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Voronoi densities for bicylists: adaptation for finite object size and speed

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Abstract. Density is one of the most relevant variables in a traffic flow description. For objects in 2 dimensions, density can be determined by the space that is allocated to each of the objects. This paper introduces a new way of computing the space available for a bicyclist, accounting for speed and accounting for the non-zero size of a bicycle. This changes local densities. The proposed method modifies the Voronoi densities, and assigns space to a bicycle. We assign space to bicycle A if it has a closer proximity to any point of bicycle A than any point of any other bicycle. The proximity is determined by the distance and the angle in relation to velocity of the bicycle. Specific proximity functions need to be formulated and calibrated to match cyclist behavior. This method helps to define a density level for cyclists, which in turn can for instance lead to a better indication of a Level of Service.

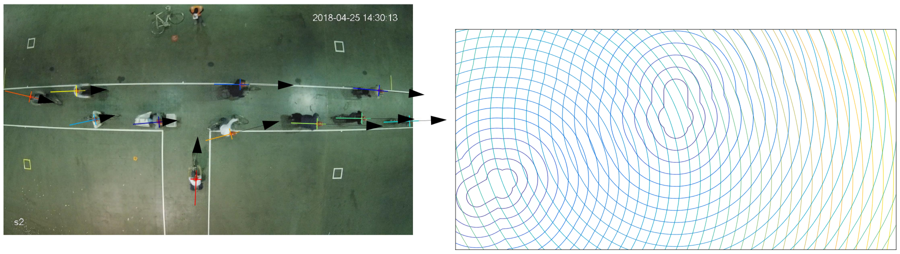
Keywords: Bicyclist traffic, Density computation, Voronoi areas, Traffic flow, Voronoi densities

1 Introduction

To properly describe bicycle traffic in a macroscopic way, the macroscopic variables flow, density and speed need to be defined. We want to define local density, i.e. the density perceived by a cyclist since this is relevant for traffic operations. This quantity explains how bicyclists behave. Densities form the basis for a dynamic macroscopic traffic models, hence a good representation of densities is essential.

Whereas for one-dimensional traffic (e.g., car traffic), the definition of density is relatively straightforward, for 2-dimensional traffic it is more complex. We refer to [4] for an overview of applying different densities to pedestrian movements.

For cyclists, this concept is different. Often, pedestrians are considered as point particles. The shape of a cyclist is – compared to their spacing – so irregular that it substantially differs from a circular (or point) particle. For instance, it would be possible to be very close to the side of a cyclist, but being at a same distance in front of the midpoint would be impossible due to its size. Also speed plays a role in the dynamics. A cyclist is expected to adapt his speed more



(a) Marked hats (b) Interacting cost fields

Fig. 1. The data used for illustration of the method

on what is happening in front than what is happening behind, particularly for higher speeds. Following that reasoning, space should not simply be allocated to the cyclist which is closest by, but also speed and direction should be taken into account.

This paper formulates a concept on how the space can be allocated, to in the end get densities. Two concepts are proposed: first, we include the size of particles, which is presented in earlier works as set Voronoi diagrams, e.g. [5], but is now for the first time applied to road traffic. Second, an effect of speed is introduced. As illustration bike sizes and anisotropic cost functions are posed. The aim of the paper is to present this concept. For this, simplified the cost functions and bike sizes are being assumed. Having said so, we do illustrate the ideas presented here with real-world data, collected in an experiment with cyclists [2]. From the data, we use the position of the head, as well as the heading direction and the speed; we use average size of a bike, and simplify its shape to a cross. Fig. 1(a) gives an impression of the data used.

The remainder of the paper is organised as follows. First, we give a background on the Voronoi densities. Then, chapter 3.1 discusses the proposed methodology. The paper finishes with a discussion and conclusions.

2 Background: Voronoi densities

Voronoi diagrams can be used in determining densities. Earlier work [1] extensively discussed the differences between local densities and global densities. Global density is the overall density, for instance determined by dividing the number of people over the area. This might vary on a smaller scale. For this reason, the concept of Voronoi diagrams is applied to pedestrian traffic. This computes the space which is available to everyone; space is allocated to the closest pedestrian. As addition, one can also have a maximum area of influence, and not assign space if it is further from any pedestrian than for instance maximum radius. As said, the concept does not deal with objects of extended size or anisotropy, which is what we introduce in this paper.

3 Proposed method

This section introduces the proposed method, first commenting on the physical space, and then on the velocities.

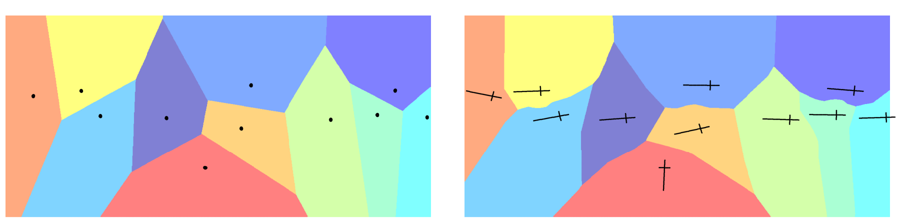
3.1 Accounting for physical space

The Voronoi diagrams use the distance to a (mid)point as basis for their space allocation. If we apply this to the center of the bikes, we get the allocation of space as shown in Fig. 2(a). However, cyclists have a physical size, and we can account for that. The base principle that we apply is that we no longer see a cyclist as one point, but instead take its physical size into account. Let's consider an example in which the space is allocated to the closest object, while considering *any point* of that object. For this, we need a distance function, or as we will call it in the remainder of the paper, a *cost function*. In this section, we consider the Euclidean distance. We now allocate space as follows. For a point in space, we find all costs to all points of all bikes. We identify the bike to which the point with the lowest overall cost belongs. We then allocate the space to that bike.

The example is of two interacting bikes as shown in Fig. 1(b). The lines show the isocost lines to a bike. They hence already include the concept of getting to the closest point of a bike. For any point in space, it is now considered which bike is closest. In the figure, that translates into checking for which bike the cost of getting to it is lowest. The space is then allocated to that bike.

To clarify the difference with an ordinary Voronoi assignment let's consider the following. The point in space $\mathcal{P}$ has a distance to the midpoint of cyclist a and b, denoted d^a_{mid} and d^b_{mid}. If $d^a_{\text{mid}} < d^b_{\text{mid}}$, the space would under normal space allocation be assigned to bike a. However, it can occur that the shortest distance to *any point* of bike a (for instance a's handle bar) is further than the shortest distance to *any point* of bike b (for instance b's rear wheel), so $d^a_{\text{shortest}} > d^b_{\text{shortest}}$.

Applying this concept to the data set (see Fig. 2(b)), we see that the boundaries have changed compared to Fig. 2(a). The boundaries between the space of the various cyclists follow the shape of the bicycles, and are no longer straight.



(a) Using one center point

(b) Using the full physical size

Fig. 2. Extended Voronoi spaces: space allocated to the bicycles based on proximity

This is due to the non-linearity in the distance. We also see that all areas are "continuous", i.e. there are no islands or concavities. The proof is as follows.

Suppose a point in space a is closest to cyclist c, then all points on the line a-c (being the shortest line from a to c) are closer to cyclist c than to another cyclist c'. Namely, if there were a point b on the line a-c which is closer to c' than to c, then $d_{ac} = d_{ab} + d_{bc} > d_{ab} + d_{bc'} > d_{ac'}$ (due to triangular inequality). This is inconsistent with the assumption, and hence it is proven by contradiction that the areas are "continuous". Following the same proof, it can be concluded that all resulting areas are convex.

Accounting for speed While cycling, bicyclists are likely to react more on what is happening in front of them than what is behind and this effect is higher for increasing speed. Therefore, the cost function as defined in the previous section might be different to objects in front of the bicycle than behind. We hence define a cost function which is anisotropic, with lower cost rates (i.e., cost per unit of distance) in front of the bicyclist (hence objects feel as if they were closer) and higher cost rates behind (hence the objects feel as if they were farther away). This cost function is speed and direction dependent; at zero speed the effect is negligible, whereas at high speed, the effect will be stronger.

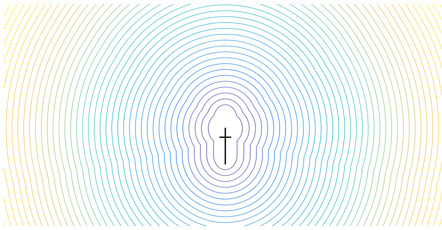
We will illustrate this principle with two different cost functions. First, an ellipse cost function is considered. Recall that for all points on an ellipsoid the sum of the distance to two (center) points is equal. In this case, we take as center points the mid point of the cyclist and a point ahead of the cyclist, with a distance depending on the cyclist's speed. Fig. 3(a) shows isocost lines which are further away in front of the bike than behind. However, they merely are shifted and the costs increase at a same level in front and behind, and therefore we conclude that another functional form is better suited.

The second functional form is introduced as a function. It prescribes the cost C between the point in space and a (specific, to be iterated over the bike) point of a bicycle. The cost is based on Euclidean distance d and angle ϑ between the movement direction of the bicycle and the direction of the segment connecting the point in space to the point of the bicycle:

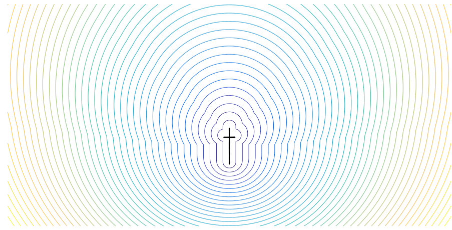
$$C = \frac{d}{\alpha + \cos(\vartheta)} \tag{1}$$

This function has a lower cost if objects are in the line of travel. Parameter α indicates the strength of the directional component, and should be tuned with speed. Qualitatively, we reason for low speeds, α is expected large, giving equal importance to objects in front and behind. For higher speeds, α can reduce asymptotically to 1, giving more importance to objects in front. This equation is in line with the equation indicating the relative effect of objects for the social force model [3].

Fig. 3(b) shows isocost lines for the function (for simplicity, we fixed α=3), still accounting for the the physical size of a bicycle. Note that the lines in the rear are much closer together and hence other objects at the rear are perceived

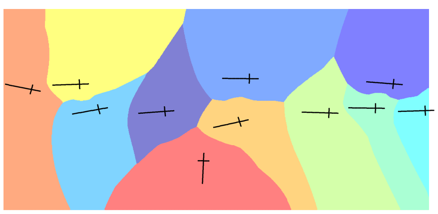


(a) Ellipse cost function

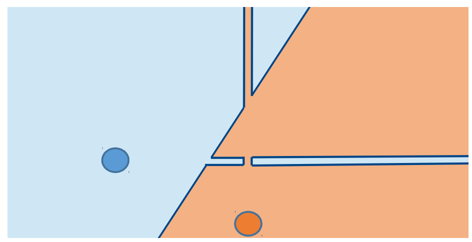


(b) Function defined in equation 1

Fig. 3. Isocost lines incorporating physical size and speed effects



(a) Voronoi areas



(b) Example of non continuous areas

Fig. 4. Cost fields of two bicycles

with lower cost. Fig. 4(a) shows the resulting areas assigned to each of the bikes. For instance the boundary between the orange bicycle at the rear and the blue bicycle at the bottom curves heavily, allowing much more space to orange bicycle than previously. This is in line with the concepts that the space behind a cyclist is less relevant, hence the blue bicyclist has less use of the space than the orange one.

For the adjusted Voronoi space which accounts for physical space, we proved in section 3.1 that the areas should be "continuous". This cannot be proven without loss of generality for the Voronoi space which accounts for motion due to the angle dependent function, as the next example will show. Consider for instance that all distances straight in front of a bicycle are very low cost (the closer the lower the cost), and always lower than any cost which are off the straight line. For all other angles, an Euclidean distance is taken as cost function. Fig. 4(b) shows the assignment of space for this cost function using a single point as base (i.e., not accounting for the physical space). It shows that the line straight ahead is assigned to one cyclist, but since these lines cross, there are several, distinct and not continuous areas belonging to each cyclist.

4 Discussion and conclusion

We addressed the issue of assigning space to individual travellers in order to come to density measures. Earlier work showed differences between local and global densities. The method using Voronoi diagrams turns out to be relevant in determining densities. In order to make this method more accurate for cyclists, in this paper, we have extended this method to include (1) physical size of the objects and (2) speed of cyclists. For the speeds, we explicitly formulated a way to account for the effect that cyclist might adapt their behavior more to objects in front than to objects behind.

The paper proposes a mathematical concept of assigning space to individuals. It is flexible for the specific cost functions, indicating what cyclists consider relevant distances and angles for their objects. The ultimate proof of the functions is in the test whether the allocated space is in line with the speed cyclist choose. Alternatively, (virtual reality) experiments can be done to test where the attention of cyclists is to find what determines and/or limits their speed.

The paper showed some mathematical properties of the areas assigned to each of the cyclist. If anisotropy is included, these areas do not need to be continuous. One might expect that a cyclist adapts his behavior to the space that he expects to be able to use. It is questionable whether space which is shielded by another cyclist is considered accessible – it can be in a dynamic environment. If one wants to exclude this space, one can still use the cost principles introduced here, and combine this with a different assignment methodology. Instead of simply assigning space to the cyclist which has the lowest cost for that area, one can gradually expand the area assigned to each cyclist. Then, one can assign space to a cyclist only if there is a direct line-of-sight between the cyclist and the point in space. Consequently, there can also be areas in space which are not assigned to any cyclist.

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