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Tidal divides

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ABSTRACT

The Wadden Sea consists of a series of tidal basins which are connected to the North Sea by tidal inlets. Each lagoon has as boundaries the mainland coast, the barrier islands on both side of the tidal inlet, and the tidal divides behind the two barrier islands. Behind each Wadden Island there is a tidal divide separating two adjacent tidal basins. The locations of the tidal divides in the Wadden Sea are not fixed. Especially after a human interference, a tidal divide can move and thereby influences the distribution of area between the basins, which is important for the morphological development of the basins. Two types of tidal divides can actually be identified behind each island: the hydraulic tidal divide and the morphological tidal divide. These do not necessarily have the same position, and their location is an indicator for the equilibrium of the adjacent basins. This paper describes theoretical analyses meant for improve the insights into the location of the tidal divides and their movements, as are observed in the Dutch part of the Wadden Sea in the last 80 years.

INTRODUCTION

In a series of tidal basins, like the Wadden Sea, a tidal watershed or tidal divide forms the separation between two adjacent tidal basins. Tidal divides are characterized by lower flow velocities and an elevated bed level. They are important for the morphological development of the basins, because by moving they change the surface area of the adjacent basins. In this paper we study the movement of the tidal divides in the Dutch Wadden Sea in the last 80 years by analyzing flow velocities from DELFT3D-model results and bathymetries collected by Rijkswaterstaat. A simplified case is used to study the factors influencing the position of the tidal divide. First, we will give a clear description of a tidal divide, which will be used throughout this paper.

METHODS

Movement of tidal divides

First, clear definitions of the tidal divides are needed for determining positions of the tidal divides in different years. We make a distinction between hydraulic and morphological tidal divides. A hydraulic tidal divide is the separation between the basins in terms of drainage. It can be defined as the line where the standard deviation of the flow velocity is minimal. In reality, we do not see such a line, but a transitional area with lower flow velocities consisting of interconnected tidal flats. However, a line is desired to be able to use the hydraulic tidal divide as a basin boundary for example in computations. The morphological tidal divide is an elevated spine-like structure in bed, see Figure 1 (inset). The centre-line of the spine-shape is the line representation of the morphological tidal divide; this is not necessarily the highest point in the bed, as can been seen from the morphological tidal divide behind Ameland.

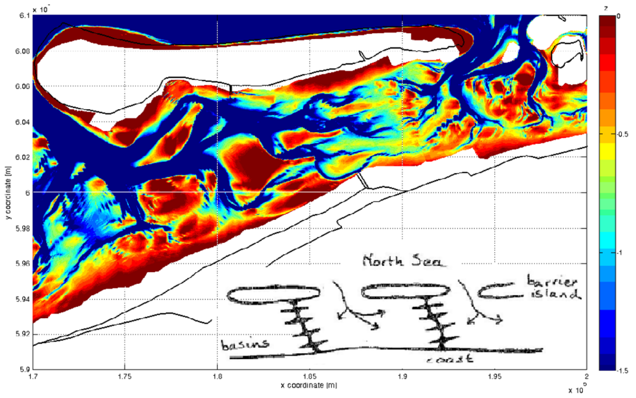


Figure 1. The spine-shaped morphological tidal divide behind Ameland

Simplified case

In order to obtain more understanding about the behaviour of the hydraulic tidal divide we analyse a simple case: The region between two adjacent tidal inlets, i.e. the area behind a barrier island, is considered as a prismatic channel. The problem is then simplified to the tidal propagation through a channel with constant water depth. We consider the propagation of a single tidal component. At one side of the channel the water level is then described by a cosine function and at the other side of the channel the amplitude as well as the phase may be different.

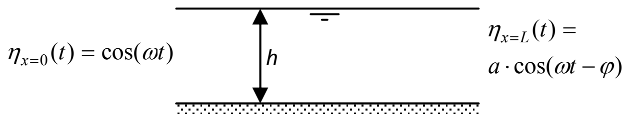


Figure 2. Sketch of the simplified case

We now ask our self the question will there be a hydraulic tidal divide, i.e. a place in the channel where the amplitude of the flow velocity is minimal? Which factors determine whether or not a hydraulic tidal divide is present? What is the position of the hydraulic tidal divide? Which factors determine the position of the hydraulic tidal divide?

First we try to answer the questions using an analytical approach. To do this we solve the linear equation for tidal propagation with linearized bottom friction (*Wang et al.*, 2011, *Vroom*, 2011):

$$\frac{\partial^2\eta}{\partial t^2} - c^2\frac{\partial^2\eta}{\partial x^2} + \kappa\frac{\partial\eta}{\partial t} = 0$$

Herein

η	Water level [m]
t	Time [s]
c	Propagation velocity of tidal wave [m/s]
x	Coordinate along the channel (island) [m]
κ	Linearized friction factor [s^{-1}]

The solution of this equation satisfying the boundary conditions as shown in Figure 1 is:

$$\tilde{\eta}(x) = \frac{\sinh[p(L-x)]}{\sinh(pL)} + \frac{\sinh(px)}{\sinh(pL)} a \cdot e^{-i\varphi}$$

With:

$\tilde{a} = a \cdot e^{-i\varphi}$, the complex amplitude at the channel end x=L.

$$p = \mu + ik = ik_0\sqrt{1 - i\sigma}$$

$$\sigma = \frac{\kappa}{\omega}$$

Herein

L	Length of channel / island [m]
a	Amplitude ratio between the two ends of the channel [-]
φ	Phase lag between the two ends of the channel
ω	Frequency of the tidal wave [rad/s]
μ	Damping factor [-]
k	Wave number [-] (= 2π/λ)
k_0	Frictionless wave number [-] (=ω/c)

The solution for the flow velocity is:

$$u(x,t) = \frac{i\omega}{ph}\left(\frac{\cosh[p(L-x)]}{\sinh(pL)} - \frac{\cosh(px)}{\sinh(pL)} a \cdot e^{-i\varphi}\right) \cdot e^{i\omega t}$$

$$\tilde{u}(x) = \frac{i\omega}{ph}\left(\frac{\cosh[p(L-x)]}{\sinh(pL)} - \frac{\cosh(px)}{\sinh(pL)} a \cdot e^{-i\varphi}\right)$$

The further elaboration of this equation algebraically is done by neglecting the bottom friction (σ=0). The amplitude of the flow velocity is then

$$\hat{u}(x) = \frac{\omega}{k_0 h}\sqrt{\left(\frac{\cos[k_0(L-x)] - a\cos(k_0 x)\cos\varphi}{\sin(k_0 L)}\right)^2 + \left(\frac{a\cos(k_0 x)\sin\varphi}{\sin(k_0 L)}\right)^2}$$

Solving the equation that the derivative of the amplitude of the flow velocity is zero yields:

$$\tan(2k_0 x) = \frac{\sin(2k_0 L) - 2a\cos\varphi\sin(k_0 L)}{\cos(2k_0 L) - 2a\cos\varphi\cos(k_0 L) + a^2}$$

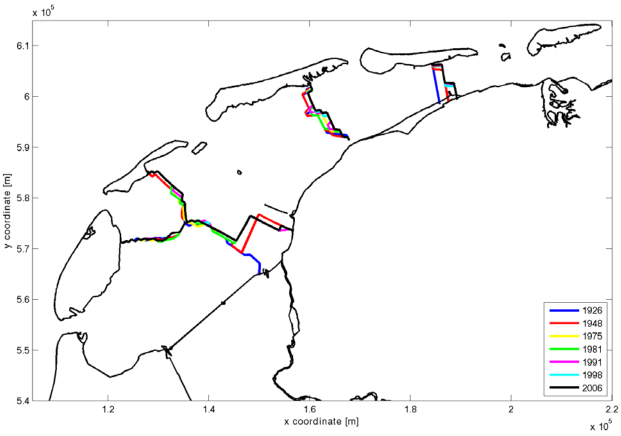
This is the equation determining the location of the hydraulic tidal divide for the frictionless case.

RESULTS

Movement of tidal divides

The positions of the hydraulic and morphological tidal divides in the Dutch Wadden Sea are determined for different years according to the definitions described in the previous section.

Regarding the hydraulic tidal divide, the largest changes are observed near the Afsluitdijk. The simulation for 1926, including the Afsluitdijk, shows that the hydraulic tidal divide has not moved yet. The standard deviation of the flow velocity is lowest at this position, but also very low at the later position of the hydraulic tidal divide. Probably, this used to be a local hydraulic tidal divide, splitting the drainage area of two branches of the Vlie



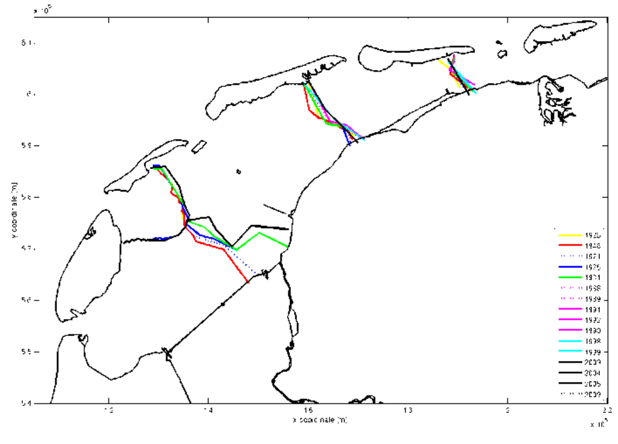


Figure 3 (top): Hydraulic tidal divides in the Dutch Wadden Sea, note that the 1926 simulation also includes the Afsluitdijk and therefore resembles the 1932 situation (post closure).

Figure 4 (bottom): Morphological tidal divides in the Dutch Wadden Sea basin. The Eierlandse Gat basin has slightly increased in size, due to an eastward expansion of the hydraulic tidal divide between Eierlandse Gat and Vlie. Although the Eierlandse Gat basin is relatively close to the Afsluitdijk, the movement of the tidal divides bordering the basin is very small. The tidal divide behind Terschelling shows a shift in eastward direction.

Looking at the lines representing the morphological tidal divides in Figure 4, we can see that the morphological tidal divide between Marsdiep and Vlie adjusts much slower than the hydraulic tidal divide. It is remarkable that for the situation just after closure, the hydraulic and morphological tidal divide have more or less the same position. Apparently, the elevated bed level of the morphological tidal divide hinders the movement of the hydraulic tidal divide first. Due to higher flow velocities at the morphological tidal divide, the topographic high erodes and the hydraulic tidal divide can move. When the morphological tidal divide has moved close to the position of the hydraulic tidal divide, the bed level can get higher again. For the other tidal divides we study in this paper, we see a smaller movement of the tidal divides. Because the movement is more gradual, the morphological tidal divide can keep pace with the hydraulic tidal divide more easily.

Simplified case

For the simplified case we can make a distinction between a frictionless derivation and a derivation including linearized friction.

Frictionless case

Figure 5 shows the position of the hydraulic tidal divide in the rectangular channel in case without friction.

The following observations are made from the analytical solution:

If the tidal amplitudes at the two ends are the same, a tidal divide exists and is located in the middle for the case that there is no phase difference between the two ends. This is the trivial case that the tides at the two ends are exactly the same.

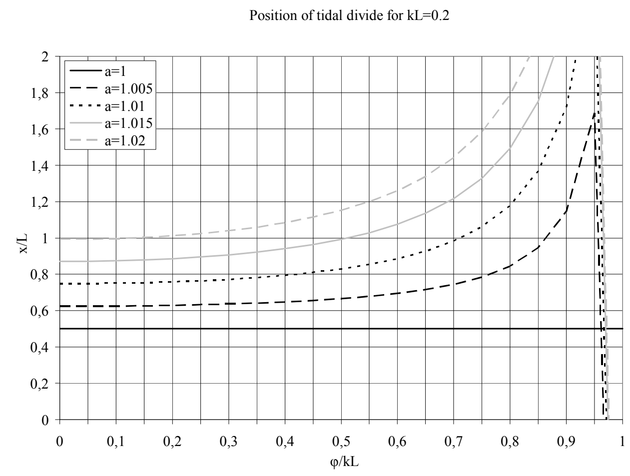


Figure 5: Position of the tidal divide versus the relative phase difference (kL=0.2) for increasing wave amplitude a (lighter gray lines). Note that a tidal divide is only present if x/L is in the realistic domain between 0 and 1.

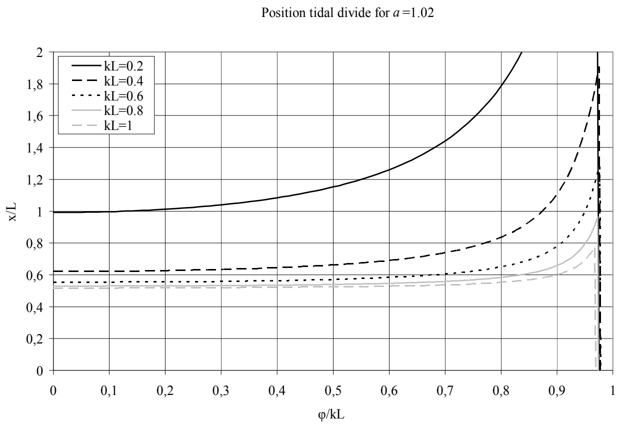


Figure 6. Position of the tidal divide versus the relative phase difference for increasing relative channel length (kL) (lighter gray lines).

It is remarkable that the phase difference has no influence on the position of the tidal divide in case the incoming waves have equal amplitudes. But the phase difference does have an influence on the absolute value of the minimum of the flow velocity amplitude in this case. The phase difference has to be smaller than *kL*, if it is equal to *kL* a purely propagating wave will be present.

Even more remarkable is the influence of the amplitude difference between the two ends. The tidal divide is closer to the side with larger tidal amplitude than the side with smaller amplitude. According to the linear solution the amplitude ratio has even more influence on the existence and the position of tidal divides than the relative phase difference. This means that it is not the propagation of the tidal wave but the spatial variation of the tidal range that has the most influence on the location of the tidal divide behind a barrier island.

There is not always a tidal divide. If the tidal amplitudes at the two ends are not equal there is a limit to the phase difference between the two ends for the existence of a tidal divide. When the relative phase difference between the two ends becomes larger than the limit, no tidal divide can be found (no solution for x in the realistic domain 0<x<L). The limit for the phase difference becomes smaller if the amplitude increases in the direction of tidal propagation. Interpreted for the Wadden Sea case this means that a tidal divide only can exists if the propagation of the tidal wave on the sea side of a barrier island is much faster than the propagation in the basin behind the island. In other words, the tidal divide can only exist if the Wadden Sea is relatively shallow.

The phase difference limit for the existence of the tidal divide becomes larger if the island is longer and/or the basin behind the island is shallower. It seems thus that the barrier islands should have lengths above a certain limit depending on the characteristics of the tide and the tidal propagation at the North Sea side.

Case with linearized friction

In Figure 7 the contribution of the bottom friction term on the importance of the phase difference and the amplitude ratio is clearly visible. If the bottom friction is small, the amplitude ratio is dominant over the phase difference. With larger friction the phase difference is the governing mechanism for determining the position of the hydraulic tidal divide. In Figure 8 we see that with the inclusion of the bottom friction, the phase difference does have an influence on the position of the tidal divide, even if the incoming waves have equal amplitudes. For larger relative channel lengths, the limits for the phase difference and amplitude ratio are less strict.

DISCUSSION

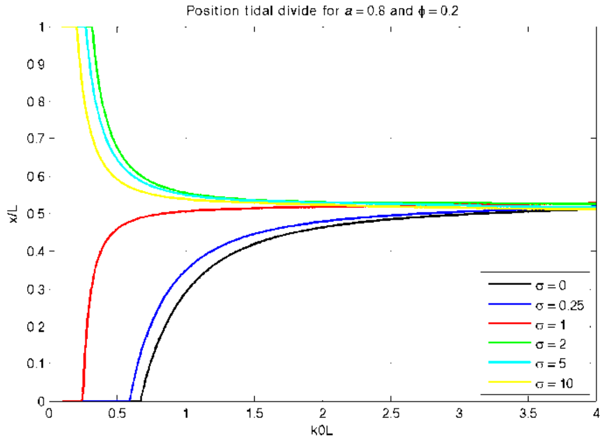
The determination of the position of the line representing the hydraulic and morphological tidal divide will always be somewhat arbitrary, which means the exact position of the line will differ from person to person. This is influencing the surface areas of the basins and can be important for the determination of the equilibrium state of the basins. The movement of the tidal divides however, can also be observed from the bathymetric data itself and the observations made in this paper are not strongly dependent on this line representation.

CONCLUSIONS

The movement of the tidal divides in the Dutch Wadden Sea show an interrelated behavior of the hydraulic and morphological tidal divide. After large changes in the basins, the morphological tidal divide might be high enough to hinder the movement of the hydraulic tidal divide. After erosion of the topographic high, the hydraulic tidal divide can move more freely, afterwards the morphological tidal divide will follow.

In order to obtain more insights into the locations of the tidal divides a theoretical analysis has been carried out for a simplified case. The tidal propagation behind an island is schematised into a simple channel flow with two open boundaries representing the two inlets where the tidal variation of water level is represented by a single tidal component with different amplitudes and phases at the two boundaries. The hydraulic tidal divide is defined at the location where the amplitude of the flow velocity is minimal. The analytical solution of the linear tidal propagation equation shows that a tidal divide does not always exist. It can only exist when the island is relatively long and / or the back barrier basin is relatively shallow. The analytical solution shows further surprisingly that the difference in tidal amplitude between the two ends of the channel (island) has more influence on the location of the tidal divide than the phase difference in case of small bottom friction. When the tidal amplitudes at the two ends are the same and bottom friction is neglected, the tidal divide is always in the middle, independent of the phase difference. When friction is included, the phase difference becomes more important. It is further surprising that the tidal divide moves to the end with larger tidal amplitude when the amplitudes at the two ends are not equal. The limit for the phase difference between the two ends for existence of tidal divide becomes smaller if the amplitude increases in the direction of tidal propagation. If the difference between the amplitudes at the two ends is too large and / or the phase difference between the two ends is too large no tidal divide exists because the amplitude of the flow velocity will be monotonously changing along the channel.

Our final conclusion is that the variation of the tidal amplitude as well as the direction of the tidal wave propagation on the sea side of the islands have influence on the location of the hydraulic tidal divides in the Wadden Sea, but which parameter is more dominant is depending on the water depth - wave height ratio.



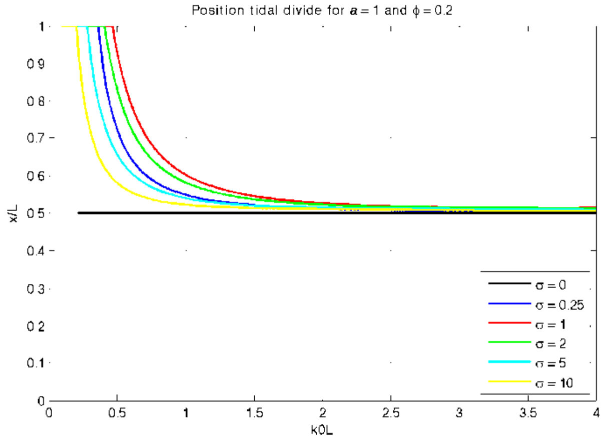


Figure 7 (top): Position of the tidal divide versus the relative channel length for an amplitude ratio of 0.8 and phase difference of 0.2 for different values of the bottom friction factor σ

Figure 8 (bottom): Position of the tidal divide versus the relative channel length for equal amplitudes and a phase difference of 0.2 for different values of the bottom friction factor σ

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