Master thesis:

Double curved precast load bearing concrete elements

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Preface

This is my thesis for the partial fulfilment of the Master track Building Engineering of the Faculty of Civil Engineering and Geosciences at Delft University of Technology.

The master thesis revolves around the topic ‘load bearing double curved prefabricated concrete elements’. The focus in this thesis is on the production of these elements and the improvement of a flexible formwork: a formwork that can easily be adjusted in every desired double curved shape to enable mass production of uniquely shaped double curved elements. This report presents the results of my research, of the models that have been made and of the concrete experiments that have been preformed during the research.

I would like to thank my gradation committee; Prof. dr. ir. J.C. Walraven, Ir. H.R. Schipper, Dr. ir. K.J. Vollers, Dr. ir. S. Grünewald for their useful help and knowledge during my research project.
Also I want to thank Dr. ir. P.C.J. Hoogenboom for helping me with the models that were made, and Edwin Scharp and Ton Blom for their guidance during the concrete experiment performed at the Stevin Laboratory.

Bas Janssen
Delft, March 2011

Summary

Construction of free form double curved surfaces

Figure 1: Der Neue Zollhof, Frank Owen Gehry [34]

Figure 2: Sydney opera house [45]

In a period in history when labour was relatively cheap, various free form surfaces -mainly shells- made of concrete were build. These double curved surfaces needed a double curved formwork, of which the construction process was very time consuming. The relatively cheap labour, however, allowed the still profitable production of shells in that period. The construction of these building has stagnated. The main reason is the increased labour cost over time. Lately, most double curved surfaces are only applied in projects with a high profile and above-average budget. An example of this is the project 'Der Neue Zollhof' of architect Frank Owen Gehry, which construction finished in 1999 (Figure 1). This building has a double curved façade. Techniques used nowadays to construct these double curved shapes can be described as statical. The formwork that is needed to cast the concrete for these double curved elements can only be used for one single element shape. Examples of these types of formworks are a custom made wooden mould or EPS foam CNC-milled into the desire shape. This makes the construction very expensive, since a formwork is more economical if it can be reused many times. Some free form building could only be build after modification and rationalization of the shape, to be able to construct it in an economically feasible way. An example of this is the Sydney Opera (Figure 2). The double curved surface was split up into elements that could be prefabricated. The shape of the surface after that was adapted, so that all of the elements could be made using the same formwork. Instead of a lot of different uniquely shaped formworks, one formwork could be reused many times, making this project feasible.
In recent years the demand for free form buildings is rising. This is partly due to the new digital architecture. The new generation of architects is educated in creating almost any possible free form shape by making use of a 3D CAD software. The manufacturing of these uniquely shaped elements yet still does not follow this development in an equal pace.

Flexible formwork

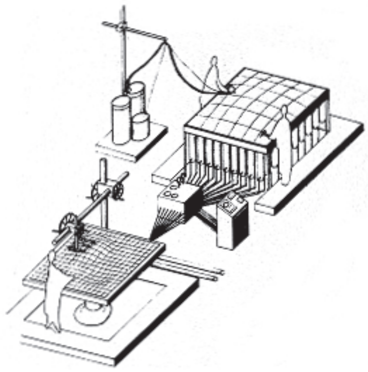


Figure 3: Flexible mould by Renzo Piano [5]

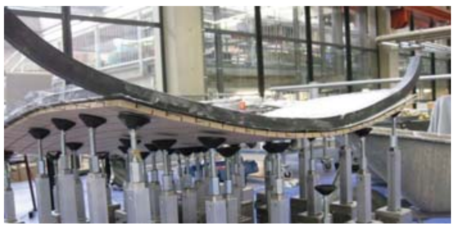


Figure 4: Actuator mould, tested by K. Huyghe and A. Schoofs [9]

A feasible method for mass production of uniquely shaped double curved elements could perhaps be formed by a flexible formwork: a formwork that can be adjusted in every desired shape. It consists of a flexible layer that can be deformed into the desired curved surface by adjusting, for example, pistons, actuators or pins.
The concept of a flexible formwork has been designed and named in many literatures over the years. In the sixties Renzo Piano made a drawing of his concept of a flexible formwork, Figure 3. In literature the idea of it is further discussed and tested. The adjustable supports could be placed against each other, a pin bed, or with interspaces. There has to be an elastic material on top of these supports, which must deform into the desired shape, to make the mould surface. Recently K. Huyghe and A. Schoofs [9] have executed a series of

experiments with a flexible formwork, build earlier by D. Rietbergen and Dr. Ir. K. Vollers. In Figure 4 a picture of their alignment is shown. Tests that they have done demonstrated that it is very important to choose the stiffness of the elastic layer right. During one test the elastic layer was chosen too stiff, as can be seen in Figure 4 were the flexible layer does not touch all of the supports. The elastic layer and therefore also the produced element did not get the desired shape. If the flexible layer is chosen too flexible the displacement between the supports becomes too large, leading to an inaccurate shape. From these tests it can be concluded that proper knowledge of the behaviour of the elastic layer is needed to be able to produce the desired shaped elements. Another aspect is the moment in time of deformation.
The flexible mould could be deformed before the concrete is poured, or with the fresh concrete already on top of it. Deforming of the mould with the fresh concrete on top of it means that the weight of the concrete helps to push the mould into the desired shape. Deforming of the mould without concrete gives large forces at the supports to pull the mould into its form. Another advantage of starting with a horizontal mould is that there is no contra mould needed, the thickness of the element can be controlled easily and the pouring is relatively easy and quick.

Research objective

The aim of this master thesis is to improve the prefabrication process of double curved load bearing concrete elements. K. Huyghe & A. Schoofs [9] have already shown that it is possible to produce thin cladding panels with an actuator mould. The purpose of this thesis is to solve a number of open-ended questions regarding the flexible formwork, and to enhance it further into a feasibly system. A second goal is to also make the system applicable for thick load bearing concrete elements. The main focus in this thesis is on the behaviour of the elastic layer. Predicting the behaviour of the deformed layer makes an accurate production possible.

Modelling the flexible layer

Single curved element
First, the behaviour of a single curved element is treated. Figure 5 shows what the flexible layer does when it is too stiff, it releases from some of the supports. In Figure 6 the behaviour of the strip is shown if the stiffness is very low, the local displacements between the supports become to large.

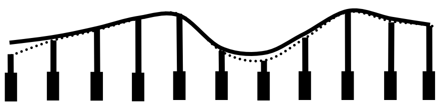


Figure 5: Mould is too stiff

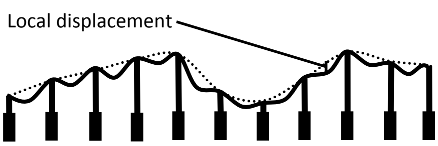


Figure 6: Mould is too flexible

By making use of the Euler-Bernoulli equation the behaviour of the single curved element is modelled. Assumption is that the support reactions must be compression. This makes sure that the elastic layer is pushed, with the weight of the concrete, to the supports. A difficult-to-realize connection between the elastic layer and the support is not needed in that case.
This results in a model that describes the behaviour of this single curved strip. The height of the supports can be filled in, as well as the

elasticity of the flexible layer and the maximum local displacement. The model gives the required thickness of the layer, to fulfil the conditions: all of the supports must be touched and there has to be limited local deflection between the supports.

Double curved mould, plate model

To be able to predict the behaviour of the elastic layer for double curved elements, a model is proposed that describes this behaviour if the flexible layer is a plate, see Figure 7.
The general fourth order plate differential equations are solved by using the differential method, by using the partial directives.
This results in an Excel sheet in which the plate geometry can be filled in. The plate characteristics such as thickness and elasticity can be varying. The model calculates the normal stresses in the plate and the support reactions. With the support reactions in can be checked if the plate is pushed to the supports, or releases at some supporting points.

Figure 7: Plate model

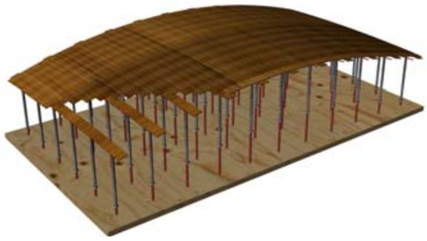


Figure 8: Strip model

Double curved mould, strip model

Another possibility is to build the mould surface out of strips instead of a plate, see Figure 8. An advantage is that the strips can slide among each other, and so the extension in a double curved shape does no longer lead to large normal stresses. A disadvantage is that an additional material is needed that covers the strips.
The strips are aligned in two directions. The bottom layer is at the supports on the short direction of the formwork, see Figure 8. The top layer is placed on top of the bottom layer and forms the actual mould surface. The top strips are only connected to the bottom layer in the middle, to make sure that the strips can slide alongside each other. The behaviour of the strips is of a single curved element. So the model that describes a single curved beam is used to build an Excel sheet that predicts all of the support reactions. In this model the thickness and the width of the strips can be varied, to see what the reaction forces become. In this case also only compression forces at the supports are favourable. The concrete weight will push the mould into its form then. The model also gives the maximum displacement between the supports.

Cement type	
Cem 1 52,5 R	26,865 kg
Filler	
Fly ash	7,380 kg
Adjuvant	
Glenium	0,190 kg
Aggregates (dry)	
Sand 0,125-0,250	8,479 kg
Sand 0,250-0,5	11,305 kg
Sand 0,5-1	16,958 kg
Sand 1-2	14,131 kg
Sand 2-4	5,653 kg
Water	10,783 kg
Total	45 l

Table 2: Mixture two

Laboratory experiments

Single curved mould surface

To verify the single curved model that was proposed and to investigate some important parameters of the flexible formwork, such as the behaviour of the flexible border and the minimum radius of the mould, tests have been performed on a single curved model.
By using the flexible formwork in the way that the concrete is poured on a horizontal surface, Figure 9, and is deformed after a while, Figure 10, the deformation time of the mixture becomes important. This is the time that the concrete is stable to keep its stability when it is in a slope (curved mould surface) but it is plastic enough to be able to deform with the mould. Table 1 is an overview of the single curved mould tests that are preformed. Concrete mixture two (Table 2) was used in these tests.

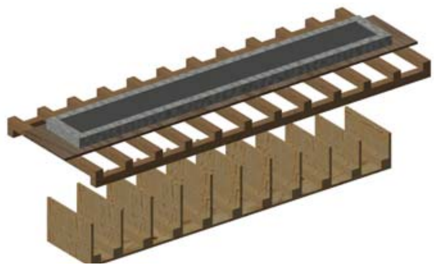


Figure 9: Concrete is poured horizontal in the mould

Figure 10: Deformed mould and concrete

	Deform time (min)	Flexibler layer	Thickness layer (mm)	Curvature (M)	Border
Test 1	50	Wood	3,8	5	polyethylene SG 40 extra firm
Test 2	50	Steel	1	5	Cold foam HR SG 55
Test 3	50	Wood	3,8	2,5	polyethylene SG 40 extra firm
Test 4	50	Wood	3,8	1,5	polyethylene SG 40 extra firm
Test 5	30	Wood	3,8	1,5	polyethylene SG 40 extra firm

Table 1: Overview single curved mould tests

Double curved mould surface

A set up was made to test the two double curved models that had been proposed, and to see if it is possible to produce double curved elements by using the principle deforming the fresh concrete after pouring it horizontal. The set up consisted of six by eleven supports, a screw-thread and nuts with a hollow pipe to make the support point adjustable in height, see Figure 11. A frame held the supports at a horizontal level, to pour the concrete at. This frame could be lowered with a crane to deform the mould, see Figure 12 for a sequence of the production process.

With this set up three tests were carried out. Test one, with a plate as mould layer, failed due to buckling of the plate (Figure 12). Test two, with the strips mould, using two different geometries (Figure 13 and Figure 14), gave satisfying double curved concrete elements.

Figure 11: Double curved test alignment

Figure 12: Buckling of the edge at the plate surface

Figure 13: Double curved strip surface test one

Figure 14: Double curved strip surface test two

Production sequence double curved element (Figure 15)

Picture	***Time***	***Action***
1.	t = 0 min.	The mould is ready for the concrete. The mould is horizontal and rests on a plate which is temporary supported. At t = 0 adding water to the mixture, start hardening of the concrete.
2.	t = 8 min.	Pouring the concrete into the mould.
3.	t = 15 min.	Smoothening of the fresh concrete.
4.	t = 49 min.	Lifting the formwork to be able to remove the temporary struts.
5.	t = 47 min.	Deforming of the mould.
6.	t = 48 min.	The end of deformation process. The element has the final shape.
7.	t = 1 day	After one day of hardening the element is removed from the mould.
8.	t = 1 day	The element above its mould.
9.	t = 1 day	The final element on the supporting points.

Figure 15: Sequence of producing double curved elements

Conclusions and recommendations

After the tests it became clear that the strip mould was the best alignment for the flexible layer. It was possible to produce double curved elements on this flexible formwork, see Figure 16 for an example of this. The method of deforming the fresh concrete after it had been poured horizontally succeeded.

The models that have been proposed appear to describe the behaviour of the flexible layer well. With these models the support reaction could be checked. If all of these forces are compression, the mould could lay loosely on the supports. If there appear tension to be forces, a connection is needed which can transfer these tension forces.

Figure 16: Double curved element at the flexible formwork

Foam, polyether, was used as a border for the formwork. It fulfilled the function of withstanding the horizontal concrete pressure, but it was also flexible enough to follow the deformations of the mould. Due to the porosity of the material it needed a protection layer. The concrete can not be poured directly to the foam. The moisture in the concrete would otherwise be absorbed by the foam. In this test the foam therefore was sealed using a silicone sealant. Due to the sealant, the elements got a rough surface. Another material still has to be found that will protect the foam, but this material should be flexible enough to follow the deformed mould and must give a smooth surface to the concrete.

The time to deform the mould was investigated with some tests. Further research is needed to find the relation between the concrete mixture, the radius of the mould and the right moment in time when the mould should be deformed.

Table of content

1. Introduction

1.1. Construction of free form double curved buildings

In the age that labour was relative cheap 'free form' buildings made of concrete, mainly shells, where build with time consuming and labour intensive formwork. This formwork was a temporary support for the concrete, and had to have also the free form shape of the building. Over the years the labour cost increased and one was looking for ways to shorten the construction time, and with this the construction costs. By making use of formwork elements the building process was optimized as much as possible. These formwork elements had standard shapes to speed up the building process and where reusable to reduce the costs.
Due to the standard shape of the formwork, elements mainly have rectangular shapes and so also buildings were build. The production process and the rentable space of a building were optimized as much as possible. Higher costs prevent the construction of shells.
During the last years the demand for free form buildings is rising again. This is partly due to the improved and frequent use of computers. With the use of 3D drawing software the architect can design and drawn every free form shape. The biggest problem to make this building is the formwork for the concrete with boundaries of economical feasibility. This formwork has to be double curved to follow the free form shape of the building.
An example of a recently build free form building is Der Neue Zollhof, Frank Owen Gehry in Dusseldorf (1999), see Figure 1-1. This building consists partly of precast elements. To make the precast double curved elements a formwork of EPS foam was used Figure 1-2. The shape of the element was CNC-milled out of the foam. Because each element had another unique shape, for every element a different formwork had to be milled. This made it a time consuming and expensive project. Another recently build project with a double curved shape is the EPFL Rolex Learning centre, Sanaa in Switzerland (2010), Figure 1-3. The floor of the building is made of concrete and has the same double curved shape as the roof. To make the double curved floor, temporary scaffolding with a wooden formwork was needed to support the concrete (Figure1-4). This wooden formwork was also CNC-milled. This was also an expensive project.
Above, two ways of building a double curved surface are mentioned. Both ways, CNC-milling of EPS foam and temporary scaffolding with a CNC-milled wooden formwork, are expensive and time consuming ways.

Figure 1-1: Der Neue Zollhof, Frank Owen Gehry [34]

Figure 1-2: EPS formwork [1]

Figure 1-3: EPFL Rolex Learning center, Sanaa [35]

Figure 1-4: Formwork EPFL [36]

1.2. Problem description

The number of buildings with double curved shapes is rising, partly because of the grown freedom of drawing and designing of architects by making use of modern computer techniques. The biggest problem that hinders this development is the manufacturing of different double curved elements, which have an unique shape, in an economic feasibly way. The formwork that is needed for the concrete elements might have different shapes for each element. The most often applied techniques to build the double curved formwork are not adjustable. A formwork is made for each unique element, and can consist of a wooden/steel mould or of milled foam. This formwork can be used only for one shaped element; this makes the existing techniques expensive and the production process time consuming.

1.3. Research objective

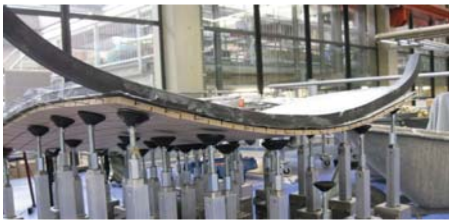


Figure 1-5: Actuator mould tested by K. Huyghe and A. Schoofs [9]

The aim of this master thesis is to improve the prefabrication of double curved load bearing concrete elements. K. Huyghe & A. Schoofs [9] have already shown that it is possible to produce thin cladding panels with an actuator mould (Figure 1-5). The purpose of this thesis is to solve more aspects of the flexible formwork, and to make it a feasibly system. Also make the system applicable for thick load bearing concrete elements.
One aspect that is investigated in this thesis is the elasticity of the flexible layer. As seen Figure 1-5 the flexible layer is to stiff, it does not touch all of its supports. In this thesis a model is proposed which describes the behaviour of it. Whit this model the needed elasticity for a free form element can be determined. Tests are carried out to verify the model.

In this master thesis the focus of producing double curved elements lies on the flexible formwork. There is also research and experiments on going to produce double curved elements with other methods, such as milling of EPS [1], fabric formwork [26] and vacuumatic [16]. It was decided to focus on the flexible formwork, it is believed that the future of free form elements lies in the improvement of a flexible formwork, and that is the starting point for this research.

1.4. Approach

The research objective is achieved by the following Chapters:

Chapter 2: Survey of literature
First some reference buildings with double curved surfaces are given; main question for each building is how the double curved surfaces are made. The existing techniques of making double curved formworks are given and the development of the flexible formwork is described.

Chapter 3: Ways of using the flexible formwork.
A flexible formwork could be used in several ways. This chapter describes the possibilities and explains the chosen method in this thesis.

Chapter 4: Design parameters of the flexible formwork
This chapter gives an overview of the aspect that plays a role at the flexible formwork and at double curved surfaces.

Chapter 5: Flexible mould layer, single curved
A model is proposed that describes the behaviour of the flexible mould layer that is single curved. This model is needed to makes sure that the flexible formwork deforms in the desire way and makes sure that the elements gets the right shape.

Chapter 6: Concrete
This chapter deals with the material which the elements are made of, concrete. First some general properties are given, and see which are important in this case. The main aspect is the deform time, the time when the mould is deformed. This time depends on the fresh concrete behavior.

Chapter 7: Reinforcement
Concrete needs reinforcement to be able to take up tension forces. Due to the radius current use reinforcement bars are not applicable. This chapter describes the other options to reinforce the concrete.

Chapter 8: Single curved mould tests
At a single curved mould test are performed to check the model of Chapter 5. Also in this chapter some parameters of the flexible mould are tested and seen what the best parameter is for this application.

Chapter 9: Flexible mould layer, double curved
To be able to describes the behaviour of the flexible layer when it is double curved, two models are proposed in this chapter. Before tests are carried out, first the behaviour of the flexible layer has to be known, to make sure that the right material and parameters are chosen.

Chapter 10: Double curved tests
A flexible mould is build and tests are performed to see if the models describe the behaviour of the flexile layer in a correct way. Also the tests are performed to see if it is possible to produce double curved elements with the flexible formwork.

Chapter 11: Conclusions and recommendations
This section gives the conclusions of the thesis and the recommendation that follows from it.

2. Survey of literature

This chapter provides a perspective of the research. First, reference projects are mentioned. The existing methods of constructing double curved panels are described. Then the development of the method which is tried to improve with this research, the flexible mould, is discussed.

2.1. Reference projects

2.1.1. Introduction

This section gives an overview of the development of free form building/constructions. For every project is given how it is build, is it constructed at site, or with the use of prefabricated elements.
An important aspect of each project is how they made the double curved shape surfaces and how the connections between the elements are designed if they use prefabricated elements.

2.1.2. Buildings

Philips Pavillion [9] [38]

Place	Brussels, Belgium
Year	1958
Architect	Le Corbusier
Structural engineer	Iannis Xenakis
Precast/in-situ	Precast concrete elements placed on temporary scaffolding
Formwork	Stabilized sand on which the precast elements are poured at. Each element had a unique shape.

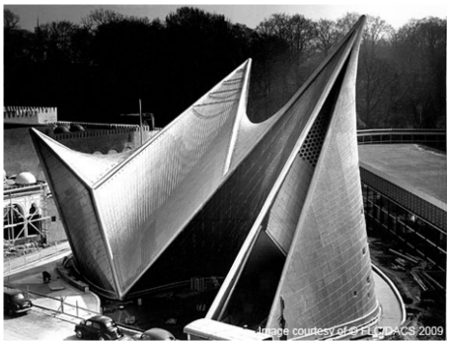


Figure 2-1: Philips Pavilion [37]

Der Neue Zollhof [1] [38]

Place	Dusseldorf, Germany
Year	1999
Architect	Frank Owen Gehry
Structural engineer	AG Hubertus Zimmerling
Precast/in-situ	Precast and in-situ
Formwork	EPS foam that is milled into the 3D shape.
Concrete	High strength concrete with steel reinforcement.

Figure 2-2: Der Neue Zollhof [34]

Jubilee Church [38]

Place	Rome, Italy
Year	2004
Architect	Richard Meier
Structural engineer	Arup
Precast/in-situ	Precast, elements are placed with a special crane and tensioned afterwards.
Formwork	One formwork for all of the elements. All of the elements have the same shape.
Concrete	Self-cleaning concrete

Figure 2-3: Jubilee Church [39]

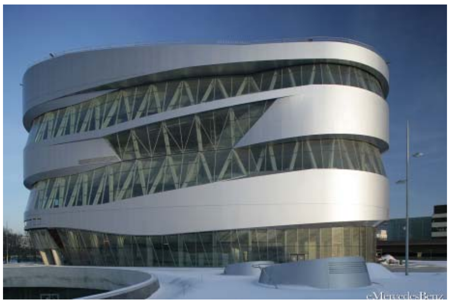


Figure 2-4: Mercedes Benz museum [40]

Mercedes Benz museum [1]

Place	Stuttgart, Germany
Year	2006
Architect	Un Studio
Structural engineer	Werner Sobek Ingenieurs
Precast/in-situ	In-situ (column precast)
Formwork	Wooden formwork, by Peri Gmbh
Concrete	Fair-face concrete B55, with steel reinforcement


Figure 2-5: Funen [41]

Funen [41]

Place	Amsterdam, the Netherlands
Year	2009
Architect	NLarchitects
Structural engineer	Ingenieursbureau Zonneveld bv
Precast/in-situ	In-situ
Formwork	Wooden formwork on a temporary scaffolding


Figure 2-6: EPFL Rolex Learning center [35]

EPFL Rolex learning center [10]

Place	Lausanne, Swiss
Year	2010
Architect	Sanaa
Structural engineer	Walther Mory Maier Bauingenieure AG
Precast/in-situ	In-situ
Formwork	Wooden formwork on a temporary support


Figure 2-7: Arnhem central station [42]

Station Arnhem central [1]

Place	Arnhem, the Netherlands
Year	In construction
Architect	UN Studio
Structural engineer	Arup
Precast/in-situ	Partial precast and partial in-situ
Formwork	Not yet applied, likely wooden formwork for the parts with small radius, milled EPS foam for the strong curved parts
Concrete	Fair-face concrete wit steel reinforcement

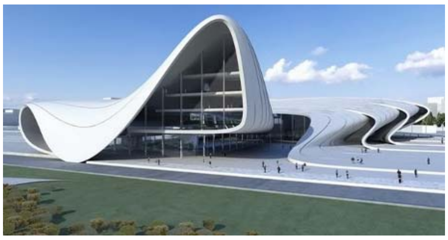


Figure 2-8: Haydar Aliyev Cultural Center [43]

Haydar Aliyev Cultural Center [43]

Place	Baku, Azerbaijan
Year	In construction
Architect	Zaha Hadid
Structural engineer	Adams Kara Taylor Engineers
Precast/in-situ	Precast panels, Steel trusses that supports the panels
Formwork	The geometry is maximum optimized that the elements have limited different shapes. Likely for each shape a formwork is made.

2.1.3. Shells

Orvieto Hangars [44]

Figure 2-9: Orvieto Hangars [44]

Place	Orvieto, Italy
Year	1935
Architect / Structural engineer	Pier Luigi Nervi
Precast/in-situ	Precast
Formwork	9 types reusable formworks

Sydney Opera House [5] [38]

Figure 2-10: Sydney opera house [45]

Place	Sydney, Australia
Year	1961
Architect	Jørn Urzon
Structural engineer	Ove Arup & Partners
Precast/in-situ	Precast
Formwork	Steel formwork. The shells are made in a way that as many as possible elements had the same shape, so the less different formworks where needed.
Connection	A bolted connection and cables that are post tensioned

Centre National Sportif et Culturel [18]

Figure 2-11: Centre National sportif et culturel [46]

Place	Kirchberg, Luxemburg
Year	1980
Architect / Structural engineer	Taillibert
Precast/in-situ	In-situ
Formwork	Steel scaffolding for one shell (reusable), with on top of it a wooden formwork.
Concrete	Concrete B35 with steel reinforcement. Because of the steep border concrete was developed with special rheological characteristics.

American Air Museum [10]

Figure 2-12: American Air museum [47]

Place	Duxford, United States of America
Year	1997
Architect	Foster and Partners
Structural engineer	Ove Arup and Partners
Precast/in-situ	Precast element, placed on temporary steel scaffolding.
Formwork	All elements have the same shape, so it is plausible that one mould is used.
Connection	'Wet' connection between the elements, reinforcement and in-situ concrete.

2.1.4. Canopies

Figure 2-13: The Shawnessy LRT station [48]

The Shawnessy LRT station canopies [48]

Place	Calgary, Canada
Year	2003
Architect / Structural engineer	CPV Group Architects & Engineers Ltd.
Precast/in-situ	Precast
Formwork	Steel plate formwork that could be reused for the shells.
Concrete	UPHC (Ductal) with poly-vinyl alcohol fibers
Connection	Bolted connection which is injected with grout.

Figure 2-14: Canopy at sanatorium Zonnestraal [49]

Canopy at sanatorium Zonnestraal

Place	Hilversum, the Netherlands
Year	2005
Architect	Henket en Partners
Structural engineer	ABT
Precast/in-situ	Precast
Formwork	Wooden formwork
Concrete	UHPC Ceracem/BSI B150 with steel fibers
Connection	Bolted connection

Figure 2-15: Toll station for the Millau bridge [50]

Toll station for the Millau Bridge [10]

Place	Millau, France
Year	2005
Architect	Michel Herbert
Precast/in-situ	Precast. Placed on temporary scaffolding.
Formwork	Steel formwork by PERI. The elements have the same shape, so also the formwork, but the elements are rotated.
Concrete	UHPC with steel fibers
Connection	Bolted connection and post tension cables through the elements.

2.1.5. Infrastructure

Figure 2-16: Spencer Dock Bridge [51]

Spencer Dock Bridge [10]

Place	Dublin, Ireland
Year	2008
Architect	Future Systems
Structural engineer	Arup
Precast/in-situ	In-situ
Formwork	Milled EPS mould, from Nedcam the Netherlands
Concrete	Normal concrete with reinforcement.

2.1.6. Conclusions

Before a small selection of projects, in time order that they where build, is given. All projects have curved, and most of them, have double curved surfaces. One of the first constructions with double curved surfaces were shells. This is due to the good mechanical performance of a shell. The shells were build in a time labour and time consuming way. During the years ways are tried to find to reduce the labour and so the costs. Usually this means concessions to the free form shape of the building. Examples of this are the American Air museum and the Sydney Opera House. The prefabricated elements all have the same shape, and could so be produced with the same mould. This reduces the cost enormously, but influenced the shape of the building. The Sydney Opera House was designed in a more free form way. The free form shape of the building means that all of the elements needed a unique formwork, which made the project too expensive.
As a result of the increasing computer techniques in time, architects use 3D computer software to drawn and design a free form surface. This free form shapes means little repetition and therefore many differently shaped elements. The techniques to produce these unique shaped elements did not follow this development, and makes the construction of it very difficult and so expensive. The Arnhem Central Station is an example of this. The free form shape is very expensive to build; it took a long time to find a constructor who could and wanted to build it. Here some concluding remarks are given.

Prefabricated or in-situ

Due to the better labour conditions and the high quality of the final product, a lot of work is nowadays prefabricated in a factory. Also the production of ultra high strength concrete can be better controlled in a factory. This phenomenon is also recognized with free form buildings. The problem with free form buildings is that the elements are double curved and mostly have unique shapes, so standard moulds are not suitable. Besides that double curved buildings are more expensive than standard ones. To be able to manufacture high quality double curved elements, which all have a unique shape it is logical to prefabricate them in a factory.

Formwork

In some projects it is seen that a free form shape is constructed out of elements with similar shape. In this case the double curved formwork can be reused many times. This makes it profitable to make a reusable formwork. In projects where the elements have unique shapes, other solutions where chosen. In some projects EPS foam was used that was CNC-milled to obtain the right shape. For every unique element another milled formwork is needed. Another possibility is to make the wooden formwork at the site on temporary scaffolding; Funen and Rolex learning centre are examples of this.

Connection
The most common solution for the connection between the elements is a bolted connection with post tension cables. The bolt connection connects the elements. With post tension cables a compression force is introduced in the elements, which makes sure that the elements cooperate structurally.

Construction
With or without the use of prefabricated elements mostly temporary scaffolding is needed to support the elements or the wooden formwork. With relative small constructions, e.g. the Shawnessy LRT station canopies, a whole element can be placed at once in its final position.

2.2. Existing techniques

2.2.1. Introduction

This section describes the existing techniques to make double curved elements. A brief description is given and the advantages and disadvantages are mentioned.

Figure 2-17: Timber mould [51]

2.2.2. Timber mould

With the use of timber any shape mould can be made. The formwork surface is mostly plywood or fibreboard. With good surface protection the moulds or parts of it could be reused. Rectangular moulds are easy to make. Double curved shaped moulds can be made, but require a lot of knowledge of the woodworkers. In some cases CNC-cutting or milling is used to make sure that the wood gets a curved shape. Double curved moulds are laborious to adapt to another shape, and therefore not very suitable for mass production of non-repetitive double curved elements.

Figure 2-18: Steel mould [52]

2.2.3. Steel mould

Instead of wood steel can be used for formworks. Due to the higher costs of a steel mould compared to timber, the formwork has to be reused as often as possible. Steel moulds are therefore mostly used in the precast industry. A double curved mould can be made of steel, but is only economical if it can be reused many times. So it is not suitable for unique shaped elements.

Figure 2-19: Foam formwork for Der Neue Zollhof [54]

2.2.4. Foam formwork

With the use of CNC computer technique a foam block, mostly expanded or extruded polystyrene (EPS/XPS), can be milled or by wire cutting shaped into the desired form. These blocks need a temporary support to resist the concrete pressure, see Figure 2-19, a wall element for Der Neue Zollhof. The moulds can be reused, if the same shape is needed. In cases where all of the elements have unique shapes this formwork technique is an expensive process, because for every element a unique foam formwork is needed. The foam itself could be reused by melting, but the residues of the concrete makes that difficult.

2.2.5. Inflatable/fabric/vacuumatic formwork

With a textile and air pressure, double curved formworks could be made. The tightening and the shape of the textile determine the shape of the concrete element. Research is going on at the TU in Eindhoven [16] and ETH Zürich [26] (Diederik Veenendaal). In Eindhoven a blob is made by applying a membrane that is inflated with air, and shotcrete on top of it. The disadvantage of this technique is the limitation of the feasible forms. With inflatable membranes only segments with circle parts can be made. This results in convex or cylindrical forms. The disadvantage with vacuumatic is that a contra mould is needed. The fabric dimensions and the contra mould are specifically for one form so the mould is not reusable, only parts of it.

Figure 2-20: A fabric formwork for a column [54]

2.2.6. Flexible formwork

In Section 2.3 the principle of the flexible mould will be explained in further detail. In short, it is a system of actuators with, on top of that, a flexible layer. The actuators can be adjusted for the desire height so a double curved surface could be made. The big advantage of this system is that it can be formed in almost every shape and it is reusable. In Section 2.3 the (academic) development of the system is described.

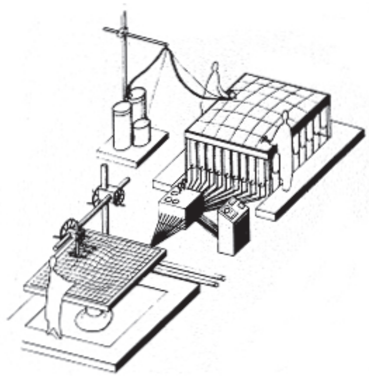


Figure 2-21: Flexible mould by Renzo Piano [5]

2.2.7. Conclusion

With all of the described techniques double curved elements can be made. The reusability of most of the systems for unique shaped elements can be doubted. The only system where, with the same mould material in a small effort a different form can be produced is the flexible formwork. In order to change the shape the pistons have to be adjusted.

2.3. Product development flexible mould

The economical solution for producing elements which all have unique shapes is a flexible formwork; a formwork that could be adjusted in every desirable shape.

In the sixties Renzo Piano mentioned this idea, see a picture of his idea in Figure 2-21. A scale model of an element could be placed into a machine that determines the height at each point. A system enlarges the element and transfers the data to pistons. The mould which is formed by an elastic layer on top of the pistons can be used to produce concrete elements. This system has never been build. Present-day the data for the height of the pistons could be obtained directly from a 3D computer model.

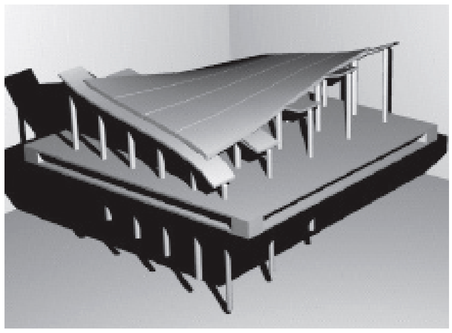


Figure 2-22: Strip mould [11]

H. Janssen [11] came with the idea of a strip mould, Figure 2-23. This mould consist of pins with are adjustable in height covered with strips in two directions. A rubber mat is placed on top of the strips, which gives the element a smooth surface. A problem with this layout is to find the best distance between the pistons and to choose the right stiffness of the strip layers.

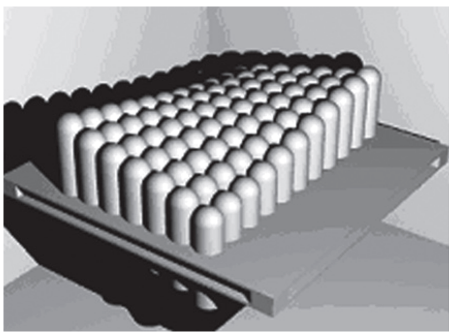


Figure 2-23: Pin mould [20] [21]

Another option is to place pins against each other, Figure 2-22. M. Quack [20] describes this principle and M. Roosbroeck [21] tested this idea. On top of the pins a flexible layer is placed which assumes that the elements have a smooth surface. The test element produced by M.

Figure 2-24: Element produced by M. Roosbroeck on a pin bed mould [21]

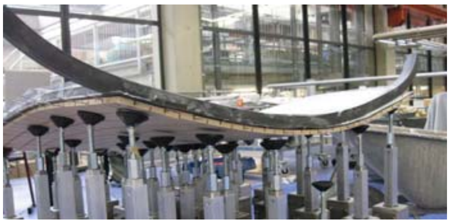


Figure 2-25: Actuator mould tested by K. Huyghe and A. Schoofs [9]

Roosbroeck (Figure 2-24) is did not have a smooth surface, the pins are visible. It was also difficult to realize a constant thickness of the elements.
Recently, K. Huyghe & A. Schoofs [9] have done some experiments with a prototype of an adjustable mould, Figure 2-25. This prototype was originally developed by Dr. Ir. K. Vollers and D. Rietbergen [28] to produce double curved glass facades. The prototype consists of actuators which are adjustable in height by a computer script. On top of the actuator a flexible layer is placed which is the formwork for the concrete. K. Huyghe & A. Schoofs have produced double curved façade elements with this system. To avoid the problem of producing the elements with constant thickness when the mould is in the curved end position, the concrete was poured when the mould was in a horizontal position. After a short period resting of the concrete the mould was deformed, by adapting the height of the actuators.
The research of K. Huyghe & A. Schoofs leaves aspects unsolved. As an example they choose the stiffness of the flexible layer without an accurate model of the plate behavior. Figure 2-25 shows that the bottom of the mould is too stiff, at some positions the flexible layer does not connect with the actuators.

2.4. Patents

A number of researchers and inventors have patented their ideas and inventions for a flexible formwork. A small selection of patented parts of the flexible formwork is discussed in this section.

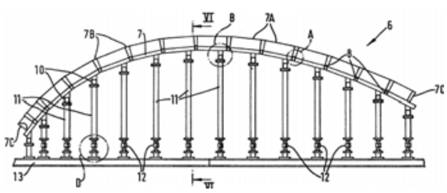


Figure 2-26: Flexible formwork by H. Vidal [27]

In the Patent of Henri Vidal [27], dating from 1987, a flexible formwork for single curved concrete elements is described, see Figure 2-26. The vertical pillars are adjustable in height, to adjust the formwork. The mould surface consists of hinged members to follow the height of the pillars and made the curved formwork. The mould surface is connected to the pillars with hinges. The pillars can slide horizontally at the connection with the bottom plate to take up the length adjustments of the curved mould surface. Since the purpose of our research is to manufacture double curved elements, the system described in this patent is not sufficient, although parts of it might be useful.

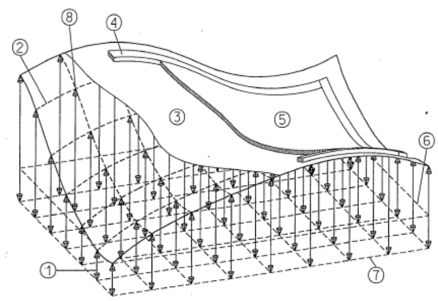


Figure 2-27: Flexible formwork by F. Kosche [12]

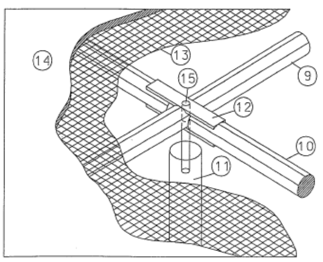


Figure 2-28: Detail of the connection mould surface and support point of F. Kosche [12]

In 2005 Florian-Peter Kosche [12] described in his patent a system to produce double curved elements, see Figure 2-27 for a drawing of his idea. The support points are adjustable by computer controlled actuators. Because of the double curved surface a plate can not serve as mould surface. To overcome that problem Kosche used a 'Gitterrost': a grid of strips perpendicular to each other. A detail of this grid is shown in Figure 2-28. The strips are situated in a grid and are connected to each other and to the supports by a sliding connection. This connection makes sure that the length changes of a curved surface compared to the original horizontal position can be taken up. On top of the strips a metal gauze is placed. This membrane must deform with the strips and make the double curved surface. This membrane must be stiff enough to make sure that it does not deflect too much at the middle. On top of the metal gauze a rubber mat is placed to pour the concrete at. The patent does not give a solution for the problem of in-plane stresses in the top layers, nor does it describe the structural behaviour of that top layer in sufficient detail. It does however offer interesting ideas which will prove to be useful in our work.

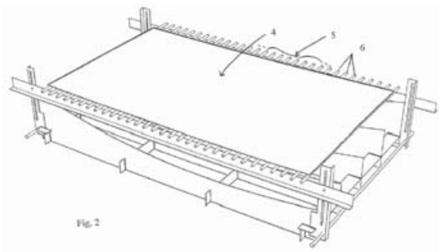


Figure 2-29: Initial horizontal surface [28]

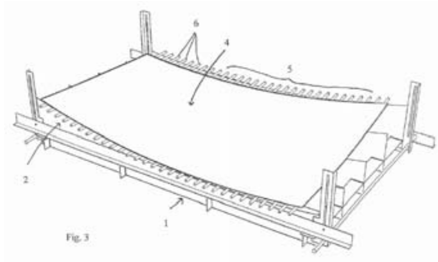


Figure 2-30: Curved surface [28]

The tests that are executed by K. Huyghe & A. Schoofs [9] are based on the ideas that are patented by Dr. Ir. K. Vollers and D. Rietbergen in 2010 [28] [29]. The Patent [28] describes the deforming of a flat glass plate into a double curved one, patent [29] does it for a viscous-liquid. In both patents the material is in a horizontal start position (Figure 2-29). The glass plate is heated to become plastically, when it can be deformed by lowering in on the curved supports (Figure 2-30). The viscous material is party hardened to make sure that it does not flow away, and then it is lowered to the curved supports (Figure 2-30). The materials can further harden in the final, deformed position.
Patent [29] describes ways to build the elastic mould surface for viscous-liquids. In patent [28] the glass plate is placed directly on rods, so an additional surface is not needed. The patents also describe ways to build the curved supports: curved panels in one direction and steel rods in the other, or pistons/actuators. Much of the work in our research builds upon the ideas of Vollers and Rietbergen, although the necessary modelling of the structural behaviour of the bending rods was not described in the patent in detail.

2.5. Conclusion

To meet the demands of architects concerning aesthetics and tolerances, and to keep up with new working methods, by using 3D computer models to make free form surfaces, a formwork for double curved elements has to be developed, which can be reused and adapted into every form. Several researchers and inventors have been working on this concept in the past, although the specific combination of a flexible concrete formwork and the exact structural modelling of the behaviour is new and has not been fully worked out yet by others. In this thesis the focus is on further developing the flexible formwork by addressing a.o. this aspect.

3. Ways of using the flexible mould

3.1. Introduction

A flexible mould system could be used in several ways. The flexible mould could be deformed first, so a double curved formwork is created. The concrete is then poured on the deformed mould. Another option is to position the flexible mould in a horizontal position; all pistons are set on the same height. The concrete is poured on the horizontal mould surface. After a short period of resting of the concrete the mould is deformed with the fresh concrete on top of it. The formwork could be semi-open, the formwork has a bottom mould with borders at the sides and the top side is open. Or the formwork could be closed; a top mould closes the formwork.
This chapter describes the different possibilities, the pro and contras and this chapter explains the chosen system in this thesis.

3.2. Semi-open formwork

3.2.1. Introduction

In this set up a flexible layer is used as bottom for the formwork with borders at the sides. The top side of the formwork is open. The actuators are situated below that layer. The mould layer could consist of different materials, but has to have the deformability to deform in the desire shape. This system could be used in two ways: deforming the mould before the concrete is poured on it, or with the fresh concrete on top of it.

Figure 3-1: Semi-open formwork

3.2.2. Deforming the mould before pouring the concrete

In this case the flexible mould is deformed to the desire position, and then the concrete is poured on it. The advantage is that the form of an element can be set directly at the flexible layer; the borders can be placed directly on the deformed mould. The deformed flexible mould is the shape of the element.
The disadvantage is that it is difficult to cast concrete at a constant thickness. When using spacers it is possible to set the thickness at some positions, but it is difficult to get the curvature between these points. Also a solid connection between the flexible layer and the actuator is needed. The flexible layer must be pulled at some places into its deformed position. This connection must transfer only the vertical forces. Because of the curvature of the mould, the deformed mould has an increased arch length compared to the original horizontal shape. This means that the vertical connection is at another position at the mould, see Figure 3-2. The support points stay on the same place. Such a glide connection is possible, but is hard to realise.

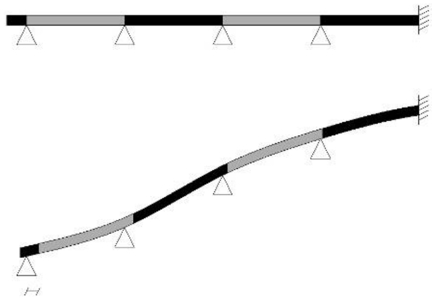


Figure 3-2: Shifting supporting points
Top: horizontal mould,
Bottom: deformed mould

3.2.3. Deforming the mould after pouring the concrete

K. Huyghe & A. Schoofs [9] used the flexible mould by this way. The flexible mould is set in a horizontal start position. The concrete is poured on this horizontal surface. After a short period of resting the mould with the fresh concrete on top of it is deformed. The mould and the concrete on top of it get the desired deformed shape.

An advantage of using the flexible mould in this way is the easy finishing and levelling of the fresh concrete. Because the concrete is poured on a horizontal surface it is easy to level the concrete, and to get a constant thickness of the element. Another advantage is the weight of the concrete; it helps to push the flexible mould into the desire shape. A glide connection, which is needed when the mould is deformed before the concrete is poured, is not needed. The flexible mould can be placed loosely on top of the support points. Only one fixed support point is needed to fixate the mould at one place.
A disadvantage of this system is the deforming of the fresh concrete. The fresh concrete must rest a while to make sure that it does not flow downwards from the curvature of the deformed mould. But it can not rest to long that it is solid and it cracks due to the deformation of the mould. This is also linked to the thickness of the concrete layer. The thicker the layer the larger the stresses are, at a given curvature due to the deformation process. It is also difficult to position the border of the formwork on the right place on this way. The border has to be on the horizontal mould but on the deformed end position.
K. Huyghe & A. Schoofs have shown that it is possible to produce thin façade elements with this method.

3.3. Closed formwork

3.3.1. Introduction

In this set up there is a flexible layer on the bottom and the top side. In this way there is a closed formwork. Because of the top mould, the fresh concrete can be poured directly into the deformed formwork. The best is to use a self compacting mixture to make sure that the concrete flows everywhere. When pouring the fresh concrete directly into the deformed shape, the concrete is not disturbed due to a deformation process.

3.3.2. Closed formwork with spacers

Figure 3-3: Closed formwork with spacers

Spacers on top of the bottom layer carry the top layer of the mould; these spacers remain in the hardened element. This will disturb the appearance of the final element. Because the mould is first deformed, there is no concrete weight to push the mould into the deformed position. A difficult connection is needed between the mould and the elastic layer, see Section 3.2.2. Or ballast could be placed on top of the top mould surface to deform the mould into its deformed shape. Also the fixation of the top layer with spacer is difficult to realize. The top layer must have exactly the same shape as the bottom layer in order to obtain an element at constant thickness.

3.3.3. Closed formwork with double actuators

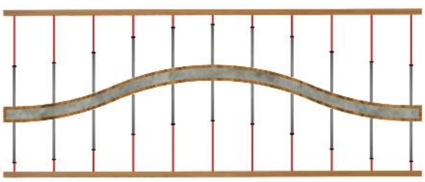


Figure 3-4: Double actuator formwork

At the top and bottom sides are actuators to bring the mould in the right shape. With this method it is also easy to make elements with a variation in thickness, because the top and bottom actuators could have different heights. A disadvantage of this system is that the connection between the actuators and the flexible layer is difficult (Section 3.2.2). Without the weight of the concrete the moulds must be positioned in the right shape. This means that the connections must transfer tension forces. Because of the horizontal shifting of the mould surface this is difficult to realize.

3.4. Conclusions

The most accurate way of producing double curved elements is to use a closed formwork. This set up does not disturb the fresh concrete, and the thickness of the elements is easy to control. A big disadvantage of this system is the difficult connection between the flexible layer and the actuators. This connection must transfer the vertical forces, but must be able to slide on the mould surface. This is due to the difference in arch length of the horizontal and the deformed mould; if these movements are prevented large horizontal forces appear.
In this thesis the principle of a semi-open formwork is used, with the deforming of the mould after pouring of the concrete.
This set up make use of the weight of the concrete to deform the flexible layer into the desire shape. The flexible layer could lay loosely on the supports. K. Huyghe & A. Schoofs [9] have shown that it is possible to produce thin façade elements with this method. First the focus lies on improving their results and methodology.
The prime objective of this thesis is to make double curved load bearing elements. Because the focus is on improving the flexible formwork, and therefore thin panels, the idea is to use the thin panels as a predalle [28].
A predalle is a thin prefabricated panel with an in-situ top layer of concrete. The thickness of the prefabricated plate is about 50-80 millimetres. A lattice girder protrudes out of the plate, with makes sure that the plate can be lifted. To predalle are places at the building side on it's finally position and is mostly temporary supported. The temporary supported is needed to carry the load of fresh concrete. After placing the couple and additional reinforcement the top lay of concrete is poured. The advantage of this system is the easy connection between the partly prefabricated elements (in-situ layer of concrete) and the fast building speed due to the lost formwork.
This system could also be used with double curved panels. The panels are in a factory prefabricated with a flexible mould. The double curved panels are then transported to the side and placed in their final position. After placing the additional reinforcement the in-situ layer of concrete is poured.
A disadvantage of this system is the additional in-situ concrete, which is difficult to get at the desire level at the building side. This is mainly difficult when there are stile slopes or a lot of curvatures.
The biggest advantage is that no difficult connection is required between the elements. The in-situ layer of concrete transfers the forces between the elements. When making use of only prefabricated elements the connection between the elements is difficult to realize. This connection is also undesirable visible and require additional finishing.

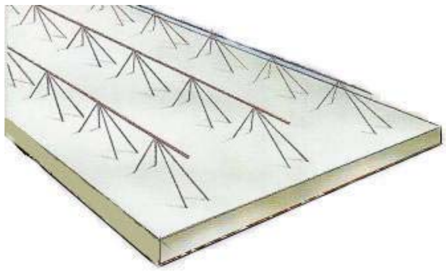


Figure 3-5: Predalle [56]

4. Design parameters of the flexible formwork

4.1. Introduction

In this chapter the design parameters of the flexible mould and for double curved surfaces are described. These parameters are the flexible formwork itself (the actuators, elastic layer and the border of the formwork), the concrete, the reinforcement, the connections between elements and the panellization.

4.2. Flexible formwork

4.2.1. Introduction

The flexible formwork consists of three main parts: the actuators (supports), the elastic layer and the border of the formwork. The next sections discuss these parts.

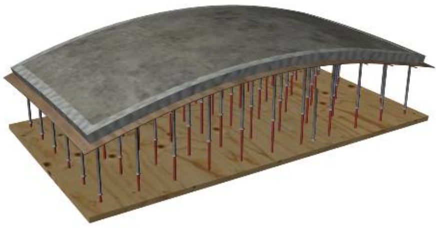


Figure 4-1: Flexible formwork

4.2.2. Actuators

The actuators are the support points for the elastic layer. These points have to be adjustable in height, to allow the formwork to be accurately deformed in a flexible manner. To be able to set the support points automatically in the desired height, actuators could be used. If linked to a computer, it can be set automatically in the desired height; K. Huyghe & A. Schoofs used this method. In this way the formwork could be set fast and accurate in the desire height. A drawback of the actuators is that they are expensive and vulnerable. The use of these actuators is outside of the scope of this thesis; in the test a simplified method is used, a screw-thread and nuts with a hollow pipe.

4.2.3. Flexible layer

The elastic layer has to have the right elasticity to follow the support points and so get the desired shape. But it can not be too low that the displacement between the supports became too large. In Chapter 5 a model is proposed that describes this behaviour in a single curved mould and Chapter 9 describes this behaviour for a double curved mould.

4.2.4. Border of the formwork

Introduction

On top of the flexible layer a border has to be to define the element. The way of using a flexible formwork determines the properties of the border. It was decided to use the method deforming the mould after pouring of the concrete. With this method the concrete is poured on the horizontal mould and the mould is deformed with the concrete on top of it. This leaves two ways of using the border. A border is placed on the horizontal mould and serves as border for the fresh concrete. Before deforming of the mould, the border is removed. The concrete must keep itself into its shape. Another option is to use a flexible border. This flexible border must deform with the mould. On this way

Figure 4-2: Border in torsion

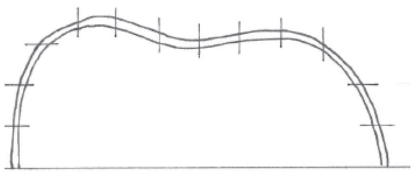


Figure 4-3: Slicing a shape with the vertical and horizontal method [10]

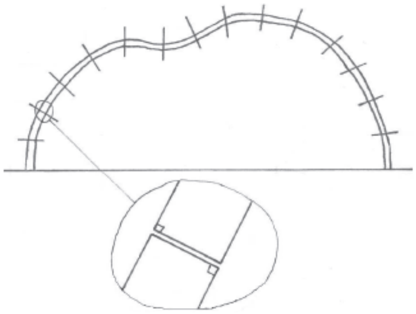


Figure 4-4: Slicing with the perpendicular method [10]

the fresh concrete has a horizontal resistance during the deformation of the mould.
A flexible border is the best option, because of the sensitivity of the fresh concrete it is desirable that the concrete is supported during deforming and final hardening.
The shape of the border depends how a free form shape is split up into elements. E. den Hartog [10] describes two methods of slicing: a combination of horizontal and vertical (compared to the whole building), Figure 4-3 or perpendicular, Figure 4-4. When using the perpendicular method, the border is always perpendicular to the flexible mould surface. This is a better starting point to use for mass production with the flexible formwork composed to the other method. The border could be quadrangle and used for every element. When using the method as shown in Figure 4-3 the border has to have another shape for every element. The edge of the border is different for all of the elements. A disadvantage of the perpendicular method is that the border is also in torsion (Figure 4-2). As is shown in Figure 4-2 the torsion is relative low compared to the curved shape of the border, so if a material is found that could bend into the desire way, the torsion could also be taken up.

Material
The material which is needed as border must be flexible enough to follow the deformed mould, but must also be stiff enough to withstand the horizontal concrete pressure.
Some materials which have these properties are rubber and foam. Rubber (or rubber-alike) is a very flexible material, which can be bought as a solid material or be casted in every shape by mixing two liquids into polyurethane. The rubber (when it is hardened) has a smooth surface which can act as the mould surface. Disadvantages of the rubber are the relatively high costs, and the processing time. When using the liquids first, a contra mould has to be made to cast the liquids in.
Foam is normally used in a.o. mattresses and pillows. This material is very flexible and can be ordered at relatively low costs, with every desirable (rectangular) format, on the internet e.q. [62]. Two types of foam, polyether and cold foam have been used in the tests described in this thesis. Cold foam has a high resilience, and is normally used for applications with high use, compared to polyether which has a lower resilience. The resilience of cold foam could be so high that the border is not flexible enough, or the resilience of polyether is so low so the border is too weak. Both types of foam are tested, see Chapter 8.
The hardness of the foam is measured with the shore value. This shore value is measured with a durometer. A durometer test is based on the penetration of a type of indentor into the material. The indentor is applied with a force at the material. It’s penetration into the material determines the shore value. This penetration depends on the elastic modulus, the viscoelastic behavior of the material and the geometry of the indentor, so each type of indentor has its own scale [1]. An overview of the types of shore indentors and the material at which they can be used is seen in Table 4-1.

To test the shore value of the foam and appropriate scale from Table 4-1 should be used.

Type (Scale)	Typical Examples of Materials Tested	Durometer Hardness (Typical Uses)
A	Soft vulcanized rubber, natural rubber, nitriles, thermoplastic elastomers, flexible polyacrylics and thermosets, wax, felt, and leathers	20–90 A
B	Moderately hard rubber, thermoplastic elastomers, paper products, and fibrous materials	Above 90 A Below 20 D
C	Medium-hard rubber, thermoplastic elastomers, medium-hard plastics, and thermoplastics	Above 90 B Below 20 D
D	Hard rubber, thermoplastic elastomers, harder plastics, and rigid thermoplastics	Above 90 A
DO	Moderately hard rubber, thermoplastic elastomers, and very dense textile windings	Above 90 C Below 20 D
M	Thin, irregularly shaped rubber, thermoplastic elastomer, and plastic specimens	20–85 A
O	Soft rubber, thermoplastic elastomers, very soft plastics and thermoplastics, medium-density textile windings	Below 20 DO
OO	Extremely soft rubber, thermoplastic elastomers, sponge, extremely soft plastics and thermoplastics, foams, low-density textile windings, human and animal tissue	Below 20 O
CF	Composite foam materials, such as amusement ride safety cushions, vehicle seats, dashboards, headrests, armrests, and door panels	See Test Method F1957

Table 4-1: Types of shore indentors [1]

For example, for the types Shore A and Shore D the relation between the elasticity and the shore value is given by [19]:

$$Log(E) = 0.0235S - 0.6403$$
$$S = S_a \text{ For } 20 < S_a < 80 \qquad (4.1)$$
$$S = S_D + 50 \text{ For } 30 < S_D < 85$$

If the shore value is measured with a durometer the elasticity can be determined by (4.1). Or, if the elasticity is known, the shore value can be determined with the same equations. In Table 4-2 the relation between E, Sa and Sd is shown.

E (MPa)	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60
Sa	28	40	48	53	57	60	63	65	67	70	82	90	95	99	
Sd											32	40	45	49	53

Table 4-2: Elasticity versus shore value

For the foam used in this thesis the shore value unfortunately could not be determined. The foam was ordered at a consumer store [62], where it is normally sold as filling in pillows. The store could not give the shore value or the elasticity of it. A durometer was not available to measure the shore value. If for further research the shore value has to be known, the foam that is used in this thesis (Polyether SG 40 extra firm and Cold foam HR SG 55) could be ordered at the shop [62] and the shore value of it can be measured with a durometer. Some samples of the used foam are still available in our lab. The abbreviation SG stands for Soortelijk Gewicht (specific weight), which might give an indication for comparison with other foams.

A disadvantage of the foam is the porosity of the surface. The fresh concrete can not be poured directly to the foam. The moisture of the concrete is absorbed into the foam and the hardened element is difficult to demould. The foam can be protected with a foil. The foil gives the hardened concrete a very smooth surface. A disadvantage is the wrinkles of the foil when the mould is deformed, especially with small radius and in corners. Another way to protect the foam is to seal it. This is a time consuming process and gives a rough surface, see test results Section 10.4.2. The best option is to use a spray that can be easily applied, and forms a smooth protecting layer on the foam.

Connection with the flexible layer

The border must be fixed to the flexible layer. This must be a solid connection. The border must remain in place when the concrete applies a horizontal load. To keep the system flexible the border has to be placed on different positions to be able to make unique elements. A solid connection is a glued one. This connection limits the dimensions and the shape of the elements. The foam can be removed from the flexible layer, but the glue damages the foam. An option is to glue the foam on the largest outside position to the flexible layer, and place insert blocks of foam into the outside ring to make the different shaped elements (Figure 4-5).
The connection does not need to be a solid. The foam could be placed to the flexible layer by clamps. These clamps should be placed at a certain distance from each other. For an element a large amount of clamps are needed. The seam between the foam and the flexible layer has to be sealed, for the leaking of the fresh concrete, which makes the system also less reusable.

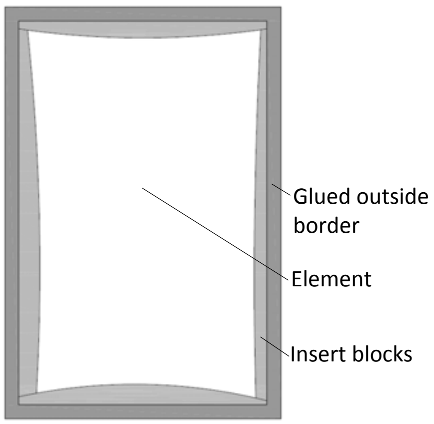


Figure 4-5: Glued outside border with insert blocks

Placements of the borders

Due to the method of using the flexible formwork, deforming the mould with the partly hardened concrete on top of it, the border has to be placed on the horizontal mould in the deformed position. A curved element with straight edges (Continues line Figure 4-6) has on a horizontal surface curved edges (Dotted line Figure 4-6). The border has to be placed on this curved line. A common used technique in the precast industry is to use laser to mark the places of the edges and for example the reinforcement of the anchors on the formwork [13]. This technique could also be used for the flexible formwork to project the places of the borders. With a computer program the places of the horizontal position of the border could be determined, to get the border on the right place on the deformed position.

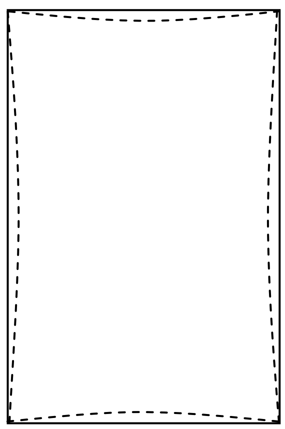


Figure 4-6: Straight edges at curved mould, gives curved edge at horizontal mould

4.3. Concrete

Due to the composition of concrete, and the variations in quantities and components of it, infinity concrete mixtures can be made with all kind of properties. For the flexible formwork the time after pouring of the concrete when the mould can be deformed is the important parameter, called hereafter the “deform time”. At this time the concrete has hardened enough to keep its shape and stability, but is weak enough to take up the deformation of the mould. Because the mould is used in the precast industry this time should be as soon as possible after pouring of the concrete, to speed up the production process. In Chapter 6 the concrete properties and the deform time are discussed.

4.4. Reinforcement

In curved elements the reinforcement also has to have the same curvature. Because of the method of production, the reinforcement has to curve with the concrete. Normally reinforcement steel rods would be too stiff to follow the curvature, the reinforcement pops out of the concrete. Instead of traditionally reinforcement, new methods could be used, such as fibre reinforced concrete or strands. Chapter 7 describes these possibilities.

4.5. Connection between the elements

A disadvantage of the use of prefabricated elements is the needed connection between the elements. This connection must transfer the forces between the elements. E. den Hartog [10] provides an overview of different connection types to connect structural prefabricated elements, see Table 4-3. These types of connections are normally used with standard rectangular shaped elements, but are also suitable for double curved element. A big disadvantage of most of the connection types is the visibility of it. Only the post-tensing connection leaves a small seam between the elements, and is the best solution for the overall visibility. The other connection systems leave bigger seams or the mortal leave a different in colour behind.

In this thesis the flexible formwork is developed for thin panels, which could be used as load bearing elements by using them as predalle. The prefabricated elements are placed beside each other on a temporary support, after placing additional reinforcement a top layer of in-situ concrete is poured, which connects the elements.

	Wet connection	Bolted connection	Post-tensioning connection	Welded connection	Glued connection	Fibre joint
Type/kind	The seam between the elements is reinforced and filled with mortar.	Provisions at the elements makes it possible to bold the elements together.	Through ducts tendens are placed. These tendens are tensioned.	Provisions at the elements makes it possible to weld the elements together.	By glue the elements are connected.	Instead of reinforcement at the wet connection a fiber mixtere is used.
Connection strength	Strong	Strong, but stiffness and moment capacity difficult to ensure.	Strong and increased tensile capacity of the shell due to compression of the	Strong	Strong, but long term performance not completely known.	Strength and durability properties not completely known.
Load introduction	The loads are introduced equally spread over the edge of the elements.	High peaks stresses can occur, but should be avoided with proper a design.	High peak forces in outer elements where tension tendons are anchored can occur.	No peak stresses in the concrete should occur.	The loads are introduced equally spread over the edge of the elements.	The loads are introduced equally spread over the edge of the elements.
Adjustment possibility	Enough room for adjustment due to space between the elements.	Depending on the design, a good design has enough room for adjustment.	Not very large	Small	Not very large, but depends on the thickness of the layer of glue.	Sufficient
Suitable for use with UHPFRC	Not especially suitable, due to large difference in strength of mortar and concrete.	Suitable	Very suitable, due to high compression strength of the concrete.	Suitable	Very suitable, due to homogeneity of the concrete.	Very suitable
Ease of assembly	It is labour intensive to place reinforcement and mortar, but the placement of the elements is easy.	When connection is designed properly the assembly is easy.	Complicated due to temporary fixing that is needed and the application of post-tensioning.	Difficult, due to welding on site.	Edges have to be roughened and cleaned carefully before the application of the epoxy layer.	Mortar should be placed in a relative narrow joint and the fibre may not bend during assembly.
Assembly speed	Slow, mortar need time to harden and gain strength and the placement of the mortar is weather dependent.	Fast and weather independent assembly possible with a good design.	Slow, all elements have to be placed and fixed temporarily before tension can be applied.	When the weld is made the connection is strong, but careful preparation before welding is essential.	Slow, the edge have to be treated before the epoxy can be applied and the epoxy need some time to harden and gain strength.	Slow, mortar needs some time to harden and gain strength.
Durability	Good	Good, when protected properly against corrosion and fire.	Good	Good, when protected properly against corrosion and fire	Still some research to long term performance needed.	Not known, research needed.

Table 4-3: Overview of the properties of different connection systems [10]

4.6. Panellization

Panellization is the technique where a surface is split up into elements. This process is always needed when prefabricated elements are used. In rectangular buildings a storey height is mostly also the height of the elements. The width of the element is determined by the transport dimensions. Mostly prefabricated elements are transported by truck so the road limits these dimensions.
By a free form shape the panellization is more difficult. The dimensions of the elements are also limited by the maximum transport dimensions. But the number a structure can be split up into elements is infinite, for example the Haydar Aliyer Cultural Center (Figure 4-7). With this kind of shapes the architect wants a fluent seam between the elements over the building. There are also preconditions such as: the maximum length and width, and the maximum angle between two edges (the optimum shape for the flexible formwork are squares). A minimum of differently shaped elements minimize the number of moulds. This panellization is an optimization process and requires a lot of mathematic knowledge.
Evolute [57] came up with a plug-in for Rhinoceros that can perform this optimization process. Given a free form shape, with set preconditions, the plug-in gives, after some steps, an optimum panellization. This tool can be used to penalize a free form shape. See Figure 4-8 for a result of the panellization process. The different colours display the different shape, and number of curvature elements.

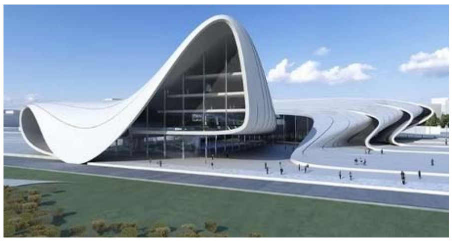


Figure 4-7: Haydar Aliyev Cultural Center [43]

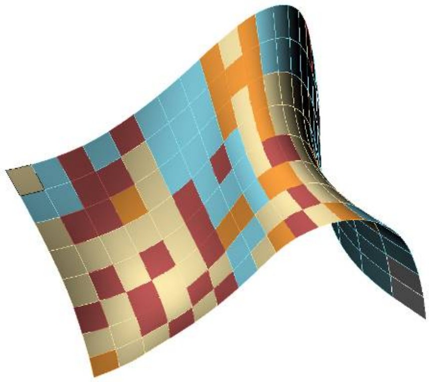


Figure 4-8: A free form shaped building penalized with Evolute tool [57]

5. Flexible mould layer, Single curved

5.1. Introduction

During the tests of K. Huyghe & A. Schoofs [9] it became clear that it was difficult to choose the right material and thickness for the flexible layer. As seen in Figure 5-1, the flexible layer does not touch all of its supports. The flexible layer was too stiff. They used a MDF plate, which was carved at both sides to reduce the stiffness of the plate. This weakening helps to decrease the stiffness, but it was more a trial and error process.
To be able to predict the behaviour of the elastic layer a model is proposed that determines the required stiffness for the elastic layer to make sure that is follows the desire shape. In this chapter a model is made for an in one direction curved element, a beam. In Chapter 9 the model is extended to a double curved model.

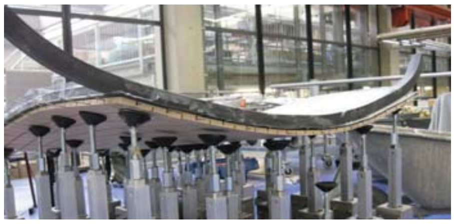


Figure 5-1: Actuator mould [9]

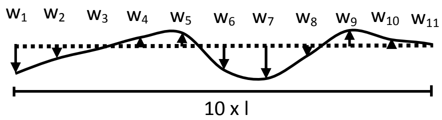


Figure 5-2: Desired shape mould

5.2. Mechanical model

For the description of the flexible behaviour of the elastic layer, a single curved model is made first. In Figure 5-2 an example of a curved shape is drawn with eleven supports. The displacement w is the distance at the supports from the neutral line to the final position. Figure 5-3 shows what happens if the flexible layer is to stiff. The mould is not connected to the supports. The continued line is the mould and the dotted line is the desire shape.
If the flexible layer is too weak, the local displacement becomes to big (Figure 5-4).

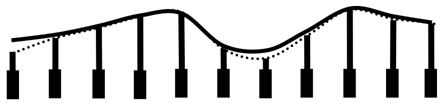


Figure 5-3: Mould is to stiff

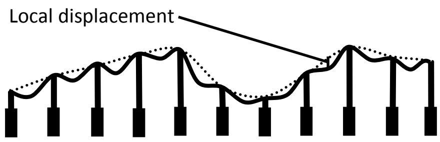


Figure 5-4: Mould is too flexible

For the model a line is drawn between the outer supports, Figure 5-5. The displacement of the other supports is related to the line between these supports. The displacement is positive downwards. In Figure 5-6 the mechanical model of the system is given, with the moved support points.
This is a statically indeterminate system. By replacing the middle supports by an unknown force the system became statically determinate (Figure 5-7).

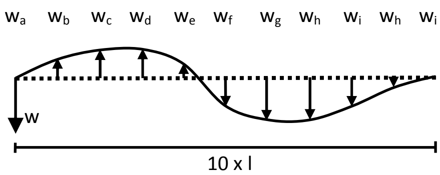


Figure 5-5: Mould shape and numbering supports

In these equations, only the bending part EI is taken into account. This is valid if there is no horizontal connection between the flexible layer and the supports. If there would be a horizontal connection at the support the membrane part EA has to be also taken account.

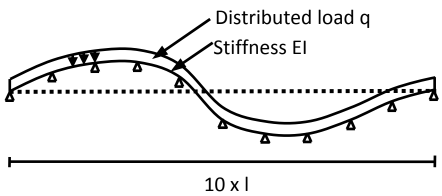


Figure 5-6: Mechanics model

This system can be solved using the force method. At every middle support point the displacement due to the distributed load q is equal to the displacement due to the support forces.
So at point B the equation becomes:

$$w_b = w_q - (w_b + w_c + w_d + w_e + w_f + w_g + w_h + w_i + w_j) \tag{5.1}$$

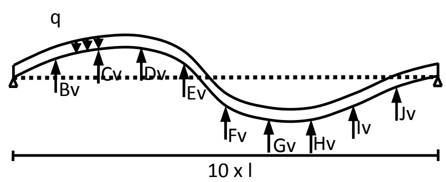


Figure 5-7: Statistic determine model

This step has to be executed for every support. Nine equations are obtained with nine unknowns, with which the reaction force could be determined.

For the bending of a beam due to a distributed load (Euler-Bernoulli differential equation):

$$EI\frac{\partial^4 w(x)}{\partial x^4} = q \tag{5.2}$$

This results in the displacement at a place x [7]:

$$w_q = \frac{qx}{24EI}(L^3 - 2L^2 + x^3) \tag{5.3}$$

The equations for the forces become [7]:

$$w_{fL} = \frac{Fbx}{6EIL}(2L(L-x) - b^2 - (L-x)^2); \tag{5.4}$$

If the place x is left of the force.

$$w_{fR} = \frac{Fa(l-x)}{6EIL}((2Lb) - b^2 - (l-x)^2) \tag{5.5}$$

If the place x is right of the force.

For the equation in B the equations (5.4) and (5.5) have to be filled in nine times, for each reaction force B t/m J. See Appendix B for an elaboration.
Beware that in maple the positive axis is upwards, while in the schematization (Figure 5-5) the positive direction is downwards.

For example, reaction force B became:

$$B_v = \frac{1}{1531316}\frac{1}{l^3}\left(-171589ql^4 + \begin{pmatrix} 1494552w_B - 1438728w_C + \\ 628776w_D - 168480w_E + \\ 45144w_F - 12096w_G + \\ 3240w_H - 864w_I + 216w_J \end{pmatrix} EI\right) \tag{5.6}$$

So a reaction force depends on: the length between the supports (l), the disturbed load q, the stiffness (EI) and the displacement of the supports (w_b t/m w_j)

A starting assumption is that the moulds must touch all supports, so the reaction force has to be bigger than zero. If this is the case, the elastic layer could lay loosely on the supports; the compression force pushes it downwards. If there are tensions forces the mould will come of the support. Then a glue connection between the elastic layer and the support point is needed, a connection that only transfer the vertical load, not the horizontal ones.

In this maple model (called hereafter the 'model'), Appendix B, the known displacement, the q load (layer of concrete), the elasticity of the layer and the distance between the supports is filled in. Along with an equation for the local displacement between the supports the required thickness of the layer h is calculated.

After choosing the thickness of the elastic layer the displacements, moment and shear line can be calculated with the model. Also the length of the curved mould is calculated, this is the length that the mould in the horizontal start position needs to have.

5.3. Verification of the model

The Maple model is verified with Matrixframe to verify if the maple model describes the beam behaviour in a correct way.

l = 500 mm.
q = 0.6 N/mm
h = 20 mm

***Supports displacement** (mm)*

wb = -31
wc = -59
wd = -81
we = -95
wf = -100
wg = -95
wh = -81
wi = -59
wj = -31

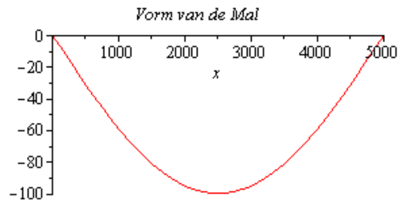


Figure 5-8: Displacement

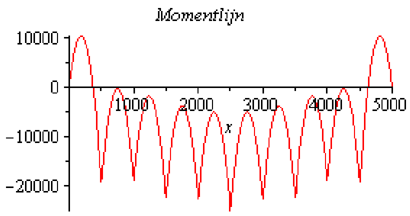


Figure 5-9: Moment line

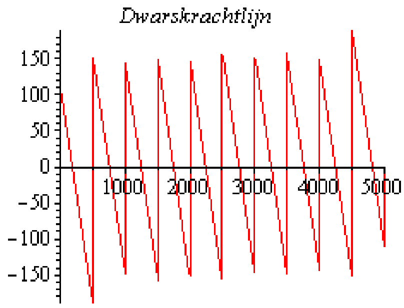


Figure 5-10: Shear force line

Reaction force			***Field moment***			***Support moment***		
	Maple	Matrix		Maple	Matrix		Maple	Matrix
Av	63	-63	M1	3331	3331			
Bv	334	-334	M2	-35405	-35405	MB	-43385	-43385
Cv	311	-311	M3	-59492	-59492	MC	-69458	-69457
Dv	331	-331	M4	-73278	-73277	MD	-89780	-89779
Ev	275	-275	M5	-83252	-83252	ME	-94419	-94419
Fv	368	-368	M6	-83252	-83252	MF	-111540	-111539
Gv	275	-275	M7	-73278	-73277	MG	-94419	-94419
Hv	331	-331	M8	-59492	-59492	MH	-89780	-89779
Iv	311	-311	M9	-35405	-35405	MI	-69458	-69457
Jv	334	-334	M10	3331	3331	MJ	-43385	-43385
Kv	63	-63						

The values calculated with maple are the same as with a Matrixframe. So it is assumed that the model describes behaviour of a single curved element in a correct way.

5.4. Case study; Funen

The model that is proposed provides as a result the required stiffness of the elastic layer for producing single curved elements with a flexible formwork. This model is applied on a recently build project, to see if it could be made with elements which where prefabricated with a flexible formwork.

5.4.1. Introduction

The Funen is a residential building in Amsterdam, the Netherlands. The roof structure is a curved surface. As seen in Figure 5-12, the formwork is made at the side, and consisted of wooden plates with a temporary steel support. The concrete roof structure has an overall thickness of 250 millimetres.
The roof is single curved. The isocurves in Figure 5-13 indicate that in one direction they are straight lines, and in the other direction they are curved. This is also seen at the temporary support; see the beams in the direction of the straight isocurves. The minimum radius is 11 meters; which is in the top corner of the building, element A, Figure 5-14.

Figure 5-11 Funen, Amsterdam [41]

Figure 5-13: Formwork Funen [41]

5.4.2. Panellization

In Figure 5-13 the load bearing walls with the roof structure is given. The walls are house separations and are over the length of the building. These walls carry the roof. In case of building the roof out of element, the elements have to span between these walls to. The lengths of the elements are then 5 meters for the outside elements and 7 meters for the middle ones. These dimensions are within the transport limits. The maximum width of the elements is 3 meters. This gives the segmentation given in Figure 5-14.

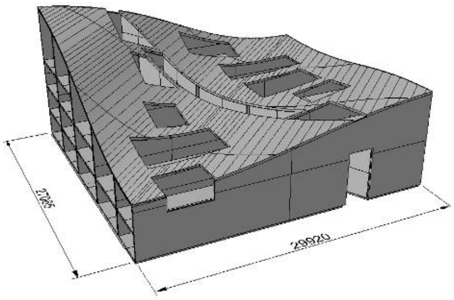


Figure 5-13: Roof with isocurves and bearing walls (mm)

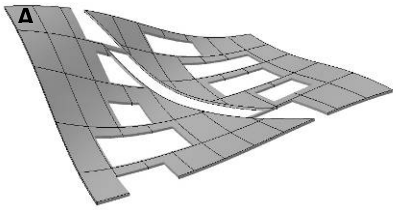


Figure 5-14: Segmentation Roof

5.4.3. Flexible mould layer

The panellized elements are prefabricated with the actuator mould. The predalle principle is applied, so the under skin, 50 millimetres, is prefabricated and the top layer, 200 millimetres, is cast at the site. The decisive element, element A, which has the smallest radius, is considered (10 meters). In Figure 5-15 the dimensions and the direction of the curvature is drawn. Due to the single curvature of the roof, the point's perpendicular to the curvature have the same height. In Figure 5-15 all of the points on the thin line have the same height as the point on the curvature, thick line.

The set up of the pistons is possible in two ways, in a square pattern (Figure 5-16) or at the isocurves (Figure 5-17).

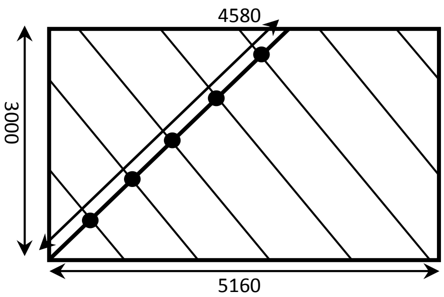


Figure 5-15: Dimension and isocurves element A

Square pattern (Figure 5-16
By using a square pattern the curvature goes by the diagonal. If choosing 500 millimetres between the actuators, the length of the span is 763 millimetres and has 7 supports points. With this set up 70 actuator are needed. The curvature is upwards in the element, so the displacement is negative.

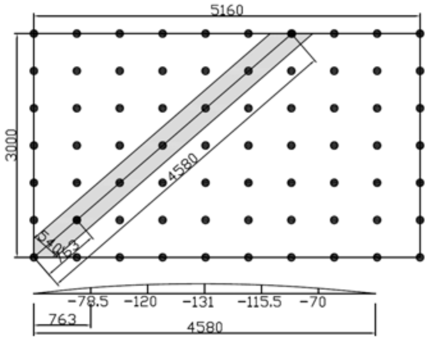


Figure 5-16: Square actuator

Appendix C shows the maple sheets of this set up.
Figure 5-16give the height of the support points. For the flexible layer timber is used with an elasticity modulus of 7000 N/mm^2. The maximum local displacement is set at one millimetre.
The needed thickness of the elastic layer became:
10.87 < h > 15.86 (mm)

Isocurves pattern (Figure 5-17
By using the isocurves pattern the beam spans nine supports. The distance between the supports is 572 millimetres. The total amount of actuators is 59.

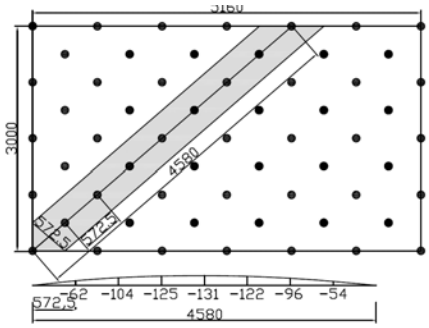


Figure 5-17 Isocurve actuator

The model gives for the thickness of the layer (with the same parameters as the square pattern, only changing the height of the supports see Figure 5-17en appendix C for the sheets):
8.31 < h > 15.26 (mm)

Conclusions
By using the same plate thickness the total of actuators is at the isocurve configuration less. So the isocurve configuration is the best option for this element of the Funen building.

5.5. Conclusion

The model that is proposed describes the behaviour of the flexible layer that is single curved in a correct way. With this model the required thickness of the flexible layer can be determined if the shape is known. The model has as starting point that no tension force acts between the layer and the support points. The flexible layer is pushed against the support and no structural connection between it is needed.

6. Concrete

Concrete is a suitable material to use for double curved elements, the fluid concrete can be poured in every shape (if necessary in a closed formwork). The concrete itself is not an issue for making double curved elements, but the formwork is. Concrete consists of a composition of different materials (cement, aggregates, water and additions), which can be combined in an infinite number of mixtures, with different kind of properties. First, general aspects of concrete mixtures are treated, then the deform time, which is the main aspect for the flexible formwork, is discussed.

6.1. Concrete mixtures

6.1.1. Self compacting concrete

Self compacting concrete is very fluid, under its own weight the fresh concrete compacts into a horizontal position, also through a very dense reinforcement. Due to self compacting concrete the work environment improves. The compacting of concrete is a very unpleasant and unhealthy activity. If the flexible formwork is used in the way that it would be first deformed and then the concrete was poured on, self compacting concrete would not be suitable. The fresh mixture is so fluid, that it flows out of the deformed mould. The way that the flexible formwork is used: pour the concrete in a horizontal position and the mould will be deformed after a while, this process makes it very suitable for a self compacting mixture. However the strength development of the mixture must be fast to make the mixture quick stable to be able to deform it. The total throughput time must be as short as possible.

6.1.2. High strength concrete

Over the last years the maximum strength of hardened concrete has grown. In former times a mixture B35 was a high strength mixture (a compressive strength of at least 35 N/mm^2), now mixtures of B200 are possible. A mixture with a compression strength of at least 65 N/mm^2 is called high strength concrete (HPC), and ultra high performance concrete (UHPC) has a compression of at least 115 N/mm^2.

Advantages of (U)HPC are:

- High strength which make slender and lighter constructions possible.
- Fast strength development.
- An optimal packing of the grains which results in durable structures, and it is even applicable in an aggressive environment.
- The consistency of the mixture can be self compacting.

Disadvantages of (U)HPC are:

- Due to the density and the high strength matrix it is a brittle material. Fibers (steel/synthetic/glass) are needed to overcome this aspect, so all (U)HPC must contain fibers.
- Because of fibers and other components, the mixture became very expensive. This is not compensated through the reduction in

thickness of the elements. In general, the higher the strength class the more expensive the mixture. Compared to other expensive cladding materials however (U)HPC might be economical.

6.1.3. Standard mixtures

There are some ‘standard’ (U)HPC mixtures on the market. D. Hartog [10] has made an overview of some of these mixtures including their properties, see Table 6-1.

		35/45 [16]	**C90/105** [16]	**ECC** [34]	**SIFCON** [1],[27]	**SIFCON** [1],[27]	**Ductal** [4],[52] (no thermal treatment)	**Ductal** [4],[52] (thermal treatment)	**BSI/ CERACEM** [4],[41], [51]	**Hybrid fibre concrete** [1], [13]	**CEMTEC** [1],[37]
Compressive strength	N/mm2	45	105	70	-	69	100-140	150-180	175	100-130	220
Tensile strength	N/mm2	2,2	3,5	3,7	28	14-16	5	8	-	10-12	20
Bending tensile strength	N/mm2	-	-	-	-	29	15-20	22	17	40	60
Modulus of Elasticity	N/mm2	30000	36667	-	-	-	45000	5000	6400	-	55000
Relative density	kg/m3	2405	-	-	-	-	2350	2500	2800	-	2915
Water-binder ratio		0,4	-	0,45	0,26	0,35	0,2	0,2	0,14	0,2	0,2
Types of fibre				Synthetic fibres	Bulk fibre	Fibre mat	Steel fibres + wollastonite	Steel fibres + wollastonite	Straight steel fibres	two different types of steel fibres	three different type of steel fibres
Length	mm			20	-	240	13-15	13-16	20	6-13 and 30-60	-
Diameter	mm			0,05	-	0,33	0,2	0,2	0,16-0,3	0,16-0,2 and 0,5-0,71	-
Average fibre volume				2%	5-20%	3-5%	2%	2%	3,5%	3%	11%

Table 6-1: Characteristic of some mixtures [10] (references in this table are in D. Hartog [10])

6.1.4. Conclusions

Because of the high strength (development), the self compacting, and overall slenderness of a element (U)HPC seems the best mixture for double curved elements. Also the fibres in the mixture could make additional reinforcement superfluous (Chapter 6). A disadvantage is that the fibre mixture is expensive. Because the fibre mixture is expensive, there is tried to make the system work with normal strength mixtures. A mixture with lower strength could also be self compacting and so applicable for the flexible formwork. If the system works with a low strength mixture, it should also do with a (U)HPC mixture. The addition of fibres should make the concrete more stable, and increase the tensile strength. These aspects help during the deformation of the formwork and the fresh concrete.

6.2. Fresh concrete; rheological behavior

Due to the method of using the flexible mould a 'new' aspect of concrete became important; the deform time. Normally the fresh concrete is poured into the formwork, and left alone to harden. When it is hardened the formwork is removed and the concrete is one solid element. By using the flexible formwork, the mould with the fresh concrete is deformed after resting of the concrete. This section describes this deform time of a mixture.

For determining the deform time of a mixture the behaviour of the fresh concrete is important, this is called the rheology: 'The science of the deformation and flow of matter' [24]. The formwork with the fresh concrete is deformed, so the fresh concrete gets a slope, from the curvature of the mould. The concrete must have enough resistance to be stable at this slope. But the concrete must be plastic enough to follow the deformation and the curvature of the mould.

6.2.1. Rheology concrete

The behaviour or the workability of fresh concrete is described by the Bingham [8] equation:

$$\tau = \tau_0 + \mu\dot{\gamma} \qquad (6.1)$$

With:

$\tau =$ shear stress (Pa) applied to the fresh concrete

$\dot{\gamma} =$ shear strain rate (s^{-1})

$\tau_0 =$ yield stress (Pa); a measure of the force necessary to start a movement of the concrete; flow resistance.

$\mu =$ plastic viscosity; a measure of the resistance of the concrete against an increased speed of movement.

The yield stress and the plastic viscosity characterize the flow properties of the fresh concrete.

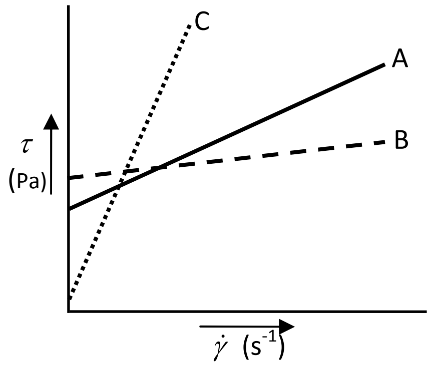


Figure 6-1: Bingham Diagram. Yield stress and plastic viscosity of different fresh concretes [30]

In Figure 6-1 for some concretes characteristic Bingham lines are drawn [30]. The τ_0 is the point where the line crosses the vertical line. Line A indicated a normal concrete. Mixture B has a relative high yield value; it takes a lot effort to bring the mixture into movement. Once it flows, it flows fast out, due to the low plastic viscosity. Line C indicates a self compacting mixture. The yield stress is almost zero, and the plastic viscosity is relative high.

The measurement of these characteristic values are possible on different ways, two instruments are presented here, the BML viscometer and the modified slump method.


Figure 6-2: BML Viscometer [58]

BML viscometer

The devise has a cylindrical bucket for the concrete and an impeller, which is above the bucket at Figure 6-2. After filling the bucket with the fresh concrete, the bucket will rotate at a fixed speed. Then the impeller is lowered, due to the inertia of the fresh concrete a torsion moment is applied on it. This torsion moment is measured. The

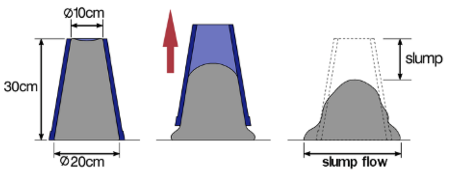


Figure 6-3: Slump test [59]

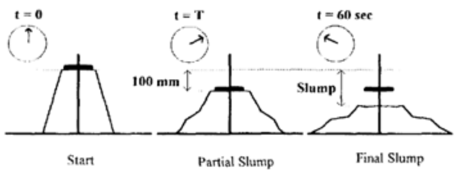


Figure 6-4: Modified slum test [8]

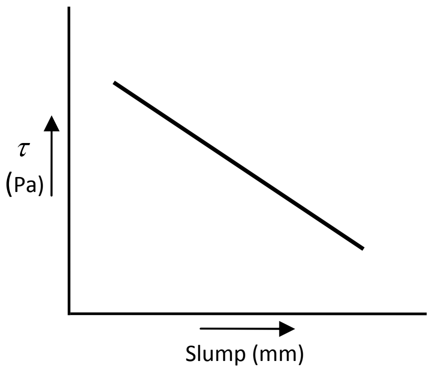


Figure 6-5: Slump versus yield stress

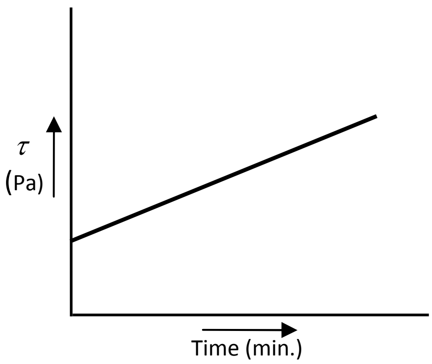


Figure 6-6: Time versus yield stress

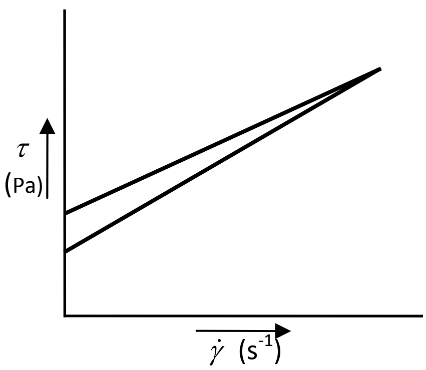


Figure 6-7: Example of a 'thixotropic loop'

rotation speed is lowered and the torsion moment is measured again. With these values the Bingham diagram can be drawn.

Modified slump method

With a normal slump test the processability of the fresh concrete is tested.
In Figure 6-3 the method is displayed. The cone with a height of 300 millimetres is filled from the top side. When it is full it is removed. The distance that the concrete is sagged is the slump value.
With the slump value the yield stress can be calculated according [8]:

$$\tau_0 = \frac{\rho}{374}(300 - s) + 212 \quad (6.2)$$

The relation between the yield stress and the slump is linear. An increase of the slump, gives a decrease of the yield stress, Figure 6-5.
To be able to determine the plastic viscosity, the slump test is modified [8], Figure 6-4. A plate is attached to a bar; the plate drops with the concrete and can lower until 100 millimetres below the top. The time that the plate drops to the 100 millimetres is measured, with this time the viscosity can be calculated. Also the final slump can be measured.
The plastic viscosity can be calculated with:

$$\mu = \rho T * 1.08 * 10^{-3}(S - 175) \quad \text{for } 200 < S < 260 \text{ mm.} \quad (6.3)$$

$$\mu = 25 * 10^{-3} \rho T \quad \text{for } S < 200 \text{ mm.} \quad (6.4)$$

With 120 < Slump < 260 mm.

Due to the hardening of the concrete the slump of a mixture will decrease in time. So demoulding the slump at time intervals gives different slump values over time and corresponding yield values. Over time the slump value decreases so the yield stress increases linear, Figure 6-6.
This effect of decreasing of the yield stress is partly due to the thixotropic behaviour of the fresh concrete. A concrete is called thixotropic 'if it seems to flocculate rather quickly at rest and it becomes apparently more and more fluid while flowing during typically several tens of seconds' [22].
This behaviour is called the 'thixotropic loop' [22] and is seen in Figure 6-7. If the shear rate is increased by a viscometer the yield stress also increase. After decreasing the shear rate, the yield stress had a lower value as with the increasing. At the same shear rate the yield stress had become smaller.
The increasing yield stress in time is given by the linear relation: [22]

$$\tau_0(t) = \tau_0 + A_{thix} t \quad (6.5)$$

$$A_{thix} = \frac{\tau_o}{T} \quad (6.6)$$

With:
A_{thix} = The flocculation rate (Pa/s)
T = Thixotropy model parameter relative to the rate of flocculation

6.2.2. Sloped stability

Due to the curvature of the mould the fresh concrete gets a slope. The stability of the concrete on the slope is given by [14]:

$$\tau_o^{crit} = \rho g h \sin(\theta) \tag{6.7}$$

So a concrete mixture has (on a time) a slump value, this value gives a yield stress, and with this stress the maximum slope can be determined at which the mixture is still stable. Or vice versa, at a given slope the maximum slump value can be determined.

$$\tau_0 \geq \tau_o^{crit}$$

$$\frac{\rho}{347}\left(300 - s\right) + 212 \geq \rho g h \sin(\theta) \tag{6.8}$$

$$s \leq -\left(gh\sin(\theta) - 212\right)347 + 300$$

So the slump value on a specific time gives the maximum slope that the fresh concrete can have. Table 6-2 for the relation with the thickness of the concrete layer of 50 millimetres.

Slump (mm)	280	270	260	250	240	230	220	210	200	190	180	170
θ max (°)	17	21	25	28	32	36	41	45	50	56	62	71

Table 6-2: Slump versus slope for a thickness of 50 mm.

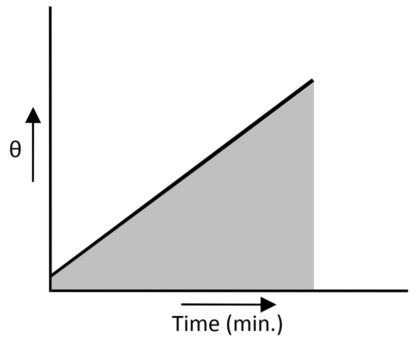


Figure 6-8: Time versus slope

Over time the slump value decreases, due to the hardening of the concrete, so the maximum slope that can be made becomes larger, see Figure 6-8. The gray surface in the figure give the area that the concrete is stiff enough to be stable on the slope.

The maximum slope or curvature that the mould can adopt depends on the concrete mixture and thickness of the concrete layer.
There is no empirical model that describes the hardening of the concrete in the fresh state. The slump values at time intervals can not therefore be calculated, and have to be determined by a test.
Also different mixtures have other slump values so no general values can be given by the graphs.

6.2.3. Crack limit

Considering the stability of the mixture, it is best to wait as long as possible with the deformation of the mould and the fresh concrete. If the concrete is rested sufficiently, the slump value is very low, a large slope is possible.
Due to the hardening of the concrete, the fresh concrete becomes solid. When the deforming of the concrete takes place to long after pouring, the concrete is hardened and it will crack when it is deformed.
When the concrete is just mixed, it is very fluid, and so also very deformable, a large slope should be possible. Due to the lack of stiffness, the concrete will flow in this case down from the slope.
During hardening of the fresh concrete the deformability of the concrete decreases. The radius of the deformed mould that the fresh concrete could have increases, so the slope decreases. This effect is represented by the curved line in Figure 6-9. The gray area in the picture is the deformable region. The deformable area that is

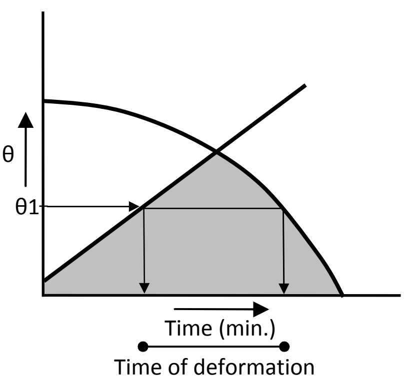


Figure 6-9: Deformable area

determined for the stability is limited by the crack limitation. This graph depends on the composition of the concrete mixture and the specific weight of it.
So for a given slope θ1, the time of deformation can be determine. A large time of deformation appear at a small slope. The larger the slope is the smaller the time of deforming become how precise this time has to be known.

6.3. Fresh concrete; Tests

By trying to find the deform time theoretically, only a graph of the deformable area is found, Figure 6-9. The deform time, the lines and the gray area, is tried to be determined with tests.

6.3.1. Slump test 1

Objective of the test
This test is to determine the deform time of a concrete mixture. By using slump tests the slump can be determined at time steps. With these slump values the yield stress and so the maximum slope can be determined.

Test facility
The slump of a mixture has to be known at different times. After mixing of the concrete it has to be poured into its formwork, in this case the slump cone. The concrete has to be poured directly into the cone; it can not rest a while into a form and then transfer into the slump cone. When it is transferred form its temporary form into the slump cone, the stability and the yield stress of the fresh concrete changes. So a good measurement is not possible anymore. To be able to determine the slump value at different time points a lot of slump cones are needed. If the slump value is wanted to know every fifteen minutes and it takes about five ours to get a good representation of it, twenty slump cones are needed. The time steps, and the total time that needed to get a satisfy representation depends on the mixture, and is difficult to predict before a test.
This amount of cone slumps was not available. So the moulds are made from a PVC pipe. The pipe is sown into 20 rings that are the formwork. The moulds have an inner diameter of 112 millimetres and a height of 50 millimetres. These pipes are placed on a wooden plate, Figure 6-10. The pipes are vertically removed in time steps of 30 minutes. After removing the mould the stability/stiffness of the concrete was evaluated and the diameter was measured.
A big disadvantage of this method is the vertical form of the pipes. A cone has a bevel edge. This edge does not disturb the stability of the mixture during the demoulding, what the vertical edge does. Also the measured slump value can not be related to the standard slump value. So the yield stress can not be calculated according equation (6.2).
The test is used to be able to judge the stability of the mixture at time points.
Also six cubes of 100 by 100 millimetres are made for testing the cube compressive strength after 7 and 28 days.

Figure 6-10: PVC pipes

Cement type	
Cem 1 52,5 R	23,670 kg
Filler	
Fly ash	29,835 kg
Quartz sand	15,795 kg
Adjuvant	
Premia 196	1,845 kg
Aggregates (dry)	
Sand 0,125-0,250	4,860 kg
Water	13,488 kg
Total	45 l
W/b Factor	0,62

Table 6-3: Concrete mixture one

Concrete mixture

For this test concrete mixture one was used. This was not a fiber mixture to make the production method as applicable as possible. The addition of fibers makes the mixture and so the final product more expensive. The composition of the mixtures can be seen in Table 6-4.

Test results

After pouring the concrete in the moulds (Figure 6-11), the mould where removed after each other with time steps of 30 minutes. After 6,5 hours of pouring, the mixture was still very unstable. Figure 6-12 shows that the concrete flows to a circle with a diameter of about 200 millimetres. The diameter of the PVC pipes 112 millimetres. The concrete flows easily and did not had any signs of stability.

Table 6-5 shows the cubic compression strength of the mixture, measured after seven and twenty eight days. The cubic's where 100 by 100 by 100 millimetres.

Conclusions

- After 6.5 hours the mixture was not stiff enough. So this mixture is not applicable for our application. An explanation for the long time to become stiff is the missing of 'large' sand in the mixture. Without sand grains the mixture could not form a stable inner skeleton.
- As is shown in Figure 6-11, some of the concrete is leaking between the PVC pipe and the wooden plate. This is because the sawed PVC pipes are not totally straight, and the PVC pipe is to light. To avoid this problem in the next tests the border between the wooden panel and the PVC pipe is sealed with kneading clay.

Figure 6-11: PVC pipes after filling with concrete

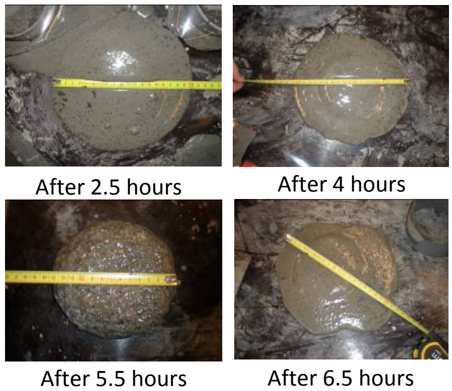


Figure 6-12: Concrete after removing the PVC moulds

Sample	After 7 day's fc (N/mm2)	After 28 day's fc (N/mm2)
1	52	
2	50	
3	51	
4		78
5		79
6		77
fcm	**51**	**78**

Table 6-4: Compressing strength mixture one

6.3.2. Slump test 2

After test one it was clear that mixture one was not suitable for using at the flexible formwork, in the application of deforming the mould after pouring of the concrete. Another mixture is chosen, mixture two. See Table 6-5 for the composition of it. The cubic compression strength of the mixture (100 by 100 by 100 millimetres) after seven and twenty eight days is shown in Table 6-6.

To see if this mixture has better properties a test mixture was made. After mixing air bubbles came out of the fresh mixture. This indicates that the mixture is self compacting. But the strength development was very fast; see Figure 6-13 that shows the shape of the mixture after 45 minutes of hardening. The strength development of this mixture is much faster as mixture one.
This test was only to determine the stability of the mixture. Because the slump test is modified, with the slump value the yield stress can not be calculated. Also the test does not indicate the deformability of a mixture.

Figure 6-13: Removing of the mould 45 minutes after pouring.

Cement type	
Cem 1 52,5 R	26,865 kg
Filler	
Fly ash	7,380 kg
Adjuvant	
Glenium	0,190 kg
Aggregates (dry)	
Sand 0,125-0,250	8,479 kg
Sand 0,250-0,5	11,305 kg
Sand 0,5-1	16,958 kg
Sand 1-2	14,131 kg
Sand 2-4	5,653 kg
Water	10,783 kg
Total	45 l

W/b Factor 0,41

Table 6-6 Concrete mixture two

Sample	After 7 day's fc (N/mm2)	After 28 day's fc (N/mm2)
1	70	
2	69	
3	69	
4		81
5		83
6		83
fcm	**69**	**82**

Table 6-5: Compressing strength mixture two

6.3.3. Deform test

Objective of the test
By using the same mixture as with the slump tests, the deformability of the mixture is investigated at different time steps.

Test facility
On a flexible vivak plate a flexible border of polyether are glued. The mould had a surface of 400 by 100 millimetres. At time steps the middle of the mould is lifted. When lifting the middle, the mould is deformed and should simulate a curved shape. When the fresh concrete flows the lifting is stopped. Whit this height of the middle and the length of the mould the radius of the mould can be determined. To do this at some time points after pouring (with different hardening) the deformability of the mixture related to the time after pouring can be determined.

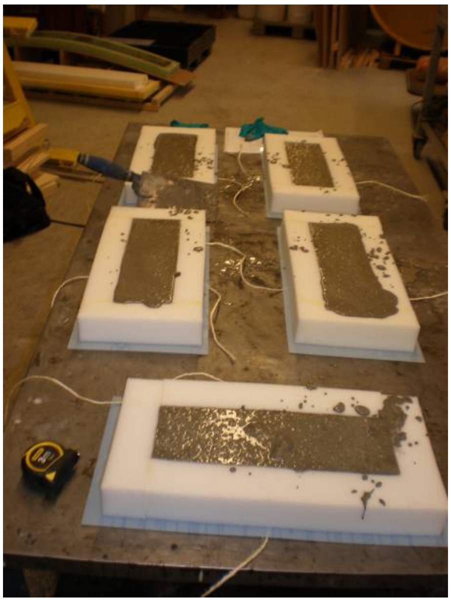


Figure 6-14: Flexible moulds

Test results
The mould was too stiff. When lifting the middle the whole mould was lifted. After applying a tension force at the ends the mould was deformed.
The deformed shape was not a smooth curve, but more of straight lines. Also because of the relative small length of the mould, lifting of the middle gives a small radius. Lifting the mould 10 millimetres the mould should get a radius of 2 meters. So it was not possible to lift the mould, especially at the beginning, only a couple millimetres. From these test no conclusions could be drawn concerning the hardening time and deformability.

Figure 6-14: Deformed mould

Because of scale effects it is very difficult to do down scaled experiments with a deformable mould. By using a mould of 400 millimetres, the middle has to be lifted only a couple millimetres to get realistic radius. This was not possible.

6.3.4. Conclusions

With the slump value the maximum slope of the fresh concrete can be determined. These values have to be determined at different time points, so many slump cones are needed, but they were not available. A modified slump test is preformed. With this test only the stability of a mixture was determined. The deform test that is carried out later did not give satisfying results. During the slump tests it was found that the Mixture two was stable after 45 minutes. This deform time was later used in full scale tests. It could be that it has hardened too much, but that could not be determined with scale tests.
It is possible that, with exact the same composition of the mixture, the mixture reacts in a different way the next time. So the slump test is performed parallel with the next tests to find the best deform time for every test.

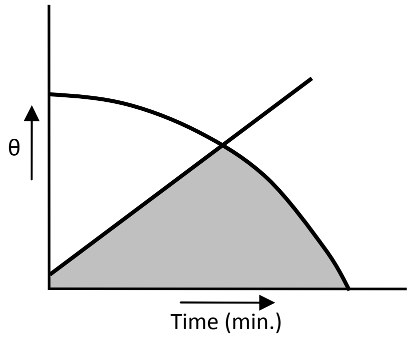


Figure 6-16: Deformable area

6.4. Conclusions

By making double curved elements, a self compacting mixture could be used. This gives the best work conditions and the easiest pouring. A self compacting mixture has a high plastic viscosity so it is very fast stable. This makes it a useful property for the flexible formwork.
By making use of the rheology of the fresh concrete, the deform time could be determined. The deform time is the gray area of Figure 6-16. The exact values of this graph can not be determined. With a theoretical approach the strength of the fresh concrete can not be determined. The tests give a stable mixture after 45 minutes for mixture two. With scale tests it was not possible to determine the slump at time steps, and the deformability of the fresh mixture.
In Chapter 8 tests are carried out on a realistic scale. With these test the graph is tried to determined. The best is to deform the concrete as soon as possible, the further it hardens the more cracks appeal.

7. Reinforcement

7.1. Introduction

Due to the low tensile stress of concrete, an additional material is needed to increase the load bearing capacity of concrete, especially when it is loaded in tension/bending. A material that fulfils this function is steel. Steel rods are placed in the tension zones of a concrete element. The combination of concrete and steel makes reinforced concrete a very good building material.
Because of the curvature of double curved elements, the reinforcement has to have a curvature too. In the situation where a double curved mould is used to pour the concrete in, the reinforcement can be bend before placement, Figure 7-1. A steel rod up to 16 millimetres can be bent with the use of a machine [60]. This makes it possible to make prefabricated reinforcement that can be placed in the formwork for a double curved element.
By using the method of deforming the flexible mould with the partly hardened concrete, it influences also the reinforcement. If steel rods are used, they must be placed at the horizontal mould. When the mould is deformed, the reinforcement must also deform with the mould and the concrete.
In Section 3.4 it is explained that the double curved panels made with the flexible mould are used as a predalle. This means that a thin skin is prefabricated with the use of the flexible mould. The prefabricated double curved panels are placed on a temporary support structure at the building site. On top of these panels the formwork is placed and the top layer of concrete is poured. This process requires curved reinforcement for the top layer. The reinforcement can be bent with a machine before it is placed.
The prefabricated double curved panels only need to be reinforced for their own weight due to the lifting of the panels, and for the formwork function for the in-situ concrete. This reinforcement is less than needed for full load bearing elements.
The next section treats the flexibility of a steel rod. With a given curvature of the element the maximum diameter of the steel rod is determined.
After that other methods of reinforcing are treated.

Figure 7-1: Curved Reinforcement [60]

Figure 7-2: Reinforcement for the EPFL Rolex learning center [61]

7.2. Flexibility steel reinforcement rod

The model that is proposed in Chapter 5 is adapted to be able to describe the behaviour of a steel rod. For the schematization in Table 7-1 a steel rod of two meters is used. A rod has six support points, the spacers, with a distance of 400 millimetres are positioned between them. For a given radius, the heights of the support points can be determined. The load on the rod is the weight of the rod.
In the reality, the rod is placed inside the concrete. So it has a 'full' support over the length and a distributed load of the concrete over the length. When the deforming takes place, the concrete is still in a fresh state. The behaviour is not like a liquid, and not as a solid.
This effect is favourable for the rod in compare to a rod without concrete, the concrete helps to bend the rod. To take this effect not into account is at the safe side.
In Appendix D the maple model is given that allows calculating the reaction forces at the support points.
In Table 7-1 an overview is provided for different radius and for a curvature upwards and downwards. As seen in the table, the thicker the diameter of the steel rod is, the larger the reaction forces are. And the smaller the radius is, the larger the forces are.
A negative reaction force means a tension force at the support. Due to the concrete on top of the supports a small tension force can be allowed.

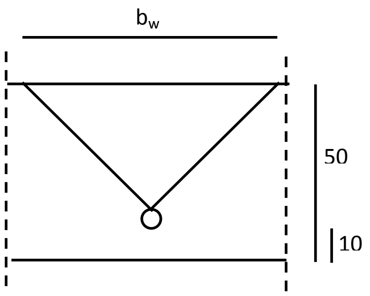


Figure 7-3: Position of the reinforcement

Assume a situation as in Figure 7-3; the concrete has a thickness of 50 millimetres. The length between two supports is 400 millimetres, so at the middle supports over a length of 400 millimetres there is a concrete weight. The force of the fresh concrete on the rod is:

$$F = L * b_w * h_b * \frac{1}{2} * \gamma_b \qquad (7.1)$$

$$F = L * h_b^2 * \gamma_b$$

$$F = 400 * 40^2 * 2400 * 10^{-8} \qquad (7.2)$$

$$F = 15.36N$$

Taking into account the partly fresh state of the concrete, say a maximum tension force, in this case, is allowed of 10 N.
This means that a rod with a diameter of 6 millimetres can be used with a maximum radius of 10 meters (with a concrete layer of 40 millimetres on top of it), a rod having a diameter of 4 millimetres for a radius of 2.5 meters, and a rod round 3 for a radius of 1.5 meters.

To be able to produce large double curved elements with the flexible mould, only very thin reinforcement rods are suitable. Also a fact is that the rods could float into the fresh concrete, and so the final place of it is very unpredictable. With a thin reinforcement bar the load bearing capacity of the elements is low. The load bearing capacity depends on the numbers of bars, total surface of it, and their position.

R (m)	Form	D (mm)	Fa (N)	Fb (N)	Fc (N)	Fd (N)	Fe (N)	Ff (N)
10		2	0,1	0,0	0,2	0,2	0,0	0,1
		3	0,3	**-0,4**	0,7	0,7	**-0,4**	0,3
		4	0,8	**-1,7**	1,9	1,9	**-1,7**	0,8
		6	3,6	**-10,0**	8,6	8,6	**-10,0**	3,6
		8	11,0	**-33,0**	25,9	25,9	**-33,0**	11,0
		10	26,4	**-82,2**	61,9	61,9	**-82,2**	26,4
10		2	0,0	0,2	0,0	0,0	0,2	0,0
		3	**-0,1**	0,9	**-0,3**	**-0,3**	0,9	**-0,1**
		4	**-0,5**	2,6	**-1,1**	**-1,1**	2,6	**-0,5**
		6	**-2,9**	12,0	**-6,9**	**-6,9**	12,0	**-2,9**
		8	**-9,8**	36,6	**-22,8**	**-22,8**	36,6	**-9,8**
		10	**-24,5**	87,7	**-57,1**	**-57,1**	87,7	**-24,5**
5		2	0,1	**-0,2**	0,3	0,3	**-0,2**	0,1
		3	0,5	**-1,1**	1,2	1,2	**-1,1**	0,5
		4	1,5	**-3,9**	3,4	3,4	**-3,9**	1,5
		6	6,9	**-21,0**	16,3	16,3	**-21,0**	6,9
		8	21,5	**-67,8**	50,3	50,3	**-67,8**	21,5
		10	51,8	**-167,1**	121,4	121,4	**-167,1**	51,8
5		2	0,0	0,4	**-0,1**	**-0,1**	0,4	0,0
		3	**-0,3**	1,6	**-0,7**	**-0,7**	1,6	**-0,3**
		4	**-1,1**	4,8	**-2,7**	**-2,7**	4,8	**-1,1**
		6	**-6,2**	23,0	**-14,6**	**-14,6**	23,0	**-6,2**
		8	**-20,2**	71,4	**-47,2**	**-47,2**	71,4	**-20,2**
		10	**-49,9**	172,7	**-116,6**	**-116,6**	172,7	**-49,9**
2,5		2	0,2	**-0,4**	0,5	0,5	**-0,4**	0,2
		3	0,9	**-2,6**	2,2	2,2	**-2,6**	0,9
		4	2,8	**-8,5**	6,6	6,6	**-8,5**	2,8
		6	13,9	**-44,2**	32,4	32,4	**-44,2**	13,9
		8	43,5	**-140,9**	101,3	101,3	**-140,9**	43,5
		10	105,7	**-345,6**	246,0	246,0	**-345,6**	105,7
2,5		2	**-0,1**	0,7	**-0,3**	**-0,3**	0,7	**-0,1**
		3	**-0,8**	3,1	**-1,8**	**-1,8**	3,1	**-0,8**
		4	**-2,5**	9,4	**-5,9**	**-5,9**	9,4	**-2,5**
		6	**-13,2**	46,2	**-30,7**	**-30,7**	46,2	**-13,2**
		8	**-42,3**	144,5	**-98,3**	**-98,3**	144,5	**-42,3**
		10	**-103,8**	351,2	**-241,3**	**-241,3**	351,2	**-103,8**
1,5		2	0,5	**-1,4**	1,2	1,2	**-1,4**	0,5
		3	2,5	**-7,5**	5,6	5,6	**-7,5**	2,5
		4	7,6	**-24,2**	17,5	17,5	**-24,2**	7,6
		6	38,3	**-123,7**	87,7	87,7	**-123,7**	38,3
		8	120,4	**-392,4**	275,9	275,9	**-392,4**	120,4
		10	293,4	**-959,6**	672,3	672,3	**-959,6**	293,4
1,5		2	**-0,4**	1,7	**-1,0**	**-1,0**	1,7	**-0,4**
		3	**-2,3**	8,0	**-5,2**	**-5,2**	8,0	**-2,3**
		4	**-7,3**	25,1	**-16,8**	**-16,8**	25,1	**-7,3**
		6	**-37,6**	125,7	**-86,0**	**-86,0**	125,7	**-37,6**
		8	**-119,2**	395,9	**-272,9**	**-272,9**	395,9	**-119,2**
		10	**-291,5**	965,1	**-667,5**	**-667,5**	965,1	**-291,5**

Table 7-1: Reaction forces at a steel rod of 2 meters

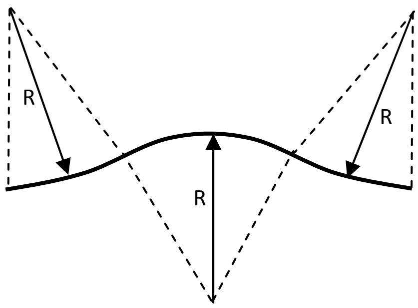


Figure 7-4: Radius R

7.3. Other reinforcement methods

This section deals with other methods that can be used to reinforce concrete. Mainly the bending flexibility of the chosen systems is discussed. The reinforcement must be placed into the (horizontal) fresh concrete and must be deformed with the fresh concrete when the mould is deformed.

7.3.1. Fibres

With the development of high strength concrete, concrete become brittle. Fibres that are mixed through the fresh concrete are used to overcome that problem. The fibres are mostly of steel or plastic. The fibres provide a good crack control, also in the fresh state of the concrete. This is a favourable effect for the flexible mould, during the deformation of the concrete tension stresses appear at the top side of the concrete. The fibres in the fresh mixture are able to prevent cracks that could appear due to tensile stresses. They also improve the tensile strength of the concrete due to the tensile capacity of the fibres; however this is normally lower as steel rod reinforcement. It's also difficult to predict the strength of the concrete because of the random orientation of the fibres in the mixture.

7.3.2. Net-reinforcement

The reinforcement could be placed into nets to make the placement easier and also increasing the load bearing capacity, due to orientation of the rods/wire/fibres. These nets could be of traditional steel rods (where the flexibility of the rods is limited, see Section 7.2), wires (e.g. DUCON [21]) or fibres (e.g. SIMCON-neu [21]). The wires and fibres nets are very flexible, so suitable for the application of the flexible mould.

A drawback of net reinforcement is that it is only suitable for developable surfaces. A developable surface is a surface that can be developed into a plane form without cutting and/or stretching their middle surface [21]. A single curved surface is developable, so net reinforcement could be used. A double curved surface can not be developable, so a net reinforcement is not suitable for the application of double curved elements.

7.3.3. Strands

Instead of nets, strands or wires could be used as reinforcement in one direction. This could be in combination with (post) tensioning. Due to the flexibility of the strands, it can deform with the fresh concrete. The reverse makes the placement very difficult. The strands are so flexible that positioning at a constant level at the bottom of the formwork is very difficult. An unknown factor is the bond of the strands and the pull out strength of it. Also due to the small surface of it, all lot of strands are needed.

7.3.4. Carbon reinforcement

Carbon reinforcement is mostly used as an external reinforcement, and is used to strengthen existing structures [16]. Due to the flexibility of it, it is suitable for double curved elements. The disadvantage is that the external reinforcement is placed after hardening. So the element must be strong enough to be able to lift out of the mould. The external carbon reinforcement also affects the visibility of the concrete element.

7.4. Conclusions

Steel bars that normally are used to reinforce concrete, never needs to be flexibly. In most applications straight bars are used. If a curved element is made (and curved reinforcement is needed), the reinforcement is bent before it is placed. By using the flexible mould in the way that it is deformed with the fresh concrete and the reinforcement in it, the reinforcement has to be flexible.
Current used traditional steel rods are too thick to be able to bend in the concrete and follow the curved shape of the mould. If an element has a radius of 10 meters a rod of maximum 6 millimetres could be used. The smaller the radius the smaller the maximum thickness of the rods has to be.
The double curved surfaces make the use of nets bars, wires or fibres, not applicable, the surfaces has to be developable to be able to use a net reinforcement.
Fibres into the concrete could be a suitable method of reinforcing the double curved elements. The fibres could be mixed into the fresh concrete. When the mould with the fresh concrete is deformed the fibres are small enough to follow the deformations of the mould and the concrete.
By using the double curved elements as a predalle, fibre reinforced concrete or thin rod reinforcement could fulfil. If the elements are lifted with enough lifting points (the span is limited), and the span of the temporary scaffolding is limited (for the in-situ concrete) the forces in the plate are limited and the predalle could carry the load. On top of the predalle, beforehand bent reinforcement can be placed, before the top layer of in-situ concrete is poured. On this way the total element could be sufficiently reinforced and fulfil its load bearing function.
Additional research is needed to determine the best flexible but also strong and stiff reinforcement that is needed for the flexible formwork.

8. Single curved mould tests

8.1. Introduction

With these tests some dependent variables; the flexible layer, the border, and the radius of the flexible formwork are investigated. To test the dependent variables it is not necessary to build a double curved formwork. On a single curved model the dependent variables are tested and with the results of these tests double curved tests can be carried out. To make the tests not to complicate the tests are carried out without actuators. A frame of beams supports the formwork in horizontal direction. In this position the concrete is poured, Figure 8-1. When the concrete has rest a while, the concrete is stiff enough to keep its shape but is plastic enough to be able to deform, the mould is deformed. The mould is placed on wooden panels, which have a different in height to represent the radius of the formwork. The concrete gets the desired shape during hardening at this deformed position.
In these test concrete mixture two is used (Table 6-2). For this mixture the deform time is determined in Chapter 6 and is 45 minutes after pouring. This time is different for every mixture; to focus on the dependent variables the same mixture is chosen for all of the tests. Because of the radius of the mould traditional thick reinforcement is not applicable. In the tests small rods with a diameter of 3 millimetres are used to make sure that the element can be unmolded. This reinforcement is not sufficient to give a significant load bearing capacity.

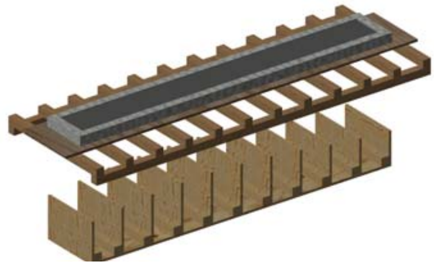


Figure 8-1: Concrete is poured horizontal at the mould

Figure 8-2: Deformed mould and concrete

The dependent variables that are investigated:

- **Flexible mould layer**
  In Chapter 5 a model is proposed that describe the elastic behaviour of the flexible layer on a single curved mould. A starting assumption is that the flexible plate must be resting on all of its supports. With the model the required elasticity of the mould is determined. Wood and steel are tested as material for the flexible layer. The thickness of these materials is determined with the model. The model is verified with these tests.

- **Radius of the mould**
  The radius of the mould is reduced to find the limit of radius that can be made with this method. First a radius of 5 meters is applied, than 2.5 and finally the mould has a radius of 1.5 meters.

- **Border of the formwork**
  The border of the formwork has to be flexible to be able to follow the radius of the formwork, but also be stiff enough to withstand the concrete pressure. Two different foam materials, polyether and cold foam are tested to determine if they fulfils the criteria.

Cement type	
Cem 1 52,5 R	26,865 kg
Filler	
Fly ash	7,380 kg
Adjuvant	
Glenium	0,190 kg
Aggregates (dry)	
Sand 0,125-0,250	8,479 kg
Sand 0,250-0,5	11,305 kg
Sand 0,5-1	16,958 kg
Sand 1-2	14,131 kg
Sand 2-4	5,653 kg
Water	10,783 kg
Total	45 l

Table 8-1: Mixture two

The tests on a single curved mould that are performed are shown in Table 8-2.

	Deform time (min)	Flexibler layer	Thickness layer (mm)	Radius (M)	Border
Test 1	50	Wood	3,8	5	polyether SG 40 extra firm
Test 2	50	Steel	1	5	Cold foam HR SG 55
Test 3	50	Wood	3,8	2,5	polyether SG 40 extra firm
Test 4	50	Wood	3,8	1,5	polyether SG 40 extra firm
Test 5	30	Wood	3,8	1,5	polyether SG 40 extra firm

Table 8-2: Overview single curved mould tests

8.2. Test 1

Objective of the test

With the first test the model that is proposed in Chapter 5 is verified. The radius of the mould is relative low, to make sure that the test succeeds.

Test facility

Due to the uncertainty that the fresh concrete behaves in a different way as tested in Chapter 6, the slump tests with the PVC pipes is also performed during these single curved mould tests. The concrete is poured almost simultaneously in the PVC pipes and in the mould. Every 15 minutes a PVC pipe is removed to judge the stability of the mixture, and to verify that the best time to deform the mould is 45 minutes after pouring.

Figure 8-3: Horizontal formwork

A wooden frame represents the curved shape of the final element, which is seen at the right side of Figure 8-3. The height of these panels can be seen in Figure 8-4. The curved line and so also the final element has a radius of five meters.

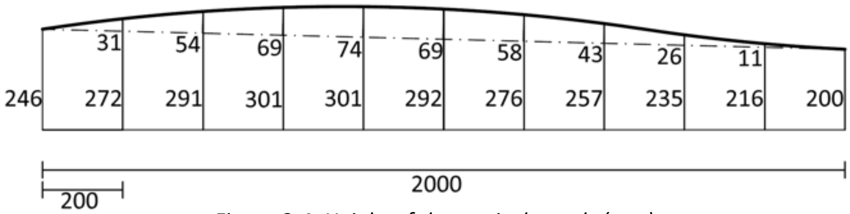


Figure 8-4: Height of the vertical panels (mm)

The elastic plate which must be deformed with the mould is placed horizontally on a timber frame, as is seen on the left side of Figure 8-3. The thickness of the wooden plate is 3.8 millimetres which is determined with the model; see Appendix E - test 1 for the sheet of it. The model proposes for the maximum thickness of the plate 3.7 millimetres. The thinnest plate that was available was 3.8 millimetres, so at one place a small tension forces should appear, see Appendix E - test 1. These tension forces mean that the plate does not touch the supporting panel at that point.

The polyether border was glued on the wooden plate. The border was 50 millimetres thick, so the maximum thickness of the element is also 50 millimetres. A foil is placed into the formwork to seal it for leaking of liquids.

Four reinforcing rods are placed at a distance of 10 millimetres from the bottom to reinforce the element, see Figure 8-3.

Test results

The third slump test that was performed, 45 minutes after pouring of the concrete by lifting the PVC pipe gave a stable concrete mixture. The mould could be deformed, by placing it on the vertical panels; this was done 50 minutes after pouring. The result of the deforming of the mould can be seen in Figure 8-5 and 8-6. The mould touches all of the panels, so it curved in the desire shape. It was expected that at one end the mould did not touch the panel, due to a small tension force. In the model the plate stops at the vertical panel, but in the test it has an overhang. This explains why it touches the panel.
After one day of hardening the element is unmolded. At Figure 8-7 and 8-9 the unmolded element can be seen at the vertical panels.

Figure 8-5: Just deformed concrete

Figure 8-6: Just deformed concrete

Conclusions

- The produced element has the desirable shape, which is seen in Figure 8-7 and does not have any cracks directly after deforming and after one day of hardening.
- Due to the overhang at the test, the model could not be verified with the test results at the end of the formwork. At the middle the mould touch all of the supports. It can be assumed that the model describes the behaviour in a correct way.
- The concrete deformed nicely into the radius of five meters. No cracks are visible at the unmolded element. The overall thickness of the unmolded element was constant; the concrete did not flow downwards from the slope during and after the deformation.
- The reinforcement with a diameter of 3 millimetres stays into the concrete when it was deformed. During the deformation it could happen that it is floated into the fresh concrete. These mean that the rods do not have a constant distance from the bottom any more. This could be tested by sawing the element at some places and measure the height of the rods compared to the bottom of the element. This could not be done at the lab, so it could not be checked.
- The border of polyether was able to follow the deformed shape of the elastic layer. It was also stiff enough to withstand the horizontal pressure of the concrete. The produced element had satisfied squared corners, see Figure 8-9.
- Figure 8-8 shows that the border of the element has many disturbances. This does not come from the polyether border, but from the foil with was placed between the mould and the concrete. The foil folded in the edges due to the radius of the formwork. A way to solve this problem is to seal the polyether border and the seam of it with the wooden plate to prevent leaking of liquid. In that way the foil is not needed and a smooth border is possible. In the next tests does not matter that the border is not smooth. The following tests have other goals, so the foil is applied.

Figure 8-7: Hardened element at the supports

Figure 8-8: Edge of the element

Figure 8-9: Hardened element at the supports

8.3. Test 2

Figure 8-10: Steel plate formwork

Figure 8-11: Formwork filled with concrete

Figure 8-12: Element at the supports

Objective of the test

With this test another elastic material is tested, steel. Also the model can be verified, because the elasticity of steel is different as for wood.

Test facility

In these test the same radius and shape as test one is used, see Figure 8-3 for the height of the vertical panels.
As flexible layer a steel plate is used, the model proposed for the thickness of it one millimetre, see Appendix E – test two.
For the border of the formwork cold foam was used, this was also glued at the steel plate. Cold foam is stiffer as the polyether used in the first test. So it must be checked if the foam deformed nicely. A foil seals the formwork for leaking of liquids. The same limited steel rods are used in this test as used in test one.
In Figure 8-10 the formwork before pouring of the concrete is seen, and in Figure 8-11 the horizontal formwork filled with concrete.

Test results

In this test he slump test is carried out parallel as well. Again after 45 minutes the mixture was stable enough, and after 50 minutes the concrete element was deformed, see Figure 8-12 and 8-13 for a figure just after the deformation of the mould and the fresh concrete. As predicted with the model the plate touches all of the supports, the reaction forces are predicted as compression ones. After demoulding the element met the expectations, Figure 8-14 and 8-15 shows the element after demoulding.

Conclusions

- The steel plate behaves as predicted in the model. The element has the desirable shape. A steel plate is a good alternative for a wooden plate, but it is much expensive. So a steel plate is favourable if it can be reused many times.
- No cracks where visible in the element after deforming and after one day of hardening.
- The cold foam border deforms in the desirable shape, despite the stiffer behaviour of the polyether. Because the polyether is cheaper, polyether is recommended as border material. The additional stiffness of the cold foam is not needed.

Figure 8-13: Mould after deforming

Figure 8-14: Demoulded element at the supports

Figure 8-15: Demoulded element at the supports

8.4. Test 3

Objective of the test

Aim of the test is to find the minimum radius of the elements which can be produced with the production principle deforming of the mould and concrete after partly hardening of the concrete.

Figure 8-16: Just poured concrete

Test facility

The radius that the element has is 2.5 meters; see Figure 8-17 for the height of the vertical panels.

In this set up, there are six supports instead of eleven. If a set up was chosen with eleven supports the wooden plate has to be maximum 2.5 millimetres thick, to meet the condition that there may not be any tension forces at the supports. A wooden plate with this thickness was not available. By using six supports the plate could be 3.8 millimetres thick, see Appendix E - test 3. The just poured concrete is seen in Figure 8-16.

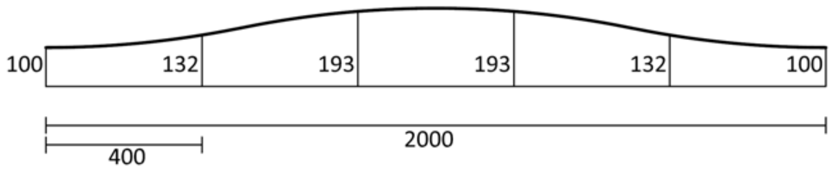


Figure 8-17: Height of the vertical panels (mm)

Figure 8-18: Just deformed concrete

Test results

After 50 minutes of resting, the element is deformed. Figure 8-18 shows the element just after the deformation and in Figure 8-19 the element is seen in the mould after one day of hardening.

In Figure 8-20, 8-21, 8-22 the element is seen after demoulding. Due to the deforming and also after one day of hardening no cracks appear.

Figure 8-19: Deformed element after one day of hardening

Conclusions

- It is possible to produce elements with a radius of 2.5 meters with the principle deforming of the fresh concrete after partly hardening.
- Due to the small radius the foil which is between the mould and the concrete causes wrinkles at the surface of the element, see Figure 8-22. To prevent that by the element with a radius of 1.5 meters the border of polyether is sealed.

Figure 8-20: Demoulded element on the supports

Figure 8-21: Height of the element

Figure 8-22: Border of the element

8.5. Test 4

Objective of the test

In test 3 it succeeded to produce an element with a radius of 2.5 meters. In these test the radius is decreased, to find the minimum radius that can be produced.

Test facility

The height of the vertical panels is shown in Figure 8-23, which represent the radius of 1.5 meters. The maple sheet has as an output that the thickness of the elastic layer must be smaller than 3.76 millimetres (Appendix E-test 4). The smallest available thickness was 3.8 millimetres. With this thickness the maple sheet gives a small tension force at the second supports from the left en right.

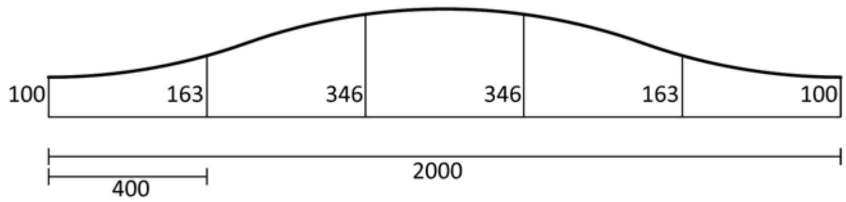


Figure 8-23: Height of the panels (mm)


Figure 8-24: Mould filled with concrete

Test results

After pouring the concrete horizontal (Figure 8-24), the mould is deformed after 50 minutes of resting, just as test one, two and three. After deforming of the concrete no cracks appeared. As expected the flexible plate did not touch the support at two places. To make sure that the radius of 1.5 meters was formed, the formwork was pushed into the desire shape (Figure 8-25) by a rope.

After one day of hardening a crack was visible at the top side of the element when the element was still in the mould (Figure 8-27). After demoulding the cracks was clearly visible, Figure 8-28.


Figure 8-25: Rope to pull to mould into its shape


Figure 8-26: Hardened element

Conclusions

- With the thickness of 50 millimetres, deforming after 50 minutes, and the used concrete mixture it is not possible to produce elements with a radius of 1.5 meters. A small crack appeared. It could be that the minimum radius that can be produced with this method is exceeded or that the deforming takes place to long after pouring.
- The computer model describes the behaviour of the flexible formwork in a correct way. It predicted that with the used plate thickness at two supports a small pull force should appear.
- The small reinforcement rods that are used the make sure that the element can be demoulded, stays into the concrete. It was not possible to check the position of it, and so measure if they where shifted into the fresh concrete.
- The border of polyether deformed nicely and follows the deformation in a correct way, see Figure 8-26.
- The polyether border was sealed, which gave the element a rougher surface, Figure 8-31 and 8-32. But the appearance of it is better as the wrinkles in the element as in test 3.

Figure 8-27: Crack, element in mould, top

Figure 8-28: Crack, element out of the mould, top side

Figure 8-29: Demoulded element

Figure 8-30: Element with a curvature of 1.5 meters

Figure 8-31: Element

Figure 8-32: Edge of the element

Figure 8-33: Height of the element

Figure 8-34: Formwork after removing the element

8.6. Test 5


Figure 8-35: Deformed concrete, top side

Objective of the test

To investigate if an element with a radius of 1.5 meters can be made, it the deform time is shortened compared to test four. In test 4 the concrete is deformed after 50 minutes of hardening. Due to this deforming of the fresh concrete the hardened concrete has a crack. Purpose of the test is to see if the deform time is shortened an element can be produced without cracks.

Test facility

The same test set up as test four is used. The mould, Figure 8-24, and the same panels, Figure 8-23 are used. The only difference is that the mould and the concrete are deformed earlier, after 30 minutes of pouring.


Figure 8-38: Demoulded element


Figure 8-36: Deformed element in the formwork at the supports

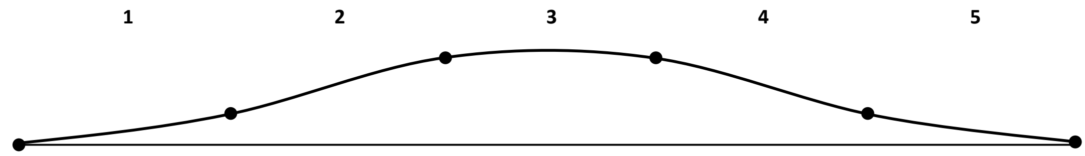

Figure 8-37: Element on its side


Figure 8-39: Demoulded element at the supports


Figure 8-40: Demoulded element at the supports

Test results

After 30 minutes of resting the element is deformed. Directly after the deformation process no cracks occur, Figure 8-35. Also after one day of hardening no cracks are visible, Figure 8-36 and 8-38.
The shortening of the deform time has a positive effect on the formation of cracks. But the earlier the concrete is deformed, the lower the yield value is, the sooner the fresh concrete slides down from the curvature. This effect was not visible during the deformation process. To investigate this phenomenon the thickness of the element is measured, see Figure 8-41. Note, it is not sure that the thickness of the element after pouring of the concrete was at a constant level. The concrete is levelled with a view. From the measurements of the thickness it is seen that the left side, section 1 is thicker that the mean, in the middle it is thinner, and the right side is thicker than the mean. Abnormalities of this came due to the levelling of the fresh concrete. In general it can be said that the concrete has slide a little bit downwards from the slope, but this effect is minimum.



Thickness	5,10	5,00	4,90	4,70	4,70	4,65	4,55	4,60	4,55	4,55	4,60	4,65	4,75	4,75	4,65	4,55	4,55	4,65	4,80	4,80	4,80
Mean	4,71	cm																			
Deviation from mean	0,39	0,29	0,19	-0,01	-0,01	-0,06	-0,16	-0,11	-0,16	-0,16	-0,11	-0,06	0,04	0,04	-0,06	-0,16	-0,16	-0,06	0,09	0,09	0,09

Min. Thickness	4,55	cm
Max. Thickness	5,10	cm
Difference thickness	0,55	cm

Figure 8-41: Thickness of the element

Conclusions

Due to shortening of the deform time no cracks appeared after the deforming process. The mixture that is used give the best deform time 30 minutes after pouring. The overall thickness of the element after hardening is acceptable.

8.7. Conclusions

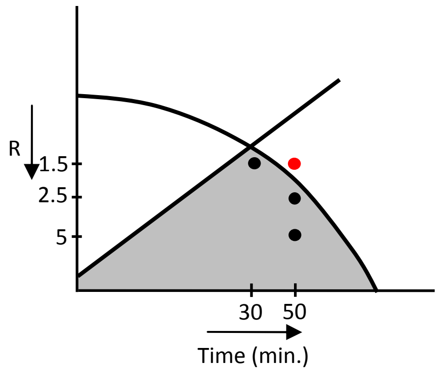


Figure 8-42: Deformable area (Not on scale)

After the tests that are performed the deform times are set out in the graph, see Figure 8-42, made at Chapter 6. (Here the radius is plotted, how smaller the radius how larger the slope). The red point is the test that fails, cracks where visible after hardening. When the deforming of the mould takes place earlier, 30 minutes after pouring, there where no cracks visible. So the black dots are tests with acceptable performances. This graph with possible radius of the mould is only valid when the same mixture is used.

The model that is proposed is able to predict the behaviour of the flexible formwork in a correct way and can be used to determine the right material and thickness of the flexible layer.

It can be assumed that the minimum radius that can be produced for an element thickness of 50 millimetres 1.5 meters is.

Polyether is a good material as border of the formwork. The material is able to follow the deformed mould, but is also stiff enough to withstand the concrete pressure. To get a nice concrete surface the polyether had to be sealed.

9. Flexible mould layer, double curved

9.1. Introduction

In Chapter 5 a model is proposed that describes the behaviour a single curved layer. Tests have confirmed that the model describes the behaviour in a correct way. To go on with double curved mould tests, first the behaviour of the double curved layer had to be known. Two models are proposed in this chapter; a model where the flexible layer is a plate (Figure 9-1), and one for a strip model (Figure 9-2).

Figure 9-1: Plate model

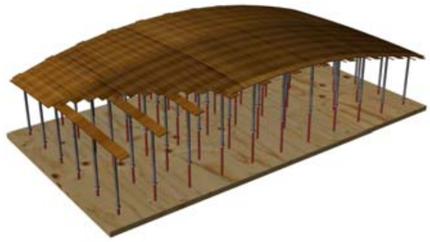


Figure 9-2: Strip model

9.2. Plate model

9.2.1. Introduction

In this model a plate is the mould surface. The plate is supported by pistons which are placed in a square. These pistons are the places where the support reactions are calculated. If there are compression forces at the supports, the plate, with the load of the concrete on top of it, touches all of the pistons. The mould and the concrete deform in this way into the desire shape.

9.2.2. General mechanical equations

The general plate equations are [25]:

$$\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2\partial y^2}+\frac{\partial^4 F}{\partial y^4}=E\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{9.1}$$

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{h}{D}\left(\frac{p}{h}+\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-2\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right) \tag{9.2}$$

With:

$$D=\frac{Eh^3}{12\left(1-v^2\right)} \tag{9.3}$$

In these equation x,y are the coordinate of the axis, w the vertical displacement, h the thickness of the layer, E the elasticity modulus, v the viscosity, p the equal distributed perpendicular force on the plate per unit of square and F a tension function dependent of x and y.

In this case the displacements are known. The desire shape of the element and from that the height of the pistons, so in (9.1) the right side is known. With this equation the force F can be calculated in the plate. Then F and w are known, p the vertical force mesh can be calculated with (9.2). This is the total vertical force in a cell. So p minus the external concrete load gives the reaction force at the pistons. To make sure that the plate touches the pistons, the reaction force has to be a compression one.

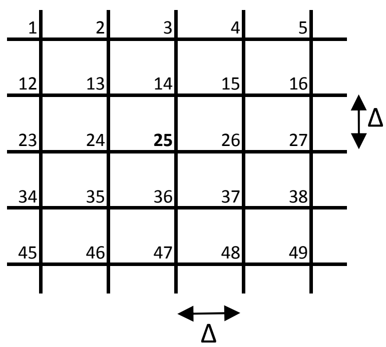


Figure 9-3: Numbering used

To solve these equations the difference method is used [13]. This method describes the connection of one cell with the neighbours by the use of the partial derivatives; see Appendix F for the partial derivatives needed in this case. Because a cell depends on its

neighbours this is an iterative process, so iterations have to be carried out to found the balance values. In a cell the displacement has to be known, so a cell is equal to the pistons surface. So Δ is the distance between the pistons, and the cell surface.
Figure 9-3 shows the used numbering. Knot 25 is the centre knot; all the partial derivatives in this point are needed to solve the differential equations.

9.2.3. Formula of F

In the general plate equation (9.1) the partial derivatives can be replaced by the formulas described in Appendix F, one of these equation is:

$$\frac{\partial^2 F}{\partial x^4}\Big|_{25} = \frac{1}{\Delta^4}\left(F_{23} - 4F_{24} + 6F_{25} - 4F_{26} + F_{27}\right) \tag{9.4}$$

These partial directives are filled in the general equitation:

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{9.5}$$

That gives for the wanted force F_{25}:

$$\begin{gathered}
\frac{1}{\Delta^4}\left(F_{23} - 4F_{24} + 6F_{25} - 4F_{26} + F_{27}\right) + \\
2\left[\frac{1}{\Delta^4}\left(F_{13} - 2F_{14} + F_{15} - 2F_{24} + 4F_{25} - 2F_{26} + F_{35} - 2F_{36} + F_{37}\right)\right] \\
+\frac{1}{\Delta^4}\left(F_3 - 4F_{14} + 6F_{25} - 4F_{36} + F_{47}\right) \\
= E\left[\left(\frac{1}{4\Delta^2}\left(w_{13} - w_{15} - w_{35} + w_{37}\right)\right)^2 - \left(\frac{1}{\Delta^2}\left(w_{24} - 2w_{25} + w_{26}\right)\frac{1}{\Delta^2}\left(w_{14} - 2w_{25} + w_{36}\right)\right)\right] \\
\Leftrightarrow \\
\frac{1}{\Delta^4}\left(F_3 + 2F_{13} - 8F_{14} + 2F_{15} + F_{23} - 8F_{24} + 20F_{25} - 8F_{26} + F_{27} + 2F_{35} - 8F_{36} + 2F_{37} + F_{47}\right) \\
= E\left[\frac{1}{\Delta^4}\left(\frac{1}{16}\left(w_{13} - w_{15} - w_{35} + w_{37}\right)^2 - \left(w_{24} - 2w_{25} + w_{26}\right)\left(w_{14} - 2w_{25} + w_{36}\right)\right)\right] \\
\Leftrightarrow
\end{gathered}$$

$$F_{25} = \frac{E\begin{pmatrix}\frac{1}{16}w_{13}^2 + \frac{1}{16}w_{15}^2 - 4w_{25}^2 + \frac{1}{16}w_{35}^2 + \frac{1}{16}w_{37}^2 - \frac{1}{8}w_{13}w_{15} - \frac{1}{8}w_{13}w_{35} + \frac{1}{8}w_{13}w_{37} - w_{14}w_{24} + 2w_{14}w_{25} - w_{14}w_{26} \\ +\frac{1}{8}w_{15}w_{35} - \frac{1}{8}w_{15}w_{37} + 2w_{24}w_{25} - w_{24}w_{36} + 2w_{25}w_{36} + 2w_{25}w_{26} - w_{26}w_{36} - \frac{1}{8}w_{35}w_{37}\end{pmatrix} - F_3 - 2F_{13} + 8F_{14} - 2F_{15} - F_{23} + 8F_{24} + 8F_{26} - F_{27} - 2F_{35} + 8F_{36} - 2F_{37} - F_{47}}{20} \tag{9.6}$$

In Excel the plate is modelled. In this case a plate of six by eleven supports. A grid is made and the given deformations (w) are filled in. Below the w another grid is made for the F, see Table 9-1. The middle cells have the formula of (9.6). The F in the cell depends on the given w of that cell, the w of the neighbour cells and the neighbours of F, as is seen in equation (9.6). At the border the formula of F_{25} is not valid

because the w does not have a deformation at the neighbour of the plate. From [13] is found that the border of F is zero and an additional fictive border is added with the value zero. Because of the dependence of the F with the neighbours iterations have to be carried out, in Excel with F9. If F remains constant a balance is found.
The total Excel sheet is given in Appendix H. Table 9-2 gives the parameter field and Table 9-1 the deflection and the calculated F.

Deflection w (mm) Given from the geometry

61	112	156	186	204	208	200	179	145	100	51
63	116	161	192	210	216	209	189	155	111	61
61	117	163	195	214	220	214	194	162	118	66
56	114	161	194	214	221	215	196	165	121	68
48	107	155	188	209	217	212	194	164	121	67
35	97	145	179	201	209	206	188	159	117	61

F (N)

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-251820	-517587	-689467	-775081	-817328	-796676	-697881	-494764	-236567	0	0
0	0	-466415	-948260	-1268940	-1431762	-1474401	-1415257	-1217751	-885047	-426932	0	0
0	0	-479934	-969851	-1297148	-1463699	-1502313	-1421877	-1225222	-898452	-435506	0	0
0	0	-274315	-553647	-722037	-811510	-837160	-791446	-692364	-518455	-259940	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9-1: Deflection w (mm) and F (N)

delta	200	mm
h	4	mm
D	61943	N/mm
E	11000	N/mm2
poisson	0,23	
Concrete load		
q concrete	0,000024	N/mm
d Concrete	50	mm
F Concrete	48	N

Table 9-2: Parameter field

9.2.4. Normal stresses

For the normal stresses in the plate applies:

$$n_{xx;25} = h\frac{\partial^2 F}{\partial y^2} = \frac{h}{\Delta^2}\left(F_{14} - 2F_{25} + F_{36}\right)$$

$$n_{yy;25} = h\frac{\partial^2 F}{\partial x^2} = \frac{h}{\Delta^2}\left(F_{24} - 2F_{25} + F_{26}\right) \qquad (9.7)$$

$$n_{xy;25} = h\frac{\partial^2 F}{\partial x \partial y} = \left(\frac{\dfrac{F_{37} - F_{35}}{2\Delta} - \dfrac{F_{15} - F_{13}}{2\Delta}}{2\Delta}\right)\frac{h}{4\Delta^2}\left(F_{13} - F_{15} - F_{35} + F_{37}\right)$$

In a new grid the stresses are calculated, see Table 9-3 as an example for the normal stresses. The sum of the x stresses in y-direction has to be zero and the sum of the y stresses in x-direction also. This is the case in the model.

Nxx(N/mm)

	0,0	-25,2	-51,8	-68,9	-77,5	-81,7	-79,7	-69,8	-49,5	-23,7	0,0
	0,0	3,7	8,7	11,0	11,8	16,0	17,8	17,8	10,4	4,6	0,0
	0,0	20,1	40,9	55,1	62,5	62,9	61,2	51,2	37,7	18,2	0,0
	0,0	21,9	43,8	60,3	68,4	69,3	63,7	54,0	39,3	18,4	0,0
	0,0	6,9	13,7	14,7	15,9	17,2	16,1	16,0	13,8	8,4	0,0
	0,0	-27,4	-55,4	-72,2	-81,2	-83,7	-79,1	-69,2	-51,8	-26,0	0,0
Σ↓	0	0	0	0	0	0	0	0	0	0	0

Table 9-3: Normal stresses Nxx(N/mm)

9.2.5. Formula of P

For equation (9.2) the same process, filling in the partial directives and determine the parameter at the middle cell, can be done to determine the force p.

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{h}{D}\left(\frac{p}{h}+\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-2\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right)$$

$$\Leftrightarrow$$

$$\frac{1}{\Delta^4}\left(w_3+2w_{13}-8w_{14}+2w_{15}+w_{23}-8w_{24}+20w_{25}-8w_{26}+w_{27}+2w_{35}-8w_{36}+2w_{37}+w_{47}\right)=$$

$$\frac{h}{D}\left[\begin{array}{l}\frac{p}{h}+\left(\frac{1}{\Delta^2}\left(F_{14}-2F_{25}+F_{36}\right)\frac{1}{\Delta^2}\left(w_{24}-2w_{25}+w_{26}\right)\right)+\left(\frac{1}{\Delta^2}\left(F_{24}-2F_{25}+F_{26}\right)\frac{1}{\Delta^2}\left(w_{14}-2w_{25}+w_{36}\right)\right)\\-2\left(\frac{1}{4\Delta^2}\left(F_{13}-F_{15}-F_{35}+F_{37}\right)\right)\left(\frac{1}{4\Delta^2}\left(w_{13}-w_{15}-w_{35}+w_{37}\right)\right)\end{array}\right]$$

$$\Leftrightarrow$$

$$p_{25}=\frac{1}{8}\frac{1}{\Delta^4}\left(\begin{array}{l}D\left(\begin{array}{l}8w_3+16w_{13}-64w_{14}+16w_{15}+8w_{23}-64w_{24}\\+160w_{25}-64w_{26}+8w_{27}+16w_{35}-64w_{36}+16w_{37}+8w_{47}\end{array}\right)+\\h\left(\begin{array}{l}F_{15}w_{35}-8F_{24}w_{14}+16F_{24}w_{25}-64F_{25}w_{25}+F_{37}w_{13}-8F_{26}w_{36}-\\F_{15}w_{13}-F_{37}w_{35}+16F_{25}w_{36}+16F_{26}w_{25}-F_{13}w_{35}-8F_{14}w_{24}-\\F_{13}w_{15}+F_{15}w_{15}-F_{35}w_{37}+F_{35}w_{35}+16F_{25}w_{24}+16F_{14}w_{25}-\\F_{15}w_{37}-8F_{24}w_{36}+16F_{25}w_{14}+F_{13}w_{37}-F_{35}w_{13}-F_{37}w_{15}+\\F_{35}w_{15}-8F_{14}w_{26}-8F_{36}w_{26}+16F_{36}w_{25}+F_{13}w_{13}+16F_{25}w_{26}-\\8F_{26}w_{14}-8F_{36}w_{24}+F_{37}w_{37}\end{array}\right)\end{array}\right) \quad (9.8)$$

This formula of p is only valid for the middle cells; the cells A in Table 9-4. This is because of p (9.8) needs two neighbours cells of w and F. At the border these cells does not exist. For each lettered border cell different boundary conditions are valid. Due to symmetry the opposite corner has the same formula. In Appendix G these edges are determined.

D	E	B	E'	D'
E'	F	C	F	E'
B''	C'''	A	C'''	B'''
E'	F	C''	F	E'
D''	E'	B'	E'	D''

Table 9-4: Numbering of Cells

Total force at the supports

A grid is made for the p. To get the force at P, the calculated p has to be multiplied by the cell surface, Δ^2 for the middle cell's.

P (N)

-26	-131	-383	-426	-546	-498	-515	-451	-259	-55	0
-111	-54	124	157	131	250	274	349	144	6	-92
-108	169	631	766	864	772	928	653	526	129	-87
-68	279	702	879	962	968	870	686	563	164	-65
-23	139	294	211	234	261	210	208	185	-847	-37
35	-143	-380	-424	-564	-470	-597	-395	-361	-162	0

Negative is tension force at the piston

Table 9-5: Force P (N)

This force plus the concrete force gives the support reaction at the pistons.

F at piston (plate and concrete) (N)

22	-83	-335	-378	-498	-450	-467	-403	-211	-7	48
-63	-6	172	205	179	298	322	397	192	54	-44
-60	217	679	814	912	820	976	701	574	177	-39
-20	327	750	927	1010	1016	918	734	611	212	-17
25	187	342	259	282	309	258	256	233	-799	11
83	-95	-332	-376	-516	-422	-549	-347	-313	-114	48

Negative is tension force at the piston

Table 9-6: Total Force at pistons (N)

See Appendix H for the whole sheet of the model. Fist the parameter field is given, where the parameters: delta (distance between the supports), h (thickness of the layer), E (elasticity), poison and the thickness of the concrete could be changed. By changing these parameters another elastic layer could be determine with the model. The displacement of the plate at the supports can be filled in. The model gives after carry out iteration until a constant value of F and p the force F, the normal stresses and the reaction forces at the supports.

9.2.6. Verification of the model

To check if the model predicts the correct behaviour of a plate the model is verified with two methods, a load case in Timoshenko [25] and a program written by De Wit [33].

Timoshenko

A case is found in [25] where a square plate is subjected to a concentrated load. The deformation in this case is of a sinus form in both directions. Timoshenko gives the deflection at the middle (9.9) and the normal stresses (9.10).

To set in the model the deformation in the sinus form the load p and the normal stresses can be determine. This can be compared with the results of Timoshenko, see Table 9-7 for the results of it.

$$w_0 = 0.802a\sqrt[3]{\frac{qa}{Eh}} \tag{9.9}$$

$$n_{xx} = 0.396\sqrt[3]{\frac{q^2Ea^2}{h^2}} \tag{9.10}$$

De Wit

Jelle de Wit wrote a computer program that describes the forces in cold bend glass panels. In this program a square plate is modelled. Three corners are clamped and one is moved downwards perpendicular to the plate. The program gives the stresses into the plate. These stresses can be checked with the ones in the model, to give the model the same deformations as in the program and set the parameters at the same value. See Table 9-7 for the results of this comparison.

Comparison

Delta	h	E	n	a	p	Wmid			Nxx max		
						Model	Timo	Delta	Model	Timo	Delta
mm	mm	N/mm2	-	mm	N/mm2	mm	mm		N/mm2	N/mm2	
500	5	11000	11	2500	0,0000	25	23,1	8%	0,32	0,58	78%
500	1	11000	11	2500	0,0000	25	22,7	9%	0,32	0,56	72%
500	10	11000	11	2500	0,0001	25	24,3	3%	0,32	0,64	98%
250	5	11000	11	1250	0,0005	25	23,1	8%	1,29	2,31	78%
100	5	11000	11	500	0,0210	25	23,1	8%	8,09	14,42	78%
50	5	11000	11	250	0,3352	25	23,1	8%	32,35	57,68	78%
25	5	11000	11	125	5,3638	25	23,1	8%	129,41	230,73	78%
5	5	11000	11	25	3352,3882	25	23,1	8%	3235,27	5768,23	78%
500	5	11000	11	2500	0,0000	25	23,1	8%	0,32	0,58	78%
500	5	11000	11	2500	63,3397	2500	2852,3	14%	3235,27	8815,70	172%
100	5	11000	11	500	1,2714	100	90,7	9%	129,40	222,68	72%
100	5	11000	11	500	0,0017	10	10,0	0%	1,29	2,73	111%
100	5	11000	11	500	0,0004	5	6,1	22%	0,32	1,01	213%
100	5	11000	11	500	0,0000	1	3,0	203%	0,01	0,25	1826%
50	5	11000	21	500	0,0004	5	6,1	22%	0,3	1,01	226%

L	h	E	w	Nxx Max			Nxx Min			Nxy Max		
				Model	de "wit"	Delta	Model	de "wit"	Delta	Model	de "wit"	Delta
mm	mm	N/mm2	mm	N/mm2	N/mm2		N/mm2	N/mm2		N/mm2	N/mm2	
100	5	11000	30	40	40	0%	-21	-15	29%	13	11	15%
100	5	11000	60	160	156	3%	-82	-44	46%	52	36	31%
200	5	11000	30	10	10	0%	-5	-4	20%	3	3	13%
200	5	11000	60	40	39	3%	-21	-12	43%	13	7	46%
1000	5	11000	60	1,6	1,5	6%	-0,8	-0,5	38%	0,5	0,4	20%
1000	1	11000	12	0,08	0,08	0%	-0,03	-0,03	0%	0,02	0,02	0%
1000	20	11000	200	18	17	6%	-9	-6	33%	5	4	20%

Table 9-7: Comparisons between The model, Timoshenko and de Wit

Conclusion

The deformation calculated with Timoshenko is in the same order obtained with the model. So the model describes p in a satisfying way. This is also the main parameter, because the plate must touch the pistons at any time.

On the other hand the stresses are in the model much smaller than calculated with Timoshenko. This is a remarkable phenomenon because the stresses and p are calculated with F, so p gives good results, but the stresses do not.

The maximum stresses are in the model in the same order as in the program of De Wit. There are a larger difference for the minimum stresses and the shear stresses.

The explanation for these differences could be the large cell surfaces, delta (Δ). Because Δ has to have the same length as the distance between the pistons, the Δ become large. It is preferred to have many cells, and a small Δ, that gives a fine mesh. But this is not possible because the displacements are only known at the pistons.

With a test, see Chapter 10, the model is tested, and verified if the model predicts the behaviour of a plate in a satisfying way.

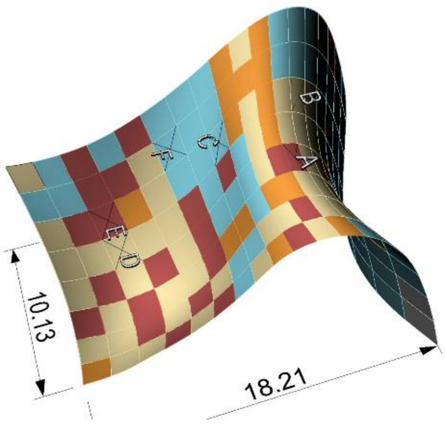


Figure 9-4: Free form building, (Evolute [57])

9.2.7. Case study

To see how the model works with geometry of a surface, a surface of a free form building is used, see Figure 9-4. The model is received from Evolute [57]; a company that makes software that subdivides a surface into elements. At a given surface and preconditions, for example: maximum length and width, angle of the edges, the software gives the optimum element distributing.

The different colours in Figure 9-4 give different kinds of curved elements.

Colour	*Curved surface*
Blue	Cylinder
Gray	Paraboloid
Yellow	Torus
Red	Cubic

From the lettered elements in Figure 9-4 the geometry is taken and put into the model. The elements are rotated and a reference surface is taken coupled at the two diagonals of the elements. For each element there are two geometries, with the reference surface above and below the element. In Appendix I this geometry is given for the surfaces A, B, C, D and F. The minimum radius of the surface is given above the table in the Appendix. Next to the geometry the pistons reactions are given, that are calculated with the model. A negative one is a tension force, preferable these tension forces are avoided.

Conclusions

No element with a realistic wooden plate thickness of 5 millimetres could be made without tension forces at the pistons. The thinner the plate, the lower the tension forces appear. This model does not consider the local displacement (the displacement between the support points), so attention must be paid that the stiffness is not too small.

When enlarging the distance Δ, at the same plate dimensions, so the less cell's, the force deviation stays the same. Only the forces increases, that is logical because the total force remains constant. Some elements have tension force all over the plate surface. If these plates are made with the plate mould, a connection is needed which transfers the tension force from the plate to the piston. These connections may only transfer the vertical force, not the horizontal ones. To test the model and see if it is possible to make double curved elements with this plate piston mould, an element is chosen which only have tension forces at the border of the plate. The tension forces at the border of the plate could be easily taken up.

9.3. Strip model

9.3.1. Introduction

Instead of a plate, strips could be used to make the mould surface. The advantage of using strips is that they can slide along each other. The extensions who came into a plate which is double curved could be transfer by the strips. The disadvantage is that an additional material is needed above the strips to make a smooth mould surface. This material has to be flexible to adjust into the double curved shape, but stiff to withstand the concrete pressure. This section discusses the model that is made to predict the support reactions; in Chapter 10 tests are preformed to verify the model.

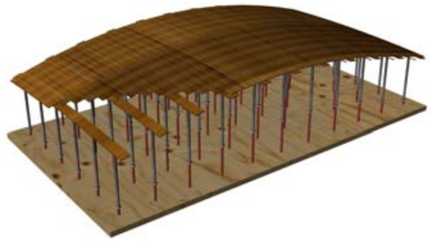


Figure 9-5: Strip model

9.3.2. Alignment

The plate is replaced by strips, as seen in Figure 9-5. These strips are arranged in two directions. The bottom layer is directed in the short direction of the formwork and only at the supports. The top layer is in the longest direction and lies against each other on top of the bottom layer to form the mould surface. The top layer of strips is only connected in the middle to the bottom strip. There may not be another solid connection between the layers of strips. If there is, the top layer can not slide along each other and the hole works as a 'solid' plate.

9.3.3. Mechanical description

The behaviour of both layers of strips is as a single curved element. So the top layer of strips is a beam with eleven supports and the bottom layer with six supports. The model that described at Chapter 5 could be adjusted to these cases.

First the parameters are given in the Excel model that could be changed. This is the distance between the supports, the Elasticity, width and the thickness of the top and bottom strips, and the thickness of the concrete that is on top of it, see Table 9-9.
The geometry of the treated element can be filled in (Table 9-8), and the relative height of each cell is determined in both directions, Table 9-8 (short and long mould direction). The model made in Chapter 5 makes use of the relative height of each point related to the line between the outside points.

Geometry (mm)

Z-as positive downwards

239	188	144	114	96	92	100	121	155	200	249
237	184	139	108	90	84	91	111	145	189	239
239	183	137	105	86	80	86	106	138	182	234
244	186	139	106	86	79	85	104	135	179	232
252	193	145	112	91	83	88	106	136	179	233
265	203	155	121	99	91	94	112	141	183	239

Table 9-8: Geometry (mm)

Parameters			
l	200	mm	Distance between the supports
Lower strip			
Eo	11000	N/mm2	
ho	3,8	mm	Thickness
bo	50	mm	With
EI_o	2514967	N/mm2	
Upper strip			
Eb	11000	N/mm2	
hb	3,8	mm	Thickness
bb	50	mm	With
EI_b	2514967	N/mm2	
Concrete			
γb	2400	kg/m3	
d	50	mm	Thickness concrete
qb	0,06	N/mm	Concrete weight at top strip
Qt	2400	N	Total force

Table 9-9: Parameters Strip mould

In longitudinal direction relative height (Compared to the line between the outside supports)

0,0	-52,0	-97,0	-128,0	-147,0	-152,0	-145,0	-125,0	-92,0	-48,0	0,0
0,0	-53,2	-98,4	-129,6	-147,8	-154,0	-147,2	-127,4	-93,6	-49,8	0,0
0,0	-55,5	-101,0	-132,5	-151,0	-156,5	-150,0	-129,5	-97,0	-52,5	0,0
0,0	-56,8	-102,6	-134,4	-153,2	-159,0	-151,8	-131,6	-99,4	-54,2	0,0
0,0	-57,1	-103,2	-134,3	-153,4	-159,5	-152,6	-132,7	-100,8	-55,9	0,0
0,0	-59,4	-104,8	-136,2	-155,6	-161,0	-155,4	-134,8	-103,2	-58,6	0,0

In transverse direction relative height (Compared to the line between the outside supports)

0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
-7,2	-7,0	-7,2	-7,4	-6,6	-7,8	-7,8	-8,2	-7,2	-7,6	-8,0
-10,4	-11,0	-11,4	-11,8	-11,2	-11,6	-11,6	-11,4	-11,4	-11,2	-11,0
-10,6	-11,0	-11,6	-12,2	-11,8	-12,4	-11,4	-11,6	-11,6	-10,8	-11,0
-7,8	-7,0	-7,8	-7,6	-7,4	-8,2	-7,2	-7,8	-7,8	-7,4	-8,0
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0

Table 9-10: Relative height in both directions (mm)

The top lay of strips has eleven supports; the formulas find to determine the reaction forces in Chapter 5 can be used in this case. A grid is made in Excel and the formulas are filled in. For example for the support reaction in point B, the second point from the left is found in Chapter 5:

$$B_v = \frac{1}{1531316}\frac{1}{l^3}\left(-171589ql^4 + \begin{pmatrix} 1494552w_B - 1438728w_C + \\ 628776w_D - 168480w_E + \\ 45144w_F - 12096w_G + \\ 3240w_H - 864w_I + 216w_J \end{pmatrix} EI\right) \quad (9.11)$$

This formula is for the cells in the second column of the grid. See Appendix B for the formulas for the other support reactions. After filling in these formulas the reaction forces are seen in Table 9-11, with the parameters of Table 9-9 and the geometry of Table 9-8.

Reaction force (N) at top strips

2,8	12,2	17,2	7,9	15,6	9,4	13,1	12,0	14,5	11,5	3,8
2,2	13,7	15,0	11,5	10,9	13,1	10,5	15,1	10,5	14,8	2,6
1,2	15,9	13,7	10,8	13,4	9,5	15,1	9,9	13,7	15,0	2,0
0,7	17,1	12,8	11,3	12,3	11,7	13,0	10,3	13,9	15,4	1,6
0,9	16,0	15,0	9,0	13,4	11,4	12,9	10,7	12,3	17,7	0,6
-0,8	20,2	11,6	9,1	16,6	6,0	18,3	7,7	11,4	20,7	-0,9

Table 9-11: Support reactions top strip (N)

The reaction force of the top strip is given by a constant q-load to the bottom strips. So the force of the top strip is divided by the length of the strips, this gives the q-loads in Table 9-12.

q-load at bottom strips due to top strips (N/mm)

0,06	0,24	0,34	0,16	0,31	0,19	0,26	0,24	0,29	0,23	0,08
0,04	0,27	0,30	0,23	0,22	0,26	0,21	0,30	0,21	0,30	0,05
0,02	0,32	0,27	0,22	0,27	0,19	0,30	0,20	0,27	0,30	0,04
0,01	0,34	0,26	0,23	0,25	0,23	0,26	0,21	0,28	0,31	0,03
0,02	0,32	0,30	0,18	0,27	0,23	0,26	0,21	0,25	0,35	0,01
-0,02	0,40	0,23	0,18	0,33	0,12	0,37	0,15	0,23	0,41	-0,02

Table 9-12: q-load of the top strips (N/mm)

The model of Chapter 5 is adapted for a beam with six supports and the reaction forces of the lower strip are determined with this model; see Appendix J for this model.

This model gives for example support point B, the second supporting point:

$$Bv=-\frac{1}{13376}\frac{\begin{pmatrix}-2561q^{1}-11688q^{2}+11448q^{3}\\+1169q^{4}-240q^{5}+168q^{6}\end{pmatrix}l^{4}+\begin{pmatrix}132096w_{B}-127104w_{c}\\+55296w_{D}-13824w_{E}\end{pmatrix}EI}{l^{3}} \tag{9.12}$$

Filling these formulas in a new grid gives for the reaction forces:

Reaction force (N) Bottom strips

-2,6	28,9	20,2	10,0	25,9	9,0	25,4	8,5	20,2	29,5	-3,2
12,7	62,2	70,2	50,3	53,5	58,6	52,7	68,6	52,3	66,2	16,4
3,8	62,7	53,0	42,2	50,9	35,9	56,4	37,2	52,5	59,6	6,0
1,5	65,3	50,6	44,7	49,3	47,4	50,8	41,1	53,8	57,6	5,1
7,2	77,0	67,7	43,3	65,2	52,2	62,5	48,9	58,3	82,9	5,5
1,2	19,4	22,2	13,7	19,2	10,5	20,7	19,5	16,3	19,9	3,5

A positive force is a compression force at the pistons

Table 9-13: Support reactions bottom strip (N)

With the two grids of the support reactions (Table 9-12 and Table 9-14) it can be determine if there appear tension forces at the supports.

To determine the local displacements, a grid is made that calculates the local deformation at the middle between two supports. In the maple model the displacement of the support point is set to zero. The displacement of the middle can then be calculated. See Appendix J for this maple sheet for the formulas of short direction, six supports, and Table 9-15 for the result of this. The maximum displacement of the middle can be read.

Local displacement (mm) In the middle of the supports

0,24	0,06	0,11	0,10	0,10	0,10	0,10	0,11	0,06	0,24	
0,22	1,02	1,36	0,74	1,15	0,88	0,98	1,09	1,08	1,01	0,29
0,24	0,06	0,11	0,10	0,10	0,10	0,10	0,11	0,06	0,24	
0,02	0,31	0,24	0,27	0,17	0,25	0,24	0,26	0,18	0,34	0,02
0,24	0,06	0,11	0,10	0,10	0,10	0,10	0,11	0,06	0,24	
0,03	0,68	0,52	0,43	0,55	0,38	0,61	0,37	0,58	0,60	0,07
0,24	0,06	0,11	0,10	0,10	0,10	0,10	0,11	0,06	0,24	
0,04	0,26	0,30	0,22	0,17	0,34	0,14	0,26	0,27	0,26	0,05
0,24	0,06	0,11	0,10	0,10	0,10	0,10	0,11	0,06	0,24	
-0,02	1,55	1,05	0,72	1,30	0,60	1,39	0,70	0,94	1,64	-0,04
0,24	0,06	0,11	0,10	0,10	0,10	0,10	0,11	0,06	0,24	

Max 1,64 mm

Table 9-14: Local displacement (mm)

In Appendix K the total Excel model is given. The dimensions and elasticity of the top and bottom strip could be varied, the distance between the pistons and the thickness of the concrete layer. As a result the model gives the reaction forces, and so it can determine if the strips touch the pistons. Also the displacement at the middle between the supports can be determined.

9.3.4. Verification of the model

The maple sheet is in Chapter 5 verified with Matrixframe, so it assumed that the maple models described the beam behaviour in a correct way.
With tests in Chapter 10 this strip model is tested. With these results the model can be verified.

9.4. Comparison plate en strip model

Element D (mainly single curved, slightly curved in the short direction Figure 9-6) is compared with the two models. In the plate model the thickness of the wooden plate is 3.8 millimetres. In the strip model, both strips are 3.8 millimetres thick and 50 millimetres width.

9.4.1. Geometry Element D (mm)

61	112	156	186	204	208	200	179	145	100	51
63	116	161	192	210	216	209	189	155	111	61
61	117	163	195	214	220	214	194	162	118	66
56	114	161	194	214	221	215	196	165	121	68
48	107	155	188	209	217	212	194	164	121	67
35	97	145	179	201	209	206	188	159	117	61

Table 9-15: Geometry Element D (mm)

Figure 9-6: Element D

9.4.2. Plate model

Forces at the supports (N)

22	**-83**	**-335**	**-378**	**-498**	**-450**	**-467**	**-403**	**-211**	**-7**	48
-63	**-6**	172	205	179	298	322	397	192	54	**-44**
-60	217	679	814	912	820	976	701	574	177	**-39**
-20	327	750	927	1010	1016	918	734	611	212	**-17**
25	187	342	259	282	309	258	256	233	**-799**	11
83	**-95**	**-332**	**-376**	**-516**	**-422**	**-549**	**-347**	**-313**	**-114**	48

Table 9-16: Plate model (N)

9.4.3. Strip model

Reaction force (N) Top strips

2,4	12,0	18,2	7,2	16,1	9,0	13,3	12,0	15,0	11,1	3,7
1,8	13,7	15,6	11,4	10,7	13,3	10,2	15,6	10,3	15,0	2,3
0,6	16,3	14,0	10,5	13,7	9,0	15,6	9,5	14,0	15,2	1,5
0,0	17,6	13,0	11,1	12,3	11,6	13,1	10,0	14,3	15,7	1,1
0,2	16,4	15,6	8,5	13,6	11,3	13,1	10,5	12,4	18,3	**-0,1**
-1,7	21,3	11,6	8,5	17,4	5,0	19,4	7,0	11,3	21,9	**-1,8**

Reaction force (N) Bottom strips

-4,2	30,1	20,5	8,5	27,0	7,4	26,5	6,7	20,5	30,8	**-5,0**
11,3	62,3	73,1	49,5	53,4	59,3	52,5	70,9	52,3	67,0	15,6
1,4	64,3	54,3	41,4	51,6	34,1	58,1	35,5	53,8	60,7	3,9
-1,3	67,4	51,5	44,3	49,8	47,5	51,5	40,1	55,2	58,3	2,9
4,8	79,6	70,2	41,4	67,1	51,8	63,8	47,9	59,2	86,4	2,8
0,2	19,1	22,8	12,8	19,2	9,1	21,0	19,6	16,0	19,7	2,9

Table 9-17: Strip Model (n)

9.4.4. Conclusion

It is clear that the forces at the strip model are much smaller than obtained with the plate model. Also the sum of tension forces is smaller. It seems that the strip model is in theory a better way of making the mould surface.
With tests, see the next chapter, the models will be tested.

10. Double curved mould tests

10.1. Introduction

After the single curved mould tests some conclusions could be made. The single curved model describes the behaviour of the flexible layer in a correct way. The best time to deform the concrete was 30 minutes for a radius of 1.5 meters and 45 minutes for radius of 2.5 meters and more.
Next, double curved tests are carried out. Goal of these tests are to see if the models made in Chapter 9 describe the flexible layer in a correct way and see if it is possible to make double curved elements with the method of deforming the fresh concrete. During these tests the radius was chosen to be about the 2.5 meters so the 45 minutes deform time are held.

10.2. Producing of the double curved elements

On the next page, a picture slideshow is shown. The text below refers to the pictures about the sequence of making the double curved elements. These sequence is followed at the tests describes in this chapter.

Picture	***Time***	***Action***
1.	t = 0 min.	The mould is ready for the concrete. The mould is horizontal and rest on a plate which is temporary supported. At t = 0 adding water to the mixture, start hardening of the concrete.
2.	t = 8 min.	Pouring the concrete into the mould.
3.	t = 15 min.	Smoothening of the fresh concrete.
4.	t = 20 min.	Finishing pouring of the concrete, covering the fresh concrete with a foil to prevent dehydration of the concrete.
5.	t = 40 min.	Making the mould ready for the deformation. Hooking with straps the temporary horizontal formwork to the crane.
6.	t = 44 min.	Lifting the formwork to be able to remove the temporary struts.
7.	t = 45 min.	Start of lowering the horizontal formwork.
8.	t = 45 min.	Start of the deformation process. Thee fist deformation of the element is seen into the middle. The time to start this process depends on the mixture. The applied deformation time is tested earlier.
9.	t = 46 min.	Further lowering of the temporary horizontal formwork. Further deforming of the mould.
10.	t = 46 min.	Further deforming of the mould.
11.	t = 47 min.	Further deforming of the mould.
12.	t = 47 min.	Further deforming of the mould.
13.	t = 47 min.	Further deforming of the mould.
14.	t = 48 min.	Further deforming of the mould.
15.	t = 48 min.	The end of deformation process. The element has the final shape.
16.	t = 50 min.	Deformed element.
17.	t = 1 day	After one day of hardening the element is removed from the mould.
18.	t = 1 day	Lifting of the element with a crane.
19.	t = 1 day	Lifting of the element with a crane.
20.	t = 1 day	Lifting of the element with a crane.
21.	t = 1 day	Removing of the mould.
22.	t = 1 day	The element above its mould.
23.	t = 1 day	Removing of the flexible mould to be able to place the element on its supports.
24.	t = 1 day	The final element on its supports.

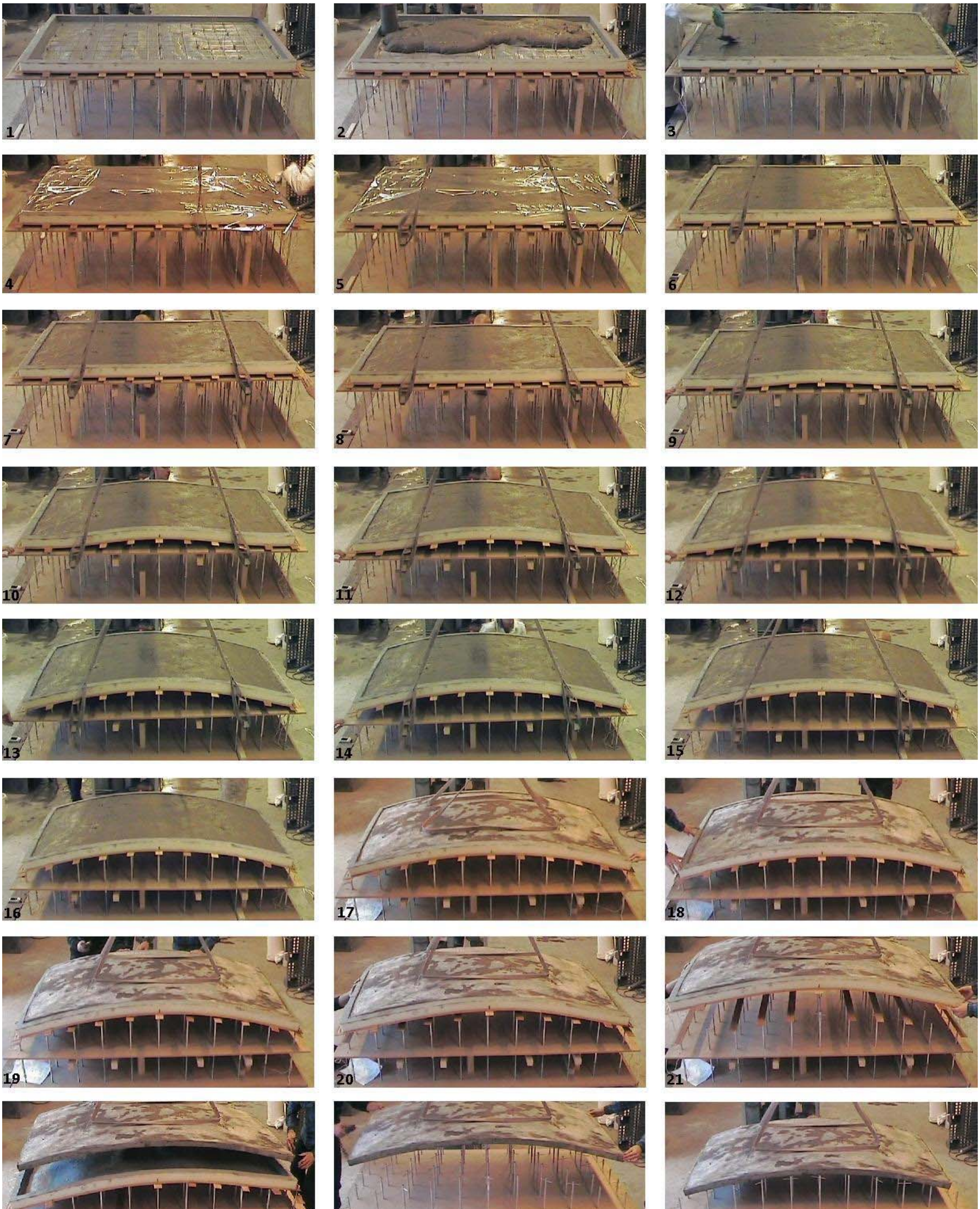


Figure 10-1: Sequence of making the double curved elements (strip mould element D, section 10.4.1)
(Pictures sequence from left to right, and top to bottom)

10.3. Plate model test

10.3.1. Element D

Objective of the test

Main goal of this test is to verify the double curved plate model, made in Chapter 9.2. It is also to see if it is possible to make double curved elements with the principle pouring the concrete horizontal and deforming the fresh concrete after partly hardening.

Figure 10-2: Horizontal mould

Figure 10-3: Deformed mould

Test facility

Overall goal is to improve the actuator formwork. In order to make the test not to complicate the actuators are omitted. Instead of making use of wooden panels, a system is used which could adapt to multiple forms. Also, it was clear that the horizontal pouring must take place above the right deformed position; in the single curved test it was difficult to place the mould with the fresh concrete on the right place on top of the panels. Because of the measurements of the double curved mould it should be harder. Figure 10-2 and Figure 10-3 gives a view of the used mould.

Following the idea of Mr. Schipper a stud with a bolt is used to set up the support points at the right height, Figure 10-4. A hollow tube is place on the stud and rest on the bolt. By twisting the bolt the support point can be set at the desire height. The studs are screwed vertical in a wooden plate.

To be able to place the mould in a horizontal position, a bar is placed in the top all of the tubes at the same height. The tube goes through a wooden plate. To lift this plate and let it rest on wooden struts the mould can be set in a horizontal position. The bar in the tube prevent that the tube slides touch the hole in the plate. The supporting plate with concrete could be lowered with the use of a crane and set to the deformed position.

In total 66 of these support points are used, in a set up of six times eleven. With a distance between each support point of 200 millimetres, in both directions, the total dimension of the mould is 2 by 1 meters.

Figure 10-4: Stud and bolt

Figure 10-5: Mould at horizontal position

On top of the tubes the mould plate is placed, this is a wooden plate (Figure 10-6). This plate lies loose on the support points. Goal of this test is to produce element D (see Table 10-2 for the height of the supports). This element is relatively small double curved. In the longest direction it is strongly curved, the radius is small. The other direction has a large radius. With the needed height of the supports the model (Appendix H) gives with a plate thickness of 3.8 millimetres acceptable support reactions (Table 10-3). As seen in the Table 10-3, there are tension forces obtained. Starting point was that they were tried to avoid. But it was not possible to find a double curved shape where they do not appear. To choose an element where the tension forces are obtained at the border supports, these tension forces could be easily omitted (through holes in the plate at the supports a string is placed which can be attracted, Figure 10-8).

Figure 10-6: Plate on top of the tubes

61	112	156	186	204	208	200	179	145	100	51
63	116	161	192	210	216	209	189	155	111	61
61	117	163	195	214	220	214	194	162	118	66
56	114	161	194	214	221	215	196	165	121	68
48	107	155	188	209	217	212	194	164	121	67
35	97	145	179	201	209	206	188	159	117	61

Table 10-2: Support height Element D

22	**-83**	**-335**	**-378**	**-498**	**-450**	**-467**	**-403**	**-211**	**-7**	48
-63	**-6**	172	205	179	298	322	397	192	54	**-44**
-60	217	679	814	912	820	976	701	574	177	**-39**
-20	327	750	927	1010	1016	918	734	611	212	**-17**
25	187	342	259	282	309	258	256	233	**-799**	11
83	**-95**	**-332**	**-376**	**-516**	**-422**	**-549**	**-347**	**-313**	**-114**	48

Table 10-3: Support reactions Element D

Figure 10-7: Sealed edge of polyether

Figure 10-8: Strings at the edge to be able to take the tension forces at the edge supports

As border of the mould polyether is used. The single curved test demonstrated that this is a good border material. The border has a height and width of 50 millimetres. The polyether is glued to the wooden plate and sealed to prevent the moisture of the concrete to be absorbed into the polyether. This is done also to make the demoulding of the element easier, and make the mould reusable.

To be able to use the same deform time, the same mixture as in the single curved test is applied, mixture two (see Chapter 6).
Also the same reinforcement is used, a net of wires with a diameter of 3 millimetres. This net is placed at 15 millimetres above the bottom of the element. This reinforcement is too little to make the element load bearing. The reinforcement is only to be able to demould the element. See Figure 10-9 for a picture of the horizontal mould with the reinforcement net.

Test results
Chapter 10.2 gives a slide show of the production of a double curved element. The element that is shown is not the element produced in this section, but the one in Section 10.4. The sequence of producing is the same at this test. The fresh concrete is deformed after 45 minutes of hardening. As seen in Figure 10-11 and Figure 10-12 the fresh concrete deforms nicely. No cracks were visible and no flowing of fresh concrete appears.

As expected with the model, the plate did not touch all of its supports at the edges (Figure 10-14). The strings that are through the hole are tightening to take up the tension force at the supports. After a few where fixing the plate begin to buckle between the support points, Figure 10-15, Figure 10-16 and Figure 10-17. After demoulding of the element this buckling effect is clearly seen into the border of the element, Figure 10-18 and Figure 10-19. This is unsatisfying.

Figure 10-9: Reinforcement

Figure 10-10: Horizontal mould

Figure 10-11: Just deformed element

Figure 10-12: Just deformed element no cracks are visible

Figure 10-13: Bottom of the mould, middle supports

Figure 10-14: Outside supports, Plate does not touch the supports

This buckling effect is explained by the Eulerse-buckling [7] (buckling of the border):

$$F_e = \frac{\pi^2 EI}{l^2}$$

$$\sigma_{xx} bh = \frac{\pi^2 E \frac{1}{12} bh^3}{l^2}$$

$$\sigma_{xx} = \frac{\pi^2 E h^2}{12 l^2} \qquad (10.1)$$

$$\sigma_{xx} = \frac{\pi^2 * 11000 * 3.8^2}{12 * 200^2}$$

$$\sigma_{xx} = 3.26 N / mm^2$$

The maximum normal stress is (Appendix H):

$$N_{xx} = 83 N / mm$$

$$n_{xx} = 83/3.8 = 21.8 N / mm^2 \qquad (10.2)$$

The stress in the plate is much larger that the critical buckling stress, so this explains the buckling. The critical buckling stress is so low, that this will happen almost always.

Figure 10-15: Edge after tensioning

Figure 10-16: Buckling of the edge

Figure 10-17: Buckling of the edge

Conclusions

- Due to relative large normal stress into the plate, the plate buckles when the plate is deformed and pushed into the desire shape.
- This phenomenon occurs at this, relative small double curved element. It will not be possible to produce with a plate as flexible layer double curved elements.
- The model that is made describes the plate behaviour in a correct way. It showed on the right places where the tension forces (the plate does not thoughts its supports) should occur.
- The alignment with the studs and bolts works well. Also pouring the concrete horizontal above the alignment and only lowering with a crane works satisfying.
- The border of polyether deformed nicely and makes sure that the border of the element is perpendicular.

Figure 10-18: Unmolded element placed on the supports

Figure 10-19: Buckled edge

Figure 10-20: Mould surface

10.4. Strip mould test

10.4.1. Element D

Objective of the test

After Section 10.3 it becomes clear that a plate is not suitable for a double curved mould layer. In this section the test that is carried out with the strip mould are described. The aim is to see if it is possible to produce double curved elements with the strip mould, see Figure 10-21 and 10-22 for a view of the strip mould. Also the model that describes the strip mould (Section 9.3) is verified. The same shape of an element (element D) is produced to be able to compare the elements made with the plate and the strip mould.

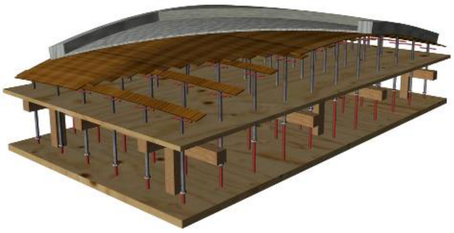


Figure 10-21: Cross section Strip mould

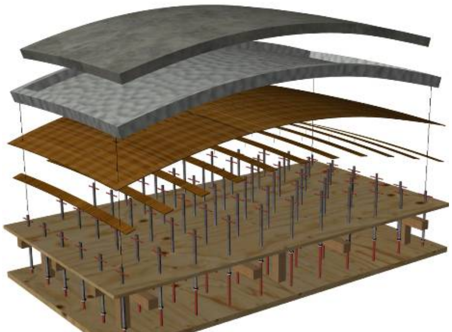


Figure 10-22: Strip Mould

Test facility

The bottom part of the mould, the studs, bolts and tubes are the same as at the plate tests. The plate is replaced by the strips.

The bottom layer of strips is in the short direction of the mould. These strips are only connected to the tubes in the middle by a nail (Figure 10-23). This connection makes sure that the strips stays on the tubes and counteract the horizontal movements. The connection is only in the middle to make sure that the strips can move horizontally at the ends and take up on this way the needed extensions. These strips, as well as the top layer, have a width of 50 millimetres and a thickness of 3.8 millimetres.

The top layer of strips is only connected in the middle by a nail to the bottom layer. At the other points the top layer is loosely placed on the bottom layer. This is done to make sure that the top layer of strips can slide along each other. In this way the extensions due to a double curved surface can be taken up.

The disadvantage of the strips is that an additional layer is needed as bottom mould layer. The concrete can not be poured directly on to the strips. The concrete could flow through the seams between the strips. Also the deformed strips could disturb the element. The same material that is used for the borders, polyether is used as bottom layer. This bottom layer is 10 millimetres thick and is sealed as well as the border (Figure 10-26). As seen in Table 10-4 at some edges a small tension force is expected. So holes are made with strings to be able to take up this tension forces (Figure 10-25).

Reaction force (N) at top strips

2,4	12,0	18,2	7,2	16,1	9,0	13,3	12,0	15,0	11,1	3,7
1,8	13,7	15,6	11,4	10,7	13,3	10,2	15,6	10,3	15,0	2,3
0,6	16,3	14,0	10,5	13,7	9,0	15,6	9,5	14,0	15,2	1,5
0,0	17,6	13,0	11,1	12,3	11,6	13,1	10,0	14,3	15,7	1,1
0,2	16,4	15,6	8,5	13,6	11,3	13,1	10,5	12,4	18,3	**-0,1**
-1,7	21,3	11,6	8,5	17,4	5,0	19,4	7,0	11,3	21,9	**-1,8**

Reaction force (N) Bottom strips

-4,2	30,1	20,5	8,5	27,0	7,4	26,5	6,7	20,5	30,8	**-5,0**
11,3	62,3	73,1	49,5	53,4	59,3	52,5	70,9	52,3	67,0	15,6
1,4	64,3	54,3	41,4	51,6	34,1	58,1	35,5	53,8	60,7	3,9
-1,3	67,4	51,5	44,3	49,8	47,5	51,5	40,1	55,2	58,3	2,9
4,8	79,6	70,2	41,4	67,1	51,8	63,8	47,9	59,2	86,4	2,8
0,2	19,1	22,8	12,8	19,2	9,1	21,0	19,6	16,0	19,7	2,9

Table 10-4: Reaction force element D

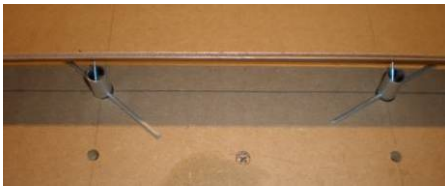


Figure 10-23: Bottom strip with nail

Figure 10-24: Bottom of the strip mould

Figure 10-25: Border of Polyether

Figure 10-26: Mould at horizontal position

Test results

The same concrete mixture and reinforcement (10.3.1) is used to be able to compare the elements and results.
See Section 10.2 for a slide show of the total production of this element.
After 45 minutes resting the concrete element is deformed, see Figure 10-27 for the deformed element. Deforming of the concrete went well. No visible cracks appeal and the concrete did not flow away.
The mould deformed easily into the desire curved shape, Figure 10-27, 10-28 and 10-29. As expected from calculations from the model at some places the strips do not touches the supports, so the ropes are tightening.
After one day of hardening the element is unmolded. After placing the element back to the supports (Figure 10-31 and 10-32) it become clear that the overall shape of the element was good. The surface quality was less. Due to the rough sealed polyether the surface of the element was also very rough, see Figure 10-33. Also the polyether expanded at several places and folded; this is clearly seen at the element (Figure 10-30).

Figure 10-27: Just deformed element

Figure 10-28: Curved border

Conclusions

- The overall shape of the element satisfies.
- The strip mould behaves well as flexible layer for making double curved elements.
- The strip model describes the behaviour of the double curved flexible layer in a correct way.
- Due to the strips the mould curved perfect into the desire way. Also after tightening of the ropes the border is smoothly curved.
- Due to the sealed surface of the polyether the element becomes very rough. Sealing is needed to make the demoulding possible and to prevent that the moisture of the concrete to be absorbed into the polyether. To make the surface of the element satisfying another surface finishing method is required: protection the polyether in a different way, or by use of another flexible material.

Figure 10-29: Bottom of the mould

Figure 10-30: Bottom of the element, distribution of the surface

Figure 10-31: Unmolded element

Figure 10-32: Unmolded element

Figure 10-33: Bottom side of the element

10.4.2. Element A

Figure 10-34: Element A

Objective of the test

After the last test it became clear that the strip mould performs well. An additional test is performed to see if it is also possible to produce a lager double curved element. For the shape element A is chosen from the free form building, Figure 9-4. The element is scaled to enlarge the radius and make use of the full capacity of the alignment (the height of the studs is the maximum distance between the highest and lowest point of the element), see Table 10-5 for the height of the supports. The element has a maximum radius of about two meters.

Heights of the supports

95	72	51	39	35	37	46	61	84	113	144
173	144	120	103	93	92	96	109	130	156	190
217	183	154	133	121	116	119	130	148	175	210
225	186	155	131	117	112	113	123	141	168	205
197	155	123	99	85	78	81	91	109	138	177
135	92	60	35	21	15	19	32	53	86	127

Reaction force (N) at top strips

4,5	18,0	6,5	12,8	11,3	14,2	10,3	13,6	7,3	17,0	4,7
7,1	11,5	10,8	14,1	8,0	16,6	9,8	10,1	15,8	7,9	8,2
6,3	13,7	11,1	7,7	17,6	9,2	11,5	13,8	11,5	9,0	8,6
8,9	7,0	16,4	8,5	12,5	13,7	11,0	11,4	15,5	4,9	10,3
9,5	6,3	16,2	7,2	17,6	6,6	15,6	12,3	10,3	9,3	9,0
10,6	2,9	20,7	4,8	16,8	9,1	12,5	15,9	5,0	14,5	7,2

Reaction force (N) Bottom strips

0,6	**-8,8**	16,1	**-10,6**	18,7	**-6,5**	12,2	13,6	**-6,3**	12,8	0,1
50,6	73,0	68,2	84,3	57,4	91,2	61,0	63,1	82,1	57,4	52,2
20,1	51,1	37,8	24,2	66,6	32,1	42,5	52,4	42,4	31,2	28,8
33,1	24,6	61,7	29,0	40,7	48,4	37,4	38,2	57,8	16,0	38,8
62,5	38,3	91,0	48,0	93,0	52,1	84,9	69,5	52,6	49,6	47,5
-9,2	17,4	**-2,3**	10,3	2,5	13,7	**-1,1**	12,6	8,3	20,5	0,9

Table 10-5: Element A,
Height of the supports and reaction forces

Test facility

Exactly the same alignment as used for the last test is used. The bolts are screwed to set the support points at the right height.
After the last test the bottom of polyether was stretched so much that the layer bulged. The bottom layer is cut open and some polyether is removed, see Figure 10-35. The seam between the foam is sealed to make it a whole.
The reinforcement net, which also is used at the other test, is weakened to make sure that it can follow the double curvature. A net can only make a developable surface, which a double curved is not. In longitudinal direction the transverse bar are cut at the ends of the net in half. This makes the net flexible and hopefully it can follow the curvature. In Figure 10-41 the red line is the line where the rods are cut to weaken the plate.

Figure 10-35: Stretched bottom layer of polyether after the first test

Test results

This element is also deformed after 45 minutes. See Figure 10-39 for a slide show of the producing of this element, and Figure 10-37 and 10-43 for a picture just after the deformation of the concrete.
During the deformation process the mould deformed in the desire way. As shown by Figure 10-36, 10-38, 10-40 and 10-42 the strips curved nicely. As expected from the predictions, at some border supports the mould does not touch the support. With a sting the mould is pushed downwards at the support to give the element the desire shape. After the deformation process few little cracks appeared. Because the concrete is still pretty liquid, the cracks could be smoothened.
After one day of hardening the element is removed from the mould. Also at this test the overall shape of the element satisfy, no cracks where visible, only the concrete surface did not satisfy; the surface is too rough. Figures 10-44 till 10-50 are pictures of the hardened element.

Figure 10-36: Bottom of the mould

Figure 10-37: Just deformed concrete

Figure 10-38: Bottom of the mould

Figure 10-39: Sequence of producing the element

Figure 10-40: Edge

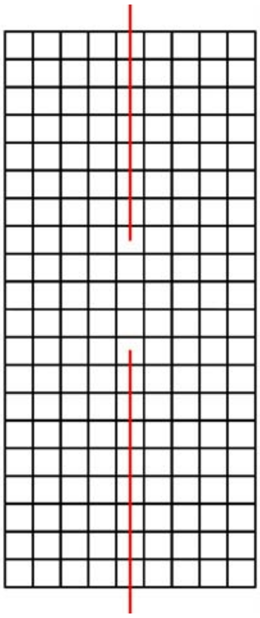


Figure 10-41: Weakened reinforcement net

Figure 10-42: Edge

Figure 10-43: Just deformed element

Figure 10-44: Unmolded element

Conclusions

- It is possible to produce double curved elements with the method: deforming the partly hardened concrete.
- The model describes the strip behaviour in a correct way.
- The overall shape of the element satisfies. The surface of the element is to rough, this is due to the sealed polyether.
- The radius is too small for the weakened reinforcement net. At some places the spacer is pushed out of the element. This is also due to the polyether bottom layer. The spacer is pushed easily into this soft material, when using a wooden plate the spacer pushed at the plate, and can not come out of the element.

Figure 10-45: Mould side

Figure 10-46: Top side

Figure 10-47: Mould side

Figure 10-48: Element at supports

Figure 10-49: Top side

Figure 10-50: Bottom side

10.5. Conclusions

After the tests it was clear that the strip mould is the best configuration for the flexible layer. Due to the strains in a plate, the plate buckles and so the mould does not get the desire shape. In a strip mould the strips could slide alongside each other and so take up the extensions which appear in a double curved surface.

Figure 10-51: Produced double curved element

It is possible to produce a double curved concrete element with the principle of deforming the fresh concrete which is partly hardened.

The two models that were proposed in Chapter 9, for a plate and the strip configuration, describe the behavior of the elastic layer in a correct way. At the expected support points there appear tension forces; the plate did not touch some of the support points.

The big disadvantage of the strip mould is the additional flexible layer that covers the strips. In the executed tests this was a layer of sealed polyether. The sealed surface was very rough, which was seen afterwards into the element. Additional tests had to be carried out to find a material that can protect the polyether of another elastic material has to be found.

11. Conclusions and recommendations

11.1. Conclusions

- It was demonstrated that it is possible to produce double curved concrete elements with a flexible mould using the strip configuration. In the setup two layers of strips (perpendicular to each other) were placed on top of pistons, which can be set in the desired height. The elements are made with the principle of deforming the partly hardened concrete. The concrete is poured when the mould is in a horizontal position; the pistons are all set at the same height. After a short period, the concrete is hardened enough to be stable on a slope, the flexible mould is deformed, and the fresh concrete gets the desired double curved shape.

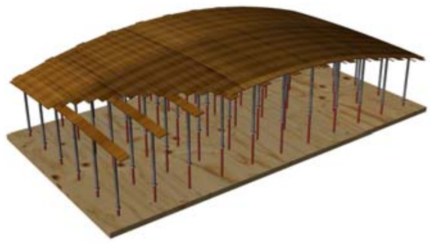


Figure 11-1: Strip model

- The bottom layer of the formwork, the flexible layer, is placed on top of the pistons, and must deform smoothly to be able to make sure that the elements get the desire shape. A thin plate does not fulfil this function; with a plate only developable surfaces or elements with very limited double curvature (large radii) are producible. When a free form double curved shape with smaller radii is made, the plate buckles, and the mould does not get the right shape.
  After building the plate out of strips, Figure 11-1, the flexible mould deformed in the desired way. The strips can slide alongside each other. On this way a smooth surface is formed.

- The models for a single curved surface, double curved surface and strip configuration that are proposed in this thesis describe the behavior of the elastic layer in a correct way. Using these models it is possible to verify in advance whether a shaped element can be produced. Given a thickness and elasticity of the elastic layer the support reactions are proposed. If all of these reaction forces are compressive, the elastic layer is pushed to the supporting points. In this way the mould surface and the element get the desire shape.

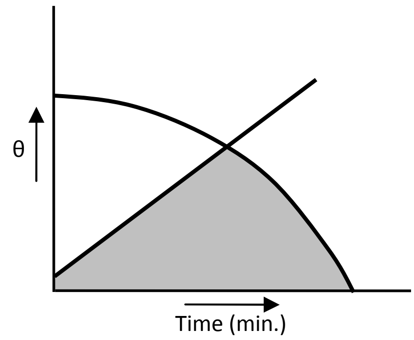


Figure 11-2: Deformable area

- The time during which the horizontal mould can be deformed depends on the composition of the concrete mixture. In Figure 11-2 the relation between the hardening time of the concrete and the slope that can be made is shown schematically. This relation depends on the slope stability, and the resting of the concrete. Directly after mixing the fresh concrete is still fluid, it flows down from a slope. The resting of the concrete makes it stable. The longer the resting period of the mixture, the more stable it gets, the larger the slope of the mould is that can be produced. However, the hardening of the mixture finally leads to a solid concrete which cracks when it is deformed. This gives the gray deformable region area in the picture.

11.2. Recommendations

- The deform time depends mainly on the slope stability of the mixture, the yield stress of it. The deformation of the concrete could best take place as soon as possible. In a fresher state the concrete restores the best. With enough slump cones, this value can be determined at different time points. So it is possible to point out when the mould could be deformed, for a mixture and a specific radius of the formwork. It requires a lot of tests to find the exact values of the graph (Figure 11-2). The graph with values is only valued for one mixture. It is recommended to carry out further research to determine the deform time of a specific mixture or find a general relation.

- During the test it became clear that polyether is a suitable material for the border. It is soft enough to deform, but it is stiff enough to resist the concrete pressure. It remains perpendicular to the mould when the concrete is poured against it.
  The polyether could serve also as a bottom layer for the strip mould, but the finishing of the surface is problematic. Because of the porosity of the polyether, moisture of the concrete mixture can absorb into it. Before doing the test the polyether was therefore first protected with sealant. The finishing of the sealed surface however appeared to be very rough, and so was the produced element afterwards. It is advised to find another material to protect the polyether and gives the concrete a smooth surface, or to find another material as flexible layer.

Figure 11-3: Double curved element at the flexible mould

- The produced elements have limited reinforcement, due to the problems of deforming the fresh concrete together with traditional steel reinforcement. The reinforcement must be flexible enough to deform along with the fresh concrete and the mould. For the tests very thin reinforcement was used, which could follow the radius. However, the diameter of this reinforcement was too limited to serve in load bearing elements. Other methods have to be investigated and tested to enlarge the load bearing capacity of the elements.

- During the research in this thesis the focus was on improving the flexible formwork for thin panels. To make the elements load bearing, these elements are used as a predalle (Dutch: ‘breedplaat’). A next step could be to make thick panels with the flexible formwork. In that situation, the main research problem might become the strains and consequently the cracks during the deformation process of the fresh partly hardened concrete. The thicker the panels, the larger the necessary strains are. The maximum thickness of the double curved panels that can be made for a radius with the flexible formwork have to be found.

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Appendices

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Appendix B. Maple model, single curved model

Ligger op 11 steunpunten
Bas Janssen

Alle waardes in:
N en mm.

Algemene belastingformules

restart;

Verdeelde belasting

$wq := \frac{q \cdot x}{24 \cdot EI} \cdot (L^3 - 2 \cdot L \cdot x^2 + x^3);$

$$\frac{1}{24} \frac{q x (L^3 - 2 L x^2 + x^3)}{EI} \quad (1.1)$$

Puntlast

Als x, de plaats van de steunpunt links van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfL := \frac{F \cdot b \cdot x}{6 \cdot EI \cdot L} \cdot (2 \cdot L \cdot (L - x) - (b)^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F b x (2 L (L - x) - b^2 - (L - x)^2)}{EIL} \quad (1.2)$$

Als x, de plaats van de steunpunt rechts van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfR := \frac{F \cdot a \cdot (L - x)}{6 \cdot EI \cdot L} \cdot ((2 \cdot L \cdot b) - b^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F a (L - x) (2 L b - b^2 - (L - x)^2)}{EIL} \quad (1.3)$$

Verplaatsingen in de steunpunten

Vergelijking in B

$x := 1 : L := 10 \cdot 1 : wBq := wq :$

$F := Bv : b := 9 \cdot 1 : wBb := wfL : F := Cv : b := 8 \cdot l : wBc := wfL : F := Dv : b := 7 \cdot l : wBd := wfL :$
$F := Ev : b := 6 \cdot l : wBe := wfL : F := Fv : b := 5 \cdot l : wBf := wfL : F := Gv : b := 4 \cdot l : wBg := wfL :$
$F := Hv : b := 3 \cdot l : wBh := wfL : F := Iv : b := 2 \cdot l : wBi := wfL : F := Jv : b := 1 \cdot l : wBj := wfL :$
$eq1 := wB = wBq - wBb - wBc - wBd - wBe - wBf - wBg - wBh - wBi - wBj;$

$$wB = \frac{327}{8} \frac{q l^4}{EI} - \frac{27}{10} \frac{Bv l^3}{EI} - \frac{14}{3} \frac{Cv l^3}{EI} - \frac{35}{6} \frac{Dv l^3}{EI} - \frac{63}{10} \frac{Ev l^3}{EI} - \frac{37}{6} \frac{Fv l^3}{EI} - \frac{83}{15} \frac{Gv l^3}{EI} - \frac{9}{2} \frac{Hv l^3}{EI} - \frac{19}{6} \frac{Iv l^3}{EI} - \frac{49}{30} \frac{Jv l^3}{EI} \quad (2.1)$$

Vergelijking in C

$x := 2 \cdot 1 : L := 10 \cdot 1 : wCq := wq :$
$F := Bv : a := l : b := 9 \cdot l : wCb := wfR : F := Cv : a := 2 \cdot l : b := 8 \cdot l : wCc := wfR : F := Dv : b := 7 \cdot l : wCd := wfL :$
$F := Ev : b := 6 \cdot l : wCe := wfL : F := Fv : b := 5 \cdot l : wCf := wfL : F := Gv : b := 4 \cdot l : wCg := wfL :$

$F := Hv : b := 3 \cdot l : wCh := wfL : F := Iv : b := 2 \cdot l : wCi := wfL : F := Jv : b := 1 \cdot l : wCj := wfL :$

$eq2 := wC = wCq - wCb - wCc - wCd - wCe - wCf - wCg - wCh - wCi - wCj;$

$$wC = \frac{232}{3} \frac{q l^4}{EI} - \frac{14}{3} \frac{Bv l^3}{EI} - \frac{128}{15} \frac{Cv l^3}{EI} - \frac{329}{30} \frac{Dv l^3}{EI} - \frac{12 Ev l^3}{EI} - \frac{71}{6} \frac{Fv l^3}{EI} - \frac{32}{3} \frac{Gv l^3}{EI} - \frac{87}{10} \frac{Hv l^3}{EI} - \frac{92}{15} \frac{Iv l^3}{EI} - \frac{19}{6} \frac{Jv l^3}{EI} \quad (2.2)$$

Vergelijking in D

$x := 3 \cdot 1 : L := 10 \cdot 1 : wDq := wq :$

$F := Bv : a := 1 \cdot l : b := 9 \cdot l : wDb := wfR : F := Cv : a := 2 \cdot l : b := 8 \cdot l : wDc := wfR : F := Dv : a := 3 \cdot l : b := 7 \cdot l : wDd := wfR :$
$F := Ev : a := 4 \cdot l : b := 6 \cdot l : wDe := wfL : F := Fv : a := 5 \cdot l : b := 5 \cdot l : wDf := wfL : F := Gv : a := 6 \cdot l : b := 4 \cdot l : wDg := wfL :$
$F := Hv : a := 7 \cdot l : b := 3 \cdot l : wDh := wfL : F := Iv : a := 8 \cdot l : b := 2 \cdot l : wDi := wfL : F := Jv : a := 9 \cdot l : b := 1 \cdot l : wDj := wfL :$
$eq3 := wD = wDq - wDb - wDc - wDd - wDe - wDf - wDg - wDh - wDi - wDj;$

$$wD = \frac{847}{8} \frac{q l^4}{EI} - \frac{35}{6} \frac{Bv l^3}{EI} - \frac{329}{30} \frac{Cv l^3}{EI} - \frac{147}{10} \frac{Dv l^3}{EI} - \frac{33}{2} \frac{Ev l^3}{EI} - \frac{33}{2} \frac{Fv l^3}{EI} - \frac{15 Gv l^3}{EI} - \frac{123}{10} \frac{Hv l^3}{EI} - \frac{87}{10} \frac{Iv l^3}{EI} - \frac{9}{2} \frac{Jv l^3}{EI} \quad (2.3)$$

Vergelijking in E

$x := 4 \cdot 1 : L := 10 \cdot 1 : wEq := wq :$
$F := Bv : a := 1 \cdot l : b := 9 \cdot l : wEb := wfR : F := Cv : a := 2 \cdot l : b := 8 \cdot l : wEc := wfR : F := Dv : a := 3 \cdot l : b := 7 \cdot l : wEd := wfR :$
$F := Ev : a := 4 \cdot l : b := 6 \cdot l : wEe := wfR : F := Fv : a := 5 \cdot l : b := 5 \cdot l : wEf := wfL : F := Gv : a := 6 \cdot l : b := 4 \cdot l : wEg := wfL :$
$F := Hv : a := 7 \cdot l : b := 3 \cdot l : wEh := wfL : F := Iv : a := 8 \cdot l : b := 2 \cdot l : wEi := wfL : F := Jv : a := 9 \cdot l : b := 1 \cdot l : wEj := wfL :$
$eq4 := wE = wEq - wEb - wEc - wEd - wEe - wEf - wEg - wEh - wEi - wEj;$

$$wE = \frac{124 q l^4}{EI} - \frac{63}{10} \frac{Bv l^3}{EI} - \frac{12 Cv l^3}{EI} - \frac{33}{2} \frac{Dv l^3}{EI} - \frac{96}{5} \frac{Ev l^3}{EI} - \frac{59}{3} \frac{Fv l^3}{EI} - \frac{272}{15} \frac{Gv l^3}{EI} - \frac{15 Hv l^3}{EI} - \frac{32}{3} \frac{Iv l^3}{EI} - \frac{83}{15} \frac{Jv l^3}{EI} \quad (2.4)$$

Vergelijking in F

$x := 5 \cdot 1 : L := 10 \cdot 1 : wFq := wq :$
$F := Bv : a := 1 \cdot l : b := 9 \cdot l : wFb := wfR : F := Cv : a := 2 \cdot l : b := 8 \cdot l : wFc := wfR : F := Dv : a := 3 \cdot l : b := 7 \cdot l : wFd := wfR :$
$F := Ev : a := 4 \cdot l : b := 6 \cdot l : wFe := wfR : F := Fv : a := 5 \cdot l : b := 5 \cdot l : wFf := wfR : F := Gv : a := 6 \cdot l : b := 4 \cdot l : wFg := wfL :$
$F := Hv : a := 7 \cdot l : b := 3 \cdot l : wFh := wfL : F := Iv : a := 8 \cdot l : b := 2 \cdot l : wFi := wfL : F := Jv : a := 9 \cdot l : b := 1 \cdot l : wFj := wfL :$
$eq5 := wF = wFq - wFb - wFc - wFd - wFe - wFf - wFg - wFh - wFi - wFj;$

$$wF = \frac{3125}{24} \frac{q l^4}{EI} - \frac{37}{6} \frac{Bv l^3}{EI} - \frac{71}{6} \frac{Cv l^3}{EI} - \frac{33}{2} \frac{Dv l^3}{EI} - \frac{59}{3} \frac{Ev l^3}{EI} - \frac{125}{6} \frac{Fv l^3}{EI} - \frac{59}{3} \frac{Gv l^3}{EI} - \frac{33}{2} \frac{Hv l^3}{EI} - \frac{71}{6} \frac{Iv l^3}{EI} - \frac{37}{6} \frac{Jv l^3}{EI} \quad (2.5)$$

Vergelijking in G

$x := 6 \cdot 1 : L := 10 \cdot 1 : wGq := wq :$

$F := Bv : a := 1\cdot l : b := 9\cdot l : wGb := wfR : F := Cv : a := 2\cdot l : b := 8\cdot l : wGc := wfR : F := Dv : a := 3\cdot l : b := 7\cdot l : wGd := wfR :$
$F := Ev : a := 4\cdot l : b := 6\cdot l : wGe := wfR : F := Fv : a := 5\cdot l : b := 5\cdot l : wGf := wfR : F := Gv : a := 6\cdot l : b := 4\cdot l : wGg := wfR :$
$F := Hv : a := 7\cdot l : b := 3\cdot l : wGh := wfL : F := Iv : a := 8\cdot l : b := 2\cdot l : wGi := wfL : F := Jv : a := 9\cdot l : b := 1\cdot l : wGj := wfL :$
$eq6 := wG = wGq - wGb - wGc - wGd - wGe - wGf - wGg - wGh - wGi - wGj;$

$$wG = \frac{124\, q\, l^4}{EI} - \frac{83}{15}\frac{Bv\, l^3}{EI} - \frac{32}{3}\frac{Cv\, l^3}{EI} - \frac{15\, Dv\, l^3}{EI} - \frac{272}{15}\frac{Ev\, l^3}{EI} - \frac{59}{3}\frac{Fv\, l^3}{EI} - \frac{96}{5}\frac{Gv\, l^3}{EI} - \frac{33}{2}\frac{Hv\, l^3}{EI} - \frac{12\, Iv\, l^3}{EI} - \frac{63}{10}\frac{Jv\, l^3}{EI} \qquad (2.6)$$

Vergelijking in H

$x := 7\cdot 1 : L := 10\cdot 1 : wHq := wq :$
$F := Bv : a := 1\cdot l : b := 9\cdot l : wHb := wfR : F := Cv : a := 2\cdot l : b := 8\cdot l : wHc := wfR : F := Dv : a := 3\cdot l : b := 7\cdot l : wHd := wfR :$
$F := Ev : a := 4\cdot l : b := 6\cdot l : wHe := wfR : F := Fv : a := 5\cdot l : b := 5\cdot l : wHf := wfR : F := Gv : a := 6\cdot l : b := 4\cdot l : wHg := wfR :$
$F := Hv : a := 7\cdot l : b := 3\cdot l : wHh := wfR : F := Iv : a := 8\cdot l : b := 2\cdot l : wHi := wfL : F := Jv : a := 9\cdot l : b := 1\cdot l : wHj := wfL :$
$eq7 := wH = wHq - wHb - wHc - wHd - wHe - wHf - wHg - wHh - wHi - wHj;$

$$wH = \frac{847}{8}\frac{q\, l^4}{EI} - \frac{9}{2}\frac{Bv\, l^3}{EI} - \frac{87}{10}\frac{Cv\, l^3}{EI} - \frac{123}{10}\frac{Dv\, l^3}{EI} - \frac{15\, Ev\, l^3}{EI} - \frac{33}{2}\frac{Fv\, l^3}{EI} - \frac{33}{2}\frac{Gv\, l^3}{EI} - \frac{147}{10}\frac{Hv\, l^3}{EI} - \frac{329}{30}\frac{Iv\, l^3}{EI} - \frac{35}{6}\frac{Jv\, l^3}{EI} \qquad (2.7)$$

Vergelijking in I

$x := 8\cdot 1 : L := 10\cdot 1 : wIq := wq :$
$F := Bv : a := 1\cdot l : b := 9\cdot l : wIb := wfR : F := Cv : a := 2\cdot l : b := 8\cdot l : wIc := wfR : F := Dv : a := 3\cdot l : b := 7\cdot l : wId := wfR :$
$F := Ev : a := 4\cdot l : b := 6\cdot l : wIe := wfR : F := Fv : a := 5\cdot l : b := 5\cdot l : wIf := wfR : F := Gv : a := 6\cdot l : b := 4\cdot l : wIg := wfR :$
$F := Hv : a := 7\cdot l : b := 3\cdot l : wIh := wfR : F := Iv : a := 8\cdot l : b := 2\cdot l : wIi := wfR : F := Jv : a := 9\cdot l : b := 1\cdot l : wIj := wfL :$
$eq8 := wI = wIq - wIb - wIc - wId - wIe - wIf - wIg - wIh - wIi - wIj;$

$$wI = \frac{232}{3}\frac{q\, l^4}{EI} - \frac{19}{6}\frac{Bv\, l^3}{EI} - \frac{92}{15}\frac{Cv\, l^3}{EI} - \frac{87}{10}\frac{Dv\, l^3}{EI} - \frac{32}{3}\frac{Ev\, l^3}{EI} - \frac{71}{6}\frac{Fv\, l^3}{EI} - \frac{12\, Gv\, l^3}{EI} - \frac{329}{30}\frac{Hv\, l^3}{EI} - \frac{128}{15}\frac{Iv\, l^3}{EI} - \frac{14}{3}\frac{Jv\, l^3}{EI} \qquad (2.8)$$

Vergelijking in J

$x := 9\cdot 1 : L := 10\cdot 1 : wJq := wq :$
$F := Bv : a := 1\cdot l : b := 9\cdot l : wJb := wfR : F := Cv : a := 2\cdot l : b := 8\cdot l : wJc := wfR : F := Dv : a := 3\cdot l : b := 7\cdot l : wJd := wfR :$
$F := Ev : a := 4\cdot l : b := 6\cdot l : wJe := wfR : F := Fv : a := 5\cdot l : b := 5\cdot l : wJf := wfR : F := Gv : a := 6\cdot l : b := 4\cdot l : wJg := wfR :$
$F := Hv : a := 7\cdot l : b := 3\cdot l : wJh := wfR : F := Iv : a := 8\cdot l : b := 2\cdot l : wJi := wfR : F := Jv : a := 9\cdot l : b := 1\cdot l : wJj := wfR :$
$eq9 := wJ = wJq - wJb - wJc - wJd - wJe - wJf - wJg - wJh - wJi - wJj;$

$$wJ = \frac{327}{8}\frac{q\, l^4}{EI} - \frac{49}{30}\frac{Bv\, l^3}{EI} - \frac{19}{6}\frac{Cv\, l^3}{EI} - \frac{9}{2}\frac{Dv\, l^3}{EI} - \frac{83}{15}\frac{Ev\, l^3}{EI} - \frac{37}{6}\frac{Fv\, l^3}{EI} - \frac{63}{10}\frac{Gv\, l^3}{EI} - \frac{35}{6}\frac{Hv\, l^3}{EI} - \frac{14}{3}\frac{Iv\, l^3}{EI} - \frac{27}{10}\frac{Jv\, l^3}{EI} \qquad (2.9)$$

Oplossen Reactiekrachten

$sol := solve(\{eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9\}, \{Bv, Cv, Dv, Ev, Fv, Gv, Hv, Iv, Jv\}) : assign(\%) :$

Reactie kracht in A en K

$$Av := \frac{\frac{1}{2}\cdot(10\cdot l)^2\cdot q - 9\cdot l\cdot Bv - 8\cdot l\cdot Cv - 7\cdot l\cdot Dv - 6\cdot l\cdot Ev - 5\cdot l\cdot Fv - 4\cdot l\cdot Gv - 3\cdot l\cdot Hv - 2\cdot l\cdot Iv - 1\cdot l\cdot Jv}{10\cdot l} :$$

$$Kv := \frac{\frac{1}{2}\cdot(10\cdot l)^2\cdot q - 9\cdot l\cdot Jv - 8\cdot l\cdot Iv - 7\cdot l\cdot Hv - 6\cdot l\cdot Gv - 5\cdot l\cdot Fv - 4\cdot l\cdot Ev - 3\cdot l\cdot Dv - 2\cdot l\cdot Cv - 1\cdot l\cdot Bv}{10\cdot l} :$$

(3.1)

$Bv;$
$Cv;$
$Dv;$
$Ev;$
$Fv;$
$Gv;$
$Hv;$
$Iv;$

$$-\frac{1}{151316}\frac{1}{l^3}\left(-171589\, q\, l^4 - 168480\, wEEI + 45144\, wFEI + 3240\, wHEI - 12096\, wGEI - 864\, wIEI + 1494552\, wBEI - 1438728\, wCEI + 628776\, wDEI + 216\, wJEI\right)$$

$$\frac{1}{75658}\frac{1}{l^3}\left(72941\, q\, l^4 - 336960\, wEEI + 90288\, wFEI + 6480\, wHEI - 24192\, wGEI - 1728\, wIEI + 719364\, wBEI - 1061664\, wCEI + 803604\, wDEI + 432\, wJEI\right)$$

$$-\frac{1}{151316}\frac{1}{l^3}\left(-12960\, wIEI - 152779\, q\, l^4 + 48600\, wHEI + 3240\, wJEI - 181440\, wGEI + 677160\, wFEI - 1619304\, wEEI + 2168472\, wDEI - 1607208\, wCEI + 628776\, wBEI\right)$$

$$\frac{1}{75658}\frac{1}{l^3}\left(-24192\, wIEI + 75449\, q\, l^4 + 90720\, wHEI + 6048\, wJEI - 338688\, wGEI + 810084\, wFEI - 1085856\, wEEI + 809652\, wDEI - 336960\, wCEI + 84240\, wBEI\right)$$

$$-\frac{1}{724}\frac{-864\, wIEI - 725\, q\, l^4 + 3240\, wHEI + 216\, wJEI - 7752\, wGEI + 10392\, wFEI - 7752\, wEEI + 3240\, wDEI - 864\, wCEI + 216\, wBEI}{l^3}$$

$$\frac{1}{75658}\frac{1}{l^3}\left(-336960\, wIEI + 75449\, q\, l^4 + 809652\, wHEI + 84240\, wJEI - 1085856\, wGEI + 810084\, wFEI - 338688\, wEEI + 90720\, wDEI - 24192\, wCEI + 6048\, wBEI\right)$$

$$-\frac{1}{151316}\frac{1}{l^3}\left(-1607208\, wIEI - 152779\, q\, l^4 + 2168472\, wHEI + 628776\, wJEI - 1619304\, wGEI + 677160\, wFEI - 181440\, wEEI + 48600\, wDEI\right.$$

$$-12960\,wCEI + 3240\,wBEI)$$

$$\frac{1}{75658}\,\frac{1}{l^3}\,(-1061664\,wIEI + 72941\,q\,l^4 + 803604\,wHEI + 719364\,wJEI - 336960\,wGEI + 90288\,wFEI - 24192\,wEEI + 6480\,wDEI - 1728\,wCEI + 432\,wBEI) \quad \textbf{(3.2)}$$

Steunpunts verplaatsingen
De in te voeren verplaatsingen van de steunpunten

Zakking t.o.v. de hartlijn, positieve waarde is een zakking omlaag gericht

$wB := -15 : wC := -29 : wD := -40 : wE := -48 : wF := -50 : wG := -48 : wH := -40 : wI := -29 : wJ := -15 :$

Mal eigenschappen

$EI := \frac{1}{12} \cdot b \cdot h^3 \cdot E :$

$E := 7000 :$

Afmetingen steunpunten(actuatoren)

$l := 200 :$

$b := l :$ (actuatoren staan in een vierkant)

Belasting op de mal, beton laag

$\gamma b := 2400 :$

$d := 50 :$

$\text{evalf}\left(q = \gamma b \cdot b \cdot d \cdot 10^{-8}\right); assign(\%);$

$$q = 0.2400000000 \quad \textbf{(5.1)}$$

Maximale Lokale doorbuiging van de mal

$wu := 1 :$

Rand voorwaarde

Steunpunts krachten moeten groten zijn dan 0, dan laat de mal niet los op de steunpunten

$eq10 := Av \geq 0 : eq11 := Bv \geq 0 : eq12 := Cv \geq 0 : eq13 := Dv \geq 0 : eq14 := Ev \geq 0 : eq15 := Fv \geq 0 :$
$eq16 := Gv \geq 0 : eq17 := Hv \geq 0 : eq18 := Iv \geq 0 : eq19 := Jv \geq 0 : eq20 := Kv \geq 0 :$

Eis voor de lokale doorzakking van de mal, tussen twee steunpunten

$eq21 := wu \geq \frac{1}{384} \, \frac{q \cdot l^4}{EI} :$

$eq22 := h \geq 0 :$

Oplossen mal dikte

$sol := solve(\{eq10, eq11, eq12, eq13, eq14, eq15, eq16, eq17, eq18, eq19, eq20, eq21, eq22\}, \{h\});$

$$\{2.046528216 \leq h, h \leq 6.418611919\} \quad \textbf{(7.1)}$$

Te kiezen mal dikte

De te kiezen mal dikte, moet voldoen aan bovenstaande randvoorwaarde

$h := 5 :$

$wu := \frac{1}{384} \, \frac{q \cdot l^4}{EI};$

$$0.06857142857 \quad \textbf{(8.1)}$$

$evalf\left(Izz = \frac{1}{12} \cdot b \cdot h^3\right);$

$$Izz = 2083.333333 \quad \textbf{(8.2)}$$

Steunpuntsreacties

$Av, Bv, Cv, Dv, Ev, Fv, Gv, Hv, Iv, Jv, Kv,$

$$18.02175417$$

$$48.93197514$$

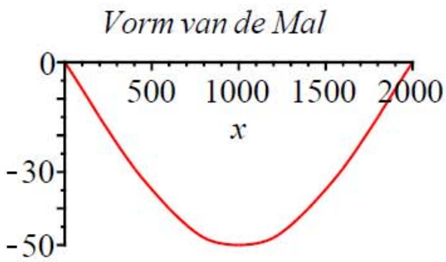
57.33459945
31.60462707
71.43439227
25.34530387
71.43439227
31.60462707
57.33459945
48.93197514
18.02175417 **(9.1)**

Doorbuigings grafiek

$x := 'x' :$
$F := Bv : a := 1 \cdot l : b := 9 \cdot l : wBL := wfL : wBR := wfR : F := Cv : a := 2 \cdot l : b := 8 \cdot l : wCL := wfL : wCR := wfR :$
$F := Dv : a := 3 \cdot l : b := 7 \cdot l : wDL := wfL : wDR := wfR : F := Ev : a := 4 \cdot l : b := 6 \cdot l : wEL := wfL : wER := wfR :$
$F := Fv : a := 5 \cdot l : b := 5 \cdot l : wFL := wfL : wFR := wfR : F := Gv : a := 6 \cdot l : b := 4 \cdot l : wGL := wfL : wGR := wfR :$
$F := Hv : a := 7 \cdot l : b := 3 \cdot l : wHL := wfL : wHR := wfR : F := Iv : a := 8 \cdot l : b := 2 \cdot l : wIL := wfL : wIR := wfR :$
$F := Jv : a := 9 \cdot l : b := 1 \cdot l : wJL := wfL : wJR := wfR :$

$w1 := wq - wBL - wCL - wDL - wEL - wFL - wGL - wHL - wIL - wJL :$
$w2 := wq - wBR - wCL - wDL - wEL - wFL - wGL - wHL - wIL - wJL :$
$w3 := wq - wBR - wCR - wDL - wEL - wFL - wGL - wHL - wIL - wJL :$
$w4 := wq - wBR - wCR - wDR - wEL - wFL - wGL - wHL - wIL - wJL :$
$w5 := wq - wBR - wCR - wDR - wER - wFL - wGL - wHL - wIL - wJL :$
$w6 := wq - wBR - wCR - wDR - wER - wFR - wGL - wHL - wIL - wJL :$
$w7 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHL - wIL - wJL :$
$w8 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHR - wIL - wJL :$
$w9 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHR - wIR - wJL :$
$w10 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHR - wIR - wJR :$

with(plots) :
$W1 := plot(w1, x = 0 \cdot l..1 \cdot l) : W2 := plot(w2, x = 1 \cdot l..2 \cdot l) : W3 := plot(w3, x = 2 \cdot l..3 \cdot l) : W4 := plot(w4, x = 3 \cdot l..4 \cdot l) :$
$W5 := plot(w5, x = 4 \cdot l..5 \cdot l) : W6 := plot(w6, x = 5 \cdot l..6 \cdot l) : W7 := plot(w7, x = 6 \cdot l..7 \cdot l) : W8 := plot(w8, x = 7 \cdot l..8 \cdot l) :$
$W9 := plot(w9, x = 8 \cdot l..9 \cdot l) : W10 := plot(w10, x = 9 \cdot l..10 \cdot l) :$
display({W1, W2, W3, W4, W5, W6, W7, W8, W9, W10}, title ='Vorm van de Mal');



Verplaatsingen in het midden van de steunpunten
$x := 0.5 \cdot l : w1; x := 1.5 \cdot l : w2; x := 2.5 \cdot l : w3; x := 3.5 \cdot l : w4; x := 4.5 \cdot l : w5; x := 5.5 \cdot l : w6; x := 6.5 \cdot l : w7; x := 7.5 \cdot l : w8; x := 8.5 \cdot l : w9;$
$x := 9.5 \cdot l : w10;$

-7.36211114
-22.24009433
-34.7660808
-44.5341521
-49.5608836
-49.5608830
-44.5341512
-34.7660812
-22.2400942
-7.36211112 **(10.1)**

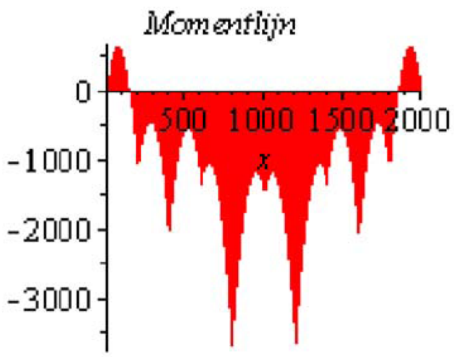
Momentlijn

$x := 'x' :$
$\theta 1 := diff(-w1, x) : \theta 2 := diff(-w2, x) : \theta 3 := diff(-w3, x) : \theta 4 := diff(-w4, x) : \theta 5 := diff(-w5, x) :$
$\theta 6 := diff(-w6, x) : \theta 7 := diff(-w7, x) : \theta 8 := diff(-w8, x) : \theta 9 := diff(-w9, x) : \theta 10 := diff(-w10, x) :$

```
m1 := diff(θ1, x)·EI : m2 := diff(θ2, x)·EI : m3 := diff(θ3, x)·EI : m4 := diff(θ4, x)·EI : m5 := diff(θ5, x)·EI :
m6 := diff(θ6, x)·EI : m7 := diff(θ7, x)·EI : m8 := diff(θ8, x)·EI : m9 := diff(θ9, x)·EI : m10 := diff(θ10, x)·EI :
```

```
v1 := diff(m1, x) : v2 := diff(m2, x) : v3 := diff(m3, x) : v4 := diff(m4, x) : v5 := diff(m5, x) :
v6 := diff(m6, x) : v7 := diff(m7, x) : v8 := diff(m8, x) : v9 := diff(m9, x) : v10 := diff(m10, x) : θ11 := diff(-wq, x) :
m11 := diff(θ11, x)·EI :
```

```
with(plots) :
M1 := plot(m1, x = 0 ..l, 'M', filled = true) : M2 := plot(m2, x = 1·l ..2·l, filled = true) :
M3 := plot(m3, x = 2·l ..3·l, filled = true) : M4 := plot(m4, x = 3·l ..4·l, filled = true) :
M5 := plot(m5, x = 4·l ..5·l, filled = true) : M6 := plot(m6, x = 5·l ..6·l, filled = true) :
M7 := plot(m7, x = 6·l ..7·l, filled = true) : M8 := plot(m8, x = 7·l ..8·l, filled = true) :
M9 := plot(m9, x = 8·l ..9·l, filled = true) : M10 := plot(m10, x = 9·l ..10·l, filled = true) :
M11 := plot(m11, x = 0 ..10·l) :
display({M1, M2, M3, M4, M5, M6, M7, M8, M9, M10}, title ='Momentlijn');
```



Veldmomenten (Als op het domein de dwarskracht niet door 0 gaat, klopt het veldmoment niet!)

```
x :='x': x1 := solve(v1 = 0) : x := x1 : M1max := m1;
x :='x': x2 := solve(v2 = 0, x) : x := x2 : M2max := m2;
x :='x': x3 := solve(v3 = 0, x) : x := x3 : M3max := m3;
x :='x': x4 := solve(v4 = 0, x) : x := x4 : M4max := m4;
x :='x': x5 := solve(v5 = 0, x) : x := x5 : M5max := m5;
x :='x': x6 := solve(v6 = 0, x) : x := x6 : M6max := m6;
x :='x': x7 := solve(v7 = 0, x) : x := x7 : M7max := m7;
x :='x': x8 := solve(v8 = 0, x) : x := x8 : M8max := m8;
x :='x': x9 := solve(v9 = 0, x) : x := x9 : M9max := m9;
x :='x': x10 := solve(v10 = 0, x) : x := x10 : M10max := m10;
Mmax := max(M1max, M2max, M3max, M4max, M5max, M6max, M7max, M8max, M9max, M10max) :
```

676.632547

-447.22449

-537.75841

-1052.56589

-1168.60155

-1168.60150

-1052.565930

-537.75830

-447.22429

676.63308 **(11.1)**

Steunpuntsmometen

```
x := 1·l : M1min := m1; x := 2·l : M2min := m2; x := 3·l : M3min := m3;
x := 4·l : M4min := m4; x := 5·l : M5min := m5; x := 6·l : M6min := m6;
x := 7·l : M7min := m7; x := 8·l : M8min := m8; x := 9·l : M9min := m9;
Mmin := min(M1min, M2min, M3min, M4min, M5min, M6min, M7min, M8min, M9min, 0) :
```

-1195.64917

-2204.90333

-1347.23755

-3768.64639

-1503.17680

-3768.646300

-1347.237600

-2204.90330

-1195.64900 **(11.2)**

Dwarskrachtlijn

$x := 'x':$

Reactie krachten

$x := 0 : vL := 0 : vR := v1 : Av := vR - vL; vA := [[x, vL], [x, vAR]] :$
$x := 1 \cdot l : vL := v1 : vR := v2 : Bv := vR - vL; vB := [[x, vL], [x, vR]] :$
$x := 2 \cdot l : vL := v2 : vR := v3 : Cv := vR - vL; vC := [[x, vL], [x, vR]] :$
$x := 3 \cdot l : vL := v3 : vR := v4 : Dv := vR - vL; vD := [[x, vL], [x, vR]] :$
$x := 4 \cdot l : vL := v4 : vR := v5 : Ev := vR - vL; vE := [[x, vL], [x, vR]] :$
$x := 5 \cdot l : vL := v5 : vR := v6 : Fv := vR - vL; vF := [[x, vL], [x, vR]] :$
$x := 6 \cdot l : vL := v6 : vR := v7 : Gv := vR - vL; vG := [[x, vL], [x, vR]] :$
$x := 7 \cdot l : vL := v7 : vR := v8 : Hv := vR - vL; vH := [[x, vL], [x, vR]] :$
$x := 8 \cdot l : vL := v8 : vR := v9 : Iv := vR - vL; vI := [[x, vL], [x, vR]] :$
$x := 9 \cdot l : vL := v9 : vR := v10 : Jv := vR - vL; vJ := [[x, vL], [x, vR]] :$
$x := 10 \cdot l : vL := v10 : vR := 0 : Kv := vR - vL; vK := [[x, vL], [x, vR]] :$

18.02175416

48.93197512

57.33459942

31.6046271

71.4343922

25.3453040

71.4343921

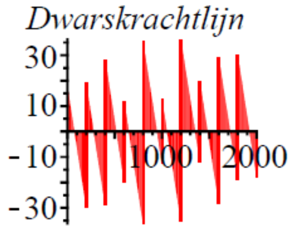
31.6046271

57.3345996

48.9319753

18.0217539 **(12.1)**

$x := 'x': with(plots) :$
$V1 := plot(v1, x = 0..l, 'V', filled = true) : V11 := plot(vA) :$
$V2 := plot(v2, x = 1 \cdot 1..2 \cdot l, filled = true) : V22 := plot(vB) :$
$V3 := plot(v3, x = 2 \cdot l..3 \cdot l, filled = true) : V33 := plot(vC) :$
$V4 := plot(v4, x = 3 \cdot l..4 \cdot l, filled = true) : V44 := plot(vD) :$
$V5 := plot(v5, x = 4 \cdot l..5 \cdot l, filled = true) : V55 := plot(vE) :$
$V6 := plot(v6, x = 5 \cdot l..6 \cdot l, filled = true) : V66 := plot(vF) :$
$V7 := plot(v7, x = 6 \cdot l..7 \cdot l, filled = true) : V77 := plot(vG) :$
$V8 := plot(v8, x = 7 \cdot l..8 \cdot l, filled = true) : V88 := plot(vH) :$
$V9 := plot(v9, x = 8 \cdot l..9 \cdot l, filled = true) : V99 := plot(vI) :$
$V10 := plot(v10, x = 9 \cdot l..10 \cdot l, filled = true) :$
$V100 := plot(vJ) : V110 := plot(vK) :$
$display(\{V1, V11, V2, V22, V3, V33, V4, V44, V5, V55, V6, V66, V7, V77, V8, V88, V9, V99, V10, V100, V110\}, title = 'Dwarskrachtlijn');$



Controle q-last
$diff(v1, x);$

-0.2400000000 **(12.2)**

Spanningen in de mal

$x := 'x':$

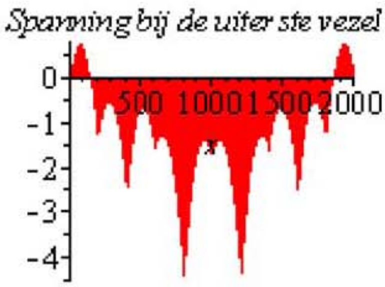
$W := evalf\left(\frac{1}{6} \cdot b \cdot h2\right);$

$s1 := \frac{m1}{W} : s2 := \frac{m2}{W} : s3 := \frac{m3}{W} : s4 := \frac{m4}{W} : s5 := \frac{m5}{W} : s6 := \frac{m6}{W} : s7 := \frac{m7}{W} : s8 := \frac{m8}{W} : s9 := \frac{m9}{W} : s10 := \frac{m10}{W} :$

$with(plots) :$
$S1 := plot(s1, x = 0..l, '\sigma', filled = true) : S2 := plot(s2, x = 1 \cdot 1..2 \cdot l, filled = true) : S3 := plot(s3, x = 2 \cdot l..3 \cdot l, filled = true) :$
$S4 := plot(s4, x = 3 \cdot l..4 \cdot l, filled = true) : S5 := plot(s5, x = 4 \cdot l..5 \cdot l, filled = true) : S6 := plot(s6, x = 5 \cdot l..6 \cdot l, filled = true) :$
$S7 := plot(s7, x = 6 \cdot l..7 \cdot l, filled = true) : S8 := plot(s8, x = 7 \cdot l..8 \cdot l, filled = true) : S9 := plot(s9, x = 8 \cdot l..9 \cdot l, filled = true) :$
$S10 := plot(s10, x = 9 \cdot l..10 \cdot l, filled = true) :$

display({*S1*, *S2*, *S3*, *S4*, *S5*, *S6*, *S7*, *S8*, *S9*, *S10*}, *title* ='Spanning bij de uiterste vezel');

833.3333333



Maximale spanningen in mal

$M := max(Mmax, abs(Mmin));$

3768.64639

$\sigma max := \frac{M}{W};$

4.522375668 **(13.2)**

Bepaling kromtestraal

$\kappa := \frac{M}{EI};$

0.0002584214667 **(13.3)**

Minimale kromte straal

$R := \frac{1}{\kappa};$

3869.647567 **(13.4)**

(1)

Rek in de ligger

Oorspronkelijke booglente:

l;

200 **(14.1)**

De nieuwe booglengte per ligger

Student-*Calculus1*:-*ArcLength*(w1, x = 0·1 ..1 · 1) : % := *evalf*(%) : *l1n* := %;
Student-*Calculus1*:-*ArcLength*(w2, x = 1·1 ..2 · 1) : % := *evalf*(%) : *l2n* := %;
Student-*Calculus1*:-*ArcLength*(w3, x = 2·1 ..3 · 1) : % := *evalf*(%) : *l3n* := %;
Student-*Calculus1*:-*ArcLength*(w4, x = 3·1 ..4 · 1) : % := *evalf*(%) : *l4n* := %;
Student-*Calculus1*:-*ArcLength*(w5, x = 4·1 ..5 · 1) : % := *evalf*(%) : *l5n* := %;
Student-*Calculus1*:-*ArcLength*(w6, x = 5·1 ..6 · 1) : % := *evalf*(%) : *l6n* := %;
Student-*Calculus1*:-*ArcLength*(w7, x = 6·1 ..7 · 1) : % := *evalf*(%) : *l7n* := %;
Student-*Calculus1*:-*ArcLength*(w8, x = 7·1 ..8 · 1) : % := *evalf*(%) : *l8n* := %;
Student-*Calculus1*:-*ArcLength*(w9, x = 8·1 ..9 · 1) : % := *evalf*(%) : *l9n* := %;
Student-*Calculus1*:-*ArcLength*(w10, x = 9·1 ..10 · 1) : % := *evalf*(%) : *l10n* := %;

200.5619692
200.4902974
200.3033468
200.1640990
200.0145522
200.0145522
200.1640990
200.3033468
200.4902974
200.5619692 **(14.2)**

Oosrponkelijke Totale lengte:

$lto := 10 \cdot l;$

2000 **(14.3)**

Nieuwe lengte:

$ltn := l1n + l2n + l3n + l4n + l5n + l6n + l7n + l8n + l9n + l10n;$

2003.068529 **(14.4)**

Totale verlenging

$\Delta L := ltn - lto;$

3.068529 **(14.5)**

Rek totale ligger (promiel):

$$\varepsilon t := \frac{ltn - lto}{lto} \cdot 1000;$$

1.534264500 **(14.6)**

Rek per ligger deel (promiel):

$$\varepsilon 1 := \frac{l1n - l}{l} \cdot 1000;\ \varepsilon 2 := \frac{l2n - l}{l} \cdot 1000;\ \varepsilon 3 := \frac{l3n - l}{l} \cdot 1000;$$
$$\varepsilon 4 := \frac{l4n - l}{l} \cdot 1000;\ \varepsilon 5 := \frac{l5n - l}{l} \cdot 1000;\ \varepsilon 6 := \frac{l6n - l}{l} \cdot 1000;$$
$$\varepsilon 7 := \frac{l7n - l}{l} \cdot 1000;\ \varepsilon 8 := \frac{l8n - l}{l} \cdot 1000;\ \varepsilon 9 := \frac{l9n - l}{l} \cdot 1000;$$
$$\varepsilon 10 := \frac{l10n - l}{l} \cdot 1000;$$

2.809846000

2.451487000

1.516734000

0.8204950000

0.07276100000

0.07276100000

0.8204950000

1.516734000

2.451487000

2.809846000 **(14.7)**

Appendix C. Maple Sheet, Funen

7 supports, square pattern

Algemene belastingformules

restart;

Verdeelde belasting

$wq := \frac{q \cdot x}{24 \cdot EI} \cdot (L^3 - 2 \cdot L \cdot x^2 + x^3);$

$$\frac{1}{24} \frac{q\, x\, (L^3 - 2\, L\, x^2 + x^3)}{EI} \quad (1.1)$$

Puntlast

Als x, de plaats van de steunpunt links van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfL := \frac{F \cdot b \cdot x}{6 \cdot EI \cdot L} \cdot (2 \cdot L \cdot (L - x) - (b)^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F\, b\, x\, (2\, L\, (L - x) - b^2 - (L - x)^2)}{EI\, L} \quad (1.2)$$

Als x, de plaats van de steunpunt rechts van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfR := \frac{F \cdot a \cdot (L - x)}{6 \cdot EI \cdot L} \cdot ((2 \cdot L \cdot b) - b^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F\, a\, (L - x)\, (2\, L\, b - b^2 - (L - x)^2)}{EI\, L} \quad (1.3)$$

Verplaatsingen in de steunpunten

Vergelijking in B

$x := 1 : L := 6 \cdot 1 : wBq := wq :$
$F := Bv : b := 5 \cdot 1 : wBb := wfL :$
$F := Cv : b := 4 \cdot l : wBc := wfL :$
$F := Dv : b := 3 \cdot l : wBd := wfL :$
$F := Ev : b := 2 \cdot l : wBe := wfL :$
$F := Fv : b := 1 \cdot l : wBf := wfL :$
$eq1 := wB = wBq - wBb - wBc - wBd - wBe - wBf;$

$$wB = \frac{205}{24} \frac{q\, l^4}{EI} - \frac{25}{18} \frac{Bv\, l^3}{EI} - \frac{19}{9} \frac{Cv\, l^3}{EI} - \frac{13}{6} \frac{Dv\, l^3}{EI} - \frac{31}{18} \frac{Ev\, l^3}{EI} - \frac{17}{18} \frac{Fv\, l^3}{EI} \quad (2.1)$$

Vergelijking in C

$x := 2 \cdot 1 : L := 6 \cdot 1 : wCq := wq :$
$F := Bv : a := 1 \cdot l : b := 5 \cdot l : wCb := wfR :$
$F := Cv : a := 2 \cdot l : b := 4 \cdot l : wCc := wfR :$
$F := Dv : b := 3 \cdot l : wCd := wfL :$
$F := Ev : b := 2 \cdot l : wCe := wfL :$
$F := Fv : b := 1 \cdot l : wCf := wfL :$
$eq2 := wC = wCq - wCb - wCc - wCd - wCe - wCf;$

$$wC = \frac{44}{3} \frac{q\, l^4}{EI} - \frac{19}{9} \frac{Bv\, l^3}{EI} - \frac{32}{9} \frac{Cv\, l^3}{EI} - \frac{23}{6} \frac{Dv\, l^3}{EI} - \frac{28}{9} \frac{Ev\, l^3}{EI} - \frac{31}{18} \frac{Fv\, l^3}{EI} \quad (2.2)$$

Vergelijking in D

$x := 3 \cdot 1 : L := 6 \cdot 1 : wDq := wq :$
$F := Bv : a := 1 \cdot l : b := 5 \cdot l : wDb := wfR :$
$F := Cv : a := 2 \cdot l : b := 4 \cdot l : wDc := wfR :$
$F := Dv : a := 3 \cdot l : b := 3 \cdot l : wDd := wfR :$
$F := Ev : a := 4 \cdot l : b := 2 \cdot l : wDe := wfL :$
$F := Fv : a := 5 \cdot l : b := 1 \cdot l : wDf := wfL :$
$eq3 := wD = wDq - wDb - wDc - wDd - wDe - wDf;$

$$wD = \frac{135}{8} \frac{q\, l^4}{EI} - \frac{13}{6} \frac{Bv\, l^3}{EI} - \frac{23}{6} \frac{Cv\, l^3}{EI} - \frac{9}{2} \frac{Dv\, l^3}{EI} - \frac{23}{6} \frac{Ev\, l^3}{EI} - \frac{13}{6} \frac{Fv\, l^3}{EI} \quad (2.3)$$

Vergelijking in E

$x := 4 \cdot 1 : L := 6 \cdot 1 : wEq := wq :$
$F := Bv : a := 1 \cdot l : b := 5 \cdot l : wEb := wfR :$
$F := Cv : a := 2 \cdot l : b := 4 \cdot l : wEc := wfR :$
$F := Dv : a := 3 \cdot l : b := 3 \cdot l : wEd := wfR :$
$F := Ev : a := 4 \cdot l : b := 2 \cdot l : wEe := wfR :$
$F := Fv : a := 5 \cdot l : b := 1 \cdot l : wEf := wfL :$
$eq4 := wE = wEq - wEb - wEc - wEd - wEe - wEf;$

$$wE = \frac{44}{3} \frac{q\, l^4}{EI} - \frac{31}{18} \frac{Bv\, l^3}{EI} - \frac{28}{9} \frac{Cv\, l^3}{EI} - \frac{23}{6} \frac{Dv\, l^3}{EI} - \frac{32}{9} \frac{Ev\, l^3}{EI} - \frac{19}{9} \frac{Fv\, l^3}{EI} \quad (2.4)$$

Vergelijking in F

$x := 5 \cdot l : L := 6 \cdot l : wFq := wq :$
$F := Bv : a := 1 \cdot l : b := 5 \cdot l : wFb := wfR :$
$F := Cv : a := 2 \cdot l : b := 4 \cdot l : wFc := wfR :$
$F := Dv : a := 3 \cdot l : b := 3 \cdot l : wFd := wfR :$
$F := Ev : a := 4 \cdot l : b := 2 \cdot l : wFe := wfR :$
$F := Fv : a := 5 \cdot l : b := 1 \cdot l : wFf := wfR :$

$eq5 := wF = wFq - wFb - wFc - wFd - wFe - wFf;$

$$wF = \frac{205}{24}\frac{q\,l^4}{EI} - \frac{17}{18}\frac{Bv\,l^3}{EI} - \frac{31}{18}\frac{Cv\,l^3}{EI} - \frac{13}{6}\frac{Dv\,l^3}{EI} - \frac{19}{9}\frac{Ev\,l^3}{EI} - \frac{25}{18}\frac{Fv\,l^3}{EI} \quad (2.5)$$

Oplossen Reactiekrachten

$sol := solve(\{eq1, eq2, eq3, eq4, eq5\}, \{Bv, Cv, Dv, Ev, Fv\}) : assign(\%) :$

Reactie kracht in A en K

$$Av := \frac{\frac{1}{2}\cdot(6\cdot l)^2\cdot q - 5\cdot l\cdot Bv - 4\cdot l\cdot Cv - 3\cdot l\cdot Dv - 2\cdot l\cdot Ev - 1\cdot l\cdot Fv}{6\cdot l} :$$

$$Gv := \frac{\frac{1}{2}\cdot(6\cdot l)^2\cdot q - 5\cdot l\cdot Fv - 4\cdot l\cdot \mathrm{Ev} - 3\cdot 1\cdot \mathrm{Dv} - 1\cdot 1\cdot \mathrm{Cv} - 1\cdot l\cdot Bv}{6\cdot l} :$$

Steunpunts verplaatsingen
De in te voeren verplaatsingen van de steunpunten

Zakking t.o.v. de hartlijn, positieve waarde is een zakking omlaag gericht
$wB := -78.5 :$
$wC := -120 :$
$wD := -131 :$
$wE := -115.5 :$
$wF := -70 :$

Mal eigenschappen

$EI := \frac{1}{12}\cdot b\cdot h^3\cdot E :$
$E := 7000 :$
Afmetingen steunpunten(actuatoren)
$l := 763.3333333 :$
$b := 540 :$
Belasting op de mal, beton laag
$\gamma b := 2400 :$
$d := 50 :$
$\mathrm{evalf}\left(q = \gamma b\cdot b\cdot d\cdot 10^{-8}\right); assign(\%);$

$$q = 0.6480000000 \quad (5.1)$$

Maximale Lokale doorbuiging van de mal
$wu := 1 :$

Rand voorwaarde

Steunpunts krachten moeten groten zijn dan 0, dan laat de mal niet los op de steunpunten
$eq10 := Av \geq 0 :$
$eq11 := Bv \geq 0 :$
$eq12 := Cv \geq 0 :$
$eq13 := Dv \geq 0 :$
$eq14 := Ev \geq 0 :$
$eq15 := Fv \geq 0 :$
$eq16 := Gv \geq 0 :$

Eis voor de lokale doorzakking van de mal, tussen twee steunpunten
$eq17 := wu \geq \frac{1}{384}\frac{q\cdot l^4}{EI} :$
$eq18 := h \geq 0 :$

Oplossen mal dikte

$sol := solve(\{eq10, eq11, eq12, eq13, eq14, eq15, eq16, eq17, eq18\}, \{h\});$

$$\{10.87651818 \leq h, h \leq 15.87689754\} \quad (7.1)$$

Te kiezen mal dikte

De te kiezen mal dikte, moet voldoen aan bovenstaande randvoorwaarde
$h := 15;$

$$15 \quad (8.1)$$

$wu := \frac{1}{384}\frac{q\cdot l^4}{EI};$

$$0.3812377461 \quad (8.2)$$

```
evalf(Izz = 1/12 · b·h^3);
```

$$Izz = 1.51875\ 10^5 \qquad (8.3)$$

Steunpuntsreacties

```
Av;
Bv;
Cv;
Dv;
Ev;
Fv;
Gv;
```

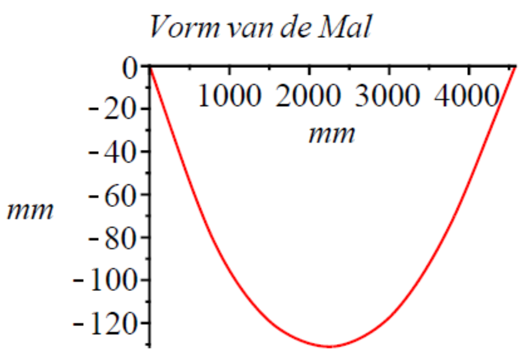
30.55885168
797.7939331
411.2103128
474.5228544
506.4923932
649.8026192
165.9940862 **(9.1)**

Doorbuigings grafiek

```
x :='x' :
F := Bv : a := 1·l : b := 5·l : wBL := wfL : wBR := wfR :
F := Cv : a := 2·l : b := 4·l : wCL := wfL : wCR := wfR :
F := Dv : a := 3·l : b := 3·l : wDL := wfL : wDR := wfR :
F := Ev : a := 4·l : b := 2·l : wEL := wfL : wER := wfR :
F := Fv : a := 5·l : b := 1·l : wFL := wfL : wFR := wfR :

w1 := wq − wBL − wCL − wDL − wEL − wFL :
w2 := wq − wBR − wCL − wDL − wEL − wFL :
w3 := wq − wBR − wCR − wDL − wEL − wFL :
w4 := wq − wBR − wCR − wDR − wEL − wFL :
w5 := wq − wBR − wCR − wDR − wER − wFL :
w6 := wq − wBR − wCR − wDR − wER − wFR :

with(plots) :
W1 := plot(w1, x = 0·l..1·l) :
W2 := plot(w2, x = 1·l..2·l) :
W3 := plot(w3, x = 2·l..3·l) :
W4 := plot(w4, x = 3·l..4·l) :
W5 := plot(w5, x = 4·l..5·l) :
W6 := plot(w6, x = 5·l..6·l) :
display({W1, W2, W3, W4, W5, W6}, title ='Vorm van de Mal', labels = [mm, mm]);
```



9 supports, isocurves pattern

Alle waardes in:
N en mm.

Algemene belastingformules

restart;

Verdeelde belasting

$wq := \frac{q \cdot x}{24 \cdot EI} \cdot (L^3 - 2 \cdot L \cdot x^2 + x^3);$

$$\frac{1}{24} \frac{q\, x\, (L^3 - 2\, L\, x^2 + x^3)}{EI} \tag{1.1}$$

Puntlast

Als x, de plaats van de steunpunt links van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfL := \frac{F \cdot b \cdot x}{6 \cdot EI \cdot L} \cdot (2 \cdot L \cdot (L - x) - (b)^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F\, b\, x\, (2\, L\, (L - x) - b^2 - (L - x)^2)}{EI\, L} \tag{1.2}$$

Als x, de plaats van de steunpunt rechts van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfR := \frac{F \cdot a \cdot (L - x)}{6 \cdot EI \cdot L} \cdot ((2 \cdot L \cdot b) - b^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F\, a\, (L - x)\, (2\, L\, b - b^2 - (L - x)^2)}{EI\, L} \tag{1.3}$$

Verplaatsingen in de steunpunten

Vergelijking in B

$x := 1 : L := 8 \cdot 1 : wBq := wq :$
$F := Bv : b := 7 \cdot 1 : wBb := wfL :$
$F := Cv : b := 6 \cdot l : wBc := wfL :$
$F := Dv : b := 5 \cdot l : wBd := wfL :$
$F := Ev : b := 4 \cdot l : wBe := wfL :$
$F := Fv : b := 3 \cdot l : wBf := wfL :$
$F := Gv : b := 2 \cdot l : wBg := wfL :$
$F := Hv : b := 1 \cdot l : wBh := wfL :$
$eq1 := wB = wBq - wBb - wBc - wBd - wBe - wBf - wBg - wBh;$

$$wB = \frac{497}{24} \frac{q\, l^4}{EI} - \frac{49}{24} \frac{Bv\, l^3}{EI} - \frac{27}{8} \frac{Cv\, l^3}{EI} - \frac{95}{24} \frac{Dv\, l^3}{EI} - \frac{47}{12} \frac{Ev\, l^3}{EI} - \frac{27}{8} \frac{Fv\, l^3}{EI} - \frac{59}{24} \frac{Gv\, l^3}{EI} - \frac{31}{24} \frac{Hv\, l^3}{EI} \tag{2.1}$$

Vergelijking in C

$x := 2 \cdot 1 : L := 8 \cdot 1 : wCq := wq :$
$F := Bv : a := 1 \cdot l : b := 7 \cdot l : wCb := wfR :$
$F := Cv : a := 2 \cdot l : b := 6 \cdot l : wCc := wfR :$
$F := Dv : b := 5 \cdot l : wCd := wfL :$
$F := Ev : b := 4 \cdot l : wCe := wfL :$
$F := Fv : b := 3 \cdot l : wCf := wfL :$
$F := Gv : b := 2 \cdot l : wCg := wfL :$
$F := Hv : b := 1 \cdot l : wCh := wfL :$
$eq2 := wC = wCq - wCb - wCc - wCd - wCe - wCf - wCg - wCh;$

$$wC = \frac{38\, q\, l^4}{EI} - \frac{27}{8} \frac{Bv\, l^3}{EI} - \frac{6\, Cv\, l^3}{EI} - \frac{175}{24} \frac{Dv\, l^3}{EI} - \frac{22}{3} \frac{Ev\, l^3}{EI} - \frac{51}{8} \frac{Fv\, l^3}{EI} - \frac{14}{3} \frac{Gv\, l^3}{EI} - \frac{59}{24} \frac{Hv\, l^3}{EI} \tag{2.2}$$

Vergelijking in D

$x := 3 \cdot 1 : L := 8 \cdot 1 : wDq := wq :$
$F := Bv : a := 1 \cdot l : b := 7 \cdot l : wDb := wfR :$
$F := Cv : a := 2 \cdot l : b := 6 \cdot l : wDc := wfR :$
$F := Dv : a := 3 \cdot l : b := 5 \cdot l : wDd := wfR :$
$F := Ev : a := 4 \cdot l : b := 4 \cdot l : wDe := wfL :$
$F := Fv : a := 5 \cdot l : b := 3 \cdot l : wDf := wfL :$
$F := Gv : a := 6 \cdot l : b := 2 \cdot l : wDg := wfL :$
$F := Hv : a := 7 \cdot l : b := 1 \cdot l : wDh := wfL :$
$eq3 := wD = wDq - wDb - wDc - wDd - wDe - wDf - wDg - wDh;$

$$wD = \frac{395}{8} \frac{q\, l^4}{EI} - \frac{95}{24} \frac{Bv\, l^3}{EI} - \frac{175}{24} \frac{Cv\, l^3}{EI} - \frac{75}{8} \frac{Dv\, l^3}{EI} - \frac{39}{4} \frac{Ev\, l^3}{EI} - \frac{69}{8} \frac{Fv\, l^3}{EI} - \frac{51}{8} \frac{Gv\, l^3}{EI} - \frac{27}{8} \frac{Hv\, l^3}{EI} \tag{2.3}$$

Vergelijking in E

$x := 4 \cdot 1 : L := 8 \cdot 1 : wEq := wq :$
$F := Bv : a := 1 \cdot l : b := 7 \cdot l : wEb := wfR :$
$F := Cv : a := 2 \cdot l : b := 6 \cdot l : wEc := wfR :$
$F := Dv : a := 3 \cdot l : b := 5 \cdot l : wEd := wfR :$
$F := Ev : a := 4 \cdot l : b := 4 \cdot l : wEe := wfR :$
$F := Fv : a := 5 \cdot l : b := 3 \cdot l : wEf := wfL :$
$F := Gv : a := 6 \cdot l : b := 2 \cdot l : wEg := wfL :$
$F := Hv : a := 7 \cdot l : b := 1 \cdot l : wEh := wfL :$
$eq4 := wE = wEq - wEb - wEc - wEd - wEe - wEf - wEg - wEh;$

$$wE = \frac{160}{3} \frac{q\, l^4}{EI} - \frac{47}{12} \frac{Bv\, l^3}{EI} - \frac{22}{3} \frac{Cv\, l^3}{EI} - \frac{39}{4} \frac{Dv\, l^3}{EI} - \frac{32}{3} \frac{Ev\, l^3}{EI} - \frac{39}{4} \frac{Fv\, l^3}{EI} - \frac{22}{3} \frac{Gv\, l^3}{EI} - \frac{47}{12} \frac{Hv\, l^3}{EI} \tag{2.4}$$

Vergelijking in F

$x := 5\cdot 1 : L := 8\cdot 1 : wFq := wq :$
$F := Bv : a := 1\cdot l : b := 7\cdot l : wFb := wfR :$
$F := Cv : a := 2\cdot l : b := 6\cdot l : wFc := wfR :$
$F := Dv : a := 3\cdot l : b := 5\cdot l : wFd := wfR :$
$F := Ev : a := 4\cdot l : b := 4\cdot l : wFe := wfR :$
$F := Fv : a := 5\cdot l : b := 3\cdot l : wFf := wfR :$
$F := Gv : a := 6\cdot l : b := 2\cdot l : wFg := wfL :$
$F := Hv : a := 7\cdot l : b := 1\cdot l : wFh := wfL :$
$eq5 := wF = wFq - wFb - wFc - wFd - wFe - wFf - wFg - wFh;$

$$wF = \frac{395}{8}\frac{q\,l^4}{EI} - \frac{27}{8}\frac{Bv\,l^3}{EI} - \frac{51}{8}\frac{Cv\,l^3}{EI} - \frac{69}{8}\frac{Dv\,l^3}{EI} - \frac{39}{4}\frac{Ev\,l^3}{EI} - \frac{75}{8}\frac{Fv\,l^3}{EI} - \frac{175}{24}\frac{Gv\,l^3}{EI} - \frac{95}{24}\frac{Hv\,l^3}{EI} \quad (2.5)$$

Vergelijking in G

$x := 6\cdot 1 : L := 8\cdot 1 : wGq := wq :$
$F := Bv : a := 1\cdot l : b := 7\cdot l : wGb := wfR :$
$F := Cv : a := 2\cdot l : b := 6\cdot l : wGc := wfR :$
$F := Dv : a := 3\cdot l : b := 5\cdot l : wGd := wfR :$
$F := Ev : a := 4\cdot l : b := 4\cdot l : wGe := wfR :$
$F := Fv : a := 5\cdot l : b := 3\cdot l : wGf := wfR :$
$F := Gv : a := 6\cdot l : b := 2\cdot l : wGg := wfR :$
$F := Hv : a := 7\cdot l : b := 1\cdot l : wGh := wfL :$
$eq6 := wG = wGq - wGb - wGc - wGd - wGe - wGf - wGg - wGh;$

$$wG = \frac{38\,q\,l^4}{EI} - \frac{59}{24}\frac{Bv\,l^3}{EI} - \frac{14}{3}\frac{Cv\,l^3}{EI} - \frac{51}{8}\frac{Dv\,l^3}{EI} - \frac{22}{3}\frac{Ev\,l^3}{EI} - \frac{175}{24}\frac{Fv\,l^3}{EI} - \frac{6\,Gv\,l^3}{EI} - \frac{27}{8}\frac{Hv\,l^3}{EI} \quad (2.6)$$

Vergelijking in H

$x := 7\cdot 1 : L := 8\cdot 1 : wHq := wq :$
$F := Bv : a := 1\cdot l : b := 7\cdot l : wHb := wfR :$
$F := Cv : a := 2\cdot l : b := 6\cdot l : wHc := wfR :$
$F := Dv : a := 3\cdot l : b := 5\cdot l : wHd := wfR :$
$F := Ev : a := 4\cdot l : b := 4\cdot l : wHe := wfR :$
$F := Fv : a := 5\cdot l : b := 3\cdot l : wHf := wfR :$
$F := Gv : a := 6\cdot l : b := 2\cdot l : wHg := wfR :$
$F := Hv : a := 7\cdot l : b := 1\cdot l : wHh := wfR :$
$eq7 := wH = wHq - wHb - wHc - wHd - wHe - wHf - wHg - wHh;$

$$wH = \frac{497}{24}\frac{q\,l^4}{EI} - \frac{31}{24}\frac{Bv\,l^3}{EI} - \frac{59}{24}\frac{Cv\,l^3}{EI} - \frac{27}{8}\frac{Dv\,l^3}{EI} - \frac{47}{12}\frac{Ev\,l^3}{EI} - \frac{95}{24}\frac{Fv\,l^3}{EI} - \frac{27}{8}\frac{Gv\,l^3}{EI} - \frac{49}{24}\frac{Hv\,l^3}{EI} \quad (2.7)$$

(2.8)

Oplossen Reactiekrachten

$sol := solve(\{eq1, eq2, eq3, eq4, eq5, eq6, eq7\}, \{Bv, Cv, Dv, Ev, Fv, Gv, Hv\}) : assign(\%) :$

Reactie kracht in A en K

$$Av := \frac{\frac{1}{2}\cdot(8\cdot l)^2\cdot q - 7\cdot l\cdot Bv - 6\cdot l\cdot Cv - 5\cdot l\cdot Dv - 4\cdot l\cdot Ev - 3\cdot l\cdot Fv - 2\cdot l\cdot Gv - 1\cdot l\cdot Hv}{8\cdot l} :$$

$$Iv := \frac{\frac{1}{2}\cdot(8\cdot l)^2\cdot q - 7\cdot l\cdot Hv - 6\cdot l\cdot Gv - 5\cdot 1\cdot Fv - 4\cdot 1\cdot Ev - 3\cdot 1\cdot Dv - 2\cdot l\cdot Cv - 1\cdot l\cdot Bv}{8\cdot l} :$$

Steunpunts verplaatsingen

De in te voeren verplaatsingen van de steunpunten

Zakking t.o.v. de hartlijn, positieve waarde is een zakking omlaag gericht

$wB := -62 :$
$wC := -104 :$
$wD := -125 :$
$wE := -131 :$
$wF := -122 :$
$wG := -96 :$
$wH := -54 :$

Mal eigenschappen

$EI := \frac{1}{12}\cdot b\cdot h^3\cdot E :$
$E := 7000 :$

Afmetingen steunpunten(actuatoren)

$l := 572.5 :$
$b := 572.5 :$

Belasting op de mal, beton laag

$\gamma b := 2400 :$
$d := 50 :$
$\text{evalf}(q = \gamma b\cdot b\cdot d\cdot 10^{-8}); assign(\%);$

$$q = 0.6870000000 \quad (5.1)$$

Maximale Lokale doorbuiging van de mal

$wu := 1 :$

Rand voorwaarde

Steunpunts krachten moeten groten zijn dan 0, dan laat de mal niet los op de steunpunten
$eq10 := Av \geq 0:$
$eq11 := Bv \geq 0:$
$eq12 := Cv \geq 0:$
$eq13 := Dv \geq 0:$
$eq14 := Ev \geq 0:$
$eq15 := Fv \geq 0:$
$eq16 := Gv \geq 0:$
$eq17 := Hv \geq 0:$
$eq18 := Iv \geq 0:$

Eis voor de lokale doorzakking van de mal, tussen twee steunpunten
$eq19 := wu \geq \frac{1}{384} \frac{q \cdot l^4}{EI}:$
$eq20 := h \geq 0:$

Oplossen mal dikte

$sol := solve(\{eq10, eq11, eq12, eq13, eq14, eq15, eq16, eq17, eq18, eq19, eq20\}, \{h\});$
$\{8.317862379 \leq h, h \leq 15.26732121\}$ **(7.1)**

Te kiezen mal dikte

De te kiezen mal dikte, moet voldoen aan bovenstaande randvoorwaarde
$h := 15;$
15 **(8.1)**

$wu := \frac{1}{384} \frac{q \cdot l^4}{EI};$

0.1705145387 **(8.2)**

$evalf\left(Izz = \frac{1}{12} \cdot b \cdot h^3\right);$

$Izz = 1.610156250\ 10^5$ **(8.3)**

Steunpuntsreacties

$Av;$
$Bv;$
$Cv;$
$Dv;$
$Ev;$
$Fv;$
$Gv;$
$Hv;$
$Iv;$

8.004926091
607.7366373
417.0142526
336.3348755
381.2482429
426.4361326
380.9737468
517.6353817
71.07580635 **(9.1)**

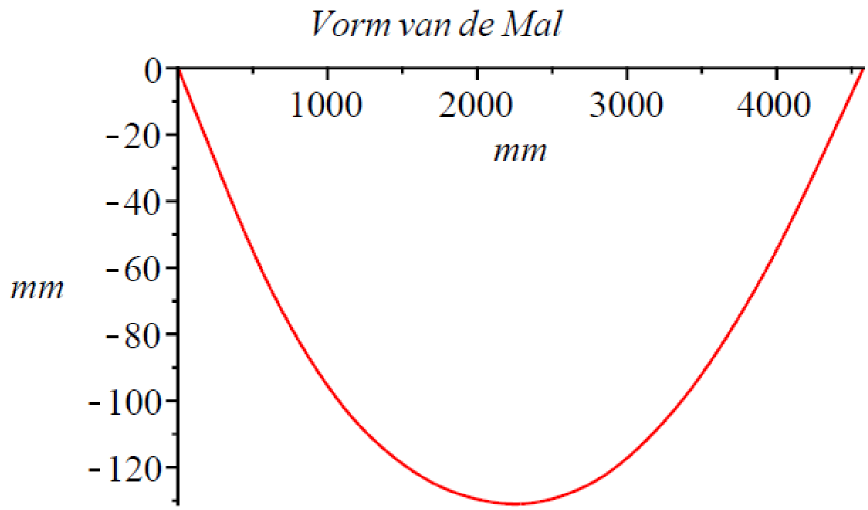
Doorbuigings grafiek

$x := 'x':$
$F := Bv : a := 1 \cdot l : b := 7 \cdot l : wBL := wfL : wBR := wfR:$
$F := Cv : a := 2 \cdot l : b := 6 \cdot l : wCL := wfL : wCR := wfR:$
$F := Dv : a := 3 \cdot l : b := 5 \cdot l : wDL := wfL : wDR := wfR:$
$F := Ev : a := 4 \cdot l : b := 4 \cdot l : wEL := wfL : wER := wfR:$
$F := Fv : a := 5 \cdot l : b := 3 \cdot l : wFL := wfL : wFR := wfR:$
$F := Gv : a := 6 \cdot l : b := 2 \cdot l : wGL := wfL : wGR := wfR:$
$F := Hv : a := 7 \cdot l : b := 1 \cdot l : wHL := wfL : wHR := wfR:$

$w1 := wq - wBL - wCL - wDL - wEL - wFL - wGL - wHL:$
$w2 := wq - wBR - wCL - wDL - wEL - wFL - wGL - wHL:$

```
w3 := wq − wBR − wCR − wDL − wEL − wFL − wGL − wHL :
w4 := wq − wBR − wCR − wDR − wEL − wFL − wGL − wHL :
w5 := wq − wBR − wCR − wDR − wER − wFL − wGL − wHL :
w6 := wq − wBR − wCR − wDR − wER − wFR − wGL − wHL :
w7 := wq − wBR − wCR − wDR − wER − wFR − wGR − wHL :
w8 := wq − wBR − wCR − wDR − wER − wFR − wGR − wHR :

with(plots) :
W1 := plot(w1, x = 0·l..1·l) :
W2 := plot(w2, x = 1·l..2·l) :
W3 := plot(w3, x = 2·l..3·l) :
W4 := plot(w4, x = 3·l..4·l) :
W5 := plot(w5, x = 4·l..5·l) :
W6 := plot(w6, x = 5·l..6·l) :
W7 := plot(w7, x = 6·l..7·l) :
W8 := plot(w8, x = 7·l..8·l) :
display({W1, W2, W3, W4, W5, W6, W7, W8}, title = 'Vorm van de Mal', labels = [mm, mm]);
```



Appendix D. Flexibility steel reinforcement

Algemene belastingformules

restart;

Verdeelde belasting

$wq := \frac{q \cdot x}{24 \cdot EI} \cdot (L^3 - 2 \cdot L \cdot x^2 + x^3);$

$$\frac{1}{24} \frac{q x (L^3 - 2 L x^2 + x^3)}{EI} \quad (1.1)$$

Puntlast

Als x, de plaats van de steunpunt links van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfL := \frac{F \cdot b \cdot x}{6 \cdot EI \cdot L} \cdot (2 \cdot L \cdot (L - x) - (b)^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F b x (2 L (L - x) - b^2 - (L - x)^2)}{EI L} \quad (1.2)$$

Als x, de plaats van de steunpunt rechts van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfR := \frac{F \cdot a \cdot (L - x)}{6 \cdot EI \cdot L} \cdot ((2 \cdot L \cdot b) - b^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F a (L - x) (2 L b - b^2 - (L - x)^2)}{EI L} \quad (1.3)$$

Verplaatsingen in de steunpunten

Vergelijking in B

$x := 1 : L := 5 \cdot 1 : wBq := wq :$
$F := Bv : b := 4 \cdot 1 : wBb := wfL :$
$F := Cv : b := 3 \cdot l : wBc := wfL :$
$F := Dv : b := 2 \cdot l : wBd := wfL :$
$F := Ev : b := 1 \cdot l : wBe := wfL :$
$eq1 := wB = wBq - wBb - wBc - wBd - wBe;$

$$wB = \frac{29}{6} \frac{q l^4}{EI} - \frac{16}{15} \frac{Bv l^3}{EI} - \frac{3}{2} \frac{Cv l^3}{EI} - \frac{4}{3} \frac{Dv l^3}{EI} - \frac{23}{30} \frac{Ev l^3}{EI} \quad (2.1)$$

Vergelijking in C

$x := 2 \cdot 1 : L := 5 \cdot 1 : wCq := wq :$
$F := Bv : a := l : b := 4 \cdot l : wCb := wfR :$
$F := Cv : a := 2 \cdot l : b := 3 \cdot l : wCc := wfR :$
$F := Dv : b := 2 \cdot l : wCd := wfL :$
$F := Ev : b := 1 \cdot l : wCe := wfL :$
$eq2 := wC = wCq - wCb - wCc - wCd - wCe;$

$$wC = \frac{31}{4} \frac{q l^4}{EI} - \frac{3}{2} \frac{Bv l^3}{EI} - \frac{12}{5} \frac{Cv l^3}{EI} - \frac{34}{15} \frac{Dv l^3}{EI} - \frac{4}{3} \frac{Ev l^3}{EI} \quad (2.2)$$

Vergelijking in D

$x := 3 \cdot 1 : L := 5 \cdot 1 : wDq := wq :$
$F := Bv : a := 1 \cdot l : b := 4 \cdot l : wDb := wfR :$
$F := Cv : a := 2 \cdot l : b := 3 \cdot l : wDc := wfR :$
$F := Dv : a := 3 \cdot l : b := 2 \cdot l : wDd := wfR :$
$F := Ev : a := 4 \cdot l : b := 1 \cdot l : wDe := wfL :$
$eq3 := wD = wDq - wDb - wDc - wDd - wDe;$

$$wD = \frac{31}{4} \frac{q l^4}{EI} - \frac{4}{3} \frac{Bv l^3}{EI} - \frac{34}{15} \frac{Cv l^3}{EI} - \frac{12}{5} \frac{Dv l^3}{EI} - \frac{3}{2} \frac{Ev l^3}{EI} \quad (2.3)$$

Vergelijking in E

$x := 4 \cdot 1 : L := 5 \cdot 1 : wEq := wq :$
$F := Bv : a := 1 \cdot l : b := 4 \cdot l : wEb := wfR :$
$F := Cv : a := 2 \cdot l : b := 3 \cdot l : wEc := wfR :$
$F := Dv : a := 3 \cdot l : b := 2 \cdot l : wEd := wfR :$
$F := Ev : a := 4 \cdot l : b := 1 \cdot l : wEe := wfR :$
$eq4 := wE = wEq - wEb - wEc - wEd - wEe;$

$$wE = \frac{29}{6} \frac{q l^4}{EI} - \frac{23}{30} \frac{Bv l^3}{EI} - \frac{4}{3} \frac{Cv l^3}{EI} - \frac{3}{2} \frac{Dv l^3}{EI} - \frac{16}{15} \frac{Ev l^3}{EI} \quad (2.4)$$

Oplossen Reactiekrachten

$sol := solve(\{eq1, eq2, eq3, eq4\}, \{Bv, Cv, Dv, Ev\}) : assign(\%) :$

Reactie kracht in A en F

$$Av := \frac{\frac{1}{2} \cdot (5 \cdot l)^2 \cdot q - 4 \cdot l \cdot Bv - 3 \cdot l \cdot Cv - 2 \cdot l \cdot Dv - 1 \cdot l \cdot Ev}{5 \cdot 1} :$$

$$Fv := \frac{\frac{1}{2} \cdot (5 \cdot l)^2 \cdot q - 4 \cdot l \cdot Ev - 3 \cdot l \cdot Dv - 2 \cdot l \cdot Cv - 1 \cdot l \cdot Bv}{5 \cdot l} :$$

$Bv;$
$Cv;$
$Dv;$
$Ev;$

$$-\frac{1}{418} \frac{-432\, wE\, EI - 473\, q\, l^4 + 4128\, wB\, EI + 1728\, wD\, EI - 3972\, wC\, EI}{l^3}$$

$$\frac{1}{418} \frac{3972\, wB\, EI + 407\, q\, l^4 - 1728\, wE\, EI + 4404\, wD\, EI - 5856\, wC\, EI}{l^3}$$

$$-\frac{1}{418} \frac{5856\, wD\, EI - 407\, q\, l^4 - 3972\, wE\, EI + 1728\, wB\, EI - 4404\, wC\, EI}{l^3}$$

$$\frac{1}{418} \frac{3972\, wD\, EI + 473\, q\, l^4 - 4128\, wE\, EI + 432\, wB\, EI - 1728\, wC\, EI}{l^3} \qquad \textbf{(3.1)}$$

Steunpunts verplaatsingen
De in te voeren verplaatsingen van de steunpunten

Zakking t.o.v. de hartlijn, positieve waarde is een zakking omlaag gericht
$wB := -83 :$
$wC := -248 :$
$wD := -248 :$
$wE := -83 :$

Staaf eigenschappen

$EI := evalf\left(\frac{\pi}{64} \cdot d^4 \cdot E\right) :$
$E := 210000 :$

Afmetingen steunpunten
$l := 400 :$

Belasting van de staaf
$\gamma s := 7800 :$
$\text{evalf}\left(qs = \frac{1}{4} \cdot \pi \cdot d^2 \cdot \gamma s \cdot 10^{-8}\right) : assign(\%) :$
$q := qs :$

Te kiezen staaf diameter

De te kiezen staaf diameter

$d := 3;$

Steunpuntsreacties

$Av;$
$Bv;$
$Cv;$

Dv,
Ev,
Fv,

2.456026122

−7.545386165

5.640709555

5.640709557

−7.545386165

2.456026120 **(7.1)**

Appendix E. Maple sheets of the single curved mould tests

Test 1

The filled in data after solving the support reactions at Appendix B:

Steunpunts verplaatsingen
De in te voeren verplaatsingen van de steunpunten

Zakking t.o.v. de hartlijn, positieve waarde is een zakking omlaag gericht
$wB := -31$:
$wC := -54$:
$wD := -69$:
$wE := -76$:
$wF := -75$:
$wG := -66$:
$wH := -49$:
$wI := -26$:
$wJ := -9$:

Mal eigenschappen

$EI := \frac{1}{12} \cdot bm \cdot h^3 \cdot E$:
$E := 11000$:
Afmetingen steunpunten(actuatoren)
$l := 200$:
$bm := 400$:
$bb := 200$:
Belasting op de mal, beton laag
$\rho b := 2500$:
$d := 60$:
evalf($q = \rho b \cdot bb \cdot d \cdot 10^{-8}$); $assign(\%)$;

$$q = 0.3000000000 \quad \textbf{(5.1)}$$

Maximale Lokale doorbuiging van de mal
$wu := 1$:

Rand voorwaarde

Steunpunts krachten moeten groten zijn dan 0, dan laat de mal niet los op de steunpunten
$eq10 := Av \geq 0$:
$eq11 := Bv \geq 0$:
$eq12 := Cv \geq 0$:
$eq13 := Dv \geq 0$:
$eq14 := Ev \geq 0$:
$eq15 := Fv \geq 0$:
$eq16 := Gv \geq 0$:
$eq17 := Hv \geq 0$:
$eq18 := Iv \geq 0$:
$eq19 := Jv \geq 0$:
$eq20 := Kv \geq 0$:

Eis voor de lokale doorzakking van de mal, tussen twee steunpunten
$eq21 := wu \geq \frac{1}{384} \frac{q \cdot l^4}{EI}$:
$eq22 := h \geq 0$:

Oplossen mal dikte

$sol := solve(\{eq10, eq11, eq12, eq13, eq14, eq15, eq16, eq17, eq18, eq19, eq20, eq21, eq22\}, \{h\})$;

$$\{1.505033595 \leq h, h \leq 3.705744091\} \quad \textbf{(7.1)}$$

Te kiezen mal dikte

De te kiezen mal dikte, moet voldoen aan bovenstaande randvoorwaarde
$h := 3.8$;

$$3.8 \quad \textbf{(8.1)}$$

$wu := \frac{1}{384}\frac{q \cdot l^4}{EI};$

$$0.06212806003 \qquad \textbf{(8.2)}$$

$evalf\left(Izz = \frac{1}{12} \cdot b \cdot h^3\right);$

$$Izz = 914.5333333 \qquad \textbf{(8.3)}$$

Steunpuntsreacties

Av;
Bv;
Cv;
Dv;
Ev;
Fv;
Gv;
Hv;
Iv;
Jv;
Kv;

−1.851709460
100.3918528
49.15098805
63.00419552
58.83223041
61.66688639
54.50022956
110.5117949
14.35058794
41.18785297
48.25509092 **(9.1)**

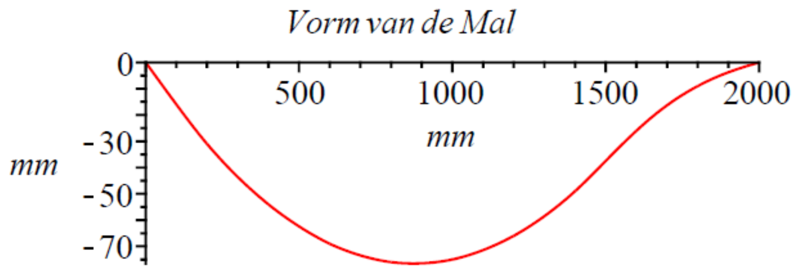
Doorbuigings grafiek

$x :=' x' :$
$F := Bv : a := 1 \cdot l : b := 9 \cdot l : wBL := wfL : wBR := wfR :$
$F := Cv : a := 2 \cdot l : b := 8 \cdot l : wCL := wfL : wCR := wfR :$
$F := Dv : a := 3 \cdot l : b := 7 \cdot l : wDL := wfL : wDR := wfR :$
$F := Ev : a := 4 \cdot l : b := 6 \cdot l : wEL := wfL : wER := wfR :$
$F := Fv : a := 5 \cdot l : b := 5 \cdot l : wFL := wfL : wFR := wfR :$
$F := Gv : a := 6 \cdot l : b := 4 \cdot l : wGL := wfL : wGR := wfR :$
$F := Hv : a := 7 \cdot l : b := 3 \cdot l : wHL := wfL : wHR := wfR :$
$F := Iv : a := 8 \cdot l : b := 2 \cdot l : wIL := wfL : wIR := wfR :$
$F := Jv : a := 9 \cdot l : b := 1 \cdot l : wJL := wfL : wJR := wfR :$

$w1 := wq - wBL - wCL - wDL - wEL - wFL - wGL - wHL - wIL - wJL :$
$w2 := wq - wBR - wCL - wDL - wEL - wFL - wGL - wHL - wIL - wJL :$
$w3 := wq - wBR - wCR - wDL - wEL - wFL - wGL - wHL - wIL - wJL :$
$w4 := wq - wBR - wCR - wDR - wEL - wFL - wGL - wHL - wIL - wJL :$
$w5 := wq - wBR - wCR - wDR - wER - wFL - wGL - wHL - wIL - wJL :$
$w6 := wq - wBR - wCR - wDR - wER - wFR - wGL - wHL - wIL - wJL :$
$w7 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHL - wIL - wJL :$
$w8 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHR - wIL - wJL :$
$w9 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHR - wIR - wJL :$
$w10 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHR - wIR - wJR :$

$with(plots) :$
$W1 := plot(w1, x = 0 \cdot l..1 \cdot l) :$
$W2 := plot(w2, x = 1 \cdot l..2 \cdot l) :$
$W3 := plot(w3, x = 2 \cdot l..3 \cdot l) :$
$W4 := plot(w4, x = 3 \cdot l..4 \cdot l) :$
$W5 := plot(w5, x = 4 \cdot l..5 \cdot l) :$

W6 := *plot*(*w6*, *x* = 5·*l*..6·*l*) :
W7 := *plot*(*w7*, *x* = 6·*l*..7·*l*) :
W8 := *plot*(*w8*, *x* = 7·*l*..8·*l*) :
W9 := *plot*(*w9*, *x* = 8·*l*..9·*l*) :
W10 := *plot*(*w10*, *x* = 9·*l*..10·*l*) :
display({*W1*, *W2*, *W3*, *W4*, *W5*, *W6*, *W7*, *W8*, *W9*, *W10*}, *title* = 'Vorm van de Mal', *labels* = [*mm*, *mm*]);



Verplaatsingen in het midden van de steunpunten
$x := 0.5 \cdot l : w1;$
$x := 1.5 \cdot l : w2;$
$x := 2.5 \cdot l : w3;$
$x := 3.5 \cdot l : w4;$
$x := 4.5 \cdot l : w5;$
$x := 5.5 \cdot l : w6;$
$x := 6.5 \cdot l : w7;$
$x := 7.5 \cdot l : w8;$
$x := 8.5 \cdot l : w9;$
$x := 9.5 \cdot l : w10;$

−15.98091496

−43.56023867

−62.40538562

−73.44548602

−76.43994782

−71.42200555

−58.49931230

−37.45801646

−16.29588391

−3.73569952 **(10.1)**

Rek in de ligger

Oorspronkelijke booglente:
l;

200 **(11.1)**

De nieuwe booglengte per ligger
Student-Calculus1:-ArcLength(w1, x = 0·1..1·1) : % := *evalf*(%) : *l1n* := %;
Student-Calculus1:-ArcLength(w2, x = 1·1..2·1) : % := *evalf*(%) : *l2n* := %;
Student-Calculus1:-ArcLength(w3, x = 2·1..3·1) : % := *evalf*(%) : *l3n* := %;
Student-Calculus1:-ArcLength(w4, x = 3·1..4·1) : % := *evalf*(%) : *l4n* := %;
Student-Calculus1:-ArcLength(w5, x = 4·1..5·1) : % := *evalf*(%) : *l5n* := %;
Student-Calculus1:-ArcLength(w6, x = 5·1..6·1) : % := *evalf*(%) : *l6n* := %;
Student-Calculus1:-ArcLength(w7, x = 6·1..7·1) : % := *evalf*(%) : *l7n* := %;
Student-Calculus1:-ArcLength(w8, x = 7·1..8·1) : % := *evalf*(%) : *l8n* := %;
Student-Calculus1:-ArcLength(w9, x = 8·1..9·1) : % := *evalf*(%) : *l9n* := %;
Student-Calculus1:-ArcLength(w10, x = 9·1..10·1) : % := *evalf*(%) : *l10n* := %;

202.3919272

201.3332474

200.5728655

200.1346881

200.0146064

200.2140186

200.7347234

201.3193311

200.7399444

200.2101075 **(11.2)**

Oosrponkelijke Totale lengte:
$lto := 10 \cdot l;$

2000 **(11.3)**

Nieuwe lengte:
$ltn := l1n + l2n + l3n + l4n + l5n + l6n + l7n + l8n + l9n + l10n;$

2007.665460 **(11.4)**

Totale verlenging
$\Delta L := ltn - lto;$

7.665460 **(11.5)**

Rek totale ligger (promiel):
$$\varepsilon := \frac{ltn - lto}{lto} \cdot 1000;$$

3.832730000 **(11.6)**

Test 2

Steunpunts verplaatsingen
De in te voeren verplaatsingen van de steunpunten

Zakking t.o.v. de hartlijn, positieve waarde is een zakking omlaag gericht
$wB := -31$:
$wC := -54$:
$wD := -69$:
$wE := -76$:
$wF := -75$:
$wG := -66$:
$wH := -49$:
$wI := -26$:
$wJ := -9$:

Mal eigenschappen

$EI := \frac{1}{12} \cdot bm \cdot h^3 \cdot E$:
$E := 210000$:
Afmetingen steunpunten(actuatoren)
$l := 200$:
$bm := 400$:
$bb := 200$:
Belasting op de mal, beton laag
$\rho b := 2500$:
$d := 60$:
evalf(q = $\rho b \cdot bb \cdot d \cdot 10^{-8}$); *assign*(%);

$$q = 0.3000000000 \qquad \textbf{(5.1)}$$

Maximale Lokale doorbuiging van de mal
$wu := 1$:

Rand voorwaarde

Steunpunts krachten moeten groten zijn dan 0, dan laat de mal niet los op de steunpunten
$eq10 := Av \geq 0$:
$eq11 := Bv \geq 0$:
$eq12 := Cv \geq 0$:
$eq13 := Dv \geq 0$:
$eq14 := Ev \geq 0$:
$eq15 := Fv \geq 0$:
$eq16 := Gv \geq 0$:
$eq17 := Hv \geq 0$:
$eq18 := Iv \geq 0$:
$eq19 := Jv \geq 0$:
$eq20 := Kv \geq 0$:

Eis voor de lokale doorzakking van de mal, tussen twee steunpunten
$eq21 := wu \geq \frac{1}{384} \frac{q \cdot l^4}{EI}$:
$eq22 := h \geq 0$:

Oplossen mal dikte

sol := *solve*({*eq10*, *eq11*, *eq12*, *eq13*, *eq14*, *eq15*, *eq16*, *eq17*, *eq18*, *eq19*, *eq20*, *eq21*, *eq22*}, {*h*});

$$\{0.5631239403 \leq h, h \leq 1.386542614\} \qquad \textbf{(7.1)}$$

Te kiezen mal dikte

De te kiezen mal dikte, moet voldoen aan bovenstaande randvoorwaarde
$h := 1;$

$$1 \quad \mathbf{(8.1)}$$

$$wu := \frac{1}{384}\frac{q \cdot l^4}{EI};$$

$$0.1785714286 \quad \mathbf{(8.2)}$$

$$evalf\left(Izz = \frac{1}{12} \cdot b \cdot h^3\right);$$

$$Izz = 16.66666667 \quad \mathbf{(8.3)}$$

Steunpuntsreacties

$Av;$
$Bv;$
$Cv;$
$Dv;$
$Ev;$
$Fv;$
$Gv;$
$Hv;$
$Iv;$
$Jv;$
$Kv;$

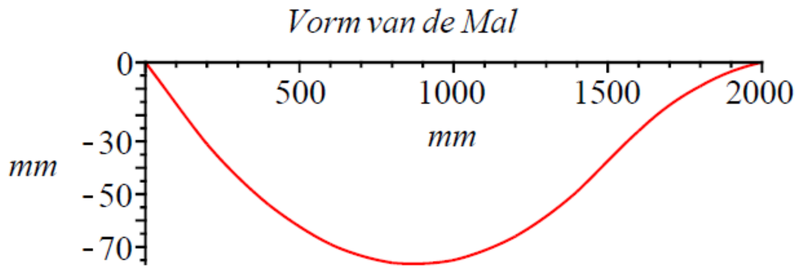
14.78418350
79.29489942
54.82040234
61.42349124
59.48563272
60.63397790
57.97845568
77.95219937
42.71274683
58.69681329
32.21719784 **(9.1)**

Doorbuigings grafiek

$x := 'x' :$
$F := Bv : a := 1 \cdot l : b := 9 \cdot l : wBL := wfL : wBR := wfR :$
$F := Cv : a := 2 \cdot l : b := 8 \cdot l : wCL := wfL : wCR := wfR :$
$F := Dv : a := 3 \cdot l : b := 7 \cdot l : wDL := wfL : wDR := wfR :$
$F := Ev : a := 4 \cdot l : b := 6 \cdot l : wEL := wfL : wER := wfR :$
$F := Fv : a := 5 \cdot l : b := 5 \cdot l : wFL := wfL : wFR := wfR :$
$F := Gv : a := 6 \cdot l : b := 4 \cdot l : wGL := wfL : wGR := wfR :$
$F := Hv : a := 7 \cdot l : b := 3 \cdot l : wHL := wfL : wHR := wfR :$
$F := Iv : a := 8 \cdot l : b := 2 \cdot l : wIL := wfL : wIR := wfR :$
$F := Jv : a := 9 \cdot l : b := 1 \cdot l : wJL := wfL : wJR := wfR :$

$w1 := wq - wBL - wCL - wDL - wEL - wFL - wGL - wHL - wIL - wJL :$
$w2 := wq - wBR - wCL - wDL - wEL - wFL - wGL - wHL - wIL - wJL :$
$w3 := wq - wBR - wCR - wDL - wEL - wFL - wGL - wHL - wIL - wJL :$
$w4 := wq - wBR - wCR - wDR - wEL - wFL - wGL - wHL - wIL - wJL :$
$w5 := wq - wBR - wCR - wDR - wER - wFL - wGL - wHL - wIL - wJL :$
$w6 := wq - wBR - wCR - wDR - wER - wFR - wGL - wHL - wIL - wJL :$
$w7 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHL - wIL - wJL :$
$w8 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHR - wIL - wJL :$
$w9 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHR - wIR - wJL :$
$w10 := wq - wBR - wCR - wDR - wER - wFR - wGR - wHR - wIR - wJR :$

```
with(plots) :
W1 := plot(w1, x = 0·l..1·l) :
W2 := plot(w2, x = 1·l..2·l) :
W3 := plot(w3, x = 2·l..3·l) :
W4 := plot(w4, x = 3·l..4·l) :
W5 := plot(w5, x = 4·l..5·l) :
W6 := plot(w6, x = 5·l..6·l) :
W7 := plot(w7, x = 6·l..7·l) :
W8 := plot(w8, x = 7·l..8·l) :
W9 := plot(w9, x = 8·l..9·l) :
W10 := plot(w10, x = 9·l..10·l) :
display({W1, W2, W3, W4, W5, W6, W7, W8, W9, W10}, title='Vorm van de Mal', labels = [mm, mm]);
```



Verplaatsingen in het midden van de steunpunten

```
x := 0.5·l : w1;
x := 1.5·l : w2;
x := 2.5·l : w3;
x := 3.5·l : w4;
x := 4.5·l : w5;
x := 5.5·l : w6;
x := 6.5·l : w7;
x := 7.5·l : w8;
x := 8.5·l : w9;
x := 9.5·l : w10;
```

−15.69398662

−43.4894674

−62.2767118

−73.3322512

−76.3228514

−71.3049081

−58.3860804

−37.3293426

−16.2251134

−3.44877150 **(10.1)**

Test 3

Algemene belastingformules

restart;

Verdeelde belasting

$wq := \frac{q \cdot x}{24 \cdot EI} \cdot (L^3 - 2 \cdot L \cdot x^2 + x^3);$

$$\frac{1}{24} \frac{q\,x\,(L^3 - 2\,L\,x^2 + x^3)}{EI} \quad (1.1)$$

Puntlast

Als x, de plaats van de steunpunt links van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfL := \frac{F \cdot b \cdot x}{6 \cdot EI \cdot L} \cdot (2 \cdot L \cdot (L - x) - (b)^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F\,b\,x\,(2\,L\,(L - x) - b^2 - (L - x)^2)}{EI\,L} \quad (1.2)$$

Als x, de plaats van de steunpunt rechts van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfR := \frac{F \cdot a \cdot (L - x)}{6 \cdot EI \cdot L} \cdot ((2 \cdot L \cdot b) - b^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F\,a\,(L - x)\,(2\,L\,b - b^2 - (L - x)^2)}{EI\,L} \quad (1.3)$$

Verplaatsingen in de steunpunten

Vergelijking in B

$x := 1 : L := 5 \cdot 1 : wBq := wq :$
$F := Bv : b := 4 \cdot 1 : wBb := wfL :$
$F := Cv : b := 3 \cdot l : wBc := wfL :$
$F := Dv : b := 2 \cdot l : wBd := wfL :$
$F := Ev : b := 1 \cdot l : wBe := wfL :$
$eq1 := wB = wBq - wBb - wBc - wBd - wBe;$

$$wB = \frac{29}{6} \frac{q\,l^4}{EI} - \frac{16}{15} \frac{Bv\,l^3}{EI} - \frac{3}{2} \frac{Cv\,l^3}{EI} - \frac{4}{3} \frac{Dv\,l^3}{EI} - \frac{23}{30} \frac{Ev\,l^3}{EI} \quad (2.1)$$

Vergelijking in C

$x := 2 \cdot 1 : L := 5 \cdot 1 : wCq := wq :$
$F := Bv : a := l : b := 4 \cdot l : wCb := wfR :$
$F := Cv : a := 2 \cdot l : b := 3 \cdot l : wCc := wfR :$
$F := Dv : b := 2 \cdot l : wCd := wfL :$
$F := Ev : b := 1 \cdot l : wCe := wfL :$
$eq2 := wC = wCq - wCb - wCc - wCd - wCe;$

$$wC = \frac{31}{4} \frac{q\,l^4}{EI} - \frac{3}{2} \frac{Bv\,l^3}{EI} - \frac{12}{5} \frac{Cv\,l^3}{EI} - \frac{34}{15} \frac{Dv\,l^3}{EI} - \frac{4}{3} \frac{Ev\,l^3}{EI} \quad (2.2)$$

Vergelijking in D

$x := 3 \cdot 1 : L := 5 \cdot 1 : wDq := wq :$
$F := Bv : a := 1 \cdot l : b := 4 \cdot l : wDb := wfR :$
$F := Cv : a := 2 \cdot l : b := 3 \cdot l : wDc := wfR :$
$F := Dv : a := 3 \cdot l : b := 2 \cdot l : wDd := wfR :$
$F := Ev : a := 4 \cdot l : b := 1 \cdot l : wDe := wfL :$
$eq3 := wD = wDq - wDb - wDc - wDd - wDe;$

$$wD = \frac{31}{4} \frac{q\,l^4}{EI} - \frac{4}{3} \frac{Bv\,l^3}{EI} - \frac{34}{15} \frac{Cv\,l^3}{EI} - \frac{12}{5} \frac{Dv\,l^3}{EI} - \frac{3}{2} \frac{Ev\,l^3}{EI} \quad (2.3)$$

Vergelijking in E

$x := 4 \cdot 1 : L := 5 \cdot 1 : wEq := wq :$
$F := Bv : a := 1 \cdot l : b := 4 \cdot l : wEb := wfR :$
$F := Cv : a := 2 \cdot l : b := 3 \cdot l : wEc := wfR :$
$F := Dv : a := 3 \cdot l : b := 2 \cdot l : wEd := wfR :$
$F := Ev : a := 4 \cdot l : b := 1 \cdot l : wEe := wfR :$
$eq4 := wE = wEq - wEb - wEc - wEd - wEe;$

$$wE = \frac{29}{6}\frac{q\,l^4}{EI} - \frac{23}{30}\frac{Bv\,l^3}{EI} - \frac{4}{3}\frac{Cv\,l^3}{EI} - \frac{3}{2}\frac{Dv\,l^3}{EI} - \frac{16}{15}\frac{Ev\,l^3}{EI} \quad \textbf{(2.4)}$$

Oplossen Reactiekrachten

$sol := solve(\{eq1, eq2, eq3, eq4\}, \{Bv, Cv, Dv, Ev\}) : assign(\%) :$

Reactie kracht in A en F

$$Av := \frac{\frac{1}{2}\cdot(5\cdot l)^2\cdot q - 4\cdot l\cdot Bv - 3\cdot l\cdot Cv - 2\cdot l\cdot Dv - 1\cdot l\cdot Ev}{5\cdot 1} :$$

$$Fv := \frac{\frac{1}{2}\cdot(5\cdot l)^2\cdot q - 4\cdot l\cdot Ev - 3\cdot l\cdot Dv - 2\cdot l\cdot Cv - 1\cdot l\cdot Bv}{5\cdot l} :$$

Steunpunts verplaatsingen
De in te voeren verplaatsingen van de steunpunten

Zakking t.o.v. de hartlijn, positieve waarde is een zakking omlaag gericht
$wB := -32 :$
$wC := -93 :$
$wD := -93 :$
$wE := -32 :$

Mal eigenschappen

$EI := \frac{1}{12}\cdot bm\cdot h^3\cdot E :$
$E := 11000 :$
Afmetingen steunpunten(actuatoren)
$l := 400 :$
$bm := 400 :$
$bb := 200 :$
Belasting op de mal, beton laag
$\gamma b := 2500 :$
$d := 50 :$
$\text{evalf}(q = \gamma b\cdot \text{bb}\cdot \text{d}\cdot 10^{-8}); assign(\%);$

$$q = 0.2500000000 \quad \textbf{(5.1)}$$

Maximale Lokale doorbuiging van de mal
$wu := 2 :$

Rand voorwaarde

Steunpunts krachten moeten groten zijn dan 0, dan laat de mal niet los op de steunpunten
$eq10 := Av \geq 0 :$
$eq11 := Bv \geq 0 :$
$eq12 := Cv \geq 0 :$
$eq13 := Dv \geq 0 :$
$eq14 := Ev \geq 0 :$
$eq15 := Fv \geq 0 :$

Eis voor de lokale doorzakking van de mal, tussen twee steunpunten

$$eq21 := wu \geq \frac{1}{384}\frac{q\cdot l^4}{EI} :$$

$eq22 := h \geq 0 :$

Oplossen mal dikte

$sol := solve(\{eq10, eq11, eq12, eq13, eq14, eq15, eq21, eq22\}, \{h\});$

$$\{2.832581674 \leq h, h \leq 4.502997721\} \quad \textbf{(7.1)}$$

Te kiezen mal dikte

De te kiezen mal dikte, moet voldoen aan bovenstaande randvoorwaarde

$h := 3.8;$

$$3.8 \quad \textbf{(8.1)}$$

$wu := \frac{1}{384}\frac{q \cdot l^4}{EI};$

$$0.8283741336 \quad \textbf{(8.2)}$$

$evalf\left(Izz = \frac{1}{12} \cdot b \cdot h^3\right);$

$$Izz = 1829.066667 \quad \textbf{(8.3)}$$

Steunpuntsreacties

$Av;$
$Bv;$
$Cv;$
$Dv;$
$Ev;$
$Fv;$

59.92433436

45.15451974

144.9211460

144.9211457

45.15451974

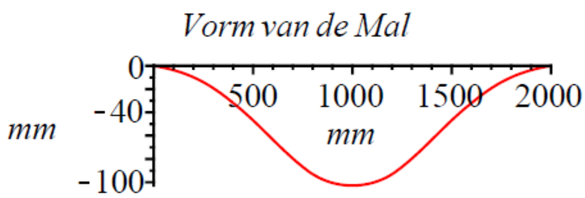
59.92433444 **(9.1)**

Doorbuigings grafiek

```
x:='x':
F := Bv : a := 1·l : b := 4·l : wBL := wfL : wBR := wfR :
F := Cv : a := 2·l : b := 3·l : wCL := wfL : wCR := wfR :
F := Dv : a := 3·l : b := 2·l : wDL := wfL : wDR := wfR :
F := Ev : a := 4·l : b := 1·l : wEL := wfL : wER := wfR :

w1 := wq − wBL − wCL − wDL − wEL :
w2 := wq − wBR − wCL − wDL − wEL :
w3 := wq − wBR − wCR − wDL − wEL :
w4 := wq − wBR − wCR − wDR − wEL :
w5 := wq − wBR − wCR − wDR − wER :

with(plots) :
W1 :=plot(w1, x = 0·l..1·l) :
W2 :=plot(w2, x = 1·l..2·l) :
W3 :=plot(w3, x = 2·l..3·l) :
W4 :=plot(w4, x = 3·l..4·l) :
W5 :=plot(w5, x = 4·l..5·l) :
display({W1, W2, W3, W4, W5}, title='Vorm van de Mal', labels = [mm, mm]);
```



Verplaatsingen in het midden van de steunpunten

$x := 0.5 \cdot l : w1;$

$x := 1.5 \cdot l : w2;$
$x := 2.5 \cdot l : w3;$
$x := 3.5 \cdot l : w4;$
$x := 4.5 \cdot l : w5;$

$$-9.88507420$$
$$-63.3427826$$
$$-102.7735465$$
$$-63.3427814$$
$$-9.88507390 \quad \textbf{(10.1)}$$

Rek in de ligger

Oorspronkelijke booglente:
$l;$

$$400 \quad \textbf{(11.1)}$$

De nieuwe booglengte per ligger
$Student\text{:-}Calculus1\text{:-}ArcLength(w1, x = 0 \cdot 1 .. 1 \cdot 1) : \% := evalf(\%) : l1n := \%;$
$Student\text{:-}Calculus1\text{:-}ArcLength(w2, x = 1 \cdot 1 .. 2 \cdot 1) : \% := evalf(\%) : l2n := \%;$
$Student\text{:-}Calculus1\text{:-}ArcLength(w3, x = 2 \cdot 1 .. 3 \cdot 1) : \% := evalf(\%) : l3n := \%;$
$Student\text{:-}Calculus1\text{:-}ArcLength(w4, x = 3 \cdot 1 .. 4 \cdot 1) : \% := evalf(\%) : l4n := \%;$
$Student\text{:-}Calculus1\text{:-}ArcLength(w5, x = 4 \cdot 1 .. 5 \cdot 1) : \% := evalf(\%) : l5n := \%;$

$$401.5143878$$
$$404.6666305$$
$$400.6587865$$
$$404.6666305$$
$$401.5143878 \quad \textbf{(11.2)}$$

Oosrponkelijke Totale lengte:
$lto := 5 \cdot l;$

$$2000 \quad \textbf{(11.3)}$$

Nieuwe lengte:
$ltn := l1n + l2n + l3n + l4n + l5n;$

$$2013.020824 \quad \textbf{(11.4)}$$

Totale verlenging
$\Delta L := ltn - lto;$

$$13.020824 \quad \textbf{(11.5)}$$

Test 4

The filled in data after solving the support reactions at test 3:

Steunpunts verplaatsingen
De in te voeren verplaatsingen van de steunpunten

Zakking t.o.v. de hartlijn, positieve waarde is een zakking omlaag gericht
$wB := -54:$
$wC := -158:$
$wD := -158:$
$wE := -54:$

Mal eigenschappen

$EI := \frac{1}{12} \cdot bm \cdot h^3 \cdot E:$
$E := 11000:$
Afmetingen steunpunten(actuatoren)
$l := 400:$
$bm := 400:$
$bb := 200:$
Belasting op de mal, beton laag
$\gamma b := 2500:$
$d := 50:$
$\text{evalf}(q = \gamma b \cdot bb \cdot d \cdot 10^{-8}); assign(\%);$

$$q = 0.2500000000 \quad \textbf{(5.1)}$$

Maximale Lokale doorbuiging van de mal
$wu := 2:$

Rand voorwaarde

Steunpunts krachten moeten groten zijn dan 0, dan laat de mal niet los op de steunpunten
$eq10 := Av \geq 0:$
$eq11 := Bv \geq 0:$
$eq12 := Cv \geq 0:$
$eq13 := Dv \geq 0:$
$eq14 := Ev \geq 0:$
$eq15 := Fv \geq 0:$

Eis voor de lokale doorzakking van de mal, tussen twee steunpunten
$eq21 := wu \geq \frac{1}{384} \frac{q \cdot l^4}{EI}:$
$eq22 := h \geq 0:$

Oplossen mal dikte

$sol := solve(\{eq10, eq11, eq12, eq13, eq14, eq15, eq21, eq22\}, \{h\});$

$$\{2.832581674 \leq h, h \leq 3.762783394\} \quad \textbf{(7.1)}$$

Te kiezen mal dikte

De te kiezen mal dikte, moet voldoen aan bovenstaande randvoorwaarde
$h := 3.8;$

$$3.8 \quad \textbf{(8.1)}$$

$wu := \frac{1}{384} \frac{q \cdot l^4}{EI};$

$$0.8283741336 \quad \textbf{(8.2)}$$

$evalf\left(Izz = \frac{1}{12} \cdot b \cdot h^3\right);$

$$Izz = 1829.066667 \quad \textbf{(8.3)}$$

Steunpuntsreacties

Av;
Bv;
Cv;
Dv;
Ev;
Fv;

74.61703410

−3.390955069

178.7739208

178.7739212

−3.390955069

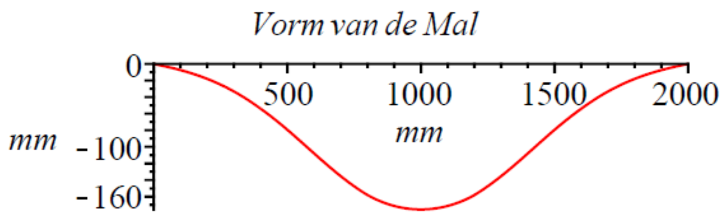
74.61703400 **(9.1)**

Doorbuigings grafiek

$x := 'x'$:
$F := Bv : a := 1 \cdot l : b := 4 \cdot l : wBL := wfL : wBR := wfR$:
$F := Cv : a := 2 \cdot l : b := 3 \cdot l : wCL := wfL : wCR := wfR$:
$F := Dv : a := 3 \cdot l : b := 2 \cdot l : wDL := wfL : wDR := wfR$:
$F := Ev : a := 4 \cdot l : b := 1 \cdot l : wEL := wfL : wER := wfR$:

$w1 := wq - wBL - wCL - wDL - wEL$:
$w2 := wq - wBR - wCL - wDL - wEL$:
$w3 := wq - wBR - wCR - wDL - wEL$:
$w4 := wq - wBR - wCR - wDR - wEL$:
$w5 := wq - wBR - wCR - wDR - wER$:

with(*plots*) :
W1 := *plot*(*w1*, *x* = 0 ·*l* ..1 · *l*) :
W2 := *plot*(*w2*, *x* = 1 · *l* ..2 ·*l*) :
W3 := *plot*(*w3*, *x* = 2 ·*l* ..3 ·*l*) :
W4 := *plot*(*w4*, *x* = 3 ·*l* ..4 ·*l*) :
W5 := *plot*(*w5*, *x* = 4 ·*l* ..5 ·*l*) :
display({ *W1*, *W2*, *W3*, *W4*, *W5*}, *title* = 'Vorm van de Mal', *labels* = [*mm*, *mm*]);



Verplaatsingen in het midden van de steunpunten
$x := 0.5 \cdot l : w1;$
$x := 1.5 \cdot l : w2;$
$x := 2.5 \cdot l : w3;$
$x := 3.5 \cdot l : w4;$
$x := 4.5 \cdot l : w5;$

−17.96402291

−107.7309437

−175.3919716

−107.7309431

−17.96402270 **(10.1)**

Rek in de ligger

Oorspronkelijke booglente:
l;

400 **(11.1)**

De nieuwe booglengte per ligger
Student:-*Calculus1*:-*ArcLength*(w1, x = 0·1 ..1· 1) : % := *evalf*(%) : *l1n* := %;

$Student\text{:-}Calculus1\text{:-}ArcLength(w2, x = 1\cdot 1\,..2\cdot 1) : \% := evalf(\%) : l2n := \%;$
$Student\text{:-}Calculus1\text{:-}ArcLength(w3, x = 2\cdot 1\,..3\cdot 1) : \% := evalf(\%) : l3n := \%;$
$Student\text{:-}Calculus1\text{:-}ArcLength(w4, x = 3\cdot 1\,..4\cdot 1) : \% := evalf(\%) : l4n := \%;$
$Student\text{:-}Calculus1\text{:-}ArcLength(w5, x = 4\cdot 1\,..5\cdot 1) : \% := evalf(\%) : l5n := \%;$

404.1489050

413.4214242

402.0465649

413.4214240

404.1489050 **(11.2)**

Oosrponkelijke Totale lengte:
$lto := 5\cdot l;$

2000 **(11.3)**

Nieuwe lengte:
$ltn := l1n + l2n + l3n + l4n + l5n;$

2037.187223 **(11.4)**

Totale verlenging
$\Delta L := ltn - lto;$

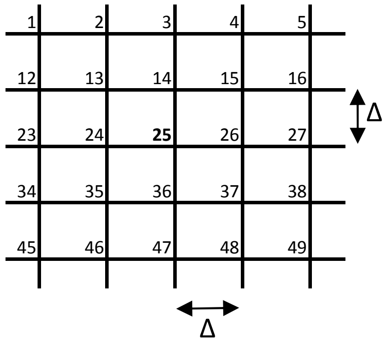
37.187223 **(11.5)**

Rek totale ligger (promiel):
$$\varepsilon := \frac{ltn - lto}{lto}\cdot 1000;$$

18.59361150 **(11.6)**

Appendix F. Partial derivatives

With the use of the below numbering the Partial derivatives are calculated [13].



The partial derivatives of F in the x direction are:

$$\frac{\partial^2 F}{\partial x^2}|_{24} = \frac{\frac{F_{25}-F_{24}}{\Delta} - \frac{F_{24}-F_{23}}{\Delta}}{\Delta} = \frac{1}{\Delta^2}\left(F_{23} - 2F_{24} + F_{25}\right)$$

$$\frac{\partial^2 F}{\partial x^2}|_{25} = \frac{1}{\Delta^2}\left(F_{24} - 2F_{25} + F_{26}\right)$$

$$\frac{\partial^2 F}{\partial x^3}|_{24-25} = \frac{\frac{1}{\Delta^2}\left(F_{24} - 2F_{25} + F_{26}\right) - \frac{1}{\Delta^2}\left(F_{23} - 2F_{24} + F_{25}\right)}{\Delta} \tag{1.1}$$

$$= \frac{1}{\Delta^3}\left(-F_{23} + 3F_{24} - 3F_{25} + F_{26}\right)$$

$$\frac{\partial^2 F}{\partial x^3}|_{25-26} = \frac{1}{\Delta^3}\left(-F_{24} + 3F_{25} - 3F_{26} + F_{27}\right)$$

$$\frac{\partial^2 F}{\partial x^4}|_{25} = \frac{\frac{\partial^2 F}{\partial x^3}|_{25-26} - \frac{\partial^2 F}{\partial x^3}|_{24-25}}{\Delta} = \frac{1}{\Delta^4}\left(F_{23} - 4F_{24} + 6F_{25} - 4F_{26} + F_{27}\right)$$

The partial derivatives in the y and x-y directions become:

$$\frac{\partial^2 F}{\partial y^2}|_{25} = \frac{1}{\Delta^2}\left(F_{14} - 2F_{25} + F_{36}\right) \tag{1.2}$$

$$\frac{\partial^2 F}{\partial y^4}|_{25} = \frac{1}{\Delta^4}\left(F_{3} - 4F_{14} + 6F_{25} - 4F_{36} + F_{47}\right)$$

$$\frac{\partial^2 F}{\partial x \partial y}\bigg|_{25} = \frac{\frac{F_{37} - F_{15}}{2\Delta} - \frac{F_{35} - F_{13}}{2\Delta}}{2\Delta} = \frac{1}{4\Delta^2}\left(F_{13} - F_{15} - F_{35} + F_{37}\right)$$

$$\frac{\partial^2 F}{\partial x^2}\bigg|_{14} = \frac{1}{\Delta^2}\left(F_{13} - 2F_{14} + F_{15}\right)$$

$$\frac{\partial^2 F}{\partial x^2}\bigg|_{36} = \frac{1}{\Delta^2}\left(F_{35} - 2F_{36} + F_{37}\right)$$

$$\frac{\partial^3 F}{\partial x^2 \partial y}\bigg|_{14-25} = \frac{\frac{\partial^2 F}{\partial x^2}\big|_{25} - \frac{\partial^2 F}{\partial x^2}\big|_{14}}{\Delta} = \frac{1}{\Delta^3}\left(-F_2 - F_4 + 2F_3 + F_{24} - 2F_{25} + F_{26}\right)$$

$$\frac{\partial^3 F}{\partial x^2 \partial y}\bigg|_{25-36} = \frac{\frac{\partial^2 F}{\partial x^2}\big|_{36} - \frac{\partial^2 F}{\partial x^2}\big|_{25}}{\Delta} = \frac{1}{\Delta^3}\left(-F_{24} + 2F_{25} - F_{26} + F_{35} - 2F_{36} + F_{37}\right)$$

$$\frac{\partial^4 F}{\partial x^2 \partial y^2}\bigg|_{25} = \frac{\frac{\partial^3 F}{\partial x^2 \partial y}\big|_{25-36} - \frac{\partial^3 F}{\partial x^2 \partial y}\big|_{14-25}}{\Delta}$$

$$= \frac{1}{\Delta^4}\left(F_{13} - 2F_{14} + F_{15} - 2F_{24} + 4F_{25} - 2F_{26} + F_{35} - 2F_{36} + F_{37}\right) \tag{1.3}$$

The partial derivatives for w are exactly in the same form as for F, so the F can be replaced for a w to get the partial derivatives for w.

Appendix G. Border's of P

To determine the formula for the edges of the plate, the method of using different plate stiffness is used [13]. To set the stiffness of parts that lay outside the plate boundaries zero, a corner differential equation can be made. See at the right site the used numbering of the cells.

D	E	B	E'	D'
E'	F	C	F	E'
B''	C'''	A	C'''	B'''
E'	F	C''	F	E'
D''	E'	B'	E'	D''

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{h}{D}\left(\frac{p}{h}+\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}\right)$$

$$\Leftrightarrow$$

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left(p+n_{xx}\frac{\partial^2 w}{\partial x^2}+n_{yy}\frac{\partial^2 w}{\partial y^2}-2n_{xy}\frac{\partial^2 w}{\partial x \partial y}\right)$$

$$\Leftrightarrow$$

$$\frac{\begin{array}{l} g_{14}(w_3-2w_{14}+w_{25})+f_{24}(w_{23}-2w_{24}+w_{25})-2f_{25}(w_{24}-2w_{25}+w_{26})- \\ 2g_{25}(w_{14}-2w_{25}+w_{36})+f_{26}(w_{25}-2w_{26}+w_{27})+g_{36}(w_{25}-2w_{36}+w_{47})+ \\ 2D_1(w_{25}-w_{26}+w_{15}-w_{14})+2D_2(w_{25}-w_{14}+w_{13}-w_{24})+ \\ 2D_3(w_{25}-w_{24}+w_{35}-w_{36})+2D_4(w_{25}-w_{36}+w_{37}-w_{26}) \end{array}}{\Delta^4}= \tag{1.1}$$

$$h\left[\begin{array}{l}\frac{p}{h}+\left(\frac{1}{\Delta^2}(F_{14}-2F_{25}+F_{36})\frac{1}{\Delta^2}(w_{24}-2w_{25}+w_{26})\right)+\left(\frac{1}{\Delta^2}(F_{24}-2F_{25}+F_{26})\frac{1}{\Delta^2}(w_{14}-2w_{25}+w_{36})\right) \\ -2\left(\frac{1}{4\Delta^2}(F_{13}-F_{15}-F_{35}+F_{37})\right)\left(\frac{1}{4\Delta^2}(w_{13}-w_{15}-w_{35}+w_{37})\right)\end{array}\right]$$

With

$$\begin{array}{ll} f_{24}=\frac{D_7 D_2}{D_7+D_2}+\frac{D_8 D_3}{D_8+D_3} & f_{24}=\frac{D_7 D_2}{D_7+D_2}+\frac{D_8 D_3}{D_8+D_3} \\ f_{25}=\frac{D_2 D_1}{D_2+D_1}+\frac{D_3 D_4}{D_3+D_4} & f_{25}=\frac{D_2 D_1}{D_2+D_1}+\frac{D_3 D_4}{D_3+D_4} \\ f_{26}=\frac{D_1 D_{12}}{D_1+D_{12}}+\frac{D_4 D_{11}}{D_4+D_{11}} & f_{26}=\frac{D_1 D_{12}}{D_1+D_{12}}+\frac{D_4 D_{11}}{D_4+D_{11}} \end{array} \tag{1.2}$$

The numbering of the cells is:

1	2	3	4	5	
		D6	**D5**		
12	13	14	15	16	
	D7	**D2**	**D1**	**D12**	
23	24	25	26	27	
	D8	**D3**	**D4**	**D11**	
34	35	36	37	39	
		D9	**D10**		
44	45	46	47	48	

Cell's B

$$
\begin{array}{lll}
f_{24} = \frac{1}{2}D & g_{14} = 0 & D_1 = D_2 = D_5 = D_6 = D_7 = D_{12} = 0 \\
 & g_{25} = 0 & n_{yy} = 0 \\
f_{25} = \frac{1}{2}D & g_{36} = D & \\
f_{26} = \frac{1}{2}D & &
\end{array}
\tag{1.3}
$$

$$
\frac{\begin{array}{l}\frac{1}{2}D(w_{23}-2w_{24}+w_{25})-D(w_{24}-2w_{25}+w_{26})+\frac{1}{2}D(w_{25}-2w_{26}+w_{27})+D(w_{25}-2w_{36}+w_{47})+\\ 2D(w_{25}-w_{24}+w_{35}-w_{36})+2D(w_{25}-w_{36}+w_{37}-w_{26})\end{array}}{\Delta^4}=
$$

$$
h\left[\begin{array}{l}\frac{p}{h}+\left(\frac{1}{\Delta^2}\left(F_{14}-2F_{25}+F_{36}\right)\frac{1}{\Delta^2}\left(w_{24}-2w_{25}+w_{26}\right)\right)+\\ -2\left(\frac{1}{2\Delta^2}\left(F_{24}-F_{26}-F_{35}+F_{37}\right)\right)\left(\frac{1}{2\Delta^2}\left(w_{24}-w_{26}-w_{35}+w_{37}\right)\right)\end{array}\right]
\tag{1.4}
$$

$$\Leftrightarrow$$

$$
p=\frac{1}{2}\frac{1}{\Delta^4}\left(\begin{array}{l}D(w_{23}-8w_{24}+16w_{25}-8w_{26}+w_{27}+4w_{35}-12w_{36}+4w_{37}+2w_{47})+\\ h\left(\begin{array}{l}-2F_{36}w_{24}-F_{35}w_{24}-F_{37}w_{35}+F_{37}w_{35}-2F_{36}w_{26}-F_{35}w_{37}+F_{35}w_{35}+4F_{36}w_{25}\\ +F_{35}w_{26}+F_{37}w_{24}-F_{37}w_{26}\end{array}\right)\end{array}\right)
\tag{1.5}
$$

Cell's C

$$
\begin{array}{ll}
D_5 = D_6 = 0 & \\
f_{24} = D & g_{14} = 0 \\
f_{25} = D & g_{25} = 0 \\
f_{26} = D & g_{36} = D
\end{array}
\tag{1.6}
$$

$$
\frac{\begin{array}{l}D(w_{23}-2w_{24}+w_{25})-2D(w_{24}-2w_{25}+w_{26})-\\ 2D(w_{14}-2w_{25}+w_{36})+D(w_{25}-2w_{26}+w_{27})+D(w_{25}-2w_{36}+w_{47})+\\ 2D(w_{25}-w_{26}+w_{15}-w_{14})+2D(w_{25}-w_{14}+w_{13}-w_{24})+\\ 2D(w_{25}-w_{24}+w_{35}-w_{36})+2D(w_{25}-w_{36}+w_{37}-w_{26})\end{array}}{\Delta^4}=
$$

$$
h\left[\begin{array}{l}\frac{p}{h}+\left(\frac{1}{\Delta^2}\left(F_{14}-2F_{25}+F_{36}\right)\frac{1}{\Delta^2}\left(w_{24}-2w_{25}+w_{26}\right)\right)+\left(\frac{1}{\Delta^2}\left(F_{24}-2F_{25}+F_{26}\right)\frac{1}{\Delta^2}\left(w_{14}-2w_{25}+w_{36}\right)\right)\\ -2\left(\frac{1}{4\Delta^2}\left(F_{13}-F_{15}-F_{35}+F_{37}\right)\right)\left(\frac{1}{4\Delta^2}\left(w_{13}-w_{15}-w_{35}+w_{37}\right)\right)\end{array}\right]
\tag{1.7}
$$

$$\Leftrightarrow$$

$$
p=\frac{1}{8}\frac{1}{\Delta^4}\left(\begin{array}{l}D\left(16w_{13}-48w_{14}+16w_{15}+8w_{23}-64w_{24}+152w_{25}-64w_{26}+8w_{27}+16w_{35}-64w_{36}+16w_{37}+8w_{47}\right)\\ +h\left(\begin{array}{l}-8F_{24}w_{36}+16F_{26}w_{25}+16F_{25}w_{24}-8F_{24}w_{14}+16F_{25}w_{25}-64F_{25}w_{25}+16F_{36}w_{25}-8F_{36}w_{24}\\ +16F_{25}w_{14}-8F_{36}w_{26}+F_{35}w_{35}+F_{35}w_{15}+F_{37}w_{13}-F_{35}w_{37}-F_{37}w_{15}-F_{37}w_{35}+F_{37}w_{37}\\ -8F_{26}w_{36}-F_{35}w_{13}-8F_{26}w_{14}+16F_{25}w_{36}+16F_{24}w_{25}\end{array}\right)\end{array}\right)
\tag{1.8}
$$

Cell's D

$$D_1 = D_2 = D_3 = D_5 = D_6 = D_7 = D_8 = D_9 = D_{12} = 0$$
$$n_{xx} = 0 \quad (1.9)$$
$$n_{yy} = 0$$
$$g_{14} = 0 \qquad f_{24} = 0$$
$$g_{25} = 0 \qquad f_{25} = 0$$
$$g_{36} = \frac{1}{2}D \quad f_{26} = \frac{1}{2}D$$

$$\frac{\frac{1}{2}D(w_{25} - 2w_{26} + w_{27}) + \frac{1}{2}D(w_{25} - 2w_{36} + w_{47}) + 2D(w_{25} - w_{36} + w_{37} - w_{26})}{\Delta^4} =$$
$$h\left[\frac{p}{h} - 2\left(\frac{1}{\Delta^2}(F_{37} - F_{26} - F_{36} + F_{25})\right)\left(\frac{1}{\Delta^2}(w_{37} - w_{26} - w_{36} + w_{25})\right)\right] \quad (1.10)$$
$$\Leftrightarrow$$
$$p = \frac{1}{2}\frac{1}{\Delta^4}\begin{pmatrix} D\left(6w_{25} - 6w_{26} + w_{27} - 6w_{36} + 4w_{37} + w_{47}\right) + \\ h\left(4F_{37}w_{25} - 4F_{37}w_{36} + 4F_{37}w_{37} - 4F_{37}w_{26}\right) \end{pmatrix} \quad (1.11)$$

Cell's E

$$D_1 = D_2 = D_5 = D_6 = D_7 = D_8 = D_{12} = 0$$
$$n_{yy} = 0 \quad (1.12)$$
$$f_{24} = 0 \qquad g_{14} = 0$$
$$f_{25} = \frac{1}{2}D \qquad g_{25} = 0$$
$$g_{36} = D$$
$$f_{26} = \frac{1}{2}D$$

$$\frac{\begin{array}{l} -2\frac{1}{2}D(w_{24} - 2w_{25} + w_{26}) + \frac{1}{2}D(w_{25} - 2w_{26} + w_{27}) + D(w_{25} - 2w_{36} + w_{47}) + \\ 2D(w_{25} - w_{24} + w_{35} - w_{36}) + 2D(w_{25} - w_{36} + w_{37} - w_{26}) \end{array}}{\Delta^4} =$$
$$h\begin{bmatrix} \frac{p}{h} + \left(\frac{1}{\Delta^2}(F_{14} - 2F_{25} + F_{36})\frac{1}{\Delta^2}(w_{24} - 2w_{25} + w_{26})\right) + \\ -2\left(\frac{1}{2\Delta^2}(F_{24} - F_{26} - F_{35} + F_{37})\right)\left(\frac{1}{2\Delta^2}(w_{24} - w_{26} - w_{35} + w_{37})\right) \end{bmatrix} \quad (1.13)$$
$$\Leftrightarrow$$
$$p = \frac{1}{2}\frac{1}{\Delta^4}\begin{pmatrix} D(6w_{24} - 15w_{25} + 8w_{26} - w_{27} - 4w_{35} + 12w_{26} - 4w_{37} - 2w_{47}) + \\ h\left(2F_{36}w_{24} - 4F_{36}w_{25} + 2F_{36}w_{26} - F_{37}w_{37} + F_{37}w_{26} + F_{37}w_{35} - F_{37}w_{24}\right) \end{pmatrix} \quad (1.14)$$

Cell's F

$$D_5 = D_6 = D_7 = D_8 = 0 \tag{1.15}$$

$$\begin{aligned} &f_{24} = 0 \quad g_{14} = 0 \\ &f_{25} = D \quad g_{25} = D \\ &f_{26} = D \quad g_{36} = D \end{aligned}$$

$$\frac{\begin{aligned} &-2D(w_{24} - 2w_{25} + w_{26}) - \\ &2D(w_{14} - 2w_{25} + w_{36}) + D(w_{25} - 2w_{26} + w_{27}) + D(w_{25} - 2w_{36} + w_{47}) + \\ &2D(w_{25} - w_{26} + w_{15} - w_{14}) + 2D(w_{25} - w_{14} + w_{13} - w_{24}) + \\ &2D(w_{25} - w_{24} + w_{35} - w_{36}) + 2D(w_{25} - w_{36} + w_{37} - w_{26}) \end{aligned}}{\Delta^4} = \tag{1.16}$$

$$h\left[\begin{aligned} &\frac{p}{h} + \left(\frac{1}{\Delta^2}\left(F_{14} - 2F_{25} + F_{36}\right)\frac{1}{\Delta^2}\left(w_{24} - 2w_{25} + w_{26}\right)\right) + \left(\frac{1}{\Delta^2}\left(F_{24} - 2F_{25} + F_{26}\right)\frac{1}{\Delta^2}\left(w_{14} - 2w_{25} + w_{36}\right)\right) \\ &-2\left(\frac{1}{4\Delta^2}\left(F_{13} - F_{15} - F_{35} + F_{37}\right)\right)\left(\frac{1}{4\Delta^2}\left(w_{13} - w_{15} - w_{35} + w_{37}\right)\right) \end{aligned}\right]$$

$$p = \frac{1}{8}\frac{1}{\Delta^2}\left(\begin{aligned} &D\left(16w_{13} - 48w_{14} + 16w_{15} - 48w_{24} + 144w_{25} - 64w_{26} + 8w_{27} + 16w_{35} - 64w_{36} + 16w_{37} + 8w_{47}\right) \\ &h\left(\begin{aligned} &-8F_{36}w_{24} + 16F_{36}w_{25} - 8F_{36}w_{26} + 16F_{25}w_{14} + 16F_{25}w_{36} - 8F_{26}w_{14} + 16F_{26}w_{25} - 8F_{26}w_{36} \\ &-F_{37}w_{15} - F_{37}w_{35} + F_{37}w_{37} + 16F_{25}w_{26} + 16F_{25}w_{24} - 64F_{25}w_{25} \end{aligned}\right) \end{aligned}\right) \tag{1.17}$$

Appendix H. Plate Model

Parameters

delta	200	mm
h	4	mm
D	61943	N/mm
E	11000	N/mm2
poisson	0,23	
Beton belasting		
q beton	0,000024	N/mm
D beton	50	mm
F beton	48	N

Deflection w (mm) Given from the geometry

61	112	156	186	204	208	200	179	145	100	51
63	116	161	192	210	216	209	189	155	111	61
61	117	163	195	214	220	214	194	162	118	66
56	114	161	194	214	221	215	196	165	121	68
48	107	155	188	209	217	212	194	164	121	67
35	97	145	179	201	209	206	188	159	117	61

F (N)

0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-251820	-517587	-689467	-775081	-817328	-796676	-697881	-494764	-236567	0	0
0	0	-466415	-948260	-1268940	-1431762	-1474401	-1415257	-1217751	-885047	-426932	0	0
0	0	-479934	-969851	-1297148	-1463699	-1502313	-1421877	-1225222	-898452	-435506	0	0
0	0	-274315	-553647	-722037	-811510	-837160	-791446	-692364	-518455	-259940	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

Nxx(N/mm)

	0,0	-25,2	-51,8	-68,9	-77,5	-81,7	-79,7	-69,8	-49,5	-23,7	0,0
	0,0	3,7	8,7	11,0	11,8	16,0	17,8	17,8	10,4	4,6	0,0
	0,0	20,1	40,9	55,1	62,5	62,9	61,2	51,2	37,7	18,2	0,0
	0,0	21,9	43,8	60,3	68,4	69,3	63,7	54,0	39,3	18,4	0,0
	0,0	6,9	13,7	14,7	15,9	17,2	16,1	16,0	13,8	8,4	0,0
	0,0	-27,4	-55,4	-72,2	-81,2	-83,7	-79,1	-69,2	-51,8	-26,0	0,0
∑↓	0	0	0	0	0	0	0	0	0	0	0

Nyy(N/mm)

											∑→
0	0	0	0	0	0	0	0	0	0	0	0
-25	-1	9	9	4	6	8	10	6	-2	-24	0
-47	-2	16	16	12	10	14	14	13	-3	-43	0
-48	-1	16	16	13	12	12	13	14	-3	-44	0
-27	-1	11	8	6	7	5	7	8	0	-26	0
0	0	0	0	0	0	0	0	0	0	0	0

Nxy

												∑→
	-6	-13	-11	-6	-3	-1	3	8	12	12	6	0
	-12	-24	-20	-12	-5	0	6	13	20	22	11	0
	-6	-11	-9	-6	-2	2	4	6	8	10	5	0
	5	10	9	6	2	0	-3	-6	-9	-9	-4	0
	12	24	20	12	5	-1	-7	-13	-20	-22	-11	0
	7	14	11	6	3	-1	-4	-7	-11	-13	-6	0
∑↓	0	0	0	0	0	0	0	0	0	0	0	

p (N/mm2)

-0,003	-0,007	-0,019	-0,021	-0,027	-0,025	-0,026	-0,023	-0,013	-0,003	-0,001
-0,006	-0,001	0,003	0,004	0,003	0,006	0,007	0,009	0,004	0,000	-0,005
-0,005	0,004	0,016	0,019	0,022	0,019	0,023	0,016	0,013	0,003	-0,004
-0,003	0,007	0,018	0,022	0,024	0,024	0,022	0,017	0,014	0,004	-0,003
-0,001	0,003	0,007	0,005	0,006	0,007	0,005	0,005	0,005	-0,021	-0,002
0,004	-0,007	-0,019	-0,021	-0,028	-0,024	-0,030	-0,020	-0,018	-0,008	0,002

P (N)

-26	-131	-383	-426	-546	-498	-515	-451	-259	-55	0
-111	-54	124	157	131	250	274	349	144	6	-92
-108	169	631	766	864	772	928	653	526	129	-87
-68	279	702	879	962	968	870	686	563	164	-65
-23	139	294	211	234	261	210	208	185	-847	-37
35	-143	-380	-424	-564	-470	-597	-395	-361	-162	0

Negative is tension force at the piston

F at piston (plate and concrete) (N)

22	-83	-335	-378	-498	-450	-467	-403	-211	-7	48
-63	-6	172	205	179	298	322	397	192	54	-44
-60	217	679	814	912	820	976	701	574	177	-39
-20	327	750	927	1010	1016	918	734	611	212	-17
25	187	342	259	282	309	258	256	233	-799	11
83	-95	-332	-376	-516	-422	-549	-347	-313	-114	48

Negative is tension force at the piston

Appendix I. Case study, Free form building

Standard parameters

h	5	mm
E	11000	N/mm2
Delta	200	mm

Displacement is positief downwoard

Displacements (mm)

Reaction force at the piston (N)

Element A

Rmin 1600 mm

Displacements (mm):

201	185	170	161	158	160	166	177	193	214	236
257	236	219	207	200	199	202	211	226	245	269
288	264	243	228	220	216	218	226	239	258	283
294	266	244	227	217	213	214	221	234	253	280
274	244	221	204	194	189	191	198	211	232	260
230	199	176	158	148	144	147	156	171	195	224

Reaction force at the piston (N):

-199	**-337**	**-718**	**-827**	**-649**	**-464**	**-520**	**-429**	**-273**	**-4**	48
907	**-463**	**-906**	**-627**	**-104**	137	104	49	12	36	713
1932	**-22**	**-769**	**-188**	83	672	548	176	48	217	1195
2333	272	**-540**	**-49**	513	506	594	349	72	405	1333
1410	156	**-237**	**-277**	59	166	188	128	174	3878	824
94	**-353**	**-409**	**-938**	**-766**	**-881**	**-675**	**-535**	**-559**	**-22**	48

Displacements (mm):

99	115	130	139	142	140	134	123	107	86	64
43	64	81	93	100	101	98	89	74	55	31
12	36	57	72	80	84	82	74	61	42	17
6	34	56	73	83	87	86	79	66	47	20
26	56	79	96	106	111	109	102	89	68	40
70	101	124	142	152	156	153	144	129	105	76

Reaction force at the piston (N):

295	433	814	923	745	560	616	525	369	100	48
-811	559	1002	723	200	**-41**	**-8**	47	84	60	**-617**
-1836	118	865	284	13	**-576**	**-452**	**-80**	48	**-121**	**-1099**
-2237	**-176**	636	145	**-417**	**-410**	**-498**	**-253**	24	**-309**	**-1237**
-1314	**-60**	333	373	37	**-70**	**-92**	**-32**	**-78**	1615	**-728**
2	449	505	1034	862	977	771	631	655	118	48

Element B

Rmin 3600 mm

Displacements (mm):

217	209	202	198	195	195	197	201	207	215	226
240	229	222	218	216	215	217	222	228	236	245
250	238	232	228	226	225	227	232	238	246	255
248	237	231	226	224	224	226	230	236	244	254
234	224	218	213	211	211	214	218	224	233	242
208	200	194	190	188	188	190	195	201	209	220

Reaction force at the piston (N):

3	0	**-28**	19	**-49**	**-14**	**-15**	**-15**	4	16	48
185	**-35**	27	48	105	42	34	79	37	120	122
307	**-51**	**-35**	25	73	59	54	52	8	100	178
288	**-73**	8	31	70	55	42	32	17	47	192
173	**-52**	57	25	76	36	102	58	17	1260	118
18	**-5**	**-14**	**-14**	**-17**	**-14**	**-47**	21	**-2**	4	48

Reaction force at the piston (N):

93	96	124	77	145	110	111	111	92	80	48
-89	131	69	48	**-9**	54	62	17	59	**-24**	**-26**
-211	147	131	71	23	37	42	44	88	**-4**	**-82**
-192	169	88	65	26	41	54	64	79	49	**-96**
-77	148	39	71	20	60	**-6**	38	79	381	**-22**
78	101	110	110	113	110	143	75	98	92	48

Displacements (mm):

83	91	98	102	105	105	103	99	93	85	74
60	71	78	82	84	85	83	78	72	64	55
50	62	68	72	74	75	73	68	62	54	45
52	63	69	74	76	76	74	70	64	56	46
66	76	82	87	89	89	86	82	76	67	58
92	100	106	110	112	112	110	105	99	91	80

h 3,8

Reaction force at the piston (N):

81	84	103	69	118	94	95	93	79	66	48
-55	119	71	53	17	57	63	36	64	3	**-11**
-155	111	109	70	34	35	41	52	81	11	**-56**
-142	139	91	57	29	49	52	57	72	36	**-64**
-47	129	56	69	36	59	24	49	77	300	**-9**
70	87	92	94	95	95	117	66	84	74	48

Element C

Rmin 5700 mm

288	267	249	234	221	211	205	201	200	203	207
260	243	230	219	209	203	200	200	203	208	216
237	224	216	207	202	199	199	203	209	218	227
219	214	205	200	198	199	203	209	219	232	240
208	204	199	197	198	203	210	220	233	248	255
201	198	197	198	203	211	221	235	251	265	271

55	57	55	50	58	51	62	56	42	62	48
46	15	27	89	**-5**	42	39	33	73	**-20**	58
77	**-157**	212	**-13**	94	49	**-5**	86	40	49	53
31	197	1	47	46	45	83	**-6**	22	103	38
46	48	27	37	**-7**	53	31	30	**-16**	**-393**	34
49	45	64	53	59	55	49	55	64	65	48

41	39	41	46	38	45	34	40	54	34	48
50	81	69	7	101	54	57	63	23	116	38
19	253	**-116**	109	2	47	101	10	56	47	43
65	**-101**	95	49	50	51	13	102	74	**-7**	58
50	48	69	59	103	43	65	66	112	**-90**	62
47	51	32	43	37	41	47	41	32	31	48

12	33	51	66	79	89	95	99	100	97	93
40	57	70	81	91	97	100	100	97	92	84
63	76	84	93	98	101	101	97	91	82	73
81	86	95	100	102	101	97	91	81	68	60
92	96	101	103	102	97	90	80	67	52	45
99	102	103	102	97	89	79	65	49	35	29

h 3,8

46	44	44	47	44	46	42	44	51	43	48
50	65	53	30	70	51	52	52	36	77	44
34	151	**-17**	74	26	47	74	32	48	46	47
51	**-11**	75	52	49	47	34	75	58	26	53
48	45	56	55	74	46	55	57	75	**-26**	54
48	49	40	44	43	46	47	45	42	40	48

Element D

Rmin 3100 mm

219	168	124	94	76	72	80	101	135	180	229
217	164	119	88	70	64	71	91	125	169	219
219	163	117	85	66	60	66	86	118	162	214
224	166	119	86	66	59	65	84	115	159	212
232	173	125	92	71	63	68	86	116	159	213
245	183	135	101	79	71	74	92	121	163	219

81	214	526	584	728	674	693	615	371	122	48
189	114	**-116**	**-153**	**-107**	**-275**	**-296**	**-408**	**-128**	34	163
190	**-173**	**-749**	**-909**	**-1036**	**-906**	**-1121**	**-758**	**-617**	**-120**	163
141	**-312**	**-837**	**-1056**	**-1158**	**-1168**	**-1040**	**-808**	**-664**	**-158**	134
79	**-129**	**-336**	**-210**	**-244**	**-286**	**-213**	**-216**	**-191**	**-1809**	97
8	221	526	581	752	642	791	546	502	245	48

31	82	126	156	174	178	170	149	115	70	21
33	86	131	162	180	186	179	159	125	81	31
31	87	133	165	184	190	184	164	132	88	36
26	84	131	164	184	191	185	166	135	91	38
18	77	125	158	179	187	182	164	134	91	37
5	67	115	149	171	179	176	158	129	87	31

15	**-118**	**-430**	**-488**	**-632**	**-578**	**-597**	**-519**	**-275**	**-26**	48
-93	**-18**	212	249	203	371	392	504	224	62	-67
-94	269	845	1005	1132	1002	1217	854	713	216	-67
-45	408	933	1152	1254	1264	1136	904	760	254	-38
17	225	432	306	340	382	309	312	287	**-687**	**-1**
88	**-125**	**-430**	**-485**	**-656**	**-546**	**-695**	**-450**	**-406**	**-149**	48

Element E

Rmin 3000 mm

162	142	129	120	117	118	123	132	145	162	181
134	117	106	99	98	101	109	119	133	151	171
119	104	94	90	90	94	102	114	129	147	168
117	104	95	91	92	96	104	116	132	150	171
128	115	106	102	102	107	115	126	141	158	178
151	138	128	123	123	126	133	143	157	172	192

2	106	161	298	240	244	207	178	149	94	48
216	**-316**	**-267**	**-366**	**-134**	**-141**	**80**	**-31**	**-38**	64	119
454	**-294**	**-684**	**-393**	**-342**	**-261**	**-232**	**-71**	**-52**	13	146
464	**-112**	**-441**	**-494**	**-183**	**-271**	**-230**	**-180**	12	37	119
259	**-124**	**-273**	**-159**	**-220**	**-13**	7	**-18**	51	**-790**	104
54	105	205	252	197	216	167	171	109	100	48

94	**-10**	**-65**	**-202**	**-144**	**-148**	**-111**	**-82**	**-53**	2	48
-120	412	363	462	230	237	16	127	134	32	**-23**
-358	390	780	489	438	357	328	167	148	83	**-50**
-368	208	537	590	279	367	326	276	84	59	**-23**
-163	220	369	255	316	109	89	114	45	**-100**	**-8**
42	**-9**	**-109**	**-156**	**-101**	**-120**	**-71**	**-75**	**-13**	**-4**	48

38	58	71	80	83	82	77	68	55	38	19
66	83	94	101	102	99	91	81	67	49	29
81	96	106	110	110	106	98	86	71	53	32
83	96	105	109	108	104	96	84	68	50	29
72	85	94	98	98	93	85	74	59	42	22
49	62	72	77	77	74	67	57	43	28	8

h 3,8

83	10	**-24**	**-121**	**-79**	**-82**	**-60**	**-38**	**-18**	20	48
-69	283	265	315	166	162	33	91	96	33	**-2**
-230	278	555	362	315	264	236	136	117	66	**-18**
-240	160	392	417	220	265	237	201	79	55	2
-98	151	258	188	211	89	75	85	40	**-60**	7
46	14	**-56**	**-90**	**-48**	**-66**	**-30**	**-35**	11	11	48

Delta 333
h 5

162	132	119	118	129	151	181
123	100	91	96	110	135	168
119	100	93	98	115	140	172
151	131	123	127	140	162	192

-9	339	497	535	448	307	209
778	**-1452**	**-1587**	**-557**	**-591**	**-131**	571
916	**-1055**	**-1239**	**-1003**	**-291**	**-3181**	529
105	604	888	752	674	434	232

38	68	81	82	71	49	19
77	100	109	104	90	65	32
81	100	107	102	85	60	28
49	69	77	73	60	38	8

276	**-72**	**-230**	**-268**	**-181**	**-41**	58
-511	1719	1854	823	857	398	**-304**
-649	1322	1505	1270	558	**-461**	**-262**
161	**-338**	**-621**	**-486**	**-407**	**-167**	35

Element F

Rmin 5200 mm

202	199	200	203	208	217	227	241	257	276	295
208	204	200	200	202	207	213	223	236	253	269
220	213	206	202	200	201	205	211	220	231	247
237	228	217	209	204	201	201	204	209	216	229
261	246	233	222	213	207	202	201	203	207	215
290	272	255	239	226	217	209	205	202	203	206

49	35	65	56	49	63	61	63	57	46	48
31	83	**-12**	49	48	76	**-11**	22	**-3**	110	35
44	52	16	69	29	21	75	35	79	**-77**	70
24	131	**-18**	21	64	26	43	72	54	**-46**	77
61	**-10**	26	59	31	68	**-5**	**-8**	51	550	68
56	62	60	47	45	54	53	68	51	67	48

98	101	100	97	92	83	73	59	43	24	5
92	96	100	100	98	93	87	77	64	47	31
80	87	94	98	100	99	95	89	80	69	53
63	72	83	91	96	99	99	96	91	84	71
39	54	67	78	87	93	98	99	97	93	85
10	28	45	61	74	83	91	95	98	97	94

47	61	31	40	47	33	35	33	39	50	48
65	13	108	47	48	20	107	74	99	**-14**	61
52	44	80	27	67	75	21	61	17	173	26
72	**-35**	114	75	32	70	53	24	42	142	19
35	106	70	37	65	28	101	104	45	335	28
40	34	36	49	51	42	43	28	45	29	48

Appendix J.Maple sheet, 6 supports

Support reaction and local displacement

N en mm.

Algemene belastingformules

restart;

Verdeelde belasting

$fAC := -\frac{1}{48 \cdot EI} \cdot \left(8 \cdot Ra \cdot (x^3 - L^2 x) + Q \cdot x \cdot \left(\frac{8 \cdot d^3}{L} - \frac{2 \cdot b \cdot c^2}{L} + \frac{c^3}{L} + 2 \cdot c^2\right)\right);$

$fCD := -\frac{1}{48 \cdot EI} \cdot \left(8 \cdot Ra \cdot (x^3 - L^2 x) + Q \cdot x \cdot \left(\frac{8 \cdot d^3}{L} - \frac{2 \cdot b \cdot c^2}{L} + \frac{c^3}{L} + 2 \cdot c^2\right) - 2 \cdot Q \cdot \frac{(x-a)^4}{c}\right);$

$fDB := -\frac{1}{48 \cdot EI} \cdot \left(8 \cdot Ra \cdot (x^3 - L^2 x) + Q \cdot x \cdot \left(\frac{8 \cdot d^3}{L} - \frac{2 \cdot b \cdot c^2}{L} + \frac{c^3}{L}\right) - 8 \cdot Q \cdot \left(x - \frac{a}{2} - \frac{b}{2}\right)^3 + Q \cdot (2 \cdot b \cdot c^2 - c^3)\right);$

$d := L - \frac{b}{2} - \frac{a}{2};$

$Ra := \frac{Q \cdot d}{L}:$

$Q := q \cdot c:$

$$-\frac{1}{48} \frac{8\, Ra\, (x^3 - L^2 x) + Q\, x \left(\frac{8\, d^3}{L} - \frac{2\, b\, c^2}{L} + \frac{c^3}{L} + 2\, c^2\right)}{EI}$$

$$-\frac{1}{48} \frac{8\, Ra\, (x^3 - L^2 x) + Q\, x \left(\frac{8\, d^3}{L} - \frac{2\, b\, c^2}{L} + \frac{c^3}{L} + 2\, c^2\right) - \frac{2\, Q\, (x-a)^4}{c}}{EI}$$

$$-\frac{1}{48} \frac{8\, Ra\, (x^3 - L^2 x) + Q\, x \left(\frac{8\, d^3}{L} - \frac{2\, b\, c^2}{L} + \frac{c^3}{L}\right) - 8\, Q \left(x - \frac{1}{2}\, a - \frac{1}{2}\, b\right)^3 + Q\, (2\, b\, c^2 - c^3)}{EI}$$

$$L - \frac{1}{2}\, b - \frac{1}{2}\, a \qquad (1.1)$$

Puntlast

Als x, de plaats van de steunpunt links van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfL := \frac{F \cdot b \cdot x}{6 \cdot EI \cdot L} \cdot (2 \cdot L \cdot (L - x) - (b)^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F\, b\, x\, (2\, L\, (L - x) - b^2 - (L - x)^2)}{EI\, L} \qquad (1.2)$$

Als x, de plaats van de steunpunt rechts van de onbekende puntlast (steunpuntsreactiekracht) zit

$wfR := \frac{F \cdot a \cdot (L - x)}{6 \cdot EI \cdot L} \cdot ((2 \cdot L \cdot b) - b^2 - (L - x)^2);$

$$\frac{1}{6} \frac{F\, a\, (L - x)\, (2\, L\, b - b^2 - (L - x)^2)}{EI\, L} \qquad (1.3)$$

Verplaatsingen in de steunpunten

Vergelijking in B

$x := 1 : L := 5 \cdot 1 :$

$F := Bv : b := 4 \cdot 1 : wBb := wfL :$

$F := Cv : b := 3 \cdot l : wBc := wfL :$

$F := Dv : b := 2 \cdot l : wBd := wfL :$

$F := Ev : b := 1 \cdot l : wBe := wfL :$

$a := 0 : b := \frac{1}{2} \cdot l : c := \frac{1}{2} \cdot l : q := q1 : wq1 := fDB :$

$a := \frac{1}{2} \cdot l : b := \frac{3}{2} \cdot l : c := l : q := q2 : wq2 := fCD :$

$a := \frac{3}{2} \cdot l : b := \frac{5}{2} \cdot l : c := l : q := q3 : wq3 := fAC :$

$a := \frac{5}{2} \cdot l : b := \frac{7}{2} \cdot l : c := l : q := q4 : wq4 := fAC :$

$a := \frac{7}{2} \cdot l : b := \frac{9}{2} \cdot l : c := l : q := q5 : wq5 := fAC :$

$a := \frac{9}{2} \cdot l : b := 5 \cdot l : c := \frac{1}{2} \cdot l : q := q6 : wq6 := fAC :$

$wBq := wq1 + wq2 + wq3 + wq4 + wq5 + wq6 :$

$eq1 := wB = wBq - wBb - wBc - wBd - wBe;$

$$wB = \frac{71}{480} \frac{q1\, l^4}{EI} + \frac{663}{640} \frac{q2\, l^4}{EI} + \frac{59}{40} \frac{q3\, l^4}{EI} + \frac{79}{60} \frac{q4\, l^4}{EI} + \frac{91}{120} \frac{q5\, l^4}{EI} + \frac{191}{1920} \frac{q6\, l^4}{EI} - \frac{16}{15} \frac{Bv\, l^3}{EI} - \frac{3}{2} \frac{Cv\, l^3}{EI} - \frac{4}{3} \frac{Dv\, l^3}{EI} - \frac{23}{30} \frac{Ev\, l^3}{EI} \qquad (2.1)$$

Vergelijking in C

$x := 2 \cdot 1 : L := 5 \cdot 1 :$

$F := Bv : a := l : b := 4 \cdot l : wCb := wfR :$

$F := Cv : a := 2\cdot l : b := 3\cdot l : wCc := wfR :$
$F := Dv : b := 2\cdot l : wCd := wfL :$
$F := Ev : b := 1\cdot l : wCe := wfL :$

$a := 0 : b := \frac{1}{2}\cdot l : c := \frac{1}{2}\cdot l : q := q1 : wq1 := fDB :$

$a := \frac{1}{2}\cdot l : b := \frac{3}{2}\cdot l : c := l : q := q2 : wq2 := fDB :$

$a := \frac{3}{2}\cdot l : b := \frac{5}{2}\cdot l : c := l : q := q3 : wq3 := fCD :$

$a := \frac{5}{2}\cdot l : b := \frac{7}{2}\cdot l : c := l : q := q4 : wq4 := fAC :$

$a := \frac{7}{2}\cdot l : b := \frac{9}{2}\cdot l : c := l : q := q5 : wq5 := fAC :$

$a := \frac{9}{2}\cdot l : b := 5\cdot l : c := \frac{1}{2}\cdot l : q := q6 : wq6 := fAC :$

$wCq := wq1 + wq2 + wq3 + wq4 + wq5 + wq6 :$
$eq2 := wC = wCq - wCb - wCc - wCd - wCe;$

$$wC = \frac{127}{640}\frac{q1\,l^4}{EI} + \frac{59}{40}\frac{q2\,l^4}{EI} + \frac{4517}{1920}\frac{q3\,l^4}{EI} + \frac{67}{30}\frac{q4\,l^4}{EI} + \frac{79}{60}\frac{q5\,l^4}{EI} + \frac{167}{960}\frac{q6\,l^4}{EI} - \frac{3}{2}\frac{Bv\,l^3}{EI} - \frac{12}{5}\frac{Cv\,l^3}{EI} - \frac{34}{15}\frac{Dv\,l^3}{EI} - \frac{4}{3}\frac{Ev\,l^3}{EI} \quad (2.2)$$

Vergelijking in D

$x := 3\cdot 1 : L := 5\cdot 1 :$
$F := Bv : a := 1\cdot l : b := 4\cdot l : wDb := wfR :$
$F := Cv : a := 2\cdot l : b := 3\cdot l : wDc := wfR :$
$F := Dv : a := 3\cdot l : b := 2\cdot l : wDd := wfR :$
$F := Ev : a := 4\cdot l : b := 1\cdot l : wDe := wfL :$

$a := 0 : b := \frac{1}{2}\cdot l : c := \frac{1}{2}\cdot l : q := q1 : wq1 := fDB :$

$a := \frac{1}{2}\cdot l : b := \frac{3}{2}\cdot l : c := l : q := q2 : wq2 := fDB :$

$a := \frac{3}{2}\cdot l : b := \frac{5}{2}\cdot l : c := l : q := q3 : wq3 := fDB :$

$a := \frac{5}{2}\cdot l : b := \frac{7}{2}\cdot l : c := l : q := q4 : wq4 := fCD :$

$a := \frac{7}{2}\cdot l : b := \frac{9}{2}\cdot l : c := l : q := q5 : wq5 := fAC :$

$a := \frac{9}{2}\cdot l : b := 5\cdot l : c := \frac{1}{2}\cdot l : q := q6 : wq6 := fAC :$

$wDq := wq1 + wq2 + wq3 + wq4 + wq5 + wq6 :$
$eq3 := wD = wDq - wDb - wDc - wDd - wDe;$

$$wD = \frac{167}{960}\frac{q1\,l^4}{EI} + \frac{79}{60}\frac{q2\,l^4}{EI} + \frac{67}{30}\frac{q3\,l^4}{EI} + \frac{4517}{1920}\frac{q4\,l^4}{EI} + \frac{59}{40}\frac{q5\,l^4}{EI} + \frac{127}{640}\frac{q6\,l^4}{EI} - \frac{4}{3}\frac{Bv\,l^3}{EI} - \frac{34}{15}\frac{Cv\,l^3}{EI} - \frac{12}{5}\frac{Dv\,l^3}{EI} - \frac{3}{2}\frac{Ev\,l^3}{EI} \quad (2.3)$$

(2.4)

Vergelijking in E

$x := 4\cdot 1 : L := 5\cdot 1 :$
$F := Bv : a := 1\cdot l : b := 4\cdot l : wEb := wfR :$
$F := Cv : a := 2\cdot l : b := 3\cdot l : wEc := wfR :$
$F := Dv : a := 3\cdot l : b := 2\cdot l : wEd := wfR :$
$F := Ev : a := 4\cdot l : b := 1\cdot l : wEe := wfR :$

$a := 0 : b := \frac{1}{2}\cdot l : c := \frac{1}{2}\cdot l : q := q1 : wq1 := fDB :$

$a := \frac{1}{2}\cdot l : b := \frac{3}{2}\cdot l : c := l : q := q2 : wq2 := fDB :$

$a := \frac{3}{2}\cdot l : b := \frac{5}{2}\cdot l : c := l : q := q3 : wq3 := fDB :$

$a := \frac{5}{2}\cdot l : b := \frac{7}{2}\cdot l : c := l : q := q4 : wq4 := fDB :$

$a := \frac{7}{2}\cdot l : b := \frac{9}{2}\cdot l : c := l : q := q5 : wq5 := fCD :$

$a := \frac{9}{2}\cdot l : b := 5\cdot l : c := \frac{1}{2}\cdot l : q := q6 : wq6 := fAC :$

$wEq := wq1 + wq2 + wq3 + wq4 + wq5 + wq6 :$

$eq4 := wE = wEq - wEb - wEc - wEd - wEe;$

$$wE = \frac{191}{1920}\frac{q1\,l^4}{EI} + \frac{91}{120}\frac{q2\,l^4}{EI} + \frac{79}{60}\frac{q3\,l^4}{EI} + \frac{59}{40}\frac{q4\,l^4}{EI} + \frac{663}{640}\frac{q5\,l^4}{EI} + \frac{71}{480}\frac{q6\,l^4}{EI} - \frac{23}{30}\frac{Bv\,l^3}{EI} - \frac{4}{3}\frac{Cv\,l^3}{EI} - \frac{3}{2}\frac{Dv\,l^3}{EI} - \frac{16}{15}\frac{Ev\,l^3}{EI} \quad (2.5)$$

Oplossen Reactiekrachten

sol := *solve*({*eq1*, *eq2*, *eq3*, *eq4*}, {*Bv*, *Cv*, *Dv*, *Ev*}) : *assign*(%) :

Reactie kracht in A en F

$$Av := \frac{\frac{1}{2}\cdot 5\cdot l\cdot l\cdot\left(\frac{1}{2}\cdot(q1+q6)+(q2+q3+q4+q5)\right)-4\cdot l\cdot Bv-3\cdot l\cdot Cv-2\cdot l\cdot Dv-1\cdot l\cdot Ev}{5\cdot l}:$$

$$Fv := \frac{\frac{1}{2}\cdot 5\cdot l\cdot l\cdot\left(\frac{1}{2}\cdot(q1+q6)+(q2+q3+q4+q5)\right)-4\cdot l\cdot Ev-3\cdot l\cdot Dv-2\cdot l\cdot Cv-1\cdot l\cdot Bv}{5\cdot l}:$$

Bv;

$$-\frac{1}{13376}\,\frac{1}{l^3}\,(240\,q4\,l^4-60\,q5\,l^4+42\,q6\,l^4-1109\,q3\,l^4+132096\,wB\,EI-127104\,wC\,EI+55296\,wD\,EI-13824\,wE\,EI-11688\,q2\,l^4-2561\,q1\,l^4) \quad (3.1)$$

Cv;

$$\frac{1}{13376}\,\frac{1}{l^3}\,(1169\,q4\,l^4-240\,q5\,l^4+168\,q6\,l^4+11448\,q3\,l^4+127104\,wB\,EI-187392\,wC\,EI+140928\,wD\,EI-55296\,wE\,EI+1109\,q2\,l^4-630\,q1\,l^4) \quad (3.2)$$

Dv;

$$-\frac{1}{13376}\,\frac{1}{l^3}\,(-127104\,wE\,EI-168\,q1\,l^4+240\,q2\,l^4-1169\,q3\,l^4-11448\,q4\,l^4-1109\,q5\,l^4+630\,q6\,l^4+55296\,wB\,EI+187392\,wD\,EI-140928\,wC\,EI) \quad (3.3)$$

Ev;

Hoogte van de Steunpunten

Z-as positief omlaag gericht

wA := 0 :
wB := 0 :
wC := 0 :
wD := 0 :
wE := 0 :
wF := 0 :

Steunpuntsreacties

Bv;
Cv;
Dv;
Ev;

$$-\frac{1}{13376}\,\frac{240\,q4\,l^4-60\,q5\,l^4+42\,q6\,l^4-1109\,q3\,l^4-11688\,q2\,l^4-2561\,q1\,l^4}{l^3}$$

$$\frac{1}{13376}\,\frac{1169\,q4\,l^4-240\,q5\,l^4+168\,q6\,l^4+11448\,q3\,l^4+1109\,q2\,l^4-630\,q1\,l^4}{l^3}$$

$$-\frac{1}{13376}\,\frac{-168\,q1\,l^4+240\,q2\,l^4-1169\,q3\,l^4-11448\,q4\,l^4-1109\,q5\,l^4+630\,q6\,l^4}{l^3}$$

$$\frac{1}{13376}\,\frac{-42\,q1\,l^4+60\,q2\,l^4-240\,q3\,l^4+1109\,q4\,l^4+11688\,q5\,l^4+2561\,q6\,l^4}{l^3} \quad (5.1)$$

x :='x' :
$F := Bv : a := 1\cdot l : b := 4\cdot l : wBL := wfL : wBR := wfR :$
$F := Cv : a := 2\cdot l : b := 3\cdot l : wCL := wfL : wCR := wfR :$
$F := Dv : a := 3\cdot l : b := 2\cdot l : wDL := wfL : wDR := wfR :$
$F := Ev : a := 4\cdot l : b := 1\cdot l : wEL := wfL : wER := wfR :$

$q := q1 : a := 0 : b := \frac{1}{2}\cdot l : c := \frac{1}{2}\cdot l : wq1L := fAC : wq1M := fCD : wq1R := fDB :$

$q := q2 : a := \frac{1}{2}\cdot l : b := \frac{3}{2}\cdot l : c := l : wq2L := fAC : wq2M := fCD : wq2R := fDB :$

$q := q3 : a := \frac{3}{2}\cdot l : b := \frac{5}{2}\cdot l : c := l : wq3L := fAC : wq3M := fCD : wq3R := fDB :$

$q := q4 : a := \frac{5}{2}\cdot l : b := \frac{7}{2}\cdot l : c := l : wq4L := fAC : wq4M := fCD : wq4R := fDB :$

$q := q5 : a := \frac{7}{2}\cdot l : b := \frac{9}{2}\cdot l : c := l : wq5L := fAC : wq5M := fCD : wq5R := fDB :$

$q := q6 : a := \frac{9}{2}\cdot l : b := 5\cdot l : c := \frac{1}{2}\cdot l : wq6L := fAC : wq6M := fCD : wq6R := fDB :$

w1 := wq1M + wq2L + wq3L + wq4L + wq5L + wq6L − wBL − wCL − wDL − wEL :
w2 := *wq1R* + *wq2M* + *wq3L* + *wq4L* + *wq5L* + *wq6L* − *wBL* − *wCL* − *wDL* − *wEL* :

$w3 := wq1R + wq2M + wq3L + wq4L + wq5L + wq6L - wBR - wCL - wDL - wEL:$
$w4 := wq1R + wq2R + wq3M + wq4L + wq5L + wq6L - wBR - wCL - wDL - wEL:$
$w5 := wq1R + wq2R + wq3M + wq4L + wq5L + wq6L - wBR - wCR - wDL - wEL:$
$w6 := wq1R + wq2R + wq3R + wq4M + wq5L + wq6L - wBR - wCR - wDL - wEL:$
$w7 := wq1R + wq2R + wq3R + wq4M + wq5L + wq6L - wBR - wCR - wDR - wEL:$
$w8 := wq1R + wq2R + wq3R + wq4R + wq5M + wq6L - wBR - wCR - wDR - wEL:$
$w9 := wq1R + wq2R + wq3R + wq4R + wq5M + wq6L - wBR - wCR - wDR - wER:$
$w10 := wq1R + wq2R + wq3R + wq4R + wq5R + wq6M - wBR - wCR - wDR - wER:$

$x := 0.5 \cdot l : w1;$

$$\frac{0.08671875000\, q1\, l^4}{EI} + \frac{0.5666666667\, q2\, l^4}{EI} + \frac{0.7750000000\, q3\, l^4}{EI} + \frac{0.6833333333\, q4\, l^4}{EI} + \frac{0.3916666667\, q5\, l^4}{EI} + \frac{0.05130208333\, q6\, l^4}{EI} + \frac{0.00004361044657\,(240\, q4\, l^4 - 60\, q5\, l^4 + 42\, q6\, l^4 - 1109\, q3\, l^4 - 11688\, q2\, l^4 - 2561\, q1\, l^4)}{EI} - \frac{0.00005887410287\,(1169\, q4\, l^4 - 240\, q5\, l^4 + 168\, q6\, l^4 + 11448\, q3\, l^4 + 1109\, q2\, l^4 - 630\, q1\, l^4)}{EI} + \frac{0.00005170952951\,(-168\, q1\, l^4 + 240\, q2\, l^4 - 1169\, q3\, l^4 - 11448\, q4\, l^4 - 1109\, q5\, l^4 + 630\, q6\, l^4)}{EI} - \frac{0.00002959280303\,(-42\, q1\, l^4 + 60\, q2\, l^4 - 240\, q3\, l^4 + 1109\, q4\, l^4 + 11688\, q5\, l^4 + 2561\, q6\, l^4)}{EI}$$

$x := 1.5 \cdot l : w3;$

$$\frac{0.1841145833\, q1\, l^4}{EI} + \frac{1.341666667\, q2\, l^4}{EI} + \frac{2.025000000\, q3\, l^4}{EI} + \frac{1.850000000\, q4\, l^4}{EI} + \frac{1.075000000\, q5\, l^4}{EI} + \frac{0.1414062500\, q6\, l^4}{EI} + \frac{0.0001024845494\,(240\, q4\, l^4 - 60\, q5\, l^4 + 42\, q6\, l^4 - 1109\, q3\, l^4 - 11688\, q2\, l^4 - 2561\, q1\, l^4)}{EI} - \frac{0.0001541940789\,(1169\, q4\, l^4 - 240\, q5\, l^4 + 168\, q6\, l^4 + 11448\, q3\, l^4 + 1109\, q2\, l^4 - 630\, q1\, l^4)}{EI} + \frac{0.0001401764354\,(-168\, q1\, l^4 + 240\, q2\, l^4 - 1169\, q3\, l^4 - 11448\, q4\, l^4 - 1109\, q5\, l^4 + 630\, q6\, l^4)}{EI} - \frac{0.00008130233253\,(-42\, q1\, l^4 + 60\, q2\, l^4 - 240\, q3\, l^4 + 1109\, q4\, l^4 + 11688\, q5\, l^4 + 2561\, q6\, l^4)}{EI}$$

$x := 2.5 \cdot l : w5;$

$$\frac{0.1940104167\, q1\, l^4}{EI} + \frac{1.458333333\, q2\, l^4}{EI} + \frac{2.416666667\, q3\, l^4}{EI} + \frac{2.416666667\, q4\, l^4}{EI} + \frac{1.458333333\, q5\, l^4}{EI} + \frac{0.1940104167\, q6\, l^4}{EI} + \frac{0.0001105836324\,(240\, q4\, l^4 - 60\, q5\, l^4 + 42\, q6\, l^4 - 1109\, q3\, l^4 - 11688\, q2\, l^4 - 2561\, q1\, l^4)}{EI} - \frac{0.0001837868820\,(1169\, q4\, l^4 - 240\, q5\, l^4 + 168\, q6\, l^4 + 11448\, q3\, l^4 + 1109\, q2\, l^4 - 630\, q1\, l^4)}{EI} + \frac{0.0001837868820\,(-168\, q1\, l^4 + 240\, q2\, l^4 - 1169\, q3\, l^4 - 11448\, q4\, l^4 - 1109\, q5\, l^4 + 630\, q6\, l^4)}{EI} - \frac{0.0001105836324\,(-42\, q1\, l^4 + 60\, q2\, l^4 - 240\, q3\, l^4 + 1109\, q4\, l^4 + 11688\, q5\, l^4 + 2561\, q6\, l^4)}{EI}$$

$x := 3.5 \cdot l : w7;$

$$\frac{0.1414062500\, q1\, l^4}{EI} + \frac{1.075000000\, q2\, l^4}{EI} + \frac{1.850000000\, q3\, l^4}{EI} + \frac{2.025000000\, q4\, l^4}{EI} + \frac{1.341666667\, q5\, l^4}{EI} + \frac{0.1841145833\, q6\, l^4}{EI} + \frac{0.00008130233254\,(240\, q4\, l^4 - 60\, q5\, l^4 + 42\, q6\, l^4 - 1109\, q3\, l^4 - 11688\, q2\, l^4 - 2561\, q1\, l^4)}{EI} - \frac{0.0001401764354\,(1169\, q4\, l^4 - 240\, q5\, l^4 + 168\, q6\, l^4 + 11448\, q3\, l^4 + 1109\, q2\, l^4 - 630\, q1\, l^4)}{EI} + \frac{0.0001541940789\,(-168\, q1\, l^4 + 240\, q2\, l^4 - 1169\, q3\, l^4 - 11448\, q4\, l^4 - 1109\, q5\, l^4 + 630\, q6\, l^4)}{EI} - \frac{0.0001024845494\,(-42\, q1\, l^4 + 60\, q2\, l^4 - 240\, q3\, l^4 + 1109\, q4\, l^4 + 11688\, q5\, l^4 + 2561\, q6\, l^4)}{EI}$$

$x := 4.5 \cdot l : w9;$

$$\frac{0.05130208333\, q1\, l^4}{EI} + \frac{0.3916666667\, q2\, l^4}{EI} + \frac{0.6833333333\, q3\, l^4}{EI} + \frac{0.7750000000\, q4\, l^4}{EI} + \frac{0.5666666667\, q5\, l^4}{EI} + \frac{0.08671875000\, q6\, l^4}{EI} + \frac{0.00002959280303\,(240\, q4\, l^4 - 60\, q5\, l^4 + 42\, q6\, l^4 - 1109\, q3\, l^4 - 11688\, q2\, l^4 - 2561\, q1\, l^4)}{EI} - \frac{0.00005170952951\,(1169\, q4\, l^4 - 240\, q5\, l^4 + 168\, q6\, l^4 + 11448\, q3\, l^4 + 1109\, q2\, l^4 - 630\, q1\, l^4)}{EI} + \frac{0.00005887410287\,(-168\, q1\, l^4 + 240\, q2\, l^4 - 1169\, q3\, l^4 - 11448\, q4\, l^4 - 1109\, q5\, l^4 + 630\, q6\, l^4)}{EI} - \frac{0.00004361044657\,(-42\, q1\, l^4 + 60\, q2\, l^4 - 240\, q3\, l^4 + 1109\, q4\, l^4 + 11688\, q5\, l^4 + 2561\, q6\, l^4)}{EI} \quad (6.5)$$

Appendix K. Excel sheet Strip model

Stippen mal

Parameters

l	200	mm	Distance between the supports

Lower strip

Eo	11000	N/mm2	
ho	3,8	mm	Thickness
bo	50	mm	With
EI_o	2514967	N/mm2	

Upper strip

Eb	11000	N/mm2	
hb	3,8	mm	Thickness
bb	50	mm	With
EI_b	2514967	N/mm2	

Concrete

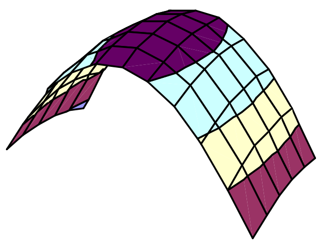
γb	2400	kg/m3	
d	50	mm	Thickness concrete
qb	0,06	N/mm	Concrete weight at top strip
Qt	2400	N	Total force

Geometry (mm)

Z-as positive downwards

239	188	144	114	96	92	100	121	155	200	249
237	184	139	108	90	84	91	111	145	189	239
239	183	137	105	86	80	86	106	138	182	234
244	186	139	106	86	79	85	104	135	179	232
252	193	145	112	91	83	88	106	136	179	233
265	203	155	121	99	91	94	112	141	183	239

Shape element



(piciture gives the shape, not the real ratio's)

In longitudinal direction relative height (Compared to the line between the outside supports)

0,0	-52,0	-97,0	-128,0	-147,0	-152,0	-145,0	-125,0	-92,0	-48,0	0,0
0,0	-53,2	-98,4	-129,6	-147,8	-154,0	-147,2	-127,4	-93,6	-49,8	0,0
0,0	-55,5	-101,0	-132,5	-151,0	-156,5	-150,0	-129,5	-97,0	-52,5	0,0
0,0	-56,8	-102,6	-134,4	-153,2	-159,0	-151,8	-131,6	-99,4	-54,2	0,0
0,0	-57,1	-103,2	-134,3	-153,4	-159,5	-152,6	-132,7	-100,8	-55,9	0,0
0,0	-59,4	-104,8	-136,2	-155,6	-161,0	-155,4	-134,8	-103,2	-58,6	0,0

In transverse direction relative height (Compared to the line between the outside supports)

0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
-7,2	-7,0	-7,2	-7,4	-6,6	-7,8	-7,8	-8,2	-7,2	-7,6	-8,0
-10,4	-11,0	-11,4	-11,8	-11,2	-11,6	-11,6	-11,4	-11,4	-11,2	-11,0
-10,6	-11,0	-11,6	-12,2	-11,8	-12,4	-11,4	-11,6	-11,6	-10,8	-11,0
-7,8	-7,0	-7,8	-7,6	-7,4	-8,2	-7,2	-7,8	-7,8	-7,4	-8,0
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0

Reaction force (N) at top strips

2,8	12,2	17,2	7,9	15,6	9,4	13,1	12,0	14,5	11,5	3,8
2,2	13,7	15,0	11,5	10,9	13,1	10,5	15,1	10,5	14,8	2,6
1,2	15,9	13,7	10,8	13,4	9,5	15,1	9,9	13,7	15,0	2,0
0,7	17,1	12,8	11,3	12,3	11,7	13,0	10,3	13,9	15,4	1,6
0,9	16,0	15,0	9,0	13,4	11,4	12,9	10,7	12,3	17,7	0,6
-0,8	20,2	11,6	9,1	16,6	6,0	18,3	7,7	11,4	20,7	-0,9

q-load at bottom strips due to top strips (N/mm)

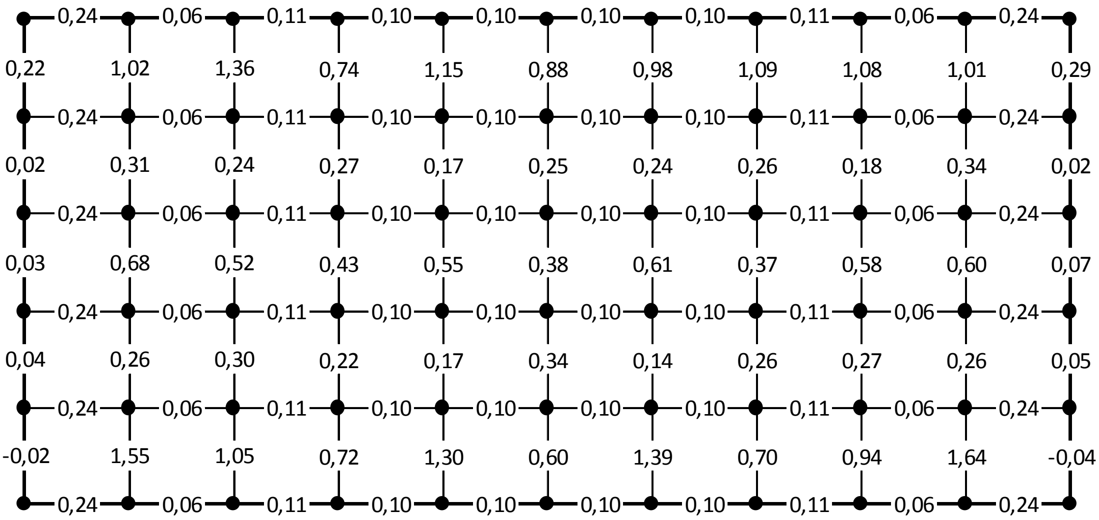
0,06	0,24	0,34	0,16	0,31	0,19	0,26	0,24	0,29	0,23	0,08
0,04	0,27	0,30	0,23	0,22	0,26	0,21	0,30	0,21	0,30	0,05
0,02	0,32	0,27	0,22	0,27	0,19	0,30	0,20	0,27	0,30	0,04
0,01	0,34	0,26	0,23	0,25	0,23	0,26	0,21	0,28	0,31	0,03
0,02	0,32	0,30	0,18	0,27	0,23	0,26	0,21	0,25	0,35	0,01
-0,02	0,40	0,23	0,18	0,33	0,12	0,37	0,15	0,23	0,41	-0,02

Reaction force (N) Bottom strips

-2,6	28,9	20,2	10,0	25,9	9,0	25,4	8,5	20,2	29,5	-3,2
12,7	62,2	70,2	50,3	53,5	58,6	52,7	68,6	52,3	66,2	16,4
3,8	62,7	53,0	42,2	50,9	35,9	56,4	37,2	52,5	59,6	6,0
1,5	65,3	50,6	44,7	49,3	47,4	50,8	41,1	53,8	57,6	5,1
7,2	77,0	67,7	43,3	65,2	52,2	62,5	48,9	58,3	82,9	5,5
1,2	19,4	22,2	13,7	19,2	10,5	20,7	19,5	16,3	19,9	3,5

A positive force is a compression force at the pistons $\sum$ **Q** **2400** **N**

Local displacement (mm) In the middle of the supports



Max 1,64 mm