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Schematizing rainfall events with multivariate depth-duration dependence

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Accurately modelling rainfall events is crucial for flood risk assessment and stormwater infrastructure design. However, transforming statistical characteristics of events into relevant rainfall patterns is challenging due to the natural variability of rainfall. Two commonly used methods to schematize rainfall events have limitations: the nested storm profile overestimates the resulting flow by assuming complete dependence between different durations, while determining the critical event duration by simulating each duration separately assumes independence and underestimates the flow. To overcome these limitations, this study presents a method that models the dependence between different rainfall durations using a Gaussian copula and combines this with marginal rain statistics to create a probabilistic model for the rain event. The SCS Curve Number approach is used to model the resulting flow, and a first-order reliability method (FORM) is applied to determine the critical combination of durations within an event. The findings of this study show that the rainfall events generated using the proposed method result in comparable flows to those produced by conventional design events. While this may not make the model a preferred choice for standard applications, it can still be valuable for flood risk assessments as it provides a probabilistic model that better captures critical rainfall patterns.

Keywords: Reliability analysis, rainfall-runoff modelling, multivariate dependence, Gaussian copula

1. Introduction

Accurate rainfall-runoff modelling is critical for designing and evaluating water systems, as it provides an estimate of how the system responds to rainfall events. This requires reliable and representative temporal rainfall patterns, known as hyetographs, to serve as boundary condition for simulating the hydrological process. A rainfall pattern can take an infinite number of shapes, therefore, a joint probability model can be employed to make a more informed estimate of this parameter. Copulas are a suitable method for this purpose (Nelsen, 2007) and are commonly used in hydrology. For example, to model hydrological aspects such as the dependence between rainfall intensity and duration (e.g., Zhang and Singh, 2007; Ariff et al., 2012; Jun et al., 2017). Others have used it to model the dependence between other hydrological variables such as river flows (Favre et al., 2004). Most applications use bivariate copulas, as these provide the greatest flexibility in modelling dependence. For modelling dependence between more than two variables, the Gaussian copula is a suitable model as demonstrated by (Renard and Lang, 2007) through several hydrological examples.

The current study aims to investigate whether a reliability analysis approach can lead to a better estimate of a design rainfall storm. The approach connects different rainfall durations into an event through the Gaussian copula providing greater flexibility in defining rainfall pattern. The First Order Reliability Method (FORM) is used to estimate the exceedance probabilities of flows. The generated rainfall events are simulated using a simple Soil Conservation Service (SCS) catchment model (Cronshey, 1986), which converts the rainfall patterns into hydrographs.

Given the importance of rainfall-runoff modelling, this study provides a valuable contribution to the field by comparing rainfall patterns from design guidelines to patterns from a reliability analysis. The results demonstrate the potential of the reliability analysis approach to estimate design rainfall storms, while also showing the limitations and effort involved with the approach.

2. Area of research and hydrology

This study examines the Avon River in Christchurch, New Zealand, whose catchment area of about 100 km^2 is predominantly urbanized. The analysis calculates representative flows for three distinct areas: full catchment, a middle-sized portion, and a smaller upstream section:

Table 1. Different catchments with their SCS parameters.

	Area	CN	lagtime
Full catchment	100 km^2	70	6 h.
Middle catchment	20 km^2	70	2 h.
Upper catchment	3 km^2	98	30 min.

Rainfall is measured at Christchurch botanical gardens, which has a relatively long record starting in 1962. The New Zealand National Institute of Water and Atmospheric Research (NIWA) has derived rainfall statistics for this location (Carey-Smith et al., 2018). While the original rainfall records are used for determining the correlations between durations (see Sect. 3), the marginal statistics for each rainfall duration are adopted from NIWA. The rainfall depths for different annual recurrence intervals (ARIs) and different durations, are shown in Fig. 1.

The rainfall patterns are converted to flows using the SCS Curve Number approach. This method is commonly used in hydrology to estimate the runoff from rainfall events in a catchment (Cronshey, 1986). It considers what part of the rainfall becomes run-off, based on a curve number (CN), a dimensionless parameter that represent soil type and land use. The remaining runoff is translated to the downstream river discharge by using a SCS Unit Hydrograph.

2.1. *Rainfall patterns*

When defining rainfall events from these statistics, a temporal distribution is needed to describe

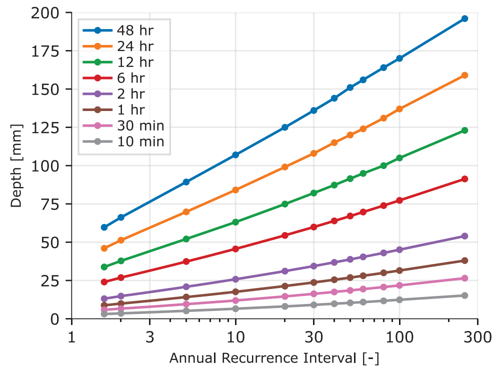

Fig. 1. ARIs for different rainfall depths per duration, based on NIWA's statistics for Christchurch Botanical Gardens.

the pattern with which the rainfall duration fills the rainfall depth. Such a temporal distribution is called a hyetograph. Fig. 2 illustrates the three options considered in this study:

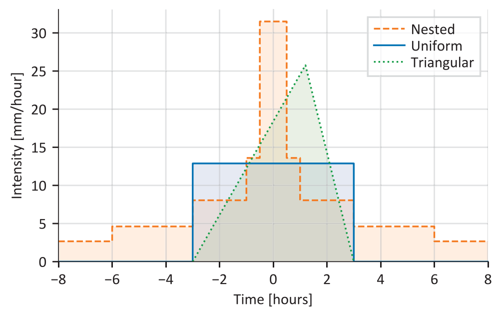

Fig. 2. Different hyetographs considered in this study.

(1) A rainfall pattern with a uniformly applied average intensity over the duration.
(2) A nested rainfall pattern, which is a design storm constructed for a specific annual recurrence interval. It involves the combination of a series of storms with increasing duration and decreasing intensity (all related to the ARI), "nested" within each other to form a single storm. The resulting storm has the specific ARI for each nested duration.
(3) A triangular rainfall pattern, similar to the one

used by the Christchurch City Council (CCC, 2022), which involves assigning a locally representative temporal pattern to an event. In this case, a triangle shape is used with the peak (i.e., double the average intensity) occurring at 70% of the duration.

In relation to a specific duration, such as 6 hours, the uniform and nested hyetographs are the most non-conservative and conservative options, respectively.

The uniform distribution assumes the lowest intensity for the other durations in the event, with the intensity during the most intense two hours being equal to the average intensity of 13 mm/hour. Any other pattern would have a higher intensity of more than 13 mm/hour for these two hours. Therefore, the uniform pattern assumes the lowest dependence between different rainfall durations. Conversely, the nested pattern assumes a specific annual recurrence interval (ARI) to be achieved for each duration within the event, assuming full dependence between different rainfall durations.

Although the considered patterns are relatively simple (a triangle and blocks), there are infinite possibilities for modelling a hyetograph as required by practitioners. In terms of dependence between durations, all patterns lie somewhere on the spectrum between the uniform and nested patterns.

3. Event selection and dependence modelling

3.1. *Event selection*

To model rainfall events based on the dependence between durations, it is necessary to determine these dependencies by analyzing the historical rainfall data from Christchurch Botanical Gardens. To achieve this, we considered the accumulated rainfall within 1, 2, 6, 12, 24, and 48-hour durations, following these steps:

(1) For each duration, we selected the maximum accumulated rainfall within a 96-hour time window.
(2) We selected the duration that had the event with the most extreme rainfall (highest Annual Recurrence Interval or ARI, based on the High Intensity Rainfall Design System (HIRDS) statistics) for each month.
(3) For the other durations, we determined the maximum accumulated rainfall that overlapped with the duration of the main event.

This approach resulted in one event per month, from which we determined the correlations between different durations.

3.2. *Dependence modelling*

The study uses a Gaussian copula to model the dependence structure, which is a multivariate probability density function with uniform marginals on the [0, 1] interval (Nelsen, 2007). Different types of copula have varying degrees of dependence for different parts of the interval, similar to scatter plots with the same correlation (see for example Anscombe's quartet (Anscombe, 1973).). The Gaussian copula is chosen due to its ability to model more than two variables, which is necessary for this study. To fit the Gaussian copula, the rainfall events are transformed to the [0, 1] range using the cumulative density function of the marginal statistics, rather than the ranks of each event. This is because combining different durations in a single event does not provide a selection similar to annual maxima or peaks over threshold. For example, the 48-hour duration rainfall being the highest ARI in the month does not necessarily imply that the 1-hour duration rainfall during that specific event is also the monthly maximum for that duration.

Figure 3 displays the cross-correlations between all event durations and their representation in the dependence model. The blue dots in the plot show the selected events. The scatter density is higher in the lower left quadrant than in the upper right quadrant, mainly because combining different durations in the monthly maximum event yields this result. During extreme rainfall events, an event is often a combination of multiple high-ARI realizations of the marginals, and the extreme events are located in the upper right corner of all panels. While the correlations are weaker in the upper right quadrant compared to the full plot, most extreme events are concentrated in the up-

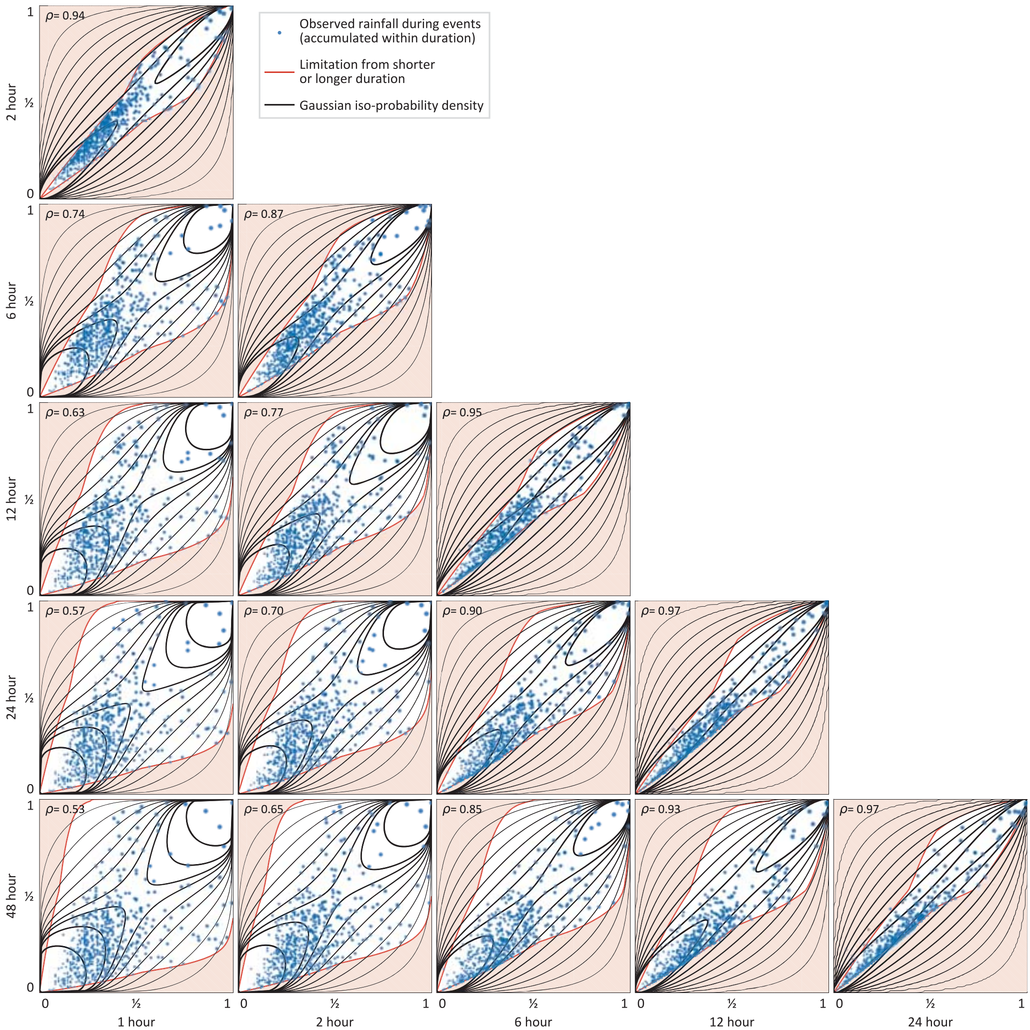

Fig. 3. Scatter plots with events, correlation bounds, and iso-PDF lines of the Gaussian copula.

per right corner. Ideally, the dependence structure with weaker correlations in the upper tail could be modeled by a copula such as the Clayton copula (Clayton, 1978). However, the Clayton copula cannot model different correlation strengths for more than two variables, making it unsuitable for this study. The Gaussian copula seems however sufficiently representative, given the strong correlations in the far upper right corner. The Pearson product moment correlations (ρ) are shown in the top left of each panel. The iso-probability densities of the Gaussian copula are represented by the black contours, while the red areas indicate the bounds within which the durations' depths can vary. These limitations arise because a 2-hour event cannot have a higher rainfall depth than twice that of the 1-hour event, and the same 2-hour event cannot have a smaller rainfall depth than the 4-hour event rainfall depth. These depth limitations are then converted to ARIs and ranks, resulting in the upper left bounds and lower right bounds, respectively.

3.3. *Exceedance probability calculation using FORM*

The dependence model from the last section consists of six dependent random variables, each representing the probability of a specific rainfall duration. Each event has a rainfall depth realization for each duration from which the catchment flow is calculated. We are interested in the annual exceedance probabilities of these flows. Calculating this is not trivial, given the probabilistic model. To do so, we employ the First Order Reliability Method, or FORM.

FORM, is a probabilistic method used in reliability engineering and probabilistic risk assessment to estimate a failure or exceedance probability. It is a mathematical method that aims to find the 'design point', which is the most likely combination of random variables that leads to failure or the exceedance of a specific value. We used the FORM-implementation from the Python module OpenTURNS (Baudin et al., 2016). To illustrate how it works in our application, consider the following steps:

(1) Define the random variables that affect the 'limit state' (in our case, the discharge from the catchment). These random variables are the marginal distributions for the different rain durations, and the correlation between them (the Gaussian Copula).
(2) FORM needs a limit state function that defines when failure occurs. In our case, the nested storm model is used to convert the different rainfall durations to a hyetograph and the SCS Unit Hydrograph model to convert this into a hydrograph. By comparing the maximum discharge from the hydrograph to a given discharge the limit state is determined.
(3) The random variables are transformed to standard normal space, and a an optimization method is used to find the realization of the joint probability distribution (i.e., the marginals plus the dependence model) with the highest probability, that leads to at least the critical discharge (i.e., exceeds the limit state).

The failure probability is calculated by linearizing the failure domain in the design point, which is generally less accurate than for example Monte Carlo or numeric integration, but faster as it requires fewer limit state function evaluations (in our case, hydrological model calculations). The FORM-procedure uses the Gaussian copula to model the dependence between the marginals. However, this does not automatically account for the limitations imposed by the different rainfall durations, where the black copula-contours in Fig. 3 cover the red areas. While the design points will be found in the upper right corner, where the Gaussian copula representation seems good, invalid combinations of rainfall depths need to be corrected to get valid profiles. For duration $D = [1, 2, 6, 12, 24, 48]$, the depth of a shorter duration cannot be greater than the depth of a longer duration, so d_i is set to $\min(d_i, ..., d_6)$. Conversely, the intensity I of a longer duration cannot be greater than that of a shorter duration, so I_i is set to $\min(I_6, ..., I_i)$. These corrections reduce the rainfall depths and the resulting flow. This penalty ensures that the final design point is not an invalid depth combination and remains outside the red areas in Fig. 3.

4. Results

Based on the hydrology described in Sect. 2 and the probabilistic approach in Sect. 3, hyetographs, hydrographs, and exceedance probability curves were calculated for the three catchments. A 100-year average recurrence interval (ARI) flow was calculated using each method to compare the three different hyetographs (Nested, triangular, and this study's FORM approach). Figure 4 shows these results. From left to right, the three catchments, from top to bottom, the hyetograph, hydrograph, and exceedance frequency curves. The nested hyetograph with full dependence is called 'Nested', while the hyetograph determined with the FORM-method (which is also a nested profile) is called 'FORM'.

The nested hyetograph (assuming full dependence) leads to the largest flows for the two largest catchments, while for the smallest catchment the triangular profile gives the highest flow because

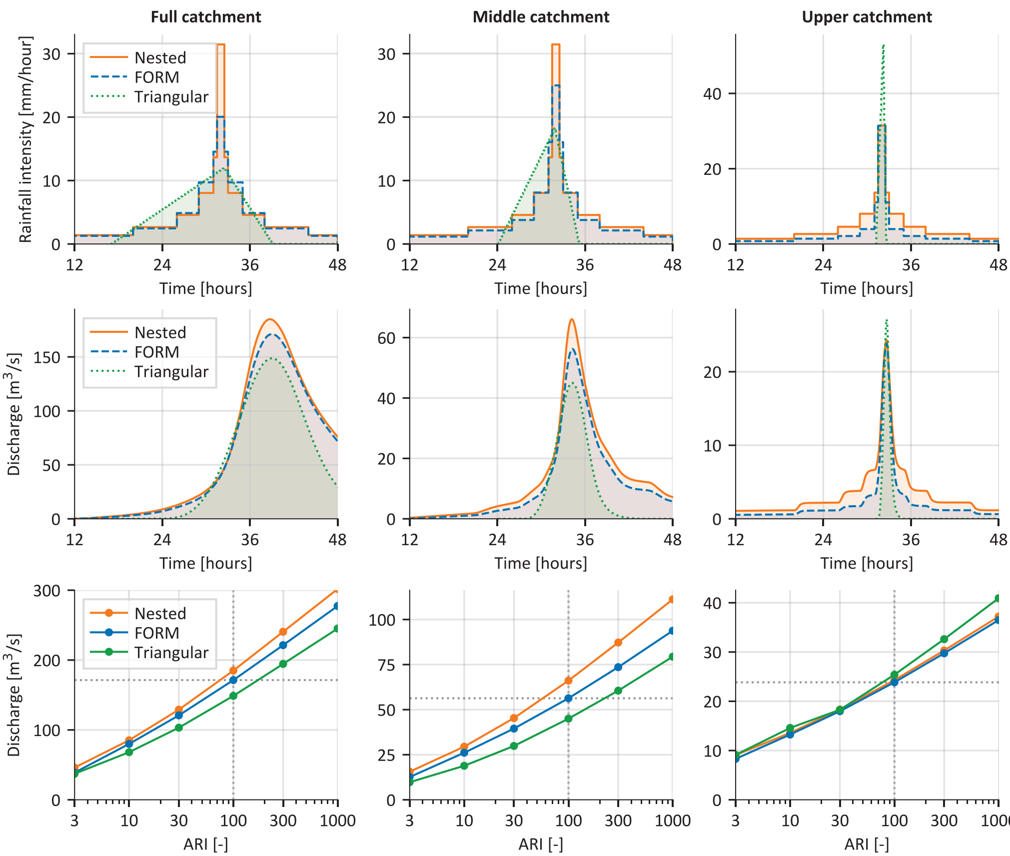

Fig. 4. Resulting hyetographs (top), hydrographs (middle), and exceedance frequency curves (bottom).

the triangular profile with a (critical) duration of 1.5 hours results in a greater than 100 ARI rainfall depth for the 1-hour duration. Both the triangular profile and the FORM-profile concentrate their probability mass around the critical duration. The upper catchment is small and modelled as largely impervious, meaning all rainfall will directly run-off without delay from the soil. This reduces the impact of the longer durations on the catchment, as antecedent conditions (filled soil storage) have little effect. This is another reason that the triangular hyetograph gives relatively high flows for this catchment.

The ARIs of the different durations within the event are shown in Table 2. The duration-ARIs for the FORM-solution are the so-called design points; the most likely combination of random variables to exceed a certain value (the 100-year flow in this case). For the nested hyetograph, all ARIs equal 100 year, by definition of the nested profile. For the triangular hyetograph the combination depends on the profile assigned to the critical duration. As a triangle is more convex than a nested profile, a shorter duration within the profile might represent a higher ARI than the critical duration's ARI. The difference between the ARIs of the nested and FORM profile are overall small. This is mainly because of the high correlations between durations: the adjacent durations all have a correlation coefficient greater than 0.9, and even the weakest correlation between 1-hour and 48-hours is greater than 0.5.

Table 2. ARIs for rainfall durations within the design rain events, for the different catchments and hyetographs.

	Full catchment			Middle catchment			Upper catchment		
Duration	Nested	Triangular	FORM	Nested	Triangular	FORM	Nested	Triangular	FORM
1 h	100	3	16	100	10	36	100	157	98
2 h	100	7	31	100	26	63	100	45	75
6 h	100	36	78	100	120	81	100	5	27
12 h	100	102	92	100	84	68	100	2	16
24 h	100	85	86	100	22	57	100	1	12
48 h	100	26	76	100	8	50	100	1	10

5. Discussion and final remarks

In this study, a joint-probability approach is used to generate rainfall events and their resulting discharge probabilities. The probabilistic model involves six marginals for the 1, 2, 6, 12, 24, and 48-hour duration accumulated rainfall, which are connected through a Gaussian copula. To simulate the generated rainfall events, the study employs a SCS curve number and SCS unit hydrograph approach. The exceedance probabilities of flows are calculated using the First Order Reliability Method (FORM).

5.1. *Probabilistic model*

The presented probabilistic model uses a Gaussian copula to describe the dependence between rainfall durations. For a better representation of the temporal variability, the phase between different duration could be included as well. When applied to a larger catchment, the spatial correlation could be included as well. The Gaussian copula provides the possibility of adding additional marginals representing these extra characteristics. However, some ad-hoc modifications were made to the model such that it abides the limitations imposed by different rainfall durations (the red areas in Fig. 3. The employed model gives a better mathematical representation than the considered alternatives (the nested and triangular profile) but it does have some caveats which would ideally be solved before extending the model.

5.2. *Efforts versus gains*

A comparison between the FORM-results and the nested and triangular results shows relatively similar flows and similar rainfall depths around the critical durations. The nested profile gives the highest flows for the first two (largest) catchments. This is expected as it is the most conservative representation of a X-year event. However, the triangular profile gives a higher flow than the nested profile: The 11-hour critical duration for the middle-sized catchment results in a 120-year ARI rainfall depth for the 6-hour duration. Similarly, the nested-profile event will have a larger than 100-year ARI because there is no full dependence between durations. These inconsistencies result from defining what an X-year event is without a) defining a singular quantity such as discharge from which this should be determined, b) providing the model to calculate this quantity, and c) defining the probabilistic model behind it. Each of these activities requires knowledge and time. So while wrong from a probabilistic point of view, following a standard approach like the nested or triangular storm does make sense from a practical point of view.

5.3. *Impact for safety and reliability*

The observed flow reductions of 10% to 20% are significant but should be considered within the broader context of hydrological uncertainty. Rainfall statistics derived from historical records can quickly become outdated due to new extreme weather events (NIWA, 2023). Uncertainties in catchment characteristics, such as infiltration capacity and antecedent soil moisture, have a comparable impact on discharge as the demonstrated effects from different hyetographs. Climate change projections further contribute to uncertainty, influencing hydrological conditions over the design lifespan (Pörtner et al., 2022). While we recognize that conservatism is not the optimal response to uncertainty, it is understandable that a combination of hydrological uncertainty and practical matters such as liability, might cause a practitioner to favor a better-safe-than-sorry approach such as a nested storm over a complex but more accurate approach such as the one presented in this article. The probabilistic model definition can however be useful as research method to more accurately quantify the magnitude of historical rainfall events, or to project them on a future climate.

5.4. *Conclusions and implications*

This study presents an alternative approach to model temporal rainfall patterns based on statistics of rainfall durations and their dependence within a rain event. The approach is compared to standard hyetographs, such as the nested and a triangular hyetograph. Our results show that flows calculated from the rainfall events schematized with this method are smaller than those of the nested hyetograph, and larger than the flow of the triangular hyetograph. The differences are relatively small, mainly because the correlations between durations are strong. The study was limited to a single event in a single catchment. Adding more temporal or spatial variation is likely needed to get an accurate event description for larger, real-world, catchments. Nevertheless, using the presented probabilistic model that includes the observed correlations can improve the accuracy of rainfall and discharge estimates, and through this inform more effective flood risk assessments and water infrastructure design.

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