Spin polarization of two-dimensional electrons determined from Shubnikov-de Haas oscillations as a function of angle

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Recent experiments in the two-dimensional electron systems in silicon metal-oxide-semiconductor field effect transistors have shown that the in-plane magnetic field H_{sat} required to saturate the conductivity to its high-field value, and the magnetic field H_s needed to completely align the spins of the electrons, are comparable. By small-angle Shubnikov-de Haas oscillation measurements that allow separate determinations of the

spin-up and spin-down subband populations, we show to an accuracy 5% that $H_{sat} = H_s$.

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Dilute two-dimensional electron (or hole) systems, which display unexpected metallic behavior, are currently the subject of great interest.¹ These strongly interacting systems also exhibit enormous magnetoresistances in response to a magnetic field applied parallel to the electron plane:^{2–6} with increasing field, the resistivity rises sharply and saturates to a constant value for $H > H_{sat}$; the saturation field H_{sat} is of the order of several tesla and varies with temperature and electron density.

Two experiments have recently shown that the behavior of the resistance is related to the spin polarization of the electrons. On the assumption that the product gm^* of the interaction-enhanced g factor g and effective mass m^* measured in a small field does not change in a strong in-plane magnetic field, Okamoto et al.⁷ found that the saturation field H_{sat} is close to the field at which the two-dimensional (2D) electron system in silicon metal-oxide-semiconductor field effect transistors (MOSFET's) becomes fully spin-polarized. Similar conclusions were drawn by Vitkalov et al.,⁸ who demonstrated that the frequency of small-angle Shubnikov–de Haas (SdH) oscillations versus filling factor ν in high fields $H > H_{sat}$, is double the frequency at low fields $H < H_{sat}$. This signals a decrease of the density of states by a factor of 2 and the complete depopulation of one of the spin subbands when $H > H_{sat}$. The equivalence between the field H_{sat} at which the resistance saturates and the field H_s required to obtain complete alignment of the electron spins, was established in these experiments to an accuracy of $\approx 10\%$ to 15%. By using a modification of the small-angle Shubnikov-de Haas method of measurement,⁸ which allows a determination of the population of each spin subband separately, we show in the present paper that $H_{sat} = H_s$ with an accuracy of $\approx 5\%$.⁹

The method is based on the following considerations. The populations of the spin-up and spin-down subbands are governed by the Zeeman energy, which is determined by the total magnetic field. The splitting of the Landau levels is controlled by the normal component of the magnetic field H_{\perp} : $\hbar \omega_c = e H_{\perp} / mc$. By rotating the 2D electron plane relative to the direction of the magnetic field, we change the

normal component of the magnetic field and, therefore, the Landau-level spitting in each band. For a fixed total magnetic field H, the sizes of the Fermi circles $k_F^{\uparrow,\downarrow}$, formed by the spin-up and spin-down electrons, are different and constant. The frequency of the SdH oscillations with $1/H_{\perp}$ is proportional to the area of the Fermi circle and therefore the density of spin-up (or spin-down) 2D carriers Thus, the ratio of the frequencies of the SdH oscillations, due to the spin-up and spin-down Fermi circles, yields the ratio of populations of the spin subbands for a given value of the in-plane magnetic field. Analyzing the data obtained by this method, we show that the saturation field H_{sat} is the same as the field required for complete spin polarization with an accuracy of 5%.

Measurements were taken on a silicon MOSFET; the mobility μ at 0.1 K was 26000 V/(cm² s). Contact resistances were minimized by using a split-gate geometry, which allows a higher electron density in the vicinity of the contacts than in the 2D system under investigation. Standard ac four-probe techniques were used to measure the resistance with ac currents in the linear regime, typically below 5 nA, at a frequency of 3 Hz. Data in high magnetic fields up to 20 T, were obtained at the National Magnetic Field Laboratory in Tallahassee, Florida. The sample was mounted at the end of a low-temperature probe on a rotating platform. The sample was rotated in constant magnetic field by a stepper motor. The electron density n_s was fixed during the sample rotation and measurements were taken at a temperature of about 100 mK. The longitudinal and the Hall voltages were detected simultaneously.

For small angles ϕ between the electron plane and the magnetic-field direction, the Hall resistance is proportional to the normal component of the magnetic field H_{\perp} : $R_{xy} = H_{\perp} / (n_s e)$, with $H_{\perp} = H \sin \phi$. The Hall coefficient does not depend on the degree of the spin polarization of the 2D electrons in silicon MOSFET's. This was demonstrated in recent experiments¹⁰ with an accuracy $\approx 5\%$ for electron densities below $n_s = 2.75 \times 10^{11}$ cm⁻². We note that the proportionality of R_{xy} with H_{\perp} is sufficient to guarantee the correct *ratio* between the two periods of the SdH oscilla-



FIG. 1. (a) The small-angle Shubnikov-de Haas oscillations of the two-dimensional system of electrons in a silicon MOSFET at density 3.72×10^{11} cm⁻² in the presence of a magnetic field of 18 T, which is close to the saturation field H_{sat} for this density. (b) The Fourier transform of the data shown in (a); the inset is a schematic band diagram, corresponding to the complete spin polarization at $H=H_s$.

tions, regardless of whether the constant of proportionality varies with in-plane magnetic field.

The angle between the magnetic field and the 2D plane was also determined from the known gearing number of the stepper motor. Both methods give the same ratio of frequencies with an accuracy of about 3%. The filling factor was calculated using the relation $\nu = n_s \Phi_0 / H_{\perp}$.

In a constant total magnetic field of 18 T, the longitudinal resistivity is shown in Fig. 1(a) plotted as a function of filling factor ν for electron density $n_s = 3.72 \times 10^{11}$ cm⁻². Clean periodic Shubnikov-de Haas oscillations are observed with a period $\Delta \nu = 2$. Taking into account the twofold valley degeneracy of 2D electrons in silicon MOSFETs,¹¹ the period $\Delta \nu$ =2 is found to correspond to quantum oscillations in a single spin band. In other words, at H=18 T and $n_s=3.72$ $\times 10^{11}$ cm⁻², the electrons are spin polarized completely. The sharp dip at $\nu = 3$ corresponds to valley spitting. The Fourier spectrum of these oscillations is shown in Fig. 1(b). The main peak in the spectrum corresponds to the period $\Delta \nu = 2$, which is clearly dominant in Fig. 1(a). The two smaller maxima at higher frequencies are second and third harmonics. Valley splitting, oscillations of the Fermi energy, and various other effects may be responsible for these



FIG. 2. (a) The small-angle Shubnikov-de Haas oscillations of the two-dimensional system of electrons in a silicon MOSFET at density 9.28×10^{11} cm⁻² in the presence of a magnetic field of 18 T. (b) The Fourier transform of the data shown in part (a); the inset is a schematic band diagram, corresponding to partial spin polarization of the 2D electrons at $H < H_s$.

higher-order peaks. The schematic band diagram in Fig. 1(b) shows the population of the spin-up and spin-down bands for full spin polarization and $H=H_s$, the condition that obtains at the density shown. A similar pattern of oscillations with period $\Delta \nu = 2$ is observed for a lower electron density $n_s = 2.75 \times 10^{11} \text{ cm}^{-2}$.

Shubnikov-de Haas oscillations are shown in Fig. 2(a) for the same magnetic field H=18 T for a substantially higher density $n_s = 9.28 \times 10^{11}$ cm⁻². In contrast with Fig. 1, the oscillations do not have a single period. Instead they demonstrate a beating pattern. The main frequencies that create the beats are easily identified in Fig. 2(b), which shows the Fourier components of the oscillations. The schematic band diagram in Fig. 2(b) shows the population of the spin-up and spin-down bands corresponding to the high-density case, where $H < H_{sat}$ and the electrons are partially polarized. The two main peaks of Fig. 2(b) are at frequencies 0.167 and 0.330, corresponding to quantum oscillations in the spindown and spin-up subbands. They are proportional to the Fermi energies (and therefore the populations) of the spindown and spin-up bands. The Fourier spectrum contains several additional peaks associated with obvious and strong nonlinearity of the SdH spectrum. A complete interpretation of the different peaks will require detailed analysis.



FIG. 3. (a) The ratio $f^{\downarrow}/f^{\uparrow}$ vs the electron density n_s in a magnetic field H=18 T, applied at a small angle relative to the plane of a silicon MOSFET; the line is drawn to guide the eye. (b) Open circles denote the field H_{sat} at which the conductivity saturates to its high-field value, as shown in the inset. The closed circles are the fields H_s required to achieve the full spin polarization of the electrons, and calculated using Eq. (3). The solid line is a linear fit to the data for H_{sat} .

The ratio of the frequencies of the oscillations in the two spin subbands is proportional to the ratio of the densities of spin-down and spin-up electrons: $f^{\downarrow}/f^{\uparrow} = n_s^{\downarrow}/n_s^{\uparrow}$. The ratio $f^{\downarrow}/f^{\uparrow}$ determined from our experiments is plotted in Fig. 3(a) for different total electron densities n_s . At high electron density $n_s = 9.28 \times 10^{11}$ cm⁻², the 2D electron system is partially spin-polarized at H = 18 T and the ratio $f^{\downarrow}/f^{\uparrow}$ is ≈ 0.5 . Decreasing the total electron density n_s reduces both the spin-up and the spin-down populations, yielding a smaller $f^{\downarrow}/f^{\uparrow}$. Finally, for $n_s = 3.72 \times 10^{11}$ cm⁻² and below, the spin-down subband is depopulated completely at 18 T, and the 2D system is fully spin polarized. The ratio $f^{\downarrow}/f^{\uparrow}$ is equal to zero for $n_s < n_{sat}(18 \text{ T}) = 3.72 \times 10^{11} \text{ cm}^{-2}$.

The magnetic field H_s required to achieve full spin polarization of the electron system can be calculated from the following simple considerations. The Fermi energy of the spin-up and spin-down electrons, measured relative to the bottom of each subband, is

$$\boldsymbol{\epsilon}^{\uparrow,\downarrow} = \boldsymbol{\epsilon}_F^0 \pm \mu g H/2 = n/(2D) \pm \mu g H/2, \tag{1}$$

where $\epsilon_F^0 = n/(2D)$ is the Fermi energy of each band when H=0, and the density-of-states *D* of the electrons in two dimensions is constant.¹¹ The ratio

$$\frac{f^{\downarrow}}{f^{\uparrow}} = \frac{n^{\downarrow}}{n^{\uparrow}} = \frac{\epsilon^{\downarrow}D}{\epsilon^{\uparrow}D} = \frac{1 - (g\,\mu HD)/n}{1 + (g\,\mu HD)/n}.$$
(2)

The condition for full spin polarization in a magnetic field H_s is that $n^{\downarrow}=0$, thus $\epsilon^{\downarrow}=0=n/(2D)-\mu g H_s/2$, and $n/D = \mu g H_s$. Substitution¹² into Eq. (2) yields

$$\frac{f^{\downarrow}}{f^{\uparrow}} = \frac{(1 - H/H_s)}{(1 + H/H_s)},$$

an expression that is valid when $H < H_s$ (H = 18 T in our experiments). The field H_s can thus be calculated from the expression

$$H_s = \frac{(1+f^{\downarrow}/f^{\uparrow})}{(1-f^{\downarrow}/f^{\uparrow})}H.$$
(3)

The solid circles shown in Fig. 3(b) denote H_s calculated for the data of Fig. 3(a), using Eq. (3). Values of H_{sat} , denoted by the open circles, are obtained from the saturation of the in-plane magnetoconductivity, as illustrated in the inset to Fig. 3(b). The line is a fit to the data for H_{sat} . In the narrow range where the two data sets overlap, $H_{sat} \approx H_s$ within about 5%.

In summary, small-angle Shubnikov–de Haas measurements, taken at a fixed total magnetic field of 18 T, as a function of the angle θ between the magnetic-field direction and the electron plane, have allowed a separate determination of the populations of the spin-down and spin-up subbands. Analysis of the data yields the field H_s required to achieve the complete spin polarization of the electrons. We find that H_s is the same as the field H_{sat} , which signals the saturation of the conductivity with an accuracy of 5% at n $> 3 \times 10^{11}$ cm⁻².

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- 12 Note that this step entails the assumption that the g value is independent of magnetic field.