

PREFERENCE-BASED DESIGN IN ARCHITECTURE

PREFERENCE-BASED DESIGN IN ARCHITECTURE

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Music is the pleasure the human soul experiences from counting without it being aware that it is counting.

— Gottfried Leibniz

PREFACE

The Age of Enlightenment, which roughly took place between 1650 and the French Revolution, is held to be the source of critical ideas, such as the centrality of freedom, democracy and reason as primary values of society. It was a departure from the Middle Ages of religious authority, guild-based economic systems, and censorship of ideas toward an era of rational discourse and personal judgment, republicanism, liberalism, naturalism, scientific method, and modernity.

Open Design methodology, which is based upon the doctoral thesis of van Loon [1998] entitled 'Interorganisational Design', is centered around these values as, according to Open Design philosophy:

The design process of creating a new building or a new urban area should, as far as possible, be open and transparent. All stakeholders are to be treated equally. Powerless stakeholders and laymen get the same 'rights' in the design process as powerful stakeholders and experts. Manipulation and abuse of knowledge power has to be avoided where possible. (*Lex A. van Gunsteren, valedictory lecture 2003*)

Open Design makes use of decision-oriented mathematical models where the input is a reflection of each decision maker's interests in the design, i.e. their preferences, and the output is the result of applying mathematical algorithms to the input. Thus, the process becomes both transparent, as the model is a glass box, and non-manipulative, as the model owner refrains from incorporating his own personal preferences (see Figure 1).

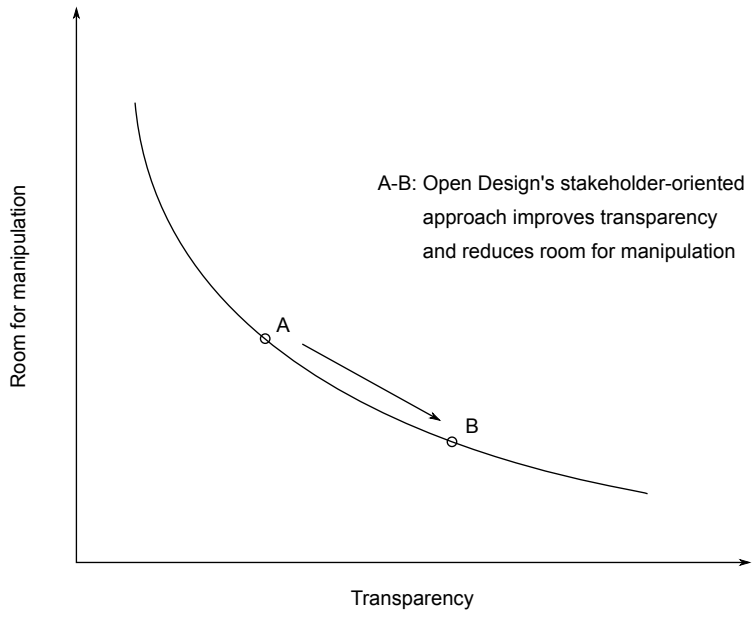


Figure 1: The Open Design approach improves transparency and reduces room for manipulation.

Incorporating decision makers' preferences into mathematical models means that preferences need to be modeled mathematically. This is not new. The Pythagorean discovery that the pitch of a note depends on the length of the string which produces it, and that concordant intervals in the scale are produced by simple numerical ratios was the first successful reduction of quality to quantity, the first step towards the mathematization of human experience. In the domain of architecture, Le Corbusier centered his design philosophy on systems of harmony and proportion. Le Corbusier's faith in the mathematical order of the universe was closely bound to the golden ratio¹ and the Fibonacci series, which he described as 'rhythms apparent to the eye and clear in their relations with one another'.

Van den Doel [1978] mentions three schools of thought on the issue of incorporating preferences: the Pigovian, the Bersonian and the Paretian approach. From these, van Loon [1998, p. 84] chooses the Paretian approach using the following motivation:

The Paretian approach is eminently suitable for optimisation in interorganisational design. It avoids utility measurement, which is difficult to perform, but does not lapse into the subjective evaluation of utility.

At that time, avoiding preference measurement was a valid motivation as classical methodologies for measuring preference lack a mathematical foundation. A new theory enabling preferences to be taken into account in a mathematically correct way has been developed by Barzilai [2004, 2005]. This thesis addresses the challenge of properly integrating preferences in Open Design methodology using Barzilai's theory, because early experiments [Binnekamp et al., 2006, pp. 345-350] showed difficulties in properly doing so.

The relevance of this thesis is two-fold:

1. The *quality* of decisions is improved, since only *feasible* designs are taken into consideration. In the current architectural practice by contrast, designs that are initially chosen turn out to be

¹ Two quantities are in the golden ratio if the ratio of the sum of the quantities to the larger one equals the ratio of the larger one to the smaller.

infeasible more often than not, making it necessary to introduce all kinds of rather arbitrary modifications.

2. The *acceptance* of decisions is improved, since *all* decision makers can see that their interests are genuinely taken into account.

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ACRONYMS

AHP	Analytical Hierarchy Process
ELECTRE	ELimination Et Choix Traduisant la REalité
FSI	Floor Space Index
GP	Goal Programming
GSI	Ground Space Index
KLM	Koninklijke Luchtvaart Maatschappij
LAMSADE	Laboratoire d'Analyse et Modélisation de Systèmes pour l'Aide à la Décision
LP	Linear Programming
MAVF	Multi Attribute Value Function
MCDA	Multiple Criteria Decision Analysis
LMOP	Linear Multi Objective Programming
MCSM	Multi Criterion Simplex Method
OMA	Office for Metropolitan Architecture
OSR	Open-Space Ratio
OR	Operations Research
PBD	Preference-Based Design
PFM	Preference Function Modeling
PROMETHEE	Preference Ranking Organisation METHod for Enrichment Evaluations
SMA	Stedelijk Museum Amsterdam

INTRODUCTION

Design in the domain of architecture is a complex process where success or failure depend on overcoming many difficulties. A substantial amount of these difficulties relate to two prominent characteristics of choice making in architecture: 1) multiple designs can fit into one intended purpose, which raises the question: how to choose the design that fits best, and 2) a multitude of decision makers have a say in the design process, which is the problem of group choice making.

It is the field of decision theory which is concerned with the problems of identifying the best choice to take. Its practical application is called decision analysis, which aims at finding tools, methodologies and software to help people, or groups of people, make better choices.

An investigation of classical decision analysis reveals a wide variety of methodologies which, however, produce contradictory results. This indicates that they all, with possibly one exception, must be incorrect! In physics, one can ask a group of people to individually perform an experiment, say determining the ratios of the weights of a set of objects, and expect each of them to return with the same results (apart from some minor measurement error). In other words, in the material sciences we would not consider using non-equivalent methodologies that produce conflicting results. In the social sciences however, of which decision theory is a part, this is commonplace. As in physics, in decision theory, of all non-equivalent methodologies no more than one can be correct.

The scientific foundation of selection (choice) is preference measurement. The purpose of measurement, i.e. representing variables by scales, is to enable the application of mathematical operations to these scale values. Scales are mappings from empirical objects to mathematical objects that reflect specific empirical operations which characterize a given property to corresponding operations in the mathematical system. In order for such mathematical operations to be applicable,

it is necessary to specify which property (e.g. length or mass) of the objects is being measured since it is not possible to ‘add objects’ without knowing whether what is being added is their mass, length, temperature, etc. The only property of relevance in the context of the mathematical foundations of decision theory is preference regardless of how it is named, including utility, value, etc.

How can we determine the correctness of a decision analysis methodology? Since the only property of relevance in the context of the mathematical foundations of decision theory is preference, decision making is founded on preference measurement. The correctness of a decision analysis methodology is determined by the correctness of the scales used for measuring preference. Recall that the purpose of representing variables by scales is to enable the application of mathematical operations to these scale values. As such, scales can be classified by the type of mathematical operations that they enable. Barzilai [2004, 2005] has shown that all of the classical models of the theory of measurement generate scales to which the operations of addition and multiplication are not applicable. Classical decision theory methodologies produce meaningless numbers, rendering them useless. The methodology developed by Barzilai [2005] called Preference Function Modeling (PFM), by contrast, has a mathematical foundation allowing the operations of addition and multiplication on the scales it generates.

Rather than following the classical theory of decision making and integrating preference in Operations Research (OR) techniques, our Open Design group uses Linear Programming (LP) models to solve design problems in the domain of architecture [Binnekamp et al., 2006]. Each stakeholder represents his interest as an objective function to be optimized. A linear program is then constructed where the constraints are these individual objective functions. The individual objective functions are aggregated in the linear program. The end result is a single objective function (as is typically the case in multi-objective optimization) that aims to reflect the goals of all decision makers. In addition to the limitation to a linear program, the feasible region is typically empty and negotiations outside the mathematical

model are required in order to re-adjust the parameters of this linear program.

As the Open Design group discovered in their applications, the technique of LP with negotiable constraints is poorly equipped to help decision makers in finding the design solution within the feasible set they prefer most. It can only produce single-criterion design solutions, which satisfy only one criterion of only one decision maker. This technique, therefore, does not extend naturally to group decision making. Our research group has attempted to integrate preferences into LP by assuming a linear [Binnekamp et al., 2006, pp. 345-350] or exponential [de Graaf and van Gunsteren, 2002] relationship between decision variable values and a decision maker's preference ratings, but several problems remain: 1) the constraints divide all possible solutions into either feasible or infeasible ones; black or white, no gray which could eventually be acceptable to decision makers; 2) the overall preference of a solution is determined as the weighted sum of the preference rating of that solution on all criteria, which is merely an approximation; 3) the results produced are still single-criterion design solutions, thereby not extending to group decision making.

What is needed is a methodology to find the design that is both feasible and most preferred by all decision makers. PFM offers a correct model for the measurement of preference and for the selection of the most preferred solution. In its current form however, PFM is an evaluation methodology, helping decision makers to choose the most preferred design alternative from a set of already existing alternatives. In the domain of architecture a design methodology is needed, where the design alternatives are not known *a priori*.

Can PFM be used as the basis for such a design methodology, which removes the limitations of using the weighted sum approximation to determine the overall preference rating and enables extension to group decision making? The Preference-Based Design (PBD) procedure proposed in this thesis offers such a design methodology. From the LP technique, it takes the definition of a design alternative as a combination of decision variable values and its feasibility determined by design constraints and allowed decision variable value ranges. Each decision maker can decide in which of the design's decision

variables he or she is interested and express his or her preference by rating three values for the design variables concerned. The decision maker thus rates a *synthetic alternative* – an alternative associated with a value for a single decision variable, regardless of other decision variables and regardless of its feasibility.

Throughout this thesis, it is assumed that the values of all variables where preference is to be measured are known. Some of these variables are directly controlled by the designer while the values of other variables are indirectly dependent on the values of those variables that are under the direct control of the designer. All variables are referred to as decision variables (or decision criteria) regardless of whether they are controlled by the designer directly or indirectly.

Combinations of design decision variable values that satisfy the design constraints constitute feasible alternatives. These are evaluated using PFM's algorithm, which enables finding the design that is both feasible and most preferred by all decision makers as a group. This, in principle, resolves the issue. However, rating only three design decision variable values on preference entails a resolution problem as no intermediate values will be considered.

Enhancing the resolution means finding preference ratings associated with intermediate design decision variable values. This can be done by fitting a curve through the three coordinates defined by the preference ratings associated with design decision variables. Testing Lagrange polynomials and cubic spline curves for this purpose did not produce satisfactory results. They oscillate between their roots (knots), so they can take negative values representing negative preference ratings. This difficulty can be avoided by asking the decision maker to *construct* a cubic Bézier curve to approximate the relationship between preference ratings and decision variable values. Such a curve (see Figure 2) is defined by four points; end points (x_0, y_0) , (x_3, y_3) , and control points (x_1, y_1) and (x_2, y_2) . The x coordinates of the end points are used to represent the range of allowed design decision variable values and the y coordinates the associated preference ratings. The decision maker can then use the control points to shape the curve, thereby relating intermediate design decision variable values to preference ratings. This is a more satisfactory approach

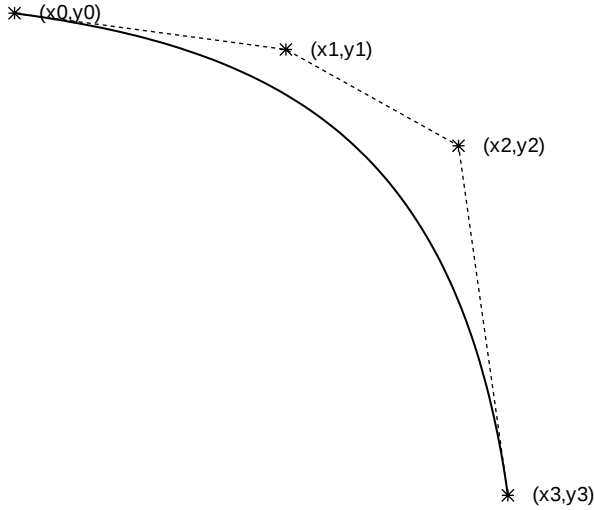


Figure 2: A cubic Bézier curve defined by four points; end points (x_0, y_0) , (x_3, y_3) , and control points (x_1, y_1) and (x_2, y_2) .

as it enables control over the derivatives of the end points of the curve. By dividing the curve in an acceptable number of segments the number of combinations of decision variable values is increased thereby solving the resolution problem.

The proposed PBD methodology is applied to three cases: 1) North Sea International Airport, 2) Amsterdam Museum and 3) Tilburg urban development.

The North Sea International Airport case resulted in a slightly different outcome using PBD as compared to using the constraint method. However, the solution obtained using the constraint method was the result of manipulation by the model owner. The PBD procedure produced a comparable result without any interference by the

model owner. The result is thus a more pure reflection of the decision maker's preferences.

The Amsterdam Museum case showed that, starting from one and the same bill of requirements, the application of LP with negotiable constraints and the application of PBD yield different outcomes. As before, this can be explained by the observation that PBD represents a more pure reflection of the decision makers' preferences.

The Tilburg urban development case shows that the Bézier curve is easy to work with and appeals to decision makers. The outcome of the PBD procedure was considered to be plausible and satisfactory by the decision makers of this project.

The following conclusions will be drawn:

1. The PBD procedure is not an extension of LP with negotiable constraints, but an independent design methodology.
2. PBD is a design methodology leading to a design which represents a pure reflection of the decision makers' preferences without any interference from the part of design experts, as is emphasized in the Open Design philosophy.
3. Outcomes from the PBD procedure are both plausible and satisfactory to the decision makers.
4. The PBD procedure is an excellent starting point in any complex architectural design problem.

Part I

FOUNDATIONS & ANALYSIS

DIFFICULTIES OF CHOICE MAKING IN ARCHITECTURE

Nowadays the profession of architects is dominated by the view that architecture is mainly a matter of aesthetics and that 'form follows form' as opposed to 'form follows function' which adheres to the principle that the shape of a building or object should be primarily based upon its intended function or purpose.

An example of form follows function can be found in the domain of car design where, after the introduction of the streamlined Chrysler Airflow in 1935 (see Figure 3), the car industry halted serious aerodynamic research. Car makers realized that optimal aerodynamic efficiency would result in a body shape that would hardly be distinguishable from other car makers which would not be good for unit sales.

An example of form follows function in the domain of architecture is the Van Nelle factory in Rotterdam, designed by Van Der Vlugt in collaboration with Mart Stam and built between 1927 and 1929 (see Figure 4). The factory occupies three volumes of decreasing height, one of eight levels for tobacco, a coffee section of five levels with a double-height entresol, and a three-level tea department. These three factory zones adjoin a main service route and are connected by bridges (see Figure 5) to a row along the water of dispatch and storage spaces. The bridges connecting the three volumes exemplify the form follows function principle as their position and size are derived from their purpose to transport goods from the factory zones to the service route. Yet, interestingly enough, these bridges are almost considered its design hallmark.

For an example of form follows form in the domain of architecture see Figure 6, showing the entries for the design competition for the extension of the Amsterdam museum of contemporary art. Although all designs are based upon the same bill or requirements, all are



Figure 3: Chrysler advertising comparing the Airflow to a streamlined train.



Figure 4: Van Nelle factory, overview.



Figure 5: Van Nelle factory, bridges connecting the three volumes.

completely different. In other words: form follows form means that a multitude of design alternatives can be generated, each fitting the same purpose but all being completely different.

Unlike car and aircraft design, where physics plays a major role in the design process, in architecture people's views and ideas can play a far more dominant role. Physics only plays a minor role because architecture is mainly focused on architectural space (rooms, corridors, halls, etc.).

This chapter aims to elaborate on and illustrate the above mentioned issues of choice making in architecture using two examples:

- The renovation and extension of the Amsterdam museum of contemporary art – the Stedelijk Museum Amsterdam (SMA);
- The development of the new office for the VPRO broadcasting company.

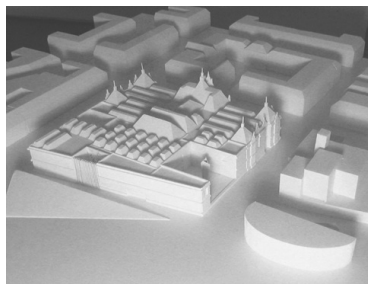
2.1 RENOVATION OF THE STEDELIJK MUSEUM AMSTERDAM

The following reconstruction is largely based on Sanders et al. [2003].

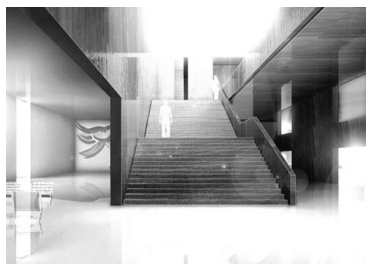
The Stedelijk Museum Amsterdam (SMA), designed by the architect A.W. Weissman, opened its doors in 1895. The museum was founded by a group of Amsterdam citizens. In the period 1945 to 1962, during the time that Willem Sandberg was the managing director, it established an international reputation as an institute focusing on the cutting edge of modern and contemporary art.

The museum's strengths are its contemporary art collection which approaches that of the Museum of Modern Art, Centre Pompidou and the Tate. It has a reputation for setting trends and for its openness and dynamism. Its main building is well located in the very center of Amsterdam and its archive and library are of high quality.

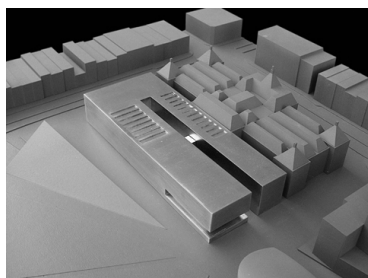
Its weaknesses are closely related to the deteriorating condition of the building. The building has climate control problems and has had to move part of its collection to a secondary location outside the city center. The museum has been considered to lack a coherent vision and to focus only on the quantitative aspects of exhibitions,



(a) Exterior Claus and Kaan architects.



(b) Interior Claus and Kaan architects.



(c) Exterior Dirrix architects.



(d) Interior Dirrix architects.



(e) Exterior Henket architects.



(f) Interior Henket architects.

Figure 6: Entries for the design competition for the extension and renovation of the Amsterdam museum of contemporary art.

not qualitative aspects. Visitor numbers are declining and as a result the museum has a hard time finding (financial) support within the municipality to extend and renovate the building.

2.1.1 *Overview of plans made*

In 1991, the municipality decided to ask four architects to make plans for extending the existing building and commissions Venturi to make final plans based on a budget of approximately 15 million euros. Although Venturi finished the final design in 1994, the municipality decided not to go ahead with these plans as they required a budget of approximately 35 million euros. In 1995 the municipality commissioned the Portuguese architect Siza to make a design (Siza I) for the extension which he finished in 1996, requiring a budget of approximately 25 million euros. As the existing building, by that time, also needed renovating and the floorspace for exhibitions was too limited, the museum, in collaboration with the municipality developed five alternatives for extending and renovating the museum. The municipality realized that its preferred alternative required a budget of 90 million euros and decided that part of the budget needed to be financed by other parties than the municipality. They commissioned Siza to make a design (Siza II) based on that alternative. The museum, in particular its staff, was disappointed with Siza II as essential elements of its organization are moved to the secondary location, a large amount of floorspace is allocated for commercial activities and because of logistic problems. They decided not to go ahead with Siza and devised a new plan named 2A/B but even this plan was not approved as the financial consequences of it were unclear. The process then stopped.

To resolve this stalemate situation, the Open Design group became involved and solved the problem by modeling the problem mathematically [Binnekamp et al., 2006, pp. 363-366]. This model contained detailed geometrical information about all the rooms in the existing building and contained formulas linking floorspace to costs. This model allowed the museum staff and management to find out which bill of requirements would meet the budgetary and functional con-



Figure 7: Exterior impression of the winning design for the extension and renovation of the Amsterdam museum of contemporary art.

straints. In other words, this model was a tool they used to come up with a feasible bill of requirements, both financially and geometrically. This bill of requirements was later used for the design competition, mentioned earlier in this chapter. The winning design is shown in Figures 7 and 8.

2.1.2 Conclusion

This case shows two major benefits of using a mathematical model: an increase in both the quality and the acceptance of the decision made. Before the model was introduced, design decisions were made that turned out to be either infeasible or unacceptable for the museum staff. The model was used to generate two alternatives: 1) maximizing the usage of the main location, as desired by the museum staff and 2) minimizing costs, as desired by the municipality. The model thus



Figure 8: Interior impression of the winning design for the extension and renovation of the Amsterdam museum of contemporary art.

enabled finding solutions that were both feasible and acceptable. However, each of these two design alternatives fully satisfies only a *single* criterion of *one* decision maker, either the museum staff or municipality. In other words, the methodology used does not extend naturally to group decision making. From a Multiple Criteria Decision Analysis (MCDA) viewpoint we are interested in a multi-criteria solution, satisfying *all* decision makers as much as possible thus supporting group decision making. More detailed information on MCDA can be found in Chapter 4. The survey of MCDA methodologies in Chapter 5 includes the methodology behind the mathematical model used for the SMA.

2.2 DEVELOPMENT OF THE NEW OFFICE FOR THE VPRO

The new office for the VPRO broadcasting company, called Villa VPRO was completed in 1997. The dissatisfaction of the most important stakeholder – the people who have to work in the building – has been extensively documented in a booklet published three years after commissioning [Paans, 2000] as well as in the press. How the design

team developed innovative solutions has been described by Roelofs [2001] and, looking at how the project was managed, by our Open Design group [Binnekamp et al., 2006, pp. 137-150]. The essence of the development process is reproduced below.

The design by MVRDV architects was based on an audacious architectural concept, which required innovative solutions from all parties involved. The main characteristic feature of the design was the architectural open space concept: open floor areas with open views from one floor to another. Two of the architects involved – Maas and Van Rijs – had previously worked at the Office for Metropolitan Architecture (OMA) of Rem Koolhaas, who had applied a similar open space concept in his design for the competition in 1993 for the Bibliotheque Jussieu in Paris.

The following key issues would have to be resolved for the realization of the open space concept [Roelofs, 2001]:

- First, there is the issue of fire protection and escape routes. Once ignited, a fire could spread through the building very quickly. Corridors with fire doors would clearly be in conflict with the open space concept.
- Second, the daylight distribution in the building constituted a serious problem. The daylight in some working locations would not meet the prevailing regulations for daylight at the working place at all.
- Third, certain areas would have to be protected against too much sunlight.
- Fourth, the installations for ventilation and heating would have to be designed in such a way that all the connected open spaces would be properly ventilated and heated.
- Finally, noise hindrance and acoustics are critical in such an open, connected space. A broadcasting company is quite different from, say, a software development firm where people are quiet behind their computer screens. A lot of verbal communication and telephone conversations are inherent to the mission of a broadcasting organization such as the VPRO.

The first four of these issues were addressed successfully, the fifth one, noise hindrance and acoustics, was not. It was considered sufficient to provide for some quiet rooms and for an extra budget, which would allow corrective measures to be taken after commissioning, such as the application of noise damping materials at critical locations. Not addressing this issue adequately made it unsuited to its very purpose: providing an adequate working place for an organization of (top) programme makers for television and radio. The architects and management persisted in their view that the design reflected the practical requirements of the users, who in turn maintained that quite the opposite was true. The result has been that most of the people who have to work in the building are extremely dissatisfied and disappointed.

Immediately after the commissioning of the building in June 1997, a stream of serious complaints from the users about noise and lack of privacy began. Employees started to correct the situation right away by building their own 'walls' with cupboards, boxes and curtains (Fig. 9 and Fig. 10).

The fact that the key issue of noise and acoustics – and to a certain extent also the lack of privacy – was largely ignored and played down during the design phase of the project was not just a coincidence. The ambition of realizing a daring architectural concept brought with it that anything that could kill it was taboo: not open for discussion because of too painful consequences.

The architects could not ignore the other four key issues. Fire protection and escape routes concern personal safety which no one is prepared to compromise. Daylight distribution and sun protection affect the very nature of the work of an architect: playing with space and light. Installations for heating and ventilation simply cannot be left out.

Noise hindrance and privacy, by contrast, do not affect safety and are subjective in the sense that different individuals perceive them differently. They are, therefore, linked to the mission and culture of the organization concerned.



Figure 9: Interior VPRO office after the building was completed.



Figure 10: Interior VPRO office after modification by the user.

2.2.1 *Conclusion*

The design team successfully addressed the challenges posed by the first four key issues, which can be considered quite an achievement. However, with respect to the fifth key issue, the Dutch artist Lucebert's observation 'Alles van waarde is weerloos' ('All things of value are defenseless') applies to the values – preferences – of the users. These values being subjective, and according to Lucebert defenseless, the architects could decide not to address the key issue related to noise hindrance and privacy, thus ignoring the interests of the users. This case shows the difficulty of properly taking into account each decision maker's interests or preferences in order to achieve the resulting design to be acceptable to all concerned.

2.3 CONCLUSIONS

Both cases show the difficulty, in the domain of architecture, of properly taking into account each decision maker's interests or preferences in order to achieve the resulting design to be acceptable to all decision makers as a group.

PREFERENCE MEASUREMENT

As shown in Chapter 2, in the domain of architecture we face the problem of multiple stakeholders having to choose the design that best fits their interests as a group.

The scientific foundation of selection (choice) is preference measurement. This brings us into the domain of preference measurement.

The survey in Chapter 5 will reveal a wide variety of methodologies involving preference measurement which, however, produce contradictory results. This indicates that they all, with possibly one exception, must be incorrect.

This chapter describes the elements of preference measurement theory (problems and solutions) providing the reader with the means to evaluate the correctness of preference measurement methodologies.

3.1 PREFERENCE MEASUREMENT THEORY

Preference measurement underpins economic theory, the theory of games and decision theory. Recent research has revealed errors at the foundations of these theories and other disciplines, including the inapplicability of the operations of addition and multiplication on utility scale values. The essence of this research is reproduced in this section which is based on Barzilai [to appear June 2010].

Whether psychological properties can be measured was an open question in 1940 when the committee appointed by the British Association for the Advancement of Science in 1932 'to consider and report upon the possibility of Quantitative Estimates of Sensory Events' published its Final Report [Ferguson et al., 1940]. An Interim Report, published in 1938, included 'a statement arguing that sensation intensities are not measurable' as well as a statement arguing that sensation intensities are measurable. These opposing views were not reconciled in the Final Report.

To re-state the opposing views in current terminology measurement has to be defined formally.

3.1.1 *The mathematical modeling framework*

To clarify what is meant by ‘the mathematical modeling of measurement’ some terminology is required. By an empirical system E we mean a set of empirical objects together with operations (i.e. functions) and possibly the relation of order which characterize the property under measurement. A mathematical model M of the empirical system E is a set with operations that reflect the empirical operations in E as well as the order in E when E is ordered. A scale s is a mapping of the objects in E into the objects in M that reflects the structure of E into M . See Figure 11.

The purpose of modeling E by M is to enable the application of mathematical operations on the elements of the mathematical system M : As Campbell [1920, pp. 267-268] eloquently states, ‘the object of measurement is to enable the powerful weapon of mathematical analysis to be applied to the subject matter of science’.

The Principle of Reflection is an essential element of modeling that states that operations within the mathematical system are applicable *if and only if* they reflect corresponding operations within the empirical system.

3.1.2 *Foundational errors*

The position that psychological variables cannot be measured was supported by Campbell’s view on the role of measurement in physics which elaborated upon Helmholtz’s earlier work on the mathematical modeling of physical measurement [von Helmholtz, 1887].

Consider Guild’s statement in support of the position that mathematical operations are not applicable to non-physical variables as summarized in Ferguson et al. [1940, p.345] in the context of measurement of sensation:

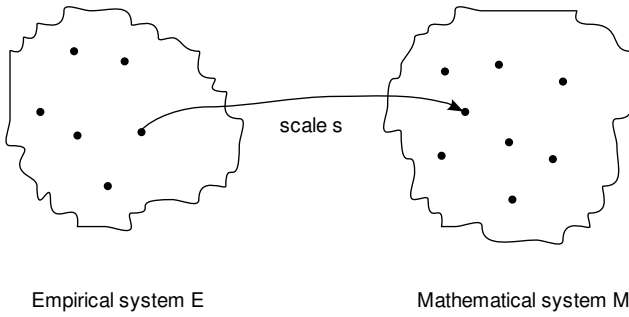


Figure 11: A scale is a mapping of the objects in the empirical system into the objects in the mathematical system.

I submit that any law purporting to express a quantitative relation between sensation intensity and stimulus intensity is not merely false but is in fact meaningless unless and until a meaning can be given to the concept of addition as applied to sensation. No such meaning has ever been defined. When we say that one length is twice another or one mass is twice another we know what is meant: we know that certain practical operations have been defined for the addition of lengths or masses, and it is in terms of these operations, and in no other terms whatever, that we are able to interpret a numerical relation between lengths and masses. But if we say that one sensation intensity is twice another nobody knows what the statement, if true, would imply.

This position, as well as the opposing position, were based on incorrect arguments concerning the applicability of mathematical operations to non-physical variables as will be explained in Section 3.1.3.

The task of constructing a model for preference measurement is addressed by von Neumann and Morgenstern [1944] indirectly in the context of measurement of individual preference. While the operation of addition as applied to length and mass results in scales that are unique up to a positive multiplicative constant, physical variables such as time and potential energy to which standard mathematical operations do apply are unique up to an additive constant and a positive multiplicative constant. (If s and t are two scales then for time or potential energy $t = p + q \times s$ for some real numbers p and $q > 0$ while for length or mass $t = q \times s$ for some $q > 0$.) Seeking an empirical operation that mimics the 'center of gravity' operation, they identified the now-familiar utility theory's operation of constructing lotteries on 'prizes' to serve this purpose.

Stevens [1946], elaborating upon von Neumann and Morgenstern's concepts, proposed a uniqueness-based classification of 'scale type' and the focus on the issues of the possibility of measurement of psychological variables and the applicability of mathematical operations to scale values has moved to the construction of 'interval' scales,

i.e. scales that are unique up to an additive constant and a positive multiplicative constant.

It might be claimed that the characterization of scale uniqueness by $t = p + q \times s$ implies the applicability of addition and multiplication to scale values for fixed scales, but this claim requires proof. There is no such proof, nor such claim, in the literature because this claim is false [Barzilai, to appear June 2010].

3.1.3 *Reconstructing the foundations*

Since the purpose of modeling is to enable the application of mathematical operations, we classify scales by the type of mathematical operations that they enable. We use the terms *proper* scales to denote scales where the operations of addition and multiplication are enabled on scale values, and *weak* scales to denote scales where these operations are not enabled.

In order for the operations of addition and multiplication to be applicable, the mathematical system M must be (i) a field if it is a model of a system with an absolute zero and an absolute one, (ii) a one-dimensional vector space when the empirical system has an absolute zero but not an absolute one, or (iii) a one-dimensional affine space which is the case for all non-physical properties with neither an absolute zero nor absolute one.

The one-dimensional affine space, is the algebraic formulation of the familiar straight line of elementary (affine) geometry so that for the operations of addition and multiplication to be enabled on models that characterize subjective properties, the empirical objects must correspond to points on a straight line of an affine geometry. In an affine space, the difference of two points is a vector and no other operations are defined on points.

The operation of addition is defined on point differences, which are vectors. Multiplication of a vector by a scalar is defined and the result is a vector. In the one-dimensional case, and only in this case, the ratio of a vector divided by another non-zero vector is a scalar.

The expression $\frac{a-b}{c-d} = k$ where a, b, c, d are points on an affine straight line and k is a scalar is used in the construction of proper

scales. The number of points in the left hand side of this expression can be reduced from four to three (e.g. if $b = d$) but it cannot be reduced to two and this implies that pairwise comparisons cannot be used to construct preference scales where the operations of addition and multiplication are enabled.

It follows that Campbell's argument is correct with respect to the application of The Principle of Reflection and the identification of addition as a fundamental operation, but that argument does not take into account the role of the multiplication operation and the modified forms of addition and multiplication when the models correctly account for the degree of homogeneity of the relevant systems. Note also that it is not sufficient to model the operation of addition since, except for the natural numbers, multiplication is not repeated addition: In general, and in particular for the real numbers, multiplication is not defined as repeated addition but through field axioms.

In summary, the fundamental issue of applicability of the operations of addition and multiplication to scale values was not resolved by von Neumann and Morgenstern's utility theory and the mathematical foundations of economic theory and other social sciences need to be corrected to account for the conditions that must be satisfied for the mathematical operations of linear algebra and calculus to be applicable.

In order for the operations of addition and multiplication to be applicable on preference scale values the mathematical system must be a one-dimensional affine space. Addition and multiplication are not applicable in von Neumann and Morgenstern's utility model, which underlies utility theory, because its axioms are not the axioms of a one-dimensional affine space. This is also the case for later formulations of utility theory.

3.2 CONCLUSIONS

Recalling the main question to solve as stated in the beginning of this chapter, the methodology to use should have a sound mathematical foundation for measuring preferences.

Recall that the purpose of measurement is to enable the application of mathematics to the variables under measurement and that Barzilai therefore classifies measurement scales by the mathematical operations that are enabled on the resultant scales and scale values. He defines proper scales as scales to which the operations of addition and multiplication (including subtraction and division) are applicable. Those proper scales that also enable order and the application of the limit operation of calculus are termed strong scales. All other scales are termed weak.

In other words, to evaluate any methodology involving preference measurement on whether it has a mathematical foundation we initially only need to look at the scales used for measuring preference. If the operations of addition and multiplication are being applied where they are not applicable, the numbers generated are meaningless.

It is important to note that, according to utility theorists, utility is a normative theory (see [Thurston, 2001, Section 1] and [Edwards, 1992, p. 254]). Specifically, Coombs et al. [1970, p. 123] state that 'utility theory was developed as a descriptive theory'. However, von Neumann and Morgenstern's utility theory, as well as its later variants, are mathematical theories and since mathematical theories do not dictate assumptions to decision makers, there is no basis in mathematical logic nor in modern utility for the claim that utility theory is normative or prescriptive.

NOTES ON MULTIPLE CRITERIA DECISION ANALYSIS

Multiple Criteria Decision Analysis (MCDA) is described by Belton and Stewart [2003, p. 2] as ‘a collection of formal approaches which seek to take explicit account of multiple criteria in helping individuals or groups explore decisions that matter’.

As shown in Chapter 2, the main question to solve is: how to select the design that meets all decision makers’ interests best, taking into account each design’s attributes.

Given that deciding is choosing and criteria are interests we can expect, that MCDA approaches should help solve the problems we face in architecture.

4.1 MULTIPLE CRITERIA DECISION ANALYSIS FRAMEWORK

The designs to choose from are the *alternatives* and in order to select the most preferred alternative we need to find out which alternative is preferred over the other by decision makers, i.e. rating an alternative’s performance. As decision makers have difficulties judging the performance of an alternative as a whole, different attributes of the alternatives are taken into account, termed *criteria*. These criteria may be defined in a tree-like structure, using main criteria, sub-criteria, sub-sub-criteria and so on. The relative importance of criteria and decision makers are incorporated using *weights*. The overall preference rating (performance) of an alternative is then determined by an algorithm that takes into account each alternative’s performance on each criterion and its weight.

The above is summarized in the following generic formal procedure:

1. Specify the alternatives.
2. Specify the decision maker’s criteria tree.

3. Rate the decision maker's preferences for each alternative against each leaf criterion.
4. To each leaf criterion assign the decision maker's weight.
5. Use an algorithm to yield an overall preference scale.

The procedure for a group of decision makers is identical to the procedure for a single decision maker, with the exception that each decision maker has to rate his preference for the alternatives on each criterion.

4.1.1 *Notes on preference measurement*

The third step in the above procedure is very important as it involves preference measurement, which underpins economic theory, the theory of games and decision theory. Recent research, reproduced in Chapter 3 [Barzilai, to appear June 2010], has revealed errors at the foundations of these theories and other disciplines, including the inapplicability of the operations of addition and multiplication on scale values. This research provides the means to evaluate the correctness of MCDA methodologies with respect to the scales used for measuring preference.

Recall that Barzilai classifies measurement scales by the mathematical operations that are enabled on the resultant scales and scale values. He defines *proper scales* as scales to which the operations of addition and multiplication (including subtraction and division) are applicable. Those proper scales that also enable order and the application of the limit operation of calculus are termed *strong scales*. All other scales are termed *weak scales*.

In other words, to evaluate any methodology involving preference measurement on whether it has a mathematical foundation we initially only need to look at preference scales. If the operations of addition and multiplication are being applied where they are not applicable the numbers generated are meaningless.

In contrast, Stevens [1946] proposed a uniqueness-based classification of scales into nominal, ordinal, interval and ratio scales:

NOMINAL SCALE Nominal scales represent the most unrestricted assignment of numerals. The numerals are used only as labels or type numbers, and words or letters would serve as well.

ORDINAL SCALE Ordinal scales arise from rank ordering. A classic example of an ordinal scale is Mohs scale of mineral hardness which characterizes the scratch resistance of various minerals through the ability of a harder material to scratch a softer material.

INTERVAL SCALE Interval scales are scales that are unique up to additive and (positive) multiplicative constants, i. e. the uniqueness of the set of all possible scales is characterized by $t = p + q \times s$.

RATIO SCALE Ratio scales are scales that are unique up to a multiplicative constant, i. e. the uniqueness of the set of all possible scales is characterized by $t = q \times s$.

Concerning ordinal scales, we note that in the case of ordinal systems, the mathematical image M of the empirical system E is only equipped with order and the operations of addition and multiplication are not applicable in M .

It is important to note that, as shown in Chapter 3 the operations of addition and multiplication are, in general, not applicable to measurement scales as proposed by Stevens, including to ratio and interval scales. In fact, these operations are not applicable to any measurement scales that are based on the models of classical measurement theory.

Stevens' classification, which has become a key element of the classical theory of measurement is used in a number of the models surveyed in Chapter 5. However, it does not distinguish between weak, proper, and strong scales and is the source of errors.

4.1.2 *Notes on yielding an overall preference scale*

Some of the MCDA models surveyed in Chapter 5 use the weighted arithmetic mean to yield an overall preference scale. However, the output of this procedure depends on the units by which the scales are

	Criterion		Weighted sum
	Opportunities	Salary (\$/Yr)	
Position 1	15	50 000	20 009
Position 2	20	45 000	18 012
Weight	0.6	0.4	

Table 1: Overall rating for two job positions using the arithmetic mean: prefer position 1.

measured. This is an unacceptable property of any mapping used to aggregate scales. As a consequence, we can construct examples where a change of units leads to order reversals in the overall ranking.

Consider a person having to decide between two job positions. Position 1 pays \$50 000 per year but does not offer as many opportunities for development as position 2 which pays \$45 000 per year. Assume that this person rates position 1 at 15 and position 2 at 20 on the opportunities criterion. Also assume that the opportunities criterion is weighted at 0.6 and the salary criterion at 0.4. Using the arithmetic mean to determine the overall rating of each position shows that position 1 is preferred over position 2 (see Table 1). However, if we change the unit for the salary criterion from \$/Yr to \$K/Yr the order is reversed and position 2 is preferred over position 1 (see Table 2).

It is important to distinguish coefficients in a linear expression, which apply to scales, from importance which applies to criteria. Length is an example of a criterion to which a weight can be attached to express the relative importance of the criterion 'length' as compared to other criteria. Length can be measured in kilometers or meters, millimeters, etc., depending on the scale used. Not making this distinction, i.e. interpreting the ratios of coefficients in a linear expression as relative importance is a fundamental error. Since the magnitude of coefficients ratio depends on the units in which the preference on different criteria is measured, it cannot correspond to relative importance: if a criterion - say the price of a house - is consid-

	Criterion		Weighted sum
	Opportunities	Salary (\$K/Yr)	
Position 1	15	50	29
Position 2	20	45	30
Weight	0.6	0.4	

Table 2: Overall rating for two job positions using the arithmetic mean: prefer position 2.

ered an ‘important’ criterion and the preference on that criterion is measured in dollars, it cannot become 1,000 times ‘less important’ by changing the unit to thousands of dollars. Criteria weights must be independent of unit change. The coefficients in linear aggregation can be interpreted as trade-off rates but this assumes that these rates are constant and decision makers have great difficulty expressing these rates.

4.1.3 Notes on group decision making

The common view in the classical literature, based on Arrow’s Impossibility Theorem [Arrow, 1963], is that group decision making cannot be modeled mathematically. However, Arrow’s theorem is a negative result, indicating that a solution cannot be found when following a given path. In the case of Arrow’s theorem the path is the way in which he formulates a group decision making problem as an *ordinal* voting system. Arrow’s theorem does not apply to group decision making problems that are different from its formulation. In other words, his theorem does not show that group decision making cannot be modeled mathematically, but only that, if formulated in a very specific way, namely as a voting system, this cannot be done.

4.2 CONCLUSIONS

To evaluate any MCDA methodology involving preference measurement on whether it has a mathematical foundation we initially only need to examine the preference scales using Barzilai's classification of scales. The operations of addition and multiplication are not applicable to measurement scales as proposed by Stevens.

Employing the weighted arithmetic mean to yield an overall preference scale is a mathematical modeling error. Equivalent descriptions of the problem yield different solutions.

There is no proof in literature that group decision making cannot be done. Arrow only showed that, if group decision making is formulated in a very specific way, namely as an ordinal voting system, this cannot be done.

SURVEY OF CURRENT MCDA APPROACHES

This chapter presents some multi-criteria methodologies as described by Belton and Stewart [2003, p. 9]:

- Value measurement methodologies
- Goal, aspiration or reference level methodologies
- Outranking methodologies

Chapter 4, which builds upon Chapter 3 provides the means to evaluate each of these methodologies.

5.1 VALUE MEASUREMENT METHODOLOGIES

Value function methods synthesize assessments of the performance of alternatives against individual criteria, together with inter-criteria information reflecting the relative importance of the different criteria, to give an overall evaluation of each alternative indicative of the decision makers' preferences [Belton and Stewart, 2003, p. 119].

These methods are based on evaluating alternatives in terms of an additive preference function.

Once an initial model structure and a set of alternatives for evaluation have been identified, the next step is to elicit the information required by the methodology. There are two types of information, sometimes referred to as intra-criterion information and inter-criterion information, or alternatively as scores and weights [Belton and Stewart, 2003, p. 121].

The overall evaluation of an alternative is determined by its value score on each bottom-level criterion and by the cumulative weight of that criterion.

In the next subsections, the main value measurement approaches will be investigated:

- Multi Attribute Value Function (MAVF)
- Analytical Hierarchy Process (AHP)
- Preference Function Modeling (PFM)

5.1.1 MAVF

The MAVF makes use of the following value function:

$$V(a) = \sum_{i=1}^m w_i v_i(a) \quad (5.1)$$

Where $V(a)$ is the overall value or performance of alternative a , v_i the value score reflecting alternative a 's performance (score) on criterion i and w_i the weight assigned to reflect the importance of criterion i .

Initial steps

The initial steps in using the MAVF are to develop a hierarchy of criteria (value tree) and to identify the alternatives.

Eliciting scores

Belton and Stewart [2003, pp. 121-123] describe the eliciting of scores as the process of assessing the value derived by the decision maker from the performance of alternatives against the relevant criteria. That is, the assessment of the partial value functions, $v_i(a)$ in the above structure. They state that 'these values need to be assessed on an interval scale of measurement, i.e. a scale on which the *difference* between points is the important factor'. However, as mentioned in Chapter 4, interval scales, as proposed by Stevens [1946], do not enable the operations of addition and multiplication and differences of interval scale values are undefined.

Recall that for the operations of addition and multiplication to be applicable to psychological scale values, the objects measured

must be points in a one-dimensional affine space. The operations of addition and multiplication are not enabled on scales constructed on the basis of classical measurement theory for in this theory no model corresponds to a one-dimensional affine space. The applicability of these operations has nothing to do with scale uniqueness, i.e. 'scale type' such as interval or ratio scales [Barzilai, to appear June 2010, p. 23].

They describe three ways to assess scores once the reference points of the scale have been determined:

DEFINITION OF A PARTIAL VALUE FUNCTION. This relates value (preference) to performance in terms of a measurable attribute reflecting the criterion of interest (such as the number of work places an office building offers). The first step in defining a partial value function is to construct a scale which is closely related to the decision maker's values. For example, in assessing the environmental impact of different cars, the CO₂ emission (g/km) of each car may serve as an appropriate indicator. Note that this assumes the decision maker's preference rating is linearly related to the attribute value. Also note that this defines an interval scale and that, as mentioned in Chapter 4, interval scales do not enable the operations of addition and multiplication.

CONSTRUCTION OF A SCALE WITH A NON-PHYSICAL VALUE. In this case, the performance of alternatives is assessed by reference to descriptive pointers to which numerical values are assigned (such as rating a design on a scale ranging from 'very good' to 'very poor'). Note that such a scale is in essence an ordinal scale as it only allows to determine whether an alternative is rated equally, higher or lower in comparison to other alternatives. Ordinal scales do not enable the operations of addition and multiplication.

DIRECT RATING OF THE ALTERNATIVES. In this case, no attempt is made to define a scale which characterizes performance independently of the alternatives being evaluated. The decision maker simply specifies a number, or identifies the position on a

visual analogue scale, which reflects the value of an alternative in relation to the specified reference points (such as assigning a score of 0 to the building with the worst layout, a score of 100 to the one with the best layout, and the others relative to these). However, this defines an interval scale which does not allow the operations of addition and multiplication as shown in Chapter 4.

When assessing the value function directly, the decision maker should begin by determining whether [Belton and Stewart, 2003, p. 123]:

- The value function is monotonically increasing against the natural scale – i.e. the highest value of the attribute is most preferred, the lowest least preferred.
- The value function is monotonically decreasing against the natural scale – i.e. the lowest value of the attribute is most preferred.
- The value is non-monotonic – i.e. an intermediate point on the scale defines the most preferred or least preferred point.

Eliciting weights

The weight assigned to a criterion is described by Belton and Stewart [2003, p. 135] as a scaling factor which relates scores on that criterion to scores on all other criteria and that thus, if criterion *A* has a weight which is twice that of criterion *B* this should be interpreted that the decision maker values 10 value points on criterion *A* the same as 20 value points on criterion *B* and would be willing to trade one for the other. They refer to these weights as *swing weights* to distinguish them from the, as they claim, less well defined concept of *importance weights*.

They define swing weights as follows: Consider a problem in which alternatives are assessed according to 3 criteria, C_1 to C_3 , with the scores for any alternative being represented by the set of values (c_1, c_2, c_3) . Eliciting swing weights, assuming that the first criterion is the

most highly ranked, requires the decision makers to specify values for X such that an alternative defined by $(X, 0, 0)$ is valued equally to the alternatives $(0, 100, 0)$ and $(0, 0, 100)$. Suppose $(50, 0, 0)$ is considered to be of equal value to alternative $(0, 0, 100)$, then $w_1 \times 50 = w_3 \times 100$ that is $w_3 = 0.5 \times w_1$.

It can be noted that swing weights, to some extent, correspond with the way in which weights are derived from pairwise comparison matrices as described by Barzilai [1997a]. In this procedure the decision maker has to use weights to make pair-wise trade-offs between criteria. This is done for as many pairs as possible, after which the weights are determined.

As mentioned in Chapter 4, people interpret the coefficients of a linear aggregation rule as weights representing relative importance, and this is an error. The coefficients in linear aggregation can be interpreted as trade-off rates but this assumes that these rates are constant and people have great difficulty expressing these rates.

Determining overall evaluations

The overall evaluation of an alternative is determined by first multiplying its value score on each bottom-level criterion by the cumulative weight of that criterion and then adding the resultant values. If the values relating to individual criteria have been assessed on a 0 to 100 scale and the weights are normalized to sum to 1 then the overall values will lie on a 0 to 100 scale.

The use of the MAVF to solve a decision making problem can be illustrated by means of a simple example in the domain of architecture.

The museum's problem

Recall the problem the Stedelijk Museum Amsterdam (SMA) faces as described in Chapter 2. Its current building no longer meets today's requirements. One of the main problems is a shortage of floorspace. The museum tried to cope with this problem by moving part of its organization to a building elsewhere in the city resulting in efficiency problems. To solve these problems the museum wishes to extend and renovate the existing building and, as it relies on public money, has

Alternative	Criterion	
	Expenditure	Centralizing
Design A	100	0
Design B	28	56
Design C	0	100

Table 3: Scores for each design on each criterion: museum example (MAVF).

to do so within limitations set by the municipality. The municipality wants to minimize the money spent on extending and renovating the building whereas the museum wants to maximize the use of the main location (existing building including its extension).

An architect made three preliminary designs all involving the complete renovation of the existing building:

- Design A involving extending the existing building with 11 500 square meters costing € 82 000 000.
- Design B involving extending the existing building with 14 000 square meters costing € 95 000 000.
- Design C involving extending the existing building with 16 000 square meters costing € 100 000 000.

Assume that the decision makers agree on defining a partial value function for rating the alternatives and that they use local scales. The attribute information of each design then determines the score of each design on each criterion. For instance, the performance of design B on the centralizing criterion is $\frac{14000-11500}{16000-11500} \times 100 = 56$ (on a 0 to 100 scale). Each design's performance on each criterion is shown in Table 3:

Assume that the decision makers agree on using the swing weight method to determine the weights. They agree that the expenditure goal is more important than the centralizing goal. Having established the rank order for the criteria weights, the next step is to assign values to them. They agree that an alternative with a score of 50

Alternative	Overall preference rating
Design A	$0.67 \times 100 + 0.33 \times 0 = 67$
Design B	$0.67 \times 28 + 0.33 \times 56 = 37$
Design C	$0.67 \times 0 + 0.33 \times 100 = 33$

Table 4: Overall rating for each design: museum example (MAVF).

on the expenditure goal and 0 on the centralizing goal is valued equally to an alternative scoring 0 on the expenditure goal and 100 on the centralizing goal. Let w_1 represent the weight attached to the expenditure goal and w_2 the weight attached to the centralizing goal. Then $w_1 \times 50 = w_2 \times 100$, that is, $0.5 \times w_1 = w_2$. Normalizing means that $w_1 + w_2 = 1$ so, $w_1 + 0.5 \times w_1 = 1$ then $w_1 = 0.67$ and $w_2 = 0.33$.

We can then use Equation 5.1 to determine the overall preference rating for each alternative as shown in Table 4.

Discussion

It has been suggested by von Winterfeldt and Edwards [1986] that, if the problem has been well structured, then the value functions should be regular in form – i.e. no discontinuities. They go further to argue that all value functions should be linear or close to linear and suggest that the analyst should consider restructuring a value to replace non-monotonic value functions by one or more monotonic functions. Belton and Stewart [2003, p. 124], however, caution against over-simplification of the problem by inappropriate use of linear value functions. They warn that the default assumption of linearity, which is often made, may generate misleading answers.

It should be noted that the concepts of continuity and linearity pre-suppose the application of addition and multiplication which, as was already pointed out, do not apply to scales constructed by von Winterfeldt and Edwards or Belton and Stewart.

It is not always possible to find a physical attribute which captures a criterion. In such circumstances it is necessary to construct a scale

for a non-physical attribute. In constructing such a scale, Belton and Stewart [2003, p. 128] emphasize that it is necessary to use an interval scale of measurement. However, Barzilai [to appear June 2010, p. 9] has shown that the operations of addition and multiplication are not applicable on scale values for any scale constructed on the basis of this theory regardless of their 'scale type' including 'ratio scales' and 'interval scales' rendering the input, and therefore the output, of Equation 5.1 meaningless.

Conclusion

Belton and Stewart [2003, p. 160] emphasize that they see Multiple Criteria Decision Analysis (MCDA) methodologies as tools for learning, a sounding board against which decision makers can test their intuition, not as a means of providing an 'answer' which is in some way 'objective' or 'right'. Because Barzilai has shown that the MAVF has no mathematical foundation, even its use as a methodology for learning or as a sounding board becomes questionable.

5.1.2 *AHP*

The AHP¹, a method for MCDA developed by Saaty [1980], has in its implementation many similarities with the MAVF approach. Both approaches are based on evaluating alternatives in terms of an additive preference function.

Initial steps

As with the MAVF approach, the initial steps in using the AHP are to develop a hierarchy of criteria and to identify the alternatives.

¹ The concept of decomposition of criteria into a sub-criteria tree (i.e., the generation of operational sub-criteria) was first proposed by Miller in his 1966 doctoral dissertation Miller [1966] (see also Miller [1969, 1970]).

Eliciting scores

Belton and Stewart [2003, p. 152] describe the major factors which differentiate the AHP from the MAVF approach from a practical viewpoint: the use of pairwise comparisons in comparing alternatives with respect to criteria (scoring) and in comparing criteria within sub-criteria (weighting), and the use of ratio scales for all judgments.

They then describe the procedure for eliciting scores [Belton and Stewart, 2003, pp. 153-154]:

For example, in comparing alternative office locations with respect to the criterion *quality of life*, decision makers would be asked: 'Thinking only about the *quality of life* of the locations, which of the two alternatives p or q do you prefer?' The decision makers are then asked to indicate the strength of their preference for p over q on the following five-point scale [Saaty, 1980]:

- 1 Equally preferred
- 2 Weak preference
- 3 Strong preference
- 4 Demonstrated preference
- 5 Absolute preference

Note that this is a verbal scale which has no mathematical basis and which does not enable the operations of addition and multiplication and in fact this structure is not a scale by the formal definition of a scale.

Once all pairs of alternatives have been compared in this way, the numeric values corresponding to the judgments made are entered into a pairwise comparison matrix. All diagonal entries are by definition equal to 1. The method interprets the above numerical scale of strengths of preference in a ratio sense. Thus if alternative p is preferred to alternative q , with strength of preference given by $a_{pq} = S$ (where a_{pq} is the entry in the p -th row and the q -th column of the comparison matrix), then the comparison of q with p is the reciprocal of that value, i.e. $a_{qp} = \frac{1}{S}$.

The aim in the AHP is to find the set of value scores v_1, \dots, v_n , such that the matrix values a_{pq} are approximated as closely as possible by the corresponding ratios $\frac{v_p}{v_q}$. The standard AHP method of doing this is to extract the eigenvector corresponding to the maximum eigenvalue of the pairwise comparison matrix.

Note that the applicability of addition and multiplication must be established before these operations are used to compute AHP eigenvectors. In order for addition and multiplication to be applicable on preference scale values, the alternatives must correspond to points on a straight line in an affine geometry [Barzilai, 2005]. Since the ratio of points on an affine straight line is undefined, preference ratios, which are the building blocks of AHP scales, are undefined. In addition, pairwise comparisons cannot be used to construct affine straight lines. The fact that eigenvectors are unique up to a multiplicative constant does not imply the applicability of addition and multiplication.

Eliciting weights

The next step is to compare all criteria which share the same parent criterion using the same pairwise comparison procedure, deriving a vector indicating the relative contribution of the criteria to the parent (analogous to the weights in the MAVF approach).

Determining overall evaluations

The judgments are aggregated by working upwards from the bottom of the hierarchy, as with the multi-attribute value function.

The previously used museum's problem can be used to illustrate use of the AHP to solve a decision making problem.

The museum's problem revisited

The first step in the AHP is to decide on the relative importance of the objectives by comparing each pair of objectives and rating them. Assume that the museum and municipality decide that the expenditure goal ($p = 1$) is *strongly* more important than the centralizing goal ($q = 2$). Then, using the five-point scale, $a_{p,q} = 5$ and the entry

Goal	Goal	
	Expenditure	Centralizing
Expenditure	1	5
Centralizing	$\frac{1}{5}$	1

Table 5: Weights for each goal: museum example (AHP).

Goal	Goal		
	Expenditure	Centralizing	Average
Expenditure	0.83	0.83	0.83
Centralizing	0.17	0.17	0.17
			$\Sigma = 1$

Table 6: Normalized weights for each criterion: museum example (AHP).

in the 1st row and the 2nd column of the comparison matrix, then the comparison of q with p is the reciprocal of that value, $a_{q,p} = \frac{1}{5}$ resulting in Table 5:

The normalized weights for the expenditure and centralizing goal are $\frac{1}{1+5} = 0.17$ and $1 - 0.17 = 0.83$ respectively (Table 6).

The second step is to rate each alternative on each criterion using the same pair wise comparison procedure. Assume that the municipality decides on the ratings on the expenditure criterion as shown in Table 7. Normalizing produces the results shown in Table 8.

Assume that the museum decides on the ratings on the centralizing criterion as shown in Table 9. Normalizing produces the results shown in Table 10.

Using the normalized weights and ratings the overall preference rating for each design is determined as shown in Table 11.

Alternative	Alternative		
	Design A	Design B	Design C
Design A	1	5	7
Design B	$\frac{1}{5}$	1	5
Design C	$\frac{1}{7}$	$\frac{1}{5}$	1

Table 7: Scores for each design on the expenditure goal: museum example (AHP).

Alternative	Alternative			Average
	Design A	Design B	Design C	
Design A	0.74	0.81	0.54	0.70
Design B	0.15	0.16	0.38	0.23
Design C	0.11	0.03	0.08	0.07
				$\Sigma = 1$

Table 8: Normalized scores for each design on the expenditure goal: museum example (AHP).

Alternative	Alternative		
	Design A	Design B	Design C
Design A	1	$\frac{1}{7}$	$\frac{1}{9}$
Design B	7	1	$\frac{1}{7}$
Design C	9	7	1

Table 9: Scores for each design on the centralizing goal: museum example (AHP).

Alternative	Alternative			Average
	Design A	Design B	Design C	
Design A	0.06	0.02	0.09	0.05
Design B	0.41	0.12	0.38	0.22
Design C	0.53	0.86	0.80	0.73
				$\Sigma = 1$

Table 10: Normalized scores for each design on the centralizing goal: museum example (AHP).

Alternative	Overall preference rating
Design A	$0.83 \times 0.70 + 0.17 \times 0.05 = 0.59$
Design B	$0.83 \times 0.23 + 0.17 \times 0.22 = 0.23$
Design C	$0.83 \times 0.07 + 0.17 \times 0.73 = 0.18$

Table 11: Overall rating for each design: museum example (AHP).

Discussion

There has been extensive debate about the AHP [Belton and Stewart, 2003, pp. 157-159]. It would take too long for the purpose of this chapter to cover all the issues raised in detail, but the main points of concern and debate will be briefly summarized below.

Pairwise comparisons (i.e. comparing two alternatives at a time) and ratios of alternatives cannot be used in the construction of preference scales to which the operations of addition and multiplication are applicable. In order for addition and multiplication to be applicable on preference scale values, the alternatives must correspond to points on a straight line in an affine geometry [Barzilai, 2005]. Since the ratio of points on an affine straight line is undefined, preference ratios, which are the building blocks of AHP scales, are undefined. In addition, pairwise comparisons cannot be used to construct affine straight lines. The fact that eigenvectors are unique up to a multiplicative constant does not imply the applicability of addition and multiplication.

Concerns have been expressed about the appropriateness of the conversion from the semantic to the numeric scale used by Saaty as a measure of strength of preference. As already mentioned, a semantic or verbal scale has no mathematical basis and does not enable the operations of addition and multiplication and in fact this structure is not a scale by the formal definition of a scale.

Normalization is done in various stages of AHP computations. Normalization is a mathematical operation and, as any other mathematical procedure, its use in any methodology must be justified, but this operation has not been justified in the AHP literature [Barzilai, 2001, p. 3].

Many AHP errors are reviewed in Barzilai [1997a, 1998a,b, 2001] (see also the references there).

Conclusion

The AHP is plagued by many flaws and these flaws are fundamental. Claims that these flaws are easily corrected have been identified as being false [Barzilai, 2001, p. 5].

The AHP claims to measure preference on ratio scales. This is a fundamental error because in order for the operations of addition and multiplication to be applicable on preference scale values, the scales must be affine. But in this case preference ratios are undefined because the ratios of points in an affine space are undefined. Belton and Gear [1983] suggested a change of AHP normalization which does not result in affine scales and therefore does not solve this problem. Dyer [1990, p. 250] states that the AHP 'generates rank orderings that are not meaningful' and that '[a] symptom of this deficiency is the phenomenon of rank reversal' which is a circular argument: the only AHP deficiency presented in his paper is rank reversal. Dyer also states that the AHP's multiple methodological flaws can be corrected by 'its synthesis with the concepts of multi-attribute utility theory' but utility theory suffers from its own flaws. Kirkwood [1996, p. 53] relies on Dyer and Sarin [1979, p. 820] which repeats the common error that the coefficients of a linear value function correspond to relative importance. Furthermore, 'difference measurement' which is the topic of Dyer and Sarin is an incorrect model of preference measurement.

5.1.3 PFM

A new theory of (preference) measurement has been developed by Barzilai [2004, 2005]. The main results of this new theory are the construction of measurement scales to which linear algebra and calculus are applicable. Based on this theory, a practical methodology for constructing proper preference scales, PFM, and a software tool that implements it, Tetra, have been developed. For details on PFM scale construction see Barzilai [1997b] and the axioms at Barzilai [to appear June 2010]. The Tetra Online Reference can be found at: <http://scientificmetrics.com/TetraReference>. The Tetra Quickstart Guide can be found at: <http://scientificmetrics.com> and contains an example.

Initial steps

As with the above approaches, the initial steps in using PFM are to develop a hierarchy of criteria and to identify the alternatives.

Eliciting scores

Each decision maker's preferences for each alternative against each criterion are rated by first establishing reference alternatives:

- A 'bottom' reference alternative which is rated at 0
- A 'top' reference alternative which is rated at 100

Then, the other alternatives are rated relative to these reference alternatives on the scale established.

Eliciting weights

To each leaf criterion each decision maker attaches a weight.

Determining overall evaluations

The PFM algorithm is used to yield an overall preference scale.

Its use in solving a decision making problem can, again, be illustrated by means of the before used museum's problem.

The museum's problem revisited

Assume that, using the rating procedure as described above, the decision makers rate the alternatives on each criterion as shown in Table 12.

The decision makers agree on attaching a weight of 67 to the expenditure criterion and a weight of 33 to the centralizing criterion.

Using the above ratings and weights, the PFM algorithm yields the overall preference scale as shown in Table 13.

Alternative	Criterion	
	Expenditure	Centralizing
Design A	100	0
Design B	28	56
Design C	0	100

Table 12: Scores for each design on each criterion: museum example (PFM).

Alternative	Overall preference rating
Design 1	55
Design 2	37
Design 3	45

Table 13: Overall rating for each design: museum example (PFM).

Discussion

As mentioned before, Barzilai [2004, 2005] developed a new theory of measurement. The main result of this theory is that there is only one model of strong measurement for preference. It also follows from *the Principle of Reflection*² that all the models of the classical theory of measurement generate weak scales to which the operations of addition and multiplication do not apply.

This new theory of measurement irrefutably invalidates common Operations Research (OR) methodologies. In other words, apart from PFM, there is no proof in literature that the mathematical operations of addition and multiplication apply to scales that are based on those common OR methodologies.

² For a detailed discussion see *The Principle of Reflection* in Barzilai [2006, section 6.8]

Conclusion

Based on a new theory of measurement, PFM is the only decision theoretical methodology that enables the construction of measurement scales to which linear algebra and calculus are applicable.

5.1.4 *Conclusions on value measurement methodologies*

Barzilai [2005] has established that the operations of addition and multiplication are not applicable to measurement scales (e.g. ratio scales and interval scales) that are based on common OR methodologies. He also established that the uniqueness of strong preference scale sets implies that all correct methodologies for constructing strong preference scales that assume no measurement error must produce identical scale sets, i.e. these methodologies must be equivalent. Conversely, of all non-equivalent methodologies, at most one can be correct.

5.2 GOAL, ASPIRATION OR REFERENCE LEVEL METHODOLOGIES

Value measurement methodologies help decision makers to choose the most preferred (design) alternative from a set of already existing alternatives and are therefore classified as evaluation methodologies. However, as mentioned in Chapter 2, in the domain of architecture we face the problem that a multitude of design alternatives can fit an intended purpose. It would be too time-consuming to ask an architect to make drawings of every conceivable design that fits an intended purpose and then analyze them by enumeration using the previously mentioned methodologies. What we need, therefore, is a *design* methodology where the alternatives to be evaluated are not known *a priori*.

To clarify the issue of such a design methodology we can use the concept of a *design space* as described by Dym and Little [2004, pp. 97-98]:

A design space is a mental construct of an intellectual space that envelops or incorporates all of the potential

solutions to a design problem. As a broad concept, the utility of the notion of a design space is limited to its availability to convey a feel for the design problem at hand. The phrase *large design space* conveys an image of a design problem in which (1) the number of potential designs is very large, perhaps even infinite, or (2) the number of design variables is large, as is the number of values they can assume.

Mathematically, a design solution or design alternative is represented by a combination of design variable values. Design variables are design attributes, e. g. a building's lettable floorspace, number of floors, the ratio between lettable and gross floorspace. The variables express each decision maker's interests in the design. By means of optimization a combination of variables (design attributes) can be found representing a design alternative. An example of such a design alternative could be a building having a floorspace of 10 000 square meters occupying 5 floors and having a ratio between lettable and gross floor space of 85%. Constraints, goals, and objectives are used in the optimization process.

Zeleny [1982, pp. 225-226] describes the conceptual and technical differences between constraints, goals, and objectives:

A **CONSTRAINT** is a fixed requirement which cannot be violated in a given problem formulation. Constraints divide all possible solutions (combinations of variables) into two groups: feasible and infeasible.

A **GOAL** is a fixed requirement which is to be satisfied as closely as possible in a given problem formulation.

AN **OBJECTIVE** is a requirement which is to be followed to the greatest extent possible (either by minimization or maximization) given the problem's constraints.

The use of constraints, goals and objectives means that we do not need to define the alternatives to be evaluated *a priori*, only the design

variables, constraints and objectives which classifies goal, aspiration or reference level methodologies as *design methodologies*.

The following methodologies are investigated:

- Linear Programming (LP)
- Goal Programming (GP)
- Linear Multi Objective Programming (LMOP)

5.2.1 LP as used in Open Design

An LP model can be stated in what is called the *canonical form*:

$$\text{Minimize } Z = \sum_{j=1}^n c_j x_j \quad (5.2)$$

subject to:

$$\sum_{j=1}^n a_{ij} x_j \geq b_i \quad \text{for } i = 1, \dots, m$$

and

$$x_j \geq 0 \quad \text{for } j = 1, \dots, n$$

Where the x_1, x_2, \dots, x_n are non-negative *decision variables* or unknowns and the c_1, c_2, \dots, c_n are contribution coefficients that represent the marginal contribution to Z for each unit of their respective decision variable. This LP model seeks a single objective or single goal of minimizing the *objective function* or Z function. In the model the objective function is subject to a set of m *constraints*. In the constraints, the a_{ij} , where $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$ are *technological coefficients* that represent the per unit usage of the x_j of the right-hand-side coefficients of b_i . In this model the n decision variables are required to be non-negative.

In LP models, the coefficients $b_1, b_2, b_3, \dots, b_m$ are considered to be fixed. Often they represent physical constraints that cannot be changed, such as the dimensions (b_1, b_2, \dots, b_m) of land available to grow various vegetables (x_1, x_2, \dots, x_n) .

The Open Design group [Binnekamp et al., 2006] utilizes LP models to solve design problems in the domain of architecture by considering each of the model's constraints as expressing a decision maker's interests concerning the design attributes. Additionally, a limited degree of group decision making is enabled by considering the coefficients b_1, b_2, \dots, b_m to be negotiable. A major advantage of this approach is that it enables to determine whether the constraint conditions of all decision makers can be satisfied by looking at whether or not the model's set of feasible solutions is empty. If the feasible set remains empty following all negotiations, the problem is unsolvable; should the feasible set be non-empty, the decision makers need to find the design solution which they prefer most, within the feasible set.

In an attempt to find the most preferred design solutions, each stakeholder's objective is translated into an objective function. This yields as many optimization models – design solutions – as there are objective functions. Stakeholders then have to negotiate the solutions.

The use of an LP model to solve a decision making problem can be illustrated by means of the previously used museum's problem.

The museum's problem revisited

The municipality, wanting to minimize the money spent on renovating the building, wants to spend no more than € 90 000 000. The museum, wanting to maximize the use of the main location, wants to allocate at least 35 000 square meters floorspace to the existing building including its extension. The museum's total floor space requirement is 40 000 square meters, renovating the old building costs € 2 600 per square meter and extending it costs € 3 000 per square meter. The existing building has a capacity of 20 000 square meters.

If x_1 represents the number of square meters floorspace renovated, x_2 the number of square meters extended and x_3 the number of square meters needed elsewhere in the city and if the before mentioned

constraints and the municipality's objective of wanting to minimize the money spent ($z_1 = 2\,600x_1 + 3\,000x_2$) is used then the LP model is:

$$\begin{array}{rcllcl}
 \text{Min } 2\,600x_1 & + & 3\,000x_2 & & & \\
 \text{s.t. } 2\,600x_1 & + & 3\,000x_2 & & \leq & 90\,000\,000 \\
 & x_1 & + & x_2 & \geq & 35\,000 \\
 & x_1 & + & x_2 & + & x_3 \geq & 40\,000 \\
 & x_1 & & & & \leq & 20\,000
 \end{array}$$

This model has an empty feasible set which means that the decision makers have to negotiate the constraints. Assume that the municipality and museum agree to relax their constraints by 10%, then the model changes:

$$\begin{array}{rcllcl}
 \text{Min } 2\,600x_1 & + & 3\,000x_2 & & & \\
 \text{s.t. } 2\,600x_1 & + & 3\,000x_2 & & \leq & 100\,000\,000 \\
 & x_1 & + & x_2 & \geq & 31\,500 \\
 & x_1 & + & x_2 & + & x_3 \geq & 40\,000 \\
 & x_1 & & & & \leq & 20\,000
 \end{array}$$

This model has a non-empty feasible set and the optimal solution is to completely renovate the existing building and to build an extension of 11 500 square meters. This would cost € 86 500 000. However, if we choose to optimize using the museum's objective of maximizing the use of the main location ($z_2 = x_1 + x_2$) the model changes:

$$\begin{array}{rcllcl}
 \text{Max } x_1 & + & x_2 & & & \\
 \text{s.t. } 2\,600x_1 & + & 3\,000x_2 & & \leq & 100\,000\,000 \\
 & x_1 & + & x_2 & \geq & 31\,500 \\
 & x_1 & + & x_2 & + & x_3 \geq & 40\,000 \\
 & x_1 & & & & \leq & 20\,000
 \end{array}$$

Objective being maximized	Value obtained	
	z_1	z_2
Minimize the money spent (z_1)	86 500 000	31 500
Maximize the use of the main location (z_2)	100 000 000	36 000

Table 14: Results of two optimization runs (LP)

The optimal solution of this model is different from the previous with regards to the size of the extension which is 16 000 square meters for this model. The associated costs are € 100 000 000. The results of each run are shown in Table 14.

This tendency to extreme values, which is a typical feature of linear programming formulations, is quite evident, so that compromise solutions are not immediately obvious [Belton and Stewart, 2003, p. 71].

Discussion

Although LP can be used as a design methodology it has the following major limitations: 1) single objective optimization 2) a harsh distinction between feasible and infeasible solutions and, 3) the linearity requirement.

SINGLE OBJECTIVE OPTIMIZATION An important feature of all optimization models is that there is only one objective function. In other words: it can only produce single-criterion design solutions, which fully satisfy no more than one of only a single decision maker's interests. Therefore, this technique does not extend naturally to group decision making. When there are multiple criteria or multiple decision makers, each criterion or decision maker is associated with its own objective function. In that case there are multiple optimization models, one for each criterion or decision maker, each having its own solution. So, although this technique helps decision makers to find feasible design solutions, it does not help them to select the most preferred

solution from these. Decision makers have to find it by means of negotiation while, as stated at the beginning of this chapter, the purpose of using an MCDA methodology is to mathematically support decision making. In other words, the math is lost. More importantly, the negotiations will not involve compromise solutions as each solution fully satisfies only one decision maker (multiple single-criterion solutions). An approach to overcome this problem is to use the *constraint method* which operates by optimizing one objective while all of the others are constrained to some value. In using this method our research group [Binnekamp et al., 2006, pp. 351-358] discovered that the whole process of choosing values for the constraints is completely arbitrary and still relies on unstructured negotiation.

HARSH DISTINCTION: FEASIBLE OR INFEASIBLE As mentioned before, the constraints divide all possible solutions (combinations of decision variables) into two groups: feasible or infeasible. In other words, a solution where € 100 000 000 is spent is considered feasible, whereas a solution where € 1 more is spent is considered infeasible. This is not a satisfactory representation of a stakeholder's preferences as it may well be that the stakeholder agrees to spending a little more if that would produce a non-empty feasible set. Our research group has made an attempt to remove this limitation where preferences were integrated into LP by assuming a linear relationship between decision variable values and a decision maker's preference ratings [Binnekamp et al., 2006, pp. 345-350]. However, the overall preference of a solution is determined as the weighted sum of the preference rating of that solution on all criteria which is an approximation. Furthermore, the relationship between decision variable values and a decision maker's preference ratings is usually not linear.

LINEARITY REQUIREMENT As mentioned in the previous paragraph, the relationship between decision variable values and a decision maker's preference ratings is usually not linear. Our research group [de Graaf and van Gunsteren, 2002] has also attempted to remove this limitation by assuming an exponential relationship between decision variable values and a decision maker's preference ratings.

This, however, did not remove the limitation of an LP model of only producing single-criterion design solutions.

Conclusion

Although LP-optimization constitutes a design methodology, the optimization framework does not extend naturally to group decision making. The use of constraints to express a decision maker's interests results in a too harsh distinction between feasible and infeasible solutions. The requirement of linearity can be removed by adopting the assumption of non-linear preference behavior which is plausible in many cases. The drawbacks of single-objective optimization and the harsh distinction of feasibility, however, are fundamental.

5.2.2 GP

The technique of GP aims to remove the limitation of LP not being able to handle decision problems involving multiple objectives. Charnes and Cooper [1961] suggest that each constraint that makes up an LP model is a separate function, called a *functional*. These functionals are viewed as individual objectives or goals to be attained. In effect, the b_i are a set of objectives or goals that must be satisfied in order to have a non-empty feasible set.

Referring to these functionals as goals, Charnes and Cooper suggest that goal attainment is achieved by minimizing their absolute deviation. They illustrate how that deviation can be minimized by placing the variables representing deviation directly in the objective function of the model. This allows multiple goals to be expressed in a model as follows:

$$\text{Minimize } Z = \sum_{i \in m} (d_i^+ + d_i^-) \quad (5.3)$$

subject to:

$$\sum_{j=1}^n a_{ij}x_j - d_i^+ d_i^- = b_i \quad \text{for } i = 1, \dots, m$$

and

$$d_i^+, d_i^-, x_j \geq 0 \quad \text{for } i = 1, \dots, m; \text{ for } j = 1, \dots, n$$

As a way of prioritizing goals in the objective function of the GP model Charnes and Cooper stated the *weighted GP model* as:

$$\text{Minimize } Z = \sum_{i \in m} (w_i^+ d_i^+ + w_i^- d_i^-) \quad (5.4)$$

Where w_i^+ and w_i^- are non-negative constants representing the *relative weight* to be assigned to the respective positive and negative deviation variables.

The use of a weighted GP model to solve a decision making problem can be illustrated by means of the previously used museum's problem.

The museum's problem revisited

Two objectives were identified: minimizing the money spent on extending and renovating the building and maximizing the use of the main location.

The deviations from the goals can be defined as follows³:

d_1 = under achievement of the expenditure goal

d_2 = under achievement of the centralizing goal

Assume that the decision makers agree on assigning a weight of 0.67 ($w_1 = 0.67$) to the expenditure goal and a weight of 0.33 ($w_2 = 0.33$) to the centralizing goal, then the goal programming model is:

³ In this example the implicit assumption is that a decision maker's preference is linear and rising until the aspiration level is achieved and constant thereafter.

$$\begin{array}{rcllcl}
 \text{Min } 0.67d_1 & + & 0.33d_2 & & \\
 \text{s.t. } 2600x_1 & + & 3000x_2 & - & d_1 = 90\,000\,000 \\
 & & x_1 & + & x_2 = 35\,000 \\
 & & x_1 & + & x_2 + x_3 \geq 50\,000 \\
 & & x_1 & & \leq 20\,000
 \end{array}$$

The optimal solution is to build an extension of 12 667 square meters. This means that the expenditure goal is fully met and the centralizing goal falls 2 333 square meters short of the set goal of 35 000.

Discussion

Recall that in order for addition and multiplication to be applicable on preference scale values, the alternatives must correspond to points on a straight line in an affine geometry. Archimedean goal programming, as well as other methodologies, has no mathematical foundation because it does not satisfy this condition. Moreover, according to Ignizio [1976, p. 183] ‘there are numerous approaches [for assigning weights], each of which can lead to different results.’ The results then depend on an arbitrary choice of weights: Since at most one approach can be correct, all others must be incorrect and it is not known which one, if any, is correct. Also, according to Ignizio [1976, p. 185] ‘weights must be positive numbers where such numbers reflect the importance associated with the minimization of a deviation variable assigned to a given objective.’ Recall that the interpretation of coefficient ratios as relative importance is a fundamental error.

In Archimedean goal programming piecewise linear functions and weighted sums are assumed to reflect decision makers’ preferences in order to enable the application of linear programming. In other words, decision makers’ preferences are dictated by goal programming researchers rather than by the decision makers.

Conclusion

Although GP, as LP, can be considered a design methodology, it is, like LP, prone to produce single criterion solutions which makes it unsuitable for group decision making. Furthermore, the solution produced depends on the norm used to handle the objective function. It also suffers from the other limitations mentioned under LP: 1) the harsh division of solutions (combinations of variables) into two groups: feasible or infeasible without any further classification and 2) the linearity requirement. More fundamentally, Archimedean goal programming has no mathematical foundation.

5.2.3 *LMOP*

Neither linear programming nor goal programming can handle multiple objectives. However, such problems can be dealt with by LMOP and compromise programming. LP and GP are viewed as special cases of these two methodologies, suitable only for specific and limited applications.

LMOP can be used to find non-dominated solutions given the set of functions to be optimized (minimized or maximized), i.e., *objectives*; and the set of functions to be satisfied (in terms of their predetermined values), i.e., *constraints*. A dominated solution means that there exists at least one other feasible solution such that one of the objective functions is improved. A non-dominated or Pareto optimal solution is one in which any further improvement in any one of the n objectives can be achieved only at the price of 'worsening' the value of at least one of the remaining objective functions [Zeleny, 1982, p. 53]. The Multi Criterion Simplex Method (MCSM) is used to identify all non-dominated corner points. Some of its modifications help to identify non-dominated segments or faces of a feasible set.

It would take too long for the purpose of this chapter to explain the MCSM in detail, but its essence can be explained by means of graphical analysis.

Consider the following problem as used by Zeleny [1982, p. 228]:

$$\begin{aligned} \text{Max } f_1(x) &= 4x_1 + x_2 \\ \text{Max } f_2(x) &= x_2 \end{aligned} \quad (5.5)$$

$$\text{s.t. } X = \left\{ \begin{array}{llll} 2x_1 + x_2 & \leq & 20 \\ \frac{5}{6}x_1 + x_2 & \leq & 10 \\ x_1 + x_2 & \geq & 5 \end{array} \right. \quad (5.6)$$

and the non-negativity conditions $x_1, x_2 \geq 0$.

This problem is represented graphically in Figure 12. Take a look at point D in the interior of X . Observe that D is a dominated point: both functions can be improved in value at any point in the northeast direction from D . All these dominated points are feasible.

The only non-dominated points of X are designated by the heavily drawn boundary connecting points A , B and C . Points A , B , and C are non-dominated corner points, and segments AB and BC are the remaining regions of non-dominance.

The MCSM is explained in detail in [Zeleny, 1982, pp. 231-248].

To illustrate its use the previously mentioned problem of the museum is used.

The museum's problem revisited

Two objectives were identified: minimizing the money spent on extending and renovating the building and maximizing the use of the main location.

The problem can be formulated as an LMOP model:

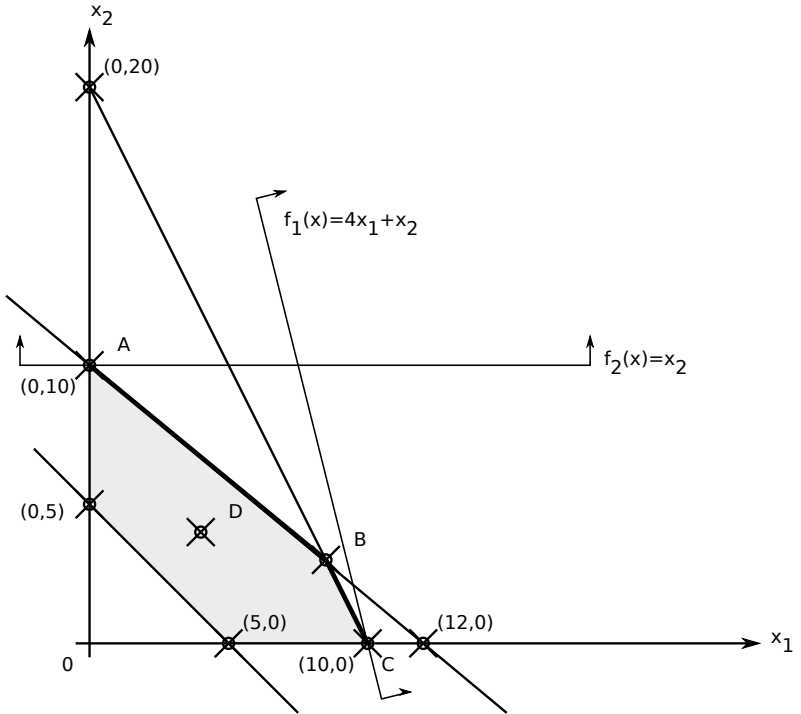


Figure 12: Drawing of constraints in two dimensions: the non-dominated set.

Solution	Value obtained	
	Expenditure z_1 in euros	Use of main location z_2 in m ²
x^1	86 500 000	31 500
x^2	100 000 000	36 000

Table 15: Results of optimization run (MCSM).

$$\begin{array}{rcllcl}
 \text{Min } 2\,600x_1 & + & 3\,000x_2 & & & \\
 \text{Max } x_1 & + & x_2 & & & \\
 \text{s.t. } 2\,600x_1 & + & 3\,000x_2 & & \leq & 100\,000\,000 \\
 x_1 & + & x_2 & & \geq & 31\,500 \\
 x_1 & + & x_2 & + & x_3 & \geq & 40\,000 \\
 x_1 & & & & & \leq & 20\,000
 \end{array}$$

The MCSM method reveals that this problem has two different non-dominated corner points shown in Table 15.

Although the MCSM used does not identify non-dominated segments or faces of a feasible set⁴, it is not hard to imagine that the remaining regions of non-dominance for this problem are represented by a face, as this problem has three decision variables. This face represents compromise solutions, but the methodology does not enable choosing from the compromise solutions the solution which is preferred most.

Discussion

It is important to notice that the corner point solutions obtained are identical to the solutions obtained when using LP, no corner point

⁴ Note that as a result resolving the two solutions in Table 15 with the solutions in Figure 12 is problematic.

solutions were found that could not have been found using LP which is due to the simplicity of the example. Applying this methodology to real life, more complex, problems with a multitude of constraints and objective functions will probably yield non-dominated corner points that could not have been found using LP.

Conclusion

Like LP and GP, LMOP can be considered a design methodology having the advantage of enabling finding compromise solutions. However, it does not offer an algorithm to choose between these solutions. Although LMOP enables finding compromise solutions, it also suffers from the limitations mentioned under LP: 1) the harsh division of solutions (combinations of variables) into two groups: feasible and infeasible without any further classification and 2) the linearity requirement.

5.2.4 *Conclusions on goal, aspiration or reference level methodologies*

The goal, aspiration and reference level methodologies described in this section can serve as design methodologies. However, both LP and GP produce or are prone to produce single-criterion solutions, rendering them unfit for solving group decision making problems. Although LMOP is able to produce compromise solutions it does not offer an algorithm to choose between them: it will still have to rely on negotiations. All suffer from the following limitations: 1) the harsh division of solutions (combinations of variables) into two groups: feasible and infeasible without any further classification and 2) the linearity requirement – decision variable values and associated preference ratings are assumed to be linearly related.

5.3 OUTRANKING METHODOLOGIES

Outranking methodologies differ from the value function methodologies in that there is no underlying aggregative value function. The output of an analysis is not a value for each alternative, but an out-

ranking relation on the set of alternatives. An alternative a is said to outrank another alternative b if, taking into account all available information regarding the problem and the decision maker's preferences, there is a strong enough argument to support a conclusion that a is at least as good as b and no strong argument to the contrary [Belton and Stewart, 2003, p. 233].

The two most prominent outranking approaches, the EElimination Et Choix Traduisant la REalité (ELECTRE) family of methods, developed by Roy and associates at Laboratoire d'Analyse et Modélisation de Systèmes pour l'Aide à la Décision (LAMSADE), University of Paris Dauphine, and Preference Ranking Organisation METHod for Enrichment Evaluations (PROMETHEE), proposed by Brans from the Free University of Brussels, are investigated in this section.

5.3.1 ELECTRE

The family of ELECTRE methods differ according to the degree of complexity, or richness of the information required or according to the nature of the underlying problem [Belton and Stewart, 2003, p. 234]. This section will focus on ELECTRE I, the earliest and simplest of the outranking approaches, which provides a good basis for understanding the underlying concepts. The remaining ELECTRE methods (ELECTRE II and ELECTRE III) will be discussed in the discussion section.

Initial steps

The starting point for most outranking methods, including ELECTRE is a decision matrix describing the performance of the alternatives to be evaluated with respect to identified criteria.

Eliciting scores

ELECTRE is usually based on a concise set of criteria, typically around 6-10. Alternatives are rated using a 5-point verbal scale: Very Low (VL), Low (L), Average (Av), High (H), Very High (VH). A higher rating indicates a higher preference [Belton and Stewart, 2003, p. 234].

Eliciting weights

Criteria are weighed such that a higher value indicates a greater 'importance'. Unlike the weights used in value functions, however, these do not represent trade-offs. Their psychological interpretation is, in fact, not well-defined, although they have been interpreted as a form of 'voting power' allocated to each criterion [Belton and Stewart, 2003, p. 234].

Determining overall evaluations

The ELECTRE methods are based on the evaluation of two indices, namely the *concordance* index and the *dis-concordance* index, defined for each pair of options a and b . The concordance index, $C(a, b)$, measures the strength of support in the information given, for the hypothesis that a is at least as good as b . The dis-concordance index, $D(a, b)$, measures the strength of evidence against this hypothesis. Note that the notion of strength of support is undefined.

There are no unique measures of concordance and dis-concordance and a number have been used. In ELECTRE I the concordance index is defined by:

$$C(a, b) = \frac{\sum_{i \in Q(a, b)} w_j}{\sum_{j=1}^m w_j} \quad (5.7)$$

where $Q(a, b)$ is the set of criteria for which a is equal or preferred to (i.e. at least as good as) b .

That is, the concordance index is the proportion of criteria weights allocated to those criteria for which a is equal or preferred to b . The index takes on values between 0 and 1, such that higher values indicate stronger evidence in support of the claim that a is preferred to b . A value of 1 indicates that a performs at least as well as b on all criteria (so that a dominates or is equivalent to b).

The dis-concordance index is determined by first defining a veto threshold for each criterion i , say t_i , such that a cannot outrank b if

the score for b on any criterion exceeds the score for a on that criterion by an amount equal or greater than its threshold. That is:

$$D(a, b) = \begin{cases} 1 & \text{if } z_i(b) - z_i(a) > t_i \text{ for any } i \\ 0 & \text{otherwise} \end{cases} \quad (5.8)$$

The concordance and dis-concordance indices for each pair of options can be used to build an outranking relation. This process starts by specifying concordance and dis-concordance *thresholds*, C^* and D^* respectively. Alternative a is defined as outranking alternative b if the concordance coefficient $C(a, b)$ is greater than or equal to the threshold C^* and the dis-concordance coefficient $D(a, b)$ is less than or equal to D^* . The values of C^* and D^* are specified for a particular outranking relation and they may be varied to give more or less severe outranking relations - the higher the value of C^* and the lower the value of D^* , the more severe the outranking relation, that is the more difficult it is for one alternative to outrank another. If the outranking relation is made too severe, then almost all pairs of alternatives will be deemed to be 'incomparable'; while if the outranking relation is not severe enough then too many alternatives will outrank too many others (i.e. most are deemed to be essentially equally good in the light of the current information). Since neither of these outcomes is particularly useful, it is a matter of experimentation to find a C^* large enough (but not too large) and a D^* small enough (but not too small) in order to define an informative and useful outranking relation [Belton and Stewart, 2003, pp. 236-238]. The above process is represented in Figure 13.

The museum's problem revisited

Recall that an architect made three preliminary designs all involving the complete renovation of the existing building:

- A design involving extending the existing building with 11 500 square meters costing € 82 000 000.

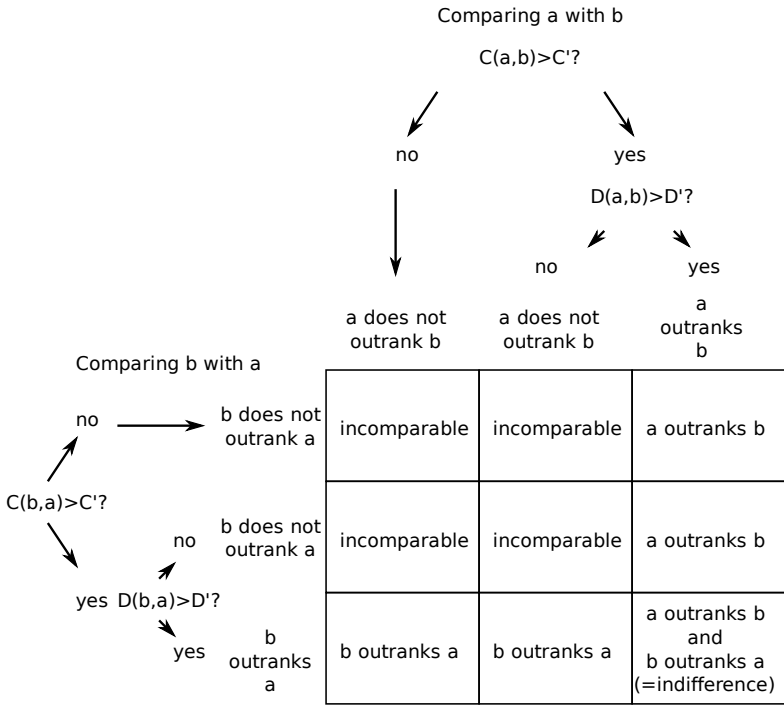


Figure 13: Building an outranking relation.

Alternative	Criterion	
	Expenditure	Centralizing
Design A	VH	VL
Design B	L	Av
Design C	VL	VH
Weights	67	33

Table 16: Scores and weights: museum example (ELECTRE)

Alternative	Alternative		
	Design A	Design B	Design C
Design A	1	0.67	0.67
Design B	0.33	1	0.67
Design C	0.33	0.33	1

Table 17: Concordance indices: museum example (ELECTRE).

- A design involving extending the existing building with 14 000 square meters costing € 95 000 000.
- A design involving extending the existing building with 16 000 square meters costing € 100 000 000.

The municipality (interested in minimizing expenditure) and museum (interested in maximizing centralizing) agree on the preference ratings and weight distribution as shown in Table 16. This table generates the concordance indices shown in Table 17.

Assume that both decision makers agree on a veto threshold of 3, in other words, a cannot outrank b if the score for b on any criterion exceeds the score for a on that criterion by an amount equal or greater than 3. This results in the discordance indices shown in Table 18

Alternative	Alternative		
	Design A	Design B	Design C
Design A	0	0	1
Design B	1	0	0
Design C	1	0	0

Table 18: Disconcordance indices: museum example (ELECTRE).

in which an entry of 1 in cell (a,b) indicates that the alternative in row a cannot outrank the alternative in column b .

The outranking relation can then be built using the concordance and dis-concordance indices. When setting $C^* = 0.3$ and $D^* = 0.8$ design A outranks design B but design B outranks design C while design C outranks design B. To resolve this we can strengthen the outranking relation by increasing D^* to 0.5. As a result design A still outranks design B but again, design B outranks design C while design C outranks design B. To strengthen the outranking relation even more we can set C^* to 0.4 which results in design A outranking design B and design B outranking design C.

Discussion

The usefulness of the outranking relation completely depends on the values chosen for the concordance and dis-concordance thresholds, C^* and D^* , respectively. The experimental process to find suitable values for these is completely arbitrary.

As the choice of procedure to make use of the outranking relation depends on the nature of the problem, this can only mean that the output for decision aid or support also varies according to the procedure used.

Most importantly, all members of the ELECTRE family suffer from a fundamental problem. The verbal scales used at most supply ordinal information. Ordinal scales do not enable the mathematical opera-

tions of addition and multiplication. As all members of the ELECTRE family carry out these operation on ordinal scales, their output is meaningless.

Conclusion

The main problem with all members of the ELECTRE family lies in the process of eliciting scores where the mathematical operations of addition and multiplication are performed on numbers obtained using either a partial value function, a non-physical scale or by direct rating of the alternatives. Nowhere in the literature on ELECTRE is there any proof that this is allowed.

5.3.2 PROMETHEE

The PROMETHEE method, developed by Brans and colleagues (Brans et al. [1984]; Brans and Vincke [1985]; Brans et al. [1986]) is another outranking approach.

Initial steps

The starting point, as with ELECTRE, is a decision matrix describing the performance of the alternatives to be evaluated with respect to identified criteria.

Eliciting scores

Rating each alternative on each criterion can be done by defining a partial value function, a non-physical value scale or by directly rating the alternatives.

Eliciting weights

As with ELECTRE, criteria are weighed such that a higher value indicates a greater 'importance'.

Determining overall evaluations

Belton and Stewart [2003, pp. 252-255] describe how the overall evaluations are determined: The next step in the PROMETHEE method is to define what Brans calls a preference function for each criterion. Rather than the specification of indifference and preference thresholds the *intensity of preference* for option a over b , $P_i(a, b)$, is described by a function of the difference in performance levels on that criterion for the two alternatives, i.e. on $z_i(a) - z_i(b)$. Note that this terminological concept of intensity of preference is undefined. The preference function takes on values between 0 and 1 and a number of suggested shapes are illustrated in Figure 14. The decision maker selects the desired shape of function, and specifies any parameters that are then needed. As shown in the figure, the functions are symmetric around a difference of zero. For positive differences ($z_i(a) > z_i(b)$), the function value gives $P_i(a, b)$ while $P_i(b, a) = 0$. Conversely, when $z_i(a) < z_i(b)$, $P_i(a, b) = 0$ and the chosen function from Figure 14 generates the required value for $P_i(b, a)$.

The next step is to determine a preference index for a over b as a measure of support for the hypothesis that a is preferred over b . The preference index $P(a, b)$ is defined as a weighted average of preferences on the individual criteria:

$$P(a, b) = \frac{\sum_{i=1}^m w_i P_i(a, b)}{\sum_{i=1}^m w_i} \quad (5.9)$$

The preference index is thus also defined to be between 0 and 1. As with the ELECTRE methods, the weights do not represent scaling factors, but some notation of global importance.

The preference index thus defines a valued outranking relation, which, as in ELECTRE III, is exploited to determine an ordering of the alternatives. Two further indices, the positive outranking flow, and the negative outranking flow are defined as follows:

The positive outranking flow for a :

$$Q^+(a) = \sum_{b \neq a} P(a, b) \quad (5.10)$$

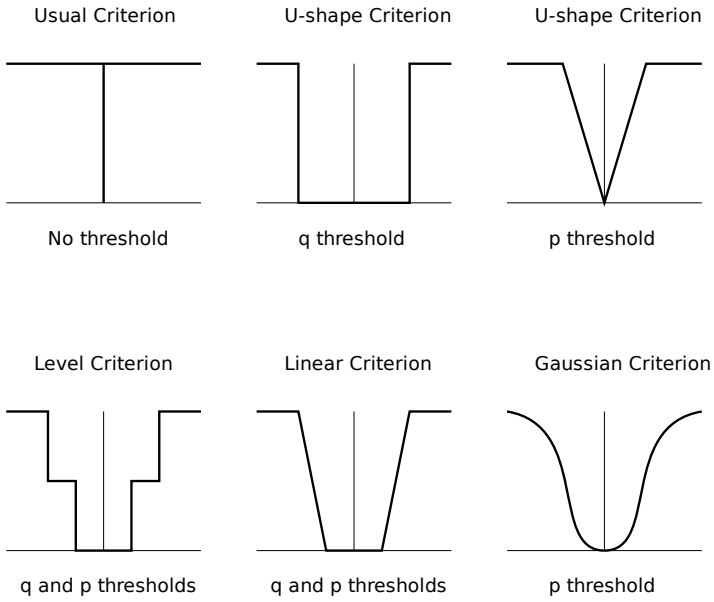


Figure 14: PROMETHEE preference functions.

The negative outranking flow for a :

$$Q^-(a) = \sum_{b \neq a} P(b, a) \quad (5.11)$$

where the sums are taken over all alternatives under consideration.

The positive outranking flow expresses the extent to which a outranks all other options. The negative outranking flow expresses the extent to which a is outranked by all other options. A complete pre-order of alternatives is derived from the 'net flow' for each alternative defined as:

$$Q(a) = Q^+(a) - Q^-(a) \quad (5.12)$$

Then a outranks b if $Q(a) > Q(b)$, with indifference if $Q(a) = Q(b)$.

The museum's problem is used again to illustrate the above procedure.

The museum's problem revisited

Recall that the architect made three preliminary designs all involving the complete renovation of the existing building:

- A design involving extending the existing building with 11 500 square meters costing € 82 000 000.
- A design involving extending the existing building with 14 000 square meters costing € 95 000 000.
- A design involving extending the existing building with 16 000 square meters costing € 100 000 000.

The municipality (interested in minimizing expenditure) and museum (interested in maximizing centralizing) agree on the preference rating and weight distribution as shown in Table 19. They both use the linear criterion function. This information is then used to generate the preference indices as shown Table 20.

Alternative	Criterion	
	Expenditure	Centralizing
Design A	100	0
Design B	28	56
Design C	0	100
Weights	67	33

Table 19: Scores and weights: museum example (PROMETHEE)

Alternative	Alternative		
	Design A	Design B	Design C
Design A	0	0.67	0.67
Design B	0.33	0	0.62
Design C	0.33	0.33	0

Table 20: Preference indices: museum example (PROMETHEE).

Alternative	Flows		
	Leaving flow	Entering flow	Net flow
Design A	1.340	0.660	0.680
Design B	0.950	1.000	-0.050
Design C	0.660	1.290	-0.630

Table 21: Leaving, entering and net flows: museum example (PROMETHEE).

The leaving and entering flows are shown in Table 21. As an example: the leaving flow for design A is the sum of the preference indices of the first row of this table ($0 + 0.67 + 0.67 = 1.34$). The entering flow is the sum of the preference indices of the first column of this table ($0 + 0.33 + 0.33 = 0.66$).

The resulting outranking relationships show that design A is preferred over design B which is in turn preferred over design C.

Discussion

The PROMETHEE method combines the simplicity and transparency of the early ELECTRE methods with some of the increased sophistication of preference modeling incorporated in ELECTRE III. However, in common with ELECTRE III, the distillation process can yield results which are counter intuitive (see, for example, Roy and Bouyssou [1993]; de Keyser and Peeters [1996]).

More fundamental, there is no proof in literature that the mathematical operations of addition and multiplication apply to the scales used by PROMETHEE to rate alternatives. In other words, the methodology has no mathematical foundation.

Conclusion

The conclusion that PROMETHEE sometimes produces counter intuitive results is related to its lack of a mathematical foundation which, as

was the case with members of the ELECTRE family of methods, renders its output meaningless.

5.3.3 *Conclusions on outranking methods*

The major drawbacks of outranking methods arise from the many rather non-intuitive inputs that are required, such as: concordance and dis-concordance threshold levels; indifference, preference and veto thresholds; and the preference functions of PROMETHEE. Such drawbacks assume particular importance when efforts are made to extend the outranking methods to produce explicit preference orderings over the full set of alternatives. The impacts of the various inputs are difficult to appreciate intuitively, and the algorithms themselves tend to be complicated for decision makers fully to understand. As a result of this, results can be counter-intuitive, with unexpected changes in rank orderings arising in response to changes in the threshold levels or to addition or deletion of alternatives [Belton and Stewart, 2003, pp. 258-259].

More fundamentally, none of the scales used by these methods enable the operations of addition and multiplication which means that they all produce meaningless numbers.

5.4 CONCLUSIONS

As mentioned in Chapter 2, the main question to solve is: how to select the design that meets all decision makers' interests best taking into account each design's attributes.

We therefore need a methodology that: 1) extends to group decision making and 2) has a mathematical foundation for measuring preference.

This survey has shown that none of the discussed goal, aspiration or reference level methodologies extends to group decision making. This leaves us with value measurement and outranking methodologies which, with the exception of PFM as shown by Barzilai, all lack a correct mathematical foundation.

Part II
SYNTHESIS

Design is, for a large part, a process of making choices. Choosing between the possible options for a given design question is fundamentally an issue of preference. As such, methods of preference measurement and preference-based selection should be applicable to design. In this chapter, we propose a design methodology in which design choices are preference-based.

When faced with group design decision making problems, our Open Design group tries to solve these using Linear Programming (LP) models. However, fundamental limitations are encountered in using these models, as described in Chapter 5: 1) only allowing single objective optimization thus satisfying only *one* interest of *one* decision maker thereby not extending to group decision making, 2) the constraints divide all possible solutions into either feasible or infeasible ones; black or white, no gray which could eventually be acceptable to decision makers thereby poorly reflecting a decision maker's preferences.

This chapter proposes a Preference-Based Design (PBD) methodology which removes these limitations. Its cornerstones are: 1) using only the design optimization framework of LP, and 2) using Preference Function Modeling (PFM) to incorporate preferences. The methodology will be illustrated using the North Sea International Airport case – the dilemma of Amsterdam Airport Schiphol of whether or not to move the airport to an artificial island in the North Sea (see 6.3.1).

6.1 THE PREFERENCE-BASED DESIGN CONCEPT

In order for the operations of addition and multiplication to be applicable on preference scale values the mathematical system must be a one-dimensional affine space. Addition and multiplication are not applicable in any measurement model based on axioms other

than those of a one-dimensional affine space. As a result, any decision methodology where addition and multiplication are used on non-affine scale values, has no mathematical foundation. Not only are all the models in the classical theory (see [Krantz et al., 1971] and [Roberts, 1979]) non-affine but since 2005, Barzilai's challenge to the community failed to produce a single such model [Barzilai, to appear June 2010, §10].

The survey of Chapter 5 shows that the only Multiple Criteria Decision Analysis (MCDA) evaluation methodology enabling the operations of addition and multiplication on preference scale values is PFM. All other evaluation methodologies lack such a foundation. However, PFM is an evaluation methodology, whereas in the domain of architecture a design methodology is needed.

Of the three design methodologies surveyed in Chapter 5 – LP, Goal Programming (GP), and Linear Multi Objective Programming (LMOP) – none extend naturally to group decision making. This is a pre-requisite for a design methodology in architecture. However, the optimization framework of these methodologies, i.e. a set of constraints defining a set of feasible alternatives, can be used to express the interests or criteria of each decision maker involved in the design process.

The concept of PBD is to 1) use constraints for expressing each decision maker's interests or criteria in terms of allowed decision variables value ranges and relationships between decision variables in order to define all feasible alternatives and 2) use PFM to select from these the alternative with the highest overall preference rating. A design alternative is then a combination of decision variable values and its feasibility is defined by the constraints.

To illustrate this, consider the design process of a new office building. The new owner is interested in the investment costs (decision variable x_1) while the tenant is interested in the rent (decision variable x_2). Assume that the new owner wants to have an initial yield (first year rent divided by the investment costs) of at least 8% and the tenant is not willing to pay more than \$ 150 per square meter. Assume that the investment costs are at least \$1 400 per square meter and

that the tenant needs 1 000 square meters. The set of feasible design alternatives is then defined by the following design constraints:

$$\begin{array}{rclcl}
 -0.08x_1 & + & 1\,000x_2 & \geq & 0 \\
 & & x_2 & \leq & 150 \\
 0.001x_1 & & & \geq & 1\,400
 \end{array}$$

The combination of decision variables $x_1 = 1\,400\,000$ and $x_2 = 112$ is an example of a feasible design alternatives as it satisfies all constraints. For this problem the combination $x_1 = 1\,875\,000$ and $x_2 = 150$ is another example of a feasible design alternative.

6.2 PROBLEMS AND SOLUTIONS

The first problem we encounter is that, if the alternative designs are defined as combinations of decision variable values and their feasibility by the constraints then, providing the feasible set is non-empty, the number of feasible alternatives will be infinitely large¹ and evaluating each by exhaustive enumeration using a computer will be fruitless.

6.2.1 Discretization of the solution space

One solution to limit the number of alternatives to evaluate is to make use of discretization. For instance, when designing a new office building the new owner may wish that it is at least 250 meters from the nearest railway station but no more than 1 000 meters. Less than 250 meters and the noise of passing trains is considered too loud, more than 1 000 meters and it will take personnel too long to still be able to walk to the station from the office within an acceptable amount of time. Assume that these are the only constraints for this design problem, then the feasible set can be represented by a line

¹ Unless the constraints define just one feasible point which is uncommon in real life situations.

segment containing an infinite number of solutions with regards to the distance from the railway station. By means of discretization the segment could be divided into, for instance, three segments and the set of feasible solutions would be limited to 250/500/750/1 000 meters.

6.2.2 *Introduction of synthetic alternatives*

Although, by making use of discretization, the number of alternatives will be finite, the constraints divide all possible solutions into two groups: feasible or infeasible. Solutions are not further distinguished which means that we only know whether an alternative is feasible or infeasible. In other words, there is no way of choosing from the feasible alternatives the most preferred one which is a requirement the PBD. To resolve this problem, information needs to be provided on the preference rating associated with different decision variable values. Going back to the previous example, without knowing the preference rating associated with each of the four feasible solutions (distances) it will be impossible to choose between them.

Recall the process of utilizing PFM (single decision maker):

1. Specify the alternatives.
2. Specify the decision maker's criteria tree.
3. Rate the decision maker's preferences for each alternative against each leaf criterion as follows:
 - a) For each criterion establish reference alternatives.
 - b) Rate the preference for the other alternatives relative to these reference alternatives on the scale established.
4. To each leaf criterion assign decision maker's weight.
5. Use the PFM algorithm to yield an overall preference scale.

Step 3 in the above procedure requires the decision maker to rate an alternative against different criteria. This is what makes PFM an

evaluation methodology as the alternatives to evaluate exist, in the sense that they can be judged on different criteria or properties as a whole. In the domain of architecture, however, a design methodology is needed, where the design alternatives are not known *a priori* and can therefore not be judged as a whole.

The problem of not knowing the alternatives in advance is solved by introducing *synthetic alternatives*. A synthetic alternative is an alternative associated with a value for a *single* decision variable value, regardless of other decision variables and regardless of its feasibility. An example of a synthetic alternative is an office building having five stories, regardless of other decision variables and regardless of its feasibility, given that one of the decision makers is interested in the criterion or decision variable 'number of stories'. Another synthetic alternative could be an office building having ten stories. Of course other decision variables – criteria – are likely to also play a role. For example, if another decision maker is interested in the decision variable investment costs then a building costing \$1 000 000 would also be a synthetic alternative as would be a building costing \$1 250 000.

Every decision maker needs to rate each decision variable on *at least* three values as pairwise comparisons cannot be used to construct preference scales where the operations of addition and multiplication are enabled as explained in Chapter 3.

The three design variable values are in essence value judgments of the decision maker and correspond, to some extent, to the quantitative goals of goal programming. However, in goal programming the decision maker needs to state only one value for each goal whereas, in the PBD procedure, the decision maker needs to rate at least three values. In our experiments [van Gunsteren, 2005] we call these ideal, acceptable and walk-out values and are usually based on both the decision maker's preference and experience.

Step 3 of the PBD procedure will thus be as follows:

Rate the decision maker's preferences for at least three values for each decision variable as follows:

- a) For each decision variable establish (synthetic) reference alternatives.

- b) Rate the preference for alternatives associated with the other decision variable values relative to these reference alternatives on the scale established.

Going back to the railway station example, the owner may define a bottom synthetic alternative (regardless of its feasibility) associated with the value for the decision variable (or criterion) 'distance' of 250 meters (too close) at 0. Another synthetic alternative associated with 1 000 meters (too far away) is also rated at 0. The owner could then define a top synthetic alternative (again, regardless of its feasibility) associated with a decision variable value of 500 meters at 100. The other synthetic alternative associated with 750 meters can then be rated on the scale established.

Step 1 and 2 in PFM's procedure also need to be modified as the alternatives are defined as combinations of decision variable values and not known *a priori*. So, instead of specifying the alternatives and criteria we need the decision maker to specify in which decision variable(s) he/she is interested including allowed ranges.

As decision variables in the PBD procedure are considered to be synonymous to criteria each decision maker has to assign weights to decision variables.

The way in which decision variables relate to each other determine the design constraints. These determine whether or not an alternative, as defined as a combination of decision variable values, is feasible. Only feasible alternatives are evaluated using the PFM algorithm to yield an overall preference scale of all feasible alternatives.

Given the design variables the decision makers are interested in and allowed values (at least three per design variable) all possible combinations constitute alternatives and are generated using a computer procedure. The total number of alternatives generated equals the number of design variable values the decision makers are interested in to the power of the number of design variables the decision makers are interested in. Within this procedure they are evaluated on whether they meet the design constraints so that only feasible alternatives combinations can be filtered out which are subsequently fed to the PFM software to yield an overall preference scale.

The above adaptation leads to the following PBD procedure (note that the numbering has changed as step 1 and 2 of the PFM procedure have merged into one step):

1. Specify the decision variable(s) the decision maker is interested in.
2. Rate the decision maker's preferences for at least three values for each decision variable as follows:
 - a) For each decision variable establish (synthetic) reference alternatives.
 - b) Rate the preference for alternatives associated with the other decision variable values relative to these reference alternatives on the scale established.
3. To each decision variable assign decision maker's weight.
4. Determine the design constraints.
5. Combine decision variable values to generate design alternatives and use the design constraints to test their feasibility.
6. Use the PFM algorithm to yield an overall preference scale of all feasible alternatives.

6.3 ILLUSTRATION: SCHIPHOL'S DILEMMA

To illustrate the PBD methodology, the design problem of the extension of Amsterdam Airport Schiphol is used. After a short introduction to Schiphol's dilemma each step of the PBD procedure is described.

6.3.1 *Schiphol's dilemma*

The dilemma for Schiphol is the following [van Gunsteren, 2005]:

- To maintain its position as a main port, the number of flight movements per year should be above a certain threshold.



Figure 15: North Sea International Airport as envisioned by Royal Haskoning & Van Oord in 2008.

- To keep the environmental effects within acceptable limits, in particular noise hindrance but also air pollution, the number of flight movements should be kept below a certain level.

This dilemma could be resolved by the North Sea Island option: moving the take-off and landing of airplanes to an artificial island in the North Sea which is connected by a train shuttle to the present airport. Preliminary studies of such an artificial island by Royal Haskoning & Van Oord are shown in Figure 15 and Figure 16.

Van Gunsteren solved this problem using the constraint method within Open Design methodology. As discussed in Chapter 5, this process has significant arbitrary elements in the sense that its final solution relies on unstructured negotiation.

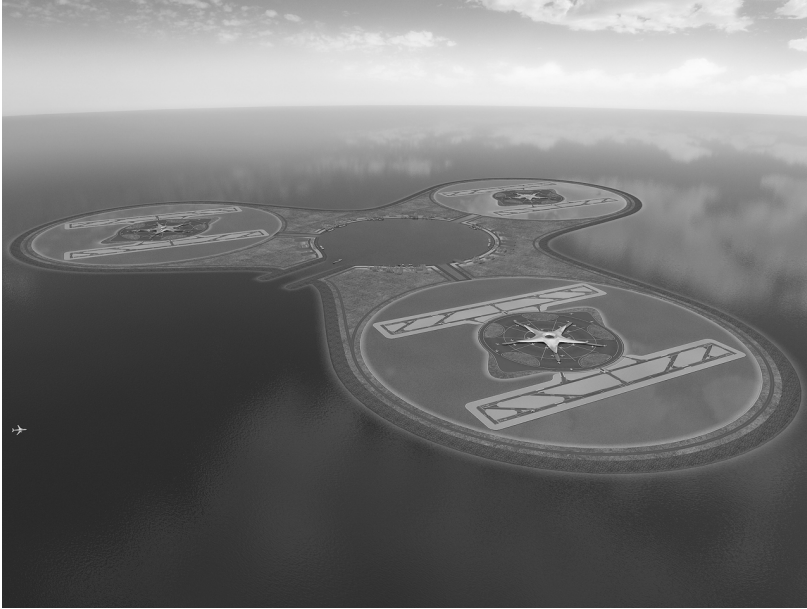


Figure 16: North Sea International Airport as envisioned by Royal Haskoning & Van Oord in 2008, close-up.

6.3.2 Using PBD methodology to solve this problem

To test the PBD methodology an experiment was carried out where colleague experts from our university were asked to play the role of the decision makers for this design problem (Workshop, July, 2007).

Step 1: Specify the decision variables the decision maker is interested in

The following decision makers are identified along with the decision variables they are interested in:

- The Ministry of Finance is interested in the investment (decision variable i in billion dollars).
- The airlines are interested in the time (decision variable t in hours) passengers would have to spend in the shuttle.
- The Ministry of Environment is interested in the distance (decision variable d in kilometers) between the island and the shore.
- The airport is interested in the number of flight movements (decision variable f in 100k flight movements).

Step 2: Rate the decision maker's preference for at least three values for each decision variable value

The Ministry of Finance rates a (synthetic) alternative that would cost 15 billion dollars at 100 and an alternative that would cost 40 billion dollars at 0. A third alternative costing 20 billion dollars is rated at 20.

The airlines rate an alternative that requires passengers to spend 0.5 hours in the shuttle at 100 and an alternative that requires them to spend 0.9 hours at 0. A third alternative that would require them to spend 0.7 hours is rated at 45.

The Ministry of Environment rates an alternative that has a distance of 40 kilometers between the island and the shore at 100 and an alternative that has a distance of 20 kilometers at 0. A third alternative that has a distance of 30 kilometers is rated at 70.

The airport rates an alternative with 10 ×100k flight movements at 100 and an alternative with 6 ×100k flight movements at 0. A third alternative with 8 ×100k flight movements is rated at 20.

Step 3: To each decision variable assign decision maker's weights

For this experiment all decision variables are weighted equally.

Step 4: Determine the design constraints

For this experiment two design constraints were used. The first relates the distance between the island and the shore (decision variable d in kilometers) and the time passengers have to spend in the shuttle (decision variable t in hours) using a postulated shuttle speed of 120 kilometers per hour:

$$\frac{d}{120} \leq t \quad (6.1)$$

The second design constraint relates the number of flight movements (decision variable f in 100k flight movements), the distance between the island and the shore (decision variable d in kilometers) and the investment (decision variable i in billion dollars). Given that building an island for 600k flight movements at a distance of 10 kilometer from the shore would cost 15 billion dollars, the investment increases with 0.15 billion dollars per 100k flight movements more than 600k and increases with 0.2 billion dollars per kilometer more distance from the shore than 10 kilometer:

$$15 + 0.15(f - 6) + 0.2(d - 10) \leq i \quad (6.2)$$

Step 5: Combine decision variable values to generate design alternatives and use the design constraints to test their feasibility

All alternatives were analyzed on their feasibility and the feasible ones were fed into the PFM software. For instance, the combination of $i = 15$, $t = 0.5$, $d = 40$ and $f = 10$ satisfies the first design constraint

as it will take the shuttle 0.33 hours to travel 40 kilometers which is less than 0.5 hours:

$$\frac{40}{120} \leq 0.5 \quad (6.3)$$

However, it fails the second design constraint as it will cost more than 15 billion dollars to build the island at 40 kilometers from the shore while supporting $10 \times 100k$ flight movements:

$$15 + 0.15(10 - 6) + 0.2(40 - 10) \not\leq 15 \quad (6.4)$$

Step 6: Use the PFM algorithm to yield an overall preference scale of all feasible alternatives

Table 22 shows the top 10 (out of 36) feasible alternatives (combinations of decision variable values that satisfy the design conditions), ordered on the overall preference rating yielded by the PFM algorithm. From this table it can be concluded that alternative 27 requiring the passengers to spend 0.5 hours in the shuttle at a distance of 40 kilometers from the shore with $10 \times 100k$ flight movements costing 40 billion dollars is both feasible and the most preferred alternative.

6.4 REMAINING PROBLEMS AND SOLUTIONS

Table 22 also shows the drawback of using only three discrete values for each decision variable. Alternatives 27 and 24 differ only with respect to the value of decision variable d (40 and 30 kilometers respectively) and the associated preference rating. The investment is the same. One would expect that the real investment associated with alternative 24 would be lower, because building the island 10 kilometers closer to the shore would reduce the construction costs, which in turn might promote alternative 24 to first place. In other words, why is alternative 24 combined with the value of 40 billion dollars in second place, and not the equivalent alternative with the value of 20 billion dollars for the investment?

Alt.	Decision variable				Pref.
	<i>i</i>	<i>t</i>	<i>d</i>	<i>f</i>	
27	40	0.5	40	10	59
24	40	0.5	30	10	57
18	40	0.7	40	10	56
26	40	0.5	40	8	54
15	40	0.7	30	10	51
21	40	0.5	20	10	50
25	40	0.5	40	6	50
36	40	0.5	20	10	50
9	40	0.9	40	10	50
23	40	0.5	30	8	49

Table 22: Top 10 feasible alternatives with associated decision variable values and preference ratings.

Equation 6.2 can be used to determine the investment as a function of the number of flight movements and distance. For alternative 24 the investment would be:

$$i = 15 + 0.15(10 - 6) + 0.2(30 - 10) = 23.6 \quad (6.5)$$

As a result the values $f = 10$ and $d = 30$ cannot be combined with the value of 20 billion dollars for the investment as 23.6 billion dollars is larger than 20 billion dollars. In other words the combination $f = 10, d = 30, i = 20$ fails the second design constraint.

This is in essence a resolution problem and can be dealt with by:

- Asking the decision makers to rate their preference for more decision variable values.
- Deriving a curve that relates the decision maker's preference ratings to decision variable values.

The first approach is an iterative fine tuning approach requiring the decision makers to invest time in finding a satisfactory combination of decision variable values. The second approach aims to find the satisfactory combination in one go, but takes up more computer time.

6.4.1 *Asking decision makers to rate more synthetic alternatives*

This approach requires each decision maker to rate his preference for more decision variable values. For an illustration of this approach see Figure 17 showing three values for the investment decision variable on the x -axis (15/20/40) and associated preference ratings on the y -axis (100/20/0). This defines three coordinates (15, 100), (20, 20) and (40, 0). The effect of asking the decision maker to rate his preference for more decision variable values is shown in Figure 18.

A similar approach is to use a more iterative procedure whereby the outcome of the first preference rating may lead a decision maker to focus on a range of values close to the decision variable value associated with the most preferred alternative. For instance, if the

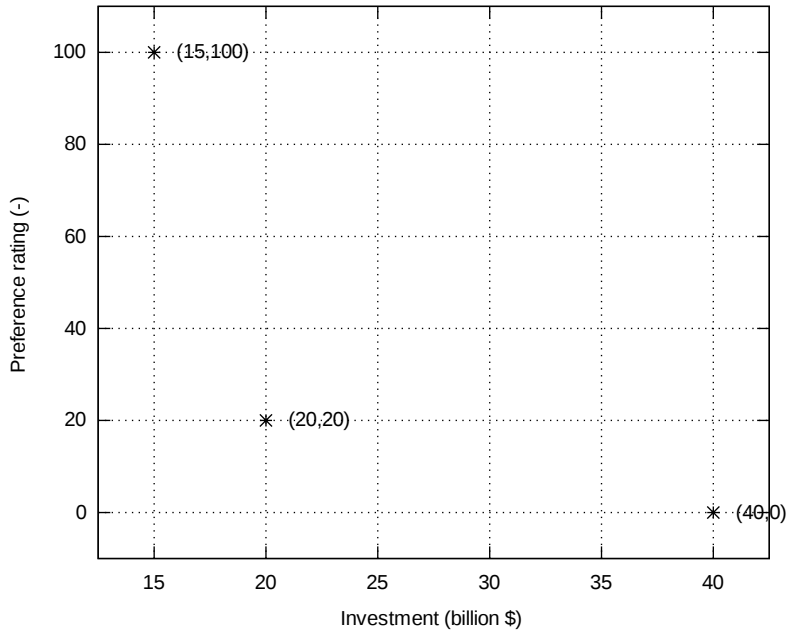


Figure 17: Values for the investment variable and associated preference ratings.

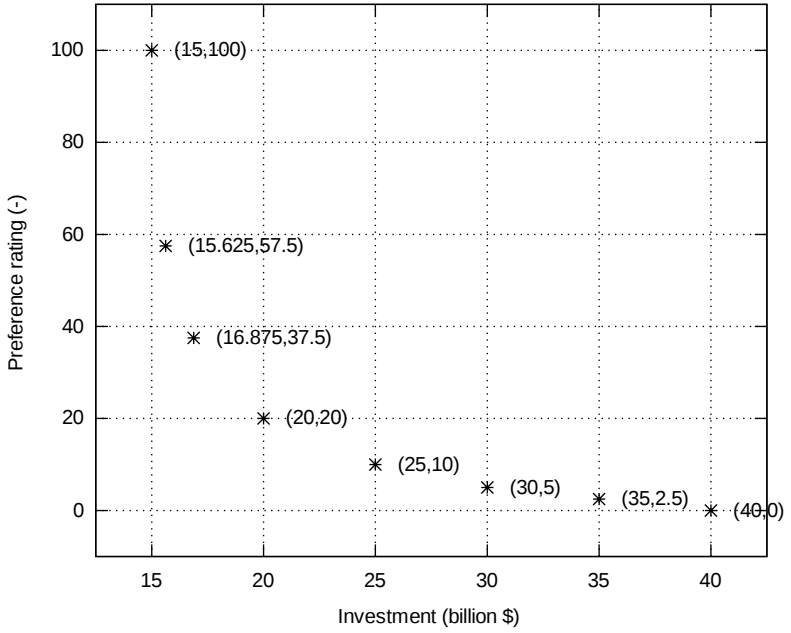


Figure 18: Effect of asking the decision maker to rate more decision variable values.

highest rated alternative is associated with an investment of 20 billion dollars, the decision maker could rate his preference for alternatives associated with 15 and 25 billion dollars. Other decision makers could decide to do the same after which the overall preference rating can be determined again.

In this way, only the relevant part of the feasible set need be explored to a depth/resolution necessary to find a satisfying solution.

6.4.2 *Deriving a curve corresponding with the decision maker's preference ratings for three decision variable values*

The time required by each decision maker to rate his preference for more decision variable values and the time required by the iterative procedure is a serious drawback. This can be removed by means of curve fitting: to fit a curve through the decision variable value - preference rating coordinates. Curves commonly used in Operations Research (OR) for this purpose are Lagrange polynomials and cubic spline curves.

Finding the Lagrange curve relating the decision maker's preference ratings to decision variable values

The Lagrange interpolating polynomial is the polynomial $P(x)$ of degree $\leq (n - 1)$ that passes through the n points $(x_1, y_1 = f(x_1))$, $(x_2, y_2 = f(x_2))$, ..., $(x_n, y_n = f(x_n))$, and is given by:

$$P(x) = \sum_{j=1}^n P_j(x) \quad (6.6)$$

where

$$P_j(x) = y_j \prod_{k=1, k \neq j}^n \frac{x - x_k}{x_j - x_k} \quad (6.7)$$

The formula was first published by Waring [1779], rediscovered by Euler in 1783, and published by Lagrange in 1795.

The quadratic form of the Lagrange polynomial interpolates three points, (x_0, y_0) , (x_1, y_1) , and (x_2, y_2) . Equations 6.6 and 6.7 can be rewritten explicitly for $n = 3$ points:

$$\begin{aligned}
 P(x) &= \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}y_1 \\
 &+ \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}y_2 \\
 &+ \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}y_3
 \end{aligned} \tag{6.8}$$

Equation 6.8 can be used to find the curve defined by the three decision variable value - preference rating coordinates for the investment variable, $(15, 100)$, $(20, 20)$, and $(40, 0)$ as shown in Figure 19. Lagrange polynomials oscillate between their roots (knots), therefore they can take negative values. In this case, the preference rating would be negative for investment values between 22 and 40 billion dollars which is not what the decision maker intended, making the Lagrange curve not a suitable representation of the decision variable value - preference rating curve.

Finding the cubic spline corresponding with the decision maker's preference ratings for possible decision variable values

Another option is to use cubic spline interpolation. A cubic spline is a function constructed by piecing together cubic polynomials $p_k(x)$ on different intervals $[x_k, x_{k+1}]$. It has the form:

$$f(x) = \begin{cases} p_1(x) & x_1 \leq x < x_2 \\ p_2(x) & x_2 \leq x < x_3 \\ \vdots & \vdots \\ p_{m-1}(x) & x_{m-1} \leq x \leq x_m \end{cases} \tag{6.9}$$

Consider points $(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)$, with $x_1 < x_2 < \dots < x_m$. A cubic spline is constructed by interpolating a cubic polynomial

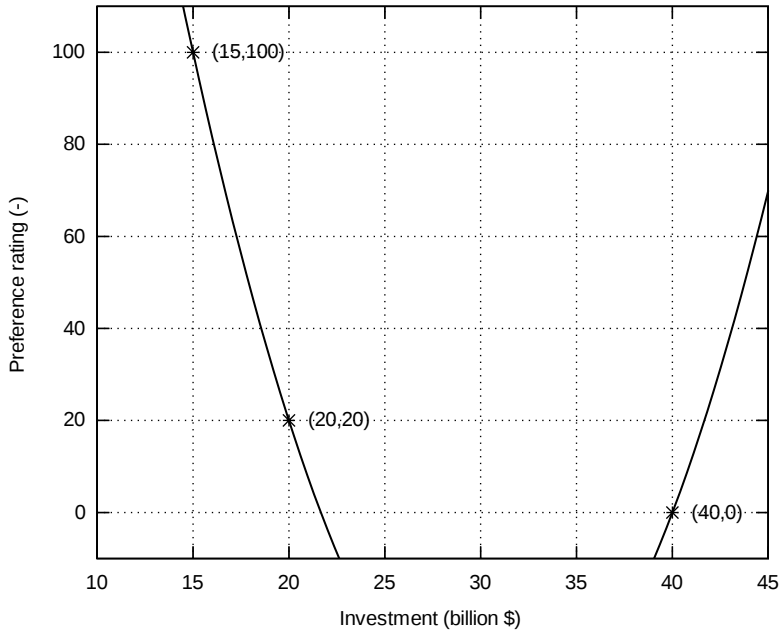


Figure 19: The Lagrange curve defined by three decision variable value - preference rating coordinates for the investment variable.

p_k between each pair of consecutive points (x_k, y_k) and (x_{k+1}, y_{k+1}) according to the following constraints:

Each polynomial passes through its respective end points:

$$p_k(x_k) = y_k \text{ and } p_k(x_{k+1}) = y_{k+1} \quad (6.10)$$

First derivatives match at interior points:

$$\frac{d}{dx}p_k(x_{k+1}) = \frac{d}{dx}p_{k+1}(x_{k+1}) \quad (6.11)$$

Second derivatives match at interior points:

$$\frac{d^2}{dx^2}p_k(x_{k+1}) = \frac{d^2}{dx^2}p_{k+1}(x_{k+1}) \quad (6.12)$$

Second derivatives vanish at the end points:

$$\frac{d^2}{dx^2}p_1(x_1) = 0 \text{ and } \frac{d^2}{dx^2}p_{m-1}(x_m) = 0 \quad (6.13)$$

The above conditions specify a system of linear equations that can be solved for the cubic spline.

The cubic spline defined by the three decision variable value - preference rating coordinates for the investment variable, $(15, 100)$, $(20, 20)$, and $(40, 0)$ is shown in Figure 20. As with the Lagrange polynomials, cubic splines oscillate between their roots (knots), and can also take negative values. It can be concluded that the cubic spline is also not a suitable representation of the curve.

6.4.3 *Letting the decision maker construct the curve relating preference ratings to decision variable values*

The proposed solution to find intermediate coordinates, is by asking the decision maker to construct a curve to approximate the relationship between preference ratings and decision variable values. To avoid

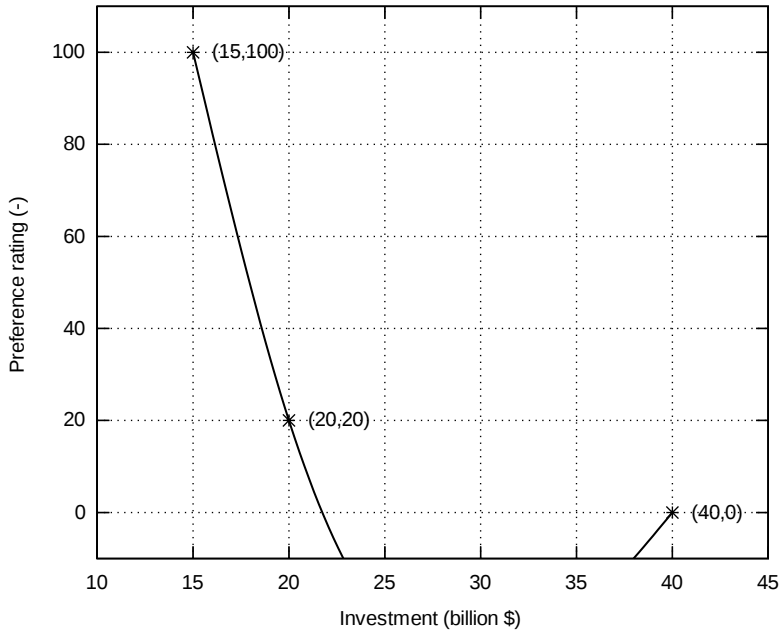


Figure 20: The cubic spline defined by three decision variable value - preference rating coordinates for the investment variable.

negative preferences, the derivatives at the endpoints must be controllable by the decision maker. A Bézier curve is used for this purpose, as this is also considered an intuitive way of constructing a curve.

Note the difference between this approach and the before mentioned curve fitting approaches. Instead of *fitting* a curve through a set of coordinates, this approach requires the decision maker to actually *construct* the curve by positioning the end points and shaping the curve using its control points.

A Bézier curve in its most common form is a simple cubic equation that can be used in any number of useful ways. Bézier curves were separately and simultaneously developed for car body design at French manufacturers by Pierre Bézier at Renault and Paul de Castel-jau at Citroën. In vector graphics, Bézier curves are an important tool used to model smooth curves that can be scaled indefinitely.

A cubic Bézier curve (see Figure 21) is defined by four points. Two are endpoints: (x_0, y_0) is the origin endpoint, (x_3, y_3) is the destination endpoint. The points (x_1, y_1) and (x_2, y_2) are the control points.

The curve as shown in Figure 21 could be used to define a preference curve where the preference rating (along the y -axis) decreases against the decision variable value (along the x -axis). Allowed decision variable values range between x_0 and x_3 and the preference ratings range between y_0 and y_3 . The control points (x_1, y_1) and (x_2, y_2) are used to control the curve's slope. Note that the coordinates associated with the control points have no meaning in real life, they only serve to shape the curve and do not represent decision variable values and associated preference ratings.

Similarly, Figure 22 shows an example of how a Bézier curve could be used to define a preference curve where the preference rating increases against the decision variable value.

Figure 23 can be used when the most preferred decision variable value lies between allowed decision variable values. An example is the distance of an office to the nearest railway station discussed earlier. In this case coordinate (x_4, y_4) represents the decision variable value that is most preferred and is defined indirectly (not input) by manipulating the two control points.

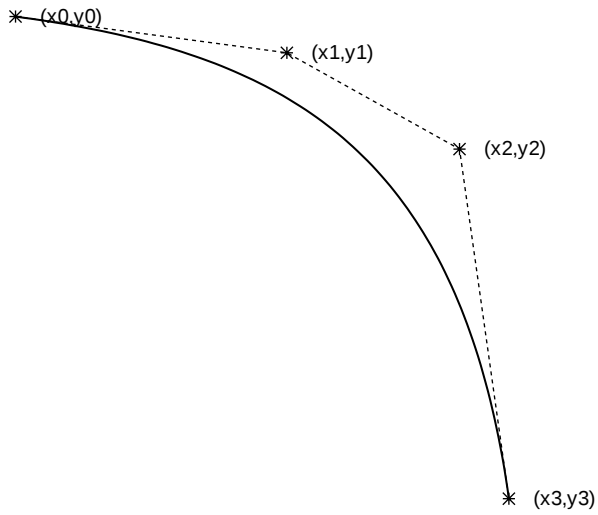


Figure 21: A cubic Bézier curve defined by four points; endpoints (x_0, y_0) , (x_3, y_3) , and control points (x_1, y_1) and (x_2, y_2) .

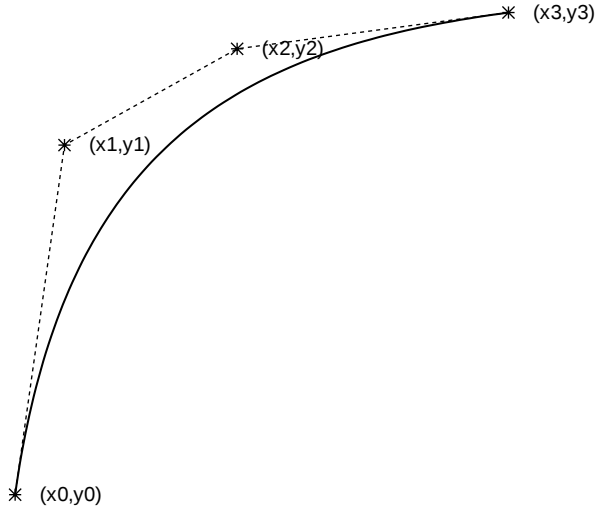


Figure 22: Defining a Bézier curve where the preference rating (y -axis) increases against the decision variable value (x -axis).

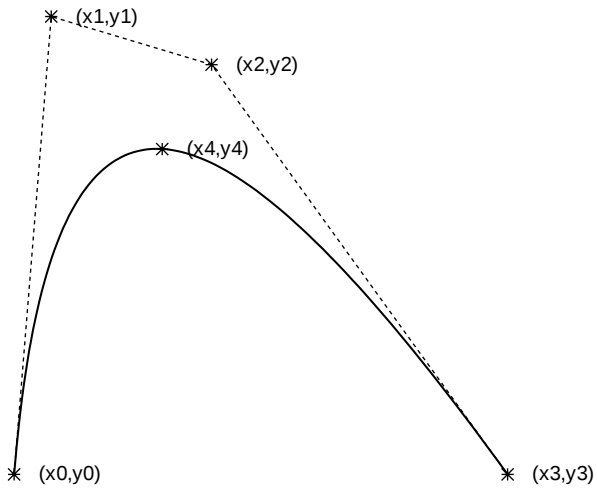


Figure 23: Defining a Bézier curve where the preference rating (y -axis) peaks between allowed the decision variable value (x -axis).

This means that the preference rating step of the PBD procedure changes as follows.

Step 2: Rate the decision maker's preferences for each decision variable by using a Bézier curve

Assume that instead of asking the decision makers to rate their preference for three decision variable values they were asked to construct a Bézier curve to relate preference ratings to the decision variable values.

Figure 24 shows the Bézier curve as defined by the Ministry of Finance, interested in the investment (decision variable i). First the decision maker has to decide on the coordinate of the origin endpoint (15, 100) and the destination endpoint (40, 0). The decision maker then uses the two control points to shape the curve until its slope corresponds to how the decision maker relates decision variable values to preference ratings. The curve shows that the Ministry of Finance's preference rating sharply drops from 100 to 20 as the investment variable increases from 15 to 20 billion dollars.

Figure 25 shows the Bézier curve as defined by the airlines, interested in the travel time (decision variable t). The curve shows that the airlines' preference rating drops almost linearly from 100 to 45 as the travel time variable increases from 0.5 to 0.7 hours.

Figure 26 shows the Bézier curve as defined by the Ministry of Environment, interested in the distance (decision variable d). The curve shows that the Ministry of Environment's preference rating drops rather gradually from 100 to 70 as the distance variable decreases from 40 to 30 kilometers.

Figure 27 shows the Bézier curve as defined by the airport, interested in the flight movements (decision variable f). The curve shows that the airport's preference rating sharply drops from 100 to 20 as the flight movements variable decreases from $10 \times 100k$ to $8 \times 100k$ flight movements.

The information contained in these figures is summarized in Table 23.

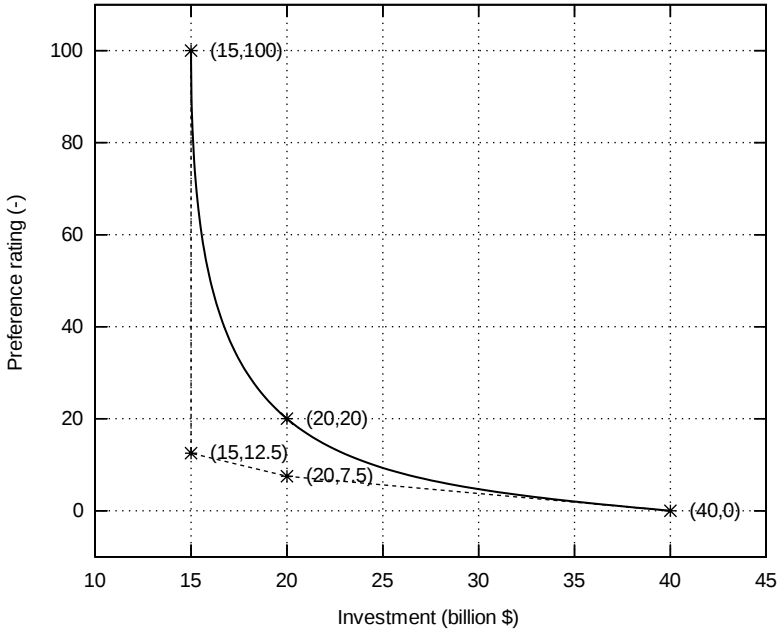


Figure 24: Investment variable value vs. preference rating.

Point	Decision variable			
	<i>i</i>	<i>t</i>	<i>d</i>	<i>f</i>
(x_0, y_0)	(15, 100)	(0.5, 100)	(20, 0)	(6, 0)
(x_1, y_1)	(15, 12.5)	(0.7, 48)	(26.5, 60)	(9, 15)
(x_2, y_2)	(20, 7.5)	(0.7, 40)	(27.5, 67)	(9, 55)
(x_3, y_3)	(40, 0)	(0.9, 0)	(40, 100)	(10, 100)

Table 23: End points (x_0, y_0) and (x_3, y_3) and control points (x_1, y_1) and (x_2, y_2) for all Bézier curves.

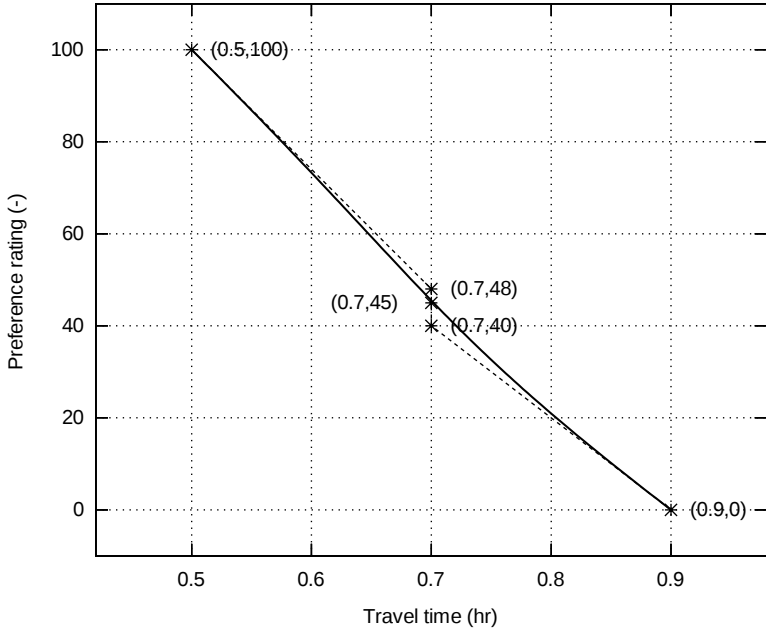


Figure 25: Travel time variable value vs. preference rating.

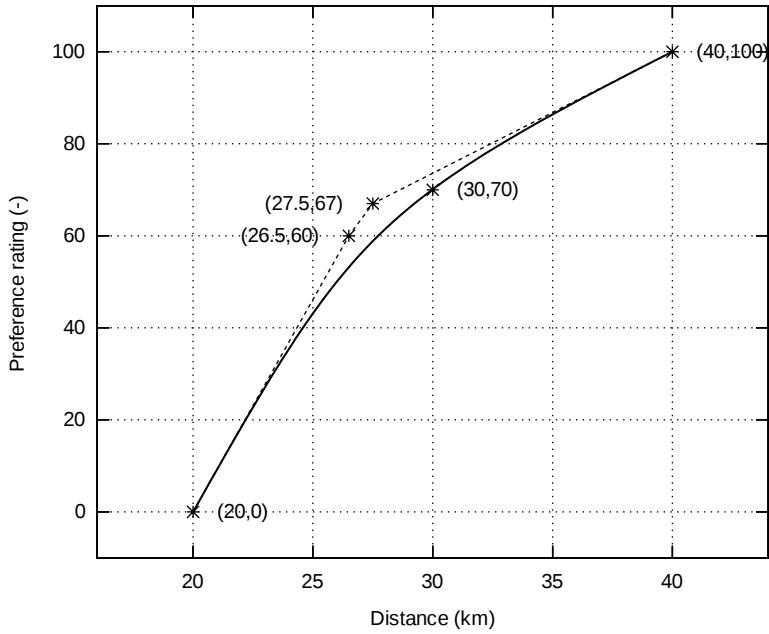


Figure 26: Distance variable value vs. preference rating.

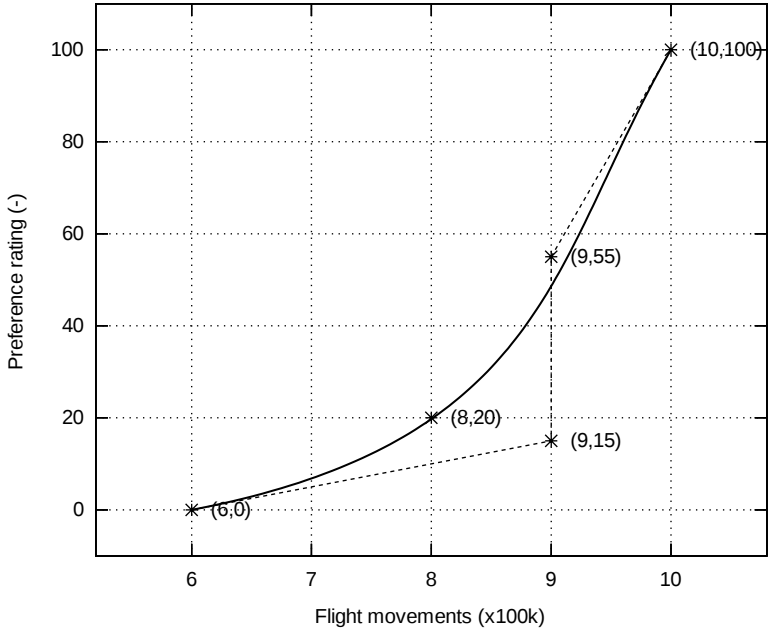


Figure 27: Flight movements variable value vs. preference rating.

A cubic Bézier curve drawn over the interval $0 \leq t \leq 1$ is produced by a relation which has its x and y coordinates, respectively, specified by the cubic polynomial functions:

$$x = x_0(1-t)^3 + 3x_1t(1-t)^2 + 3x_2t^2(1-t) + x_3t^3 \quad (6.14)$$

and

$$y = y_0(1-t)^3 + 3y_1t(1-t)^2 + 3y_2t^2(1-t) + y_3t^3 \quad (6.15)$$

These equations can then be used to find coordinates along the curve using different values for t ($0 \leq t \leq 1$).

For example: the x -coordinate for decision variable i for $t = 0.5$ is given by Equation 6.14 (using the information in Table 23):

$$\begin{aligned} x &= x_0(1-t)^3 + 3x_1t(1-t)^2 + 3x_2t^2(1-t) + x_3t^3 \\ &= 15(1-0.5)^3 + 3 \times 15 \times 0.5(1-0.5)^2 \\ &\quad + 3 \times 20 \times 0.5^2(1-0.5) + 40 \times 0.5^3 \\ &= 20 \end{aligned}$$

Similarly, the y -coordinate is given by Equation 6.15:

$$\begin{aligned} y &= y_0(1-t)^3 + 3y_1t(1-t)^2 + 3y_2t^2(1-t) + y_3t^3 \\ &= 100(1-0.5)^3 + 3 \times 12.5 \times 0.5(1-0.5)^2 \\ &\quad + 3 \times 7.5 \times 0.5^2(1-0.5) + 0 \times 0.5^3 \\ &= 20 \end{aligned}$$

Each Bézier curve can then be divided in, say, ten segments yielding eleven points on each curve by setting t to 0, 0.1, 0.2, 0.3, 0.5, 0.6, 0.7, 0.8, 0.9 and 1. By comparison, in the previous procedure we only had three points. The coordinates obtained using Equation 6.14 and 6.15 for each decision variable are shown in Table 24.

Having increased the number of combinations of decision variable values we can now go to step 5 and 6 of the PBD procedure.

t	Decision variable							
	i		t		d		f	
	x_t	y_t	x_t	y_t	x_t	y_t	x_t	y_t
0	15.00	100.00	0.50	100.00	20.00	0.00	6.00	0.00
0.1	15.16	76.14	0.55	85.64	21.80	16.49	6.81	5.23
0.2	15.68	56.72	0.60	73.47	23.38	30.27	7.47	11.84
0.3	16.62	41.23	0.64	63.03	24.82	41.82	8.00	19.71
0.4	18.04	29.16	0.67	53.86	26.25	51.62	8.42	28.72
0.5	20.00	20.00	0.70	45.50	27.75	60.13	8.60	38.75
0.6	22.56	13.24	0.73	37.50	29.43	67.82	9.02	49.68
0.7	25.78	8.37	0.76	29.41	31.40	75.19	9.26	61.39
0.8	29.72	4.88	0.80	20.77	33.74	82.69	9.49	73.76
0.9	34.44	2.26	0.85	11.12	36.58	90.80	9.73	86.67
1	40.00	0.00	0.90	0.00	40.00	100.00	10.00	100.00

Table 24: Eleven coordinates along all Bézier curves.

Step 5: Combine decision variable values to generate design alternatives and use the design constraints to test their feasibility

The information in Table 24 is used to generate 14 641 possible combinations of decision variable values (the number of decision variable values to the power of the number of decision variables). Then, all alternatives are analyzed on their feasibility and the feasible ones are fed into the PFM software.

Step 6: Use the PFM algorithm to yield an overall preference scale of all feasible alternatives

Table 25 shows the top 10 of feasible alternatives (combinations of decision variable values that satisfy the design conditions) ordered on the overall preference rating yielded by the PFM algorithm. From this table it can be concluded that alternative 396 requiring the passengers to spend of 0.5 hours in the shuttle at a distance of 31.4 kilometers from the shore with $10 \times 100k$ flight movements costing 20 billion dollars is both feasible and the most preferred alternative.

6.5 EVALUATION OF THE PBD PROCEDURE

The result obtained using Bézier curves to enhance the resolution is more satisfying as the value of 20 billion dollars for the investment variable is closer to the calculated value of 19.88 billion associated with the construction of an island at a distance of 31.4 kilometers accommodating $10 \times 100k$ flight movements obtained using Equation 6.5 earlier. Remember that before introducing the Bézier curves, only three decision variable values were used resulting in a value of 40 billion dollars being combined with the construction of an island at 30 kilometers from the shore. It will be clear that the higher the number of segments each curve is divided in, the more satisfying the result will be with regards to getting closer to the value of 19.88 billion dollars.

Table 24 can be used to find the preference rating associated with the decision variable values of alternative 396 for each decision maker as shown in Table 26.

Alt.	Decision variable				Pref.
	i	t	d	f	
396	20	0.5	31.40	10	80.144
1430	22.56	0.5	40	10	79.969
385	20	0.5	29.43	10	79.281
1419	22.56	0.5	36.58	10	78.677
374	20	0.5	27.75	10	78.318
1429	22.56	0.5	40	9.73	78.255
1551	22.56	0.55	40	10	78.151
1408	22.56	0.5	33.74	10	77.852
363	20	0.5	26.25	10	77.157
1397	22.56	0.5	31.40	10	77.083

Table 25: Top 10 of feasible alternatives with associated decision variable values and preference ratings.

Decision variable	Value	Rating
Investment i [\$ billion]	20.00	20.000
Travel time t [hours]	0.50	100.000
Distance d [km]	31.40	75.190
Flight movements f [$\times 100k$]	10.00	100.000
Overall preference rating		80.144

Table 26: Decision variable values and preference ratings for the highest rated alternative.

As mentioned before, the Ministry of Finance's preference rating sharply drops from 100 to 20 as the investment variable increases from 15 to 20 billion dollars. The curve as defined by the airport also shows a sharp drop in preference rating as the number of flight movements deviates from the most preferred value. On the other hand, the curves of the two other decision makers show a far less sharp drop in preference rating as soon as their decision variable deviates from the most preferred value. Therefore one would expect an alternative associated with an investment close to 15 billion dollars and 1 000 000 flight movements to be selected as the highest rated alternative. Looking at alternative 396 this is true with respect to the number of flight movements, but not true with respect to the investment.

This can be explained by looking at the second design constraint, see Equation 6.2. Consider an alternative associated with an investment of 15 billion dollars:

$$15 + 0.15(f - 6) + 0.2(d - 10) \leq 15 \quad (6.16)$$

In order to satisfy this constraint the number of flight movements f needs to be equal to 6 and the distance d equal to 10 (both need to be larger than or equal to zero). As the Ministry of Environment's preference rating vs. decision variable value curve shows that the lowest allowed value for the distance equals 20, any alternative associated with an investment of 15 billion dollars is infeasible. In other words, as the Ministry of Finance will need to 'give in' anyway, it makes sense that alternative 396 has the highest overall preference as, although it is not the highest rated alternative by the Ministry of Finance, it is for the airlines and the airport and is also highly rated by the Ministry of Environment.

6.5.1 PBD methodology as compared to the constraint method

To compare the solution obtained using PBD with the solution using the constraint method proposed by van Gunsteren [2005], we need to determine the overall preference rating associated with the solution

Decision variable	van Gunsteren		PBD	
	Value	Rating	Value	Rating
Investment i [\$ billion]	20.60	18.048	20.00	20.000
Travel time t [hours]	0.65	59.325	0.50	100.000
Distance d [km]	35.00	86.369	31.40	75.190
Flight movements f [$\times 100k$]	10.00	100.000	10.00	100.000
Overall preference rating		73.521		80.144

Table 27: Comparison of alternatives.

he obtained. For this, we need to determine the preference rating associated with each decision variable value obtained in his final run: $i = 20.6$, $t = 0.65$, $d = 35$ and $f = 10$. Using the coordinates of the end and control points of the curves we can use Equation 6.14 to determine the value for t associated with each decision variable value. Using the obtained value for t , we can use Equation 6.15 to find the associated preference rating. For instance, using the coordinates of the control and endpoints of the curve associated with decision variable i , setting t to 0.53 produces $x = 20.6$ and $y = 18.048$. The PFM algorithm can then be used to yield the overall preference rating associated with the solution van Gunsteren obtained compared to the overall preference rating obtained using the PBD procedure. The results are shown in Table 27.

Professionals from practice involved in this experiment considered the solution proposed by van Gunsteren close to the solution obtained using the PBD procedure. In the judgment of these professionals the difference was insignificant. However, his solution is the result of manipulation by the model owner with respect to the constraint value for the distance d associated with the Ministry of Environment.

The model was run several times, selecting different decision variables to optimize, and changing constraint values from walk-out to ideal. It appeared that i could be set to a value halfway between

acceptable and ideal values, both t and d to acceptable values and f to the ideal value. Considering these results, van Gunsteren then decided to satisfy in particular the Ministry of Environment by setting the constraint value for the distance d to a value halfway between acceptable and ideal values, but not doing so for the airlines interested in the travel time t . His reasoning was that increasing the distance from 30 to 35 kilometer constitutes a major improvement for the Ministry of Environment as opposed to a diminishable improvement for the airlines if the travel time would decrease from 0.7 to 0.6 hours.

We can therefore conclude that the PBD produces a result similar to the result using the constraint method, but without relying on negotiation or suffering from manipulation, in other words, without losing the math. We can also conclude that, although the quality of the decision is almost the same, the acceptance of the solution obtained using the PBD procedure will be higher as it does not require any manipulation by the model owner.

6.6 SUMMARY OF THE PBD PROCEDURE

The above leads to the following PBD procedure, incorporating the use of Bézier curves to relate decision variable values to preference ratings:

1. Specify the decision variable(s) the decision maker is interested in.
2. Rate the decision maker's preferences for each decision variable as follows:
 - a) For each decision variable establish (synthetic) reference alternatives which define the endpoints of a cubic Bézier curve:
 - i. Define a 'bottom' reference alternative, the alternative associated with the value for the decision variable that is least preferred, rated at 0. This defines the origin endpoint of the curve, (x_0, y_0) .

- ii. Define a 'top' reference alternative, the alternative associated with the value for the decision variable that is most preferred, rated at 100. This defines the destination endpoint of the curve, (x_3, y_3) .
 - b) Rate the preference for alternatives associated with the other decision variable values relative to these reference alternatives by manipulating the two control points (x_1, y_1) and (x_2, y_2) .
3. To each decision variable assign decision maker's weight.
4. Determine the design constraints.
5. Combine decision variable values to generate design alternatives and use the design constraints to test their feasibility.
6. Use the PFM algorithm to yield an overall preference scale of all feasible alternatives.

Required graphical interface

The above procedure needs to incorporate a graphical interface so that a decision maker can quickly find the Bézier curve that approximates the relation between decision variable values and preference ratings. As an illustration how this interface could work we take the example of the Ministry of Finance. We could start with an arbitrarily chosen curve as shown in Figure 28.

The first step for the decision maker would be to change the scale of the x-axis to reflect the range of allowed decision variable values, in this case 15 to 40 (billion \$). The result of this is shown in Figure 29.

In the second step, the decision maker moves the endpoints to the desired positions (15, 0 to 15, 100 and 40, 100 to 40, 0) to reflect that the preference rating decreases as the investment increases. Finally, the decision maker moves the control points (20, 80 to 15, 12.5 and 20, 80 to 20, 7.5) to express how intermediate decision variable values are rated on preference resulting in Figure 30.

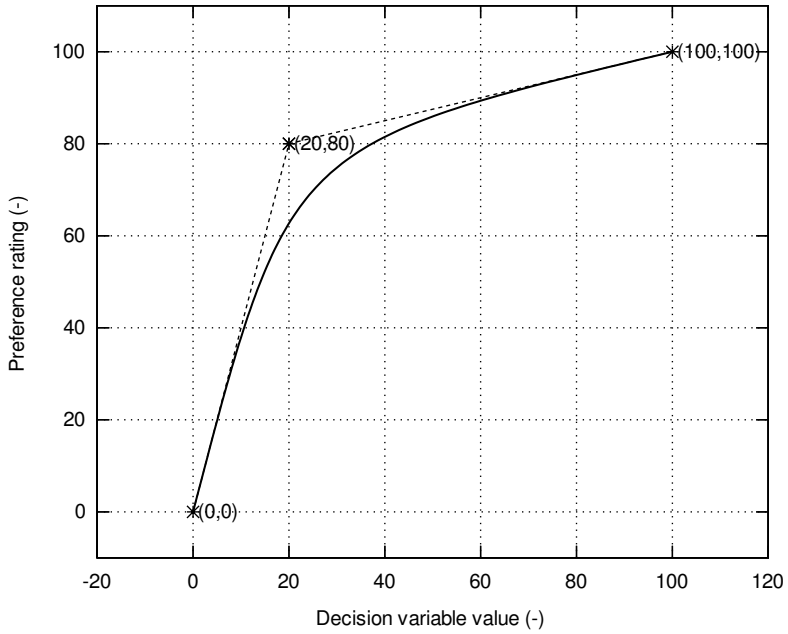


Figure 28: Arbitrary starting point for manipulating the Bézier curve.

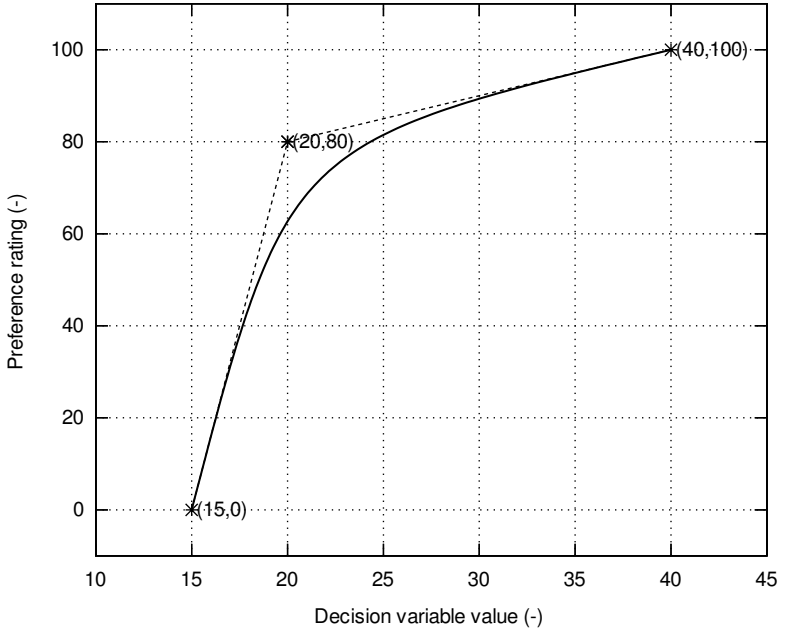


Figure 29: Bézier curve after the scale for the x-axis has been changed.

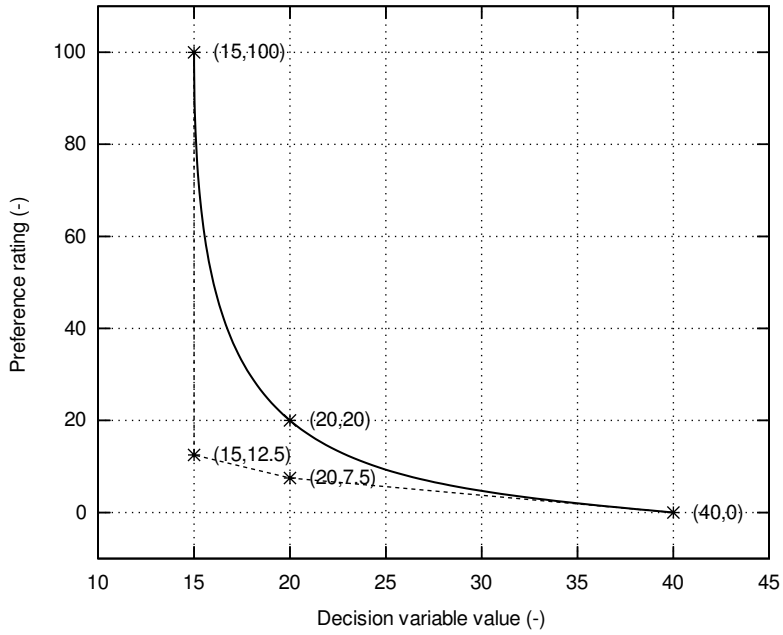


Figure 30: Bézier curve after the endpoints and control points have been moved.

6.7 CONCLUSIONS

The PBD proposed in this chapter fulfills both requirements as stated in Chapter 5 as, being built upon PFM, it extends to group decision making and has a sound mathematical foundation for measuring preference. It also removes all limitations of using either LP, GP or LMOP as it removes the harsh division of solutions into feasible or infeasible and the linearity requirement by introducing curves to represent how decision variable values relate to preference ratings.

Applying the PBD methodology to the North Sea International Airport case successfully produced results. The outcome is slightly different as compared to using the constraint method. However, the solution obtained using the constraint method was the result of manipulation by the model owner. The PBD procedure produced a comparable result without any interference by the model owner. The result is thus a better reflection of the decision maker's preferences.

This chapter serves to illustrate the application of the Preference-Based Design (PBD) methodology described in Chapter 6 on problems in the domain of architecture.

Devising an interface to let the decision maker construct a Bézier curve as described in Chapter 6 is beyond the scope of this thesis. Instead, a set of pre-determined Bézier curves as shown in Appendix A is used so that decision makers can quickly relate preference ratings to decision variable values. A drawback of using this limited amount of curves is that, because they are pre-determined, they only approximately reflect a decision maker's preferences.

The following cases will serve to further illustrate the PBD methodology:

- The Stedelijk Museum Amsterdam (SMA)
- The Tilburg area development

7.1 THE STEDELIJK MUSEUM AMSTERDAM CASE

See Chapter 2 for an overview of this case. The main problem to solve: which allocation of functions to the primary location (existing building and extension) and secondary location is feasible and most desirable?

Specifying the decision variable(s) the decision maker is interested in

The following decision makers are identified along with the decision variables they are interested in:

- The municipality interested in the investment (decision variable i) which is allowed to range between 71.2 and 73.4 million euros.

Space type	Var.	Space requirement (m ²)	
		Minimal	Ideal
Exhibitions	p_1	7 901	10 154
Workshops	p_2	1 354	1 438
	s_1	693	714
Education	p_3	941	1 378
	s_2	56	84
Depots	p_4	525	721
	s_3	7 423	9 960
Installations	p_5	6 264	7 031
	s_4	1 207	1 648
Public	p_6	3 167	3 990
Offices	p_7	1 624	2 230
	s_5	168	281
Total		31 742	40 048

Table 28: Minimal and ideal values for the allocation of floor space for the Amsterdam museum case.

- The museum staff interested in the floor space allocated to 7 functions on the primary ($p_1 \dots p_7$) and 5 functions on the secondary ($s_1 \dots s_5$) location which are allowed to range between values as shown in Table 28.

Determining the Bézier curves as defined by the decision makers

The set of pre-determined Bézier curves in Appendix A is used so that decision makers can quickly relate preference ratings to the decision variable values as shown in Tables 29 and 30.

Var.	Curve	End and control points	
		(x'_0, y'_0)	(x'_1, y'_1)
p_1	10	(7 901, 0)	(9 929, 10)
p_2	3	(1 354, 0)	(1 371, 80)
s_1	2	(693, 0)	(695, 90)
p_3	3	(941, 0)	(1 028, 80)
s_2	2	(56, 0)	(59, 90)
p_4	8	(525, 0)	(662, 30)
s_3	2	(7 423, 0)	(7 677, 90)
p_5	2	(6 264, 0)	(6 341, 90)
s_4	2	(1 207, 0)	(1 251, 90)
p_6	8	(3 167, 0)	(3 743, 30)
p_7	3	(1 624, 0)	(1 745, 80)
s_5	2	(168, 0)	(179, 90)
i	3	(73 400 000, 0)	(73 180 000, 90)

Table 29: End points (x_0, y_0) and control points (x_1, y_1) for all Bézier curves for the museum case.

The lower and upper values as shown in Table 28 then represent the x-coordinates of the end points for each decision variable. Because the x-coordinates of the end and control points of the pre-determined curves are known, the x-coordinates of the control points of the curves associated with each decision variable can be determined.

As the y-coordinates of the end and control points for the pre-determined curves and the curves associated with the decision variable values both range from 0 to 100, the y-coordinates of the end and control points for both are equal. The only information needed from the decision maker are the x-coordinates for the end points x'_1 and x'_3 and the curve chosen.

Var.	Curve	End and control points	
		(x'_2, y'_2)	(x'_3, y'_3)
p_1	10	(9 929, 10)	(10 154, 100)
p_2	3	(1 371, 80)	(1 438, 100)
s_1	2	(695, 90)	(714, 100)
p_3	3	(1 028, 80)	(1378, 100)
s_2	2	(59, 90)	(84, 100)
p_4	8	(662, 30)	(721, 100)
s_3	2	(7 677, 90)	(9 960, 100)
p_5	2	(6 341, 90)	(7031, 100)
s_4	2	(1 251, 90)	(1 648, 100)
p_6	8	(3 743, 30)	(3 990, 100)
p_7	3	(1 745, 80)	(2 230, 100)
s_5	2	(179, 90)	(281, 100)
i	3	(73 180 000, 90)	(71 200 000, 100)

Table 30: End points (x_3, y_3) and control points (x_2, y_2) for all Bézier curves for the museum case.

t	Decision variable					
	p_1		p_2		s_1	
	x_t	y_t	x_t	y_t	x_t	y_t
0.00	7901	0	1354	0	693	0
0.33	9336	10	1368	57	695	64
0.67	9920	36	1390	83	701	90
1.00	10154	100	1438	100	714	100

Table 31: Four coordinates along Bézier curves for the museum case (decision variables p_1 , p_2 and s_1).

Each Bézier curve is divided in three segments yielding four points on each curve by setting t to 0, 0.333, 0.667 and 1 using Equation 6.14 and 6.15 for each decision variable. The reason for dividing the curve into only three segments is to keep the number of combinations to evaluate relatively low. This will, however, result in a low resolution.

Assigning weights to decision variables

For this experiment all decision variables are weighted equally.

Determining the design constraints

For this experiment the first design constraint relates the floorspace to the investment and depends on the characteristics of the candidate solution:

If the total amount of floorspace on the central location is below the capacity of the existing building (14 142 m²):

$$2580 \sum_{i=1}^7 p_i + 1400 \sum_{i=1}^5 s_i \leq i \quad (7.1)$$

t	Decision variable							
	p_3		s_2		p_4		s_3	
	x_t	y_t	x_t	y_t	x_t	y_t	x_t	y_t
0.00	941	0	56	0	525	0	7423	0
0.33	1015	57	59	64	624	24	7686	64
0.67	1129	83	66	90	675	50	8344	90
1.00	1378	100	84	100	721	100	9960	100

Table 32: Four coordinates along Bézier curves for the museum case (decision variables p_3, p_4, s_2 and s_3).

t	Decision variable					
	p_5		s_4		p_6	
	x_t	y_t	x_t	y_t	x_t	y_t
0.00	6264	0	1207	0	3167	0
0.33	6344	64	1253	64	3582	24
0.67	6542	90	1367	90	3795	50
1.00	7031	100	1648	100	3990	100

Table 33: Four coordinates along Bézier curves for the museum case (decision variables p_5, p_6 and s_4).

<i>t</i>	Decision variable					
	<i>p</i> ₇		<i>s</i> ₅		<i>i</i>	
	<i>x</i> _{<i>t</i>}	<i>y</i> _{<i>t</i>}	<i>x</i> _{<i>t</i>}	<i>y</i> _{<i>t</i>}	<i>x</i> _{<i>t</i>}	<i>y</i> _{<i>t</i>}
0.00	1 624	0	168	0	73 400 000	0
0.33	1 727	57	180	64	73 171 852	64
0.67	1 884	83	209	90	72 601 481	90
1.00	2 230	100	281	100	71 200 000	100

Table 34: Four coordinates along Bézier curves for the museum case (decision variables *p*₇, *s*₅, and *i*).

If it exceeds the capacity of the existing building:

$$(2580 - 2950) \times 14\,142 + 2950 \sum_{i=1}^7 p_i + 1\,400 \sum_{i=1}^5 s_i \leq i \tag{7.2}$$

The second constraint relates the floorspace on the central location to the capacity of the existing building and extension:

$$\sum_{i=1}^7 p_i \leq 14\,142 + 24\,007 \tag{7.3}$$

The third relates the floorspace on the secondary location to the capacity of the building on the secondary location:

$$\sum_{i=1}^5 s_i \leq 13\,000 \tag{7.4}$$

The information gained from dividing each curve in three segments is used to generate 67 108 864 possible combinations of decision variable values (the number of decision variable values to the power of the

number of decision variables). A script is used to combine decision variable values that satisfy the earlier mentioned design constraints to yield feasible alternatives and feed these to the Preference Function Modeling (PFM) software.

Using the PFM algorithm to yield an overall preference scale

The decision variable values, and preference ratings, associated with the highest rated feasible alternative as determined using the PFM algorithm is shown in the first column of Table 35 (rated at 46). The second column shows the preference rating (34) associated with the solution obtained using Linear Programming (LP) maximizing the usage of the primary location. The preference ratings associated with each decision variable value were obtained as described in Section 6.5.

This comparison, also looking at the individual preference ratings, shows even more clearly the advantages of using PBD instead of using LP with negotiable constraints. The solution obtained using the PBD procedure is a result of maximizing overall preference (multi-criterion) while the solution obtained using LP is a result of maximizing the usage of the primary location (single criterion). This is why, although the solution obtained using LP has a higher amount of floorspace allocated to the primary location, its overall score is lower than that of the solution obtained using the PBD procedure. Also, the LP algorithm does not take into account the individual preference ratings for each space type, it 'simply' maximizes the total floorspace allocated to the primary location. This is why the individual preference ratings associated with the design variables, with the exception of p_2 , p_3 , and p_4 are equal to zero. This tendency to extreme values, already mentioned in Chapter 5, is a typical feature of linear programming formulations, making it difficult to find compromise solutions. In contrast, the very nature of PBD allows finding such solutions enhancing both the quality of the decision (functionality) and the acceptance of the solution obtained as it involves no manipulation.

Space type	Var.	Methodology			
		PBD		LP	
		Value	Rating	Value	Rating
Exhibitions	p_1	7901	0	7901	0
Workshops	p_2	1438	100	1438	100
	s_1	714	100	693	0
Education	p_3	941	0	1009	54
	s_2	84	100	56	0
Depots	p_4	721	100	721	100
	s_3	7423	0	7423	0
Installations	p_5	6264	0	6264	0
	s_4	1253	64	1207	0
Public	p_6	3167	0	3167	0
Offices	p_7	1624	0	1624	0
	s_5	180	64	168	0
Total costs	i	73 400 000	0	73 400 000	0
Overall rating			46	34	

Table 35: Two feasible alternatives with associated decision variable values and preference ratings for the Amsterdam museum case.

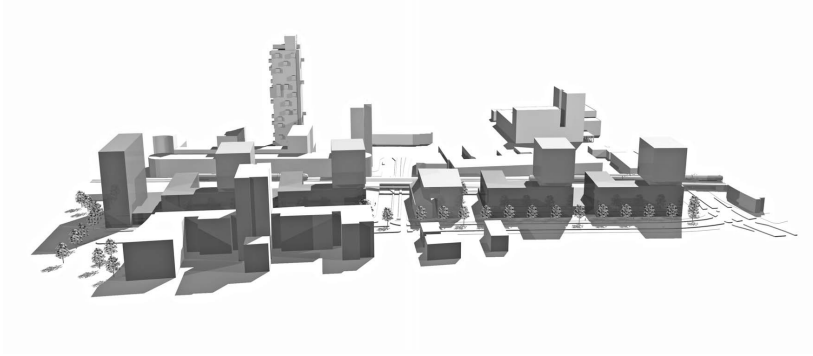


Figure 31: A design by BDP. Khandekar to develop part of the railway zone connecting the city center of Tilburg to the northerly residential areas.

7.2 THE TILBURG AREA DEVELOPMENT CASE

Tilburg is one of the larger cities within the province of Noord Brabant, the Netherlands, and serves as a public transport hub connecting some major cities within the province: Breda, Den Bosch and Eindhoven. The railway zone running through Tilburg acts as a barrier between the city center to the South and the residential areas to the North and plans are being developed to solve this problem. An example of such a plan is shown in Figure 31.

The idea is that by developing a mix of different functions within the railway zone the barrier is removed so that the center is connected to the northerly residential areas. The question then arises: What mix of functions is both feasible and most desirable?

Whereas the desirability aspect is determined by the municipality's wishes with respect to what amount of what function is most desirable, the feasibility aspect is determined by the actual plot size and density indicators (Floor Space Index (FSI)/Ground Space Index (GSI)/Open-Space Ratio (OSR)).

Space type	Var.	Unit	Space requirement	
			Lower	Upper
Student houses	h_1	-	80	180
Starter houses	h_2	-	60	140
Remaining houses	h_3	-	60	80
Offices	o	m ²	28 000	40 000
Cultural facilities	c	m ²	30 000	80 000
Open space	s	-	0.30	0.45

Table 36: Lower and upper values for the allocation of space for the Tilburg area case.

Specifying the decision variable(s) the decision maker is interested in

The municipality's urban design department is interested in the composition of the new area. The design department defines the composition by allowed lower and upper decision variable values for each function; the amount of square meters floorspace for three types of houses (h_1, h_2, h_3), offices (o), cultural facilities (c) and the percentage open space (s):

Determining the Bézier curves as defined by the decision makers

The set of pre-determined Bézier curves in Appendix A is used so that decision makers can quickly relate preference ratings to the decision variable values as shown in Tables 37 and 38.

The lower and upper values as shown in the previous table then represent the x-coordinates of the end points for each decision variable. Because the x-coordinates of the end and control points of the pre-determined curves are known, the x-coordinates of the control points of the curves associated with each decision variable can be determined.

Var.	Curve	End and control points	
		(x'_0, y'_0)	(x'_1, y'_1)
h_1	9	(80,0)	(160,20)
h_2	9	(60,0)	(124,20)
h_3	6	(60,0)	(70,50)
o	11	(40 000,0)	(28 000,0)
c	9	(30 000,0)	(70 000,20)
s	4	(0.30,0)	(0.345,70)

Table 37: End points (x'_0, y'_0) and control points (x'_1, y'_1) for all Bézier curves for the area development case.

As the y-coordinates of the end and control points for the pre-determined curves and the curves associated with the decision variable values both range from 0 to 100, the y-coordinates of the end and control points for both are equal. The only information needed from the decision maker are the x-coordinates for the end points x'_1 and x'_3 and the the curve chosen. The decision maker associates curve 22 with decision variable h , curve 12 with decision variable o and f and curve 9 with decision variable s . The resulting coordinates of all control and end points of the Bézier curves associated with each decision variable are shown in Table 37 and 38.

Each Bézier curve is then divided in five segments yielding six points on each curve by setting t to 0, 0.2, 0.4, 0.6, 0.8, and 1 using Equation 6.14 and 6.15 for each decision variable. The reason for dividing the curve into only five segments is to keep the number of combinations to evaluate relatively low. This will, however, result in a low resolution.

Assigning weights to decision variables

For this experiment all decision variables are weighted equally.

Var.	Curve	End and control points	
		(x'_2, y'_2)	(x'_3, y'_3)
h_1	9	(160, 20)	(180, 100)
h_2	9	(124, 20)	(140, 100)
h_3	6	(70, 50)	(80, 100)
o	11	(28 000, 0)	(28 000, 100)
c	9	(70 000, 20)	(80 000, 100)
s	4	(0.345, 70)	(0.45, 100)

Table 38: End points (x'_3, y'_3) and control points (x'_2, y'_2) for all Bézier curves for the area development case.

t	Decision variable					
	h_1		h_2		h_3	
	x_t	y_t	x_t	y_t	x_t	y_t
0.0	80	0	60	0	60	0
0.2	119	10	91	10	65	25
0.4	144	21	111	21	68	42
0.6	159	36	123	36	72	58
0.8	170	61	132	61	75	75
1.0	180	100	140	100	80	100

Table 39: Six coordinates along all Bézier curves for the area development case (decision variables h_1 , h_2 , and h_3).

t	Decision variable					
	o		c		s	
	x_t	y_t	x_t	y_t	x_t	y_t
0.0	40 000	0	30 000	0	0.30	0
0.2	34 144	1	49 600	10	0.32	34
0.4	30 592	6	62 000	21	0.34	57
0.6	28 768	22	69 600	36	0.36	72
0.8	28 096	51	74 800	61	0.40	85
1.0	28 000	100	80 000	100	0.45	100

Table 40: Six coordinates along all Bézier curves for the area development case (decision variables o , c , and s).

Determining the design constraints

For this experiment two design constraints are used. The first relates the floorspace of each function and the required open space to the available amount of land:

$$\begin{aligned}
 &0.1(100h_1) + 0.16667(80h_2) + 0.2(120h_3) \\
 &\quad + 0.1o + 0.25(40h_1 + 35h_2 + 28h_3) \\
 &\quad + 0.33c + 100500s + 40h_1 + 40h_2 + 28h_3 \\
 &\quad + 0.3(o + (40h_1 + 35h_2 + 28h_3) + c) \\
 &\leq 100500 \quad (7.5)
 \end{aligned}$$

The second relates the required amount of open space per house to the available amount of open space:

$$85h_1 + 85h_2 + 75h_3 \leq 100500s \quad (7.6)$$

Combining decision variable values that satisfy the design constraints

The information in Tables 39 and 40 is used to generate 46 656 possible combinations of decision variable values (the number of decision variable values to the power of the number of decision variables). Then, all alternatives were analyzed on their feasibility and the feasible ones were fed into the PFM software.

Using the PFM algorithm to yield an overall preference scale

Table 41 shows the top 10 of feasible alternatives (combinations of decision variable values that satisfy the design conditions) ordered on the overall preference rating yielded by the PFM algorithm. From this table it can be concluded that alternative 4 245, and alternative 9 188 are both feasible and most preferred. The preference ratings associated with the decision variable values are shown in Table 42. As can be seen, these alternatives only differ with respect to h_1 , and h_2 and the preference rating associated with an alternative having 119 student houses and 140 starter houses is equal to an alternative having 180 student houses, and 91 starter houses. This explains why both have the same overall preference rating.

7.3 CONCLUSIONS

As we have seen in Chapter 6, the North Sea International Airport case resulted in a slightly different outcome using PBD as compared to using the constraint method. The tunnel length is 35 kilometer in the solution obtained using the constraint method aimed at satisfying the Ministry of Environment. Its length is 31 kilometer according to the PBD procedure. However, the solution obtained using the constraint method was the result of manipulation by the model owner. The PBD procedure produced a comparable result without any interference by the model owner. The result is thus a better reflection of the decision maker's preferences.

The Amsterdam Museum case showed that, starting from one and the same bill of requirements, application of LP with negotiable constraints and PBD yield different outcomes. Again, this can be

Alt.	Decision variable						Pref.
	h_1	h_2	h_3	o	c	s	
4245	119	140	80	28 000	49 600	0.32	68.343
9188	180	91	80	28 000	49 600	0.32	68.343
5884	144	140	75	28 000	49 600	0.32	67.646
5692	144	132	80	28 000	49 600	0.32	66.636
5850	144	140	72	28 000	49 600	0.32	66.403
3973	119	132	80	28 000	49 600	0.34	66.186
4159	119	140	72	28 000	49 600	0.34	65.692
9148	180	91	75	28 000	49 600	0.32	65.283
4202	119	140	75	28 000	49 600	0.32	65.283
5813	144	140	68	28 000	49 600	0.32	65.199

Table 41: Top 10 of feasible alternatives with associated decision variable values and preference ratings for the area development case.

Decision variable	Alt. 4245		Alt. 9188	
	Value	Rating	Value	Rating
Student houses h_1 [-]	119	10	180	100
Starter houses h_2 [-]	140	100	91	10
Remaining houses h_3 [-]	80	100	80	100
Offices o [m ²]	28 000	100	28 000	100
Cultural facilities c [m ²]	49 600	10	49 600	10
Open space s [-]	0.32	34	0.32	34
Overall preference rating	68.343		68.343	

Table 42: Decision variable values and preference ratings for two the highest rated alternatives.

explained by the observation that PBD represents a better reflection of the decision makers' preferences.

The Tilburg urban development case shows that the Bézier curve is easy to work with and appeals to the decision makers concerned. The outcome of the PBD procedure was considered to be plausible and satisfactory by the decision makers concerned.

CONCLUSIONS

The Preference-Based Design (PBD) methodology proposed in this thesis fulfills the requirements outlined in Chapter 5 as it, being built upon Preference Function Modeling (PFM), 1) extends to group decision making and 2) has a correct mathematical foundation for measuring preference. It also removes the limitations of using either Linear Programming (LP), Goal Programming (GP) or Linear Multi Objective Programming (LMOP) as it avoids single objective optimization and it removes the harsh division of solutions into feasible or infeasible ones.

The following conclusions are drawn:

1. The PBD procedure is not an extension of LP with negotiable constraints, but an independent design methodology.
2. PBD is a design methodology leading to a design which represents a more pure reflection of the decision makers' preferences without any interference from the part of design experts, as is emphasized in the Open Design philosophy. As a result the quality of the design decision is higher and the outcome more acceptable.
3. Experiments show that the results obtained using the PBD procedure are both plausible and satisfactory to the decision makers.
4. The PBD procedure is an excellent starting point in many complex architectural design problems.

8.1 FUTURE RESEARCH

A drawback of using a limited amount of Bézier curves as described in Chapter 7 is that they, because they are pre-determined, do not

purely reflect a decision maker's preferences. Future research aimed at devising a user friendly interface so that the decision maker can directly shape the preference curve is desirable.

A limitation of the PBD procedure is that it requires generating alternatives by combining *all* values for *all* decision variables and then filtering from these the feasible alternatives using the design conditions. This makes it a 'brute force' approach. As the number of possible combinations equals the number of decision variable values to the power of the number of decision variables, the number of combinations will be very large for more complex problems as these normally have a greater number of decision variables. Filtering and evaluating each combination will then take up (too) much computer time.

In general, given function values, a non-linear optimization algorithm proceeds by computing from the current point or points the next one higher value. In the PBD procedure Bézier curves and PFM algorithm are functions. The first outputs a preference rating given a decision variable value (or vice-versa). The second outputs the overall preference rating given a set of preference ratings and weights. Therefore, given the control and end points of all Bézier curves and PFM's algorithm, an optimization algorithm can be used to directly compute the best design (at least approximately). We then have a design methodology which takes into account each decision maker's preferences. Recall that in fact the 'design' part of the LP process is due to its optimization step.

Part III
APPENDIX



SAMPLE PREFERENCE CURVES

A set of pre-determined curves is used so that decision makers can quickly relate preference ratings to the decision variable values. The curves as shown in Figure 32 to Figure 42 could be used to define a preference curve where the preference rating (along the y -axis) increases or decreases (mirror image) against the decision variable value (along the x -axis). Allowed decision variable values range between x_0 and x_3 and the preference ratings range between y_0 and y_3 . The control points (x_1, y_1) and (x_2, y_2) are used to control the curve's slope. This also allows for defining curves should an intermediate point between allowed decision variable values be most preferred, see Figure 43 to Figure 53. In this case coordinate (x_4, y_4) represents the decision variable value that is most preferred and is defined by two control points.

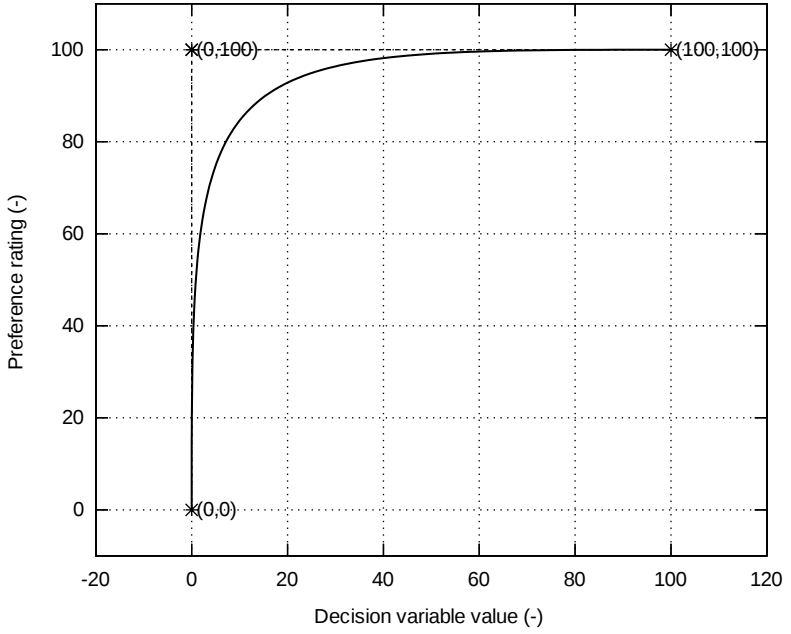


Figure 32: Preference curve 1.

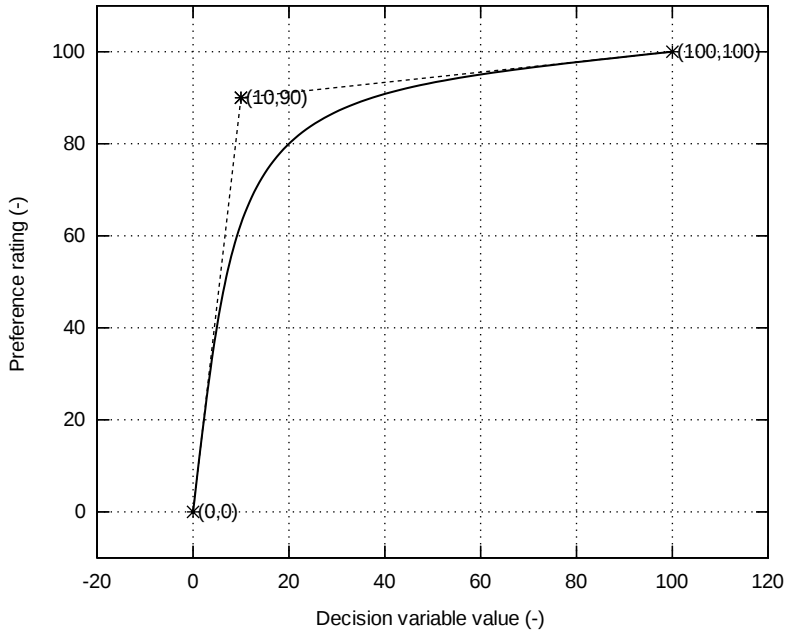


Figure 33: Preference curve 2.

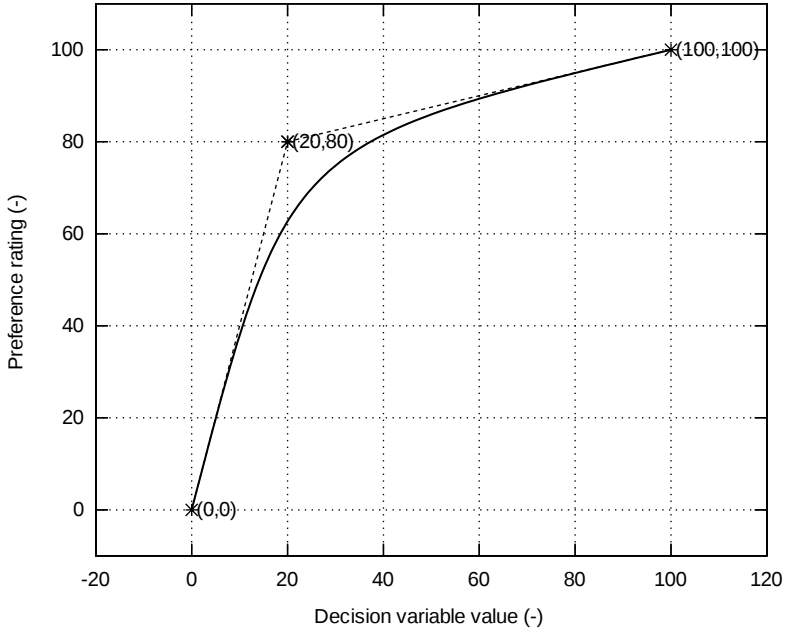


Figure 34: Preference curve 3.

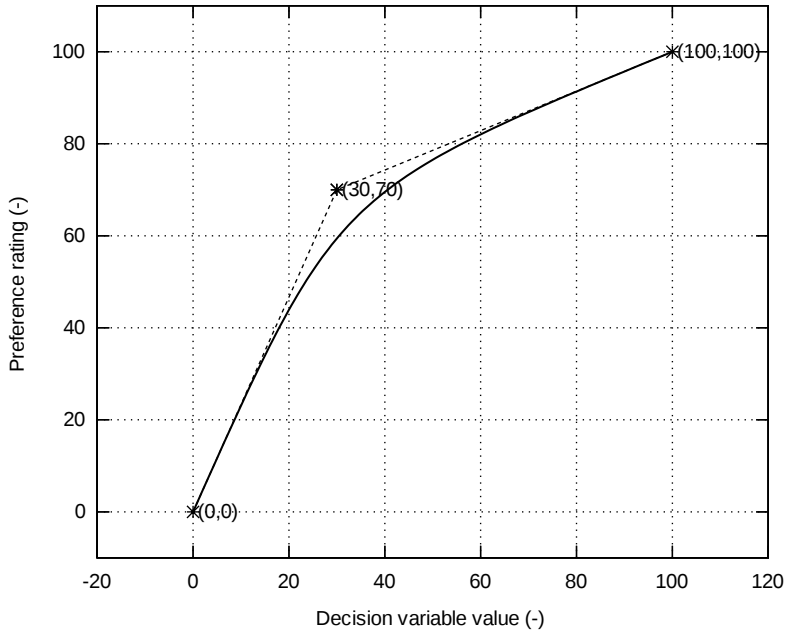


Figure 35: Preference curve 4.

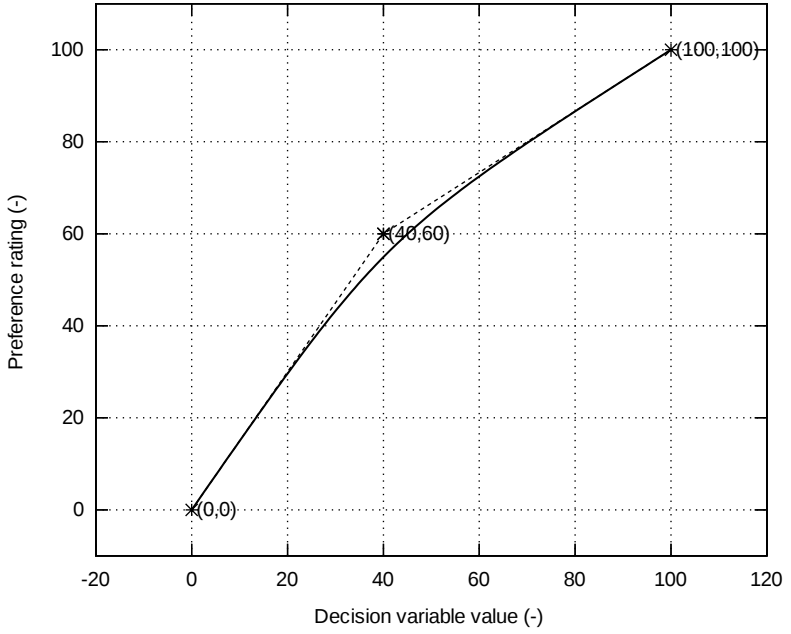


Figure 36: Preference curve 5.

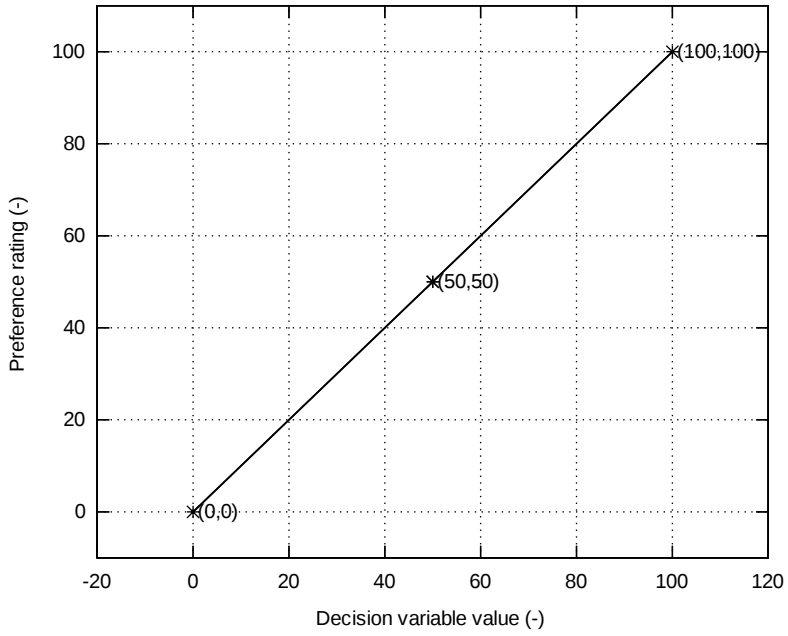


Figure 37: Preference curve 6.

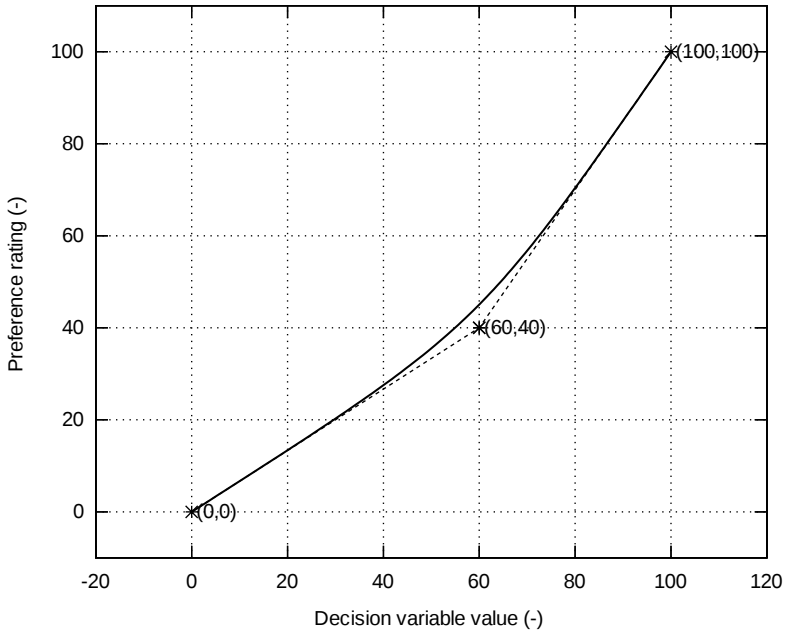


Figure 38: Preference curve 7.

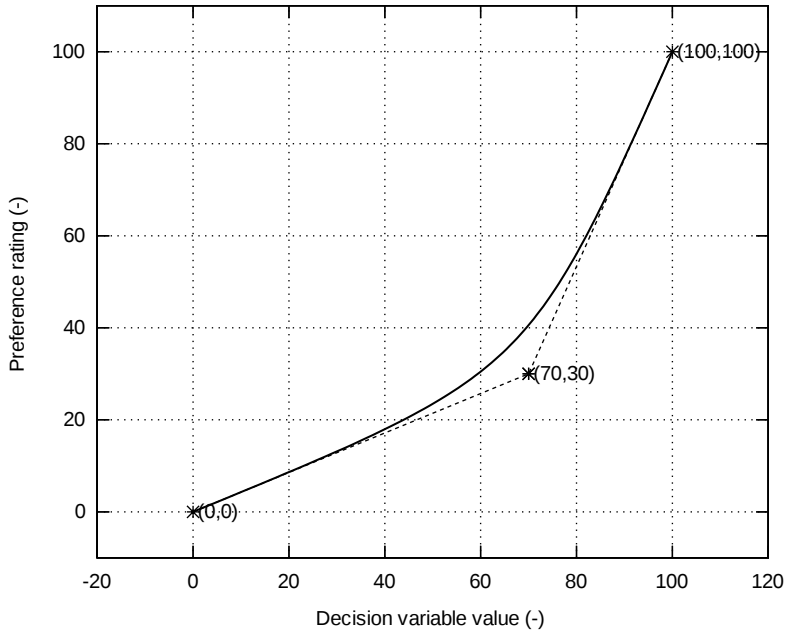


Figure 39: Preference curve 8.

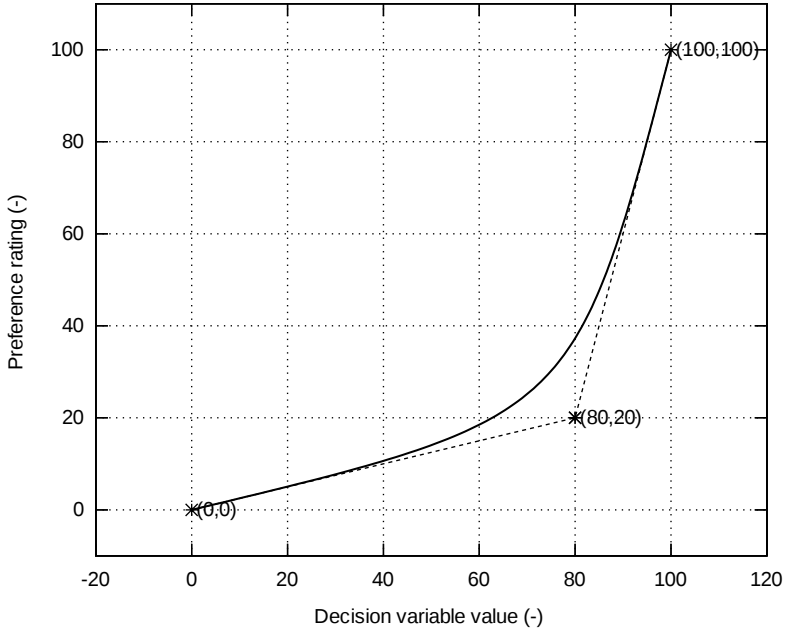


Figure 40: Preference curve 9.

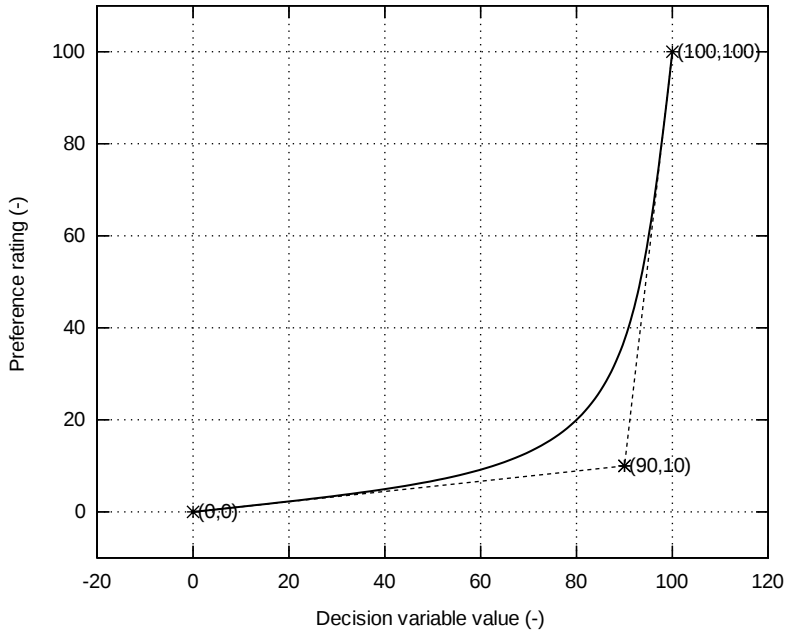


Figure 41: Preference curve 10.

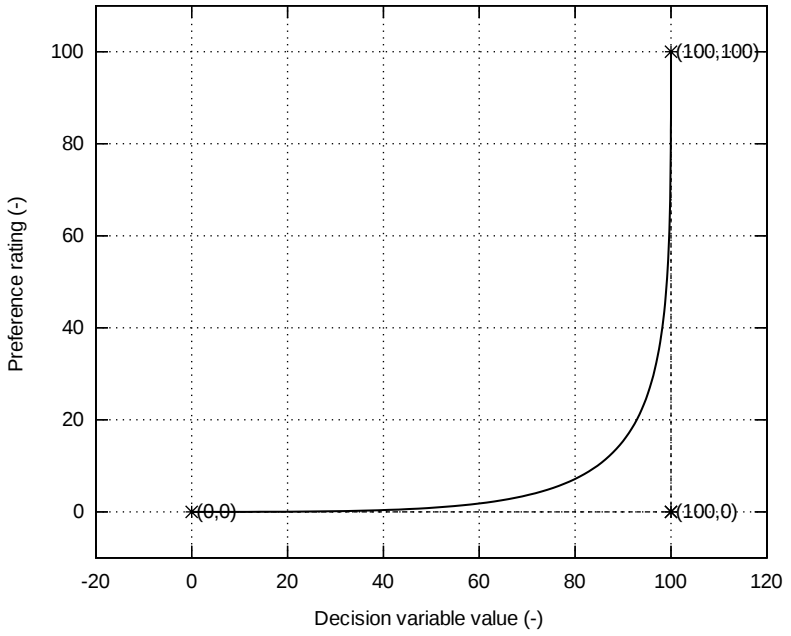


Figure 42: Preference curve 11.

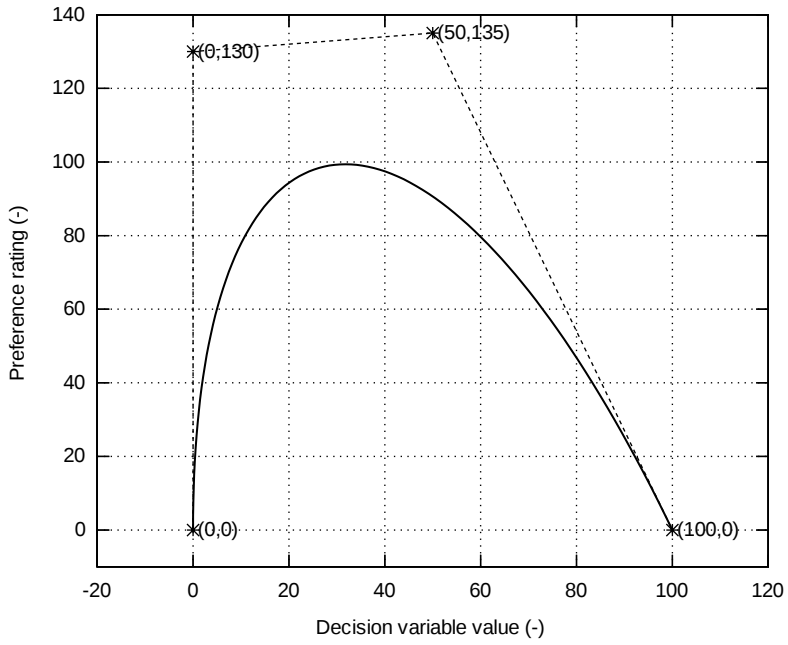


Figure 43: Preference curve 12.

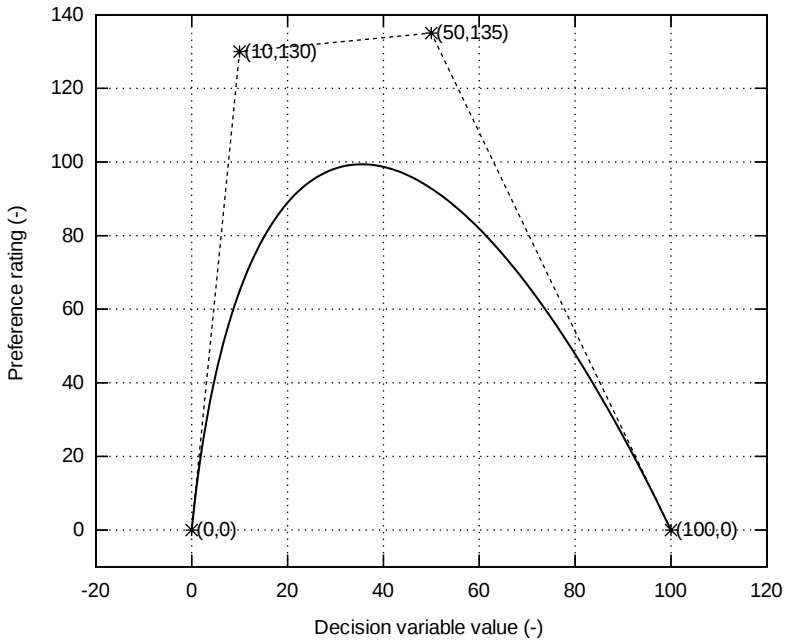


Figure 44: Preference curve 13.

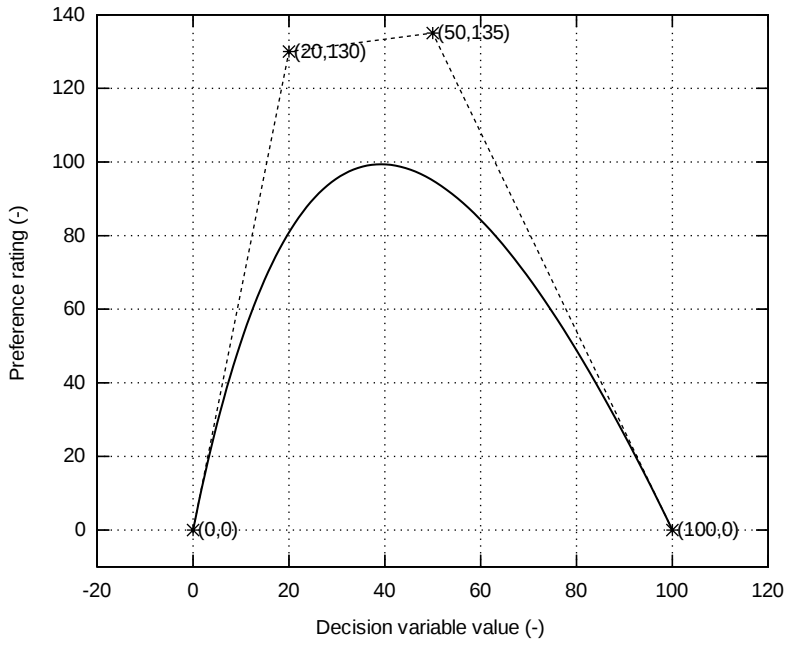


Figure 45: Preference curve 14.

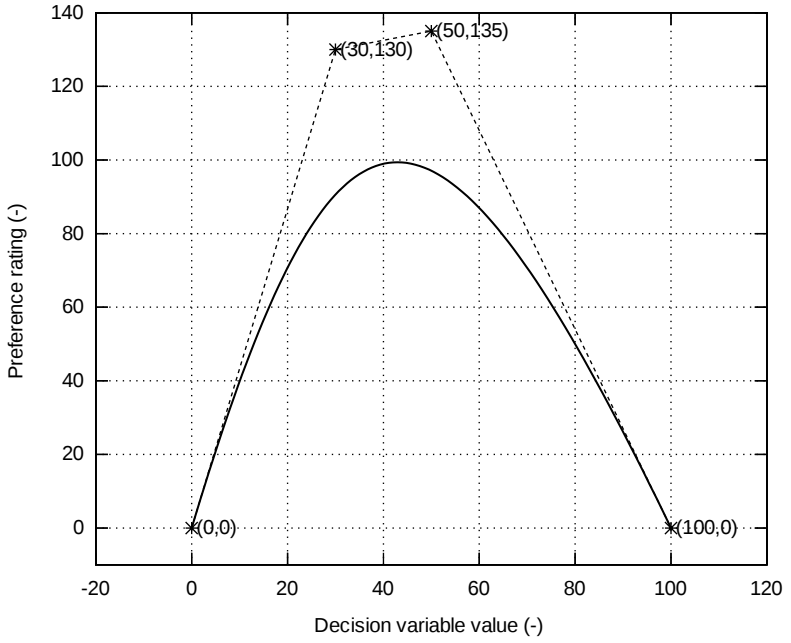


Figure 46: Preference curve 15.

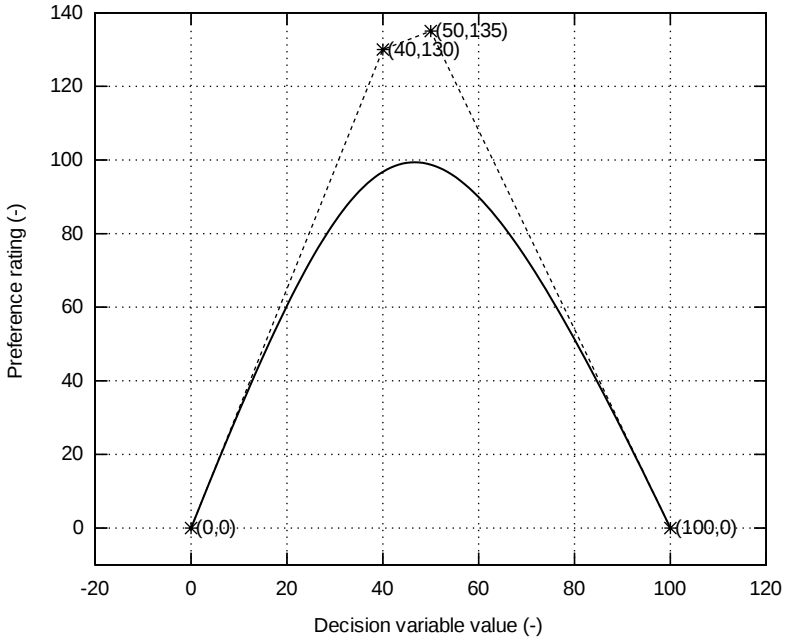


Figure 47: Preference curve 16.

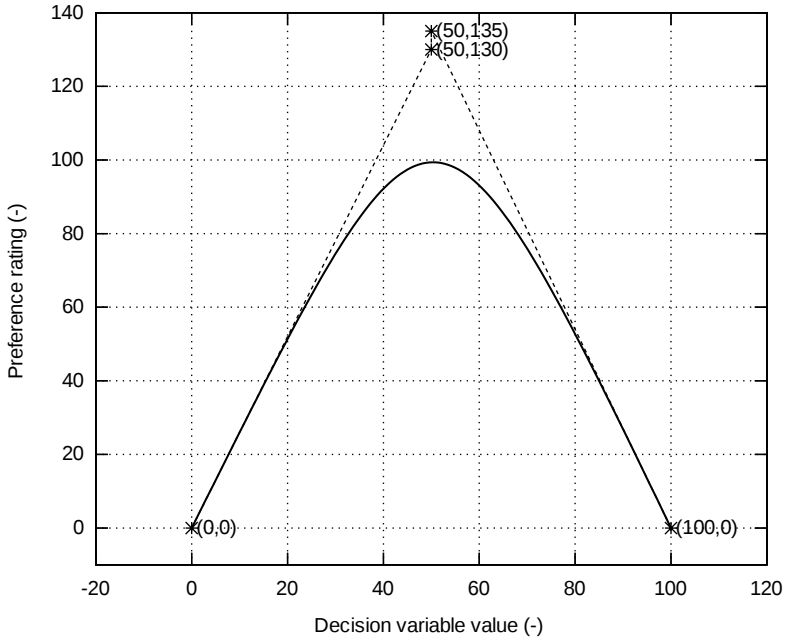


Figure 48: Preference curve 17.

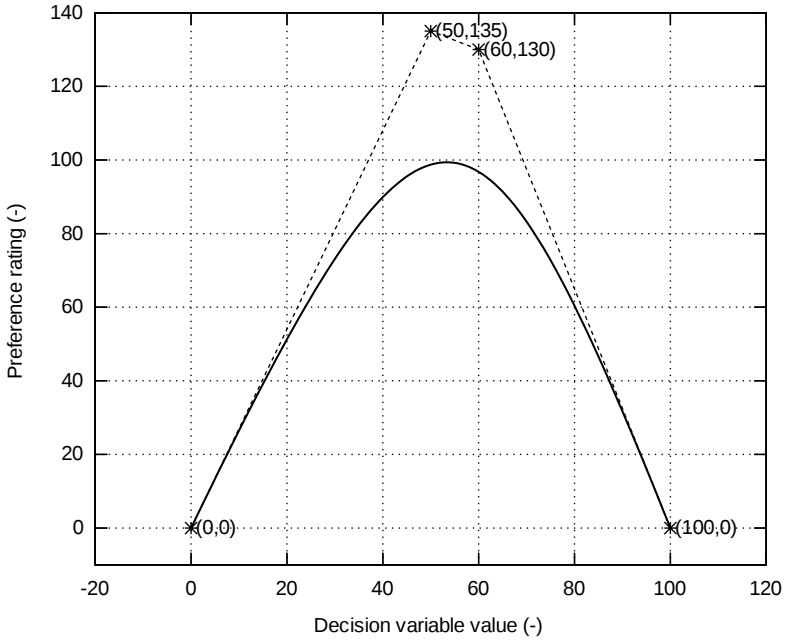


Figure 49: Preference curve 18.

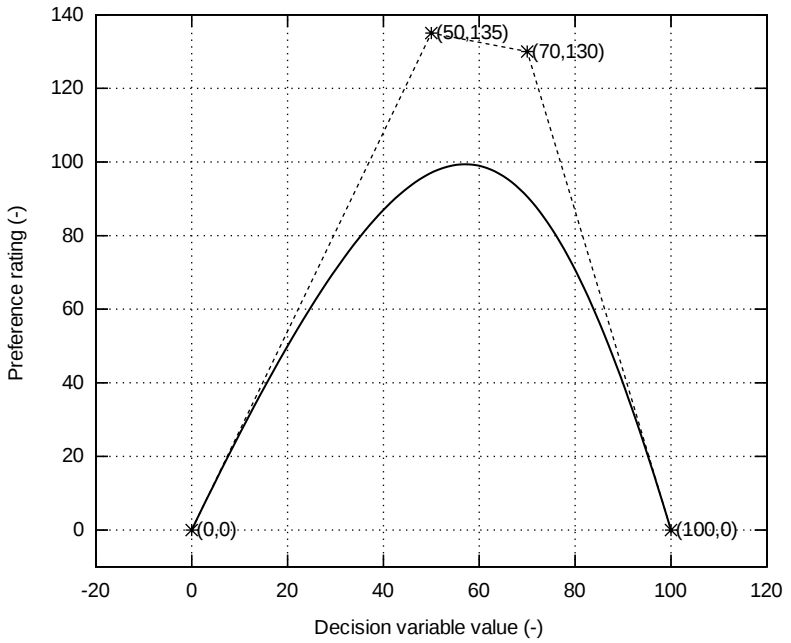


Figure 50: Preference curve 19.

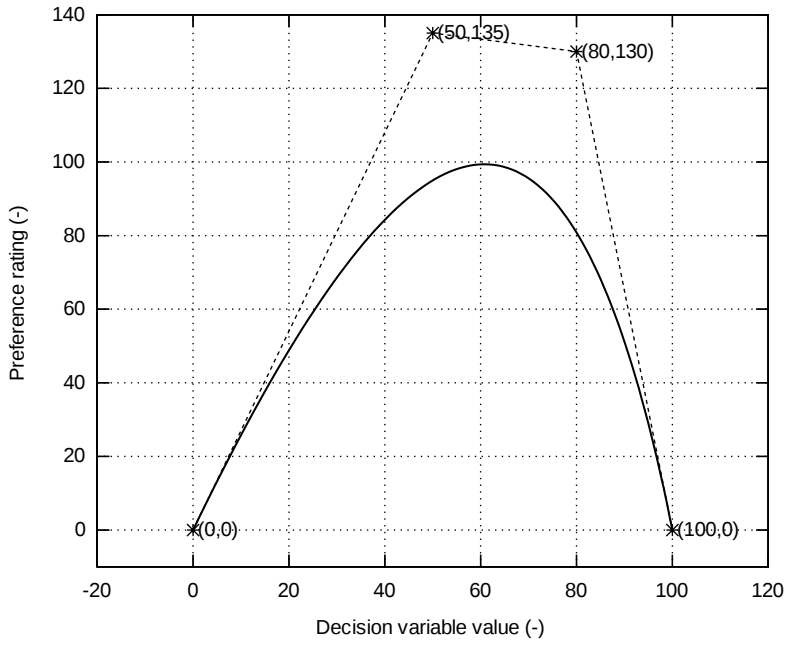


Figure 51: Preference curve 20.

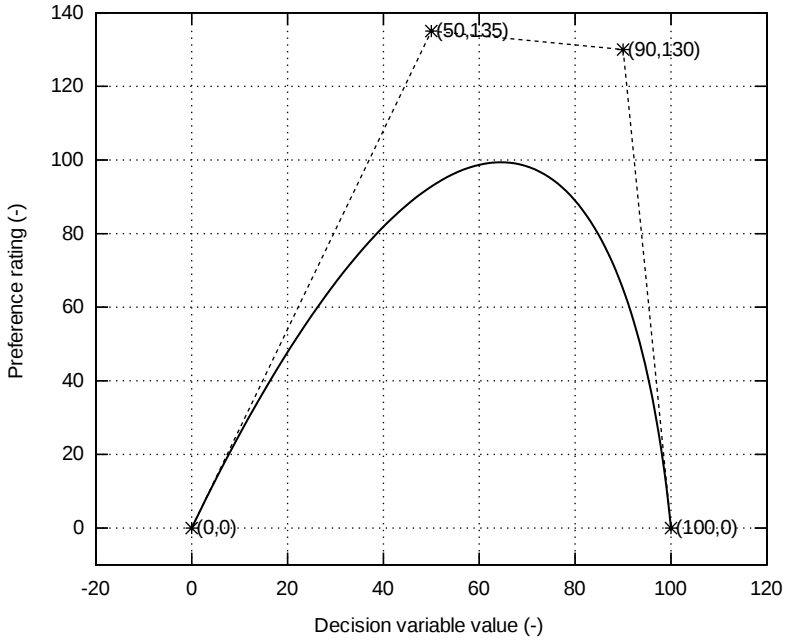


Figure 52: Preference curve 21.

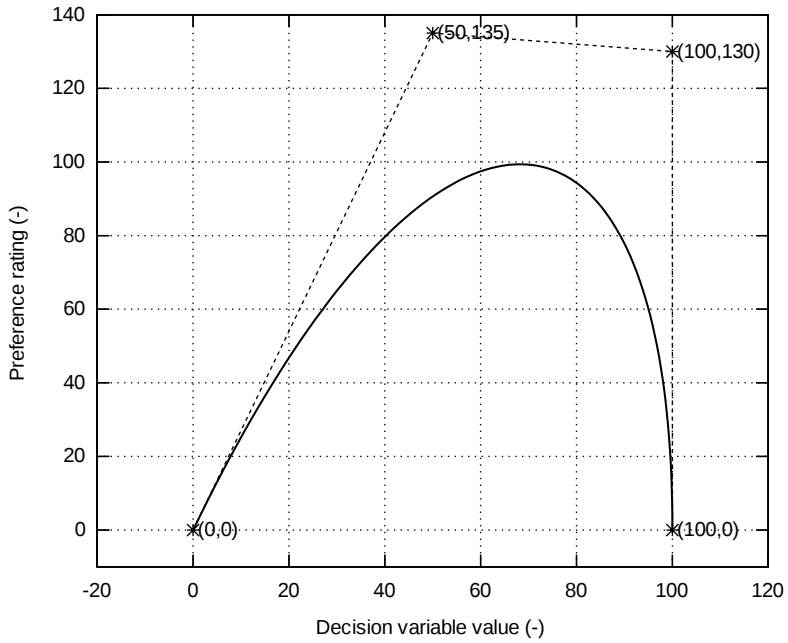


Figure 53: Preference curve 22.

BIBLIOGRAPHY

- K.J. Arrow. *Social Choice and Individual Values*. Wiley, New York, 1963. (Cited on page 35.)
- J. Barzilai. Deriving Weights from Pairwise Comparison Matrices. *Journal of the Operational Research Society*, 48(12):1226–1232, 1997a. (Cited on pages 41 and 50.)
- J. Barzilai. A New Methodology for Dealing with Conflicting Engineering Design Criteria. In *Proceedings of the 18th Annual Meeting of the American Society for Engineering Management*, pages 73–79, 1997b. (Cited on page 51.)
- J. Barzilai. On the Decomposition of Value Functions. In *Operations Research Letters*, volume 22, pages 159–170, 1998a. (Cited on page 50.)
- J. Barzilai. Understanding Hierarchical Processes. In *Proceedings of the 19th Annual Meeting of the American Society for Engineering Management*, pages 1–6, 1998b. (Cited on page 50.)
- J. Barzilai. Notes on the Analytic Hierarchy Process. In *Proceedings of the NSF Design and Manufacturing Research Conference*, 2001. (Cited on page 50.)
- J. Barzilai. Notes on Utility Theory. In *Proceedings of the IEEE International Conference on Systems, Man, and Cybernetics*, pages 1000–1005, 2004. (Cited on pages ix, 2, 51, 53, and 182.)
- J. Barzilai. Measurement and Preference Function Modelling. In *International Transactions in Operational Research*, volume 12, pages 173–183, 2005. (Cited on pages ix, 2, 46, 50, 51, 53, 54, and 182.)
- J. Barzilai. Decision Theory in Engineering Design. In K.E Lewis, W. Chen, and L.C. Schmidt, editors, *Decision Making in Engineer-*

- ing Design*, chapter 6, pages 43–47. ASME Press, 2006. (Cited on page 53.)
- J. Barzilai. Preference Function Modeling: The Mathematical Foundations of Decision Theory. In M. Ehrgott, J.R. Figueira, and S. Greco, editors, *Trends in Multiple Criteria Decision Analysis*, chapter 1, pages 1–37. Springer, to appear June 2010. (Cited on pages 23, 27, 32, 39, 44, 51, and 86.)
- V. Belton and A.E. Gear. On a Shortcoming of Saaty's Method of Analytic Hierarchies. *Omega*, 11(3):228–230, 1983. (Cited on page 51.)
- V. Belton and T.J. Stewart. *Multiple Criteria Decision Analysis: An Integrated Approach*. Kluwer Academic Publishers, 2003. (Cited on pages 31, 37, 38, 40, 43, 44, 45, 50, 59, 69, 70, 71, 76, and 81.)
- R. Binnekamp, L.A. van Gunsteren, and P.P. van Loon. *Open Design, a Stakeholder-oriented Approach in Architecture, Urban Planning, and Project Management*. IOSPress, Amsterdam, 2006. (Cited on pages ix, 2, 3, 15, 18, 57, 60, and 182.)
- J.P. Brans and P. Vincke. A Preference Ranking Organization Method: the PROMETHEE Method for Multiple Criteria Decision Making. *Management Science*, (31):647–656, 1985. (Cited on page 75.)
- J.P. Brans, B. Mareschal, and P. Vincke. PROMETHEE: A New Family of Outranking Methods in Multicriteria Analysis. In J.P. Brans, editor, *Operational Research '84*, pages 477–490, 1984. (Cited on page 75.)
- J.P. Brans, B. Mareschal, and P. Vincke. How to select and how to rank projects: The PROMETHEE method. *European Journal of Operational Research*, (24):228–238, 1986. (Cited on page 75.)
- N.R. Campbell. *Physics: The Elements*. Cambridge University Press, Cambridge, 1920. (Cited on page 24.)
- A. Charnes and W.W. Cooper. *Management Models and Industrial Applications of Linear Programming*. Wiley, New York, 1961. (Cited on page 61.)

- C.H. Coombs, R.M. Dawes, and A. Tversky. *Mathematical Psychology: An Elementary Introduction*. Prentice-Hall, 1970. (Cited on page 29.)
- James S. Dyer. Remarks on the Analytic Hierarchy Process. *Management Science*, 36(3):249–258, 1990. (Cited on page 51.)
- James S. Dyer and Rakesh K. Sarin. Measurable Multiattribute Value Functions. *Operations Research*, 27(4):810–822, 1979. (Cited on page 51.)
- C.L. Dym and P. Little. *Engineering Design, a Project-based Introduction*. Wiley, New York, 2004. (Cited on page 54.)
- W. Edwards, editor. *Utility Theories: Measurements and Applications*. Kluwer, 1992. (Cited on page 29.)
- A. Ferguson, C.S. Myers, R.J. Bartlett, H. Banister, F.C. Bartlett, W. Brown, N.R. Campbell, K.J.W. Craik, J. Drever, J. Guild, R.A. Houstoun, J.O. Irwin, G.W.C. Kaye, S.J.F. Philpott, L.F. Richardson, J.H. Shaxby, T. Smith, Thouless R.H., and W.S. Tucker. Quantitative Estimates of Sensory Events. In *Advancement of Science*, volume 2, pages 331–349, 1940. (Cited on pages 23 and 24.)
- James P. Ignizio. *Goal Programming and Extensions*. Lexington Books, 1976. (Cited on page 63.)
- Craig W. Kirkwood. *Strategic Decision Making*. Duxbury, 1996. (Cited on page 51.)
- D.H. Krantz, R.D. Luce, P. Suppes, and A. Tversky. *Foundations of Measurement*, volume 1. Academic Press, New York, 1971. (Cited on page 86.)
- J.R. Miller. *The Assessment of Worth: A Systematic Procedure and Its Experimental Validation*. PhD thesis, M.I.T., 1966. (Cited on page 44.)
- J.R. Miller. Assessing Alternate Transportation Systems. Memorandum RM-5865-DOT, The RAND Corporation, 1969. (Cited on page 44.)

- J.R. Miller. *Professional Decision-Making*. Praeger, 1970. (Cited on page 44.)
- P. Paans. *Villa VPRO, 3 jaar verder*. VPRO, Hilversum, 2000. In Dutch. (Cited on page 17.)
- F.S. Roberts. *Measurement Theory*. Addison-Wesley, Reading, MA, 1979. (Cited on page 86.)
- W. Roelofs. Conflict Resolving Methods for Concept Integration in Innovative Design Processes. Master's thesis, Delft University of Technology, 2001. In Dutch. (Cited on page 18.)
- B. Roy and B. Bouyssou. *Aide Multicritere d'Aide a la Decision: Methodes et Cas*. Economica, Paris, 1993. (Cited on page 80.)
- T.L. Saaty. *The Analytic Hierarchy Process*. McGraw-Hill, New York, 1980. (Cited on pages 44 and 45.)
- Martijn Sanders, Victor Halberstadt, and John Leighton. Het Stedelijk Museum: Terug naar de top. In Dutch, 2003. (Cited on page 13.)
- J. Van den Doel. *Demokratie en Welvaartstheorie*. Samson, 1978. (Cited on page ix.)
- R.P. de Graaf and L.A. van Gunsteren. Non-linearity of Stakeholder's Preferences in Open Design. In *IFORS 2002 Conference 'OR in a Globalised, Networked World Economy'*, Edinburgh, Jul 8th - 12th 2002. (Cited on pages 3, 60, and 183.)
- L.A. van Gunsteren. Open Design Methodology in Governmental Decision Making: The Case of Schiphol Airport Amsterdam. *Mathematical and Computer Modelling*, 2005. (Cited on pages 89, 91, and 119.)
- H. von Helmholtz. Zählen und Messen erkenntnis-theoretisch betrachtet. *Philosophische Aufsätze*, pages 17–52, 1887. English translation by C.L. Bryan, *Counting and Measuring*, van Nostrand, Princeton, NJ, 1930. (Cited on page 24.)

- W. de Keyser and P. Peeters. A Note on the use of PROMETHEE Multicriteria Methods. *European Journal of Operations Research*, (89): 457–461, 1996. (Cited on page 80.)
- P.P. van Loon. *Interorganisational Design, a New Approach to Team Design in Architecture and Urban Planning*. PhD dissertation, Delft University of Technology, Faculty of Architecture, 1998. (Cited on pages vii and ix.)
- J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behaviour*. Princeton University Press, Cambridge, 1944. (Cited on page 26.)
- D. von Winterfeldt and W. Edwards. *Decision Analysis and Behavioral Research*. Cambridge University Press, Cambridge, 1986. (Cited on page 43.)
- S.S. Stevens. On the Theory of Scales of Measurement. *Science*, (103): 677–680, 1946. (Cited on pages 26, 32, and 38.)
- Deborah L. Thurston. Real and Misconceived Limitations to Decision Based Design with Utility Analysis. *Journal of Mechanical Design*, 123(2):176–182, 2001. (Cited on page 29.)
- E. Waring. Problems Concerning Interpolations. In *Philos. Trans. R. Soc. London*, number 69, pages 59–67, 1779. (Cited on page 101.)
- M. Zeleny. *Multiple Criteria Decision Making*. McGraw-Hill, New York, 1982. (Cited on pages 55, 64, and 65.)

CURRICULUM VITAE

Ruud Binnekamp (1964) graduated at the Faculty of Architecture of Delft University of Technology in 1993 on the application of Open Design methodology on the renovation of the former Koninklijke Luchtvaart Maatschappij (KLM) head office in The Hague. He was employed by two different consultancy firms in the construction industry. Since 2000 he is Assistant Professor in the Chair of Computer Aided Design and Planning of the Faculty of Architecture of Delft University.

SAMENVATTING

VOORKEUR-GEBASEERD ONTWERPEN IN DE BOUWKUNDE

Bouwkundige ontwerpprocessen zijn complex doordat een veelheid van factoren bepaalt of een project slaagt of mislukt. Een aanzienlijk deel van deze factoren heeft betrekking op twee prominente kenmerken van besluitvormingsprocessen in de bouw: 1) meerdere ontwerpen kunnen eenzelfde doel dienen, waarbij de vraag zich aandient: hoe het ontwerp te kiezen dat het doel het beste dient, en 2) een veelheid van beslissers heeft een stem in het ontwerpproces. Dit is een probleem van groepbesluitvorming.

Besluitvormingstheorie betreft het vaststellen van de beste keuze. De praktische toepassing hiervan is gericht op het vinden van instrumenten, methoden en software om mensen, of groepen van mensen, te helpen in het maken van hun keuzes.

De bestaande besluitvormingsmethoden vertonen een grote verscheidenheid en leveren tegenstrijdige uitkomsten. Dit impliceert dat ze allemaal, op mogelijk één uitzondering na, incorrect moeten zijn. Men kan een groep mensen vragen om individueel een natuurkundig experiment uit te voeren, bijvoorbeeld de bepaling van de verhoudingen van de gewichten van een reeks van voorwerpen en mag dan verwachten dat elk van hen met hetzelfde resultaat terugkomt (afgezien van kleine meetfouten). Met andere woorden, in de natuurkunde wordt het bestaan van niet-equivalente methodologieën die tegenstrijdige uitkomsten opleveren niet geaccepteerd. In de sociale wetenschappen waar voornoemde besluitvormingsmethoden worden gebruikt, wordt dit wel geaccepteerd doordat de onjuistheid niet volgt uit de verkregen tegenstrijdige uitkomsten. Wat geldt in de natuurkunde, moet logischerwijs ook gelden voor besluitvormingsmethoden in de sociale wetenschappen: van alle niet-gelijkwaardige methodieken kan er niet meer dan één correct zijn.

Hoe kunnen we de juistheid bepalen van een besluitvormingsmethode anders dan door naar de uitkomsten te kijken? De wetenschappelijke grondslag van selectie (keuze) is voorkeursmeting. De juistheid van een besluitvormingsmethode wordt bepaald door de juistheid van de gebruikte schalen voor het meten van voorkeur. Het doel van het weergeven van variabelen via schalen is om de toepassing van wiskundige operaties mogelijk te maken. Schalen kunnen worden ingedeeld naar de aard van wiskundige operaties die zij toestaan. Barzilai [2004, 2005] heeft aangetoond dat alle bestaande meetschalen de operaties van optellen en vermenigvuldigen niet toestaan. Klassieke besluitvormingsmethoden produceren slechts betekenisloze getallen. De methode, ontwikkeld door Barzilai [2005], Preference Function Modeling (PFM), heeft een correcte wiskundige basis. PFM is derhalve de aangewezen methode om voorkeuren te meten.

Onze Open Design groep [Binnekamp et al., 2006] volgt niet de klassieke besluitvormingsstheorie voor de integratie van voorkeuren. In Open Design worden Lineaire Programmering (LP) modellen gebruikt om bouwkundige ontwerpproblemen op te lossen. Elk van de randvoorwaarden van het model drukt het belang van een beslisser uit betreffende een eigenschap van het ontwerp. Daarnaast wordt in beperkte mate groepsbesluitvorming mogelijk gemaakt door sommige randvoorwaarden als onderhandelbaar te beschouwen. Een groot voordeel van deze aanpak is dat deze het mogelijk maakt te bepalen of aan de randvoorwaarden van alle beslissers kan worden voldaan. Als de verzameling van toegestane oplossingen leeg blijft na alle onderhandelingen, is het probleem onoplosbaar. Indien die verzameling niet leeg is kunnen de beslissers de ontwerp oplossing selecteren door onderhandeling.

Toepassing van deze techniek van LP met onderhandelbare randvoorwaarden bracht aan het licht dat deze slechts in beperkte geschikt is om beslissers te helpen bij het vinden van de ontwerp oplossing met de hoogste groepsvoorkeur. De LP techniek kan alleen ontwerp oplossingen genereren gericht op één criterium van slechts één beslisser. In essentie is deze techniek is dus niet gericht op groepsbesluitvorming. Onze onderzoeksgroep heeft geprobeerd om voorkeursmeting te integreren in LP door uit te gaan van een lineaire [Binnekamp

et al., 2006, pp. 345-350] of exponentiële [de Graaf and van Gunsteren, 2002] relatie tussen waarden van beslissingsvariabelen (eigenschappen van het ontwerp) en voorkeurscores. Toch blijven ook dan sommige problemen bestaan: 1) de randvoorwaarden verdelen alle mogelijke oplossingen in ofwel toegestaan of niet toegestaan, zwart of wit, geen grijs dat uiteindelijk acceptabel zou kunnen zijn voor de beslissers, 2) de voorkeurscore van een oplossing wordt bepaald als het gewogen gemiddelde van de voorkeurscores van deze oplossing op alle criteria, hetgeen een benadering is, 3) de uitkomsten zijn nog steeds ontwerp oplossingen gericht op één criterium van één besluitvormer, niet op groepbesluitvorming.

Het gaat er om een methode te vinden die leidt tot een ontwerp dat zowel haalbaar is als de hoogste voorkeur heeft van alle beslissers. PFM biedt een correct model voor de meting van voorkeur en voor de selectie van de oplossing met de hoogste voorkeur. In zijn huidige vorm is PFM een evaluatiemethode. Dat wil zeggen PFM helpt beslissers het alternatief te kiezen met de hoogste voorkeur uit een verzameling van reeds bekende alternatieven. Voor toepassing binnen de bouwkunde is echter een ontwerpmethodologie nodig. Dat wil zeggen een methode die leidt tot (ontwerp)alternatieven die *a priori* niet bekend zijn.

De ontwerpmethodologie Preference-Based Design (PBD) als beschreven in dit proefschrift, biedt een dergelijke ontwerpmethodologie. Van de LP-techniek, gebruikt PBD de definitie van een ontwerpalternatief als een combinatie van waarden van ontwerpvariabelen en de haalbaarheid bepaald door randvoorwaarden en toegestane marges van de ontwerpvariabelen. Elke beslisser kan bepalen in welke van de ontwerpvariabelen hij of zij is geïnteresseerd en zijn of haar voorkeur geven voor drie waarden van de betrokken ontwerpvariabele. De beslisser beoordeelt dus een alternatief bekeken vanuit een waarde voor een ontwerpvariabele, onafhankelijk van andere ontwerpvariabelen en ongeacht de haalbaarheid. Verschillende ontwerpvariabelen vertegenwoordigen daarmee verschillende criteria. Combinaties van waarden voor ontwerpvariabelen die voldoen aan de ontwerp randvoorwaarden vormen toegestane alternatieven. Deze worden geëvalueerd aan de hand van het PFM algoritme. Dat maakt het mogelijk het ontwerp

te vinden dat zowel haalbaar is als de hoogste voorkeur heeft van alle beslissers. Hiermee is het probleem in principe opgelost. Echter, de beoordeling van slechts drie waarden per ontwerpvariabele op voorkeur brengt een resolutieprobleem met zich mee doordat er geen tussenliggende waarden worden beoordeeld.

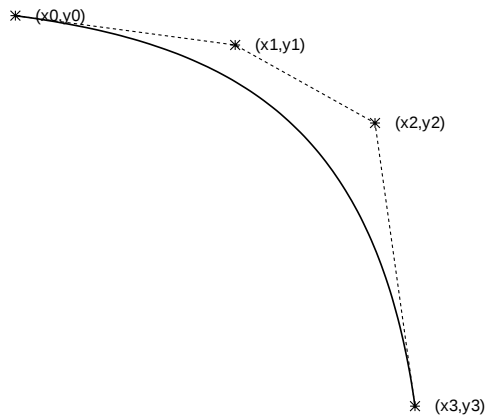
Verbetering van de resolutie impliceert het vinden van voorkeurscores betreffende tussenliggende waarden van de ontwerpvariabelen. Dit kan worden gedaan worden door het aanbrengen van een curve door de drie coördinaten van de voorkeurscores, dat wil zeggen door interpolatie. Tests voor dit doel met Lagrange polynomen en cubische splines leverden geen bevredigende resultaten. Beide oscilleren tussen hun wortels (knopen). Dit probleem kan worden vermeden door de beslisser te vragen om een cubische Bézier curve te construeren als weergave van de relatie tussen zijn voorkeurscores en de waarden van de ontwerpvariabelen. Een dergelijke curve (Figuur 54) wordt bepaald door twee punten en de afgeleiden in die twee punten. Het resolutieprobleem is hiermee opgelost.

De voorgestelde PBD methode is toegepast op drie cases: 1) Noordzee eiland voor een internationaal vliegveld, 2) Stedelijk Museum Amsterdam en 3) Spoorzone Tilburg.

De case van het Noordzee eiland resulteerde in een vrijwel gelijke uitkomst met PBD als de uitkomst verkregen met de op LP gebaseerde constraintmethode. De oplossing verkregen met de constraintmethode geeft een resultaat beïnvloedt door de modeleigenaar. De PBD procedure, daarentegen, levert een vergelijkbaar resultaat zonder inmenging van de modeleigenaar. Het resultaat is een zuivere weerspiegeling van de voorkeuren van de beslissers, hetgeen van belang is voor de acceptatie.

De case van het Stedelijk Museum Amsterdam levert verschillende uitkomsten voor de twee toegepaste methoden: LP met onderhandelbare randvoorwaarden en PBD. Dit eveneens als gevolg van het feit dat het PBD resultaat een zuivere weerspiegeling is van de voorkeuren van de beleidsmakers.

De Tilburg case toont dat gebruikers van mening zijn dat Bézier curves gemakkelijk zijn om mee te werken. Door de gebruikers werden



Figuur 54: Een cubische Bézier curve gedefinieerd door vier punten; eindpunten (x_0, y_0) , (x_3, y_3) , en controlepunten (x_1, y_1) en (x_2, y_2) .

de uitkomsten als aannemelijk gezien en het gebruik van de Bézier curves als gebruikersvriendelijk.

De volgende conclusies worden getrokken:

1. De PBD procedure is niet een uitbreiding van LP met onderhandelbare randvoorwaarden, maar een zelfstandige ontwerpmethodologie.
2. PBD is een ontwerpmethodiek die leidt tot een model waarin de voorkeuren van de beslissers zuiver zijn afgebeeld zonder inmenging van experts.
3. Resultaten uit de PBD procedure zijn zowel aannemelijk als bevredigend voor de beslissers.
4. De PBD procedure is een uitstekend startpunt in elk complex bouwkundig ontwerpprobleem.