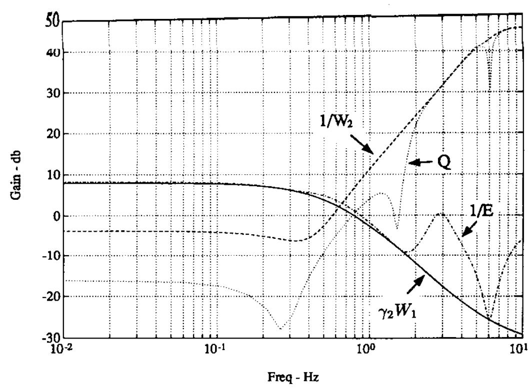

Fig. 7. Control design step: $k = 2$, $\gamma_2 W_1$, $1/E$, $1/W_2$, and Q.

here is known to be more conservative than the μ measure of Doyle, it is affine in the performance index and admits reasonable optimization strategies. In particular, a relaxation algorithm is proposed and shown to produce a sequence of controllers which have monotonically improving robust performance. Several important properties of the identification criterion were discussed, including its tendency to force accurate curve fitting in the vicinity of the critical point in the Nyquist plane.

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Accurate Identification for Control: The Necessity of an Iterative Scheme

Ruud J. P. Schrama

Abstract—**If approximate identification and model-based control design are used to accomplish a high-performance control system, then the two procedures have to be treated as a joint problem. Solving this joint problem by means of separate identification and control design procedures practically entails an iterative scheme. A frequency-response identification technique and a robust control design method are used to set up such an iterative scheme. Its utility is illustrated by an example.**

I. Introduction

Many control design techniques rest on the availability of a model. It is often taken that an appropriate model can be derived prior to the control design. Traditionally, a nominal model $\hat{P}$ is estimated from plant data, and subsequently a compensator $C_{\hat{P}}$ is designed for $\hat{P}$. Since $\hat{P}$ is just an approximate description of the plant P, the compensator $C_{\hat{P}}$ must be robust. This has motivated the development of identification techniques that estimate an upper bound on the model error as in [9] and [6]. With this upper bound, a controller $C_{\hat{P}}$ can ideally be designed to achieve some robust performance. However, this robust performance can be a *high* performance only if the nominal model $\hat{P}$ has been chosen with care.

In this note, we focus on the derivation of a nominal model $\hat{P}$ for high-performance control design. Accordingly, a nominal model $\hat{P}$ is said to be appropriate, if it gives rise to a controller $C_{\hat{P}}$, that achieves similar high performances for P and $\hat{P}$. Thus, the performance of the model-compensator pair $\hat{P}, C_{\hat{P}}$ must be robust in view of the plant P. This is accomplished, if the feedback system composed of the nominal model $\hat{P}$ and the model-based compensator $C_{\hat{P}}$ approximately describes the feedback system containing the plant P and the same compensator $C_{\hat{P}}$. In this perspective, the quality of a nominal model $\hat{P}$ depends on its compensator $C_{\hat{P}}$.

Now suppose we derive an approximate model first, and after that, we design a compensator. Then, in the approximation stage, we have to select a nominal model $\hat{P}$ without knowing fully the quality of each candidate model. The exact quality of the selected nominal model $\hat{P}$ will remain unknown until the second stage of control design has been completed. In order that the model-compensator pair $\hat{P}, C_{\hat{P}}$ approximately describes the plant-compensator

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pair $P, C_{\hat{P}}$, we have to treat the approximation and the control design as a *joint problem* instead of two individual problems. An iterative scheme is required to solve this joint problem by means of the separate stages of approximate identification and model-based control design.

The need of an iteration has been motivated already in, e.g., [15] and [12], and it is also advocated in philosophical terms in [1]. However, to our knowledge, approximation of feedback properties rather than approximation of the plant itself has not been raised as a motivation before. Several iterative schemes of identification and control design have been proposed in the literature. In [11], such an iteration is used to build prefilters for a control-relevant, open-loop, prediction-error identification. Instead of using one open-loop dataset, we take new data at each identification stage while the plant is operated under feedback by the previously designed compensator. This is closely related to adaptive control, but the iterative scheme enables an analysis of the interaction between the identification and control design stages [2]. In the latter reference, prediction-error identification and LQG/LTR control design are combined in an iteration that focuses on robust stability. The iterations of [3] and [7] use prediction-error identification and address LQ-performance.

We tackle the joint problem by an iteration of repeated frequency-response identification and robust control design. In this note, we delineate our iterative scheme, and we focus on the identification part in particular. For a full discussion, the reader is referred to [14]. In Section II, we discuss the robust control design method, which leads to the problem of feedback-relevant approximate identification from closed-loop data. Then in Section III we frame the identification problem in terms of coprime factorizations. Section IV contains an example of the proposed iteration and the final section provides some concluding remarks.

II. Control Design and Approximation

From [4], we adopt the following control design paradigm. The feedback configuration of interest is depicted in Fig. 1. The transfer matrix, which maps col (r_2, r_1) into col $(\hat{y}, \hat{u})$, is denoted $T(\hat{P}, C)$, i.e.,

$$T(\hat{P}, C) = \begin{bmatrix} \hat{P}(I + C\hat{P})^{-1}C & \hat{P}(I + C\hat{P})^{-1} \\ (I + C\hat{P})^{-1}C & (I + C\hat{P})^{-1} \end{bmatrix}. \quad (1)$$

The model-based controller $C_{\hat{P}}$ is derived from $\hat{P}$ as

$$C_{\hat{P}} = \arg\min_{C} \| T(\hat{P}, C) \|_\infty. \quad (2)$$

The resulting controller is robust in the sense that it anticipates stable factor perturbations (see [4] and [16] for details). Moreover, $C_{\hat{P}}$ pursues traditional design specifications such as a small sensitivity at the lower frequencies and a small complementary sensitivity at the higher frequencies [10].

If $\| T(\hat{P}, C_{\hat{P}}) \|_\infty$ is small, then the nominal performance is high. The performance for the actual plant P can be examined through

$$\| T(P, C_{\hat{P}}) \|_\infty \le \| T(\hat{P}, C_{\hat{P}}) \|_\infty + \| T(P, C_{\hat{P}}) - T(\hat{P}, C_{\hat{P}}) \|_\infty. \quad (3)$$

The term on the left reflects the performance of the controlled plant. $\| T(\hat{P}, C_{\hat{P}}) \|_\infty$ is the minimum achieved in (2); and $\| T(P, C_{\hat{P}}) - T(\hat{P}, C_{\hat{P}}) \|_\infty$ is the "worst-case" performance degradation due to the fact that $C_{\hat{P}}$ has been designed for the nominal model $\hat{P}$ rather than for the plant P.

The feedback systems corresponding to $T(P, C_{\hat{P}})$ and $T(\hat{P}, C_{\hat{P}})$ have similar performances if $\| T(P, C_{\hat{P}}) - T(\hat{P}, C_{\hat{P}}) \|_\infty$ is small. At the same time, $\| T(\hat{P}, C_{\hat{P}}) \|_\infty$ must be made as small as possible in order to achieve a high performance. As the latter is pursued in the control design stage, cf. (2), we would like to minimize $\| T(P, C_{\hat{P}}) - T(\hat{P}, C_{\hat{P}}) \|_\infty$ in the preceding approximation stage. And since $C_{\hat{P}}$ is not known *a priori*, the approximation and control design have to be treated as a joint problem.

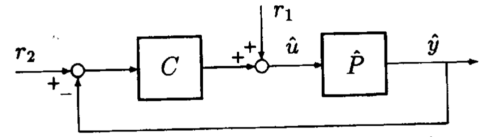

Fig. 1. Feedback configuration for control design.

We propose the following iterative scheme to tackle the joint problem. In the ith step, we obtain data from the plant while it operates under feedback by C_{i-1}. The nominal model $\hat{P}_i$ is derived according to

$$\hat{P}_i = \arg\min_{\tilde{P} \in \mathscr{P}} \| T(P, C_{i-1}) - T(\tilde{P}, C_{i-1}) \|_\infty \quad (4)$$

where $\mathscr{P}$ is the set of candidate models. This minimizes the performance degradation for C_{i-1}. Subsequently, $\hat{P}_i$ is used to construct C_i as in (2), which produces a small nominal performance term $\| T(\hat{P}_i, C_i) \|_\infty$. Then, this controller is applied to the plant P and new data can be collected.

In a straightforward application of the identification in (4) and the control design in (2), we would encounter the following problem. Since by (2), C_i is based solely on the nominal model $\hat{P}_i$, the "new" compensator C_i may be completely different from the "old" compensator C_{i-1}. And although $T(\hat{P}_i, C_{i-1})$ approximately describes $T(P, C_{i-1})$ [see (4)], this does not necessarily hold if C_{i-1} is replaced by C_i. Consequently, the degradation $\| T(P, C_i) - T(\hat{P}_i, C_i) \|_\infty$ can be very large, despite the fact that C_i is maximally robust in view of the achieved nominal performance. In order to provide for a small performance degradation, we have to introduce weighting functions in the control design of (2).

In this note, we just use an adjustable scalar weight α_i. The controller C_i is designed as

$$C_i = \arg\min_{C} \| T(\alpha_i \hat{P}_i, C/\alpha_i) \|_\infty. \quad (5)$$

This causes C_i to maximize robustness for a nominal performance level associated with α_i. The resulting designed feedback system will have its bandwidth close to the crossover frequency of $\alpha_i \hat{P}_i$ [10]. Thus, a large α_i corresponds to a high nominal performance, and it can be adjusted to cause only a slight improvement upon C_{i-1}. Thereby, we keep the performance degradation small at each step of the iteration. By gradually increasing the weight during the iteration, we end up with a large weight and a high-performance controller for the plant.

The identification problem that has to be solved at each iteration step is

$$\hat{P}_i = \arg\min_{\tilde{P} \in \mathscr{P}} \| T(\alpha_i P, C_{i-1}/\alpha_i) - T(\alpha_i \tilde{P}, C_{i-1}/\alpha_i) \|_\infty. \quad (6)$$

As there exists no identification technique that can be used to solve (6), we replace the above H_∞ (or L_∞) approximation by an L_2 approximation. The rationale for this replacement is that the L_2 approximation will yield a reasonably good nominal model in L_∞ sense, provided that the error-term is sufficiently smooth. This observation is backed up by the result in [5] on the L_∞ consistency of L_2 estimators. The L_2-identification problem is discussed in the next section.

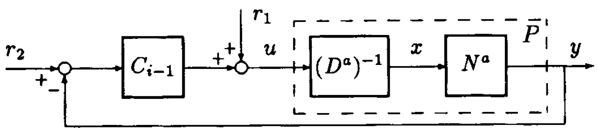

Fig. 2. Feedback configuration for identification.

III. Framework for Identification

We consider the case in which the plant P is controlled by C_{i-1} as in Fig. 2. In order to simplify the notation, we take $\alpha_i = 1$. The problem of interest is to identify a nominal model $\hat{P}_i$ from measurements of the plant's input u and output y such that

$$\hat{P}_i = \arg\min_{\tilde{P}\in\mathscr{P}} \| T(P, C_{i-1}) - T(\tilde{P}, C_{i-1}) \|_2. \qquad (7)$$

With $P \notin \mathscr{P}$ the minimization of (7) from u and y combines all problems that are encountered in approximate identification and closed-loop identification. Therefore, direct application of standard identification methods to u and y will not yield the desired $\hat{P}_i$ (see [13] for a discussion). In order to solve (7), we represent the plant P by a right coprime factorization (definitions are found in [16]), which is dual to the representation used in [8].

We assume that the plant P is stabilized by the controller C_{i-1}. Since C_{i-1} is known from the previous design step, it can be used to parameterize the set of all stabilized systems by their right coprime factorizations. This result is dual to the parameterization of all stabilizing compensators [16]. One of these factorizations corresponds to the unknown plant P. Hence, P can be written as

$$P = (N_a + D_c R)(D_a - N_c R)^{-1} \qquad (8)$$

where R is stable, the pairs (N_c, D_c), (N_a, D_a) are coprime factorizations satisfying $C_{i-1} = N_c D_c^{-1}$ and $P_a = N_a D_a^{-1}$, and P_a is just an auxiliary model, that is stabilized by C_{i-1}. Next we define

$$N^a \doteq N_a + D_c R; \qquad D^a \doteq D_a - N_c R \qquad (9)$$

so that $N^a(D^a)^{-1}$ is a right coprime factorization of P by virtue of (8). With this representation of P we can obtain the following two results.

Lemma 3.1: Let the feedback system of Fig. 2 be stable and let controller C_{i-1} be known. Further let (N_a, D_a) be a right coprime factorization of an auxiliary model P_a, that is stabilized by C_{i-1}. Then the variable x of Fig. 2 can be reconstructed from u and y via

$$x = (D_a + C_{i-1}N_a)^{-1}(u + C_{i-1}y). \qquad (10)$$

Proof: From Fig. 2, we have $y = N^a x$ and $u = D^a x$, and by straightforward calculation we obtain $x = (D^a + C_{i-1}N^a)^{-1}(u + C_{i-1}y)$. With the equality

$$D^a + C_{i-1}N^a = D_a + C_{i-1}N_a \qquad (11)$$

in which the right-hand side follows from substituting (9) on the left-hand side, we arrive at (10). ∎

Theorem 3.2: Let the assumptions of Lemma 3.1 hold. Then, the frequency response of $T(P, C_{i-1})$ can be estimated from u and y.

Proof: With the use of $P = N^a(D^a)^{-1}$ and (11), the transfer function $T(P, C_{i-1})$ can be rewritten as

$$T(P, C_{i-1}) = \begin{bmatrix} N^a(D^a)^{-1}[(D^a + C_{i-1}N^a)(D^a)^{-1}]^{-1} \\ [(D^a + C_{i-1}N^a)(D^a)^{-1}]^{-1} \end{bmatrix} \cdot [C_{i-1} \quad I]$$
$$= \begin{bmatrix} N^a \\ D^a \end{bmatrix} (D_a + C_{i-1}N_a)^{-1} [C_{i-1} \quad I]. \qquad (12)$$

The terms $(D_a + C_{i-1}N_a)$ and $[C_{i-1} \; I]$ are known, and thus their frequency responses can be calculated. Further, the frequency responses of N^a and D^a can be estimated from $y = N^a x$ and $u = D^a x$ and x reconstructed as in Lemma 3.1. Together, these frequency responses make up an estimate of the frequency response of $T(P, C_{i-1})$. ∎

In [13], it is shown that Lemma 3.1 and Theorem 3.2 do also hold in case the plant output y is contaminated by an unmeasurable noise. That is, x can still be reconstructed from u and y, and the identification of N^a and D^a from u, y and x turns out to be an open-loop identification problem.

With Theorem 3.2, we have access to the frequency response of $T(P, C_{i-1})$ and thus $\hat{P}_i$ can be identified from (7). This frequency-domain identification problem is not trivial, because $\tilde{P}$ appears in $T(\tilde{P}, C_{i-1})$ in a multiple and nonlinear fashion. In [14] an algorithm is developed that solves (7) by estimating $\hat{P}_i$ in terms of coprime factors.

IV. Example

The plant P under investigation is a real rational continuous-time system of order 9: $P(s) = n(s)/d(s)$ with

$$n(s) = 6.599 \cdot 10^{-5}s^9 - 2.552 \cdot 10^{-3}s^8 - 0.1264s^7 - 0.2836s^6 - 4.195s^5 + 6.983s^4 - 13.74s^3 + 215.2s^2 + 144.0s + 1057$$

$$d(s) = s^9 + 2.401s^8 + 32.68s^7 + 54.78s^6 + 347.2s^5 + 351.2s^4 + 1256s^3 + 488.8s^2 + 635.3s + 105.9.$$

The iterative scheme started from open-loop, i.e., $C_0 = 0$. The identified nominal models $\hat{P}_i$ are of order 5, and the controllers C_i are of order 4. The design objective is to reduce the sensitivity at the lower frequencies. Pretending that the plant P is unknown, we cannot tell *a priori* what performance is attainable with a reduced-order controller.

The number of performed iteration steps is 19. The log-magnitude Bode diagrams of the nominal models $\hat{P}_1$, $\hat{P}_{12}$, and $\hat{P}_{19}$ have been draw in Fig. 3 together with that of the plant P. The curves corresponding to P and the open-loop nominal model $\hat{P}_1$ are indiscernible at the frequencies where the magnitude of P is high. The other two nominal models show a good match only in the frequency range from 1 to 2 rad/s. Based on Fig. 2, $\hat{P}_{12}$ and $\hat{P}_{19}$ should be marked as bad nominal models. Similar observations apply to the phase plots.

The scalar design weight α_i has been increased during the iteration: $\alpha_1 = 1$, $\alpha_{12} = 5.4$, and $\alpha_{19} = 9.2$. For completeness, we mention that the design from $\hat{P}_1$ would have resulted in a destabilizing controller if α_{19} had been used instead of α_1.

The sensitivity $(I + C_i P)^{-1}$ has been depicted in Fig. 4 for the controllers C_1, C_{12}, and C_{19}. These curves show that a reduction of the sensitivity at lower frequencies has been realized at the expense of some increase at higher frequencies. For comparison, we have also designed controllers from the plant P itself. The controller C_P, which has order 4 also, has been designed with the scalar weight α_{19}. The resulting sensitivity $(I + C_P P)^{-1}$ shows a great resemblance to $(I + C_{19}P)^{-1}$. From this we conclude that the nominal model $\hat{P}_{19}$ is very well-suited to high-performance control design in the sense that the resulting model-based controller C_{19} is as good as the "plant-based" controller C_P. Lastly, we remark that $\hat{P}_{19}$ exhibits the worst open-loop match, and at the same time it is the best nominal model for high-performance control design.

V. Concluding Remarks

We observed that approximate identification and model-based control design have to be treated as a joint problem if they are

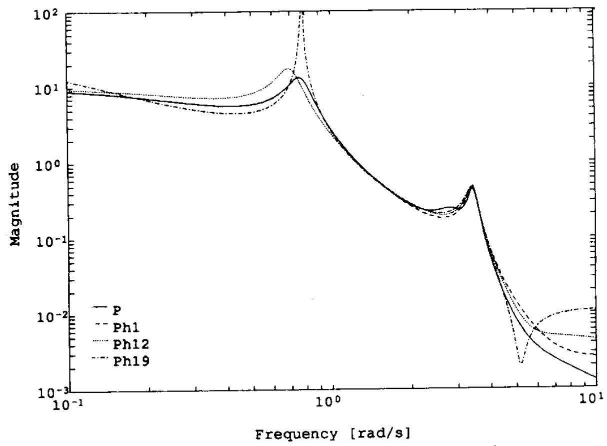

Fig. 3. Bode plots of P(——), $\hat{P}_1$(- -), $\hat{P}_{12}$(· · ·), and $\hat{P}_{19}$(- · -).

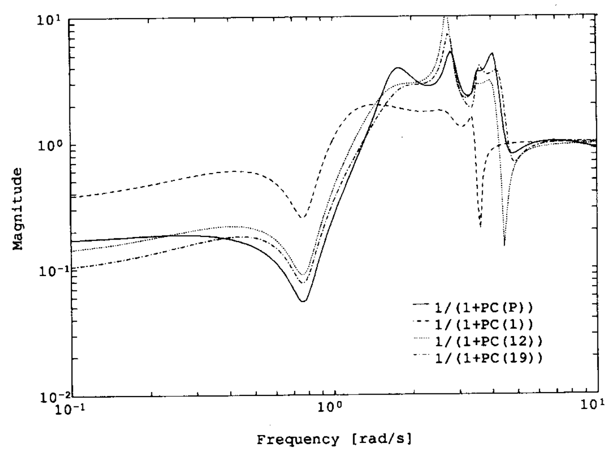

Fig. 4. Bode plots of the sensitivities $(I + C_P P)^{-1}$(——), $(I + C_1 P)^{-1}$ (- -), $(I + C_{12} P)^{-1}$(· · ·), and $(I + C_{19} P)^{-1}$(- · -).

combined to achieve a high-performance control system. Solving this joint problem with individual identification and control design methods requires an iterative approach.

The proposed iterative scheme is based on a robust control design method. Each identification step uses the previously designed controller to obtain new data from the plant. The associated identification problem has been solved by means of a coprime factorization of the unknown plant. An example has given evidence of the utility of the iterative scheme. It also illustrated the need of an iteration, since a good controller is required for the identification of an appropriate nominal model for high-performance control design. As an additional pay-off, the iteration reveals the performance that is attainable for the unknown plant.

A drawback of our iteration is that the identification stage focuses on the "old" compensator. In order to speed up the iteration, the identification should anticipate the "new" compensator. This is a topic for future investigations, together with the application of the same identification framework in case of time-domain data and other control design methods.

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A Comparison of Classical Stochastic Estimation and Deterministic Robust Estimation

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Abstract—**This note compares the formulation and solution of two linear parameter estimation problems. The basic distinction in the prob-**

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