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The macroscopic fundamental diagram explained by a walking experiment in class

Victor L. Knoop[1]*, Andreas Hegyi[1] and Dorine Duives[1]

Abstract
In traffic flow theory, there is a relationship between the number of vehicles in a certain area and their average speed, the so-called macroscopic fundamental diagram (MFD). Related control concepts involve limiting the inflow into an area, so-called perimeter control. This may increase both the flow inside the controlled area and the outflow rate, reducing overall delays. This control concept is taught in classes on traffic flow theory around the globe. This paper presents an experiment that can be run in class to create the MFD with a group of students. The basic idea is that we have a confined area where participating students have to walk a predefined route. We repeat this experiment several times (runs of 2–4 minutes each), each time with a different, pre-defined number of students being simultaneously in the area (the accumulation). The number of students is controlled by metering the inflow of participants. Exit flow rates are determined for each run by dividing the number of exiting participants by the run time. One run will hence yield one observed combination of number of participants and run time. From this, an accumulation and exit flow rate can be computed. The Macroscopic Fundamental Diagram (MFD) is then created by plotting the exit flow rates and accumulation of all runs. Students can this way derive this MFD from data. Moreover, they also experience during the walking how delays change with accumulation, and thereby the experiment intuitively teaches them the concepts of the MFD and perimeter control. The paper describes the experiment, and provides tools (software, routes) for repeating the experiment with other groups.

Keywords Traffic flow theory, Macroscopic fundamental diagram, Pedestrian experiment, Teaching

1 Introduction

In almost a century, the field of traffic engineering has significantly matured. Whereas in the early days there were individual researchers (not seldom mathematicians) developed theories on traffic, traffic engineering has received a prominent place in educational programs at many universities. The need is evident: by educating capable traffic engineers, these traffic engineers can design, improve and manage transportation systems.

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Educational programs and methods vary among universities and between levels (BSc, MSc, and even PhD programs). And whether we like it or not, the teaching style and method are often a main driver for the interest of the students, and hence for their study efforts and ultimately their level of understanding. It would be in benefit traffic engineering to have some inspiring methods to illustrate the concepts.

In the past decades, some good study/text books of traffic engineering and in particular about traffic flow theory have been published. Seminal books by May [1] and Daganzo [2], have been used for a long time, the first one being more practical and the second one more theoretically profound. The joint effort of combining expertise by the Traffic Flow Theory Committee is an interesting combination of insights [3], yet more modern concepts like a macroscopic fundamental diagram are not included. More recently, a text book by Treiber et al. [4] has been published, explaining traffic flow theory from the point of view of physics. The book by Jin [5] has a more fundamentally mathematical approach. Elefteriadou et al. [6] chooses a more engineering approach. The text book by Ni [7] focuses on the models for describing traffic. The (open access) text book by Knoop [8] is leaning towards the physical phenomena.

Whereas there are text books for students, this might not be enough to fully stimulate the academic interest. In-class experiments might help to get students fascinated and understand the concepts. Most traffic flow theory books are centered around car traffic. Experiments with cars are impossible to do in class because of their size. However, if the same traffic flow theory concepts can be shown by pedestrians, and this can be done with students as participants. In the past other experiments have been done with students, yet we are unaware of the scientific reports on these. This paper will present such an experiment that the authors developed and carried out.

In this education method the students will create an MFD from data they produce. By analyzing the data they will discover the shape of the MFD, i.e., the increase and decrease of outflow as a function of accumulation. More importantly, they will physically experience how perimeter control indeed can decrease travel times. This paper is different from a regular research paper on traffic, which presents a new theory or control measure improving traffic flow. Instead, we share set-up of a walking experiment that can be repeated, and discuss the experiences of the experiment as performed by the authors.

The remainder of the paper is set up as follows. First, Sect. 2 describes the setup of the experiment. Next, Sect. 3 describes the results, where we reflect on the extent to which the experiment fulfilled the intended goal, i.e., does it show the concept of the MFD to participants. Note that it is beyond the aim of the experiment (and hence the paper) to extract traffic engineering quantities from the experiments The experiment can be repeated as is, for which also tools are being suggested and shared. Section 4 discusses some variations to – if needed – adapt the experiment for specific groups. Finally, Sect. 5 presents the conclusions.

2 Setup

This section describes the setup of the experiment. First, sect. 2.1 presents the background of the MFD. Section 2.2 then presents the theoretical concepts for a walking experiment. Section 2.3 discusses the design of the actual experiment.

2.1 Background of the MFD

In traffic flow theory, the concept of the macroscopic fundamental (MFD) diagram is an active concept of study since the paper by [9] introducing the theoretical concept, which is supported by the empirical proof in [10]. It basically boils down to the following. The average speed v of vehicles in a pre-defined area depends on the number of vehicles in that area A: $v = v(A)$. If there are few cars in a city, they can drive at their free flow speed. If the number of cars increase, their speed decreases. The exit rate P (also referred to as performance) depends on the product of speed and number of vehicles: $P = P(v \cdot A)$ and hence $P = P(v(A) \cdot A)$. Since the speed decreases to 0 in the limit of a high number of vehicles, the exit rate will ultimately also reduce with increasing density, ultimately to 0 once the speed is 0. This implies, that there exists a maximum exit rate, and a corresponding number of vehicles in the area for which this maximum is achieved. It has been already postulated by [9] that it would be beneficial for the overall delay if the entering vehicles would be metered at the boundary because then the inner area could have a higher exit rate, decreasing the overall delay (including the delays induced at the boundary). This concept of limiting the number of vehicles inside an area is called perimeter control, and has been subject of intensive study for the past decade and a half, e.g. in Haddad and Geroliminis [11], Keyvan-Ekbatani et al. [12,13].

2.2 Conceptual background of the experiment

The aim of the experiment is to show the existence of the MFD in some conceptual shape, i.e., students should see a traffic performance that first increases and then decreases with increasing densities in the area. This can be achieved by having students walk in an area that is limited in space.

In order to explain the concepts here in the paper, we will define the following quantities. We introduce them here, but details of the meaning of some quantities will follow later:

- Area size S (m^2)
- Performance P (ped/min), outflow from the area per unit of time, measured in pedestrians per minute
- Accumulation A (ped), the number of pedestrians in the area
- Density k (ped/m^2), the number of pedestrians per unit of space, aggregated over the area, measured in pedestrians per square meter
- Task length: number of tasks in someones trip
- Actual path length L_R (m) length of the route passing all tasks
- Shortest path length L_S (m) shortest path length in someones path

- Average walking speed v (m/s) in the area, and unhindered walking speed v^{free} (m/s)
- Efficiency η: indicator of the performance level at a certain density compared to the performance with that density level if there was no hindrance/interaction ($0 \leq \eta \leq 1$)

As long as there is no hindrance, the performance increases linearly with the density, or accumulation. In an equation, we can state:

$$P = \frac{Av}{L_R} \tag{1}$$

Note that efficiency loss can come from either of the following 2 elements: an increase of actual route length or a decrease in speed. . The efficiency factor is the factor determining the decrease in performance.

$$P = A\frac{v}{L_R} = A\eta\frac{v^{\text{free}}}{L_S} \tag{2}$$

So, the efficiency can be expressed as:

$$\eta = \frac{L_S}{L_R} \cdot \frac{v}{v^{\text{free}}}. \tag{3}$$

Once a pedestrian wants to move to a certain location and finds congestion in their path, they can themselves choose to reduce speed or move around the congestion.

The key of the experiment is that efficiency reduces to such an extent that the performance will actually *decrease* with increasing accumulation, i.e., that the increase of performance due to the more people walking (and producing in parallel) is being offset by a larger decrease in efficiency.

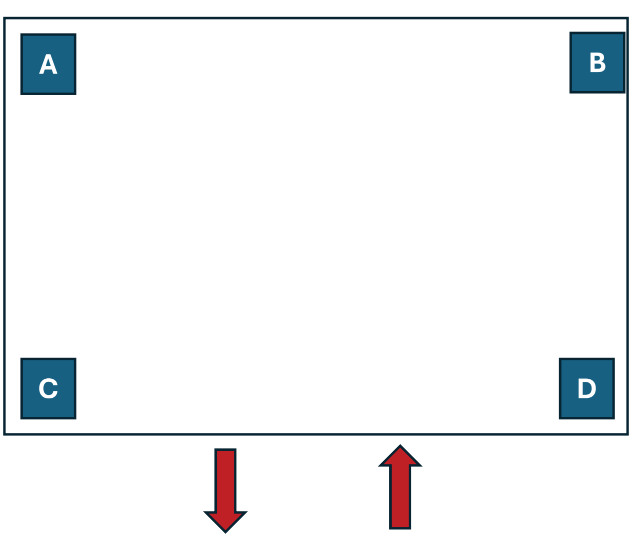


Fig. 1 Lay-out of the experiment. The participants perform a small task one or more of the four stools A, B, C, or D

In the proposed experimental setting each pedestrian will have a pre-specified, individual path. Essentially, there are 2 mechanisms that limit the performance:

(1) people walking or waiting where others want to walk;
(2) limited capacity of the activities (including exit) that pedestrians need to undertake;

For the sake of simplicity, point (1) combines all hindrance of other pedestrians inside the area, regardless whether it is due to people waiting or walking.

The above derivation allows us to get insight into how performance changes as function of accumulation. That is, in order to get a decrease in performance with increasing accumulation, the relative decrease of efficiency (due to lower speed and longer paths) should be larger than the relative increase in accumulation (refer to equation (2)). In practice, this means the walking conflicts with other participants (i.e., the effect of mechanism 1) should be quite sizable, and more than just "some hindrance."

2.3 Designing the experiment

In a rectangular area, 4 stools are placed: 1 in each of the corners, as shown in Fig. 1. We ask participants to follow a route from the entrance (in the middle on the long edge) and follow a predefined route along a set of stools. At each of the stools, they are requested to wait for 2 seconds. This waiting time lengthens the time they remain in the experimental area, and hence increases the interactions with the other participants.

Routes are randomly pre-generated, and consist of 10–12 visits to stools before exiting, chosen stochastically. The first stool to be visited always is a stool at the nearby side (C or D) to allow for a relatively easy entry. Then, which stool to visit next depends on the current stool. With 50% probability, the following stool is the stool diagonally opposing the current one. There are two diagonally opposing pairs of stools, and the streams connecting these two cross each other, creating interactions (walking conflicts) between participants. The other 50% probability is equally divided between the remaining 2 stools. Participants get a route to follow on a card (Fig. 2). The tool to create the routes and cards is made available [14].

We keep track of how many people enter and exit the area. The entry to area is controlled by a person at the entrance, giving access to people when this is allowed. This access is based on the number of people that are in the area, and basically access is controlled by a so-called bang-bang algorithm. That means, as long as the number of people in the area is below the preset accumulation,

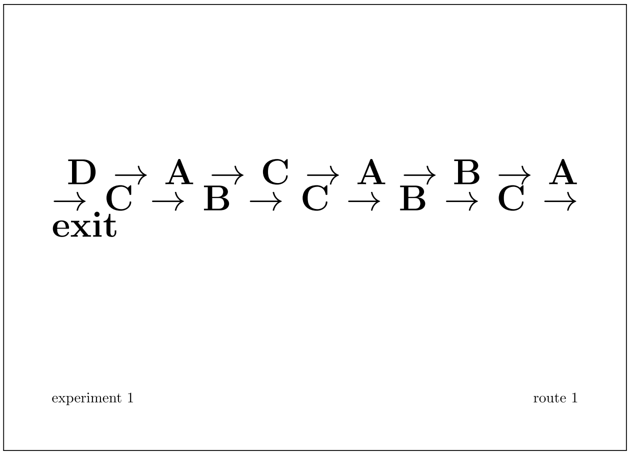


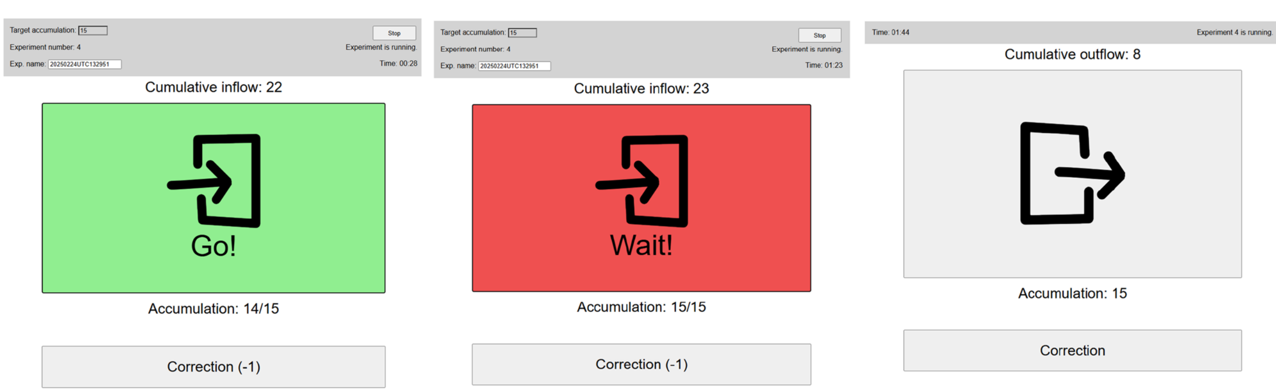
Fig. 2 An example of a pre-defined route given to one of the participants

access is allowed without restriction. If it is equal to or greater than the target accumulation, the participants have to wait to get in. Only once a person leaves, and the number of people in the area hence decreases to below the threshold, the next person is allowed in.

Outflow is determined by the rate of measured exits. To determine the MFD, we will relate this to the number of participants that is in the area, the accumulation. This accumulation is (after a start-up phase ideally) constant throughout one run of the experiments. That means that one run yields one data point, being one combination of accumulation (i.e., the preset value which is not exceeded) and outflow rate in that experiment. We start the measurements of a run after allowing for a warm-up phase to settle into an equilibrium outflow. We rerun the experiment for different values of the preset value of accumulation to get various points of the MFD.

Participants that leave the area, join the queue of participants waiting to re-enter the area later in the same experiment run. This way, the experiment can continue with a lower number of participants than if we would allow a participant to enter only once. After exiting, the participant gets another route (to avoid the participant gets to know the route and special tricks for the route), and rejoins the queue to enter. The required number of participants is slightly higher than the target accumulation, because we require the participants after exiting to prepare and re-enter. Also we always like to have people waiting, so we can ensure the desired inflow. Inversely, if there is inflow possible and there are no people waiting, the accumulation is decreasing, which conflicts with the setup of the experiment.

To record the operations, two types of measurements are taken. One type of measurements provides details: a camera is installed which views the participants from the top. The participants wear red hats, which makes it easy to recognize pedestrians in the images. The idea of using red hats has been widespread over the past 2 decades, e.g. [15–17]; they are especially well distinguishable in HSV color space. Using these video data, all accumulations, chosen paths, speeds etc. can be retrieved. A second type of measurements only gives entry and exit times. We developed an app Hegyi and Knoop [14] that can track the number of pedestrians in the area. It is a web-based interface that shows information (the number of participants currently in the area) on two smart phones simultaneously: one at the exit and one at the entrance (see Fig. 3). It gets its information from two users. On each phone, a button is placed. One has an entry-button, to be pressed when a participant enters the area, the other has an exit button, to be pressed when a participant leaves the area. The two phones are connected with virtually no delay. Moreover, at the phone near the entrance, a threshold accumulation could be set, such that the app indicates to the person controlling the inflow, whether



(a) Entry screen if target accumulation has not been reached yet

(b) Entry screen if target accumulation has been reached

(c) Exit screen showing cumulative exits and time since the start of the experiment

Fig. 3 Screens of the app

someone could be allowed in. The web-server logs inflow, outflow and accumulation at a 0.1 s precision. The code is available and runs on any web-server that allows PHP scripts.

During the experiment, one person logs (and hence automatically counts) the participants leaving the area, and the other one logs the participants entering the area. The same person, at the entrance, is performing the control, stopping participants flowing into the area if the accumulation is above the preset threshold (see Fig. 3b for a screen shot).

The data of the coupled phones give inflow and outflow (i.e., performance) and the accumulation. These data suffice to extract the single data point for an experiment. Doing this for various runs (with various threshold levels of accumulation) gives several data points, which creates the MFD. This data is in principle available directly after the experiment, so the (points creating the) MFD can further be visualized after each run of the experiment has finished.

For the experiment, in particular for using results in a scientific communication, an application to the TU Delft ethics committee was prepared and approved (registered under ID 4510). Key points were that participation was voluntarily and no personal data from participants was recorded or shared. Videos were blurred before recording, and even blurred images with people are not made publicly available.

3 Results

In this section we describe the results. We first comment on the results of running the experiment itself: how did the experiment go. Only at the end of the section, we will show the results of the created MFD (traffic flow results).

3.1 Running the experiment

Participants in the experiment were participants (and teaching and assisting staff) to the summer school on multi-modal traffic management, organized by Delft University of Technology. The experiment was conducted on 2 July 2024. There were around 25–30 participants for the runs, and we chose for an area of 2 m × 3 m in size (see Fig. 4). We ran the experiments for target accumulation values of 3, 6, 10, 12, 14, 18, and 20 peds (i.e. densities of 0.5, 1, 1.7, 2, 2.3, 3 and 3.3 peds/m^2). The runs took 2–4 minutes each.

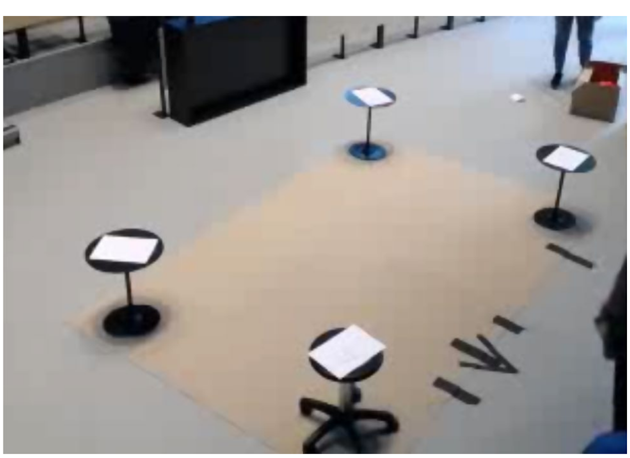


Fig. 4 Implementation of the experiment

For the data collection by video we used a high tripod (a high vantage point is useful) and included the time in the video. The video itself was also blurred in order to not record personal information due to privacy issues. Note moreover that the counting data proved to be sufficiently rich to create MFDs, even though the data collection was not flawless (see next paragraph).

The counters were susceptible to a user error. The app did not provide feedback whether the button properly was clicked, so it lacked for instance a haptic feedback (vibration if the button was pressed). Especially the person counting the exits did suffer from the fact that he sometimes was unsure whether the button was pressed, and would press again. This occurred mainly if someone exits, the exit-counter pressed the button, then someone else could be allowed in and that happens quite quickly. Then, since the app had virtually no latency, the person counting at the exits would not see that the accumulation would decrease and increase by one, and it looks like the press of the button had no effect. Responding to that, he would press again. In a revised version of the app (after the experiment, but before publishing the code) a cumulative counter has been added to the displays of the entry and exit counters, respectively. The cumulative counters work independently, so they can provide visual feedback of each button press.

Consequently, some errors were made during the experiment. In some runs, especially with larger threshold number of people in the area, the number of people in the area at the end of run was sometimes 1–2 lower than the number of people that should have been let in. So indeed, the registered number of people was sometimes 1–2 persons lower than the number of people actually in the area. Once this error occurs, the system will not recover by itself, so the error persists until the end of the run. At the end of each run, the number of people in the area was counted, and reinitialized for the next run.

3.2 Traffic properties

Cumulative flows plots can quite easily be derived from the counting data. An example of the curves shown in Fig. 5. The figure also shows with arrows an illustration of the instantaneous accumulation and the outflow over a period of time. The outflow sometimes goes in batches, and is sometimes more smooth. This depends on the number of people finishing their task at the same time.

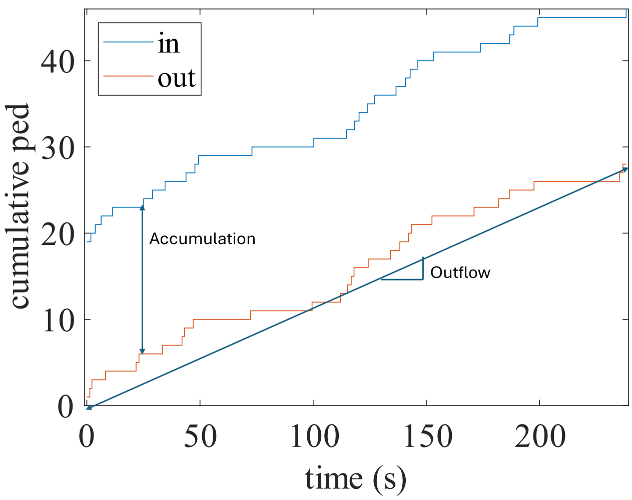


Fig. 5 Cumulative flow curves

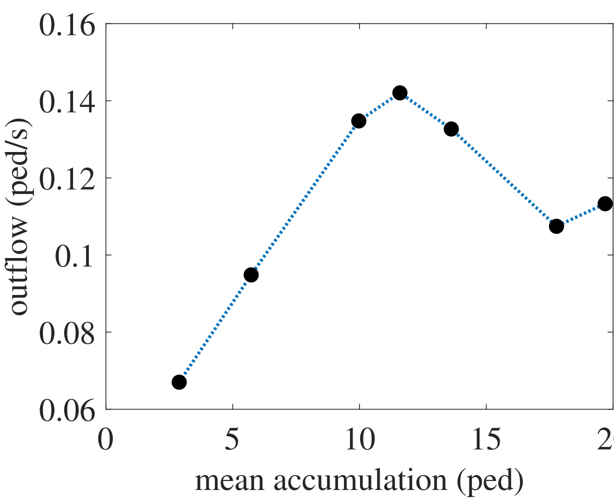


Fig. 6 The MFD resulting from the experiment

With longer routes (i.e., the time required for the route length much longer than the entry time for the threshold number of participants in the area), a typical pattern is as follows. At the start, all participants quickly move into the area. Then, they all follow their route. Then, since the routes are of similar lengths, they finish at a similar time, and they will exit shortly after each other. While exiting, a new group of people will enter in a short time. Hence, this operates almost like groups which are together in the area, then leave and another group enters. This will flatten out over time due to natural fluctuations in route length and travel time. Even though this is not restricting the outcomes of the research, this might feel undesirable. A way to address this is to start with the threshold people already in the area, and instruct them not to start at the beginning of their route, but somewhere within their route.

For the participants it would be desirable to see the results directly after a run of the experiment, so the MFD could be built in real time. That follows directly from the number of exited participants divided by the run time of the experiment. Besides the quantitative insights, in our experiment the performance loss as consequence of the overcrowding was evident also for the participants because they could simply notice fewer people per unit of time flowing out. Hence, even without the data, the experiment was successful in showing the concept.

The MFD resulting from the experiment we carried out is shown in Fig. 6. It follows nicely the shape as expected for an MFD, with first an increase of performance with increasing. This was presented after the experiments to the participants.

4 Discussion

In this paper, we have shown a setup that can show the working of the macroscopic fundamental diagram in an interactive experiment. There are some recommendations for further refinement. First, the experiment benefits from a start with an area filled up to the threshold level, with participants in different stages of their routes (i.e., just started, midway, almost finished ...). This gives less fluctuations in the outflow.

To reach densities of more than of 3 ped/m^2, some encouragement was needed. This is above a critical accumulation, so people prefer not to enter. We had to indicate on beforehand that it would be busy indeed and people should still enter. Theoretically, for even higher accumulations the MFD should go down to 0 performance. This is not likely to be achievable without really encouraging people to enter a crowded area, like it was a very crowded train.

Counting the in- and outflows is susceptible to errors and there is no mechanism to correct the accumulation error that this causes. Besides adding the cumulative counter on the displays (as done in the revised version of the counter code), adding haptic feedback on the phones, could help to reduce the counting errors. Besides, another, more simple way of keeping the accumulation at the target level is to make the inflow and outflow self-regulated: The first person in the queue to enter can only enter once being tapped by a person exiting. This way, the number of people in the area remains constant. Besides, a log on the people exiting would be necessary to find the outflow rate, being the performance.

In the ideal case, each run of the experiment has the same total number of people exiting, and this number is large enough to remove fluctuations. The log of the exits would ideally not only give the times of individual people leaving, but also the total number of people left.

A very low-tech way of introducing this is to simply keep count of the number of people that exited (cumulatively), and keep a stopwatch running. This information is also visible in the current app. Whenever the number of exited people exceeds N, stop the stopwatch. Dividing N by the recorded time period required to exit will give the outflow rate, i.e. the performance for that run.

We also performed the experiment with a different audience. The aim for this audience was to understand the effects on crowding on the MFD, but there was no learning objective in terms of working with flows or cumulative flow curves, nor time for the audience to work with the data. We hence choose to run the experiment in this low-tech setting: self-organized constant accumulation and recording only start and end time. The resulting MFD looks very similar.

The design can be adapted so the experiment still works with a lower number of participants by adapting the size of the area. The minimum number of participants always is at least the threshold number of participants, added with the number of participants that moves from exit to start.

5 Conclusions

We have created an experiment that can be done with students in which they will experience that the performance of a "transport network" improves and degrades with the accumulation. The experiment has been tested and works: students indeed did empirically experience the delays, which shows the effects even without further computations. Moreover, the data shows a very nicely shaped macroscopic fundamental diagram. The setup is simple and the tools are made available. The experiment is flexible can be adapted to suit smaller and larger groups. Last but not least, the experiment is also fun to do for the participants.

Author contributions
The authors thank Winnie Daamen and Alexandra Gavriilidou for their insightful contributions in discussing this experiment.

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Data availability
As mentioned in the body text, the relevant materials (app, code) are available at Hegyi and Knoop [14].

Declarations

Competing interests
There are no conflicts of interest to mention.

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