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A Fast Converging Boundary Element Method for the Scattering by Perfectly Conducting Non-orientable Objects

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Abstract

The electric field integral equation can describe scattering by closed and open surfaces, surfaces containing junctions, and even non-orientable surfaces. The boundary element discretisation of this equation results in linear systems whose condition number grows as the square of the inverse mesh size. This eventually leads to systems that in practice cannot be solved, not even when using powerful iterative solvers such as GMRES and efficient matrix compression algorithms such as the fast multipole algorithm or an H-matrix based low rank representation. As a remedy, Calderón preconditioners are used to significantly reduce the number of iterations required to reach an acceptable solution. This type of preconditioners are available for open and closed surfaces, and recently also for surfaces containing junctions. In this contribution, a Calderón type preconditioner will be constructed for the electric field integral equation applied to non-orientable surfaces such as the Moebius strip. It is based on a redundant representation for the induced current, and a block-diagonal preconditioning strategy. Numerical experiments corroborate the correctness and efficiency of this approach.

1 Introduction

The electric field integral equation can model the scattering of time-harmonic electromagnetic fields by perfectly conducting objects. It is often preferred over other approaches (notably the magnetic field integral equation) because it is applicable in a wide variety of situations: scattering by surfaces that can either be open or closed, surfaces that contain junctions, and even surfaces that are non-orientable, such as the Moebius strip. The solution to the electric field integral equation can be approximated by the boundary element method: the induced surface current is approximated by a linear combination of basis functions subordinate to a triangular mesh on the surface and the equation is tested by the same set of basis functions. This process results in a linear system that can be solved for the unknown expansion coefficients.

Because the single layer boundary integral operator at the heart of the electric field integral equation acts as an integrator on solenoidal currents and as a differentiator on non-solenoidal currents, the condition number grows as the square of the inverse mesh size. More importantly, the solution time required by Krylov iterative solvers such as GMRES grows accordingly. This phenomenon is dubbed the dense grid breakdown of the electric field integral equation and precludes its solution when high accuracy is desired or when small geometric details are included in the design.

To counteract this problem, Calderón preconditioners have been designed. For closed surfaces, the self-regularising property of the single layer boundary operator is clear from the Calderón identities, which provide a relation the single layer and double layer boundary operators. For open surfaces, more theoretical observations allow to conclude that the spectral condition number may grow at most as logarithm of the mesh size [5].

For surfaces containing junctions, submitting to quotient-space paradigm allows the formulation of integral equations that are singular, and whose solution yields one of an infinite number of equivalent radiating surface currents [2]. For geometries that allow decoupling the various sides of the surface by making cuts along its boundary, a block diagonal preconditioner has been introduced in [4]. This preconditioner has been analysed in [3] for the Laplace equation and for geometries of type A as defined in that paper.

For non-orientable surfaces like the Moebius strip, it is impossible to define a continous field of normals. Because of this, we cannot construct a global Calderón preconditioner. Here, this problem will be resolved by covering the Moebius strip by orientable open surfaces. This entails introducing redundant (non-radiating) degrees of freedom, but allows for the construction of an efficient block diagonal preconditioner for the discretised single layer boundary operator, resulting in a small number of iterations required by solvers like GMRES in order to reach an accurate approximate solution. This is similar to what is done for surfaces that contain junctions, but unlike for type the A geometries from [3], the boundary of the orientable open surfaces is no longer fully contained in the boundary of Γ. Fortunately, this poses no obstacle as long as sufficient overlap is maintained in choosing the covering.

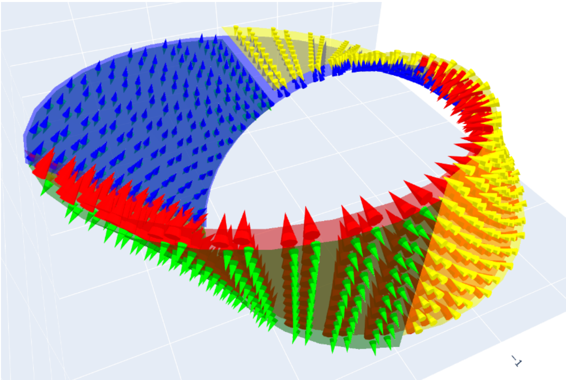

Figure 1. The Moebius strip Γ is covered by orientable open surfaces Γ_i. For this example, four overlapping surfaces are used, which are depicted in different colours. For clarity, the corresponding normals are also displayed. Note that, as a consequence of Γ's non-orientability, it is not true that these normals are opposite to each other at all points where the Γ_i overlap.

2 Equations and Discretisation

Consider the Moebius strip Γ in Fig. 1. The idea is to cover Γ by $\cup_i \Gamma_i$ such that each Γ_i is oriented by a consistent normal field n_i, and such that each point in the interior of Γ is in the interior of at least one of the Γ_i as well. This guarantees that each current u on Γ can be represented by a (in general not unique) sum of currents u_i on the Γ_i. This means, in particular, that their radiated fields -and hence the traces of those radiated fields- are identical. We will make use of this insight to devise our method.

Take an incident time-harmonic electromagnetic field varying with frequency ω. The space surrounding Γ is occupied by a material characterised by permittivity ε and permeability μ or equivalently by wave number $\kappa = \omega\sqrt{\varepsilon\mu}$ and impedance $\eta = \sqrt{\mu/\varepsilon}$.

The boundary element method is applied to compute an approximate solution to the scattering problem: the surfaces Γ_i are imbued with a triangular mesh $\Gamma_{i,h}$ that agrees with the triangular mesh on Γ_j where the two surfaces overlap. Let $\mathrm{RWG}(\Gamma_{i,h})$ be the space of Rao-Wilton-Glisson functions subordinate to $\Gamma_{i,h}$, and with vanishing binormal components on $\partial\Gamma_{i,h}$. We are looking for a current $(u_i)_i \in \prod_i \mathrm{RWG}(\Gamma_{i,h})$ such that, for all test currents $(k_i)_i \in \prod_i \mathrm{RWG}(\Gamma_{i,h})$, it holds that

$$\eta \sum_i < n_i \times k_i, T_{ij} u_j >_{\Gamma_i} = \sum_i < n_i \times k_i, n \times e^{inc} >_{\Gamma_i} \quad (1)$$

with $< \cdot, \cdot >_{\Gamma_i}$ the L^2 pairing on Γ_i, and

$$\begin{aligned} T_{ij}(u_j)(x) = &- \imath \kappa n_i \times \int_{\Gamma_j} \frac{e^{-\imath\kappa|x-y|}}{4\pi|x-y|} u_j(y) dy \\ &+ \frac{1}{\imath\kappa} n_i \times \operatorname{grad} \int_{\Gamma_j} \frac{e^{-\imath\kappa|x-y|}}{4\pi|x-y|} \operatorname{div}_{\Gamma_j} u_j(y) dy. \end{aligned}$$

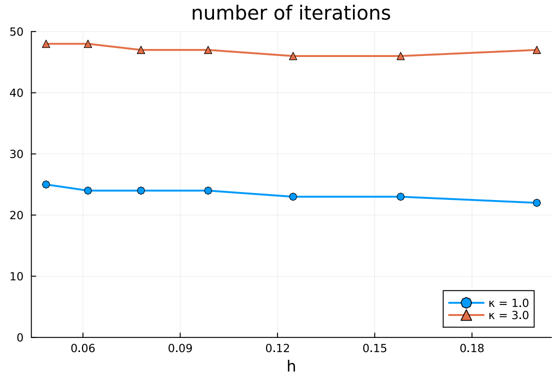

Figure 2. The number of iterations required for GMRES to produce a solution up to a relative error of $2.0 \cdot 10^{-4}$ using our preconditioner is small compared to the number of DoFs and only depends mildly on h.

Using the standard basis $\left(f_m^{(i)}\right)_{m,i}$ for this boundary element space, this results in the linear system

$$\begin{pmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} & \mathbf{T}_{13} & \mathbf{T}_{14} \\ \mathbf{T}_{21} & \mathbf{T}_{22} & \mathbf{T}_{23} & \mathbf{T}_{24} \\ \mathbf{T}_{31} & \mathbf{T}_{32} & \mathbf{T}_{33} & \mathbf{T}_{34} \\ \mathbf{T}_{41} & \mathbf{T}_{42} & \mathbf{T}_{43} & \mathbf{T}_{44} \end{pmatrix} \begin{pmatrix} \mathsf{u}_1 \\ \mathsf{u}_2 \\ \mathsf{u}_3 \\ \mathsf{u}_4 \end{pmatrix} = \begin{pmatrix} \mathsf{e}_1 \\ \mathsf{e}_2 \\ \mathsf{e}_3 \\ \mathsf{e}_4 \end{pmatrix} \quad (2)$$

Since on each of the Γ_i a consistently oriented field of normals n_i has been chosen, it is possible to build the following block diagonal preconditioner:

$$\begin{pmatrix} \mathbf{N}_{11}^{-T} & & \\ & \ddots & \\ & & \mathbf{N}_{44}^{-T} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{T}}_{11} & & \\ & \ddots & \\ & & \tilde{\mathbf{T}}_{44} \end{pmatrix} \begin{pmatrix} \mathbf{N}_{11}^{-1} & & \\ & \ddots & \\ & & \mathbf{N}_{44}^{-1} \end{pmatrix}, \quad (3)$$

with $(\mathbf{N}_{ii})_{m,n} =< n_i \times f_m^{(i)}, g_n^{(i)} >_{\Gamma_i}$, and $(\tilde{\mathbf{T}}_{ii})_{m,n} =< n_i \times g_m^{(i)}, T_{ii} g_n^{(i)} >_{\Gamma_i}$, and $\left(g_m^{(i)}\right)_{m,i}$ the Buffa-Christiansen basis functions on $(\Gamma_{i,h})_i$ [1].

3 Numerical Results

Consider an incident wave with signature $e^{inc}(x) = (1,0,0)^T \exp(-\imath\kappa x_3)$. The geometry from Fig. 1 is imbued with a triangular mesh generated with target mesh size h. In the overlapping regions these meshes are conforming but not necessarily oppositely oriented. It is important that each subsurface Γ_i is given a consistently oriented field of normals and that each edge of Γ_h is in the interior af at least one of the Γ_i. The induced current is computed for $\kappa = 1.0m^{-1}$ and $\kappa = 3.0m^{-1}$.

For each value of h, the number of iterations is recorded and plotted in Fig. 2. No values for the number of iterations required to solve the traditional (single-trace) electric field integral equation are included because, even for the coarsest mesh (at $h = 0.2m$), GMRES fails to converge, stressing the importance of an efficient preconditioner.

To verify correctness of the solution, the solution of the method presented here is projected onto the single-trace boundary element space and compared to the traditional solution (Fig. 3) as computed by LU decomposition, showing complete agreement.

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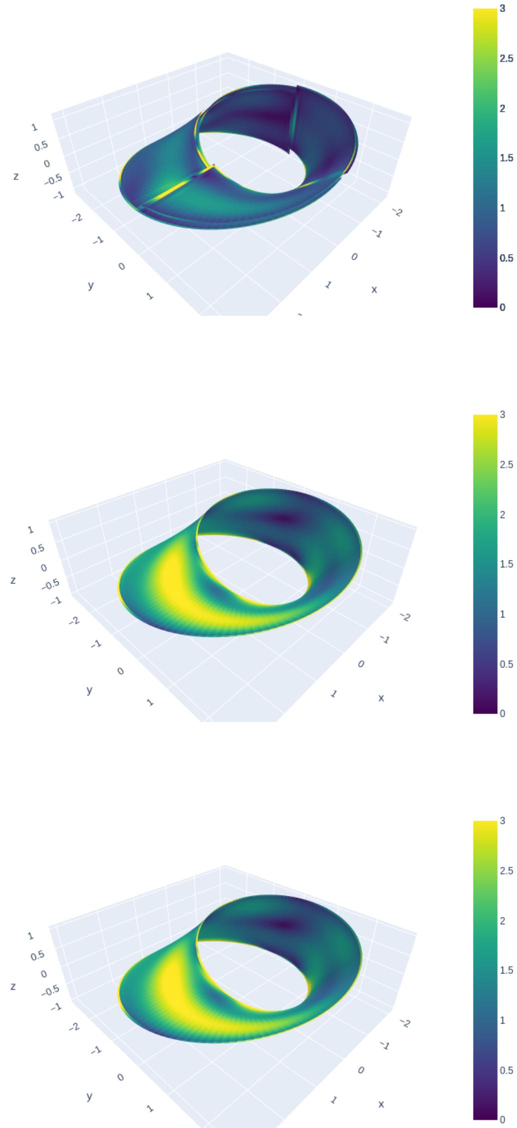

Figure 3. Induced currents computed by a redundant approach (top), the corresponding projection onto the single-trace space (middle), and the solution of a classic single-trace equation (bottom). Note that even though the representative for the radiating part of the current clearly depends on the covering $(\Gamma_i)_i$, its projection and thus its radiated field agrees with the classic solution.