQ1: What patterns and homogeneous periods can be identified in (Dutch) railway passenger demand? *(Chapter 2)*
To answer this question, we developed and applied a hierarchical clustering approach to identify patterns and homogeneous periods within passenger demand in Chapter 2. Our approach began with evaluating the applicability of an existing bus-demand clustering method to railway passenger data. This existing method utilises normalised origin-destination (OD) matrices as input, where the entry for each OD pair represents the share of all passengers travelling between that origin and destination. Consequently, this method captures the demand structure but ignores the passenger volumes. To provide a more comprehensive analysis, we complemented this with a clustering method based on regular (non-normalised) OD matrices that incorporates both the demand structure and the passenger volumes. Both clustering techniques were applied to divide the day into contiguous time periods for each workday. Subsequently, these daily periods were clustered to identify similarities and differences across workdays.

We applied these methods to a case study covering a large part of the railway network in the Netherlands. Using the realised demand data of the Netherlands Railways (NS) from 2019, we created OD matrices for each ordinary workday as the basis for our clustering analyses. Applying these methods revealed notable differences depending on the type of OD matrix used. The normalised OD matrix method divided the day into two periods: a morning period and an afternoon/evening period. This reflects the asymmetric nature of demand, with passengers typically travelling to destinations in the morning and returning in the afternoon or evening.

In contrast, the method using regular OD matrices identified a finer-grained division of the day into 9 to 10 periods. These included distinct pre-morning peak, morning peak, midday off-peak, afternoon peak, and evening periods. Furthermore, both the morning and afternoon peaks were further subdivided into three phases: the start, the hyper peak, and the end. The periods identified through regular OD matrices showed greater compactness in terms of passenger volumes and average kilometres travelled, making them more appropriate for informing line plan and timetable design.

Comparing demand patterns across different workdays indicated a high degree of similarity between days. One notable exception was the peak periods on Fridays, which differed significantly from those of Monday to Thursday. This suggests that Friday peak hours could benefit from a distinct service plan to better reflect the specific demand characteristics. However, as the variations within individual workdays were more pronounced than the differences between days, the other research subquestions focus on addressing the differences in demand within the day.

RQ2: Which line plan variations could be considered when creating a multi-period line plan to better serve the varying demand? ***(Chapter 3)***

To better accommodate varying passenger demand throughout the day, this research subquestion focused on developing a multi-period line plan rather than a fully cyclic or fully acyclic one. In a multi-period line plan, the day is segmented into several periods, each with its own cyclic line plan. The findings from RQ1 support this approach, indicating that passenger demand remains relatively homogeneous within certain time periods. Consequently, there is no need to alter the line plan during these intervals. Using the same line plan within each period improves the schedule's memorability and reduces planning efforts compared to a fully acyclic line plan, since it requires creating line plans for a limited number of periods instead of generating a plan for the entire day.

Several line plan variations can be considered when creating a multi-period line plan. These include the selection of routes, stopping patterns, frequencies, and the use of asymmetric lines. Since passenger flows and station-specific demand change throughout the day, adjusting routes and stopping patterns enables the plan to better respond to temporal demand variations. Modifying frequencies allows the service to accommodate fluctuations in passenger volumes, providing higher frequencies for larger passenger groups and thereby increasing the attractiveness of the line plan. Additionally, incorporating asymmetric lines can effectively address spatial imbalances in demand observed during some periods. An asymmetric line operates with different frequency and/or stopping pattern in each direction.

In Chapter 3, we proposed a mixed-integer linear programming model that includes all these line plan variations. The model's objective is to minimise the total Generalised Journey Time (GJT) of the passengers, thereby creating an attractive line plan. Making changes to the line plan throughout the day also brings a certain cost to the railway undertaking (RU), as it complicates the creation of the rolling stock circulations and makes rescheduling during disruptions more challenging. To balance these trade-offs, the model uses the ε-constraint method to generate multiple line plans with varying amounts of line plan adjustments. This approach enables planners at the RU to evaluate and compare different schedules, helping them determine whether the benefits of introducing additional line plan changes justify the associated operational costs.

The proposed model and solution method were tested on a case study based on part of the Dutch railway network. The results showed that allowing for changes to the line plan during the day could reduce the total GJT by up to 1.9% when symmetric lines were used, and by as much as 4.3% when asymmetric lines were allowed. In general, line plans with asymmetric lines consistently achieved lower total GJT than those with symmetric lines, given the same costs and number of adjustments. Additionally, the incremental GJT savings diminished as the number of adjustments increased. The model initially optimised frequencies throughout the day, which is especially beneficial for reducing the GJT since all passengers on a given line benefit from higher frequencies. Once the frequency adjustments were made, the model began to make changes to the stopping patterns. These changes typically yielded smaller GJT improvements because they shorten travel times for some passengers while increasing them for others. Consequently, the diminishing returns in GJT savings highlight the importance for planners of the RU to carefully determine the optimal number of line plan changes for their specific context. Overall, the case study demonstrated that the included line plan variations enable the creation of a multi-period line plan that effectively adapts to varying passenger demand.

RQ3: How can a multi-period timetable be created for a multi-period line plan? ***(Chapter 4)***

In the multi-period line plan, the day is segmented into several periods, each with its own cyclic line plan. As demonstrated in Chapter 3, this approach effectively adapts to varying passenger demand throughout the day. The next step is to create a multi-period cyclic timetable for this multi-period line plan, such that the timetable is cyclic within each period. By repeating the timetable every hour during periods with homogeneous demand, we enhance the memorability of the timetable compared to a fully acyclic timetable.

In Chapter 4, we presented a mixed-integer linear programming model to construct multi-period timetables. This model takes as input information about the railway network and a multi-period line plan, where for each line a specific time-period is given during which the line is operated. The resulting timetable is required to be cyclic within each period, provide a good transition between the periods, and be feasible by ensuring that trains using the same infrastructure maintain at least a minimum headway. To facilitate the transition between cyclic periods, the trains departing during the first cycle(s) of the new period are allowed to deviate from the cyclic departure times or may be cancelled if necessary. The model aims to create a timetable that is attractive for the passengers by minimising avoidable journey time, deviations from regular departure time intervals, and by penalising shifts from cyclic departure times and train cancellations during the transition phases to enhance memorability. These four objectives are combined into a weighted sum by normalising each objective and setting weights reflecting the relative importance of each objective.

The model has been successfully tested on a case study based on a section of the Dutch railway network. In this case study, the demand was considered homogeneous within three distinct periods: the morning peak, midday off-peak, and afternoon peak, each served by a period-specific line plan with varying stopping patterns. The results demonstrated the model's capability to generate different feasible timetables, by varying the weights in the objective function. Analysis of the timetables revealed that placing insufficient weight on avoiding train cancellations led to poor-quality schedules during transition periods, while yielding only minimal improvements in avoidable journey time. Furthermore, a sensitivity analysis highlighted the trade-off between minimising avoidable journey time (which minimises passenger in-vehicle time), and achieving an equal departure time distribution (which minimises the deviations from the passengers' desired departure times). When trains with incompatible frequencies share the same infrastructure, some degree of uneven departure intervals becomes necessary. If the objective strongly prioritised minimising avoidable journey time, this uneven distribution persisted across all stations on the route. Conversely, when greater weight was given to maintaining equal departure intervals, additional dwell and run time were used to achieve more regular departures where feasible.

RQ4: How will the introduction of a multi-period timetable affect the railway demand? *(Chapter 5)*
To assess whether the multi-period timetable can serve the varying demand better than the traditional fixed cyclic timetable, Chapter 5 examined the potential impact on railway demand using a case study based on part of the Dutch railway network. The clustering method developed in Chapter 2 was used to identify periods with homogeneous demand, based on which the models described in Chapters 3 and 4 created a multi-period line plan and timetable for the case study. Next, the passengers' GJT was calculated for both the multi-period timetable and a reference timetable, which was based on the timetable operated in that part of the network in 2023.

Analysis of the case study showed that the multi-period timetable could reduce the average GJT, and increase demand by up to 1.5%. These results were shown to be robust to changes in the value of the elasticity parameter. However, improvements were not uniform across all OD pairs. For some OD pairs the GJT increased significantly in the multi-period timetable, with the largest percentage increase being 533%. This was due to the fact that some stations were only served in one direction in the multi-period timetable. Passengers travelling in an unserved direction either have to first travel away from their intended destination and transfer before continuing in the correct direction, or they must pass their destination without stopping and then travel backwards to reach it. Both situations significantly increase the overall journey time. Thus, while in general the multi-period timetable could reduce the average GJT and attract more passengers, this was not the case for each OD pair. Instead, the service was improved for OD pairs that had a lot of passengers, sometimes at the expense of those with fewer passengers.

Main RQ: How can railway line plans and timetables be designed to effectively serve the varying demand?
As indicated in the conclusions from the research questions, multi-period line plans and timetables can effectively serve the varying passenger demand throughout the day while offering greater memorability for passengers than strictly acyclic timetables. In a multi-period line plan and timetable, the schedule is repeated hourly within each period, so only one hourly schedule per period needs to be created. This significantly reduces the planning effort compared to developing a complete day schedule with many hourly variations according to a fully acyclic timetable.

Throughout Chapters 2 to 5, we presented a comprehensive set of methods that together facilitate the design of railway line plans and timetables tailored to varying passenger demand. The methodological framework presented in Chapter 5 details this approach. The process involves four stages: identifying periods with homogeneous passenger demand, developing a multi-period line plan, constructing a multi-period timetable for this line plan, and evaluating the resulting timetable against a reference timetable. The clustering algorithm introduced in Chapter 2 enables the segmentation of the day into periods with homogeneous passenger demand. Subsequently, the optimisation models from Chapters 3 and 4 support the creation of multi-period line plans and timetables, respectively. Chapter 5 described a method to evaluate the multi-period timetable, which uses the change in GJT compared to the reference timetable and time elasticities to estimate the new demand. Following the analysis of changes in GJT and demand, a decision is made either to accept the timetable or to revisit the line planning or timetabling stages to refine the solution further. This iterative process enables the creation of multi-period cyclic timetables that effectively reduce average GJT and increase the total passenger demand, thereby improving the overall service performance.

6.2 Implications for practice

In this thesis, we have demonstrated that multi-period line plans and timetables can serve varying passenger demand better than the fixed cyclic schedules currently used in the Netherlands. However, adopting such a timetable would have significant implications for several stakeholders, including passengers, the railway undertaking (RU), and policymakers. In this section, we outline the key implications for these three groups.

Passengers
The multi-period timetable is better aligned with the passenger demand throughout the day than a regular hourly timetable repeated over the entire day, while using the same resources. Therefore, most passengers will experience lower journey times under the multi-period timetable. However, this comes at the cost of the service for some lesser used OD pairs. The line planning model presented in Chapter 3 currently favours OD pairs with larger passenger volumes, so the service for these OD pairs is improved while the service of OD pairs with fewer passengers is sometimes greatly deteriorated. This results in that many passengers will experience shorter trips, while some passengers also experience a significant increase in their journey time. However, as the multi-period timetable can improve the service while using the same resources, the costs of the train services remain the same. Hence, the ticket prices or the financial burden on taxpayers does not increase.

Regarding the memorability of the timetable, the multi-period timetable is harder to remember than a fully cyclic schedule. However, it is also easier to remember than a fully acyclic timetable as there is regularity within certain periods of the day. The memorability of the schedule also depends on the user group. Regular users of train services are most likely to remember the schedule. During the thesis, we have mainly focused on changing the timetable during the day as the differences between days are limited. If demand on different days is similar, the same timetable can be used. Hence, passengers who regularly make the same trip around the same time can still easily remember the train schedule that is relevant to them. On the other hand, when a different schedule is created for different weekdays or if people regularly do the same trip, but not around the same time, the memorability of the schedule can decrease significantly. Passengers that do not regularly use train services, or do many different types of trips, likely have to consult a trip planner before conducting the trip anyway, so the reduced memorability likely is not an issue for them. One group that could struggle with a more complex timetable are those who lack digital skills or the ability to look up a travel advice. As the timetable becomes harder to memorise, passengers will increasingly have to rely on online trip planners, which could be problematic for elder or tech-illiterate people.

Railway undertaking
The introduction of a multi-period timetable offers a promising opportunity to enhance the profitability of the RU. This thesis has demonstrated that such a timetable can more effectively accommodate passenger demand throughout the day without increasing the total number of

train kilometres. By enabling the RU to make better use of its rolling stock and personnel, the multi-period timetable allows for a more efficient allocation of resources. Consequently, serving a higher demand with the same resources can lead to increased profitability for the RU.

However, this improvement comes with notable challenges, particularly in the planning process. Unlike a fully cyclic timetable, which relies on a single line plan and timetable applicable throughout the day, a multi-period timetable requires the development of multiple line plans and timetables. Although various software tools exist to support this task, it remains impossible to generate a comprehensive nationwide line plan and timetable with the press of a single optimisation button. Human planners must still invest considerable effort to produce a timetable that can be executed. Furthermore, the creation of rolling stock schedules becomes more complicated, especially when asymmetric frequencies are introduced within the multi-period framework. Moreover, when contingency plans are used during disruptions, more plans have to be created as there are more variations in the timetable throughout the day. As a result, adopting a multi-period timetable will increase the workload and complexity faced by planners within the RU.

Given these challenges, it is crucial to strike a balance in the number of line plan and timetable adjustments permitted. As discussed in Chapter 3, each successive modification to the line plan tends to yield diminishing savings in Generalised Journey Time. While asymmetric line frequencies provide greater flexibility to align services with passenger demand, they further complicate rolling stock scheduling. Additionally, significant fluctuations in the timetable throughout the day may make it harder for passengers to comprehend and utilise the service, potentially deterring some from travelling by train. To address this risk, the RU must prioritise passenger accessibility and ease of use. This can be achieved by offering user-friendly route planning applications and ensuring that staff are readily available at stations to assist travellers who may be less comfortable with digital tools. By combining a flexible timetable with effective passenger support, the RU can maximise the benefits of the multi-period approach while minimising its drawbacks.

Policymakers

The transition to a multi-period timetable introduces important considerations for policymakers. In the Netherlands, the national government grants concessions to RUs for specific parts of the railway network. A concession includes various agreements regarding key parameters such as service levels (for instance, how frequently stations should be served and the overall punctuality of trips) and ticket pricing. Through this concession framework, policymakers have significant influence over the flexibilities granted to the RU during the line plan and timetable creation processes. Agreements can be established to ensure that railway services remain fair and accessible.

In this thesis, the existing concession agreements have not been taken into account. For example, the current concession mandates that on weekdays almost all stations must be served at least twice per hour in both directions during the day. The line plans developed within this research do not comply with these established concession requirements, for example by serving some stations only in one direction. However, permitting such flexibilities allows for resources to be reallocated from low-demand OD pairs to high-demand OD pairs, enabling more passengers to benefit from faster journeys, although at the expense of fewer passengers on less frequented routes. Therefore, a transition to a multi-period timetable will necessitate negotiations between policymakers and the RU to determine which flexibilities should be permitted.

Some key questions to consider include: Should stations be allowed to be served in only one direction? And what minimum level of service should be guaranteed at each station?

At the same time, prescribed service levels may constrain the RU's ability to optimise profitability, since increasing service frequency or coverage can raise operational costs without proportional increases in revenue. It is therefore crucial that concession conditions strike a careful balance between maintaining sufficient service quality for all passengers and ensuring financial viability. Achieving this balance will provide the RU with sufficient flexibility to operate profitably while fulfilling its public service obligations.

6.3 Recommendations for future research

We see several areas for future research that are interesting to explore to further optimise the multi-period timetable and ensure that it can be used in practice.

An important direction for future research is the development of a method to determine the optimal number of periods to use throughout the day. In Chapter 2 we have provided a method to identify the maximum number of periods that need to be considered throughout the day based on demand variations. However, using all these periods in a multi-period line plan and timetable may not be ideal. Firstly, some of these periods were quite short, with the shortest period lasting only 30 minutes. As it takes time to transition between schedules, using periods that are this short may not be practical. The size of the network also influences the minimum practical period length, as networks with longer routes require more time to switch between timetables. Secondly, the number of periods also affects the planning efforts needed to create a schedule. When more periods are taken into account, more line plans and timetables have to be created. Thirdly, a higher number of periods reduces the memorability of the timetable for the passengers. One of the main reasons for introducing the multi-period timetable is that it is easier for passengers to remember than a completely acyclic timetable, since during certain parts of the day the same timetable is used. However, if the timetable changes often (up to every 30 minutes), than this advantage diminishes significantly. It would be valuable to determine a method for finding the optimal number of periods to consider which takes these factors into account.

Another important aspect that was not addressed in this thesis is how passengers would perceive and experience using a multi-period timetable. We have assumed that a multi-period timetable is easier to remember than completely acyclic timetables or other hybrid timetables that have both cyclic and acyclic characteristics, because the multi-period timetable has several periods during which the timetable is completely cyclic. It would be interesting to investigate how passengers experience using a multi-period timetable and compare its memorability with the memorability of other types of hybrid timetables. Additionally, the memorability of the timetable could also be significantly reduced if different timetables are used for different days of the week. This thesis focused on variations within the day because the demand analysis in Chapter 2 showed that the demand patterns across different weekdays were quite similar. However, it should be noted that this analysis used demand data from before the Covid-19 pandemic. Since then, the passenger demand in the Netherlands has shifted, where on Tuesdays and Thursdays more people travel to work compared to other weekdays due to an increase in teleworking from home. As differences in demand between the weekdays have increased, it might be logical to develop different line plans and timetables for different days. While such an approach could improve the alignment with demand, it would reduce the timetable memorability for passengers travelling on multiple days during the week.

Another direction for future research is how the introduction of a multi-period timetable affects the other planning stages beyond the line planning and timetabling stages. When a RU switches from a cyclic timetable to a multi-period timetable, extra effort is needed to create the timetable, as a line plan and timetable have to be designed for each period throughout the day. However, the multi-period timetable potentially also complicates the rolling stock planning and crew scheduling, and rescheduling trains after a disruption. To fully assess the costs and operational impact of implementing a multi-period timetable, these added complexities also need to be taken into account. Understanding these effects will provide RUs with a more comprehensive view of the trade-offs involved and help them determine whether adopting a multi-period timetable is beneficial.

Lastly, future research is needed to create multi-period timetables for large-scale networks, such as the entire Dutch railway network. In this thesis, we have used a busy but relatively small part of the Dutch railway network to test our models. Due to its limited size, we assumed that transitions between cyclic timetables can be realised within one hour. However, for larger networks, where minimum journey times for trains to drive from origin to destination may exceed one hour, this assumption no longer holds. Future research should explore whether extending the transition period during which trains have greater flexibility is sufficient, or if different measures are needed. Additionally, solving large-scale planning problems for entire national networks may require specialised heuristics, making this another interesting direction for future research.

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Summary

Passenger demand for railway services fluctuates considerably throughout the day, varying not only in volume but also in travel purposes and hence destinations. Despite these variations, in many European countries, including the Netherlands, line plans and timetables are operated that remain nearly fixed throughout the day. These cyclic schedules have several benefits for passengers, including that they are easy to remember and therefore easy to use, and that they provide consistent transfer opportunities throughout the day. However, as cyclic railway timetables are often based on peak hour demand, the service offered during the off-peak may be suboptimal. Furthermore, railway undertakings (RUs) must use their limited resources (e.g., train fleet and personnel) as effectively as possible, especially when they face financial challenges. Therefore, RUs may seek to tailor their line plans and timetables more closely to fluctuating demand.

This thesis aims to design line plans and timetables that effectively accommodate the varying demand while preserving the benefits of cyclic timetables as much as possible. To achieve this, we divided the day into periods during which the passenger demand is homogeneous. Within these periods, the same line plan and (cyclic) timetable can be used, as the demand is essentially the same. By using the same line plan and timetable during certain periods, the memorability of the timetable can be improved compared to a completely acyclic timetable. In this thesis, we introduce methods and optimisation models to analyse the passenger demand to find homogeneous periods and based on these periods create multi-period line plans and timetables.

First, to identify patterns and homogeneous periods in railway demand, Chapter 2 introduces a hierarchical clustering method specifically tailored to railway data. We begin by assessing the transferability of an existing clustering method developed for bus demand to railway origin-destination (OD) data. Since this method relies on normalised OD matrices, it does not consider variations in the demand volumes. To better capture railway demand characteristics, we enhance this technique by using regular OD matrices, thereby incorporating both demand volume and structure. The clustering technique is applied in two ways: to identify contiguous homogeneous periods during the day, and to determine periods with similar demand patterns across different weekdays. Testing these methods on realised OD data from the Dutch railway network reveals substantial differences between the periods based on regular OD matrices and those based on normalised OD matrices. The periods based on regular OD matrices are more compact in terms of passenger volumes and average kilometres travelled, and therefore more suitable to use as input for designing a service plan. Additionally, the demand patterns across most workdays are similar, although the peak periods on Fridays are less pronounced. Based on these findings, the subsequent chapters focus on designing line plans and timetables that adapt to demand variations throughout the day.

Second, Chapter 3 explores which line plan variations could be included in the multi-period line planning problem. In a multi-period line plan, each time period has its own tailored line plan to reflect the specific demand during that interval. Chapter 3 introduces a mixed-integer

linear programming model for designing an optimal multi-period line plan. This model extends existing literature by incorporating a comprehensive set of line planning decisions, including route selection, stopping patterns, frequencies, and asymmetric lines. For asymmetric lines, the stopping patterns and/or frequencies can differ between directions on the same route. The model's objective is to minimise the passengers' Generalised Journey Time (GJT), accounting for in-vehicle time as well as frequency-based waiting and transfer times, subject to a fixed budget of allowed train kilometres. Using the ε-constraint method, Pareto optimal solutions are identified with varying degrees of modifications to the line plan throughout the day. The model is tested on a case study of part of the Dutch railway network. Results demonstrate that adapting the line plan throughout the day can reduce total GJT by up to 4.3%. Higher GJT savings are achievable with asymmetric lines than with symmetric ones, as the former allow for better handling of spatially imbalanced passenger demand. Overall, the results highlight the significant potential of moving beyond fixed line plans to improve service quality.

Third, we investigate how to create timetables for multi-period line plans that combine the memorability of cyclic timetables with the flexibility of acyclic ones. To achieve this, Chapter 4 introduces the concept of multi-period cyclic timetables, and a mixed-integer linear programming model to design such timetables. This multi-period cyclic timetable maintains a cyclic schedule within each period while ensuring feasible transitions between successive periods. The model aims to create a timetable that is both attractive and easy to remember for passengers. This is done by minimising a weighted sum of train journey times, deviations from regular departure intervals, and penalties for deviating from the cyclic schedule during transitions. The model is validated on a case study with changing stopping patterns, for which it successfully generates an optimal timetable. In this timetable, the transition between cyclic schedules can be done without cancelling trains or shifting trains from the new cyclic times. Additionally, a sensitivity analysis of the different objectives reveals a trade-off between minimising journey times and departure interval irregularity.

Fourth, we evaluate the impact of implementing a multi-period railway timetable on passengers' GJT and passenger demand in Chapter 5. For a case study based on part of the Dutch railway network, we compare the passengers' GJT under a traditional cyclic timetable with that under a multi-period timetable. Changes in GJT are then translated into demand adjustments through incremental elasticity analysis. The results show that adopting a multi-period timetable can reduce the average GJT and increase passenger demand by up to 1.5%. These results are shown to be robust to value changes of the elasticity parameter. Furthermore, the chapter presents a methodological framework for designing and evaluating multi-period timetables. This framework features an integrated feedback loop, which enables iterative improvements to both the line plan and timetable based on evaluation insights.

In summary, this thesis presents a comprehensive approach to designing railway line plans and timetables that effectively respond to fluctuating passenger demand throughout the day, while being easier to remember than completely acyclic timetables. Key contributions include a novel clustering method to identify homogeneous demand periods, and mixed-integer linear programming models for designing multi-period line plans and timetables that balance memorability with operational flexibility. Furthermore, the impact of multi-period timetables on passenger GJT and demand is rigorously evaluated, supported by a methodological framework that enables iterative refinement through feedback loops. This comprehensive framework offers valuable insights and practical tools for RUs seeking to enhance service quality and resource efficiency in the face of varying demand.

Samenvatting

De reizigersvraag naar treindiensten fluctueert aanzienlijk gedurende de dag, niet alleen qua reizigersaantallen, maar ook qua reisdoel en dus bestemming. Ondanks deze schommelingen worden in veel Europese landen, waaronder Nederland, dienstregelingen gebruikt die gedurende de dag vrijwel hetzelfde blijven. Deze cyclische dienstregelingen bieden verschillende voordelen voor reizigers, waaronder dat ze gemakkelijk te onthouden en dus gemakkelijk te gebruiken zijn, en dat ze gedurende de hele dag dezelfde overstapmogelijkheden bieden. Echter, hierdoor kan het aanbod tijdens de daluren suboptimaal zijn, omdat cyclische dienstregelingen vaak gebaseerd zijn op de reizigersvraag tijdens de spitsuren. Bovendien moeten spoorwegmaatschappijen hun beperkte middelen (zoals materieel en personeel) zo effectief mogelijk inzetten, vooral wanneer zij met financiële uitdagingen worden geconfronteerd. Daarom kunnen spoorwegmaatschappijen ernaar streven hun lijnvoeringen en dienstregelingen beter af te stemmen op de fluctuerende vraag.

Dit proefschrift heeft als doel lijnvoeringen en dienstregelingen te ontwerpen die effectief inspelen op de wisselende vraag, terwijl de voordelen van cyclische dienstregelingen zoveel mogelijk behouden blijven. Om dit te bereiken is voorgesteld om de dag op te delen in periodes waarin de reizigersvraag homogeen is. Binnen deze periodes kan dezelfde lijnvoering en (cyclische) dienstregeling gebruikt worden, aangezien de vraag ongeveer hetzelfde is. Door tijdens bepaalde periodes dezelfde cyclische lijnvoering en dienstregeling te gebruiken, kan de dienstregeling makkelijker onthouden worden dan een volledig acyclische dienstregeling. In dit proefschrift introduceren we methoden en optimaliseringsmodellen om de reizigersvraag te analyseren om zo homogene periodes vast te stellen en op basis hiervan multiperiode lijnvoeringen en dienstregelingen op te stellen.

Ten eerste is een hiërarchische clustermethode geïntroduceerd, speciaal afgestemd op spoorwegdata, om patronen en homogene periodes in de vraag naar spoorvervoer te identificeren. Eerst hebben we beoordeeld of een bestaande clustermethode die is ontwikkeld voor de vraag naar busvervoer ook geschikt is voor herkomst-bestemming (HB) data van het spoorvervoer. Aangezien deze methode gebaseerd is op genormaliseerde HB-matrices, houdt deze geen rekening met variaties in de vraagvolumes. Om de kenmerken van de vraag naar spoorvervoer beter in kaart te brengen, hebben we deze methode verbeterd door gebruik te maken van reguliere HB-matrices, waardoor zowel het vraagvolume als de structuur van de vraag worden meegenomen. De clustertechniek is op twee manieren toegepast: om aaneengesloten homogene periodes gedurende de dag te identificeren en om periodes met vergelijkbare vraagpatronen gedurende de week te bepalen. Het testen van deze methoden op gerealiseerde HB-data van het Nederlandse spoorwegnetwerk laat aanzienlijke verschillen zien tussen de periodes op basis van reguliere HB-matrices en die op basis van genormaliseerde HB-matrices. De periodes op basis van reguliere HB-matrices zijn compacter in termen van reizigersvolumes en gemiddeld afgelegde kilometers, en daarom geschikter om te gebruiken als input voor het ontwerpen van een dienstregeling. Bovendien zijn de vraagpatronen op de meeste werkdagen vergelijkbaar, hoewel de piekperiodes op vrijdag minder uitgesproken zijn. Op basis van deze bevindingen richten de volgende hoofdstukken zich op het ontwerpen van lijnvoeringen en dienstregelingen die zich aanpassen aan vraagvariaties gedurende de dag.

Ten tweede is onderzocht welke variaties in de lijnvoering kunnen worden opgenomen in het multiperiode lijnvoeringsprobleem. In een multiperiode lijnvoering heeft elke periode zijn eigen op maat gemaakte lijnvoering om de specifieke vraag tijdens die periode te bedienen. Hoofdstuk 3 introduceert een gemengd-geheeltallig lineair programmeringsmodel voor het ontwerpen van een optimale multiperiode lijnvoering. Dit model breidt de bestaande literatuur uit door een uitgebreide reeks lijnvoeringsbeslissingen op te nemen, waaronder routekeuze, stoppatronen, frequenties en asymmetrische lijnen. Voor asymmetrische lijnen kunnen de stoppatronen en/of frequenties verschillen tussen de richtingen op dezelfde route. Het doel van het model is het minimaliseren van de gegeneraliseerde reistijd (GRT) van de reizigers, rekening houdend met de tijd in het voertuig en de op frequentie gebaseerde wachttijden en overstaptijden, binnen een vast budget van toegestane treinkilometers. Met behulp van de ε-constraint-methode worden Pareto-optimale oplossingen geïdentificeerd met verschillende hoeveelheden aanpassingen aan de lijnvoering gedurende de dag. Het model is getest op een casestudy van een deel van het Nederlandse spoorwegnet. De resultaten tonen aan dat aanpassing van de lijnvoering gedurende de dag de totale GRT met 4.3% kan verminderen. Met asymmetrische lijnen kunnen grotere GRT-besparingen worden gerealiseerd dan met symmetrische lijnen, omdat de ruimtelijk onevenwichtige reizigersvraag beter bediend kan worden met asymmetrische lijnen. Over het algemeen wijzen de resultaten op het aanzienlijke potentieel van het afstappen van een vaste lijnvoering om de kwaliteit van de dienstverlening te verbeteren.

Ten derde hebben we onderzocht hoe we dienstregelingen kunnen maken voor multiperiode lijnvoeringen die de onthoudbaarheid van cyclische dienstregelingen combineren met de flexibiliteit van acyclische dienstregelingen. Om dit te bereiken, introduceert Hoofdstuk 4 het concept van multiperiode cyclische dienstregelingen en een gemengd-geheeltallig lineair programmeringsmodel om zulke dienstregelingen te ontwerpen. Deze multiperiode cyclische dienstregeling handhaaft een cyclische dienstregeling binnen elke periode en zorgt tegelijkertijd voor haalbare overgangen tussen opeenvolgende periodes. Het model is bedoeld om een dienstregeling te creëren die zowel aantrekkelijk als gemakkelijk te onthouden is voor reizigers. Dit wordt gedaan door het minimaliseren van een gewogen som van rij- en halteertijden, afwijkingen van regelmatige vertrektijdintervallen en boetes voor afwijkingen van de cyclische dienstregeling tijdens overgangen. Het model is gevalideerd aan de hand van een casestudy met veranderende stoppatronen, waarvoor succesvol een optimale dienstregeling gegenereerd is. In deze dienstregeling kan de overgang tussen de verschillende cyclische dienstregelingen worden uitgevoerd zonder treinen te annuleren of treinen te verschuiven ten opzichte van de cyclische tijden in de nieuwe periode. Bovendien blijkt uit een gevoeligheidsanalyse van de verschillende doelstellingen dat er een afweging moet worden gemaakt tussen het minimaliseren van de rij- en halteertijden enerzijds en de onregelmatigheid van de vertrektijden anderzijds.

Ten vierde hebben we de impact van de invoering van een multiperiode dienstregeling op de GRT van reizigers en de reizigersvraag geëvalueerd. Voor een casestudy op basis van een deel van het Nederlandse spoorwegnet hebben we de GRT van reizigers onder een traditionele cyclische dienstregeling vergeleken met die onder een multiperiode dienstregeling. Veranderingen in GRT zijn vervolgens vertaald naar vraagaanpassingen met behulp van incrementele elasticiteitsanalyse. De resultaten tonen aan dat de invoering van een multiperiode dienstregeling de gemiddelde GRT kan verminderen en de reizigersvraag tot 1.5% kan doen laten toenemen. Deze resultaten blijken robuust te zijn ten opzichte van waardeveranderingen van de elasticiteitsparameter. Verder is in dit hoofdstuk een methodologisch kader gepresenteerd voor het ontwerpen en evalueren van multiperiode dienstregelingen. Dit kader omvat een geïntegreerde feedbackloop, die iteratieve verbeteringen van zowel de lijnvoering als de dienstregeling op basis van evaluatie-inzichten mogelijk maakt.

Samengevat presenteert dit proefschrift een uitgebreide aanpak voor het ontwerpen van lijnvoeringen en dienstregelingen die effectief inspelen op de fluctuerende reizigersvraag gedurende de dag, terwijl ze gemakkelijker te onthouden zijn dan volledig acyclische dienstregelingen. Belangrijke bijdragen zijn onder meer een nieuwe clustermethode om homogene vraagperiodes te identificeren, en gemengd-geheeltallige lineaire programmeringsmodellen voor het ontwerpen van multiperiode lijnvoeringen en dienstregelingen die een evenwicht bieden tussen onthoudbaarheid en operationele flexibiliteit. Bovendien is de impact van multiperiode dienstregelingen op de GRT en de vraag van reizigers grondig geëvalueerd, ondersteund door een methodologisch kader dat iteratieve verfijning mogelijk maakt door middel van feedbackloops. Dit uitgebreide kader biedt waardevolle inzichten en praktische hulpmiddelen voor spoorwegondernemingen die de kwaliteit van hun dienstverlening willen verbeteren en hun middelen efficiënter willen gebruiken in het licht van een wisselende vervoervraag.

About the author

Renate van der Knaap was born in 1996 in Naaldwijk, the Netherlands. She started her Bachelor's degree in Business Economics at Tilburg University in 2014. During her studies, she participated in an Erasmus exchange at the Athens University of Economics and Business in Greece, where a course on linear programming sparked her interest in mathematical optimisation. To gain admission to the Master's programme in this field, she also completed several courses from the Econometrics bachelor's curriculum before graduating cum laude with her BSc degree in 2019. Renate went on to complete her Master's degree in Business Analytics and Operations Research from Tilburg University in 2021, again graduating cum laude. Her MSc thesis was written during an internship at the Dutch railway undertaking NS, where she became acquainted with the complex and fascinating challenges involved in planning public transport.

Inspired by her MSc thesis work, Renate joined TU Delft's department of Transport & Planning in September 2021 to start her PhD research. Throughout her doctoral studies, she continued collaborating with NS, researching methods to develop more flexible railway line plans and timetables that incorporate demand-driven train service variations. She was an active member of both the Digital Rail Traffic lab and the Smart Public Transport Lab within the department.

Throughout her PhD research, Renate actively contributed to various departmental activities. She supervised two MSc students, guiding one through writing their thesis and supporting another with an additional research project. She also organised the monthly department SPARKS meetings and represented the interests of PhD candidates within the management team of Transport & Planning. Additionally, Renate managed the website of the Smart Public Transport Lab, successfully overseeing a complete migration to a new digital platform.

Publications

Journal papers

1. **van der Knaap, R. J. H.**, van Oort, N., & Goverde, R. M. P. (2026). Evaluating the Impact of Multi-Period Railway Timetabling on Passenger Demand. *(Under revision)*.
2. **van der Knaap, R. J. H.**, van Oort, N., de Bruyn, M., & Goverde, R. M. P. (2026). Multi-period line planning for varying railway passenger demand with asymmetric lines. *(Under revision)*.
3. **van der Knaap, R. J. H.**, van Oort, N., & Goverde, R. M. P. (2025). Multi-period railway timetabling to serve time-dependent demand. *Journal of Rail Transport Planning & Management*, 35, 100536.
4. **van der Knaap, R. J. H.**, de Bruyn, M., van Oort, N., Huisman, D. & Goverde, R. M. P. (2024). Clustering railway passenger demand patterns from large-scale origin-destination data. *Journal of Rail Transport Planning & Management*, 31, 100452.

Conference contributions

1. **van der Knaap, R. J. H.**, van Oort, N., & Goverde, R. M. P. (2025) Balancing flexibility and predictability: Evaluating the impact of multi-period timetabling on railway demand. *16th International Conference on Advanced Systems in Public Transport (CASPT)*, Kyoto, July 2025.
2. **van der Knaap, R. J. H.**, van Oort, N., & Goverde, R. M. P. (2025) Multi-period railway timetabling to serve time-dependent demand. *RailDresden 2025: 11th International Conference on Railway Operations Modelling and Analysis*, Dresden, April 2025. **Received Best Paper Award.**
3. **van der Knaap, R. J. H.**, van Oort, N., & Goverde, R. M. P. (2024) Multi-period railway timetabling to serve time-dependent demand. *International Conference on Optimization and Decision Science (ODS 2024)*, Badesi, September 2024.
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6. **van der Knaap, R. J. H.**, de Bruyn, M., van Oort, N., Huisman, D. & Goverde, R. M. P. (2023) Extracting Railway Passenger Demand Patterns from Origin-Destination Data for Developing Demand-Oriented Service Plans. *RailBelgrade 2023: 10th International Conference on Railway Operations Modelling and Analysis*, Belgrade, April 2023.
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Other

1. **van der Knaap, R. J. H.** (2024) Optimizing timetables for varying demand: A MILP model for multi-period timetabling. *TiSEM Seminar: Operations Research at Tilburg University*, November 2024.
2. **van der Knaap, R. J. H.** (2024) Wiskundige Renate promoveert op de optimale dienstregeling voor treinen. *Interview on Dutch national radio show 'Met het Oog op Morgen'*, August 2024.
3. **van der Knaap, R. J. H.** (2021) Cyclic railway timetabling using an iterative SAT approach with a feedback mechanism. *MSc Thesis, Tilburg University.*

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Summary

Cyclic railway timetables offer passengers a memorable and easy-to-use service, but they lack flexibility in responding to variations in passenger demand throughout the day. To maintain memorability while better adapting train services to fluctuating demand, this dissertation introduces multi-period cyclic timetables. It develops methods to partition the day into demand-homogeneous periods and optimisation models to design tailored line plans and cyclic timetables for each period, thereby improving the alignment between railway services and passenger demand.

About the Author

Renate van der Knaap conducted her PhD research at Delft University of Technology's Department of Transport & Planning. She holds a Master's degree in Business Analytics and Operations Research and a Bachelor's degree in Business Economics, both from Tilburg University.

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