

# Towards an Accurate Stress Dependant Time & Frequency Domain VE Response Model for Bituminous Binders

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**ABSTRACT:** Linear viscoelastic properties of bituminous binders for short loading times are analyzed using dynamic mechanical analysis methods. Dynamic Shear Rheometer (DSR) test with parallel plate (PP) configuration is widely used for this purpose. Due to the complex stress distribution over the cross-section of the test sample, application of this setup for nonlinear material characterization, however, has been limited. Cone-and-Plate (CP) setup on the other hand provides a relatively uniform shear stress distribution. Due to this geometrical advantage a new CP configuration were developed and used to conduct constant-stress response measurements. Application of the setup for response measurement was first verified by comparing the CP and PP results obtained at low stress levels. The effects of the non-uniform stress distribution on fatigue results were also evaluated. Using material properties obtained from dynamic tests, simulation of time-domain relaxation tests with short loading times were in good agreement with experimentally obtained relaxation test results at low stress levels. Binder response at higher stress levels exhibits a nonlinear, stress dependent, behavior.

**KEY WORDS:** DSR, cone and plate, fatigue, viscoelastic, dissipated energy.

## 1 INTRODUCTION

Depending on the applied stress and strain levels, bituminous binders posses both linear and nonlinear viscoelastic behavior. At small loads, binders generally behave linear viscoelastic whereas at high loads they exhibit a nonlinear viscoelastic response. Insights obtained from asphalt-micromechanics study showed the existence of a wide range of stresses and strains in the binder domain with in asphalt mixtures (Kose et al. 2000, Masad et al. 2001, Huurman et al. 2009). The range of stresses in which nonlinear viscoelastic behavior is evident for various binders has been reported elsewhere (Cheung and Cebon, 1997; Airey et al. 2002).

Material response for short loading times (typically 0.1 to 1 second) is usually obtained from dynamic tests. Using frequency-time domain inter-conversion, the corresponding time domain material functions can then be obtained (Schapery, 1999). Commonly a Dynamic Shear Rheometer (DSR) test setup with parallel plate geometries is used to conduct tests in frequency domain. The strain/stress levels used in this setup have to be kept very low so that the linear viscoelastic response of the material is obtained. For nonlinear viscoelastic response characterization, however, the use of DSR has its own limitation. This is due to the difficulties in interpreting the associated non-uniform shear stress distribution effect on the response of the material in the parallel plate configuration.

In this article, a newly introduced Cone-and-Plate (CP) setup has been used to characterize

the binder in the linear and nonlinear viscoelastic ranges. First, frequency domain response measurements in the linear viscoelastic range were conducted using the CP and the PP geometries. The results were compared. The effects of the complex shear distribution on the fatigue life were evaluated by conducting fatigue tests using the two geometries. For low stress response measurements, time-frequency domain relationships were verified by using frequency domain response data to simulate time-domain relaxation data. Response results showing stress dependent behavior at high stress levels were also obtained.

## 2 MATERIAL AND EXPERIMENTS

### 2.1 Materials

The following three different materials were tested in the laboratory.

- 40-60 penetration grade bitumen
- 70-100 penetration grade bitumen
- Bituminous mastic with filler binder ratio of 1. The mastic was made from a Bitumen grade penetration 70/100 and a Wigro 60K limestone filler with 25% chalk hydrate.

### 2.2 Dynamic Shear Rheometer (DSR)

For all the binders, DSR frequency sweep tests in the linear viscoelastic range were conducted at various temperatures. The 8 mm diameter and 25 mm diameter PP geometries have been used for lower and higher temperatures respectively. For the newly introduced CP setup, 8 mm and 25 mm diameter geometries with a cone angle ( $\alpha$ ) of  $26.6^\circ$  and  $2.29^\circ$  respectively were used. As recommended in the original cone and plate geometry setup for viscosity measurement, the tips of the cones were trimmed to reduce test result variability. Based on the maximum expected size of the particles in the binder, 63 microns in the filler, the tip of the cone were trimmed to obtain a gap of 65 microns. Figure 1 illustrates the PP and CP configurations.

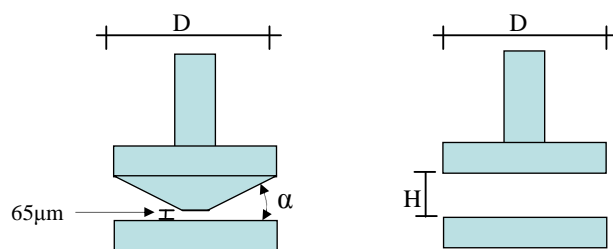


Figure1: CP geometry (left) and PP geometry (right).

### 2.3 Time Temperature Superposition Principle

For response data in the linear viscoelastic range, description of the binder response for a wide range of frequencies and temperatures is usually made using a Time-Temperature superposition principle. Using this principle shifting with respect to frequency can be applied to the complex modulus and phase angle until the curves merge into a single smooth function resulting in a master curve. The shift factor at each temperature required to form the master curve describes the temperature dependency of the material. In this article, the

Williams-Landel-Ferry (WLF) equation is used for obtaining the shift factors (Ferry, 1980):

$$\text{Log } a_T = \frac{-C_1(T - T_0)}{C_2 + (T - T_0)} \quad (1)$$

Where:

C1 and C2= constants,

T= temperature [<sup>0</sup>C],

T<sub>0</sub>= reference temperature [<sup>0</sup>C],

a<sub>T</sub>= shift factor

#### 2.4 Frequency-Time domain inter-conversion

Dynamic mechanical properties of a linear viscoelastic material are obtained from laboratory tests with a steady-state sinusoidal excitation. For numerical applications, the time domain material functions need to be obtained from the measured dynamic response data. Numerous analytical expressions are available for frequency-time domain inter-conversion (Ferry 1980; Tschoegl 1989; Schapery 1999). The inter-conversion method using a Prony series model is widely applicable. Even though the model involve a number of model parameters, its computational efficiency for numerical applications is remarkable. The model expression for the creep shear compliance and relaxation shear modulus of a visco-elastic material can be given as;

$$J(t) = J_g + \sum_{i=1}^n J_i (1 - e^{-t/\tau_i}) \quad (2)$$

$$G(t) = G_r + \sum_{i=1}^n G_i \cdot e^{-t/\tau_i} \quad (3)$$

Where J<sub>g</sub>, J<sub>i</sub>, and τ<sub>i</sub> in Equation 2 are referred as the glassy shear compliance, retardation strength and retardation times respectively. Similarly in Equation 3; G<sub>r</sub>, G<sub>i</sub> and τ<sub>i</sub> refer the rubbery shear modulus, relaxation strength and relaxation times respectively. The corresponding expression in frequency domain can be obtained using a Laplace transform and are given as;

$$J'(\omega) = J_g + \sum_{i=1}^n \frac{J_i}{1 + \tau_i^2 \omega^2} \quad ; \quad J''(\omega) = \sum_{i=1}^n \frac{J_i \tau_i \omega}{1 + \tau_i^2 \omega^2} \quad (4)$$

$$G'(\omega) = G_r + \sum_{i=1}^n \frac{G_i \tau_i^2 \omega^2}{1 + \tau_i^2 \omega^2} \quad ; \quad G''(\omega) = \sum_{i=1}^n \frac{G_i \tau_i \omega}{1 + \tau_i^2 \omega^2} \quad (5)$$

Where J'(ω), J''(ω), G'(ω), G''(ω) represent the storage shear compliance, loss shear compliance, storage shear modulus, and loss shear modulus respectively.

#### 2.5 Dissipated energy

Dissipated energy has been used as a fatigue resistance indication parameter for viscoelastic materials (Hopman and Pronk.1990, Bahia et al.2001). In a cyclic test, the amount of energy

loss in one unit volume during one complete cycle can be obtained by integrating the increment of work done  $\tau d\gamma$  over a complete cycle of period T (Equation 6).

$$\Delta W = \int_0^T \tau \frac{d\gamma}{dt} dt \quad (6)$$

For a time varying uniformly distributed stress signal,  $\tau = \tau_0 \sin \omega t$ , and a strain response,  $\gamma = \gamma_0 \sin(\omega t - \delta)$ , integration of Equation 6 yields the following expression for energy loss per unit volume per cycle.

$$\Delta W = \pi \tau_0 \gamma_0 \sin \delta \quad (7)$$

For a uniform stress distribution, the energy loss per unit volume per cycle is the same for any differential area over the cross-section of the test sample. However, in the shear fatigue testing using DSR machine, the shear stress distribution across the radius of the test sample at any given time varies from zero at the center to  $\tau_0$  at the periphery of the test sample. This implies the dissipated energy per volume obtained using Equation 7 varies for different differential areas located along the radius of the cylindrical test sample. The total dissipated energy for the system can be obtained through integration. The mean dissipated energy per unit volume per cycle for a cylindrical shape DSR test sample will then become;

$$\Delta W = \frac{1}{2} \pi \tau_0 \gamma_0 \sin \delta \quad (8)$$

The Dissipated Energy Ratio (DER) obtained by dividing the cumulative dissipated energy at the  $n^{\text{th}}$  cycle by the dissipated energy at the  $n^{\text{th}}$  cycle, can be used to evaluate the fatigue evolution process of a material (Pronk et al.1990).

$$DER = \frac{\sum_{i=1}^n \Delta W_i}{\Delta W_n} \quad (9)$$

Where:

DER = Dissipated Energy Ratio,

$\Delta W_i$  = Dissipated Energy per cycle per unit volume at the  $i^{\text{th}}$  cycle,

$\Delta W_n$  = Dissipated energy at the  $n^{\text{th}}$  cycle,

In this method, the fatigue life to crack propagation ( $N_p$ ), used as a fatigue resistance indication parameter marking the transition from the crack initiation to crack growth, can be obtained from the plot of DER vs the number of load cycles (Pronk 1995).

### 3 RESULTS

#### 3.1 Frequency sweep test results

DSR frequency sweep test results obtained for temperatures ranging from  $-10^{\circ}\text{C}$  to  $50^{\circ}\text{C}$  were used to construct master curves using time temperature superposition (TTS) principle. The

master curve data, complex modulus and phase angle, can be described with Christensen Anderson (CA) Model given in Equation 10.

$$|G^*(\omega_r)| = G_g \left[ 1 + \left( \frac{\omega_c}{\omega_r} \right)^{\frac{\log 2}{R}} \right]^{-\frac{R}{\log 2}} ; \delta(\omega_r) = 90 \cdot \left( 1 + \left( \frac{\omega_c}{\omega_r} \right)^{\frac{\log 2}{R}} \right)^{-1} \quad (10)$$

Where,  $G^*(\omega_r)$  is the complex shear modulus;  $\omega_r$  is the reduced frequency in rad/s,  $\delta(\omega_r)$  is the phase angle in degrees,  $\omega_c$  is the reduced frequency in rad/s where the phase angle equals 45 degree; also known as the crossover frequency and R is the regression parameter defined as the Rheological index.

Figure 2 presents the master curve data obtained for B40-60 binder. The data were obtained from PP and CP test configurations.

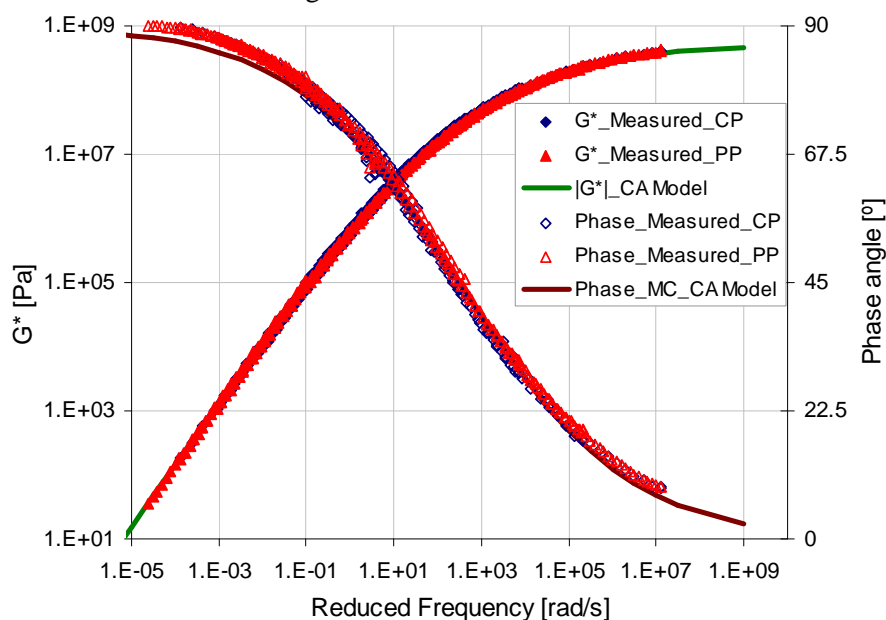


Figure 2: Master curve for B40-60 (obtained from CP and PP geometry test results).

Figure 2 shows that the data obtained in the linear viscoelastic range using the CP and PP geometries are in good agreement. A similar quality of fit is obtained for the B70-100 bitumen. For mastic tests at high temperatures, 40°C and above, the CP results didn't give good results and differed significantly from the PP results. The Low temperature results (30°C and below) were in good agreement with PP results. For reasons of clarity, the master curve plots for the B70-100 and mastic are not included here. However, the WLF and CA model parameters for obtaining the master curves for all the materials are given in Table 1. The good agreement between the results of the CP and standard PP geometries implies the new CP geometries can be used to conduct response measurements. The CP geometry will especially be advantageous for measurements where constant stress distributions are sought. In the CP setup uncertainties associated with the non-uniform stress distribution effect on the obtained response data are minimal.

Table 1: CA model parameters and WLF factors ( $T_{ref}=20^{\circ}\text{C}$ )

Parameters		B40-60	B70-100	Mastic
CA Model	$G_g$ [MPa]	480	420	903
	$\omega_c$ [rad/s]	303.564	305.00	36.66
	R [-]	1.268	1.175	1.17
WLF factors	C1 [-]	15.718	15.079	31.033
	C2 [-]	135.546	128.319	238.125

### 3.1 Fatigue results

Stress controlled fatigues testing at high stress levels were conducted using the PP and CP geometries. For B70-100 at stress levels of 0.3 Mpa and 0.4Mpa, figure 3 illustrates the dissipated energy ratio vs load cycles plot obtained using the two test geometries at  $20^{\circ}\text{C}$ .

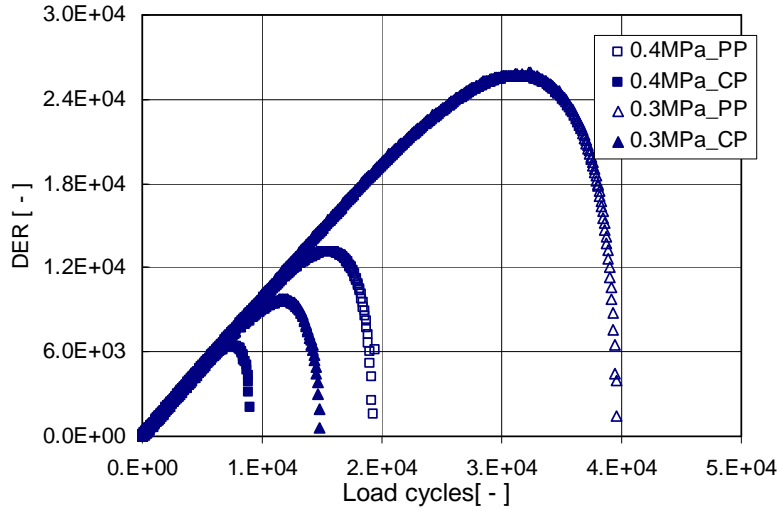


Figure 3. Dissipated energy ratio at  $20^{\circ}\text{C}$  for two stress levels (CP and PP setups).

Defining the fatigue life,  $N_p$ , as the intersection point obtained by drawing a tangent line through the initial linear part of the DER plot and horizontal tangent line drawn through the maximum point of the curve, it can be seen that the fatigue life cycles obtained using the PP geometry is greater than that obtained from the CP geometry. Referring to the theoretical discussion in section 2.5, the mean dissipated energy per cycle per volume for the test specimen in CP setup is twice as much as the PP setup. Higher initial dissipated energy per cycle per volume implies early crack initiation and hence lower fatigue life. The longer fatigue life, a factor of 2 to 3.5 higher as compared to the CP results, obtained for the PP results can thus be mainly attributed to the non-uniform stress distribution effect leading to lower energy dissipation per cycle per volume of the sample in the PP setup.

### 3.2 Time-frequency domain relationship

Time domain shear relaxation and creep tests have been conducted. These results have been compared with simulated results. The time domain simulation was made based on material

functions obtained from frequency domain tests. Collocation method (Schapery,1961) were used to obtain the Prony model parameters from the frequency domain master curve data. The Prony model parameters for the materials are given in Table 2. In the shear relaxation test, linearly increasing shear strains were applied. The step time, the time range to reach the maximum applied strain magnitude, has been varied from 1 second to 10 seconds. Figure 4 and Figure 5 presents representative results obtained for the simulated and measured relaxation data at 20<sup>0</sup>C.

Table 2: Prony series parameters at T<sub>ref</sub>= 20<sup>0</sup>C

Materials	i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Er
B40-60	$\tau_i$	3.87E-6	1.50E-5	5.82E-5	2.25E-4	8.73E-4	3.38E-3	1.31E-2	5.08E-2	1.97E-1	7.63E-1	2.96	11.45	44.4		0
	$G_i$	120.61	11.23	67.36	30.46	28.66	13.33	6.82	2.54	8.29E-1	2.42E-1	6.53E-2	1.39E-2	2.6E-3		
B70-100	$\tau_i$	4.22E-6	1.78E-5	7.50E-5	3.16E-4	1.33E-3	5.62E-3	2.37E-2	1.0E-1	4.22E-1	1.78	7.50	31.62	133.4		0
	$G_i$	107.52	12.02	69.34	30.27	26.95	11.60	5.04	1.53	4.32E-1	9.28E-2	2.16E-2	3.18E-3	9.7E-4		
B70-100 Mastic	$\tau_i$	1.29E-7	5.25E-7	2.14E-6	8.71E-6	3.55E-5	1.45E-4	5.89E-4	2.40E-3	9.77E-3	3.98E-2	1.62E-1	6.60E-1	2.69	10.95	8.0
	$G_i$	99.33	21.05	109.86	87.82	130.81	123.04	118.79	87.96	56.07	28.42	11.85	4.01	1.15	0.272	E-2

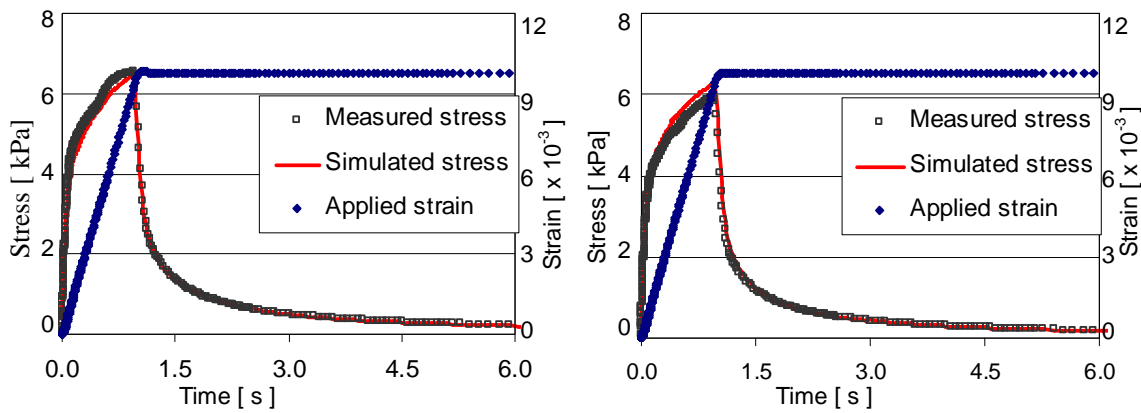


Figure 4: Measured and simulated relaxation tests; step time =1s (left; B40-60, right;B70-100)

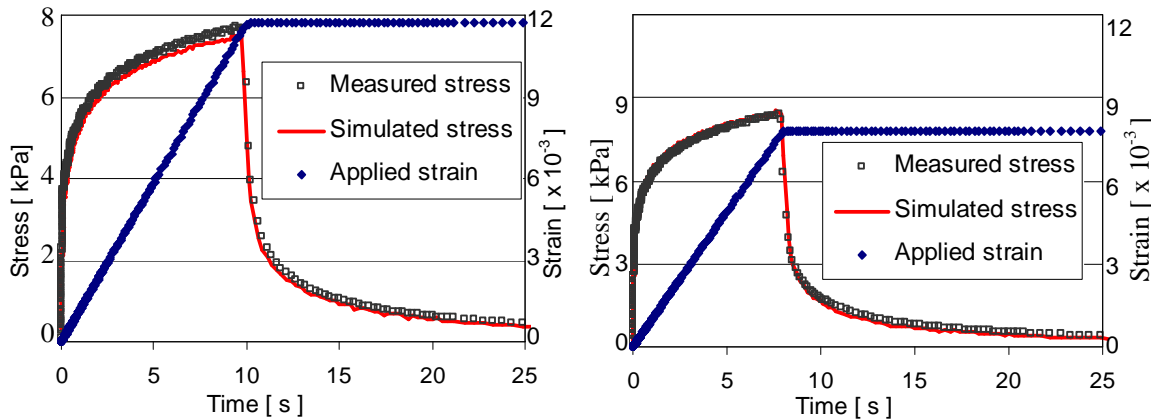


Figure 5: Measured and simulated relaxation tests; step time=10s (left; B40-60, right;B70-100)

Figure 4 and Figure 5 showed that the time domain response of the binder can be simulated well using material properties obtained from frequency domain tests. In some cases, the model prediction for the strains applied in longer step times appear more accurate than the ones obtained for shorter step times. In other cases, the reverse situations have been observed;

more accurate in shorter step times and less accurate for longer steps time. This can mainly be due to the quality of master curve fits, quality of data and unavoidable result variations while conducting different tests. In general, the quality of the simulation results for step loading times less than 10 seconds are quite impressive. The practical implications for this result would be that for fatigue-based pavement performance prediction models, where loading times are usually short, frequency domain response results provides adequate material response data. However, for models that require input with higher loading times, like permanent deformation, additional time domain tests with longer loading times might be necessary.

### 3.2 High Stress Measurements

Stress controlled frequency sweep measurements at various stress levels were conducted at each temperature. After each stress increment, a linear range frequency sweep measurements were conducted to monitor damage incurred as a result of the previously applied stress. Figure 6 typically shows the stress dependent behavior of two binders at 20°C. For other temperatures the stress dependent behaviors of B70-100 binder and Mastic are presented in figure 7.

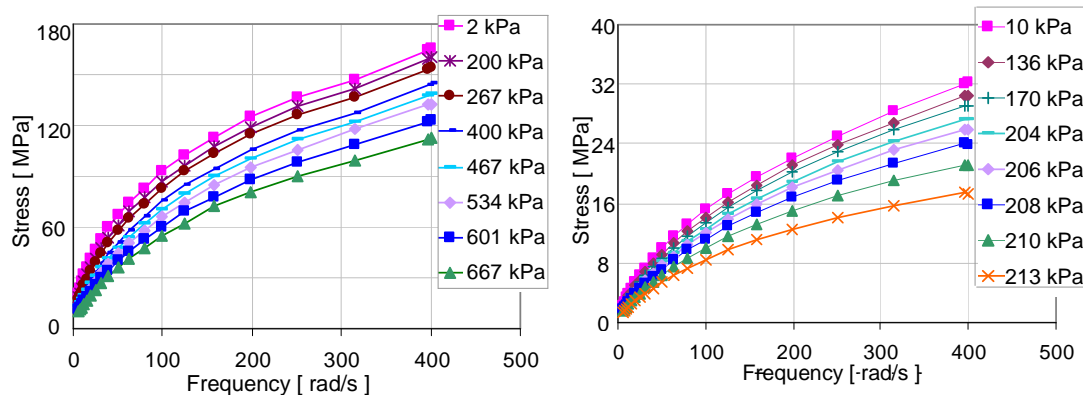


Figure 6: Stress dependent behavior at 20°C (Left; mastic, Right; B70-100 binder)

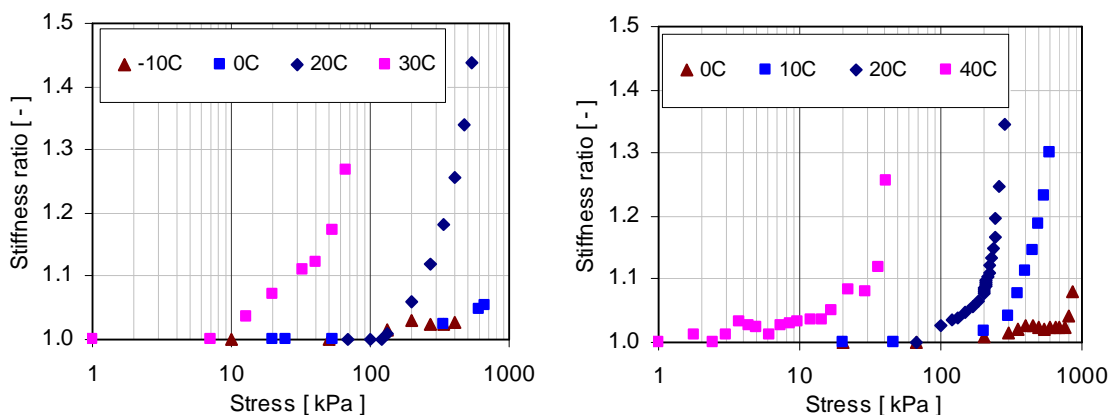


Figure 7: Nonlinear behavior at different temperatures (Left, Mastic; Right, B70-100 binder).

In Figure 7, the stiffness ratio is obtained by dividing the linear complex modulus value by the complex modulus value obtained at each stress level. It can be seen that for lower



temperatures, 20<sup>0</sup>C and below, nonlinearity becomes significant for shear stresses values of 100 kPa or more. At higher temperatures nonlinearity begins at a much smaller stress level; less than 10 kPa.

#### 4 CONCLUSIONS AND RECOMMENDATIONS

- A new CP test configuration has been used to measure the response of binders.
- Applicability of the CP configuration for response measurement has been verified by comparing CP test results with data obtained from the standard tests with a PP setup.
- Response test results obtained for all the binders with CP configuration are generally comparable with results obtained from PP configurations. However, mastic response results at high temperatures with a CP setup have a significant difference with the corresponding results obtained using the PP setup. Hence results from the 25 mm diameter CP setup for high temperature ranges (40<sup>0</sup>C and above) were excluded from the analysis.
- The effects of the non-uniform shear stress distribution on the fatigue results in the PP geometry were evaluated. The differences in results between the two setups, in terms of number of load cycles to failure, were significant. This is in agreement with the energy dissipation theory discussed in section 2.5. The CP setup dissipates energy at a higher rate and hence resulted in a shorter fatigue life. The test sample in the PP geometry dissipates energy at various rates over the cross-section, decreasing from maximum value at the periphery to a minimum at the center, resulting in a higher fatigue life of the test sample.

Modeling in the linear viscoelastic range;

- Dynamic material properties were used to simulate time-domain tests. The Prony series model was applied for frequency-time domain inter-conversion.
- Simulated tests have shown a good agreement with the time-domain experimental data. This implies that for short loading times, material response information obtained from frequency domain tests can be sufficient for making time-domain simulations.

Nonlinear behavior;

- At high stress levels, the response of the binders show stress dependency
- For lower temperature ranges, 20<sup>0</sup>C and below, nonlinearity becomes significant for shear stresses values of 100 kPa or more. At high temperature ranges nonlinearity begins at a much smaller stress level; less than 10 kPa.

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