

# BEAM OR TRUSS MECHANISM FOR SHEAR IN CONCRETE

*Problems converting a beam into a truss*

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# Preface

This research project on determining the difference between a concrete beam and a concrete truss is carried out as a Master's thesis project. The thesis is part of the study Civil Engineering with specialization Structural Engineering - Concrete Structures at Delft University of Technology. This thesis has been performed in collaboration with Royal HaskoningDHV.

The success of this thesis would not have been possible without the contributions of my graduation committee. In particular my daily supervisors Kees Blom and Rob Vergoossen, whose inspiring guidance and discussions contributed to the content of the research. Furthermore I would like to thank my colleges at Royal HaskoningDHV, especially Marcel het Hart for the tips and tricks of TNO DIANA. Lastly I would like to thank Professor A.W. Beeby (2011†) which provided the basis of the research.

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# Abstract

An unpublished study by Prof. A.W. Beeby shows the differences in strength capacity between a reinforced concrete beam without shear reinforcement and the same beam with a cut-out section at the middle of the span. The cut-out section exists at the bottom part of the beam while the reinforcement still remains. It remarkably turns out that the strength capacity of the beam with the cut-out section is 1.6 times larger compared to the reference beam.

The reference beam fails by a flexural shear crack which does not arise in the beam with the cut-out section. On the occasion of Beeby's experiments and the lack of a simple physical model for a flexural shear crack this thesis has the objective to clarify the difference between a beam and a truss mechanism and the failure due to flexural shear cracks. The study is based on a simple supported beam without shear reinforcement subjected to a concentrated load with a three point bending test with a slenderness-ratio of 2.45 and a reinforcement ratio of 0.89%.

An analytical study describes the difference between the beam and a simplified truss mechanism. Linear analyses show the differences in stress distributions and deflections. The study shows the same difference in strength capacity as the experiments of Beeby. In addition quite a difference is revealed in the displacements of both mechanism. The truss mechanism shows a larger deformation compared to the beam mechanism.

Finite element modelling with DIANA has been used to gain better insight in the difference of the strength capacity. The models use a total strain fixed crack model. The Hordijk-curve describes the tensile properties and an ideal relation describes the compressive properties. The decrease of the poisson ratio and the shear resistance around a crack have been taken into account by a damaged based shear retention model and a damaged based crack model.

The finite element models show differences of the strength capacity within the same level of Beeby's experiments. The force mechanism in both systems is different before the flexural shear crack arises in the beam. After a flexural shear crack occurs both mechanism seems to change into a similar truss mechanism, but detailed analyses show important deviations from this expectation.

Variation of the cut-out dimensions shows that a too small gap results in a flexural shear crack and a too large gap in the failing of the cantilever part. Gaps between these limits all change into a truss mechanism which reaches the same level of failure load as the basic truss. The decrease of stiffness of the beam results in more compressive stresses in the truss mechanism preventing the occurrence of the shear crack. If the shear crack does not occur in the beam it results in a higher strength capacity.

A dedicated shaped beam which has initially exactly the shape of a shear cracked beam without the concrete part below the crack, has a different strength capacity compared to a regular shear cracked beam. The dedicated shaped beam proves that the crack shape itself has no influence on the ability of converting into a truss. It turns out that in the regular beam it is impossible to develop a perfect truss mechanism after a flexural shear crack due to the concrete that is still present beneath the crack. The concrete beneath the crack causes a different stress distribution in the top of the beam compared to the dedicated shaped beam without this concrete. A hypothesis is given for the failure of the shear crack.

The acquired knowledge of the influence of the concrete beneath the crack and the stiffness of the beam allows other design possibilities. It is possible to design a concrete truss, if among other, the yielding of steel, crushing of the concrete and the deformation capacity of the truss are taken into account. Further research is for instance possible for unbonded reinforcement and beams with a descending height.

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# *INTRODUCTION*

*INTRODUCTION*  
*LITERATURE STUDY*  
*BASIC PRINCIPLES*

# 1 Introduction

A part of the existing structures, especially concrete bridges, does not meet the current standard for new structures. The standard evolves by new methods and theories. The increase in quantity and frequency of traffic loads and the increase of traffic lanes change the load which the structure has to resist.

Several structure checks revealed a lack of shear capacity in the majority of the concrete bridges. In 2007 Rijkswaterstaat started a research program together with TU Delft and TNO on this topic. After the first phase of the research 1.179 structures had been appointed for possible inadequate life span[1]. During the research several hidden capacities were found which indicate that there is still enough safety in the existing structures.

The problem within the shear capacity is that there is no simple physical model for shear failure, resulting in the low boundary of the current norms. On the other hand there is a universally accepted view for flexure failure which meets the experimental test results. The shear failure has a lot of influence factors which determine the failure mechanism. The researches on shear failure have a long history, but there is still no widely accepted physical model for shear. Some of the critical influence factors are the span to depth ratio, the kind of loading, the reinforcement quality and amount, the laterally restrain factor and the concrete quality.

An unpublished research is the study of Prof. dr. ir. A.W. Beeby in 2000[6]. He showed the difference in capacity between a truss and a beam with the same dimensions. An impression is shown in Figure 1.1. It is non-intuitive that the truss has 1.5 times the capacity of the beam. According Beeby, the beam is unable to act as a truss resulting by the cracks out of the bending of the beam. But what are the limits and how is the transition between these mechanisms?

The conclusion of Beeby for the different failure loads is the inability of the beam to transform into a truss by the cracks which occur by bending. The cracks caused by bending of the beam, cut the line of compression of the truss, which ensure that the beam is not able to convert into a truss. But what determines the real difference of these failure modes and why is the beam unable to transfer into a truss? According to the research, a diagonally tensile failure crack occurs before beam can perform as a truss.

The results of the experiment are not modelled and no further information is available on the limits between the bending and truss mechanism. But what are the limits for this difference and how do beams react with other parameters of these mechanisms?

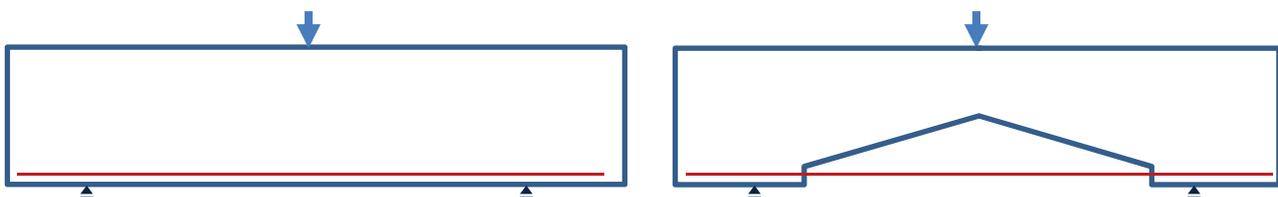


Figure 1.1 Beam and truss model of the Beeby experiment

## 1.1 Research question

The not availability of simply physical model for shear and the remarkable research gives the basis for these research. This results in the following research question:

*Why is a concrete beam on shear unable to transform into a truss mechanism?*

First is tried to find the difference between a truss and beam mechanism followed with the finding the answer of the research question.

To answer this question the research and the document are build up in different steps with each their own targets.

### Step 1: Introduction

In this step the research question is formed and the available theories on this topic are summarized. Some basic principle are explained to avoid uncertainty in this research.

### Step 2: Analytical models

In step 2 the differences between a beam and truss mechanism are reviewed with analytical models. It describes the difference in force and stress distribution and the belonging deflection with analytical models. These analytical models give a basis and simple inside in the difference between the mechanisms. The failure limits are based on the limits of the Eurocode.

### Step 3: Finite element models

With the analytic models an insight is given in the problem. With Finite element models it is possible to have a deeper look into the stress and displacement developments in the beam. Second order effects are taken into account.

This step starts with the explaining of the chosen parameters, important for finite element calculations. With these parameters the basic beam and truss models are reviewed. With the knowledge of this basic models the dimensions of the models are varied to find the boundaries.

### Step 4: Flexural shear crack

The finite element model gives some boundaries for the shear problem. In step 4 this shear problem is looked more into detail to find the properties of the occurrence of the shear crack and the failure mechanism of the crack

### Step 5: Truss design

With the knowledge of the shear crack and truss parameters it is possible for designing with a truss. In this step a deeper look is taken and a further variation is used to show the boundaries.

### Step 6: Conclusion

All these step together give some conclusions and recommendations.

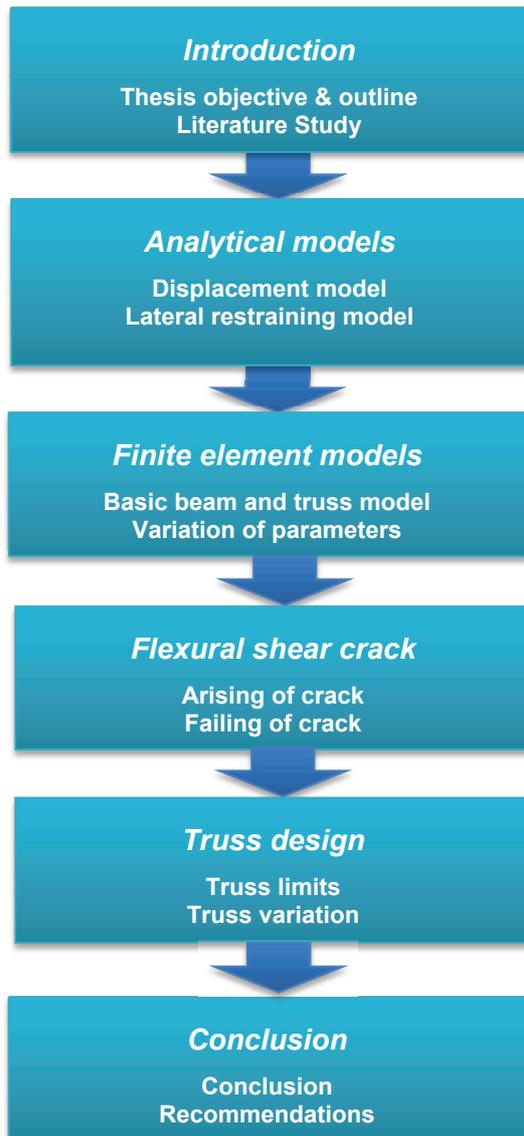


Figure 1.2 Thesis Scheme

## 1.2 Scope

The target of the research is finding the limits between the shear failure of the bending mechanism and the strut mechanism.

This is a fundamental research, so safety factors will not be taken into account. The research will start with beam dimensions from the Beeby research and goes further with other dimensions.

No shear reinforcement will be used to gain a good insight in the difference of the beam mechanism and the truss mechanism of the total beam. The shear reinforcement would create another truss mechanism which has another failure mechanism.

To analyse the structure the finite element program DIANA will be used. For the modelling the "Guidelines for non-linear finite element analysis of concrete structures" [7] will be used.

## 1.3 Structure of document

The structure follows the steps of the researched as described in Paragraph 1.1. This is shown schematically in Figure 1.2.

The first part consist of an introduction to the subject and gives an overview of the available literature.

The analytical models give a basic comparison of the beam and truss mechanism.

With finite element models these mechanism are more looked into detail and variation of the parameters is reviewed

Out of these models, an explanation is given for the flexural shear crack. How does the crack occur and what determines it failing mechanism.

The models also give boundaries for the truss design which is further varied.

The study finishes with a conclusion and recommendation.

## 2 Literature study

The literature study gives an outline of the available theory and experiments. The study starts with the describing of the Beeby experiment, followed by the basic mechanic relations, Bernoulli regions, failing limits and the shear and crack models.

### 2.1 Experiment Beeby

The Beeby experiment shows the difference in shear failure mechanism between a normal beam and beams with specific cut-outs of the concrete. The experiments show that in specific situations the beam with less material has a higher resistance capacity. But other structures will fail at the same way.

#### 2.1.1 Tests and results

The experiment of Beeby consists of 8 tested beams with different dimensions. Figure 1.1 shows the dimensions of the tested beams. All the beams are tested with a point load in the middle of the span. Beams A, B and G are tested to give reference material. Beam C is tested with a cut-out of the concrete, but with the reinforcement still there. The same holds for beam D and E, but the cut-out sections have a different shape. Beam H and I have artificial crack formers. The straight and curved lines in Figure 1.1 give the lines where a layer is poured into the concrete which takes care that the concrete create no tensile interaction there.

The results are summarized in Table 2-1. The beams A, B and G are the reference beams and all fail by with diagonal tensile crack, the normal failure mechanism for this beam.

Beam C shows a remarkable result with 1.5 times the capacity before failing of the structure. Less material results in a higher restraining result.

Beam D and E also have external reinforcement for which a high result is also expected, both beams have the same kind of failure mechanism as the normal beams, but even fail by a lower load. An assumption for this difference is the length of the cut-out sections.

Beam H and I are used to give an indication what influences the difference. With the layers which prevent the bending cracks to cross the compressive strut. This works, because the beams H and I both have a high failing load.

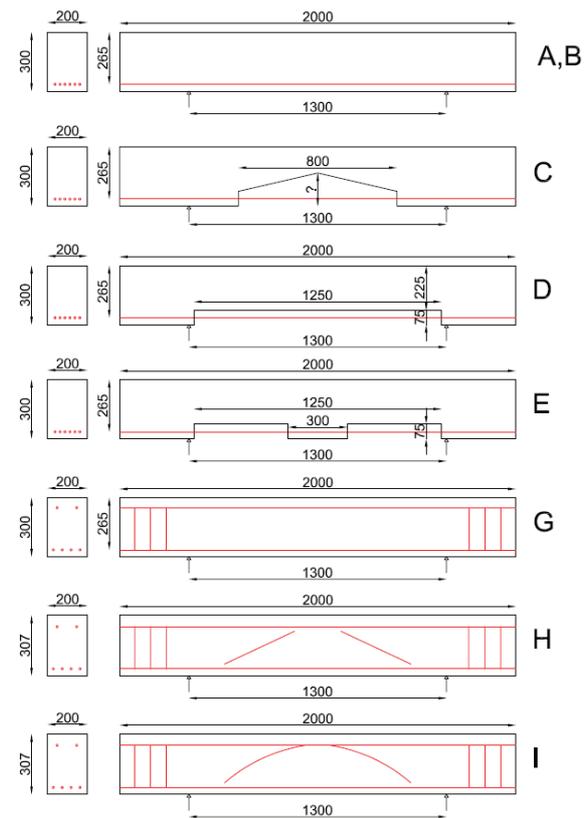


Figure 1.1 Tested beams

#### 2.1.2 Truss and beam mechanism

A remarkable test result is the difference between a beam and a truss-beam[6]. The truss-beam has around 1.5 times the capacity of the normal beam.

The working of the truss mechanism is depending on the tension in the reinforcement. In a theoretical truss mechanism the tension stresses are constant in the bottom truss while in a beam the tensile stresses in the reinforcement are proportional with the bending moment, with the assumption that the reinforcement takes all the tensile stresses. This principle is shown schematically in Figure 2.3.

Table 2-1

Beam	Ult. Load [kN]	Top Reinf.	Bottom Reinf.	Cube Strength [ $N/mm^2$ ]	Details	Failure mode
A	135	-	6 $\emptyset$ 10	50	Normal	Shear crack
B	123	-	6 $\emptyset$ 10	50	Normal	Shear crack
C	202	-	6 $\emptyset$ 10	50	Triangular cut-out	Bond failure
D	72.5	-	6 $\emptyset$ 10	50	Exposed Reinforc.	Shear crack
E	70	-	6 $\emptyset$ 10	50	Exposed Reinforc.	Shear crack
G	150	2 $\emptyset$ 10	4 $\emptyset$ 12	51.6	Normal	Shear crack
H	190	2 $\emptyset$ 10	4 $\emptyset$ 12	54.1	Straight crack	Crushing concrete
I	210	2 $\emptyset$ 10	4 $\emptyset$ 12	52.3	Curved crack	Crushing concrete

The test results show that this truss mechanism does not occur even when a shear crack occurs and the beam has enough deformation capacity. The research describes some possibilities for failing.

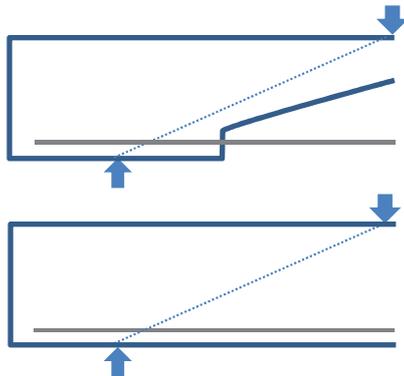


Figure 2.2 Truss-beam and normal beam

The shear cracks are a result of the bending cracks which result from the bending moment, because the beam wants to act as bending beam. When a shear crack occurs the steel is able to deform in such a way that a truss mechanism could occur. The beam fails suddenly after a shear crack occurs without reaching the theoretical truss failure, why this does not happen is not clear.

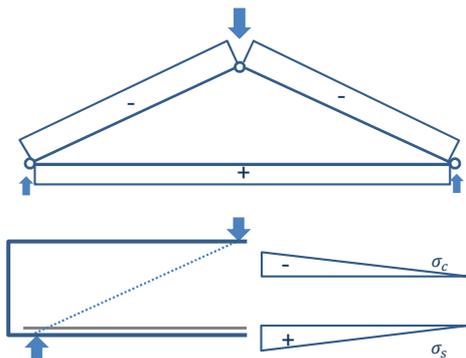


Figure 2.3 schematically truss and beam stresses

One of the possible failure mechanisms is a bond failure beyond the support, but with extra bond length the shear failure still occurs.

Another failure mechanism could be the rotation capacity of the hinge at the top of the beam, but this kind of failure does not occur in the cut-out beam, so enough rotation capacity is available in the beam.

The way of failure according Beeby is the failing by tensile stresses in the top of the beam. Beeby [6] describes a model in which the beam will fail by tension in the top layer of the beam when the shear crack influences the compressive strut.

Before a crack occurs the compression strut will have a uniform distribution of the stresses, as shown in the left picture in Figure 2.5.

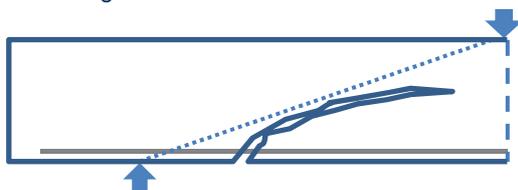


Figure 2.4 Shear crack

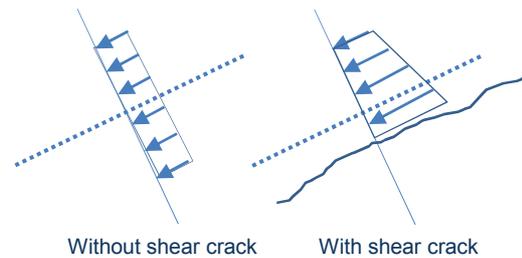


Figure 2.5 Compressive stresses truss mechanism

When a shear crack occur (Figure 2.4) Beeby describes that the stress distribution in the compressive strut will redistribute, with the assumption that it is not possible to have compressive stresses over the shear crack. The new distribution of the stresses causes higher stresses close to the crack. (Right in Figure 2.5). This distribution will create a moment in the beam which results in tensile stresses in the top of the beam.

When the tensile capacity of the concrete is reached there will be tensile failure at the top, which will lower the compressive centreline, increase the moments, followed with the increase of the tensile stresses. This results in failing of the beam.

### 2.1.3 Other cracks

With the difference in failing between the normal beam and the truss beam it is expected that beam D and E also have a higher failing load. But beam D and E fail like a normal beam by a shear crack. Beeby explains this difference by the width of the cut-outs. Beam D and E both have cut-out sections which are close to the supports and probably cross the concrete compression strut.

Beam H and I have artificial cracks. These artificial cracks are formed by layers which are poured into the concrete. These artificial cracks prevent that the concrete shear cracks will influence the compressive struts. The test result shows that the artificial cracks result in higher failure of the load. The failure is by crushing of the concrete instead of shear failure. Also in this situation the truss mechanism occurs.

### 2.1.4 Conclusion study

Leaving concrete out on specific places or creating artificial cracks results in higher failure loads. In a normal beam the truss mechanism does not occur. In the normal beam bending/shear cracks occur which result in failing of the beam without occurrence the truss mechanism. Why this does not happen is not clear.

## 2.2 Basis relations

This paragraph gives the basic relations which are important to give a relation between the loads on the beam, stresses in the beam and the deformations and displacement which occur in the beam. In Figure 2.7 the basic relation are schematically shown. The displacements are related to the deformations with the kinematic relations, like the constitutive and equilibrium relation do that for the deformations and stresses and stresses and loads.

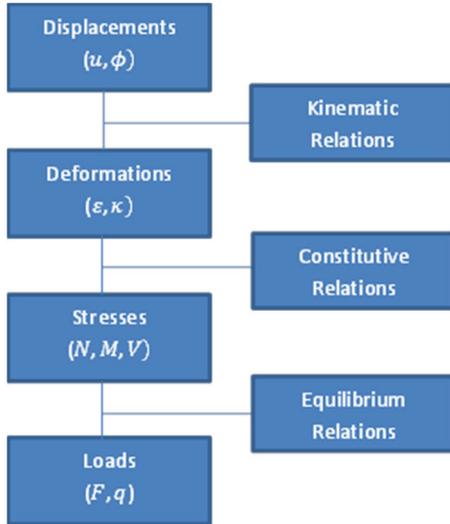


Figure 2.7 Basic relations

### 2.2.1 Euler Bernoulli bending beam

With some simple assumptions these relations become applicable for simple calculations. According to the Euler Bernoulli beam, the deformation of a beam is formed by the strain in longitudinal direction of the fibers.[8] A key assumption for the Bernoulli beam is that the plane cross-sections remain planar and normal to the beam axis in a beam which is subjected to bending. Out of these assumptions the following relations are derived.

#### Kinematic relations

The kinematic relations give a relation between the displacements and deformation. Within the Euler Bernoulli beam the next kinematic relations hold:

$$\begin{aligned} \varepsilon &= \frac{du}{dx} \\ \varphi &= -\frac{dw}{dx} \\ \kappa &= \frac{d\varphi}{dx} = -\frac{d^2w}{dx^2} \end{aligned} \quad [2-1]$$

#### Constitutive relations

The constitutive relations relate the stresses with the deformations. In the linear elastic state, shown in Figure 2.6, holds Hooke's law:

$$\sigma = E * \varepsilon \quad [2-2]$$

With this law the normal force in longitudinal direction are related with:

$$N = EA\varepsilon \quad [2-3]$$

With the assumption of Bernoulli that plane cross-sections remain planar and normal to the beam axis if the

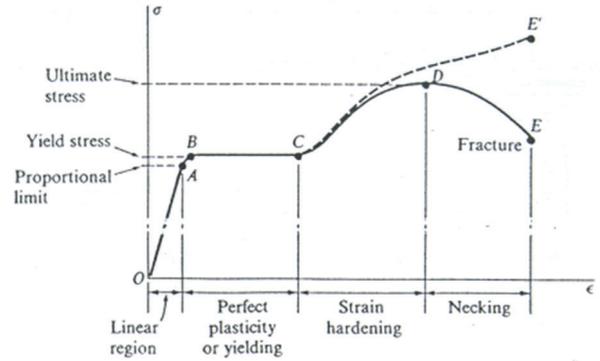


Figure 2.6 Conventional stress-strain diagram

beam is subjected to bending the moment in that cross-section is related to the curvature with:

$$M = EI\kappa \quad [2-4]$$

#### Equilibrium relations

In static situations the construction is stable. For a stable construction all the forces in all the directions have to be in equilibrium. This results in the next equations:

$$\begin{aligned} \frac{dV}{dx} &= q_z \\ \frac{dM}{dx} &= V \\ \frac{d^2M}{dx^2} &= -q_z \end{aligned} \quad [2-5]$$

All these relation together give a relation for the deflection and bending for a relatively slender structure:

$$-EI \frac{d^4w}{dx^4} + q_z = 0 \quad [2-6]$$

The min or plus sign is depending on the direction of the forces, but the relations stay the same.

Assumptions for the Bernoulli theory:

- Plane cross-sections remain planar and normal to the beam axis in a beam subjected to bending.
- Shear strains are approximately zero.
- Displacements are small.
- Beam has a straight longitudinal axis.
- Symmetric cross section about the y-axis.
- The beam is in linear elastic phase.

#### Timoshenko beam

The Timoshenko beam is an extension of the Euler Bernoulli beam. The extension is the adding of an extra degree of freedom, which causes shear stresses in the beam. The Bernoulli assumption is relaxed for the Timoshenko beam. The plane in the cross-sections remains not straight because an extra rotation is possible around the neutral axes. This results in the next Timoshenko beam formula:

$$-EI \frac{d^4w}{dx^4} + q_z = -\frac{EI}{\kappa * A * G} \frac{d^2q}{dx^2} \quad [2-7]$$

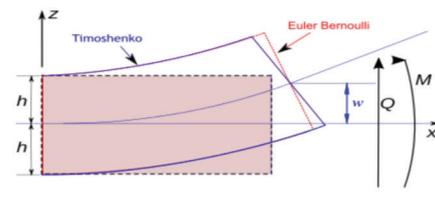


Figure 2.8 Difference Euler Bernoulli and Timoshenko

### 2.2.2 Shear stress for linear elastic stage

The bending of the beam causes shear stresses in the beam. A derivation of the shear stresses for the linear elastic stage is shown in the paragraph [8]. There are two kinds of shear stresses in a beam. Longitudinal and cross-section shear stresses, shown in Figure 2.9.

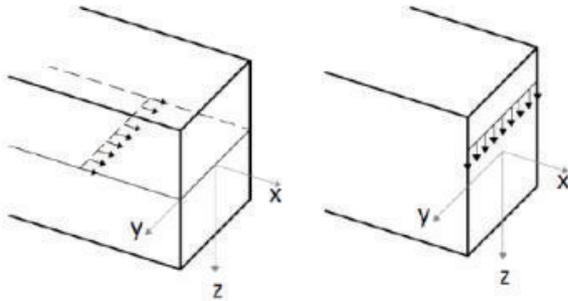


Figure 2.9 Longitudinal and cross sectional shear stresses

#### Shear stresses in longitudinal direction:

For explaining the stresses the cross-section in Figure 2.10 is analysed. By bending of the beam normal stresses occur in the beam according linear elastic calculation:

$$\sigma(z) = \frac{N}{A} + \frac{M_z z}{I_{zz}} \quad [2-8]$$

The stresses change over the length of the beam, this give a change in stresses, which cause shear stresses:

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta \sigma(z)}{\Delta x} = \frac{d\sigma(z)}{dx} = \frac{d}{dx} \left( \frac{N}{A} + \frac{M_z z}{I_{zz}} \right) \quad [2-9]$$

If the beam is prismatic and the normal force is constant it's rewritten to:

$$\begin{aligned} \frac{d\sigma(z)}{dx} &= \left( \frac{1}{A} \frac{dN}{dx} + \frac{z}{I_{zz}} \frac{dM_z}{dx} \right) \\ \frac{d\sigma(z)}{dx} &= \frac{V_z z}{I_{zz}} \end{aligned} \quad [2-10]$$

The normal stresses need to be in equilibrium. The marked part in Figure 2.11 is the rupture part, which need to be in equilibrium. Rupture occurs when this part is not in equilibrium.

The horizontal equilibrium for the rupture part:

$$\sum F_x = -N^a + (N^a + \Delta N^a) + s_x^a \Delta x = 0 \quad [2-11]$$

In which:

$s_x^a$  is the shear stress in longitudinal direction per cross section width.

$$N^a = \int_{A^a} \sigma(z) dA \text{ and}$$

$A^a$  is the area of the rupture part.

When  $\Delta x \rightarrow 0$  the shear stress become:

$$s_x^a = - \frac{dN^a}{dx} \quad [2-12]$$

For which:

$$\frac{dN^a}{dx} = \frac{d}{dx} \int_{A^a} \sigma(z) dA = \int_{A^a} \frac{d\sigma(z)}{dx} dA = \int_{A^a} \frac{V_z z}{I_{zz}} dA \quad [2-13]$$

If  $V_z$  and  $I_{zz}$  are constant for the cross-section this is rewritten to:

$$\frac{dN^a}{dx} = \frac{V_z}{I_{zz}} \int_{A^a} z dA = \frac{V_z S_z^a}{I_{zz}} \quad [2-14]$$

With this, the relation between the shear force and the longitudinal shear stresses becomes:

$$s_x^a = - \frac{V_z S_z^a}{I_{zz}} \quad [2-15]$$

With this expression the longitudinal (average over the width) shear stress,  $\tau_{gem}$ , can be calculated:

$$\tau_{gem}^a = \frac{s_x^a}{b^a} = - \frac{V_z S_z^a}{b^a I_{zz}} \quad [2-16]$$

In practice it is common to use the absolute value of these stresses:

$$\tau = \left| \frac{V_z S_z^a}{b^a I_{zz}} \right| \quad [2-17]$$

This holds if:

- The x-as passes the NC
- The z-direction is one of the main directions.
- The beam is prismatic
- The normal force is constant

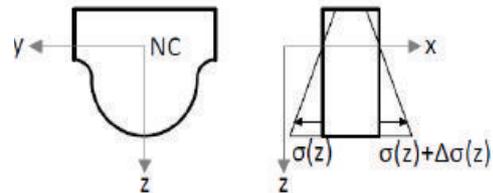


Figure 2.10 Cross section and slice of beam

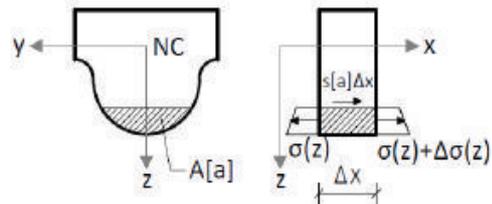


Figure 2.11 Longitudinal shear plane

#### Shear stresses in cross-section:

The shear stresses in the cross-section are related to the longitudinal shear stresses. To show this a rectangle block with the dimensions  $\Delta x \Delta y \Delta z$  is analysed. The block is shown in Figure 2.12. If the moment around A is taken (in the centre of gravity), only the longitudinal and cross section shear stresses are left for the equilibrium.

With the x/z- plane, shown in Figure 2.13, the moment equilibrium follows:

$$\sum T_y |A = +(\sigma_{zx} * \Delta x \Delta y) \Delta z - (\sigma_{xz} * \Delta y \Delta z) \Delta x = 0 \quad [2-18]$$

This results in:

$$\sigma_{zx} = \sigma_{xz} \quad [2-19]$$

The same relation holds for the other directions:

$$\begin{aligned} \sum T_x |A| &= 0 \\ \sum T_z |A| &= 0 \end{aligned} \quad [2-20]$$

This gives:

$$\begin{aligned} \sigma_{zy} &= \sigma_{yz} \\ \sigma_{xy} &= \sigma_{yx} \end{aligned} \quad [2-21]$$

Out of the moment equilibrium of a small rectangle element follows that the shear stresses of two perpendicular directions are equal.

$$\sigma_{ij} = \sigma_{ji} \text{ with } i, j = x, y, z \text{ and } i \neq j$$

**Conclusion:**

The longitudinal shear stress and de cross-section shear stress are the same if the planes are perpendicular and are:

$$\tau = \left| \frac{V_z S_z^a}{b^a I_{zz}} \right| \quad [2-22]$$

This holds if:

- The x-as passes the NC.
- The z-direction is one of the main directions.
- The beam is prismatic.
- The normal force is constant.
- The stresses are equal spread over the width
- The material is in linear elastic state.

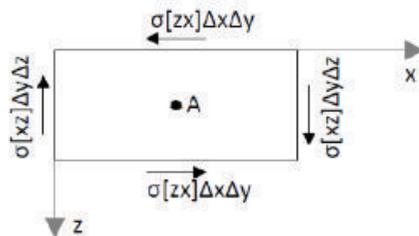


Figure 2.13 Moment equilibrium x/z-plane

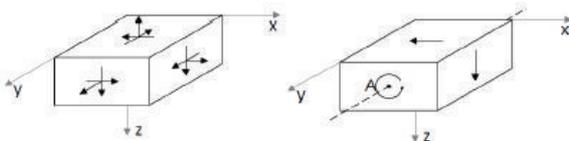


Figure 2.12 Stress resultants for  $\Delta x \Delta y \Delta z$  and moment A

### 2.3 B- and D- regions

For introduction to the stresses in a structure the difference is often made between Bernoulli regions (B - regions) and discontinues regions (D-regions).

B-regions are section in which the Bernoulli theory, described in 2.2.1, holds. In these regions the principle stress lines are almost straight for the compression and tensile member.

The D-regions are the discontinue regions where the constant or slowly changing stresses are disturbed. For instance by corners or force introduction. The most accepted theory about the size of d-regions is the St-Venant theory. St-Venant describes the discontinue zone as the disturbance length is equal to the width over which the forces have to distribute to a stress pattern according the Bernoulli theory, shown in Figure 2.15.

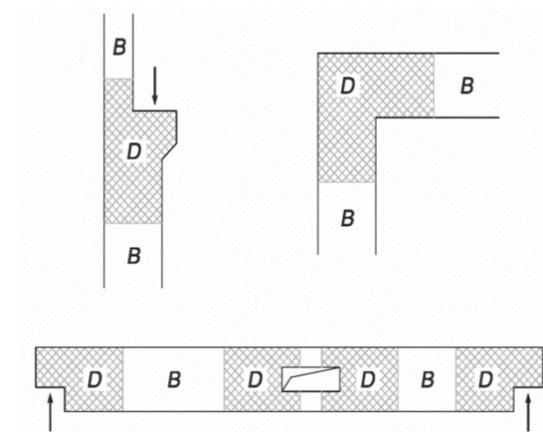


Figure 2.14 B-regions and D-regions

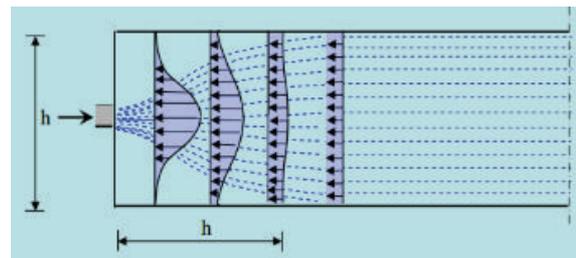


Figure 2.15 St-Venant disturbance length

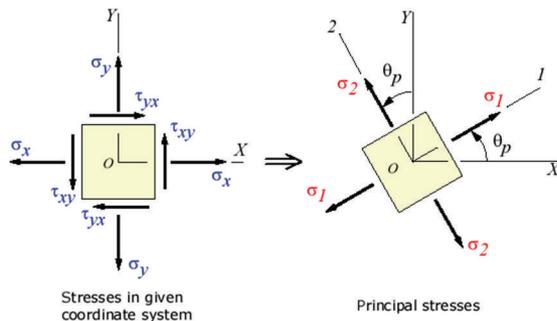


Figure 2.16 Principal stresses

### 2.3.1 Relation stresses and principal stresses [9]

The stresses in the mechanics are mostly given as stresses in a given coordinate system which meets the coordinate system of the structure element. The stresses consist for a small element out of normal stresses and shear stresses.

The stresses on the planes differ from the direction of the planes. The stresses of the planes of an element relate to each other with the following formula:

$$\sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} \quad [2-23]$$

If the element is turned in such a way that there are only normal stresses and no shear stresses the principal stresses are found. The principal is shown in Figure 2.16. These are the extreme normal stresses in the element, which determine the failure strength. These normal stresses are the maximum stresses out of equation [2-24] and are describes according:

$$\begin{aligned} \sigma_{\bar{x}\bar{x}} &= \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \frac{1}{2}(\sigma_{xx} - \sigma_{yy}) \cos 2\alpha + \sigma_{xy} \sin 2\alpha \\ \sigma_{\bar{y}\bar{y}} &= \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) - \frac{1}{2}(\sigma_{xx} - \sigma_{yy}) \cos 2\alpha - \sigma_{xy} \sin 2\alpha \\ \sigma_{\bar{x}\bar{y}} &= -\frac{1}{2}(\sigma_{xx} - \sigma_{yy}) \sin 2\alpha - \sigma_{xy} \cos 2\alpha \end{aligned} \quad [2-24]$$

Most of the failure models are based on this principal stresses.

#### Mohr's circle

A graphical form of this transformation is the Mohr circle, shown in Figure 2.17. The circle makes it possible to determine the stresses on the different planes and finding the maximum shear stresses.

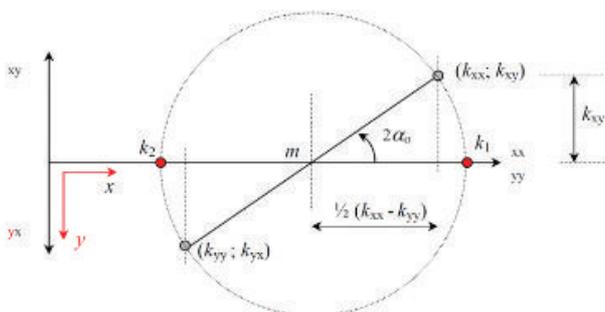


Figure 2.17 Mohr's Circle

### 2.3.2 Failure mechanisms

Failure occurs if a stresses exceeds a limit. For the limits are multiple theories. Most failure models are based on the principal stresses. Two common used models are the Von Mises failure model and the Tresca's failure model. Both models are described shortly in this paragraph.

#### Von Mises model:

The Von Mises model is based on the deformation due to the deviator stress component. The deviator stresses are the stress differences of the isotropic stresses. Von Mises models that failure only occurs if the stresses in different direction differs too much.

In formula form:

$$\frac{1}{3}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \leq s_{max}^2 \quad [2-25]$$

The criterion  $s_{max}$  is based on a uniaxial test. The limit stresses in the uniaxial test are  $\sigma_1 = f_y$ ;  $\sigma_2 = 0$ ;  $\sigma_3 = 0$ .

This results in:

$$s_{max}^2 = \frac{2}{3}f_y^2$$

With rewriting the formula the Von Mises criterion becomes:

$$\frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \leq \frac{1}{3}f_y^2 \quad [2-26]$$

This Von Mises criterion is visually presented in Figure 2.19.

#### Tresca's model:

The Tresca's model is based on a maximum for the shear stress criterion. If the yield stress in denoted with  $f_y$  Tresca's limit value  $c$  becomes:

$$c = \frac{1}{2}f_y \quad [2-27]$$

For a 2d-plane this becomes:

$$|\sigma_1 - \sigma_2| \leq 2c \quad [2-28]$$

For a 3d forms this results in:

$$\begin{aligned} |\sigma_1 - \sigma_2| &\leq 2c \\ |\sigma_2 - \sigma_3| &\leq 2c \\ |\sigma_3 - \sigma_1| &\leq 2c \end{aligned} \quad [2-29]$$

#### Comparison:

This criterion creates a hexagon limit. This is shown in Figure 2.19. In a plane, where  $\sigma_3 = 0$ , the Von Mises and Tresca's criteria are an ellipse and a hexagon. As shown in Figure 2.18 the Von Mises and Tresca's curve are fitting together. The differences are small. Tresca uses the shear stress to limit, while Von Mises uses the deviatoric stresses to limit. Von Mises is mostly used, because the curve is most suitable to use in a computer model. This failing models are theoretically and do not make a difference in tensile and compressive stress limits.

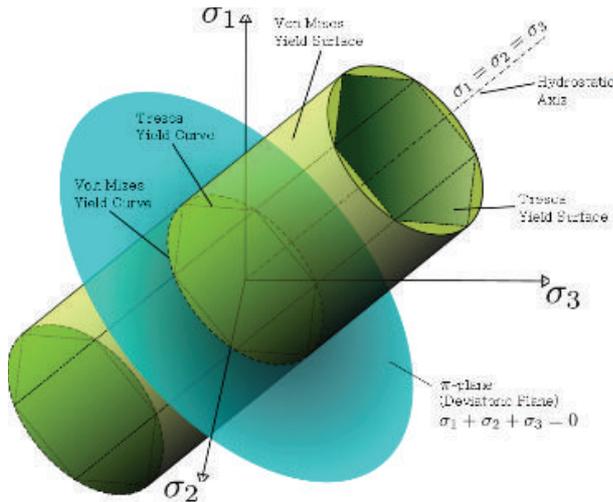


Figure 2.19 Von Mises and Tresca's criterion

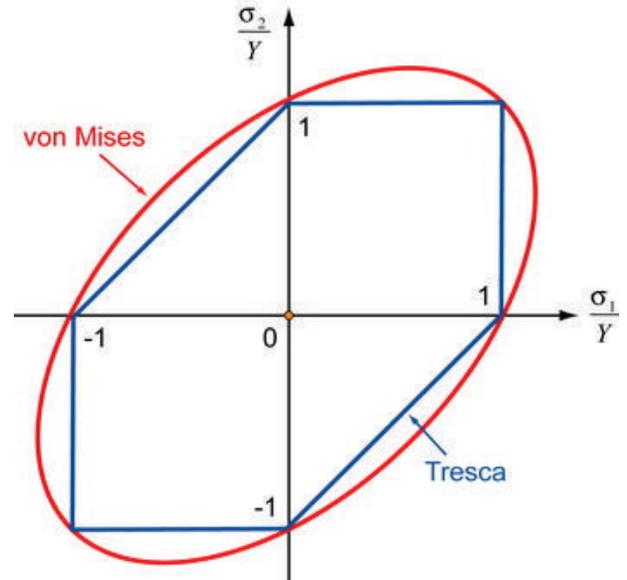


Figure 2.18 Von Mises and Tresca's criterion  $\sigma_3 = 0$

## 2.4 Concrete crack formation

Reinforced concrete has the capacity of the concrete for the compression and the capacity of the steel for the tension. But for distributing the forces the concrete first has to deform. In this paragraph a short and simple explanation of the principle is explained.

### 2.4.1 Crack formation

For a simple explanation a slender reinforced beam with reinforcement is used. In Figure 2.20 the force-elongation diagram of a reinforced concrete tensile member is shown, the different steps in diagram are described shortly. Before the concrete cracks the concrete and the reinforcement have the same elongation and the force is distributed equally, this is the uncracked stage. When the concrete reaches the maximal tensile stresses the concrete will crack and the steel will absorb the tensile stresses to create force equilibrium, the crack formation stage has started. The first crack occurs and by bond the stresses are distributed into the concrete. After the first crack occurs a second crack will occur at the weakest place in the concrete. This takes a while till no cracks have to occur to have enough deformation capacity. When no new cracks occur the tension is all in the steel

and the stabilized cracking stage is started. The stabilized cracking stage is able to resist more by the increasing of the tensile stresses and deformation in the steel. The limit is reached when the reinforcement starts yielding. The same holds for a beam in bending, but this happens in the tensile part of the beam.

## 2.5 Failure mechanisms

For shear are different fail mechanism, a short list of the possible failures is presented.

### Flexural failure

Flexure failure occurs when vertical cracks develop in the maximum bending moment ratio, it will fail by yielding of the steel or crushing of the concrete. The failure mode is shown in Figure 2.21.

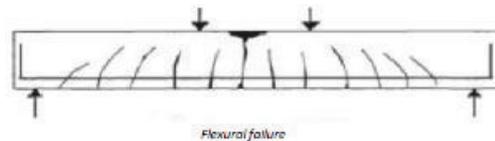


Figure 2.21 Flexural failure

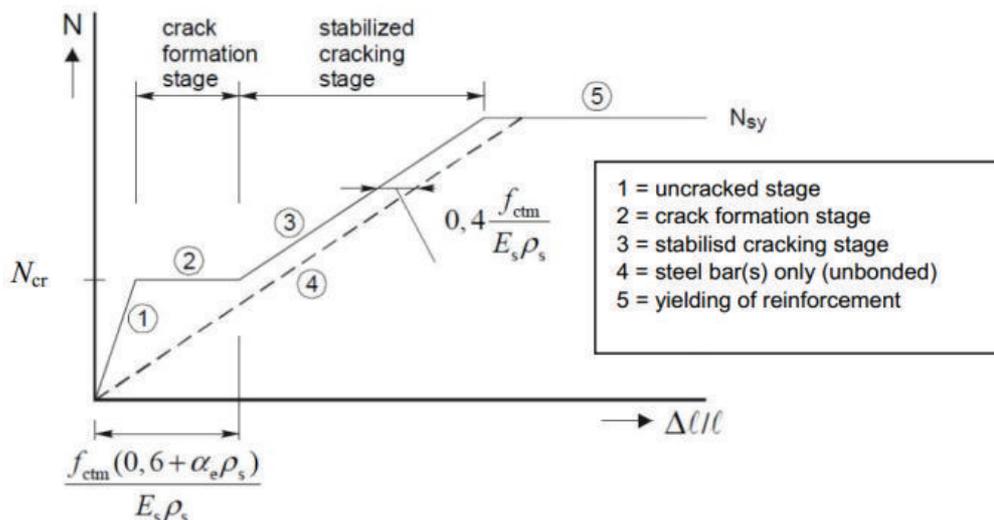


Figure 2.20 Force- elongation diagram of concrete

### Diagonal Failure

Flexural shear cracks starts in the tip of a bending crack and extends this crack. The flexural shear crack has 3 failure modes, shown in Figure 2.23. With diagonal tension failure the shear crack propagate through the beam until it becomes unstable. Within the shear tension failure, the crack propagates along the reinforcement and anchorage failure occurs. The shear compression failure is a mode in which the compressive stresses in the top become too large and fails. [10]

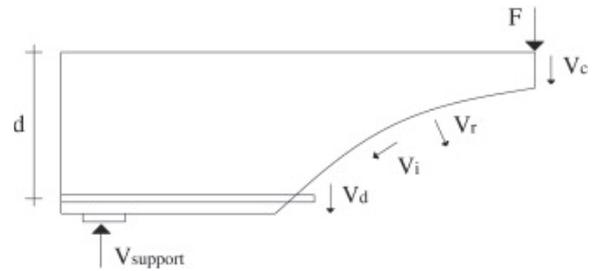


Figure 2.22 Schematic representation of shear transfer mechanisms

### Shear transfer in cracked concrete

A reinforced concrete beam with a critical diagonal tension crack has 4 mechanisms for transferring the shear force[11]. The mechanisms are shortly explained. There are several researches on this topic which each have another contribution of the shear forces between the mechanisms.

#### Shear in uncracked zone

In the uncracked zone there are still shear stresses present, shown as  $V_c$  in Figure 2.22. When cracks occur, the compressive stresses in the uncracked zone increase and the shear stresses could also increase.

#### Aggregate interlock [12]

If the surface of the crack is rough, there is resistance for slip if the protruding aggregate particles are larger than the crack width. The amount of transfer depends on aggregate size, concrete type, crack width and compressive strength. The aggregate interlock is shown as  $V_i$  in Figure 2.22.

#### Residual tensile stresses

According the stress-strain curve of concrete it can be observed that there are still some small tensile stresses in the concrete, shown as  $V_r$  in Figure 2.22. The tensile stresses could occur is there is no complete crack, but the crack consists of a lot of small cracks in the concrete. There is still some contact at some points.

#### Dowel action

The reinforcement is able to resist a part of the shear force, shown as  $V_d$  in Figure 2.22. The reinforcement is subjected to bending and shear, which result in dowel action in the reinforcement. The dowel action increases with the amount of longitudinal reinforcement. In FE-models this mechanism is commonly not modelled, for modelling this effect, the reinforcement bars should be modelled as beams rather than trusses.

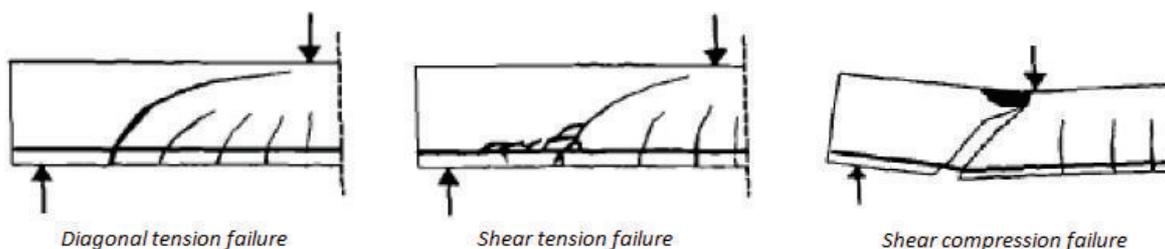


Figure 2.23 Diagonal failure mechanisms

## 3 Basic principles

### 3.1 Introduction

In the literature some different kind of definitions are used for several phenomena. The definition of a term differs in the literature, whereby the meaning is not clear. To clarify this kind of difference the definitions, some terms are described in this chapter to show of the term are used in this study.

### 3.2 Compressive membrane action

Compressive membrane action or arching are both used in the literature to describe some form of compressive action to restrain the load.

In this study compressive membrane action is used as a form in which the external force is passed through the construction by the use of compressive stresses as main form.

In concrete the compressive membrane action occurs by the difference in tensile and compressive strength of the concrete. By cracks another force mechanisms occurs which has force equilibrium by the compressive forces. The principle of compressive membrane action is shown in Figure 3.1.

The lateral restraining is necessary to gain enough stiffness to gain compressive membranes.

The truss mechanism is a special form of compressive membrane action. The compressive struts are clearly present in the truss and the lateral restraining is given by the horizontal tensile stresses in the horizontal strut as shown in figure

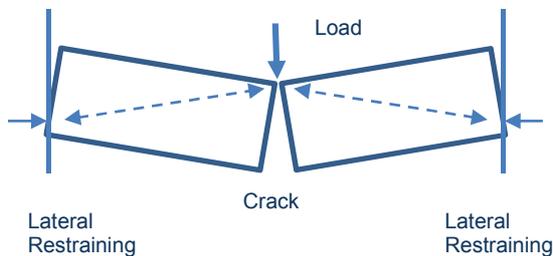


Figure 3.1 Compressive membrane action

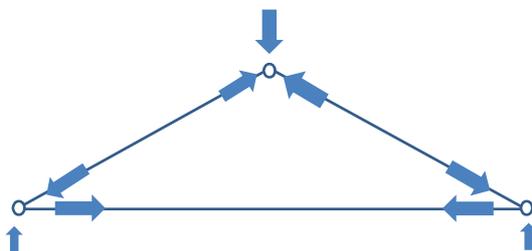


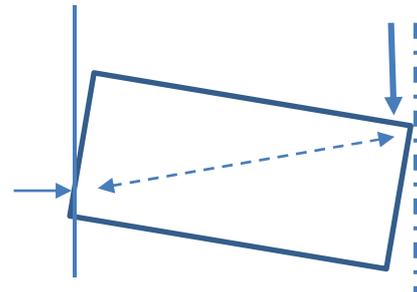
Figure 3.2 Truss mechanism

### 3.3 Lateral restraining

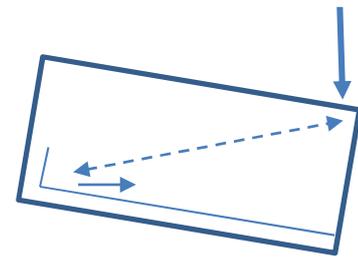
Lateral restraining of a beam is used for several forms in different literature. In this study the lateral restraining is used for the resistance of horizontal deformation of the beam. Difference is made in internal lateral restraining and external lateral restraining.

External lateral restraining is the resistance of the horizontal deformation of the beam by the surrounding situation of the beam. Like for instance a beam between 2 stiff walls, as shown in Figure 3.3. Most of the literature only described the external lateral restraining as lateral restraining

Internal lateral restraining is the resistance of horizontal deformation by the properties of the beam itself. For instance the reinforcement gives resistance to horizontal deformation, as shown in Figure 3.3.



External lateral restraining



Internal lateral restraining

Figure 3.3 Lateral restraining

# *ANALYTIC MODEL*

*BEAM OR TRUSS  
DEFLECTION  
LATERAL RESTRAINING  
FORCES AND STRESSES*

## 4 Beam and truss

The experiment of Beeby gives an indication for the way of force distribution of a beam with and without cut-out section. This difference is probably due to the difference in beam mechanism and truss mechanism. To find the difference between these mechanisms 2 theoretical models are used to find the differences between these systems.

Paragraph 4.1 starts with the basic properties of the concrete beam which will be used for comparing the mechanisms. Paragraph 4.2 describes a small introduction in the mechanisms which result in the compared models, described in paragraph 4.3. An indication for the optimal dimensions of the concrete strut is given in paragraph 4.4.

All formula used for the calculations are shown in this report. The total calculation is found in Appendix A.

### 4.1 Basic properties

The basic properties for comparing the mechanisms are based on the parameters used in the Beeby experiment. These parameters are used in the research if no further information is given.

#### Dimensions of the standard beam:

$$l = 1300 \text{ mm}$$

$$h = 300 \text{ mm}$$

$$d = 265 \text{ mm}$$

$$w = 200 \text{ mm}$$

$$A_s = 6\phi 10 \text{ mm} = 471 \text{ mm}^2$$

$$f_{ck} = 50 \frac{\text{N}}{\text{mm}^2}$$

$$f_y = 500 \frac{\text{N}}{\text{mm}^2}$$

$$E_c = 33.000 \frac{\text{N}}{\text{mm}^2}$$

$$E_s = 210.000 \frac{\text{N}}{\text{mm}^2}$$

The dimension parameters are shown in Figure 4.1

### 4.2 The beam and truss mechanism

To show the differences a comparison is made between a beam and truss with the same dimensions. The truss is formed in the beam, to compare the same span and depth parameters.

#### 4.2.1 Truss mechanism

A truss mechanism is shown in Figure 4.2. It consists of 3 struts which are connected by hinges. The reinforcement is used as tensile strut at the bottom and the two diagonal struts consist of concrete. The principle of a truss mechanism is that there are only normal forces in the struts and no shear and moment forces. External forces can only act on the hinges. The stresses in each truss are constant. The stresses cause deformation of the trusses which result in deflection of the total mechanism. The deformation of the mechanism is shown in Figure 4.2.

#### 4.2.2 Beam mechanism

A beam mechanism is shown in Figure 4.2. In a beam the stresses are spread over the beam height, which causes tensile stresses at the bottom and compressive stresses at the top. When the concrete cracks the reinforcement takes a part of the tensile stresses. The stresses cause also shear stresses to create equilibrium in the beam. The deformation of the beam mechanism is shown in Figure 4.2.

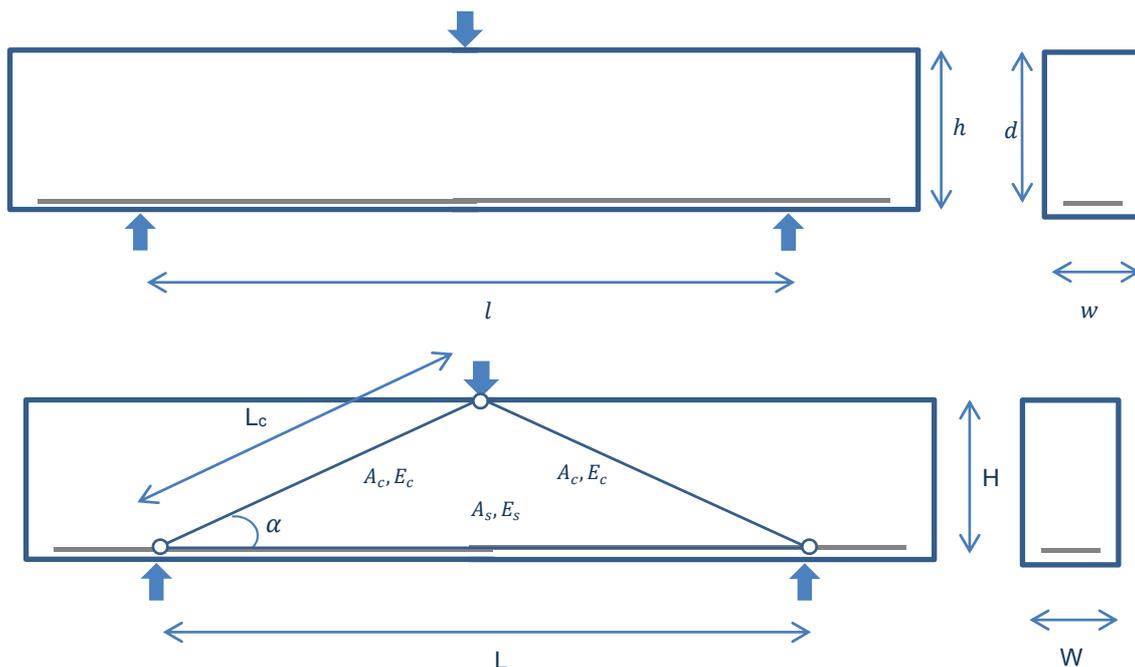


Figure 4.1 Beam and truss mechanism

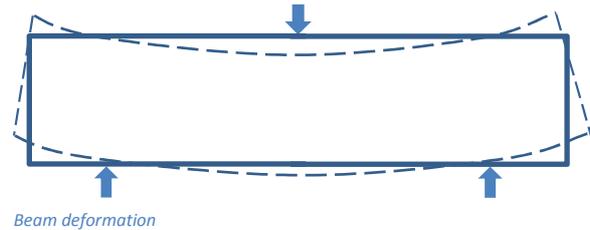
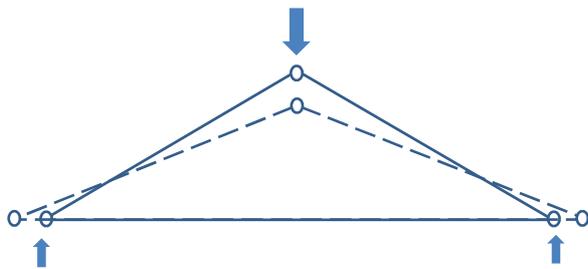
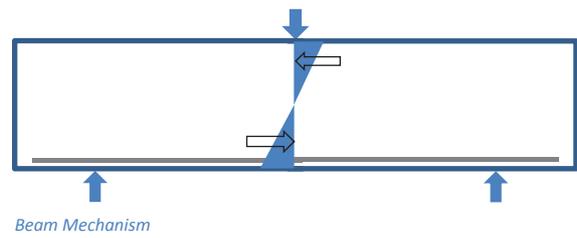
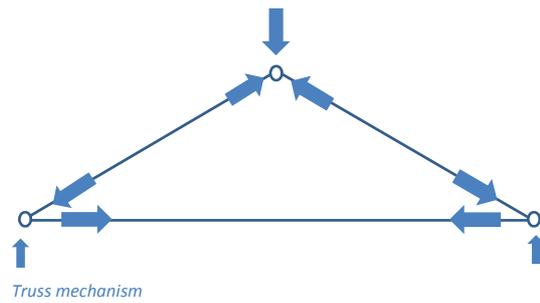


Figure 4.2

### 4.3 Compared models

The basic models give an indication for the real beam action. But in reality the boundary conditions and failure mechanisms has influence on the reactions of the mechanism. Four models are compared and shown in Figure 4.3.

#### 4.3.1 Truss models

The first model is the theoretical truss mechanism. The boundary conditions only take care of the vertical reaction forces. The struts have to form equilibrium to restrain the external forces.

The second model is a theoretical laterally restrained truss mechanism. In this model the struts cannot deform horizontally at the bottom. This gives less deformation and a higher stiffness of the mechanism.

In reality the hinges of the truss mechanism are not 100% moment free and the truss is not 100% laterally

restrained. So the reality will lie between these two models.

#### 4.3.2 Beam models

The beam mechanism acts differently in an uncracked and a cracked phase. In the uncracked phase the deformations of the steel and concrete are the same, which results in a small contribution of the steel.

After a certain loading the concrete will crack and the deformation of the steel will be higher and so the contribution of the steel. The steel takes almost all the tensile stresses and the concrete is used as compression zone in the top.

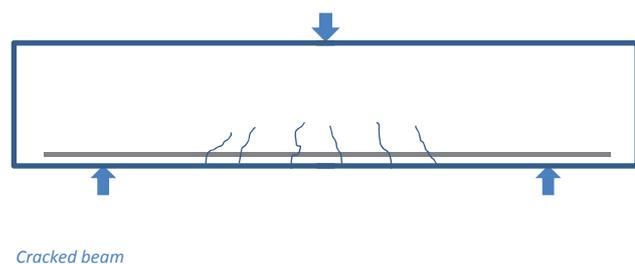
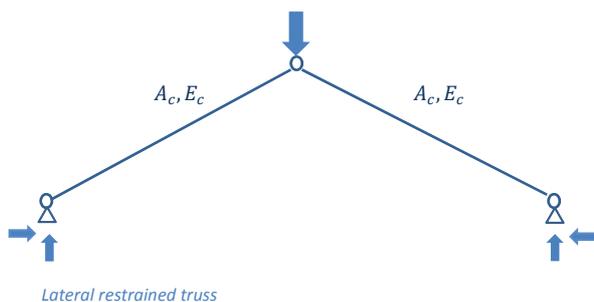
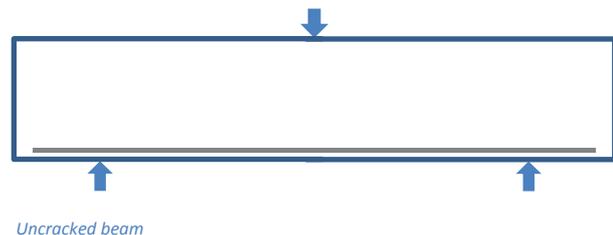
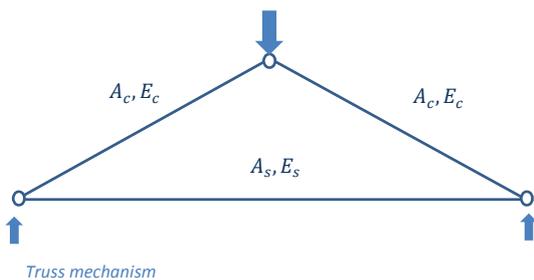


Figure 4.3 Compared models

#### 4.4 Dimensions of concrete strut

The truss mechanism has to fit into the beam to give a realistic model. But what are the dimensions of the concrete compressive strut?

In this paragraph a simple model is described which gives an indication of the width of the compressive strut.

First some assumptions are used to gain a usable model:

- The beam is 100% laterally restrained.
- The capacity of the concrete is  $f_{ck}$ .
- In the strut only acts normal force.
- The concrete stresses are equally spread in the compressive strut.
- Self weight of the concrete is not taken into account.
- Deformations are not taken into account.

The simplified model is shown in Figure 4.4. The concrete truss depth, ( $t$ ), influences the angle, ( $\alpha$ ), of the truss.

A higher strut gives a smaller angle and smaller angle causes higher concrete stresses in of the strut according to the horizontal force equilibrium.

But a higher concrete truss depth makes it possible to resist a higher force by having more area to gain stresses.

With the model an optimal concrete truss depth is found. The capacity of the strut is based on:

$$N_{strut} = s * \cos(\alpha) * width * f_{ck} \quad [4-1]$$

With follows from:

$$\begin{aligned} N_{truss} &= A_{truss} * f_{ck} \\ A_{truss} &= t * width \\ t &= s * \cos(\alpha) \end{aligned}$$

The force in the strut follows from force equilibrium of the truss:

$$F_{strut} = \left(\frac{F}{2}\right) / \sin(\alpha) \quad [4-2]$$

The maximum resistance of the concrete compressive strut is found when the force reached the capacity:

$$\begin{aligned} F_{strut} &= N_{truss} \\ \frac{F/2}{\sin(\alpha)} &= s * \cos(\alpha) * w * f_{ck} \end{aligned}$$

With this the maximum resisting force is found:

$$F_{max} = 2 * s * \cos(\alpha) * w * f_{ck} * \sin(\alpha) \quad [4-3]$$

With:

$$\tan(\alpha) = \frac{d}{L/2} = \frac{h-s}{L/2} \quad [4-4]$$

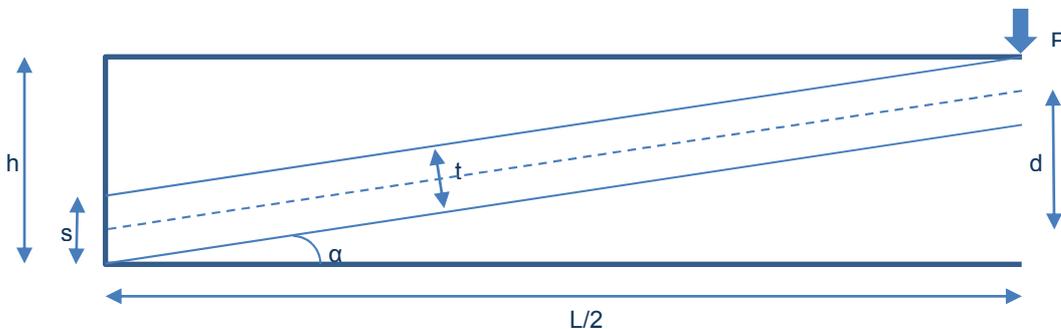


Figure 4.4 Compressive strut model

#### 4.4.1 Application

To give an impression of the model an example is shown with the properties:

$$L = 2m$$

$$h = 0,4m$$

$$width = 1m$$

$$f_{ck} = 20 \frac{N}{mm^2}$$

Table 4-1 the properties and forces are shown for different dimensions of the strut width. This is graphically shown in Figure 4.5.

It shows that the capacity of the strut is increasing with larger strut, while the total capacity of the truss system has a maximum of external force resistance by the increasing load for the horizontal equilibrium.

Table 4-1 Strut dimensions and forces

$s$ [m]	$d$ [m]	$t$ [m]	$N_{strut}$ [N]	$F_{max}$ [N]
0,00	0,4	0,000	0	0
0,02	0,38	0,019	374	265
0,04	0,36	0,038	753	509
0,06	0,34	0,057	1136	731
0,08	0,32	0,076	1524	928
0,10	0,3	0,096	1916	1100
0,12	0,28	0,116	2311	1246
0,14	0,26	0,135	2710	1363
0,16	0,24	0,156	3112	1452
0,18	0,22	0,176	3516	1510
0,20	0,2	0,196	3922	1538
0,22	0,18	0,217	4330	1534
0,24	0,16	0,237	4740	1497
0,26	0,14	0,257	5150	1428
0,28	0,12	0,278	5560	1324
0,30	0,1	0,299	5970	1188
0,32	0,08	0,319	6380	1017
0,34	0,06	0,339	6788	813
0,36	0,04	0,360	7194	575
0,38	0,02	0,380	7598	303

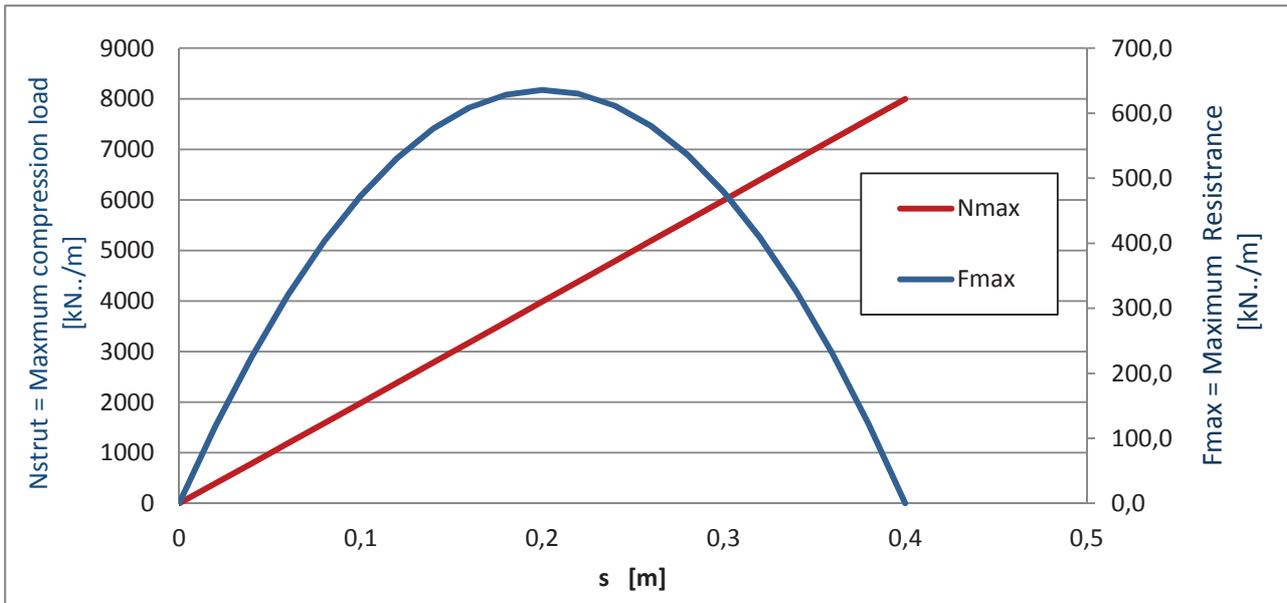


Figure 4.5 Capacity strut and truss model

#### 4.4.2 Parameter influences

The example shows the result for a specific beam. The maximum force resistance,  $F_{max}$ , depends on the concrete compressive strut dimensions, the span and the concrete strength. The maximum resistance is found when the derivative of the force resistance is equal to zero. The maximum of equation [4-3] is found with:

$$\frac{dF_{max}}{ds} = 0$$

Out of this follows, after some simplification, the strut height:

$$s_{max} = \frac{L^2 - L\sqrt{L^2 + 4h^2} + 4h^2}{4h} \quad [4-5]$$

The maximum width of the concrete strut,  $t_{max}$ , follows out of the relation:  $t = s * \cos(\alpha)$ .

$$t_{max} = \frac{L^2 - L\sqrt{L^2 + 4h^2} + 4h^2}{4h * \sqrt{\frac{4 * \left(h - \frac{L^2 - L\sqrt{L^2 + 4h^2} + 4h^2}{4h}\right)^2}{L^2} + 1}} \quad [4-6]$$

Both,  $t_{max}$  and  $s_{max}$ , only depend on the height and the span length. The concrete strength and width of the strut only determine the maximum value of the resistance, but

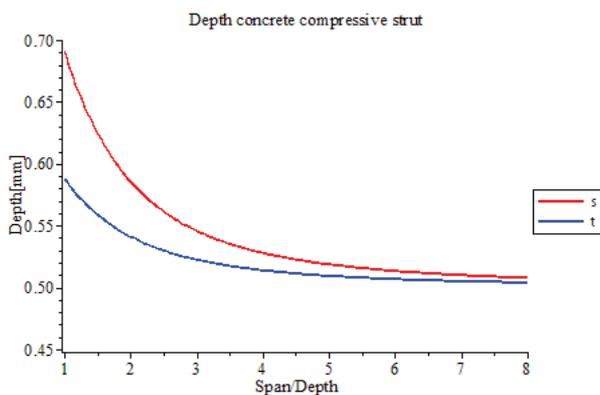


Figure 4.7 Depth compressive strut

not the dimensions for this maximum resistance.  $t_{max}$  and  $s_{max}$  are both shown graphical with the span to depth ratio ( $l/h$ ) as variable in Figure 4.7.

The figure shows that the maximum strut width goes to  $0.5h$  for higher span to depth ratios.

#### 4.4.3 Evaluation

This model for the concrete strut gives just an indication of the dimensions. Alternative forms are also possible. Some forms are shown in Figure 4.6. The form will depend on the strut-reinforcement interaction and the surrounding of the beam. The strut-reinforcement interaction depends on the layout and the sort of reinforcement. With the assumptions of 100% laterally restraining and the equally spread load it gives a good indication but not the exact solution. Also the deflections are not taken into account and these have influence on the result. The width of  $0.5h$  will be used for the models in the next chapters.

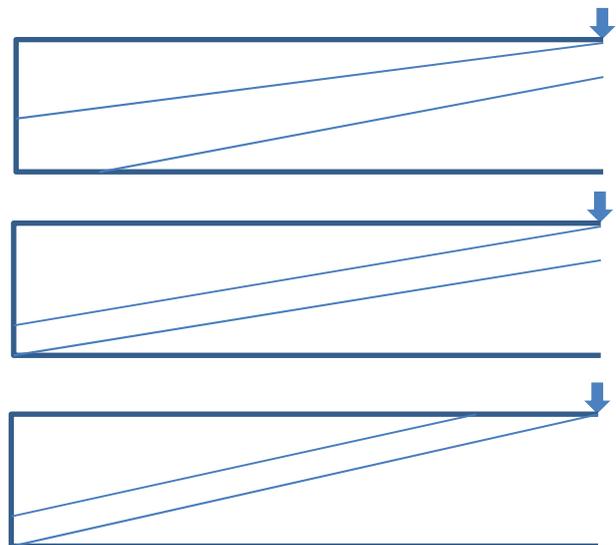


Figure 4.6 Compression strut alternatives

## 5 Forces and stresses

The truss and the beam mechanisms fail differently. A beam will most of the time fail by bending or shear, while a truss will fail by yielding of the steel or crushing of the concrete.

First the structures stress limit will be reviewed with simple checks of stresses. Safety factors are not taken into account to reach the failure stresses as good as possible with the simplified codes formulas.

### 5.1 Self-weight difference

Explaining the Beeby experiment the first thought is the difference in self weight. In this paragraph this is checked. The cut-out model is shown in Figure 5.1.

The self-weight of concrete is roughly  $24 \frac{kN}{m^3}$ . With the dimensions of the standard beam the weight becomes:

$$\begin{aligned} W_{beam} &= \rho_c * A_c * l \\ &= 24 \frac{kN}{m^3} * (0.2m * 0.3m) * 1.3m \\ &= 1.87 kN \end{aligned}$$

The dimensions of the cut out section are not exactly known. With the assumption that the maximum depth is 150mm and the started depth is 100mm the weight of the cut-out section is:

$$\begin{aligned} W_{cutout} &= \rho_c * A_c * l \\ &= 24 \frac{kN}{m^3} * (0.125m * 0.2m) * 0.8m \\ &= 0.48 kN \end{aligned}$$

The difference in weight between the normal beam and truss beam is 0.48 kN.

When comparing this difference with the failing load of 125 kN for the normal beam and 200 kN for the truss beam it can be concluded that the selfweight of the concrete of the cut-out section has a very small contribution. So this is not taken into account in the calculations.

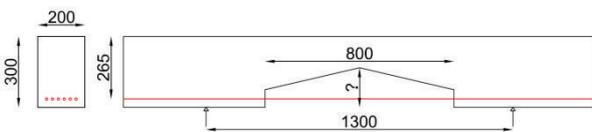


Figure 5.1 Dimensions truss beam

### 5.2 Verification with EC

To get an indication of the stresses and forces in the beam a first control is done with simple mechanisms and control of stresses out of the EC. All the safety and design factors are ignored.

#### 5.2.1 Truss mechanism

The truss mechanism has its limits in the concrete struts and the steel strut. The steel strut will fail by yielding of the steel and the concrete strut will fail by crushing of the concrete. In formula form:

$$\sigma_c = \frac{N_c}{A_c} = \frac{\left(\frac{F}{2}\right)}{\sin(\alpha)} \leq f_{ck} \quad [5-1]$$

$$\sigma_s = \frac{N_s}{A_s} = \frac{\left(\frac{F}{2}\right)}{\tan(\alpha)} \leq f_{yk} \quad [5-2]$$

Assumption still is that the hinges got no resistance and no failure mode.

#### Capacity

The steel strut will fail first with the standard beam properties and the assumption that the concrete strut width is half the beam height. This is shown for the standard beam subjected with a force of  $F = 100 kN$  in Figure 5.2. The figure shows the safety factor for every span. This is the present force divided by the capacity.

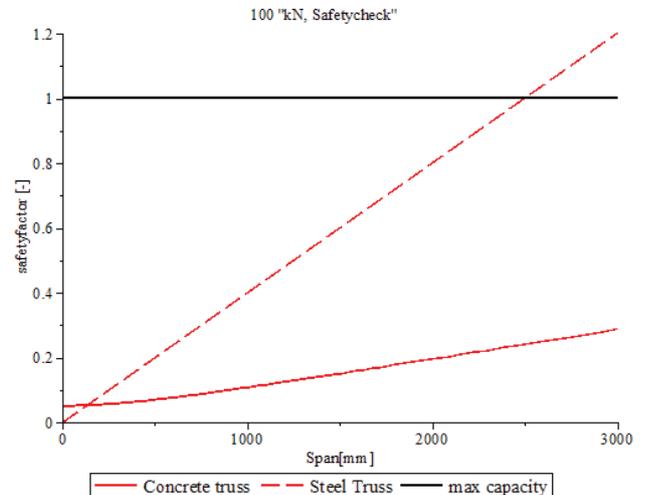


Figure 5.2 Capacity of truss mechanism

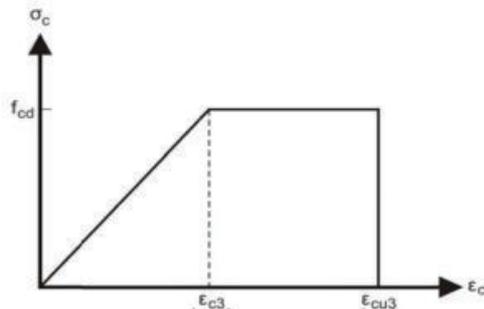
### 5.2.2 Beam mechanism

The main resistance of a concrete beam consist of the moment resistance and the shear resistance of the beam according the Eurocode.

#### Moment resistance

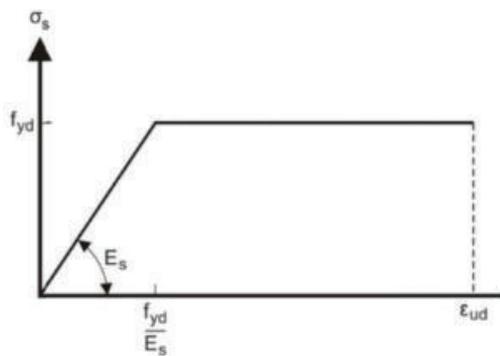
Concrete structures are designed on such a way that yielding of the reinforcement will occur before crushing of the concrete. This gives a warning mechanism before the real failure of the structure.

According the codes a beam will be check by moment at the maximum moment in the beam. The code is based on the Bernoulli theory in which the plane remains straight. The maximum moment resistance is based on a cracked beam in which the steel tensile member takes care of all the tensile stresses and a concrete compressions zone takes care of the compression.



$$\begin{aligned} \epsilon_{cu3} &= 3,5 \text{ ‰} \\ \epsilon_{c3} &= 1,75 \text{ ‰} \end{aligned}$$

Bi-linear stress-strain diagram concrete compression



$$\epsilon_{ud} = 45 \text{ ‰}$$

Stress-strain diagram reinforcement steel

Figure 5.4

The maximum moment according this theory is[13]:

$$M_{Rd} = A_s f_{yk} d \left(1 - 0.52 \rho \frac{f_{yk}}{f_{ck}}\right) \quad [5-3]$$

Assumptions taken:

- The concrete and steel properties are based on schematic diagrams which are shown in Figure 5.4.
- The plane with maximum moment remains straight.

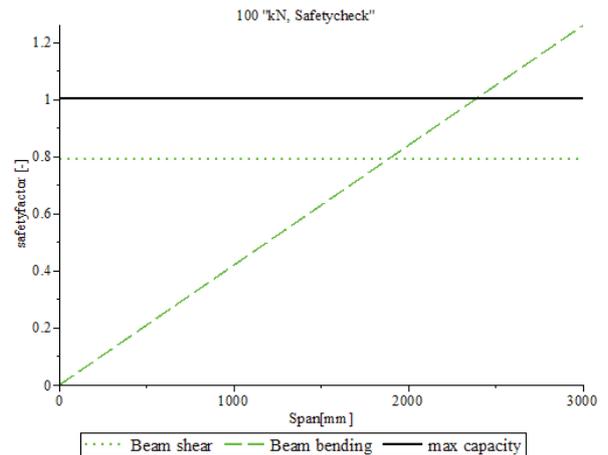


Figure 5.3 Capacity of beam mechanism

#### Shear resistance

The shear resistance is a mechanism which is difficult to describe with an exact failure mechanism.

The formulas used in several codes are all empirical. The empirical formula according EC2 without using shear reinforcement:

$$V_{Rd,c} = \left[ \left( \frac{0,18}{\gamma_c} \right) k (100 \rho_l f_{ck})^{\frac{1}{3}} \right] b_w d \quad [5-4]$$

With:

$$V_{Rd,c,min} = v_{min} * b_w * d$$

$$v_{min} = 0,035 k^{3/2} f_{ck}^{1/2}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2,0$$

$$\rho_l = \frac{A_{sl}}{b_w d} \leq 0,02$$

Equation [5-4] is depending on a lot of variables of the beam, but not on the span length.

#### Capacity beam

For the standard beam subjected to force of 100kN the shear resistance will not be reached, because it is not depending on the span length. The moment resistance safety check is increasing with the span length for this force. In the truss mechanism the steel strut is the weakest of the struts, and this will fail before crushing of the concrete strut.

### 5.2.3 Capacity beam and truss

To compare the beam and truss capacity the safety checks depending on the span length are plotted together in one plot shown in Figure 5.5. It shows clearly that this beam does not fail by shear but fails by bending mechanism with an increasing span length for this force. In the truss mechanism the steel strut is the weakest of the struts, and this will fail before crushing of the concrete strut.

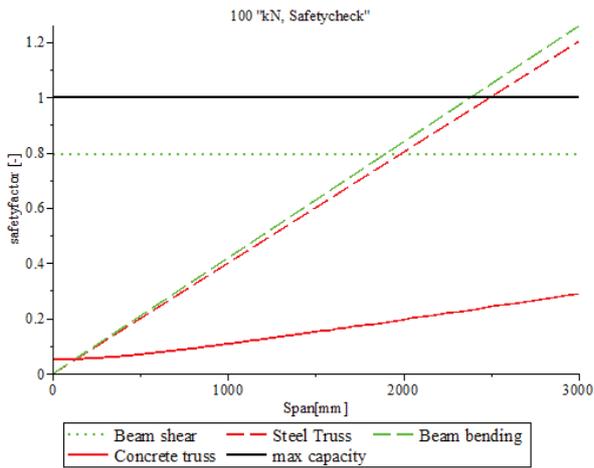


Figure 5.5 Safety check beam and truss mechanism

### 5.2.4 Force as variable

A beam is designed for a known span and an unknown force. So it is more convenience to use the force as variable. This difference is shown for the standard beam (Beeby beam) in the top figure of Figure 5.7. This graph shows clearly the difference in failure mechanism of a beam and truss mechanism.

#### Beeby experiment

The simplified model shows a same kind of difference as the Beeby experiment. The failure of the beam is by shear stress according the EC is 126 kN and the failure for the truss is the yielding of the steel for 192 kN. In Table 5-1 the ultimate loads of the model are compared with the ultimate loads from the Beeby experiment.

Table 5-1

	Shear limit EC	Steel yielding truss	Truss /Beam - Ratio
Model	126 kN	192 kN	1.52
	Normal beam	Truss beam	
Beeby	120 – 135 kN	202 kN	1.68– 1.49

This table shows that the models describe the different failure mechanisms in the beam of the Beeby experiment. The ratios are close to each other and the absolute values. The models give a proper indication for the kind of mechanism which occurs in the beam and truss beam of the Beeby experiment.

#### Span to depth ratio

The truss/beam-ratio of about 1.5 is reached for this specific span in the experiment. With other span to depth ratios the ratio between the extra capacity and the truss mechanism changes. For some different spans the safety check is done for a variable force which results are shown in Figure 5.7. With a low span ( $l = 500\text{mm}$ ) it is shown that the shear limit is reached first. In the truss mechanism this shear failure cannot occur which results in a higher failure loading. With a large span ( $l = 2500\text{mm}$ ) it shows that the limit is caused by the bending of the beam. The bending of the beam failure lies always close to the tensile member failure of the truss mechanism.

To show the difference in limit between the ratio between the truss mechanism and the shear mechanism this ratio is plotted for a different  $a/d$  – ratios in Figure 5.6. The other parameters are the same as the standard beam properties. The relations show that the benefit of truss mechanism in the beam is limited to span to depth ratio of about 3.5.

The truss mechanism is used for modelling the forces which lead direct to the support, but this is limit till a specific distance of  $2d$  in the EC, and still then is taken partly into account. This is because the truss mechanism does not always occur in the beam as shown in the Beeby experiment. In some span to depth ratio the shear crack has influence at the truss mechanism. If the load is close to the support it will not fail in shear. Increasing the distance the shear crack will occur, but a truss mechanism occurs after the beam is failed by shear crack. Increasing the distance even more a shear crack occur but no truss mechanism occurs after the shear crack. Increasing the distance even more will result in bending failure instead of shear failure.

### 5.2.5 Conclusion

The force model describes the different in loading capacity of the mechanisms. It shows that the failure mechanism for a truss mechanism is by yielding of the steel and the beam mechanism fails by shear. The shear influences the truss mechanism in the beam, but it does not describe why the truss mechanism does not occur in the beam.

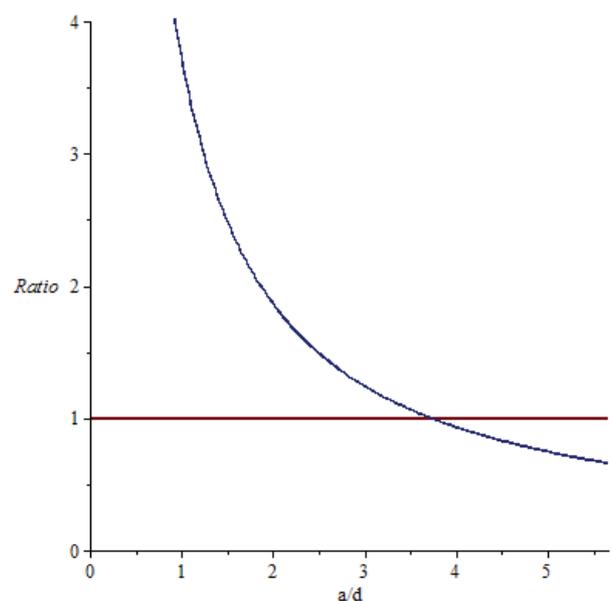


Figure 5.6 Truss/beam ratio for span to depth ratio

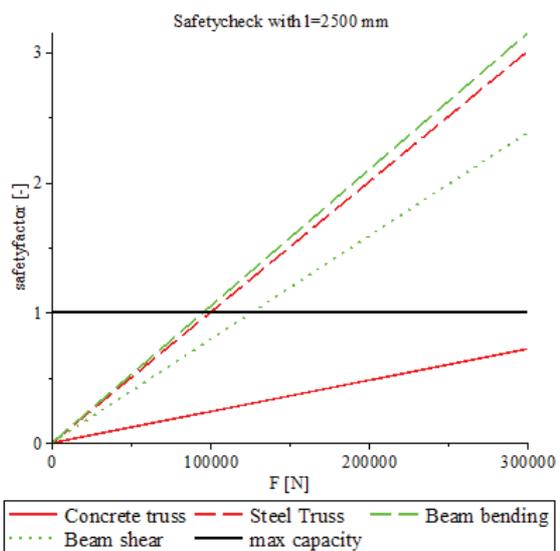
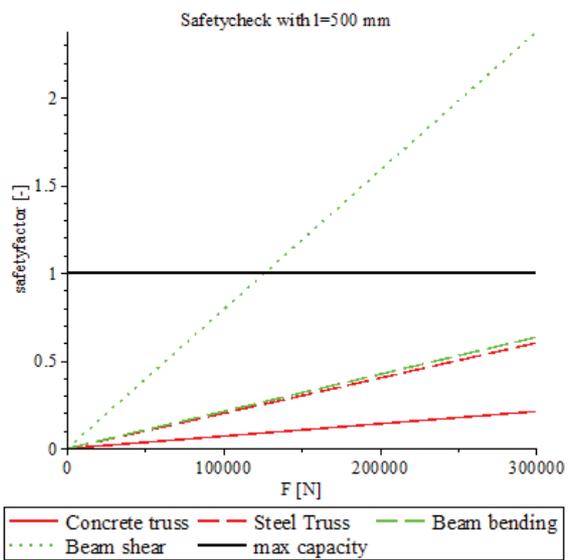
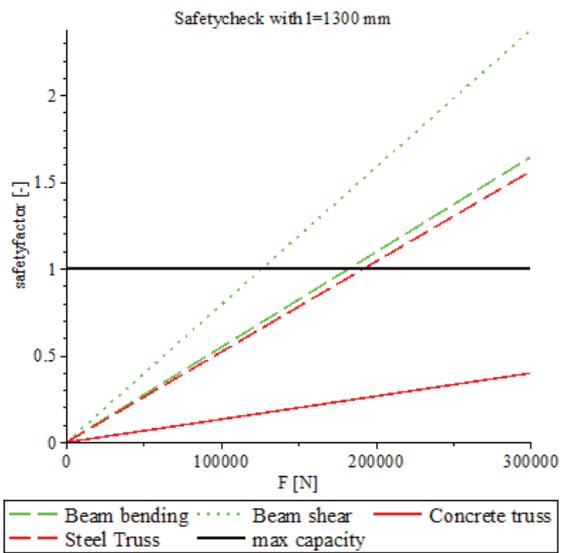


Figure 5.7 Safety factor for variable force and different spans

## 6 Deflection

The truss mechanism deforms by deformation of the struts, while the deformation of the beam is bending of the total structure. This chapter deals with the deflection of the different models.

### 6.1 Truss deflection

In this paragraph the deflection for a normal truss and a laterally restrained truss are discussed.

#### 6.1.1 Deflection of truss mechanism

The truss deforms due to the normal forces in the struts. For the dimensions of the trusses a certain concrete area  $A_c$  is taken into account with the assumption that this works as the concrete compression truss. As assumption the relation out of paragraph 4.4 is used, which assumes that the compressive member height is half the beam height.

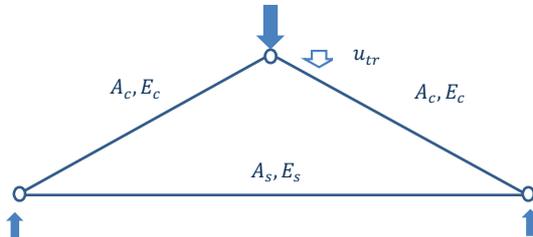


Figure 6.1 Truss mechanism

The shortening of the concrete compressive truss:

$$L_{c,def} = \Delta L_c + L_c \quad [6-1]$$

With:

$$\Delta L_c = -\frac{L_c N_c}{E_c A_c}$$

$$N_c = \left(\frac{F}{2}\right) / \sin(\alpha)$$

$$A_c = \frac{h}{2} * width$$

The elongation of the steel tension truss:

$$L_{s,def} = \Delta L_s + L_s \quad [6-2]$$

With:

$$\Delta L_s = \frac{L_s N_s}{E_s A_s}$$

$$A_s = \phi_s * n_s$$

Assumptions:

- Deformation gain no extra forces in the trusses.
- Only normal forces cause deformation.
- The deformation is according Hooke's law.
- The trusses remain straight.
- The concrete truss dimensions are assumed.

With the changed strut dimensions only one possible deformation is possible. The new height becomes:

$$H_{tr,def} = \sqrt{L_{c,def}^2 - \left(\frac{L_{s,def}}{2}\right)^2} \quad [6-3]$$

This gives a deflection of:

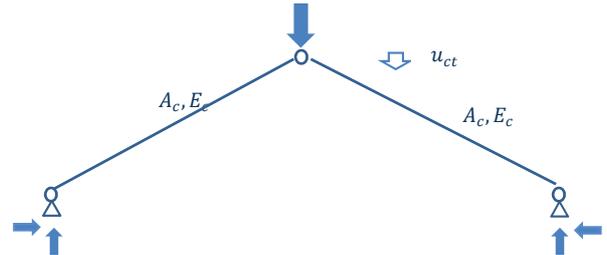
$$u_{tr} = H - H_{tr,def} \quad [6-4]$$

#### 6.1.2 Deflection of laterally restrained truss

The laterally restrained truss acts like the other truss, but without the deformation of the tensile strut. Only the deformation of the concrete struts is taken into account. Equation [6-1] still holds, but there is no elongation of the steel.

Assumptions:

- Deformation gains no extra forces in the struts.
- Only normal forces cause deformation.
- The deformation is according Hooke's law.
- The struts remain straight.
- The concrete strut dimensions are assumed.



With the changed dimension only one possible deformation is possible. The new height becomes:

$$H_{ct,def} = \sqrt{L_{c,def}^2 - \left(\frac{l}{2}\right)^2} \quad [6-5]$$

This gives a deflection of:

$$u_{ct} = H - H_{ct,def} \quad [6-6]$$

#### 6.1.3 Differences in truss deflection

The differences in deflection become clear when the results are shown graphically, Figure 6.2. The basic beam parameters are used to show the difference, with a variable span length. The real deflection of the truss mechanism will lie between these values.

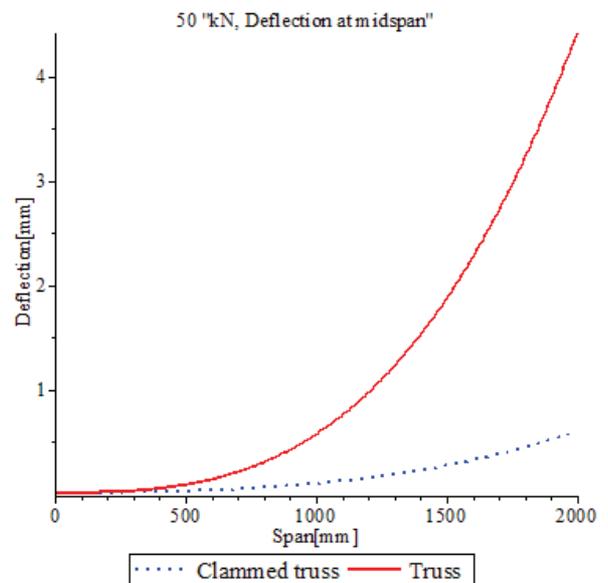


Figure 6.2 Deflection truss and lateral restrained truss

## 6.2 Beam deflection

In this paragraph the deflection for an uncracked and cracked beam are discussed.

### 6.2.1 Deflection of uncracked beam

The deflection of the beam is determined with the Bernoulli theory. For the uncracked beam model only the concrete is taken into account.

With the basic relation of the Bernoulli theory the deflection is related to the loading, according paragraph 2.2.1:

$$EI * w(x)'''' = -q(x) \quad [6-7]$$

and the boundary conditions:

$$q = 0 \text{ N/mm}$$

$$\text{At: } x = 0$$

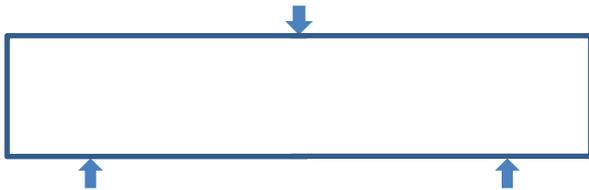


Figure 6.3 Uncracked beam

$$w_1 = 0 \text{ and } M_1 = 0$$

$$\text{At: } x = 0.5l$$

$$w_1 = w_2, \varphi_1 = \varphi_2, M_1 = M_2$$

$$\text{and } V_1 + F = V_2$$

$$\text{At: } x = l$$

$$w_2 = 0 \text{ and } M_2 = 0$$

This gives the deflection at the middle of the span:

$$u_b = \frac{1}{48} \frac{Fl^3}{EI} \quad [6-8]$$

Assumptions:

- Bernoulli theory is used.
- For the moment of inertia and the modulus of elasticity only the concrete is taken into account.

### 6.2.2 Deflection of partly cracked beam

When a beam is subjected to a force, cracks will occur if the tensile stresses of the concrete pass the concrete tensile strength. For this model it is assumed that the beam acts as a cracked beam when the cracking moment is passed. The moment which belongs to this cracking moment is:

$$M_{cr} = W * f_{ctm} \quad [6-9]$$

When the concrete cracks the reinforcement takes all the tensile stresses and a concrete compressive zone rises at the top of the structure. In the bending beam parts are cracked and parts are uncracked. The resistance and thereby the rotation differs for each section.

For the calculation of the deflection of the cracked beam the rotation is interpolated between the cracked and uncracked part of the beam[14]

For the uncracked part holds:

$$\kappa_I = \frac{M}{(EI)_I}$$

With:

$$(EI)_I = E_c * \frac{1}{12} * w * h^3$$

For the cracked part holds:

$$\kappa_{II} = \frac{M}{(EI)_{II}}$$

$$(EI)_{II} = E_c * \left( \frac{1}{12} wx^3 + (wx) \left( \frac{1}{2} x \right)^2 + \alpha_e A_s (d - x)^2 \right)$$

With:

$$\text{Compressive height: } x = \left( -\alpha_e \rho + \sqrt{(\alpha_e \rho)^2 + 2\alpha_e \rho} \right) d$$

$$\alpha_e = E_s / E_c$$

The Eurocode[15] gives a distribution factor for pure bending. It includes the tension stiffening.

$$\zeta = 1 - \beta \left( \frac{M_{cr}}{M} \right)^2$$

The curvature is calculated by interpolation with the distribution factor:

$$\kappa^* = \zeta \kappa_{II} + (1 - \zeta) \kappa_I$$

The maximum moment is used ( $0.25 * F * l$ ) to give the upper boundary for the deformation. This gives a proper estimation most of the time, because the sections in the middle of the span have the highest contribution to the deformation.

The deflection at the middle of the span for the uncracked beam becomes:

$$u_{b,cr} = \frac{1}{48} \frac{4M_{max}l^2}{EI} = \frac{4}{48} \kappa^* l^2 \quad [6-10]$$

Assumptions:

- Only use of the maximum moment
- Creep and shrinkage are not taken into account
- Interpolation of curvature

The calculation gets a higher accuracy if the moment and curvature are calculated for every slide of the beam, but this way of calculation gives a proper approximation[14].

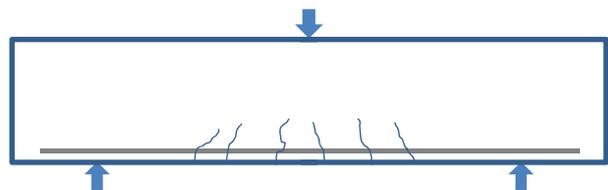


Figure 6.4 Cracked beam

### 6.2.3 Differences in beam deflection

The differences between the uncracked and cracked beam are shown in Figure 6.6. Until the cracking moment is reached the models act the same. When the beams reach the cracking moment the cracked beam gets the first crack and the beam drops by the extra rotation capacity. The model of the cracked beam models the deflection smooth after the first crack. In reality the occurrence of multiple cracks will give some disturbed deflection. The real deflection of a concrete beam lies close to the cracked beam model, so this model is used in further comparison, as shown in Figure 6.5.

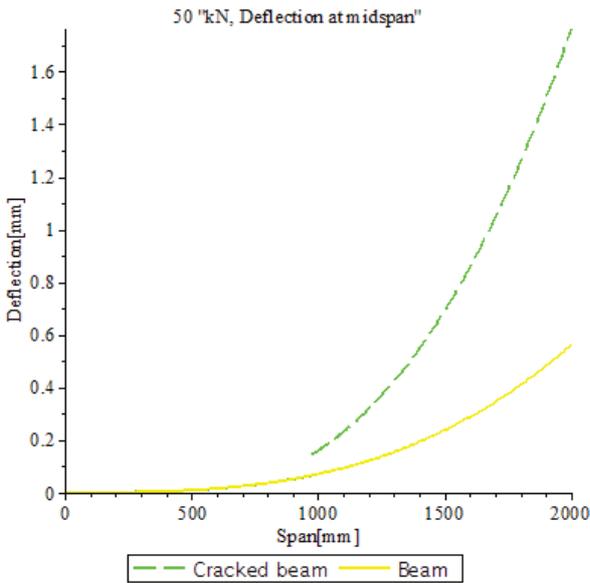


Figure 6.6 Deflection uncracked and cracked beam

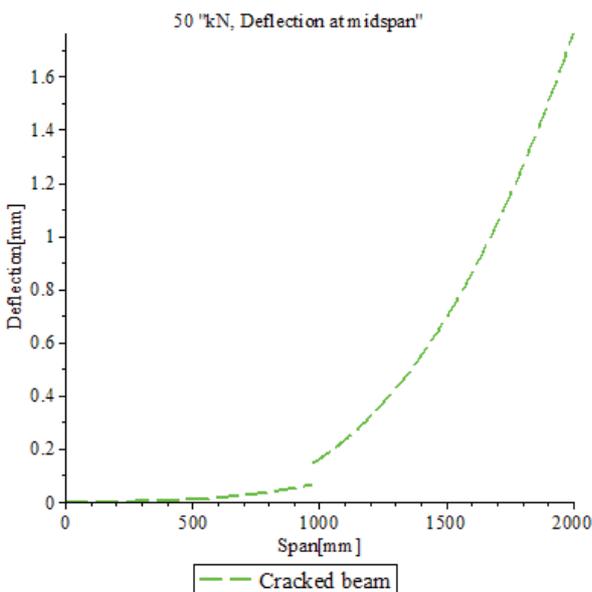


Figure 6.5 Beam deflection

### 6.3 Conclusion

The deflections of the truss and beam models are shown together in Figure 6.7. The difference is shown for 50 kN till a span of 2m and for 100 kN till a span of 3m. It shows that the deflection of the truss mechanism, without laterally restraining, is a lot higher as both beam mechanisms. The latterly restraining has a large influence on the stiffness of the truss mechanism. The real truss mechanism will lie between the truss and lateral restrained truss model.

This gives the following conclusions:

- The beam mechanism will handle the force if the lateral restraining is not enough, because the beam has a higher capacity for the same deflection.
- The laterally restraining of the truss has a lot influence on the deflection properties.

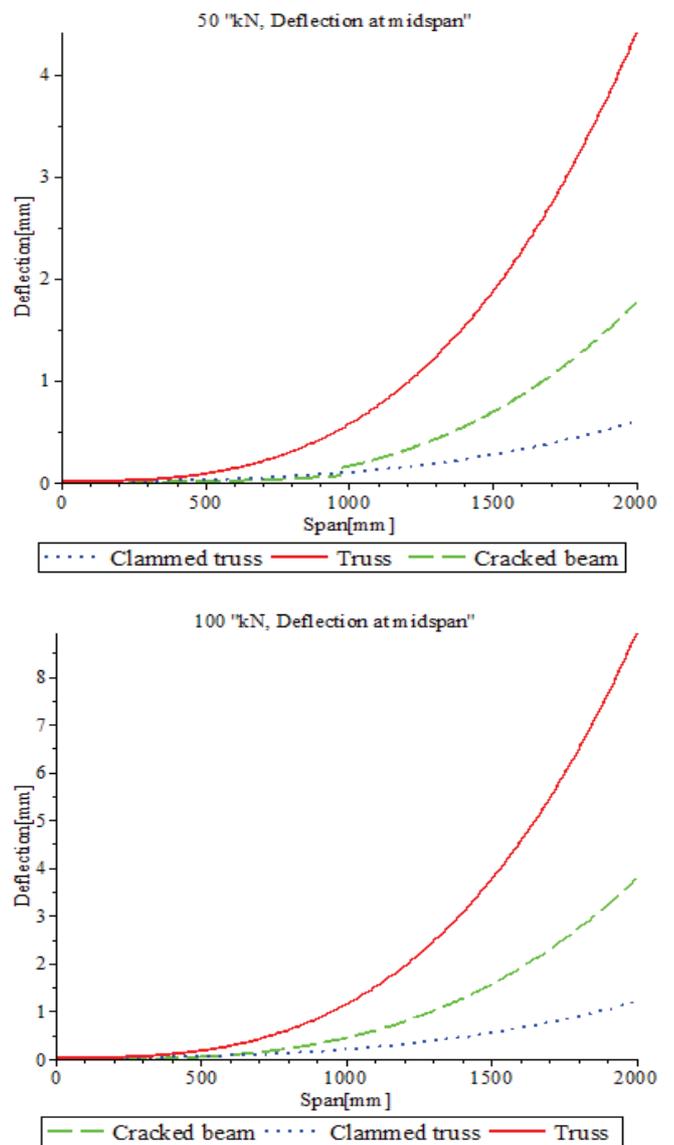


Figure 6.7 Deflection of all models for 50kN and 100kN with different spans

# 7 Lateral restraining

## 7.1 Internal lateral restraining

The 100% laterally restrained truss gives a higher resistance than a beam. But a 100% laterally restrained truss is practical not possible. But is it possible to get enough stiffness to reach the cracked beam model? In this paragraph is shown if it is possible to get enough stiffness from the reinforcement. This is modelled with the model shown in Figure 7.1.

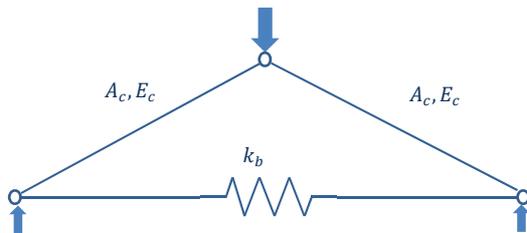


Figure 7.1 Spring model

Assumptions taken in the model:

- The spring has linear stiffness  $k_b$ .
- Only normal forces cause deformation.
- Deformation gains no extra forces in the struts.
- The struts remain straight.
- The concrete strut dimensions are assumed.

The relation between force and displacement is described as:

$$F = k * \Delta L \quad [7-1]$$

With:

$$k = \frac{E_s A_s}{l_s} \quad [7-2]$$

$\Delta L$ : Spring displacement  
 $k$ : Spring stiffness

### 7.1.1 The deflection

The shortening of the concrete compressive truss stays the same as equation [6-1]. The elongation of the spring truss becomes:

$$\Delta L_{sp} = \frac{N_s}{k_b}$$

With  $k_b$  consisting of the reinforcement in the beam. Again the new height is calculated with:

$$H_{bspr} = \sqrt{L_{c,def}^2 - \left(\frac{L + \Delta L_{sp}}{2}\right)^2}$$

This gives the deflection:

$$u_{kb} = H - H_{bspr}$$

Table 7-1

Span [mm]	Force [kN]	$k_b * 10^5$ [N/mm <sup>2</sup> ]	$A_s$ [mm <sup>2</sup> ]	$n$ $\phi 16$	c. t. c. [mm]	S [mm]
1300	50	3,18	1969	10	15	-1
1300	100	2,39	1480	8	19	3
1300	200	2,25	1393	7	22	6
6000	50	0,50	1429	8	19	3
6000	100	0,59	1686	9	17	1
6000	200	0,951	2717	14	10	-6

### 7.1.2 Spring properties

The deflection is calculated with the spring stiffness from the reinforcement of the standard beam with the properties according paragraph 4.1

For restraining the same forces with the truss model as least the same stiffness as the cracked beam should be reached by the truss.

To search if it is possible to gain enough stiffness from the reinforcement the needed stiffness is calculated which result in the same deflection as the cracked beam with the standard dimensions. This calculation is shown in Appendix A.

The needed spring stiffness,  $k_b$ , is found for  $u_{crack} = u_{kb}$  for different spans and forces and is shown in Table 7-1.

From the stiffness the reinforcement is determined which is needed to get enough stiffness to reach the same kind of deflection of a cracked beam.

$$A_s = \frac{k_b * l_s}{E_s}$$

$$n = \frac{A_s}{A_{\phi 16}} \text{ (Rounded up)}$$

$$ctc = \frac{w - 2\phi}{n - 1} \text{ (Rounded down)}$$

$$s = ctc - \phi$$

The reinforcement needed for reaching the stiffness to have the same deflections as the cracked beam is very high.

In this comparison only the reinforcement is increased, but not taken into account that by increasing the reinforcement, the cracking deflection will decrease, so even more reinforcement is needed to reach the stiffness. Only one iteration step already shows that reinforcement amount reach values which are impossible for practice. The values are shown in Table 7-1.

When the displacements of the beam and truss mechanism are plotted with the reinforcement amount,  $A_s$  as variable it is shown that it is never possible to give the truss model more stiffness then the beam model. The calculation is found in Appendix B. The beam is always stiffer than the truss.

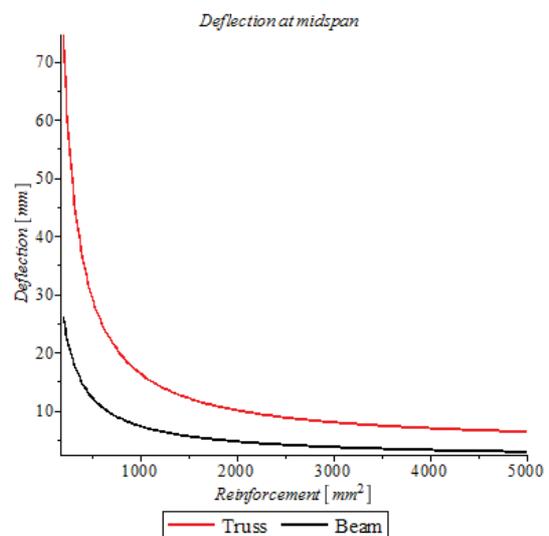


Figure 7.2 Deflection with variable reinforcement

### 7.1.3 Plate reinforcement

With an increasing reinforcement amount it is not possible to reach enough stiffness for acting the beam as a truss. But is it, for instance, possible that reinforcement outside the beam area contributes to the lateral restraining stiffness?

To answer this question a theoretical simplification is used to give an indication if this is possible. The model consists of a beam with the same properties as the standard beam, but is extended with plates on both sides, shown in Figure 7.4. This model is based on some assumptions:

- Bending stiffness (moment of inertia) consist only of the beam part of the plate.
- Spring stiffness is given by all the reinforcement in the plate.
- The beam width and plate with are respectively 200mm and 1000mm.

It is not realistic that only the beam gives bending stiffness and that the reinforcement in the plate all works for the lateral restraining, but this model gives an indication if it is even possible.

The required reinforcement to get the same stiffness as the normal beam stiffness is calculated for the spans of 1300mm and 6000 mm and shown in Table 7-2.

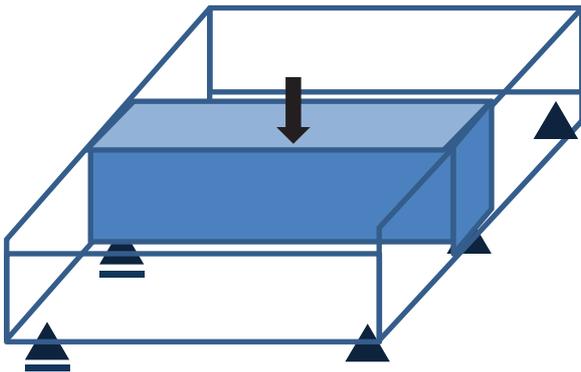


Figure 7.4 Beam-plate model

In this calculation it is assumed that all the extra reinforcement is added outside the beam, so that it does not contribute to the stiffness of the beam, but that it only contributes to the spring stiffness of the truss system.

The results show that the needed reinforcement is realistic. It shows that it is realistic that the reinforcement outside the bending stiffness area can contribute to the truss mechanism.

Table 7-2

Span [mm]	Force [kN]	$k_b * 10^5$ [N/mm]	$A_s$ total [mm]	$A_s$ Beam [mm]	$A_s$ Plate [mm]	$n \phi 16$	c. t. c. [mm]	s[mm]
1300	50	3,18	1969	471	1498	8	105	89
1300	100	2,39	1480	471	1009	6	147	131
1300	200	2,25	1393	471	922	5	184	168
6000	50	0,50	1429	471	958	5	184	168
6000	100	0,59	1686	471	1215	7	122	106
6000	200	0,951	2717	471	2246	12	66	50

The real reinforcement probably should be higher to gain enough stiffness because the plate has simply more bending stiffness than the bending stiffness of the beam. This makes it proper that the reinforcement should be higher as well.

This simplified model is not a realistic model, but it shows that it plausible that enough stiffness can be reached to act as a truss.

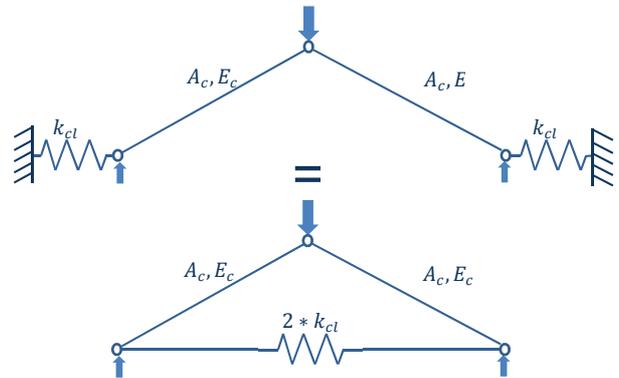


Figure 7.3 Lateral restrained Stiffness

## 7.2 External lateral restraining

The laterally restrained truss by the boundary conditions is an unrealistic model. But what is the influence of the laterally restrained section?

To model the laterally restraining a model with springs is used, the upper model in Figure 7.3. If both springs have the same properties it will act the same as the model in the lower model of Figure 7.3. This second model is used for calculations for this system. Again the question if it is possible to gain enough lateral restraining that results in a truss mechanism in the beam.

### 7.2.1 The deflection

The shortening of the concrete compressive strut stays the same as equation [6-1].

The elongation of the spring truss becomes:

$$\Delta L_{sp} = \frac{N_s}{2 * k_b}$$

With the assumptions that:

- Deformations gain no extra forces in the trusses.
- Only normal forces cause deformation.
- The trusses remain straight.
- The concrete truss dimensions are assumed.

Again the new height could be calculated with:

$$H_{clspr} = \sqrt{L_{c,def}^2 - \left(\frac{L + \Delta L_{clspr}}{2}\right)^2}$$

This gives a deflection of:

$$u_{clspr} = H - H_{clspr}$$

### 7.2.2 Spring properties

The standard beam properties will be used again. The springs are a simplification of the boundary conditions of the beam. In this paragraph a simplified model is used to get some feeling of these boundary conditions. The model is shown in Figure 7.5. For this model the spring stiffness is determined for which the displacement is the same as the cracked beam. So  $k_{cl}$  is found for  $u_{crack} = u_{kb}$  for different spans and forces. This is shown in Table 7-3.

Table 7-3

Span: [mm]	Force [kN]	$k_{cl} * 10^5$
1300	50	0,89
1300	100	0,81
1300	200	0,79
6000	50	0,28
6000	100	0,31
6000	200	0,39

For the laterally restraining beams the deformation of an uncracked beam is taken to give an indication for the stiffness.

With equation [6-8] and [7-1] the laterally restraining stiffness,  $k_{cl}$ , could be found:

$$k_{cl} \geq \frac{F}{u} = \frac{F}{\frac{1}{48} \frac{FL^3}{EI}} = \frac{48EI}{L^3}$$

In Table 7-4 the limits are given for the boundary conditions of the used model for the calculated values of  $k_{cl}$ , with  $E_c = 33000 \text{ N/mm}^2$ .

Table 7-4

$k_{cl} * 10^5$	Max l for: $b = 300$ $h = 500$	Max l for: $b = 300$ $h = 1000$	Min $I_{zz}$ for: $l = 2000$ (* $10^6$ )
[N/mm]	[mm]	[mm]	[mm <sup>4</sup> ]
0,89	382	763	5394
0,81	394	788	4897
0,79	397	793	4812
0,28	560	1120	1709
0,31	544	1087	1867
0,39	503	1005	2364

In reality the stiffness need to be higher because the supports of the restraining beams also have settlement, but this gives an indication that it is possible to give some lateral restraining.

## 7.3 Conclusion

Reaching lateral restraining stiffness with reinforcement in the beam resulting in a truss mechanism is not possible for a beam. Increasing the amount of reinforcement for reaching enough truss stiffness results in increasing of the cracked beam stiffness. The beam mechanism will takes all the forces before the truss mechanism could work properly.

For a simplified plate model it is possible to reach enough lateral restraining by adding reinforcement outside the beam area which provides the bending stiffness. Reinforcement outside of the beam stiffness area could contribute to the spring stiffness of the truss mechanism. Lateral restraining of the boundary conditions of the beam is possible. This is effective for a stiff boundary construction, but is very difficult to determine the effective lateral restraining stiffness.

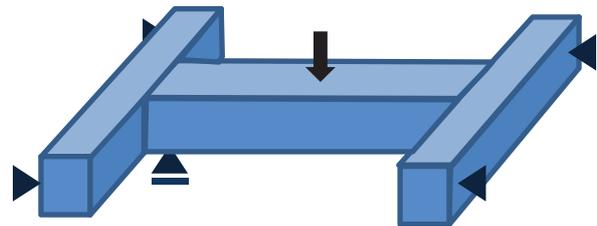


Figure 7.5 Simplified boundary conditions

## 7.4 Total restraining

The stiffness of the truss depends on several properties. The reinforcement stiffness and the laterally restraining stiffness are just some of the mechanisms which give the truss stiffness.

The truss stiffness consists out of several mechanisms which all give some stiffness. All the schematic mechanisms in the previous paragraphs give a higher deflection for the truss than in practice will occur.

A schematically total mechanism of a truss is shown in Figure 7.6. All the springs describe a part of the structure resistance.

Shortly the different resistance are described:

$k_{rconc}$ : A hinge is never 100% moment free. In all hinges occurs some resistance when the truss is changing in shape.

$k_{conc}$ : This is the resistance of the concrete truss. Elongation or shortening for only normal force is  $k_{conc} := \frac{EA}{L_c}$ , this is already simplified.

$k_{reinf}$ : The resistance of the reinforcement in the beam as discussed in paragraph 7.1.

$k_{clam}$ : The horizontal restraining consists of the clamping between the other structures. This is discussed in paragraph 7.2.

$k_{hor}$ : Also the surrounding of the beam influences the stiffness. For example the model from paragraph 7.1.3.

$k_{ver}$ : In vertical direction the beam need also support, which has stiffness. The stiffness of the vertical support partly determines the total deflection of the beam.

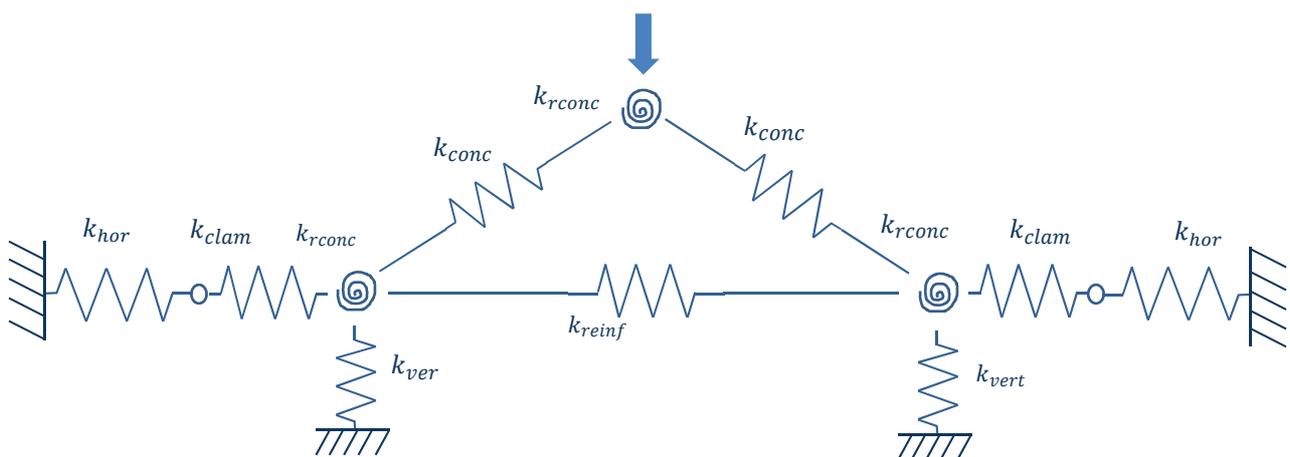


Figure 7.6 Schematic truss resistance

# 8 Cracking or resisting

Bending of the beam causes cracks, while the truss mechanism gives compression in the concrete struts. But when a concrete strut cracks by the bending mechanism, is it then able to convert to a compressive strut? In this chapter the beam and truss development with an increasing force is compared with a simple model to show how the models are related to each other. The calculation of the model is added in Appendix D.

## 8.1 Cracking of the beam

In the Bernoulli theory, in which planes remains plane when it is subjected to bending, the compressive height of the beam stays the same for a changing moment. The concrete beam mechanism cracks to convert to a beam which results in a concrete compressive height and the reinforcement taking care of all the tensile stresses. The swift from an uncracked section to a cracked section occurs when the concrete stress limit is reached. When the beam is cracked a new equilibrium is created, shown in Figure 8.1.

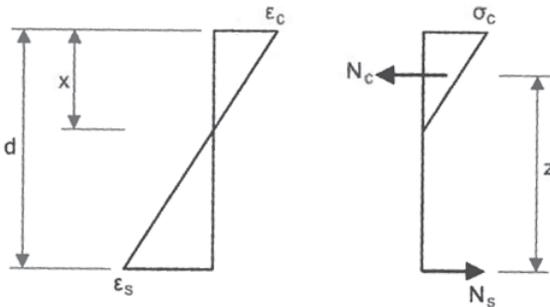


Figure 8.1 Force equilibrium crack

This result in the next formula[13]:

$$\frac{x}{d} = -\alpha_e \rho + \sqrt{(\alpha_e * \rho)^2 + 2\alpha_e \rho} \quad [8-1]$$

The formula shows that the compression zone height,  $x$ , is not depending on the loading, but only on the beam properties. The crack occurs when the concrete tensile strength is exceeded. This happens for bending according the Bernoulli theory if:

$$M_{cr} = W * f_{ctm} \quad [8-2]$$

If the force subjected to the beam increase, the part of the beam which is cracked is also increasing. When  $M_{cr}$  is reached the beam will crack in that section. This is shown in Figure 8.3.

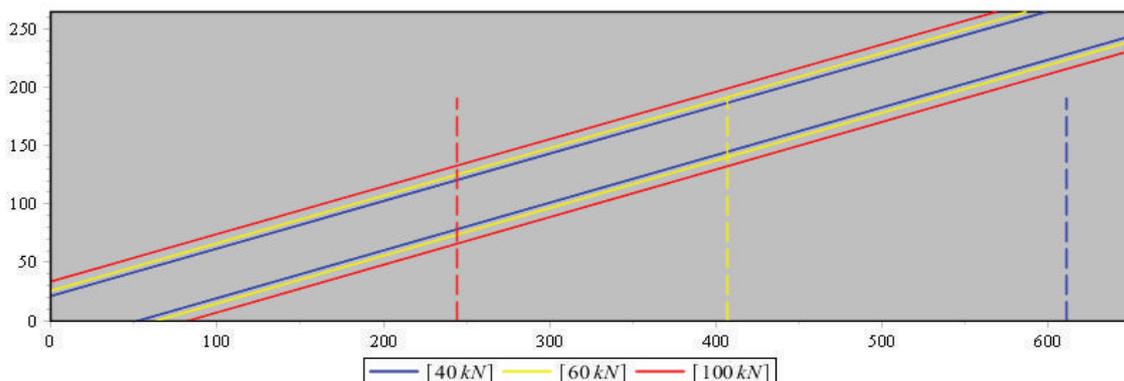


Figure 8.2 Truss and crack development model



Figure 8.3 Moment in beam

The modelling of the cracked beam gives a shear crack on the plane where the cracking moment is reached. The depth of the concrete cracks will reach till the concrete compression zone.

### 8.1.1 Compression diagonal

In the truss mechanism the concrete strut takes care of the compression. In this model it is assumed that the compression is equally spread over the area of the strut. The strut area in the model depends on the force on the beam. It is assumed that the stresses in the strut will reach the concrete compression strength.

The concrete area is then calculated with:

$$A_c = \frac{N_c}{f_{ck} * \beta}$$

With:

$$N_c = \frac{F}{2 \sin(\alpha)}$$

$$f_{ck} = 50 \frac{N}{mm^2}$$

$$\beta = 0.7$$

$\beta$  gives an estimation for the distribution of the stresses in the strut. It is mechanical not possible to have somewhere the total compressive stress and just next to it no stresses. This factor gives an indication for the distribution of the stresses.

Another important assumption is that the width and height of the strut are the same:

$$h_{strut} = w_{strut} = \sqrt{A_c}$$

### 8.1.2 Result

The comparing of these models is shown in Figure 8.2. This is done for 3 different forces. 40 kN is just higher as the cracking moment in the middle of the beam. 100 kN is given to show the results which is close to shear failure load in experiments and 60 kN for an indication between this values.

It shows that the strut dimensions are growing with increasing force. The planes which are cracked approach the support. With the bending crack of 60kN it is imaginable that the cracks bends, but for the crack of 100kN this is harder.

The model is very simplified but gives an indication on which area the mechanisms interrupt each other. Some limits are discussed.

The cracks occur by tensile stresses while the concrete strut is full of compression stresses. The model shows that cracks interrupt the compressive strut, which is not possible out of mechanical view.

According to the Bernoulli theory and crack model the crack depth and compression zone is the same for every plane in the beam. But in reality the crack are not straight, but curved by the influence of shear stresses also the real depth is difficult to predict.

### 8.1.3 Conclusion

This model gives a simple comparison of the development of the truss and beam mechanism. It shows that the mechanisms interrupt each other. The model is limited because the interaction of the mechanisms is not taken into account and the reality of cracking pattern and truss dimensions will differ.

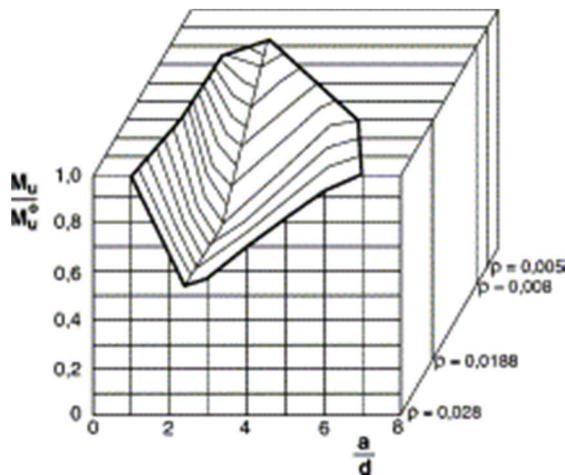


Figure 8.5 Kani's Valley

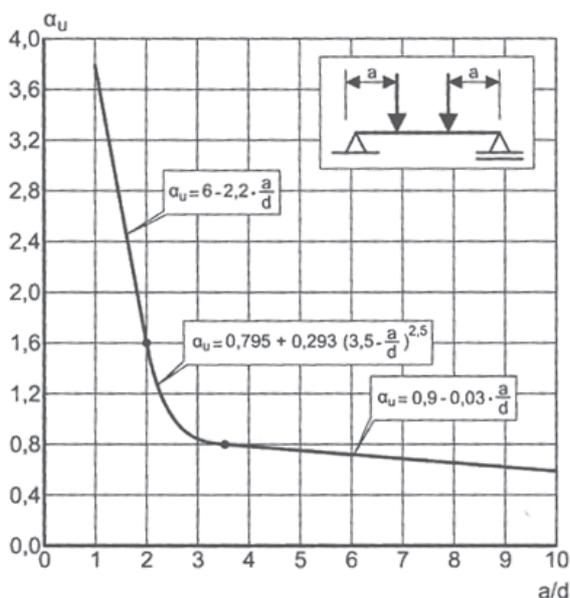


Figure 8.4 Factor  $\alpha_u$  for equation [8-3]

## 8.2 Shear crack

The importance of the interaction of both systems is clear. A further look is taken here into the shear problems to give some boundaries for the shear crack and to give a deeper inside look into the stresses developing in a beam.

Different researches conclude that shear has several influence factors.

The shear capacity depends on the following properties:

- Concrete quality
- Longitudinal reinforcement ratio
- Width of the cross-section
- Height of the cross-section
- Position of the loading

The concrete quality has a relation with all the physical properties of the concrete, so it is logic that a higher concrete quality gives a higher resistance for shear loading.

An increasing reinforcement results in smaller crack width, which result in higher concrete tensile stresses and increasing of crack friction by aggregate interlock.

The width and height of the cross-section increase the area to transfer the shear stresses. The width gives a proportional contribution, while the height is disproportionate to the capacity. According to crack mechanics a long crack is more sensitive to crack propagation than a small crack.

### 8.2.1 Position of loading

The position of the loading has a lot of influence at the capacity. Several researches give different boundaries. All researches give 3 areas, only the limits differ.

If a load is close to the support a part of the load is directly leaded to the support. When the load has a high distance to the support the beam will fail by the bending mechanism. In the area between the failing in bending mechanism and the leading direct to the support shear failure can occur.

The most common shear resistance formula is from Rafla[13]:

$$V_{u,m} = \alpha_u d^{-0.25} \sqrt{f_{cm}} \sqrt{\rho_l * 10^2} b d \quad [8-3]$$

The formula from Rafla is empirical and it is a result of 442 test specimens. All the influence factors are parameters for the resistance. The factor  $\alpha_u$  depends on the position of the loading. In Figure 8.4 the factor alpha is plotted for a variable distance to the support. It clearly shows that close to the support the shear resistance is increasing a lot by the direct distribution to the support.

Kani[16] was the first one who did experimental research to the limits of shear failure and the other failing mechanisms. Out of experimental research the graph (Figure 8.5) is created. It shows the ratio of  $M_{u,V}$  and  $M_{u,fl}$ . If the ratio is higher than one, the failure will be by yielding of the steel. The most important from the graph is the valley in which the ratio is lower than one. In that valley the beam will fail on shear failure instead of yielding of the steel. This area is interesting for researching the shear failure mechanism.

Leonhardt and Walther[17] further investigated to a specific part of the Kani's Valley. Out of the experiments follows that for  $a/d < 3$  holds that the beam will not fail by the shear crack. After continuing of the shear crack a truss mechanism occurs. The experiments show a shear failure mechanism for  $3 < a/d \leq 7$ . When increasing the span to  $a/d \geq 7$  the beam will fail by yielding of the steel instead of a shear failure. The result of this study does meet the Kani's Valley.

### 8.2.2 Shear failure position

The Beeby experiment is in the deepest point of the Kani's Valley, with a reinforcement ratio of 0.88%. According to Kani's Valley is also fails on shear failure. But the theories for the area till  $a/d = 3$  differ about the mechanisms which occur in the beam. The limits for direct loading to the support differ. The Leonhardt and Walther[17] research result in a truss mechanism till  $a/d < 3$  while other documents[13] give  $a/d < 2.5$  as limit. The crack will get stucked in the force introduction zone by the multiaxial compression stresses in that area. The Kani's Valley shows that for  $a/d = 2.5$  the possibility for shear failure is the highest one. The Beeby experiment shows that for  $a/d = 2.49$  shear failure occurs.

The theories differ, but the difference is in the mechanism which occurs after shear failing. Some theories say that the truss mechanism occur after shear failing, where other say it fails. The limits for this changing are different and unknown, but in some specific situations the beam is able to transform into a truss mechanism. But what are the limits for this transforming?

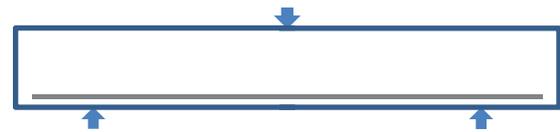


Figure 8.6 Beam mechanism

### 8.2.3 Principle stresses

To get a better insight into the stress mechanism a deeper look is taken into the principle stresses in this paragraph. The force in the middle of the beam (Figure 8.6) causes moment and shear forces in the beam. The shear forces cause shear stresses which result in changing directions of the principle stresses. The principle stresses follow from the equations described in paragraph 2.3.1. Between the force introduction points there is no vertical force. So for this model the next formulas hold:

$$\begin{aligned} \sigma_1 &= \frac{\sigma_x}{2} + \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} \\ \sigma_2 &= \frac{\sigma_x}{2} - \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} \\ \tan(2\theta_1) &= \frac{\tau_{xy}}{\frac{1}{2}\sigma_x} \end{aligned} \quad [8-4]$$

This gives stress lines in which there is no shear stress and only normal stresses: the trajectories of principal stresses. Figure 8.7 shows a schematically picture of these lines. By the rotation of the principle the concrete strut also changes from a straight strut to an arch. The calculations for this model are added in Appendix E.

### 8.2.4 Uncracked phase

For an uncracked beam the stresses could be approached with the Bernoulli theory. This will hold for the B-areas, but not for the D-areas as described in 2.3.

For the shear stresses for a rectangle cross section holds according the Bernoulli theory[8]:

$$\tau_{xy} = -\frac{3}{2} * \frac{V}{2wh} \left( 1 - 4 * \frac{z^2}{h^2} \right)$$

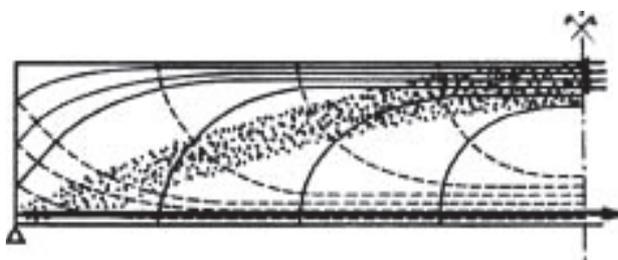
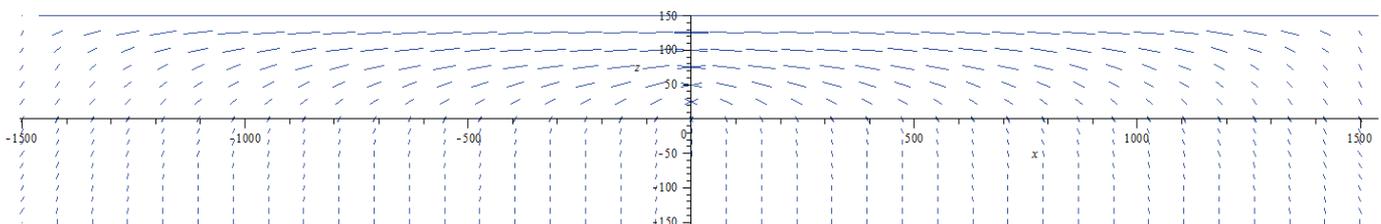


Figure 8.7 Trajectories and arch



Direction of principle compressive stresses according Bernoulli theory

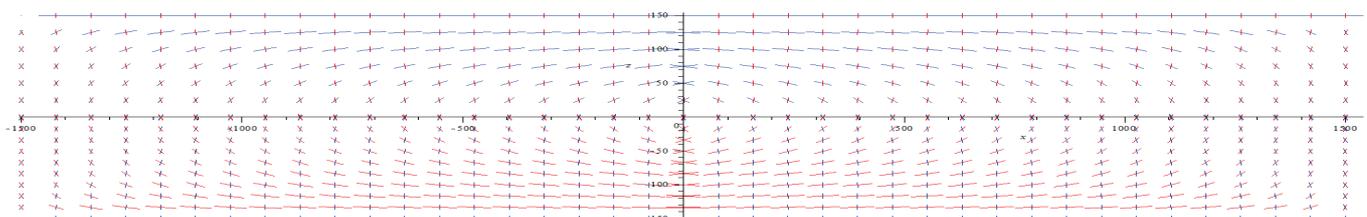


Figure 8.8 Direction of principle compressive and tension stresses according Bernoulli theory

For the stresses following out of the moment:

$$\sigma_{xx} = \frac{M(x) * z}{I_{zz}}$$

For this particularly beam holds, with the x/z-as in the middle of the beam at centre of gravity of the total beam.

The next holds:

$$V = \frac{1}{2} F$$

$$M(x) = \left(1 - \frac{|x|}{\frac{1}{2}L}\right) * \frac{1}{4} * F * L$$

With this, the principle stresses of the beam can be calculated:

$$\tau_{xy}(x, z) = -\frac{3}{2} * \frac{0.5F}{wh} \left(1 - 4 \frac{z^2}{h^2}\right)$$

$$\sigma_{xx}(x, z) = \frac{\left(\left(1 - \frac{|x|}{\frac{1}{2}L}\right) * \frac{1}{4} * F * L * z\right)}{\frac{1}{12} * w * h^3}$$

The rotation of the principle stresses are found with equation [2-24]. The compression principal stress directions for the total beam are shown in top picture of Figure 8.8. The tensile stresses are perpendicular to the compression stresses. Both stress directions are shown in the bottom of Figure 8.8. The complete calculation sheet is found in Appendix E.

### 8.2.5 Cracked phase

When the principle stresses reach the maximum compression or tensile stresses the concrete will fail locally. Failure limits are described in paragraph 2.3.2. When a crack occur the principle stress directions and volume change. This changing changes the crack direction. This follows out of experimental result and final element methods, but it is very difficult to predict the exact crack pattern with a simple analytic model, that is why this is not done here. This changing of principle stresses direction causes the changing of crack direction, as shown in Figure 8.9.

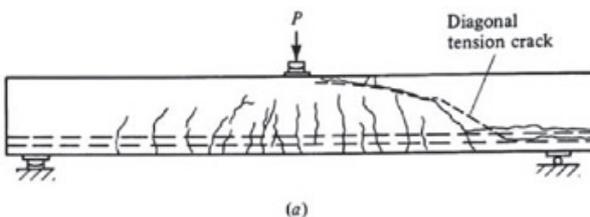


Figure 8.9 Changing crack direction

### 8.2.6 Conclusion

The shear stresses change the direction of the principle stresses, which causes an arch instead of a truss mechanism. When the principle stresses pass the limits cracks occur. The cracks change the principle stress direction and size, which causes a rotation of crack direction. The model in this paragraph does not take into account the cracks. Finite element programs are necessary to show this.

## 9 Conclusion

This research is started to find a reason for the difference in failure by a beam and truss beam out of the experiments from Beeby. Why is the beam unable to act like a truss mechanism?

The differences in displacements show that there is a difference between a partly cracked and uncracked concrete beam. The cracked beam gives the best approximation for the deflection of the beam mechanism. The deflection of the truss mechanism is larger than the beam mechanism. If the truss mechanism has a 100% lateral restraining it becomes stiffer than the beam, but total horizontal restraining is not possible. This difference shows that the lateral restraining has a lot of influence on the truss deflection.

When comparing the beam and truss mechanism it shows that the beam mechanism handles the same force with a smaller deformation. This shows that the cracked beam is always stiffer than the truss.

The lateral restraining has a lot of influence on the stiffness of the beam. With a simple model it is shown that it is not possible to get enough lateral restraining stiffness by increasing the reinforcement. The increasing of the reinforcement causes increasing of the stiffness of the truss, but also increasing of the stiffness of the beam. The beam will always be stiffer as the truss mechanism.

Another theoretical model shows that reinforcement in a plate can contribute to the lateral restraining, but only with the assumption that a beam part provides bending stiffness and all the reinforcement completely contributes to the lateral restraining. With this construction the truss mechanism could reach the stiffness of the beam.

This stiffness could also be reached by external lateral restraining.

The comparison of the mechanics of both mechanisms also shows differences. In a beam the stresses are distributed in the beam and the shear stresses result in changing directions of the principal stresses. This stress development changes when cracks occur. This is difficult to show this with simple analytical calculation.

The shear mechanism is based on empirical experiments and it is not possible to predict the exact crack pattern.

In the truss mechanism the stresses rise in the struts till one of the strut limits is reached. Yielding of the steel strut will be the fail mechanism of the truss.

A model is designed to show the development of the truss and beam mechanism together. This mechanism shows that the growing concrete strut is crossed by the crack of the beam mechanism. This is mechanical not possible. This model shows that the interaction between the mechanisms is important to declare the research question.



# *FINITE ELEMENT MODELS*

*INTRODUCTION*

*BASIC MODELS*

*PARAMETER STUDY*

## 10 Introduction

The analytical analysis gives a basic description of the truss and beam model of Beeby. With a finite element models (FEM) it is possible to look more into detail and to approach the reality better. With a FEM program it is possible to use non-linear analyses and take into account the second order effects.

### 10.1 Target Finite element modelling

The target of using FEM is to analyse the beams of the Beeby experiment to investigate the difference in stress development and failure modes. After analysing the Beeby beams it is possible to vary with the parameters to get the boundaries of the truss and beam mechanism. The FEM program makes it possible to see the developments in the beam during loading and the development of the mechanisms.

### 10.2 Principle of finite element methods

The principle of finite elements methods is the dividing of the structure in a great amount of elements. The elements are connected by nodes which have at least the same deformations. The stress developing in the structure is based on the stiffness parameters of every element, which are based on the assigned properties. With integration methods the developed stresses and strains are calculated for every element. Interpolation is used between the points to approach the exact solution.

Using finite element programs has advantages and disadvantages.

Some advantages are:

- Complicated calculation are possible
- Complicated shapes are possible
- The use of non-linear material properties
- Easy to change the structure

Some disadvantages are:

- Time consuming creating and verifying
- Very sensitive to boundary conditions
- Difficult to check the results

### 10.3 Beeby beams

Not all dimensions are known from the truss model tested by Beeby, especially the dimensions of the cut-out section. Only the span of the cut-out section is known,  $1300\text{mm}$ . In the FEM analyse this dimensions are assumed:

- The concrete height at the top is  $100\text{mm}$
- The height at the sides is equal to twice the reinforcement depth,  $70\text{mm}$

The dimensions of the truss model are shown in Figure 10.1.

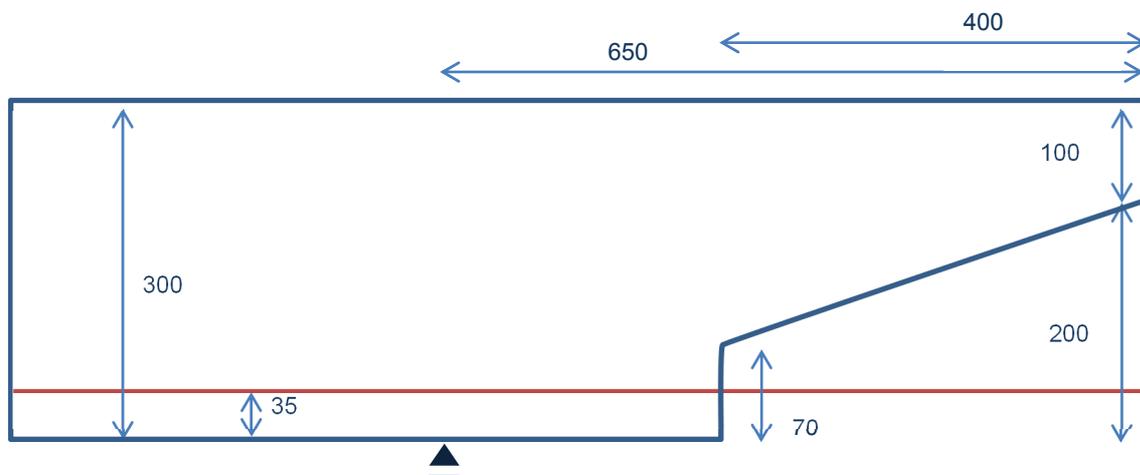


Figure 10.1 Modelled truss dimensions [mm]

# 11 Basic models

The FEM analysis starts with the description of the constructing and calculation methods used in the FEM program. The results are described in the next paragraph after which the results are verified with the results of the Beeby experiment and the analytic solution. The verified models are compared to each other to describe the differences in mechanical mechanism.

## 11.1 Constructing models

The FEM program uses several parameters to model the concrete properties. The “guidelines for Non-linear finite element analysis of concrete structures” [7] is used to gain the proper parameters.

### 11.1.1 Used program

For the FEM analysis the following programs are used:  
For pre and post processing:

- iDiana Release 9.4.4

For calculations:

- Diana Release 9.4.4

A summary of the basic data and calculation files is found in Appendix F.

### 11.1.2 FEM Models Diana

The beams are modelled as 2d models, because the depth is small in ratio with the span and the cross-section and loading do not vary over the depth. The force is acting in the same direction as the 2d-plane. The use of a 2d-model instead of a 3d-model reduces the calculation time. Also by symmetry of the structures a symmetry axes is used to reduce the elements and thereby the calculation time of the FEM calculation. The schematical geometry of the models are shown with the boundary conditions in Figure 10.1.

### 11.1.3 Elements

#### Plane stress elements

The concrete is modelled with plane stress elements. Stress elements are used because it is plausible that the stresses do not vary over the depth of the beam. CQ16M elements are used with 3 by 3 gauss integration scheme. These are eight-node quadrilateral isoparametric plane stress elements, schematically shown in Figure 11.2. Each node has 2 degrees of freedom (vertical and horizontal displacement). This form is based on quadratic

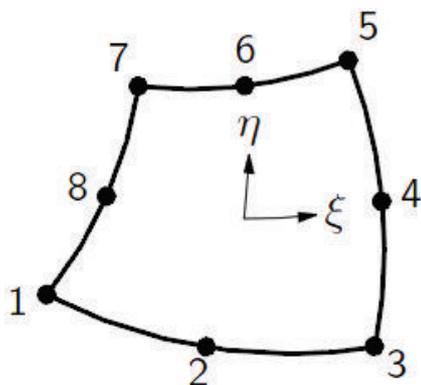


Figure 11.2 CQ16M element

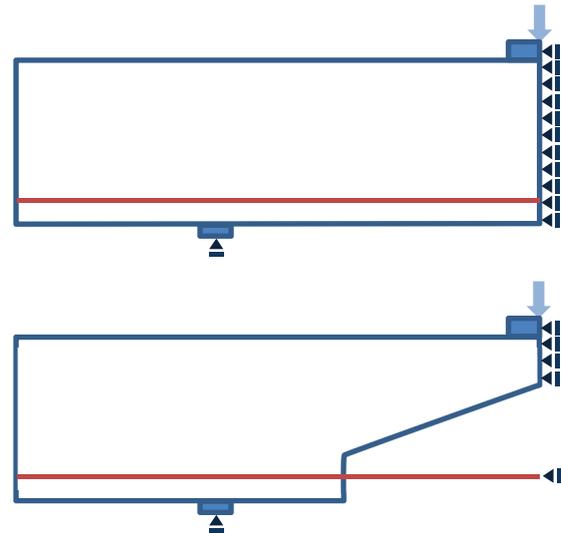


Figure 10.1 Geometry FEM-model

interpolation and gauss integration. The polynomial for the displacements  $\xi$  and  $\eta$  is expressed as [18]:

$$u_i(\xi, \eta) = a_0 + a_1\xi + a_2\eta + a_3\xi\eta + a_4\xi^2 + a_5\eta^2$$

#### Embedded reinforcement

The reinforcement in the beam and the reinforcement in the truss embedded in the concrete are both modelled with embedded reinforcement elements. This embedded reinforcement is added to the model without reducing the cross-section properties of the concrete, but adds stiffness of the reinforcement steel. The strains in the reinforcement are calculated with the displacements of the mother element. Perfect bond is assumed with the embedded reinforcement.

#### Truss elements

The embedded reinforcement is not possible for the external reinforcement. The external reinforcement is modelled with one L2TRU straight truss element. The nodes have only one degree of freedom. The deformation of the truss element is only possible with axial elongation; there are no bending and shear stresses. The polynomial for the displacement  $\xi$  is expressed as [18]:

$$u_i(\xi) = a_0 + a_1\xi$$

#### Force introduction

The force introduction is realised with steel plates to introduce the force to avoid high local stresses in the concrete. The force is displacement based with the introduction point in the middle of the span on top of the steel plate.

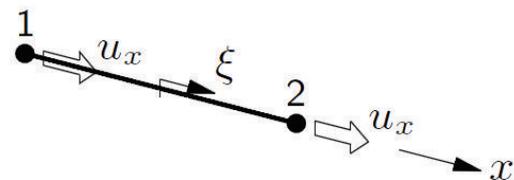


Figure 11.3 L2TRU element

### 11.1.4 Material properties

Important for approaching a valid result is choosing the correct material parameters which match the real material properties. This paragraph describes the chosen parameters and the used models.

#### Concrete properties

The only known property of the Beeby experiment is the mean compressive strength of  $50 \text{ N/mm}^2$ . This property is used as mean and characteristic compressive strength:

$$f_{ck} = f_{cm} = 50 \frac{\text{N}}{\text{mm}^2}$$

The concrete tensile strength is based on this value with some basic parameters[7]:

$$f_{ctm} = f_{ctk0,m} * \left(\frac{f_{cm}}{f_{ck0}}\right)^{\frac{2}{3}} = 4,1 \frac{\text{N}}{\text{mm}^2}$$

$$E_{ci} = E_{c0} * \left(\frac{f_{cm}}{f_{cm0}}\right)^{\frac{1}{3}} = 37.620 \frac{\text{N}}{\text{mm}^2}$$

With the basic parameters from [7]:

$$\nu_c = 0,15$$

$$E_{c0} = 22000 \frac{\text{N}}{\text{mm}^2}$$

$$f_{ctk0,m} = 1,40 \frac{\text{N}}{\text{mm}^2}$$

$$f_{ck0} = f_{cm0} = 10 \frac{\text{N}}{\text{mm}^2}$$

#### Cracking model

The crack modelling of concrete is very sensitive to the material properties, element size and type of cracks in the structure.

A total strain fixed crack model is used to model the tensile and compressive properties of concrete in one model, because it is the most suitable to model and show the shear cracks[19]. This is a smeared crack model in which the cracks are spread over the element. The crack occurs in a element in perpendicular to the tensile principle stress direction. The fixed part of the model holds this direction of the crack if the principle stress direction changed. In a total strain fixed crack model the crack direction stays the same when the principle stresses direction changes. In Appendix G detailed description is given.

#### Tensile strength

For modelling the tensile strength the Hordijk curve is used, shown in Figure 11.4 [18], according [7]. The ultimate tensile strength  $f_{ctm}$  is used for  $f_t$ . For  $h$ , the

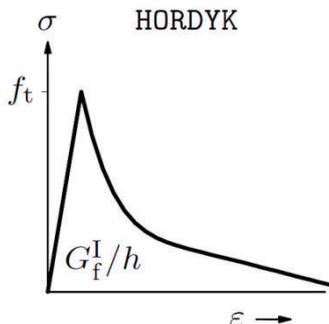


Figure 11.4 Hordijk curve

crack bandwidth, the standard value of DIANA is used, which is  $h = \sqrt{A}$ . For rectangle elements this gives the element size as advised according [7]. With changing cross section and mesh dimensions this is a proper approach for the crack bandwidth.

The fracture energy,  $G_F$ , is based on:

$$G_F = G_{F0} \left(\frac{f_{cm}}{f_{cm0}}\right)^{0.7} \quad [7]$$

$G_{F0}$  varies for by the maximum aggregate size and has a lot of influence on the maximum resistance of the beam[19].

With variation of the fracture energy, described in Appendix G., the value  $G_F = 0.09$  shows the best shear crack and is thereby used.

When a crack rises in the structure the shear transfer and poison ratio decrease near the crack. The shear retention is modelled with the damaged based shear retention model. This shear curve reduces the shear resistance after a crack occurs in the structure. This was also an important advise according[19]. In the relation  $G_{red} = \beta G_0$  the factor  $\beta$  reduces with increasing crackwidth. The poison ratio also decreases with increasing damage. This is modelled with the damaged based crack model in DIANA.

#### Compressive model

The concrete compressive strength is modelled as ideal, shown in Figure 11.5, with:

$$f_c = f_{ck} = 50 \frac{\text{N}}{\text{mm}^2}$$

$$E_s = 200.000 \frac{\text{N}}{\text{mm}^2}$$

If large areas reach the compressive strength, this modelling will not be realistic.

#### Steel properties

From the Beeby experiment the steel strength and amount is given:

$$f_{yk} = 500 \frac{\text{N}}{\text{mm}^2}$$

$$A_s = 6\emptyset 10 = 471 \text{mm}^2$$

The steel properties are modelled with a yield stress according Von Mises. The Von Mises stress model is based on:

$$E_s = 200.000 \text{ N/mm}^2 \quad [7]$$

$$f_{yk} = 500 \frac{\text{N}}{\text{mm}^2}$$

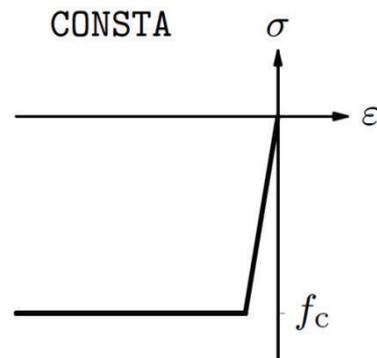


Figure 11.5 Ideal compression curve

### 11.1.5 Calculation method

For the numerical calculations the modified Newton Raphson method is used with an energy based convergence criterion. The modified Newton Raphson method gives the most stable results for this problem, comparison is showed in Appendix G. The load is applied with a displacement based node. The line search function is used to increase the convergence rate.

### 11.1.6 Mesh size

A fine mesh is chosen for both models to investigate the crack patterns. In the beam model all the elements are squares. The truss model is based on the same principle, but in the middle concrete section the height of the elements changes by the changing form. This form is chosen so that the number of elements does not change over this part of the structure. The mesh is shown in Figure 11.6 with the constrain directions. The constraining on the top is the displacement based force introduction.

### 11.1.7 Vertical reinforcement

In the truss model a small piece of vertical reinforcement is used near the support, shown in Figure 11.7. This reinforcement was needed for leading all the stresses to the support, without this vertical reinforcement the embedded reinforcement was not able to restrain the force in the horizontal strut. The structure would fail by cracks around the full length of the embedded reinforcement bar. This shows that the interaction between the concrete and the reinforcement is important for the stress developing and restraining in the truss model.

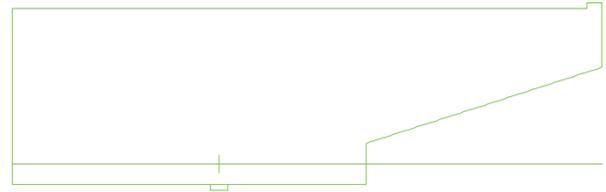


Figure 11.7 Vertical reinforcement

### 11.1.8 Variation of parameters

In a finite element program the chosen parameters determine the results. Variation of the parameters shows the influence. Variation of a lot of parameters is used to give a proper failure mechanism which approaches the values. With variation the used parameters are chosen. The properties are kept the same during the research to investigate the differences between the models instead of the parameters.

Variation of parameters is done to show the influence. Different fracture energy models are used to gain a proper crack development and failure load. For the compressive strength relation also a linear relation is used. Only some small different where found. During the research also mesh and calculation methods are varied to show the influences and the use of different load steps to gain convergence in the calculations.

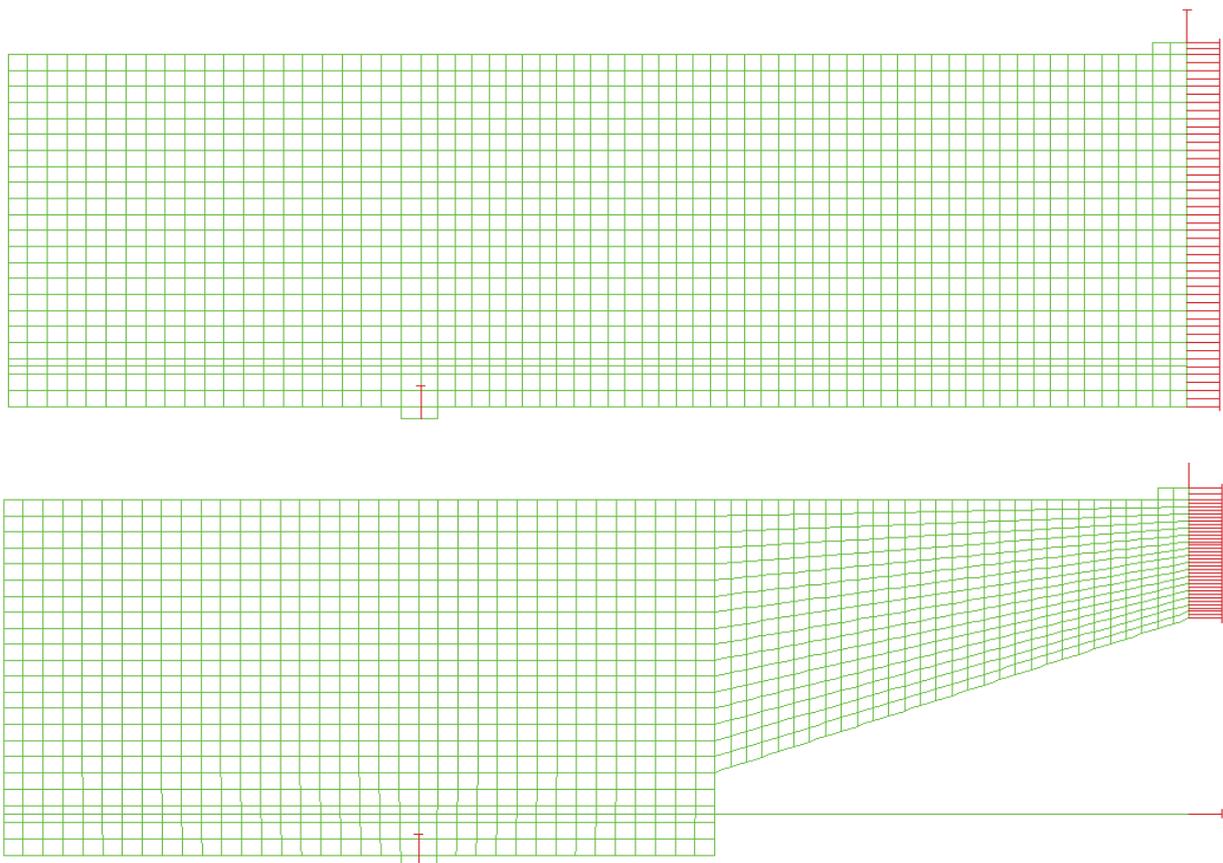


Figure 11.6 Beam mesh and truss mesh

## 11.2 Analysis Basic models

In this paragraph the basic FEM models are analysed. First a description of the different legends, followed with the analysis of the strain and stress development in the beam and truss mechanism.

### 11.2.1 Legends

For comparing and analysing the models uniform legends are used.

#### Strain legend

The cracks in the concrete result in failure, for instance the shear failure. The crack model of Hordijk is used to model the cracks. This Hordijk curve is used a basis for the legend. The legend based is shown in Figure 11.8. The colours present the different regions in the used Hordijk tension restraining model.

The peak is reached when the tensile stress is reached with linear elastic relation:

$$\epsilon_{peak} = \frac{f_t}{E_c} = \frac{4.1 \frac{N}{mm^2}}{37620 \frac{N}{mm^2}} = 0.109 * 10^{-3}$$

The ultimate strain, till which there is still a little tensile resistance, is given by:

$$\begin{aligned} e_{ult} &= 5.163 \frac{G_f^I}{h f_t} \\ &= 5.163 * \frac{0.09 \frac{N}{mm}}{14mm * 4.1 \frac{N}{mm^2}} = 0.8 * 10^{-2} \frac{1}{mm} \end{aligned}$$

When the ultimate strain,  $e_{ult}$ , is passed, no tensile stresses are remaining and a crack arise. Along this limits, some other limits are chosen to show the differences, like a separate colour for compression strains.

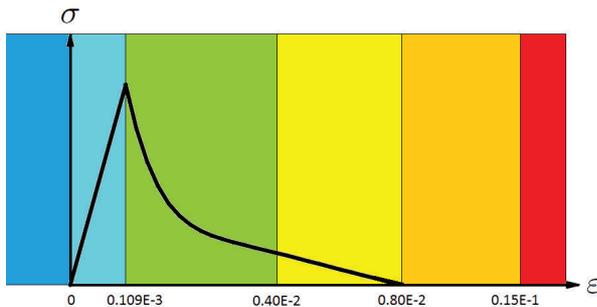


Figure 11.8 Strains legend

#### Stress legend

To compare the different beams a standard legend is used. This legend is shown in Figure 11.9. The tensile stresses are divided in 2 colour scales. The maximum tensile stress is  $f_t = 4.1 \frac{N}{mm^2}$ . With dividing it into 2 scales it gives the possibility to show the difference between a starting tensile stress and a stress which is close to the cracking limit. The compressions stresses are divided in steps of  $2.5 \frac{N}{mm^2}$  till  $10 \frac{N}{mm^2}$  which makes it possible to show the development of the compression areas. The steps above these are divided steps of  $10 \frac{N}{mm^2}$  and  $20 \frac{N}{mm^2}$  to limit the number of colors, but make it possible to show the further development of stresses and the stresses which are close to the limit compression strength of  $50 \frac{N}{mm^2}$ .



Figure 11.9 Stress legend [N/mm<sup>2</sup>]

### 11.2.2 Beam model

The basic beam model gives the normal shear crack. In Figure 11.10 the force-displacement diagram is shown. It shows that the maximum force resistance is 131 kN. The diagram is divided in several stages to describe the development and changes in the beam. To analyse the different stages the stress and strain development are shown in Figure 11.11. The force displacement diagram is explained together with the strain and stress development of the different stages.

First stage A, the linear elastic stage. In this stage the concrete and steel have the same deformations and there are no cracks present. The stresses are equally spread over the height.

Stage B is entered when the first crack occurs in the beam when the concrete tensile strength is passed in the bottom fibre at the middle of the span. The mechanical scheme changes to a cracked cross-section which has equilibrium by the compression stresses in the concrete top and tensile stresses in the reinforcement.

The cracks keep occurrence till the steel elongation of the steel in the cracks is able to resist the force. This stage is called the crack formation stage.

The first crack is shown in the plot of step 0.50mm where a vertical crack occurs in the middle of the beam. More cracks occur, resulting in a crack pattern as shown in step 0.75mm.

In stage C no new cracks occur, this is the stabilised crack stage. In this stage the force equilibrium stays the same, with tensile stresses in the reinforcement and compressive stresses in the top.

This stage goes to stage D when a shear crack occurs in the beam and force increasing is no longer possible. The displacement still grows in this stage, but the maximum force does not.

The development of the shear crack is clearly shown in Figure 11.11 in the steps 1.5 till 2.0. In stage E the constructions fails by the shear crack and the resisting force decreases.

Together stage D and E form the shear failure mode. In this model the shear failure is able to find equilibrium again in stage F.

In stage F the new equilibrium exists of a truss mechanism. This mechanism fails before a new resisting force is reached. The occurrence of the compression truss is clearly showed with the  $s_2$  stresses in step 2.0 – 2.74 in which the compression stresses are growing along the compression strut.

The beam model start with acting as a Bernoulli beam till the first bending crack occurs. After the crack formation stage has finished the loading increases till a shear failure occurs. In this model the shear crack does not failure at once and a truss mechanism occurs after failing of the shear crack. This shear crack is very depended on the parameters. With other parameters it gave a more sudden failure and the truss did not always occur after failing by shear. But in all models the shear failure was causing the force limit.

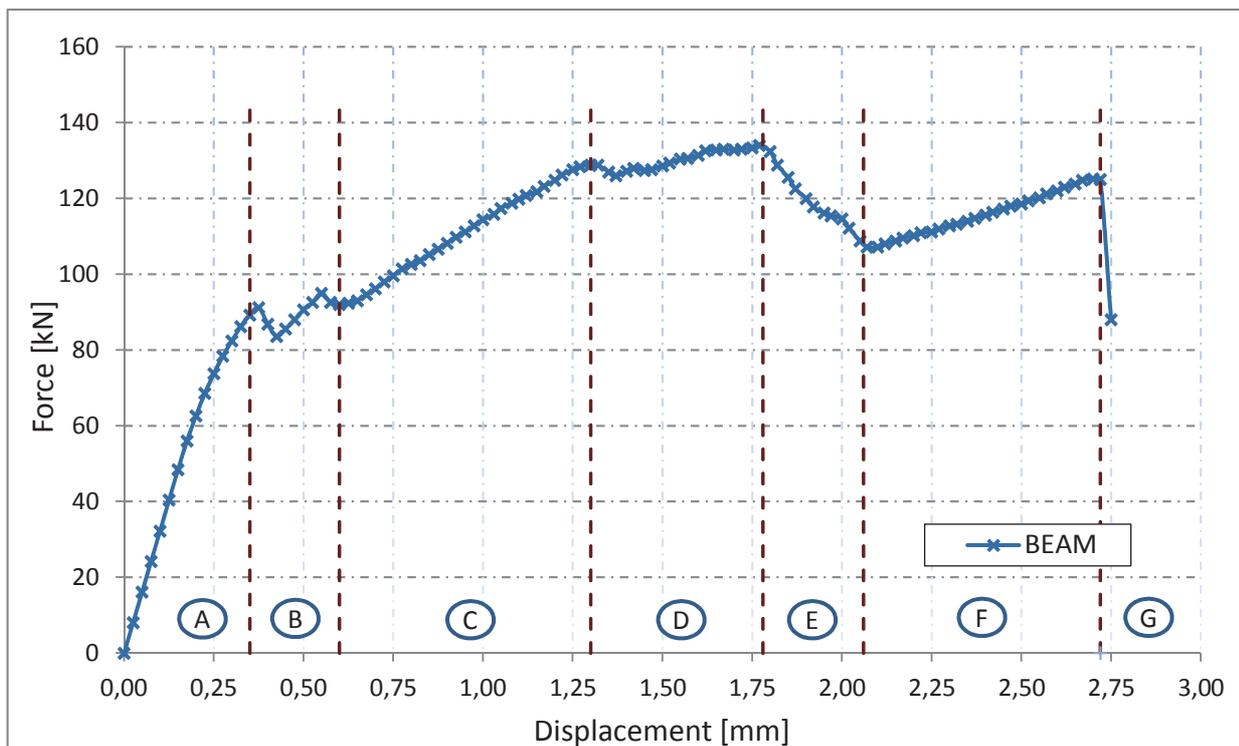


Figure 11.10 Force displacement diagram of beam

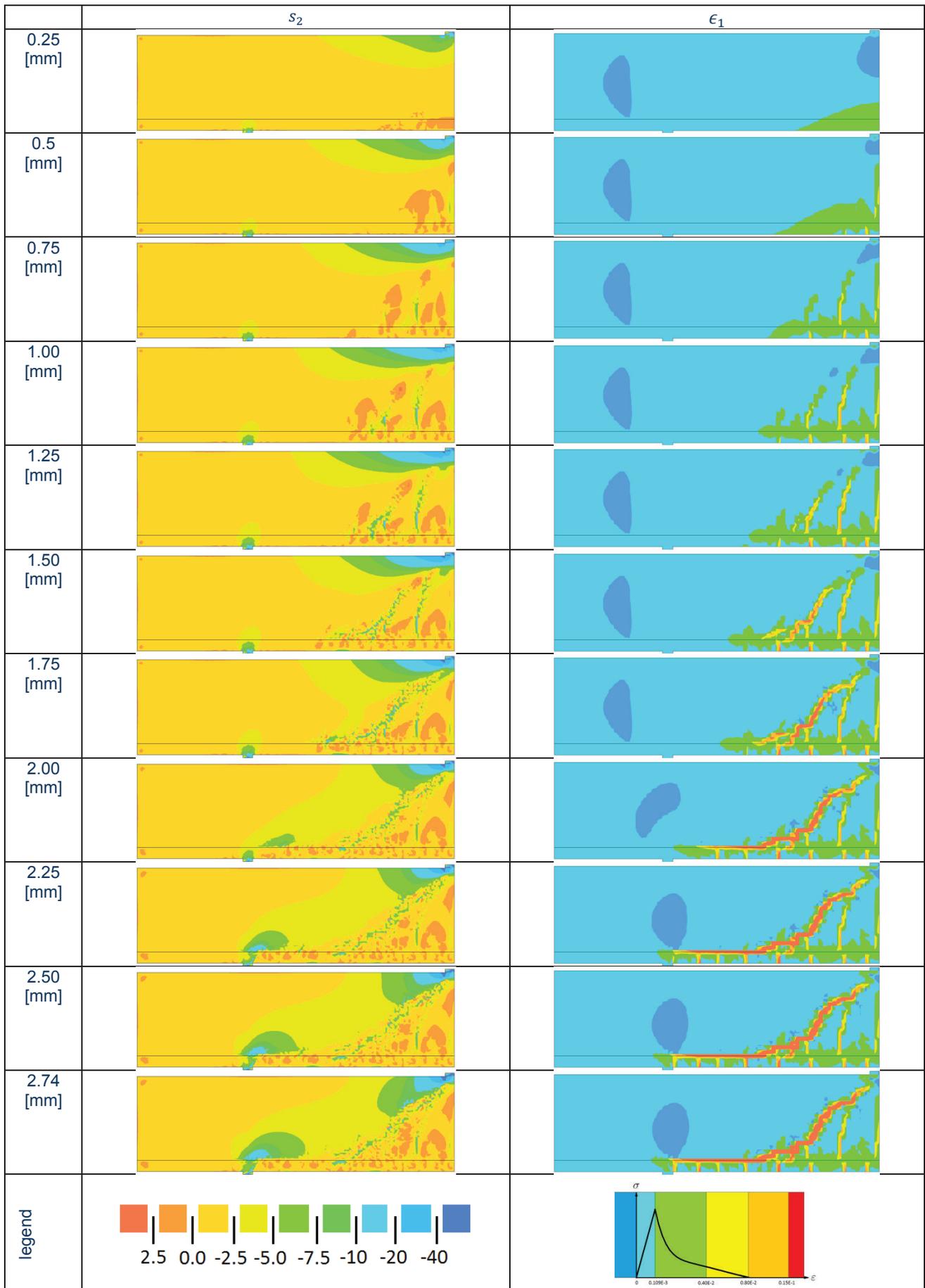


Figure 11.11 Stress and strain development beam

### 11.2.3 Truss model

This paragraph describes the results of the standard truss model of the Beeby experiment. Figure 11.12 shows the force-displacement diagram of the truss model. The diagram is divided in several stages. Together with the strain and stress development shown in Figure 11.13 an analysis is made of the development and failure mode of the truss model. The force-displacement diagram shows that the failure load is 180 kN

The truss starts with the linear elastic state. Stage A ends when a bending crack occurs in the middle of the top structure. In the bottom fibres of the top concrete part the tensile stress limit is passed and a crack occurs. The crack is shown in stage 0.5mm of Figure 11.13 where  $\epsilon_1$  shows a crack at the middle of the structure.

In stage B this crack grows and two compression points occur, one in the top of the structure and one at the corner of the gap, shown in step 1.0 mm in Figure 11.13. This gives an indication that a compression strut is occurrence between these points.

In stage C the cracks occur around the reinforcement, like in the crack formation stage of the beam. Also tensile stresses are occurrence at the top of the concrete above the support. Besides there are some compression stresses which move from the left to the right of the support. This strains and cracks around the reinforcement make it possible to gain enough deformation to act like a hinge. This is clearly shown in Figure 11.13 in the differences between the strains of step 1.5mm and 2.0mm, where the strains around the reinforcement have developed.

In stage D all the strains and stresses are increasing the compression truss is developing. The stress developing shows clearly that the strut is not straight, but goes along the corner of the cut out section.

In stage E the construction fails. A concrete cracks occur at the top of the beam directly above the support, high compressive stresses occur around the support. The stresses around the support are not in equilibrium after the crack and the structure fails. Probably the structure is now unable to bring the compression forces into the reinforcement steel. The reinforcement steel is in this stage still not yielding, but is close to the yielding stress.

The truss mechanism occurs after a crack in the top and activation of the reinforcement. The truss mechanism keeps fails when the construction is unable to transform the compressive stresses to the reinforcement and the support.

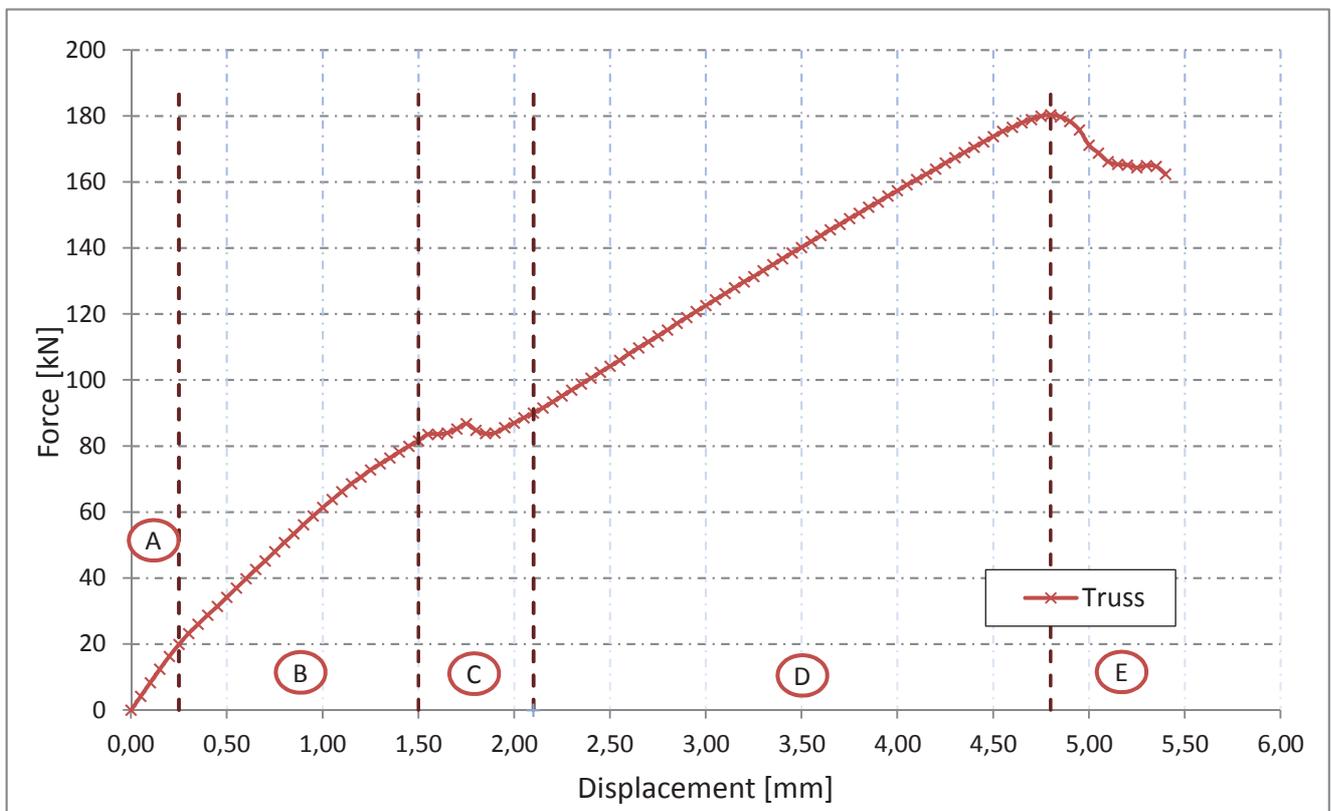


Figure 11.12 Force-displacement diagram truss

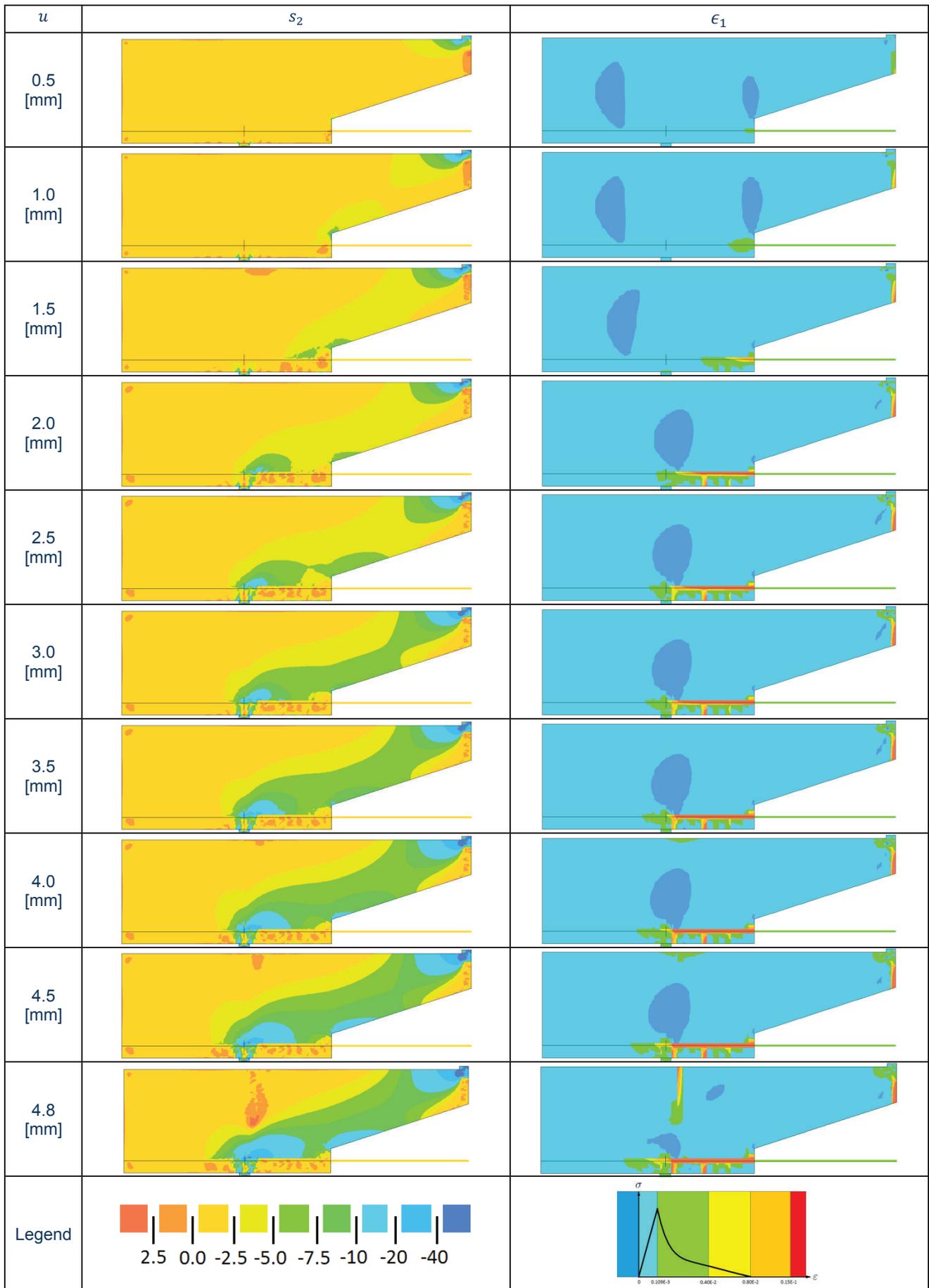


Figure 11.13 Stress and strain development truss

## 11.3 Verifying models

### 11.3.1 Verifying with Beeby

The FEM-models are designed to approach the results from the Beeby experiments. Some of the Beeby dimensions are unknown and assumed in paragraph 10.3. With this chosen dimensions the parameters of the FEM model are chosen that the failing mechanisms and failure loads are close the Beeby experiment.

The differences in failure load are showed in Table 11-1. It shows that the beam failure loads, all by shear, are close to each other. This gives a proper indication for the kind of failure of the beam. The experiment, the analytical model and the FEM-model show the same kind of failure mechanism with approximately the same failure load. This shows that the FEM-model is a proper approximation of the Beeby Experiment. The occurrence of the truss mechanism after failing of the beam is not taking into account in the analytical model and in the Beeby experiment the shear failure is taken as the failure mode. The failing off shear is the main failure mechanism.

Table 11-1 Failure loads different models

	Beam [kN]	Truss [kN]	Ratio
<b>Beeby Experiment</b>	120 – 135	202	1.68– 1.49
<b>Analytical Model</b>	126	192	1.52
<b>FEM Model</b>	131	180	1.38

The truss failure loads differs more, but are in the same range. The ratio for the FEM model is lower. The comparing of the failure mechanism of the truss is difficult. In the Beeby experiment the failure mechanism was not clear. The failure mechanism was bond failure near the support, and the stresses in the reinforcement were close to yielding. In the analytical model the failure was the yielding of the reinforcement steel. In the FEM-model the failure mechanism is also not very clear. By deformation of the truss a vertical crack occurs straight above the support at the top of the structure. This cracks results in

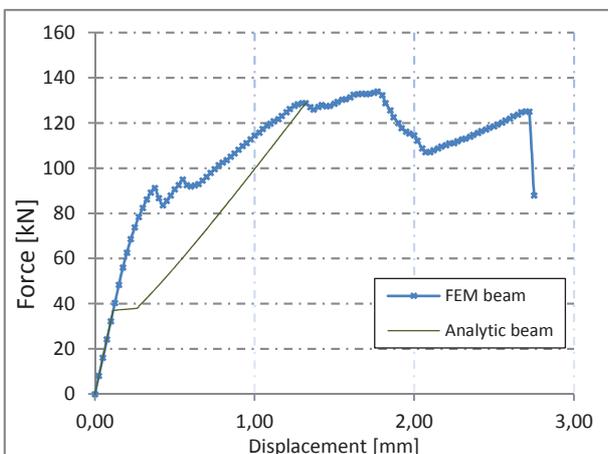


Figure 11.15 Force displacement diagram FEM and analytic beam

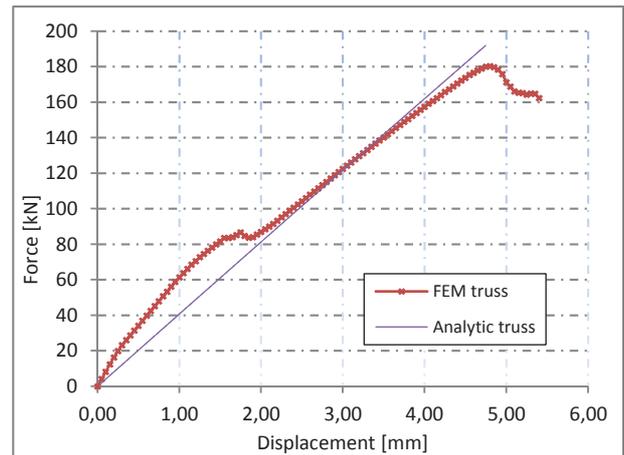


Figure 11.14 Force displacement diagram FEM and analytic truss

a hinge above the support. The structure is unable to handle the stress changes and fails.

The failures are close to each other, both the FEM and the Beeby model fail just before the yielding stress is reached, which is the failure load of the analytical model. This gives an indication that the FEM-model gives a proper approximation for the truss model from the Beeby experiment. That the different models show different failure mechanisms shows that the failure mechanism is not totally clear.

### 11.3.2 Verifying with analytic solutions

Besides the failure mechanism the force displacement diagrams of the analytical and FEM model are compared to verify the FEM model.

The force-displacements diagrams of the beam model are shown together in Figure 11.15. The analytic beam and FEM show a different relation between the force and the displacement. The crack formation stage of the FEM model is with a higher force than the analytical model. The explanation for this difference is the assumptions in the analytical model that the bending cracks occurs entirely at once when the tensile stress limit is reached at the bottom fibre of the beam.

In the FEM model tensile stresses are possible after the tensile stress limit is reached, while in the analytic model no tensile stresses are possible when the limit is passed, this declares the differences.

The force-displacement diagrams of the truss models are shown in Figure 11.14. The analytical truss is almost a straight line, while the FEM-model shows some variation. The differences in the first part are declarable by the activation of the reinforcement, as declared in paragraph 11.2.3. In this part the structure changes to a truss mechanism. When this activation of the strut is realised the analytic and FEM truss act almost the same. The differences between the analytical and FEM model in the second part are declarable by the assumptions that the strut only has normal stresses in the analytical model, while in the FEM model also bending stresses cause deformation.

### 11.3.3 Conclusion

The verifications showed that the FEM models give a proper approach for the Beeby experiments. The failure loads are close to each other and the force displacement diagrams of the analytic and FEM models give declarable differences. Only the failure mode of the truss mechanism does not give an equivalent result.

## 11.4 Comparison beam and truss

The FEM analyses show clearly the differences in stress development and force mechanisms of the beam and truss mechanism.

The beam will first act as Bernoulli beam, till the first crack occurs in the beam. Some bending cracks together form a shear crack which results in this case in a truss mechanism.

The truss model develops as a truss after a crack is raised in the top structure. A truss from the top to the support occurs when the reinforcement is activated.

In Figure 11.16 both the FEM force-displacement diagrams are shown. The difference in stiffness in both systems is seen directly. The beam starts with a lot higher stiffness and reaches the crack formation stage with a lot smaller deformation.

The stiffness of the beam after the crack formation stage is almost the same as the stiffness of the truss, visible in the almost same slope of

The beam and the truss both form a truss mechanism. This is also shown clearly in the graph by the same slope in the last part of both models. The beam model fails with a smaller force and deformation because it is not able to find force equilibrium in the truss mechanism. The stress developing of the truss mechanisms is almost the same, shown in Figure 11.17.

Probably this difference in failure is by the shape of the shear crack. An increasing force in the truss results in larger compressive area in the top. In the truss mechanism there is a vertical crack in the top of the structure, which is able to transform horizontal compression stresses over the crack, while in the beam the shear crack result in an almost horizontal crack which is much more sensitive to horizontal sliding.

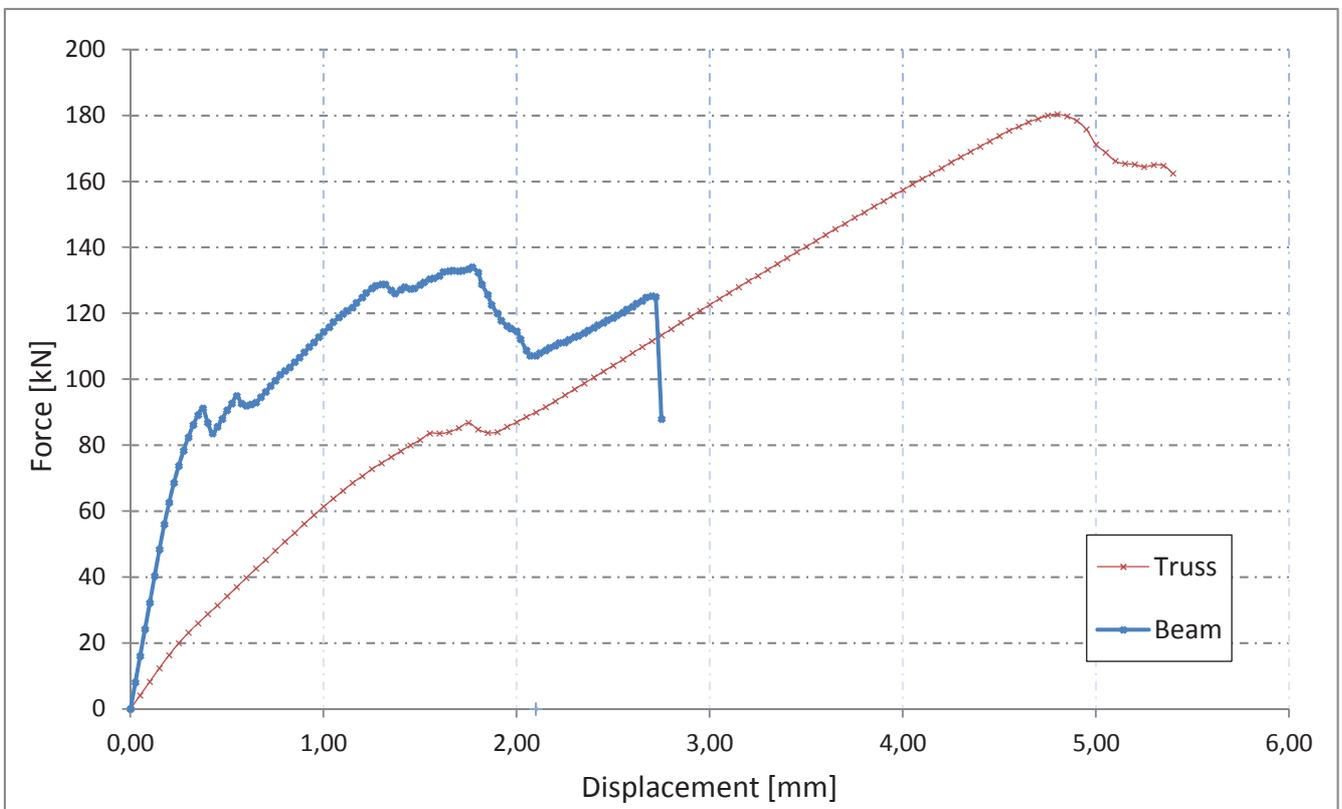


Figure 11.16 Force displacement diagram beam and truss

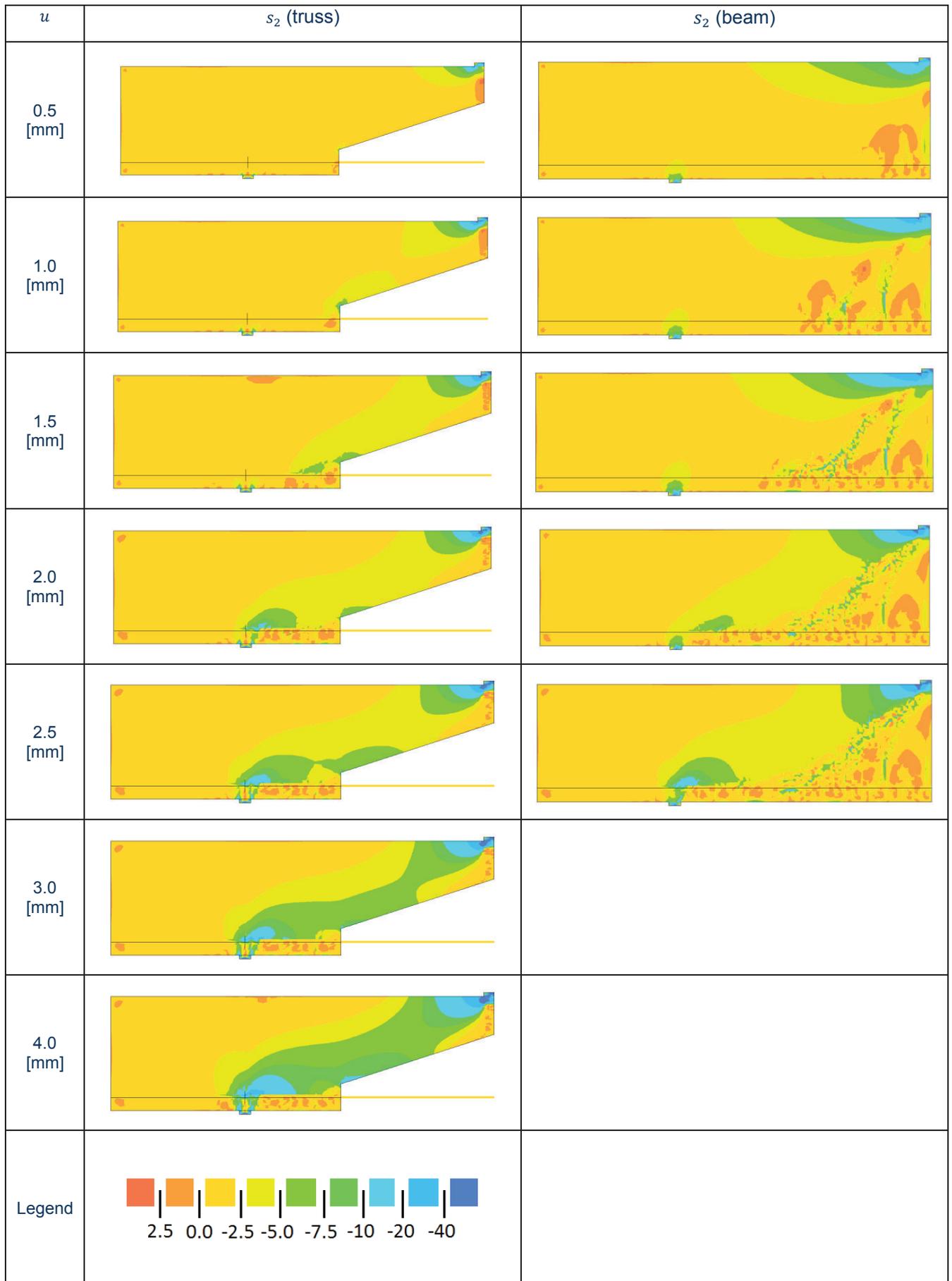


Figure 11.17 Comparison stress development beam and truss mechanism

## 11.5 Conclusion basic models

DIANA is used for the FEM analysis. Within the modelling of the structures a lot of parameters are assumed to approach the reality as good as possible. Some influence factors are the mesh-size, element type and crack model. In addition to this some parameters are unknown from Beeby's experiment, which are assumed during the modelling.

The beam model shows a shear failure with a maximum force resistance of 131 kN which is in the same range as the analytical model and the Beeby experiment. The model shows that the beam first acts as a Bernoulli beam till the first bending crack occurs. After occurrence of some more bending cracks the beam fails by a shear crack. This shear crack makes it possible to convert to a truss mechanism, but this mechanism fails again before a new force limit is reached. The failing of this truss mechanism is probably by horizontal sliding over the shear crack in the concrete compression top.

The truss FEM model starts with the occurrence of a concrete compressive strut. But this occurs from the top till the corner of the cut-out section. The truss transforms to a support – top strut when the reinforcement is activated. The failing of the truss is not very clear. The steel is close to yielding, a crack is occurrence straight above the support at the top of the structure and high compression stresses occur around the support. Probably the truss is unable to transform the high compressive stresses to the reinforcement, the connections fails between steel and the reinforcement and the truss mechanisms fails.

The differences between the beam and truss models in the FEM-analyses are similar to the analytic models. The beam is a lot stiffer then the truss mechanism. The shear crack does not occur in the truss, whereby the force limit is higher. The deformation capacity of the truss mechanism is important to realize this. The deformation of the truss mechanism is significant higher as the beam mechanism.

The difference in failure load after both occurrence a strut mechanism is declarable by the shear crack which is influencing the truss mechanism. Probably this difference can be explained by the shape of the shear crack. In the truss mechanism there is a vertical crack in the top of the structure, which is able to transform horizontal compression stresses over the crack, while in the beam the shear crack result in an almost horizontal crack which is much more sensitive to horizontal sliding in the compressive area in the top of the structure.

Both the analytical and the FEM analysis show the difference in failure load and failure mechanism which were the results of the Beeby experiment. It shows that with less material a higher force mechanism occurs. The beam mechanisms fails by a shear failure, while the truss mechanism fails by transferring the compressive stresses into the reinforcement steel.

## 12 Parameter study

The basic models show an obvious difference. But what determines this difference? In this chapter the dimensions of the gap are varied to explore the influences of the gap. Does it give an explanation for the differences between the beam and the truss and does it explain the failure after a shear crack?

First the height of the gap is varied, followed with the span and the corner of the gap. The models show different failure mechanisms which are arranged in different categories.

### 12.1 Gap height

The first variation is the height of the gap. By variation of this height, the angle of the cut out section changes and the height of the compression hinge in the top. The variable parameter is shown in Figure 12.1. The remaining properties are kept the same.

The results of the FEM calculations are shown in Figure 12.3. The graphs show hardly any differences between the gap heights. All the models act like the basic truss model. Only the first phase of loading shows a remarkable difference. The difference is caused by the translation from a beam to a truss. For this translation a crack occurs in the middle of the beam. This crack forms a hinge in the top of the beam. A lower gap height gives more resistance for the occurrence of this crack. This explains the high stiffness for small gap height in the first phase of loading, but after the occurrence of the hinge the model will act like the basic truss. The differences in failure loads are due to the differences in failure loads caused by the different mesh.

### 12.1.1 Conclusion

The gap height has no influence in the failure mechanism and no remarkable influence on the failure load. The gap height has only influence on the stiffness before the cracking moment is reached. When the cracking moment is reached a crack occurs in the middle of the span and a hinge occurs at the middle of the span. The mechanism is the same as the basic truss after the crack in the middle.

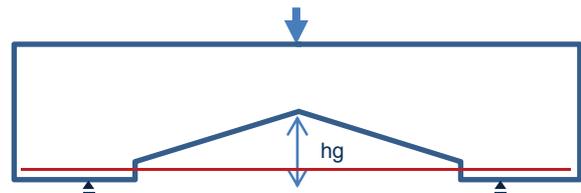


Figure 12.1 Gap height variation

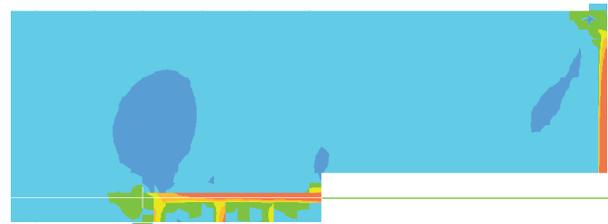


Figure 12.2 Crack at middle span for  $hg=70$

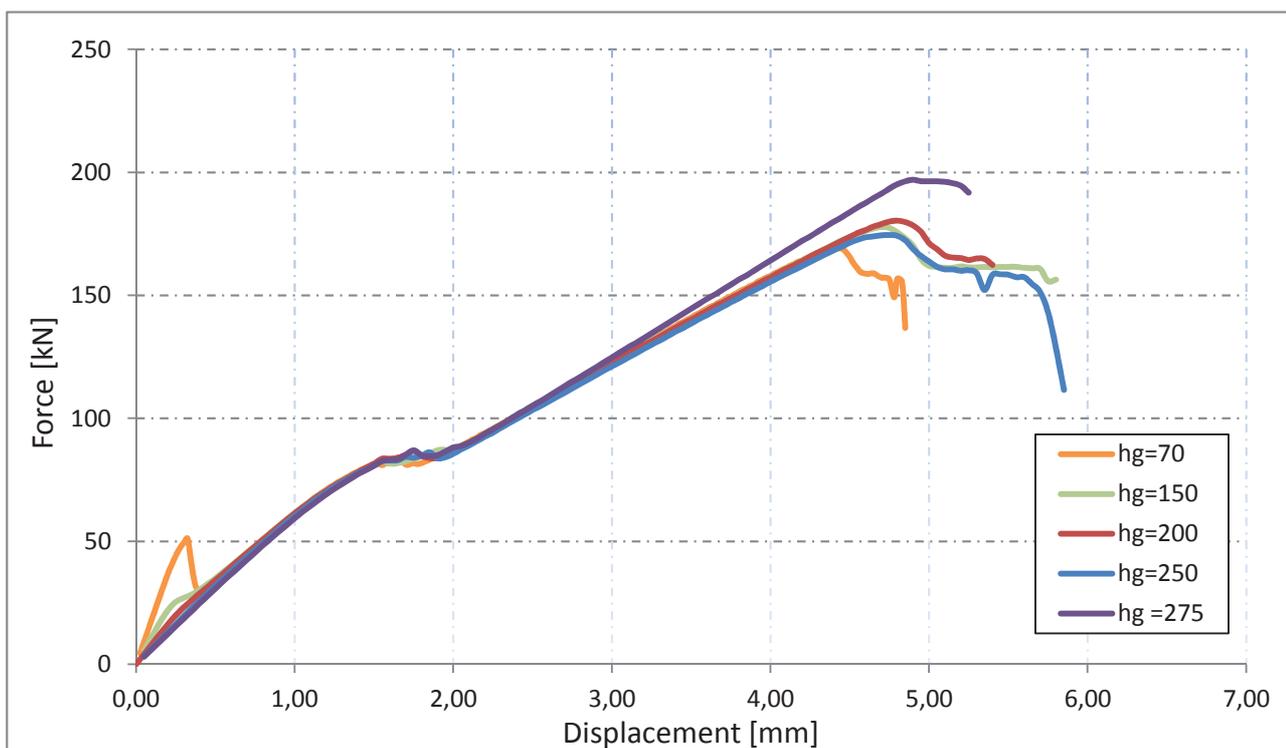


Figure 12.3 Force-displace diagram for gap height variation

## 12.2 Variation of gap length

In this paragraph the influence of the gap length is explored. The basic truss variant has a gap length of 800mm.

### 12.2.1 Larger gap length

First the influence of a larger gap length is regarded. The length is increased in steps of 100mm. The belonging force-displacement curves are shown in Figure 12.4.

The graphs shows that for larger lengths the force resistance is the same. Expect for the large lengths which cross the line which is needed for the truss. By crossing this line it is impossible to transform into a truss mechanism. This inability to change into a truss results in another failure mechanism. If the gap passes the line, a cantilever part occurs. The cantilever part fails by a crack as shown in Figure 12.6. The beam is unable to transform into a truss.

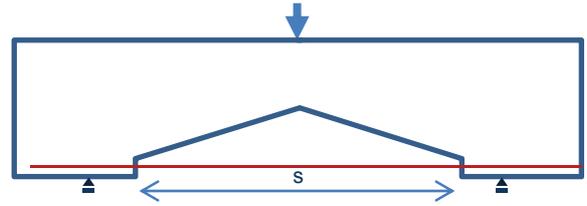


Figure 12.5 Gap length variation



Figure 12.6 Crack at cantilever part

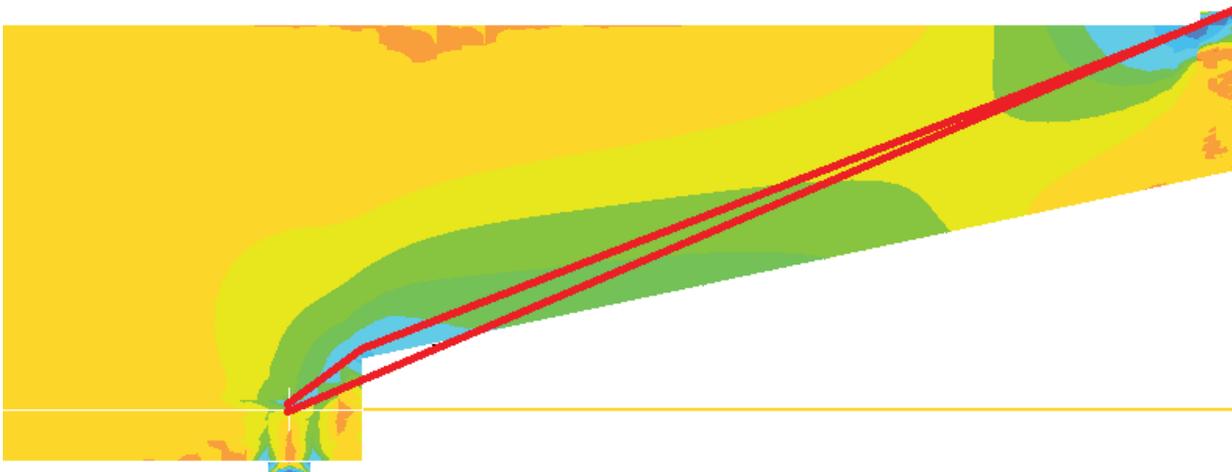
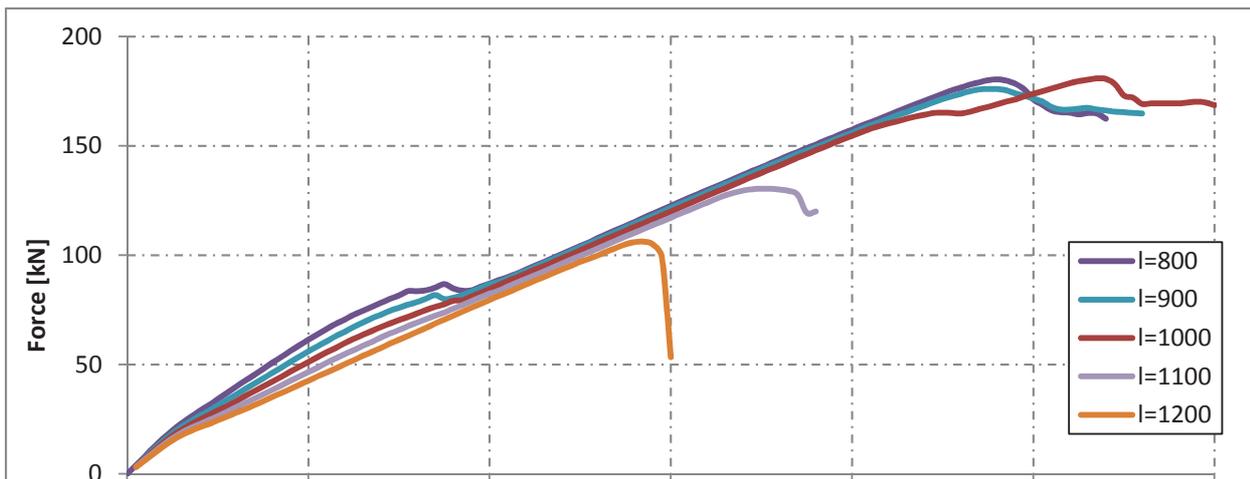


Figure 12.7 Stress plot gap length 1200mm with compressive struts [N/mm<sup>2</sup>]

### 12.2.2 Smaller gap length

The influence of a smaller gap length is described in this paragraph. Figure 12.9 shows the force-displacement curves.

The first obvious difference is the higher stiffness in the first phase of loading. It shows a higher stiffness for smaller spans.

With smaller gaps the models act more like a beam mechanism. This seems logical because the model shape is approaching the beam.

The reinforcement is activated with a displacement of about 2mm. With the activation of the reinforcement it changes to a truss mechanism. The models with a gap smaller than 250mm do not reach the real truss mechanism. All these models show a flexural shear crack.

The models with a shear crack show a lower failure load. These models have a smaller gap. With a gap length of 250mm the model will act as truss, while the model with 200mm gap length fails before the truss limit is reached. The small gaps have a flexural shear crack which result in an unstable truss or the impossibility to change to a truss mechanism. Figure 12.8 shows the crack pattern for the 200mm and 250mm gap length models. This figure shows that for small lengths a shear crack occurs.

A small gap length results in a flexural shear crack and a large gap length in a truss. The difference in failure load depends on the presence of a flexural shear crack. A shear crack results in failure, while the absence of a shear crack results in a truss mechanism. So the crack determines the occurrence of the truss mechanism.

Figure 12.8 shows that the reinforcement is activated in both models. All the models with a 200mm or smaller gap length have a shear crack. The larger gap lengths do not have a shear crack. A too small gap length has to less deformation capacity, and thereby the stiffness too high, which result in a flexural shear crack which makes it possible to deform more. If only the deformation capacity determines the occurrence of the shear crack the crack is expected in a later phase for longer gap lengths. But the models do not show this. After a certain decrease of stiffness something happens which ensure that the shear

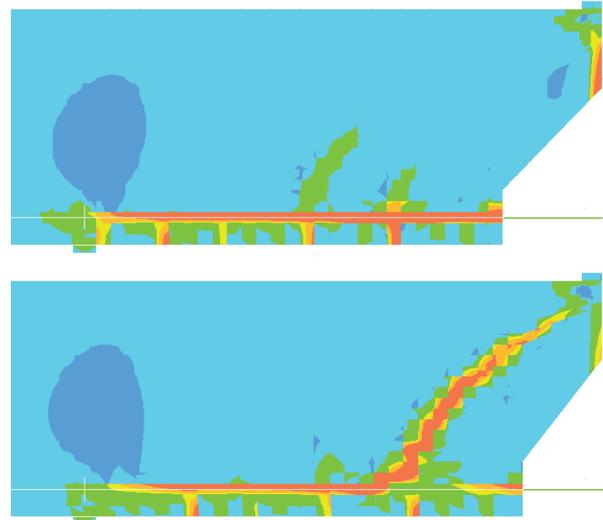


Figure 12.8 Difference crack pattern between 200mm and 250mm gap span

crack does not occur. This difference is more looked into detail in Paragraph 13.1.

### 12.2.3 Conclusion

The gap length influences the capacity and force mechanism of beam models. The stiffness is lower for high lengths and the transformation to a truss is possible. The limit for large gap lengths is the passing of the compressive truss.

For small lengths the stiffness is higher and the reinforcement activations results in bending cracks and shear cracks. With small lengths this results in lower failure loads. The truss is not able occur. Not only the deformation capacity results in failure, because a little bit larger gap does not results in a flexural shear failure in a later phase as expected if the stiffness was the only difference.

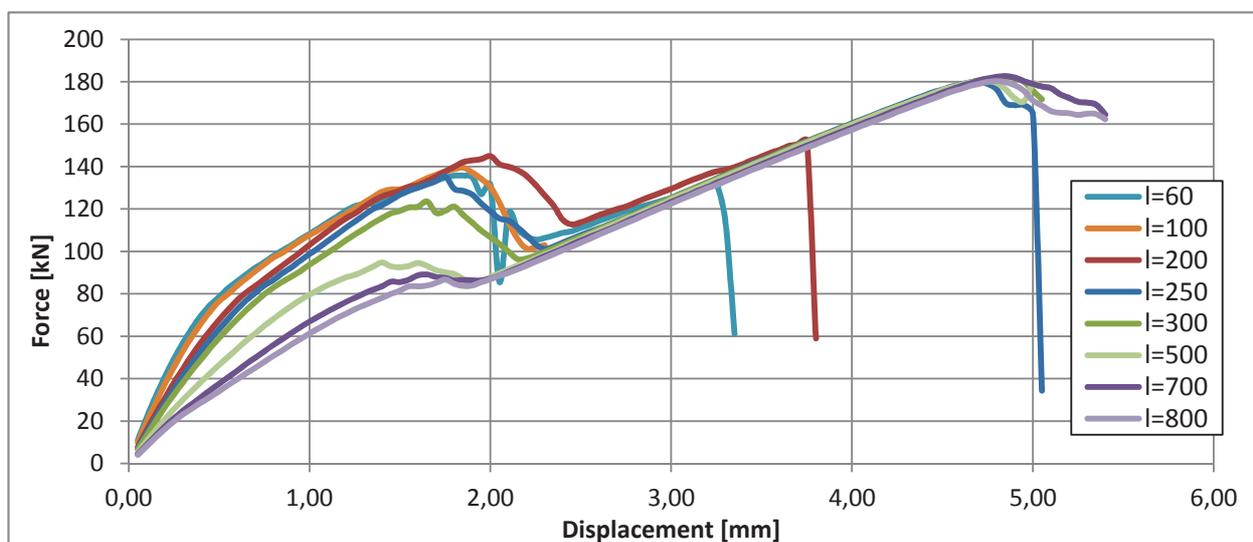


Figure 12.9 Force-displacement diagram for gap span variation

### 12.3 Limits cut out section

The height and length of the gap are varied. This paragraph give the limits of the gap.

The height of the gap is kept constant, 250mm, because the influence is small, as shown in paragraph 12.1. The length ( $s$ ) and height of the corner of the gap ( $d_1$ ) are varied, as shown in Figure 12.10.

For finding the boundaries of the truss mechanism models with different dimensions are used. Different beam dimensions are modelled to find the boundaries of the truss system. For each model  $s$  and  $d_1$  are varied. The results for all the models divides in several categories which give the same kind of results.

Figure 12.13 gives an overview of the results. Every dot represents one model variation. The dot is represents the corner of the gap of the representing model. With a straight line to the bottom and a line to the middle height of 250mm the dimensions of the model are fixed.

The results are divided in four categories, represented with the different colours:

- Truss mechanism without shear crack (Green)
- Truss mechanism does occur with shear crack (Blue)
- Flexural shear crack failure (Yellow)
- Failure of cantilever part (Blue)

Every category is discussed separately in the next paragraphs.

#### 12.3.1 The truss without shear crack

In this category the truss occurs and reaches the same kind of failure load as the basic truss. There are no large cracks and the truss occurs after activation of the reinforcement. An example of the stress distribution and crack pattern is shown in Figure 12.11. All these models fail by a crack in the top of the beam, straight above the support. If a shear crack has this shape the beam is able to transform to a truss. In Figure 12.13 this category is pictures as the green dots.

#### 12.3.2 The truss with shear crack

This category with the developing shear cracks has the same failure load and failure mechanism as the basic truss. But these occurs a shear crack, but this does not influence the truss. The shear crack always end beneath the cantilever part and has no influence in the truss mechanism. An example of the stress distribution and

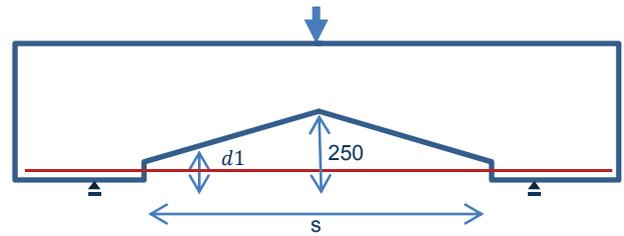


Figure 12.10 Gap size variation [mm]

crack pattern is shown in Figure 12.12. In Figure 12.13 this category is pictures as the blue dots.

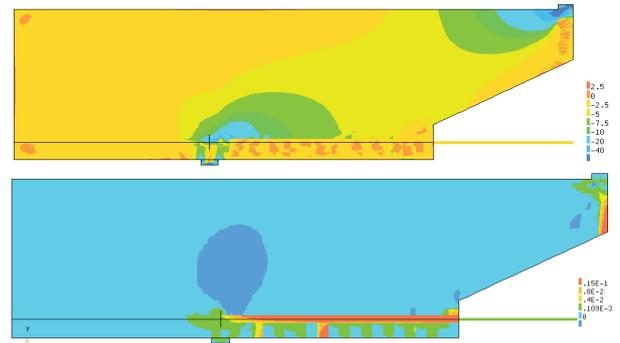


Figure 12.11 Stress and crack development truss without shear crack

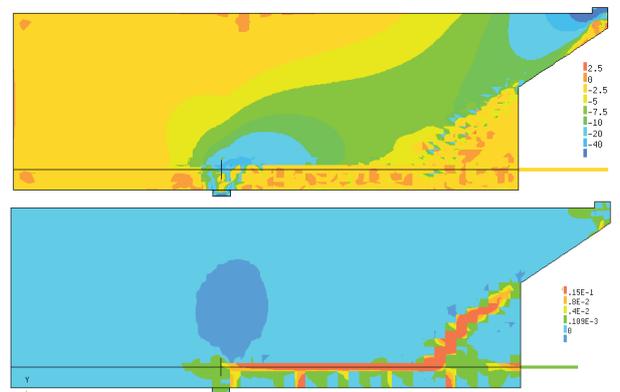


Figure 12.12 Stress and crack development truss with shear crack

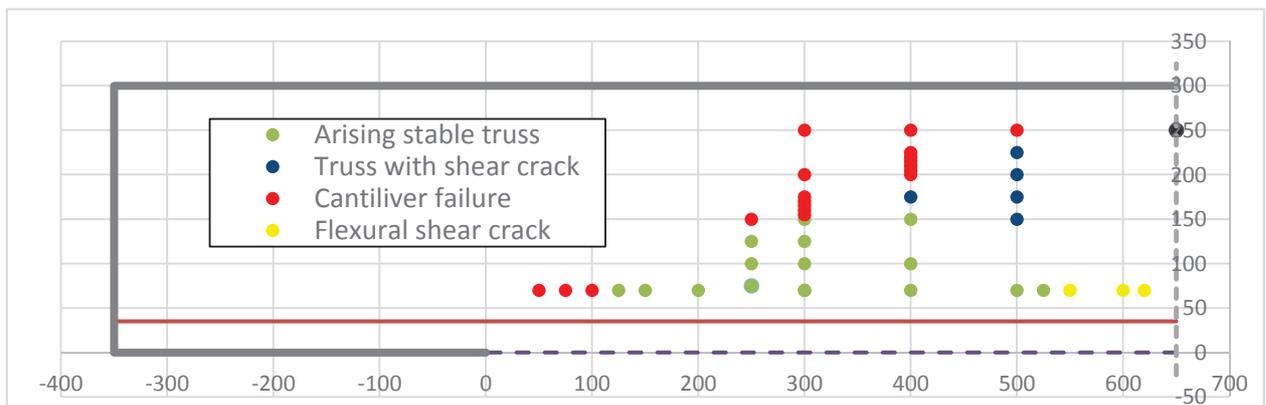


Figure 12.13 Overview and catearisation different models

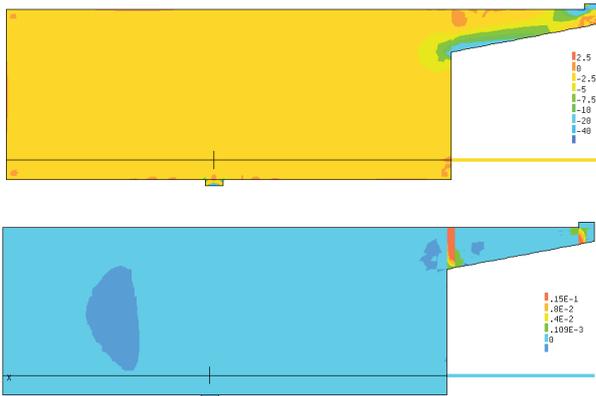


Figure 12.14 Stress and crack development truss failure of cantilever part

### 12.3.3 Failure of cantilever part

In this category the truss mechanism does not occur. The models fail by failing of the cantilever part in the structure. The stress and crack pattern are shown in Figure 12.14. This category, the red dots in Figure 12.13, shows that the truss is not possible if the gap passes the strut line. This line lies between the force introduction point and the reinforcement point just above the support. When it passes this line, the truss is not able to develop a straight compressive strut, which results in failing of the cantilever part. A truss mechanism occurs in the cantilever part, which fails in combination with the occurrence moment, as explained in Figure 12.14. The figure shows graphically the failure. A truss develops in the cantilever part. A moment develops by the external position of the truss it results in a moment which results in failure.

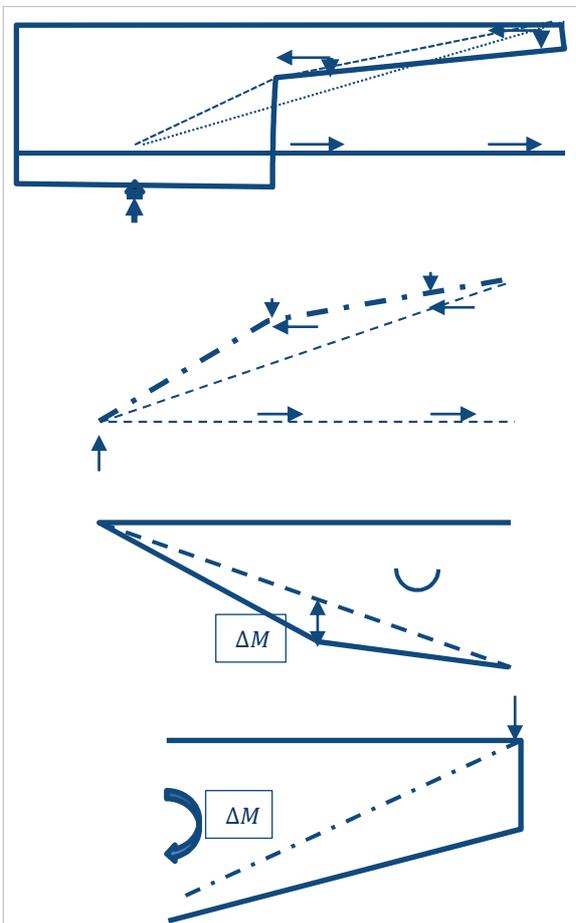


Figure 12.15 Cantilever mechanism

### 12.3.4 Shear crack

This category consist of beams which fails by a flexural shear crack before a new force limit is reached. In Figure 12.13 this category is pictures as the yellow dots. The models in this category have small gap length. This gives a high stiffness which is close to the basic beam. These beams do not have enough deformation capacity, as explained in paragraph 12.2.2, to change to a truss. Most of the models in this category show a development of a truss in the stress development after the crack, just like in the basic beam. But it is not clear why the flexural shear crack results in failure.

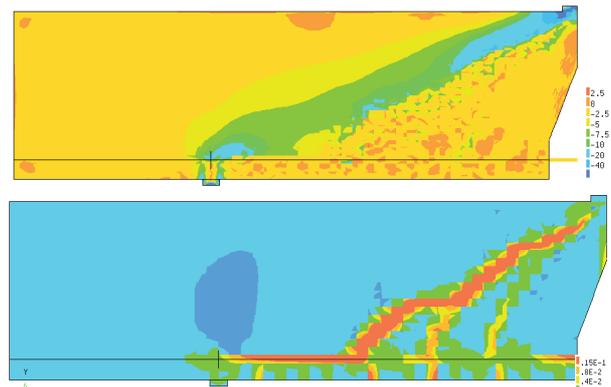


Figure 12.16 Stress and crack development flexural shear crack



# *FLEXURAL SHEAR CRACK*

*LIMITS SHEAR CRACK*  
*FLEXURAL SHEAR CRACK*



# 13 Limits shear crack

This chapter discusses the boundaries of the shear crack into more detail. When does a shear crack occur and why does it fail?

## 13.1 Occurrence of shear crack

The gap length variation shows a boundary between the green and yellow dots in Figure 12.13. What determines the occurrence of the shear crack or the change in force mechanism without a shear crack?

A flexural shear crack occurs when there is not enough deformation capacity to resist the force with the cracked beam mechanism. Figure 13.1 shows the boundary between the occurrence of the shear crack and the truss mechanism. In the bottom picture the truss occurs while in the top picture, with just a smaller gap, a flexural shear crack occurs which results in a lower force limit.

### 13.1.1 Gap length 200mm and 250mm

The variation of the gap length shows the boundary between the occurrence of a flexural shear crack and the occurrence of a truss without a shear crack. The boundary lies between the gap lengths of 200mm and 250mm. First the force-displacement curves are compared followed with the stress developing.

#### Force-displacement curves

Figure 13.2 compares the curves. In the first part of loading the mechanisms act the same. The difference is in the crack formation zone. In this zone the 250mm gap shows a translation to the truss mechanism with a smaller force and a smaller displacement. The 250mm beam activates the reinforcement earlier than the 200mm beam. The 200mm shows a shear crack before it transforms to a truss mechanism, but this truss mechanism does not reach the same kind of failure load as the 200mm model.

The shapes of the trusses after the shear crack are different, so it is not possible to compare the truss mechanisms after the crack. The flexural shear crack gives the truss another boundary then the gap of the 200mm gap beam.

The force displacement diagrams shows that the 250mm beam is less stiff and changes to a truss mechanism before a flexural shear crack occurs. The flexural shear crack does not occur. To see what determines the

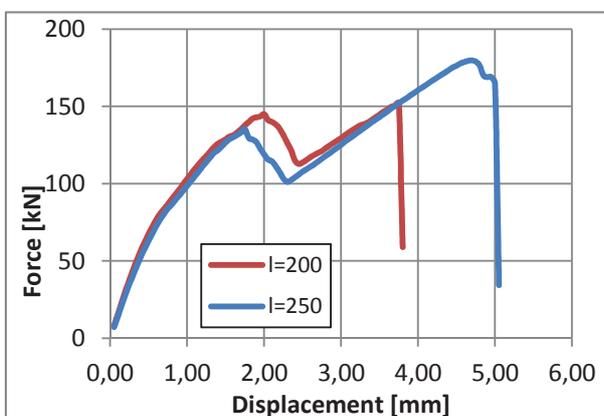


Figure 13.2 Force displacement diagram Gap length 200mm and 250mm

difference the stress development before the shear crack is compared.

#### Stress developing

The stress development is compared for the 200mm and 250mm models. Figure 13.3 shows the stress development of both beams for different loading steps. The stress  $\sigma_{xx}$  is used to compare the models.

The stress development is compared in different sections. The sections are pictured in the top of Figure 13.3 by the red lines. The sections are on the same location in both models to make a proper comparison.

The stress development is shown next to each other to make for each section. On the vertical axis the distance from the top, and on the horizontal axis the stress  $\sigma_{xx}$  [N/mm<sup>2</sup>] are plotted. The scales differ per section.

The stresses are compared for the load steps 1,25 mm, 1,75mm and 2.0mm. Figure 12.8 shows that these displacements steps are just before the models change into a truss mechanism. The stresses before the shear crack are interesting to see the difference in the occurrence of the shear crack. Comparing the stresses after this change is not suitable because both trusses have a different shape.

The stress comparison is explained for each section: Section A shows a comparable stress development, no remarkable differences are visible.

In section B the load step of 2,0 mm shows a difference in the 250mm beam. The compressive stresses are more spread to a lower part of the beam.

Section C shows also this change. The compressive stresses are more spread to the bottom in the 250mm model instead of the high compressive stresses in the top of the section in the 200mm model.

Section D shows the developing of the compressive stresses in both beams. In the 250mm model the change of stress distribution is visible, but there is no real difference visible. This is probably declarable by the almost occurrence of the shear crack at the next, shown as the green crack shape strain development in 250mm model in Figure 13.1.

Section E and F show for both beams a comparable stress development with no remarkable differences.

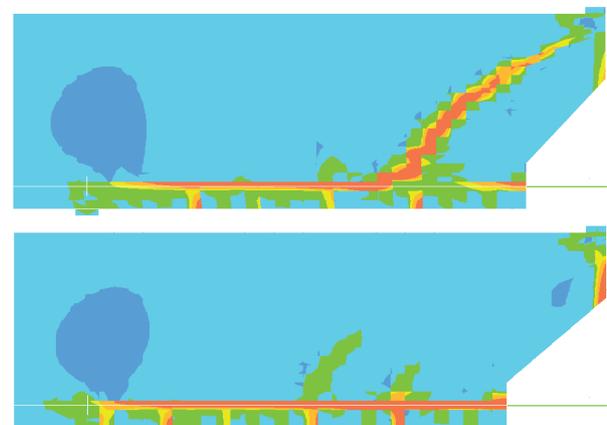


Figure 13.1 Difference crack pattern between 200mm and 250mm gap span

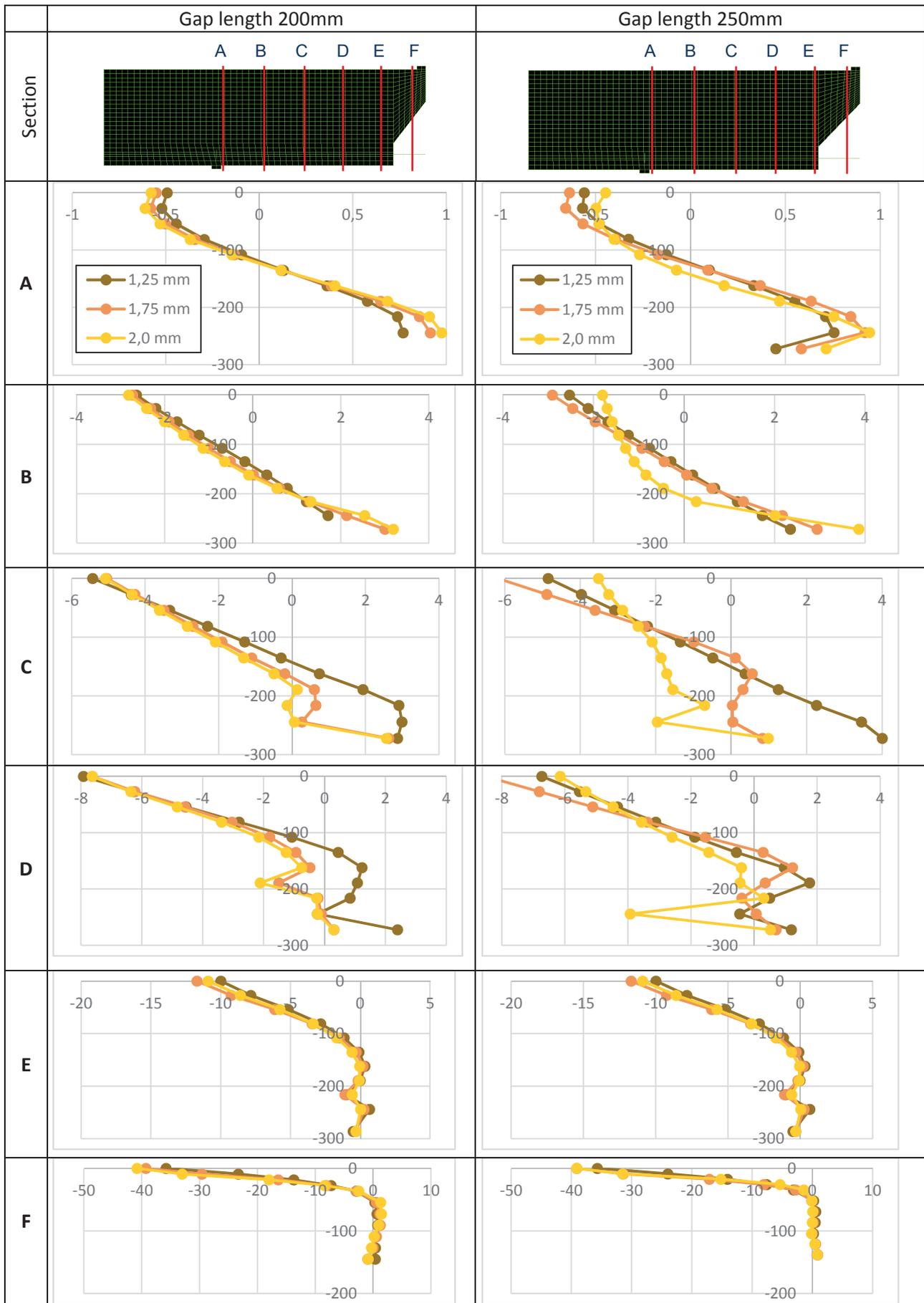


Figure 13.3 Stress ( $S_{xx}$ )  $\left[ \frac{N}{mm^2} \right]$  over the height [mm] per crosssection of 200mm and 250mm gap length

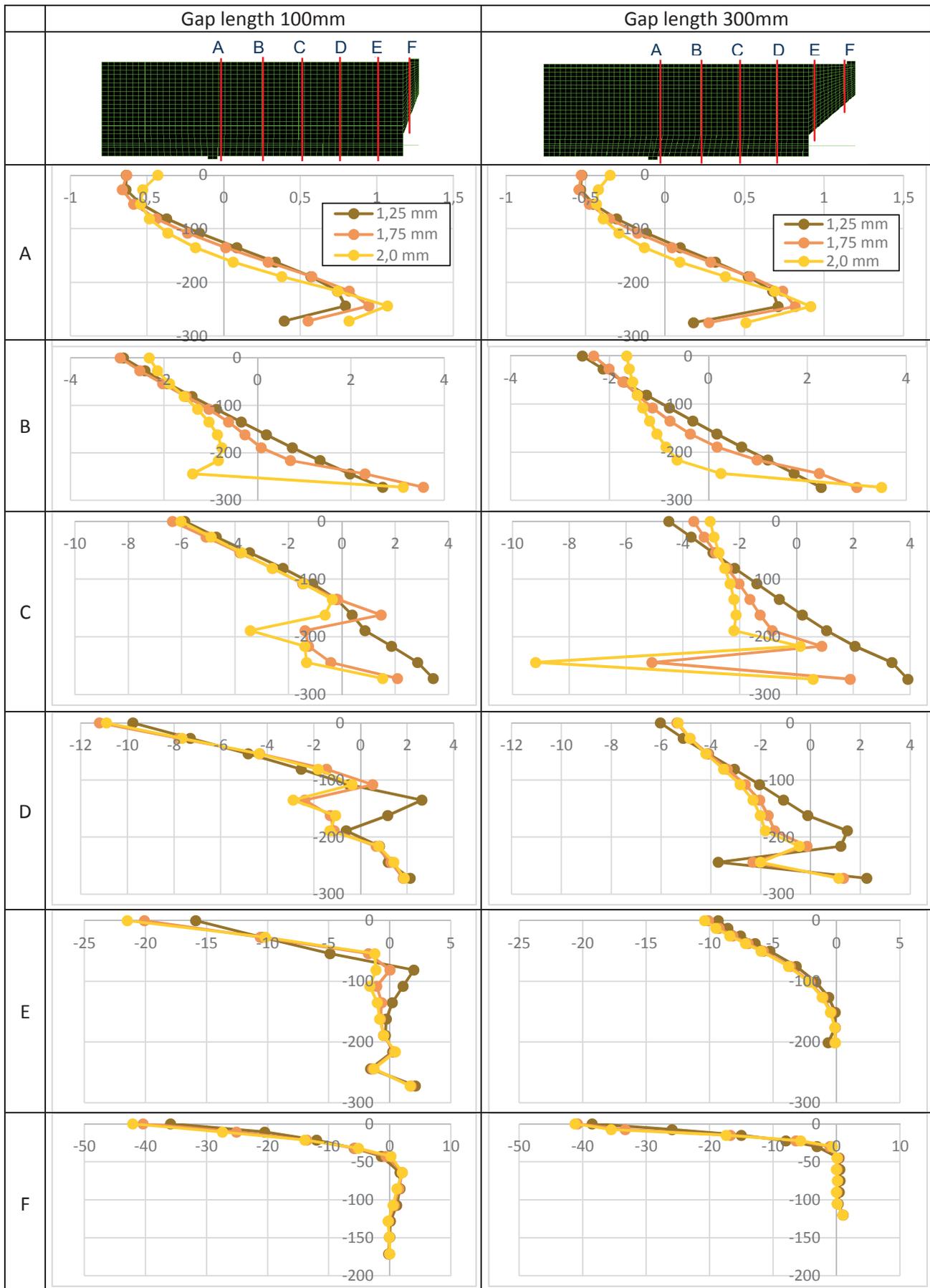


Figure 13.4 Stress ( $S_{xx}$ )  $\left[\frac{N}{mm^2}\right]$  over the height [mm] per crosssection of 100mm and 300mm gap length

### 13.1.2 Gap length 100mm and 300mm

The difference in stress development between the 200mm and 250mm beams shows a difference in compressive stress distribution. The differences around the crack are too small to give a proper comparison. In this paragraph the gap of 100mm and 300mm are compared to show if the stress distribution is better notable.

The force displacement curves are shown in Figure 13.6. It shows that the stiffness of the 100mm model is a little bit higher than the 300mm model. The 100mm model is not able to transform to a stable truss mechanism and fails immediately after the shear crack. The displacement for which the mechanisms change to a truss mechanism are almost the same. The difference of the 200mm and 250mm models are probably declarable by finite element differences.

The stresses are compared in the same way as in the previous paragraph and shown in Figure 13.4. Again the red lines in the top of the figure show the sections location. The load steps 1,25 mm, 1.75mm and 2.0mm are used to compare the stress development before a shear crack occurs in the beam. After the crack two different truss models occur.

Section A and B both show the same kind of stress developing. No remarkable differences are shown here. The same holds for the section F, where both models have high compressive stresses in the top.

Section C, D and E show all remarkable differences. In the 300mm model the compressive stress distribution spreads the height, with an increasing compressive depth. The tensile stresses in the bottom are in the same range. The larger compressive stresses in the truss models is a remarkable difference. With this compressive stresses it is understandable that the crack does not jumps into the concrete, but only occurs to activate the reinforcement. In the 100mm model the compressive stresses only occur and stay in the top. In the bottom part is it now possible for the crack to occur. There are not enough compressive stresses to prevent this.

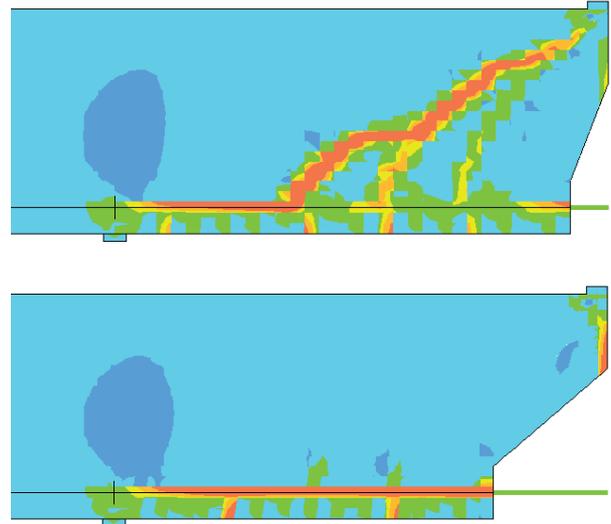


Figure 13.5 Difference crack pattern between 100mm and 300mm gap span

### 13.1.3 Conclusion

Decreasing the stiffness enough gives the structure enough deformation capacity to avoid the flexure shear crack. The flexure shear crack does not occur because the higher deformations results in higher compressive stresses in the strut mechanism. This makes it impossible for the flexural shear crack to jump into the concrete. The bending cracks, which result in flexural shear crack, do not occur by the higher compressive stresses. By the occurrence of the truss mechanism the force mechanism changes in the beam.

It could be stated that the flexural shear crack is the winning of the crack in a competition between the developing of the compression stresses of the truss mechanism and the bending cracks of the cracked beam mechanism. By decreasing the stiffness less cracks are needed and more compressive stresses develop.

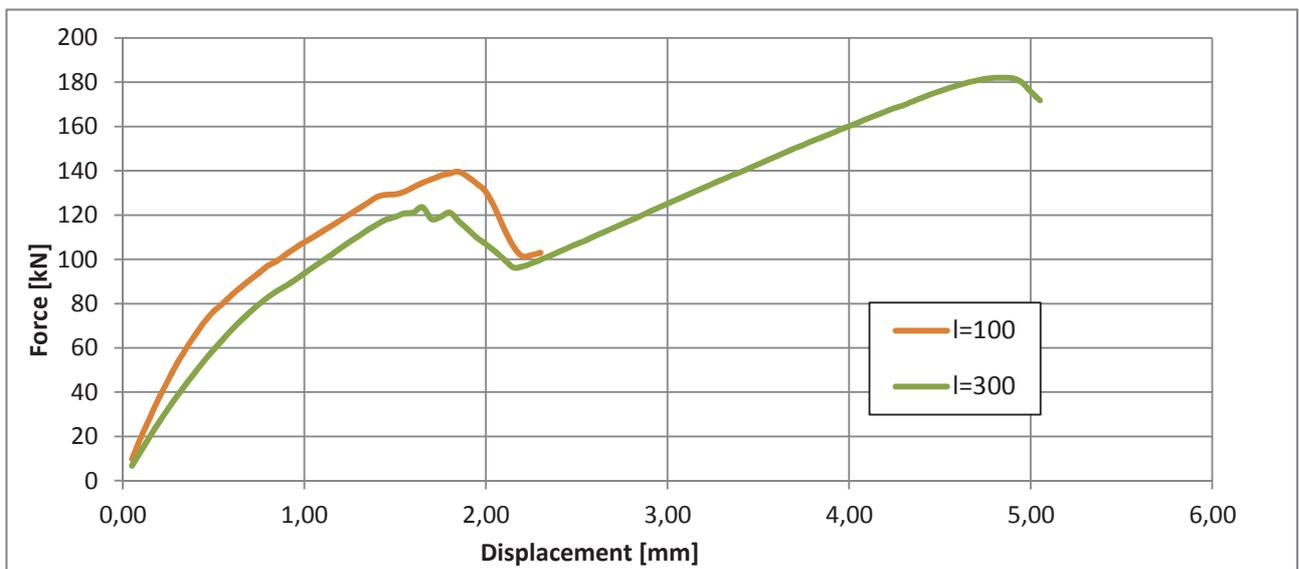


Figure 13.6 Force displacement diagram Gap length 100mm and 300mm

## 13.2 Failing flexural shear crack

The failing after a shear crack has different forms. Some models show the occurrence of a truss mechanism after the crack, while other fail immediately after the shear crack. The models with a flexural shear crack fail in three different ways:

- Beam fails immediately after a flexural shear crack.
- A flexural shear crack occurs in the beam and the model changes to a truss. This truss is not able to reach the same kind of failure load as a truss without a shear crack.
- Beam changes to a truss and transforms to a truss mechanism and fails in the same way as the truss mechanism.

Allmost all the models fail by the second category. The basic beam changes to a truss, but fails before the optimum of the truss mechanism is reached. This paragraph describes the influence of the shape of the crack and the influence of the concrete beneath the crack.

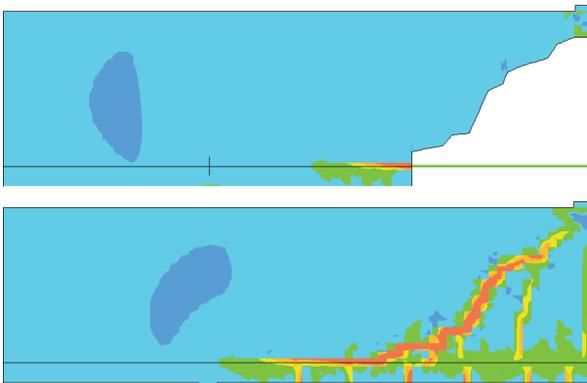


Figure 13.7 Cracked basic beam with mesh overlay of truss with crack shape

### 13.2.1 Truss with crack pattern

The previous models show that the shear crack pattern is different than the smooth cut out section of the truss mechanisms. Does the shear crack pattern influence the truss mechanism? The influence of the crack pattern is investigated with a truss model with the same shape as the shear crack which occurs in the basic beam.

Figure 13.7 shows the crack pattern of the beam mechanism and the truss model with the crack shape. It shows that the truss follows the crack line. With this model it is possible to investigate the relation of the crack shape on the failure mechanism.

The force displacement curves of the basis beam, the basic truss and the truss with the crack shape are shown in Figure 13.8. It clearly shows that the cracked shape truss acts almost the same as the basic truss.

Before the shear crack the models are not comparable. After the flexural shear crack of the basic beam it follows the line of both trusses, but the line of the basic lies a bit higher than the truss lines. This difference is well known as the tension stiffening. This is the tension stiffening of the concrete under the crack.

With reaching the same force limit with the cracked shape truss as the basic truss it can be concluded that the cracked shape has no influence on the inability of the beam mechanism transforming into a stable truss. But what determines this difference?

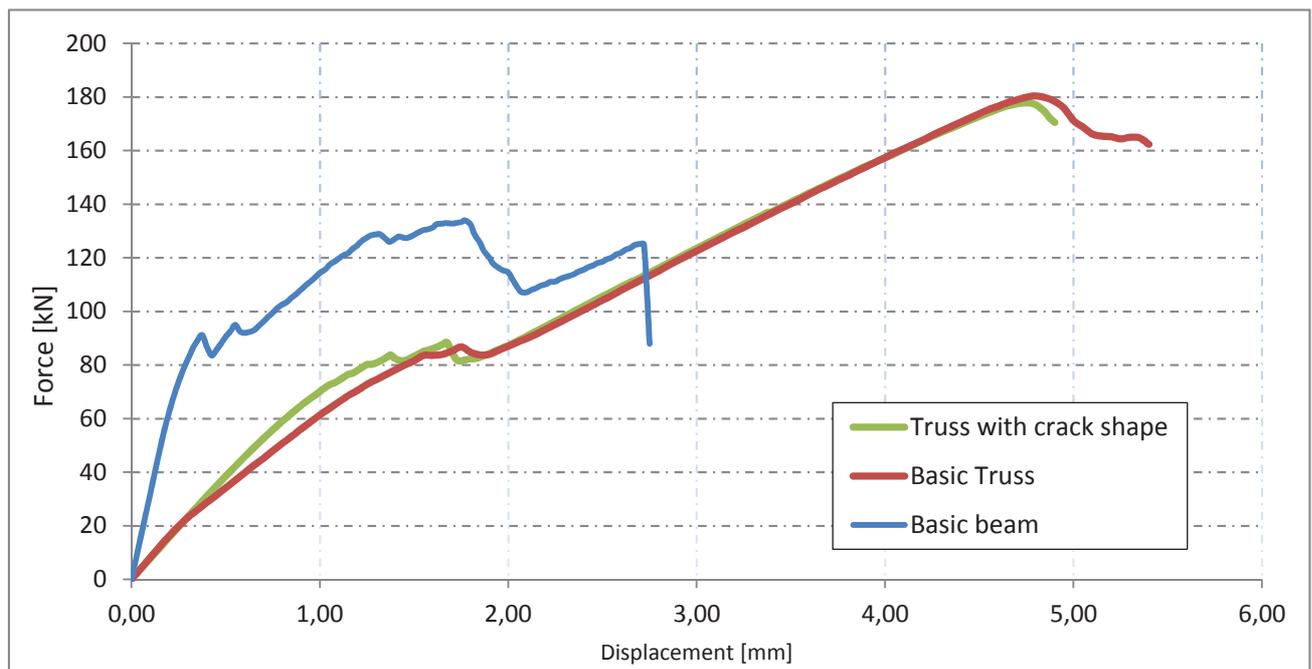


Figure 13.8 Force displacement graphs of basic beam, basic truss and truss with crack shape

### 13.2.2 Stress distribution

The force-displacement graphs do not show remarkable differences after the crack. The next step is comparing the stress distribution after the crack. The stress distribution just before failing of the beam model is shown in Figure 13.9. Some small differences are visible, but the overall stress distribution seems to be the same. With this stress distribution the beam fails, while the truss mechanism is able to increase the resisting load (visible in Figure 13.10).

The principle stresses show some small differences. The stresses are looked more into detail in Figure 13.10. By comparing the models with the stress  $\sigma_{xx}$ . The stress development is compared in different sections, figured in the top of Figure 13.10 by the red lines.

The stress development for each section is shown next to each other. The figure shows the stress over the height. The scale differs per section.

The steps used to compare the stresses are 2,00 mm, 2,25mm and 2,50mm. For this steps, shown in Figure 13.8, both models acts as truss mechanism. Before these steps the beam models do not act as a truss mechanism. Comparing these stresses is not suitable.

The stress comparison is explained per section:

Section A shows a difference in the first load step. The beam mechanism is not fully changed to a truss mechanism, visible in the force-displacement graph. The change to the high compressive stresses in the beam mechanism shows probably the last part of activation of the reinforcement. The truss mechanism already has this form.

Section B shows difference in the lower part of the models. The compressive stresses in the truss are higher than in the beam model. Both models show a stress level of around zero at the bottom of the beam. This is declarable by the cracking around the reinforcement to activate the reinforcement. Another difference is the higher tensile stresses in the top part of the beam.

Section C shows smaller compressive stresses in the truss than in the beam. The stress development in this section shows some differences. In the truss mechanism the stresses slowly increase, while in the beam mechanism a change in stress distribution is visible. The differences in the bottom of the beam are declarable by the differences in crack and thereby the stress distribution under the reinforcement.

Section D, the first line which crosses the crack, shows that the compressive stresses are linear distributed in the beam and that there are still some tensile stresses in the concrete under the crack. In the truss mechanism the stresses are more spread over the height.

In section E the same kind of difference is shown as in section D. In the beam model the stresses are linear distributed with large compressive stresses in the top and no compressive stresses above the crack. The truss shows only compressive stresses and the highest are in the bottom instead of the top. The distribution shows a remarkable difference.

Section F shows hardly any differences, the compressive stresses are high in the top of the beam.

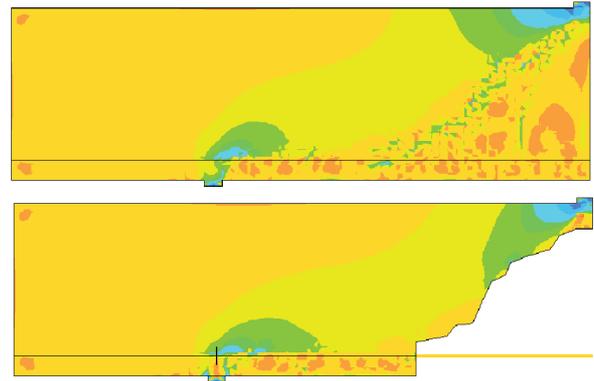


Figure 13.9 Principle stress  $\sigma_2$  for beam and crack shape truss model

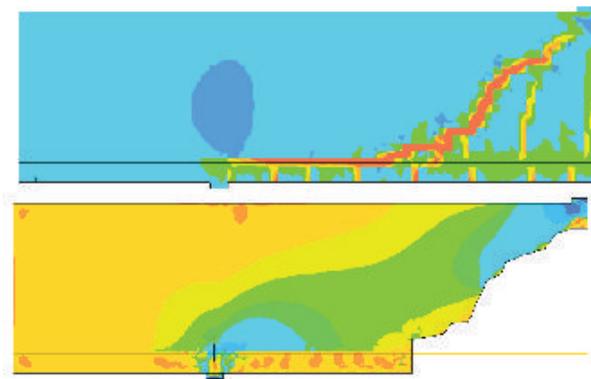


Figure 13.10 Crack pattern beam and stress distribution cracked shape model

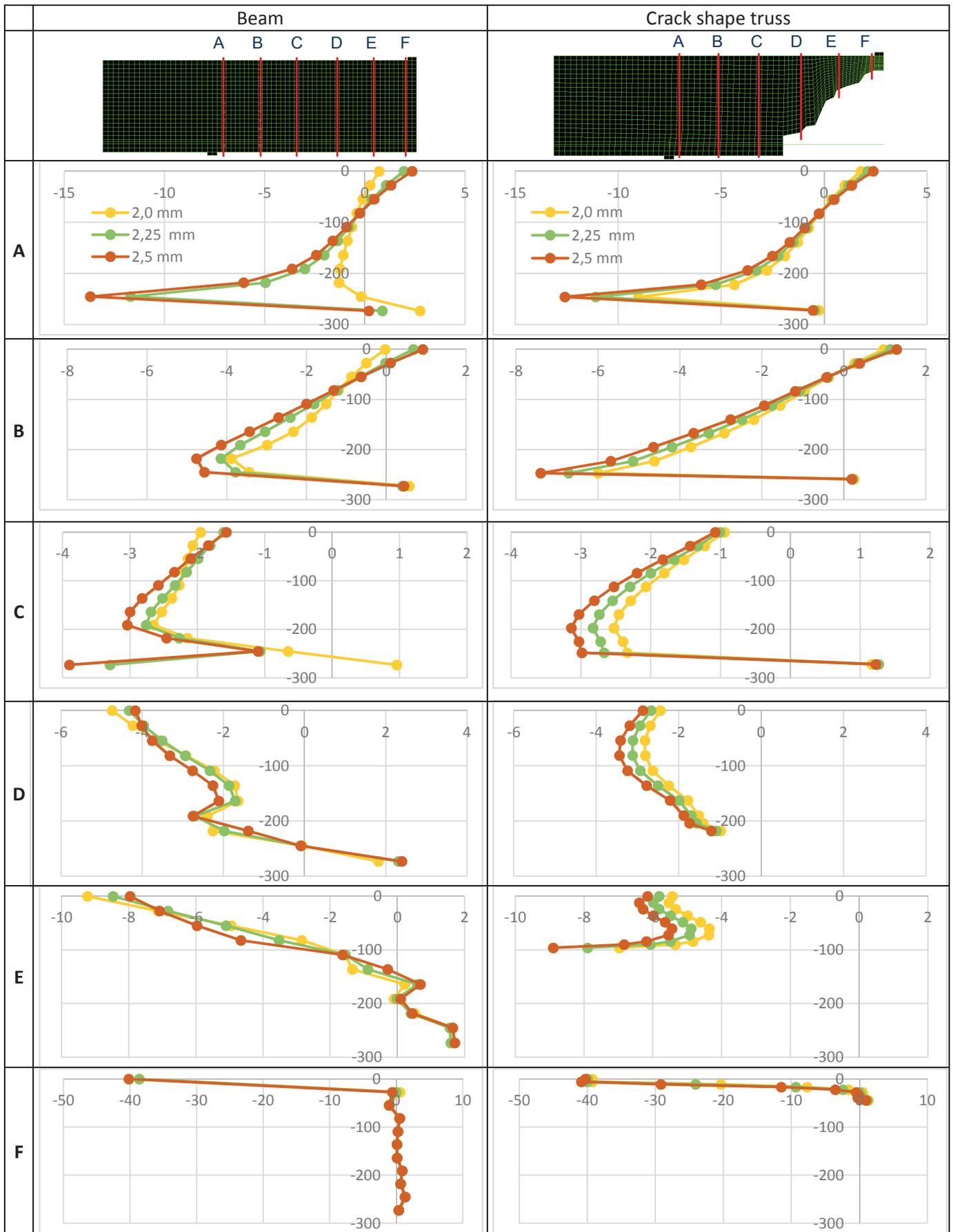


Figure 13.11 Stress ( $S_{xx}$ )  $\left[\frac{N}{mm^2}\right]$  over the height [mm] per crosssection of basic beam and crack shape truss

### 13.2.3 Conclusion

The truss mechanism after a flexural shear crack and the truss mechanism of the cracked shape truss are not identical. It is concluded that the crack shape has no influence on the inability of the beam to develop an ideal truss mechanism due to the concrete beneath the flexural shear crack.

The stress distribution over the height of the compression strut differs after the crack between the beam and the truss model. The truss model shows a more or less uniform distribution of the stresses. The beam model shows a stress distribution with a high gradient over the height. High compressive stresses at the top and low compressive stresses at the bottom. Figure 13.12 shows this difference.

It is concluded that a beam is unable to develop a perfect truss mechanism. The concrete beneath the shear crack causes another stress distribution above the shear crack. Probably this difference in stress distribution causes the lower failure load of the truss mechanism which develops in the beam after the flexural shear crack has appeared. This is not proven, but a hypothesis is proposed.

### Proposed hypothesis:

The difference of stress distribution in the top of the beam results due to the connection with the concrete beneath the crack. The concrete beneath the crack tends to deform in a different shape compared to the concrete above the crack. The concrete above the crack wants to shorten to resist the compressive stresses in the top. The concrete beneath the crack wants to extend to resist the tensile stresses in the concrete. These tensile stresses in the concrete are introduced by the bonding with the reinforcement.

The deformation difference at the top of the flexural shear crack is not possible. The deformation at the top of the crack will be resisted by bond. As a consequence shear stresses are introduced. Shear stresses occur around the top of the crack and in the crack where transmission is still possible. These shear stresses due to bond do not occur in a perfect truss mechanism.

The models of this study fail suddenly. With the knowledge of this study the interaction of the shear stresses over and around the crack are very important. To find the actual failure mechanism additional research is required on the interaction of the stresses and deformations over and around the crack.

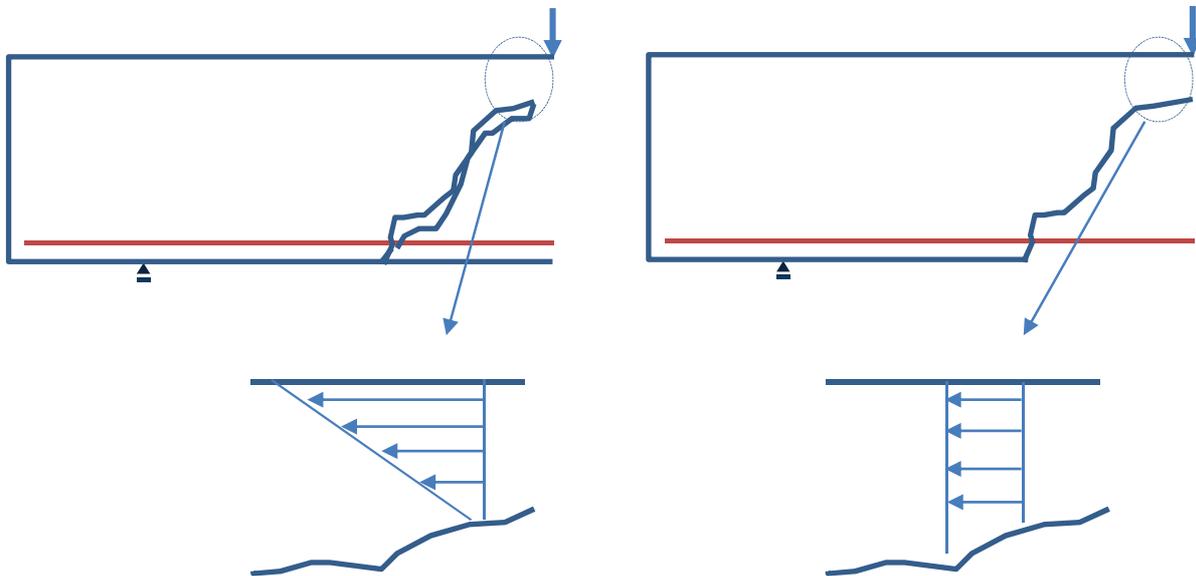


Figure 13.12 Stress distribution above crack

# 14 Flexural shear crack

With the knowledge of the previous chapter a description of the flexural shear crack is given in this chapter. How does the crack occur and what determines the failing of a flexural shear crack?

## 14.1 Occurrence shear crack

This paragraph gives a description of the occurrence of a flexural shear crack. These cracks occur the area where the force directly results in the support and the area where the beam fails by bending failure. Typical for a flexural shear failing is the sudden occur of a large crack.

The occurrence of the crack depends on the stiffness of the beam. A flexural shear crack is the occurrence of a bending crack which curves to the middle due to the compressive stresses of the truss mechanism. The cracks jump into the concrete to gain enough deformation capacity to restrain the force. This happens if the bending cracks not give enough deformation capacity. This always happen at the last crack.

The flexural shear crack occurs when the compressive stresses of the truss mechanism are too low to prevent the crack before the force mechanism is changed to a truss mechanism. It could be stated that the flexural shear crack is a competition between the developing of the compression stresses of the truss mechanism and the bending cracks of the cracked beam mechanism. A flexural shear crack is the innig of the cracked beam.

In the change from beam mechanism to cracked beam mechanism a lot of bending cracks occur. The bending cracks change the force mechanism. If the stresses of the truss mechanism are not high enough a flexural shear crack is occurs. But if the stresses in the strut already developed it is impossible for a crack to go through this compressive strut and no flexure shear crack occurs.

### 14.1.1 Crack shape

Typical of a flexural shear crack is the curvature to the middle of the beam. Experiences show that the bending cracks are straight when the moment is constant and the cracks are curved when there is a varying moment. An example is shown in Figure 14.2. The compressive stresses of the truss mechanism explain the curvature of these cracks. This is schematically shown in Figure 14.3.



Figure 14.2 Crack pattern 4 point loaded beam (Sumajouw and Rangan, 2006)

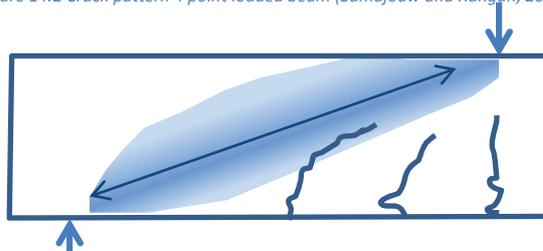


Figure 14.3 Explanation crack pattern

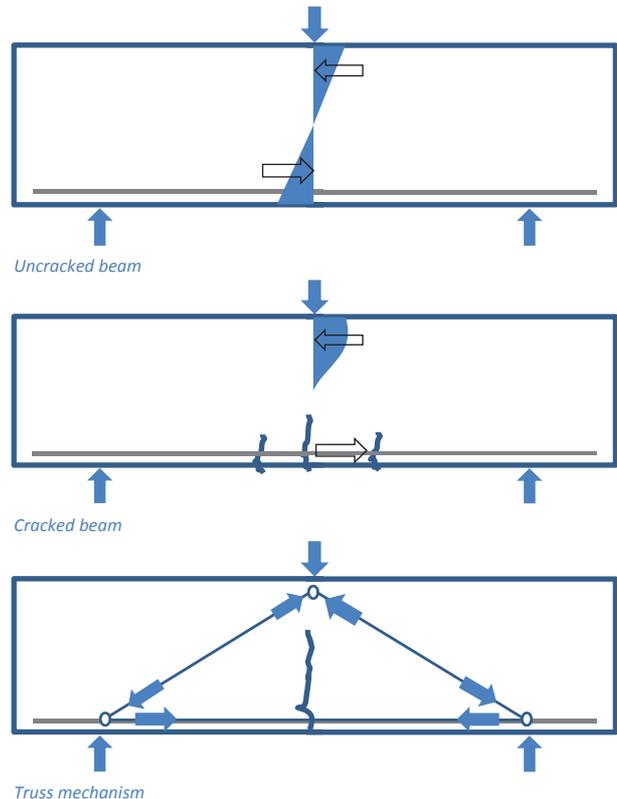


Figure 13.1 Force mechanisms

The cracks occur when the beam mechanism changes from uncracked force distribution to cracked distribution. Cracks make it possible to gain stresses in the reinforcement. The cracks reach till the compression zone. This crack cannot grow in the area with the compressive stresses and the crack curves to the middle. The cracks grow till the reinforcement has enough deformation freedom to restrain the forces.

### 14.1.2 Stiffness differences

The stiffness is important for the differences between the force mechanisms. Forces are transferred to the support by deformation of the structure. The most stiffen mechanism acts first. This is in the beam the uncracked beam mechanism. In this mechanism which the stresses are spread over the height by the Bernoulli theory. It holds till it cracks. Then the cracked beam mechanism is the most stiffen system. The cracked beam mechanism holds till crushing of the concrete, yielding failure or shear crack occurs. With this failure the truss mechanism is the next system which could occur. This truss mechanism will occur when the stiffness of the beam is decreased enough and the truss shape is possible. The truss after a flexural shear crack most of the times converts to an unstable truss by the concrete beneath the crack.

### 14.1.3 Truss mechanism after shear crack

After a flexural shear crack the beam tries to convert to a truss mechanism because the truss is the most stiffen mechanism after the cracked beam mechanism. After the flexural shear crack the beam mechanism partly changes to a truss mechanism. After a flexural shear crack the stress distribution above the shear crack differs from the stresses in the truss mechanism. The beam mechanism is unable to transform to a total truss mechanism due to the presence of the concrete beneath the flexural shear crack.

The presence of the concrete beneath the crack results in another stress distribution above the crack. Probably this difference causes the lower failure load of the truss which occurs in the beam after the flexural shear crack. This is not proven, but a hypothesis is proposed.

The stress and deformation direction differ above and beneath the crack. But the requirement at the top of the crack is that the displacement is the same. This requirement is satisfied by the occurrence of shear stresses over and around the crack. These shear stresses probably results in the growing of the crack, which results in the failing of the beam by the inability of finding a equilibrium. This is just a hypothesis, which should be proved.

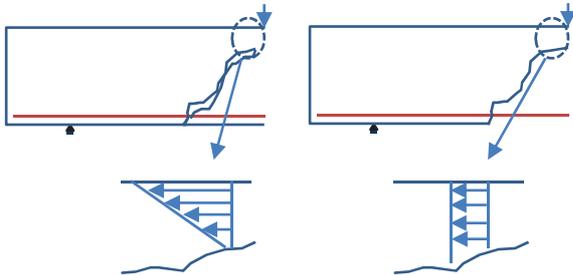


Figure 14.4 Stes distribution above crack

### 14.1.4 Explaining Beeby beams

The experiment of the Beeby could be explained each beam. Beam one fails by the flexural shear crack, which makes it impossible to change to a truss, as described in Paragraph 14.1.3. The same holds for beam G.

Beam C give a stable truss mechanism, which results in a higher force resistance.

In Beam D and E the compressive strut is not possible to occur by the size of the gap. The gap passes the line which is necessary for the occurrence of the truss, described in paragraph 12.3.3.

In Beam H and beam I a small bending cracks results in small bending cracks. This bending cracks immediately have a lot of deformation capacity by the artificial cracks. The beam transforms to a stable truss mechanism without the sudden occurrence of a shear crack. The stress development will be the same as the truss mechanism and will not result in failure mechanism of the flexural shear crack.

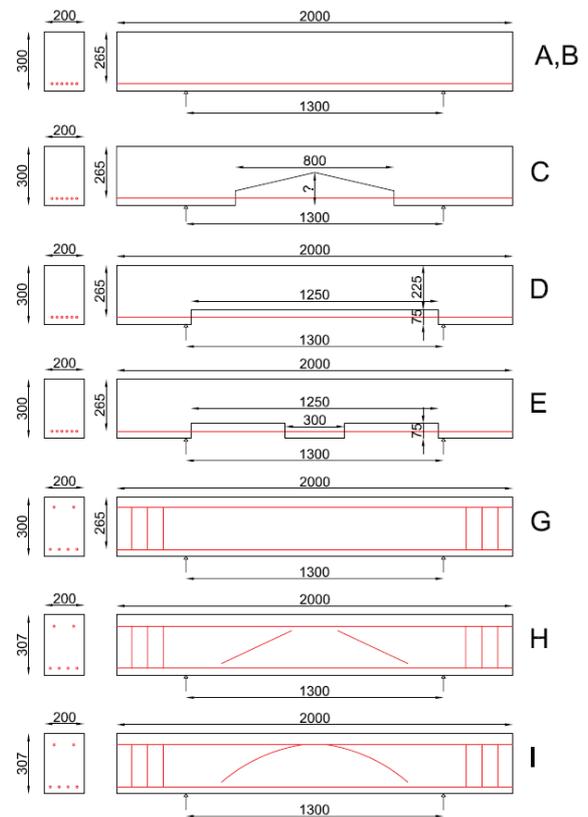


Figure 14.5 Tested beams

# *TRUSS DESIGN*

## *TRUSS DESIGN* *TRUSS VARIATION*



# 15 Designing truss

This chapter describes the boundaries of the truss mechanism. What are the limits to design a truss mechanism which can result in a higher force resistance?

## 15.1 Designing truss mechanism

The previous chapters showed that a flexural shear crack does not occur if a gap is introduced in the structure. The gap prevents the occurrence of a shear crack which results in failure. The stiffness of the beam decreases by leaving a part of the concrete out. Lowering the stiffness increases the compressive stresses in the strut. The increased compressive stresses prevent the occurrence of a flexural shear crack. The result is a higher force resistance. With this knowledge it is possible to design a truss. The truss mechanism has also certain limits, which determine its failure load.

## 15.2 Gap limits

The gap decreases the stiffness which is required for a truss mechanism. But the size of the gap has certain limits.

### 15.2.1 External reinforcement

An important condition for the truss mechanism is the external reinforcement. The external reinforcement deals with the tensile stresses in the truss mechanism. So the external reinforcement is a requirement for the truss.

### 15.2.2 Too large gap

If the gap passes the strut line, shown in Figure 15.2, the truss is not able to occur. The cantilever part of the beam will fail by the occurrence moment as explained in paragraph 12.3.3. It will fail by a crack in the top. The failure mechanism of the crack is shown in Figure 15.3.

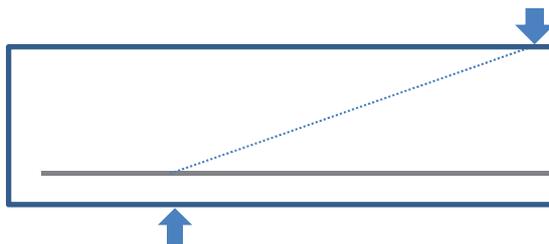


Figure 15.2 Truss compressive line

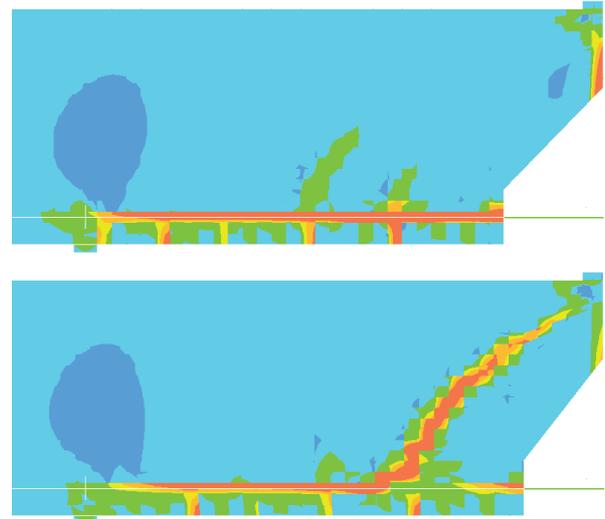


Figure 14.1 Difference crack pattern between 200mm and 250mm gap span

### 15.2.3 Too small gap

A too small gap does not decrease the stiffness enough for preventing a shear crack. The shear crack makes it impossible to change to a stable strut as explained in paragraph 14.1.3.

The limit for the standard dimensions is between a gap of 200mm and 250mm. The gap length for the standard dimensions has to be at least 250mm. This gives a ratio of

$$\frac{250\text{mm}}{1300\text{mm}} = 0.192$$

A larger gap length gives a higher reliability for the occurrence of the truss. With the results of the FEM models a rule of thumb is given for the minimum gap length to prevent shear crack in the beam. This is the rounded value of the standard beam:

$$\frac{\text{Gap length}}{\text{Span}} = 0.2$$

This is an assumption only based on the FEM results. For using this value this must first be verified with other models and real lab tests.

### 15.2.4 The height of the gap

Paragraph 12.1 showed that the height of the gap has no influence on the force resistance if the compressive height is large enough. For the designing the visual cracks has to be taken into account. The height of the gap has to be large enough to prevent too large cracks for the visual effect.

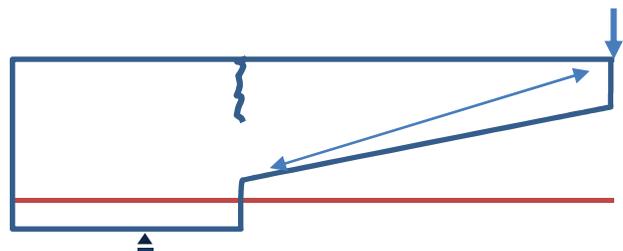


Figure 15.3 Passing strut line of gap failure

### 15.3 Limits truss mechanism

The limits of the truss determine the force resistance of the truss. This paragraph deals with these limits.

The truss mechanism has different failure mechanisms which result in different limits:

- *Yielding of reinforcement*

The reinforcement steel in the tension strut of the truss mechanism limits the truss mechanism. It depends on the amount and quality of the steel.

- *Crushing concrete*

The crushing of the concrete appears when the concrete strength is reached. The compressive strut is not able to restrain the high compressive forces and fails.

- *Cracks by lack of deformation capacity*

The truss mechanism requires relative high deformation and rotation capacity for restraining the load. The rotation of the hinge above the support is limited by the concrete. The concrete rotation is limited by the present reinforcement. A crack will occur above the support or at the left side of the beam. This depends on the length of the cantilever part at the left and the height of the beam. Both failure mechanisms are shown in Figure 15.4.

Why this crack results in failure is not clear. Probably the hinge above the support is now not able to reach force equilibrium by the compressive strut and the tensile strut of the truss.

- *Reinforcement limits*

The models use models for the reinforcement. The bending stiffness and the bonding with the concrete have certain limits. These are not taken into account in the model, but could result in failure.

### 15.4 Boundaries of limits

The truss design limits are based on the results from this research. So it is important to know these boundaries.

#### 15.4.1 Specific parameters

The limits on the specific models with the chosen parameters. The basic beam has a span of 1300mm with a height of 300mm without shear reinforcement subjected to a concentrated load in the middle of the span. The truss mechanism is modelled with different gap sizes, but the main dimensions are kept the same as the beam.

#### 15.4.2 Analytical + FEM calculations

Analytical and FEM calculations are used to model the basis models. Within this calculation parameters are chosen to approach the concrete properties as good as possible. Verifying the models with a real experiment is a good possibility.

Some assumptions are:

- Reinforcement modelled as truss.
- Full bonded reinforcement.
- Hordijk tensile stress relation.
- Dowel action is not taken into account.

### 15.5 Conclusion truss mechanism

It is possible to design a truss if all the limits are taken into account. The gap dimensions should not pass the compressive strut line. The gap needs to be large enough to decrease the stiffness to prevent the occurrence of a flexural shear crack. For the minimum gap length the rule of thumb is:

$$\frac{\text{length gap}}{\text{span}} = 0.2$$

If the truss occurs, the limits for the truss are the yielding of the reinforcement, the crushing of the concrete and the lack of deformation capacity.

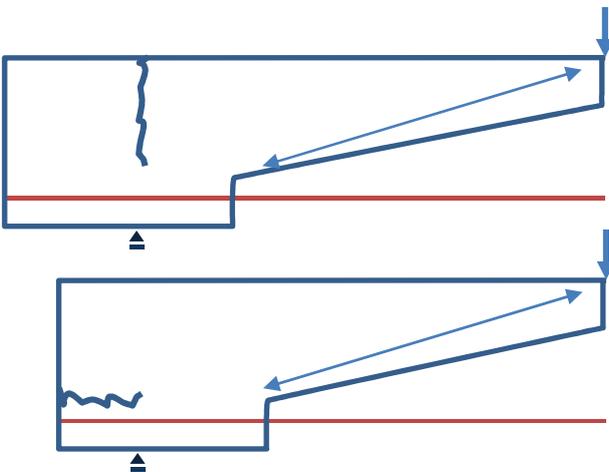


Figure 15.4 Cracks by lack of deformation capacity

# 16 Truss variation

The truss limit in the previous chapter gives the limits based on the basic truss variations. This chapters varies the basic parameters to show the differences and verify the limits of the truss. The basic parameters are given in Table 16-1.

Table 16-1 Basic parameters

Height	300mm
Depth	200mm
Length	1300mm
Reinf. amount	471mm <sup>2</sup>
Reinf. ratio	0,89%
Force	Point load

## 16.1 Longer beam

The basic beam has an  $a/d$  ratio of 2.45. The longer beam has an  $a/d$  ratio of 3.0. The adapted parameters are shown in Table 16-2. The length changed, but also the reinforcement amount. These amount is changed to gain a shear failure instead of a bending failure.

Table 16-2 Basic parameters long beam

Height	300mm
Depth	200mm
Length	1600mm
Reinf amount	1050mm <sup>2</sup>
Reinf ratio	1,98%
Force	Point load

### 16.1.1 Calculation methods

For the longer beam the same calculations methods are used as for the basic beam and truss models. For the analytic models the formulas of Chapter 5 are used. For the FEM calculation the methods and parameters are used as described in Chapter 11. An overview of the force differences is given in Table 16-3.

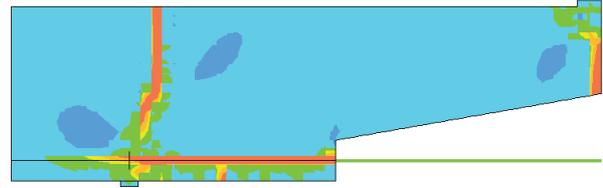


Figure 16.1 Fail mechanism long truss (gap length 1000mm)

Table 16-3 Force limit basic and longer models

	Beam [kN]	Truss [kN]	Ratio
<b>Basic Analytical model</b>	126	192	1.5
<b>Basic FEM model</b>	131	180	1.4
<b>Longer Analytical model</b>	165	346	2.0
<b>Longer FEM model</b>	140	140	1.0

### 16.1.2 Analytical and FEM differences

The analytical models show a difference in the shear capacity of the beam. This difference is not shown in the FEM results. In Figure 16.2 the force displacement graphs of the longer span beam and trusses are shown. It shows that the truss models are not able to reach a higher force limit.

The truss models of the FEM models do not show the same kind of difference as the analytic model for the longer beam and truss mechanism. The truss models shows a failure by the lack of deformation capacity, shown in Figure 16.1. The lack of deformation capacity is not taking into account with the analytic model. This lack of deformation capacity occurs with a displacement of 3.5mm in stead of 4.7mm as the basic truss. With the flexibility of the longer cantilever part a higher deformation capacity is expected. The difference is deformation capacity is probable declarable by the stiffness of the reinforcement. In the longer beam a higher bending stiffness around the support is ceated by the higher reinforcement amount. This shows that the bending stiffness and thereby the deformation capacity are important for the capacity of the truss mechanism. This, together with the reinforcement layout influences the force resistance. An interesting point for further research.

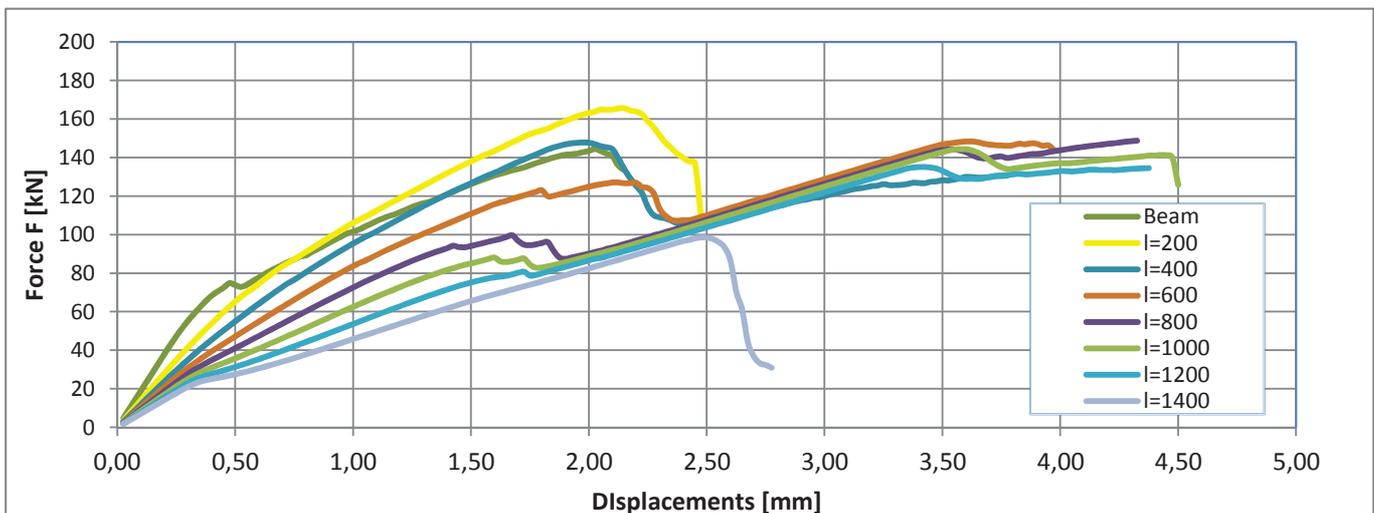


Figure 16.2 Force displacement graph longer span

The longer model does not show a higher force resistance, but it is used for reviewing the other boundaries of the truss system.

The longer models show that a too large gap also results in failure. This limit lies between the gap span of 1200mm and 1400mm. The beam cannot transform into a truss if the gap passes the truss line, shown in Figure 16.3.

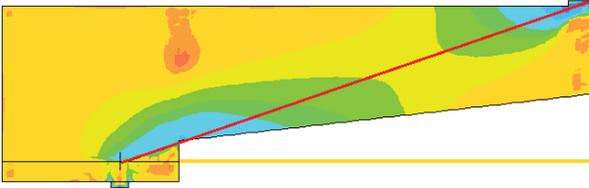


Figure 16.3 Gap length 1400mm, passes needed line

The graph in Figure 16.2 show also limits in span length between the occurrence of the truss and the failing by a flexural shear crack. With this longer beam the boundary lies between a gap length of 200mm and 400mm. The 400mm does not show a stable truss, but there does not occur a shear crack. The ratio for no shear crack is with this length:

$$\frac{\text{length gap}}{\text{span}} = \frac{400\text{mm}}{1600\text{mm}} = 0.25$$

This matches the same limit as described in paragraph 15.2. The exact limit is still not clear, further research is needed to found the exact relation.

### 16.1.3 Conclusion

The longer model does not show a higher force resistance. This is due the lack of deformation capacity, which is probably declarable by the higher bending stiffness through the increased reinforcement amount. This is an interesting point for further research. The gap length limits for the occurrence of a truss show the same kind of boundaries. For the exact boundaries further research is necessary.



Figure 16.4 Crack pattern with extra reinforcement

## 16.1 Reinforcement ratio

The longer beam shows a failure by deformation capacity. The stiffness of the reinforcement is perhaps the problem. To investigate the influence of the extra reinforcement the reinforcement amount is doubled in this paragraph.

Table 16-4 Basic parameters

Height	300mm
Depth	200mm
Length	1300mm
Reinf amount	942mm <sup>2</sup>
Reinf ratio	1,78%
Force	Point load

### 16.1.1 Calculation methods

The same calculations methods are used as for the basic beam and truss models. For the analytic models the formulas of Chapter 5 are used. The FEM calculation methods and parameters are described in Chapter 11. The dimensions of the truss and beam are taken the same as the basic models. The parameters are shown in Table 16-4.

### 16.1.2 Analytical and FEM differences

The only difference in the analytic and the FEM models is the reinforcement amount. The difference in analytical limits and finite element limits is shown in Table 16-5.

Table 16-5 Force limit basic and extra reinforcement models

	Beam [kN]	Truss [kN]	Ratio
Basic Analytical model	126	192	1.5
Basic FEM model	131	180	1.4
Extra reinforcement Analytical model	159	384	2.4
Extra reinforcement FEM model	180	182	1

Figure 16.5 shows the force displacement graphs of the models with extra reinforcement. It shows that the truss and the beam with the extra reinforcement are stiffer. With the extra reinforcement the crack formation zone occurs with a higher force. The stiffness difference between the truss and the beam with extra reinforcement show the same kind of difference as the basic models.

The beam with extra reinforcement fails by a shear crack, shown in Figure 16.4. The crack pattern is almost the same as the beam with the normal reinforcement. In the analytical solution for shear resistance the reinforcement ratio has a relation as given in equation [5-4]:

$$V_{Rd,c} = \left[ \left( \frac{0,18}{\gamma_c} \right) k (100 \rho_l f_{ck})^{\frac{1}{3}} \right] b_w d \quad [5-4]$$

The difference in shear capacity in the FEM models is a bit larger than the factor  $\sqrt[3]{2}$  in the analytical solution.

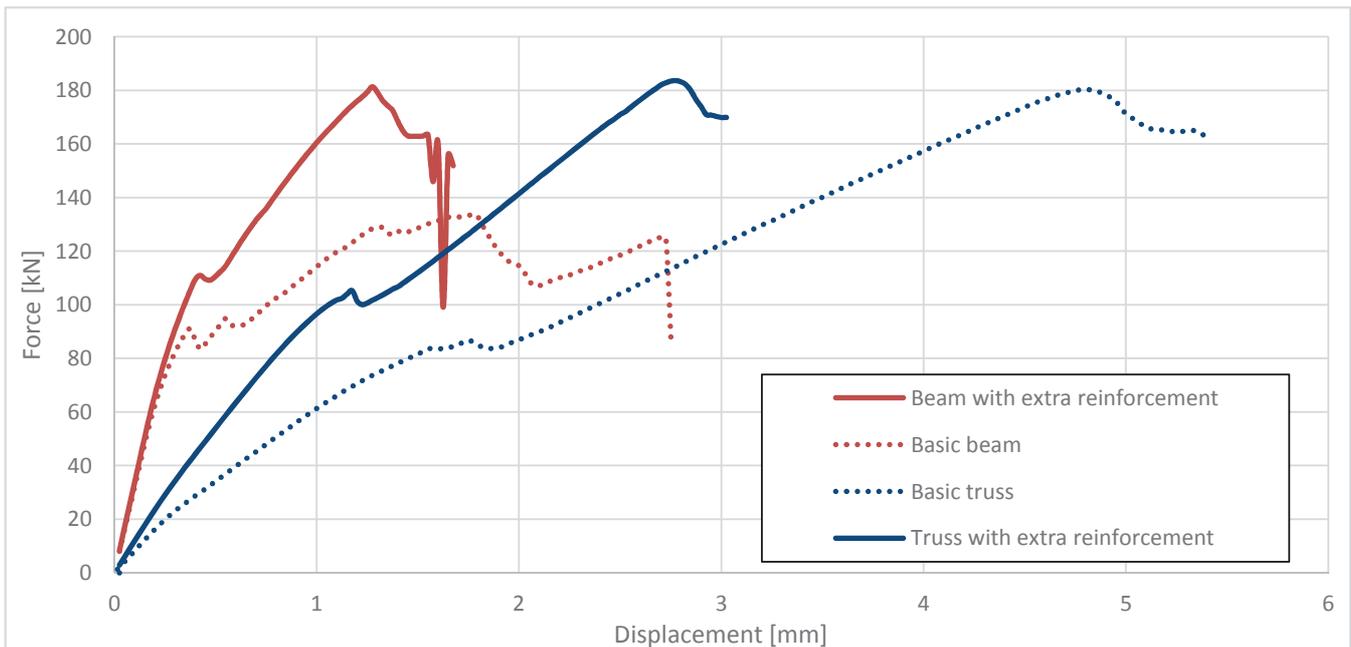


Figure 16.5 Force displacement diagrams extra reinforcement

The failure load of both the truss models is around 180 kN. Both models do not fail by the yielding of the steel. The displacements differ for each truss for the occurrence of the crack. In both beams this crack occurs above the support at the top of the beam. This lack of deformation capacity is not taken into account with the analytic model, this declares the high difference between the analytical model and the finite element model. In the normal truss this lack of deformation capacity occurs around 4.7mm instead of the 2.7mm in the truss with extra reinforcement. The bending stiffness is proportional to the deformation. This is clearly show in the force displacement graphs of the trusses. By a different crack pattern the deformation in this zone differs. After the crack the difference stays proportional till the end of the graph. The stiffness of the reinforcement determines the crack at the top of the structure.

### 16.1.3 Conclusion

Together with the longer beam it could be concluded that the deformation capacity is based on the span and the amount of reinforcement. With a higher reinforcement ratio the limit for deformation capacity is reached with a smaller deformation, but with the same kind of force limit.

This shows that for a truss mechanism it is interesting to have a deeper look into the reinforcement layout properties of the cantilever part at the outside of the span. The reinforcement needs enough bonding settlement, but also whis results in a higher stiffness which decreases the force resistance of the truss.

## 16.1 Distributed load

The basic beam is subjected to a concentrated force. In this paragraph the influence of a distributed load is described. All the other dimensions kept the same.

Table 16-6 Basic parameters distributed load

Height	300mm
Depth	200mm
Length	1300mm
Reinf amount	471mm <sup>2</sup>
Reinf ratio	0,89%
<b>Force</b>	<b>Distributed load</b>

### 16.1.1 Calculation methods

The distributed force needs other calculation methods to determine the limits. In a distributed load the form of the truss mechanism change. The varying shear stress converts the form of the theoretical truss to a theoretical arch.

The analytical limit for the shear resistance in the beam stays the same as given in Equation [5-4]:

$$V_{Rd,c} = \left[ \left( \frac{0,18}{\gamma_c} \right) k (100 \rho_l f_{ck})^{\frac{1}{3}} \right] b_w d \quad [16-1]$$

Only the shear force differs in the beam by the distributed load. This difference is shown in Figure 16.6. The maximum shear force in the beam is  $V_{max} = 0.5 q l$ .

In the analytic truss model only the failure by yielding of the steel is taken into account.

### Force based FEM calculation

For the basic models are displacement based calculations. With a distributed load this is not possible. Force based calculation are used, but the stability of force based calculation is not fine. The force displacement curves shows that the model has difficulties to pass the crack formation zone, because a large step is needed. The automatic Arc length method is used to solve this problem. This calculates the steps, which make it possible to calculate snap back curvatures. The calculation file, .DCF is found in Appendix H. The differences between the analytical and the FEM model are shown in Table 16-7.

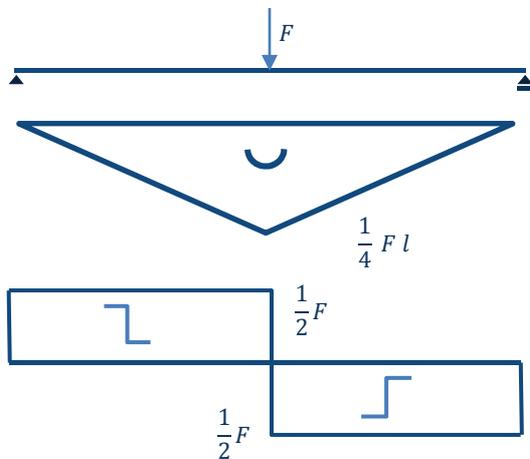


Figure 16.6 Moment and shear force lines of concentrated force and distributed force

Table 16-7 Force limit basic and longer models

	Beam [kN/m]	Truss [kN/m]	Ratio
<b>Analytical model</b>	227	295	1.3
<b>FEM model</b>	220	280	1.2

### 16.1.2 Analytical and FEM differences

Figure 16.8 shows the force displacement diagrams of the distributed force. In comparison to the previous graphs this shows more unstable lines. This is caused by the force based calculations. The arch form of the truss mechanism is shown in the stress distribution in Figure 16.7.

The beam and truss mechanism, arch is this type, show again a difference in failure mechanism. With the distributed load the truss mechanism needs more freedom to transform to an arch. Different beam dimensions are modelled to find the boundaries of the truss system. For each model the gap length and the height of the corner are varied. The results for all the models are divided in several categories which have the same kind of results. An overview of the results is shown in Figure 16.9. Every dot represents one model. The dot represents the corner of the gap of the representing model. With a straight line to the bottom and a line to the middle height of 250mm the dimensions of the model are fixed.

The results are divided in three categories, represented as the different colours:

- Stable arch mechanism without shear crack
- Flexural shear crack failure
- Failure of cantilever part

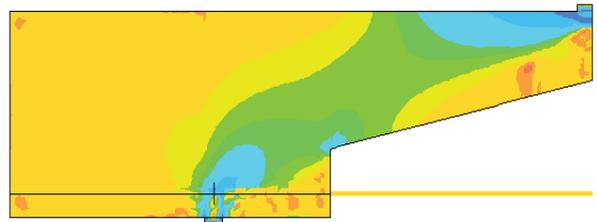
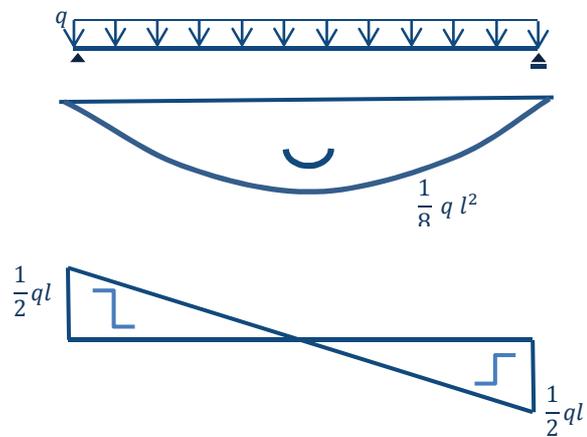


Figure 16.7 Stress development distributed load



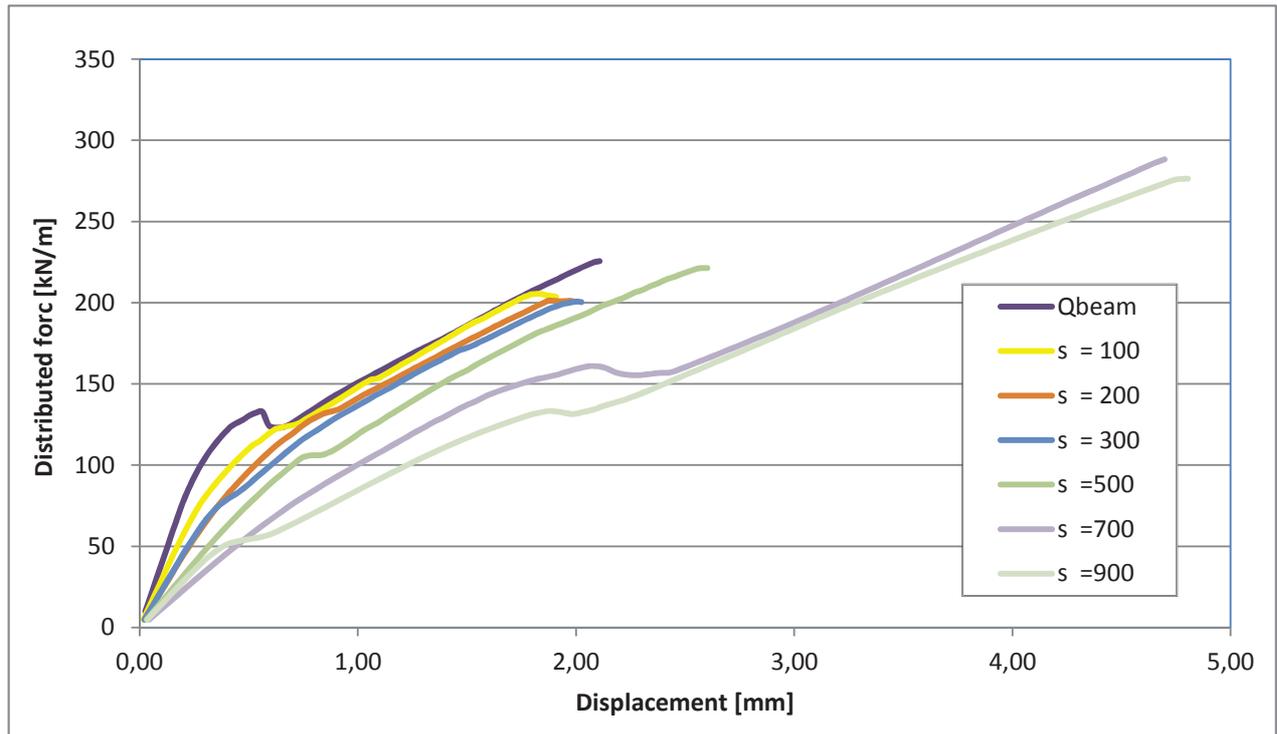


Figure 16.8 Force displacement curves distributed load

A stable arch occurs if the gap length is minimal 700mm. This give a ratio of  $\frac{\text{length gap}}{\text{span}} = \frac{700\text{mm}}{1300\text{mm}} = 0.54$ . For the exact boundary further research is needed.

A too large gap results also in the inability of transforming in a truss. The failure of the cantilever part occurs, just as with the basic beams. But it is not the truss line which should be crossed, but the arch line. The exact line depends on the force and length of the beam.

### 16.1.3 Conclusion

The distributed load beam is also able to reach a higher force resistance with a gap in the beam. The gap need to be larger than the basis truss and the truss is in shape of an arch instead of a truss.

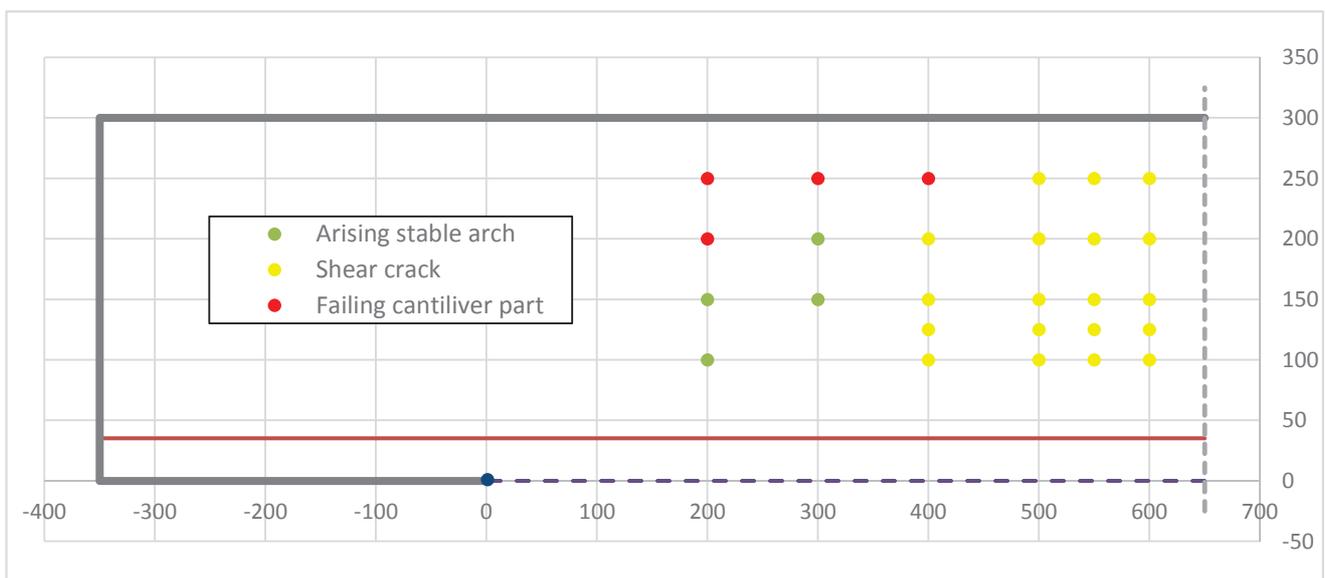


Figure 16.9 Overview and categorisation different models distributed load

## 16.2 Smooth steel

The flexural shear crack occurs by the bending cracks which occur in the beam by the lack of deformation capacity. Is it possible to design a part of the beam with smooth steel to prevent bending cracks to occur?

All the properties of the beam are kept the same, only the reinforcement is now smooth in certain areas. The smooth length is varied to show the influence of the smooth.

Height	300mm
Depth	200mm
Length	1300mm
Reinf amount	471mm <sup>2</sup> Partly smooth
Reinf ratio	0,89%
Force	Point load

### 16.2.1 Calculation methods

In the FEM models the bonded part of the reinforcement is modelled with the normal embedded reinforcement. This embedded reinforcement is fully bonded. The smooth part is increased in the modelling. The part which is smooth is given as  $u$  and shown in Figure 16.10. This smooth part is modelled as a truss element. Embedded reinforcement without bond did not give a proper result. The connection between the bonded and smooth part failed. Therefore the smooth part is modelled as truss element.

### 16.2.2 Analytical and FEM differences

For some of the models the force displacement graphs are shown in Figure 16.12. In all the smooth models a large crack occurs in the middle of the beam which results in a hinge in the middle of the beam. In the beam with the larger smooth parts a crack occurs in addition to the crack in the middle. All these cracks start at the interaction point of the smooth reinforcement and the bonded reinforcement. An example of this kind of crack is shown in Figure 16.11.

The crack occurs on the spot where the smooth truss is connected with the full bonded embedded reinforcement. Large stresses occur in this connection point. In reality these stresses probably will spread over a larger area by the slipping of the concrete. An alternative for improved modelling would be the use of an interface element. This could model the slip behaviour of the reinforcement.

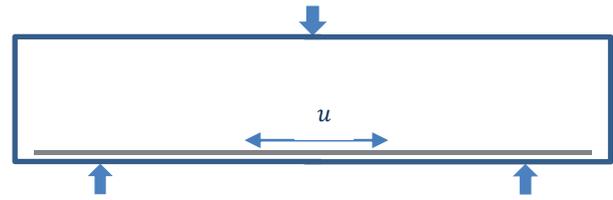


Figure 16.10 Smooth part  $u$



Figure 16.11 Crack pattern smooth reinforcement,  $u=700mm$

No crack occurs if the smooth part is increased to almost above the supports. No crack occurs at the point where the bonded and smooth part are connected. A stable truss mechanism occurs, with a large crack in the middle of the beam.

### 16.2.3 Conclusion smooth reinforcement

The modelling of smooth reinforcement has certain limits. The modelling shows that it is possible for the occurrence of a truss in a beam with only one crack in the middle of the span. Simplifications as full bonded and fully smooth do not exist in reality. The models show that when there are no bending cracks there is no shear failure. It confirms that shear cracks occur out of bending cracks.

The smooth reinforcement steel gives an interesting possibility for further research. Further research is needed to model the slip of the reinforcement and investigate the possibilities with the smooth steel.

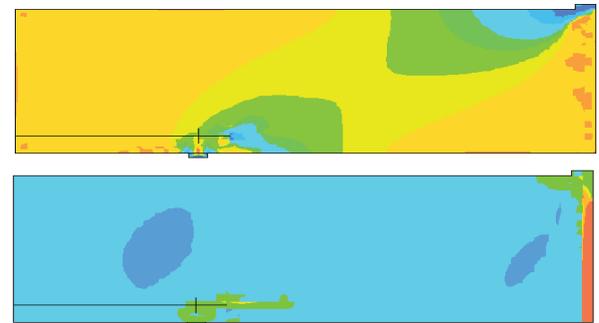


Figure 16.13 Stress development and crack pattern  $u=1200$

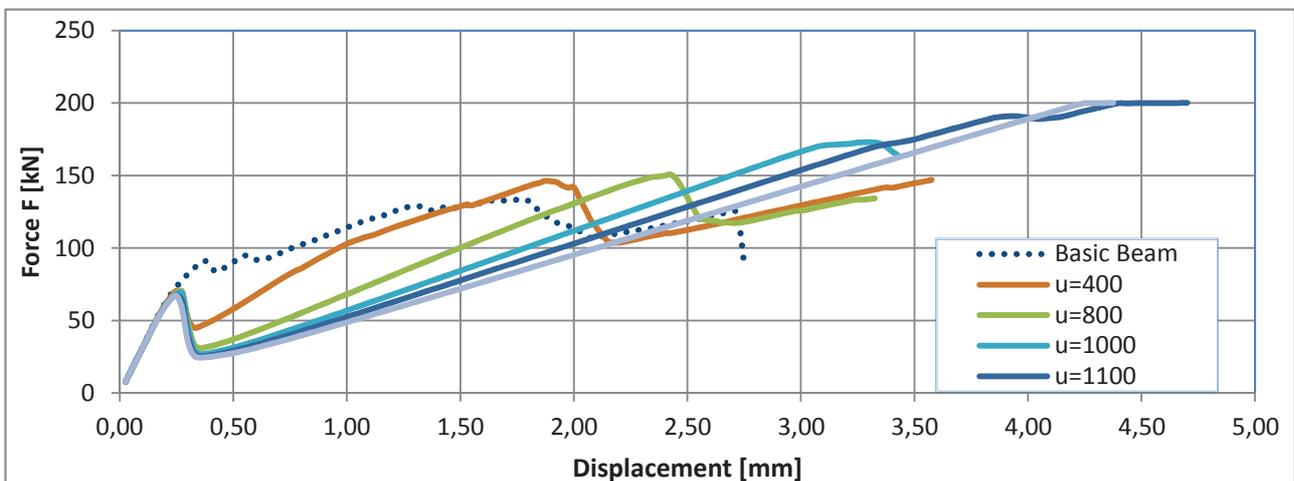


Figure 16.12 Force-displacement graphs beams with smooth reinforcement

### 16.3 Descending beam

Decreasing the stiffness gains more stresses in the truss mechanism. Another possibility for decreasing the stiffness is a descending height to the end of the beams. PhD student Yuguang Yang did a experiments with this kind of beams.

By decreasing the height at the ends, the stiffness is decreased and the compressive zone is forced downwards. By the decreasing height the bending stiffness is decreased which results in an earlier occur of bending cracks. On the other hand results the decrease of stiffness in more compressive stresses in the truss mechanism, which prevent the occurrence of the shear crack.

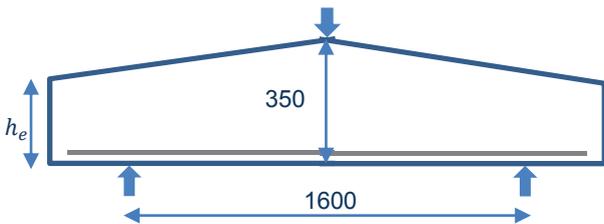


Figure 16.14 Variation of end height [mm]

#### 16.3.1 Finite element models

With DIANA different models with descending heights are explored. The height at the ends is varied with steps of 50mm, Table 16-8 shows the variation of the height at the ends.

The dimensions of the beam are the same as tested by the PhD student. Only the reinforcement ratio and concrete strength are different. The data were not known at the moment of constructing the models.

Figure 16.15 presents the force-displacement graphs of the descending beams. It shows that both the stiffness and failure loads differ. Table 16-8 gives an overview of the different shear crack shapes with the corresponding failure loads of the FEM models. This shows clearly the increase of bending cracks in the direction of the support by the decrease of bending stiffness. The flexural shear crack occurs in the most outer bending crack present in the beam.

The maximum failure load increases between the reference beam and the descending beam with a height at the end of 200mm. It shows an increase from 184kN to 197 kN. The increase is not huge, but visible in multiple models.

Table 16-8

Side height [mm]	F max [kN]	
350	184	
300	171	
250	196	
200	198	
150	191	
100	179	
50	160	

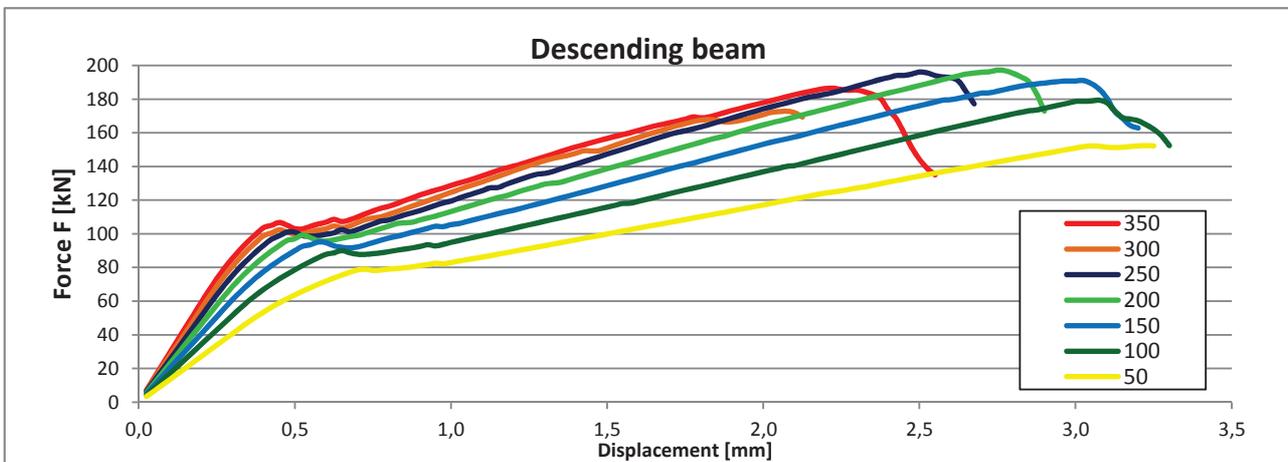


Figure 16.15 Force-displacement diagram descending beams

### 16.3.2 Lab experiment

Friday 15-11-2013 two beams are tested in the Stevinlab. One reference beam and one descending beam, shown in Figure 16.16. The height of the beams is 350mm and the descending beam has a height of around 170mm at the end.

The failure load of the reference beam was 126kN and the failure load of the descending beam was 144kN. This shows a clear increase of failure load with a descending beam. Of course this are just 2 beams and there will be some variation in the concrete properties, but it gives a clear indication.

The normal beam shows a flexural shear crack, as expected. The beam was not loaded further. The crack is shown in Figure 16.17. The descending beam shows, as expected, a lot more bending cracks which approach the supports. The bending cracks are shown in Figure 16.18.



Figure 16.16 Beam and descending beam before testing

The failure of the descending beam occurs suddenly. A shear crack occurs and the concrete jumps of. The result is shown in Figure 16.19. The results after a crack shows the breaking free of the reinforcement.

It clearly shows that stiffness could influence the force resistance.

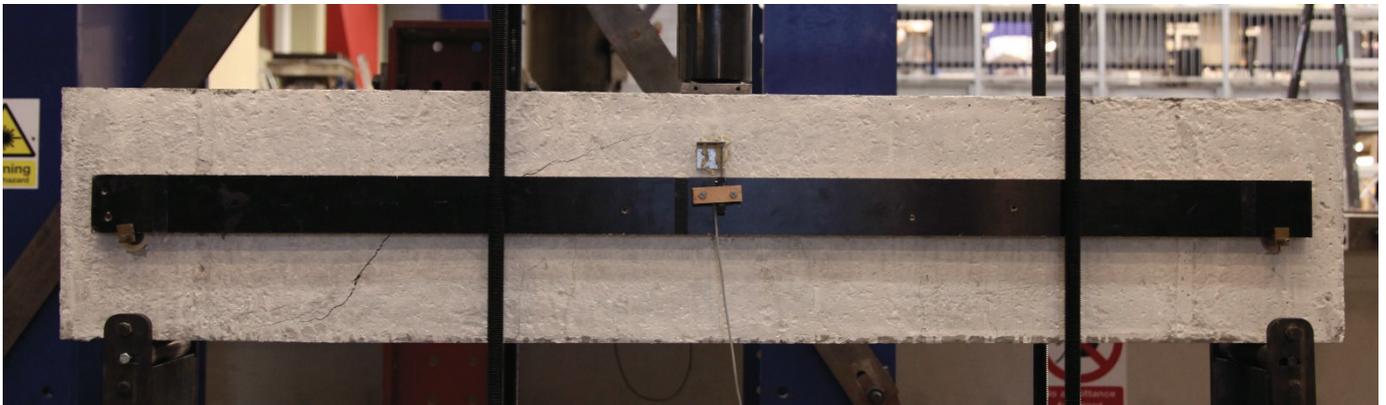


Figure 16.17 Flexural shear crack ( $F = 124kN$ )



Figure 16.18 Bending cracks descending beam



Figure 16.19 Descending beam after failure

# *CONCLUSION*

*REFLECTION ON MODELS*

*CONCLUSION*

*RECOMMENDATIONS*



# 17 Conclusion

## 17.1 Conclusion

The objective of this study has been to clarify the difference in strength capacity between a beam which fails on shear and the truss of the Beeby experiment. Why is a concrete beam on shear unable to transform into a perfect truss mechanism?

The following conclusion holds for a simply supported concrete beam without shear reinforcement subjected to a concentrated load with a three point bending test with a slenderness-ratio of 2.45 and a reinforcement ratio of 0.89%.

The analytic and finite element models confirm the difference in strength capacity of the Beeby research. The truss mechanism has other limits than the beam mechanism. The analytic models show that the cracked beam mechanism always has a higher stiffness compared to the truss mechanism.

In the beam mechanism a flexural shear crack occurs. This crack occurs due to the lack of deformation capacity of the cracked beam mechanism. The crack starts with a bending crack and curves to the middle of the beam to avoid the compressive strut of the truss mechanism.

Variation of the gap length shows shear cracks for small gap lengths and truss mechanisms for large gap lengths. The flexural shear crack occurs when the compressive stresses of the truss mechanism are too low. The compressive stresses prevent the crack to develop before the force mechanism is changed into a truss mechanism. It could be stated that the occurrence of a flexural shear crack is a competition between the developing of the compression strut and the bending cracks. Decreasing the stiffness of the beam, for instance with a gap, increases the compression stresses in the truss which prevent the occurrence of a flexural shear crack.

A beam with a flexural shear crack is not able to transform into a mechanism which is the same as the perfect truss mechanism due to the concrete beneath the flexural shear crack. The concrete beneath the crack causes another stress distribution above the crack in comparison with the truss model.

The inability of a beam to transform into a truss does not depend on the shape of the crack. A beam with a shear crack and a truss with the shape of the shear crack show different force mechanism.

The different stress distribution above the shear crack does not show the failure such as crushing of the concrete or the occurrence of a crack. A hypothesis has been developed, based on the results of this thesis. Further research is required to verify this hypothesis. The hypothesis describes that the failure occurs due to deformation boundaries at the peak of the crack. The stress and deformation direction differ above and beneath the crack. The requirement at the peak of the crack is an equal displacement. To fulfil this requirement shear stresses occur over and around the crack due to bond.

The shear stresses provide a change of stress distribution to meet the displacement requirement.

The shear crack arises by the lack of deformation capacity. With this knowledge it is possible to design a truss with less stiffness. Adding a gap in the middle of the beam results in the decrease of stiffness which results in higher compressive stresses in the strut. This higher compression stresses prevent the arising of the shear crack. The mechanism changes to a truss mechanism by the activation of the reinforcement.

Designing a truss has limits. The gap dimensions have limits to be able to develop the compressive truss. The minimum gap length to span ratio is at least 2.0 and may not pass the strut line. The deformation capacity is determined by the reinforcement layout and amount. For longer spans this becomes critical. The truss principle is also possible for uniform distributed loads.

There are also other design opportunities which are interesting for further research. One design opportunity is the use of smooth reinforcement. Bonded reinforcement results in bending cracks which could result in a flexural shear crack. This prevents the occurrence of a flexural shear crack. Another possibility to decrease the stiffness is a beam with descending height. A small experiment showed an increase of force resistance of 15%.

## 17.2 Reflection on models

The conclusion is based on models with chosen parameters. This paragraph gives a summary and reflection of the used models. The beams of Beeby's experiment were the start of this research. Not all data of the experiment is known. With the known data the best approximation has been made to give a proper indication for the Beeby beams. This research uses analytical and finite element models for approaching the reality.

Analytic models are used to give an indication for the differences between strength capacity of a beam and a truss. The analytic models are simplified models to approach the reality. Only linear elastic relations are used.

The hinges in the truss mechanism are modelled without resistance, while in the concrete provide some resistance. Deformation of the truss mechanism is only based on linear deformation of the struts. Bending of the struts is not taken into account.

The beam models use a linear stress distribution in the uncracked phase. In the cracked phase the concrete restrains the compressive stresses and the reinforcement resists all the tensile stresses. The deformation is based on interpolation of the cracked and uncracked deformation parts of the beam.

In the analytic models only linear analysis is used and no physical non linearity's are involved. These assumptions give a simplification of the reality, but give a proper indication for the differences.

In the finite element models a lot of parameters are chosen. The material properties are based on the properties of the concrete and the reinforcement like the concrete strength, the modulus of elasticity and the yielding strength of the steel.

The concrete properties are based on a fixed crack model, which uses the Hordijk Curve for the tensile relation and an ideal relation for the compressive properties. The fracture energy models the crack resistance of the concrete. The parameters are varied to give a reliable result. The exact relation could never be reached with the used simplifications. Improved results should be possible when the force-displacement diagrams of the Beeby experiment were available because this study shows the importance of the stiffness.

Another simplification is the modelling of the reinforcement. The beam and truss are modelled with full smooth reinforcement and truss elements for the external reinforcement. In reality the reinforcement is not fully bonded and the external reinforcement has bending stiffness. Thereby the dowel action and the bending stiffness of the external reinforcement are not taken into account.

Finite element models are sensitive to parameter and mesh changes. The variation of these parameters shows some differences in failing loads, but the failure mechanisms show hardly any difference.

The parameters are chosen as careful as possible, but differences with reality are unavoidable. It is useful to know this when interpreting the results.

## 17.3 Recommendations

### Verifying results

This study used specific models and parameters. Models give an approach of the reality. Verifying this result with other programs could confirm the results. Verifying with real tests gives the best verification.

### Other dimensions

The results are based on the basic dimensions of the Beeby beams and some small variations. Further research could investigate the influence of other dimensions.

### Form of flexural shear crack

The form of the flexural shear crack depends on the bending cracks and the stresses of the compressive strut. A model could be developed to model the shape of this crack.

### Failing of flexural shear crack

The thesis shows that a beam on shear is unable to transform into a perfect truss mechanism due to the presence of the concrete beneath the crack. This concrete causes another stress distribution in the part above the crack. A hypothesis is given for the failure mechanism, this could be investigated in further research. Especially the influence of the shear stresses over the crack should be analysed more into detail.

### Truss limits

The basic limits of the truss mechanism are known. But the exact boundaries are only based on the used parameters. Further research is needed, especially on the deformation capacity.

### Model properties

The parameters of the models are chosen to approach the reality as good as possible. But the crack models are very sensitive to differences. Proving and investigating the hypothesis with other model properties and real beams is interesting for further research.

### Investigate other possibilities

The differences between the stiffness of the beam mechanism and truss mechanism could be used on different ways. Other possibilities which are interesting for further research are unbonded reinforcement, descending height and perhaps flexible concrete.

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# Appendix A

restart : clc : clear :

## Truss and beam model

### Parameters:

```
> F := F : #N
l := l : #mm
h := 300 : #mm
d := 265 : #mm
width := 200 : #mm
hc :=  $\frac{h}{3}$  : #mm
bc := width : #mm
fyk := 500 :
fyd := fyk :
φs := 10 : #mm
Nuφs := 6 :
As := Nuφs · evalf( $\frac{1}{4} \cdot \text{Pi} \cdot \phi_s^2$ ) : #mm2
Ec := 33000 : #  $\frac{N}{\text{mm}^2}$ 
Es := 200000 : #  $\frac{N}{\text{mm}^2}$ 
fck := 50 :
fcd := 1 · fck :
fctm := evalf( $0.3 \cdot f_{ck}^{(\frac{2}{3})}$ ) ;
```

$$A_s := 471.2388981$$

$$f_{ctm} := 4.071626424$$

(1.1)

### Truss mechanism:

```
> α := evalf( arctan(  $\frac{d}{(0.5 l)}$  ) ) :
Lc := evalf( sqrt(  $(\frac{l}{2})^2 + (d)^2$  ) ) : #mm
Actruss := hc · bc : #mm2
Ls := l : #mm
Nc :=  $\frac{(\frac{F}{2})}{\sin(\alpha)}$  : #N
σc :=  $\frac{N_c}{A_{ctruss}}$  :
εc :=  $\frac{N_c}{E_c \cdot A_{ctruss}}$  :
Δc :=  $-\frac{N_c \cdot L_c}{E_c \cdot A_{ctruss}}$  :
Lcon := Lc + Δc :
Ns :=  $\frac{(\frac{F}{2})}{\tan(\alpha)}$  :
σs :=  $\frac{N_s}{A_s}$  :
εs :=  $\frac{N_s}{E_s \cdot A_s}$  :
Δs :=  $\frac{N_s \cdot L_s}{E_s \cdot A_s}$  :
Lste := Ls + Δs :
```

$$H_{def} := \text{sqrt}\left(L_{con}^2 - \left(\frac{L_{ste}}{2}\right)^2\right);$$

$$u_{midtruss} := d - H_{def}; \#mm$$

$$H_{def2} := \text{sqrt}\left(L_{con}^2 - \left(\frac{L_s}{2}\right)^2\right);$$

$$u_{clam} := d - H_{def2}; \#mm:$$

$$N_c := 0.0009433962265 \cdot Fl \cdot \sqrt{1 + \frac{2.8090000000 \cdot 10^5}{\rho^2}}$$

$$N_s := 0.0009433962265 \cdot Fl$$

(2.1)

### Truss with springs mechanism:

#### Beam stiffness parameter

$$L_{con} := L_{con};$$

$$\Delta_{kb} := \frac{N_s}{k_{rei}};$$

$$L_{kb} := l + \Delta_{kb};$$

$$H_{def} := \text{sqrt}\left(L_{con}^2 - \left(\frac{L_{kb}}{2}\right)^2\right);$$

$$u_{kb} := d - H_{def}; \#mm$$

#### Clamming stiffness parameter

$$L_{con} := L_{con};$$

$$\Delta_{kcl} := \frac{N_s}{2 \cdot k_{clam}};$$

$$L_{kcl} := l + \Delta_{kcl};$$

$$H_{def} := \text{sqrt}\left(L_{con}^2 - \left(\frac{L_{kcl}}{2}\right)^2\right);$$

$$u_{kcl} := d - H_{def}; \#mm$$

### Beam mechanism:

#### Algemeen

##### Invoer

$$\rho := \frac{A_s}{width \cdot d};$$

$$kv := \min\left(\text{evalf}\left(1 + \text{sqrt}\left(\frac{200}{d}\right)\right), 2.0\right);$$

$$\rho l := \rho;$$

$$\gamma_c := 1.0;$$

$$\rho := 0.008891299962$$

(4.1.1.1)

$$M_{ma} := \frac{1}{4} \cdot F \cdot l;$$

$$M_{Rd} := A_s \cdot f_{yd} \cdot d \cdot \left(1 - 0.52 \cdot \rho \cdot \frac{f_{yd}}{f_{cd}}\right);$$

$$V_{Rd} := \text{evalf}\left(\left(\left(\frac{0.18}{\gamma_c}\right) \cdot kv \cdot (100 \cdot \rho l \cdot f_{ck})^{\left(\frac{1}{3}\right)}\right) \cdot width \cdot d\right);$$

$$M_{ma} := \frac{1}{4} \cdot Fl$$

$$M_{Rd} := 5.955229469 \cdot 10^7$$

$$V_{Rd} := 63155.33664$$

(4.1.1)

#### Uncracked beam:

$$I_{zz} := \frac{1}{12} \cdot width \cdot h^3;$$

$$u_{beam} := evalf\left(\frac{1}{48} \cdot \frac{F \cdot l^3}{E_c \cdot I_{zz}}\right) :$$

#### Cracked Beam:

$$\alpha_e := evalf\left(\frac{E_s}{E_c}\right) :$$

$$x := (-\alpha_e \cdot \rho + \sqrt{(\alpha_e \cdot \rho)^2 + 2 \cdot \alpha_e \cdot \rho}) \cdot d :$$

$$W := \frac{1}{6} \cdot width \cdot h^2 :$$

$$M_{cr} := W \cdot f_{ctm} :$$

$$M_{cr} := 1.221487927 \cdot 10^7$$

(4.3.1)

$$\kappa 1 := \frac{1}{12} \cdot width \cdot h^3 :$$

$$\kappa l := \frac{M_{ma}}{E_c \cdot I_{z1}} :$$

$$I_{z2} := \frac{1}{12} \cdot width \cdot x^3 + width \cdot x \cdot \left(\frac{1}{2} x\right)^2 + \alpha_e \cdot A_s \cdot (d - x)^2 :$$

$$\kappa 2 := \frac{M_{ma}}{E_c \cdot I_{z2}} :$$

$$\beta := 0.5 : \# \text{FACTOR UIT EUROCODE}$$

$$\xi := 1 - \beta \cdot \left(\frac{M_{cr}}{M_{ma}}\right)^2 :$$

$$\kappa := \xi \cdot \kappa 2 + (1 - \xi) \cdot \kappa l :$$

$$u_{crack} := \frac{4}{48} \cdot \kappa \cdot l^2 :$$

#### Stiffness parameters

##### Beam stiffness

$$\kappa := u_{kb} : u_{crack} :$$

$$l := 2000;$$

$$F := 50 \cdot 1000 : eq1 := \frac{u_{kb}}{u_{crack}} = 1 : eq2 := k_{rei} \geq 0 : sol := solve(\{eq1, eq2\}, \{k_{rei}\});$$

$$F := 100 \cdot 1000 : eq1 := \frac{u_{kb}}{u_{crack}} = 1 : eq2 := k_{rei} \geq 0 : sol := solve(\{eq1, eq2\}, \{k_{rei}\});$$

$$F := 200 \cdot 1000 : eq1 := \frac{u_{kb}}{u_{crack}} = 1 : eq2 := k_{rei} \geq 0 : sol := solve(\{eq1, eq2\}, \{k_{rei}\});$$

$$l := 2000$$

$$sol := \{k_{rei} = 1.536150407 \cdot 10^5\}$$

$$sol := \{k_{rei} = 1.398203733 \cdot 10^5\}$$

$$sol := \{k_{rei} = 1.380974438 \cdot 10^5\}$$

(5.1.1)

$$l := 6000;$$

$$F := 50 \cdot 1000 : eq1 := \frac{u_{kb}}{u_{crack}} = 1 : eq2 := k_{rei} \geq 0 : sol := solve(\{eq1, eq2\}, \{k_{rei}\});$$

$$F := 100 \cdot 1000 : eq1 := \frac{u_{kb}}{u_{crack}} = 1 : eq2 := k_{rei} \geq 0 : sol := solve(\{eq1, eq2\}, \{k_{rei}\});$$

$$F := 200 \cdot 1000 : eq1 := \frac{u_{kb}}{u_{crack}} = 1 : eq2 := k_{rei} \geq 0 : sol := solve(\{eq1, eq2\}, \{k_{rei}\});$$

$$l := 6000$$

$$sol := \{k_{rei} = 50485.16245\}$$

$$sol := \{k_{rei} = 59438.70933\}$$

$$sol := \{k_{rei} = 95129.09140\}$$

(5.1.2)

$$\kappa := l' :$$

## Clamming stiffness

```

> u_kcl: u_crack:
> l := 2000;
F := 50·1000 : eq1 :=  $\frac{u_{kcl}}{u_{crack}} = 1$  : eq2 :=  $k_{clam} \geq 0$  : sol := solve({eq1, eq2}, {k_clam});
F := 100·1000 : eq1 :=  $\frac{u_{kcl}}{u_{crack}} = 1$  : eq2 :=  $k_{clam} \geq 0$  : sol := solve({eq1, eq2}, {k_clam});
F := 200·1000 : eq1 :=  $\frac{u_{kcl}}{u_{crack}} = 1$  : eq2 :=  $k_{clam} \geq 0$  : sol := solve({eq1, eq2}, {k_clam});

```

l := 2000

sol := {k\_clam = 76807.52034}

sol := {k\_clam = 69910.18664}

sol := {k\_clam = 69048.72187}

(5.2.1)

```

> l := 6000;
F := 50·1000 : eq1 :=  $\frac{u_{kcl}}{u_{crack}} = 1$  : eq2 :=  $k_{clam} \geq 0$  : sol := solve({eq1, eq2}, {k_clam});
F := 100·1000 : eq1 :=  $\frac{u_{kcl}}{u_{crack}} = 1$  : eq2 :=  $k_{clam} \geq 0$  : sol := solve({eq1, eq2}, {k_clam});
F := 200·1000 : eq1 :=  $\frac{u_{kcl}}{u_{crack}} = 1$  : eq2 :=  $k_{clam} \geq 0$  : sol := solve({eq1, eq2}, {k_clam});

```

l := 6000

sol := {k\_clam = 25242.58122}

sol := {k\_clam = 29719.35466}

sol := {k\_clam = 47564.54569}

(5.2.2)

```

> l := 'l';
F := 'F';

```

## Comparison

```

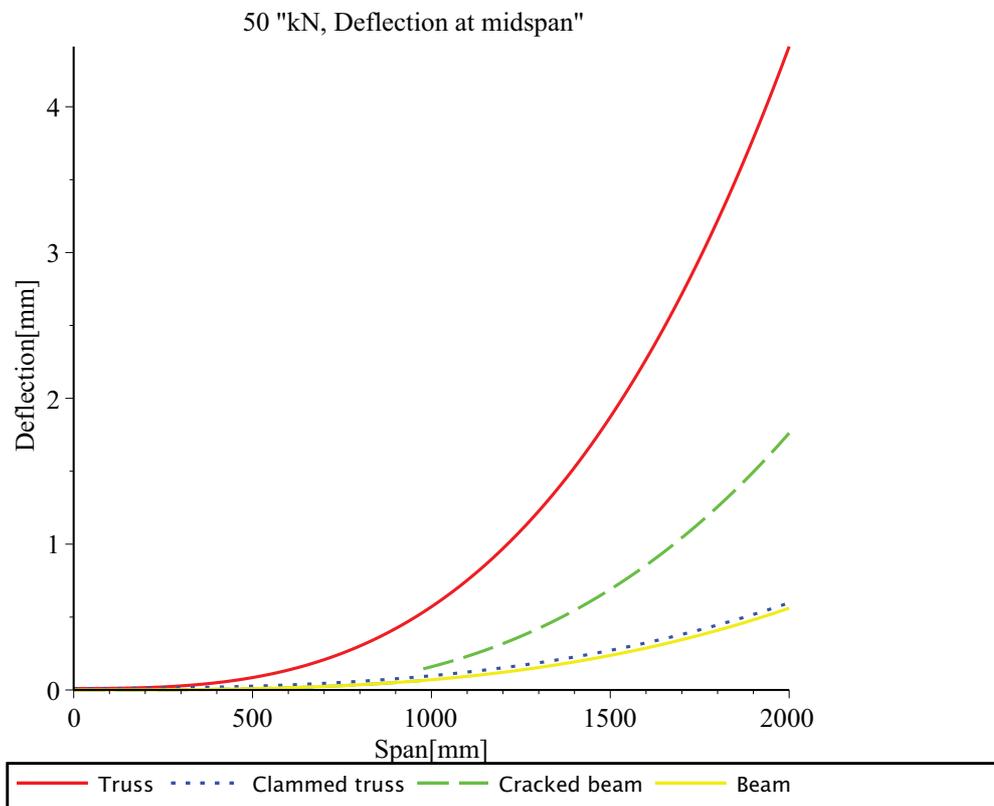
> #k_beam :=  $\frac{200000000 \cdot 10}{l}$  : #  $\left[ \frac{N}{mm} \right]$ 
#k_clam :=  $\frac{150000000}{l}$  : #  $\left[ \frac{N}{mm} \right]$ 
u_midtruss : u_kb : u_clam : u_beam : u_crack :  $\sigma_c$  : f_cd :  $\sigma_s$  : f_yd : M_ma : M_Rd : V_Rd : V_ma := 0.5 F:
> with(plots) : lmax2 := 2000 : #mm
k_rei :=  $\frac{E_s \cdot A_s}{L_s}$  : F := 50·1000 :
l_cr :=  $\frac{M_{cr}}{M_{ma}} \cdot l$  :
A1 := plot( u_beam, l=0 ..lmax2, title =  $\frac{F}{1000}$  "kN, Deflection at midspan", color=[yellow], linestyle=[solid], legend=["Beam"], legendstyle
= [font=["HELVETICA", 8], location=bottom] ) : A2 := plot( u_midtruss, l=0 ..lmax2, color=[red], linestyle=[solid], legend
=["Truss"] ) : A3 := plot( u_clam, l=0 ..lmax2, color=[blue], linestyle=[dot], legend=["Clammed truss"] ) : AA := plot( u_beam, l=0
..l_cr, color=[green], linestyle=[dash] ) : AB := plot( u_crack, l=l_cr..lmax2, color=[green], linestyle=[dash], legend
=["Cracked beam"], ) :
A4 := plot( u_kb, l=0 ..lmax2, color=[blue], linestyle=[dot], legend=["Spring beam"] ) :
display({A1, A2, A3, AA, AB}, labels=["Span[mm]", "Deflection [mm]"], labeldirections=["horizontal", "vertical"]);
#display({A1, A3}, labels=["Span[mm]", "Deflection [mm]"], labeldirections=["horizontal", "vertical"]);
#display({A2, A3}, title =  $\frac{F}{1000}$  "kN, Deflection at midspan", labels=["Span[mm]", "Deflection [mm]"], labeldirections=["horizontal",
"vertical"]);
# display({AA, AB, A1}, title =  $\frac{F}{1000}$  "kN, Deflection at midspan", labels=["Span[mm]", "Deflection [mm]"], labeldirections=["horizontal",

```

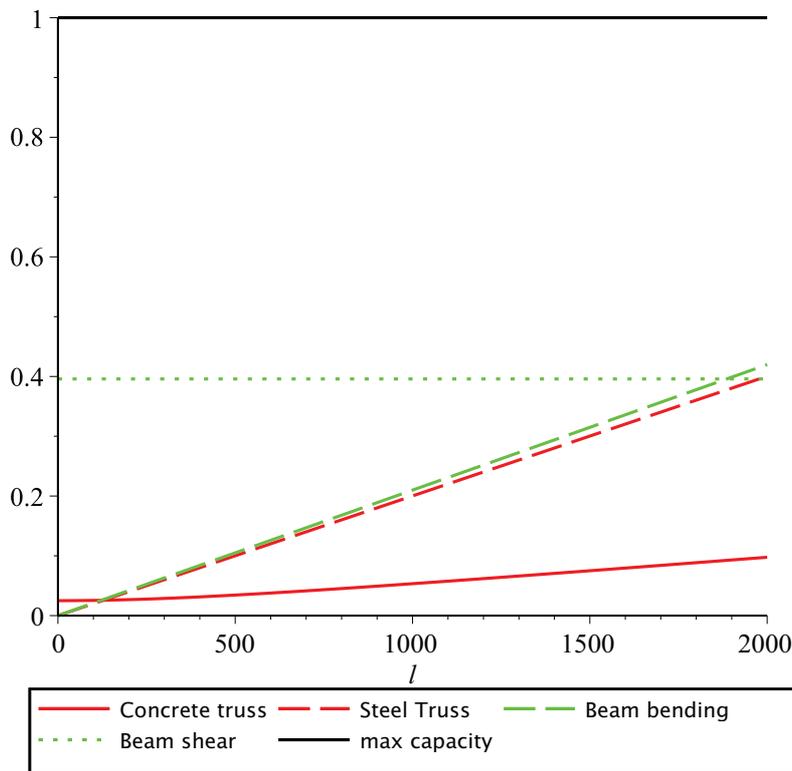
```

"vertical"]);
#display({A2, A4});
AS0 := plot(1, 0 ..lmax2, color=[black], linestyle=[solid], legend=["max capacity"], title =  $\frac{F}{1000}$  "kN, Safetycheck", legendstyle=[font
=["HELVETICA", 8], location = bottom] );
AS1 := plot( $\frac{\sigma_c}{f_{cd}}$ , l=0 ..lmax2, color=[red], linestyle=[solid], legend=["Concrete truss"] );
AS2 := plot( $\frac{\sigma_s}{f_{yd}}$ , l=0 ..lmax2, color=[red], linestyle=[dash], legend=["Steel Truss"] );
AS3 := plot( $\frac{M_{ma}}{M_{Rd}}$ , l=0 ..lmax2, color=[green], linestyle=[dash], legend=["Beam bending"] );
AS4 := plot( $\frac{V_{ma}}{V_{Rd}}$ , 0 ..lmax2, color=[green], linestyle=[dot], legend=["Beam shear"] );
display({AS0, AS1, AS2, AS3, AS4});

```



50 "kN, Safetycheck"



```

> F := 100·1000 :
l_cr :=  $\frac{M_{cr}}{M_{ma}}$  · l;
M_ma ;
B1 := plot( u_beam, l=0 ..lmax2, title =  $\frac{F}{1000}$  "kN, Deflection at midspan", color=[yellow], linestyle=[solid], legend=["Beam"], legendstyle
= [font=["HELVETICA", 10], location=bottom] ) : B2 := plot( u_midtruss, l=0 ..lmax2, color=[red], linestyle=[solid], legend
=["Truss"] ) : B3 := plot( u_clam, l=0 ..lmax2, color=[blue], linestyle=[dot], legend=["Clammed truss"] ) : BA := plot( u_beam, l=0
..l_cr, color=[green], linestyle=[dash] ) : BB := plot( u_crack, l=l_cr..lmax2, color=[green], linestyle=[dash], legend
=["Cracked beam"], ) :
display( {B1, B2, B3, BA, BB}, labels=["Span[mm]", "Deflection [mm]"], labeldirections=["horizontal", "vertical"]);
AS0 := plot( 1, 0 ..lmax2, color=[black], linestyle=[solid], legend=["max capacity"], title =  $\frac{F}{1000}$  "kN, Safetycheck" ) :
AS1 := plot(  $\frac{\sigma_c}{f_{cd}}$ , l=0 ..lmax2, color=[red], linestyle=[solid], legend=["Concrete truss"] ) :
AS2 := plot(  $\frac{\sigma_s}{f_{yd}}$ , l=0 ..lmax2, color=[red], linestyle=[dash], legend=["Steel Truss"] ) :
AS3 := plot(  $\frac{M_{ma}}{M_{Rd}}$ , l=0 ..lmax2, color=[green], linestyle=[dash], legend=["Beam bending"] ) :
AS4 := plot(  $\frac{V_{ma}}{V_{Rd}}$ , 0 ..lmax2, color=[green], linestyle=[dot], legend=["Beam shear"] ) :

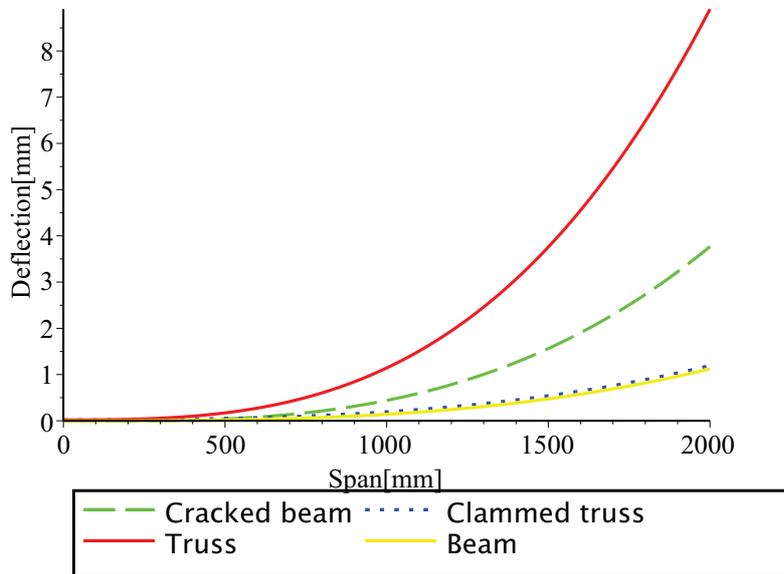
display( {AS0, AS1, AS2} );
display( {AS0, AS3, AS4} );

```

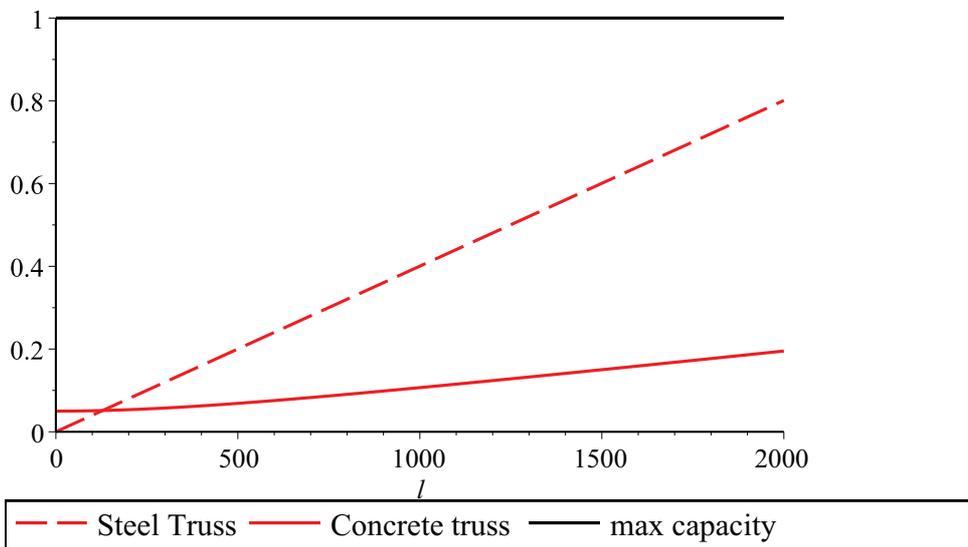
display({AS0, AS1, AS2, AS3, AS4});

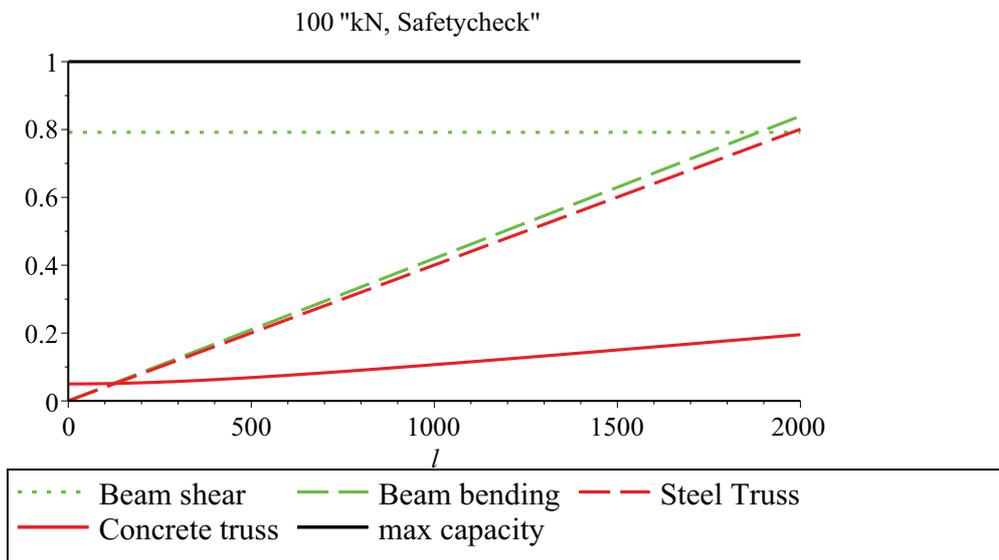
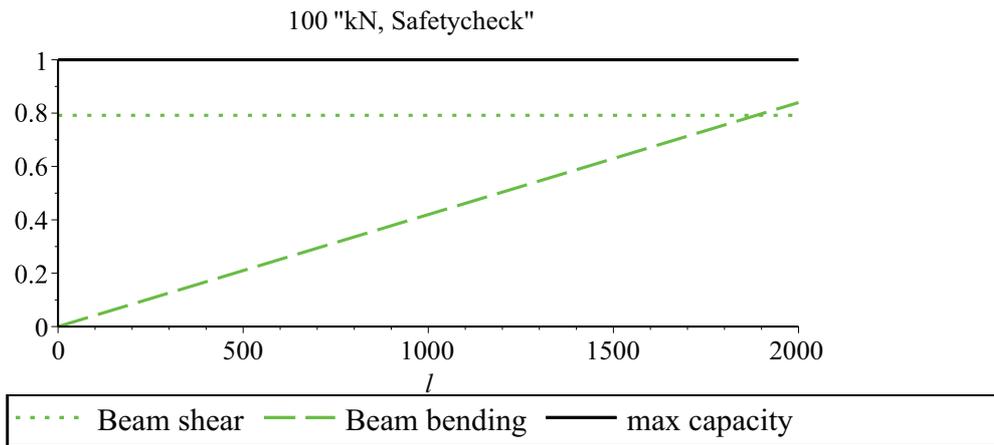
$$l_{cr} := 488.5951708$$
$$25000 l$$

100 "kN, Deflection at midspan"



100 "kN, Safetycheck"





```

> F := 200-1000 :
u_midtruss := u_midtruss
u_trusscc := u_clam :
u_beamc := u_beam :
l_cr := M_cr / M_ma ;
C1 := plot( u_beamc l=0 ..lmax2, title = F/1000 "kN, Deflection at midspan", color=[yellow], linestyle=[solid], legend=["Beam"], legendstyle
=[font=["HELVETICA", 10], location=bottom] ) : C2 := plot(u_midtruss l=0 ..lmax2, color=[red], linestyle=[solid], legend
=["Truss"]) : C3 := plot(u_clam l=0 ..lmax2, color=[blue], linestyle=[dot], legend=["Clammed truss"]) : CA := plot(u_beamc, l=0
..l_cr, color=[green], linestyle=[dash]) : CB := plot(u_crack, l=l_cr..lmax2, color=[green], linestyle=[dash], legend
=["Cracked beam"], ) :
display({C1, C2, C3, CA, CB}, labels=["Span[mm]", "Deflection [mm]"], labeldirections=["horizontal", "vertical"]);
AS0 := plot(1, 0 ..lmax2, color=[black], linestyle=[solid], legend=["max capacity"], title = F/1000 "kN, Safetycheck") :

```

```

AS1 := plot( (sigma_c / f_cd, l=0 ..lmax2, color=[red], linestyle=[solid], legend=["Concrete truss"] ) ):
AS2 := plot( (sigma_s / f_yd, l=0 ..lmax2, color=[red], linestyle=[dash], legend=["Steel Truss"] ) ):
AS3 := plot( (M_ma / M_Rd, l=0 ..lmax2, color=[green], linestyle=[dash], legend=["Beam bending"] ) ):
AS4 := plot( (V_ma / V_Rd, 0 ..lmax2, color=[green], linestyle=[dot], legend=["Beam shear"] ) ):

display( {AS0, AS1, AS2, AS3, AS4} );

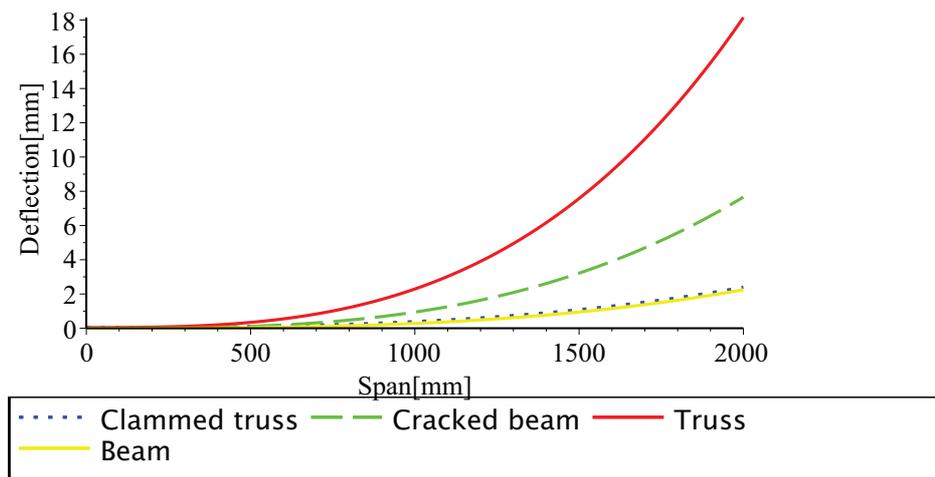
```

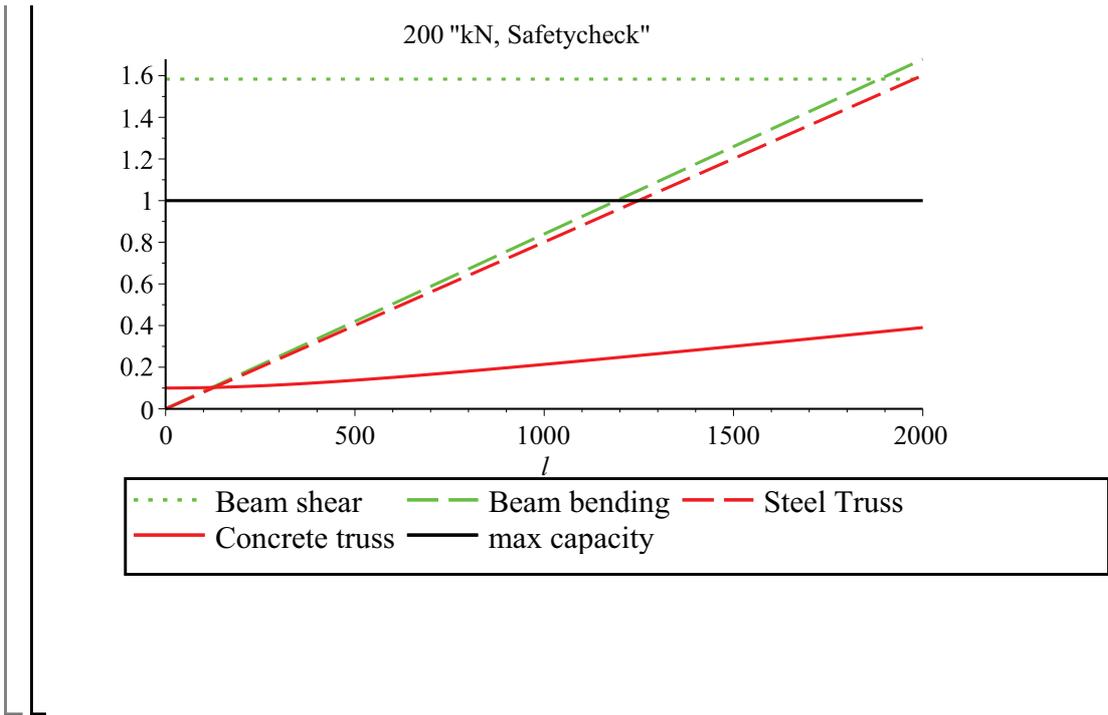
$u_{midtrussc} := 265$

$$-\frac{1}{2} \left( 4 \left( 0.5000000000 \sqrt{l^2 + 2.80900 \cdot 10^5} - 1.429388222 \cdot 10^{-7} l \sqrt{1 + \frac{2.809000000 \cdot 10^5}{l^2}} \sqrt{l^2 + 2.80900 \cdot 10^5} \right)^2 - (l + 0.000002001948970 l^2)^2 \right)^{1/2}$$

$l_{cr} := 244.2975854$

200 "kN, Deflection at midspan"





**Variable Force F:**

```

> l := 'l';
   F := 'F';
   Mma :=
   Mma := 1/4 · F · l;
   σcl := σc;
   Mmal := Mma;
   MRd1 := MRd;

```

$$l := l$$

$$\frac{1}{4} Fl$$

$$M_{ma} := \frac{1}{4} Fl$$

$$\sigma_{cl} := 4.716981132 \cdot 10^{-8} Fl \sqrt{1 + \frac{2.8090000000 \cdot 10^5}{l^2}}$$

$$M_{mal} := \frac{1}{4} Fl$$

$$M_{Rd1} := 5.955229469 \cdot 10^7$$

(7.1)

(7.2)

```

> F := 'F';
   Fmax := 300 · 1000 :
   l := 500 :
   ub1 := ubeam :
   um1 := umidtruss :
   ukb1 := ukb :
   ucl1 := uclam :
   ucr1 := ucrack :
   σcl := σc :
   fcd :
   σs1 := σs :
   fyd :
   Mmal := Mma :
   MRd1 := MRd :
   VRd :

```

$$V_{ma} := 0.5 F :$$

$$F_{cr} := \frac{4 \cdot M_{cr}}{l} :$$

$$l_{cr} := \frac{M_{cr}}{M_{ma}} \cdot l :$$

> AS0 := plot(1, 0 ..Fmax, color=[black], linestyle=[solid], legend=["max capacity"], title="Safetycheck with l=500mm") :

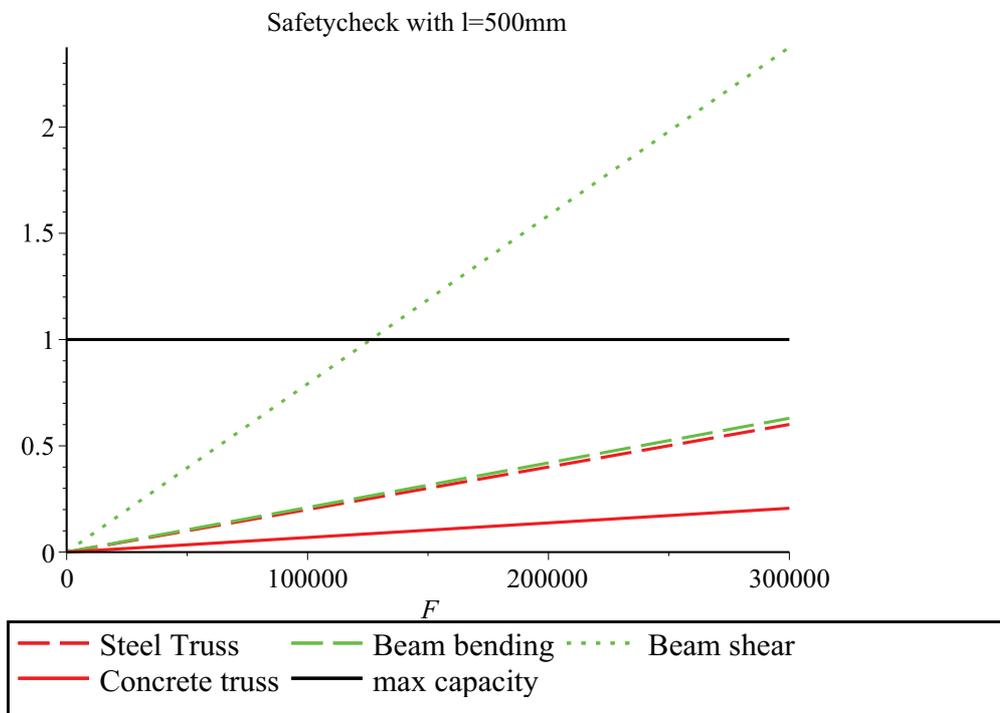
$$AS1 := \text{plot} \left( \frac{\sigma_{cl}}{f_{cd}}, F=0 ..Fmax, color=[red], linestyle=[solid], legend=["Concrete truss"] \right) :$$

$$AS2 := \text{plot} \left( \frac{\sigma_{sl}}{f_{yd}}, F=0 ..Fmax, color=[red], linestyle=[dash], legend=["Steel Truss"] \right) :$$

$$AS3 := \text{plot} \left( \frac{M_{ma1}}{M_{Rd1}}, F=0 ..Fmax, color=[green], linestyle=[dash], legend=["Beam bending"] \right) :$$

$$AS4 := \text{plot} \left( \frac{V_{ma}}{V_{Rd}}, F=0 ..Fmax, color=[green], linestyle=[dot], legend=["Beam shear"] \right) :$$

display({AS0, AS1, AS2, AS3, AS4});



> F := 'F':  
Fmax := 300·1000 :  
l := 'l':  
M<sub>ma1</sub> := M<sub>ma</sub> :  
l := 2500 :

$$u_{b2} := u_{beam} : u_{m2} := u_{midtruss} : u_{kb2} := u_{kb} : u_{cl2} := u_{clam} : u_{ct2} := u_{crack} : \sigma_{c2} := \sigma_c : f_{cd} : \sigma_{s2} := \sigma_s : f_{yd} : M_{ma2} := M_{ma} : M_{Rd2} := M_{Rd} : V_{Rd} :$$

$$V_{ma2} := 0.5 F : F_{cr} := \frac{4 \cdot M_{cr}}{l} : l_{cr} := \frac{M_{cr}}{M_{ma}} \cdot l :$$

AS0 := plot(1, 0 ..Fmax, color=[black], linestyle=[solid], legend=["max capacity"], title="Safetycheck with l=2500 mm") :

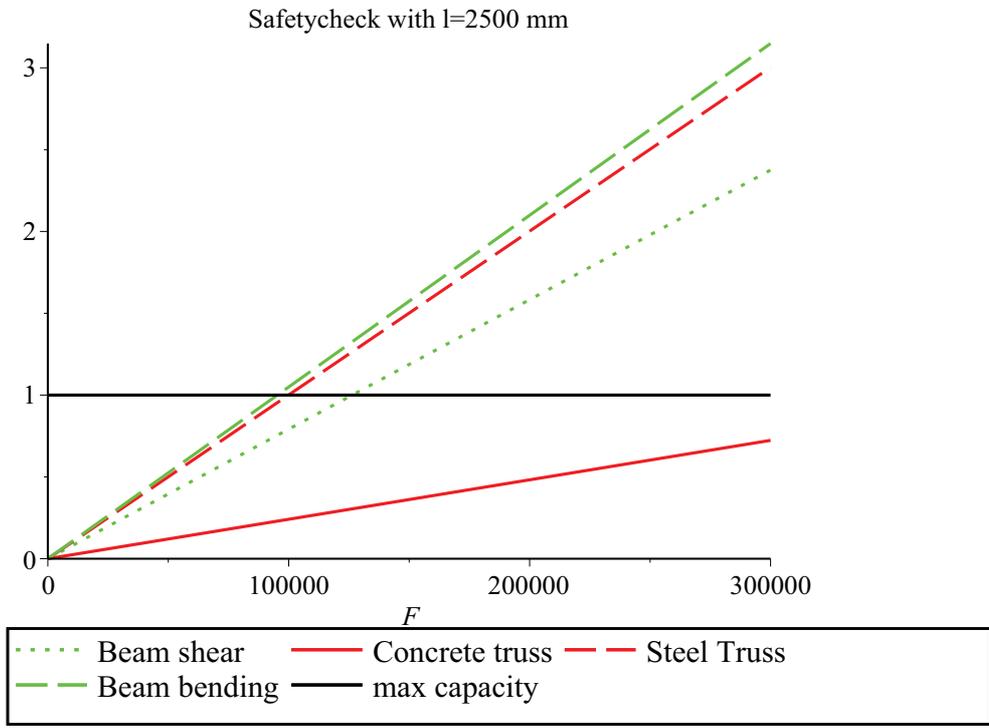
$$AS1 := \text{plot} \left( \frac{\sigma_{c2}}{f_{cd}}, F=0 ..Fmax, color=[red], linestyle=[solid], legend=["Concrete truss"] \right) :$$

```

AS2 := plot( (sigma_s2 / f_yd, F=0 ..Fmax, color=[red], linestyle=[dash], legend=["Steel Truss"] ) :
AS3 := plot( (M_ma2 / M_Rd2, F=0 ..Fmax, color=[green], linestyle=[dash], legend=["Beam bending"] ) :
AS4 := plot( (V_ma / V_Rd, F=0 ..Fmax, color=[green], linestyle=[dot], legend=["Beam shear"] ) :

display({AS0, AS1, AS2, AS3, AS4});

```



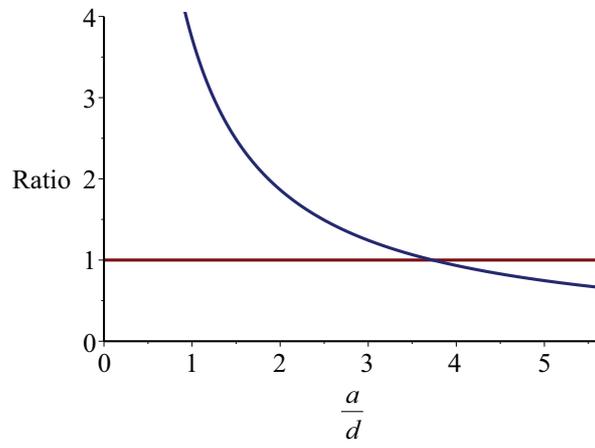
```

> l := 'l':
> Ratio := (V_ma / V_Rd) = 1 :
solution := solve({Ratio}, {F}) :
assign(solution) :
F1 := F :
F := 'F':

Yield := (sigma_s / f_yd) = 1 :
solution1 := solve({Yield}, {F}) :
assign(solution1) :
F2 := F :
Delta := (F2 / F1) :
lmax := 3000 :
# plot([1, Delta], l=0 ..lmax, y=0 ..4) :
l := 'ad':
Delta := (Delta - 0.5) / d :
admax := (lmax * 0.5) / d :
d := 'd':

```

```
plot([1, Δ2], l=0 ..admax, y=0 ..4 , labels = [ $\frac{a}{d}$ , "Ratio"]);
```



# Appendix B

restart : clc : clear :

## Stiffness comparing for variable reinforcement amount

### ► Parameters:

### ▼ Truss mechanism:

$$\alpha := \text{evalf}\left(\arctan\left(\frac{d}{(0.5 l)}\right)\right) :$$

$$L_c := \text{evalf}\left(\text{sqrt}\left(\left(\frac{l}{2}\right)^2 + (d)^2\right)\right) : \#mm$$

$$A_{ctruss} := hc \cdot bc : \#mm^2$$

$$L_s := l : \#mm$$

$$\begin{aligned} > N_c := \frac{\left(\frac{F}{2}\right)}{\sin(\alpha)} ; \sigma_c := \frac{N_c}{A_{ctruss}} ; \epsilon_c := \frac{N_c}{E_c \cdot A_{ctruss}} ; \Delta l_c := -\frac{N_c \cdot L_c}{E_c \cdot A_{ctruss}} ; L_{con} := L_c + \Delta l_c : \end{aligned}$$

$$N_s := \frac{\left(\frac{F}{2}\right)}{\tan(\alpha)} ; \sigma_s := \frac{N_s}{A_s} ; \epsilon_s := \frac{N_s}{E_s \cdot A_s} ; \Delta l_s := \frac{N_s \cdot L_s}{E_s \cdot A_s} ; L_{ste} := L_s + \Delta l_s :$$

$$H_{def} := \text{sqrt}\left(L_{con}^2 - \left(\frac{L_{ste}}{2}\right)^2\right) ; u_{midtruss} := d - H_{def}$$

$$H_{def2} := \text{sqrt}\left(L_{con}^2 - \left(\frac{L_s}{2}\right)^2\right) ; u_{clam} := d - H_{def2} : \#mm :$$

$$N_c := 2.874015998 \cdot 10^5$$

$$\sigma_c := 14.37007999$$

$$N_s := 2.830188678 \cdot 10^5$$

$$\sigma_s := \frac{2.830188678 \cdot 10^5}{A_s}$$

$$u_{midtruss} := 265 - \sqrt{2.318204727 \cdot 10^6 - \frac{1}{4} \left(3000 + \frac{4245.283017}{A_s}\right)^2}$$

(2.1)

### ▼ Beam mechanism:

#### ▼ Algemeen

## Invoer

$$\begin{aligned} > \rho &:= \frac{A_s}{(\text{width} \cdot d)}; \\ kv &:= \min\left(\text{evalf}\left(1 + \text{sqrt}\left(\frac{200}{d}\right)\right), 2.0\right); \\ \rho l &:= \rho; \\ \gamma &:= 1.0; \\ & \rho := \frac{1}{53000} A_s \end{aligned}$$

(3.1.1.1)

$$\begin{aligned} > M_{ma} &:= \frac{1}{4} \cdot F \cdot l; \\ M_{Rd} &:= A_s \cdot f_{yd} \cdot d \cdot \left(1 - 0.52 \cdot \rho \cdot \frac{f_{yd}}{f_{cd}}\right); \\ V_{Rd} &:= \text{evalf}\left(\left(\left(\frac{0.18}{\gamma}\right) \cdot kv \cdot \left(100 \cdot \rho l \cdot f_{ck}\right)^{\left(\frac{1}{3}\right)}\right) \text{width} \cdot d\right); \end{aligned}$$

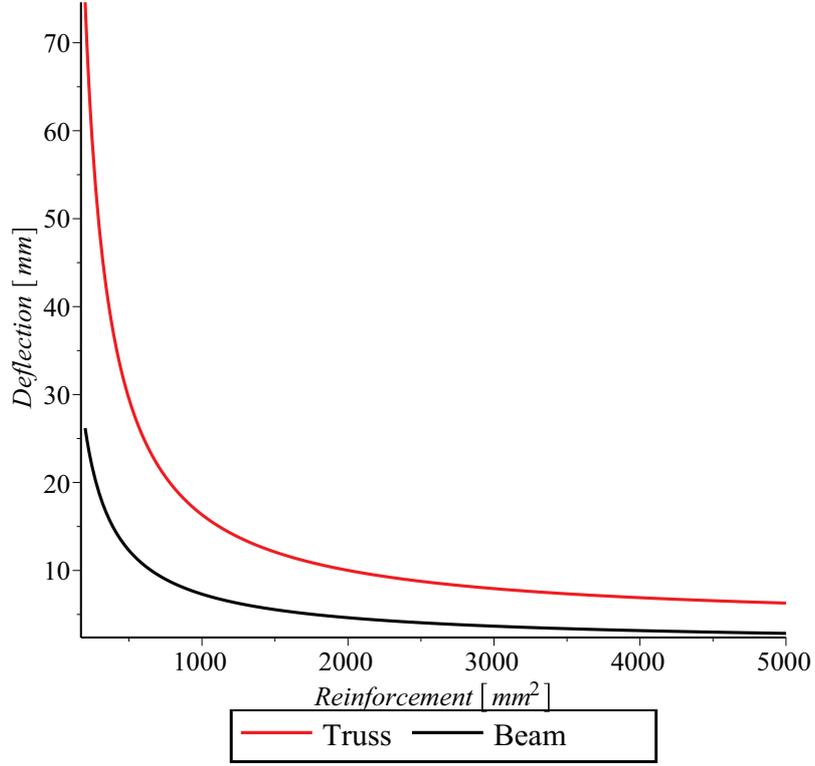
## Cracked Beam:

$$\begin{aligned} > \alpha_e &:= \text{evalf}\left(\frac{E_s}{E_c}\right); \\ x &:= \left(-\alpha_e \cdot \rho + \text{sqrt}\left(\left(\alpha_e \cdot \rho\right)^2 + 2 \cdot \alpha_e \cdot \rho\right)\right) \cdot d; \\ W &:= \frac{1}{6} \cdot \text{width} \cdot h^2; \\ M_{cr} &:= W \cdot f_{ctm}; \\ > Iz_1 &:= \frac{1}{12} \cdot \text{width} \cdot h^3; \kappa l := \frac{M_{ma}}{E_c \cdot Iz_1}; \\ Iz_2 &:= \frac{1}{12} \cdot \text{width} \cdot x^3 + \text{width} \cdot x \cdot \left(\frac{1}{2}x\right)^2 + \alpha_e \cdot A_s \cdot (d-x)^2; \kappa 2 := \frac{M_{ma}}{E_c \cdot Iz_2}; \\ > \beta &:= 0.5 \text{ :# FACTOR UIT EUROCODE} \\ \xi &:= 1 - \beta \cdot \left(\frac{M_{cr}}{M_{ma}}\right)^2; \kappa := \xi \cdot \kappa 2 + (1 - \xi) \cdot \kappa l; u_{\text{crack}} := \frac{4}{48} \cdot \kappa \cdot l^2; \\ > \text{with}(\text{plots}) : \\ A_{smax} &:= 5000; \end{aligned}$$

$AS0 := \text{plot}(u_{\text{crack}}, A_s = 200 .. A_{smax}, \text{color} = [\text{black}], \text{linestyle} = [\text{solid}], \text{legend} = [\text{"Beam"}], \text{title} = \text{Deflection at midspan}) :$

$ASI := \text{plot}(u_{\text{midtruss}}, A_s = 200 .. A_{smax}, \text{color} = [\text{red}], \text{linestyle} = [\text{solid}], \text{legend} = [\text{"Truss"}], \text{labels} = [\text{Reinforcement [mm}^2\text{]}, \text{Deflection [mm]}], \text{labeldirections} = [\text{"horizontal"}, \text{"vertical"}]) :$   
 $\text{display}(\{AS0, ASI\});$

*Deflection at midspan*



# Appendix C

restart : clc :

## Selfweight cracks

```
> M_max := 1/8 * 24 * w * h * l^2 ;  
M_cr := 1/6 * w * h^2 * 4000 ;  
verhouding := M_max / M_cr ;  
h := 1 ;  
verhouding ;  
l_ma := 20 ;  
plot( {verhouding, 1}, l = 0 .. l_ma labels = [ Slenderness, Mselfweight / Mcrack ] );
```

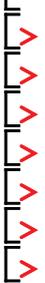
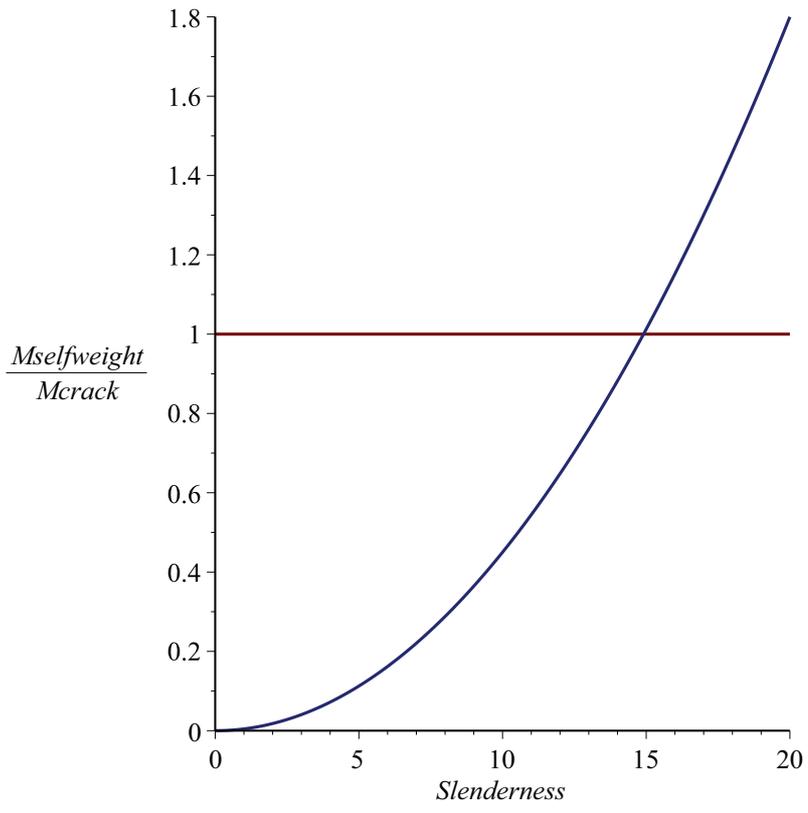
$$M_{cr} := \frac{2000}{3} w h^2$$

$$verhouding := \frac{9}{2000} \frac{l^2}{h}$$

$$h := 1$$

$$\frac{9}{2000} l^2$$

$$l_{ma} := 20$$



# Appendix D

restart : clc : clear :

## Truss versus beam mechanism

### Parameters:

```
> F := 'F'; #N
l := 1300 : #mm
hb := 300 : #mm
d := 265 : #mm
width := 200 : #mm
hc : #mm
bc := width : #mm
fyk := 500 :
fyd := fyk :
φs := 10 : #mm
Nuφs := 6 :
As := Nuφs · evalf( (1/4) · Pi · φs2 ); #mm2
Ec := 33000 : #  $\frac{N}{mm^2}$ 
Es := 200000 : #  $\frac{N}{mm^2}$ 
fck := 50 :
fcd := 1 · fck :
fctm := evalf( 0.3 · fck(2/3) ) :
```

$$A_s := 471.2388981$$

(1.1)

### Truss mechanism:

```
> α := evalf( arctan( (d / (0.5 l)) ) );
Lc := evalf( sqrt( ( (l/2)2 + (d)2 ) ); #mm
Ls := 1 : #mm
Nc :=  $\frac{\left(\frac{F}{2}\right)}{\sin(\alpha)}$ ; #N
BETAM := 0.7 :
```

$$A_{ctruss} := \frac{N_c}{f_{ck} \cdot BETAM} :$$

$$h_{truss} := evalf(\text{sqrt}(A_{ctruss})) :$$

$$t_{truss} := \frac{h_{truss}}{\cos(\alpha)} :$$

$$w_{truss} := \frac{h_{truss}}{\sin(\alpha)} :$$

$$\alpha := 0.3871200373$$

$$L_c := 701.9437300$$

$$N_c := 1.324422132 F$$

(2.1)

> F := 'F':

$$\rho := \frac{A_s}{(\text{width} \cdot d)} ;$$

$$\alpha_e := evalf\left(\frac{E_s}{E_c}\right) :$$

$$x := \left(-\alpha_e \cdot \rho + \text{sqrt}\left((\alpha_e \cdot \rho)^2 + 2 \cdot \alpha_e \cdot \rho\right)\right) \cdot d ;$$

$$W := \frac{1}{6} \cdot \text{width} \cdot h b^2 :$$

$$M_{cr} := W \cdot f_{ctm} ;$$

$$F_{cr} := \frac{M_{cr} \cdot 4}{l} :$$

$$M_{ma} := \frac{1}{4} \cdot F \cdot l ;$$

$$xc := \frac{W \cdot f_{ctm}}{0.5 F} ;$$

$$\rho := 0.008891299962$$

$$x := 73.88068080$$

$$M_{cr} := 1.221487927 \cdot 10^7$$

$$M_{ma} := 325 F$$

$$xc := \frac{2.442975854 \cdot 10^7}{F}$$

(2.2)

> F := 40000 :

$$M_{ma} : h_{truss1} := h_{truss} : t_{truss1} := t_{truss} : w_{truss1} := w_{truss} : xc1 := xc ;$$

$$F := 60 \cdot 1000 ;$$

$$h_{truss2} := h_{truss} : t_{truss2} := t_{truss} : w_{truss2} := w_{truss} : xc2 := xc ;$$

$$F := 100 \cdot 1000 ;$$

$$h_{truss3} := h_{truss} : t_{truss3} := t_{truss} : w_{truss3} := evalf(w_{truss}) : xc3 := xc ;$$

```

xc1 := 610.7439635
F := 60000
xc2 := 407.1626423
F := 100000
xc3 := 244.2975854

```

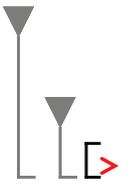
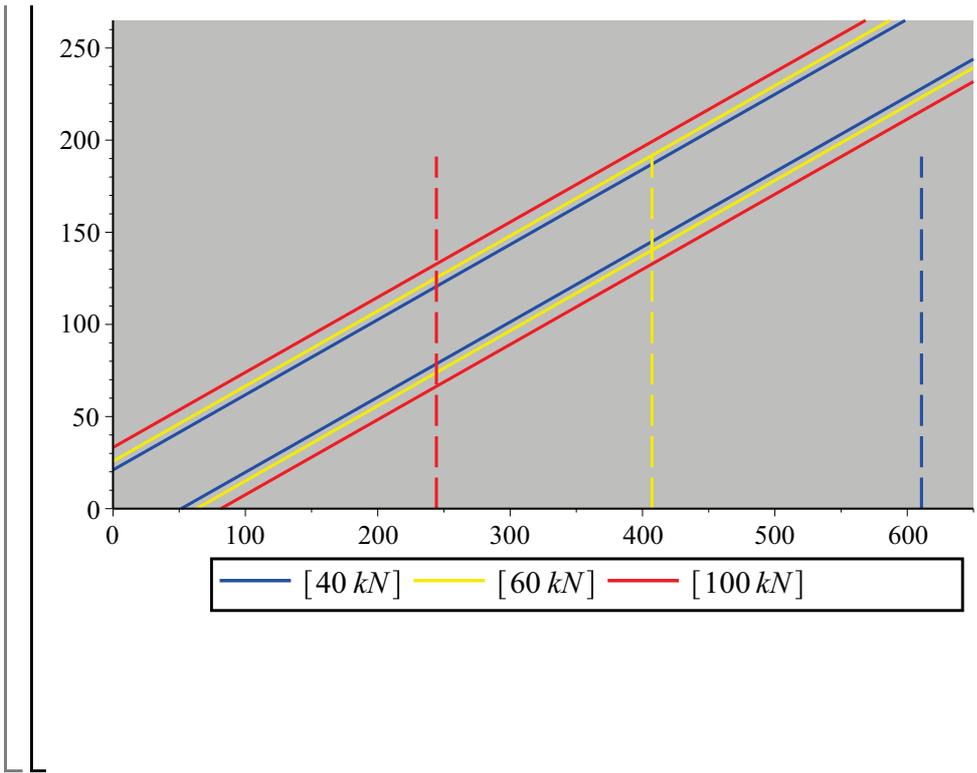
(2.3)

```

> with(plots) :
with(plottools) :
BEAM := (polygon( ([0, 0], [0.5·l, 0], [0.5·l, d], [0, d] ), color = grey, linestyle
= dash) ) :
A1 := line( [0, 0], [ 1/2, d ], color = red, linestyle = dash ) :
B1 := line( [ 0, t_truss1/2 ], [ l/2 - w_truss1/2, d ], color = blue, legend = [40 kN] ) :
B2 := line( [ w_truss1/2, 0 ], [ l/2, d - t_truss1/2 ], color = blue ) :
BC := line( ([xc1, 0], [xc1, d-x], color = blue, linestyle = dash) ) :
C1 := line( [ 0, t_truss2/2 ], [ l/2 - w_truss2/2, d ], color = yellow, legend = [60 kN] ) :
C2 := line( [ w_truss2/2, 0 ], [ l/2, d - t_truss2/2 ], color = yellow ) :
CC := line( ([xc2, 0], [xc2, d-x], color = yellow, linestyle = dash) ) :
D1 := line( [ 0, t_truss3/2 ], [ l/2 - w_truss3/2, d ], color = red, legend = [100 kN] ) :
D2 := line( [ w_truss3/2, 0 ], [ l/2, d - t_truss3/2 ], color = red ) :
DC := line( ([xc3, 0], [xc3, d-x], color = red, linestyle = dash) ) :

y1 := t_truss1 :
x1 := w_truss1 :
display( {B1, B2, BC, C1, C2, CC, D1, D2, DC, BEAM} );

```



# Appendix E

## Principle stresses according Bernoulli

### Parameters

```
> h := 300 :
    w := 200 :
    L := 3000 :
    F := 200·1000 :
```

### Formula:

```
>  $\tau_{xy} := \frac{3}{4} \cdot \frac{F}{w \cdot h} \cdot \left( 1 - 4 \cdot \frac{z^2}{h^2} \right) :$ 
```

```
 $M_x := \left( 1 - \frac{\text{abs}(x)}{0.5 L} \right) \cdot 0.25 \cdot F \cdot L :$ 
```

```
 $I_{zz} := \frac{1}{12} \cdot w \cdot h^3 :$ 
```

```
 $\sigma_{xx} := - \frac{M_x \cdot z}{I_{zz}} :$ 
```

```
 $\tan(2 \cdot \theta) := \frac{\tau_{xy}}{\left( \frac{1}{2} \sigma_{xx} \right)} :$ 
```

```
 $\theta := - \frac{\arctan\left(\frac{\tau_{xy}}{\left(\frac{1}{2} \sigma_{xx}\right)}\right)}{2} :$ 
```

```
> z := 'z': x := 'x':
```

```
with(plots) :
```

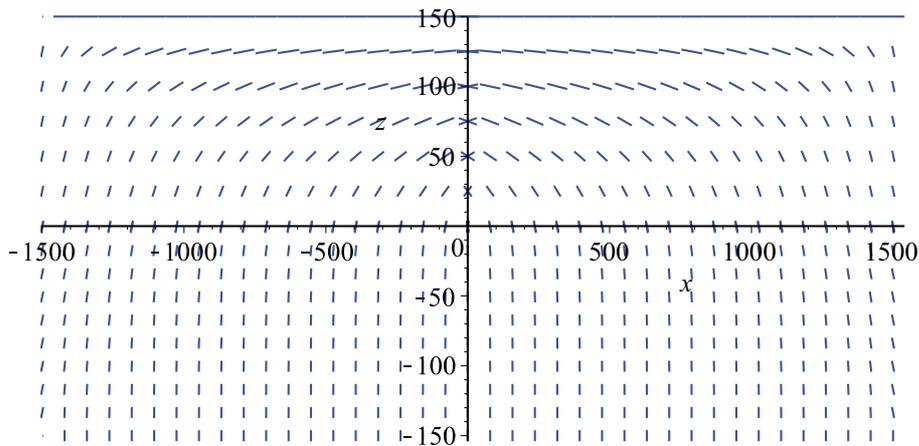
```
A := fieldplot( [ cos(θ) , sin(θ) ], x = -1500 .. 0, z = 0 .. 150, grid = [20, 7], arrows = line,
    color = blue) :
```

```
B := fieldplot( [ -cos(θ) , sin(θ) ], x = 0 .. 1500, z = 0 .. 150, grid = [20, 7], arrows = line,
    color = blue) :
```

```
C := fieldplot( [ cos(θ -  $\frac{90}{360} \cdot 2 \cdot 3.14$ ) , -sin(θ -  $\frac{90}{360} \cdot 2 \cdot 3.14$ ) ], x = 0 .. 1499, z = -150 ..
    -1, grid = [20, 10], arrows = LINE, color = blue) :
```

```
E := fieldplot( [ -cos(θ -  $\frac{90}{360} \cdot 2 \cdot 3.14$ ) , -sin(θ -  $\frac{90}{360} \cdot 2 \cdot 3.14$ ) ], x = -1500 .. 0, z =
    -150 .. -1, grid = [20, 10], arrows = line, color = blue) :
```

```
display( {A, B, C, E} );
```



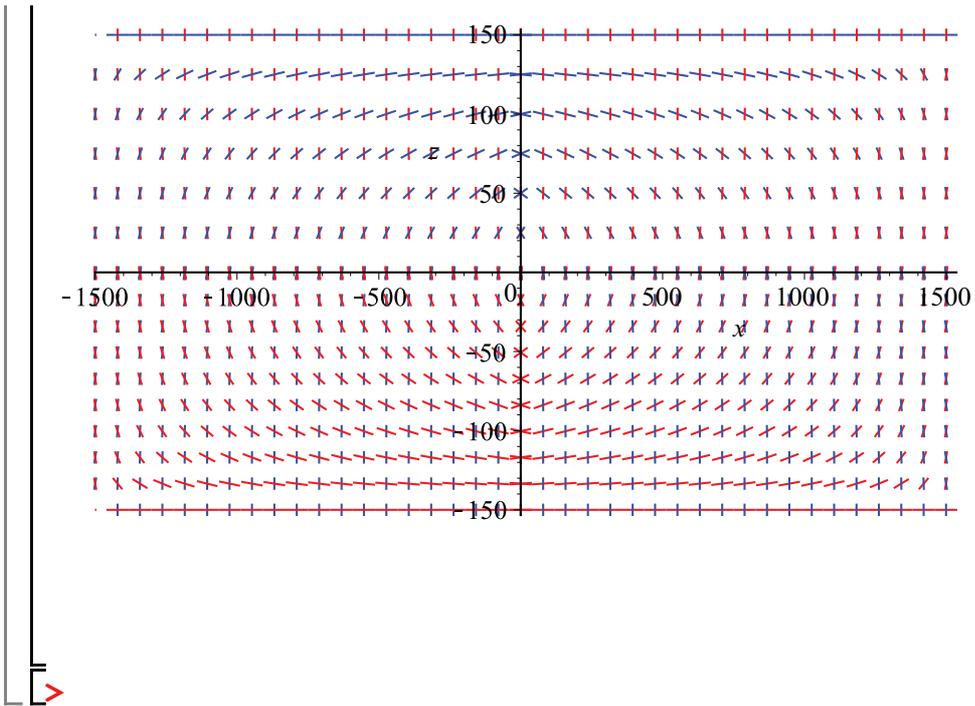
```
> F := fieldplot( [ cos(θ - 90/360 * 2 * 3.14), sin(θ - 90/360 * 2 * 3.14) ], x = -1500..0, z = 0..150,
  grid = [20, 7], arrows = line, color = red ) :
```

```
G := fieldplot( [ -cos(θ - 90/360 * 2 * 3.14), sin(θ - 90/360 * 2 * 3.14) ], x = 0..1500, z = 0
  ..150, grid = [20, 7], arrows = line, color = red ) :
```

```
H := fieldplot( [ cos(θ), -sin(θ) ], x = 0..1499, z = -150..-1, grid = [20, 10], arrows
  = LINE, color = red ) :
```

```
J := fieldplot( [ -cos(θ), -sin(θ) ], x = -1500..0, z = -150..-1, grid = [20, 10], arrows
  = line, color = red ) :
```

```
> display( {A, B, C, E, F, G, H, J} );
```



# Appendix F

Diana files

Truss.dat  
Reinf.dat  
Truss.dcf

Beam.dat  
Gauss.dat  
Beam.dcf

## BEAM.dat

FEMGEN MODEL : BEAM  
ANALYSIS TYPE : Structural 2D  
'UNITS'  
LENGTH MM  
TIME SEC  
TEMPER CELSIU  
FORCE N  
'COORDINATES' DI=2  
1 -1.666667E+01 0.000000E+00  
2 -1.666670E+01 -1.000000E+01  
3 0.000000E+00 0.000000E+00  
4 0.000000E+00 -1.000000E+01

....

4546 6.500000E+02 4.772727E+01  
4547 6.500000E+02 3.409091E+01  
4548 6.500000E+02 2.045455E+01  
4549 6.500000E+02 6.818182E+00

'ELEMENTS'  
CONNECTIVITY  
1 CQ16M 1 7 2 9 4 10 3 8  
2 CQ16M 3 10 4 12 6 13 5 11  
3 CQ16M 14 20 15 22 17 23 16 21  
4 CQ16M 16 23 17 25 19 26 18 24

....

1452 CQ16M 1534 4501 1535 4523 1557 4545 1556 4522  
1453 CQ16M 1535 4502 1536 4524 1558 4546 1557 4523  
1454 CQ16M 1536 4503 1537 4525 1559 4547 1558 4524  
1455 CQ16M 1537 4504 1538 4526 1560 4548 1559 4525  
1456 CQ16M 1538 4505 1539 4527 1561 4549 1560 4526

MATERIALS  
/5-1456 / 1  
/1-4 / 3  
GEOMETRY  
/5-1456 / 1  
/1-4 / 3

'REINFORCEMENTS'  
LOCATI  
5 BAR  
LINE -3.500000E+02 3.500000E+01 0.000000E+00  
6.500000E+02 3.500000E+01 0.000000E+00

MATERIALS  
/5 / 2  
GEOMETRY  
/5 / 2

'MATERIALS'  
1 YOUNG 3.762000E+04  
POISON 1.500000E-01  
TOTCRK FIXED  
TENCRV HORDYK  
TENSTR 4.100000E+00  
GF1 9.000000E-02  
COMCRV CONSTA  
COMSTR 5.000000E+01  
SHRCRV DAMAGE  
POIRED DAMAGE  
2 YOUNG 2.000000E+05

```
YIELD VMISES
YLDVAL 5.000000E+02
3 YOUNG 2.000000E+05
POISON 1.500000E-01
YIELD VMISES
YLDVAL 5.000000E+02
'GEOMETRY'
1 THICK 2.000000E+02
2 CROSSE 4.710000E+02
3 THICK 2.000000E+02
'GROUPS'
ELEMEN
1 CONCR / 5-1456 /
NODES
2 CONCR_N / 1 3 5 8 11 14 16 18 21 24 27-4549 /
REINFO
3 RS1 / 5 /
'SUPPORTS'
/ 14 15 20 1540-1561 4528-4549 / TR 1
/ 4 15 / TR 2
'LOADS'
CASE 1
DEFORM
15 TR 2 -0.100000E+01
'DIRECTIONS'
1 1.000000E+00 0.000000E+00 0.000000E+00
2 0.000000E+00 1.000000E+00 0.000000E+00
3 0.000000E+00 0.000000E+00 1.000000E+00
'END'
```

## GAUSS.dat

```
'ELEMENTS'  
DATA  
/ CONCR / 1  
'REINFO'  
DATA  
/ 5 / 2  
'Data'  
  1 NINTEG 3 3  
  2 NINTEG 3  
'END'
```

## BEAM.dcf

```
*FILOS
INITIA
*INPUT
READ FILE "beam.dat"
*INPUT
BEGIN READ
  APPEND
  FILE "GAUS.dat"
END READ
*NONLIN
BEGIN EXECUT
  begin iterat
    METHOD Newton modifi
    linese
    maxite 20
    begin conver
      displa off
      energy newref contin tolcon=3e-4 tolabt=1e4
      force off
    end conver
  end iterat
BEGIN LOAD
  LOADNR 1
  STEPS EXPLIC SIZES 0.025(200)
END LOAD
END EXECUT
BEGIN OUTPUT
  FILE "BEAM2"
  DISPLA TOTAL TRANSL GLOBAL
  FORCE REACTI TRANSL GLOBAL
  STRAIN CRACK GREEN
  STRAIN TOTAL GREEN GLOBAL INTPNT
  STRAIN TOTAL GREEN PRINCI INTPNT
  STRESS CRACK CAUCHY LOCAL
  STRESS TOTAL CAUCHY GLOBAL
  STRESS TOTAL CAUCHY PRINCI
  STRESS TOTAL CAUCHY VONMIS INTPNT
END OUTPUT
*END
```

## Truss.dat

FEMGEN MODEL : TRUSS  
ANALYSIS TYPE : Structural 2D

'UNITS'

LENGTH MM

TIME SEC

TEMPER CELSIU

FORCE N

'COORDINATES' DI=2

1	6.233333E+02	3.000000E+02
2	2.500000E+02	3.500000E+01
3	6.500000E+02	3.500000E+01
4	6.500000E+02	2.000000E+02
5	6.500000E+02	2.058824E+02
6	6.500000E+02	2.117647E+02

...

4094	0.000000E+00	0.000000E+00
4095	0.000000E+00	-1.000000E+01
4096	1.500000E+01	-1.000000E+01
4097	-1.500000E+01	-5.000000E+00
4098	-7.500000E+00	0.000000E+00
4099	-7.500000E+00	-1.000000E+01
4100	0.000000E+00	-5.000000E+00
4101	7.500000E+00	0.000000E+00
4102	7.500000E+00	-1.000000E+01
4103	1.500000E+01	-5.000000E+00

'ELEMENTS'

CONNECTIVITY

1 L2TRU 2 3  
2 CQ16M 4 561 5 579 23 596 22 578  
3 CQ16M 5 562 6 580 24 597 23 579  
4 CQ16M 6 563 7 581 25 598 24 580  
5 CQ16M 7 564 8 582 26 599 25 581  
6 CQ16M 8 565 9 583 27 600 26 582  
7 CQ16M 9 566 10 584 28 601 27 583

....

1303 CQ16M 3688 4017 3689 4054 3726 4090 3725 4053  
1304 CQ16M 3689 4018 3690 4055 3727 4091 3726 4054  
1305 CQ16M 3690 4019 3691 4056 3728 4092 3727 4055  
1306 CQ16M 3708 4097 4093 4099 4095 4100 4094 4098  
1307 CQ16M 4094 4100 4095 4102 4096 4103 3706 4101

MATERIALS

/ 2-511 514-1305 / 1

/ 1 / 5

/ 512 513 1306 1307 / 6

GEOMETRY

/ 2-511 514-1305 / 1

/ 1 / 7

/ 512 513 1306 1307 / 8

'MATERIALS'

1 YOUNG 3.762000E+04  
POISON 1.500000E-01  
TOTCRK FIXED  
TENCRV HORDYK  
TENSTR 4.100000E+00  
GF1 9.000000E-02  
COMCRV CONSTA  
COMSTR 5.000000E+01  
SHRCRV DAMAGE

```

POIRED DAMAGE
4 YOUNG  2.000000E+05
YIELD VMISES
YLDVAL  5.000000E+02
5 YOUNG  2.000000E+05
POISON  1.500000E-01
YIELD VMISES
YLDVAL  5.000000E+02
6 YOUNG  2.000000E+05
POISON  1.500000E-01
YIELD VMISES
YLDVAL  5.000000E+02
'GEOMETRY'
1 THICK  2.000000E+02
6 CROSSE 4.710000E+02
7 CROSSE 4.710000E+02
8 THICK  2.000000E+02
9 CROSSE 1.000000E+02
'GROUPS'
ELEMEN
1 BETON / 2-511 514-1305 /
NODES
2 BETON_N / 1 2 4-1627 1636-4092 4094 /
'SUPPORTS'
/ 3-21 561-577 1630 1633 / TR 1
/ 1630 4095 / TR 2
'LOADS'
CASE 1
DEFORM
1630 TR 2 -0.100000E+01
'DIRECTIONS'
1 1.000000E+00 0.000000E+00 0.000000E+00
2 0.000000E+00 1.000000E+00 0.000000E+00
3 0.000000E+00 0.000000E+00 1.000000E+00
'END'

```

## Reinf.dat

```
'ELEMENTS'  
DATA  
/ beton / 1  
'REINFORCEMENTS'  
LOCATI  
11 BAR  
LINE -3.510000E+02 3.500000E+01 0.000000E+00  
2.510000E+02 3.500000E+01 0.000000E+00  
elemen BETON /  
12 BAR  
LINE 0.000000E+00 5.000000E+01 0.000000E+00  
0.000000E+00 2.000000E+01 0.000000E+00  
elemen BETON /  
MATERIALS  
/ 11-12 / 4  
GEOMETRY  
/ 11 / 6  
/ 12 / 9  
DATA  
/ 11-12 / 2  
'Data'  
1 NINTEG 3 3  
2 NINTEG 3  
'END'
```

## truss.dcf

```
*FILOS
INITIA
*INPUT
READ FILE "truss.dat"
*INPUT
BEGIN READ
  APPEND
  FILE "reinf.dat"
END READ
*NONLIN
BEGIN EXECUT
  begin iterat
    METHOD Newton modifi
    linese
    maxite 20
    begin conver
      displa off
      energy newref contin tolcon=3e-4 tolabt=1e4
      force off
    end conver
  end iterat
BEGIN LOAD
  LOADNR 1
  STEPS EXPLIC SIZES 0.05(150)
END LOAD
END EXECUT
BEGIN OUTPUT
  FILE "truss"
  DISPLA TOTAL TRANSL GLOBAL
  FORCE REACTI TRANSL GLOBAL
  STRAIN CRACK GREEN
  STRAIN TOTAL GREEN GLOBAL INTPNT
  STRAIN TOTAL GREEN PRINCI INTPNT
  STRESS CRACK CAUCHY LOCAL
  STRESS TOTAL CAUCHY GLOBAL
  STRESS TOTAL CAUCHY PRINCI
  STRESS TOTAL FORCE LOCAL
  STRESS TOTAL CAUCHY VONMIS INTPNT
  STRESS TOTAL FORCE
END OUTPUT
*END
```

# Appendix G

### 1.1 Calculation method

With DIANA several numerical methods are available for the FEM analysis. The newton regular and newton modified methods are compared.

The modified newton raphson method gave the best result. The modified newton force limit is lower than the regular newton method and closer to the result from Beeby.

Both the models fail in shear. Markedly is the difference after shear failure. The crack pattern differs, which gives the modified newton a failure before the truss really evolves, while the regular newton fails like a truss mechanism after yielding of the steel. The other form of the crack gives the truss the ability to act as a truss afterwards.

The beam from the Beeby experiment fails in total after shear failure, no new limit is reached. That is why the Modified newton method is chosen as basis calculation method.

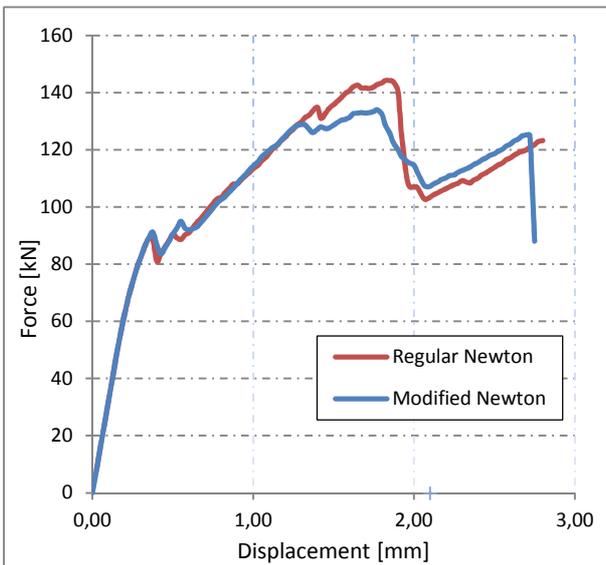
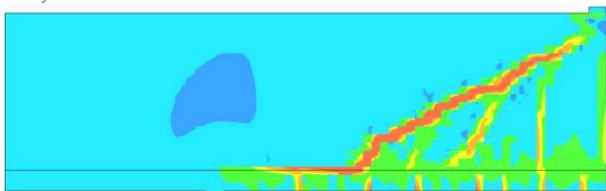


Figure 1 Compare iteration method



Modified Newton



Regular Newton

Figure 2 Different crack patterns

### 1.2 Fixed crack model

The study uses the total strain fixed crack model. This model uses a fixed smeared crack model. The smeared principle smears the crack strain out over an element instead of one crack in the element. With the smeared crack model it is possible to model the growing of the crack. In a discrete crack the crack arises suddenly in the whole element. The smeared crack model is based on the strains. This is explained in Figure 3.

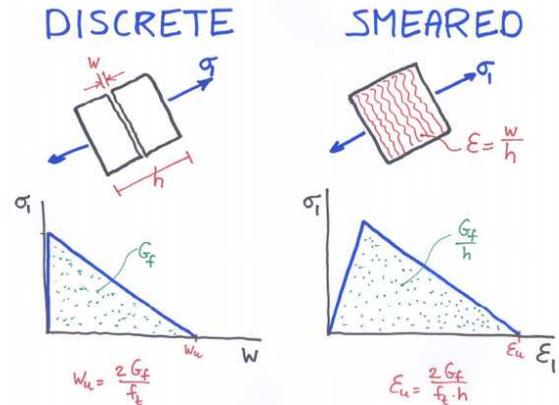


Figure 3 Discrete and smeared cracking

The fixed part of the models is the fact that the direction of the cracks is fixed in one element. This is the difference between the rotating and fixed crack model. In the fixed crack model the crack arises perpendicular to the principle tensile stress direction. The cracks remain in this direction if the principle stress direction changes. This is the difference with the rotating crack model, where the cracks rotate in the element if the principle stress direction changes. The fixed crack model is used because it gives the best approach for flexural shear crack. [2]

**Convergence norm**

In Diana the converging norm of the numeric calculation is limited on force, energy or displacement based. To select the convergence norm a comparison is made of the results. In Figure 4 the force displacement diagrams of different convergence limits are shown.

The displacement limits shows a realistic curve, in which the beam fails with a shear crack and finds it equilibrium rapidly after the beams fails. But this is logical, because the force is displacement based, that is why this is not a proper convergence limit.

In force limit shows a vibrating result. It has a form which is close to the displacement limit. After failing the force limit finds reasonably quick the new convergence limit.

The energy limit shows a smooth behaviour of the force displacement diagram. The convergence after failure needs some time to find the new equilibrium.

The energy limit gives the smoothest results and fits to the result. This is used for the calculations.

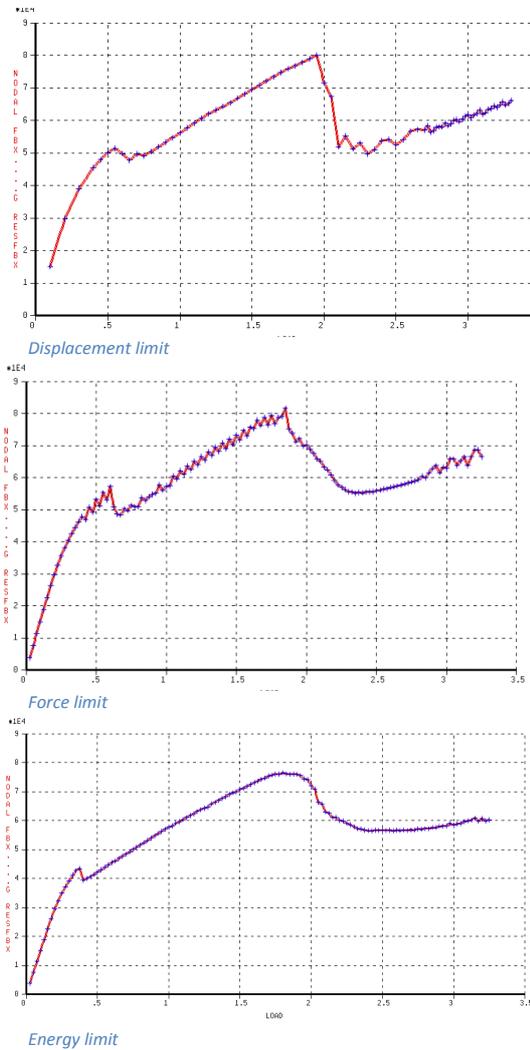


Figure 4 Converge limit

**1.3 Variation of fracture energy**

The Hordijk curve, Figure 5, is used for modelling the tensile curve. The fracture energy is one of the parameters which determine the area under the graph together with the crack bandwidth, the energy which is needed to create a crack.

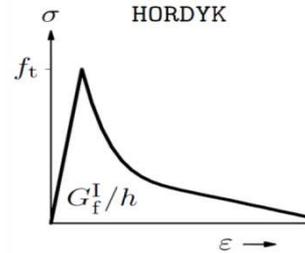


Figure 5 Hordijk Curve

With the "Guidelines for non-linear finite element analysis of concrete structures" [1] an approximation of the energy is made which depends on the maximum aggregate size. This aggregate size is not known from Beeby experiment.

$$G_F = G_{F0} * \left(\frac{f_{cm}}{f_{cm0}}\right)^{0.7}$$

With:

$$G_{F0} = 0.025 \frac{Nmm}{mm^2} \text{ for } 8mm$$

$$G_{F0} = 0.030 \frac{Nmm}{mm^2} \text{ for } 16mm$$

$$G_{F0} = 0.058 \frac{Nmm}{mm^2} \text{ for } 32mm$$

This gives:

$$G_{f,8} = 0.077 \frac{N}{mm}$$

$$G_{f,16} = 0.093 \frac{N}{mm}$$

$$G_{f,32} = 0.179 \frac{N}{mm}$$

Different values of  $G_f$  are modelled, but a small range gives the wanted result.

The range between  $0.07 \frac{N}{mm}$  and  $0.09 \frac{N}{mm}$  for  $G_f$  gives a shear failure for the beam model and a result which is close to the failings loads of the Beeby experiment. The 3 values are compared to select the one which is used for the basic comparison. The values for which the

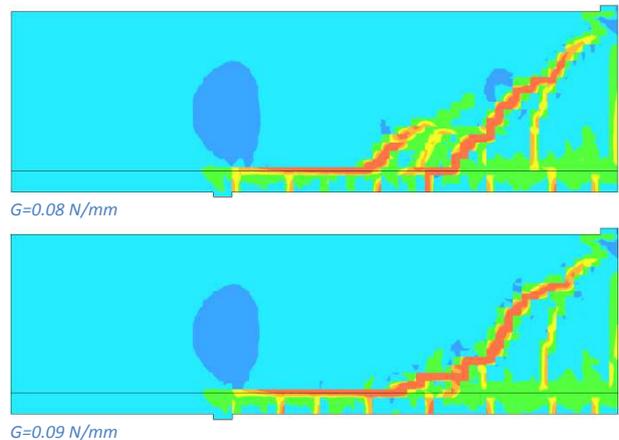


Figure 6 Difference shear crack

force-displacement diagrams are shown in Figure 8 and Figure 7.

The value of  $G = 0,07 \frac{N}{mm}$  shows that the beam model will immediately fail when a shear crack arises. In the truss model the beam had the lowest value, which is the most away to the values of the Beeby experiment.

The value of  $G = 0,08 \frac{N}{mm}$  and  $G = 0,09 \frac{N}{mm}$  give both a proper force-displacement diagram and failure load. The crack pattern difference is shown in Figure 6. The value of  $G = 0,09 \frac{N}{mm}$  gives the most realistic crack pattern. So this value will be used as basic parameter.

As described in this paragraph it is clear that the fracture energy has a lot of influence on the crack pattern and failure mode of the beam mechanism, but not a lot of influence on the truss mechanism. With variation of the dimensions it is recommended to use some different values of the fracture energy to see the influence.

## 1.4 References

1. Hendriks, Dr. ir. M.A.N., *Guidelines for non-linear finite element analysis of concrete structures (Scope: Girder Structures)*. 2011, Delft University of Technology.
2. A.T. Slobbe, M.A.N.Hendriks., J.G. Rots, *Sequentially linear analysis of shear critical reinforced concrete beams without shear reinforcement*, 2011, Elsevier (50) 2012

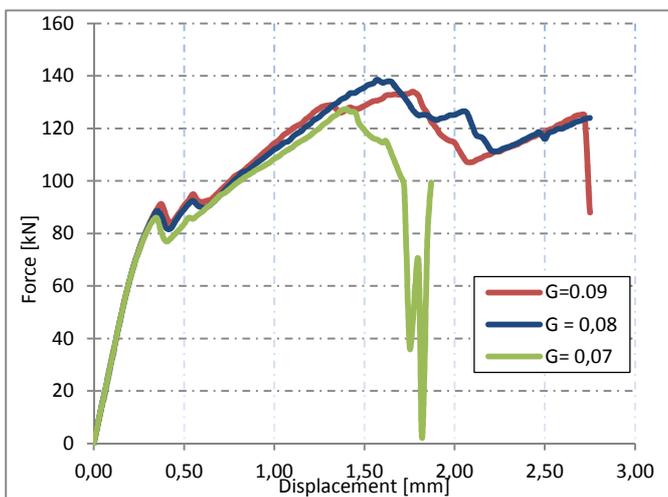


Figure 8 Variation G beam

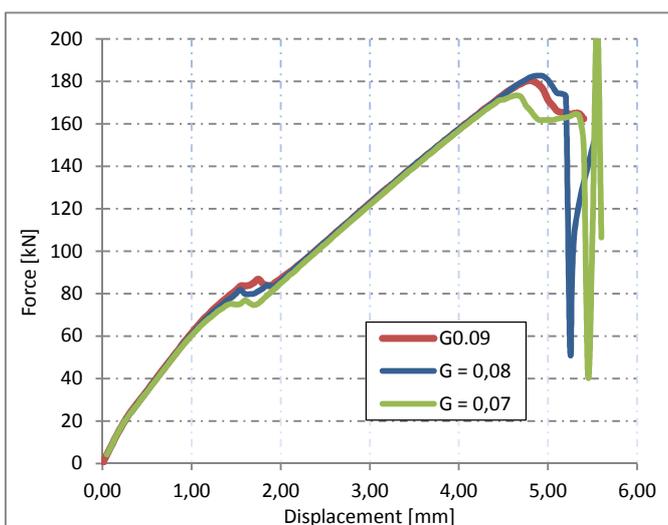


Figure 7 Variation G Truss

# Appendix H

.CDF file for distributed load

```
*FILOS
INITIA
*INPUT
*INPUT
BEGIN READ
  APPEND
  FILE "reinf.dat"
END READ
*NONLIN
BEGIN EXECUT
  begin iterat
  METHOD Newton modifi
  linese
  maxite 20
  begin conver
    displa newref contin tolcon=3e-2 tolabt=1e2
    energy off
    force off
  end conver
  end iterat
BEGIN LOAD
  LOADNR 1
  begin STEPS
  begin automa
    size=1000
    minsiz=1e-5
    maxsiz=0.005
    arclen update
  end automa
  end steps
END LOAD
END EXECUT
BEGIN OUTPUT
  DISPLA TOTAL TRANSL GLOBAL
  FORCE REACTI TRANSL GLOBAL
  STRAIN CRACK GREEN
  STRAIN TOTAL GREEN GLOBAL INTPNT
  STRAIN TOTAL GREEN PRINCI INTPNT
  STRESS CRACK CAUCHY LOCAL
  STRESS TOTAL CAUCHY GLOBAL INTPNT
  STRESS TOTAL CAUCHY PRINCI
  STRESS TOTAL CAUCHY VONMIS INTPNT
END OUTPUT
BEGIN OUTPUT
  TABULA
  DISPLA TOTAL TRANSL GLOBAL COORDI
  STRESS TOTAL CAUCHY GLOBAL INTPNT
COORDI
  FORCE REACTI TRANSL GLOBAL COORDI
END OUTPUT
*END
```

P

