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A new robust design for imperfection sensitive stiffened cylinders used in aerospace engineering

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A knock-down factor is commonly used to take into account the obvious decline of the buckling load in a cylindrical shell caused by the inevitable imperfections. In 1968, NASA guideline SP-8007 gave knock-down factors which rely on a lower-bound curve taken from experimental data. Recent research has indicated that the NASA knock-down factors are inclined to produce very conservative estimations for the buckling load of imperfect shells, due to the limitations of the computational power and the experimental skills available five decades ago. A novel knock-down factor is proposed composed of two parts for the metallic stiffened cylinders. A deterministic study is applied to achieve the first part of the knock-down factor considering the measured geometric imperfection, the other types of imperfections are considered in the second part using a stochastic analysis. A smeared model is used to achieve the implementation of the measured geometric imperfection for the stiffened cylinder. This new robust and less conservative design for the stiffened cylinders is validated by using test results.

knock-down factor, NASA guideline SP-8007, stiffened cylinder, stochastic analysis, smeared model

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1 Introduction

Launcher structures usually use thin-walled structures, such as cylindrical shells, as primary components. These thin-walled cylinders are prone to be limited in their load carrying capability by buckling. In addition, many shell type structures exhibit unstable post-buckling behavior and are highly sensitive to geometric imperfections and/or load imperfections [1,2]. In the early 1900 s, researchers observed that the measured buckling loads in tests were much lower than the corresponding numerically predicted buckling loads of a geometrically perfect cylinder using the classical buckling theory. It seems that the classical buckling theory cannot be applied to realistic structures where initial geometric imperfections are always present. Some relative improvements [3,4] have been made to the classical theory to take into account the effect of initial geometric imperfections. Among these studies [3–6], the most famous one is that of Koiter [6] who predicted accurately the imperfection sensitivity trends in 1945 that were observed experimentally. Actually, design guidelines which can quickly consider the influences of imperfection sensitivity are much more interesting to engineers. In 1965, the well-known NASA SP-8007 guideline [7] was proposed based on a collection of experimental results [8,9] for cylindrical shells. The lower-bound curve of all the test results gives the shell buckling knock-down factors which are normally less than 1 to measure the decline of the buckling load in a cylinder caused by imperfections. The load carrying capability of the

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cylinder is achieved easily by multiplying the buckling load of perfect cylinder obtained using the classical buckling theory with the corresponding knock-down factor. Here, the perfect cylinder means that there is no imperfection in the structure. At that time there was not sufficient computational power to carry out nonlinear structural analyses [10,11] which could accurately represent and predict the complex behavior of imperfection sensitive structures in the case of buckling, hence the accuracy of NASA knock-down factors relies heavily on the reliability of the experimental results. The manufacturing and experimental techniques [12] have been significantly improved since 1968. Recently, many researchers have found that the current NASA SP-8007 guideline leads to conservative structures. In addition, the knock-down factors from NASA were proposed mainly for unstiffened cylinders. Thus the EU project DESICOS was proposed with the aim of constructing a new robust design for imperfection sensitive structures which can fully exploit the potential of the load carrying capability of the thin-walled launcher structures. In this robust design approach, the novel knock-down factor is composed of two knock-down factors in contrast to one single knock-down factor as proposed by NASA.

Several types of imperfections may exist in the manufactured structures [10,13], such as material imperfections, load imperfections [1], e.g. eccentricities, thickness imperfections and geometric imperfections [14]. In addition, the hygro-thermal condition of a structure, which influences the static bending strength, can also be regarded as a imperfection [15,16]. A robust knock-down factor should take into account the effects of the realistic imperfections as much as possible. Among all the realistic imperfections, geometric imperfections are the most recognized imperfection type in engineering and some researches [1,14,17,20] have found that the buckling loads of the thin-walled structures are very sensitive to geometric imperfections. There are many ways to model geometric imperfections in the buckling analysis, e.g. measured geometric imperfections [1,2], a linear combination of the first few buckling modes to achieve an imperfection shape [17], or the application of small perturbation loads to generate initial deformations [18]. The way to use a small perturbation load on the structure to generate an initial imperfection shape is called the single perturbation load approach (SPLA) [17–20].

The outline of this paper is as follows. The new robust design and the two metallic stiffened cylinders analyzed in this study are introduced in Section 2. The first and second parts of the knock-down factor proposed in this robust design are achieved using a deterministic study and a stochastic analysis, respectively, in Sections 3 and 4. In Section 5, the novel knock-down factor is combined and compared with the NASA knock-down factors and the test results. We summarize the paper and draw conclusions in Section 6.

2 New robust design

2.1 Comparison of two designs

The knock-down factor proposed in NASA SP-8007 guideline is given by a lower-bound curve of the experimental results collected in 1965, as shown in Figure 1, where the buckling load of the perfect cylinder is scaled to 1, each dot denotes one test result, and the x axis represents the ratio of the cylinder radius over the skin thickness. The knock-down factor of a specific cylinder can be figured out using this lower-bound curve.

Based on the NASA guideline, the load carrying capability of the cylinder can be achieved:

$$F_{\text{design}} = F_{\text{perfect}} \times k_{\text{NASA}}, \quad (1)$$

where F_{perfect} is the buckling load of the perfect structure, F_{design} represents the design load, and k_{NASA} denotes the knock-down factor obtained from the NASA guideline. The NASA knock-down factor is proposed based on the experimental results collected before 1965. Nowadays, the complex behavior of imperfection sensitive structures can be accurately predicted numerically, with the development of computational powers. In this work, a novel robust design is proposed based on the numerical techniques, as given by

$$F_{\text{design}} = F_{\text{perfect}} \times k, \quad (2)$$

where

$$k = k_1 \times k_2, \quad (3)$$

where the new knock-down factor k is composed by two parts, k_1 and k_2. k_1 is obtained considering the geometric imperfection while the effects of other imperfections are taken into account in k_2.

2.2 Two tested metallic stiffened cylinder

Two configurations of metallic stiffened cylinders, specified in Table 1, were manufactured, tested and analyzed numeri-

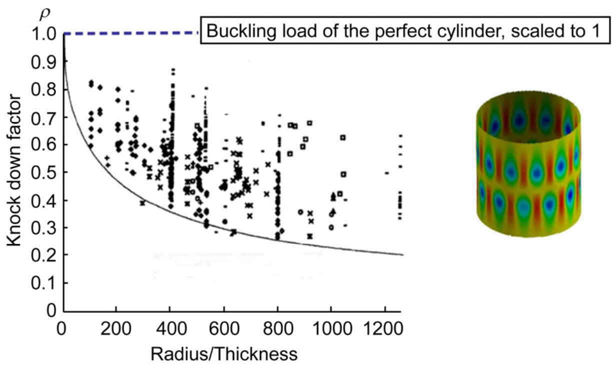

Figure 1 (Color online) Knock-down factors proposed in NASA SP-8007 guideline.

Table 1 Configurations of two stiffened cylinders analyzed in this study (MPa, mm)

Id	Material	Cylinder		Skin	Stiffener			NASA SP8007 knock-down factor
	E, μ	Radius	Height	thickness	Thickness	Height	Number	
A	70000, 0.33	400	1000	0.8	0.8	5.2	90	0.4616
B	70000, 0.33	400	1000	0.55	0.55	5.2	126	0.4387

cally. According to the NASA guideline, the knock-down factors of these two stiffened cylinders are $k_{\mathrm{NASA\_A}} = 0.4616$ and $k_{\mathrm{NASA\_B}} = 0.4387$, respectively. Two types of finite element models are constructed using ABAQUS, as shown in Figure 2. One is the stringer shell model, where all the stiffeners inside the cylindrical shell are modeled using shell elements. The other is the smeared model, the stiffeners are not modelled and the effect of the stiffeners in the analysis is taken into account using a general shell stiffness.

The three translations of the element nodes along the circumference of the cylinder bottom are constrained. A compressive load is applied uniformly at the top of the cylinder. A linear eigenvalue analysis and a nonlinear structural study are carried out respectively for perfect cylinders A and B, using both the stringer shell model and the smeared model, the results are compared in Figure 3 and Table 2. It can be seen from Figure 3 and Table 2 that the results calculated from these two models agree very well for both cylinders, excepting the first buckling mode of cylinder B. Two properties make the smeared model attractive: first, in cases where we do not care about the buckling deformation of the stiffeners we can use a relatively coarse mesh to decrease the computational cost significantly (Table 2). The complete execution time is measured on a standard PC with a single core and 4 GB physical memory; second, the real measured geometric imperfections of the unstiffened cylinders can be implemented into the smeared model of the stiffened cylinders for imperfection-sensitivity analyses due to a lack of the measured imperfection data on the stiffeners.

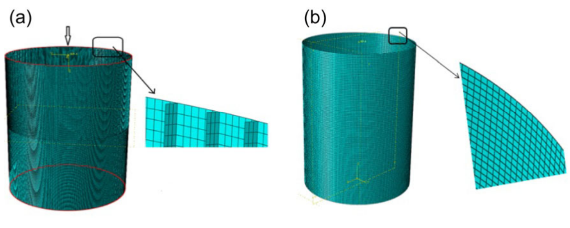

Figure 2 (Color online) Finite element models of the stiffened cylinder. (a) stringer shell model; (b) smeared model.

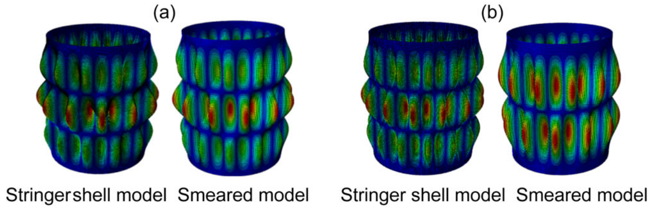

Figure 3 (Color online) The first buckling modes of two cylinders, compared between the stringer shell model and the smeared model. (a) Cylinder A; (b) cylinder B.

3 Knock-down factor k_1

The effect of geometric imperfections is considered using the knock-down factor k_1. Here, we use the single perturbation load approach and the real measured geometric imperfection to model the geometric imperfections in imperfection sensitivity analyses.

3.1 Single perturbation load approach

Single perturbation load approach, shown in Figure 4, uses a single perturbation load (SPL) to create a single buckle imperfection, which classifies as a "worst", "realistic" and "stimulating" imperfection [18–20]. A nonlinear structural analysis is applied for each value of the single perturbation load to obtain the corresponding buckling load. With the increasing of the single perturbation load, the buckling load becomes nearly constant after a given level of single perturbation load P_1 in Figure 4. This phenomenon determines the lower- bound of buckling loads, which can be used to determine the knock-down factor for a safe design, as given by

$$\mathrm{KDF} = \frac{F_1}{F_{\mathrm{perfect}}}. \tag{4}$$

Table 2 Comparison of stringer shell model and smeared model (kN)

Models	Elements	CPU time for nonlinear analysis	Linear buckling load F_{perfect}	Nonlinear buckling load for perfect cylinder
Stringer shell model	174960 S4R	7 h	Cylinder A: 205.92 Cylinder B: 103.09	Cylinder A: 193.466 Cylinder B: 102.845
Smeared model	25100 S4R	2 h	Cylinder A: 203.27 (1.29% error) Cylinder B: 103.76 (0.65% error)	Cylinder A: 192.457 (0.475% error) Cylinder B: 101.973 (0.85% error)

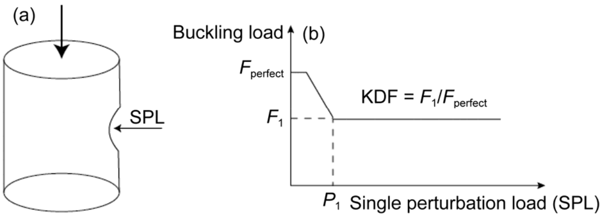

Figure 4 Sketch map of the single perturbation load approach. (a) Single perturbation load; (b) knock-down curves.

where $F_{perfect}$ is the linear buckling load of the perfect structure, F_1 is the buckling load calculated when the single perturbation load is equal to P_1.

Nonlinear buckling analyses with different values of single perturbation loads are carried out on cylinder A, and the structural response curves are plotted in Figure 5(a). Since the SPL can be applied either on the skin or on the stiffener, both cases were studied. The knock-down curves of cylinder *A* are plotted in Figure 5(b), from which it can be seen that the location of the SPL will not influence the buckling behavior. The deformations of cylinder A along the load shortening curve are given in Figure 6, for the case that the SPL is 30 N. It is also demonstrated, in Figure 5(b), that the value of the buckling load leads to constant value when the SPL is larger than 49 N, which leads to the knock-down factor k_{1_A} = 0.62 for cylinder A. In the same way, the knock-down factor of cylinder B was calculated to obtain a value of k_{1_B} = 0.64.

3.2 The measured geometric imperfections

Geometric imperfection data measured from Z15, Z17 and Z20 unstiffened cylinders are implemented into the smeared model of the stiffened cylinder. The way to produce such measured geometric imperfection is to translate the nodes directly in the finite element mesh. The knock-down curves are shown in Figure 7. The knock-down factors of cylinders A and B are roughly 0.621 and 0.638, respectively, after averaging the results obtained using three measurements. The knock-down factors obtained using the measured geometric imperfection are coincident with those (k_{1_A} = 0.62 and k_{1_B} = 0.64) achieved using the SPLA.

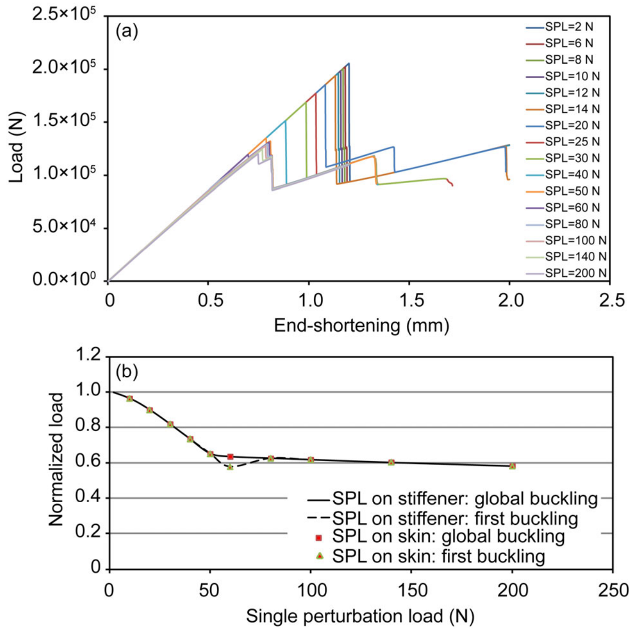

Figure 5 (Color online) End-shortening curves and knock-down curves of cylinder A. (a) End-shortening curves; (b) knock-down curves.

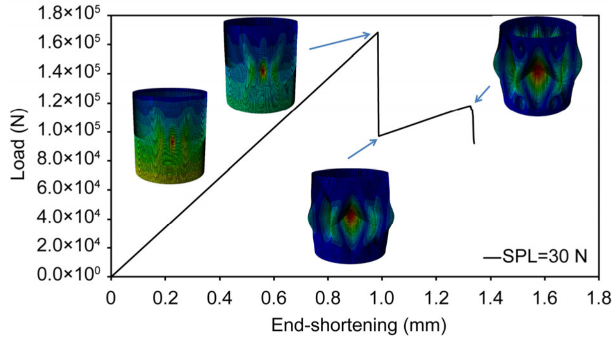

Figure 6 (Color online) Deformations of cylinder A, along the load-shortening curve.

4 Knock-down factor k_2

In stochastic analysis, the buckling behavior of the stiffened cylinder is analyzed as a probabilistic phenomenon due to the distribution of the input parameters. Then, once the

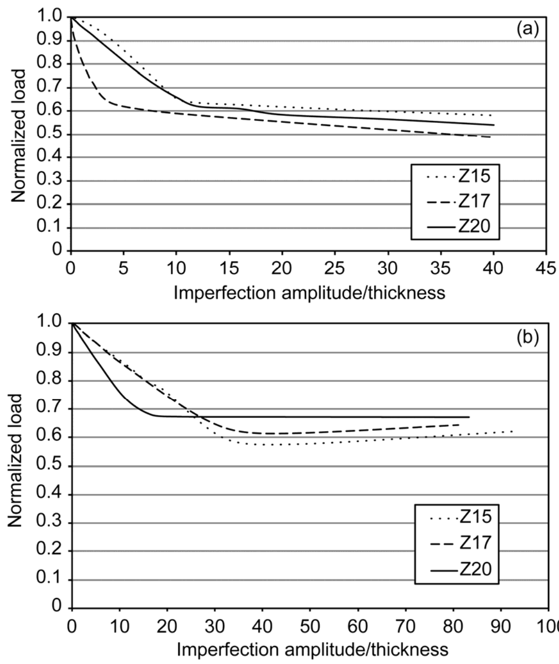

Figure 7 Knock-down curves of cylinders A and B, using the measured geometric imperfections Z15, Z17 and Z20. (a) Cylinder A; (b) cylinder B.

distribution of buckling loads is known, a lower bound can be defined at a 99% confidence level, which determines the value of the knock-down factor, as given in Figure 8.

Five input parameters, material property, thickness of the skin, thickness of the stiffener, applied compressive load on each node and measured geometric imperfections, are taken into account as random variables in the stochastic analysis. The distributions of these input parameters are all assumed to be a normal distribution with a mean equal to the initial design/measured value and a coefficient of variation (*CV*) = 5%. The ABAQUS is used in the Monte Carlo simulation. The corresponding 1000 data set of buckling load is obtained from a number of 1000 of samples of the input parameters. Two families of stochastic analyses are carried out below, according to whether taking into account the geometric imperfection as the random variable or not.

1) The geometric imperfection is not included. The knock down factors obtained here are used as the knock-down factor k_2. The coefficient of variation for the load imperfection is set to be 3%, 5% and 10%, respectively, to detect the imperfection sensitivity.

2) The geometric imperfection is included. The second family considers all of the 5 random variables to make a comparison to the proposed new knock-down factor. In each Monte Carlo simulation, the amplitude of the measured geometric imperfection is assumed to be a normal distribution. The three measured geometric imperfections (Z15, Z17, Z20) are respectively used to obtain a robust result.

The probability density function (PDF) and the cumulative distribution function (CDF) of the data are plotted against a normal distribution, respectively, to check whether they has a normal distribution. We take one simulation case of cylinder B as an example. In Figure 9, buckling loads are plotted using the histogram with a probability density curve of a normal distribution. The buckling data are plotted against a CDF of a normal distribution in Figure 10, where the data points should assume an approximate red straight line, and scatters from this straight line indicate the non-representativeness of this distribution. From Figures 9 and 10 it can be seen that the distribution of buckling loads satisfies the normal distribution. In the same way, all the data sets of buckling loads obtained from stochastic analyses for cylinders A and B satisfy the normal distribution.

The lower-bound of the confidence interval can be quickly achieved with a 99% confidence level once the distribution of the buckling loads is confirmed to be normal. From Table 3, we can conclude that the value of the knock-

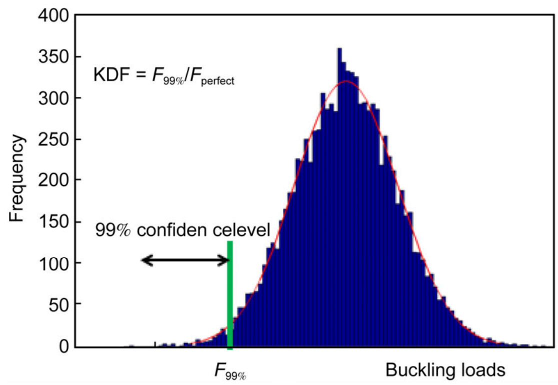

Figure 8 (Color online) Definition of the knock-down factor in stochastic analysis.

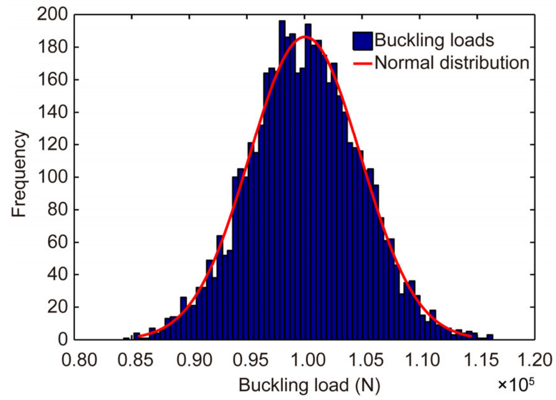

Figure 9 (Color online) Histogram of buckling loads of cylinder B, without geometric imperfections, *CV*=3%.

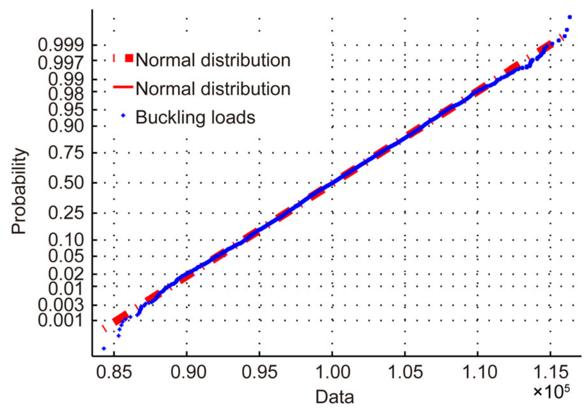

Figure 10 (Color online) Cumulative distribution of buckling loads of cylinder B, without geometric imperfections, *CV*=3%.

Table 3 Knock-down factors obtained using stochastic analyses

Simulation cases	Knock-down factors of cylinder A	Knock-down factors of cylinder B
4 input parameters, without geometric imperfection, with a different *CV* for load imperfection	k_2 = 0.90 (*CV* 3%), 0.89 (*CV* 5%), 0.87 (*CV* 10%)	k_2 = 0.86 (*CV* 3%), 0.84 (*CV* 5%), 0.81 (*CV* 10%)
5 input parameters, with geometric imperfection	0.79 (Z15), 0.62 (Z17), 0.66 (Z20)	0.63 (Z15), 0.65 (Z17), 0.63 (Z20)

down factor decreases slightly with an increasing *CV* value for the load imperfection, which demonstrates that the load imperfection has insignificant effect on the value of the knock-down factor. It can also be seen from Table 3 that the value of the knock-down factor obtained using the stochastic analysis obviously decreases if the measured geometric imperfection is taken into account as an input parameter, which indicates that the geometric imperfection is very sensitive to the buckling load.

5 Novel knock-down factors

The new combined knock-down factors for cylinders A and B are represented by the triangular markers in Figure 11 and compared with the knock-down factors obtained by NASA. For each cylinder, three new knock-down factors ($k_{\_a}$, $k_{\_b}$, $k_{\_c}$) are listed, respectively, corresponding to the three values of k_2 which were obtained using different values of the coefficient of variation for the load imperfection in the stochastic analysis. Two reduced experimental samples were manufactured and tested for each cylinder in NPU China. It can be seen from Figure 11 that all the numerical results obtained from the new robust design are below the test results but above the NASA knock-down factors, which demonstrates that the new knock-down factors are safe and less conservative than those proposed in NASA SP-8007. In Figure 11, the improvement of the new knock-down factor compared to the NASA knock-down factor is represented by the value in the bracket. The improvements indicate that the structural weight and design cost [21,22] of an aerospace structure can be significantly reduced using this new

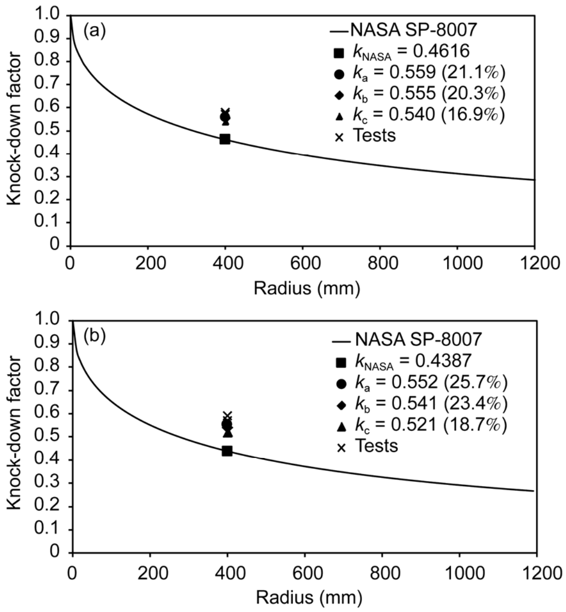

Figure 11 New combined knock-down factors of cylinders A and B, with different *CV* values for the load imperfection. (a) Cylinder A; (b) cylinder B.

Table 4 Novel knock-down factors compared with Almroth's work

Cylinders	Knock-down factors by Almroth	Knock-down factors in this study	Improvement
Cylinder A	0.469	0.55	17.3%
Cylinder B	0.461	0.54	17.1%

robust design. In addition, the novel knock-down factors are also less conservative compared to those obtained by Brush and Almroth [23] in 1975, as shown in Table 4.

The values of the new knock-down factors are low compared to the values of the knock-down factors obtained in the stochastic analysis considering the geometric imperfection as given in Table 3. The reason is that the effect of the "worst" geometric imperfection is considered in the first part k_1 of the new knock-down factor based on a lower-bound buckling value in Figure 5, however the amplitude of the geometric imperfection used in the stochastic analysis is the measured value, amplitude/thickness<3, which is obviously not the worst case as shown in Figure 7.

6 Conclusions

The buckling performances of two stiffened cylinders were studied using a deterministic study and a stochastic analysis. A smeared model of a stiffened cylinder was used to implement the real measured geometric imperfection. In order to obtain a robust and less conservative design, the new knock-down factor was derived from a combination of the knock-down factor k_1 obtained by considering the geometric imperfection and a knock-down factor k_2 achieved by taking into account other imperfections. Compared to the NASA knock-down factors and the test results, the results presented in this study demonstrate that for axially loaded stiffened cylinders the new combined knock-down factor leads to a safe and less conservative design compared to the respective NASA design. This allows for a significant reduction in structural weight and design costs.

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